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An improved model to describe the repeated loading-unloading in compression of cellular materials

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Abstract

Cellular materials, often referred as foams or structural foams when used for energy absorption, are largely used to protect people and goods in the case of shocks and impacts. The detailed knowledge of their behavior is fundamental to design components for this aim.
Peroni et al. (2008)-(2009) proposed a model able to describe the mechanical compression behavior of some polymeric material. Such model, based on the work by Rusch (1970), described the stress-strain curve as a sum of two contributions, the first for the elastic part and the second for the densification. More recently Avalle and Belingardi (2018) presented a more general model where the stress is calculated as a sum of three terms, one for the elasto-plastic phase, the second for the plateau, and a third for the densification. The model could include effects like density and strain-rate.
However, those models allow to describe only the monotonic compression behavior: in several situations repeated impacts can happen with unloading followed by further reloading. Unfortunately unloading cannot be described by a linear relation between stress and strain (as is usually considered for metals). Unloading follows a non-linear law with a variable relation between stress and strain in the successive cycles: this requires a particularly complex model.
In this work, a new model able to effectively reproduce the compression behavior of some polymeric cellular materials is presented. The model is validated and tuned on the basis of experimental tests with specimen subject to complex cycles of repeated loading and unloading. The model describes both the loading from different levels of residual compression and unloading from any value of compression level. The application to several materials justifies the generality of the method.

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1. Introduction

Accurate modeling of materials is essential in the design of innovative high-tech products such as aerial, marine, and ground transportation vehicles where virtual testing methods are widely used to accelerate their development. Virtual models allow reducing prototypes, by reducing the time to market, costs and improve products quality.

For applications where safety is of primary concern, but also in many packaging products, foams are an important class of materials used to absorb and dissipate energy in impact situations. This is due to their ability to allow for large deformations with controlled load levels, and then to dissipate the absorbed energy. Foams are derived from almost all materials by realizing a cellular structure with voids enclosed by closed or, sometimes, open cells. The obtained cellular materials have the ability to deform absorbing energy: moreover, with a suitable combination of the base material, cellular structure and density, it is possible to design a foam adapted to each specific application.

Modeling of the foam materials in terms of stress-strain characteristic, which depends on the material and cellular structure, is therefore necessary. Ideally, such models could be obtained from the properties of the base materials, and the cellular structure. However, more often such models can be obtained on the basis of a limited set of experimental tests interpolating the behavior in different situations.

A model able to describe with a higher level of fidelity many types of foams has been recently proposed by Avalle and Belingardi (2018). The model is proven to be very representative of the stress-strain curve and influence of density and strain-rate for many base materials of different nature (both polymeric and metallic, and not only). This model is aimed at providing the stress-strain characteristics in monotonic, compression conditions. However, there are cases, both in safety applications and in packaging, where, after a first impact, the kinetic energy is not fully absorbed and dissipated and secondary impacts can occur. To simulate in detail such situations, a more detailed model is necessary, able to describe the unloading and subsequent reloading at higher or lower values of force.

The paper will report about an improved model suitable to describe the loading and unloading of some materials of engineering interest. Samples of the materials were subjected to repeated uniaxial compression at different levels, recording the stress-strain curves to be fitted by the model. Considered materials for this paper are polystyrene, both expanded (EPS) and extruded (XPS), rigid polyurethane (PUR), and expanded polypropylene (EPP).

2. Phenomenological models of the stress-strain behavior of foams

A simple but effective model for the stress-strain relation between compression stress and strain of a foam was proposed by Rusch (1970):

$$\sigma(\varepsilon) = a\varepsilon^{p} + b\varepsilon^{n} \tag{1}$$

Many subsequent models tried to improve the results from the Rusch (1970) model that is either not predictive in the elastic and plateau phase, or in the densification.

Avalle et al. in (2005)-(2007) proposed an improved approximation for the elastic-plateau phase, further improved by adding the strain-rate influence by Jeong et al. (2012):

$$\sigma(e) = \left\{ A\left[1 - \exp\left(-(E/A)e(1-e)^{n}\right)\right] + B\left(\frac{e}{1-e}\right)^{n} \right\}\left[1 + (a + be)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] \tag{2}$$

In Avalle and Belingardi (2018) a model was proposed that combines contributions from both the Rusch model to describe the densification, and from the Peroni et al. (2008)-(2009) to describe the elastic-plateau phase. The Rusch model, in fact, does not properly describe the elastic phase: the derivative of the first term tends to infinity and this is not physically correct. The new proposed model, similarly to what proposed by Goga (2010) is stated as follows, for quasi-static loading:

$$\sigma(\varepsilon) = \sigma_p [1 - \exp(-m\varepsilon)] + \sigma_s \varepsilon + \sigma_D \varepsilon^n \quad (3)$$

With:

- σ_p plateau stress level
- σ_s linear hardening slope in the intermediate phase
- σ_D Rusch densification parameter
- m linear-plateau transition constant
- n Rusch densification exponent

The first term represents the elastic phase and the elastic phase transition. In fact, it is immediate to show that the derivative of (3) is:

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = m\sigma_p \exp(-m\varepsilon) + \sigma_s + n\sigma_D \varepsilon^{n-1} \quad (4)$$

Therefore, when the strain approaches zero, the slope of the stress-strain curve equals the value $m\,\sigma_p + \sigma_s = E$, initial elastic modulus of the foam. It is important to notice that the exponential model for the elastic-plastic transition is consistent with the universal law proposed by Chen et al. (2016) and Sun et al. (2011) in a series of papers for metals and other materials.

The second term can be explained by the progressive compaction of the expanded beads that make up most foams, especially polymeric. In fact, foams obtained by other manufacturing processes such as extruded polystyrene or polyurethane, typically exhibit a flat horizontal plateau and the σ_s terms equals zero.

