Multiproduct Oligopoly and Bertrand Supertraps*

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Abstract

We study oligopoly price competition between multiproduct firms, firms whose products interact in the profit function. Specifically, we focus on the impact of intra-firm product interactions on the level of equilibrium prices and profits. This impact is divided into two effects: a direct effect and a strategic effect (i.e., through the competitors' actions). We derive conditions such that, if intra-firm product interactions cause prices to decrease (increase) while holding competitors' prices fixed, then the strategic effect hurts (benefits) the firm. We also show that, under reasonable general assumptions, the strategic effect more than outweighs the direct effect, so that equilibrium profits vary in the direction opposite of the direct effect (*Bertrand supertrap*). Several instances of Bertrand supertraps are developed. For example, stronger demand complementarity or economies of scope lead to tougher price competition to an extent that may decrease profitability (even when the direct profit effect is positive). We present a number of applications of the general results, including learning curves, network effects, systems competition, bundling, switching costs, and internet cross-referencing.

Keywords: Competition, strategic complementarity, profit complementarity, profitability, economies of scope, learning curves, core competencies, network effects, systems competition, bundling, switching costs, the Internet.

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1. Introduction

Most firms produce more than one product: Ford produces cars and trucks; Kodak sells cameras and film; TWA offers air travel services along various routes; and so fourth. Not only do these firms sell different products, they sell products that "interact" with each other in the firm's profit function. For example, if some Ford-loyal consumers are undecided between buying a car and buying a truck, selling more trucks may imply selling fewer cars. For Kodak, by contrast, increasing sales of cameras is likely to imply an increase in the sales of film. As for TWA, increasing output or capacity in the Chicago-St Louis and St Louis-New York routes is likely to decrease the cost of offering air travel from New York to Chicago, another example of intra-firm products interaction.

Similarly, in a dynamic context, we can interpret a firm selling a given product in different periods as a multiproduct firm. Specifically, we can interpret the output of a given product in different periods as different outputs. In this context, interactions across products within a firm result from dynamic effects on the firm's demand or cost function. For example, increasing the output of aircraft sold today lowers Boeing's cost of selling aircraft next period. We thus have an additional class of examples of multiproduct oligopoly competition with interactions across products. Switching costs and dynamic network effects would provide additional examples within the same class.

In this paper we look at oligopoly price competition between multiproduct firms, firms whose products interact in the firm's profit function, as in the above examples. We are interested in the impact of intra-firm product interactions on the level of equilibrium prices and profits. We divide this impact into two effects: a direct effect and a strategic effect which takes place through the competitors' actions. We show that, under reasonable conditions, the strategic effect more than outweighs the direct effect, so that equilibrium profits vary in the opposite direction of the direct effect. We call this a *Bertrand supertrap*, a reference to the supercompetitive effect of product interactions on firm competition.[1]

We present several instances of Bertrand supertraps. For example, we show that firm profits are lower the greater the degree of intra-firm demand complementarity (that is, demand complementarity between the products offered by a given firm), while profits are higher with

[1]The term "Bertrand trap" has been used by some authors (Hermalin 1993) as a reference to the situation where equilibrium profits under some form of single-product competition (e.g., Hotelling) remain constant despite seemingly favorable exogenous changes. For example, if all firms' marginal cost is decreased, the strategic effect (price reduction) exactly cancels the direct effect (cost reduction).

greater demand substitutability. Moreover, increasing the degree of economies of scope, while lowering the firms' costs, may decrease profitability. Similar effects take place in the context of learning curves, network effects, systems competition, bundling, switching costs, and internet cross-referencing.[2]

In order to understand the strategic effect, consider the thought experiment whereby, as the level of product interactions within firm i is changed, the actions of firm i's competitors are set fixed at their initial equilibrium level.[3] Given this exogenous change in the level of product interaction, firm i would want to change its prices. We refer to this reaction as the *monopoly effect*. In other words, the monopoly effect is the effect on firm i's prices of a change in the degree of product interaction, keeping other firms' prices constant. If this monopoly effect is in the direction of increasing prices, then the strategic effect benefits firm i because of strategic complementarity in prices. That is, in equilibrium, firm i's competitors raise their prices, which benefits firm i. Conversely, if the monopoly effect is in the direction of decreasing prices, then the strategic effect hurts firm i. We characterize the conditions of the profit function that determine the sign of the monopoly effect. Roughly speaking, demand complementarity and economies of scope imply a price-reduction monopoly effect, and consequently a negative strategic effect. In order to make this argument precise, we must distinguish between the cases when firm i's profit function is and is not supermodular in the vector of all prices. If supermodularity holds, then the above results apply for all levels of intra-firm product interactions. If supermodularity does not hold, then the results only apply for small degrees of intra-firm product interaction. Supermodularity of the profit function may obtain when, for example, firms sell demand substitutes.

When the direct and strategic effects have opposite signs, the natural question is to determine their relative importance. We show that, for symmetric equilibria and when the entire market is covered (constant total demand), the strategic effect outweighs the direct effect: basically, the direct effect is more than "competed away." The idea that the strategic effect may cancel the direct effect is not new. For example, under symmetric Hotelling (or Bertrand) competition a (common) decrease in marginal cost leads to a strategic effect that exactly cancels

[2]The competitive price discrimination literature that addresses the issue of product choice (Katz, 1984; Champsaur and Rochet, 1989; Stole, 1995) is related to our work. However, that literature addresses the choice of the degree of product interactions, while our focus is the impact of product interaction on profits. We briefly discuss the endogenous choice of product interactions in Section 7 below. Regarding competitive price discrimination see also Borenstein (1985), Corts (1998), and Armstrong and Vickers (1999).

[3]Throughout the paper, we will take prices as the strategic variable. However, some of our results apply more generally to any strategic variable.

the direct effect of the cost reduction; likewise, a (common) increase in consumer valuations does not affect the equilibrium level of profits. What is new in our analysis is the idea that the strategic effect may outweigh the direct effect in a large range of interesting situations where the level of intra-firm product interactions changes, and where firms compete with multiple products.[4] For example, we show that an increase in economies of scope, though it leads to a decrease in total and in marginal costs (direct effect) implies a decrease in equilibrium profits (strategic effect greater than direct effect).

To conclude this introduction, we describe some of the applications of our general results.

When firms sell demand complements (e.g., cameras and film), the monopoly effect of greater complementarity is to lower prices: a lower price of a given product generates additional demand for the firm's other products. It follows (from our results) that greater complementarity induces more intense competition and lower equilibrium profits. Conversely, when firms sell demand substitutes (e.g., online and in-store grocery shopping), an increase in the degree of substitutability softens competition and leads to greater equilibrium profits.

If firms enjoy economies of scope, then the monopoly effect of an increase in intra-firm product interactions is for each firm to decrease prices. In fact, lowering the price of good j lowers the cost of good ℓ. It follows that greater economies of scope induce more intense competition to the point that equilibrium profits are lower, even though costs are lower. Conversely, an increase in the degree of diseconomies of scope implies an increase in equilibrium profits.

A related application is the case of learning curves in production. The monopoly effect is that a greater degree of learning induces firms to cut prices in order to lower future costs. Therefore, greater learning implies more intense competition, which in turn results in lower profits. This extends Cabral and Riordan (1994) to the case of any number of firms and continuous output level; it also shows that their learning curve result is part of a general theory of oligopoly competition with multiproduct firms.

Network externalities cause the monopoly effect to be in the direction of cutting prices, in order to induce a greater future network (Katz and Shapiro, 1986). It may therefore be the case that profits are lower with network externalities than without. Similarly, under systems competition (e.g., hardware/software) firms sell complementary components of a system, and

[4] Bulow et al. (1985) also point out that supposedly positive exogenous changes in one market may have negative effects in another market because of the strategic interaction between firms. The idea that the strategic effect may dominate the direct effect can also be obtained in other settings (without changes in intra-firm product interactions) with one or multiple products (for example, the results in Corts, 1998, can be interpreted in this spirit).

therefore the monopoly effect is in the direction of cutting prices in order to sell more of one component and increase demand for the other component. It follows that systems competition may result in lower equilibrium profits than the situation where there are no intra-firm product interactions.

Pure bundling can be seen as the extreme case of systems competition (one product can only be sold jointly with another). We thus provide conditions such that pure bundling implies lower profits than no bundling. This is consistent with the results in Whinston (1990), where bundling is used as an entry deterrent, and where potential competing firms offer only one product (see also Nalebuff, 2000). Demand synergies between products within a firm can be seen as related to systems competition; we show that greater demand synergies may lead to lower profits (Strauss, 1999).

We also look at other possible applications of our general results, including switching costs and internet cross-referencing.

The rest of the paper is organized as follows. The next section goes through some preliminary definitions and assumptions. Section 3 focuses on the strategic effect, discusses how the monopoly effect translates into equilibrium behavior under competition, and presents conditions on the profit functions that determine the sign of the monopoly effect. Section 4 presents conditions under which the strategic effect outweighs the direct effect. An extension of the general framework to dynamic problems is presented in Section 5. Section 6 goes through several applications. Section 7 discusses the case when the level of intra-firm product interactions is endogenous and concludes the paper.

