

# 1 Analytical and numerical prediction of the bending behaviour of textile reinforced 2 concrete sandwich beams

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## 7 Abstract

8 This paper concerns the investigation of the behaviour of sandwich beams previously tested in four point bending through  
9 analytical and numerical models. Modelling is a fundamental resource to predict the mechanical response of the element  
10 and to investigate the mechanisms that act during the evolution of the test.

11 The sandwich beams here taken into account are characterised by external textile reinforced concrete (TRC) layers and  
12 an insulation material (expanded polystyrene, EPS) able to transfer shear stresses. Bond between the layers is obtained  
13 during production thanks to an in-pressure casting technique, and no particular device is used in order to transfer shear  
14 stresses between the layers. Two beam slenderness values are taken into account.

15 An analytical and a numerical approach have been used in order to predict the experimental behaviour: concerning the  
16 analytical approach, a model based on the Stamm and Witte sandwich theory has been developed including material non-  
17 linearity; concerning the numerical analysis, a finite element (FE) model has been built in ABAQUS including material  
18 and geometry non-linearity. The assumption of perfect bond is used in both cases.

19 The non-linear analytical and finite element models have been validated, as a good agreement with experimental results  
20 has been achieved. The experimental identification of material parameters - TRC in tension, mortar in compression and  
21 EPS in tension, compression and shear - is crucial for the definition of proper constitutive laws for the models and is here  
22 presented and discussed. For both approaches, the assumptions of modelling TRC in bending as homogeneous and  
23 assuming perfect bond between TRC and EPS (even when behaviour becomes highly non-linear) have been proved to be  
24 reliable. Analytical and FEM results show that EPS non-linear behaviour and TRC membrane and bending behaviour  
25 govern the response. The FE analysis also highlights the mechanisms involved in specimen failure.

26 *Textile reinforced concrete (TRC); sandwich beam; four-point bending test; non-linear analytical model; finite element*  
27 *method.*

## 28 1 Introduction

29 Since the 1940s, sandwich constructions have been used primarily in the aircraft industry and later in the missile and  
30 spacecraft structures [1]. Starting from the 1960s, the sandwich solution was applied in other fields, including buildings;  
31 a worldwide boom in prefabricated building elements favoured the diffusion of these sandwich products [2]. In particular,  
32 panels characterised by both the inner and the outer faces formed of metal sheets that act compositely with a relatively  
33 low strength core (with suitable insulating and stiffening properties) are largely diffused. For this kind of panel, the bond  
34 between components can be obtained through a line forming process, the use of adhesive or through mechanical  
35 fastenings. According to Davies [2], these sandwich solutions are designed in such a way that they act as a composite  
36 load-bearing unit for the expected service life.

37 In the residential and commercial building industry, in Europe and North America, the use of pre-cast R/C cladding  
38 sandwich panels is largely diffused; both the structural and the insulating potential of these wall elements are exploited  
39 [3]. Generally, two external R/C layers and an inner insulation layer characterise these wall panels. Various types of shear  
40 connectors are used to link the external concrete faces; depending on their stiffness and strength, the panel behaves as  
41 non-composite, partially-composite or fully-composite [4, 5, 6]. The weight is considerably higher with respect to panels  
42 characterised by metallic faces; in fact, the thickness of each concrete layer has been reduced to 40 mm only recently [3].  
43 Hegger and Horstmann [7] proposed wall and floor sandwich panels in which both the concrete layers are made of textile  
44 reinforced concrete (TRC). This solution allowed the researchers to obtain a lightweight precast product full of design  
45 and finishing capabilities, free from corrosion problems, and characterised by good durability. The introduction of shear  
46 connectors allowed for adequate and durable sandwich action [8]. Further recent research proposed sandwich elements  
47 with advanced cementitious composite faces connected by means of adhesive bond to the insulating material without  
48 using any connector [9, 10, 11, 12, 13, 14].

49 A model able to reproduce the panel behaviour is crucial for the design of this kind of solution. In literature, analytical  
50 and numerical models are proposed to predict the bending behaviour of sandwich elements.

51 The simplest analytical model that can be applied to a sandwich beam is based on the plane section assumption. However,  
52 this approach is not suitable to predict the real response of the sandwich, as the shear deformability of the core generally  
53 plays a key role and it is not taken into account. Stamm and Witte [15] proposed an analytical model for sandwich beams,  
54 which accounts for the shear deformability of the core and considers the bending stiffness of the outer layers, that cannot  
55 be neglected. The model is based on the formulation previously proposed by Plantema [16]. In this model linear elastic  
56 materials are assumed. Shams et al. [17] implemented the Stamm and Witte analytical model to account for the non-linear  
57 behaviour of the materials. As the equations in [15] are solved for constant bending and axial stiffness, to avoid solving  
58 the differential equations with a stiffness function, the authors propose constant average stiffness values for the whole

59 length of the beam. Two ways to compute the overall stiffness are proposed by the authors: weighting the beam local  
60 stiffness basing on the deflection or basing on the internal actions (e.g. bending moment). By investigating several TRC  
61 sandwich panels (characterised by different reinforcement and slenderness), the authors could state that the average beam  
62 stiffness, weighted on the deflection, accurately assesses the load-deflection behaviour taking into account the cracking  
63 of the concrete faces. The model used for deriving the shear stiffness of the core is described in [18].

64 In finite element analysis, when accounting for material non-linearity, TRC could be modelled considering the material  
65 as homogeneous [12] or discretizing the fabric as a grid reinforcement embedded in the matrix [19, 20, 21]. Larrinaga et  
66 al. [19] demonstrated that the assumption of rigid fabric-matrix interface is sufficient in order to estimate the global  
67 behaviour of the specimens with good accuracy. In the ABAQUS environment, some authors [22, 23] modelled foam  
68 materials through crushable foam model with volumetric hardening. In particular, concerning polystyrene foam, Masso-  
69 Moreu and Mills [22] modelled extruded polystyrene (XPS) using truncated pyramidal shapes subjected to impact testing  
70 which led to a good prediction of the experimental results. Moreover, Ozturk and Anlas [23] modelled expanded  
71 polystyrene foam (EPS) under multiple compressive loading and unloading and demonstrated that it is possible to predict  
72 the force-displacement curve accurately for the first loading, while the numerical results do not match the experimental  
73 ones in case of unloading and reloading.

74 The present research is developed in the framework of a European project [24] concerning the energy retrofitting of  
75 existing buildings. A multi-layer precast panel 1.5 m wide and 3.3 m high, fastened to the existing façade through four  
76 punctual connectors, is proposed for the application on existing buildings; it is characterised by an inner insulation layer  
77 in expanded polystyrene (EPS, 100 mm thick) and by two outer faces in textile reinforced concrete (10 mm thick). EPS  
78 is used to transfer shear stresses between the two external TRC faces, thus preventing thermal bridges due to the presence  
79 of connectors. Few connectors are embedded in the full-scale panel to prevent the detachment in extreme loading  
80 conditions.

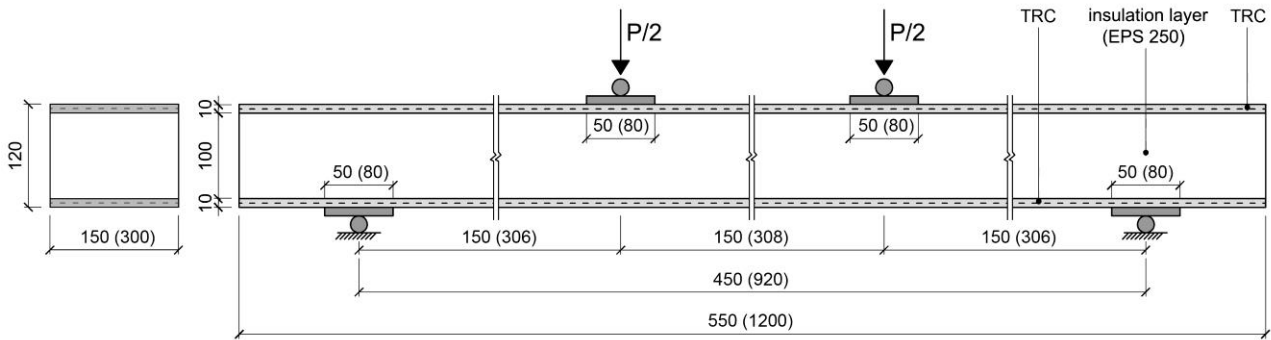
81 Specific investigations on durability of the interface between the external layer and EPS foam in case of freezing and  
82 thawing cycles have shown a reduction of the overall ductility [25].

83 Previous tests were performed on sandwich beams. The results of this campaign, deeply discussed in [26], underline the  
84 significant role played by the tangential non-linear behaviour of the EPS foam on the composite panel behaviour and  
85 allow a deeper understanding of the failure mechanisms, when different shear slenderness are considered. These  
86 experimental results are used as a reliable benchmark for the analytical and numerical models discussed in the present  
87 paper.

88 **2 Reference experimental campaign and mechanical characterisation of materials**

89 **2.1 Reference experimental campaign**

90 The experimental results, taken as a reference in the paper, concern tests performed according to a four-point loading  
91 scheme on 550 x 150 mm<sup>2</sup> (“deep”) and 1200 x 300 mm<sup>2</sup> (“slender”) sandwich beams. The test set-up is shown in  
92 Figure 1. Beams are characterised by two 10 mm thick external layers made of textile reinforced concrete (TRC)  
93 connected by a 100 mm thick insulation layer of expanded polystyrene foam (EPS250). All the details concerning the  
94 experimental campaign and test results can be found in [26], in which deep beams are named “small” (specimens S1, S2,  
95 S3 and S4) and slender beams are named “big” (specimens B1, B2, B3 and B4). In this paper, the same notation is used  
96 to identify the specimens.



97

98 **Fig. 1** Deep and slender sandwich beam geometry and test set-up (in mm). Values in parentheses represent slender  
99 specimens.

100 Sandwich beams are produced adopting an in-pressure casting technique in order to minimise the voids in the mortar and  
101 to enhance the bond between TRC layers and EPS, also because only the insulating material is used to transfer the shear  
102 between the external TRC layers [26].

