Optimal Brand Umbrella Size

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Abstract

In a framework or repeated-purchase experience goods with seller's moral hazard and imperfect monitoring, umbrella branding may improve the terms of the implicit contract between seller and buyers, whereby the seller invests in quality and buyers pay a high price. The benefits from umbrella branding may come from one of two sources: in some cases, umbrella branding leads to a softer punishment of product failure, which increases the seller's value. In other cases, umbrella branding leads to a harsher punishment of product failure, which allows for an equilibrium where buyers trust in the seller's brand. Against these benefits, one must consider the costs of umbrella branding, namely that, in some cases, a bad signal in one product kills two streams of revenue and profit. Combining costs and benefits, I determine the set of parameter values where umbrella branding is an optimal strategy.

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1 Introduction

Umbrella branding, the practice of labeling more than one product with a single brand name, is common practice among multiproduct firms in a variety of markets.[1] Examples include *Canon* cameras and photocopiers, *Colgate* toothpaste and toothbrushes, *Levi's* jeans and sneakers. To underscore the importance of the strategy of umbrella branding, Aaker and Keller (1990) quote a Nielsen report according to which "from 1977 to 1984, approximately 40% of the 120 to 175 new brands that were introduced into supermarkets annually were extensions" (p. 27).

In this paper, I propose a model that highlights the costs and benefits of umbrella branding and ultimately provides the basis for a theory of optimal umbrella size. I consider a framework or repeated-purchase experience goods with seller's moral hazard. Firms have a short-run incentive to reduce quality and save costs, as consumers can only observe quality ex post. However, there exist equilibria whereby firms refrain from cheating consumers. In these equilibria, when consumers infer that the firm has cheated them, the firm's reputation breaks down, whereby consumers no longer pay a high price for the firm's product and the firm no longer produces quality products.[2]

I show that umbrella branding may improve the terms of the "implicit contract" between firm and consumers. This may happen for two reasons. First, umbrella branding may provide for a harsher punishment on cheating sellers. For example, suppose that a low-quality product fails with probability $1 - \beta$. If a firm sells two products under different names and consumers punish each product failure separately, then the the probability that cheating is detected in a given product is given by $1-\beta$. If however the firm sells both products under the same name and the firm is punished in both products following any product failure, then the probability of punishment given that the firm shirks in both products is given by $1 - \beta^2$, which is greater than $1 - \beta$.

A second reason why umbrella branding may improve equilibrium payoff is that, if there is slack in the no-deviation constraint, consumers may be more lenient with a firm that sells two products under the same name. Specifically, there exist parameter values such that the threat of punishing a firm only when two simultaneous product failures take place is sufficient to keep the firm from shirking; and lowering the probability of punishment increases equilibrium payoff.

Generically, if the value of the discount factor is very high then the optimal

[1]The terms "brand extension" or "brand stretching" are also used. Some authors distinguish between "line extension" (when the new product is in the same class) and "brand extension," when the new product is in a different class. Given the level of generality of the model considered in this paper, this distinction is not crucial.

[2]See Klein and Leffler (1981), Shapiro (1983).

equilibrium is for the seller to umbrella brand and for consumers to punish the seller only if two simultaneous product failures take place. For lower values of the discount factor, no umbrella branding is optimal. For still lower values of the discount factor, the optimal equilibrium is for the seller to umbrella branding and for consumers to punish the seller for any product failure. Finally, if the discount factor is very low then the only equilibrium is the repetition of the static Nash equilibrium (where no quality products are sold)

■ **Related literature.** There is a fairly sizeable literature on brand extension and umbrella branding. One first explanation for brand extensions is that umbrella branding is a form of economies of scope, as it economizes on the costs of creating a new brand. Tauber (1988), for example, argues that

> World-competitive pressures and slow-growth markets keep cost containment as a primary management imperative. In this environment, growth through brand leverage will continue to flourish (p. 30).

A related idea is that brands have an intrinsic value (status or otherwise). Brands are therefore like a "public good" in the sense that the more products are sold under the same brand the greater the total value created. See Pepall and Richards (2002).

A different perspective on brand extensions is that, in a world where consumers are uncertain about product characteristics (due to horizontal or vertical differentiation), brands may play an informational role. As the *Economist* argues (July 2nd, 1994),

> Brands are created because buyers crave information. They see a huge range of products that look the same and seem to perform similar. Brands offer a route through the confusion.

Wernerfelt (1988) suggests that, if umbrella branding is costly, then it may serve as a signal of new product quality that is used by high-quality firms only. In a similar context (adverse selection), Montgomery and Wernerfelt (1992) show that, in a free-entry equilibrium, single-product and multiple-product (umbrella) brands coexist. Still in a similar context, Cabral (2000) studies the direct and feed-back reputational effects of brand extensions.