2. Preliminaries

Consider an oligopoly with I price-setting firms. Each firm offers a set J_i of products. Let p^i_j be the price of product j set by firm i, $p^i = (p^i_j)$ the vector of firm i's prices, $p = (p^i)$ the vector of all prices, p^{-i} the price vector of firm i's competitors, and p^i_{-j} the vector of firm i's prices except p^i_j. Firm i's profit function is $\Pi^i(p, s)$, where s is an exogenous parameter that measures the level of intra-firm product interaction; the role of s is at the center of this paper. Firms simultaneously set prices in a one-shot game, the equilibrium of which is given by $\widehat{p}$.

The profit function can be written as

$$\Pi^i(p,s) = \sum_{j \in J_i} p_j^i D_j^i(p,s) - C^i(D^i(p,s),s),$$

where D_j^i is the demand for product j sold by firm i, $D^i(p)$ the vector of firm i's demands, and $C^i(\cdot)$ firm i's cost of supplying D^i. Two polar cases can be considered: the case when product interactions occur only through the demand functions, and the case when product interactions occur only through the cost function.[5]

Throughout the paper we maintain the assumption that any firm i's profit is increasing in the rivals' prices (competitive markets):

ASSUMPTION 1 *(competitive markets): For each firm i,*

$$\frac{\partial \Pi^i}{\partial p_j^k} \geq 0,$$

for all $k \neq i$ and $j \in J_k$. Moreover, there is at least a $j \in J_k$ for which the inequality is strict.[6]

Assumption 1 states that a firm is never worse off when a competitor raises one of its prices. This is not a trivial assumption in the case of multiproduct firms. When intra-firm product interactions take place through the cost function, Assumption 1 holds if there are economies of scope: an increase in a competitor's price increases demand for the firm's products, which in turn unambiguously decreases costs and increases profits. Similarly, product interactions through the demand side when firms sell substitute products would also satisfy this assumption. If, however, the product interactions are through the cost function and involve diseconomies of scope, or through the demand side and involve demand complements, then Assumption 1 only holds if the competitive effect is greater (in terms of profits) than the effect of diseconomies of scope or demand complementarity. Assumption 1 also holds in the case when firms sell products that are complements within the firm but products are not complements across firms (see the example below of demand synergies). Finally, Assumption 1 holds, if in an obvious way, for small levels of intra-firm product interaction.

[5] A formal definition of these cases is presented in the next section.

[6] We assume throughout that the profit function is differentiable and continuous. Several of our results can be derived without these assumptions, using the methods presented in Milgrom and Roberts (1990) or Villas-Boas (1997).

As mentioned above, we paramaterize the profit functions by $s \in I\!R$. The role of this exogenous parameter is defined by the following assumption:

Assumption 2 *(intra-firm product interactions): For any firm i and products j and ℓ ($j \neq \ell$) sold by firm i,*

$$\frac{\partial}{\partial s}\frac{\partial^2 \Pi^i}{\partial p^i_j \partial p^i_\ell} \geq 0 \quad \text{and} \quad \left.\frac{\partial^2 \Pi^i}{\partial p^i_j \partial p^i_\ell}\right|_{s=0} = 0,$$

with strict inequalities for at least one firm i.

In order to better describe the impact of s, we introduce the concepts of profit complementarity and substitutability:

Definition 1 *(profit complements/substitutes): Products j and ℓ ($j \neq \ell$) are said to be profit complements for firm i if and only if $\frac{\partial^2 \Pi^i}{\partial p^i_j\, \partial p^i_\ell} \geq 0$. Products j and ℓ ($j \neq \ell$) are said to be profit substitutes for firm i if and only if $\frac{\partial^2 \Pi^i}{\partial p^i_j\, \partial p^i_\ell} \leq 0$.*

Given Assumption 2, we have profit complementarity for positive s and profit substitutability for negative s. An increase in the absolute value of s corresponds to an increase in the degree of product interactions within a firm. For s positive this is an increase in profit complementarity; for s negative we have an increase in profit substitutability. As we will see below, profit complementarity is implied by demand substitutability or economies of scope. Profit substitutability holds when firms sell demand complements or have diseconomies of scope.[7]

We make one final general assumption regarding interaction across products from different firms:

Assumption 3 *(strategic complementarity): For each pair of firms i, k, for all $j \in J_i$ there is a $\ell \in J_k$ such that*

$$\frac{\partial^2\, \Pi^i}{\partial\, p^i_j\ \partial\, p^k_\ell} > 0.$$

Furthermore, for any products $j \in J_i$ and $\ell \in J_k$, we have

$$\left.\frac{\partial^2 \Pi^i}{\partial p^i_j \partial p^k_\ell}\right|_{s=0} \geq 0.$$

[7]Throughout the paper, we use the term economies (and diseconomies) of scope to mean profit complementarity (substitutability) under cost interactions. This definition is related, but not identical, to the definition of economies of scope used in the literature (see Panzar, 1991).

Assumption 3 corresponds to the traditional assumption of strategic complementarity in prices when firms sell substitute products. Here, it is extended to the case in which each firm sells several products: if the price of a competitor's product is increased, and if everything else is kept constant, then the firm would like to increase at least one of its prices. This assumption also implies that, for no product interaction ($s = 0$), products across firms are substitutes or independent.

In some applications, a particular form of strategic complementarity may occur which involves all products in the market (we would have supermodularity). We call it *strong strategic complementarity* and define it formally as:

Definition 2 *(strong strategic complementarity): If, for all firms i and k and for all products $j \in J_i$ and $\ell \in J_k$*

$$\frac{\partial^2 \Pi^i}{\partial p^i_j \partial p^k_\ell} \geq 0,$$

then strong strategic complementarity holds.

When intra-firm product interactions result from firms selling substitute products, we expect strong strategic complementarity to hold, since all the products in the market are demand substitutes. On the other hand, when intra-firm interactions result from firms selling complements, strong strategic complementarity may not hold, because a firm could be selling a product that is a complement with respect to a product sold by another firm. For cost interactions, strong strategic complementarity will likely hold if there are diseconomies of scope: an increase in p^k_ℓ leads to an increase for some $j \in J_i$ of D^i_j, the firm's demand of product j, which results in an increase in the marginal cost of firm i's other products, and an increase in the optimal levels of these products' prices.

The market equilibrium $\widehat{p}(s)$ is determined by the first-order conditions for all the firms,

$$\frac{\partial \Pi^i(\widehat{p}(s), s)}{\partial p^i_j} = 0, \ \forall i, j \in J_i.$$

We assume throughout that the second-order conditions are satisfied and that the equilibrium is unique for any s.[8] From the first-order conditions for each firm i, we can write firm i's best-response functions as $p^i = b^i(p^{-i}, s)$, where b^i is a vector of functions. The monopoly effect of s,

[8] If the equilibrium is not unique, then the results below present comparative statics on the upper and lower bounds of the equilibrium set (see Milgrom and Roberts, 1990). In the applications presented below, the equilibrium is always unique.

that is, the effect of a change in s holding competitors' prices constant, can be obtained from the derivative of b^i with respect to s. Obviously, in equilibrium we also have $\widehat{p}^i(s) = b^i(\widehat{p}^{-i}(s), s), \forall i$.

Our main goal is to determine the impact on equilibrium prices, $\widehat{p}(s)$, and profits, $\Pi^i(\widehat{p}(s), s)$, of an increase in the degree of product interactions, s. By the envelope theorem, we know that

$$\frac{d\,\Pi^i(\hat{p}(s),s)}{d\,s} = \frac{\partial\,\Pi^i(\hat{p}(s),s)}{\partial\,s} + \sum_{k\neq i}\sum_{j\in J_k}\frac{\partial\,\Pi^i(\hat{p}(s),s)}{\partial\,p_j^k}\frac{d\widehat{p}_j^k(s)}{ds}.$$

The first term on the right-hand side is the *direct effect* of intra-firm product interactions on profits. The second term is the *strategic effect* (that is, through the competitors' actions). The *total effect* is simply the sum of the direct and strategic effects. We analyze the strategic effect in the next section. Section 4 compares the strategic and direct effects in order to evaluate the total effect.

3. The Strategic Effect

In this section, we first characterize the strategic effect given the monopoly effect. This analysis drives the main results in the paper. We then show how the degree of intra-firm product interactions affects the monopoly effect for the cases of interactions in demand and in costs. Finally, we link these two ideas to obtain general results on the strategic effect.

3.1. Main Idea

In this subsection, we provide conditions under which the strategic effect is positive or negative. Assumption 1 implies that, if for all $k \neq i$ and $j \in J_k$, $\frac{d\widehat{p}_j^k(s)}{ds} \geq 0$, then the strategic effect on Π^i is positive. Similarly, if for all $k \neq i$ and $j \in J_k$, $\frac{d\widehat{p}_j^k(s)}{ds} \leq 0$, then the strategic effect on Π^i is negative.