103 From the experimental results emerged that a large ductility was experienced by both deep and slender specimens; this  
104 ductility was achieved thanks to the multi-cracking of both TRC layers and the large compressive plastic strain  
105 experienced by the EPS core. The fabric position in TRC layer thickness affected the multi-cracking pattern, but not the  
106 global response. Hence, it has to be regarded at Serviceability Limit State, in which crack opening needs to be controlled,  
107 rather than at Ultimate Limit State, in which the ultimate bearing capacities is accounted for. Beams behaved as partially  
108 composite sandwich and the global non-linear response was strongly driven by the EPS plastic compressive strains. The  
109 failure was due to the tensile failure of TRC for deep specimens and to the EPS brittle cracking in the case of slender  
110 beams. Moreover, when the slenderness was higher, the mono-dimensional beam assumption seemed reliable, while not  
111 negligible strains in the load direction were registered in the case of deep beams, denoting a two dimensional behaviour  
112 of the samples.

113 The analytical and numerical models shown in this paper allow further investigating the behaviour of deep and slender  
114 beams, in order to exactly understand the failure mechanism involved during the progress of the test.

115 In sections 2.2 and 2.3, the mechanical characterisation of TRC and EPS adopted for the sandwich production is provided.

## 116 **2.2 Textile reinforced concrete: uniaxial tension and bending**

117 TRC is obtained by reinforcing a high strength fine grain mortar with an alkali-resistant glass fabric, manufactured by  
118 means of a leno-weave technique and coated with a water resin based on styrene butadiene rubber (SBR).

119 The fabric used as reinforcement, whose geometrical and mechanical characteristics are collected in [26], is characterised  
120 by a nominal strength in the warp direction of 820 MPa (computed on the equivalent cross-sectional area of the glass  
121 reinforcement).

122 The cementitious matrix used is characterised by a water to binder ratio equal to 0.225 and by a superplasticiser to cement  
123 ratio equal to 9.3%. The maximum aggregate size selected is equal to 1 mm. These properties guarantee a high flow  
124 capability and, hence, a good bond between matrix and fabric and the possibility to cast the mortar in pressure. 12

125 nominally identical specimens were used to measure the cubic compressive strength ( $f_{cc}$ ) according to EN 196-1 [27].

126 The average value is equal to 71.89 MPa and the coefficient of variation is 9.13% of the average value.

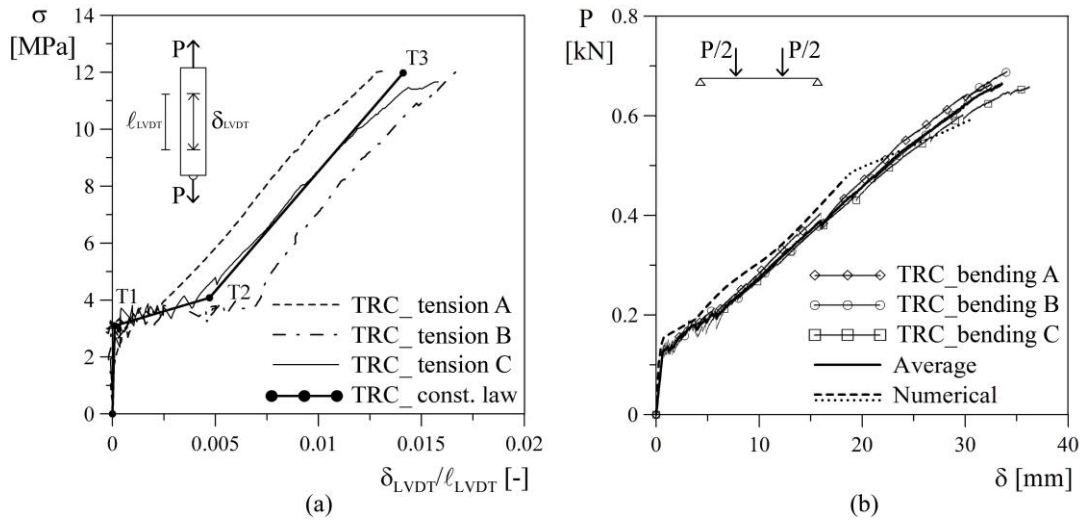
127 In order to characterise the TRC layer behaviour in tension and bending, six nominally identical specimens with  
128 dimensions 400 x 70 x 10 mm<sup>3</sup> were cast. These specimens were reinforced with one fabric placed in the middle of their  
129 thickness, with the warp aligned in the longitudinal direction.

130 Three specimens were tested in uniaxial tension, and three in bending. In both cases the tests were performed using an  
131 electromechanical press INSTRON 5867 with a maximum load capacity of 30 kN, imposing a constant cross-head  
132 displacement rate.

133 Concerning tensile tests on TRC, the same test apparatus and modalities described in [28] are used. The test results are  
134 shown in Figure 2(a) in terms of nominal stress ( $\sigma = P/A$ , with  $P$  = load and  $A$  = specimen cross section) vs. nominal  
135 strain ( $\epsilon = \delta_{LVDT} / L_{LVDT}$ ) curves.

136 Nominal strains are derived from a direct measure of axial displacement ( $\delta_{LVDT}$ ) on the specimen during tensile tests: an  
137 inductive full bridge type transducer, with a nominal displacement equal to 10 mm and a gauge length ( $L_{LVDT}$ ) equal to  
138 about 200 mm, is placed on the specimen. This choice allows the displacement measurement to not be affected by the  
139 relative sliding between specimen and clamping devices that may occur during tensile tests.

140 Bending tests on TRC specimens were performed according to a four-point bending scheme with a distance between the  
141 supports equal to 350 mm and a constant bending moment region 158 mm long . The tests were displacement controlled,  
142 with an imposed displacement rate of the machine cross-head equal to 1e-3 mm/sec. The test results are shown in  
143 Figure 2(b) in terms of load ( $P$ ) vs. machine cross-head displacement ( $\delta$ ) curves.



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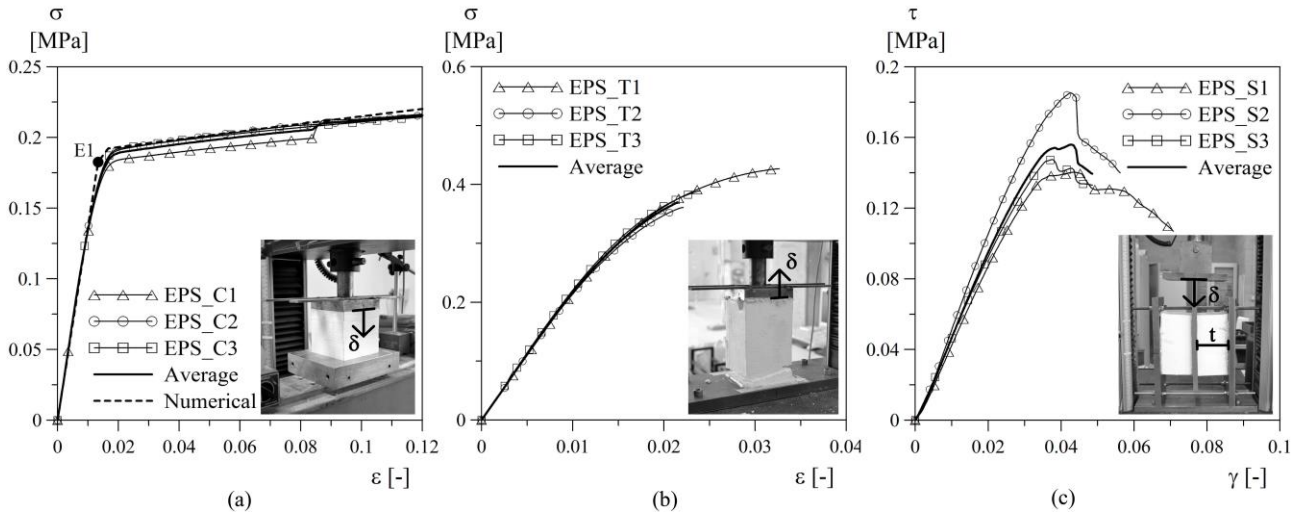
146 **Fig. 2** TRC mechanical behaviour: results of tests in uniaxial tension (a) and bending (b). “TRC\_const. law” curve and  
 147 points “T1”, “T2” and “T3” (subfigure a) are discussed in Sections 3.1 and 4.1, and “Numerical” curve (subfigure b) is  
 148 discussed in Section 4.3.1.

### 149 2.3 Expanded polystyrene: uniaxial compression, uniaxial tension and shear

150 The expanded polystyrene foam used is commercially known as EPS250. According to EN 13163 [29], it is characterised  
 151 by a compressive strength of 0.25 MPa at a strain equal to 10%.

152 Six nominally identical specimens with dimensions 100 x 100 x 150 mm<sup>3</sup> were cut from a larger mat of EPS: three were  
 153 tested in uniaxial compression and three in uniaxial tension by gluing their ends to the testing devices; all tests were  
 154 performed using the same electromechanical press described above. The tests were displacement controlled by imposing  
 155 a constant displacement rate of the machine cross-head equal to 1e-3 mm/sec. Test results are shown in Figure 3(a) and  
 156 (b) respectively for tension and compression in terms of nominal stress ( $\sigma = P/A$ , with  $P =$  load and  $A =$  unloaded  
 157 specimen cross section) versus nominal strain ( $\varepsilon = \delta/h$ , with  $\delta =$  top displacement and  $h =$  specimen height) curves. The  
 158 initial slope of the compressive  $\sigma$ - $\varepsilon$  curve, equal to 13.7 MPa, gives an estimation of the elastic modulus in compression.  
 159 It is possible to observe that the compressive behaviour is elasto-hardening, while the uniaxial tensile behaviour is elasto-  
 160 brittle, with a higher strength in tension rather than in compression.

161 Three shear tests were performed on EPS specimens in order to determine the  $\tau$ - $\gamma$  shear constitutive law. A proper test  
 162 set-up was adopted according to EN 12090 [30] following the prescriptions for the double test specimen assembly. The  
 163 test results are plotted in Figure 3(c) in terms of nominal shear stress ( $\tau = P/2A$ , with  $P =$  load and  $A =$  specimen cross-  
 164 section) vs. nominal strain ( $\gamma = \delta/t$ , with  $t =$  specimen thickness) curves. The average maximum shear stress ( $\bar{\tau}$ ) measured  
 165 is equal to 0.16 MPa. An estimation of the shear modulus  $G$  can be defined as the initial slope of the experimental  $\tau$ - $\gamma$   
 166 curve and is equal to 5.04 MPa.



