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A mechanical model of cellular solids for energy absorption

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Abstract:	Cellular materials have a variety of applications in the fields of packaging and mitigation in the case of impact of vehicles, due to their ability to protect goods by absorbing energy. To design energy absorption systems, it is necessary to use predictive models of cellular materials. The models must describe the stress-strain behavior then energy absorption characteristics can be evaluated. Moreover, it must consider affecting factors like strain-rate. Modeling the influence of the density helps designer in selecting the best foam solution. In previous works the authors already presented models able to describe the quasi-static stress-strain behavior of several cellular materials. The current paper presents a general model able to describe the mechanical characteristic of a much larger variety of cellular materials including metal foams and considers the influence of strain-rate. Among the considered materials there are the Foaminal® foam and APM® aluminum foams. The model is fitted to experimental tests with parameters identified based on experimental data. Tests include quasi-static, dynamic, and impact tests in different loading conditions. It will be shown that the proposed model is fundamentally suitable for most materials, virtually any foamed material, and it is a useful tool for designers in the mentioned areas.

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A mechanical model of cellular solids for energy absorption**

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Cellular materials, also known as foams, have a variety of applications in the field of packaging, and shock mitigation in the case of crash of vehicles, due to their ability to protect goods by absorbing energy in the case of impact while reducing the transmitted loads. To properly design energy absorption devices and systems such as bumpers, road barriers, helmets, sole paddings, packages, etc. it is necessary to use precisely predictive models of cellular materials, in order to select the most suitable foam for the considered application. The model must describe the stress-strain behavior, at least uniaxial compression, but also sometimes the tension and multiaxial loading, then energy absorption characteristics can be evaluated. Moreover, it must take into account affecting factors like the strain-rate. Secondarily, modeling the influence of the density heavily helps designer in selecting the best solution in terms of minimum weight per given energy to dissipate.

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In previous works the authors already presented more than one model able to describe the quasi-static stress-strain behavior of several cellular materials.[1, 2] *The current paper presents a very general model able to describe, with properly identified parameters, the mechanical characteristics of a much larger variety of cellular materials including metal*

foams, foam mechanical properties (like, for example, the dependence on density) and takes into account the influence of strain-rate.[3]

Among the considered materials are the Foaminal®[4] *aluminum foam and the APM®*[5] *hybrid foam. The model is fitted to experimental tests with parameters identified based on past experimental data from the authors themselves. Tests include quasi-static, dynamic, and impact tests at different loading speed and impact energy.*

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It will be shown that the proposed model is fundamentally suitable for most materials, virtually any foamed material, and it is an outstandingly useful tool for designers in the mentioned areas.

1. Introduction

Accurate modeling of materials is essential in the design of innovative high-tech products such as aerial, marine, and ground transportation vehicles where virtual testing methods are widely used to accelerate their development. Virtual models allow reducing prototypes, therefore reduce the time to market and the costs, and at the same time help improving the products quality.

For applications where safety is of primary concern, but also in many packaging products, foams are an important class of materials used to absorb and dissipate energy in impact situations as largely explained in the works from Gibson and Ashby.[6] This is due to their ability to allow for large deformations with controlled load levels, and then to dissipate the absorbed energy. Foams are derived from almost all materials by producing a cellular structure with voids enclosed by closed or, sometimes, open cells. The obtained cellular materials can deform absorbing energy: moreover, with a suitable combination of the base material, cellular structure and density, it is possible to design a foam adapted to each specific application.

