A mathematical model to infer underground thermal characteristics for the design of borehole heat exchangers

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Abstract

Geothermal exchangers are exploited as tools for the indirect analysis of the thermal properties of the underground material. Two simple inverse problems based on different heat transfer models are proposed: the first one is based on the cylindrical model of heat transfer and yields the thermal parameters of the borehole, the second one is based on a forced convective model and yields the soil thermal profile. We test our approach on sets of data collected by direct numerical simulation and from a real experiment.

Keywords: Heat Equation; Inverse Analysis; Thermal Diffusivity; Thermal Resistance; Volumetric Heat Capacity

1. Introduction

The most important world challenge in the energy field for the future is the transition to secure, clean, renewable and sustainable energy systems. Among the renewable resources, the geothermal energy plays a key role due to its main characteristics. In fact, the advantages of the geothermal applications, apart from exploiting a renewable and sustainable resource, are [12, 13]: to be environmentally friendly, for instance an average geothermal power plant releases 1/8 of the carbon emissions associated with a typical coal power plant, to have a small footprint since both low and high enthalpy applications require a much smaller amount of space with respect to the heat or power generating technologies in use today, to be independent of seasonal variations meaning a constant supply (this feature distinguishes geothermal energy from other renewable energies), to have a huge potential, in fact, estimates for the potential of geothermal power plants vary between 35 GW to 2 TW, finally, to be used in nearly all geological environments by means of low temperature geothermal applications.

In this paper, we focus on low-temperature systems based on vertical exchangers for the heating and cooling of buildings. These devices consist in a single or double U-shaped pipe placed into a borehole [8, 9] and a carrier fluid used to exchange heat with the ground material, that at suitable depth has constant temperature throughout the year. They have quite good performances but design and installation must consider numerous influencing factors, with particular reference to the underground thermal regime and the exchanger properties. These thermal characteristics of soil and exchanger are usually considered constant for the whole depth of the exchanger, even if this might not be realistic especially for the soil temperature profile. Their estimation is feasible by means of direct or indirect analyses [5, 10, 11]. The latter is the way most employed, comprising the estimation by literature that conveys average thermal features for a given lithotype, and the estimation by in-situ thermal response test (TRT) that is quite expensive and sometimes could be not enough accurate, especially in the determination of the undisturbed soil temperature. The direct estimation of the thermal parameters, instead, is accurate but it needs time-demanding and expensive laboratory analyses. They require the collection of ground samples taken from non-destructive drilling operations, which in turn mean very high drilling costs.

We propose two indirect analysis methods to infer the underground thermal properties and the temperature profile of the undisturbed soil, respectively. In other word, we want to exploit borehole exchangers as tools for relevant geological analyses. In more detail, regarding the thermal parameters estimation, the experiment consists in the monitoring and acquisition of the fluid temperature during the recovery phase after a thermal stimulation. In fact, the heat propagation into the soil is determined by the soil diffusivity, volumetric heat capacity, and the heat provided by the fluid within the exchanger. Obviously, also the borehole filling material influence the heat transfer. Thus, the temperature of the fluid conveys information about the soil thermal diffusivity, volumetric heat capacity, and the borehole thermal resistance. The heat transfer between the exchanger and the soil is described by means of the classical cylindrical source model. This model is quite accurate in absence of significant groundwater flow and for short-term evaluation. Our approach proposes a best-fit problem between the measured and computed fluid temperatures for obtaining the thermal parameters. The computed values of thermal diffusivity, heat capacity and resistance are also compared with the values obtained from direct analyses on ground samples. Regarding the undisturbed soil temperature estimation, the experiment consists in the registration of the fluid temperature from the starting of the fluid circulation for the time necessary to the fluid at the bottom of the exchanger to reach the ground level. In fact, during the ascending way the fluid temperature is modified by the heat transfer with the surrounding soil but it keeps information about its initial temperature. The fluid temperatures are described by a convective model valid for rectilinear pipes and fully developed flows. Our approach proposes again a best-fit problem between the measured and computed fluid temperatures for obtaining the undisturbed soil temperature at various depth levels. The obtained thermal profile of the soil is compared with the known soil profile.