In a framework of horizontal product differentiation, Sappington and Wernerfelt (1985) argue that umbrella branding may reduce uncertainty about a new product's attributes, a fact that increases value if consumers are risk averse. They also present empirical evidence consistent with the predictions of their model.[3] Choi and Scarpa (1992) show that a benefit of umbrella branding is to credibly deter entry.

[3]Other papers that include empirical evidence relating to brand extensions include Aaker and Keller (1990), Sullivan (1990) and Erdem (1998).

More closely related to my paper, Choi (1998) proposes a moral-hazard theory of brand extensions. In a repeated-game framework, sellers introduce high-quality products and buyers pay a high price for them. Buyers can observe quality ex post and punish sellers who cheat by introducing low-quality products; the nature of the punishment is that future introductions are no longer believed to be of high quality as before. My paper differs from Choi (1998), among other things, in that it considers the costs, not only the benefits, from umbrella branding.

Recent work that is closely related to mine includes Cai and Obara (2006) and Hakenes and Peitz (2004). Cai and Obara consider a repeated game framework similar to mine. However, they assume that sellers make a once-and-for-all investment in product quality. Moreover, the integrated firm case, which is their equivalent to umbrella branding, assumes that quality decisions are the same across all products. They therefore address a different set of issues. In particular, one of the important issues in my model is precisely whether the binding constraint under umbrella branding is to shirk on one product or to shirk in both products. Hakenes and Peitz consider a firm that sells a product in each of two periods and must decide on product quality at the beginning of the first period. Again, my approach differs from theirs in that I consider repeated quality decisions.

Two additional recent papers are Thal (2006) and Dana and Spier (2006), both of which make important contributions to the umbrella branding literatures. Thal develops a model in the line of Cabral (2000) but considers the case when the seller can choose the degree of correlation between the products sold under the same umbrella (whereas Cabral, 2000, assumes such correlation as exogenously given). Dana and Spier address the important case when consumers are small and receive imperfect private signals, which in turn makes the task of punishing shirking more difficult. In the end of Section 3 I will touch on the issue of private monitoring.

The remainder of the paper is structured as follows. In Section 2, I consider the case when the seller's quality choices are perfectly observable. In this context, I show that umbrella branding has no equilibrium effects. The result is similar to Bernheim and Whinston's (1990) irrelevance result in the context of multimarket contact. In Section 3, I consider the case of imperfect monitoring. I characterize the optimal solution under umbrella branding, compare it to the optimal solution under no-umbrella branding, and characterize the overall optimal equilibrium.

2 Deterministic quality

Consider a firm that exists for an infinite number of periods and must decide, at the beginning of each period, whether or not to spend effort into producing quality products; a product is of high quality if and only if the firm spends effort on it, at a cost ϵ.

The good in question is an experience good. Specifically, consumers observe ex-post whether a product performs well or rather breaks down. Product performance is batch specific, so all consumers observe the same performance outcome. If the product is of low quality, then it breaks down with probability one; if the product is of high quality, then it works with probability one.[4] Consumers value products that work (high-quality products) at π and products that do not work (low-quality products) at zero.

I assume that, in the eyes of consumers, a firm's history is encapsulated in the value of its brand. I moreover assume that a firm's brand can take two values: trustworthy or not trustworthy. This assumption is motivated by the belief that consumers have finite capabilities to process information, and thus encapsulate a firm's history into a coarse measure consisting of a finite number of states.[5]

Finally, I consider a simple class of consumer strategies: in each period, consumers are willing to pay a price that is a function of the firm's brand value; and upon observation of the firm's quality level they update their belief regarding the brand's trustworthiness. Throughout most of the paper, I assume this updating is deterministic. Later in the paper I also consider the possibility of consumers randomizing.

Regarding parameter values, I make the following assumption:

Assumption 1 $\epsilon < \pi$.

Assumption 1 implies that the efficient solution is for the firm to produce high-quality products: the added value it creates more than compensates for the extra cost. And to the extent that the seller can extract the consumer's surplus, it also implies that the seller's optimal solution is to produce high-quality products.

To summarize, the timing in each period is as follows. First, the firm decides whether to spend ϵ in order to produce a high-quality product. Then consumers, who classify the firm as being trustworthy or not, decide whether to buy from the firm and, if so, pay a price equal to the expected value of the product given their beliefs about product quality. Finally, consumers who purchased the product update their beliefs regarding the firm (trustworthy or not).

The above model and type of consumer strategy is essentially the model of quality provision proposed by Klein and Leffler (1981).[6] If the game were played only once, then no high-quality products would be produced. In fact, it is the firm's dominant strategy in the one-shot game not to make any effort to provide quality. However, if

[4]In the next section, I relax the assumption of deterministic quality levels.