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Modeling of the foamed materials in terms of stress-strain characteristic, which depends on the material and cellular structure, mainly synthesized by the density,[4, 7] is therefore necessary, as well as taking into account affecting factors like strain-rate,[3-10] temperature,[11] anisotropy,[12] different loading modes[13, 14] included repeated loading.[15] Ideally, such models could be obtained from the properties of the base materials, and the cellular structure as in the Gibson-Ashby model.[6] However, more often such models can be obtained on the basis of a limited set of experimental tests interpolating the behavior in different situations. The paper reports about a new model which demonstrated to be almost perfectly fitting almost all foam materials in uniaxial loading conditions, also taking into account the most important affecting factors that are strain-rate and density effect. Even if the model does not include other loading conditions, such as biaxial and triaxial, it is still very valuable since uniaxial compression is often the main stress mode. The model is applied to recent innovative aluminum foams (Foaminal® and *Advanced Pore Morphology*, APM®) after being applied to plastic based materials like expanded polypropylene and expanded polystyrene.[3]

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2. Phenomenological models of the stress-strain behavior of foams

A simple but effective model for the stress-strain relation between compression stress and strain of a foam was proposed by Rusch in 1970:[16]

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$$\sigma(\varepsilon) = a\varepsilon^{p} + b\varepsilon^{n} \qquad (1)$$

Many subsequent models tried to improve the results from the Rusch model that is not completely satisfactory and predictive in the elastic and plateau phases, while it is better in the description of the densification phase.

Avalle et al. proposed the following model for the stress-strain characteristics of various polymeric foams:[1]

$$\sigma(e) = ae^{p} + b\left(\frac{e}{1-e}\right)^{n} \qquad (2)$$

Subsequently Avalle et al.[2] proposed this improved approximation for the elastic and plateau phases:

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$$\sigma(e)=A\left\{1-\exp\left[-(E/A)e(1-e)^{n}\right]\right\}+B\left(\frac{e}{1-e}\right)^{n} \tag{3}$$

The same model was further improved by adding the strain-rate influence by Jeong et al.[17] (also referred more recently by Kim et al.[6]):

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$$\sigma(e)=\left\{A\left[1-\exp\left(-(E/A)e(1-e)^{n}\right)\right]+B\left(\frac{e}{1-e}\right)^{n}\right\}\left[1+(a+be)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0}}\right)\right] \tag{4}$$

The model proposed here combines contributions from both the Rusch model to describe the densification, and from the Avalle et al. model to describe the elastic-plateau phase.[2] The Rusch model, in fact, does not properly describe the elastic phase: the derivative of the first term tends to infinity and this is not physically correct. The new proposed model, similarly to what proposed by Goga,[19, 20] is stated as follows, for quasi-static loading:

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$$\sigma(\varepsilon)=\sigma_{p}\left[1-\exp(-m\varepsilon)\right]+\sigma_{s}\varepsilon+\sigma_{D}\varepsilon^{n} \tag{5}$$

Where:

- σ_p plateau stress level
- σ_s linear hardening slope in the intermediate phase
- σ_D Rusch densification parameter
- m linear-plateau transition constant
- n Rusch densification exponent

The first term represents the elastic phase and the elastic phase transition. In fact, it is immediate to show that the derivative of (5) is:

$$\frac{d\sigma(\varepsilon)}{d\varepsilon}=m\sigma_{p}\exp(-m\varepsilon)+\sigma_{s}+n\sigma_{D}\varepsilon^{n-1} \tag{6}$$

Therefore, when the strain approaches zero, the slope of the stress-strain curve is equal to the value $m\ \sigma_p + \sigma_s = E$, initial elastic modulus of the foam. It is important to notice that the exponential model for the elastic-plastic transition is consistent with the universal law proposed by Wagoner et al. in a series of papers for metals and other materials.[21-24] The Quasi-Plastic Elastic Second model (QPE-2) model is equivalent to the elasto-plastic and plateau parts in Eq. (6).[22]

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The second term can be explained by the progressive compaction of the expanded beads that make up most foams, especially polymeric. In fact, foams obtained by other manufacturing processes such as extruded polystyrene or polyurethane, typically exhibit a flat horizontal plateau and therefore the σ_s terms equals zero.

The third term of Eq. (5) explains the densification exactly as in the Rusch model, and it is perfectly suitable for all the foam materials considered in this work.