As discussed above, the monopoly effect (the optimal reaction to a change in s, holding competitors' prices fixed), is determined by the partial derivative $\frac{\partial b_j^i(p^{-i},s)}{\partial s}$. In order to determine the direction of the strategic effect, given the monopoly effect, one then has to relate $\frac{d\widehat{p}_\ell^k(s)}{ds}$ and $\frac{\partial b_j^i(p^{-i},s)}{\partial s}$ for all firms i and $k, j \in J_i$, and $\ell \in J_k$. This is done by the following result for profit complementarity ($s > 0$) and profit substitutability ($s < 0$).

Theorem 1 *(strategic effect): If $s > 0$ and* strong strategic complementarity *holds, then the strategic effect has the same sign as the monopoly effect, that is,* $\frac{d\widehat{p}_j^i(s)}{ds} \geq 0 (\leq 0), \forall i, j \in J_i$, *if*

$\frac{\partial b_j^i(p^{-i},s)}{\partial s} \geq 0 (\leq 0), \forall i, j \in J_i$. *If* $s < 0$ *or strong strategic complementarity does not hold, the same statement is true if* $|s|$ *is sufficiently small and the monopoly effect is represented with a strict inequality.*

Consider the thought experiment whereby all of firm i's competitors' actions are kept fixed. This is as if firm i were a monopolist. Theorem 1 states that, if an increase in s were to lead the "monopolist" firm i to increase its prices, then, in the oligopoly equilibrium, all prices increase as a result of an increase in s.

The proof of this and the following results may be found in the Appendix. Theorem 1 follows from standard supermodularity results. If profit complementarity and strong strategic complementarity hold, then the profit functions are supermodular in all prices, both within and across firms. If either profit complementarity or strong strategic complementarity do not hold, then we must make sure product interactions do not outweigh the cross-firm pricing effects; thus the condition that s be small in absolute value.

The next subsection characterizes the monopoly effect as a function of conditions on the profit function.

3.2. Monopoly Effect

This subsection derives conditions which determine the sign of the monopoly effect.[9] We then link these conditions directly to the strategic effect identified above.

As stated above, an important part of our analysis is to see how changes in s affect the marginal profit with respect to each of firm i's prices. Remember that we have profit complementarity when $s > 0$ and profit substitutability when $s < 0$.

THEOREM 2: *If* $s > 0$ *and* $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} > 0$ *(*$\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} < 0$*) for all* $j \in J_i$*, then the monopoly effect is positive (negative), that is,* $\frac{\partial b_j^i(p^{-i},s)}{\partial s} > 0$ *(*$\frac{\partial b_j^i(p^{-i},s)}{\partial s} < 0$*) for all* $j \in J_i$*. If* $s < 0$*,* $|s|$ *is small, and* $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} > 0$ *(*$\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} < 0$*) for all* $j \in J_i$*, then the monopoly effect is negative (positive), that is,* $\frac{\partial b_j^i(\widehat{p}^{-i},s)}{\partial |s|} < 0$ *(*$\frac{\partial b_j^i(\widehat{p}^{-i},s)}{\partial |s|} > 0$*) for all* $j \in J_i$*.*

[9] These conditions formalize known results from the analysis of monopoly pricing. See, for example, Tirole (1988, pp. 69-72). Readers who are familiar with these results may want to skip directly to Section 4.

This result simply states that the monopoly effect has the same sign as the impact of the product interaction parameter on each product's price marginal profit. For profit complementarity the result holds for any degree of product interactions because of the supermodularity results. The case of profit substitutability is similar, but one must make sure that the intra-firm product interactions are not so great as to reverse the effects of exogenous changes in s. This implies the condition that $|s|$ be small.

The natural next step is to derive conditions that determine the sign of $\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s}$. We consider separately the cases of demand and cost interactions.

Definition 3 *(demand interactions): Under the* demand interactions *case, firm i's profit function can be written as*

$$\Pi^i(p,s) = \sum_{j\in J_i} p^i_j D^i_j(p^i_j, sp^i_{-j}, p^{-i}) - \sum_{j\in J_i} C^i_j(D^i_j(p^i_j, sp^i_{-j}, p^{-i})),$$

with $\frac{\partial D^i_j}{\partial (sp^i_k)} > 0$ *for* $k \neq j$*. Moreover, the second derivatives of the demand and cost functions are small compared to the first derivatives.*

It can be easily checked that the conditions for demand interactions satisfy Assumption 2. When $s = 0$, the products sold by firm i are independent, given the prices of the other firms. When s is positive, the firm sells demand substitutes; when s is negative, the firm sells demand complements.[10]

We can then obtain the following result:

Proposition 1: *If* $|s|$ *is small, then, under demand interactions,* $\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} > 0$ *for all* $i, j \in J_i$.

Linking this result to Theorem 2, we can see that the monopoly effect is to increase prices if the firm sells product substitutes and to decrease prices if the firm sells product complements.

Let us consider now the case of product interactions in the cost functions.

[10]Note that our definition of demand interactions only allows for effects on D^i_j through p^i_{-j}. The results presented here are robust to the introduction of additional effects so long as they are small with respect to the main effect as in Definition 3. Assuming the second derivatives to be small allows us to concentrate on the first order effects.

Definition 4 *(cost interactions): Under the* cost interactions *case, firm i's profit function may be written as*

$$\Pi^i(p,s) = \sum_{j\in J_i} p^i_j D^i_j(p^i_j, p^{-i}) - \sum_{j\in J_i} C^i_j(D^i_j(p^i_j, p^{-i}), sD^i_{-j}(p^i_{-j}, p^{-i})).$$

Moreover, $\frac{\partial C^i_j}{\partial(sD^i_k)} < 0$, $\frac{\partial^2 C^i_j}{\partial D^i_j \partial(sD^i_k)} < 0$ *for* $k \neq j$, *and* $\left|\frac{\partial^2 C^i_j}{\partial(sD^i_k)\partial(sD^i_\ell)}\right|$ *is small compared with* $\left|\frac{\partial^2 C^i_j}{\partial D^i_j \partial(sD^i_k)}\right|$ *for* $k, \ell \neq j$.

It can be easily checked that the conditions for the cost interactions case satisfy Assumption 2. When $s = 0$, the products sold by firm i are independent. When s is positive, the firm enjoys economies of scope (see Footnote 7), while when s is negative the firm has diseconomies of scope.

We can then obtain the following result:

Proposition 2: *Under cost interactions,* $\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} < 0$ *for all* $i, j \in J_i$.

Linking this result to Theorem 2, we can see that the monopoly effect is to decrease prices if the firm has economies of scope and to increase prices if the firm has diseconomies of scope.

3.3. General Result

Building directly on Theorems 1 and 2 and on Propositions 1 and 2, we can now derive general results of the sign of strategic effect given conditions on the payoff functions. We separate the results for profit complementarity ($s > 0$) and profit substitutability ($s < 0$).

Theorem 3: *If* $s > 0$ *and strong strategic complementarity holds, then the strategic effect is positive (negative), that is,* $\frac{d\widehat{p^i_j}(s)}{ds} \geq 0$ ($\frac{d\widehat{p^i_j}(s)}{ds} \leq 0$) *for all* $i, j \in J_i$, *if* $\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} > 0$ ($\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} < 0$) *for all* $i, j \in J_i$. *In particular, the strategic effect is positive under demand substitutes and negative under economies of scope. If strong strategic complementarity does not hold the results still hold for s small.*

Theorem 4: *If* $s < 0$ *and* $|s|$ *is small, then the strategic effect is negative (positive), that is,* $\frac{d\widehat{p^i_j}(s)}{d|s|} \leq 0$ ($\frac{d\widehat{p}(s)}{d|s|} \geq 0$) *for all* $i, j \in J_i$, *if* $\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} > 0$ ($\frac{\partial^2 \Pi^i}{\partial p^i_j \partial s} < 0$) *for all* $i, j \in J_i$. *In particular, the strategic effect is negative under demand complements and positive under diseconomies of scope.*

4. Total Effect in Symmetric Markets

In the previous section we have determined the sign of the strategic effect. In other words, we have determined whether an increase in s leads to an increase or a decrease in equilibrium prices. This tells us something about the impact of s on firm profitability through the competitors' prices. However, changes in s also have a *direct* effect on profits. The question is then to determine the value of the *total* effect of a change of the degree of intra-firm product interactions on equilibrium profits.

Our analysis in this section is restricted to symmetric equilibria of symmetric markets. We make the following assumption regarding the equilibria in the symmetric markets we are considering.

Assumption 4 *(symmetric equilibrium): There exists a unique equilibrium, $\widehat{p}$, which is symmetric: $\widehat{p}^i = \widehat{p}^k$, $\forall i, k$.*

In symmetric industries all firms carry the same products, so that each firm's product j is the same as any other firm's. We also assume that, for each product, the market is fully covered in equilibrium (total demand is constant), so that the only effect of prices is to divide demand across firms:

Assumption 5 *(market fully covered): For every $j \in J_i$ and s, $\sum_{i=1}^{I} D_j^i(\widehat{p}, s) = \overline{D}_j$.*

We present two results, one for demand interactions and another for interactions in the cost function.

