Personalized Pricing and Quality Differentiation*

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January 2005

Abstract

We develop an analytical framework to investigate the competitive implications of personalized pricing (PP), whereby firms charge different prices to different consumers, based on their willingness to pay. We embed personalized pricing in a model of vertical product differentiation, and show how it affects firms' choices over quality. We show that firms' optimal pricing strategies with PP may be non-monotonic in consumer valuations. When the PP firm has a high quality both firms raise their qualities, relative to the uniform pricing case. Conversely, when the PP firm has low quality, both firms lower their qualities. Although many firms are trying to implement such pricing policies, we find that a higher quality firm can actually be worse off with PP. While it is optimal for the firm adopting PP to increase product differentiation, the non-PP firm seeks to reduce differentiation by moving in closer in the quality space. While PP results in a wider market coverage, it also leads to aggravated price competition between firms. Since this entails a change in equilibrium qualities, the nature of the cost function determines whether firms gain or lose by implementing such PP policies. Despite the threat of first-degree price discrimination, we find that personalized pricing with competing firms can lead to an overall increase in consumer welfare.

Keywords: *Personalized Pricing, Quality Differentiation, Price Competition.*

*We are grateful to Christine Parlour, Kannan Srinivasan and seminar participants at WISE 2001, ICIS 2002, CIST 2003 and, Carnegie Mellon University, for helpful comments. We also thank Ram Rao and an anonymous reviewer at the Review of Marketing Science (ROMS) for useful suggestions during the acceptance process in their Working Paper series (Vol 2, Issue 1, 2002). Finally, we are grateful to the Editor, AE and three anonymous reviewers for their comments. All errors remain our own responsibility.

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1 Introduction

Different consumers typically derive different value from the same product. Firms often respond to this heterogeneity in valuations by trying to determine what customers will pay. This is done in a variety of ways: by understanding the nature of a customer's business and how the product will be used, by asking about their budget during a negotiation or via market research using different collaborative and content filtering techniques. The information about willingness to pay is then used to provide a personalized price for the customer.

In this paper, we use the term personalized pricing, or PP, to refer to the limiting case in which a firm can implement a pricing policy based on complete knowledge of the willingness to pay of each consumer.[1] We bypass the question of how the firm acquires this knowledge. Rather, we focus on the implications this has for firm strategies. Specifically, we examine the following questions: (i) How does competition affect equilibrium product quality outcomes when firms engage in personalized pricing? (ii) Does the improvement in firms' knowledge of individual consumers alleviate or intensify price competition? (iii) What are the tradeoffs firms face in adopting PP? (iv) How does PP affect consumer welfare?

Examples of personalized pricing come from the markets for both consumer and business products. Firms selling large proprietary enterprise-level software often finalize the price through a negotiation. Vendors typically work with clients to conduct an ROI (Return on Investment) analysis to determine the benefit (in the form of cost savings or revenue enhancements) of the product to the client. Sweeney et al., describe such a collaborative development of ROI by Teradata Inc. in fostering sales of its data-warehousing technology.[2] The ROI analysis is used to price software. For instance, a California-based in-store demand planning software developer sets the price as a percentage of a mutually determined ROI. This is also common practice in the sale of enterprise telephone cost auditing software.[3]

[1]Since, the amount of information required for implementing PP is high, in practice, firms may not know valuations precisely. Hence, our results should be interpreted as the solution to an important limiting case which provides a useful benchmark – the case of perfect information.

[2]Sweeney, R., Davis, R. and M. Jeffery, "Teradata Data Mart Consolidation Return on Investment at GST", 2002, http://www.kellogg.nwu.edu/faculty/jeffery/htm/cases/Data%20Mart% %20Consolidation%20ROI%20Case%20at%20GST.pdf

[3]These anecdotes were communicated to us in conversations with Steve Acterman, Director Corporate IT Management, Volt Information Sciences, Harnish Kanani, Senior Vice President Global Services, Emagia Corporation, Tim Johnson, Account Executive, Apreo Inc.

The market for computer servers, storage devices and workstations in the Asia-Pacific region combines PP and quality differentiation. Major players such as IBM, Hewlett Packard, and Sun Microsystems use personalized discounting for different customers based on ROI, even at the same quality levels. There is also a trend towards increasing the degree of service quality and value-added software differentiation in the industry. For instance, in the UNIX platform, HP and IBM cater to the high-end market, while Sun serves the low-end market.[4] Other examples of value-based personalized pricing are found in the healthcare (Smith and Nagle, 2002) and chemicals industries.

Online retailers with their ability to collect data are well-positioned to take advantage of dynamic pricing. In a well-known example, Amazon offered different prices to different consumers on its popular DVD titles.[5] Although Amazon's experiment was short-lived due to a consumer backlash, it has since found innovative ways of implementing PP without annoying consumers, through the use of the "Gold Box". Each consumer is provided access to a prominently displayed Gold Box with their name (e.g. John Doe's Gold Box) on webpages at Amazon. Opening the Gold Box provides access to a limited number of products with special discounts that are not available outside the Gold Box. The items offered in the Gold Box are different for different consumers. This allows Amazon to charge personalized prices. This is an example of the continuing evolution of PP and an indication of the likely use of such pricing by online retailers. Chen and Iyer (2002) mention several other examples of customized pricing.[6] Wertenbroch and Skiera (2002) provide an empirical study that compares several approaches for determining consumer willingness to pay.

We consider a vertically differentiated duopoly framework in which one or both firms can perfectly identify valuations of heterogenous consumers.[7] A monopolist with such information could engage in first-degree price discrimination. As Armstrong and Vickers (2001) point out, the literature on competitive price discrimination is not as extensive as in the monopoly case. They provide an elegant framework that incorporates much of the earlier work on price competition in

[4] "Are Proprietary RISC Servers More Expensive Than Their UNIX Alternatives?", TechWise Research, Incorporated, May 1999.

[5] Morneau, J., "Dynamic Pricing: Who Really Wins?," *TechWeb* Sep. 29, 2000.

[6] These include major providers of long distance telephone service (such as AT&T, MCI and Sprint), direct marketing companies like Land's End and L.L. Bean, who have individual specific catalog prices, and financial services and banks, who engage in PP through personalized discounts on card fees. Zhang (2003) mentions Wells Fargo and MBNA in this regard.

[7] We ignore the possibility of mistargeting. Chen, Narasimhan and Zhang (2001) show that mistargeting can have an important effect, by softening price competition in the market, and qualitatively changing the incentives for competing firms engaged in individual marketing.

an environment with multiple firms. Recent work on customer recognition and behavior-based price discrimination includes Villas-Boas (1999) and Acquisti and Varian (2002). Much of the recent work on perfect price discrimination has been done either in the context of horizontal product differentiation (Vives and Thisse, 1988, Chen and Iyer, 2002, Ulph and Vulkan, 2002, Bhaskar and To, 2002) or monopoly (Aron, Sunderarajan and Viswanathan, 2003). Shaffer and Zhang (2002) consider perfect price discrimination by competing firms in a model that includes both horizontal and vertical differentiation. Desai (2001) analyzes second-degree price discrimination with both vertical and horizontal differentiation. Dellaert and Syam (2002) bring into focus the issues surrounding mass-customization via an analysis of consumer-producer interaction. We contribute to the literature in this field by incorporating perfect price discrimination in a vertically differentiated duopolistic setting. Since our paper examines the issue of how firms' knowledge of individual customers affects the nature of their strategic interactions, it complements the work done on how customers' knowledge about firms may affect firms' competitive strategies (Lal and Sarvary, 1999, and Zettelmeyer, 2000).

We derive a number of analytical results on firm pricing, quality differentiation, and consumer welfare when one or both firms have PP. First, if the firm with PP has a low quality, its optimal price is non-monotonic in consumers' willingness to pay. That is, some high valuation consumers are offered lower prices than some low valuation ones. Second, when one firm adopts PP, the other firm responds by lowering its price. This is a competitive response: a firm with PP knows the valuation of each consumer, and can therefore charge prices as low as its own marginal cost to a specific consumer. It therefore encroaches into the market share of the other firm, which responds to the increased competition by reducing its price. Third, when only one of the firms adopts PP, it is optimal for it to increase product differentiation. This can be interpreted as a move to reduce competition with the other firm. When the cost of quality is quadratic, if the low quality firm adopts PP, both firms reduce their quality levels. Conversely, when the high quality firm adopts PP, both firms increase their quality levels. We show that when both firms adopt PP, the high quality firm reduces its quality while the low quality firm raises its quality. Finally, consumer surplus falls (compared to the no PP case) if the PP firm has low quality, but rises if the PP firm has high quality. In fact, consumer surplus is highest when both firms have PP.

In addition to the above results, for a wide range of cost parameters, we demonstrate some properties of firm profit with PP. First, within this range, it is a dominant strategy for the low quality firm to adopt PP. That is, regardless of whether the high quality firm adopts PP or not, the low quality firm makes a higher profit with PP. Conversely, the high quality firm can actually be worse off with PP and should adopt PP only if the costs of quality are not too steep. This paradox emerges because in a vertical differentiation context, the other firm responds by lowering its quality. Next, if marginal costs sharply increase in quality, then both firms earn lower profits compared to the case where neither has PP. Essentially, they are trapped in a prisoner's dilemma.[8] However, if costs are not too convex, both firms increase profits when they adopt PP. Thus, our paper highlights that the cost-of-quality effect can lead to circumstances wherein firms can avoid the prisoner's dilemma situation when they both have PP.

The rest of the paper is organized as follows. Section 2 briefly describes the model. In Section 3 we show that when only one firm has PP, there are two possible equilibria, with the PP firm having either a low quality or a high quality.[9] We next consider the case of both firms using PP. In Section 4 we analyze the impact of PP on firms' profits and consumer surplus. This allows us to consider the question of when firms will adopt PP. We discuss some implications of our findings in Section 5. All proofs are relegated to the Appendix.

2 Model

We consider personalized pricing in a duopoly model of vertical differentiation.[10] Two firms compete in both the quality and price of the products they offer. Formally, we model their competition as a three-stage game. At the first stage, firms simultaneously choose the quality levels of their products. At stage 2, the firms choose their prices. When neither firm has access to PP, prices are chosen simultaneously. When only one firm has access to PP, the firm without PP chooses its price first, followed by the firm with access to PP. Personalized pricing is executed for each

[8] Shaffer and Zhang (1995), Thisse and Vives(1988) obtain similar results in models of price discrimination.

[9] We do not consider the question of which equilibrium will emerge. In our model, neither firm has the option of forcing the other into a particular equilibrium.

[10] Shaked and Sutton (1982) and Gabszewicz and Thisse (1986), building on research by Mussa and Rosen (1978), develop duopoly models of vertical differentiation and show that to reduce price competition, firms seek maximal product differentiation. Moorthy (1988) extends the basic model by incorporating variable production costs and allowing consumers the opportunity to not buy a product. This results in less than maximal product differentiation.

consumer at the point of sale. Hence, a firm which engages in PP chooses its price after a rival that has a uniform pricing policy (which must be posted and committed to before sales occur). In other words, the flexibility implied by personalized pricing incorporates an implicit assumption on flexibility in timing as well. When both firms have PP, the order of moves at stage 2 does not affect the outcome; for convenience, we again posit that prices are chosen simultaneously. Once prices are chosen, at the last stage of the game (stage 3), consumers decide which, if any, product to buy.

If a consumer purchases a product of quality q at price p, his utility is $U(\theta) = \theta q - p$, where $\theta \in [0, 1]$. A consumer has positive utility for one unit only. The type parameter θ indicates a consumer's marginal valuation for quality. For any given quality, a consumer with a higher θ is willing to pay more for the product than one with a lower θ. If either of the two products offers a positive net utility, a consumer buys the one that maximizes his surplus. Otherwise, he chooses not to buy either product. It is immediate to show in this model that, if the qualities of the firms are the same, personalized pricing adds no value—the result is Bertrand competition, with both firms pricing at marginal cost. Hence, in this paper, we consider a model in which firms first choose qualities (which will be different in equilibrium), and then prices.

Consistent with prior literature (for example, Desai, 2001), we assume that firms have a marginal cost of production which is invariant with the quantity, but depends on the quality of the product. That is, both firms have the same cost function, but depending on the quality levels they choose, their marginal costs may differ in equilibrium. Each firm has a constant marginal cost for producing the good, denoted by c. Further $c(\cdot)$ is twice differentiable, strictly increasing and strictly convex in q. That is, $c' > 0$ and $c'' > 0$. Quality in this model is a broad notion that encompasses any feature that may affect a consumer's willingness to pay for a good. These could include features intrinsic to the product itself (such as durability and functionality) or those related to the quality of the shopping experience, or the service level provided by the firm (such as warranties and customer service). Quality is observed perfectly by all consumers.