The strain-rate effect is relatively complex to describe. After examining the application of many formulations such as those proposed by Cowper and Symonds,[25] Johnson and Cook,[26] Jones,[27] Liu and Subhash,[28] and Jeong,[17] it has been verified that the three stress constants σ_P, σ_S, and σ_D of the law proposed by Eq. (4) can be effectively modified by means of a multiplying factor similar to the Cowper-Symonds[25] law, that is:

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\begin{aligned}
\sigma_P &= \sigma_{P,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_P(\rho)\\
\sigma_S &= \sigma_{S,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_S(\rho)\\
\sigma_D &= \sigma_{D,0}\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right] f_D(\rho)
\end{aligned}
\tag{7}
$$

Where:

- $\dot{\varepsilon}$ strain-rate value
- $\dot{\varepsilon}_0$ reference strain-rate value

- p strain-rate exponent
- $\sigma_{p,0}$ plateau stress level in static loading
- $\sigma_{s,0}$ linear hardening slope in the intermediate phase in static loading
- $\sigma_{D,0}$ Rusch densification parameter in static loading
- ρ density of the material
- $f_p(\rho)$ density function for the plateau stress level
- $f_s(\rho)$ density function for the intermediate phase
- $f_D(\rho)$ density function for the densification parameter

In this way, the influence of the density is also included in the formulation, as often suggested for example by Butt et al.[29] In most cases the three f functions are the same function of the density, but this is not always true: so, it is more convenient to consider the three distinct functions as reported.

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The proposed model fits very well the mechanical behavior of several foams in various loading conditions and at different densities. In the following sections the identification of the parameters for such materials, from experimental tests previously performed by the authors, are reported and discussed.

3. Experimental tests

All the experimental tests used in this work were performed by the authors and published in previous papers.[30-33] The tests used in the current analysis were obtained by the uniaxial compression of cubic or cylindrical samples.

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Materials were two different types of aluminum foams produced by the Fraunhofer Institute IFAM in Bremen. FOAMINAL is a closed cells aluminum (or zinc) foam obtained from metal powders and a foaming agent, through compaction and heating to start the expansion process.[31-32] The process allows the production of near net-shape parts. Sandwich structures can be obtained by foaming directly between aluminum face sheets (or also steel). It can be

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produced in relatively large size to obtain panels for automotive or aerospace applications. Density is typically between 0.3 kg dm^{-3} and 1 kg dm^{-3}, that is between 10% and 30% of the base material density. The second foam known as APM consists of small sphere-like metallic foam elements assembled together by a bonding medium: epoxy or polyamide resins are typically used. APM foam parts are obtained introducing the foamed spheres into a mold together with the adhesive material which is then cured to obtain the net-shape. By this process, parts with whatever complexity can be easily obtained.[33]

Cubic samples (**Fig. 1.a**) used for FOAMINAL®[31-32] had nominal side length of 41 mm. Exact values of the side lengths were measured and recorded to evaluate the relations between applied forces/shortening and stress/strain. Cylindrical samples of FOAMINAL® were also used for impact tests (diameter 25 mm, height 12.5 mm, Fig. 1.b). For the APM® aluminum foam cylindrical specimens with diameter 41 mm and height 41 mm (Fig. 1.c) have been used.[33]

A first batch of tests were quasi-static uni-axial compression tests performed with a constant speed, typically at 50 mm s^{-1} equivalent to 0.02 s^{-1} engineering strain-rate. This very low loading speed can be considered as quasi-static for all materials meaning that no strain-rate effect is present during the test. The tests were carried out until a maximum compression level was reached, up to 90% of the initial length in most cases. Load-stroke curves were recorded from which engineering stress-engineering strain curves were obtained. Detailed observations of the compression process revealed that in almost all tests: 1) the transverse area of the samples remained almost unchanged; 2) deformation occurred without visible localization, that is, it was homogeneous along the axial direction (except for the highest values of compression, usually more than 80%). These observations have the consequences that: 1) the true stress σ in the material can be considered equal to the engineering stress s, ratio of the axial load over the initial transverse area of the sample; 2) the true strain ε can be calculated

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from the engineering strain *e*, ratio of the shortening of the sample divided by its initial length, with the usual logarithmic expression:

$$\varepsilon = -\ln(1 - e) \qquad (8)$$

Please note that the minus sign before the engineering stress value e and before the logarithm sign come from the fact that when dealing with the compressive behavior of foam, it is common practice to consider as positive the compression strain. Similarly, the compression stress is considered as positive. These conventions were used in the previous Eq. (1) to (5).