Theorem 5 *(demand interactions): Consider the demand interactions case. If firms sell demand substitutes (s positive) then the total effect is positive, that is, profits increase in the degree of intra-firm demand substitutability. If firms sell demand complements (s negative) and the degree of complementarity is small ($|s|$ is small) then the total effect is negative, that is, profits decrease in the degree of intra-firm demand complementarity.*

This result states that competing firms benefit from intra-firm demand substitutability, while they are hurt by demand complementarity. That is, the strategic effect characterized in the previous section dominates, even though the direct effect could be to decrease profits under demand substitutability, or to increase profits under demand complementarity.

Suppose now that all intra-firm product interactions take place on the cost side. Our second result in this section allows for a full characterization of the total effect.

Theorem 6 *(cost interactions): Consider the cost interactions case and suppose that* $\frac{\partial}{\partial s}\frac{\partial^2 C^i}{\partial D_j^{i2}} \leq 0, \forall i, j$. *If there are economies of scope (s positive), then the total effect is negative, that is, profits decrease with the degree of economies of scope. If there are diseconomies of scope (s negative), then the total effect is positive, that is, profits increase in the degree of diseconomies of scope.*

This result states that in symmetric, fully-covered markets, competing firms are hurt by economies of scope while they benefit from diseconomies of scope. This means that the strategic effect, characterized in the previous section, dominates the direct effect. Although the direct effect of greater economies of scope may be to improve profits, the total effect involves lower profits.[11]

The above Bertrand supertrap results are obtained for total market coverage and symmetry. By continuity, small deviations from these conditions do not change the results. However, note that one may reverse the results if we have a large deviation from the above conditions. A downward sloping market demand curve may imply a more favorable total effect than we considered. For example, consider the case of economies of scope. An increase in s implies a positive direct effect (cost savings) and a negative strategic effect (lower prices). If the market demand is downward sloping, then we would expect the strategic effect to be less negative than in the fixed total demand case, perhaps to an extent that the direct effect dominates the strategic effect. Likewise, in an asymmetric oligopoly, the strategic effect on a large firm's profit is likely to be smaller; it is therefore possible that the total effect has the same sign as the direct effect. In fact, in the limit of monopoly (the extreme of an asymmetric oligopoly), the strategic effect is zero and the total effect has the same sign as the direct effect.

5. Dynamic Market Interactions

As mentioned in the introduction, we can think of multiperiod competition as a particular case of multiproduct competition. Suppose that each firm offers one product over T periods. This situation is analogous to that of a firm selling T products. Intra-firm product interactions would then correspond to dynamic interactions in the production or sale of the firm's product. Examples of this are competition with learning curves, network externalities, or switching

[11]The condition that $\frac{\partial}{\partial s}\frac{\partial^2 C^i}{\partial D_j^{i2}} \leq 0$ simply captures the idea that when there are economies (diseconomies) of scope there are no diseconomies (economies) of scale.

costs. In these situations, the results of the previous sections still apply if we look at open-loop equilibria, that is, the case when all prices are set at the beginning of the game.

In most cases, however, firms are able to change their prices at different moments in time. It thus makes sense to focus on Markov perfect equilibria, that is, equilibria such that the firms' strategies in each period are only a function of the payoff-relevant state variables.[12]

Suppose that, in each of two periods, I symmetric firms sell one product each.[13] In period t, firm i sets price p_t^i. Firm i's first-period profit is given by $\pi_1^i(p_1)$, where $p_t = (p_t^i)$. Firm i's second-period profit is given by $\pi_2^i(p_2, p_1, s)$.[14] The second period profit is discounted by the factor δ, so that firm i's net present value in the first period is $\pi_1^i(p_1) + \delta\pi_2^i(p_2, p_1, s)$.[15] We focus on the symmetric equilibrium of the game and assume that $|s|$ is small. Assumption 2 corresponds in this case to $\frac{\partial}{\partial s}\frac{\partial^2 \pi_2^i}{\partial p_1^i \partial p_2^i} \geq 0$ and $\left.\frac{\partial^2 \pi_2^i}{\partial p_1^i \partial p_2^i}\right|_{s=0} = 0$.

The second-period equilibrium is derived from $\frac{\partial \pi_2^i}{\partial p_2^i} = 0, \forall i$, from which we obtain the equilibrium price $p_2^{i^*}(p_1, s)$ for each firm i. First-period equilibrium prices, in turn, are derived from

$$\frac{\partial \pi_1^i}{\partial p_1^i} + \delta \frac{\partial \pi_2^i}{\partial p_1^i} + \delta \sum_{k \neq i} \frac{\partial \pi_2^i}{\partial p_2^k} \frac{\partial p_2^{k^*}}{\partial p_1^i} = 0. \tag{1}$$

The first two terms of (1) are exactly as in the previous sections. The third term is specific to dynamic games. It corresponds to the impact of firm i's first-period price on its competitors' second-period prices. Our main point is that, if the sum of the second and third terms has the same sign as the second term, then the results in the previous sections will still go through. To put it differently, in order for the results from the static game to extend to dynamic games, it is necessary that the third term does not outweigh the effect of the second term.

The condition that the sign of $\frac{\partial \pi_2^i}{\partial p_1^i}$ is the same as the sign of $\frac{\partial \pi_2^i}{\partial p_1^i} + \delta \sum_{k \neq i} \frac{\partial \pi_2^i}{\partial p_2^k} \frac{\partial p_2^{k^*}}{\partial p_1^i}$ is quite natural and will hold under a variety of applications. For example, under cost interactions the

[12]See Maskin and Tirole (1997) for a definition of Markov perfect equilibria. Under general conditions this set of equilibria is equal to the set of closed-loop equilibria.

[13]For the two-period horizon being considered the set of Markov perfect equilibria is equal to the set of subgame perfect equilibria.

[14]In a longer-horizon model, $\pi_2^i(p_2, p_1, s)$ would represent the net present value of profits in the second and future periods.

[15]Note that the computation of the profit function in the first period can be quite complicated because first-period demands may also depend on consumer expectations of second-period prices (as, for example, in markets with switching costs or network externalities). This effect could make first-period profits a function of the intra-firm product interaction parameter, s. We rule out this possibility in order to simplify the analysis, but the results presented here still go through if the consumer expectations effect is not too large (for example, it is zero under cost interactions or myopic consumers).

condition implies that a firm benefits from having a lower cost even though its competitors may behave more aggressively because of the firm's lower cost; under demand interactions, the condition implies that the direct impact of the first-period demand on second-period profits dominates any possible effect through the competitors' actions.

In terms of the profit function, the precise condition under which the above holds true is given by the following result:

Theorem 7 *(dynamic market interaction): Consider the symmetric equilibrium of a symmetric industry and suppose that s is close to zero. First-period prices are decreasing in s (negative strategic effect) if*

$$\frac{\partial^2 \pi_2^i}{\partial s \partial p_1^i} + \frac{I-1}{(a-b)(a+b(I-1))} \frac{\partial \pi_2^i}{\partial p_2^k} \left[a \frac{\partial}{\partial s} \frac{\partial^2 \pi_2^i}{\partial p_2^i \partial p_1^k} - b \frac{\partial}{\partial s} \frac{\partial^2 \pi_2^i}{\partial p_2^i \partial p_1^i} \right] < 0, \tag{2}$$

where $a \equiv -\frac{\partial^2 \pi_2^i}{\partial p_2^{i\,2}}$, and $b \equiv -\frac{\partial^2 \pi_2^i}{\partial p_2^i \partial p_2^k}$, with $k \neq i$. If condition (2) is not satisfied then equilibrium first-period prices are increasing in s.

Assuming that the expression in (2) has the same sign as $\frac{\partial^2 \pi_2^i}{\partial s \partial p_1^i}$ – as indeed is the case in the applications we consider below – we can now state the dynamic version of the results presented in the previous sections. Specifically, when second-period demand is increasing in first-period demand (because of, for example, switching costs or network effects), first-period prices are lower the greater the degree of switching costs or network effects. More generally, first-period prices are decreasing in the degree of dynamic demand complementarity. Conversely, first-period prices are increasing in the degree of dynamic demand substitutability.

Turning to cost interactions, if each firm's second-period costs is decreasing in its first-period output (learning curve), then first-period prices are lower the "steeper" the learning curve. Similarly, if greater production in the first period increases costs in the second period, then first-period prices are higher the greater the effect of first-period production on second-period costs.

As in Section 4, we can also look at the total effect of dynamic intra-firm interactions. Suppose, as before, that the market is fully covered in both periods. Suppose additionally that first-period prices affect the second-period profits through the difference between first-period prices, that is,

$$\pi_2^i(p_2, p_1, s) = \pi_2^i(p_2, s(p_1^i - p_1^1), ..., s(p_1^i - p_1^I)). \tag{3}$$

Then, because we are considering a symmetric equilibrium, changes in s do not affect equilibrium profits in the second period.[16] Moreover, since the market is fully covered in the first period, the total effect is determined by the first-period equilibrium prices. We thus conclude that the total effect is positive or negative under the same conditions as those derived above for the first-period strategic effect.