Given the quality levels and prices offered by the two firms, consumers make their choices. Suppose, in the benchmark case of uniform pricing, firm 1 offers (q_1, p_1), and firm 2 offers (q_2, p_2). There will be a subset of consumers (including null) who buy from each firm, 1 and 2. The profit of firm j is its market coverage times $(p_j - c(q_j))$. In the case of PP, we allow one or both firms to be exogenously equipped with a technology that perfectly reveals the consumer's type before the price is disclosed to the consumer. Both firms know which firm has PP before the game is played. While the firm offers the same quality product to all consumers, it can choose a personalized price for each consumer. In this case, firm j's profit from consumer θ is $(p_j(\theta) - c(q_j))$. Let c_j denote $c(q_j)$. In practice, implementing personalized pricing may well incur some fixed costs. However, if such costs are independent of the quality of the product being offered by the firm, they do not affect the qualitative nature of the results. For simplicity, we treat these costs as zero.[11] We consider pure strategy subgame-perfect equilibria of this three-stage game. That is, for any strategies the firms may choose at stages 1 and 2, consumers behave optimally at stage 3. Firms, in turn, not only anticipate this behavior, but also choose optimal prices, given quality levels, at stage 2. The subgame-perfect equilibrium is determined by backward induction, starting with stage 3.

Consider the case when neither firm has access to PP (we call this the no-PP case). As shown by Moorthy (1988), in equilibrium at stage 3, the firms share the market in the following manner.[12] There exist threshold consumers θ_h and θ_ℓ, such that consumers with valuations greater than a cutoff level θ_h and less than 1 purchase product h, and those with valuations between a second cutoff level θ_ℓ and θ_h purchase product ℓ. This situation is depicted in Figure 1 below. The details of the profit equations and reaction functions are provided in Section 1.1 of the Appendix.

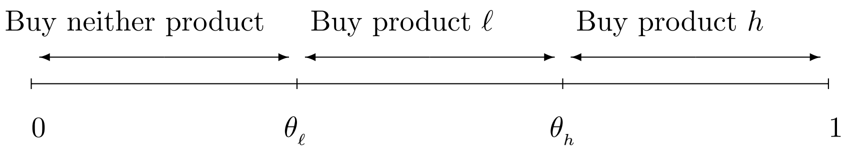

Figure 1: Consumers' purchasing decision by consumer type (θ)

The same intuition also applies in the case that one or both firms have PP. Of course, for the equilibrium to have these properties with both firms existing, it must be that $0 < \theta_\ell < \theta_h < 1$. In solving the various cases, we show that an equilibrium with these properties exists.

[11]In Section 4, we provide guidelines as to when firms should or should not invest in PP if the fixed costs are non-zero.

[12]Moorthy assumes quadratic costs, but this result depends only on consumer preferences.

3 Duopoly with Personalized Pricing

Suppose first that only one firm has PP. There are two equilibria in this case; one in which the PP firm has a lower quality than the other firm, and a second one in which the PP firm has a higher quality. We consider each of these, and then examine the case in which both firms have PP.

3.1 PP Firm Offers Low Quality

We use the superscript $^{\ell}$ to denote this case, while the subscripts h, ℓ denote the firms. In the spirit of backward induction, suppose firms choose qualities q_h, q_ℓ at stage 1, and consider stage 2 first. Let $p_h^\ell, p_\ell^\ell(\theta)$ denote the optimal prices chosen at stage 2 (as functions of q_h, q_ℓ). Similarly, let $\pi_\ell^h, \pi_\ell^\ell$ denote the profit functions, as functions of q_h and q_ℓ alone (that is, after substituting in the optimal stage 2 prices). For brevity, in the notation we often suppress the dependence of these functions on q_h, q_ℓ (this dependence is clear in the expressions exhibited below). We use q_ℓ^h and q_ℓ^ℓ to denote equilibrium qualities chosen at stage 1. A notation guide is provided in the Appendix.

We restrict attention to qualities q that satisfy $c(q) < q$. The rationale for this is as follows: the consumer with $\theta = 1$ is the one who is willing to pay the most for a given product with quality q. This consumer is willing to pay up to q for the product. If $c(q) \geq q$, a firm cannot obtain a positive market share unless it also makes a loss. Regardless of the quality it chooses at stage 1, it can always prevent a loss by charging consumers a price $p \geq q$, which ensures zero sales. Hence, we only consider qualities with $c(q) < q$. Since the cost function is convex, it is sufficient to impose this condition on the higher quality firm.

In this case, firm ℓ knows the type of each consumer, and hence can offer prices that depend on θ. It must be willing to offer a price as low as its marginal cost, $c_\ell = c(q_\ell)$, to each consumer, if necessary. Further, consistent with price discrimination, it will charge each consumer as high a price as it can. At stage 3, firm h (which does not have PP) will operate in a market segment $[\theta_h, 1]$, and firm ℓ in a market segment $[\theta_\ell, \theta_h]$. Consider first the location of the marginal consumer θ_h, who is indifferent between buying from either firm. This consumer must obtain the same utility from either product. If $p_\ell^\ell(\theta_h) > c_\ell$, then firm ℓ would lower its price for this consumer, to ensure that he strictly prefers to buy product ℓ. Hence, it must be that $p_\ell^\ell(\theta_h) = c_\ell$. Therefore, this consumer is defined by $\theta_h q_h - p_h^\ell = \theta_h q_\ell - c_\ell$, or $\theta_h = \frac{p_h^\ell - c_\ell}{q_h - q_\ell}$. For now, the qualities could be arbitrary, so define

$\theta_h = \min\{\frac{p_h^\ell - c_\ell}{q_h - q_\ell}, 1\}$. Similarly, θ_ℓ is defined as the consumer who is indifferent between buying product ℓ and not consuming at all. Again, it must be that $p_\ell^\ell(\theta_\ell) = c_\ell$, else firm ℓ could increase its profit by reducing its price for this consumer. Hence, $\theta_\ell q_\ell - c_\ell = 0$ or $\theta_\ell = \frac{c_\ell}{q_\ell} > 0$. Finally, the consumer who is exactly indifferent between not buying at all and buying from firm h is defined as $\hat{\theta} = \frac{p_h^\ell}{q_h}$. At arbitrary qualities, it may be that this leads to $\hat{\theta} > 1$, so define $\hat{\theta} = \min\{\frac{p_h^\ell}{q_h}, 1\}$.

In the pricing subgame, we further restrict attention to prices that satisfy $p \geq c(q)$ for a given quality q. No firm is willing to sell to consumers at a price less than its marginal cost (since this results in a loss). However, for some qualities, there exist equilibria in the subgame at which firm h may price below its cost, but makes zero sales. Firm h earns zero profits across these equilibria, so we consider the equilibrium in which it prices no lower than its cost, $c(q_h)$.

Now, consider stage 2. Suppose firms have chosen qualities q_h, q_ℓ at stage 1. We show that, at stage 2, the optimal price function of firm ℓ is non-monotonic in consumer type; that is, it charges some high valuation consumers less than it charges some low valuation consumers.

Proposition 1 *Suppose firms choose any qualities q_h and q_ℓ at stage 1, with associated costs $c_h = c(q_h)$ and $c_\ell = c(q_\ell)$, that satisfy (i) $q_\ell < q_h$ and (ii) $c_h < q_h$. In the equilibrium of the pricing subgame starting at stage 2, we have $0 < \theta_\ell < \theta_h \leq 1$. Further,*
(a) firm h sets a price $p_h^\ell = \max\{\frac{1}{2}(q_h - q_\ell + c_h + c_\ell), c_h\}$
(b) firm ℓ sets a price $p_\ell^\ell(\theta)$ that is non-monotonic in a consumer's valuation θ, such that some higher valuation consumers obtain lower prices than some lower valuation ones. Specifically,

$$p_\ell^\ell(\theta) = \begin{cases} \theta q_\ell & \text{if } \theta \in [\theta_\ell, \hat{\theta}] \\ p_h^\ell - \theta(q_h - q_\ell) & \text{if } \theta \in (\hat{\theta}, \theta_h] \\ c_\ell & \text{if } \theta \in [0, \hat{\theta}) \text{ or } \theta \in (\theta_h, 1]. \end{cases}$$

This situation is depicted in Figure 2. The intuition is that in the market segment $[0, \hat{\theta}]$, firm ℓ faces no competition from firm h. These consumers are not willing to buy product h at the offered quality and price. Hence, firm ℓ is able to extract their entire consumer surplus. However, consumers in the range $[\hat{\theta}, 1]$ obtain a positive utility from consuming product h as well. Hence, firm ℓ faces competition in this range, and must offer consumers at least as high a surplus as firm h, to induce them to buy product ℓ. Thus, the threat of latent competition from firm h provides these consumers with a positive surplus that is monotonically increasing in their valuations.

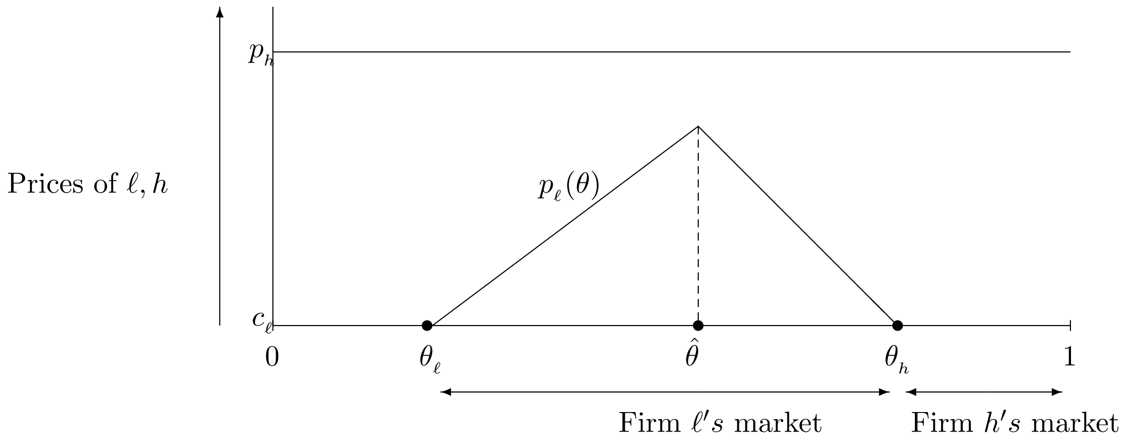

Figure 2: Prices of firms ℓ and h when firm ℓ alone has PP

Now, consider the choice of qualities at stage 1. In Lemma 1 in the appendix, we show that at the equilibrium qualities, given the prices exhibited in Proposition 1, the threshold consumer types satisfy $0 < \theta_\ell < \theta_h < 1$. Hence, we ignore the Kuhn-Tucker constraints implied by these conditions, and focus on the interior solution.

Suppose firm ℓ chooses q_ℓ, and firm h chooses q_h. Further, suppose both firms choose optimal prices (as given by Proposition 1), given the two qualities. Then, the profit functions of the two firms are:

$$\pi_\ell^\ell(q_h, q_\ell) = \int_{\theta_\ell}^{\hat{\theta}} (\theta q_\ell - c_\ell) d\theta + \int_{\hat{\theta}}^{\theta_h} (p_h^\ell - \theta(q_h - q_\ell) - c_\ell) d\theta = \frac{(p_h^\ell q_\ell - q_h c_\ell)^2}{2(q_h - q_\ell) q_h q_\ell} \tag{1}$$

$$\pi_h^\ell(q_h, q_\ell) = (p_h^\ell - c_h)\,(1 - \theta_h(p_h^\ell, q_h, c_\ell, q_\ell)) = \frac{(q_h - q_\ell - c_h + c_\ell)^2}{2(q_h - q_\ell)} \tag{2}$$

When firm ℓ adopts PP, the competitive response of firm h is to reduce its price. This is the "price competition effect." PP allows firm ℓ to set a price as low as marginal cost for a particular consumer, to induce him to buy product ℓ. This leads to an immediate increase in the market coverage of firm ℓ, both amongst low valuation consumers, and those who were previously buying product h. In response to this heightened competition from firm ℓ, firm h strategically reduces its price. This response of firm h, in turn, induces firm ℓ to lower its own quality, to reduce the competition with firm h and tap some more uncontested marginal consumers on the left. We demonstrate these effects in Lemma 2 of the Appendix, which also derives the reaction functions for the two firms.

Of course, in equilibrium, both firms change their qualities from the no-PP case. If the cost function is quadratic, we show that both firms reduce their qualities.

Proposition 2 *Suppose the cost function is quadratic; that is, $c(q) = Aq^2$. In equilibrium, when firm ℓ adopts PP, both firms reduce their qualities compared to the no-PP case. In particular, $q_h^\ell = \frac{0.388}{A}$ and $q_\ell^\ell = \frac{0.164}{A}$.*

Since analytic solutions are infeasible in the general case, we numerically solve for qualities using a cost function $c(q) = q^\alpha$, where $\alpha > 1$. In the numeric solution, we check that the constraints $0 < \theta_\ell < \theta_h < 1$ are satisfied for each α (so that each firm has a positive market share in all cases). The results are shown in Figure 3.[13] If the cost function is not too convex (in particular, $\alpha \leq 1.2$), firm h chooses a higher quality in equilibrium. Conversely, if the cost function is highly convex ($\alpha > 1.2$), it chooses a lower quality.

The intuition for this is as follows. PP allows firm ℓ to charge a price as low as its cost c_ℓ. This lets it penetrate an untapped market segment with lower valuations than it is currently serving, as well as make some headway into the market served by firm h. This is the "market coverage effect." Firm h has two competitive responses to this. First, as a result of the price competition effect, it reduces its price. Second, when costs are sufficiently convex, it reduces its quality. By moving towards the low quality firm, h increases the uncontested portion of its market. This further induces firm ℓ to reduce its own quality, to mitigate the more aggressive competition from firm h. However, if costs are almost linear (i.e., for low values of α), firm h increases its quality in equilibrium, and increases its price. Though this entails a lower market coverage, the nature of the cost function implies that the profit per unit sold is higher. This "cost of quality effect" is also critical in determining the new equilibrium qualities and prices.