The paper is organized as follows. Section 2 focuses on the computation of soil diffusivity and heat capacity, as well as borehole thermal resistance. Section 3 focuses on the reconstruction of the undisturbed soil temperature. Finally, in Section 4 we discuss all the results and we propose some further developments.

2. The estimation of the soil thermal parameters

We propose the estimation of the thermal parameters involved in the design of a borehole exchanger that exploits the following experiment. Soil and fluid inside the borehole exchanger initially are in thermal balance. Then, the soil is thermally stimulated by injecting continuously a prescribed amount of heat by means of the hot fluid circulation. When the circulation time expires, the temperature of the fluid at various depths into the device is measured until the soil recovers its undisturbed temperature. In Section 2.1, we briefly introduce the cylindrical model of heat transfer, that is suitable for describing the previous experiment, and we formulate a nonlinear fitting problem to infer the soil thermal diffusivity, volumetric heat capacity and the borehole thermal resistance. In Section 2.2, we present and discuss the results obtained by using the proposed approach.

2.1. The inverse problem based on the cylindrical model

To describe the fluid temperature inside the exchanger, we consider the cylindrical model of heat transfer [2], which formulates the heat propagation into the soil where a borehole exchanger is placed. We use the cylindrical coordinates with the exchanger placed at $r = 0$. The domain is described by $r > a$, where a is the radius of the exchanger that is centered at the origin of the coordinate system. The model describes the temperature outside an infinite medium such as soil or rock, and there is contact resistance between the cylinder and the surrounding medium. The problem formulation is:

$$\begin{cases} \dfrac{\partial u}{\partial t}(r,t) - \alpha \dfrac{\partial^2 u}{\partial r^2}(r,t) - \alpha \dfrac{1}{r}\dfrac{\partial u}{\partial r} = 0, & r > a, t > 0, \\ u(r,0) = 0, & r > a, \\ 2\pi a k \dfrac{\partial u}{\partial r}(a,t) + M c_f \dfrac{\partial v}{\partial t}(a,t) + \dfrac{1}{R} v(a,t) = Q, & t > 0, \\ u(a,t) = v(t), & t > 0 \\ u \to 0 \text{ as } r \to +\infty, \end{cases} \tag{1}$$

where u is the soil temperature, v is the fluid temperature, α, k are the thermal diffusivity and conductivity, respectively, M, c_f are the mass per unit length and the specific heat of the fluid, respectively, R is the surface thermal resistance per unit length, Q is the heat supplied per unit time and unit length. We note that problem (1) discards the variable z, meaning that the heat transfer occurs symmetrically with respect to the horizontal sections of the exchanger. Moreover, problem (1) is formulated for the soil temperature translated with respect to the undisturbed temperature of the soil T_{us}, i.e.,

$$T_s(r,t) = u(r,t) + T_{us},$$

where T_s is the real soil temperature. Our interest is focused on the temperature of the fluid within the cylinder, which can be derived from the solution of (1); in fact, as illustrated in [2], we have

$$T_f(t) = T_{us} + \frac{8\rho c Q}{\pi^3 \rho_f^2 c_f^2 \alpha} \int_0^\infty \frac{1 - exp(-\alpha t p^2/a^2)}{p^3 g(p)} \, dp, \tag{2}$$

where ρ_f is the fluid density, ρ, c are the density and the specific heat of the soil, respectively, and

$$g(p) = \left(pJ_0(p) - \left(\frac{2\rho c}{\rho_f c_f} - hp^2 \right) J_1(p) \right)^2 + \left(pY_0(p) - \left(\frac{2\rho c}{\rho_f c_f} - hp^2 \right) Y_1(p) \right)^2,$$