The third term of (5) explains the densification exactly as in the Rusch (1970) model, and it is perfectly suitable for all the foam materials considered in this work.

The strain-rate effect is relatively complex to describe. After examining the application of many formulations as done by Cowper-Symonds, Johnson-Cook, Jones and Jeong et al. (2012), it has been verified that the three stress constants σ_p, σ_s, and σ_D of the law proposed by equation (4) are modified as predicted by the Cowper-Symonds law, that is:

$$\sigma_P = \sigma_{P,0}\left[1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_P(\rho)$$
$$\sigma_P = \sigma_{S,0}\left[1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_S(\rho)$$
$$\sigma_D = \sigma_{D,0}\left[1 + \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_D(\rho) \quad (5)$$

With:

- $\dot{\varepsilon}$ strain-rate value
- $\dot{\varepsilon}_0$ reference strain-rate value
- p strain-rate exponent
- $\sigma_{p,0}$ plateau stress level in static loading
- $\sigma_{s,0}$ linear hardening slope in the intermediate phase in static loading
- $\sigma_{D,0}$ Rusch densification parameter in static loading
- ρ density of the material

- $f_P(\rho)$ density function for the plateau stress level
- $f_S(\rho)$ density function for the intermediate phase
- $f_D(\rho)$ density function for the densification parameter

In this way, the influence of the density is also included in the formulation. In most cases a power law approximation is a good description of the influence of the density.

The proposed model fits very well the mechanical behavior of several foams in various loading conditions and at different densities. In the following sections the identification of the parameters for such materials, from experimental tests previously performed by the authors, are reported and discussed.

There are situations where it is necessary to describe the loading of the foam after previous compression at a certain level, or to analyze repeated loading at different values. Since the foam behavior is highly non-linear, the behavior of the foam when loading from a strain different from the initial state is not easy to describe. Fig. 1 shows a typical situation: loading is non-linear up to a certain degree described by Eq. (6), unloading is equally non-linear following a different stress-strain relation, and leading to a residual strain; following re-loading is still non-linear following a relation similar to Eq. (3) but with different initial slope. Finally, it is known that, in each reloading, after a transition, the material tends to follow the basic characteristic as if loading-unloading has not occurred: the following reloading curves, as it is clear from Fig. 1 in light blue and green, all are overlapped to the curve corresponding to the normal loading path (curve in dark blue).

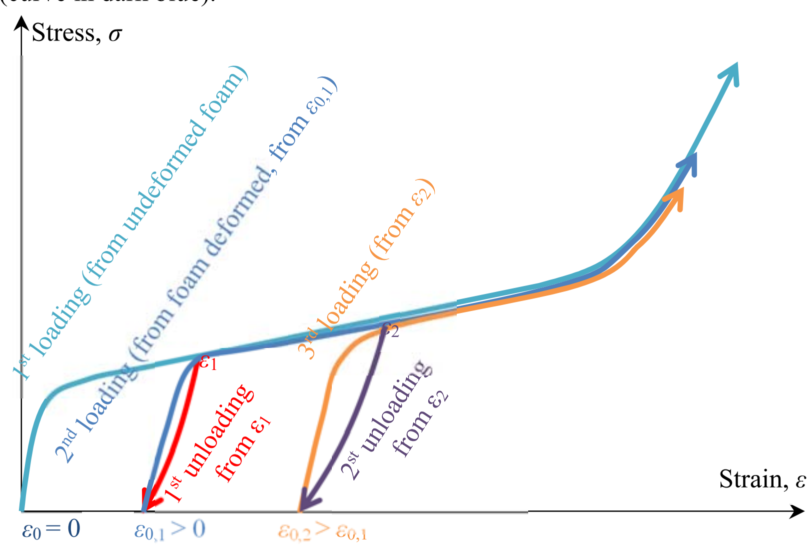


Fig. 1. Description of the loading-unloading mechanisms.

From the analysis of several experimental tests, and based on the recognized expression (3) for the loading of the foam, it appears that repeated loading can be expressed as:

$$\sigma_{i,\mathrm{loading}}(\varepsilon) = \left\{\sigma_p\left[1-\exp\left(-m\left(\varepsilon-\varepsilon_{0,i}\right)\right)\right]+\sigma_s\varepsilon+\sigma_D\varepsilon^n\right\}\left[1-\exp\left(-r\frac{\varepsilon-\varepsilon_{0,i}}{\varepsilon_{0,i}}\right)\right] \qquad (6)$$

With $\varepsilon_{0,i}$ being the value of initial strain at the *i*-th reloading, that is the residual strain after the previous unloading. The new parameter *r* accounts for the observed reduction in the initial stiffness when reloading. It is important to note that the three parameters σ_p plateau stress level, σ_s linear hardening slope, σ_D densification parameter and the *m* linear-plateau transition coefficient and *n* Rusch densification exponent do not change in any reloading.

Unloading is even more complex, especially because the curve parameters can change during each unloading.

Unloading can be expressed as follows:

$$\sigma_{i,\text{unloading}}(\varepsilon) = \sigma_i - \sigma_{p,i}\left[1 - \exp(-m_i(\varepsilon_i - \varepsilon))\right] - \sigma_{s,i}(\varepsilon_i - \varepsilon) \tag{7}$$

Where σ_i and ε_i are the values of stress and strain reached when unloading starts, $\sigma_{p,i}$ the plateau stress level at the *i*-th unloading phase, $\sigma_{s,i}$ linear hardening slope at the *i*-th unloading phase, m_i the linear-plateau transition coefficient at the *i*-th unloading phase. These three parameters change from unloading to unloading depending on the levels of stress and strain reached in the material. Their variation is not large, especially for low values of the global deformation. At higher values of strain, when densification is onset, the variation cannot in general be neglected. Unless in the case of lack of information about the variation of the parameters: in this case it is possible to use the initial parameters for a rough approximation of unloading.