[5]See Monte (2006), Wilson and Ekmekci (2006).

[6]See also Telser (1980), Shapiro (1983).

the game is repeated an infinite number of times, then there may exist equilibria such that high quality products are sold. In the present context, the quality assurance equilibrium works as follows. Let all consumers start by considering the firm's brand trustworthy, meaning consumers expect the firm to offer quality products. As long as the firm does so, that is, as long as consumers observe high quality, consumers continue to consider the brand to be trustworthy, that is, to expect high quality and pay the corresponding price $p = \pi$. If at any time consumers observe a low quality product, then they lose their trust in the firm and stop purchasing from it.

Formally, the consumer's strategy is to buy at a price π if the brand is trustworthy and not to buy if the brand is not trustworthy; and a brand is trusted if and only if it was trustworthy before the current period and high quality was observed in the current period. The firm's strategy is to make effort to offer high quality if and only its brand is considered trustworthy by consumers. One of the central results in Klein and Leffler (1981) is that, if the firm sells frequently enough, then the above is indeed a Nash equilibrium. Let δ be the firm's discount factor. Define an optimal equilibrium as a subgame perfect equilibrium that maximizes the firm's expected discounted profit. We then have the following result.[7]

Proposition 1 (Klein and Leffler) *If $\delta > \epsilon/\pi$, then the following is an optimal equilibrium: (a) the seller chooses high-quality if an only if consumers trust its brand; (b) consumers purchase from the seller (at price π) if and only if they consider the brand to be trustworthy; (c) the brand is trustworthy if and only if past products sold under that brand have performed well.*

So far, the analysis corresponds to the standard repeated-game approach to relational contracting. The only value added of the above is to cast such approach in a particular framework that features a formal rule of trust in a brand name. Next, I consider the extension of this framework in the context of a firm selling two products. I do so first in the a deterministic context and then, in Section 3, in an imperfect monitoring context.

■ **Two-product firms and umbrella branding.** I now consider the case of a firm that sells two products. Specifically, I consider an economy with a continuum of firms. Each firm is endowed with two products and faces two main decisions: At time of birth, it must decide whether to sell both products under the same name, or rather under different names. Then, in each period, it must choose the quality level of each of its products. I assume that effort is independent across products, that is, producing a high-quality product 1 costs ϵ regardless of the effort choice in the other product.

[7] The proof of this and the remaining results in the paper may be found in the Appendix.

The firm's choice of name is important because consumers cannot observe the ownership of each brand:

Assumption 2 (brand ownership) *Consumers observe brand names but not the identity of each brand owner.*

A consequence of Assumption 2 is that, if a firm does not umbrella brand, then consumers consider the firm's products are being offered by separate firms.

I now show that, under deterministic quality levels, the case I have been considering in this section, umbrella branding has no impact on firm profits.

Proposition 2 (irrelevance of umbrella branding) *Under perfect information, firm profits are the same with or without umbrella branding.*

The result is similar to Proposition 1 in Bernheim and Whiston (1990). Intuitively, adding a second product doubles a firm's value along the equilibrium path, but it also doubles the gain from shirking on offering quality products. In other words, the entire no-deviation constraint is multiplied by two, leading to the same solution as under no umbrella branding.

In the next section, I consider the case of imperfect monitoring, that is, the case when observed quality is a stochastic function of firm effort. I will show that the Bernheim-Whinston irrelevance result breaks down.

3 Stochastic quality level

Consider now the case of random quality level. As before, consumers cannot observe quality ex ante, only ex post. Now I add the assumption that (a) there is a stochastic relation between effort and quality and (b) consumers only observe quality, not effort. Specifically, I assume that if the firm makes effort, then its product works with probability α, whereas low effort leads to a product that works with probability $\beta < \alpha$. For simplicity, I now assume that consumers value a product that at π/α, so expected value given firm effort is π.

I first consider the extension to imperfect monitoring of the case of one-product firms. The problem at hand is, again, to find an equilibrium that maximizes the firm's expected discounted payoff.

Proposition 3 *If $\delta > \epsilon/(\alpha\,\pi)$, then the following is an optimal equilibrium: (a) the seller chooses high-quality if an only if consumers trust its brand; (b) consumers purchase from the seller (at price π) if and only if they consider the brand to be trustworthy; (c) the brand is trustworthy if and only if past products sold under that brand have performed well.*

Notice that Proposition 3 is nearly identical to Proposition 1, with two important differences. First, the lower bound on the discount factor is higher, thus there may be values of δ such that quality provision is an equilibrium under perfect monitoring but not under imperfect monitoring. Second, the firm's value is lower under imperfect monitoring: $(\pi-\epsilon)/(1-\delta\,\alpha)$, as opposed to $(\pi-\epsilon)/(1-\delta)$ under perfect monitoring. Notice also that, as expected, the relevant values are continuous in α, so the perfect monitoring case obtains as the limit as α converges to 1.