where $h = 2\pi R k$ is the dimensionless thermal resistance parameter, J_0, J_1 are the Bessel functions of the first kind and Y_0, Y_1 of the second kind. Problem (1) needs a refinement in the boundary condition to fully describe our experiment. Since we are interested in the soil temperature as response of a thermal stimulation with prescribed duration t_c, we modify the boundary condition in this way

$$2\pi a k \frac{\partial u}{\partial r}(a,t) + M c_f \frac{\partial v}{\partial t}(a,t) + \frac{1}{R} v(a,t) = Q\, H(t; t_c), \quad t > 0, \tag{3}$$

where H is the following step function

$$H(t; t_c) = \begin{cases} 1, & \text{if } 0 < t \le t_c, \\ 0, & \text{if } t > t_c. \end{cases}$$

The solution of problem (1) endowed with the boundary condition (3), for $t > t_c$, is

$$T_f(t) = T_{us} + \frac{8Q\rho c}{\pi^3 \alpha \rho_f^2 c_f^2} \int_0^\infty \frac{exp(-\alpha(t-t_c)p^2/a^2)\left(1 - exp(-\alpha t_c p^2/a^2)\right)}{p^3 g(p)}\, dp, \tag{4}$$

see [5] for details.

Now, the problem for the estimation of the soil parameters can be easily formulated as a least square problem, that is

$$\min_{\alpha,\rho c,h} \sum_{n=1}^{N} \left(T^M(t_n) - T_f(t_n;\alpha,\rho c,h)\right)^2, \tag{5}$$

where T^M is the fluid temperature measured in situ during the recovery, T_f is the explicit solution (4), and N is the number of measurements registered during the recovery.

2.2. Numerical results of the thermal parameters

We numerically approximate function (4) applying the mid point rule as quadrature scheme, in addition the integral domain is truncated in such a way to neglect the values of the variable p giving exponential values smaller than $\exp(-1/5)$. The solution of least square problem (5) is computed by using the Matlab function *fminsearch* that is based on the Nelder-Mead simplex method [6].

The numerical simulation is based on field measurements acquired during an experiment lasted for about 94 hours. In this test, the average amount of heat injected was $Q = 40\, Js^{-1}m^{-1}$. In the simulations, we used three time sequences of measured fluid temperatures corresponding to three depths. For each of them, the thermal parameters of the soil at that depth are available from laboratory analyses [10]. The comparison between the direct estimation of the parameters and the indirect estimation obtained by the proposed model is reported in Table 1. Some values of thermal diffusivity and volumetric heat capacity are missing in the side of the direct estimations but the comparison is still possible by means of the thermal conductivity, k. In fact, the model does not provide directly k, because $k = \alpha\rho_s c_s$. All the computed parameters have the same order of magnitude of the measured ones, even if the model tends to overestimate them. The comparison between the thermal resistance parameter of the borehole is given in Table 2. In this case, the reference values in the left column have been estimated using the average resistance value obtained from the TRT [10]. The order of magnitude between the two estimation techniques are in agreement but the proposed model tends to underestimate the thermal resistance. Nevertheless, being the soil thermal parameters and the borehole resistance subtle features to be determined via an indirect analysis, this preliminary estimation yields strongly promising results, which could be refined either modifying the numerical approximation of the problem or even the analytical model behind the computation of the fluid temperature.

Table 1: Comparison between thermal parameters (conductivity, diffusivity and volumetric capacity) obtained by laboratory analyses and those calculated by the cylindrical model.

Depth $[m]$	Direct estimation			Simulations		
	k $[10^3 \frac{J}{mKh}]$	α $[10^{-3} \frac{m^2}{h}]$	$\rho_s c_s$ $[10^6 \frac{J}{mK}]$	k $[10^3 \frac{J}{mKh}]$	α $[10^{-3} \frac{m^2}{h}]$	$\rho_s c_s$ $[10^6 \frac{J}{mK}]$
60	9.58	–	–	12.6	2.7	4.68
80	6.6	2.45	2.68	11.37	3.0	3.79
97	8.42	–	–	13.9	2.5	5.55

Table 2: Comparison between the thermal resistance parameter estimated by TRT and that calculated by the cylindrical model.