6. Applications

As suggested by the examples in the introduction, intra-firm product interactions are a fairly prevalent phenomenon. In this section, we present a series of applications of our general framework.

6.1. Economies of scope

As discussed above, the case of economies of scope is a direct application of our general results, and in a fully-covered symmetric market profits decrease with greater economies of scope. Consider the following specific example to illustrate the result. Suppose that two firms compete in two products. Each product is characterized by a Hotelling demand: there is a mass one of consumers uniformly distributed along a unit segment; consumers pay a transportation cost of t per unit of distance; and firms are located at the extremes of the segment. The demands for the two products are independent and equal to $D_j^i(p_j^i, p_j^k) = \frac{1}{2} + \frac{p_j^k - p_j^i}{2t}$ for all $i, k \neq i$, and j. Suppose also that each firm's cost function is given by $C = cq_1 + cq_2 - sq_1q_2$, where q_1 and q_2 represent the outputs of product 1 and 2 by the firm, respectively, and are equal to each of the firm demands for each product (this notation for outputs is used throughout this section). Thus, s measures the degree of economies of scope. It can be easily checked that this example satisfies the definition of cost interactions introduced earlier.

Firm i's profit function is given by

$$\Pi^i = \frac{1}{2t}\left(\sum_{j=1}^{2}(t + p_j^k - p_j^i)(p_j^i - c) + \frac{s}{2t}\prod_{j=1}^{2}\left(t + p_j^k - p_j^i\right)\right), \text{ with } k \neq i.$$

[16] This assumption is satisfied in all the examples of the applications we consider below.

Deriving the first-order conditions and solving for a symmetric equilibrium yields

$$\widehat{p}_j^i = c + t - \frac{1}{2}s.$$

Substituting in the profit function and simplifying, we get

$$\Pi^i = t - \frac{1}{4}s. \tag{4}$$

In other words, equilibrium profits are lower the greater the degree of economies of scope.

The direct effect of an increase in s is clearly positive: $\frac{\partial \pi^i}{\partial s} = q_1^i q_2^i$. Equation (4) thus implies that the strategic effect more than outweighs the direct effect – a Bertrand supertrap. This result could also be derived from our Theorems 3 and 6. In fact, it is straightforward to show that the example satisfies the theorems' conditions.

A possible real-world application of these results is telecommunications. Recently, Bell Atlantic was allowed to enter the long-distance market.[17] This event signals "a new era of competition in telecommunications markets," one where companies will offer both local and long-distance services. This raises the question of how prices and profits will change in comparison to the case when local and long-distance services are offered by separate companies. Suppose the initial situation is characterized by $2I$ firms, I competitors in long-distance and I competitors in local services, the latter being different from the former.[18] Consider a new situation when there are I firms, each offering *both* local and long-distance services. Our theoretical results suggest that, if there are cost efficiencies in offering local and long-distance, in the form modeled above, then equilibrium prices and profits will be lower in the "global" competition scenario.

6.2. Core competencies

One possible implication of the core competencies hypothesis is that profitability is greater when a firm focuses on a small set of products or services – its core competencies (Prahalad and Hamel, 1990). For example, it may be that managers cannot pay enough attention to any particular activity when the firm is involved in too many activities. This can be modeled

[17] "First Baby Bell To Gain Approval for Long Distance," *The New York Times*, December 22, 1999.

[18] There is still relatively little competition in local telecommunications. Our results would also apply to the case when local telecommunications are a monopoly.

by a cost function that exhibits diseconomies of scope: it is more costly to produce q_1 and q_2 together then it is to produce both separately. If we believe that these diseconomies of scope are valid at the margin, then we have the reverse of the case considered before: industry profits are greater when two firms produce two products each than when there are four firms, each producing one product. Even though the direct effect of multi-product firms spinning off one of their products is positive, the total effect is negative: a "focused" firm is not only more efficient but also more aggressive, to the point that, in equilibrium, price cuts outweigh cost savings.

6.3. Demand synergies

Consider now the case of "demand synergies:" greater sales of firm i's product j increases the demand for firm i's product ℓ. One important instance of this setting is internet software. Good j might be a browser plug-in and good ℓ the software necessary for web site managers to create files downloadable with that plug-in. For example, Acrobat reader and Acrobat writer would be goods j and ℓ, respectively; or RealPlayer and the software that creates files to be read with RealPlayer. In these examples, the greater the number of users of a given plug-in, the greater value web site managers have in creating files under that plug-in's format – and the greater the demand for the software necessary to create such files. These effects result in demand complementarity, and applying the results above we know that greater demand synergies may yield lower equilibrium profits.

Consider the following specific example of this situation with duopoly competition, where firms offer two products subject to these cross-market effects (Strauss, 1999). Specifically, each firm $i = 1, 2$ offers a product A (demanded by Type A buyers) and a product B (demanded by Type B buyers). Consumers of each type are uniformly distributed along Hotelling segments and firms are located at the extremes of the segments. Each Type B's valuation (for a B product) is given by r minus the cost of "traveling" to the seller, which is equal to the distance traveled. For Type A consumers, however, gross valuation for firm i's A product also includes the term $\widetilde{s}q^i_B$, where q^i_B is demand for firm i's B product, and $\widetilde{s} = -s > 0$. So, continuing with the above example, q^i_B would be the number of consumers buying firm i's plug-in and q^i_A the number of web site managers buying firm i's software.

It can be shown that the demand for product B is given by $q^i_B = D^i_B(p^i_B, p^k_B) = \frac{1+p^k_B-p^i_B}{2}$ (as in a standard Hotelling model), whereas demand for product A is given by $q^i_A = D^i_A(p^i, p^k) = \frac{1+p^k_A-p^i_A+\widetilde{s}p^k_B-\widetilde{s}p^i_B}{2}$. Finally, assuming zero costs (for simplicity), firm i's profits are given by $\Pi^i = p^i_A q^i_A + p^i_B q^i_B$. Straightforward differentiation yields $\frac{\partial D^i_A}{\partial \widetilde{s}} = (p^k_B - p^i_B)/2$, which is zero in a symmetric equilibrium, $\frac{\partial D_A^i}{\partial p_A^i} = -1/2$, which is independent of $\widetilde{s}$, and $\frac{\partial D_A^i}{\partial p_B^i} = -\widetilde{s}/2$, which is zero at $\widetilde{s} = 0$ and decreasing in $\widetilde{s}$. We thus have demand complementarity. Theorem 4 implies that, if $\widetilde{s}$ is small, then an increase in $\widetilde{s}$ has a negative strategic effect. Moreover, Theorem 5 implies that the total effect is also negative. We thus conclude that equilibrium profits are lower the greater the degree of demand synergies, a result derived in Strauss (1999).

6.4. Click and mortar

As e-commerce booms, many firms are linking their stores to their web sites. Other retailers, such as Wal-Mart and K-Mart, have formed partnerships with Internet firms. The goal of these efforts is not only to shift from store sales to online sales but rather, according to the firms, to create a synergy that will increase total sales. Some retailers claim they are already seeing growth in one selling channel increase sales in another one. The president of Sharper Image, for example, states that "all three channels [stores, Internet and catalogs] are growing at a nice pace. Cannibalism is minimal."[19]

Suppose that the alliance between stores and online sellers (bricks and clicks) extends to joint pricing decisions. (In fact, this is the right assumption if they are jointly owned.) Then our analysis of the demand synergies case would indicate that total industry profits are lower under click-and-mortar competition than in the case when stores and Internet sellers are independent. As before, the reason is that the alliance induces firms to price more aggressively: one extra online sale, for example, implies not only the extra margin from the online sale but also a fraction of an extra sale in the corresponding store.

In other cases, such as grocery shopping, there are reasons to believe that online and in-store sales are substitutes, not complements. In this case, our prediction would be the opposite, namely that industry profits are greater if retailers and online sellers are linked. In a recent article on the future of online retailer Webvan, it was stated that

> Many ... caution Webvan against dropping its prices to garner customers. The rabid price competition that was once the premise of the Internet ended up being a death sentence to many retailers ...
>
> Webvan [should not] avoid alliances with traditional retailers: they need each other ... Don't give consumers an either-or choice between offline and online purchasing

[19] "Retailers Strive for Shopping Synergy," *The Wall Street Journal*, December 20, 1999.

of consumables.[20]

Our analysis corroborates the view that online retailers such as Webvan would be better off if linked to traditional retailers, among other reasons because this would soften price competition.

6.5. Systems competition

Suppose that consumers buy systems composed of J components, e.g., a computer and a computer monitor in the case $J = 2$. Suppose moreover that I firms supply each of the J components. Each consumer buys one "system" and has a valuation v_j^i for firm i's jth component. Consumers can either mix and match components from different firms or buy a "system" from firm i. Let $\widetilde{s}$ be the benefit that consumers derive from buying a firm i-only "system", with $\widetilde{s} = -s > 0$. How do equilibrium profits vary as a function of $\widetilde{s}$? Consider first the case when $\widetilde{s} = 0$. In this case, firm i's demands for its J products are independent: $\frac{\partial D_\ell^i}{\partial p_j^i} = 0$. Consider now the case when $\widetilde{s}$ is positive. In this case, some consumers buy firm i's products as a "system." It follows that $\frac{\partial D_\ell^i}{\partial p_j^i} \leq 0$. This suggests that demand complementarity applies and, as $\widetilde{s}$ increases, firm profits decrease.