■ **Two-product firms and umbrella branding.** Analogously to Section 2, I now consider the case of a firm that sells two products. As before, the firm's strategy has two components: at time of birth, it must decide whether to sell both products under the same name, or rather under different names; and then, in each period, it must choose the quality level of each of its products. The consumers' strategy is now a little more complicated than before. In each period, the outcome is one of three cases: no failure, failure of one product, failure of both products. In each case, consumers must decide whether or not to lose trust in the brand.

As before, I look for the optimal equilibrium from the firm's point of view, that is, the equilibrium leading to the highest discounted value. I also refer to a quality provision equilibrium, or simply a quality equilibrium, as one where the seller starts off by offering quality products (unlike the one-shot game equilibrium). My main result is as follows:

Proposition 4 *For each set of values $\alpha,\beta,\epsilon,\pi$, there exist threshold values $0<\delta_1\leq\delta_2\leq\delta_3\leq 1$, such that*

- *If $\delta<\delta_1$, then no quality provision equilibrium exists, that is, the only equilibrium is the repetition of the static game equilibrium.*
- *If $\delta_1<\delta<\delta_2$, then the optimal equilibrium is for the seller to umbrella brand and consumers to lose trust following any product failure.*
- *If $\delta_2<\delta<\delta_3$, then the optimal equilibrium is for the seller not to umbrella brand.*
- *If $\delta>\delta_3$, then the optimal equilibrium is for the seller to umbrella brand and consumers to lose trust following two simultaneous product failures only.*

Moreover, there exist values ϵ_1,ϵ_2 such that

- *If $\epsilon<\epsilon_1$, then $\delta_1<\delta_2<\delta_3$*
- *If $\epsilon_1<\epsilon<\epsilon_2$, then $\delta_1=\delta_2<\delta_3$*

- *If* $\epsilon > \epsilon_2$*, then* $\delta_1 = \delta_2 = \delta_3$

Figure 1 illustrates Proposition 4. Consider first the case when ϵ is small, say $\epsilon = \epsilon_0$, so that the gains from offering a quality equilibrium are particularly high. For this value of ϵ (and for the values of π, α and β considered in the figure), we obtain three thresholds for the value of the discount factor δ. If the discount factor is very low, specifically if $\delta < \delta_1$, then no equilibrium exists where the seller offers quality products, either with or without umbrella branding. If $\delta_1 < \delta < \delta_2$, then the optimal equilibrium is for the seller to umbrella brand and for consumers to lose trust following *any* product failure. For higher values of the discount factor, $\delta_2 < \delta < \delta_3$, the optimal equilibrium is for the seller not to umbrella brand. In this case, the equilibrium in each product (and each brand) follows Proposition 3. Finally, for very high values of the discount factor, $\delta > \delta_3$, the optimal equilibrium is for the seller to umbrella brand and for consumers to lose trust following two *simultaneous* product failures only.

More generally, the optimal solution is as follows: in region UB_2, follow umbrella branding and the simultaneous failure policy; in region UB_{1+}, follow umbrella branding and the policy of punishing any failure; in region "no UB," sell the two products under separate names. Finally, in the southeast region (low δ or high ϵ) the optimal solution is to play the repeated Nash equilibrium (no quality provision).

Notice that the thresholds δ_i are only weakly increasing in i. It is quite possible that $\delta_1 = \delta_2$ or/and $\delta_2 = \delta_3$. In fact, if $\epsilon_1 < \epsilon < \epsilon_2$ then $\delta_1 = \delta_2$, so that the optimal equilibrium is either UB_2 or no umbrella branding. If $\epsilon_2 < \epsilon < \epsilon_3$, $\delta_1 = \delta_2 = \delta_3$, so that the optimal policy is either the static Nash equilibrium or UB_2.

In order to understand the intuition for the main results, it helps to re-write the no deviation constraint as follows:

$$(\pi - \epsilon) + \delta\,\rho\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta} \geq \pi + \delta\,\rho'\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta}, \tag{1}$$

where ρ is the continuation probability along the equilibrium path and ρ' the continuation probability following a deviation. Defining $\xi \equiv \frac{\epsilon}{\pi - \epsilon}$, (1) is equivalent to

$$\delta \geq \frac{\xi}{(\rho - \rho') + \rho\,\xi}$$

This conditions shows that there are two factors that determine the lowest value of δ such that a quality equilibrium exists: the continuation probability, ρ, and the difference in continuation probabilities between the equilibrium action and shirking, $\rho - \rho'$. The higher either of these is (everything else constant), the lower the lower bound on the value of δ. Moreover, the higher ξ (that is, the lower the efficiency gains from quality provision), the greater the relative importance of ρ vis-a-vis $\rho - \rho'$.