3.2 PP Firm Offers High Quality

We use the superscript ^{h} to denote this case. In this case, firm h knows the type of each consumer, and hence is willing to price as low as $p_h^h(\theta) = c_h$ if need be.[14] The threshold consumer θ_h obtains

[13] A description of the technique used to solve for the equilibrium in the no-PP case is contained in the Technical Appendix.

[14] Again, for brevity, we suppress the dependence of the optimal price functions p_h^h, p_ℓ^h on q_h, q_ℓ.

the same utility from either product.[15] If $p^h_h(\theta_h) > c_\ell$, then firm ℓ would lower its price for this consumer, to ensure that he strictly prefers to buy product ℓ. Hence, it must be that $p^h_h(\theta_h) = c_\ell$. Therefore, this consumer is defined by $\theta_h = \frac{c_h - p^h_\ell}{q_h - q_\ell}$. Similarly, θ_ℓ is defined by the consumer indifferent between buying product ℓ and not consuming at all. Hence, $\theta_\ell = \frac{p^h_\ell}{q_\ell}$. In contrast to the low-PP case, when firm h adopts PP, it charges a price monotonic in consumer valuations.

Proposition 3 *Suppose firms choose any qualities $q_h > q_\ell$ at stage 1, with associated costs $c_h = c(q_h)$ and $c_\ell = c(q_\ell)$, that satisfy (i) $q_\ell < q_h$ and (ii) $c_h < q_h$. In the equilibrium of the pricing subgame starting at stage 2, we have $0 < \theta_\ell < \theta_h \leq 1$. Further,*

(a) the optimal price of firm ℓ is lower than c_h, the marginal cost of firm h. Specifically, firm ℓ sets

$$p^h_\ell = \begin{cases} \frac{1}{2}\left(c_\ell + \frac{q_\ell}{q_h} c_h\right) & \text{if} \quad \frac{1}{2}\left(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h}\right) \leq 1 \\ \min\{c_h - (q_h - q_\ell), \frac{c_\ell + q_\ell}{2}\} & \text{otherwise.} \end{cases}$$

(b) over the market it serves, firm h charges an optimal price monotonically increasing in consumer valuations. Specifically, firm h sets

$$p^h_h(\theta) = \begin{cases} c_h & \text{if} \quad \theta \in [0, \theta_h] \\ \frac{1}{2}(c_\ell + \frac{q_\ell}{q_h} c_h) + \theta(q_h - q_\ell) & \text{if} \quad \theta \in (\theta_h, 1]. \end{cases}$$

Firm h charges a monotonically increasing price because it faces no competitive threat from firm ℓ in the region $[\theta_h, 1]$. Interestingly, firm $\ell's$ price is lower than even the marginal cost of firm h, i.e., $p^h_\ell < c_h$. This pricing policy enables it to serve a sizable segment of the market, despite being a low quality firm and not having PP.

Now consider the choice of qualities of stage 1. Incorporating the optimal stage 2 prices leads to the following profit functions for the firms:

$$\pi^h_\ell(q_h, q_\ell) = (\theta_h - \theta_\ell)(p^h_\ell - c_\ell) = \frac{(c_h q_\ell - q_h c_\ell)^2}{2 q_h q_\ell (q_h - q_\ell)} \tag{3}$$

$$\pi^h_h(q_h, q_\ell) = \int_{\theta_h}^1 (p^h_h(\theta) - c_h) d\theta = \frac{(p^h_\ell + q_h - q_\ell - c_h)^2}{2(q_h - q_\ell)} \tag{4}$$

In this case, too, the price-competition effect works in the same direction: the firm that does not have PP (here, firm ℓ) reduces its price to compete more effectively. In response to this "price-competition effect," firm h raises its quality. We demonstrate this in Lemma 4 in the Appendix.

[15] As in the low-PP case, we solve for an interior solution, with $0 < \theta_\ell < \theta_h < 1$. We show in Lemma 3 in the Appendix that the equilibrium must satisfy this condition.

Of course, in equilibrium, both firms change their qualities from the no-PP case. We first demonstrate that, with quadratic costs, both firms raise their qualities.

Proposition 4 *Suppose the cost function is quadratic; that is, $c(q) = Aq^2$. In equilibrium, when firm h adopts PP, both firms raise their qualities, compared to the no-PP case. In particular, $q_h^h = \frac{0.444}{A}$ and $q_\ell^h = \frac{0.222}{A}$.*

However, this is not true for all degrees of convexity of the cost function. As in the low-PP case, if the cost function is not too convex (in particular, $\alpha \leq 1.55$), firm ℓ chooses a lower quality in equilibrium. Conversely, if the cost function is highly convex ($\alpha > 1.55$), it chooses a higher quality (see Figure 3).

Figure 3: Equilibrium qualities of firms h (top) and ℓ (bottom) with $c(q) = q^\alpha$.

Thus, for a wide range of α, both firms increase their qualities compared to the no-PP case. Here, the market coverage effect benefits firm h, which can penetrate into the market of firm ℓ. The competitive response of firm ℓ takes two dimensions: it reduces its price (the price-competition effect), and also increases its quality (to come closer to firm h). This, in turn, induces firm h to increase its own quality, to avoid head-to-head competition. As in the low-PP case, if costs are close to linear (i.e., for low values of α), the firm without PP moves further away in quality. That is, firm ℓ reduces its quality, with a corresponding reduction in price. This results in lower market coverage, but a higher profit per unit, due to the cost-of-quality effect. Therefore, starting from the no-PP case, if the cost function is convex enough, the non-PP firm seeks to reduce quality differentiation and come closer to the PP firm in the quality space. That is, if the PP firm has a low quality, in equilibrium both firms end up with lower qualities than previously. The converse outcome occurs if the PP firm chooses high quality; that is, both firms end up with higher qualities. Further, the firm without PP offers a lower price than the corresponding price in the no-PP case.

3.3 Both Firms have PP

We denote this case with the superscript ^{b}. Suppose the firms choose qualities q_h and q_ℓ at stage 1. Then, $\theta_h = \frac{c_h - c_\ell}{q_h - q_\ell}$, $\theta_\ell = \frac{c_\ell}{q_\ell}$, and $\hat{\theta} = \frac{c_h}{q_h}$. Recall that firm h sells to consumers in the region $[\theta_h, 1]$ and firm ℓ in the region $[\theta_\ell, \theta_h]$. As in the low-PP case, $\hat{\theta}$ represents the point beyond which firms compete for consumers, so that consumers in the region $[\theta_\ell, \hat{\theta}]$ are not willing to buy good h at any price c_h or higher.[16]

Consider stage 2 of this game, where the firms choose their price schedule, given qualities q_h, q_ℓ. Let $p^b_h(\theta)$ be the optimal price charged by firm h to the consumer of type θ. This is the price at which he is exactly indifferent between buying the low quality product at c_ℓ (the lowest price firm ℓ is willing to charge) and the high quality product h at $p^b_h(\theta)$. Therefore, $\theta q_h - p^b_h(\theta) = \theta q_\ell - c_\ell$, or $p^b_h(\theta) = c_\ell + \theta(q_h - q_\ell)$. As in the high-PP case, this price is strictly increasing in θ.

Consider the price charged by firm ℓ. The pricing function is similar to the one in the low-PP case, with the one difference that firm h is willing to price as low as c_h to any consumer. Hence, the optimal price function for firm ℓ is $p^b_\ell(\theta) = \theta q_\ell$ for $\theta \in [\theta_\ell, \hat{\theta}]$ and $p^b_\ell(\theta) = c_h - \theta_h(q_h - q_\ell)$ for $\theta \in [\hat{\theta}, \theta_h]$. As before, in the latter region, the price of firm ℓ is declining in a consumer's willingness to pay. Stepping back to stage 1, we incorporate the optimal stage 2 prices into the firms' profit functions to obtain

$$\pi^b_h(q_h, q_\ell) = \int_{\theta_h}^{1} (c_\ell + \theta(q_h - q_\ell) - c_h) d\theta = \frac{(q_h - q_\ell - c_h + c_\ell)^2}{2(q_h - q_\ell)} \quad (5)$$

$$\pi^b_\ell(q_h, q_\ell) = \int_{\theta_\ell}^{\hat{\theta}} (\theta q_\ell - c_\ell) d\theta + \int_{\hat{\theta}}^{\theta_h} (c_h - \theta(q_h - q_\ell) - c_\ell) d\theta = \frac{(c_h q_\ell - q_h c_\ell)^2}{2 q_h q_\ell (q_h - q_\ell)}. \quad (6)$$

Comparing equations (23) and (12), we observe that the profit function of firm h, when both firms have PP, is exactly the same as in the case when only firm ℓ has PP. Hence, $h's$ reaction function in the two cases is the same as well. Similarly, comparing equations (24) and (18), the profit function of firm ℓ, when both firms have PP, is exactly the same as in the case when only firm h has PP.

[16] We show in the proof of Proposition 5 that the equilibrium satisfies $0 < \theta_\ell < \theta_h < 1$.

Hence, $\ell's$ reaction function in the two cases is the same as well. The analysis of the previous two cases can now be directly used when both firms have PP.

We show that when both firms have PP, both firms choose a lower quality than when only firm h has PP. Notice that this result does not depend on additional restrictions on the cost function. In comparing the qualities to the case when only firm ℓ has PP, we find that both qualities are higher when the cost function is convex enough, but lower when the cost function is not too convex. Numerically, for the function $c(q) = q^{\alpha}$, when $\alpha > 1.3$, both qualities are higher than in the low-PP case. We analytically prove this latter result for the quadratic cost function.

Proposition 5 *Consider the case in which both firms have PP.*
(i) In equilibrium, both firms offer a lower quality than in the case where only firm h has PP. That is, $q_h^b < q_h^h$ *and* $q_\ell^b < q_\ell^h$.
(ii) Suppose the cost function is quadratic, so $c(q) = Aq^2$*. Then, in equilibrium, both firms offer a higher quality than in the case where only firm* ℓ *has PP. That is,* $q_h^b = \frac{0.4}{A} > q_h^\ell$ *and* $q_\ell^b = \frac{0.2}{A} > q_\ell^\ell$.

When costs are quadratic, compared to the case when neither firm had PP, in equilibrium the high quality firm lowers its quality and the low quality firm raises its quality. Thus, both firms actually come closer to each other in quality. A subtle consequence of both firms having PP is that the increase in pricing flexibility levels the playing field. Since both firms can now price at marginal cost for the threshold customer, the price competition effect leads to intensified competition for market share. Further, both have an incentive to compete more aggressively, so the relative product differentiation between the two firms decreases, which in turn increases the market coverage of each firm. But, for increasingly convex cost structures, the additional burden of the cost-of-quality effect leaves both firms worse off. The intensified price competition implies that consumers are better off.

4 Firm Profits and Consumer Surplus

In this section, we examine which firms are likely to adopt PP, and the resultant consumer welfare. Suppose neither firm has PP. We assume that after one or both firms adopt PP, the quality rankings of the firms do not change. That is, the low quality firm, when neither firm had PP, remains the low quality firm when one or both firms have PP. Quality levels are tantamount to brand equity, and significant changes to quality are likely to be costly. This is especially true when quality rankings are reversed. By contrast, local or marginal changes to quality can be made in a continuous fashion. Hence, we now consider firm ℓ acquiring PP, or firm h acquiring PP, or both. First, consider the quadratic cost case, with $c(q) = Aq^2$. In Table 1, we exhibit equilibria under different settings for this case.

	Neither firm has PP		Low-PP case		High-PP case		Both firms have PP	
Firm	h	ℓ	h	ℓ	h	ℓ	h	ℓ
Quality	$0.410/A$	$0.199/A$	$0.388/A$	$0.164/A$	$0.444/A$	$0.222/A$	$0.4/A$	$0.2/A$
Market Coverage	0.279	0.345	0.224	0.612	0.444	0.222	0.4	0.4
Average Price	$0.227/A$	$0.075/A$	$0.201/A$	$0.056/A$	$0.247/A$	$0.074/A$	$0.2/A$	$0.06/A$
Profit	$0.016/A$	$0.012/A$	$0.011/A$	$0.018/A$	$0.022/A$	$0.006/A$	$0.016/A$	$0.008/A$
Cons. Surplus	$0.047/A$		$0.045/A$		$0.049/A$		$0.12/A$	

Table 1: Summary of equilibrium results when $c(q) = Aq^2$

The consumer surplus shown above is defined in each of the four cases as

$$CS = \int_{\theta_\ell}^{\theta_h} (\theta q_\ell - p_\ell(\theta))d\theta + \int_{\theta_h}^{1} (\theta q_h - p_h(\theta))d\theta,$$

where $p_\ell(\theta)$ and $p_h(\theta)$ are, respectively, the prices paid in equilibrium by a consumer of type θ buying good ℓ and good h.[17] For example, in the case when the PP firm has low quality, we have $p_h(\theta) = p_h^\ell$, which is independent of θ, and $p_\ell(\theta) = p_\ell^\ell(\theta)$, as given in Proposition 1.