Depth $[m]$	Direct estimation	Simulations
	h [ ]	h [ ]
60	3.14	2.15
80	2.17	1.62
97	2.76	2.53

3. The estimation of the undisturbed soil temperature

We propose the estimation of the temperature profile of the undisturbed soil by using the following experiment. The fluid remains within the exchanger for a long time, such as a month, in order to reach the thermal balance with the soil. When the circulation starts, the temperature of the fluid exiting the exchanger is measured, until the fluid that initially was at the bottom of the ascending pipe of the exchanger, reaches the ground level. From the profile of these temperatures we want to recover the soil temperature profile. In Section 3.1, we briefly introduce a convective model able to describe the heat transfer occurring between the fluid in the pipe and the soil during the experiment, and we formulate the least square problem to infer the undisturbed soil temperature. In Section 3.2, we present and discuss results obtained with this inverse analysis.

3.1. The inverse problem based on a forced convective model

The heat transfer between the carrier fluid into a geothermal exchanger and the surrounding soil occurs by forced convection. In the model we propose, being the carrier fluid mostly water, the following assumptions hold: the fluid is incompressible and Newtonian, the thermal conductivity and the viscosity are constant, there is no internal heat generation and the viscous dissipation is negligible. The fluid flow is considered dynamically and thermally fully developed. Moreover, the geometric description of the exchanger is simplified by supposing a pipe having irrelevant wall effect and in direct contact with the ground. So the borehole with the filling material and the pipe wall thickness are discarded.

We consider the ascending pipe for our experiment, however the model can be formulated in a similar way for the descending pipe. The convective heat transport in this rectilinear pipe is described by means of the first principle of Thermodynamics, that is

$$\begin{cases} \dfrac{dT_f(z)}{dz} = \dfrac{kNu}{r^2\rho c U}\left(T_s(z) - T_f(z)\right), & z \in (-H, 0), \\ T_f(\bar{z}) = T_f(\bar{z}, 0), & \bar{z} \in [-H, 0), \end{cases} \tag{6}$$

where H, r are pipe length and radius, respectively, T_s is the pipe wall temperature that is the soil temperature, $T_f(\bar{z}, 0)$ is the temperature of the fluid at the depth $\bar{z}$ and at the starting of the circulation pump, i.e., $t = 0$ that has been highlighted as second argument of T_f, k is the thermal conductivity of the fluid, U is the mean fluid velocity, ρ, c are the fluid density and specific heat, respectively, Nu is the Nusselt number. We note that the proposed model is unidimensional since we consider the mean temperature on cross sections of the pipe, and it is time independent since the flow is thermally fully developed [1]. From standard arguments on ordinary differential equations theory, the solution of (6) is

$$T_f(z) = e^{a\bar{z}}\left(T_f(\bar{z}) + a\int_{\bar{z}}^{z} T_s(\zeta)e^{a(\zeta-\bar{z})}d\zeta\right), \tag{7}$$

where $a = kNu/(r^2\rho c U)$ is constant. However, we need to take into account the time variable, because the fluid passing through a fixed depth theoretically has different temperatures for different time instants. At time t after the starting of the circulation, the fluid has traveled along a distance proportional to its mean velocity, so we can rewrite the solution formula introducing the time variable, which is not an independent variable since it depends on the space variable. Besides, we evaluate the solution at the ground level $z = 0$ that is the point where the experimental temperatures are measured. Thus, formula (7) becomes

$$T_f(0, t) = e^{aUt}\left(T_f(Ut, 0) + a\int_{Ut}^{0} T_s(\zeta)e^{a(\zeta-Ut)}d\zeta\right), \tag{8}$$

where $\bar{z} = Ut$.