2. Experimental method and equipment

2.1. Materials

Four materials were considered in this study, namely the materials were:

- Expanded polystyrene (EPS)
- Extruded polystyrene (XPS)
- Expanded polyurethane (PUR)
- Expanded polypropylene (EPP)

The tests were performed in uniaxial compression on cubic samples 50 mm side. Fig. 2 shows a sample of each examined material.

Because the main objective of the work was the identification of the proposed model, only one density was considered: the effect of density has been previously analyzed and modeled in previous papers by Avalle et al. (2001) or Avalle and Belingardi (2018). Before accomplishing the multiple loading/unloading tests, simple monotonic compression tests were performed to obtain the basics compressive behavior of the foams: in this case three different loading speeds were considered to have indications about of the strain-sensitivity of the materials.

Basic properties of the examined foams are reported in table 1. The density ratio is defined as the ratio of the foam density ρ_f divided by the base solid material ρ_s. Scatter of the material properties was in general rather limited. For each material at least 5 samples in monotonic compression were tested per each speed, whereas for multiple loading/unloading at least 3 samples were tested.

Table 1. Basic foam properties.

Material	Density, ρ_f (kg/dm³)	Approximate foam density ratio (ρ_f / ρ_s)	Average yield stress (MPa)
Expanded polystyrene (EPS)	14.29 ± 0.28	13 ‰	0.06
Extruded polystyrene (XPS)	29.18 ± 0.29	27 ‰	0.31
Expanded polyurethane (PUR)	28.12 ± 0.43	35 ‰	0.08
Expanded polypropylene (EPP)	≈70	78 ‰	0.18

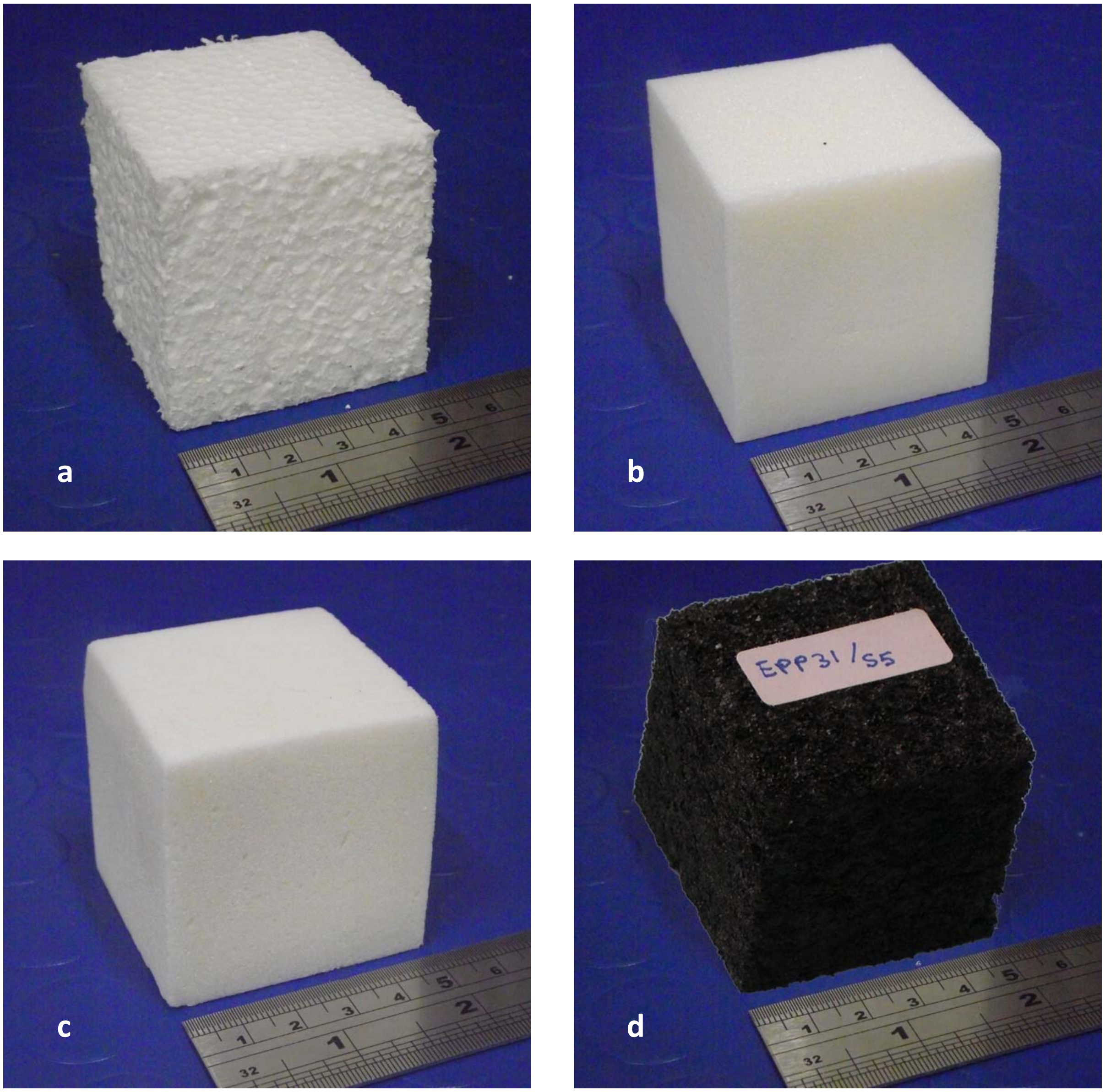


Fig. 2. Samples of the for materials considered: (a) Expanded polystyrene (EPS); (b) Extruded polystyrene (XPS); (c) Expanded polyurethane (PUR); (d) Expanded polypropylene (EPP).

