An Examination of Alternative Option Hedging Strategies in the Presence of Transaction Costs

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Abstract

Substantial progress has been made in developing option hedging models that account for transaction costs. Previous analyses of option hedging strategies in the presence of transaction costs use a Monte Carlo simulation framework in conjunction with a mean variance rule to compare different strategies. These studies being based on simulated stock price data are essentially theoretical tests. It is not known, however, how various proposed hedging strategies compare in terms of hedging precision and transaction costs when tested using actual market data. In addition, the mean variance rule is subject to certain well-known restrictive assumptions.

This thesis aims to fill two gaps in the literature, by: (1) using actual market data to examine hedging performance, and (2) using a stochastic dominance rule as an alternative hedging performance measure. I undertake two studies. The first compares hedging strategies using Monte Carlo simulation together with mean variance and stochastic dominance criteria. Simulation allows us to study the consistency of the hedging outcomes determined by criteria rules in a controlled environment. The second study is a comprehensive empirical investigation of the merits of competing option hedging strategies with transaction costs, using S&P 500 index options. Both studies examine the hedging performance of the delta-neutral hedge. Given the widely documented volatility risk in empirical data, I further supplement the empirical study with a delta–vega-neutral hedge.

Consistent with the literature, the Monte Carlo simulation demonstrates that move-based strategies are superior to time-based strategies. In contrast, empirical testing shows time-based strategies, in particular the Black-Scholes discrete time hedging strategy, are the optimal hedging strategies. Empirically, I find that a delta-neutral hedge is sufficient for a hedger to attain the optimal tradeoff between hedging precision and transaction costs paid if the hedger is using time-based strategies. I further demonstrate that a hedger can save a substantial amount of transaction costs by simply switching from a move-based strategy to a time-based strategy. A hedger is able to save an average 46% of the transaction costs associated with a poorly performing hedging strategy by simply switching to the optimal hedging strategy. I also show that mean variance and stochastic dominance comparisons are not always mutually consistent with each other; however, the differences are usually small. The rank of each strategy under either rule is highly dependent on the characteristics of the empirical distribution of the net hedging error. I also show that a stochastic dominance test provides a precise ranking of hedging performance for each hedging strategy only when there are strong dominance relationships among the strategies, that is, when the empirical density functions of net hedging error for each of the strategies are sufficiently different. The comparisons presented in my study strengthen the confidence in the mean variance rule as a performance measure in assessing hedging outcomes in the presence of transaction costs. The findings of my thesis will assist financial institutions in making informed hedging decisions when transaction costs are taken into consideration.

Declaration

This is to certify that:

(i) the thesis comprises only my original work towards the PhD except where indicated in the Preface,

(ii) due acknowledgement has been made in the text to all other material used,

(iii) the thesis is fewer than 100,000 words in length, exclusive of tables, maps bibliographies and appendices.

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Table of Contents

Tables

Figures

Acronyms and Abbreviations

AMEX	American Stock Exchange
AT	Henrotte's asset tolerance strategy
ATM	at-the-money
BS	Black–Scholes hedge at fixed time intervals
CBOE	Chicago Board of Exchange
CDF	cumulative distributions function
CPPI	constant proportion portfolio insurance
DSD	decreasing absolute risk aversion stochastic dominance
DT	delta tolerance strategy
EDF	empirical distribution function
ES	expected shortfall
FB	hedging to a fixed bandwidth around delta strategy
FSD	first-order stochastic dominance
FTSE	Financial Times Stock Exchange
GARCH	generalized autoregressive conditional heteroskedasticity
GFC	global financial crisis
HE	hedging error at maturity
ITM	in-the-money
LEAPS	Long-term Equity AnticiPation Securities
LS	Leland's hedge
LTCM	long-term capital management
MV	mean variance
NHE	net hedging error
NYSE	New York Stock Exchange
OTM	out-of-the-money
P&L	profit and loss
PDE	partial differential equation
PDF	probability density function
RSSD	risk-seeking stochastic dominance
S&P	Standard and Poor's
SD	stochastic dominance
SPDR	Standard and Poor's depository receipts
SSD	second-order stochastic dominance

TC	transaction costs
TSD	third-order stochastic dominance
VaR	value at risk
VB	hedging to a variable bandwidth around delta strategy

Chapter 1

1. Introduction

The derivatives market has expanded dramatically since the publication of Black and Scholes' (1973) and Merton's (1973) seminal papers on option pricing and dynamic hedging. The core idea of their papers is that, in a frictionless and complete market, the use of a dynamic trading strategy provides a perfectly instantaneous riskless hedge at the limit of continuous trading. Since then, there has been an explosion of new option pricing and hedging models that relax the restrictive assumptions imposed in the Black–Scholes–Merton model.

In this thesis, I consider a situation in which a trader has written a European call option. The trader is presented with a long list of possible option hedging models in the literature, which can be grouped into three major categories: stochastic volatility, stochastic interest rates and jump diffusion processes. The trader would like to identify the best way to hedge the risk exposure of the short position. Using empirical analysis, Bakshi, Cao and Chen (1997) find that, if a delta-vega-neutral strategy is implemented, the ad hoc Black–Scholes model performs no worse than other more complicated models that allow volatility, interest rates and jumps to be stochastic. Dumas, Fleming and Whaley (1998) model the volatility as a deterministic function of strike price and maturity. They show, also using empirical analysis, that the hedge ratios determined by the Black–Scholes model are more reliable than those obtained from a deterministic volatility function. Lam, Chang and Lee (2002), who examine the variance-gamma option pricing model, and Yung and Zhang (2003), who look at the Generalised AutoRegressive Conditional Heteroskedasticity (GARCH) option pricing model, conclude similarly that the Black–Scholes hedging strategy performs well relative to other, more complicated models. All these studies conclude that simpler hedging strategies work best; however, these studies do not include transaction costs in their analyses.

In the real world, trading occurs at discrete time points; therefore, the assumption of continuous trading in most hedging models is violated. When market friction such as transaction costs for each trade are taken into consideration, a perfect hedge using the underlying asset and cash (equivalently, riskless bonds or the money market account) cannot be obtained. This is because continuous trading of arbitrarily small amounts of stock is infinitely costly. As a result, perfect hedging is impossible and the optimal hedging strategy becomes preference-dependent. In addition, most option pricing and hedging models assume no transaction costs. When it comes to practical applications, transaction costs may be a major reason to reject a theoretically sound hedging strategy that can eliminate most of the risks such as jumps in asset prices, stochastic interest rates and stochastic volatility. Although many hedging studies have included transaction costs in their empirical tests, these studies assess how the introduction of transaction costs affect the performance of the more sophisticated models; they do not test the performance of alternative hedging strategies designed to reduce the *total* costs of hedging.

Several theoretical papers examine option hedging in the presence of transaction costs. There are two main classifications of hedging strategies in the literature: time-based strategies and move-based strategies. Leland (1985) is the first author to examine how both discrete time trading and transaction costs affect the cost and risk of hedging an option. Leland's hedging strategy is a time-based strategy in which the hedging portfolio is rebalanced at fixed time intervals. Instead of perfect replication at every time interval, he focuses on matching the option payoff at maturity. Leland derives a hedging strategy that is similar to the Black–Scholes strategy but with modified hedging volatility, which depends on proportional transaction costs and the hedging frequency.

Other authors consider non-constant hedging time intervals; these are classified as move-based hedging strategies. Henrotte (1993), Grannan and Swindle (1996) and Toft (1996) analyse strategies based on the percentage change in the underlying asset price (asset tolerance strategies, a subset of move-based hedging strategies). They show that under certain conditions, move-based strategies are superior to simple time-based strategies; that is, when the underlying asset is volatile, transaction costs are small and the hedger is less risk-averse. Whalley and Wilmott (1993) propose another move-based strategy, based on the movement of the delta of the option, termed the delta tolerance strategy.

The aforementioned hedging strategies may not satisfy optimality criteria. Several theoretical papers examine hedging strategies as an optimal control problem in the presence of transaction costs. These include Hodges and Neuberger (1989), Davis, Panas and Zariphopoulou (1993), and Barles and Soner (1998). By using an expected utility maximisation framework of intertemporal portfolio optimisation for an investor with exponential utility, optimal hedging strategies are derived such that no portfolio rebalancing is required when the portfolio value (or delta) is within the hedging bandwidth, and trading should only occur to adjust the hedge position to the nearest edge of the bandwidth when the hedging bandwidth is breached. Although these hedging strategies are optimal and have good empirical performance under simulation, they are rarely used in practice because of the need for preference specification and computational difficulties in deriving hedging bandwidth.

Toft (1996) first analyses the costs and risks of hedging an option discretely in the mean variance framework. He presents closed-form solutions of expected hedging error, transaction costs and variance of the cash flow for both Leland's time-based strategy and Henrotte's move-based strategy. His analysis, which can be considered as an extension of Figlewski's (1989) simulation study, indicates that a move-based strategy is superior to a time-based strategy when the volatility of the underlying asset is high, transaction costs are small and the hedger has low risk aversion.

Although an extensive list of theoretical models has developed over decades, there are relatively few empirical comparisons of different option hedging strategies in the presence of transaction costs. Only four studies have been published so far: Mohamed (1994), Martellini and Priaulet (2002), Zakamouline (2006b, 2009), which compare the performance of competing hedging strategies using a Monte Carlo simulation framework. Mohamed (1994) uses the value at risk of the hedging error, while the other three papers use a mean variance rule to assess the performance of competing hedging strategies. The reason for using a mean variance rule as an assessment tool is that the hedger faces a tradeoff between transaction costs incurred and accuracy of the hedging results. To obtain a highly precise hedging result, a hedger often incurs higher transaction costs; therefore, the hedger takes into consideration the mean and variance of the hedging error. However, the use of the mean variance rule has to satisfy one of two assumptions: (1) that the hedger has a quadratic utility function, or (2) that hedging errors are normally distributed. These assumptions are rarely satisfied in practice, casting doubt on the validity of the results of previous studies. On the other hand, Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970) introduce stochastic dominance rules and their economics application. Stochastic dominance rules are conceptually superior to the mean variance rule since they maximise investors' expected utility, using the entire distribution of asset returns for decision-making.

This research aims to fill two important gaps in the literature. First, existing empirical studies of the performance of alternative option hedging strategies with transaction costs are tested in a Monte Carlo simulation environment. In contrast to most empirical studies in option pricing and hedging, there is a lack of empirical tests of these strategies based on actual market data. Second, the comparison of alternative hedging strategies is largely based on mean variance criteria. Given the restrictive assumptions of the mean variance framework as described earlier, the validity of existing results is an open question. *Therefore, the key original contributions of this thesis to the existing literature are (1) to examine the performance of alternative hedging strategies using actual market data, and (2) to compare the results using both mean variance and stochastic dominance rules.*

Previous studies show that the empirical performance of Black–Scholes hedging is no worse than the performance of more sophisticated models. Therefore, this research examines the best use of the Black–Scholes hedging strategy in the presence of transaction costs. In other words, I use the Black–Scholes hedge ratio to form a hedging portfolio and employ hedging strategies proposed in the transaction costs literature to dynamically hedge the risk exposure of a short European call option position. The objective of the study is to search for the most efficient European call option hedging strategy in terms of transaction costs paid and hedging precision. I examine both time-based and move-based hedging strategies, comparing a total of six strategies. These strategies are the Black–Scholes hedge at fixed time intervals (BS), Leland's hedge (LS), Henrotte's asset tolerance strategy (AT), delta tolerance strategy (DT), hedging to a fixed bandwidth around delta strategy (FB), and hedging to a variable bandwidth around delta strategy (VB). The first two strategies are time-based strategies, while the last four are move-based strategies. These strategies are the same as those examined in Zakamouline's (2009) study.

It is noted that, in the option hedging literature, the term, hedging error is typically used in empirical testing without consideration of transaction costs. In this thesis, I therefore use the term *net hedging error to represent the difference between the value of the hedging portfolio and the payoff on the option at maturity, after taking transaction costs into account.*

This thesis includes two studies which solve longstanding questions in the literature: (1) how well do established hedging strategies perform when applied to actual market data? (2) does stochastic dominance test provide different conclusion from mean variance test? In my first study, I assume that the underlying risky asset is subject to proportional transaction costs but trading in a riskless bond requires no transaction costs. I use Monte Carlo simulation to investigate the performance of alternative hedging strategies in a controlled environment. I compare simulated hedging results by using both mean variance and stochastic dominance rules in the presence of transaction costs. To assess stochastic dominance, I use statistical tests proposed by Barret and Donald (2003) and Linton, Masoumi and Whang (2005).

The second study examines the performance of alternative hedging strategies using actual Standard and Poor's (S&P) 500 index option data obtained from the Ivy Option Metrics database. The sample period is from January 2, 1996 to October 31, 2010. The dataset comprises 676,358 European call options. Time-based strategies allow assessment of net hedging error at the end of each rebalancing period; however, move-based strategies are rebalanced at random time points. As a result, my hedging performance computation method differs from that of Bakshi et al. (1997) and other studies in order to consistently compare the empirical performance of the proposed hedging strategies. In my study, I assume that the hedger will hold an option position until maturity so that I can assess the total amount of transaction costs paid for each proposed hedging strategy. Therefore, the net hedging error presented in my study may be larger than the actual transaction costs paid. This is because it is rarely the case that a market maker will hold an option position until maturity. Instead, he will close his position as soon as possible to reduce his risk exposure. Therefore, the reported net hedging errors in my empirical study represent the most that a market maker could have lost if he had held his position until maturity. The proposed hedging strategies are based on the assumption that the hedger will aim to maintain a delta-neutral hedge position. In addition, empirical studies performed by other authors have emphasised the importance of maintaining a delta-vega-neutral hedging strategy in order to control volatility risk. Therefore, my empirical study considers the hedging performance of the proposed strategies while maintaining a delta-vega-neutral position whenever applicable.