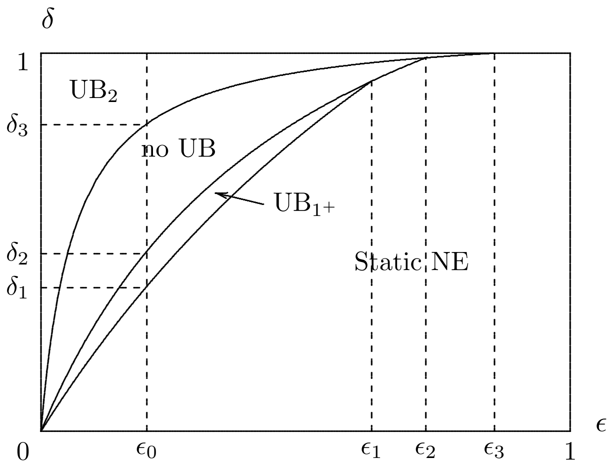

Figure 1: Optimal equilibrium as a function of cost of effort, ϵ, and the discount factor, δ (assuming $\pi = 1,\ \alpha = .9,\ \beta = .6$).

Under no umbrella branding, $\rho = \alpha$ and $\rho' = \beta$. We thus have

$$\delta \geq \frac{\xi}{(\alpha - \beta) + \alpha\,\xi}$$

Consider now the case of umbrella branding and the equilibrium whereby loss of trust follows any product failure occurrence. In the appendix I show that the binding constraint corresponds to offering low quality in both products. The no-deviation constraint is then given by

$$2\,(\pi - \epsilon) + \delta\,\rho\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta} \geq 2\,\pi + \delta\,\rho'\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta},$$

Dividing through by two, we get an expression identical to (1). The difference between no umbrella branding and the form of umbrella branding we are considering resides in the values of the continuation probabilities ρ, ρ'. Specifically, under the umbrella branding equilibrium we are considering, we have $\rho = \alpha^2$ and $\rho' = \beta^2$. The condition on δ is therefore

$$\delta \geq \frac{\xi}{(\alpha^2 - \beta^2) + \alpha^2\,\xi}$$

Notice that the continuation probability, ρ, under no umbrella branding, $\rho = \alpha$, is lower than the continuation probability under umbrella branding, $\rho = \alpha^2$. Moreover, the differential in continuation probabilities, $\rho' - \rho$, is given by $\rho' - \rho = \alpha - \beta$ under no umbrella branding and $\rho' - \rho = \alpha^2 - \beta^2$ under umbrella branding. If $\alpha + \beta < 1$, then $\alpha - \beta > \alpha^2 - \beta^2$. We thus conclude that, if $\alpha + \beta < 1$, then the quality provision set is greater under no-umbrella branding (with respect to umbrella branding and punishing any deviation). Intuitively, punishing any deviation lowers the equilibrium value (lower continuation probability) with respect to no-umbrella branding. Formally, we have $\rho = \alpha^2$ instead of $\rho = \alpha$. Moreover, if $\alpha + \beta < 1$, then punishing only one deviation increases the incentives for shirking.

If $\alpha + \beta > 1$, then we have two counteracting effects: equilibrium value is greater under no umbrella branding; but the decline in continuation probabilities is greater under umbrella branding (harsher punishment). In this case, no umbrella branding is better only if ξ is high enough, specifically, if

$$\xi \equiv \frac{\epsilon}{\pi - \epsilon} > \xi_1 = \frac{(\alpha - \beta)(\alpha + \beta - 1)}{(1 - \alpha)\,\alpha}$$

A similar intuition applies to comparison between no umbrella branding and the policy of umbrella branding and the equilibrium where only two simultaneous product failures lead to the loss of reputation. In this case, the continuation probability favors the umbrella branding equilibrium. In fact, $\rho = 1 - (1 - \alpha)^2$, which is greater than α. However, the differential in continuation probabilities favors no umbrella branding. As before, if ξ is sufficiently high then the first effect dominates, and umbrella branding leads to a higher set of values of the discount factor.

So far I have only considered the intuition for the relative size of the sets of the discount factor such that a quality equilibrium exists. Going from here to optimal equilibria is easy. In fact, the continuation probability is highest under umbrella branding and punishment of two simultaneous product failures, and lowest under umbrella branding and punishment of any product failure. It follows that, whenever more than one solution is feasible, umbrella branding and punishment of two simultaneous product failures is preferable to both umbrella branding and punishment of any product failure and no umbrella branding; and no umbrella branding is preferable to umbrella branding and punishment of any product failure.