167

168

**Fig. 3** EPS mechanical behaviour: results of tests in uniaxial compression (a), uniaxial tension (b) and shear (c).

169

“Numerical” curve and point “E1” (subfigure a) are discussed in Section 4.3.2.

170

### 3 Analytical model for sandwich beams

171

Classical Bernoulli beam theory cannot be used to predict the behaviour of a sandwich panel because of the large shear deformability of the composite. Stamm and Witte [15] proposed an analytical model to predict the behaviour of sandwich beams characterised by faces with no negligible bending stiffness if compared to the one of the whole beam: a previous model, whose solution is due to Plantema [16], was implemented by superimposing the local bending state of each external layer to the membrane state of stress of these outer layers due to sandwich global behaviour. The model is based on the following assumptions:

177

- the sandwich is characterised by faces with no negligible bending stiffness if compared to the one of the whole sandwich beam; for this reason the local bending state of each external layer is superimposed to the membrane state of stress of these outer layers due to sandwich global behaviour. The distribution of stresses considered is shown in Figure 4(a);

181

- the shear stiffness of the outer layers is large, hence their shear deformations  $\gamma_{xz}^{face}$  can be neglected; the cross sections of the outer layers thus remain planar and perpendicular to the axis even after the deformation (Bernoulli hypothesis), see Figure 4(b);

184

- the core is soft if compared with the outer faces, hence  $\sigma_x^{core}$  can be taken equal to zero, while  $\tau_{xz}^{core}$  is constant (Figure 4(a));

186

- due to the shear deformation of the core, the total cross section of the sandwich beam is not plane, but it deforms to a broken line (as shown in Figure 4(b));

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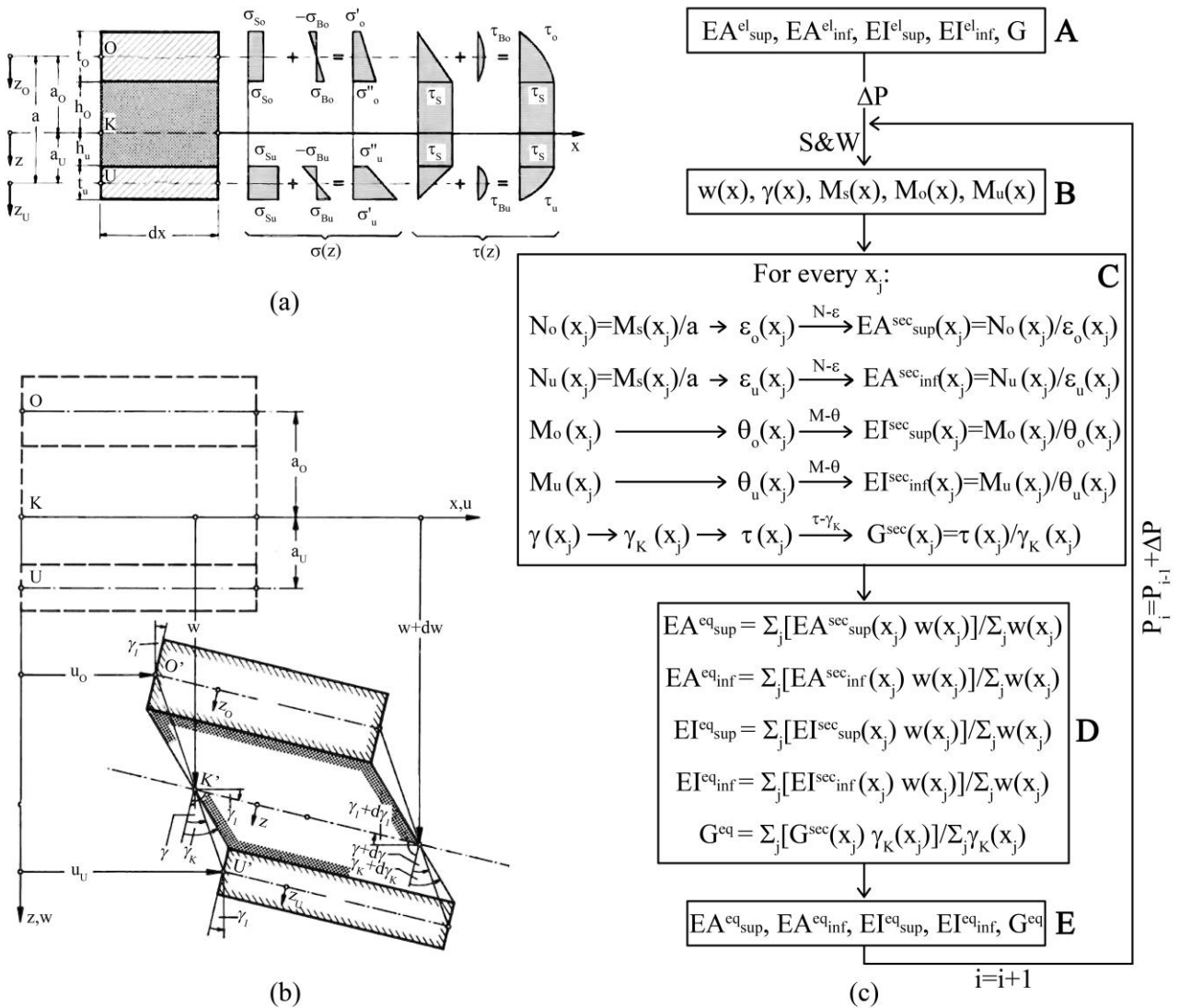
- the sandwich panel is calculated as a one-dimension structure, e.g. a beam;



189 - small displacements and deformations are considered.

190 Stamm and Witte [15] provided the linear elastic solution of the problem considering Hooke's Law for both core and  
 191 faces and taking into account different boundary and loading conditions.

192 An analytical solution has been implemented by the authors of the present paper following the same approach proposed  
 193 by Shams et al. [17] in order to take into account the non-linear material behaviour in the Stamm and Witte model. The  
 194 beam is divided into a finite number of elements with equal length and both axial and bending stiffness are assigned to  
 195 each element accounting for non-linearity according to a secant stiffness approach. The differential equations of the  
 196 Stamm and Witte model are solved by using constant and homogeneous equivalent stiffness values properly defined for  
 197 each layer. A flow chart representing the steps of the analytical model is proposed in Figure 4(c). All symbols included  
 198 in Figure 4 are collected in Table 1.



199  
 200 **Fig. 4** Stress distribution (a) and deformed configuration (b) adopted [15]; flow chart of the analytical model (c).  
 201

202 Symbols are collected in Table 1.



a	distance between the centroidal axis of the upper and lower TRC layer
$a_o$	distance between the centroidal axis of the beam and the centroidal axis of the upper TRC layer
$a_u$	distance between the centroidal axis of the beam and the centroidal axis of the lower TRC layer
b	width of the beam
$EA^{el}_{inf}$	initial elastic axial stiffness of lower TRC layer
$EA^{el}_{sup}$	initial elastic axial stiffness of upper TRC layer
$EA^{eq}_{inf}$	equivalent axial stiffness of lower TRC layer
$EA^{eq}_{sup}$	equivalent axial stiffness of upper TRC layer
$EA^{sec}_{inf}(x)$	axial stiffness of lower TRC layer defined through a secant approach at coordinate x
$EA^{sec}_{sup}(x)$	axial stiffness of upper TRC layer defined through a secant approach at coordinate x
$EI^{el}_{inf}$	initial elastic bending stiffness of lower TRC layer
$EI^{el}_{sup}$	initial elastic bending stiffness of upper TRC layer
$EI^{eq}_{inf}$	equivalent bending stiffness of lower TRC layer
$EI^{eq}_{sup}$	equivalent bending stiffness of upper TRC layer
$EI^{sec}_{inf}(x)$	bending stiffness of lower TRC layer defined through a secant approach at coordinate x
$EI^{sec}_{sup}(x)$	bending stiffness of upper TRC layer defined through a secant approach at coordinate x
G	initial shear modulus of the core material
$G^{eq}$	equivalent shear modulus of the core material
$G^{sec}(x)$	shear modulus of the core defined through a secant approach at coordinate x
h	thickness of the core
$h_o$	thickness of the part of the core above the centroidal axis
$h_u$	thickness of the part of the core below the centroidal axis
i	<i>i</i> -th step
K	point on the centroidal axis of the core
K'	point on the centroidal axis of the core in the deformed configuration
M- $\theta$	bending moment vs. curvature relationship
$M_o(x)$	bending moment of the upper TRC layer at coordinate x $M_o = \int_{-t_o/2}^{t_o/2} \sigma_{Bo}(z_o)bz dz_o$
$M_s(x)$	bending moment due to the sandwich action at coordinate x $M_s = \int_{-t_o/2}^{t_o/2} \sigma_{So}(z_o)bz dz_o + \int_{-t_u/2}^{t_u/2} \sigma_{Su}(z_u)bz dz_u$
$M_u(x)$	bending moment of the lower TRC layer at coordinate x $M_u = \int_{-t_u/2}^{t_u/2} \sigma_{Bu}(z_u)bz dz_u$
N- $\epsilon$	axial force vs. strain relationship
$N_o(x)$	axial force in the upper TRC layer due to sandwich action at coordinate x: $N_o = \int_{-t_o/2}^{t_o/2} \sigma_{So}(z_o)b dz_o$
$N_u(x)$	axial force in the lower TRC layer due to sandwich action at coordinate x: $N_u = \int_{-t_u/2}^{t_u/2} \sigma_{Su}(z_u)b dz_u$
O	point on the centroidal axis of the upper TRC layer
O'	point on the centroidal axis of the upper TRC layer in the deformed configuration
$P_i$	applied load at <i>i</i> -th step
$P_{i-1}$	applied load at <i>i+1</i> step
S&W	application of the Stamm and Witte model
$t_o$	thickness of the upper TRC layer
$t_u$	thickness of the lower TRC layer
U	point on the centroidal axis of the lower TRC layer
U'	point on the centroidal axis of the lower TRC layer in the deformed configuration
u	displacement along the x-axis
$u_o$	displacement of point O along the x-axis
$u_u$	displacement of point U along the x-axis
w	displacement of point K along the z-axis
$w(x)$	vertical displacement at coordinate x
x	coordinate along the horizontal axis
$x_j$	coordinate of the central point of the <i>j</i> -th element in which the beam is discretised
z	coordinate along the axis which is orthogonal to the centroidal axis of the beam
$z_o$	coordinate along the axis perpendicular to the centroidal axis of the upper TRC layer
$z_u$	coordinate along the axis perpendicular to the centroidal axis of the lower TRC layer
$\gamma(x)$	global rotation at coordinate x
$\gamma_K(x)$	shear strain of the core at coordinate x $\gamma_K = \frac{a}{h} \gamma$

