Costly Enforcement of Quality Standards in Decentralized Supply Chains

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Abstract

We consider a supply chain where the quality level can be observed by the buyer(s) only after the purchase is completed (experience good). If the delivered quality is below the levels agreed in the contract, the buyer(s) may take action to seek remedy, but this effort is costly. Obviously, this fact can be exploited by the seller. In the case of a single buyer, we show that the buyer may not be motivated to appropriate all the channel profits but, rather, is able to pay a higher purchase price in order to induce the seller to provide higher quality. The set of purchase prices that support trade split into different regions. We identify regions where counterintuitive behavior is exhibited, for example, the buyer prefers higher purchase price, while the seller prefers lower purchase price.

For the case of more than one buyer, we examine and contrast the behavior of two remedy regimes: In the case of individual enforcement, each buyer must work individually to enforce her own quality. In the case of joint enforcement, all buyers benefit from enforcement by a single buyer. We examine the externalities that arise in this supply chain and identify how the market share, enforcement costs and purchase price affect the quality level.

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1 Introduction

What happens when a supplier does not meet his contractual obligations? In this paper, we study a supply chain comprising a single seller and multiple buyers in which the quality level can be observed by the buyer(s) only after the purchase is completed (experience good (see, for example, Washburn et al. (2004)) and buyer(s) have to incur a transaction cost to enforce (mutually agreed upon) quality standards. For the case of more than one buyer, we study two types of enforcement regimes, individual and joint. In the individual enforcement regime, if a buyer enforces the contract, the seller refunds the purchase cost of the defective units to this buyer only. In the joint enforcement case, which is similar to a "class action" scenario, the seller must refund the purchase price of all buyers, even if only one of them goes through the trouble of enforcing the contract. Obviously, this creates a "free rider" problem amongst the buyers. We examine the externalities that arise in this supply chain and characterize the market equilibria.

The extant literature on contract theory in the context of inventory and pricing decisions in supply chains is exhaustive (see, for example, Anupindi and Bassok (1999) and Plambeck and Taylor (2006)). Cachon (2001) is an excellent survey paper on the topic. However, the literature on contracts in the context of quality is more limited (see, for example, Reyniers (1992) and Reyniers and Tapiero (1995)). Below, we briefly discuss the literature that is most relevant to our work.

Reyniers and Tapiero (1995) model the effect of contract parameters such as price rebates and after-sales warranty costs on the choice of quality of the supplier, the inspection policy of the buyer and the resulting end quality. In their model, the supplier of a part chooses technology and this choice is not observed by the buyer, who independently chooses his inspection policy. Lim (2001) extends the Reyniers and Tapiero (1995) framework to the case where this incomplete information regarding the quality of parts. Starbird (2001) considers a supplier who selects a batch size and quality level. He shows that when sampling inspection is used, penalties and rewards are substitutes for one another in motivating the supplier. Kaya and Ozer (2004) study contract manufacturing with unverifiable product quality and hidden information on quality costs. They obtain optimal two-part linear contracts depending on whether the quality is verifiable and the contract manufacturer's quality cost is public. Balachandran and Radhakrishnan (2005) study the quality implications of warranties in a supply chain in which final product consists of components made by a buyer and a supplier. Hwang et al. (2006) highlight an additional indirect cost of inspection - when the buyer employs the inspection regime, it can induce the supplier to perform unwanted inspection to reduce expected penalty for bad quality. They show that certification is preferred to appraisal being preferred when inspection technology becomes more effective.

Next, we discuss some of the common schemes that have been proposed for the study of breach of contract in the context of supply chain management. Plambeck and Taylor (2004) study situations in which parties do not always satisfy agreed upon contracts. For instance, Plambeck and Taylor (2004) compare the effects of two common schemes, Specific Performance and Expectation Damages (their work is in the context of renegotiations). Under the Specific Performance scheme, the seller is required to fix the defective units. Under the Expectation Damages scheme, the seller is required to compensate the buyer for the shortfall in revenue due to defective units. The issue of contract enforcement has also been widely studied by the legal fraternity (Epstrin (1989) and Schwartz (1990)). We study here another common scheme (studied by Reyniers and Tapiero (1995)), in which the seller is required to refund the buyer the entire purchasing cost of the defective units, which we refer to as Full Rebate. This scheme is simple and intuitive. Sappington (2005) explains how these penalties are particularly common in the electricity distribution industry.

We now briefly describe our results. For the case of a single buyer and single seller, we specify conditions under which the buyer is not motivated to "squeeze" all the profits out of the supply chain but, rather, is able to pay a higher purchase price in order to induce the seller to provide higher quality. The set of purchase prices that support trade split into different regions. We identify regions where counterintuitive behavior is exhibited, for example, the buyer prefers higher purchase price, while the seller prefers lower purchase price.

In the case of two buyers and a single seller, the nature of the interaction between the buyers determines the quality choice of the seller. For the case of individual enforcement, we identify how the the costs of enforcement, the purchase price and the market share of the buyers affect the seller's quality choice. One of the interesting results is that for some values of purchase price, if the market shares of the two buyers is similar, the quality choice to the buyer is low, while for dissimilar market shares the quality is high. On the other hand, in the case of joint enforcement, we show that the seller's quality choice is always high.