Notice that the results in the case when neither firm has PP correspond exactly to those of Moorthy (1991). The average price displayed in the table is the average of the prices paid by different consumers for the good. In the case when neither firm has PP, all consumers pay the same price. When both firms have PP, due to the intensified competition, the average price of both firms is the lowest across all cases. Further, the overall market coverage is at its highest. Hence, CS is maximized in this case.

As Table 1 shows, both firms have an incentive to adopt PP when costs are quadratic, regardless of whether the other firm also has PP. However, the firms are trapped in a prisoners' dilemma: if both firms adopt PP, their profits are each lower than in the no-PP case. Even though there is a market coverage effect which boosts market share, the deleterious impact of the price competition effect is that the average price of each sale is lower. Further, due to the cost-of-quality effect the

[17] Since this CS expression applies to each of the four cases, we omit the superscript on prices and qualities.

profits dip, leaving both firms worse off as a result. We summarize the quadratic case as follows. The proof follows directly by comparison across the columns in Table 1.

Proposition 6 *Suppose costs are quadratic, so $c(q) = Aq^2$. Then,*
(i) if one firm alone adopts PP, its profit increases, compared to the case when neither firm has PP. However, if both firms have PP, each firm has lower profits than in the no-PP case.
(ii) consumer surplus (CS) is highest when both firms have PP. Further, it is higher when only firm h has PP, compared to the cases when either firm ℓ alone or neither firm has PP.

Next, consider the case $c(q) = q^\alpha$, where $\alpha > 1$. As before, we numerically compare equilibria across the different cases. We find that consumer surplus remains highest in the case when both firms have PP. This points out the benefits of competition when there is perfect price discrimination, in contrast to the scenario where there is only a monopolist, which results in zero CS.[18]

Observation 1 *For all $\alpha > 1$, consumer surplus is higher when both firms have PP, as compared to any of the other cases.*

When both firms have PP, firm h charges a price $p_h^b(\theta) = c_\ell + \theta(q_h - q_\ell)$ to its consumers. Compare this to the price it charges when firm ℓ does not have PP: $p_h^h(\theta) = p_\ell^h + \theta(q_h - q_\ell)$. If firm ℓ now adopts PP, the greater competition leads to a lower price for consumers of firm h, and a corresponding increase in welfare. Consumer surplus (CS) falls, compared to the no personalized pricing case, if the PP firm has low quality, but rises if the PP firm has high quality. When firm ℓ has PP, it extends its market reach to a segment previously untapped, since it can price as low as marginal cost. However, a segment of firm $\ell's$ consumers receive no surplus, since they pay a price exactly equal to their willingness to pay. Conversely, if firm h has PP, it faces competition from firm ℓ throughout its market segment, and is forced to concede some surplus to consumers.

Observation 2 *For $\alpha \in [1, 4]$, it is a dominant strategy for firm ℓ to adopt PP. That is, regardless of whether firm h has PP, firm ℓ should adopt PP.*

Figure 4 demonstrates the increase in profit to firm ℓ when it adopts PP. The figure on the

[18] Bhaskar and To (2002) obtain similar results in their framework.

left illustrates the case of neither firm having PP, and the figure on the right, the case of firm h having PP. We emphasize that the cost of acquiring a resource to enable personalized pricing is not factored into this calculation. Such a cost can be incorporated as follows. The vertical gap between the dashed and solid line indicates the gain to firm ℓ from PP. It will adopt PP if and only if this gap exceeds the fixed cost of adopting PP.

Figure 4: Profit of Firm ℓ when Firm h does not (left) and does (right) have PP

Observation 3 *Regardless of whether firm ℓ has PP, firm h should adopt PP only if the cost function is not too convex. In particular, there exists an $\hat{\alpha} \in [2.5, 3]$ such that, if $\alpha > \hat{\alpha}$, and firm h adopts PP, its profits decrease.*

Figure 5 demonstrates this result. How can the profit of firm h decrease when it adopts PP? Recall that, when firm h adopts PP and firm ℓ does not have PP, firm ℓ responds by reducing its price. This induces firm h to increase its quality. Increasing quality is especially costly when the cost function is steep; indeed, it is costly enough in this case to outweigh the benefits of charging consumers according to their willingness to pay. A similar intuition holds when firm ℓ has PP. If firm h adopts PP in this situation, the new equilibrium sees both firms at a higher quality, which is correspondingly costly for firm h. Again, note that this result does not factor in a cost for implementing PP. With such a cost, firm h has even less incentive to adopt PP. Together, these results imply the following.

Figure 5: Profit of Firm h when Firm ℓ does not (left) and does (right) have PP

Observation 4 *Personalized Pricing by both firms need not lead to a prisoner's dilemma situation in which all firms are worse off. If both firms adopt PP, then, for a lower level of convexity of the cost function, both firms have higher profits compared to the case when neither firm has PP. However, the market shares increase for both firms, for all α.*

The result on profits can be seen by comparing the profits of the two firms in Figures 4 and 5, between the cases "Neither firm has PP" and "Both firms have PP." PP increases the pricing flexibility of both firms. An obvious consequence of this is more intense price competition. However, since the firms also respond by strategically changing qualities, the cost-of-quality effect plays a crucial role in determining the net change in profits. This ensures that personalized pricing does not invariably lead to a prisoner's dilemma.

5 Managerial Implications and Conclusion

The practice of personalized pricing is important in both offline and online channels. Our results show that an appropriate pricing strategy must take into account both consumers' willingness to pay and competition in a particular market segment. Ignoring either one can result in lower profits. In our model, if the low quality firm deploys PP, it is optimal for it to use a non-monotonic price schedule. Thus, some high valuation consumers are charged lower prices than some lower valuation consumers. An example of such pricing comes from the hardware industry for RISC/NT servers and high end workstations wherein, it is quite common to charge different prices to different customers, for the same quality and same quantity. Large customers are able to extract huge discounts, despite valuing the product very highly. On the other hand, smaller enterprises obtain lower discounts because no other firm competes for their demand (since the profit margins are much lower). In the latter segment, manufacturers often price according to the customers' willingness to pay and in the process, capture most of their surplus. Consequently, they are able to extract higher profits from some under-contested customers. Conversely, their margins are also squeezed by some large customers who play the firms against each other and win price concessions.

Our model also sheds light on the different product quality choices made by firms, given that one or both firms implement PP. When a low quality firm adopts PP, both firms reduce their quality levels in equilibrium. In the IT hardware industry, this is often done through stripping off some value-added customer service, such as next-day on-site repair versus same-day 8-hour repair, or a 99%-uptime guarantee versus 99.95%.[19] Conversely, if the high quality firm adopts PP, both firms should augment the quality levels of their offerings by providing additional product features or services. For instance, HP differentiates itself by providing higher quality, new generation web-based applications, as well as clustering and security management software embedded in the same hardware box.[20]

The critical issue for managers in vertically differentiated industries to keep in mind in adopting PP is the interplay between two countervailing effects: increased market coverage and intensified competition, given the convexity of the cost function. The increase in market coverage makes PP attractive. However, aggravated price competition hurts firms' profits. Further, optimal qualities of firms change. Therefore, the net effect of PP also depends on the nature of the cost function. For a lower level of convexity of the cost function, both firms have higher profits compared to the case when neither firm has PP and hence, are able to avoid the prisoner's dilemma situation. With moderately convex costs, $\alpha \in [1.5, 3]$, both low and high quality firms have an incentive to adopt PP, regardless of the other firm's actions. However, both firms are better off in the scenario where neither has PP, as compared to both having PP, resulting in a prisoner's dilemma situation. Conversely, if $\alpha > 3$, only the low quality firm will adopt PP, since the high quality firm reduces its own profit by adopting PP.

[19]The cost difference to the consumer between, say, a 99%-uptime guarantee and a 99.95% guarantee is substantial, so this difference is non-trivial.

[20]See http://www.hp.com/hpinfo/execteam/speeches/fiorina/oracleapps_02.html.

Finally, our model also demonstrates that consumers would benefit if higher quality firms adopt PP. In the event that all firms adopt PP, consumers would benefit the most. Thus we conclude that, in a competitive scenario, increasing knowledge about consumers' willingness to pay should eventually lead to an overall increase in consumer welfare.

One limitation of our paper is that we only consider a single product offering by each firm, whereas in practice firms often offer multiple products. In the extreme case, one can conceive of firms offering a personalized quality to each consumer, in addition to a personalized price. If both firms have the ability to customize product quality at no additional cost, Bertrand competition for each consumer is inevitable, and both firms will be held to zero profit. Hence, a proper study of customization must therefore incorporate additional features not considered in our model, such as horizontal differentiation (Ulph and Vulkan, 2002) or differences in the customization ability of firms (Dellaert and Syam, 2002).

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6 Appendix

Guide to Notation

Variable	Interpretation
Firm h, Firm ℓ	High quality and Low quality firm, respectively.
q_h, q_ℓ	Generic qualities of firm h and firm ℓ, respectively.
c_h, c_ℓ	Costs of firm h and firm ℓ, at respective qualities q_h and q_ℓ.
θ_h, θ_ℓ	Generic market coverage cut-offs of firm h and firm ℓ, respectively.
p_h^n, p_ℓ^n	Optimal price functions of firm h and firm ℓ, when neither firm has PP.
$p_h^\ell, p_\ell^\ell(\theta)$	Optimal price functions of firm h and firm ℓ, when PP firm has low quality.
$p_h^h(\theta), p_\ell^h$	Optimal price functions of firm h and firm ℓ, when PP firm has high quality.
$p_h^b(\theta), p_\ell^b(\theta)$	Optimal price functions of firm h and firm ℓ, when both firms have PP.
q_h^n, q_ℓ^n	Equilibrium qualities of firm h and firm ℓ, when neither firm has PP.
q_h^ℓ, q_ℓ^ℓ	Equilibrium qualities of firm h and firm ℓ, when PP firm has low quality.
q_h^h, q_ℓ^h	Equilibrium qualities of firm h and firm ℓ, when PP firm has high quality.
q_h^b, q_ℓ^b	Equilibrium qualities of firm h and firm ℓ, when both firms have PP.
π_h^n, π_ℓ^n	Profit functions of firms h and ℓ, when neither firm has PP.
$\pi_h^\ell, \pi_\ell^\ell$	Profit functions of firms h and ℓ, when PP firm has low quality.
π_h^h, π_ℓ^h	Profit functions of firms h and ℓ, when PP firm has high quality.
π_h^b, π_ℓ^b	Profit functions of firms h and ℓ, when both firms have PP firm.

Table 2: Guide to Notation

6.1 Neither Firm has PP

We briefly exhibit the closed form expressions for prices and profits when neither firm has PP. These expressions are used to numerically solve the model with the cost function $c(q) = q^\alpha$, with α varying.

Suppose the firms have chosen qualities q_h, q_ℓ at the first stage. Consider the optimal price functions at the second stage. Assume that the constraints $0 < \theta_\ell < \theta_h < 1$ are satisfied (in equilibrium, these constraints are satisfied numerically in the solutions we exhibit for all $\alpha > 1$). As Moorthy (1988) has shown, here θ_h is defined by the consumer exactly indifferent between products h and ℓ, and is given by $\theta_h q_h - p_h^n = \theta_h q_\ell - p_\ell^n$, so $\theta_h = \frac{p_h^n - p_\ell^n}{q_h - q_\ell}$. Similarly, θ_ℓ is defined by the consumer indifferent between product ℓ and not consuming at all, which yields $\theta_\ell = \frac{p_\ell^n}{q_\ell}$.

Now, the profit of firm h is $\pi_h = (1 - \frac{p_h^n - p_\ell^n}{q_h - q_\ell})(p_h^n - c_h)$. Differentiating with respect to p_h and setting the derivative equal to zero, we have $2p_h^n - p_\ell^n = q_h - q_\ell + c_h$. The second derivative is

$\frac{-2}{q_h - q_\ell} < 0$, so we have a maximum.

Similarly, the profit of firm ℓ is $\pi_\ell = (\frac{p_h^n - p_\ell^n}{q_h - q_\ell} - \frac{p_\ell^n}{q_\ell})\ (p_\ell^n - c_\ell)$. Differentiating with respect to p_ℓ and setting the derivative equal to zero, we have $-p_h^n q_\ell + 2p_\ell^n q_h = c_\ell q_h$. The second order condition for maximization is satisfied as $\frac{-2q_h}{q_h - q_\ell} < 0$. The optimal price functions at stage 2, (p_h^n, p_ℓ^n), are found by simultaneously solving the two first-order conditions, which yields

$$p_h^n = \frac{q_h(2(q_h - q_\ell) + c_\ell + 2c_h)}{4q_h - q_\ell}. \tag{7}$$

$$p_\ell^n = \frac{q_h(q_\ell + 2c_\ell) + q_\ell(c_h - q_\ell)}{4q_h - q_\ell}. \tag{8}$$

From these prices the profit functions can be derived and are given by

$$\pi_h^n(q_h, q_\ell) = \frac{c_h(-2q_h + q_\ell) + q_h(2q_h - 2q_\ell + c_\ell)^2}{(q_h - q_\ell)(4q_h - q_\ell)^2}. \tag{9}$$

$$\pi_\ell^n(q_h, q_\ell) = \frac{q_h\left(q_\ell(q_h - q_\ell + c_h) + (c_\ell(-2q_h + q_\ell)^2)\right)}{q_\ell(q_h - q_\ell)(4q_h - q_\ell)^2}. \tag{10}$$

Now, the equilibrium qualities at stage 1 are solved for in the usual way; differentiating each profit function with respect to the quality of that firm yields a reaction function, and the equilibrium qualities must simultaneously satisfy both reaction functions.