$\gamma_1(x)$	rotation of the upper and lower TRC layers at coordinate x
$\Delta P$	increment of load
$\varepsilon_o(x)$	axial strain of the upper TRC layer at coordinate x
$\varepsilon_u(x)$	axial strain of the lower TRC layer at coordinate x
$\theta_o(x)$	curvature of the upper TRC layer at coordinate x
$\theta_u(x)$	curvature of the lower TRC layer at coordinate x
$\sigma(z)$	normal stress of the sandwich beam at generalised position z
$\sigma_{Bo}$	maximum normal stress of the upper TRC layer due to $M_o$
$\sigma_{Bo}(z_o)$	maximum normal stress of the upper TRC layer due to $M_o$ at position $z_o$
$\sigma_{Bu}$	maximum normal stress of the lower TRC layer due to $M_u$
$\sigma_{Bu}(z_u)$	maximum normal stress of the lower TRC layer due to $M_u$ at position $z_u$
$\sigma'_o, \sigma''_o$	minimum and maximum normal stress of the upper TRC layer due to both $M_o$ and $M_S$
$\sigma'_u, \sigma''_u$	maximum and minimum normal stress of the lower TRC layer due to both $M_u$ and $M_S$
$\sigma_{So}$	normal stress of the upper TRC layer due to $M_S$
$\sigma_{So}(z_o)$	normal stress of the upper TRC layer due to $M_S$ at position $z_o$
$\sigma_{Su}$	normal stress of the lower TRC layer due to $M_S$
$\sigma_{Su}(z_u)$	normal stress of the lower TRC layer due to $M_S$ at position $z_u$
$\tau\text{-}\gamma_K$	shear stress vs. shear strain relationship of the core
$\tau(x)$	shear stress of the core at coordinate x
$\tau(z)$	shear stress of the beam at generalised position z
$\tau_{Bo}$	maximum shear stress of the upper TRC layer due to the bending of the layer itself
$\tau_{Bu}$	maximum shear stress of the lower TRC layer due to the bending of the layer itself
$\tau_o$	shear stresses in the upper TRC layer at position $z_o$
$\tau_S$	shear stresses of the core
$\tau_u$	shear stresses in the lower TRC layer at position $z_u$

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204

205 At the beginning, the initial elastic stiffness values of both TRC layers ( $EA^{el}_{sup}$ ,  $EA^{el}_{inf}$ ,  $EI^{el}_{sup}$  and  $EI^{el}_{inf}$ ) and the core  
206 initial shear modulus  $G$  are imposed (step **A** – Figure 4(c)). The load, initially null, is incremented by  $\Delta P$  and the values  
207 of the vertical displacement  $w$ , the rotation  $\gamma$ , the bending moments  $M_s$ ,  $M_o$  and  $M_u$  are computed according to the Stamm  
208 and Witte classical model as a function of the coordinate  $x$  along the beam length (step **B** – Figure 4(c)).  
209 Once the material behaviour becomes non-linear, membrane and bending stiffness values of each element are computed  
210 by means of a secant approach referring to generalised constitutive laws ( $M\text{-}\theta$ , with  $\theta$  = curvature, and  $N\text{-}\varepsilon$ ) for the TRC  
211 layers and to a shear ( $\tau - \gamma_K$ ) constitutive relationship for the EPS (step **C** – Figure 4(c)). A proper description of these  
212 constitutive relationships is provided in Section 3.1. Referring to a generic  $i$ -th step, for TRC layers, the stiffness values  
213 at position  $x$  are defined starting from  $M$  and  $N$  evaluated at the previous ( $i - 1$ ) step for that position. Generalised  
214 constitutive laws allow to define the corresponding  $\theta$  and  $\varepsilon$  respectively and, therefore, axial ( $EA^{sec}=N/\varepsilon$ ) and bending  
215 ( $EI^{sec}=M/\theta$ ) stiffness. A similar approach is adopted for EPS:  $\gamma$ , and hence  $\gamma_K$ , are computed from the model at step ( $i$  -  
216 1); the ( $\tau - \gamma_K$ ) constitutive law allows to define the corresponding value of  $\tau$  and, therefore, secant shear modulus at  $i$ -th  
217 step is defined as  $G^{sec} = \tau/\gamma_K$ . It is worth noting that, in the model, axial and bending actions acting on the external faces  
218 are uncoupled.  
219 Stamm and Witte differential equations ask for unique stiffness for each layer; in order to define equivalent stiffness to  
220 be used, the local stiffness values evaluated for each element are weighted according to different response parameters  
221 (steps **D-E** - Figure 4(c)). In particular, equivalent values of axial and bending stiffness ( $EA^{eq}$  and  $EI^{eq}$ ) of TRC are

222 evaluated by using the displacement  $w$  as weighting function, while equivalent shear modulus ( $G^{eq}$ ) of EPS considers  
223 shear deformation as a weighting function.

224 It is worth to underline that a strong assumption of the Stamm and Witte model is to consider the same rotation for the  
225 upper and the lower TRC layers (see Figure 4(b)).

226 Moreover, in the implemented model, global stiffness values of each layer derive from an average process, along the  
227 beam axis, of local stiffness values defined according to constitutive models. This process leads to global stiffness values  
228 that do not directly respect the generalised constitutive laws at each point.

### 229 **3.1 Generalised constitutive relationships**

230 The constitutive relationships introduced in the analytical model are described in this section.

231 The axial and the bending behaviour of TRC are described by means of generalised constitutive models and, as already  
232 discussed, no interaction is considered between them. As discussed in the following (Section 3.2, Figure 7), this  
233 assumption does not lead to any influence in considering the membrane stiffness of the TRC layers, but may cause a  
234 difference in the computation of the bending stiffness of these layers, especially in the bottom one that is subjected to  
235 tension. Nevertheless, it can be demonstrated that this difference has a negligible effect (<5%) on the global response of  
236 the sandwich.

237 Regarding the axial behaviour, axial force (N) vs. axial strain ( $\epsilon$ ) relationship is considered.

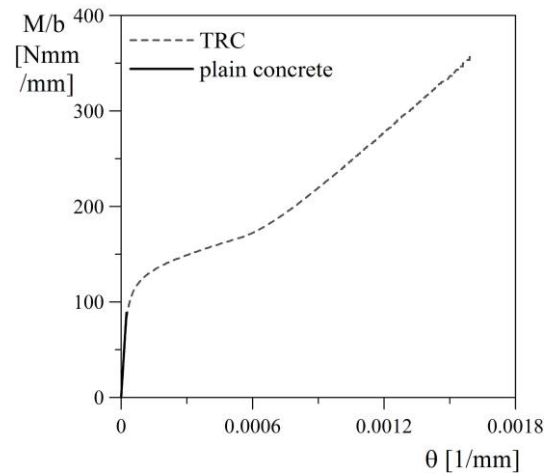
238 In compression, an elastic behaviour is assumed for TRC, considering an elastic modulus of 30 GPa (according to  
239 literature results on cement matrix characterised by similar compressive strength and maximum aggregate size [31]).

240 In tension, the stress-strain behaviour shown in Figure 2(a) is assumed for TRC (see “TRC\_const. law” curve).

241 The bending behaviour of the TRC external layers is described by  $M$ - $\theta$  diagrams built considering the plane section  
242 kinematic assumption [32].

243 In Figure 5 the  $M$ - $\theta$  relationships assigned to the bending behaviour of external layers are shown. Two relationships  
244 representing extreme situations are proposed: a lower bound situation in which the fabric contribution in bending is  
245 neglected (“plain concrete”, this represents the situation in which fabric is always on the neutral axis), and a situation in  
246 which the contribution of the fabric is accounted and spread all over the thickness of the layer (“TRC”). In both cases, the  
247 compressive behaviour is described through a parabolic-rectangular stress-strain relationship [33] considering a maximum  
248 strength  $f_{cm}$  equal to 71.86 MPa (average cubic compressive strength experimentally measured); this strength is reached  
249 at a strain equal to 0.002, while the failure takes place at a strain equal to 0.0035. In tension, when neglecting the fabric  
250 contribution, the formulation proposed by MC2010 [33] for plain concrete is adopted, considering an average tensile  
251 strength of 4.1 MPa (C50 class of concrete) and the same elastic modulus of 30 GPa assumed above. When the fabric

252 contribution is taken into account, the tensile constitutive law  $\sigma$ - $\varepsilon$  obtained from tensile tests (Figure 2(a)) is adopted to  
253 build the M- $\theta$  diagram.



254

255 **Fig. 5** M- $\theta$  constitutive relationships used in the analytical model.

256 The EPS ( $\tau$  -  $\gamma_K$ ) relationship adopted in the model is assumed to be elasto-perfectly plastic. The values of shear modulus  
257  $G$  and maximum shear stress  $\bar{\tau}$  are identified from the experimental tests presented in Section 2.3; the average values  
258 equal to 5.04 and 0.16 MPa are used respectively for  $G$  and  $\bar{\tau}$ . These values are close to those found in literature:  
259 considering the empirical correlations deduced by Gnip et al. [34], that relate the density of EPS with shear modulus and  
260 ultimate strength, a shear modulus equal to 4.64 MPa and a maximum shear stress equal to 0.144 MPa could be expected.

### 261 3.2 Analytical results

262 The results of the analytical model are shown in Figure 6 in terms of load ( $P$ ) versus displacement ( $\delta$ ) curves for both the  
263 deep and slender sandwich beams. For each beam size, two curves are proposed, depending on the M- $\theta$  relationship used  
264 in the model (plain concrete "PC" or textile reinforced concrete "TRC"). In the figure, the analytical responses are  
265 compared with the experimental results already presented and discussed in [26].