In my simulation study, I find that, with proportional transaction costs, the optimal hedging strategy is consistent with theoretical results, which is to rebalance the hedging portfolio to the nearest boundary of the hedging bandwidth (VB) under the mean variance rule. Move-based strategies are superior to time-based strategies when drift decreases and volatility increases. However, when I introduce fixed transaction costs for each trade, the hedging performance of move-based strategies is weakened. Given that the theoretical optimal hedging strategy is derived under the assumption of small proportional transaction costs and no fixed costs in each transaction, I therefore also assess the impact of fixed transaction costs on hedging performance. The introduction of fixed transaction costs increases total transaction costs paid for the hedging strategy. Nonetheless, this suggests that a hedger should switch from a time-based strategy to a move-based strategy, which allows the hedger to save substantial transaction costs for the same level of hedging precision. For example, a hedger who aims for a highly precise hedging outcome is able to save 45% of total transaction costs associated with the BS hedging strategy by simply switching to VB while maintaining the same level of hedging risk. Overall average transaction costs savings for high, moderate and low hedging precision are 28%, 13% and 12% respectively. On the other hand, stochastic dominance test results show that FB is the optimal hedging strategy when hedging precision is high and AT is optimal when the hedging precision is low. Although I find that mean variance results do not conform with stochastic dominance results in terms of the ranking of the hedging performance most of the time, both tests consistently identify the same set of top three and bottom three hedging strategies in terms of hedging performance. I also find that mean variance results depend on the choice of hedge parameters or option characteristics. In contrast, stochastic dominance results are robust with respect to those aspects.

In the second study, I investigate the empirical performance of the selected hedging strategies based on S&P 500 index option data. For a hedger who maintains a delta-neutral portfolio, I find that time-based strategies are superior to move-based strategies in the mean variance framework, and that BS is the optimal hedging strategy. Switching from the worst to the best hedging strategy saves an average of 46% of total transaction costs paid for the worst-performing strategy. Note that the average transaction costs saving is larger than the one documented in my simulation study. My stochastic dominance results demonstrate that the hedging performance of fixed bandwidth and delta tolerance strategy are no worse than time-based strategies, but it is rare that a single strategy dominates all others.

Given the emphasis in the literature on controlling volatility risk in the hedging process (e.g., Bakshi et al., 1997), my second study also tests the performance of a delta-vega-neutral hedge using the same set of hedging strategies. To control for volatility risk, a European call option with the same maturity but a different strike price from the target option is added to the existing hedging portfolio. In the empirical testing, it is assumed that the hedging portfolio is delta-vega-neutral at each rebalancing point. Consistent with the terminology in Bakshi et al. (1997), the target option is a European call option the hedger wants to hedge, and the hedging instrument is a European call option used for neutralising the vega of the target option. In other words, volatility risk is tilted back to a neutral position whenever the delta hedge is triggered. I note that not all European call options that can be used as a hedging instrument in forming a vega neutral position have daily market prices throughout the life of the hedging portfolio. The life of a hedging portfolio is equivalent to the maturity of the target option. As a result, the sample size for delta-vega-neutral hedge testing is only 249,111 observations. I also introduce early liquidation into the delta-vega-neutral hedge given the sample characteristics. I assume that transaction costs for the option are negligible or that the hedger has an existing option to form the vega-neutral hedge. Therefore, total transaction costs paid for the hedge are expected to be less than the amount paid for the delta-neutral hedge. As opposed to the delta-neutral hedge results, my hedging performance measures indicate that neither time-based nor move-based strategies consistently dominate all other strategies. In a delta-vega-neutral hedge, the average transaction costs saving is 86% of total transaction costs paid for the worst-performing strategy. It is noted that these savings are relative to the total transaction costs paid for alternative hedging strategies when a delta-vega-neutral hedge is formed. I demonstrate that the saving in a delta-vega-neutral hedge is on average 29%, 21% and 19% less than the total transaction costs saving in a delta-neutral hedge for high, moderate and low hedging precision, respectively. The adjustment of a move-based strategy based on my simple setup may result in over-hedging at each rebalancing point, and therefore the hedging performance may deteriorate. There is a potential that move-based strategies have controlled volatility risk indirectly when forming a delta-neutral hedge. Therefore, I also compare the performance of time-based strategies implemented using a delta-vega-neutral hedge and move-based strategies implemented using a delta-neutral hedge. The objective of the test is to examine whether I can obtain better or similar hedging performance from simple time-based strategies after controlling for volatility risk. The test shows that time-based strategies (particularly the BS) are optimal and a delta-neutral hedge is sufficient for a hedger to obtain the hedging precision desired at minimal transaction costs.

My research will assist financial institutions in making better informed decisions when selecting hedging strategies. Given that transaction costs are non-recoverable in the trade process, hedging performance can be improved merely through changing to an optimal hedging strategy. This benefit is clearly demonstrated in both my simulation and empirical studies. In particular, I recommend hedgers choose BS as the preferred hedging strategy regardless of risk preferences because this strategy provides the optimal tradeoff between hedging precision and transaction costs. As concerns an optimal framework for the comparison of hedging performance, I find that the results of mean variance and stochastic dominance rules are reasonably consistent and suggest that time-based strategies are empirically superior to move-based strategies. However, the stochastic dominance test I use does not consistently rank one of the six hedging strategies highest.

In the remainder of this chapter, I outline the structure of the thesis and provide an overview of each chapter. Chapter 2 contains a review of the existing literature and identifies gaps. Chapter 3 presents a survey of option pricing and hedging models with transaction costs, and provides a detailed examination of their implicit assumptions. Chapters 4 and 5 cover sample construction, methods and results for the Monte Carlo simulation study and the empirical study of S&P 500 index option hedging, respectively. Chapter 6 provides a summary and overall discussion.

1.1 Outline of the Thesis

Chapter 2: Literature Review

This chapter reviews three major strands of the literature. Each section considers both theoretical and empirical literature. First, I review the option pricing and hedging literature, starting from the situation of a frictionless market, before discussing hedging at discrete time intervals and in the presence of transaction costs. Second, I look at the trading costs literature and its empirical estimation methods. Third, I study the stochastic dominance literature starting by considering both theory and application of consistent tests of stochastic dominance. The third section also compares and contrasts the application of the well-known mean variance rule and the stochastic dominance rule in finance.

The existing literature suggests that (1) ad hoc Black–Scholes hedging performance is no worse than that of more sophisticated models after controlling for volatility risk; (2) theoretical option hedging models show that the choice of hedging strategy depends on the hedger's level of risk aversion when transaction costs are taken into account; (3) stochastic dominance rules have become increasingly popular for comparing choices under uncertainty; and (4) recent developments of statistical tests of dominance rules allow us to perform empirical test with greater power. I identified two points that remain unaddressed:

(1) existing empirical studies of the performance of alternative option hedging strategies with transaction costs have been performed in a Monte Carlo simulation environment, but there is a lack of empirical tests of these strategies based on actual market data; and
(2) comparison of alternative hedging strategies is largely based on the mean variance rule; given the restrictive assumptions in the mean variance framework, the validity of the results of those tests is questionable and it remains to be shown whether they hold when applying a more general rule such as stochastic dominance.

Chapter 3: Survey of Transaction Costs Option Pricing Models

This chapter presents a survey of option pricing and hedging models with transaction costs in detail. The first section of the survey reviews the Black–Scholes model, which is an option pricing and hedging model developed under the assumption of a frictionless market – a benchmark model in the literature. Subsequent sections explore option pricing and hedging models in the extended Black–Scholes economy, which relaxes the assumptions of continuous trading and no transaction costs in trading. The survey is grouped according to two major types of hedging strategies in the literature, namely, time-based strategies and move-based strategies. Time-based strategies are those strategies that rebalance the hedging portfolio at pre-determined regular time intervals, that is, hedging frequency in terms of time units such as days. Move-based strategies are those strategies that rebalance the hedging portfolio based on movement of the underlying asset return or option Greeks. I also study strategies that do not belong to either group. For each strategy, I discuss its model specifications, benefits and the drawbacks of implementing such strategies. In addition, I look into the literature on asset allocation with transaction costs given that the two problems are inherently similar except with regard to time horizon. Finally, the chapter outlines the hedging strategies chosen for performance assessment in chapters 4 and 5.

Chapter 4: Simulation Study

In this chapter I examine the performance of selected alternative hedging strategies using a Monte Carlo simulation framework. The benefit of using simulation is that I am able to isolate the factors that affect hedging performance of each strategy and identify how performance varies with a particular factor. In the presence of transaction costs, previous researchers compare hedging strategies using the mean variance rule. I compare hedging outcomes of alternative strategies using both mean variance and stochastic dominance rules. My objective is to identify the optimal hedging strategy in the presence of transaction costs. At the same time, I want to examine whether both performance measures provide the same conclusions and if not, how test results differ. The simulation study shows that move-based strategies are superior to time-based strategies when drift decreases and volatility increases. In particular, the variable bandwidth strategy performs best. Both mean variance and stochastic dominance rules consistently identify the same set of top three and bottom three performing hedging strategies. A hedger is able to save an average of 28%, 13% and 12% of total hedging transaction costs at high, moderate and low precision respectively.

Chapter 5: Empirical Study – The S&P 500 Index Option

This chapter provides an empirical assessment of the performance of alternative hedging strategies using S&P 500 index option data. My testing procedure is similar to that used by Bakshi et al. (1997), who analyse various option pricing models. My study does not fully replicate Bakshi et al. (1997) because I consider transaction costs. In order to study the impact of transaction costs, the hedger is assumed to rebalance the hedging portfolio according to the hedging criteria of selected hedging strategies until option maturity. I define net hedging error as the difference between the hedging portfolio value and option payoff at maturity. My calculation incorporates total transaction costs for rebalancing trades during the life of the option. My assumptions differ from those of Bakshi et al. (1997), as their calculation method only considers time-based rebalancing. It is also worth mentioning that my terminology of net hedging error is different from the hedging error in Bakshi et al. (1997) because I include transaction costs in the calculation.

This chapter is divided into two sections. The first section concerns the delta-neutral hedging performance of alternative hedging strategies. The second section focuses on the performance of delta-vega-neutral hedging. I examine delta-vega-neutral hedging because volatility risk has been widely documented in the option pricing and hedging literature. This delta-vega-neutral hedge exercise is a simple control for volatility risk, using the same set of proposed hedging strategies designed to minimise transaction costs. The delta-vega-neutral hedging portfolio is assumed to maintain delta-neutral and vega-neutral positions when delta hedge is triggered by the hedging criteria; in another words, the hedging portfolio's vega is allowed to drift at all other times. The empirical results show that, in the presence of transaction costs, a hedger can attain optimal hedging outcomes by implementing a delta-neutral hedge using time-based strategies. In terms of performance, both mean variance and stochastic dominance produce consistent results, showing that time-based strategies are superior to move-based strategies. However, the stochastic dominance rule does not suggest a single optimal hedging strategy, due to the lack of strong dominance relationship in the empirical distribution function.

Chapter 6: Conclusion and Future Research

This chapter concludes the thesis with a summary of findings and the contribution of the research to the literature. It outlines the limitations of the study and suggestions for future research.

Chapter 2

2. Literature Review

This chapter presents reviews of three major areas of literature. First, I review option pricing and hedging literature related to the frictionless market, discrete hedging and the presence of transaction costs. This section is divided into theoretical and empirical literature. Second, I look at transaction costs literature and its empirical estimation methodology. Third, I study stochastic dominance literature starting from theory to the development of consistent tests of stochastic dominance on empirical data. In addition, this section also contrasts the well-known mean variance comparison framework with the stochastic dominance rule in finance literature. From this literature review, I derive research problems that have been largely resolved to date:

- although sophisticated option models were developed to capture empirical regularities of the underlying asset, the Black–Scholes model still has superior empirical hedging performance if one is willing to add an appropriate financial instrument to hedge against the volatility risk;
- many theoretical models on option pricing and hedging in the presence of transaction costs have been proposed in the past. Theoretical results suggest that the choice of hedging strategy depends on the hedger's level of risk aversion;
- the increasing popularity of applying stochastic dominance rules in comparing choices under uncertainty given that the rules overcome some inferiority in the mean variance framework; and
- development of the statistical test of stochastic dominance rules allow us to carry out empirical study with greater power.

In addition, I identify gaps in the literature that remain unanswered, given below:

- empirical studies of the performance of option hedging strategies with transaction costs have been performed in a Monte Carlo simulation environment. In contrast to most of the empirical studies of option pricing and hedging, there is a lack of empirical testing of these strategies based on actual market data; and

- comparison of performance hedging strategies is largely based on the mean variance criteria. Given the restrictive assumptions in the mean variance framework, the validity of the results is subject to question.

Therefore, the main contributions of this research to the literature are (1) an examination of the performance of alternative hedging strategies using actual market data, and (2) comparing the results obtained using mean variance and stochastic dominance rules. The key literature on the topic of this research is summarised in Table 1 below. I have also prepared a literature roadmap to assist the reader to understand the linkage of the three strands of literature being reviewed in this chapter. The literature roadmap is set out in Figure 1.