■ **Stochastic reputation updating.** Throughout the paper, I have assumed that consumers follow a deterministic reputation procedure. Alternatively, I could consider the possibility of a random transition process. Specifically, suppose that, following one single product failure, consumers lose trust in the brand with probability θ_1, whereas two simultaneous product failures lead to reputation breakdown with probability θ_2. The situation I considered in the previous sections thus corresponds to the extreme cases when $\theta_1 = 0$ and $\theta_2 = 1$, or $\theta_1 = 1$ and $\theta_2 = 0$. What if we allow for any values $\theta_i \in [0, 1]$?

It can be shown that the optimal solution is to choose the lowest values of θ consistent with the no deviation constraints. Specifically, for very low values of δ, even if $\theta_1 = \theta_2 = 1$ no equilibrium exists other than the repetition of the static Nash equilibrium. If $\delta = \delta_1$, where δ_1 is derived in Proposition 4 and illustrated in Figure 1, setting $\theta_1 = \theta_2 = 1$ just satisfies the no-deviation constraint. As the value of δ increases, the optimal value of θ_1 is decreased, keeping $\theta_2 = 1$. For $\delta = \delta_3$, where δ_3 is derived in Proposition 4 and illustrated in Figure 1, $\theta_1 = 0$ and $\theta_2 = 1$ just satisfies the no-deviation constraint. Finally, for higher values of δ, that is, $\delta > \delta_3$, the optimal solution consist of setting $\theta_1 = 0$ and θ_2 to the lowest value consistent with the no-deviation constraint (of shirking in one product only).[8]

Intuitively, the idea is to concentrate punishment as much as possible in the event of two simultaneous product failures. The optimal solution then "convexifies" the solution determined in Proposition 4.

In many applications it may not be realistic to assume consumers play a random strategy. However, an alternative interpretation of consumer random strategies is that *each* consumer loses trust with probability θ_i. To the extent that firms are risk neutral and there is a continuum of consumers, the firm's value is the same when all consumers leave the firm with probability θ or a fraction θ leaves with probability 1.

4 Concluding remarks

Although I do not develop a full theory of optimal brand size, my results provide some pointers towards the costs and the benefits from umbrella branding. The benefits from umbrella branding may come from one of two sources: if the discount factor is sufficiently high, then umbrella branding allows for an equilibrium with a softer punishment (specifically, the equilibrium where consumers only lose trust in a brand when they observe two simultaneous product failures). If, by contrast, the discount factor is very small, then umbrella branding may allow for an equilibrium where quality is supplied at all (specifically, the equilibrium where consumers lose trust following any product failure).

Against these benefits, we must also consider the potential costs arising from umbrella branding. Under the equilibrium where any product failure is punished by loss of trust in the brand, the seller is subject to an unfortunate "domino" effect: a bad signal in one product kills two streams of revenue and profit. If the no-deviation constraint is satisfied under no umbrella branding, then the latter is the better choice. In fact, under no umbrella branding, the effect of a product failure is limited to the profit stream from the product that failed.

[8] The technical details of this derivation are available upon request.

Appendix

Proof of Proposition 1: Since quality is a deterministic function of the firm's effort, the best case for an efficient equilibrium is for consumers' to impose the harshest "punishment" on a firm that shirks (Abreu, 1986). I will thus assume that, following product failure, consumers lose their trust in the brand and never purchase again. Make the equilibrium hypothesis that an equilibrium exists whereby a firm makes effort in every period. The firm's value is then given by

$$v = \frac{\pi - \epsilon}{1 - \delta}.$$

If the firm were not to make effort in a given period, it would earn π during that period but zero in all future periods, since its brand would lose its trustworthiness. It follows that the seller's no-deviation constraint is given by $v > \pi$, or simply $\delta > \epsilon/\pi$. Given this, the consumers' beliefs along the equilibrium path are consistent, and so is their strategy. ■

Proof of Proposition 2: Given the same reasoning as in the proof of Proposition 1, I consider extreme consumer punishments. Specifically, I assume that, following *any* product failure, consumers lose their trust in the brand and never purchase again. Given this form of punishment, the binding no-deviation constraint corresponds to the firm shirking in both products. But then both the expected value along the equilibrium path and the expected payoff from shirking are twice what they are in the no-deviation of a one-product firm. ■

Proof of Proposition 3: Given Assumption 1, an optimal equilibrium is one that leads the firm to invest in quality. In order for the solution to be an equilibrium, we must impose an incentive compatibility constraint on the firm, namely that it is better off by offering high quality. Given the simple nature of the consumers' strategies, the only hope for an efficient equilibrium is that, just as in the case of perfect monitoring, consumers "punish" the firm by losing trust in the brand whenever low quality is observed. In fact, were consumers not to punish the firm, then it would be the latter's dominant strategy not to offer quality.