The results in this paper are presented in the case of complete information. For the case of incomplete information regarding the buyers enforcement cost, we refer the readers to Sheopuri and Zemel (2007) who study a class of optimization problems where the decision maker is greedy upto an (unknown) cut-off, and regrets his decision thereafter.

The remainder of the paper is organized as follows. We present the the model in Section 2. We first present the results for the case of a single buyer and single seller (Section 3). The insights that we derive for this case are useful for analyzing the case of two buyers and a single seller (Section 4). Finally, we discuss how our results generalize to N buyers and a single seller (Section 5).

2 The Model

Consider a supply chain consisting of N buyers and a single seller. Each buyer, $i = 1, 2, ..., N$ wishes to buy a given (fixed) quantity D_i of a product, with $D = \sum_{i=1}^{N} D_i > 0$. The set-up is of complete information.

The product can be produced at a quality level p, $0 \leq p \leq 1$. Once p is set by the seller, all buyers receive the same quality. Higher quality levels (low p) are valuable to the buyers but costly to the seller. We assume that the production cost of the seller is $C(p, D) = (u - \gamma p)D$, with $\gamma \leq u$. We refer to u as the cost of production and to γ as the cost of quality. Similarly, the revenue from the units to buyer i is given by $R_i(p, D_i) = (v - \omega p)D_i$, with $\omega \leq v$. We refer to v as the value of a perfect unit, and to ω as the value-loss of a defective unit.

As stated earlier, we assume that the product quality is of the "experience type" in that p can not be observed by a buyer in advance, but it becomes known to her with use. We assume that there is a standard quality level $\bar{p}$ in the market, known and agreed to by all participants, and without loss of generality we let $\bar{p} = 0$. If a buyer observes (after the purchase) that $p > 0$, she can enforce the standard by seeking remedies from the seller, but this action is costly. We elaborate on the purchase price and on the costs of enforcement below.

We treat the purchase price, T, as a given parameter of the problem, rather than a variable to be determined by the participants of the supply chain. Thus, the analysis can be used independently of the relative bargaining powers of the parties in terms of setting the price. However, we determine the preference of the buyers and sellers with respect to T, and at what levels it would be set if a given party had the power to unilaterally do so (subject to the participation constraint of the other parties). *Surprisingly, even in the simple case of one buyer and one seller, it is not always in the buyer's interest to set the price as low as possible, and vice-versa for the seller.*

We assume that the cost to buyer i to enforce the standard is c_i. The activities that induce this cost may include packaging and returning defective items, paperwork and supplier notification, contracting a third party to document a breach, initiating or escalating a conflict, legal action or arbitration, or even curtailing or terminating a business relationship. The critical factor here is that such activities are costly to the buyer. Naturally, the higher is c_i, the more reluctant is the buyer to initiate enforcement. This fact can be exploited by the seller.

The sequence of events in the model is as follows.

1. All data of the model, $v, \omega, u, \gamma, c_i, D_i$ as well as the purchase price T are known to all parties in advance.

2. First, the seller decides whether he wants to enter the market and offer the product for sale at the price T, and, each buyer decides whether she wishes to participate (individual rationality).

3. Assuming that the seller and some buyers decide to participate, the seller decides on the quality level p, supplies each buyer the quantity D_i and receives the purchase price TD_i.

4. After the purchase, all buyers discover the level of quality to be p. Based on this information, each buyer decides whether she wishes to enforce the standard.

We assume that $0 < u \leq v$ and that $0 < \gamma \leq \omega$. Let π_B^i denote the profits of buyer i and π_S denote the profits of the seller. We use the argument T for π_B^i and π_S, when we discuss the behavior of the profit function with T. Define the market efficiency due to decentralization as the ratio of the sum of the buyers' profits and seller's profits to the profits of the centralized system, i.e.,

$$\frac{\sum_{i=1}^N \pi_B^i + \pi_S}{(v-u) \cdot D}.$$

We now analyze the single buyer case. The insights that we derive from this case are useful for studying the case of two buyers.

We use the following convention. The interval (a, b), $[a, b)$ is empty if $a \geq b$.

3 The Single Buyer Case

When there is a single buyer, the individual enforcement and joint enforcement cases are equivalent. For ease of notation, we use c to denote the enforcement cost of the buyer, i.e., $c = c_1$, in this section. We first examine the behavior of the supply chain as a function of the purchase price T, ignoring the effects of the individual rationality constraints. Subsequently, we include the effects of these constraints. We first introduce some notation. Let

$$\mathcal{X} = \{T : T > \frac{c}{D}\} \text{ and } \mathcal{Y} = \{T : T + \gamma \frac{c}{DT} \geq \gamma\}.$$

Proposition 3.1. *Assume that both the buyer and seller participate. For the single buyer case,*
(i) if $T \in \mathcal{X} \cap \mathcal{Y}$, *the seller selects* $p^* = \frac{c}{DT}$, *the buyer does not enforce and the market efficiency is*

$$1 - \frac{\omega - \gamma}{v - u} \cdot \frac{c}{DT},$$

(ii) if $T \notin \mathcal{X}$, *the seller selects* $p^* = 1$, *the buyer does not enforce and the market efficiency is*

$$1 - \frac{\omega - \gamma}{v - u},$$

(iii) if $T \in \mathcal{X} \cap \mathcal{Y}'$, *the seller selects* $p^* = 1$, *the buyer does enforce and the market efficiency is*

$$1 - \frac{\omega - \gamma}{v - u} - \frac{c}{(v - u) \cdot D}.$$

Proof: We show (i). The proof of (ii) and (iii) is similar. Given p, the buyer's objective function is

$$\begin{cases} D[v - \omega p - T] & \text{no enforcement;} \\ D[v - \omega p - T(1-p)] - c & \text{enforcement.} \end{cases}$$