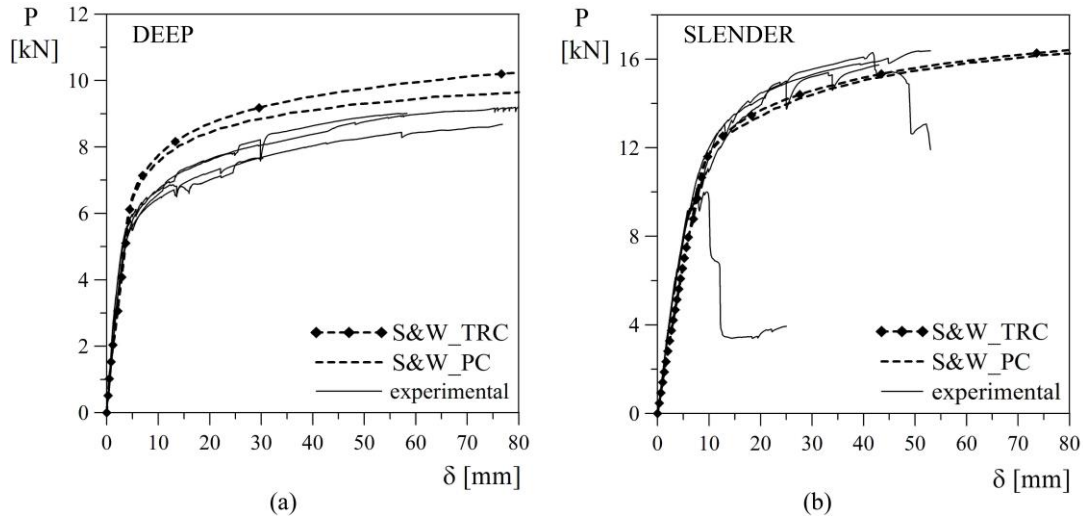
266 The small membrane compressive stress achieved in the upper TRC layer ( $\sigma_c < 8$  MPa) justifies the linear elastic  
267 assumption for this behaviour.

268 In the case of deep sandwich beams, both the predictions overestimate the specimen behaviour. For slender sandwich  
269 beams, the analytical predictions are practically overlapped to the experimental curves.

270 Even if the contribution of the fabric on the bending behaviour of TRC faces is relevant, as it can be observed in the M-  
271  $\theta$  diagram (see Figure 5), the difference between the global response either neglecting or considering TRC bending  
272 contribution is limited. This result confirms the experimental evidence [26] according to which the fabric position within  
273 the external layers has a low influence on the global sandwich response, only affecting the crack pattern of the specimen.

274 In fact, the fabric location can affect the M- $\theta$  curve reducing the post-peak significantly when placed at the TRC layer

275 extrados. This effect could locally reduce the stiffness, but the average approach selected can profit of the stiffness of the  
276 still un-cracked regions.



277

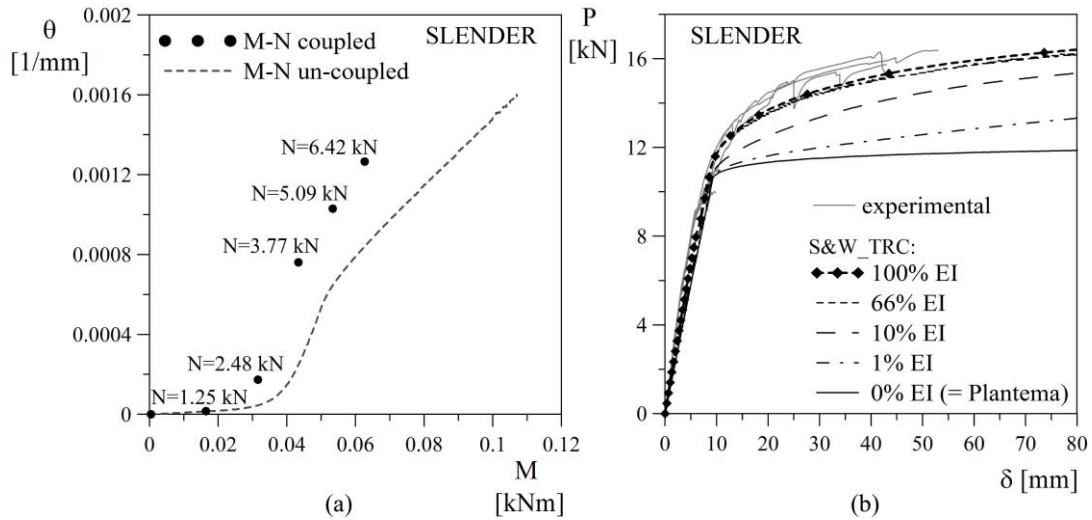
278 **Fig. 6** Analytical prediction for deep (a) and slender (b) sandwich beam compared with the experimental results [26].

279

280 In Figure 7, the effect of the M-N interaction is discussed (slender sandwich beams are taken into account). Figure 7(a)  
281 compares the relationships between moment and curvature whether or not considering the interaction between M and N.  
282 In particular, the  $\theta$ -M diagram adopted in the model that neglects this interaction is compared with some points that have  
283 been obtained considering a coupled approach. In the definition of these points, for a given value of M and N, both  
284 rotational and translational equilibrium were imposed in order to define the corresponding curvature  $\theta$ . These M-N points  
285 were directly selected from the results of “S&W\_TRC” model previously presented. This comparison shows that the un-  
286 coupled approach, which adopts the secant approach, underestimates the curvature thus causing an overestimation of the  
287 bending stiffness of about 34%.

288 The effect of a reduced bending stiffness is shown in Figure 7b, in which the sandwich responses corresponding to  
289 different bending stiffness reductions are considered. In particular, the situation with 0% EI corresponds to the Plantema  
290 solution in which the external layers just provide a membrane contribution. The case of 66% EI represents the bending  
291 stiffness reduction due to M-N interaction previously discussed. It is worth noting that this curve just considers a uniform  
292 stiffness reduction along the beam without taking into account the real membrane stress variation; therefore, it represents  
293 a lower bound of the sandwich response. For this reason and because of the small difference between the 100% EI and  
294 66% EI responses, the M-N un-coupled approach can be considered as reliable.

295 It is worth noting that the model is not able to predict the specimen failure as no failure criterion is introduced.



296

297

298

**Fig. 7** Influence of M-N interaction for slender sandwich beam: effect on curvature (a) and effect of TRC stiffness on global response (b).

299

#### 4 FE model for sandwich beams

300

A prediction of the behaviour of the sandwich beams was also performed by means of 3D numerical FE models, developed with the finite element program ABAQUS/Standard 6.12.

302

The constitutive laws used in the Abaqus finite element models for textile reinforced concrete and expanded polystyrene are presented and discussed in Sections 4.1 and 4.2. Then, the reliability of these adopted relationships is validated in

304

Section 4.3, with reference to experimental tests performed on TRC and EPS specimens, whose results have been already shown in Sections 2.2 and 2.3 respectively. The sandwich beam model description is provided in Section 4.4 and the

306

finite element analysis results are provided and discussed in Section 4.5.

307

##### 4.1 TRC layer behaviour

308

The elastic phase is defined through two parameters: the Young's modulus, assumed equal to 30 GPa according to literature results on cement matrix characterised by similar compressive strength and maximum aggregate size [31]; the Poisson's ratio, assumed equal to 0.2.

311

Plasticity is accounted by Concrete Damage Plasticity model [35], which is implemented in Abaqus. The model is a continuum plasticity-based model for concrete. It assumes that the main two failure mechanisms are tensile cracking and compressive crushing of the concrete material. The evolution of the yield (or failure) surface is controlled by two hardening variables,  $\tilde{\epsilon}_t^{pl}$  (tensile equivalent plastic strain) and  $\tilde{\epsilon}_c^{pl}$  (compressive equivalent plastic strain), linked to failure mechanisms under tension and compression loading, respectively [36, 37].

316

Because of the monotonic load considered, no damage curves were introduced; for this reason the model simply behaves as a plasticity model.

317

318 The compressive behaviour is assumed to be elasto-perfectly plastic, with a yield stress equal to 72 MPa according to  
319 experimental results presented in Section 2.2.

320 The plastic tensile behaviour is defined by a stress-strain relationship, already discussed in Section 2.2 (see Figure 2(a),  
321 “TRC\_const. law”). Some relevant points typical of the TRC tensile behaviour are highlighted in the figure on the  
322 constitutive law curve: T1 corresponds to the beginning of the multi-cracking branch; T2 represents the point after which  
323 only the contribution of fabric and tension stiffening are acting; and T3 corresponds to the brittle failure of the fabric.

324 In the Concrete Damage Plasticity model the tensile behaviour of TRC layers is assumed homogeneous over the layer  
325 thickness. The reliability of this assumption is discussed in Section 4.3.1.

## 326 4.2 EPS layer behaviour

327 The elastic phase of EPS is defined by introducing a Young's modulus equal to 13.7 MPa (Section 2.3) and a Poisson's  
328 ratio equal to 0.1.

329 To account for plasticity, Crushable Foam model with volumetric hardening implemented in ABAQUS is used [36]. The  
330 phenomenological isotropic model was originally developed, for metallic foams, by Deshpande and Flek [38]. The model  
331 assumes that the evolution of the yield surface is controlled by the volumetric compacting plastic strain experienced by  
332 the material.

333 This model has also a pressure-dependent yield surface with an elliptical shape in the meridional ( $p$ - $q$ ) stress plane (with  
334  $p$  = hydrostatic stress and  $q$  = Von Mises stress) and a Von Mises circle in the deviatoric stress plane.

335 The parameters that define this yield surface are the yield stress in uniaxial compression ( $\sigma_c^0$ ), the ratio  $k$  between the  
336 uniaxial ( $\sigma_c^0$ ) and the hydrostatic ( $p_c^0$ ) compressive strength and the ratio  $k_t$  between the hydrostatic tensile ( $p_t$ ) and the  
337 hydrostatic compressive ( $p_c^0$ ) strength.

338 The compression yield stress ratio  $k$  can vary between 0 and 3, and the hydrostatic yield stress ratio  $k_t$  has to be equal or  
339 higher than 0.

340 The evolution of the yield surface follows the volumetric hardening rule, which is controlled by the volumetric plastic  
341 strain ( $-\varepsilon_{vol}^{pl}$ ) experienced by the material. The hardening law is introduced in the model by using the uniaxial  
342 compression test data (uniaxial compression yield stress as a function of axial plastic strain), considering the fact that, in  
343 uniaxial compression,  $\varepsilon_{axial}^{pl} = \varepsilon_{vol}^{pl}$ .