6.6. Bundling

Pure bundling may be interpreted as the limit of systems competition when, for a very high $\widetilde{s}$, consumers only buy "systems" from the same firm. The above analysis suggests that competition with pure bundling leads to lower equilibrium payoffs than no bundling. As an illustration, consider the case of a double-Hotelling demand system ($J = 2$), whereby consumers are uniformly and independently distributed along two unit segments (each consumer has a location in each of the segments). Each consumer buys one pair of products. Under no bundling, equilibrium profits are the sum of two Hotelling profits. Under pure bundling, it is as if consumers were only buying one product. Since the valuations for the components are independent, it is as if firms were located at the extreme of a segment of length two. The density of consumers along this segment is triangular, with a value of one at the middle (as in the simple Hotelling game). Therefore, the equilibrium price is the same as for each of the components under no bundling which means that the equilibrium profits under pure bundling are one half of the equilibrium profits under no bundling.

[20] "Will Webvan Ever Find a Better Way to Bring Home the Bacon?," *The Wall Street Journal*, October 2, 2000.

The result that bundling makes firms more aggressive is not novel. Whinston (1990), for example, considers a model of a monopolist in a given market who leverages its power into a second market. Tying sales of the first and second products may allow the monopolist to drive rivals out of the second market (the tied good market).[21] Although the context in which Whinston looks at bundling is different from ours, the intuition for the result is the same.

6.7. Switching costs

In several markets consumers incur costs if they choose to switch sellers between periods — switching costs. This case can then be construed as a case of dynamic market interactions with intertemporal demand complementarity. That is, a lower price in the first period yields a great profit in the second period. In relation to the previous section, such a model would have two additional effects. First, the switching costs could affect the demand own-price sensitivity in the second period. Second, consumers in the first period make choices taking into account the expected prices in the second period. If these two additional effects are not too large, we then obtain that greater switching costs yield a lower present value of equilibrium profits.

Consider the following duopoly example where the first effect disappears in equilibrium, and the second effect ends up not being too large. Firms sell a given product in two periods. Each consumer buys at most one unit in each period. Consumers are uniformly distributed along a Hotelling segment, whereas firms are located at the extremes of the segment. Transportation costs are t per unit. Firms and consumers care equally about both periods and production costs are zero. A consumer who buys from firm i in the first period and wants to buy from firm $j \neq i$ in the second period has to incur a switching cost $\widetilde{s} = -s > 0$, which is assumed small compared to t.

Suppose also that the consumer's location on the Hotelling line in the second period is independent from his or her location in the first period.[22] To motivate this assumption, suppose that firms sell different incompatible word processors. A consumer's location on the Hotelling line refers to the number of work colleagues who use the same word processor: if most colleagues use Microsoft Word, then the consumer's location is close to Microsoft's. Between the first and the second period, consumers change jobs and start working with a different group of colleagues.

[21]Specifically, Whinston (1990) states that, for the monopolist, "tying represents a commitment to foreclose sales in the tied good market, which can drive its rival's profits below the point where remaining in the market is profitable" (p. 840).

[22]This is similar to the assumption in von Weizsacker (1984). Other authors (including, e.g., Beggs and Klemperer, 1989), make the opposite extreme assumption, namely that location is the same in all periods.

This effectively corresponds to being assigned a new location on the Hotelling line, one that we assume is independent from the initial one.

Given the above assumptions, we can start by determining second-period demand for each firm. A consumer who bought from firm i in the first period will be indifferent between the two firms in the second period if and only if the consumer's address, x, is such that $p_2^i + tx = p_2^k + t(1-x) + \widetilde{s}$, where x is the distance with respect to firm i. Solving for x, we get $x = 1/2 + (p_2^k - p_2^i + \widetilde{s})/(2t)$. Suppose that q_1^i consumers bought from firm i in the first period. Then firm i's demand in the second period is given by $q_1^i \left(1/2 + (p_2^k - p_2^i + \widetilde{s})/(2t)\right) + q_1^k \left(1/2 + (p_2^k - p_2^i - \widetilde{s})/(2t)\right)$. From the first order conditions in the second period for both firms one can obtain the equilibrium second period prices as a function of q_1^i (note that $q_1^k = 1 - q_1^i$) as $p_2^i = t + \frac{2q_1^i - 1}{3}\widetilde{s}$ and $p_2^k = t - \frac{2q_1^i - 1}{3}\widetilde{s}$. Second period profits for firm i as a function of q_1^i are $\pi_2^i = \frac{1}{2t}(t + \frac{2q_1^i - 1}{3}\widetilde{s})^2$.

Consider now the decisions by first-period consumers. For the marginal consumer buying product i, denoted by q_1^i (because the marginal consumer determines the demand for firm i in the first period), with probability $\frac{1}{2} + \frac{5-4q_1^i}{6t}\widetilde{s}$ the consumer will buy product i in the second period at expected price plus transportation costs equal to $\frac{5t}{4} + \frac{4q_1^i+1}{12}\widetilde{s}$. With probability $\frac{1}{2} - \frac{5-4q_1^i}{6t}\widetilde{s}$ the consumer will buy product k in the second period at expected price plus transportation costs plus switching costs equal to $\frac{5t}{4} - \frac{4q_1^i+1}{12}\widetilde{s} + \widetilde{s}$. Similarly, one can obtain the consumer payoffs if the marginal consumer buys product j. Indifference of the marginal consumer then yields

$$q_1^i = D_1^i(p_1^i, p_1^k) = \frac{t + \frac{2\widetilde{s}^2}{3t} + p_1^k - p_1^i}{2t + \frac{4\widetilde{s}^2}{3t}}.$$

If consumers are myopic, or there are no switching costs, this reduces to the static Hotelling demands. Switching costs and forward-looking consumers makes the first-period demands less price sensitive because the marginal consumers realize that by buying one product they increase the probability of paying a higher price in the next period.

Straightforward differentiation of the total profits in the first period, $\Pi^i = p_1^i q_1^i + \pi_2^i$, yields $p_1^i = p_1^k = t - \frac{2\widetilde{s}(t-\widetilde{s})}{3t}$. This means that the greater the switching costs, the lower the equilibrium prices and profits. Even though the first-period consumer expectations reduce the price sensitivity in the first period, which is an effect not considered in the previous section,[23] we still obtain the result that with demand complementarity equilibrium profits are lower.[24]

[23] As noted above, this effect completely disappears if consumers are myopic.

[24] This result is also obtained in von Weizsacker (1984), through a different mechanism, for the case of constant prices. Note that the assumption of independence of locations can be weakened without losing the result that

6.8. Network externalities

Consider a product such that consumer utility is increasing in the number of past consumer purchases – a network externality.[25] In this setting, lower prices in earlier periods generate greater demand and profits in future periods. We can then apply the results of the previous section to obtain that firms may compete so much in the earlier periods that they end up with a lower present value of equilibrium profits.

Consider the following specific duopoly example where firms sell a given durable product in two periods. A measure one of consumers buys one unit from one of the firms in the first period. A second set of consumers face the same choice in the second period. Consumers are uniformly distributed along a Hotelling segment, whereas firms are located at the extremes of the segment. In addition to the Hotelling transportation cost, second-period consumers derive an extra utility that is proportional to the number of consumers who bought the same product in the first period – a network externality.[26] Specifically, consumers in the second period derive net utility

$$U^i = \widetilde{s}q_1^i - td^i - p_2^i,$$

where U^i is the utility from buying from firm i, q_1^i is firm i's output in the first period, p_2^i is firm i's price in the second period, $\widetilde{s}$ measures the intensity of network effects with $\widetilde{s} = -s > 0$, t is the importance of product differentiation (transportation cost), and d^i is the distance from the consumer to firm i. For simplicity, assume that costs are zero.

Firms compete by simultaneously setting prices in each period. Second-period profits are given by

$$\pi_2^i = \frac{1}{2t}p_2^i \left(t - \widetilde{s} - (p_2^i - p_2^k) + 2\widetilde{s}q_1^i \right).$$

The equilibrium of the second-period pricing game is given by $p_2^i = t + \frac{2}{3}q_1^i - \frac{1}{3}\widetilde{s}$, leading to

switching costs lower equilibrium profits. However, this would mean that in the symmetric equilibrium the switching costs would have effects in the second period, a case outside the total effect result considered in the previous section.

[25] Network effects may also arise indirectly through the informational role of market shares (Caminal and Vives, 1996), and through compatibility issues (Farrell and Saloner, 1986).