Table 1 Summary of Key Literature in Option Pricing and Hedging in the Presence of Transaction Costs

Panel A: Theory

Papers	Assumptions	Main findings	Transaction costs included?
Black and Scholes (1973) Merton (1973)	Frictionless and complete market with continuous trading	Instantaneous riskless hedge can be formed through dynamic trading of option's underlying asset and riskless bonds	No
Cox, Ross and Rubinstein (1979)	Discrete time trading and no arbitrage condition	Option can be replicated using a portfolio of stocks and bonds. Risk-neutral probability measure is independent of investor's risk preference	No
Figlewski (1989) Jameson and Wilhelm (1992)	Discrete time trading	Risk arising from discrete hedge rebalancing has a statistical and economic influence on option spreads	Yes for Figlewski (1989) No for Jameson and Wilhelm (1992)
Leland (1985)	Discrete time trading and transaction costs	A time-based strategy similar to Black–Scholes but with modified hedging volatility that depends on proportional transaction costs and hedging frequency This is not a self-financing trading strategy	Yes, proportional transaction costs

Papers	Assumptions	Main findings	Transaction costs included?
Hodges and Neuberger (1989) Whalley and Wilmott (1997) Barles and Soner (1998) Lai and Lim (2009)	Transaction costs	An optimal move-based strategy is derived under a utility maximisation framework of inter-temporal portfolio optimisation for an investor with exponential utility The strategy requires no portfolio rebalancing when the portfolio delta is inside the hedging bandwidth If rebalancing is required, trading should only adjust the hedge position to the nearest edge of the hedging bandwidth	Yes, proportional transaction costs
Boyle and Vorst (1992)	Discrete time trading and transaction costs	A discrete self-financing strategy under a binomial framework	Yes, proportional transaction costs
Henrotte (1993) Grannan and Swindle (1996) Toft (1996)	Transaction costs	Analysed the conditions when move-based strategies perform better time-based strategies and vice versa	Yes, proportional transaction costs
Martellini (2000)	Discrete time trading	Generalise time-based strategy by	Yes, proportional

Papers	Assumptions	Main findings	Transaction costs included?
	and transaction costs	rebalancing different fractions of an option portfolio at different hedging frequencies	transaction costs
Gondzio, Kouwenberg and Vorst (2003)	Discrete time trading, stochastic volatility, transaction costs and trading restrictions	The resulting stochastic optimisation model stays close to a delta-vega neutralised position but with some additional slack to avoid needless transaction costs	Yes, proportional transaction costs

Panel B: Empirical test

Type of tests	Papers	Methodology	Data	Main findings	Transaction costs included?
Empirical tests of alternative option pricing models	Bakshi et al. (1997) Nandi (1998)	Stochastic interest rates, volatility and jumps model	S&P 500 index option	Black–Scholes hedge performs no worse than stochastic volatility model after controlling for volatility risk	No
	Dumas et al. (1998)	Deterministic volatility function	S&P 500 index option	Black–Scholes hedge performs no worse than the proposed model after	No

Type of tests	Papers	Methodology	Data	Main findings	Transaction costs included?
				controlling for volatility risk	
	Yung and Zhang (2003)	GARCH option pricing model	S&P 500 index option	Black–Scholes hedge performs the best after controlling for volatility risk	No
	Vähämaa (2003)	Skewness and kurtosis adjusted Black–Scholes model	FTSE 100 index option	Black–Scholes hedge performs better than the proposed model	No
Empirical tests of alternative hedging strategies	Mohamed (1994)	Value at risk comparison	Simulation short call option	Move-based strategy performs the best	Yes
	Martellini et al. (2002) Zakamouline (2006)	MV comparison	Simulation short call option	Move-based strategy performs the best	Yes
	Zakamouline (2009)	MV comparison	Simulation, exotic option	Mixed results	Yes
	Chen et al. (2011)	Average out-of-sample realised hedging errors	S&P 500 futures option and simulated data	Rule-based strategy performs the best	Yes

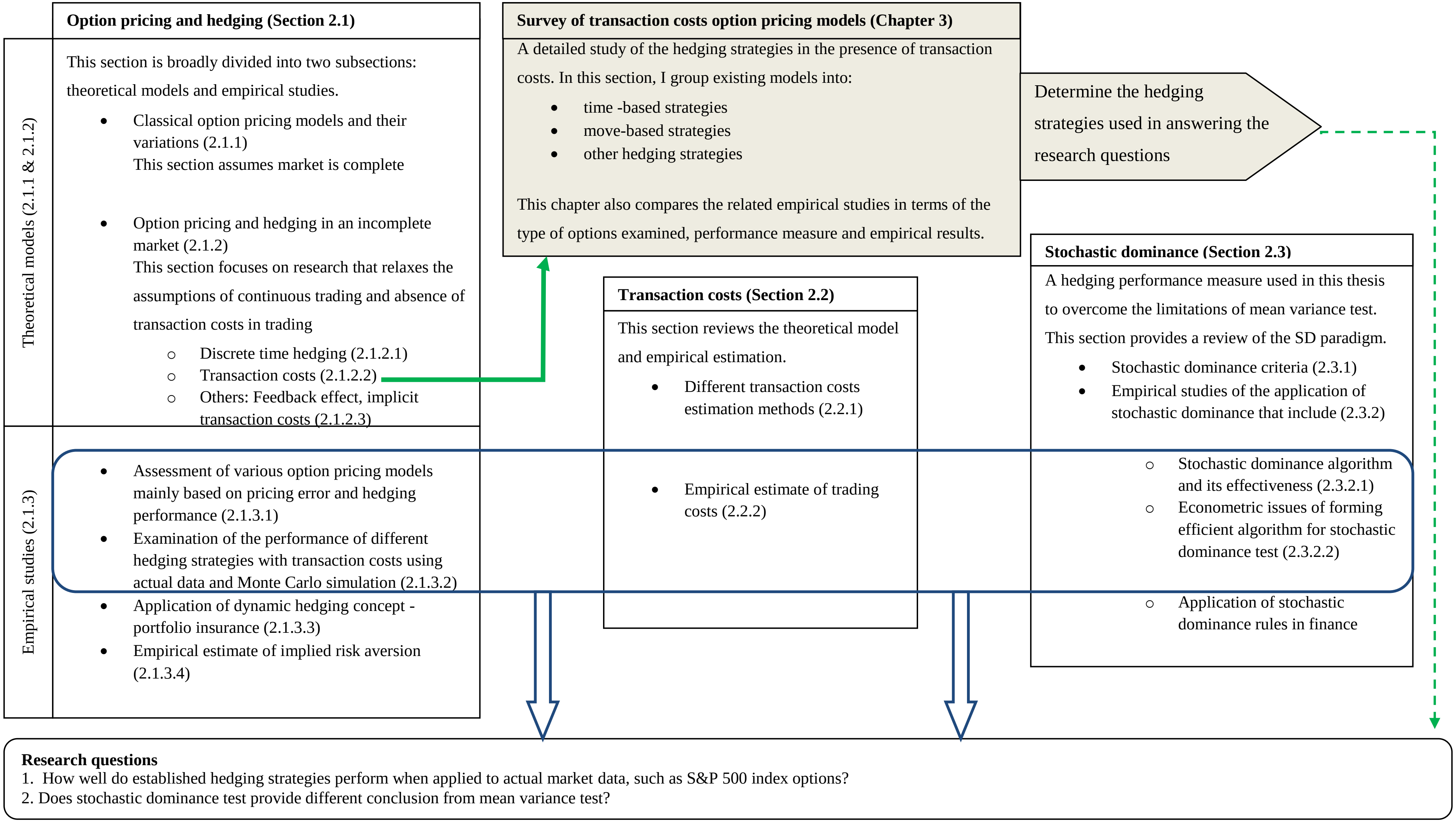

Figure 1 Literature Roadmap

2.1 Option Pricing and Hedging

The derivatives market has expanded dramatically since the early 1970s. The academic research on financial derivatives has expanded in parallel, following the fundamental insights of Black and Scholes (1973) and Merton (1973) on option pricing and dynamic hedging. In this section, I briefly review the development of option pricing and hedging literature with an emphasis on research that relaxes the assumptions of continuous trading and absence of transaction costs in trading.

2.1.1 Classical Option Pricing Models and their Variations

The theory of option pricing can be traced back as early as 1900. Bachelier (1900) assumes stock prices follow a Brownian motion process with zero drift to derive an option pricing formula. However, this assumption implies that stock prices can be negative and option prices can be greater than the underlying stock prices. More importantly, Bachelier's formula leads to the development of the option pricing theory and the ground-breaking Black and Scholes (1973) and Merton (1973) option pricing formulations. Following Bachelier, Sprenkle (1961), Boness (1964) and Samuelson (1965) provide valuation formulas of the same general form[1]. Their formulas, however, depend on the expected return or risk premium of the underlying stock, which is related to investors' risk preferences. The seminal papers of Black and Scholes (1973) and Merton (1973), hereafter Black-Sholes[2], show that in fact option price can be valued without requiring knowledge of investors' beliefs about expected returns on the underlying stock. The core idea of their papers is that, in a frictionless and complete market, the use of dynamic trading strategy provides a perfectly instantaneous riskless hedge in the limit of continuous trading. Later, Cox, Ross and Rubinstein (1979) present a discrete time binomial option pricing model, including the Black-Scholes model as a special limiting case. The binomial model shows that an option value can be replicated using a portfolio of stocks and riskless bonds. In addition, using only the no-arbitrage condition, the risk-neutral probability measure can be determined without entering any investor preferences in the dynamic hedging strategy. Harrison and Kreps (1979) illustrate that, in a discrete time model, the absence of arbitrage implies that the discounted underlying asset price process is a martingale with the risk-neutral

[1] Smith (1976) reviews the development of the explicit solutions to the option pricing problem prior to the work of Black and Scholes. Each of the works is presented with the formula and Smith highlights the improvements and differences in the basic approaches taken to solve the option pricing problem.
[2] In the literature, the model is also referred to as the Black-Scholes-Merton model.

probability measure[3]. This leads to the famous results of Harrison and Pliska (1981), known as the Fundamental Theorems of Arbitrage Pricing: i) a market is arbitrage-free if and only if there exists an equivalent martingale measure, and ii) a market is complete if and only if there exists a unique equivalent martingale measure.

The Black-Scholes model employs several restrictive assumptions:

(i) the option is an European option or the option can only be exercised at maturity,
(ii) the short term risk-free interest rate is known and is constant through time,
(iii) the stock pays no dividend,
(iv) there are no short sale restrictions,
(v) the stock returns process follows a geometric Brownian motion with constant volatility,
(vi) no transaction costs involve in purchasing or selling the underlying stock or the option, and
(vii) trading takes place continuously in time.

The later development of option pricing has centred on developing new models that relax some of these assumptions.

The Black-Scholes model remains valid when some of these assumptions are relaxed. For example, Merton (1973) shows that in the absence of dividends, it is never optimal to exercise an American call option early and so the American call can be priced as if it was a European call. Merton (1973) and Amin and Jarrow (1992) introduce stochastic interest rates into the option pricing framework by assuming the short-term risk-free interest rate can vary over the life of the option. Merton considers the option pricing problem when the underlying stock pays continuous dividend yield by modifying the stock price input in the BS model, while Thorp (1973) examines both the effect of the short sales restrictions and dividend payments. Ingersoll's (1976) model takes into consideration the differential tax rates on capital gains and ordinary income.

[3] The risk-neutral probability measure is also called the equivalent martingale measure. For any probability measure P, an equivalent martingale measure is a probability measure P* on (Ω, F) with properties: 1) P* and P are equivalent such that P*(A)>0 if and only if P (A)>0, 2). The discounted price process S becomes a martingale when P is replaced by P*.

However, the Black-Scholes solution is not valid when the stock return dynamics cannot be represented in a stochastic process with a continuous sample path and/or constant volatility. This creates volatility risk or jump risk. Under such circumstances, a Black-Scholes riskless hedge is not possible. To mitigate volatility risk or jump risk, additional restrictions on the stock price distribution are required. For instance, Merton (1976) models the underlying stock return as the mixture of continuous and jump processes. In his model, the jump component reflects the arrival of important information and is a Poisson process. Likewise, Cox and Ross (1976) assume a birth and death process for the jump part of the stock return dynamics. Bates (1991) postulates the jump process is asymmetric and systematic. Hull and White (1987) and Wiggins (1987) examine the stochastic volatility problem, but their models have no closed-form solution and require the use of numerical techniques to solve a two-dimensional partial differential equation (PDE). Later, Stein and Stein (1991) and Heston (1993) derive a closed-form solution for the price of a European call option on an asset with stochastic volatility. Some other extensions related to stock price dynamics include Bailey and Stulz's (1989) and Bakshi and Chen's (1997) option pricing models, which admit both stochastic volatility and stochastic interest rates; Duan (1995) values the option in the context of a GARCH asset return process; and Madan, Carr and Chang (1998) introduce the variance gamma process for the dynamics of the log stock price to control for the skewness and kurtosis of the return distribution, and use it to derive an analytical solution for the prices of European options.