344 The model parameters were set in order to impose the initial yielding surface to satisfy the following conditions:

- 345 • the uniaxial compressive yield stress ( $\sigma_c^0$ ) is equal to that obtained from the experimental tests in uniaxial  
346 compression at the end of the initial linear branch (average value obtained from three tests on nominally identical

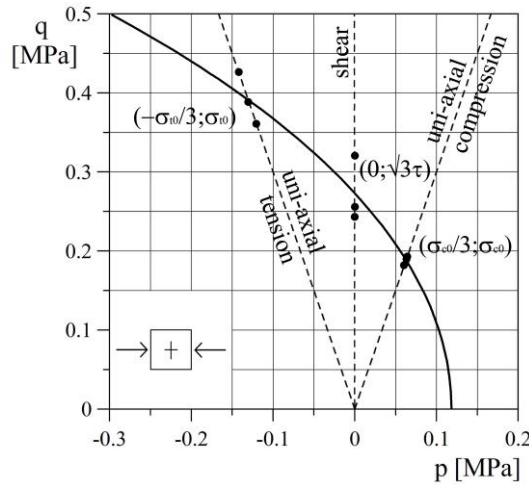


347 specimens:  $\sigma_{c_{av}}^0 = 0.188 \text{ MPa}$ ). This condition means that the initial yield surface passes through the point  
 348 ( $p = \sigma_{c_{av}}^0/3; q = \sigma_{c_{av}}^0$ );

- 349 • the uniaxial tensile yield stress ( $\sigma_t^0$ ) is equal to that obtained from the experimental tests in uniaxial tension at  
 350 the peak (average value obtained from three tests on nominally identical specimens:  $\sigma_{t_{av}}^0 = 0.392 \text{ MPa}$ ). This  
 351 condition means that the initial yield surface passes through the point ( $p = -\sigma_{t_{av}}^0/3; q = \sigma_{t_{av}}^0$ );
- 352 • the shear yield stress ( $\tilde{\tau}$ ) is equal to that obtained from the experimental shear tests (average value obtained from  
 353 three tests on nominally identical specimens:  $\tilde{\tau}_{av} = 0.160 \text{ MPa}$ ). This condition means that the initial yield  
 354 surface passes through the point ( $p = 0; q = \sqrt{3}\tilde{\tau}_{av}$ ).

355 The resulting ratios  $k$  and  $k_t$  are equal to 1.59 and 54.30 respectively; the corresponding initial yield surface plotted in the  
 356  $p$ - $q$  stress plane is shown in Figure 8 (solid line). Experimental results performed on EPS are shown in the figure through  
 357 round markers.

358 It is worth noting that the material simulated by this model has a hardening behaviour in uniaxial tension, which does not  
 359 comply with the brittle failure of EPS in tension; hence, it is necessary to verify if the EPS tensile strength is exceeded or  
 360 not, in order to check if a tensile failure of the core occurs.



361  
 362 **Fig. 8** Crushable Foam model: adopted initial yield surface (solid line) in the  $p$ - $q$  plane for  $k=1.59$  and  $k_t=54.30$ : round  
 363 markers represent experimental results; dashed lines correspond to uniaxial tension, uniaxial compression and shear  
 364 domain.

### 365 4.3 Reliability assessment of the constitutive models

366 This section aims to discuss the reliability of the constitutive models adopted and some crucial assumptions taken into  
 367 account in the FEM analysis of the sandwich beams presented.

368 In particular, two main issues are discussed: (a) the use of a homogeneous tensile behaviour of TRC layers over the  
 369 thickness even if bending moment is acting; (b) the choice of the hardening curve assumed for the EPS model.

### 370 4.3.1 Modelling of the bending behaviour of TRC layer

371 As already discussed, the tensile behaviour of TRC layers is assumed homogeneous over the thickness and, in particular,  
 372 the uniaxial tensile constitutive law derived from uniaxial tensile tests (Section 2.2) is applied to the whole concrete layer  
 373 disregarding the real fabric position.

374 The uniaxial tensile constitutive law is introduced in Abaqus referring to plastic strain. The curve adopted is reported in  
 375 Figure 2(a) (“TRC\_const. law”).

376 In order to assess the reliability of this choice, bending tests performed on TRC specimens (Section 2.2) have been  
 377 modelled in ABAQUS by considering this assumption.

378 The specimen is modelled as a solid homogeneous section and it is discretised by means of 8-node linear brick elements  
 379 (C3D8R). The characteristics of the finite element mesh related to TRC are collected in the second column of Table 2; in  
 380 particular, the number of nodes, the number of elements, the number of elements over the thickness and the maximum  
 381 aspect ratio (ratio between the longest and shortest edge of an element) are specified.

382 The load-displacement curve resulting from the simulation is superimposed to the experimental results in Figure 2(b).  
 383 Looking at the figure it is possible to observe that the numerical response is in good agreement with the experimental  
 384 behaviour of TRC in bending. This result shows that considering TRC as a homogeneous material over a thickness of 10  
 385 mm allows to adequately predict the bending behaviour of TRC.

386 **Table 2** Mesh characteristics of FE models

Model	TRC (bending)	EPS (compression)	Deep sandwich beam		Slender sandwich beam	
Nodes	4692	58956	TRC <sub>sup</sub>	10400	TRC <sub>sup</sub>	15128
			EPS	28600	EPS	41602
			TRC <sub>inf</sub>	10400	TRC <sub>inf</sub>	15128
Elements	3264	54450	TRC <sub>sup</sub>	7425	TRC <sub>sup</sub>	10890
			EPS	24750	EPS	36300
			TRC <sub>inf</sub>	7425	TRC <sub>inf</sub>	10890
Elements over the thickness	3	50	TRC <sub>sup</sub>	3	TRC <sub>sup</sub>	3
			EPS	10	EPS	10
			TRC <sub>inf</sub>	3	TRC <sub>inf</sub>	3
Max. aspect ratio	1.9	1.0	TRC <sub>sup</sub>	1.9	TRC <sub>sup</sub>	3.1
			EPS	1.9	EPS	1.1
			TRC <sub>inf</sub>	2.3	TRC <sub>inf</sub>	3.1

387  
 388 It is worth mentioning that no failure criterion is introduced in the model, therefore Abaqus code maintains the last stress  
 389 level of the constitutive curve when the maximum strain of the constitutive law is exceeded.

390 Furthermore, the change of slope observable in the numerical curve at a load of about 0.5 kN corresponds to the situation  
 391 in which the maximum strain of the constitutive law (Figure 2(a),  $\varepsilon_{max} = 1.37e-2$ ) is reached at the intrados of the specimen.  
 392 After this point the numerical curve is represented by a dotted line.

393 The numerical curve is stopped when the maximum strain of the constitutive law (Figure 2(a),  $\varepsilon_{max} = 1.37e-2$ ) is reached  
394 at the mid-height of the cross-section, where the fabric is supposed to be placed.

#### 395 **4.3.2 Modelling of the compressive behaviour of EPS**

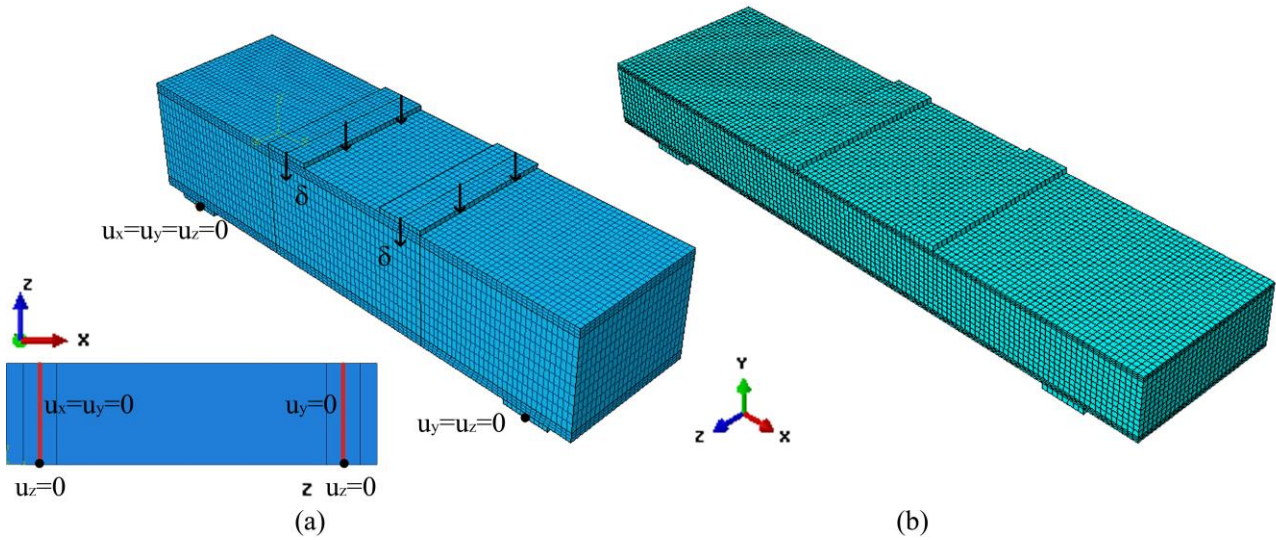
396 In order to assess the reliability of the parameters adopted for EPS and to validate the hardening law introduced, a  
397 numerical model of the uniaxial compressive tests made on EPS prisms described in Section 2.3 was performed. A solid  
398 homogeneous section discretised through 8-node linear brick elements (C3D8R) with a dimension  $3.03 \times 3.03 \times 3 \text{ mm}^3$   
399 was used; the mesh characteristics are collected in the third column of Table 2. In Figure 3(a) the numerical curve in terms  
400 of nominal stress vs. nominal strain (variables defined as in the experimental test results) is compared with experimental  
401 data and a good agreement is achieved. In the same figure point E1 is highlighted: once this point is reached, a significant  
402 change in the slope of the response is registered.

#### 403 **4.4 Sandwich beam model description**

404 Both deep and slender sandwich beams described in Section 2.1, having the size  $550 \times 150 \times 120 \text{ mm}^3$  and  
405  $1200 \times 300 \times 120 \text{ mm}^3$  respectively, are modelled in ABAQUS.