The problem for the estimation of the undisturbed soil temperature profile is formulated as a least square problem, that is

$$\min_{T_s(z)} \left(T^M(0,t) - T_f(0,t)\right)^2, \tag{9}$$

where T^M is the fluid temperature measured in situ at the ground level and at the time instant t, and $T_f(0,t)$ is the fluid temperature given by solution (8). We note that the fluid exiting the exchanger at the time instant t corresponds to the fluid that at the initial time was at depth $\bar{z}$.

3.2. Numerical approximation and results about the soil thermal profile

We numerically approximate solution (8) by the simple quadrature scheme of midpoint rule. We divide the spatial interval $[-H,0]$ into N subintervals of length h_z, consequently, the time variable inherits the spatial discretisation so the time step is $\Delta t = h_z/U$. The discretized solution at the n-th step is

$$\begin{aligned} T_f(0,t_n) =& e^{az_n}\left(T_f(z_n,0)\left(\frac{-1}{ah_z}\left(-e^{ah_z}+1\right)\right) + T_f(z_{n-1},0)\left(\frac{-1}{ah_z}\left(e^{ah_z}-1\right) + \right.\right.\\ & \left.\left. e^{ah_z}\right)\right) + \sum_{i=n-1}^{1}\left(T_f(z_i,0)\left(-e^{az_i}\left(1+\frac{1}{ah_z}\right) + e^{az_{i-1}}\frac{1}{ah_z}\right) + \right.\\ & \left. T_f(z_{i-1},0)\left(e^{az_{i-1}}\left(1-\frac{1}{ah_z}\right) + e^{az_i}\frac{1}{ah_z}\right)\right), \end{aligned}$$

where $z_n = -nh_z$, $t_n = n\Delta t$, $n = 1,\dots,N$, and the soil temperature profile T_s in formula (8) has been approximated by a linear piecewise interpolation, that in the interval $[z_n, z_{n-1}]$ is

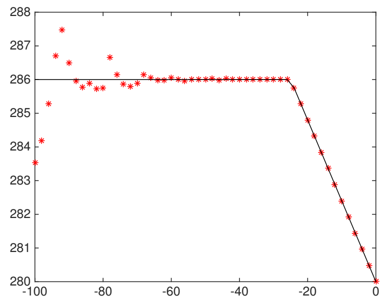
$$T_s(z) = \frac{z_{n-1}-z}{h_z}\left(T_f(z_n,0) - T_f(z_{n-1},0)\right) + T_f(z_{n-1},0). \tag{10}$$

The least square problem can be numerically reformulated in

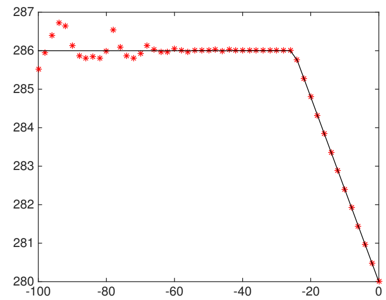
$$\min_{T_f(z_n,0)} \left(T^M(0,t_n) - T_f(0,t_n)\right)^2 + \lambda\left(z_n - z_{n-1}\right)^2, \tag{11}$$

for $n = 1,\dots,N$, where a Tikhonov regularization term has been added in order to avoid typical instabilities in the inversion of the heat equation solution due to ill-posedness. In (11), for each step n the sole unknown is the fluid temperature $T_f(z_n,0)$ at depth z_n and at the initial time, because there is thermal balance between the fluid and the soil before the experiment starts. So this reformulation is consistent with problem (9).

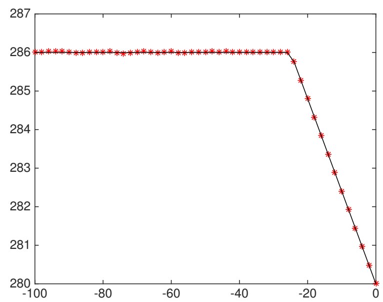
In the numerical simulations, we consider an exchanger with depth $H = 100$ m, which has been divided into $N = 50$ subintervals, meaning that the values of soil temperature have been calculated with a spatial resolution of $h_z = 2$ m.