The Black-Scholes model disagrees with reality in several ways; nevertheless, it is widely used in practice. For example, the Black-Scholes model serves as a useful approximation in pricing and it can be generalised to price other contingent claims such as options on forward contracts, as in Black (1976). For hedging purposes, the idea of a Black-Scholes instantaneous riskless hedge through dynamic investment strategies was applied to popular portfolio insurance in the 1980s, as discussed by Leland (1980), Rubinstein and Leland (1981) and Rubinstein (1985a). Another popular way of using the Black-Scholes model is to infer implied volatility of an option at a given market price as in Rubinstein (1985b). For a given range of option strike prices and maturities, one can construct an implied volatility surface and use it to price other exotic contingent claims, especially for those contingent claims on non-traded or illiquid underlying assets. The applications of the Black-Scholes model are versatile, but the users are required to understand its limitations to avoid unexpected risk.

2.1.2 Option Pricing and Hedging in an Incomplete Market

In this section, I depart from the Black-Scholes paradigm and look into option pricing and hedging in an incomplete market. In the Black-Scholes model, options are traded in a complete market. This means that an option payoff can be replicated by using assets traded in the market (i.e., stocks and bonds). However, if an investor is trading in an incomplete market, he or she is subject to risk. Given the topic I examine in this thesis, I pay careful attention to the risks arising from discrete time hedging and transaction costs involved in trading the underlying assets.

2.1.2.1 Discrete Time Trading

Since trading takes place at discrete time intervals and therefore it is clearly impossible to rebalance the hedge portfolio continuously, the hedge portfolio is no longer riskless as in the Black-Scholes model. As a result, the hedge portfolio returns become risky.

Black and Scholes (1973) claim that the discrete rebalancing risk can be across a hedged portfolio, and therefore it can be ignored if a hedging portfolio can be rebalanced frequently. In this case, market makers seem to have the advantage of diversifying the risk either though trading a variety of instruments or across many markets. In spite of this, in an empirical test, Jameson and Wilhelm (1992) demonstrate that the risk arising from discrete hedge rebalancing has a statistically and economically significant influence on option spreads. The option's gamma serves as the proxy for the marginal contribution of the risk introduced by discrete hedge rebalancing. Their results imply that the inability to rebalance an option position continuously is considered by market makers when they set bid-ask quotes in the options markets. Their results are consistent with Figlewski's (1989) simulation results that discrete rebalancing imposes relatively wide bounds on option prices on top of transaction costs.

On the other hand, Boyle and Emanuel (1980) examine the distribution of the returns on hedge portfolios when rebalancing takes place at discrete time intervals. They show that the expected hedge return is zero, but the distribution of the return is skewed and leads to biased t-statistics. Galai (1983) decomposes hedge returns into three elements: first, the riskless return on the cash position; second, the return from the discrete hedge position; and third, the return from the change in the deviation of the actual option price and model option price. He finds that the third element is the dominant factor in explaining the performance of hedging activity. The third element is linked to price changes due to stochastic volatility. His results suggest that, in any hedging scheme, controlling for stochastic volatility is more important than discrete hedging. Similarly, Hull and White (1987) examine several hedging schemes, including delta hedging, delta-gamma hedging and delta-vega hedging. They conclude that if there is only one option to be used for hedging, then it must be used for controlling volatility risk.

Discrete trading also introduces systematic risk. Black and Scholes (1973) and Boyle and Emanuel (1980) argue that option hedge returns are uncorrelated with the market even with discrete rebalancing; however, their findings are limited to discrete rebalancing with short time intervals. Glister (1997) asserts that, for longer rebalancing time intervals, Black-Scholes hedge positions exhibit substantial systematic risk that may bias the empirical tests of Black-Scholes option pricing models. However, Jarrow (1997) points out that Glister's assertion is false due to his neglect of a term in his hedging risk estimation that causes the suggested bias.

2.1.2.2 Transaction Costs

When market frictions such as transaction costs for each trade are taken into consideration, the argument of perfect replication using stocks and bonds may not hold. This is because continuous trading of arbitrarily small amount of stock at random time point is infinitely costly. Hence, it is impossible to perfectly replicate an option payoff. Consequently, trading an option is subject to hedging risk. With transaction costs, the concept of no arbitrage price of an option is replaced by a range of option prices. In the option hedging/pricing context, transaction costs can be as fixed and/or proportional to the amount of stock traded.

Leland (1985) was the first to examine how both discrete trading and transaction costs affect the cost and risk of an option hedging strategy. Leland's hedging strategy is considered a time-based strategy in which the hedging portfolio is rebalanced at fixed regular time intervals. Instead of having perfect replication at every time interval, he focuses on matching the option payoff and derives a hedging strategy which is similar to Black-Scholes but with modified hedging volatility that depends on the proportional transaction costs and the hedging frequency. However, Leland's model is not a self-financing trading strategy. Boyle and Vorst (1992) value option replicating costs through a discrete self-financing trading strategy in a binomial framework by assuming initial holding of stocks portfolio. Their model serves as an extension of Merton's (1990) two-period binomial model; however, their use of a binomial model requires the user to explicitly specify a revision interval. Hoggard, Whalley and Wilmott (1994) also consider hedging strategies that take place at a fixed time interval by looking into portfolios of European-type options. Hoggard et al. also present a nonlinear PDE for which the nonlinearity arises from the presence of transaction costs. Amster, Averbuj, Mariani and Rial (2005) extend Leland's model by assuming non-increasing transaction costs with a combination of fixed and proportional transaction costs. Martellini (2000) generalises the standard time-based strategies that rebalance different fractions of an option portfolio at different time frequencies. These strategies are based on the idea of trading portions of underlying assets at different time intervals[4].

As noted in chapter one, other authors consider non-constant hedging time intervals. Henrotte (1993), Grannan and Swindle (1996) and Toft (1996) analyse strategies based on the movement of the percentage change in the underlying asset. They show that under certain conditions, move-based strategies are superior to simple time-based strategies; that is, when the underlying asset is volatile, transaction costs are small and the hedger is less risk-averse.

The aforementioned hedging strategies may not satisfy some optimality criteria. There are two ways of defining optimality criteria. First, the strategy should maximise the expected utility of the difference between the realised and the desired cash flow at maturity. Second, the strategy has to minimise the initial cost of obtaining a terminal

[4] On average, an investor will enjoy the diversification benefit from up and down movements of the underlying asset when trading at different time intervals.

option payoff that dominates the desired payoff. The first strand of research focuses on solving the optimal control problem in the presence of transaction costs, as attempted by Hodges and Neuberger (1989). They set up the problem in the utility maximisation framework of intertemporal portfolio optimisation for an investor with exponential utility[5]. With this setting, they are able to derive optimal hedging strategy where no portfolio rebalancing is required when the portfolio value (or delta) is inside the "hedging bandwidth". Trading should only occur to adjust the hedge position to the nearest edge of the band when the hedging bandwidth is breached. Their finding is similar to that of Constantinides (1986) in his study of capital market equilibrium with transaction costs. In Constantinides' model, the investor will modify the trading frequency and volume to accommodate the transaction costs. Davis, Panas and Zariphopoulou (1993) and Clewlow and Hodges (1997) use discrete time dynamic programming to develop the numerical methods to compute optimal hedge.

Although these hedging strategies are optimal and have good empirical performance, they are rarely used in practice because of the need for preference specification and computational difficulties in deriving hedging bandwidth. Whalley and Wilmott (1997) first provide an asymptotic analysis to Hodges and Neuberger (1989), assuming the transaction costs are small. They derive an optimal hedging bandwidth which centres around the Black-Scholes delta. Similarly, Barles and Soner (1998) present another asymptotic analysis of the same model and assume that the transaction costs are small and the hedger is risk-averse. Their optimal hedging strategy is different from that of Whalley and Wilmott, depending on optimal hedging bandwidth and volatility adjustment. Zakamouline (2006) overcomes the computation drawback by presenting an analytic approximation to the solution. Lai and Lim (2009) use an alternative approach based on a cost-constrained pathwise risk minimisation method to solve for the optimal buy and sell boundaries.

The second strand of optimality criteria research focuses on super replicating an option. Bensaid, Lesne and Pages (1992) and Edirsinghe, Naik and Uppal (1993) relax the strict objective of replicating an option payoff by replacing the objective of dominating the option payoff; this is termed a super replication strategy. Their papers

[5] These models consider the indifference in expected utility between final wealth with and without option liability.

propose similar trading strategies to the utility maximisation framework, for which rebalancing occurs only when the marginal gain from revising a portfolio more than offsets the cost of trading. However, Davis and Clark (1993) conjecture – and Soner, Shreve and Cvitanic (1995) prove – that the cheapest super replication strategy is to purchase one share of the underlying asset initially and hold it until maturity. This leaves an unsatisfactory result of little economic interest for an option writer for which the option premium bound is the underlying asset price. Based on Leland’s (1985) idea, Avellaneda and Parás (1994) introduce a new option hedging strategy that super replicates an option with a non-convex payoff function in the presence of large transaction costs. The resulting hedging strategy is path-dependent, and there may be a long period of no re-hedging transactions even though the hedging strategy must be monitored at a fixed regular time interval. A more recent study by Primbs (2009) solves the super replication problem by using the first two moments of the replication error.

More recent papers solve the hedging problem with transaction costs by incorporating stochastic volatility and jump diffusion. For example, Gondzio, Kouwenberg and Vorst (2003) develop a stochastic optimisation model for hedging contingent claims that takes account of the effects of stochastic volatility, transaction costs and trading restrictions. The hedging strategy resulting from their model is similar to a delta-vega-neutral hedge, except there is some slack to avoid needless transaction costs. Xing, Yu and Lim (2012) study the influence of jump diffusion in option pricing with proportional transaction costs by maximising expected utility of terminal wealth in a stochastic optimal control setting. Nguyen and Pergamenshchikov (2015) provide a new specification for the volatility adjustment in Leland's (1985) model that incorporates stochastic volatility.

The transaction costs literature concludes that, in the presence of transaction costs, the hedging strategy a hedger chooses depends on his or her risk aversion. This is because a hedger is facing tradeoff between hedging accuracy and transaction costs expenditure.

2.1.2.3 Others: Feedback Effect, Implicit Transaction Costs

When the perfectly competitive market assumption is weakened, a dynamic hedging strategy affects the underlying asset's price process. Jarrow (1994), Frey and Stremme (1994) and Frey (1998) analyse how standard option pricing theory can be extended when there is a feedback effect from the demand of a large trade on an underlying stock price process[6]. Since hedging an option involves buying shares of the underlying asset when its price goes up and selling when it falls, the literature shows that a hedging strategy causes the underlying asset to be more volatile. As a result, a large trader is confronted with higher hedging costs. The approach used in these papers is similar: the market is split into a group of small traders who trade based on an asset's fundamental value and a single large trader who trades according to his own strategy. A modified asset price process is the result of the market equilibrium derived from the aggregate demand of the two groups.

Jarrow (2001) is one of the first attempts to incorporate liquidity risk into arbitrage pricing theory as a convenience yield; however, convenience yield only captures the inventory dimension of liquidity. Therefore, the price-taking assumption is still valid and classical arbitrage pricing theory can be applied. The model structure implies that different trade sizes have no impact on asset prices. Hence, bid-ask spread does not exist for an asset, which is not consistent with the market observations.

Alternatively, Longstaff (2001) defines liquidity as the risk of a trader being unable to liquidate the desired amounts when need arises. With this definition, Longstaff solves a continuous time partial equilibrium in which investors are limited to choose trading strategies of bounded variation[7]. Amaro de Matos and Antao (2001) derive super-replicating bounds on a European option price when it is impossible to trade the underlying asset at some points in time.

Following Jarrow (2001), with the focus on temporary imbalance in short-term supply of and demand for the underlying asset, Cetin, Jarrow and Protter (2004)

[6] Jarrow (1994) proposes a discrete time model to formalise the issue of how a large trader takes account of the feedback effect when he chooses a hedging strategy. Jarrow also presents the conditions that must be satisfied in order to forbid market manipulation. With these conditions, a unique derivative price can be obtained. Frey and Stremme (1994) extend Jarrow's result in a continuous time framework.

[7] Longstaff (2001) partial equilibrium shows that the optimal trading strategy endogenously imposed the borrowing and lending constraints on investors.

introduce a stochastic stock supply curve to model the asset prices as a function of trade size and time. They show that trading strategies that are continuous and of finite variation incur no liquidity costs and the market is approximately complete. In particular, an option's price is shown to be equal to the Black-Scholes option value with such trading strategies. However, the classical hedge will not attain the Black-Scholes value.

In research parallel to Cetin et al. (2004), Liu and Yong (2005) examine how the price impact on the underlying asset market affects the replicating cost of a European option. They derive a generalised Black-Scholes pricing PDE by incorporating price impact function[8] into the stochastic stock price process and establish the existence and uniqueness of a classical solution to this PDE. The pricing PDE shows that the option replicating cost is only affected by the price impact of traders' trading activities on stock return volatility. The difference between Liu and Yong (2005) and Cetin et al. (2004) is that the former suggest traders trade more underlying assets in the presence of price impact (i.e., greater option delta in the hedging process), while Cetin et al. suggest trading at the classical Black-Scholes delta, but each delta trade would generate an additional liquidity cost due to price impact. Cetin and Rogers (2007) study the maximisation of investor's expected utility from terminal liquidation wealth. Due to the fact that liquidity costs can prevent arbitrageurs earning unbounded profits, they show that arbitrage opportunities and the optimal strategy with respect to a given utility function can co-exist. In addition, traders appear to trade less and more cautiously in the presence of liquidity costs. Rogers and Singh (2010) emphasise the difference between price impact effects and illiquidity effects. They model liquidity cost as a nonlinear transaction costs, which is a function of the rate of change of portfolio.