2.2. Experimental setup

Tests were performed in simple uniaxial compression in two modes:

- Simple monotonic compression
- Multiple loading/unloading

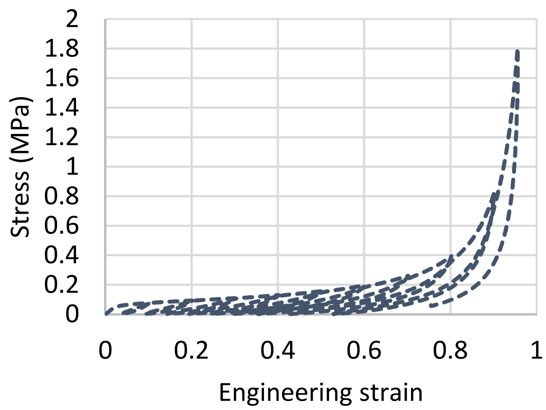
In simple monotonic compression the stroke was applied at a fixed rate until a maximum level of 90% was reached. Three low values of speed were considered due to the limitations of the used equipment: for the aims of the current paper examining higher values of speed was not considered necessary. The three values of loading speed selected were 0.1, 1.0 and 10.0 mm/s corresponding to an (initial) engineering strain-rate of 2×10^{-3}, 2×10^{-2}, and 2×10^{-1} s^{-1}.

Due to the high level of deformation, the true strain-rate cannot be constant: however, due to the rather low values of loading rate, and the limited influence of the strain-rate this variation is of negligible importance.

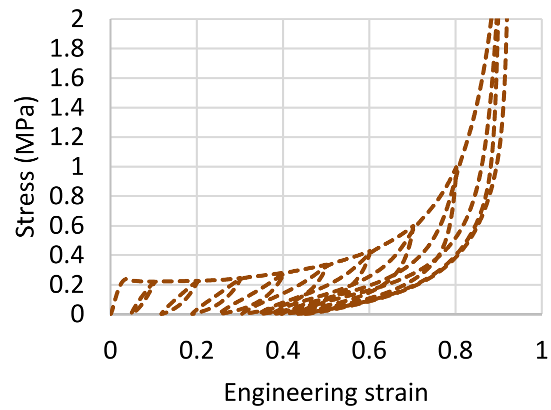
In multiple loading/unloading experiments, the following test scheme was adopted:

- Compression, in stroke control, up to the first compression level (typically 10%)
- Unloading, in stroke control down to a force level approximately zero
- Following compression phases up to multiple values of compression levels (20%, 30%...)
- After each compression phase, unloading approximately down to zero force

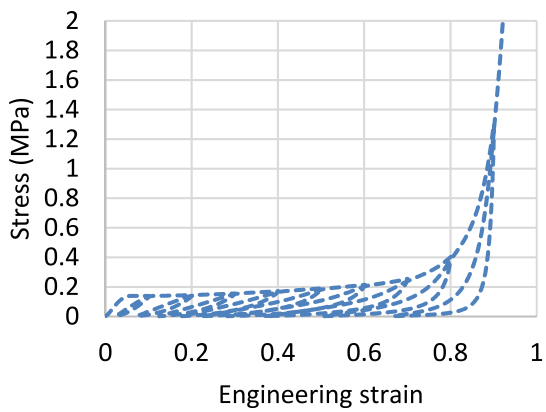
Loading/unloading were repeated, in steps of 10%, up to a maximum compression level typically of 90%. Fig. 3 shows the results from one repetition of the tests performed on each of the four materials (only one result is shown for the sake of clarity).



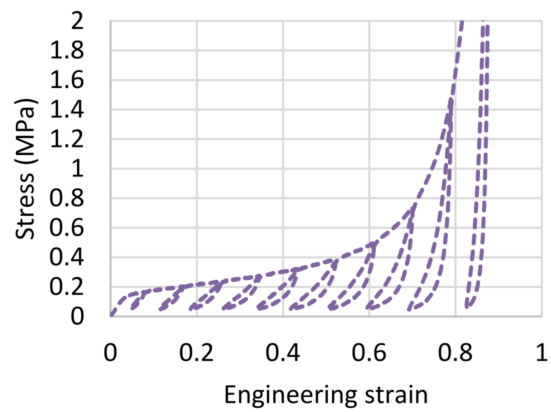
(a) Sample CR_EPS1_003



(b) Sample CR_XPS1_004



(c) Sample CR_PUR2_012



(d) Sample CR_EPP70_01

Fig. 3. Multiple loading/unloading test results with the four materials: (a) Expanded polystyrene (EPS); (b) Extruded polystyrene (XPS); (c) Expanded polyurethane (PUR); (d) Expanded polypropylene (EPP).

2.3. Testing setup

Tests were performed at low-speed and constant rate with an electromechanical testing machine Zwick Z010 TN ProLine equipped with parallel faces compression plates. The system is controlled by Zwick hardware (testControl II) and software (testXpert III) for control and data acquisition. The force was measured by a standard load cell 10 kN range, class 1 ISO 7500-1, together with the stroke measured by the position transducers of the machine.