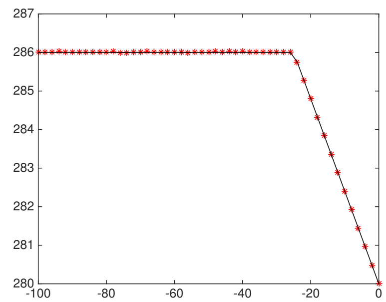
(a) $\lambda = 0$,



(b) $\lambda = 10^{-5}$,



(c) $\lambda = 5 \cdot 10^{-4}$,



(d) $\lambda = 10^{-3}$,

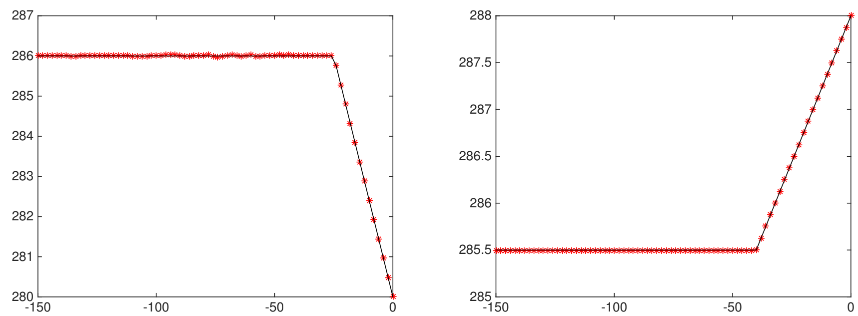
Figure 1: The known soil profile (solid black line) and the computed one (starred red line) with different Tikhonov regularization parameters.

The experiment we described to infer the soil temperature has been reproduced as a numerical experiment, because real data are not available at the moment. Thus we chose a reasonable temperature profile for the undisturbed soil in the winter season [3, 4], that is

$$T_s^n(z) = \begin{cases} -6z/25 + 280, & \text{if } z > -25, \\ 286, & \text{if } -H \leq z \leq -25, \end{cases} \tag{12}$$

and we compute the experimental fluid temperatures $T^M(0, t_n)$ by solving problem (6) with a Runge-Kutta method. We note that both the initial data and the results of the proposed method are obtained from problem (6), but the method used for the generation of the data is iterative whereas the computation of the soil thermal profile exploits the explicit formula (8). In this way, we avoid simplifications that may appear in applying consequently the direct and the inverse form of the heat problem (6).

Figure 1 shows the known soil profile and the computed one varying the regularization parameter λ. In Figure 1a, where the regularization parameter is



(a)

(b)

Figure 2: The known soil profile (solid black line) and the computed one (starred red line) with different geometrical or physical features and with $\lambda = 5 \cdot 10^{-4}$.

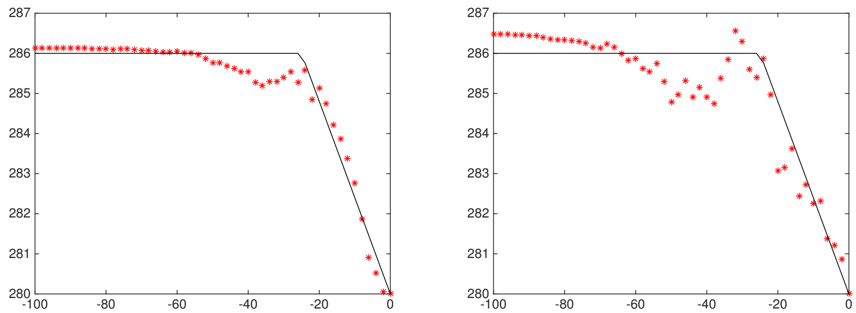
switched off, big oscillations occur for depth bigger than $H/2$. If it is too much small, as $\lambda = 10^{-5}$ in Figure 1b, the oscillations cannot be avoided even if they are smaller than in absence of regularization. Figures 1c,1d correspond to similar values of λ, $5 \cdot 10^{-4}$ and 10^{-3}, respectively, and show great correspondence between the known thermal profile of the soil (solid black line) and the computed one (starred red profile).Thus, we choose as optimal regularization parameter the smallest one, $\lambda = 5 \cdot 10^{-4}$. These qualitative evaluations are confirmed by the relative error between the two soil profiles, i.e.,