[26] Note that we are assuming that network effects only apply to the second-period consumers. The case where the network effects also apply to the first-period consumers generates similar results, as in the switching costs example.

second-period equilibrium profits of

$$\widehat{\pi}_2^i = \frac{1}{2t}\left(t + \frac{1}{3}\widetilde{s}(2q_1^i - 1)\right)^2.$$

First-period equilibrium profits are given by $p_1^i q_1^i + \widehat{\pi}_2^i$. Since $q_1^i = D_1^i(p_1^i, p_1^k) = \frac{1}{2} + (p_1^k - p_1^i)/(2t)$, we have an expression that is a function of p_1^i, p_1^k. Solving the equilibrium pricing game we get $p_1^i = t - \frac{2}{3}\widetilde{s}$ and total equilibrium profits of $t - \frac{1}{3}\widetilde{s}$, a value that is decreasing in $\widetilde{s}$ (Bertrand supertrap).

As noted above, the result that the strategic effect is negative and outweighs the direct effect could also be derived by applying our Theorem 7. In fact, simple calculations show that the second term in (2) is proportional to $\widetilde{s}$, whereas the first one is negative and independent of $\widetilde{s}$. It follows that, for a small value of $\widetilde{s}$, condition (2) holds. Moreover, π_2^i depends on first period prices through the value of q_1^i, which in turn is a function of the price difference $p_1^i - p_1^k$, that is, π_2^i can be written as in (3). It follows that total equilibrium profits are decreasing in $\widetilde{s}$.

6.9. Learning by doing

Similarly to network externalities, we can apply our general framework to a two-period model of oligopoly competition with learning-by-doing. Consider the case when, in each period, firms compete in prices. Suppose that each firm's second-period marginal cost is decreasing in its first-period output – the learning curve hypothesis. Our results imply that, in a subgame perfect equilibrium, prices and profits are lower than they would be were there no learning effects. Cabral and Riordan (1994) present a similar result in a model with two firms, infinite horizon, and discrete demand.

7. Concluding Remarks

We have examined the impact of intra-firm product interactions on the equilibrium strategies and payoffs of competing oligopolists. As illustrated in Section 6, there are many market situations where product interactions imply a direct and a strategic effect in opposite directions. Moreover, the strategic effect may outweigh the direct effect in absolute value, a situation that we call "Bertrand supertrap." In these situations, the effect of price competition is so powerful that "more is less:" stronger product interactions, which in a monopoly situation would imply greater profits, turn out to lower equilibrium profits under competition.

We have looked at the comparative statics of a *common* change in s, the degree of product interactions. However, in many situations the value of s results from the firms' decisions. For example, firms may have some discretion in choosing the degree of product substitutability/complementarity or the cost technologies that determine the degree of learning by doing or economies of scope. The natural way to model this would be to consider a two-stage game: in the first stage, firms simultaneously choose the level of s_i, one for each firm; in the second stage, firms compete in prices. The impact of s_i is now more likely to be determined by the direct effect rather than the strategic effect through the competitors' actions, because the firms' actions may be more sensitive to their own s_i.[27] A tantalizing possibility is then that the total effect of a unilateral increase in s_i is positive even though a common increase in s leads to a negative total effect, as was the case in several of the applications considered above. The direct effect dominates in the choice of s_i by each firm, while the strategic effect dominates in the overall payoff. In other words, it is possible that the two-stage (symmetric) game has the structure of a prisoner's dilemma. For example, firms may choose technologies with steep learning curves, or selling conditions that imply a high degree of intra-firm demand complementarity (e.g., pure bundling). Even though, unilaterally, firms are better off by doing so, in equilibrium they are worse off than they would be if there were no intra-firm product interactions.

The situation may be different if the meta-game is one of *sequential*, not simultaneous, choice of s_i. In particular, consider the case where Firm A, an incumbent, first chooses the value of s_A, and Firm B, a potential entrant, then decides whether to enter and which value s_B to choose.[28] Firm A's monopoly and duopoly profits are given by $\Pi^A(s_A)$ and $\pi^A(s_A, s_B)$, respectively. Firm B's duopoly profits are given by $\pi^B(s_B, s_A; \theta)$, where $\frac{\partial \pi^B}{\partial \theta} > 0$. The variable θ is public information at the time Firm B decides whether to enter, but unknown at the time Firm A chooses s_A. Suppose the prior on θ is given by the cumulative distribution function $F(\theta)$ and let $\theta^*(s_A)$ be such that $\max_{s_B} \pi^B(s_B, s_A; \theta^*(s_A)) = 0$. Firm A's expected payoff is then given by

$$F\left(\theta^*(s_A)\right) \Pi^A(s_A) + \int_{\theta^*(s_A)}^{\infty} \pi^A(s_A, s_B^*(\theta)) dF(\theta),$$

[27]Remember that the effect of s in Section 4 is for the symmetric case, that is, all s_i changing at the same time. The condition that makes the direct effect greater, when s_i alone is being considered, is related to the condition (2) above.

[28]In the particular case when s represents the bundling decision, this structure is similar to the one analyzed by Whinston (1990). This general structure can also be seen as related to Fudenberg and Tirole (1984).

where $s_B^*(\theta) = \arg\max_{s_B} \pi^2(s_B, s_A; \theta)$. This analysis suggests an extra reason why a unilateral increase in s_A may have a positive effect. Not only an increase in s_A may increase the value of the duopoly profits (as suggested above) but it may also increase Firm A's ex-ante expected payoff. First, with some probability Firm A will be a monopolist, and the total effect of s_A on $\Pi^A(s_A)$ is simply the direct effect. Second, if, as our results suggest, $\pi^B(s_B^*(\theta), s_A; \theta)$ is decreasing in s_A, then an increase in s_A implies an increase in θ^*, which in turn increases Firm A's expected payoff. In other words, precisely because greater values of s imply lower duopoly profits, an increase in s_A may have the strategic effect of deterring entry.

The above extensions of our basic framework to include the endogenous choice of s_i suggest a solution to an apparent puzzle raised by our results. Business people and business analysts are wont to stress the positive effect of strategies that lead to demand synergies, cost synergies, greater switching costs, and so forth – the very same strategies which, according to our analysis, lead to lower industry profitability. If the two-stage game has the structure of a prisoner's dilemma (simultaneous choice of s_i), then we may interpret the business advice as reflecting the fact that choosing a high s_i is a dominant strategy, a fact that is consistent with our result that higher values of s_i *by all firms* lead to lower profits. If, on the other hand, we consider the sequential choice of s_i, then our results point to the benefits that early entrants may reap from intra-firm product interactions.

APPENDIX

Proof of Theorem 1: For $s > 0$ and strong strategic complementarity each profit function is supermodular in all prices, which implies that all functions $b^i_j(p^{-i}, s), \forall i, j \in J_i$, are increasing in every element of p^{-i}. The mappings indexed by s, $p^i \to b^i(p^{-i}, s), \forall i$ are then increasing mappings in the componentwise order. Furthermore, given the monopoly effect, either the mapping is increasing or decreasing in s. Therefore, applying the results in Milgrom and Roberts (1990), the fixed point of the mapping $b^i, \forall i$ is increasing or decreasing in s, depending on whether $\frac{\partial b^i_j(p^{-i},s)}{\partial s} \geq 0, \forall i, j \in J_i$ or $\frac{\partial b^i_j(p^{-i},s)}{\partial s} \leq 0, \forall i, j \in J_i$, respectively.

If $s < 0$ or if strong strategic complementarity does not hold, one must make sure that the intra-firm product interactions are not so great as to reverse the effects of exogenous changes in s. If $|s|$ is small, then these intra-firm product interactions effects do not outweigh the effects of strategic complementarity. To see this, define the following matrix and vector:

$$H^i \equiv \left[\frac{\partial^2 \Pi^i}{\partial p^i_j \, \partial p^i_\ell}\right] \; (j, \ell = 1, \ldots, J_i),$$

$$v^{ik}_\ell \equiv \left[\frac{\partial^2 \Pi^i}{\partial p^i_j \, \partial p^k_\ell}\right] \; (j = 1, \ldots, J_i).$$

Firm i's first-order conditions are given by

$$\frac{\partial \Pi^i(\hat{p})}{\partial p^i_j} = 0, \quad j \in J_i$$

and can be rewritten as

$$p^i_j = f^i_j(p^i_{-j}, p^{-i}, s), \quad j \in J_i. \tag{i}$$

Solving with respect to p^i_j, we get

$$p^i_j = b^i_j(p^{-i}, s), \quad j \in J_i.$$

We would like to show that b^i_j is increasing in every element of p^{-i} for $s = 0$. Note that total differentiation of (i) with respect to p^i and p^k_ℓ, $(k \neq i)$ yields

$$\frac{d\,p^i_j}{d\,p^k_\ell} = -H^{i^{-1}} v^{ik}_\ell.$$

which is strictly positive for $s = 0$. In fact, $s = 0$ implies that H^i is a diagonal matrix with negative elements in the diagonal (because of Assumption 2 and of the second-order conditions) and v_ℓ^{ik} is composed of positive elements (because of Assumption 3), where at least one element is strictly positive. Then, if by the monopoly effect all $\frac{\partial b_j^i(p^{-i},s)}{\partial s}$ are different from zero at $s = 0$ then we can apply the supermodularity results. We can then obtain that at $s = 0$, if $\frac{\partial b_j^i(p^{-i},s)}{\partial s} > 0, \forall i, j \in J_i$ then $\frac{d\widehat{p}_j^i(s)}{ds} > 0, \forall i, j \in J_i$, and if $\frac{\partial b_j^i(p^{-i},s)}{\partial s} < 0, \forall i, j \in J_i$ then $\frac{d\widehat{p}_j^i(s)}{ds} < 0, \forall i, j \in J_i$. By continuity this result is also satisfied for $|s|$ small. Q.E.D.