Thus, the buyer will enforce iff

$$p > \frac{c}{DT}.$$

Anticipating this, the seller's objective is

$$\begin{cases} D[T - (u - \gamma p)] & p \leq \frac{c}{DT}; \\ D[T(1-p) - (u - \gamma p)] & o/w. \end{cases}$$

The seller's profit is maximized at $p^* = \frac{c}{DT}$ if the profit he makes at the quality level $\frac{c}{DT}$ without enforcement is not less than the profit he makes at the quality level 1, with enforcement, i.e.,

$$\gamma \frac{c}{DT} \geq \gamma - T.$$

□

We now briefly discuss the implication of the Proposition. In the region $T \notin \mathcal{X}$, the cost of enforcement is "too high" compared to the purchase price. Thus, the threat of enforcement is not credible and the seller provides the lowest quality knowing that he will not be penalized. On the other hand, in the region $T \in \mathcal{X} \cap \mathcal{Y}'$, the cost of producing a bad unit is sufficiently low and the penalties invoked on enforcement are not deterring leading again to low quality.

Next, we focus on the region $T \in \mathcal{X} \cap \mathcal{Y}$. In this case, the buyer's, seller's and total channel profits (π_T), respectively, are given by

$$\pi_B = (v - \omega \cdot \frac{c}{DT} - T) \cdot D, \tag{3.1}$$

$$\pi_S = (T - u + \gamma \cdot \frac{c}{DT}) \cdot D, \tag{3.2}$$

and

$$\pi_T \quad = \quad v - u - (\omega - \gamma) \cdot \frac{c}{DT} \cdot D.$$

Thus, for a given D and $T \in \mathcal{X} \cap \mathcal{Y}$, p^* increases with the cost of enforcement, c. Consequently, the channel's profits as well as the buyer's decrease with c. In contrast, the seller's profits increase with c. Also, given T and c, p^* decreases with D: Large customers will be less tolerant of poor quality than their small counterparts. We will examine in Section 4 how this property affects complex markets with buyers of varying sizes.

We next discuss the incentives of the buyer and the seller as a function of the purchase price T in the region $\mathcal{X} \cap \mathcal{Y}$. In order to do this, we first examine the structure of this region.

Lemma 3.1. $\exists\, t_1 \leq t_2$ *such that*
(i) $\mathcal{X} \cap \mathcal{Y} = (\frac{c}{D}, t_1] \cup [t_2, \infty)$,
(ii) if $\gamma \geq \frac{4c}{D}$, *the size of the interval* (t_1, t_2) *increases with* γ *and is given by*

$$|t_1 - t_2| = \sqrt{\gamma \cdot (\gamma - 4c/D)},$$

(iii) if $\gamma < \frac{4c}{D}$, $t_1 = t_2$ *and* $\mathcal{X} \cap \mathcal{Y} = (\frac{c}{D}, \infty)$.

Proof: Omitted. □

Combining Proposition 3.1 and Lemma 3.1, we study p^* as a function of the purchase price, T (the cases $\gamma < \frac{4c}{D}$ and $\gamma \geq \frac{4c}{D}$ are shown in Figure 1 and Figure 2 (see Appendix B) respectively. For the case of $\gamma \geq \frac{4c}{D}$, the behavior of p^* in the interval (t_1, t_2) is the result of the fact that c/D is "too small", i.e., the buyer is "too demanding". In such cases, given the stringent demand of the buyer, the seller finds it advantageous to produce the lowest level of quality $p^* = 1$ despite enforcement. Thus, the seller's quality choice need not be monotone in the purchase price. Likewise, the quality choice of the seller need not be monotone in the cost of enforcement. Figures 3 and 4 (see Appendix B) show the market efficiency as a function of T for the two cases.

We now examine the effect of the individual rationality constraints. For $T \in \mathcal{X} \cap \mathcal{Y}$, the buyer participates iff his profits are non-negative, i.e., $v - T - \omega\frac{c}{DT} \geq 0$, while the seller participates iff $T - u + \gamma\frac{c}{DT} \geq 0$.

To illustrate the managerial insights that our model demonstrates, it suffices to restrict ourselves to the case $\gamma < \frac{4c}{D}$, i.e., $\mathcal{X} \cap \mathcal{Y} = (\frac{c}{D}, \infty)$. Under this assumption, we consider the profit functions of the buyer and the seller.

Consider the seller first. Recall that the seller's profit when $T \in \mathcal{X} \cap \mathcal{Y}$ is

$$\pi_S(T) = T - u + \gamma\frac{c}{DT}.$$

In general, the region $(\frac{c}{D}, \infty)$ may be partitioned into three intervals M_1, M_2 and M_3, such that (i) if $T \in M_3$, the seller participates and his profits are increasing with the purchase price, T, (ii) if $T \in M_2$, the seller will not participate, and (iii) if $T \in M_1$, the seller will participate but his profits are *decreasing* with the purchase price, T.