6.2 PP Firm Offers Low Quality

Proof of Proposition 1

Suppose firms have chosen qualities q_h, q_ℓ at stage 1 that satisfy $q_\ell < q_h$ and $c_h < q_h$, where $c_h = c(q_h)$ and $c_\ell = c(q_\ell)$. Consider the pricing subgame that starts at stage 2.

Firm ℓ will set its price for each consumer, $p_\ell^\ell(\theta)$, as high as possible to satisfy two restrictions: (i) the consumer buys product ℓ instead of product h, and (ii) the consumer buys product ℓ, rather than not consume at all. These conditions imply

$$\theta q_\ell - p_\ell^\ell(\theta) \geq \theta q_h - p_h^\ell \quad \text{or} \quad p_\ell^\ell(\theta) \leq p_h^\ell - \theta(q_h - q_\ell)$$
$$\theta q_\ell - p_\ell^\ell(\theta) \geq 0, \quad \text{or} \quad p_\ell^\ell(\theta) \leq \theta q_\ell.$$

Further, firm ℓ must set $p_\ell^\ell(\theta) \geq c_\ell = c_\ell(q_\ell)$ for each consumer, or else it makes a loss on that consumer, and would prefer not to sell to him. Hence, we have $p_\ell^\ell(\theta) \geq c_\ell$, and $p_\ell^\ell(\theta) \leq \min\{\theta q_\ell,\ p_h^\ell - \theta(q_h - q_\ell)\}$. In the second inequality, $p_\ell^\ell(\theta) \leq \theta q_\ell$ follows from the fact that the consumer's reservation utility is zero. Further, $p_\ell^\ell(\theta) < p_h^\ell - \theta(q_h - q_\ell)$ can be interpreted as an incentive compatibility constraint for the consumer: if this is violated, he buys product h instead.

By definition, $\theta_h = \min\{\frac{p_h^\ell - c_\ell}{q_h - q_\ell}\ ,\ 1\}$. Hence, the profit of firm h, if it charges price p_h^ℓ, is $\pi_h = (1-\theta_h)\ (p_h^\ell - c_h) = (1 - \frac{p_h^\ell - c_\ell}{q_h - q_\ell})\ (p_h^\ell - c_h)$. First, suppose that $\theta_h < 1$. The first-order condition for profit-maximization, $\frac{\partial \pi_h}{\partial p_h^\ell} = 0$, directly yields its optimal price $p_h^\ell = \frac{1}{2}\ (q_h - q_\ell + c_h + c_\ell)$. Given this price for firm h, we have $\theta_h = \min\{\frac{p_h^\ell - c_\ell}{q_h - q_\ell}\ ,\ 1\} = \min\{\frac{q_h - q_\ell + c_h - c_\ell}{2(q_h - q_\ell)}\ ,\ 1\}$. Clearly, $\theta_h > 0$. Further, $\theta_h < 1$ if and only if $c_h - c_\ell < q_h - q_\ell$. In this case, all conditions for profit-maximization by firm h are satisfied (the second derivative of the profit function is easily checked), and we can ignore the boundary conditions on θ_h.

Suppose, instead, that $c_h - c_\ell \geq q_h - q_\ell$. Then, if firm h continues to price at $p_h^\ell = \frac{1}{2}\ (q_h - q_\ell + c_h + c_\ell)$, we have $p_h^\ell \leq c_h$ and $\theta_h = 1$. Since $\theta_h = 1$, firm h has no sales, so its profits are zero. As long as $\theta_h = 1$, any price charged by firm h remains a best response. From this set of prices, we set $p_h^\ell = c_h$, the marginal cost of firm h. Note that at this price, the condition $\theta_h = 1$ is satisfied, since $c_h - c_\ell \geq q_h - q_\ell$.

Now, consider $\hat{\theta} = \frac{p_h^\ell}{q_h}$. Given the price postulated for firm h, and that $c_h < q_h$ (by assumption), it is immediate that $\hat{\theta} < 1$. Further, $p_h^\ell - \theta(q_h - q_\ell) > \theta q_\ell$ for $\theta > \hat{\theta}$, and $p_h^\ell - \theta(q_h - q_\ell) < \theta q_\ell$

for $\theta < \hat{\theta}$. The pricing function for firm ℓ now follows for consumers in the range $\theta \in [\theta_\ell, \theta_h]$. Consumers outside this range are unwilling to purchase product ℓ even at the price c_ℓ; to these consumers, firm ℓ charges a price equal to c_ℓ.

We have already shown that $\theta_h \leq 1$. Now $\theta_\ell = \frac{c_\ell}{q_\ell}$, and since $c_h < q_h$, it must be that $c_\ell < q_\ell$ (by convexity of the cost function $c(\cdot)$). Hence, $0 < \theta_\ell < 1$. Finally, we need to show that $\theta_\ell < \theta_h$. If $\theta_h = 1$, this is immediate. Hence, suppose that $\theta_h < 1$; as shown earlier, this can happen only if $c_h - c_\ell < q_h - q_\ell$. In this case, $\theta_h = \frac{q_h - q_\ell + c_h - c_\ell}{2(q_h - q_\ell)}$.

Now, convexity of the cost function implies that $\frac{c_h - c_\ell}{q_h - q_\ell} > \frac{c_\ell}{q_\ell}$ when $q_h > q_\ell$. Hence,

$$\begin{aligned} 1 + \frac{c_h - c_\ell}{q_h - q_\ell} &> \frac{c_\ell}{q_\ell} + \frac{c_\ell}{q_\ell} \\ \frac{q_h - q_\ell + c_h - c_\ell}{q_h - q_\ell} &> 2\,\frac{c_\ell}{q_\ell} \\ \theta_h = \frac{q_h - q_\ell + c_h - c_\ell}{2(q_h - q_\ell)} &> \frac{c_\ell}{q_\ell} = \theta_\ell. \end{aligned}$$

■

Lemma 1 *In the equilibrium of the whole game,* $0 < \theta_\ell < \theta_h < 1$.

Proof: Suppose firms have chosen their equilibrium quality levels at period 1. Then, it cannot be that $q_\ell = q_h = 0$ (since at least one firm would choose a positive quality and earn a positive profit). Without loss of generality, assume $q_h > 0$ and $c_h < q_h$. Then there exists a $q_\ell \in (0, q_h)$ such that firm ℓ earns a positive profit. From Proposition 1, it follows that $0 < \theta_\ell < \theta_h \leq 1$.

Now, we show that $\theta_h < 1$ in the equilibrium of the whole game. The argument runs as follows. We show that, in equilibrium, $c'(q_\ell) < 1$. This will imply that there exists a q_h such that firm h makes a positive profit, which further implies that $\theta_h < 1$ (since firm h earns a zero profit if $\theta_h = 1$).

Let c'_ℓ denote $c'(q_\ell)$. Suppose $c'_\ell \geq 1$. Also, suppose that $\theta_h = 1$. Then, from Proposition 1, it must be that $c_h - c_\ell \geq q_h - q_\ell$. Suppose first that $\frac{c_h - c_\ell}{q_h - q_\ell} = 1$. Then, given its pricing scheme, the revenue of firm ℓ is obtained as the area of the triangle in Figure 2. As shown, the price of firm ℓ is linear in q_ℓ. Hence, its average price $\hat{p} = \int_{\theta_\ell}^{\theta_h} p_\ell^\ell(\theta) d\theta$ is also linear in ℓ. Further, since only the consumer with $\theta = 1$ is willing to pay q_ℓ, we can write this average price as $\hat{p} = \beta q_\ell$, where $\beta < 1$.

Now, the profit of firm ℓ can be written as $\pi_\ell = (\theta_h - \theta_\ell)(\beta q_\ell - c(q_\ell))$. Consider a small change in q_ℓ. The second term $(\beta q_\ell - c(q_\ell))$ denotes the profit per unit sold; this increases when q_ℓ falls, since its derivative is $\beta - c_\ell$, where $\beta < 1 \leq c'_\ell$.

Consider the first term, which denotes the market coverage. $\theta_\ell = \frac{c_\ell}{q_\ell}$, which falls when q_ℓ falls. Also, since $\theta_h = \frac{c_h - c_\ell}{q_h - q_\ell}$, we have $\frac{\partial \theta_h}{\partial q_\ell} = \frac{c_h - c_\ell - c'_\ell(q_h - q_\ell)}{(q_h - q_\ell)^2} = \frac{1 - c'_\ell}{q_h - q_\ell}$ since $c_h - c_\ell = q_h - q_\ell$. Hence $\frac{\partial \theta_h}{\partial q_\ell} \leq 0$ when $c'_\ell \geq 1$. That is, if q_ℓ falls slightly, firm ℓ still sells all the way up to $\theta = 1$. So, if q_ℓ falls, both terms in π_ℓ increase. Hence, it cannot be that $\theta_h = 1$ and $c'(q_\ell) \geq 1$.

Finally, if $\frac{c_h - c_\ell}{q_h - q_\ell} > 1$, it follows even more strongly that $c'(q_\ell) < 1$, since the market coverage of firm ℓ improves, and its profit per unit also improves.

Hence, it must be that $c'_\ell < 1$. Now, suppose firm h chooses a quality $q_\ell + \epsilon$. For $\epsilon > 0$ small enough, $c_h - c_\ell \approx c'_\ell \epsilon < q_h - q_\ell = \epsilon$. Hence, there exists an $\epsilon > 0$ such that $c_h - c_\ell < q_h - q_\ell$, which implies $p^\ell_h > c_h$. Substituting the value of $p^\ell_h = \frac{q_h - q_\ell + c_h + c_\ell}{2}$, we have $p^\ell_h = \frac{1}{2}(1 + \frac{c_h - c_\ell}{q_h - q_\ell})$, which implies $\theta_h < 1$. Hence, there exists a q_h such that firm h makes a positive profit. Hence, it will never choose a q_h such that $\theta_h = 1$.

Therefore, in the equilibrium of the whole game, $0 < \theta_\ell < \theta_h < 1$. ■

6.2.1 Derivation of Reaction Functions

We show (as an aside; this is used in Proof of Proposition 5) that the reaction functions are upward sloping.

Lemma 2 *(i) The reaction functions of both firms are upward-sloping in the range of quality such that $c'(q_\ell) < 1$.*
(ii) Suppose both firms offer the no-PP qualities, q^n_h, q^n_ℓ, and firm ℓ now adopts PP. Then, compared to the no-PP case: (a) if firms remain at their original qualities, q^n_h, q^n_ℓ, the optimal defensive strategy of firm h is to charge a lower price (b) Compared to the no-PP case, if firm h remains at its original quality, q^n_h, then the optimal strategy of firm ℓ is to lower its quality.

Proof of Lemma 2

(i) We first derive the reaction functions of the two firms. As shown in equations (1) and (2) of the text, the profit functions of the firms in this case are

$$\pi^\ell_\ell(q_h, q_\ell) = \int_{\theta_\ell}^{\hat{\theta}} (\theta q_\ell - c_\ell) d\theta + \int_{\hat{\theta}}^{\theta_h} (p^\ell_h - \theta(q_h - q_\ell) - c_\ell) d\theta = \frac{(p^\ell_h q_\ell - q_h c_\ell)^2}{2(q_h - q_\ell) q_h q_\ell} \quad (11)$$

$$\pi_h^\ell(q_h, q_\ell) \quad = \quad (p_h^\ell - c_h)\,(1 - \theta_h(p_h^\ell, q_h, c_\ell, q_\ell)) \quad = \quad \frac{(q_h - q_\ell - c_h + c_\ell)^2}{2(q_h - q_\ell)} \tag{12}$$

We focus on an interior equilibrium with $0 < \theta_\ell < \theta_h < 1$. We restrict attention to qualities such that $c'(q_\ell) < 1$ because we have shown that the equilibrium qualities satisfy this restriction. The reaction functions are valid for all qualities in this range: as shown in the proof of Lemma 1, if $c'(q_\ell) < 1$, there exist qualities q_h at which firm h can earn a positive profit (and therefore $\theta_h < 1$).

The first-order condition of firm ℓ is $\frac{\partial \pi_\ell^\ell}{\partial q_\ell} = 0$. Recognizing that the optimal price of firm h, p_h^ℓ, is a function of q_ℓ (from Proposition 1), we have

$$\frac{(p_h^\ell q_\ell - c_\ell q_h)(c_\ell q_h(q_h - \ 2q_\ell) + q_\ell(p_h^\ell q_h - (q_h - q_\ell)(2c_\ell' q_h + q_\ell - c_\ell' q_\ell)))}{2(q_h - q_\ell)^2\, {q_\ell}^2\, q_h} \quad = \quad 0. \tag{13}$$

Since $p_h^\ell > c_h$, and $q_h > q_\ell \Longrightarrow \frac{c_h}{q_h} > \frac{c_\ell}{q_\ell}$ (since $c(\cdot)$ is convex), the optimal quality q_ℓ is given by the solution to

$$c_\ell q_h(q_h - \ 2q_\ell) + q_\ell(p_h^\ell q_h - (q_h - q_\ell)(2c_\ell' q_h + q_\ell - c_\ell' q_\ell)) \quad = \quad 0. \tag{14}$$

The solution to this equation yields the reaction function of firm ℓ, denoted $r_\ell^\ell(q_h)$.