406 The TRC layers, the EPS layer and the steel plates are all modelled as homogeneous solid, assuming perfect bond at  
407 interfaces. The perfect bond assumed at the TRC/EPS interface, with no interface elements introduced, constitutes a strong  
408 assumption of the model, but, as no detachment was observed during experimental tests, this assumption is expected to  
409 be reliable. TRC layers, EPS layer and steel plates are discretised with 8-node linear brick elements (C3D8R). The  
410 characteristics of the finite element mesh are reported in Table 2 for both the models; in particular, the number of nodes,  
411 the number of elements, the number of elements over the thickness and the maximum aspect ratio are specified.

412 TRC and EPS layer behaviour is defined in the finite element software as specified in Sections 4.1 and 4.2, while steel  
413 plates used to reduce stress concentration in sandwich beam tests are assumed to be elastic, with a Young's modulus equal  
414 to 210 GPa and a Poisson's ratio equal to 0.3. The same boundary conditions were adopted for both panel geometries  
415 investigated. These boundary conditions are shown in Figure 9(a) taking the deep panel as a reference. The meshes of the  
416 models are represented in Figure 9(a) and (b) for the deep and slender panels respectively.



417 (a) (b)

418 **Fig. 9** FEM model of sandwich beams: geometry with constraints (a); mesh of deep (a) and slender (b) beam.

419  $\delta$  represents the imposed vertical displacement.

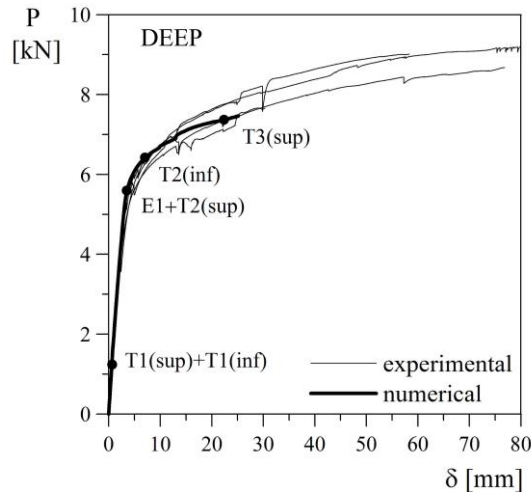
420 **4.5 FEM results**

421 **4.5.1 Deep sandwich beam**

422 The numerical result obtained for a deep sandwich beam is shown in Figure 10 in terms of load ( $P$ ) versus vertical  
 423 displacement ( $\delta$ ) curve. This curve is compared to the experimental results already presented in [26]: reliable prediction  
 424 of the experimental results is achieved.

425 In the figure, some relevant points are highlighted on the numerical curve. In particular, concerning the TRC tensile  
 426 constitutive law: point  $T1(sup) + T1(inf)$  indicates when the upper and the lower TRC layers start to crack only in one  
 427 point; points  $T2(sup)$  and  $T2(inf)$  indicate the end of the multi-cracking phase respectively for the upper and the lower  
 428 TRC layer at the intrados only in one point; point  $T3(sup)$  indicates the reaching of tensile strength at the intrados of the  
 429 upper TRC layer only in one point.

430 The numerical analysis confirms the experimental evidence that both TRC layers are cracked, which is indicative that the  
 431 deep beam was acting as a partially composite sandwich.



432

433

**Fig. 10** FEM results - vertical load vs. displacement curves for deep sandwich beam.

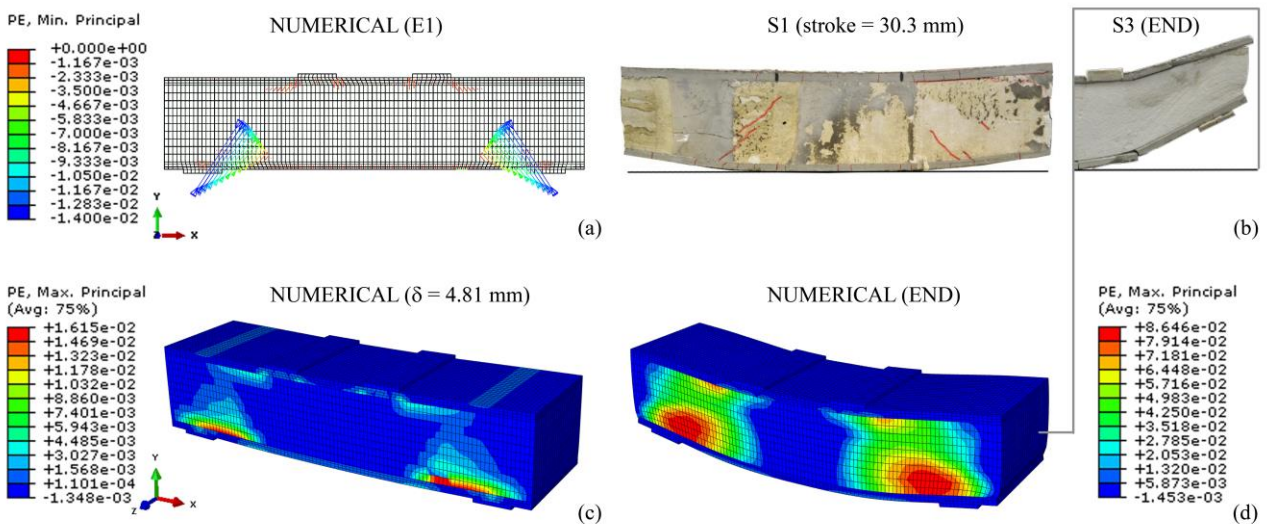
434

It is worth noting that, even if the TRC is cracked, the global response remains almost linear up to point *E1*, after which EPS foam starts to yield in compression. In fact, the significant change in the slope of the composite global response is related to the yielding of a compressive strut in the EPS layer when point *E1* is reached. This compressive strut is well visible in Figure 11(a), where the minimum principal plastic strains of EPS are shown, and in Figure 11(c) and (d), in which maximum principal plastic strains are plotted.

439

The numerical analysis stops because of the incompatibility of the deformations of EPS and TRC at the interface caused by the perfect bond assumption. EPS elements at the interface show very large deformations in order to maintain a perfect bond with TRC causing large distortions of the elements thus neglecting convergence of the numerical solution.

442



443

444

**Fig. 11** FEM model of deep sandwich beam - yielding of a compressive strut in the EPS layer: minimum principal

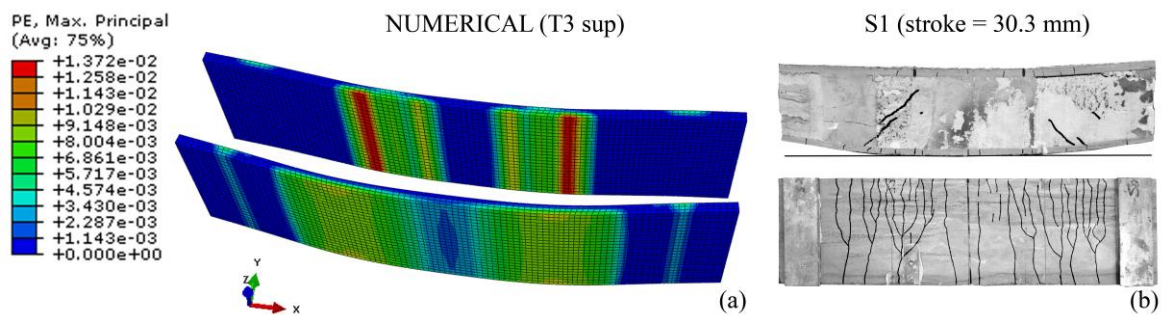
445

plastic strain at point *E1* (a), picture of specimen S1 and S3 after test (b), maximum principal plastic strain at

446

$\delta = 4.81\text{mm}$  (c) and maximum principal plastic strain at the end of the analysis (d) [COLOR PRINT]

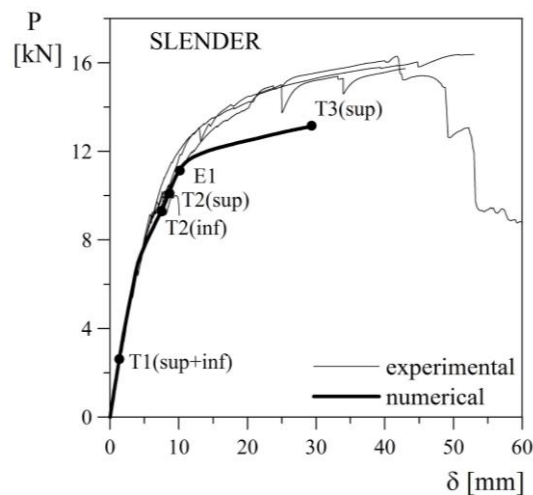
447 The multi-cracking phenomenon, observable in both the TRC layers, is shown in Figure 12 by displaying the maximum  
 448 principal plastic strains of TRC; in the same figure, the crack pattern of specimen S1 is shown (lateral and bottom view).  
 449 Comparing the pictures related to numerical analysis and experimental test, it is possible to state that the numerical  
 450 analysis well represents the regions involved in multi-cracking: in the upper TRC layer cracks form under the loading  
 451 knives, while the lower TRC layer is cracked along the whole length.  
 452 It is possible to state that the numerical solution well represents the behaviour of the composite sandwich beam, not only  
 453 in terms of load versus displacement curve, but also in terms of identification of the involved mechanisms. On the  
 454 contrary, the ductility is not adequately captured.  
 455 Nevertheless, because of the large displacement reached and the good prediction achieved, this approach can be  
 456 considered a valuable tool for the design of this kind of structures.