More recent studies of market impact on option pricing and hedging consider both permanent and temporary market impact. In addition to market impact, Guéant and Pu (2015) take into account execution costs and characterise the optimal hedging strategy with a PDE that relies on a numerical solution. Their optimal strategy is smoother than a classical delta hedging model, and their hedging strategy is affected by the type of settlement (e.g. physical delivery versus cash settlement). Motivated by a

[8] The price impact is linear in trade size.

market event that may be caused by naive option hedging, Li and Almgren (2016) illustrate that unstable pricing swings can arise if the hedging strategy is applied carelessly with discrete time steps. They provide a formulation to avoid such instability through mean variance optimisation.

2.1.3 Empirical Studies

The logic of dynamic replication in Black and Scholes (1973) and Merton (1973) is that options can be replicated by the continuous trading of a portfolio of underlying asset and cash. This means that options are redundant securities, since their value can be derived from the value of the replicating portfolio. However, in practice, the financial market is subject to frictions. The restrictive assumptions of the Black-Scholes option pricing model for the underlying asset challenge the validity of its practical applications. Substantial progress has been made in developing more realistic models[9]. While the search for a perfect model is a difficult task, empiricists question whether these models can explain the well-documented Black-Scholes empirical biases such as what causes the existence of volatility smiles, which model gives the smallest pricing errors or the best hedging performance, and how the implicit parameters vary differently from the actual market. Figlewski (1989) simulates market frictions and other trading problems such as transaction costs, indivisibilities and discrete time rebalancing. He documents the importance of market frictions in determining option prices and testing the valuation models. Bates (2003) provides a comprehensive overview of empirical option pricing research. In this section I first briefly review related literature and then focus on the empirical tests of option pricing with market frictions.

2.1.3.1 Option Pricing and Hedging

Many researchers have conducted empirical studies of the performance of various option pricing models. In terms of hedging performance, Bakshi et al. (1997) find that, if a delta-vega-neutral strategy is implemented, the Black-Scholes model performs no worse than other, more complicated models that allow volatility, interest rates and jumps to be stochastic. Dumas et al. (1998) model the volatility as a

[9] Examples include the stochastic interest rate models of Amin and Jarrow (1992) and Merton (1973); mixtures of jump and diffusion processes of Bates (1991), Cox and Ross (1976) and Merton (1976); the stochastic volatility models of Heston (1993) and Hull and White (1987), and the stochastic interest rate and volatility models of Bailey and Stulz (1989) and Bakshi and Chen (1997).

deterministic function of strike price and maturity. They show empirically that hedge ratios determined by the Black-Scholes model are more reliable than those obtained from the deterministic volatility function. Other studies show that the Black-Scholes hedging strategy performs well relative to other models: Lam, Chang and Lee (2002) examine the variance gamma option pricing model, and Yung and Zhang (2003) look at the GARCH option pricing model. All these studies use S&P 500 options data and conclude that simpler hedging strategies work the best. Alternatively, using the sample of S&P 500 index options, Bakshi and Kapadia (2003) demonstrate that the delta-hedged option portfolio underperforms and is consistent with a non-zero negative volatility premium; this implies that the portfolio has dynamically hedged against all risks except volatility risk.

On the other hand, Bakshi et al. (2000) focus on the predictions of the one-dimensional diffusion class of option model[10] and empirically test the predictions using S&P 500 index options. Their results contradict the model's prediction, implying that frequent hedge revision using the Black-Scholes delta-neutral strategy will in fact compound hedging errors[11]. While some of the contradictory option price movements can be explained by market microstructure factors and time decay impact, the one-dimensional diffusion option model is still unable to capture the inverse relationship between option price and stock price. With the use of canonical valuation (proposed by Stutzer, 1996), Alcock and Grey (2005) derive a close form hedge ratios for call and put. They demonstrate that hedging using the canonical deltas is more effective than using the Black-Scholes delta in simulation studies when the underlying process deviates from geometric Brownian motion. Gray, Edwards and Kalotay (2007) examine the pricing and hedging effectiveness of canonical valuation on Australian Stock Exchange index options. They find that, in term of pricing accuracy, unconstrained canonical estimation fails to outperform Black-Scholes estimation; however, with the incorporation of a small amount of information, the constrained canonical estimation is able to reduce

[10] Black and Scholes (1973), Merton (1973), Bakshi et al. (1997) and Dumas et al. (1998) give examples of the one-dimensional diffusion class of option model. These models predict (1) call (put) prices are monotonically increasing (decreasing) with underlying asset price, (2) option prices are perfectly correlated with the underlying price movement, and 3) options are redundant securities.

[11] Bakshi et al.'s (2000) empirical investigation demonstrates that call (put) prices do not increase (decrease) monotonically with underlying asset price, call and put prices tend to move together regardless the underlying price movement, and the adjustment in option prices can be larger than that in the underlying asset.

pricing errors drastically. In addition, the canonical approach results in superior hedging effectiveness compared to the Black-Scholes approach. Kaeck (2013) illustrates that jumps are important features of S&P 500 index data, and shows that hedging performance can be improved when jumps are included in the model.

Jameson and Wilhelm (1992) provide evidence that the inability to rebalance an option position continuously and uncertainty about the return volatility of the underlying asset account for a significant portion of option bid-ask spreads observed in the market. Similarly, Cho and Engle (1999) offer the derivative hedge theory[12] and examine the relationship between option bid-ask spread and illiquidity in the underlying market. Their empirical results support the derivative hedge theory that option market spread is positively related to the underlying market spread. However, option market duration also explains the variation in option market spread. Cho and Engle's (1999) mixed results imply that an option market maker cannot perfectly hedge his positions by using the underlying asset.

2.1.3.2 Transaction Costs

Swidler and Diltz (1992) use both option and stock price transaction data to infer the total transaction costs associated with the Black-Scholes replicating strategy. Pena, Rubio and Serna (1999) find that transaction costs proxied by bid-ask spread are a key determinant of the Spanish IBEX-35 index.

Some empirical studies that compare the performance of different hedging strategies with transaction costs focus on simulated numerical results. Mohamed (1994) uses 95% risk of loss as the criterion to determine the best strategy. Toft (1996) derives closed-form solutions for expected hedging errors, expected transaction costs and variance of the cash flow for both time-based and move-based hedging strategies. He then compares the performance of these strategies under the mean variance framework. Martellini and Phillippe (2002) consider more hedging strategies in their performance comparisons. The aforementioned studies focus on a plain vanilla European call option. These papers show that move-based strategies perform better than time-based strategies

[12] The derivative hedge theory states that, in a perfectly hedged world, the spread of an option is solely due to the illiquidity of the underlying asset rather than inventory risk or adverse selection in the options market.

when the drift of the underlying asset decreases and volatility of the underlying asset increases. Zakamouline (2009) investigates the hedging performance of different types of options and option portfolios. He shows that the ranking of alternative hedging strategies depends of the type of option position being hedge and the risk preferences of the hedger.

2.1.3.3 Portfolio Insurance

The concept of dynamic hedging strategy used in option hedging has been applied to implement portfolio insurance. Garcia and Gould (1987) empirically study the cost of portfolio insurance in terms of forgone returns for the period starting from 1963 to 1983. They demonstrate that portfolio insurance is able to protect the investor against market downfall, however, the evidence does not show that a dynamically rebalanced insured portfolio will outperform a static mix portfolio in the long run. Do and Faff (2004) assess the synthetic put and Constant Proportion Portfolio Insurance (CPPI) under both tranquil and turbulent market conditions in the Australian market. They show that the futures-based implementation of both methods are robust to market conditions in preserving the desired floor value. Annaert, Osselaer and Verstraete (2009) evaluate the portfolio insurance performance through the use of synthetic put option, stop loss strategy and CPPI by using the stochastic dominance approach on a multinational basis. Their results indicate that the 100% floor value should be preferred to low floor values and daily rebalancing strategies dominate strategies with less frequent rebalancing.

2.1.3.4 Implied Risk Aversion Estimation

Jackwerth (2000) empirically derives the implied risk aversion from option prices and realised returns on the S&P 500 index simultaneously. The implied risk aversion functions he finds are consistent with the standard assumption in economic theory, being concave in utility, during the pre-1987 crash period. However, he also observes there are partially increasing risk aversion functions, and utility functions changed to convex in shape during the post-crash period. Aït-Sahalia and Lo (2000) show that implied risk aversion is a U-shaped function of stock price. Both Jackwerth’s and Aït-Sahalia and Lo’s studies assume that the return distribution is constant over long period. Aït-Sahalia, Wang and Yarred (2001) and Rosenberg and Engle (2002) assume the conditional densities are time invariant in deriving the implied risk aversion. Consequently, the resulting risk aversion functions are somewhat inconsistent with the theory. Bliss and Panigirtzoglou (2004) use both power and utility function to estimate the representative agent's relative risk aversion at different time horizons. Their estimates are reasonable and consistent across utility functions and markets.

2.2 Transaction Costs

Transaction costs or trading costs are most often referred to in the literature as the combination of quoted bid-ask spreads, brokerage fees (or commissions), and execution costs (or market impact costs as defined in Constantinides, 1997, and Zakamouline, 2009). Many papers focus on bid-ask spreads in estimating the trading costs of equities. The bid-ask spread of the underlying asset serves as a proxy of transaction costs involved in the hedging process. This is due to the hedger purchasing the asset at the ask price and selling the asset at the bid price.

Although market impact costs have an important role in trading, the impact may not be as economically significant in the hedging context unless a trader is hedging options on small or illiquid stocks at large transaction volume. The hedgers in this context are assumed to be large traders or market makers who trade in liquid stocks. In addition, they also act as price takers in the market. Following Zakamouline (2009), the transaction costs considered in this research are proportional to the trading size, and can be inferred from the spread observed in the market. The formation of bid-ask spread in the market can be affected by order-handling costs (see Demsetz, 1968; Roll, 1984), inventory costs (see Smidt, 1971; Garman, 1976; Amihud & Mendelson, 1980; Ho & Stoll, 1981) and adverse selection costs (see Grossman & Stiglitz, 1980; Glosten & Milgrom, 1985; Kyle, 1985). Studies such as Glosten and Harris (1988), Madhavan and Smidt (1991, 1993), Hasbrouck and Sofianos (1993) and Ho and Macris (1984) explain the above factors using the empirical data.

Roll (1984) derives an implicit bid-ask spread from the first-order serial covariance of price changes under the assumption that the market is informationally efficient. Roll's model offers a simple estimation of execution costs with the use of transaction price data. The implicit spread serves as the price of immediate trade and is expected to be less than the quoted spread, which includes other factors. On the other hand, Arnott and Wagner (1990) decompose the total trading costs into two components: immediate execution costs and the opportunity cost of delaying a trade. With these two components, the total trading costs are U-shaped with respect to transaction time. Three other low frequency spread measures have been developed. Lesmond, Ogden and Trzcinka (1999) develop a spread estimate that is based on the frequency of zero stock returns. Holden (2009) and Goyenko, Holden and Trzcinka (2009) derive the effective tick estimator based on the idea that wider spreads are associated with larger effective tick sizes. Corwin and Schultz (2012) develop the bid-ask spread estimator from daily high and low prices. The high-low spread estimator is a good estimator when intraday trade and quote data are unavailable. The estimator is shown to outperform Roll's covariance estimator and the Lesmond et. al. measure.

In terms of the empirical estimate of equity transaction costs, Stoll and Whaley (1983) report quoted spread and commission costs of 2% and 9% for the largest New York Stock Exchange (NYSE) deciles and smallest deciles respectively. Bhardwaj and Brooks (1992) report median quoted spread and commission costs between 2%, for NYSE securities with prices greater than \$20, and 12.5% for securities with prices less than \$5. Schultz (2000) applies Roll’s estimator to quantify changes in transaction costs of NASDAQ stocks from 1993 to 1996, since time-stamps of trades and quotes cannot be estimated. Hasbrouck (2004) improves Roll’s model using a Bayesian Gibbs approach and applies it to futures transaction data. Hasbrouck (2009) generalises the Hasbrouck (2004) model and applies it to daily Centre for Research in Securities Prices (CRSP) US equity data; he reports that the average effective trading costs are below 1%[13] for the three highest capitalisation quartiles of equities listed on the NYSE and American Stock Exchange (AMEX) after the Great Depression.

Stoll’s (2000) study of frictions provides an overview of trading costs estimations. The quoted spread and effective spread, which reflect total friction, are 7.9 and 5.6 cents for all NYSE and AMEX stocks. Bessembinder (2003a) reports the average quoted bid-ask half spreads are 0.486% for stocks traded on NYSE and 0.739% for stocks traded on the NASDAQ in 1998. Quoted spreads have become less indicative

[13] Hasbrouck (2009) reports one-way costs.

of actual trading costs since decimalisation for both NYSE and AMEX. The effective bid-ask spread appear to a more relevant measure of transaction costs. Bessembinder (2003b) reveals that the trading costs are on average 0.33% for both NYSE and NASDAQ stocks. Alternatively, Lesmond et al. (1999) propose another transaction costs estimator based on the incidence of zero return. They report round-trip transaction costs of 1.2% and 10.3% for large and small decile firms from 1963 to 1990. Goyenko et al. (2009) report round-trip mean effective spread of 2.6%–2.9% for 400 randomly selected US stocks over 1993 to 2005. Corwin and Schultz (2012) find round-trip mean effective spreads of 2.4% for AMEX, NYSE and NASDAQ stocks from 1993 to 2006. In addition, they show that the spread between large stock and small stocks are small in the later period.