The corresponding first-order condition for firm h is $\frac{\partial \pi_h^\ell}{\partial q_h} = 0$, or

$$\frac{(q_h - q_\ell - c_h + c_\ell)(q_h - q_\ell + c_h - c_\ell - 2(q_h - q_\ell)c_h')}{4(q_h - q_\ell)^2\, {q_\ell}^2} \quad = \quad 0. \tag{15}$$

Since $q_h > q_\ell$, it cannot be that $(q_h - q_\ell - c_h + c_\ell) = 0$. Hence, the optimal quality of firm h, q_h is given by the solution to

$$q_h - q_\ell + c_h - c_\ell - 2(q_h - q_\ell)c_h' \quad = \quad 0. \tag{16}$$

Let $r_h^\ell(q_\ell)$, the solution to this equation, denote the reaction function for firm h.

Now, let ψ_ℓ denote the left-hand side of equation (13), and ψ_h the left-hand side of equation (15). We have

$$\begin{aligned}\frac{\partial \psi_\ell}{\partial q_h} \quad = \quad & \frac{(c_\ell' - 1)(c_h + c_\ell - c_h' q_h)}{4{q_h}^2} + \frac{(c_h - c_\ell)(c_h' + c_\ell')}{4(q_h - q_\ell)} - \frac{(c_h - c_\ell)^2}{4(q_h - q_\ell)^3} \\ & - \frac{(c_h' c_\ell')}{(q_h - q_\ell)} + \frac{(q_\ell - c_\ell' q_\ell)}{4{q_h}^2}.\end{aligned}$$

Now, the first and the fifth terms sum to $\frac{(c_\ell' - 1)(c_h + c_\ell - c_h' q_h - q_\ell)}{{q_h}^2} > 0$, since $c_\ell' < 1$ (as argued in Step 2 above) $c_h' > \frac{c_h}{q_h}$ (by convexity of $c(\cdot)$, and $c_\ell < q_\ell$ (else firm ℓ has zero sales). Adding the second, third and fourth terms, and simplifying, we have

$$\frac{((c_h - c_\ell) - c'_\ell(q_h - q_\ell))(-(c_h - c_\ell) + c'_h(q_h - q_\ell))}{4(q_h - q_\ell)^3}.$$

Now, convexity of $c(\cdot)$ implies that $c_h > c_\ell + c'_\ell(q_h - q_\ell)$, and $c_\ell > c_h - c'_h(q_h - q_\ell)$. Hence, this expression is also positive. Therefore, $\frac{\partial \psi_\ell}{\partial q_h} > 0$. Now, suppose firm ℓ has chosen an optimal quality, and firm h now increases q_h. Then, at the original optimal quality of firm ℓ, $\psi_\ell > 0$. Hence, to reach $\psi_\ell = 0$, firm ℓ must increase its quality. Hence, its reaction function, r^ℓ_ℓ, is upward-sloping.

Next, consider $\psi_h(q_h, q_\ell)$. We have

$$\frac{\partial \psi_h}{\partial q_\ell} = \frac{(c_h - c_\ell - c'_\ell(q_h - q_\ell))(c'_h(q_h - q_\ell) + c_\ell - c_h)}{2(q_h - q_\ell)^3}.$$

Convexity of $c(\cdot)$ directly implies $c_h > c_\ell + c'_\ell(q_h - q_\ell)$ and $c'_h > \frac{c_h - c_\ell}{q_h - q_\ell}$. Hence, $\frac{\partial \psi_h}{\partial q_\ell} > 0$. Now, suppose firm h has chosen an optimal quality, and firm ℓ now increases q_ℓ. Then, at the original optimal quality of firm h, $\psi_h(q_h, q_\ell) > 0$. Hence, to reach $\psi_h = 0$, firm h must increase its quality. Hence, its reaction function, r^ℓ_h, is upward-sloping.

(ii) (a) Suppose both firms choose the same quality levels as in the no-PP case; that is, $q_h = q^n_h$ and $q_\ell = q^n_\ell$. Given p^n_h from equation (7) and $p^\ell_h = \frac{1}{2}\,(q_h - q_\ell + c_h + c_\ell)$ from Proposition 1, we have $p^\ell_h < p^n_h$ if and only if $\frac{1}{2}(q_h - q_\ell + c_h + c_\ell) < \frac{2q_h c_h + q_h c_\ell - 2q_h q_\ell + 2q_h{}^2}{(4q_h - q_\ell)}$. Cross-multiplying and simplifying, this condition holds if and only if $(q_h c_\ell - q_\ell c_h) + (c_\ell - q_\ell)(q_h - q_\ell) < 0$. Now, $q_h > q_\ell$, and $c_\ell \leq q_\ell$ (or else firm ℓ sells zero units). Further, since $c(\cdot)$ is convex, $\frac{c_h}{q_h} > \frac{c_\ell}{q_\ell}$, so $q_h c_\ell - q_\ell c_h < 0$. Hence, $p^\ell_h < p^n_h$.

(ii) (b) Consider ψ_ℓ, the left-hand side of equation (13). Then, $\frac{\partial \psi_\ell}{\partial p^\ell_h} = \frac{1 - c'_\ell}{2q_h} + \frac{c_h - c_\ell - c'_\ell(q_h - q_\ell)}{2(q_h - q_\ell)^2}$. The argument in Step 2 of Lemma 1 is easily extended to show $c'_\ell < 1$ in equilibrium. Hence, the first term above is positive. Since $c(\cdot)$ is convex, $c_h > c_\ell + c'_\ell(q_h - q_\ell)$, so the second term is also positive. Thus $\frac{\partial \psi_\ell}{\partial p^\ell_h} > 0$.

Now, suppose the firms choose their no-PP quality levels, q^n_h, q^n_ℓ. If firm h keeps its quality level at q^n_h and reduces its price from p^n_h to p^ℓ_h, we will have $\psi_\ell > 0$. Hence, firm ℓ must reduce its quality to ensure that $\psi_\ell = 0$. ∎

6.2.2 Proof of Proposition 2

Moorthy (1991) provides the equilibrium for the no-PP case with quadratic costs; the qualities here are $q^n_h = \frac{0.4098}{A}$ and $q^n_\ell = \frac{0.199}{A}$.

Consider the case in which only firm ℓ has PP. In Lemma 2, we demonstrate the reaction functions for a general cost function. Substituting $c(q) = Aq^2$, the profit-maximization condition for the high quality firm, equation (15), reduces to $(q_h - q_\ell)(1 + Aq_\ell - 3Aq_h) = 0$, so $q_h = \frac{q_\ell}{3} + \frac{1}{3A}$ (since $q_h = q_\ell$ is obviously not profit-maximizing). The corresponding condition for firm ℓ, equation (13), is $(1 + A(q_h - q_\ell))(q_h - 2q_\ell + A(q_h - 4q_\ell)(q_h - q_\ell)) = 0$. Again, the reaction function is given by setting the second term to zero (since the first is always positive when $q_h > q_\ell$). Simultaneously solving the two reaction functions, we obtain $q_h^\ell = \frac{0.388}{A}$ and $q_\ell^\ell = \frac{0.164}{A}$, both lower than the corresponding qualities in the no-PP case. It is immediate to check that the constraints $0 < \theta_\ell < \theta_h < 1$ are satisfied.

Finally, we show that neither firm wants to leapfrog; that is, firm h does not want to choose a quality lower than that of firm ℓ, and vice versa.

For convenience, call the firms 1 and 2 at this stage. Suppose firm 1 chooses $q_1 = \frac{0.388}{A}$, and firm 2 chooses $q_2 = \frac{0.164}{A}$ in equilibrium. Note that firm 2 has PP, and firm 1 does not. In equilibrium, firm 1 is the high quality firm. However, we wish to rule out the following deviations:

(i) firm 1 chooses some $q_1 < \frac{0.164}{A}$, and becomes a lower quality firm.

(ii) firm 2 chooses some $q_2 > \frac{0.388}{A}$, and becomes a higher quality firm.

Consider (i) first. Suppose firm 2 chooses $q_2 = \frac{0.164}{A}$, and firm 1 does leapfrog, and chooses $q_1 < \frac{0.164}{A}$. In particular, let the optimal quality of firm 1, given that it has lower quality than firm 2, be denoted $q_1 = \frac{0.164-\epsilon}{A}$, where $\epsilon \geq 0$.

Note that firm 1 does not have PP, and firm 2 does. In the conjectured deviation, firm 1 has lower quality. Hence, its profit function is given by equation (3) of the text (that is, its profit is the same as that of the low quality firm, when only the high quality firm has PP). Replacing the cost functions, $c_h = Aq_h{}^2$ and $c_\ell = Aq_\ell{}^2$, we get

$$\pi_1(q_h, q_\ell) = \frac{A^2(q_h - q_\ell)q_h q_\ell}{2}.$$

Note that, in the conjectured deviation, we have $q_h = q_2 = \frac{0.164}{A}$, and $q_\ell = q_1 = \frac{0.164-\epsilon}{A}$. Substituting these into the profit function, we have

$$\pi_1(\epsilon) = \frac{(0.0138 - 0.082\epsilon)\epsilon}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.082$. Then, the profit of firm 1 is $\frac{0.0005}{A}$. The equilibrium profit of firm 1 is easily computed to be $\frac{0.011}{A}$ (this is reported in Table 1 on page 14 of the paper). Hence, there is no incentive for the high quality firm to deviate and become the lower quality firm.

Now, consider case (ii). Suppose firm 1 chooses quality $q_1 = \frac{0.388}{A}$. Suppose that firm 2 deviates to some quality higher than this, and becomes the higher quality firm. Let the new (after deviation) quality level for the firm 2 be $q_2 = \frac{0.388+\epsilon}{A}$. Since firm 2 has PP, and is now (after the deviation) the higher quality firm, its profit function for firm 2 is given by equation (4) in the text of the paper. Replacing the optimal price p_ℓ^h of the other firm, and cost functions, $c_h = A{q_h}^2$ and $c_\ell = A{q_\ell}^2$ and then simplifying, we get

$$\begin{aligned}\pi_2(q_h, q_\ell) &= \frac{(A{q_h}^2(q_\ell - 2q_h) + q_h(2q_h - 2q_\ell + A{q_\ell}^2))^2}{8{q_h}^2(q_h - q_\ell)} \\ &= \frac{(q_h - q_\ell)(2 - A(2q_h + q_\ell)^2}{8}.\end{aligned}$$

Substituting in the qualities $q_h = q_2 = \frac{0.388+\epsilon}{A}$ and $q_\ell = q_1 = \frac{0.388}{A}$, we have

$$\pi_2(\epsilon) = \frac{0.125(0.836 - 2\epsilon)^2\epsilon}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.139$. Then, the profit of firm 2 is $\frac{0.005}{A}$. This is less than its profits at the Nash equilibrium, given by $\frac{0.018}{A}$ (see Table 1). Hence, there is no incentive for the low quality firm to deviate and become the higher quality firm. ■

6.3 PP Firm Offers High Quality

Proof of Proposition 3

Suppose firm ℓ chooses a price p_ℓ^h. Then, consumers with $\theta \geq \frac{p_\ell^h}{q_\ell}$ receive a non-negative utility from product ℓ. For θ in this range, a consumer with type θ receives utility $\theta q_\ell - p_h^\ell$. Given the price of firm ℓ, the best response of firm h for each θ is the maximum price p_h^h at which the consumer obtains a weakly higher utility from product h than from (i) buying product ℓ and (ii) not buying the product at all. The first requirement yields $\theta q_h - p_h^h(\theta) \geq \theta q_\ell - p_\ell^h$, or $p_h^h(\theta) \leq p_\ell^h + \theta(q_h - q_\ell)$. The second one yields $\theta q_h - p_h^h(\theta) \geq 0$, or $p_h^h(\theta) \leq \theta q_h$. Further, we require that $p_h^h(\theta) \geq c_h$ for all θ (so that firm h never exposes itself to a loss). Putting all these together, and recalling that $\theta_\ell = \frac{p_\ell^h}{q_\ell}$, for any p_ℓ^h, the best response of firm h is: for each $\theta \in [0,1]$,

$$p_h^h(\theta) \quad = \quad \begin{cases} \max\{p_\ell^h + \theta(q_h - q_\ell),\ c_h\} & \text{if } \theta \geq \theta_\ell \\ \max\{\theta q_h,\ c_h\} & \text{otherwise.} \end{cases} \tag{17}$$

Now, suppose firm ℓ charges a price p_ℓ^h. Given the best response of firm h, what part of the market can it cover? First, note that if $p_\ell^h > q_\ell$, firm ℓ makes no sales; the consumer with the highest valuation for the good (with $\theta = 1$) is unwilling to purchase good ℓ. Hence, we restrict attention to prices for firm ℓ that satisfy $p_\ell^h < q_\ell$. This immediately implies that $\theta_\ell < 1$.