Under the equilibrium assumption that consumers punish the firm for a product that breaks down, the firm's no-deviation constraint is derived as follows. If the firm invests in quality, then its expected value is $\pi - \epsilon$, net revenues during the current period, plus $\delta\,\alpha\,v$, where v is the firm's value. By shirking today, the firm receives profits π today but only $\delta\,\beta\,v$ in the future. The no-deviation constraint is therefore

$$\pi - \epsilon + \delta\,\alpha\,v \geq \pi + \delta\,\beta\,v \tag{2}$$

Under the equilibrium hypothesis that the seller chooses quality in every period until trust is broken, firm value is given by

$$v = \pi - \epsilon + \delta\,\alpha\,v,$$

or simply

$$v = \frac{\pi - \epsilon}{1 - \delta\,\alpha}\;. \tag{3}$$

Substituting (3) for v in (2), and solving for δ, we get

$$\delta \geq \frac{\epsilon}{\pi\,(\alpha - \beta) + \beta\,\epsilon}\;, \tag{4}$$

and the result follows. ■

Lemma 1 *Under umbrella branding, the no-deviation constraint corresponding to shirking in n products ($n = 1, 2$) is given by*

$$\delta \geq \frac{\xi}{(3 - n)\,(\rho - \rho') + \rho\,\xi}$$

where ρ is the equilibrium continuation probability, ρ' the continuation probability conditional on shirking, and $\xi \equiv \frac{\epsilon}{\pi-\epsilon}$. Moreover, under no umbrella branding, the no-deviation constraint has the same form as under umbrella branding and $n = 2$.

Proof of Lemma 1: Consider first the case of no umbrella branding. The no-deviation constraint is given by

$$(\pi - \epsilon) + \delta\,\rho\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta} \geq \pi + \delta\,\rho'\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta}, \tag{5}$$

This is equivalent to

$$\begin{aligned} -\epsilon + \delta\,\rho\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta} &\geq \delta\,\rho'\,\frac{(\pi - \epsilon)}{1 - \rho\,\delta} \\ -\xi + \delta\,\rho\,\frac{1}{1 - \rho\,\delta} &\geq \delta\,\rho'\,\frac{1}{1 - \rho\,\delta} \\ \delta\,(\rho - \rho') &\geq \xi\,(1 - \rho\,\delta) \end{aligned}$$

Finally, solving with respect to ξ we get

$$\delta \geq \frac{\xi}{(\rho - \rho') + \rho\,\xi} \tag{6}$$

Consider now the case of umbrella branding and the no-deviation constraint corresponding to shirking on two products, $n = 2$. The no-deviation constraint is given by

$$2\,(\pi - \epsilon) + \delta\,\rho\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta} \geq 2\,\pi + \delta\,\rho'\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta},$$

Dividing through by 2, we get an expression identical to (5). Consider now the case of shirking on one product only. The no-deviation constraint is now given by

$$2\,(\pi - \epsilon) + \delta\,\rho\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta} \geq 2\,\pi - \epsilon + \delta\,\rho'\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta},$$

This is equivalent to

$$\begin{aligned} -\epsilon + \delta\,\rho\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta} &\geq \delta\,\rho'\,\frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta} \\ -\xi + \delta\,\rho\,\frac{2}{1 - \rho\,\delta} &\geq \delta\,\rho'\,\frac{2}{1 - \rho\,\delta} \\ 2\,\delta\,(\rho - \rho') &\geq \xi\,(1 - \rho\,\delta) \end{aligned}$$

Finally, solving with respect to ξ we get

$$\delta \geq \frac{\xi}{2\,(\rho - \rho') + \rho\,\xi} \tag{7}$$

Putting together (6) and (7), Lemma 1 follows. ■

Lemma 2 *Under umbrella branding, (a) if consumers lose trust following any product failure, then the binding no-deviation constraint is to shirk on both products; (b) if consumers lose trust following two simultaneous product failures, then the binding no-deviation constraint is to shirk on one product only.*

Proof of Lemma 2: Consider the case of umbrella branding with punishment of any product failure. In this case, $\rho = \alpha^2$. If the seller shirks in both products, then $\rho' = \rho'_2 = \beta^2$. If the seller shirks in one product then $\rho' = \rho'_1 = \alpha\,\beta$. Applying Lemma 1, we see that the lowest value of δ under two deviations is higher than the lowest value of δ under one deviation only if and only if

$$(\rho - \rho'_2) < 2\,(\rho - \rho'_1)$$

which is equivalent to

$$
\begin{array}{rcl}
(\alpha^2 - \beta^2) & < & 2\,(\alpha^2 - \alpha\,\beta) \\
(\alpha + \beta)\,(\alpha - \beta) & < & 2\,\alpha\,(\alpha - \beta) \\
(\alpha + \beta) & < & 2\,\alpha \\
\beta & < & \alpha
\end{array}
\tag{8}
$$