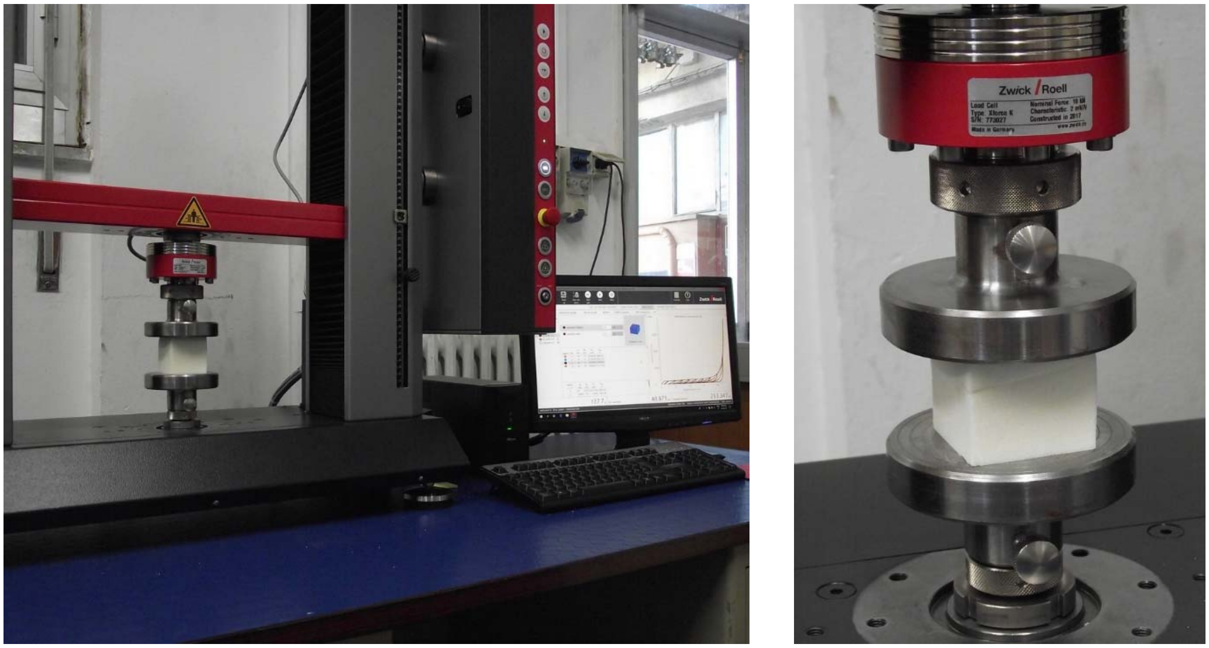


Fig. 4. Test equipment, left, and a detail of the compression setup, right.

A test procedure was programmed for the multiple loading/unloading tests using the testXpert III *Graphical Sequence Editor* software tool.

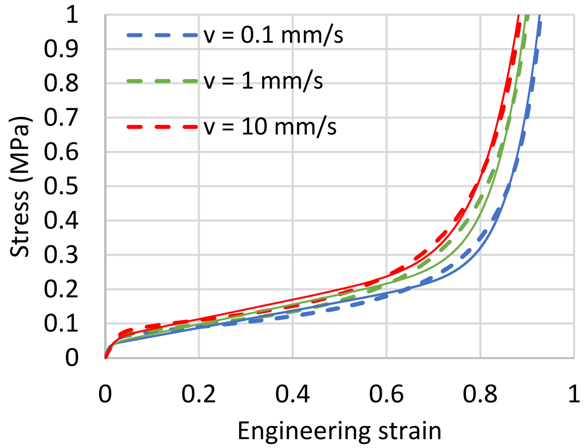
3. Experimental results

3.1. Monotonic compression

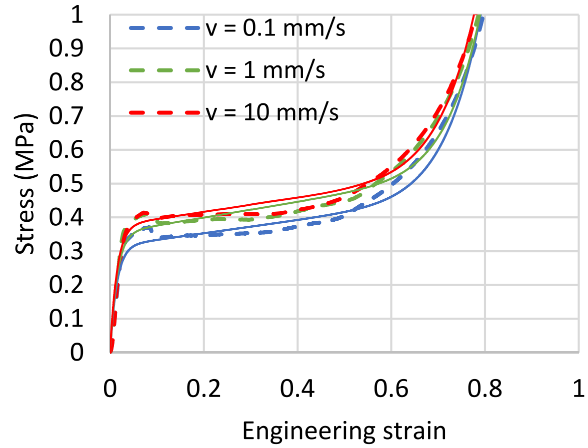
Fig. 5 shows some results of the compression tests performed on the four materials. Since the scatter between the various test repetitions was very low, in this case also, only one curve is reported per each loading rate, together with the approximation predicted by eq. (5).

The effect of the loading rate is rather low at the examined speed: however, some influence was found and reported in the following Fig. 5. The values of the constants exponent m and n together with the expression for the material parameters σ_p, σ_s, and σ_D are reported in Table 2. Values are reported for the lower loading speed only, being the influence of strain-rate quite low, at least in the considered range of speed. Regarding the values of the exponents m and n even if they show some variations, it is effective and more convenient to consider as constants.

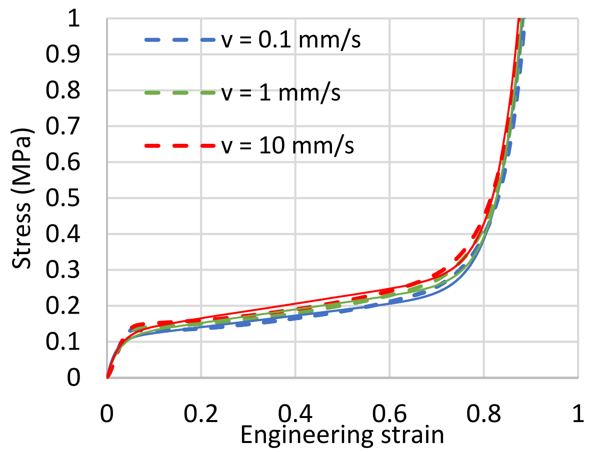
The materials show similarities with some noticeable differences: in particular, extruded polystyrene shows a first peak sometimes seen in other materials (included expanded polyurethane); as previously reported by Avalle et al. (2001), expanded polyurethane has an almost flat plateau corresponding to the lowest value of the corresponding parameter σ_s.