$$err = \sum_{n=1}^{N} \frac{|T_s^n(z_n) - T_s(z_n)|}{|T_s^n(z_n)|}.$$

Such relative errors are 0.0361 for Figure 1a, 0.0019 for Figure 1b, 0.002 for Figure 1c, 0.0164 for Figure 1d.

We also tested the model for different geometrical or physical features of the real soil profile with the chosen value for the Tikhonov parameter. In particular, in Figure 2a we increased the exchanger depth up to 150 m, maintaining the same real temperature profile of the soil. The correspondence between the starred red profile and the solid black one is full. In Figure 2b, we chose a soil thermal profile in cooling mode, so the temperature trend is reverted with respect to the previous cases; also the influence of the environmental temperature goes deeper into the soil up to 40 m of depth, meaning that the stratigraphy of the soil is made of different lithologies. Again, the computed soil profile and the known profile are in perfect agreement. The relative error confirms this qualitative correspondence, in fact, it is 0.0023 for Figure 2a and 0.0012 for Figure 2b.

Finally, we introduced a perturbation to the data for testing the response of the model in presence of significative uncertainties in the measurements. We used the reference soil profile T_s^n in (12) to generate the data and then we added



(a) $\lambda = 2 \cdot 10^{-1}$,

(b) $\lambda = 5 \cdot 10^{-2}$,

Figure 3: Perturbed measurements of the fluid temperature with random uncertainties at each point having order of 10% of the temperature range.

random perturbations at each data point with magnitude of 10% of the soil temperature range. As shown in Figure 3, the value of the regularization parameter must be increased to reproduce the correct profile mitigating the random perturbations, especially around the singular point. In Figure 3a, $\lambda = 2 \cdot 10^{-1}$ and the computed temperature profile of the soil closely follows the reference profile, in fact, the maximum gap between them is near the singular point and is less than 1 degree. On the contrary, in Figure 3b, the regularization parameter is about 1 order smaller and the correspondence between the two profiles is lost, although the global trend of the soil profile is reproduced discarding the spurious oscillations. The relative error between the two profiles is $4.8 \cdot 10^{-2}$ for Figure 3a and $8.4 \cdot 10^{-2}$ for Figure 3b.

4. Conclusions

We proposed two models for using borehole exchangers to infer soil characteristics. The first model calculates thermal parameters as diffusivity and volumetric capacity of the soil and resistance of the borehole, known the fluid temperature during the recovery phase of the thermal response test. This kind of investigation allows to confirm the stratigraphy of the soil in case it cannot be reconstructed by means of direct analyses, but it is also useful to obtain detailed thermal information about a lithology avoiding long and expensive laboratory analyses. The second model calculates the temperature profile of the undisturbed soil, known the fluid temperature exiting the exchanger during a prescribed time interval after the starting of the fluid circulation. This kind of investigation is valuable for the design of geothermal fields and to predict their potentiality that is an important issue in high-performing geothermal plants, where investors need financial protection by insurance products. In addition, the estimation of the soil temperature increases the knowledge about the physiochemical and biological processes occurring into the soil, and the gas exchange between atmosphere and soil.

We propose also some further developments. The estimation of the soil thermal parameters could be improved by introducing in the best fit functional suitable constraints given by geological settings. Moreover, the cylindrical model estimating the temperature of the fluid could be replaced by a more refined model that takes into account the mutual influence soil-exchanger. Also the estimation of the soil thermal profile could be improved by implementing a global fitting technique for the computation of the soil temperature.

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