Depending on the problem parameters, M_1 and / or M_2 may be empty. For example, in Figure 5 (see Appendix), $M_1 = (24, 27.71]$, $M_2 = \{\}$ and $M_3 = (27.71, \infty)$. However, if

$$\min\{u \cdot \frac{u}{4c/D}, \frac{4c}{D}\} > \gamma > \max\{c/D, u - c/D\},$$

all three regions are non-empty.

Next, we consider the buyer. Recall that the buyer's profit when $T \in \mathcal{X} \cap \mathcal{Y}$ is

$$\pi_B(T) = v - T - \omega\frac{c}{DT}.$$

In general, the region $(\frac{c}{D}, \infty)$ may be partitioned into four intervals, N_1, N_2, N_3 and N_4, such that (i) if $T \in N_2$, the buyer participates and his profits are decreasing with the purchase price, T, (ii) if $T \in N_1 \cup N_4$, the buyer will not participate, and (iii) if $T \in N_3$, the buyer will participate but his profits are *increasing* with the purchase price, T.

Depending on the problem parameters, any of the intervals N_1, N_2 and N_3 may be empty. For example, in Figure 5, $N_1 = \{\}$, $N_2 = (24, 30.98]$, $N_3 = (30.98, 77.63]$, $N_4 = (77.63, \infty)$. Also, if v is too low ($v < 2\sqrt{\frac{\omega c}{D}}$), then the buyer will not participate at any price ($N_2 = N_3 = \{\}$). However, if

$$v \cdot \frac{v}{4c/D} > \omega > \max\{c/D, v - c/D\} \text{ and } \gamma < \frac{4c}{D},$$

all the regions N_1, N_2, N_3 and N_4 are non-empty.

We now study the incentives of the buyer and seller simultaneously. We first show that any interval $[a, b]$ in which both the buyer and the seller participate cannot be such that the buyer's and seller's profits are decreasing with the purchase price in this interval.

Lemma 3.2. *Let $T \in \mathcal{X} \cap \mathcal{Y}$. Let $[a, b]$ be such that the buyer's profits are decreasing with T in $[a, b]$. Then, the seller's profits are increasing with T in $[a, b]$.*

Proof: The proof follows from noting that

$$\sqrt{\frac{\gamma c}{D}} = \text{argmin } \pi_S(T) \leq \text{argmin } \pi_B(T) = \sqrt{\frac{\omega c}{D}}.$$

□

Of course, if both the buyer and seller participate, the other three cases may exist, i.e., (i) the buyer's profits are increasing with T, while the seller's profits are decreasing with T, (ii) the buyer's profits are decreasing with T, while the seller's profits are increasing with T, and, (iii) both the buyer's and sellers profits are increasing with T (see Figure 5). The region (i) ($T \in M_1 \cap N_3$) is particularly interesting and counter to intuition. The region corresponds to the case where the purchase price, T is low and p^* is high (poor quality). An example might be a street vendor selling inexpensive watches. The region (ii) ($T \in M_3 \cap N_2$)

corresponds to the case where the incentives of the buyer and the seller are "normal", i.e., in this region, the buyer prefers lower purchase while the seller prefers higher purchase price. In this region, the purchase price, T is high and p^* is low (high quality). These observations are summarized in the table below that describes the seller's quality choice for the regimes where the buyer's and / or seller's profits are increasing / decreasing with the purchase price. The region where both $\pi^S(T)$ and $\pi^B(T)$ are decreasing does not exist, and the corresponding entry in the table is left vacant.

Table: Quality level decision by the seller.

	$\pi^S(T)$ Increasing	$\pi^S(T)$ Decreasing
$\pi^B(T)$ Increasing	Medium	Low
$\pi^B(T)$ Decreasing	High	

We now compare the quality level, p^* in the Full Rebate contracts above with the Specific Performance and Expectation Damages contracts studied by Plambeck and Taylor (2004). Under the first contract, if the buyer enforces the contract, the seller fixes the defective units at a per-unit cost of δ. Consequently, in this case, $p^* = \frac{c}{D\delta}$. Under the second contract, the seller compensates the buyer the revenue-loss due to defective units resulting in a quality level $p^* = \frac{c}{D\omega}$. Note that, in both these schemes, the quality level is independent of the purchase price. Also, in both cases, the buyer's profits decrease with purchase price and the seller's profits increase with purchase price (the "normal" region).

4 The Two Buyer Case

In this section, we study the case of two buyers and a single seller. We consider the individual and joint enforcement regimes. We assume, without loss of generality, that the per-unit cost of enforcement of buyer 1 is no greater than that of buyer 2, i.e.,

$$c_1/D_1 \leq c_2/D_2.$$

We refer to buyer 1 as the "demanding buyer" and to buyer 2 as the "lax buyer".

4.1 Individual Enforcement

In the individual enforcement regime, if buyer i enforces and finds the seller's units to be defective, the standard is enforced on the units supplied only to him. Since each buyers decision is independent of the others, buyer i enforces if $p > \frac{c_i}{D_i T}$.

For $i = 1, 2$, define S_i to be the set of purchase prices where the seller's profits, when only buyer i is present, are non-negative. Define S_{12} to be the set of purchase prices where the seller's profits when both buyers, 1 and 2 are present are non-negative under individual enforcement. Similarly, define B_i to be the set of purchase prices where the buyer's profits, π_B^i, when only buyer i is present are non-negative. Define B_{12} to be the set of purchase prices where the profits of both buyer 1 and buyer 2 are non-negative under individual enforcement.