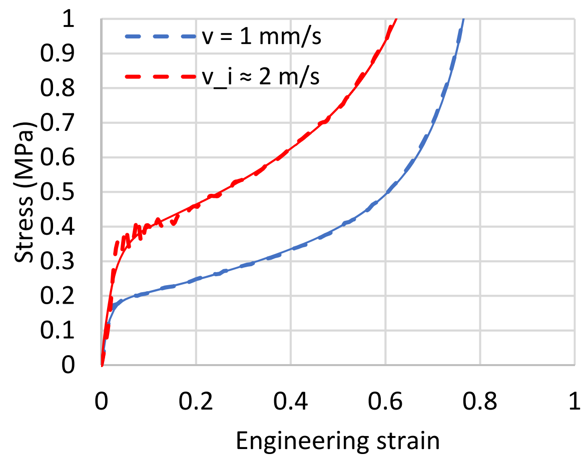
(a) Samples CS_EPS1_006, CS_EPS1_009, CS_EPS1_013



(b) Samples CS_XPS1_007, CS_XPS1_009, CS_XPS1_15



(b) Samples CS_PUR2_001, CS_PUR2_004, CS_PUR2_009



(d) Samples EPP70S5, EPP70D2

Fig. 5. Monotonic test results obtained with the four different materials: (a) Expanded polystyrene (EPS); (b) Extruded polystyrene (XPS); (c) Expanded polyurethane (PUR); (d) Expanded polypropylene (EPP). Thick dashed lines are the experimental curves, fitted model thin continuous lines.

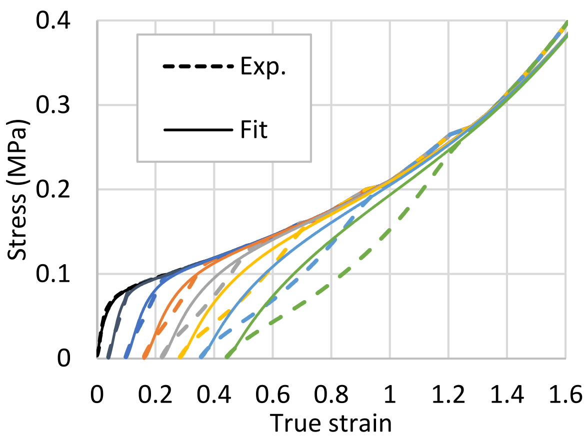
Table 2. Identified parameters of the monotonic curves of the examined materials.

Property	Expanded polystyrene (EPS)	Extruded polystyrene (XPS)	Expanded polyurethane (PUR)	Expanded polypropylene (EPP)
σ_p (MPa)	0.0623 ± 0.0017	0.196 ± 0.0136	0.124 ± 0.0173	0.182 ± 0.0027
σ_s (MPa)	0.110 ± 0.0020	0.128 ± 0.0102	0.0816 ± 0.0328	0.327 ± 0.0219
σ_D (MPa)	0.0172 ± 0.0004	0.113 ± 0.0011	0.0125 ± 0.0059	0.0664 ± 0.0102
m	59.6 ± 4.17	64.8 ± 2.54	45.1 ± 3.95	67.9 ± 15.1
n	3.717 ± 0.033	3.334 ± 0.075	5.255 ± 0.514	5.227 ± 0.255

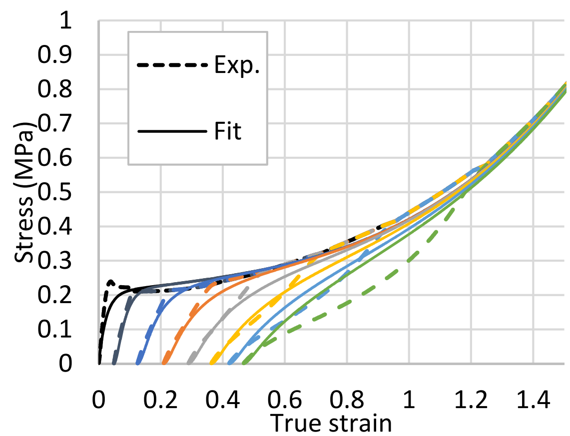
3.2. Multiple loading/unloading tests

3.2.1. Analysis of the loading process

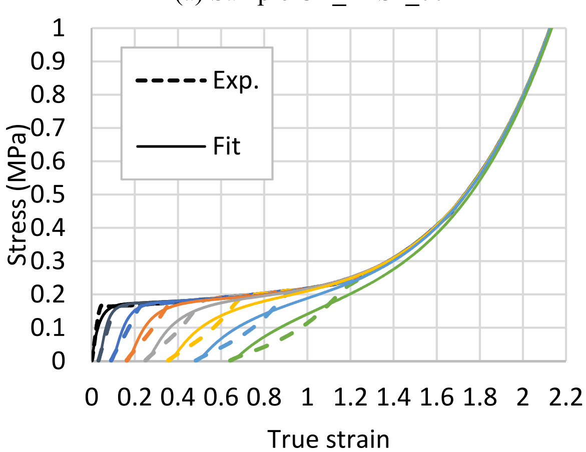
The results of the loading curves in the multiple loading/unloading tests are reported in Fig. 6 showing the fit of Eq. (6) on a single sample for each material. Repeatability was good, so only a single curve is representative of each material. This is justified by the small variation of the identified parameters reported in Table 3 summarizing the parameters computed with the four examined materials.