457  
 458 **Fig. 12** FEM model of deep sandwich beam - multi-cracking of both TRC layers: maximum principal plastic strain at  
 459 point *T3sup* (a) and pictures of specimen S1 at the end of the test (b): side (top) and intrados (bottom) view. [COLOR  
 460 PRINT]

#### 461 4.5.2 Slender sandwich beam

462 The results of the numerical analysis performed for the slender sandwich beam are shown in Figures 13, 14 and 15.  
 463

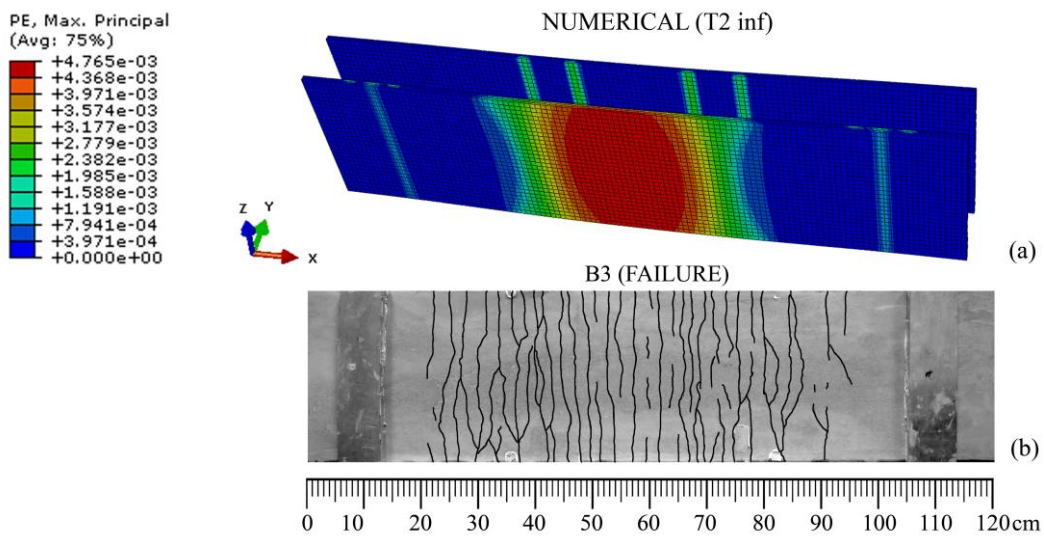


464  
 465 **Fig. 13** FEM results - vertical load vs. displacement curves for slender sandwich beam.



466 As done for deep sandwich beam, the load ( $P$ ) versus vertical displacement ( $\delta$ ) numerical curve is compared to the  
 467 experimental results, achieving quite a good agreement in terms of global response (Figure 13). A good superposition is  
 468 achieved in the initial linear phase, while the non-linear branch of the numerical response is lower than that of the  
 469 experimental curves; moreover, the model is not able to predict the ductility exploited by the beams. Looking at the  
 470 relevant points related to the material constitutive laws highlighted on the numerical response, it is possible to note that  
 471 after reaching point  $E1$  the global response becomes highly non-linear. As observed for the deep sandwich, the beam  
 472 behaves as a partially composite sandwich (Figure 14).

473



474

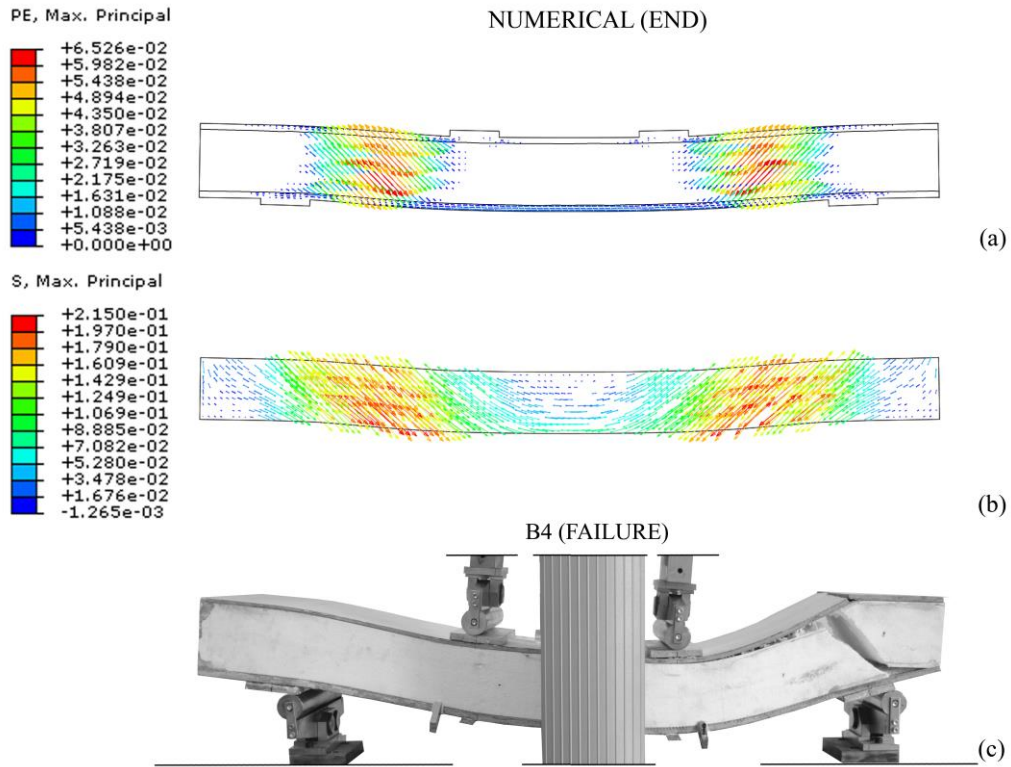
475 **Fig. 14** FEM model of slender sandwich beam - multi-cracking of both TRC layers: maximum principal plastic strain at  
 476 point  $T2inf$  (a) and picture of the intrados of specimen B3 at failure (b). [COLOR PRINT]

477

478 The yielding of a compressed strut in the EPS layer, which in the tests caused the tensile crack and the sudden failure of  
 479 the specimens (Figure 15(c)), is clearly shown in Figure 15 (a) and (b) by means of maximum principal plastic strains  
 480 and stresses.

481 Again, the numerical analysis stops because of the incompatibility of the deformations of EPS and TRC at the interface  
 482 caused by the perfect bond assumption, hence it is not possible to model the large ductility experienced by the specimens.





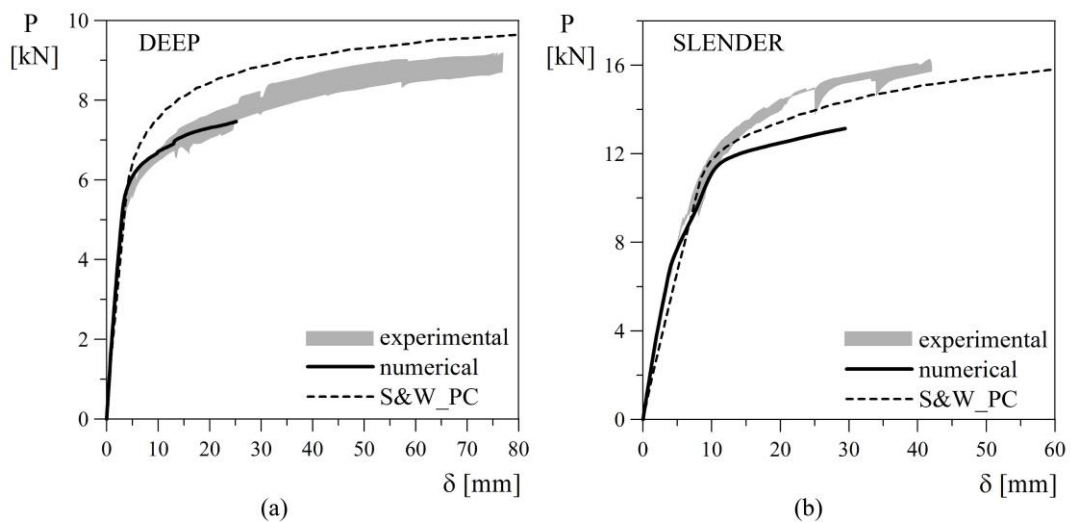
483

484 **Fig. 15** FEM model of slender sandwich beam – EPS failure: (a) maximum principal plastic strain and (b) maximum  
 485 principal stresses in the EPS layer at the end of the numerical analysis; (c) specimen B4 at failure. [COLOR PRINT]

486 **5 Comparison of results**

487 The results of the analytical and numerical analysis are summarised in Figure 16. In the same figure, the experimental  
 488 curves are plotted as a reference (grey shadow).

489



490

491 **Fig. 16** Comparison of experimental, analytical and numerical results for deep (a) and slender (b) sandwich beams.

492 Concerning the analytical response (dashed lines), it provides a reliable prediction of the results; in case of deep beam  
493 the model overestimates the maximum load of about 10% (Figure 16(a)), while for slender sandwich beam the solution  
494 is conservative (Figure 16(b)). As already discussed, the model is not able to predict the specimen failure as no failure  
495 criterion is introduced.

496 Looking at the numerical results (solid bolt lines), it is possible to state that the response well catches the experimental  
497 results especially in the case of deep specimens (Figure 16(a)), while in the case of slender sandwich beam an  
498 underestimation of the real behaviour in the plastic branch of about 15% can be observed (Figure 16(b)). In both deep  
499 and slender specimens, the ductility cannot be regarded as reliable because analysis stopped due to the large distortion of  
500 the elements at EPS-TRC interface.

## 501 **6 Conclusions**

502 The results of the analytical and numerical analysis suggest some interesting conclusions.

503 In particular, concerning the analytical modelling, the results of the analysis confirm the reliability of the assumptions  
504 imposed in order to evaluate the global response of the sandwich solution. For slender sandwich beam the analytical  
505 solution is conservative, while in case of deep beam the model overestimates the maximum load of about 10%. The results  
506 obtained through the analytical model show how the fabric position within the external layers has a low influence on the  
507 global sandwich response, which is mainly governed by the non-linear behaviour of EPS and by the membrane and  
508 bending behaviour of TRC.

509 Concerning the FE analysis, the choice of modelling TRC as a homogeneous material is reliable if the aim is to determine  
510 the global response of the solution; local information is not captured. The Crushable Foam model, with initial domain  
511 parameters identified from compressive, tensile and shear tests on EPS specimens, allows to properly predict the  
512 behaviour of EPS in compression and consequently to model the strut in the sandwich beams.

513 For both deep and slender sandwich beams, the perfect bond assumption is reliable in the initial part of the analysis and  
514 also when the sandwich behaviour becomes highly non-linear. However, this assumption does not allow to simulate the  
515 high ductility experienced by the specimens due to the large deformations that arise at TRC/EPS interface, thus creating  
516 convergence problems. The FEM model adequately predicts experimental results both in terms of global response and  
517 deformation modes. In the case of slender sandwich beam the numerical response is conservative, with an underestimation  
518 of load in the plastic branch of about 15% with respect to the real behaviour.

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