2.3 Stochastic Dominance

The concept of stochastic dominance (SD, hereafter) was introduced as majorisation theory in the early 1900s (see Karamara, 1932; Hardy, Littlewood & Polya, 1934); Sherman, 1951; Blackwell, 1951, 1953; and Lehmann, 1955). However, the development of SD theory and its application to economic and finance only begins after the publication of Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970). Since then, an extensive SD literature has developed. Bawa (1982) provides an exhaustive list of early SD publications, and Levy (1992) surveys the SD literature with a focus on contributions since 1980.

This section contains a review of the SD paradigm. I first present the SD criteria and the empirical issues related to the development of the SD algorithm, SD's test, and SD’s application in finance.

2.3.1 Stochastic Dominance Criteria

This section provides the formulation of SD criteria and various ways of defining and ordering investor’s preferences. Let us denote U_i for i=1, 2, 3 as the class of utility function where U_1 includes all utility functions such that $U' \geq 0$; U_2 includes all utility functions such that $U' \geq 0$ and $U'' \leq 0$; and U_3 includes all utility functions such that $U' \geq 0$, $U'' \leq 0$ and $U''' \geq 0$. For economic interpretation, $U' \geq 0$ assumes that the investor prefers more to less. $U'' \leq 0$ assumes that the investor is risk-averse. $U''' \geq 0$

assumes that the investor prefers positive skewness. U_3 is also a set of utility functions that is larger than the set of utility functions with decreasing absolute risk aversion (I denote the later set of utility functions as U_d). Also, assume the investor seeks to maximise his expected von Neumann-Morgenstern expected utility. Then, the following three theorems hold[14]:

Theorem 2.1 Let $F(x)$ and $G(x)$ be the cumulative distributions of two prospects. F dominates G by first-order stochastic dominance (FSD) for all $U \in U_1$ if and only if $F(x) \leq G(x)$ for all values x, and there is at least one x_0 for which a strict inequality holds.

F first-order stochastically dominates G is denoted FD_1G and is illustrated in Figure 2.

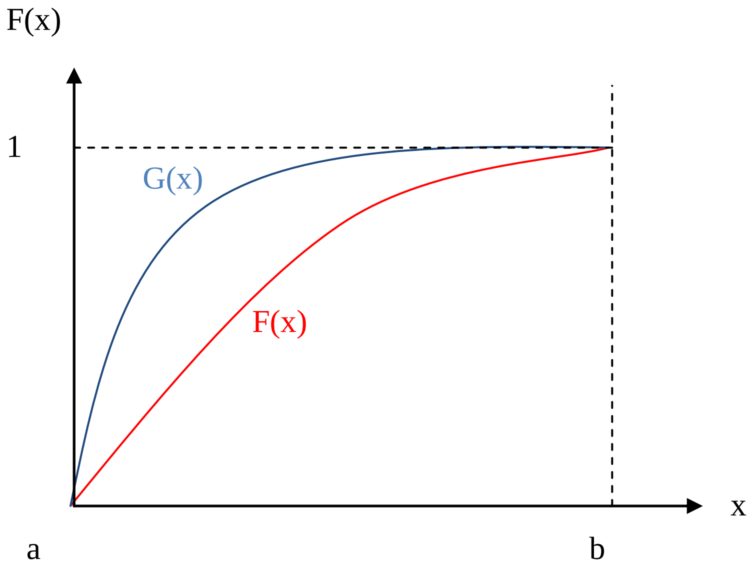

Figure 2 First Order Stochastic Dominance

Theorem 2.2 Let $F(x)$ and $G(x)$ be the cumulative distributions of two prospects whose density functions are $f(x)$ and $g(x)$ respectively. F dominates G by second-order stochastic dominance (SSD) for all $U \in U_2$ if and only if

$$I_2(x) \equiv \int_a^x [G(t) - F(t)]dt \geq 0$$

for all $x \in [a, b]$ and there is at least one x_0 for which a strict inequality hold.

[14] For example, Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970) and Whitmore (1970).

F second-order stochastically dominates G is denoted FD_2G and is illustrated in Figure 3.

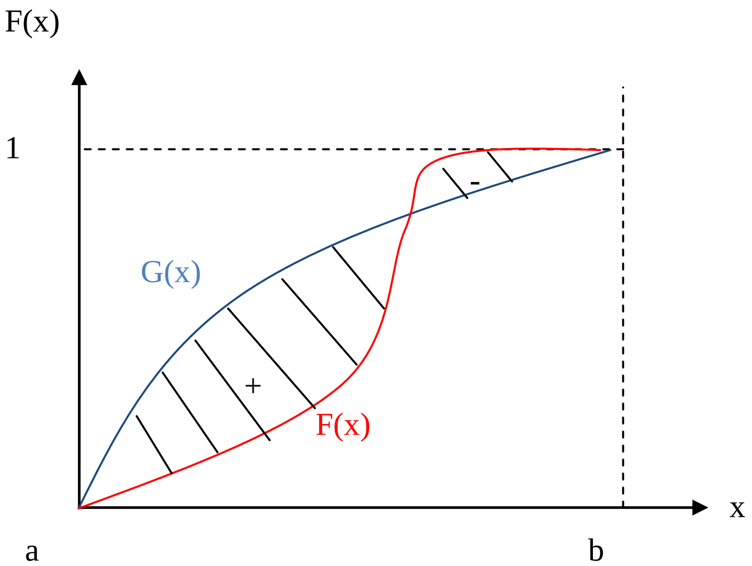

Figure 3 Second Order Stochastic Dominance

Theorem 2.3 Let $F(x)$ and $G(x)$ be the cumulative distributions of two prospects whose density functions are $f(x)$ and $g(x)$ respectively. F dominates G by third-order stochastic dominance (TSD) for all $U \in U_3$ if and only if the following two conditions hold:

(a) $I_3(x) \equiv \int_a^x \int_a^z [G(t) - F(t)]dtdz \geq 0$ for all $x \in [a, b]$,

(b) $E_F(x) \geq E_G(x)$

and there is at least one x_0 for which a strict inequality hold.

F third-order stochastically dominates G is denoted FD_3G. Although not considered here, Whitmore (1989) presents higher order SD rules.

Levy (2006) establishes various sufficient rules for SSD and TSD such that FSD => SSD => TSD. This means that more restrictive sets of assumptions must be imposed in order to derive the higher order stochastic dominance.

In addition to FSD, SSD and TSD, it is often the case that the investor has decreasing absolute risk aversion. This means that the investor is willing to pay a lower risk premium for the same amount of risk as his wealth increases. This leads to another set of criterion for all $U \in U_d$ called *Decreasing Absolute Risk Aversion Stochastic*

Dominance (DSD). Hammond (1974) provides some conclusions for DSD, however, no simple and clear rule can be derived. Given that $U_d \subseteq U_3$, it is obvious that $FD_3G \Rightarrow FD_dG$. As a result, the DSD-efficient sets are a subset of the TSD-efficient sets.

On the other hand, Levy (2006) presents the SD rule for risk-seeking investors. The set of risk-seeking utility functions has the properties of $U' \geq 0$ and $U'' \geq 0$, denoted as $\overline{U_2}$.

Theorem 2.4 Let $F(x)$ and $G(x)$ be the cumulative distributions of two prospects whose density functions are $f(x)$ and $g(x)$, respectively. F dominates G by *Risk-Seeking Stochastic Dominance* (RSSD) for all $U \in \overline{U_2}$ if and only if

$$I_2(x) \equiv \int_x^b [G(t) - F(t)]dt \geq 0$$

for all $x \in [a, b]$ and there is at least one x_0 for which a strict inequality hold.
I denote F dominates G by RSSD as $\overline{FD_2G}$.

The relationship between the various SD rules is summarised in Figure 4.

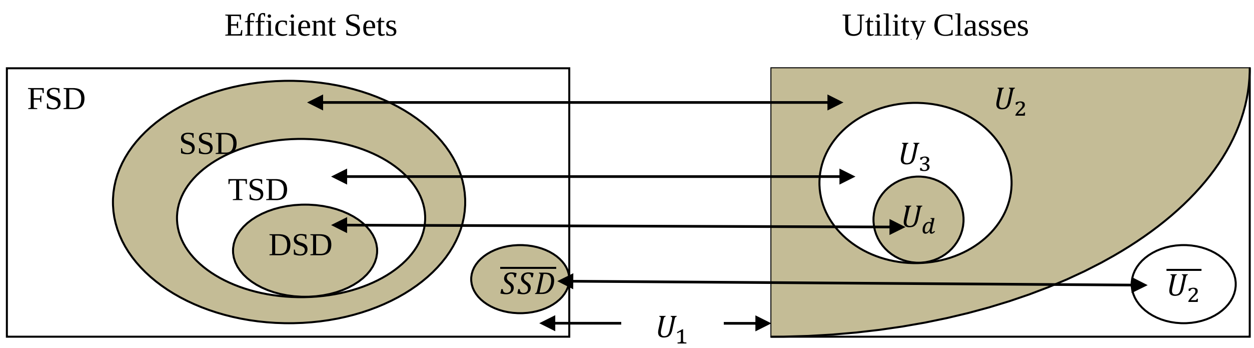

Figure 4 The Stochastic Dominance Relationship between the Different Classes of Utility Functions and the Resulting Efficient Sets

Source: (Levy (2006), Chapter 3, pp. 140)

2.3.1.1 Extensions of Stochastic Dominance

Given the complexity of considering the whole distribution of the prospects being compared, Levy (1973a) first uses the quantile of order *P* of the distribution to examine the SD conditions for log-normally distributed prospects. Levy and Kroll (1978) then formalise a set of SD rules based on the quantile approach. They show the equivalence between the cumulative distribution and the quantile formulations. Let $Q_F(P)$ and $Q_G(P)$ denote the quantiles of order P of distribution F and G such that $Pr_F[X \leq Q_F(P)] = P$ and $Q_G(P)$ are defined similarly. The FSD, SSD and TSD are restated below by using the quantile approach:

Theorem 2.5 F dominates G by FSD if and only if $Q_F(P) \geq Q_G(P)$ for all P, with a strict inequality for at least one P.

Theorem 2.6 F dominates G by SSD if and only if $\int_0^P [Q_F(t) - Q_G(t)]dt \geq 0$ for all P, with a strict inequality for at least one P.

Theorem 2.7 F dominates G by TSD if and only if $\int_0^P \int_0^t [Q_F(z) - Q_G(z)]dzdt \geq 0$ for all P, with a strict inequality for at least one P and $\int_0^1 [Q_F(t) - Q_G(t)]dt \geq 0$.

The advantage of the SD rule based on the quantile formulation is that the analysis can be easily extended among those prospects that have unknown statistical distributions of the mixed random variables. Examples of the quantile approach include Levy and Kroll (1978), who consider the FSD, SSD and TSD when the riskless asset is allowed; Levy (1985) uses it to derive the upper and lower bound of call and put option values; and Levy and Wiener (1998) develop the prospect SD corresponding to the S-shaped utility function of Kahneman and Tversky (1979), that is, the investor displays risk aversion in choices involving gains and risk seeking in choices involving losses.

Jarrow (1986) examines the relationship between arbitrage and FSD. He provides a set of conditions under which FSD implies the existence of arbitrage opportunities. If the two assets have perfect linear correlation, then there exist arbitrage opportunities if and only if there is FSD between the two assets. This relationship is proven by Kroll (1984), who states the arbitrage in terms of state-contingent SD.

2.3.1.2 Multi-Period Stochastic Dominance

The aforementioned SD criteria were developed in the single-period framework. However, investment decisions and portfolio selection process often involve multi-period considerations. Levy (1973b) and Levy and Paroush (1974) extend the one-period SD criteria to the multi-period case. With the assumption of independence over time, they show that if X_1 (the first-period return of prospect X) dominates Y_1 (the first-period return of prospect Y) and X_2 (the second-period return of prospect X) dominates Y_2 (the second-period return of prospect Y), then the two-period return X_1X_2 dominates Y_1Y_2. The result can also be generalised to an n-period return. In addition, Levy and Paroush (1974) show that dependence of the rate of return can be ignored only if multi-period additive utility function is considered. However, the result does not hold when one considers the case of one-period utility function, which is defined on terminal wealth. Levy and Levy (1982) present the multi-period SD criteria with riskless asset. These papers conclude that the size of the efficient sets decreases with the length of the investment horizons with the assumption of stationary and independent distribution. In contrast, Huang, Vertinsky and Ziemba (1978) provide a counter-example showing that the conclusion does not hold.

2.3.2 Empirical Studies

Empirical studies of the application of SD can be divided into two stages. The first stage of the literature focuses on developing an efficient algorithm, while the second stage focuses on the econometric issues of forming consistent tests[15] of SD.