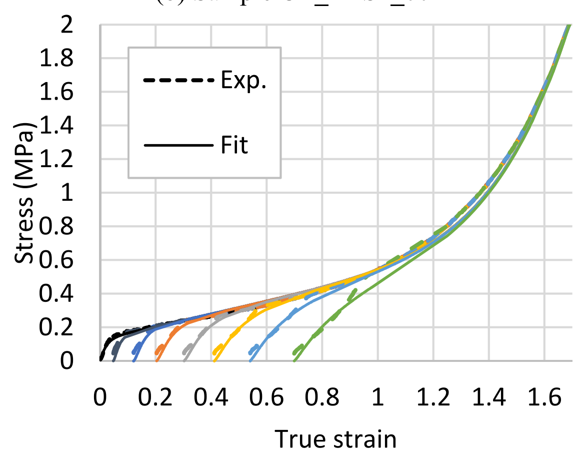
(a) Sample CR_EPS1_002



(b) Sample CR_XPS1_004



(c) Sample CR_PUR2_013



(d) Sample CR_EPP70_01

Fig. 6. Loading curves in multiple loading/unloading tests with the four materials: (a) Expanded polystyrene (EPS); (b) Extruded polystyrene (XPS); (c) Expanded polyurethane (PUR); (d) Expanded polypropylene (EPP).

Table 3. Identified parameters of the loading curves of Eq. (6).

Property	Expanded polystyrene (EPS)	Extruded polystyrene (XPS)	Expanded polyurethane (PUR)	Expanded polypropylene (EPP)
σ_p (MPa)	0.065 ± 0.0054	0.205 ± 0.0064	0.158 ± 0.0177	0.128
σ_s (MPa)	0.120 ± 0.0002	0.121 ± 0.0158	0.037 ± 0.0248	0.378
σ_D (MPa)	0.018 ± 0.0022	0.106 ± 0.006	0.02 ± 0.0063	0.038
m	42.87 ± 7.288	45.50 ± 8.369	25.94 ± 5.863	46.59
n	3.77 ± 0.0467	3.399 ± 0.0558	4.767 ± 0.4125	6.627
r	1.788 ± 0.1816	1.75 ± 0.2141	1.51 ± 0.3527	4.356

Identification of the curves with the proposed model show the good predictability with all the examined materials. The fit is particularly good especially when dealing with the smaller values of initial stress when reloading the material: especially with expanded polypropylene and polyurethane the description is quite good even if the material is reloaded from a value of true strain relatively high, around 0.6-0.7. The description of the material behavior is worse for expanded and extruded polystyrene: for these materials the reloading curves are well described only for values of the initial true strain at reloading up to 0.2-0.3. The description, for higher values of the initial true strain, is quite raw for the lower stresses even if, with increasing strain, the curves tend to the original curves and so are sufficiently descriptive.

About the differences among the various materials, as previously reported expanded polyurethane show a flat plateau confirmed by the small value of σ_s: in contrast this material has a higher densification exponent with respect to polystyrene but lower than expanded polypropylene. A significant difference lies in the r parameter of Eq. (6) between expanded polypropylene and the other materials: this reflects the observation that the reloading curves are less modified with respect to the initial, or monotonic, loading curve and, consequently, the representation of reloading is better. Correctly, the parameters identified in the multiple loading/reloading test of Eq. (6) are similar to the corresponding parameter for monotonic loading described by Eq. (3).

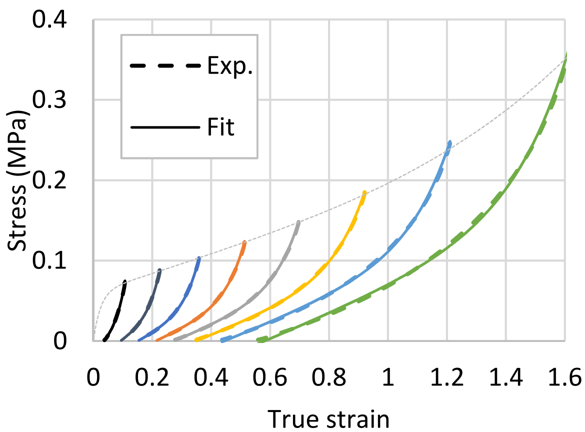
3.2.2. Analysis of the unloading process

The results of the unloading curves in the multiple loading/unloading tests are reported in Fig. 7 showing the fit of Eq. (7) on a single sample for each material. In this case also repeatability was very good, so only a single sample is representative of each material. This is justified by the small variation of the identified parameters reported in Table 4 summarizing the parameters computed with the four examined materials.

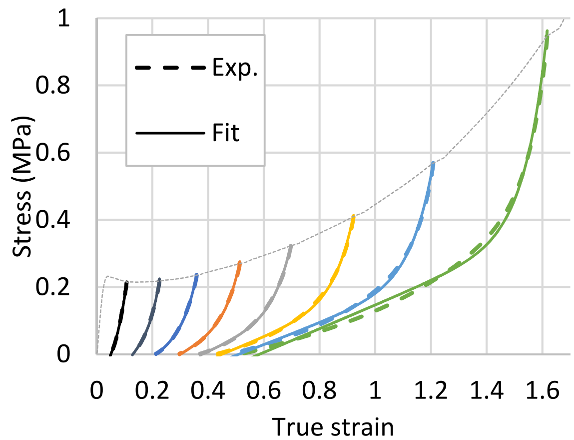
It is worth noting that in all cases, and for all materials, the description of the unloading curves is highly accurate: as matter of fact it confirms that unloading follows similar laws as loading.