Clearly, firm ℓ has no sales in the region $[0, \theta_\ell)$, where $\theta_\ell = \frac{p_\ell^h}{q_h}$. Suppose $\theta_\ell \geq \frac{c_h}{q_h}$. This implies that $p_\ell^h \geq \frac{c_h}{q_h}\, q_\ell$. If a consumer with $\theta \in [\theta_\ell, 1]$ buys product ℓ, her consumer surplus is $\theta q_\ell - p_\ell^h \leq q_\ell(\theta - \frac{c_h}{q_h})$. Firm h's best response is to choose $p_h^h(\theta)$ to match this surplus. Then, firm h will win over all consumers in the range $[\theta_\ell, 1]$, leaving firm ℓ with no sales.

Hence, in the equilibrium of the pricing subgame, it must be that $\theta_\ell < \frac{c_h}{q_h}$. From equation (17), this implies that, for consumers in the range $[0, \theta_\ell]$, the best response of firm h is c_h. Consumers in the range $[\theta_\ell, 1]$ are charged $p_h^h(\theta) = \max\{p_\ell^h + \theta(q_h - q_\ell)\ \ c_h\}$. For now, suppose $p_h^h(\theta) = p_\ell^h + \theta(q_h - q_\ell)$. We show later that, given the optimal price of firm ℓ, such a pricing policy satisfies $p_h^h(\theta) \geq c_h$ for all $\theta \geq \theta_\ell$.

Then, $\theta_h = \min\{\frac{c_h - p_h^\ell}{q_h - q_\ell}, 1\}$. There are therefore two cases to consider.

Case 1: Suppose $\frac{c_h - p_h^\ell}{q_h - q_\ell} \leq 1$, so that $\theta_h = \frac{c_h - p_h^\ell}{q_h - q_\ell}$. Then, the profit of firm from the first-order condition ℓ can be written as $\pi_\ell = (p_\ell^h - c_\ell)(\theta_h - \theta_\ell)$. The first-order condition for profit-maximization, $\frac{\partial \pi_\ell}{\partial p_\ell} = 0$, directly yields firm ℓ's optimal price function, $p_\ell^h = \frac{1}{2}\ (c_\ell + \frac{c_h}{q_h}\, q_\ell)$. Since $q_\ell < q_h$ and

$c_\ell < c_h$, it follows that $p_\ell^h < c_h$, so that the price of firm ℓ is lower than the marginal cost of firm h. Note also that from the convexity of $c(\cdot)$, it follows that $\frac{c_h}{q_h} > \frac{c_\ell}{q_\ell}$ and it follows immediately that $p_\ell^h > c_\ell$.

Now, the optimal price of firm h for consumers in the range $[\theta_\ell, 1]$ is $p_h^h(\theta) = \max\{p_\ell^h + \theta(q_h - q_\ell)\ ,\ c_h\}$. Given p_ℓ^h, we have $\theta_h = \min\{\frac{c_h - p_\ell^h}{q_h - q_\ell}, 1\} = \min\{\frac{1}{2}(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h}), 1\}$. Clearly, $\theta_h > 0$. If, in addition $\frac{1}{2}(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h}) \leq 1$, we have $\theta_h \leq 1$. In this case, the best response of firm h is to price at $p_h^h(\theta) = c_h$ for $\theta \in [0, \theta_h]$ and at $p_h^h(\theta) = p_\ell^h + \theta(q_h - q_\ell)$ for $\theta \in (\theta_h, 1]$.

Case 2: Suppose $\frac{c_h - p_\ell^h}{q_h - q_\ell} > 1$, so that $\theta_h = 1$. From Case 1 above, this can happen in the equilibrium of the pricing subgame only if $\frac{1}{2}(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h}) > 1$.

Then, the profit function of firm ℓ can be written as $\pi_\ell^h = (p - c_\ell)(1 - \theta_\ell)$, where p is the price charged by firm ℓ. Since $\theta_\ell = \frac{p_\ell^h}{q_\ell} < 1$, it is immediate that $0 < \theta_\ell < \theta_h \leq 1$.

The first-order condition suggests that the optimal price of firm ℓ is $p_\ell^h = \frac{c_\ell + q_\ell}{2}$. To ensure that firm ℓ captures all consumers in the segment $[\theta_\ell, 1]$, it must further be that the consumer with $\theta = 1$ obtains at least as high a surplus from firm ℓ as from firm h. From firm h, this consumer obtains a surplus $q_h - c_h$ (since he is offered good h at price c_h). From firm ℓ, he obtains $q_\ell - p_\ell^h$. Adding this extra restriction, the optimal price of firm ℓ in this case is $p_\ell^h = \min\{c_h - (q_h - q_\ell), \frac{c_\ell + q_\ell}{2}\}$. The optimal price of firm h is $p_h^h(\theta) = c_h$ for all consumers in the range $[0, \theta_h]$. ■

Lemma 3 *In the equilibrium of the whole game,* $0 < \theta_\ell < \theta_h < 1$.

Proof of Lemma 3: Given Proposition 3, all we need to show is that $\theta_h < 1$. The argument is similar to that in Lemma 1. Again, we show that $c'_\ell < 1$ in equilibrium. This implies that there exists a q_h such that firm h earns a positive profit. Hence, firm h will not choose a q_h such that $\theta_h = 1$.

Suppose that $c'_\ell \geq 1$. From the proof of Proposition 3, $\theta_h = 1$ implies that $\frac{1}{2}(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{qh}) \geq 1$, or $\frac{c_h - c_\ell}{q_h - q_\ell} \geq 2 - \frac{c_h}{q_h}$. First, suppose $\frac{c_h - c_\ell}{q_h - q_\ell} = 2 - \frac{c_h}{q_h}$. Then, $\theta_h - \theta_\ell = 1 - \frac{1}{2}(\frac{c_h}{q_h} + \frac{c_\ell}{q_\ell})$. The derivative of this term with respect to q_ℓ is $-\frac{c'_\ell q_\ell - c_\ell}{{q_\ell}^2} < 0$ (since $c'_\ell > \frac{c_\ell}{q_\ell}$). Hence, if q_ℓ falls slightly, the market coverage of firm ℓ increases.

Further, $p_\ell^h - c_\ell = \frac{1}{2}(\frac{c_h q_\ell}{q_h} - c_\ell)$. The derivative of this w.r.t. q_ℓ is $\frac{c_h}{q_h} - c'_\ell$. Now, the consumer with type θ_h is indifferent between good ℓ and good h. Hence, he obtains a positive surplus from good h. Since $\theta_h = 1$ by assumption, this implies $c_h \le p_h^h < q_h$. Hence, $\frac{c_h}{q_h} < c'_\ell$ when $c'_\ell \ge 1$. Hence, this term also increases if q_ℓ falls slightly.

Now, $\pi_\ell = (\theta_h - \theta_\ell)(p_\ell^h - c_\ell)$. Since both terms increase when q_ℓ falls, π_ℓ increases when q_ℓ falls. Since the argument holds at any quality such that $c'_\ell \ge 1$, it must be that $c'_\ell < 1$ in equilibrium.

Next, suppose that $\theta_h = 1$, and $\frac{1}{2}(\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h}) > 1$. It is immediate that a small reduction in q_ℓ increases the market coverage of firm ℓ. In this case, its optimal price is $p_\ell^h = \min\{c_h - (q_h - q_\ell), \frac{c_\ell + q_\ell}{2}\}$. Whichever of these two is lower, the derivative of $p_\ell^h - c_\ell$, the profit per unit, is less than or equal zero whenever $c'_\ell \ge 1$. Hence, reducing q_ℓ weakly improves the profit per unit. Hence, at equilibrium, it must be that $c'_\ell < 1$.

Given that $c'_\ell < 1$, as before, this implies that there exists a q_h at which firm h earns a positive profit; that is, $\theta_h < 1$. ■

6.3.1 Derivation of Reaction Functions

We show (as an aside; this is used in Proof of Proposition 5) that the reaction functions are upward sloping.

Lemma 4 *(i) The reaction functions of both firms are upward-sloping in the range of quality such that $c'(q_\ell) < 1$.*
(ii) Suppose both firms offer the no-PP qualities, q_h^n, q_ℓ^n, and firm h now adopts PP. Then, compared to the no-PP case: (a) if firms remain at their original qualities, q_h^n, q_ℓ^n, the optimal defensive strategy of firm ℓ is to charge a lower price (b) Compared to the no-PP case, if firm ℓ remains at its original quality, q_ℓ^n, then the optimal strategy of firm h is to increase its quality.

Proof of Lemma 4

(i) We first derive the reaction functions of the two firms. As shown in equations (3) and (4) of the text, the profit functions of the firms in this case are

$$\pi_\ell^h(q_h, q_\ell) = (\theta_h - \theta_\ell)(p_\ell^h - c_\ell) = \frac{(c_h q_\ell - q_h c_\ell)^2}{2 q_h q_\ell (q_h - q_\ell)} \tag{18}$$

$$\pi_h^h(q_h, q_\ell) = \int_{\theta_h}^1 (p_h^h(\theta) - c_h) d\theta = \frac{(p_\ell^h + q_h - q_\ell - c_h)^2}{2(q_h - q_\ell)} \tag{19}$$

We focus on an interior equilibrium with $0 < \theta_\ell < \theta_h < 1$. We restrict attention to qualities such that $c'(q_\ell) < 1$ because we have shown that the equilibrium qualities satisfy this restriction. The reaction functions are valid for all qualities in this range; as shown in the proof of Lemma 3, if $c'(q_\ell) < 1$, there exist qualities q_h at which firm h can earn a positive profit (and therefore $\theta_h < 1$).

The first-order condition for firm h is $\frac{\partial \pi_h^h}{\partial q_h} = 0$,

$$\frac{(p_\ell^h + q_h - q_\ell - c_h)}{q_h - q_\ell} \left\{ 1 - c_h' + \frac{q_\ell(c_h' q_h - c_h)}{2(q_h)^2} - \frac{p_\ell^h + q_h - q_\ell - c_h}{2(q_h - q_\ell)} \right\} = 0. \tag{20}$$

Firm h's reaction function, $r_h^h(q_\ell)$ is the solution to this equation.

The corresponding first-order condition for firm ℓ is $\frac{\partial \pi_\ell^h}{\partial q_\ell} = 0$, which gives us

$$\frac{(c_h q_\ell - c_\ell q_h)(c_\ell(q_h - 2q_\ell) + q_\ell(c_h - 2c_\ell'(q_h - q_\ell))}{(2q_\ell(q_h - q_\ell))^2} = 0. \tag{21}$$

Since $\frac{c_h}{q_h} > \frac{c_\ell}{q_\ell}$, it cannot be that $(c_h q_\ell - c_\ell q_h) = 0$. Hence, the optimal quality of firm ℓ, q_ℓ is given by the solution to

$$c_\ell(q_h - 2q_\ell) + q_\ell(c_h - 2c_\ell'(q_h - q_\ell)) = 0. \tag{22}$$

Firm ℓ's reaction function, $r_h^\ell(q_h)$ is the solution to this equation.

Now, let ψ_ℓ be the left-hand side of equation (21), the first-order condition for firm ℓ, and ψ_h the left-hand side of equation (20), the first order condition of firm h. Then, $\frac{\partial \psi_\ell}{\partial q_h} = \frac{(c_h - c_\ell - c_\ell'(q_h - q_\ell))(c_\ell - c_h + c_h'(q_h - q_\ell))}{2(q_h - q_\ell)^3}$. Convexity of $c(\cdot)$ directly implies $c_h > c_\ell + c_\ell'(q_h - q_\ell)$ and $c_h' > \frac{c_h - c_\ell}{q_h - q_\ell}$. Hence, $\frac{\partial \psi_\ell}{\partial q_h} > 0$, so that to reach $\psi_\ell = 0$, firm ℓ must increase its quality.

Next, consider $\psi_h(q_h, q_\ell)$. Recalling that p_ℓ^h is a function of q_h, q_ℓ, we have

$$\frac{\partial \psi_h}{\partial q_\ell} = \frac{1}{4} \left(\frac{(c_h - q_h c_h')(c_h - (2 - c_\ell')q_h)}{{q_h}^2} + \frac{(c_h' + c_\ell')(c_h - c_\ell)}{(q_h - q_\ell)^2} - \frac{(c_h - c_\ell)^2}{(q_h - q_\ell)^3} - \frac{(c_h' c_\ell')}{(q_h - q_\ell)} \right).$$

Consider the first term. Since $c(\cdot)$ is strictly convex, $c_h < q_h\, c_h'$. Further, by assumption, $c_\ell' < 1$. Hence, the first term is strictly positive. Adding terms 2, 3 and 4 we have

$$\frac{\partial \psi_h}{\partial q_\ell} = \frac{(c_h - c_\ell - c_\ell'(q_h - q_\ell))(c_\ell - c_h + c_h'(q_h - q_\ell))}{4(q_h - q_\ell)^3}.$$

Convexity of $c(\cdot)$ directly implies $c_h > c_\ell + c_\ell'(q_h - q_\ell)$ and $c_h' > \frac{c_h - c_\ell}{q_h - q_\ell}$. Hence, $\frac{\partial \psi_h}{\partial q_\ell} > 0$, and to reach $\psi_h = 0$, firm h must increase its quality.