Consider now the case of umbrella branding with punishment of two simultaneous product failures. In this case, $\rho = 1 - (1 - \alpha)^2$. If the seller shirks in both products, then $\rho' = \rho_2' = 1 - (1 - \beta)^2$. If the seller shirks in one product then $\rho' = \rho_1' = 1 - (1 - \alpha)\,(1 - \beta)$. Applying Lemma 1, we see that the lowest value of δ under two deviations is lower than the lowest value of δ under one deviation only if and only if

$$
(\rho - \rho_2') > 2\,(\rho - \rho_1')
$$

which is equivalent to

$$
\begin{array}{rcl}
\left(1 - (1 - \alpha)^2\right) - \left(1 - (1 - \beta)^2\right) & > & 2\left(\left(1 - (1 - \alpha)^2\right) - \left(1 - (1 - \alpha)\,(1 - \beta)\right)\right) \\
\left(2\,\alpha - \alpha^2\right) - \left(2\,\beta - \beta^2\right) & > & 2\left(\left(2\,\alpha - \alpha^2\right) - \left(\alpha + \beta - 2\,\alpha\,\beta\right)\right) \\
\left(\beta^2 - \alpha^2\right) & > & 2\left(\alpha\,\beta - \alpha^2\right),
\end{array}
$$

which is equivalent to (8). ■

Proof of Proposition 4: Generally speaking, equilibrium value is given by

$$
v = \frac{2\,(\pi - \epsilon)}{1 - \rho\,\delta},
$$

where ρ is the continuation probability. Under no umbrella branding, $\rho = \alpha$. Under umbrella branding and punishment of any product failure, $\rho = \alpha^2$. Finally, under umbrella branding and punishment of two simultaneous product failures, $\rho = 1 - (1 - \alpha)^2$. It follows that, if the respective no-deviation constraints are satisfied, equilibrium value is highest under umbrella branding and punishment of two simultaneous product failures, next under no umbrella branding, and finally under umbrella branding and punishment of any product failure. I next show when each proposed solution satisfies the no-deviation constraints.

From Lemmas 1 and 2, the lower bounds on the value of the discount factor as as follows. Under no umbrella branding, the equilibrium continuation probability is α, whereas the continuation probability is given by β. It follows that the no-deviation constraint is given by

$$\delta > \frac{\xi}{(\alpha - \beta) + \rho\, \xi}.$$

Under umbrella branding and the strategy of punishing any product failure, the continuation probability, ρ, is equal to α^2. By Lemma 2, the binding deviation is shirking in both products, so $\rho' = \beta^2$. From Lemma 1, it follows that

$$\delta > \frac{\xi}{(\alpha^2 - \beta^2) + \rho^2\, \xi}.$$

Finally, under umbrella branding and the strategy of only punishing two simultaneous occurrences of product failure, the continuation probability, ρ, is equal to $1 - (1 - \alpha)^2$. By Lemma 2, the binding deviation is shirking in both products, so $\rho' = 1 - (1 - \alpha)\,(1 - \beta)$. From Lemma 1, it follows that

$$\delta > \frac{\xi}{\left(1 - (1 - \alpha)^2\right) - \left(1 - (1 - \beta)^2\right) + \left(1 - (1 - \alpha)^2\right)\, \xi}.$$

It follows that the lower bound on the discount factor under umbrella branding with punishment of any product failure is lower than that under no umbrella branding if and only if

$$(\alpha^2 - \beta^2) + \rho^2\, \xi > (\alpha - \beta) + \rho\, \xi,$$

which is equivalent to

$$\xi < \xi_1 = \frac{(\alpha - \beta)\,(\alpha + \beta - 1)}{(1 - \alpha)\, \alpha}$$

Moreover, the lower bound on the discount factor under umbrella branding with punishment of two product failures is lower than that under no umbrella branding if and only if

$$\left(1 - (1 - \alpha)^2\right) - \left(1 - (1 - \beta)^2\right) + \left(1 - (1 - \alpha)^2\right)\, \xi > (\alpha - \beta) + \rho\, \xi,$$

which is equivalent to

$$\xi > \xi_2 = \frac{(\alpha - \beta)\,(2\, \alpha - 1)}{(1 - \alpha)\, \alpha}$$

Finally, $\alpha > \beta$ implies that $\xi_2 > \xi_1$, which leads to the characterization in the proposition. ■

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