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In dynamic impact tests the speed cannot be considered constant but it decreases progressively down to zero while all the kinetic energy of the impacting mass is absorbed by the foam sample and transformed into strain energy. Therefore, for those tests, the reported value of strain-rate is its initial value calculated as the ratio of the initial impact speed divided by the sample height. Simple analysis of the kinematic of the impact allows to compute the instantaneous speed and strain-rate during the tests. This calculation was necessary to properly evaluate the instantaneous values of strain-rate and of its effect when fitting the proposed model to the experimental tests, that is to properly compute the strain-rate coefficients expressed by Eq. (7). In this way, even if the number of dynamic test is so small, the effect of the strain-rate can be effectively evaluated because during the test the material is subjected to variable values of strain-rate (in all the examined range) and the fit is obtained only if a correct model is used. The final value of compression could not be obtained constant because of the practical difficulty in forecasting the exact amount of energy required to obtain such value of final compression: this is, however, a secondary limitation of the method with minor impact on the results.

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4. Fitting of the new model to experimental results

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4.1. FOAMINAL® aluminum foam

Tests on FOAMINAL® samples were conducted on cubic samples as described in the previous section. Different values of the foam density (from 0.3 kg dm^{-3} to 0.6 kg dm^{-3}) were examined, loading the foam samples in quasi-static (0.02 s^{-1}) and impact conditions (1000 s^{-1} initial strain-rate) as explained in the previous section. The stress-strain curves compared with the fit according to the new model are reported in **Fig. 2**: only the result of one single test is shown for each value of the density, for reasons of clarity, but in every test condition at least three samples were tested. Repeatability was very good in every test condition and all the curves were almost overlapping each other.

Fig. 3 shows the effect of the density on the model parameters. A power law approximation describes sufficiently well the effect described by Eq. (6). The exponents m and n have some scatter: an average value of m = 220 and n = 4 can be considered a convenient approximation for all values of the density.

4.2. APM® aluminum foam

Tests on APM® samples were conducted on cubic samples as described in the previous section. Different values of the foam density (from 380 g dm^{-3} to 725 g dm^{-3}) were examined, the foam samples were submitted only to quasi-static (0.02 s^{-1}) compression test conditions because APM foam, as a result of preliminary tests, did not behave effectively in impact conditions. The material tends to lose cohesion under an impact and the resistance in those unconfined conditions is rather poor. The stress-strain curves compared with the fit according to the new model are reported in **Fig. 4**: again, only the result of one single test is shown for each value of the density, but in every test condition at least three samples were tested as reported in the already mentioned paper.[14] Repeatability was also very good for this kind of foam.

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Fig. 5 shows the effect of the density on the model parameters. A power law approximation describes sufficiently well the effect described by Eq. (6). The exponents *m* and n have a relatively limited scatter as observed in Fig. 5.b so that average values of *m* = 80 and *n* = 5.8 can be considered a convenient approximation for all values of the density.

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5. Discussion: analysis of the responses and modeling

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The FOAMINAL® aluminum foams are well properly described by the proposed model of Eqs. (6)-(7). Looking at the curves reported in Fig. 2, it is possible to say that all the fits of the experimental curves are very accurate with values of the coefficient of correlation always greater than 95%. In particular the fit is extremely accurate to describe the densification phase but also the plateau. In some cases, the transition from the elastic to the plastic-plateau phase is not smooth: in some cases, in the experimental curves there is a peak at the buckling onset and then a slight decrease in the stress after yield of the aluminum cells. This cannot be modeled by the proposed equations but it can be considered as a minor detail not affecting the ability to predict especially the energy absorption characteristics of the foam. Moreover, it was observed that this slight peak is often caused by the presence of a denser wall present in some samples, i.e. some material inhomogeneities, depending on the manufacturing direction.[13] Those local small variations in the density can be neglected in design of a component made of such materials.