(ii) (a) Given p_ℓ^n from equation (8) and p_ℓ^h from Proposition 3, we have $p_\ell^h < p_\ell^n$ if and only if $\frac{c_h q_\ell + c_\ell q_h}{2q_h} < \frac{2q_h c_\ell + q_\ell c_h + q_h q_\ell - {q_\ell}^2}{(4q_h - q_\ell)}$. Cross-multiplying and simplifying, this condition reduces to $q_\ell(q_h - q_\ell)(c_h - q_h + q_h(\frac{c_h - c_\ell}{q_h - q_\ell} - 1)) < 0$. Now, $q_h > q_\ell$, and $c_h \leq q_h$ (else firm h sells zero units). Further, in equilibrium $\frac{c_h - c_\ell}{q_h - q_\ell} + \frac{c_h}{q_h} < 2$ (since $\theta_h < 1$, from Lemma 3). Multiplying throughout by q_h and rearranging, this implies that $c_h - q_h + q_h(\frac{c_h - c_\ell}{q_h - q_\ell} - 1) < 0$. Hence $p_\ell^h < p_\ell^n$.
(ii) (b) The profit function of the high quality firm is given by the equation $\pi_h^h(q_h, q_\ell) = \frac{(p_\ell^h + q_h - q_\ell - c_h)^2}{2(q_h - q_\ell)}$. Consider $r_h^h(q_\ell)$ the reaction function for firm h. Taking the partial of this expression with respect to p_ℓ^h and replacing the optimal price, gives the following $\frac{\partial}{\partial p_\ell^h}(\frac{d\pi_h^h}{dq_h}) = \frac{1}{2}(\frac{c_h - q_h c'_h}{q_h} + \frac{(c_h - c_\ell - (q_h - q_\ell)c'_h)}{(q_h - q_\ell)^2})$. The first term is clearly negative. From the convexity of the cost function, the second term is strictly less than 0, from which the inequality holds. ■

6.3.2 Proof of Proposition 4

Suppose only firm h has PP. Substituting $c(q) = Aq^2$, the profit-maximization condition of firm ℓ reduces to $(q_h - 2q_\ell)(q_h - q_\ell) = 0$, which yields the reaction function $q_\ell = \frac{q_h}{2}$ (since $q_\ell = q_h$ is not profit-maximizing). Similarly, the profit-maximization condition of firm h reduces to $(2 - A(2q_h + q_\ell))\ (2 + 3A(q_\ell - 2q_h)) = 0$. Since the reaction function of firm h is upward-sloping (from Lemma 2), it must be given by the second term in the above equation. This yields $q_h = \frac{2+3Aq_\ell}{6A}$. Now, simultaneously solving the two reaction functions gives the equilibrium qualities: $q_h^h = \frac{4}{9A} = \frac{0.444}{A}$, and $q_\ell^h = \frac{2}{9A} = \frac{0.222}{A}$. Both qualities are higher than the respective qualities in the no-PP case. Again, it is immediate to verify that the constraints $0 < \theta_\ell < \theta_h < 1$ are satisfied.

Finally, we show that neither firm wants to leapfrog; that is, firm h does not want to choose a quality lower than that of firm ℓ, and vice versa.

For convenience, call the firms 1 and 2 at this stage. Suppose firm 1 chooses $q_1 = \frac{0.444}{A}$, and firm 2 chooses $q_2 = \frac{0.222}{A}$ in equilibrium. Note that firm 1 has PP, and firm 2 does not. In equilibrium, firm 1 is the high quality firm. However, we wish to rule out the following leapfrogs:
(i) firm 1 chooses some $q_1 < \frac{0.222}{A}$, and becomes a lower quality firm.
(ii) firm 2 chooses some $q_2 > \frac{0.444}{A}$, and becomes a higher quality firm.

Consider (i) first. Suppose firm 2 chooses $q_2 = \frac{0.222}{A}$, and firm 1 does leapfrog, and chooses $q_1 < \frac{0.222}{A}$. In particular, let the optimal quality of firm 1, given that it has lower quality than firm 2, be denoted $q_1 = \frac{0.222-\epsilon}{A}$, where $\epsilon \geq 0$. Since firm 1 has lower quality, but has PP, its profit function is given by equation (1) of the paper (reproduced as equation (11) of the appendix). Replacing the optimal price p_h^ℓ for the firm without PP, and cost functions, $c_h = Aq_h{}^2$ and $c_\ell = Aq_\ell{}^2$, we get

$$\begin{aligned}\pi_1(q_h, q_\ell) &= \frac{(q_h q_\ell + Aq_h{}^2 q_\ell - q_\ell{}^2 - 2Aq_h q_\ell{}^2 + Aq_\ell{}^3)^2}{8q_h q_\ell(q_h - q_\ell)} \\ &= \frac{(1 + A(q_h - q_\ell))^2(q_h - q_\ell)q_\ell}{8q_h}.\end{aligned}$$

Substituting in the qualities $q_h = q_2 = \frac{0.222}{A}$ and $q_\ell = q_1 = \frac{0.222-\epsilon}{A}$, we have

$$\pi_1(\epsilon) = \frac{0.56(0.222 - \epsilon)\epsilon(1 + \epsilon)^2}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.121$. Then, the profit of firm 1 is $\frac{0.008}{A}$. This is less than its profits at the Nash equilibrium, given by $\frac{0.022}{A}$ (see Table 1 on page 14 of the paper). Hence, there is no incentive for the high quality firm to deviate and become the lower quality firm.

Next, consider case (ii). Suppose firm 1 chooses quality $q_1 = \frac{0.444}{A}$. Suppose that firm 2 deviates to some quality higher than this, and becomes the higher quality firm. Let the new (after deviation) quality level for the firm 2 be $q_2 = \frac{0.444+\epsilon}{A}$. Given that it has higher quality, the profit function for firm 2 is given by equation (2) of the text, or (12) of the appendix. Replacing the cost functions, $c_h = Aq_h{}^2$ and $c_\ell = Aq_\ell{}^2$ and then simplifying, we get

$$\pi_2(q_h, q_\ell) = \frac{(1 - A(q_h + q_\ell))^2(q_h - q_\ell)}{2}.$$

Substituting in the qualities $q_h = q_2 = \frac{0.444+\epsilon}{A}$ and $q_\ell = q_1 = \frac{0.444}{A}$, we have

$$\pi_2(\epsilon) = \frac{0.5(0.112 - \epsilon)^2\epsilon}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.037$. Then, the profit of firm 2 is $\frac{0.0001}{A}$. This is less than its profits at the Nash equilibrium, given by $\frac{0.006}{A}$ (see Table 1). Hence, there is no incentive for the low quality firm to deviate and become the higher quality firm. ■

6.4 Both Firms have PP

Proof of Proposition 5

As in the low-PP and high-PP cases, in the equilibrium of the whole game, $0 < \theta_\ell < \theta_h < 1$ (the argument is similar to Lemma 1, with the one difference that $\theta_h = \frac{c_h - c_\ell}{q_h - q_\ell}$; for brevity, we do not repeat it here). Hence, we ignore the constraints $0 \le \theta_\ell \le \theta_h \le 1$, and focus on the interior solution.

The profit functions of the firms in this case are:

$$\pi_h^b(q_h, q_\ell) = \int_{\theta_h}^{1} (c_\ell + \theta(q_h - q_\ell) - c_h) d\theta = \frac{(q_h - q_\ell - c_h + c_\ell)^2}{2(q_h - q_\ell)} \tag{23}$$

$$\pi_\ell^b(q_h, q_\ell) = \int_{\theta_\ell}^{\hat{\theta}} (\theta q_\ell - c_\ell) d\theta + \int_{\hat{\theta}}^{\theta_h} (c_h - \theta(q_h - q_\ell) - c_\ell) d\theta = \frac{(c_h q_\ell - q_h c_\ell)^2}{2 q_h q_\ell (q_h - q_\ell)}. \tag{24}$$

As argued in the text, the profit function of firm h is the same as in the low-PP case. Hence its reaction function is also the same as in the low-PP case, and is given by the solution to equation (16). Similarly, the profit function of firm ℓ is the same as in the high-PP case, so its reaction function is given by the solution to equation (22).

(i) From equation (16), we know that the optimal quality of firm h is the solution to the equation $\psi_h(q_h, q_\ell) = 1 - 2c_h' + \frac{c_h - c_\ell}{q_h - q_\ell} = 0$. We evaluate ψ_h at the equilibrium qualities when only firm h has PP, q_h, q_ℓ. These are the solutions to the two equations (20) and (22).

From equation (20),

$$1 - 2c_h' + \frac{c_h - c_\ell}{q_h - q_\ell} = -\frac{q_\ell(c_h' - (c_h/q_h))}{q_h} - 2 + \frac{c_h - p_\ell^h}{q_h - q_\ell} + \frac{c_h - c_\ell}{q_h - q_\ell}.$$

Now, $c_h' > \frac{c_h}{q_h}$ since $c(\cdot)$ is convex. Further, $q_h - c_h > q_\ell - c_\ell$, else firm h has zero sales. Since $p_\ell^h > c_\ell$, this yields $\frac{c_h - p_\ell^h}{q_h - q_\ell} < \frac{c_h - c_\ell}{q_h - q_\ell} < 1$. Therefore, at the qualities (q_h, q_ℓ), $\psi_h < 0$. Hence, to reach a quality at which $\psi_h(q_h, q_\ell) = 0$, firm h must decrease its quality. Since both reaction functions are upward sloping, firm ℓ will also decrease its quality.

(ii) The reaction function of firm h is the same as in the low-PP case. From Proposition 2, this is $q_h = \frac{q_\ell}{3} + \frac{1}{3A}$. Similarly, the reaction function of firm ℓ is the same as in the high-PP case. From Proposition 4, $q_\ell = \frac{q_h}{2}$. Solving these simultaneously, we have $q_h^b = \frac{0.4}{A}$ and $q_\ell^b = \frac{0.2}{A}$. The corresponding qualities in the low-PP case are $\frac{0.388}{A}$ and $\frac{0.164}{A}$. Clearly, both qualities are higher when both firms have PP.

Finally, we show that neither firm wants to leapfrog; that is, firm h does not want to choose a quality lower than that of firm ℓ, and vice versa.

First, consider firm h. For convenience, call the firms 1 and 2 at this stage. Suppose firm 2 chooses $q_2 = \frac{0.2}{A}$, which is the lower quality in equilibrium. In equilibrium, firm 1 chooses $q_1 = \frac{0.4}{A}$. However, we wish to rule out a leapfrog, whereby firm 1 chooses some $q_1 < \frac{0.2}{A}$, and becomes a lower quality firm.

To show this, suppose firm 2 chooses $q_2 = \frac{0.2}{A}$, and firm 1 does leapfrog, and chooses $q_1 < \frac{0.2}{A}$. In particular, let the optimal quality of firm 1, given that it has lower quality than firm 2, be denoted $q_1 = \frac{0.2-\epsilon}{A}$, where $\epsilon \geq 0$. Since firm 1 has lower quality, its profit function is given by equation (24). Replacing the cost functions, $c_h = A{q_h}^2$ and $c_\ell = A{q_\ell}^2$ and then simplifying, we get

$$\pi_1(q_h, q_\ell) = \frac{A^2(q_h - q_\ell)q_h q_\ell}{2}.$$

Substituting in the qualities $q_h = q_2 = \frac{0.2}{A}$ and $q_\ell = q_1 = \frac{0.2-\epsilon}{A}$, we have

$$\pi_1(\epsilon) = \frac{(0.02 - 0.1\epsilon)\epsilon}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.1$. Then, the profit of firm 1 is $\frac{0.001}{A}$. This is less than its profits at the Nash equilibrium, given by $\frac{0.016}{A}$ (see Table 1 on page 14 of the paper). Hence, there is no incentive for the high quality firm to deviate and become the lower quality firm.

Next, we show that the low quality firm has no incentive to deviate and become the higher quality firm. Suppose firm 1 chooses quality $q_1 = \frac{0.4}{A}$. Suppose that firm 2 deviates to some quality higher than this, and becomes the higher quality firm. Let the new (after deviation) quality level for the firm 2 be $q_2 = \frac{0.4+\epsilon}{A}$. Given that it has higher quality, the profit function for firm 2 is given by equation (23). Replacing the cost functions, $c_h = A{q_h}^2$ and $c_\ell = A{q_\ell}^2$ and then simplifying, we get

$$\pi_2(q_h, q_\ell) = \frac{(1 - A(q_h + q_\ell))^2(q_h - q_\ell)}{2}.$$

Substituting in the qualities $q_h = q_2 = \frac{0.4+\epsilon}{A}$ and $q_\ell = q_1 = \frac{0.4}{A}$, we have

$$\pi_2(\epsilon) = \frac{0.5(0.2 - \epsilon)^2\epsilon}{A}.$$

Maximizing this with respect to ϵ yields $\epsilon^* = 0.066$. Then, the profit of firm 2 is $\frac{0.0005}{A}$. This is less than its profits at the Nash equilibrium, given by $\frac{0.008}{A}$ (see Table 1). Hence, there is no incentive for the low quality firm to deviate and become the higher quality firm. ■