Lemma 4.1.

$$S_1 \subseteq S_{12} \subseteq S_2 \textit{ and } B_2 \subseteq B_{12} \subseteq B_1.$$

Proof: We discuss the proof of $S_1 \subseteq S_{12}$. The proof of $S_{12} \subseteq S_2$ and of $B_2 \subseteq B_{12} \subseteq B_1$ is similar. Let $T \in S_1$. Then, when only buyer 1 is present, the seller's optimal quality level, p_1^*, is such that $\pi_S^1 \geq 0$. Further, since $\frac{c_1}{D_1} \leq \frac{c_2}{D_2}$, the seller's profits when only buyer 2 is present, at the quality level p_1^* is no less than $\frac{D_1}{D} \cdot \pi_S^1 \geq 0$. Therefore, the seller's optimal profit when both buyers are present, at the quality level p_1^* is no less than $\pi_S^1 \cdot (1 + \frac{D_2}{D_1}) \geq 0$. □

We now develop the analysis similar to the single buyer case. We first study the quality level and the market efficiency ignoring the individual rationality constraints. We then study the effect of these constraints.

Let

$$\mathcal{X}_1 = \{T : T > \frac{c_1}{D_1}\} \text{ and } \mathcal{X}_2 = \{T : T > \frac{c_2}{D_2}\}.$$

Let

$$\mathcal{Y}_1 = \{T : T + \gamma\frac{c_1}{D_1 T} \geq \gamma\} \text{ and } \mathcal{Y}_2 = \{T : T + \gamma \cdot \frac{c_2}{D_2 T} \geq \gamma + \frac{c_2}{D_2} \cdot \frac{D_1}{D}\}.$$

Proposition 4.1. *Assume that both the buyers and the seller participate. For the two buyer case under individual enforcement,*

(i) if $T \in \mathcal{X}_2 \cap (\mathcal{Y}_1 \cup \mathcal{Y}_2)$, the seller selects the quality level,

$$p^*_{12} = \begin{cases} \frac{c_1}{D_1 T} & \frac{D_1}{D} \geq \frac{\gamma}{T}(\frac{c_2/D_2 - c_1/D_1}{c_2/D_2}); \\ \frac{c_2}{D_2 T} & o/w. \end{cases}$$

Buyer 1 enforces only when $p^*_{12} = \frac{c_2}{D_2 T}$; buyer 2 never enforces and the market efficiency is

$$\eta_{12} = \begin{cases} 1 - \frac{\omega-\gamma}{v-u} \cdot \frac{c_1}{D_1 T} & \frac{D_1}{D} \geq \frac{\gamma}{T}(\frac{c_2/D_2 - c_1/D_1}{c_2/D_2}); \\ 1 - \frac{\omega-\gamma}{v-u} \cdot \frac{c_2}{D_2 T} - \frac{c_1}{(v-u)\cdot D} & o/w. \end{cases}$$

(ii) if $T \in \mathcal{X}_2 \cap (\mathcal{Y}_1 \cup \mathcal{Y}_2)'$, the seller selects the quality level $p^*_{12} = 1$ and both buyers enforce; the market efficiency is given by

$$1 - \frac{\omega - \gamma}{v - u} - \frac{c_1 + c_2}{(v-u) \cdot D},$$

(iii) if $T \notin \mathcal{X}_1$, the seller selects the quality level $p^*_{12} = 1$ and neither buyer enforces; the market efficiency is given by

$$1 - \frac{\omega - \gamma}{v - u},$$

(iv) if $T \in \mathcal{X}_1 \cap \mathcal{X}_2'$, the seller selects the quality level

$$p^*_{12} = \begin{cases} \frac{c_1}{D_1 T} & \gamma \cdot (1 - \frac{c_1}{D_1 T}) \geq T \cdot \frac{D_1}{D}; \\ 1 & o/w. \end{cases}$$

Buyer 1 enforces only when $p^*_{12} = 1$; buyer 2 never enforces and the market efficiency is

$$\eta_{12} = \begin{cases} 1 - \frac{\omega-\gamma}{v-u} \cdot \frac{c_1}{D_1 T} & \gamma \cdot (1 - \frac{c_1}{D_1 T}) \geq T \cdot \frac{D_1}{D}; \\ 1 - \frac{\omega-\gamma}{v-u} - \frac{c_1}{(v-u)\cdot D} & o/w. \end{cases}$$

PROOF: We show (i). The proof of (ii), (iii) and (iv) is similar. The seller's profit as a function of his defect rate, p is piece-wise linear:

$$\begin{cases} (T-u+\gamma p)D & p \leq \frac{c_1}{D_1 T}; \\ ((T-u)+p(\gamma - T\frac{D_1}{D}))D & \frac{c_1}{D_1 T} < p \leq \frac{c_2}{D_2 T}; \\ (T-u-(T-\gamma)p)D & p > \frac{c_2}{D_2 T}. \end{cases}$$