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About the strain-rate effect on FOAMINAL® aluminum foams, it was rather difficult to model: the individual curves are well reproduced as shown in Fig. 2, but a definite trend was not obtained. Generally speaking a rough approximation can be obtained by multiplying the three density parameters in Eq. (7) by a strain-rate multiplication factor expressed by:

$$\left[1+\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p\right]$$

$$\dot{\varepsilon}_0 = 1020 \qquad (8)$$

$$p = 7.1$$

Table 1 reports the estimated values of the power law fit describing the effect of density for this foam.

As discussed in the previous section, there are no data about the strain-rate influence on the behavior of APM® examined foam. The effect of density is instead clearly identified as reported in synthesis with **Table 2**. An important observation is that for this kind of foam the plateau is flat and horizontal: the σ_s parameter is zero. This result can be justified by the nature of this type foam made of an assembly of small foamed spheres bonded together by a structural adhesive (epoxy, or polyamide in other cases).[14] The spheres have a limited cohesion so that the load cannot be sustained unless densification occurs: as a matter of fact this result is contrasting with a similar observation for polymeric foams.[3] However, in polymeric foams like expanded polypropylene, expanded polystyrene or others, where expanded beads form the foam, the cohesion between them is rather strong and causes the progressively increasing stress before densification.

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6. Conclusions

A new model to describe the mechanical stress-strain behavior, including the strain-rate sensitivity, of aluminum foams has been presented. The model has been applied and describes very well the compression behavior of FOAMINAL® and APM® foams produced by IFAM. The same model was recently proposed also for many polymeric foams with similar performances: the model can represent effectively the elastic-plastic transitions, the plateau and the densification of many foams with different densities and in various testing conditions. In particular, it is also possible to model the influence of at least two fundamental factors such as density and strain-rate.

The result is that the presented model can describe effectively the behavior of both types of foams: the model parameters can be described by a power law approximation to include the influence of the density. The proposed model is likely able to describe the structural behavior of virtually every metal foam. For the FOAMINAL® foam, where dynamic impact test results were available, the influence of the strain-rate was also included. The difference between the two foams has been captured and detailed.

Concluding, a useful tool to design energy absorbing applications based on metal foams is provided that can help to simplify the selection of the most proper foam material in structures.

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(a) (b) (c)

Fig. 1. Samples of the aluminum foams: (a) FOAMINAL® cubic; (b) FOAMINAL® cylindrical; (c) APM® cylindrical, ARALDITE® AT 1-1 epoxy resin binder.[13]

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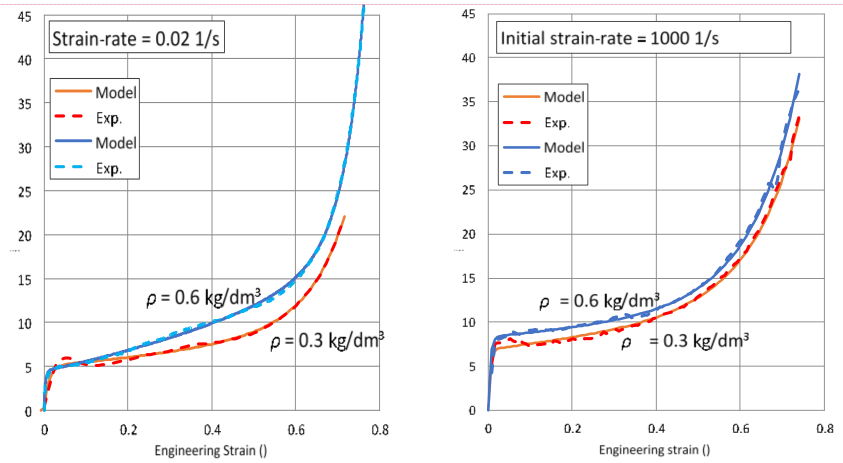

(a) (b)

Fig. 2. Typical curves of single samples of uniaxial compression tests on FOAMINAL® aluminum foams: (a) quasi static tests; (b) impact tests.