2.3.2.1 The stochastic dominance algorithm and its effectiveness

The early stage of the empirical application of SD rules focuses on identifying the efficient sets in a pool of securities returns. In order to check whether a dominance relationship exists, we need to know the precise cumulative distributions function (CDF) of the rates of return (which can be obtained from historical data). Levy and Hanoch (1970) develop the first algorithm for FSD and SSD, which can be applied to the empirical distributions of the ex-post rates of returns. The algorithms take advantage of the fact that the observed rates of return have uniform discrete distribution. For example, if we have obtained n observations, then each observation will be assigned a probability of $\frac{1}{n}$. Porter, Wart and Ferguson (1973) employ the necessary rules in developing the algorithms to reduce the number of pairwise comparisons of large numbers of portfolios.

[15] A consistent test is one for which the power of the test for an untrue hypothesis increases to one when the number of observations increases.

The algorithms for FSD and SSD are simpler than those for the TSD. Early papers on the TSD algorithm, such as Porter et al. (1973), compare two distributions only at jump points; Kearns and Burgees (1979) apply the trapezoidal rule; Aboudi and Thon (1994) propose a method of finding the crossing points without merging the data points; Bawa, Lindenberg and Rafsky (1979) use zero crossing point and lower partial moment method; Levy and Kroll (1979) employ the quantile approach. According to Levy (2006), the proposed TSD algorithms are wrong for two reasons. First, unlike the FSD and SSD, we cannot only compare the two distributions at the jump points. Fishburn and Vikson (1978) claim that the TSD integral is non-linear and therefore comparisons made at the jump points may lead to the wrong decision. Second, switching between the cumulative distribution and the quantile of a distribution does not apply to TSD. Levy, Leshno and Hechet (2004) derive a TSD algorithm based on the cumulative distribution and check the integral condition in interior points in order to overcome the shortcomings found in the existing literature.

Vickson (1977) presents the algorithms for decreasing absolute risk aversion (DSD). Vickson and Altmann (1977) apply the algorithm to 20 securities traded on the Toronto Stock Exchange and conclude that there is no material difference in stochastic ordering between TSD and DSD. Levy and Kroll (1979) demonstrate the algorithm for SD with riskless assets.

2.3.2.2 Testing for SD Rules

The early empirical studies of portfolio efficiencies and investment strategies using SD rules, such as Levy and Hanoch (1970), Levy and Sarnat (1971), Porter and Gaumnitz (1972), Porter (1973), Porter (1974), Joy and Porter (1974), Vickson and Altman (1977), Kroll (1977), Kroll and Levy (1979), Levy and Brooks (1989), treat the empirical distribution of asset returns as the true cumulative distribution of asset returns. The widespread use of empirical distribution functions (EDFs) may be due to the desirability of avoiding the assumption of the form of the distribution and the optimal properties of EDF as an estimator of the CDF (see Zacks, 1971). Porter et al. (1973) point out the potential difficulties in empirical application of the SD rules. The existing empirical studies, as mentioned above, assume (1) the historical samples represent accurate estimates of the true underlying distribution, and (2) the true distribution remains the same over time so that past data can be used to predict future outcomes.

The empirical application of the SD rules is subject to sampling errors. The historical samples of returns only represent one of the possible observations in the population. Sampling errors arise; for example, we may have X dominating Y in the population but the two empirical distributions intersect, hence no FSD relationship is found empirically. Another possibility is that there is no SD relationship in the population but it is found in the sample. As a result, the application of SD rules is subject to type I or type II error as in any other statistical analysis. Kroll and Levy (1980) consider two types of sampling errors in the use of empirical data:

(a) for two options, one option dominates another one in the population but no dominance is found in the sample (Type II statistical error); and
(b) neither option dominates in the population, but dominance is found in the sample; or one option dominates the other in the population but an opposite dominance relationship is found in the sample (Type I statistical error).

With the use of Monte Carlo simulation, Kroll and Levy (1980), Pope and Ziemer (1984) and Stein and Pfaffenberger (1986) examine the power of tests for efficiency for various distributions. The main result of these papers is that the power of the SD tests is relatively low. Stein, Pfaffenberger and Kumar (1983) derive an analytical formula for the Type I error probability for FSD when the options are sampled from the same population of returns. Tolley and Pope (1988) present the statistical tests for second-order SD using a permutation test, but the power of the test is not examined. To increase the power of the test, Ben-Horim (1990) suggests truncating the tail of the sample before applying the SD rules; he shows that truncation will greatly reduce the tail problem which has led to low power in previous studies. Nelson and Pope (1991) suggest bootstrapping as an alternative way to increase the power of the tests when dominance exists in the population. The improvement resulting from their test may stem from the smoothing process of the order statistics to avoid inadvertent intersection of the cumulative distributions.

Based on the assumptions of independent and identically distributed observations and independent prospects, McFadden (1989) establishes a closed-form statistical test for FSD using the concept of the Smirnov statistic (see Durbin, 1973). He assumes that X dominates Y in the population as the null hypothesis; however, he does not distinguish the case when Y dominates X in the sample, which poses a serious error in the statistical test. McFadden also considers the test for SSD, but its computation is complicated. Klecan, McFadden and McFadden (1991) extend McFadden's test by allowing for dependence in observations and replacing the independence with a general exchangeability amongst prospects. Kaur, Rao and Singh (1994) propose a test in the reverse form such that the null hypothesis is X does not dominate Y against the alternative hypothesis that X dominates Y. The above studies use the infimum or supremum statistics over the support of the distributions.

In contrast, Anderson (1996) and Davidson and Duclos (2000) calculate the test statistics based on a fixed and arbitrarily chosen set of grid points. Anderson employs the trapezoidal rule of approximating integrals at fixed grid points. Davidson and Duclos' approach is based on tests of inequality constraints for which multiple hypotheses at different grid point are involved. Therefore, their tests are based on multiple comparisons. Tse and Zhang (2004) compare the performance of several tests, namely, Kaur et al. (1994), Anderson (1996) and Davidson and Duclos (2000); their Monte Carlo simulation results suggest Davidson and Duclos' test is the best in terms of power. Davidson and Duclos' (2000) test has sound practicality, as it is based on a small number of comparisons; however, the test may be subject to inconsistency. Given that the comparisons are made at fixed grid points, only a subset of SD restrictions is tested. Barrett and Donald (2003) introduce a new test of SD based on a Kolmogorov–Smirnov-type test that compares the prospects at all intervals. They also assume independent samples of prospects with different sample sizes from two populations.

With the use of a subsampling method, Linton et al. (2005) relax the assumptions and allow prospects and observations to be dependent and not identically distributed in estimating the critical values of the tests. In addition, they provide a test for SD among K prospects where $K \geq 2$. Kläver (2005) further improves the Linton et al. (2005) method by using the circular block method to capture the dependence structure of sample data. Donald and Hsu (2016) construct a new Kolmogorov–Smirnov type of test for SD that is less conservative and more powerful than Barret and Donald's (2003) test. Under certain circumstances, their proposed method is also more powerful than the subsampling tests developed by Linton et al. (2005).

2.3.2.3 Mean Variance Rule versus Stochastic Dominance

Markowitz (1958) introduces the mean variance (MV) rule for portfolio selection. The rule for determining the superior performance of one portfolio over another only requires the mean and variance of the return distribution. A portfolio is said to dominate another if it has higher return for the same level of risk or the same return for lower risk. Given its simplicity, the rule has been widely applied in the portfolio management and hedging literature. However, the MV rule is subject to criticisms that its criterion is only appropriate under restrictive assumptions. These assumptions include that the investor has a quadratic utility function or asset returns are normally distributed. Pratt (1964), Arrow (1965) and Hanoch and Levy (1970) have discussed the limitations of quadratic utility functions. One of the important properties is that investors with quadratic utility functions display increasing risk aversion behaviour. In addition, there is plenty of evidence which shows that financial asset returns are non-normal (see Mandelbrot, 1963; Fama, 1965; Clark, 1973; Cornew, Crowson & Town, 1984; Bookstaber & Clarke, 1985; Brown & Warner, 1985; Helms & Martell, 1985; Richardson & Smith, 1993; Peiró, 1999). Despite the criticism, Levy and Markowitz (1979) and Kroll, Levy and Markowitz (1984) show, for various utility functions and empirical return distributions, an expected utility maximiser will be indifferent in making choices between using the direct utility maximisation and MV rule.

The SD rules are conceptually superior to the MV rule, since they maximise the investor's expected utility and utilise the information of the entire probability function. In addition, the dominance results apply to a general class of utility functions. However, the SD rules' practical applicability is limited due to the difficulty in estimating the distribution functions. In fact, these two competing rules have generated considerable literature with a focus on efficient portfolio formation. Aharony and Loeb (1977) and Ghandhi and Saunders (1981) demonstrate the advantages of using the SD rules, while

Levy (1982) demonstrates how the MV rule will fail in the truncated normal distribution. On the other hand, Ashton (1982) shows that SD is inferior to MV when dealing with alternative risk projects given the opportunity to invest in riskless assets. At the same time, a number of studies examine conditions under which the two approaches would be indifferent. These include Ali (1975), which explores the relationships between SD and MV by considering various popular distributions; Johnson and Burgess (1975) who examine how different samples affect the SD and MV tests; and Meyer (1987) who identifies the location and scale parameter condition to ensure consistency between two rules.

2.3.2.4 Applications

Stochastic dominance rules have been applied to fields including economics, finance, insurance, statistics and agriculture. In this section, I focus on their applications in finance.

Levy (1985, 1988) applies SSD rules to derive option bounds when an investor is allowed to invest in a stock or a corresponding call option. Their discrete model can easily incorporate transaction costs and taxes into the analysis.

Brooks, Levy and Yoder (1987) use the SD rule to evaluate the performance of a stock portfolio with options, for example covered calls and protective puts. They point out that it is inappropriate to use the MV rule to assess the option strategies because the return distributions for these strategies are negatively skewed. With simulation, Brooks and Levy (1993) investigate the effectiveness of portfolio insurance by examining the dominance of insured and uninsured portfolios. They find that neither a naked portfolio nor a covered portfolio dominates unless some specific utility functions are assumed.

More recent applications involve the SD statistical tests on actual market data. For example, Fisher, Wilson and Xu (1998) examine the economic significance of the term premium in real returns on US Treasury Bills; Gasbarro, Wong and Zumwalt (2006) analyse the performance of country indices represented by iShares; Cho, Linton and Whang (2007) test the Monday effect in daily stock index returns; Constantinides, Jackwerth and Perrakis (2009) show empirically that there is a mispricing of S&P 500 index options through the widespread violations of stochastic dominance; and Annaert et al. (2009) evaluate the performance of different portfolio insurance strategies using 30 years of equity data from the US, the UK, Japan, Australia and Canada. They find no dominance relationship between portfolio insurance and buy and hold strategies. Al-Khazali, Lean and Samet (2014) use SD analysis to show that Islamic indexes outperformed the conventional Dow Jones indexes during the global financial crisis (GFC), but not when the market is tranquil.

Roman, Mitra and Zverovich (2013) apply the idea of second-order SD as a choice criterion to form an enhanced indexation type of portfolio. Using three datasets, the FTSE 100, Nikkei 225 and S&P 500, they show empirically that the SSD-based model consistently outperforms the index return.

Chapter 3

3. Survey of Transaction Costs Option Pricing Models

This chapter contains a survey of option pricing and hedging models with transaction costs. For simplicity, as in the Black-Scholes world, I assume that the economy includes only two assets: a riskless asset, such as a riskless bond, money market account or cash account, and a risky one, such as stock. There are no transaction costs associated with transactions in the riskless asset, but transactions in the risky asset incur proportional transaction costs. These are the standard assumptions in the existing literature. I will mention explicitly in the following sections if the assumptions in the model depart from the standard ones. In the presence of transaction costs, perfect replication under the Black-Scholes framework is no longer possible because of infinite transaction costs incurred from hedging activities. Therefore, different hedging strategies are proposed in order to hedge the option at the cheapest cost.

In this chapter, I take the perspective of an option writer who hedges his position by holding a portfolio of underlying asset and cash accounts according to certain hedging strategies. I denote the option price at time t as $C\big(t, S(t)\big)$ and maturity as time T. $S(t)$ denotes the stock price at time t. The hedging strategy is as follows: when an option writer wants to hedge the position he uses the proceeds $C\big(t, S(t)\big)$ to set up a hedging portfolio according to some prescribed hedging strategy. If re-hedging is performed frequently, the hedging error will reduce but transaction costs will increase. Conversely, the hedger is able to reduce transaction costs paid but at the cost of larger hedging errors. Essentially, for all hedging strategies, the hedger is facing a tradeoff between reducing transaction costs and diminishing hedging errors. The goal of the option writer or hedger is to attain his objective of minimising the combination of these costs.

This survey groups the existing models according to two main categories of option hedging strategies proposed in the literature, namely, time-based and move-based strategies. Time-based hedging strategies are those for which trading occurs at a fixed regular time interval. The time interval is also termed the hedging frequency in this context. For example, once a hedger has chosen the hedging frequency, he only needs to rebalance the hedging portfolio at the pre-specified time interval until the end of his risk exposure (i.e. option maturity). On the other hand, a move-based strategy permits rebalancing the hedging portfolio according to the movement of the underlying asset price or option delta. A hedger who adopts a move-based strategy is required to monitor the market closely. In addition, I survey other hedging strategies such as stop loss and static hedging, as well as optimal trading strategies introduced in the asset allocation and portfolio management with transaction costs literature.