When $p < \frac{c_1}{D_2 T}$ (region 1), neither buyer enforces; when $\frac{c_1}{D_2 T} < p \leq \frac{c_2}{D_2 T}$ (region 2), buyer 1 enforces but buyer 2 does not; while when $p > \frac{c_2}{D_2 T}$ (region 3), both buyers enforce. Since $T \in \mathcal{Y}_1 \cup \mathcal{Y}_2$, the seller's profits at the quality levels $\frac{c_1}{D_1 T}$ or $\frac{c_2}{D_2 T}$ are higher than those at the quality level 1. Since the seller's profit function is piece-wise linear, an optimal solution for the seller is to choose either $p = \frac{c_1}{D_1 T}$ or $p = \frac{c_2}{D_2 T}$. An optimal solution is $p = \frac{c_1}{D_1 T}$ if $[T-u+\gamma\frac{c_1}{D_1 T}]D \geq [(T-u)+(\gamma - T\frac{D_1}{D})\frac{c_2}{D_2 T}]D$ or

$$\frac{D_1}{D} \geq \frac{\gamma}{T}(\frac{c_2/D_2 - c_1/D_1}{c_2/D_2}). \tag{4.3}$$

□

We next discuss the managerial implications of the Proposition. For simplicity, we restrict our discussion to the set of purchase prices $T \in \mathcal{X}_2 \cap (\mathcal{Y}_1 \cup \mathcal{Y}_2)$. We define m to be the market share of the demanding buyer, i.e.,

$$m = \frac{D_1}{D}.$$

We are interested in understanding how the market share and the costs of enforcement (c_1 and c_2) affect the quality level decision.

We first study how the market share, m affects the quality level decision for a given c_1/c_2. From our assumption that $c_1/D_1 \leq c_2/D_2$, it follows that

$$m \geq \frac{c_1}{c_1 + c_2}. \tag{4.4}$$

Rewriting inequality 4.3 in terms of m, we note that if

$$m + \frac{1}{m} \cdot \frac{\gamma}{T} \cdot \frac{c_1}{c_2} \geq \frac{\gamma}{T} \cdot (\frac{c_1}{c_2} + 1),$$

then the seller chooses $p = \frac{c_1}{D_1 T}$. For high enough values of m, the impact of the lax buyer is negligible. For low values of m ($m \approx \frac{c_1}{c_1+c_2}$), the two buyers have similar per-unit enforcement costs, i.e., $c_1/D_1 \approx c_2/D_2$. In that case, the gain from the reducing the quality from $\frac{c_1}{D_1 T}$ to $\frac{c_2}{D_2 T}$ is too small to compensate the seller for the impact of enforcement by the demanding buyer. In both cases, the result is that the seller behaves as if the entire market was demanding. The lax buyer "free rides" on the demanding buyer, and his profits are higher than would prevail if the demanding buyer did not exist. However, for intermediate values of m, the buyer may find it advantageous to "split" the market by supplying the quality level $\frac{c_2}{D_2 T}$. In that case, the demanding buyer will enforce, while the lax buyer will not. The demanding buyer "suffers" from the existence of lax buyer since he is supplied with lower quality than would have been the case if the lax buyer did not exist.

Next, we study the effect of the relative costs of enforcement of the demanding and lax buyer on the quality level decision, for a given value of the market share, m. Again, note that our assumption $c_1/D_1 \leq c_2/D_2$ implies that

$$\frac{c_1}{c_2} \geq \frac{1-m}{m}.$$

Under this assumption, the seller selects $\frac{c_1}{D_1 T}$ if

$$c_1/c_2 \geq (1 - \frac{mT}{\gamma}) \cdot \frac{m}{1-m}$$

and $\frac{c_2}{D_2 T}$ otherwise. If the cost of enforcement of the demanding buyer is low, she will enforce the contract anyway and the seller selects the quality level $\frac{c_2}{D_2 T}$.

These observations that can be summarized in the table below which illustrates the the quality choice by the seller in different regimes of the problem parameters. When the market share of the demanding buyer is low (but greater than $\frac{c_1}{c_1+c_2}$) or high, the seller's quality choice is high. However, when the market share of the demanding buyer is an intermediate value, the seller may select an inferior quality level. The quality level is also inferior if the cost of enforcement of the demanding buyer is similar to that of the lax buyer. Finally, note that the seller is induced to provide a higher quality level only when the purchase price, T is greater than a certain threshold.

Table: Quality level decision by the seller with market share of demanding buyer.

	Low	Medium	High
Market share of Demanding Buyer	High	Low	High

Table: Quality level decision by the seller with cost of enforcement of demanding to lax Buyer.

	Low	High
Cost of Enforcement of Demanding to Lax Buyer	Low	High

Table: Quality level decision by the seller with purchase price.

	Low	High
Purchase Price	Low	High

We now examine the effect of the individual rationality constraints for the region $(\mathcal{Y}_1 \cup \mathcal{Y}_2) \cap \mathcal{X}_2$. The analysis for the other regions is similar and is hence omitted.

Lemma 4.2. *Let* $T \in (\mathcal{Y}_1 \cup \mathcal{Y}_2) \cap \mathcal{X}_2$.