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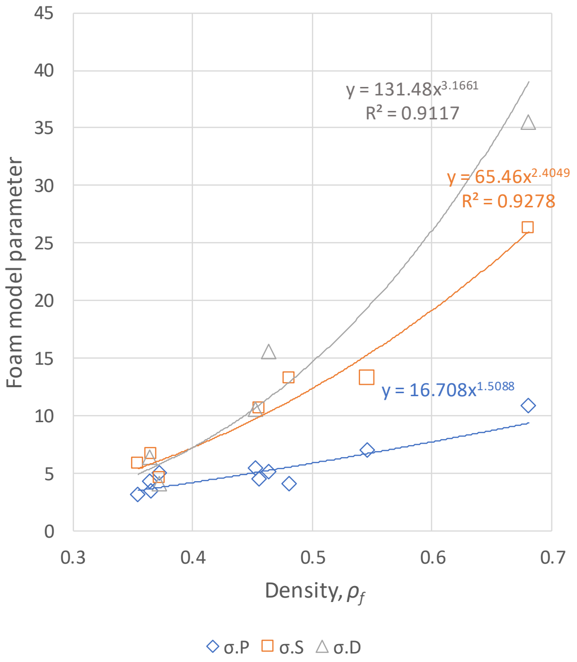
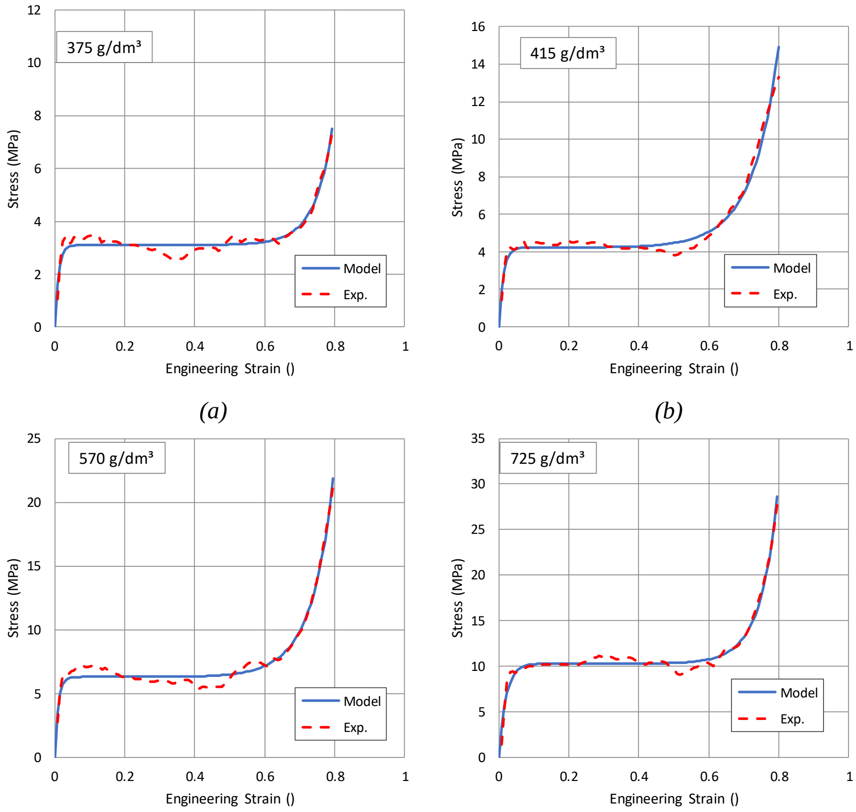


Fig. 3. Influence of the density on the model parameters of FOAMINAL® aluminum foams: influence on the stress constants. Each point is the average of at least three test repetitions.