Proof of Theorem 2: Suppose first that $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} > 0, \forall j \in J_i$. Writing firm i's first-order conditions as $p_j^i = f_j^i(p_{-j}^i, p^{-i}, s), \quad j \in J_i$, we have, because of the second order conditions, $\frac{\partial f_j^i}{\partial s} > 0$.

Suppose that $s > 0$ (profit complementarity). Then, f_j^i is increasing in all the elements of p_{-j}^i. Moreover, the mapping indexed by s, $p^i \to f^i(p^i, p^{-i}, s)$ is increasing in p^i and s in the componentwise order. Therefore, applying the results in Milgrom and Roberts (1990), the fixed point of the mapping f^i is increasing in s in the componentwise order, that is, $\frac{\partial b_j^i(p^{-i},s)}{\partial s} > 0$, for all $j \in J_i$.

Suppose now that $s < 0$ (profit substitutability). Then, at $s = 0$, $f_j^i(p_{-j}^i, p^{-i}, s) = b_j^i(p^{-i}, s)$ which implies $\frac{\partial b_j^i(p^{-i},s)}{\partial s} > 0$, for all $j \in J_i$. By continuity, the result also holds for small $|s|$.

The case when $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} < 0, \forall j \in J_i$, is analogous. Q.E.D.

Proof of Proposition 1: Differentiating Π^i with respect to p_j^i and s we get

$$\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} = \sum_{k \neq j, k \in J_i} p_k^i \frac{\partial D_j^i}{\partial (s p_k^i)} + \sum_{k \neq j, k \in J_i} (p_k^i - \frac{dC_k^i}{dD_k^i}) \frac{\partial D_k^i}{\partial (s p_j^i)} + O^2$$

where O^2 includes terms with the second derivatives of the demand or cost functions. From the first-order conditions, we get that $p_k^i - \frac{dC_k^i}{dD_k^i} > 0$ for small $|s|$. Given the definition of demand interactions, it follows that $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} > 0$ for all $j \in J_i$. Q.E.D.

Proof of Proposition 2: Differentiating Π^i with respect to p_j^i and s we get

$$\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} = - \sum_{k \neq j, k \in J_i} \left[\frac{\partial C_k^i}{\partial (s D_j^i)} D_j^{i'} + \frac{\partial^2 C_j^i}{\partial D_j^i \partial (s D_k^i)} D_k^i D_j^{i'} + \sum_{\ell \neq k, \ell \in J_i} s \frac{\partial^2 C_k^i}{\partial (s D_j^i) \partial (s D_\ell^i)} D_\ell^i D_j^{i'} \right].$$

From the definition of cost interactions, we have $\frac{\partial^2 \Pi^i}{\partial p_j^i \partial s} < 0$, for all $j \in J_i$. Q.E.D.

Proof of Theorem 5: Since total demand is constant, the equilibrium is symmetric, and there are no product interactions in costs, it follows that the only terms in the profit function that vary with s are prices. The result then follows from Theorems 3 and 4. Q.E.D.

Proof of Theorem 6: Firm i's equilibrium profits are given by $\widehat{\Pi}^i(s) = \sum_{j=1}^J \hat{p}^i_j(s) D^i_j(\widehat{p}(s)) - C(D^i(\widehat{p}(s)), s)$. This may be rewritten as

$$\widehat{\Pi}^i(s) = \sum_{j=1}^{J} \left(\widehat{p}^i_j(s) - \frac{\partial C}{\partial D^i_j}(D^i(\widehat{p}(s)), s) \right) D^i_j(\widehat{p}(s)) + \sum_{j=1}^{J} \frac{\partial C}{\partial D^i_j}(D^i(\widehat{p}(s)), s) D^i_j(\widehat{p}(s)) - C(D^i(\widehat{p}(s)), s). \tag{ii}$$

Firm i's j'th first-order condition for profit maximization can be written as

$$\left(p^i_j - \frac{\partial C}{\partial D^i_j} \right) \frac{\partial D^i_j}{\partial p^i_j} = -D^i_j.$$

Assumptions 4 and 5 imply that the right-hand side is invariant with respect to s. Cost interactions implies that the second term on the left-hand side is also invariant with respect to s. It follows that margins are invariant with respect to s. We conclude that an increase in s implies a decrease in equilibrium profits if and only if the last two terms in (ii) decrease in s, that is, if

$$\sum_{j=1}^{J} \frac{\partial^2 C}{\partial D^i_j \, \partial s} D^i_j - \frac{\partial C}{\partial s} \leq 0 \tag{iii}$$

(We only need to take partial derivatives with respect to s because the equilibrium demands do not change with s, by Assumption 5.)

With fixed costs being independent of s we have

$$\frac{\partial C^i}{\partial s}(D^i, s) = \sum_{j=1}^{J} \int_0^{D^i_j} \frac{\partial^2 C^i}{\partial D^i_j \, \partial s}(D^i_{j-}, D^i_j = t, D^i_{j+} = 0)\, dt$$

where D^i_{j-} is the vector with the elements of D^i_k for $k < j$ and D^i_{j+} is the vector with elements D^i_k for $k > j$. Assumption 2 and the Theorem assumptions that $\frac{\partial}{\partial s} \frac{\partial^2 C^i}{\partial D^i_j \partial D^i_k} < 0, \forall i, j \in J_i, k \in J_i$, and $\frac{\partial}{\partial s} \frac{\partial^2 C^i}{\partial D^{i\,2}_j} \leq 0, \forall i, j$, imply that

$$\frac{\partial^2 C^i}{\partial s \partial D^i_j} D^i_j \leq \int_0^{D^i_j} \frac{\partial^2 C^i}{\partial D^i_j \, \partial s}(D^i_{j-}, D^i_j = t, D^i_{j+} = 0)\, dt,$$

since $\frac{\partial^2 C^i}{\partial s \partial D^i_j} \leq \frac{\partial^2 C^i}{\partial D^i_j \, \partial s}(D^i_{j-}, D^i_j = t, D^i_{j+} = 0)$ for $0 \leq t \leq D^i_j$. Adding up for all product j, we conclude that (iii) holds, which in turn implies the result. Q.E.D.

Proof of Theorem 7: We are looking at a symmetric equilibrium when s is close to zero. Since there is strategic complementarity in the first period by assumption, we just need to investigate whether $\frac{\partial}{\partial s}\left(\frac{\partial \pi^i_2}{\partial p^i_1} + (I-1)\frac{\partial \pi^i_2}{\partial p^k_2}\frac{\partial p^{k^*}_2}{\partial p^i_1}\right)$, with $k \neq i$, is positive or negative. Because at $\left.\frac{\partial p^{k^*}_2}{\partial p^i_1}\right|_{s=0} = 0$, the proof reduces to computing $\frac{\partial}{\partial s}\frac{\partial p^{k^*}_2}{\partial p^i_1}$ at s close to zero. Totally differentiating the second-period first-order conditions of firm k with respect to p^i_1 and s we get

$$\sum_j \frac{\partial p^{j^*}_2}{\partial p^i_1}\frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^j_2} + \sum_j \frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^j_2}\frac{\partial}{\partial s}\frac{\partial p^{j^*}_2}{\partial p^i_1} + \frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^i_1} = 0.$$

Again, because $\left.\frac{\partial p^{k^*}_2}{\partial p^i_1}\right|_{s=0} = 0 \; \forall k$, we get

$$\left[\frac{\partial}{\partial s}\frac{\partial p^{j^*}_2}{\partial p^k_1}\right]_{j=1,I} = -\left[\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^j_2}\right]^{-1}_{j,k=1,I}\left[\frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^i_1}\right]_{k=1,I}.$$

Since we are considering a symmetric equilibrium of a symmetric industry, the matrix $\left[\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^j_2}\right]$ has the same number in diagonal element, $-a \equiv \frac{\partial^2 \pi^k_2}{\partial p^{k^2}_2}$, and the same number in every element off the diagonal, $-b \equiv \frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^j_2}$, with $k \neq j$. The inverse matrix has the same form, with diagonal elements equal to $-\frac{a+b(I-2)}{(a-b)[a+b(I-1)]}$ and off-diagonal elements equal to $\frac{b}{(a-b)[a+b(I-1)]}$. Because, by symmetry, all elements $\frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^i_1}$ with $k \neq i$ are equal, we have that $\frac{\partial}{\partial s}\frac{\partial p^{k^*}_2}{\partial p^i_1} = \frac{1}{(a-b)[a+b(I-1)]}\left(a\frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^i_1} - b\frac{\partial}{\partial s}\frac{\partial^2 \pi^k_2}{\partial p^k_2 \partial p^k_1}\right)$ for $k \neq i$, from which we can immediately obtain condition (2). Q.E.D.

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