The parameters used in Eq. (7) are of course a function of the strain, and stress, reached in the material during loading at the onset of unloading: so, they change continuously as soon as the strain and stress increases. To better evaluate the values of the parameters in Eq. (7) to be able to evaluate the material behavior while unloading it, it is useful to express the value of the parameters $\sigma_{p,i}$, $\sigma_{s,i}$, and m_i as functions of the strain ε_i reached at the onset of unloading. Based on the experimental tests it appears that a simple linear approximation is a sufficient description of the variation of such parameters. Therefore, the parameters $\sigma_{p,i}$, $\sigma_{s,i}$, and m_i of Eq. (7) can be expressed as:

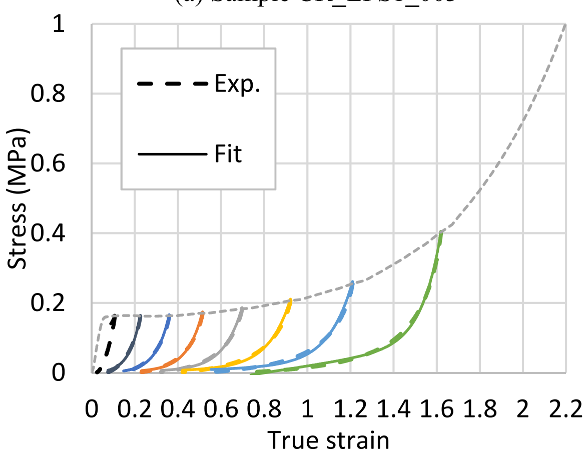
$$\begin{aligned} \sigma_{p,i} &= A_{p,u} + B_{p,u}\varepsilon_i \\ \sigma_{s,i} &= A_{s,u} + B_{s,u}\varepsilon_i \\ m_i &= A_{m,u} + B_{m,u}\varepsilon_i \end{aligned} \qquad (8)$$



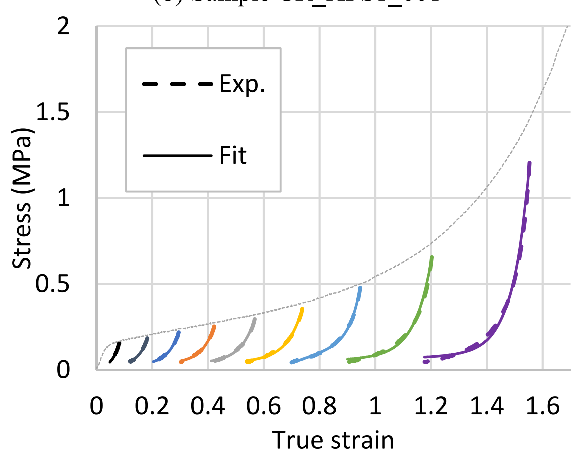
(a) Sample CR_EPS1_003



(b) Sample CR_XPS1_001



(c) Sample CR_PUR2_011



(d) Sample CR_EPP70_01

Fig. 7. Unloading curves with the four materials: (a) Expanded polystyrene (EPS); (b) Extruded polystyrene (XPS); (c) Expanded polyurethane (PUR); (d) Expanded polypropylene (EPP).

Table 4. Identified parameters of the unloading curves of Eqs. (7)-(8).

Property	Expanded polystyrene (EPS)	Extruded polystyrene (XPS)	Expanded polyurethane (PUR)	Expanded polypropylene (EPP)
$A_{p,u}$ (MPa)	0.034 ± 0.008	0.11 ± 0.0454	0.104 ± 0.0241	0.064
$B_{p,u}$ (MPa)	0.1 ± 0.006	0.23 ± 0.042	0.11 ± 0.024	0.345
$A_{s,u}$ (MPa)	0.222 ± 0.1149	0.631 ± 0.3019	0.062 ± 0.1008	0.223
$B_{s,u}$ (MPa)	−0.06 ± 0.0947	−0.246 ± 0.2808	0.007 ± 0.0528	−0.023
$A_{m,u}$ (MPa)	26.842 ± 6.094	27.409 ± 4.6976	21.204 ± 4.672	34.725
$B_{m,u}$ (MPa)	−16.012 ± 5.0783	−11.534 ± 4.3682	−8.546 ± 4.1787	−15.555

In practice, the trend is similar in almost all the four materials: the plateau stress in unloading clearly decreases with increasing initial strain: for the expanded polystyrene the increase is even more important than for the other materials. The slope instead is generally decreasing with increasing initial strain with the exception of the expanded polyurethane: it must be noted, however, that the slope is very small for this material, like in loading. The exponent of the exponential change is also strongly modified by the initial strain rate, and it decreases steadily: the transition is smoother with increasing values of compression.

4. Conclusions

In this paper a new model to describe the mechanical compression behavior of structural foams has been reported. The model is aimed to describe the complex behavior of the material subject to whatever loading path with unloading and successive reloading. Such behavior is sometimes observed in impact scenarios when the energy is not fully dissipated and, after a first larger impact, secondary impacts can occur. In these cases, it is necessary to describe the unloading from reached values of stress and strain and further reloading with stress and strain increasing again. The laws describing such phenomena are non-linear and depend on several factors.

Based on a series of dedicated experimental tests performed on some polymeric materials the proposed model has been fitted to the test results to check the validity of the model and identify the parameters. Reproducibility of the tests was quite high, and the model is able to describe such complex behaviors in a large range of situations. In particular, it is possible to accurately describe the unloading from almost any level of stress and strain reached in the material. The loading is more complex to describe and only an approximate representation was obtained: when reloading from higher values of strain, typically above 40-50% of initial strain after unloading to zero, there is a relatively large error. However, this being the best approximation of such complex behavior, it can be considered largely sufficient in most applications: especially when the alternative models are usually rough linear approximations of the stress-strain curves.

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