(a) (b)

(c) (d)

Fig. 4. Single samples of curves result of uniaxial quasi-static compression tests on APM® aluminum foams: (a) 375 g dm^{-3}; (b) 420 g dm^{-3}; (c) 570 g dm^{-3}; (d) 725 g dm^{-3}.

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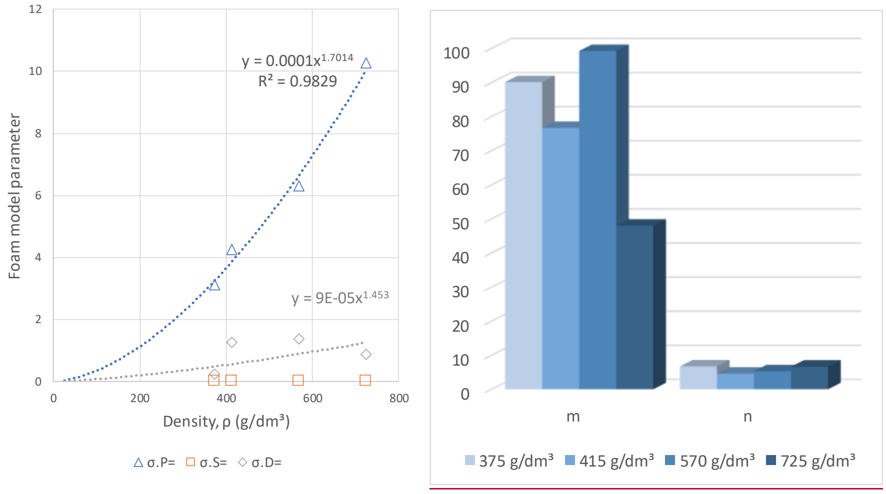

(a) *(b)*

Fig. 5. Influence of the density on the model parameters of APM® aluminum foams: (a) influence on the stress constants; (b) influence on the exponents.

Table 1. Estimated values of the parameters for the FOAMINAL® aluminum foams

Density function	σ_{i0} (MPa)	α_i (-)
$\sigma_{p0}\, f_p(\rho) = \sigma_{p0}\, \rho^{\alpha p}$	$\sigma_{p0} = 3.11$	$\alpha_p = 1.5$
$\sigma_{s0}\, f_s(\rho) = \sigma_{s0}\, \rho^{\alpha s}$	$\sigma_{s0} = 16.0$	$\alpha_s = 2.4$
$\sigma_{D0}\, f_D(\rho) = \sigma_{D0}\, \rho^{\alpha D}$	$\sigma_{D0} = 105$	$\alpha_D = 3.2$

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Table 2. Estimated values of the parameters for the APM® aluminum foams

Density function	σ_{i0} (MPa)	α_i (-)
$\sigma_{p0}\, f_p(\rho) = \sigma_{p0}\, \rho^{\alpha p}$	$\sigma_{p0} = 2.0$	$\alpha_p = 1.4$
$\sigma_{s0}\, f_s(\rho) = \sigma_{s0}\, \rho^{\alpha s}$	$\sigma_{s0} = 0.0$	$\alpha_s = 0.0$
$\sigma_{D0}\, f_D(\rho) = \sigma_{D0}\, \rho^{\alpha D}$	$\sigma_{D0} = 17$	$\alpha_D = 1.7$

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The paper describes a very general model for the uniaxial compression of metal foams, previously validated also for other polymeric and non-organic expanded materials. The model allows to describe with great precision the stress-strain curve from the elastic phase to the densification, and to take into account affecting factors such as the strain-rate and the density of the material. Identified parameters for some production aluminum foams are also reported.

M. Avalle, G. Belingardi*

A mechanical model of cellular solids for energy absorption