(i) If $\frac{D_1}{D} \geq \frac{\gamma}{T}(\frac{c_2/D_2 - c_1/D_1}{c_2/D_2})$, *buyer* $i, i = 1, 2$ *participates iff*

$$v - \omega \frac{c_1}{D_1 T} - T \geq 0,$$

and the seller participates iff

$$T - u + \gamma \frac{c_1}{D_1 T} \geq 0.$$

(ii) If $\frac{D_1}{D} < \frac{\gamma}{T}(\frac{c_2/D_2 - c_1/D_1}{c_2/D_2})$, *buyer* 1 *participates iff*

$$v - \omega \frac{c_2}{D_2 T} - T + \frac{c_2}{D_2} - \frac{c_1}{D_1} \geq 0,$$

buyer 2 participates iff

$$v - \omega \frac{c_2}{D_2 T} - T \geq 0,$$

and the seller participates iff

$$T - u + \gamma \frac{c_2}{D_2 T} - \frac{c_2}{D_2} \cdot \frac{D_1}{D} \geq 0.$$

4.2 Joint Enforcement

In the joint enforcement regime, if buyer i enforces the quality standard, and finds the units supplied to her to be defective, the seller compensates *all* the buyers for the defective units. We first derive results similar to Lemma 4.1 for the joint enforcement case.

Define S'_{12} to be the set of purchase prices where the seller's profits when both buyers, 1 and 2 are present are non-negative under joint enforcement. Define B'_{12} to be the set of purchase prices where the profits of both buyer 1 and buyer 2 are non-negative under joint enforcement. We can show the following result.

Lemma 4.3.

$$S'_{12} \subseteq S_1 \subseteq S_2 \textit{ and } B_2 \subseteq B_1 \subseteq B'_{12}.$$

We now study the incentives of the players under the joint enforcement regime with two buyers. We will show that the the seller's profit function is given by

$$\begin{cases} ((T-u)+\gamma p)D & p \leq \frac{c_1}{D_1 T}; \\ ((T-u+\frac{c_1}{D_1}) - (T-\gamma)p)D & \frac{c_1}{D_1 T} < p \leq \frac{c_2}{D_2 T}; \\ ((T-u) - (T-\gamma)p + \frac{c_1 c_2}{D_1 D_2 T p})D & p > \frac{c_2}{D_2 T}, \end{cases}$$

Thus, when $p \leq \frac{c_1}{D_1 T}$ (region 1), there is no enforcement; when $\frac{c_1}{D_1 T} < p \leq \frac{c_2}{D_2 T}$ (region 2), the enforcement is caused by the "demanding buyer"; while in the case when $p > \frac{c_2}{D_2 T}$ (region 3), both buyers have an incentive to enforce (However, each prefers that the other enforces). The seller may be tempted to select p in region 3 in order to provoke an "enforcement war" between the buyers hoping that none would enforce. Such a war can indeed be set up by the seller for certain values of the purchase price, T. Note that the "enforcement war" is equivalent to the classical game of chicken.

Proposition 4.2. *Assume that both the buyer and seller participate. Let $T \in \mathcal{X}_2$. For the case of two buyers under joint enforcement,* *(i) if $T \geq \gamma$, the seller selects the quality level $p'^{*}_{12} = \frac{c_1}{D_1 T}$, and no buyer enforces; the market efficiency is given by*

$$\eta'_{12} = 1 - \frac{\omega - \gamma}{v - u} \cdot \frac{c_1}{D_1 T}$$

(ii) if $T < \gamma$, the seller selects the quality level,

$$p^{*}_{12} = \begin{cases} \frac{c_2}{D_2 T} & \gamma - T + \frac{c_2}{D_2} - \frac{c_1}{D_1} \leq \frac{c_2}{D_2 T} \cdot (\gamma - \frac{c_1}{D_1}); \\ 1 & o/w, \end{cases}$$

buyer 1 enforces with probability $1 - \frac{c_2}{D_2 p^{*}_{12} T}$ and buyer 2 with probability $1 - \frac{c_2}{D_2 p^{*}_{12} T}$; the market efficiency is given by

$$\eta'_{12} = 1 - \frac{\omega - \gamma}{v - u} \cdot p^{*}_{12} + \frac{c_1 c_2}{D_1 D_2 T p^{*}_{12}} - \frac{c_1}{D_1} - \frac{c_2}{D_2}.$$

PROOF: The proof of (i) is a special case for $N > 1$ buyers which is provided in the Appendix as a special case of N buyers. The proof of (ii) is similar. □

Thus, when the purchase price is high enough ($T \geq \gamma$), the threat of enforcement is credible and the seller's quality choice is conservative. However, for low values of the purchase price ($T < \gamma$), the seller is tempted to provide lower quality as the punishment from being enforced is not high enough.

Lemma 4.4. *Let $T > c_2/D_2$ and $T \geq \gamma$. For the case of two buyers under joint enforcement, buyer $i, i = 1, 2$ participates iff*

$$v - \omega \frac{c_1}{D_1 T} - T \geq 0,$$

and the seller participates iff

$$T - u + \gamma \frac{c_1}{D_1 T} \geq 0.$$

5 Joint Enforcement with N Buyers

In this section, we discuss extensions on the results of two buyers to N buyers. We outline how the results in Section 4 extend to N buyers in the joint enforcement regime. We do not discuss the extension for individual enforcement, as it offers no additional insight into the problem.

Without loss of generality, we assume that $\frac{c_1}{D_1} \leq \frac{c_2}{D_2} \leq ... \leq \frac{c_N}{D_N}$. We will find the following notation useful. Let A_j be the sum of the demands of buyers $1, 2, ..., j$, i.e., $A_j = \sum_{i=1}^{j} D_i, j = 1, 2, .., N$. We will assume throughout that $T > \frac{c_N}{D_N}$ as this is the interesting case in which all the quantities, $\frac{c_1}{D_1 T}, \frac{c_2}{D_2 T}, ..., \frac{c_N}{D_N T}$ are strictly less than one.

Recall that in the individual enforcement regime, each buyers' decision is independent of the others, and hence, buyer i enforces if $p > \frac{c_i}{D_i T}$. The seller's profit is given by

$$\begin{aligned}\pi_S &= \sum_{i: p \leq \frac{c_i}{D_i T}} D_i[T - (u - \gamma p)] + \sum_{i: p > \frac{c_i}{D_i T}} D_i[T(1-p) - (u - \gamma p)] \\ &= D[T - (u - \gamma p)] - \sum_{i: p > \frac{c_i}{D_i T}} D_i T p.\end{aligned}$$

Consequently, the seller's quality choice, $p^* \in \{\frac{c_1}{D_1 T}, \frac{c_2}{D_2 T}, ..., \frac{c_N}{D_N T}, 1\}$. By enumerating the profits at each element of this set and setting $c_{N+1}/D_{N+1} = T$, we identify that the seller selects the quality level $\frac{c_k}{D_k T}, 1 \leq k \leq N+1$ if

$$T \geq \gamma D \frac{(\frac{c_j}{D_j} - \frac{c_k}{D_k})}{\frac{c_j}{D_j} A_{j-1} - \frac{c_k}{D_k} A_{k-1}}, \forall j > k$$

and

$$T \leq \gamma D \frac{(\frac{c_j}{D_j} - \frac{c_k}{D_k})}{\frac{c_j}{D_j} A_{j-1} - \frac{c_k}{D_k} A_{k-1}}, \forall j < k.$$

In that case, buyers $\{1, 2, ..., k-1\}$ enforce, while buyers $\{k, k+1, ..., N\}$ don't.

Now consider the joint enforcement regime. Clearly, in this case as well, the seller's quality choice, $p^* \in \{\frac{c_1}{D_1 T}, \frac{c_2}{D_2 T}, ..., \frac{c_N}{D_N T}, 1\}$. However, we show that when $T \geq \gamma$, the seller selects the quality level $\frac{c_1}{D_1 T}$ and no buyer enforces.

Proposition 5.1. *Let $T > c_N/D_N$ and $T \geq \gamma$. Assume that all the N buyers and the seller participate. Then, for the case of joint enforcement, the seller selects the quality level $\frac{c_1}{D_1 T}$ and no buyer enforces.*

PROOF: Clearly, when $p \leq \frac{c_1}{D_1 T}$, the sellers profit function is increasing with p. Further, in the region $\frac{c_1}{D_1 T} < p \leq \frac{c_2}{D_2 T}$, player 1 enforces (he knows no other player would enforce) and the sellers profit function is decreasing. Now consider the case $p > \frac{c_2}{D_2 T}$. Is an Audit War possible? In this case, $\exists\, r \geq 2$ such that $\frac{c_r}{D_r T} \leq p \leq \frac{c_{r+1}}{D_{r+1} T}$. Clearly, buyers $r+1, r+2, ..., N$ will never enforce. We now restrict ourselves to the set of buyers $1, 2, ..., r$. Let x_i be the probability that buyer i does not enforce. Then the probability that none of the others enforce is given by:

$$z_i = \frac{\Pi_{i=1}^{r} x_i}{x_i}.$$

The pay-offs for buyer i when none of the other buyers enforce and when at least one other buyer enforces is given in the table below.

Table: Pay-offs for buyer i

	Buyer i does not enforce	Buyer i enforces
Buyers $j, j \neq i$ don't enforce	$[(v - wp) - T]D_i$	$[(v - wp) - T(1-p) - \frac{c_i}{D_i}]D_i$
At least one buyer $j, j \neq i$ enforces	$[(v - wp) - T(1-p)]D_i$	$[(v - wp) - T(1-p) - \frac{c_i}{D_i}]D_i$

In equilibrium, buyer i is indifferent between enforcing and not enforcing, i.e, $z_i = \frac{c_i}{D_i p T}$. Now

$$x_1 = \sqrt[r-1]{\frac{z_2 z_3 ... z_r}{z_1^{r-2}}} = \sqrt[r-1]{\frac{c_2/D_2 \cdot c_3/D_3 ... c_r/D_r}{(c_1/D_1)^{r-2}}}(pT)^{-\frac{1}{r-1}},$$

and hence

$$x_1 z_1 T p = \sqrt[r-1]{\frac{c_2/D_2 \cdot c_3/D_3 ... c_r/D_r}{(c_1/D_1)^{r-2}}}(pT)^{-\frac{1}{r-1}} \frac{c_1}{D_1 p T} pT = \sqrt[r-1]{c_1/D_1 \cdot c_2/D_2 \cdot c_3/D_3 ... c_r/D_r}(pT)^{-\frac{1}{r-1}}.$$

Therefore, the seller's profit function is given by:

$$
\begin{aligned}
[(T-(u-\gamma p))x_i z_1 + (T(1-p)-(u-\gamma p)](1-x_i z_i)]D &= (T-u-(T-\gamma)p)D + Tpx_i z_i D \\
&= (T-u-(T-\gamma)p)D \\
&+ \sqrt[r-1]{c_1/D_1 \cdot c_2/D_2 \cdot c_3/D_3 ... c_r/D_r}(pT)^{-\frac{1}{r-}}
\end{aligned}
$$

which is decreasing in p (since $r > 2$). □

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