The layout of the chapter is as follows. Section 1 presents a review of the classical Black-Scholes option pricing model. Subsequent sections present reviews of the models developed under the extended Black-Scholes economy, which relaxes the assumptions of continuous trading and no transaction costs incurred in trading the underlying assets. Finally, I look into the empirical performance of alternative hedging strategies and outline the hedging strategies chosen to assess their performance in chapter 4 and 5.

3.1 Option Pricing and Hedging in a Frictionless Market

Black and Scholes (1973) show that, in a frictionless market, it is possible to replicate an option's payoff by constructing a self-financing portfolio of stocks and cash. In a Black-Scholes world, the stock price, $S(t)$, is assumed to follow a diffusion process given by

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t\,, \tag{3.1}$$

where μ and σ are the mean and volatility of the stock returns respectively, and W_t is a standard Brownian motion. The cash account, $B(t)$, earns at a constant risk-free rate of $r \geq 0$ and its corresponding diffusion process is

$$dB(t) = rBdt. \tag{3.2}$$

Assuming no arbitrage, the price of an option is given by solving the PDE

$$C_t + rSC_S + \frac{1}{2}\sigma^2 S^2 C_{SS} - rC = 0 \tag{3.3}$$

subject to a boundary condition corresponding to the payoff on the option at maturity, $C(T, S(T))$. Here, C_S is the first-order derivative of the option price with respect to stock price commonly referred to as the 'delta' of the option. C_t is the first-order derivative of the option price with respect to time to maturity, which is termed 'theta'. C_{SS} is the second-order derivative of the option price with respect to stock price and is commonly referred to as the 'gamma' of the option.

In the Black-Scholes framework, the hedging strategy at time t consists of holding Δ_t shares of stock and some cash $B(t)$,

$$\Delta_t = C_S = N(d_1), \qquad d_1 = \frac{\ln\left(\frac{S(t)}{K}\right) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}$$

$$B_t = C(t, S(t)) - \Delta_t S(t). \tag{3.4}$$

where K is the exercise price of the option and τ is the time to maturity. The hedging portfolio is rebalanced continuously.

In the presence of transaction costs, the Black-Scholes hedging strategy incurs infinite transaction costs. As a result, various hedging strategies have been proposed to overcome this problem, each resulting in a different price for the option. Figlewski (1989) shows that market imperfections result in wide bounds on equilibrium option prices. In the following sections, I denote c and k as the constant and proportional transaction costs of trading an underlying asset.

Following Dewynne, Whalley and Wilmott (1994), many transaction costs models result can be summarized in the following PDE form

$$C_t + rSC_S + \frac{1}{2}\sigma^2 S^2 C_{SS} - rC = F(S, C_{SS}). \tag{3.5}$$

where the left-hand side corresponds to the Black-Scholes PDE and the function $F(.,.)$ on the right-hand side depends upon the hedging strategy. In the Black-Scholes economy, a perfect hedge is possible and hence $F(.,.) = 0$. On the other hand, in the presence of transaction costs, the option price depends on the cost of hedging. The cost of hedging is linked to how often the hedging portfolio is rebalanced. Hence, one distinct feature of the function $F(.,.)$ is its dependence on the gamma. Gamma is a measure of the sensitivity of the change in delta with respect to the change in the underlying price, and hence it is also a measure of transaction costs.

3.2 Time-Based Strategies

A time-based strategy involves rebalancing a portfolio at a pre-determined regular time interval. This type of strategy is usually local in time, as specified in the hedging literature. A local-in-time strategy means that the investor would like to minimise the hedging risk over a short time interval and does not consider the hedging risk at any other time intervals. In contrast, a global-in-time hedging strategy considers the whole time horizon when minimising hedging risk.

3.2.1 Black–Scholes Hedging at Fixed Time Intervals

The simplest hedging strategy to alleviate the infinite transaction costs incurred in hedging portfolio rebalancing is to implement the Black-Scholes delta-hedging strategy at discrete time intervals. The idea is to divide the time interval [t, T] into fixed regular time intervals of δt, such that $\delta t = \frac{T-t}{n}$. At time t, the hedger forms a hedge with Δ_t unit of the underlying asset and $B(t)$ amount of cash. At time $t + \delta t$, the hedger will rebalance the hedging portfolio according to Black-Scholes delta. Therefore, an additional unit, $\Delta_{t+\delta t} - \Delta_t$, of the underlying asset will be purchased or sold. The hedging is repeated in the same manner for all subsequent time intervals until maturity.

When n is large, the variance of hedging error is small and transaction costs are large because of frequent hedging. The choice of n is related to the hedger's risk aversion. When a hedger is risk-averse, he will choose a large n so that he can rebalance his hedge position frequently and minimise hedging error at maturity. However, a hedging strategy with large n will involve large transaction costs. On the other hand, a less risk-averse hedger will choose a small n because he is willing to accept larger hedging error and save on transaction costs. Note that this strategy is neither local in time nor global in time. Black and Scholes (1973) and Boyle and Emanuel (1980) argue that hedging error is relatively small if one rebalances the portfolio frequently. However, frequent trades will increase the cost of the hedging portfolio significantly and the cost may exceed the price of the underlying asset. In addition, transaction costs appear to be random and tend to amplify the error in the Black-Scholes hedging strategy. On the other hand, this simple time-based strategy may benefit from a volatile market. For example, the underlying asset price movement tends to be volatile on a daily basis but the price may revert back to the last rebalancing state when the time interval is sufficiently large, that is, no or only a small amount of rebalancing of the underlying asset position is required.

3.2.2 Leland's Hedge

Leland (1985) is the first to consider the option pricing and hedging problem in the presence of transaction costs. His model setup is similar to the Black-Scholes model with the exception that the underlying asset (the risky asset) is subject to proportional round-trip transaction costs k. In addition, the hedger will rebalance the portfolio at fixed time interval δt. Leland's strategy is a local-in-time strategy. The expected hedging error (inclusion of transaction costs) and the variance of the hedging error approach zero when δt becomes small. With these features, he proposes a modified Black-Scholes hedging strategy to ensure no infinite transaction costs are incurred in hedging an option no matter how small the re-hedging interval. When transaction costs are small or the rehedging interval approaches zero, Leland's strategy is similar to Black-Scholes delta hedging but with a modified volatility as input:

$$\sigma_m^2 = \sigma^2 \left(1 + \frac{k}{\sigma}\sqrt{\frac{2}{\pi \delta t}}\, sgn(Css)\right) \tag{3.6}$$

where k is the proportional transaction costs, σ is the Black-Scholes volatility, δt is the transaction frequency and sgn is the sign function:

$$sign(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \tag{3.7}$$

Based on Leland's (1985) idea, Hoggard et al. (1993) present the following PDE:

$$C_t + rSC_S + \frac{1}{2}\sigma^2 S^2 C_{SS} - rC = k\sigma S^2 \sqrt{\frac{2}{\pi \delta t}}\, C_{SS}\, sign(C_{SS}) \tag{3.8}$$

with the boundary condition of $C\big(T, S(T)\big) = \max\,(S(T) - X, 0)$. Equation (3.8) is an example of an option hedging with transaction costs model that is expressed in the form of equation (3.5).

Similar to a Black-Scholes discrete hedge, the choice of transaction frequency δt reflects a hedger's risk-aversion level. A very risk-averse hedger will choose to rebalance as frequently as possible and select a small δt, while a relatively risk-tolerant hedger will select a large δt.

Zakamouline (2006b) explains in detail how Leland's modified volatility works in reducing the risk of a hedging strategy. The mechanism (the modified volatility) used in Leland's strategy improves the risk-return tradeoffs of the hedging portfolio. Specifically, the modified volatility allows the hedging error to be negatively correlated with transaction costs. For example, when a hedger is in a short gamma position, the option is hedged with an increased hedging volatility. This increase in volatility will decrease the absolute value of the gamma[16] in a high-gamma region and therefore reduce transaction costs because of reduced re-hedging activities. From another perspective, Leland's hedge allows for systematic gains accumulated over the hedging horizon. The systematic gains are then offset by the transaction costs incurred during the dynamic hedging process.

Let $\Lambda = \frac{k\sqrt{\left(\frac{2}{\pi}\right)}}{\sigma\sqrt{\delta t}}$, such that $\sigma_m^2 = \sigma^2(1 - \Lambda \mathrm{sgn}(\mathrm{C_{SS}}))$. Avallaneda and Paras (1994) show that Leland's model works well when the security has convex payoff or $\Lambda < 1$. In terms of transaction costs k, the condition $\Lambda < 1$ implies that $k < \sigma\sqrt{\delta t}\sqrt{\frac{\pi}{2}} \approx 1.25\sigma\sqrt{\delta t}$. This means that round-trip transaction costs should not exceed 1.25 times the standard deviation of the underlying asset price movement for a single period[17]. For some highly risky securities, Leland's model generates great hedging slippage even though the condition $\Lambda < 1$ is satisfied. On the other hand, when $\Lambda \geq 1$ or the security has a concave payoff, the authors show it is not valid to use Leland's strategy but rather suggest a new path-dependent hedging strategy, which is to delta hedge at each interval using the modified variance for certain periods and maintain static hedge in other periods when the critical events are triggered.

Albanese and Tompaidis (2008) extend Leland's model by using the risk reward analysis found in "good deal" pricing in incomplete markets[18] (see Cochrane & Saa-

[16] An option gamma is negatively related to underlying asset's volatility and spot price.
[17] This corresponds to the assumptions in Leland (1985) that the transaction costs are small and the re-hedging interval approaches zero.
[18] Their idea is based on the fact that investors like assets with high Sharpe ratios.

Requejo, 2000 and Bernardo & Ledoit, 2000). They perform the analysis from the market maker and price taker point of view. They find the optimal length of time interval between each trade and the resulting adjusted volatility are different from those in Leland (1985). The objective of a market maker is different from a price taker. A market maker is interested in minimising the risk taken for a given level of compensation so that he can set the price competitively. On the other hand, a price taker is concerned about maximising his returns, since he can only observe implied volatility from the market. For a market maker, the optimal time length is proportional to the level of transaction costs and inversely related to the volatility and risk reward factor. The optimal adjusted volatility is proportional to the square root of the transaction costs. Let τ^* be the optimal time length between each trade and A be the risk reward factor. The results are as follows:

$$\tau^* = \frac{k}{\sqrt{\pi}A\sigma} + O(k), \qquad \Lambda^* = 2\sqrt{\frac{2Ak}{\sqrt{\pi}\sigma}} + O. \tag{3.9}$$

For a price taker, the optimal time length is proportional to the square of transaction costs and inversely proportional to the volatility and the square of adjusted volatility. The optimal risk reward factor is proportional to the square of the adjusted volatility.

$$\tau^* = \frac{8k^2}{\pi\sigma\Lambda^2} + O\left({k^2}/{\Lambda^2}\right), \qquad \mathrm{A}^* = \frac{\Lambda^2\sigma\sqrt{\pi}}{8k} + O\left({\Lambda^2}/{k}\right). \tag{3.10}$$

In contrast with Leland's model, which discretely hedges an option in a continuous time framework, Boyle and Vorst (1992) consider a discrete time framework for hedging an option with transaction costs. They also point out that Leland's strategy is not self-financing and present another variance adjustment of the form

$$\sigma_L^2 = \sigma^2\left(1 + \frac{2k\sqrt{n}}{\sigma\sqrt{T}}\right), \tag{3.11}$$

when replicating a long call option, where n is the number of periods to option expiration. On the other hand, the adjusted variance for replicating a short call option is

$$\sigma_S^2 = \sigma^2\left(1 - \frac{2k\sqrt{n}}{\sigma\sqrt{T}}\right). \tag{3.12}$$

The adjusted variance is greater than that in Leland's model.

Grannan and Swindle (1996) consider another type of time-based strategy that includes Leland (1985) as a special case. The optimal strategy can be derived under two optimisation criteria: (1) minimisation of the expected square replication error given a portfolio value, and (2) minimisation of the weighted sum of portfolio value and replication error. Their optimal strategy is different from previous studies. The time interval between each re-hedging trade is not constant, but rather more trades happen towards the end of the life of the option. Their method is proved to be able to reduce replication errors relative to the constant time interval strategies in a Monte Carlo simulation setting.

3.2.3 Multi-Scale Strategy

Consider a hedger who has two options. The hedger can choose to hedge two options at every δt time interval or at every $2\delta t$ depending on how much transaction cost he is willing to pay. Martellini (2000) suggests a third method to hedge the options, which is to hedge one option at every δt time interval and another one at every $2\delta t$. He uses the concept that the hedger enjoys a diversification benefit through hedging at different time scales.

Let $\varepsilon = \delta t$. A multi-scale strategy is denoted as follows. A $w_o\varepsilon, w_1 2\varepsilon, \dots, w_n 2^n\varepsilon$ strategy for $T = 2^n m\varepsilon$ is a strategy for which the investor hedges a fraction of w_0 of the option portfolio by trading every ε, a fraction of w_1 of the option portfolio by trading every 2ε, …, and a fraction of w_n of the option portfolio by trading every $2^n\varepsilon$. w_i represents portfolio weight with rebalancing frequency $2^i\varepsilon$. If $\omega_i = 1$ for some i and $\omega_j = 0$ for $i \neq j$, the multi-scale strategy is the same as the single-scale strategy, as Leland (1985) shows. Note that the hedging ratio for every rebalancing portfolio is the same as the one in Leland (i.e., Black-Scholes hedge with adjusted volatility). Although this strategy is derived under the optimal framework, Martellini shows that the performance of the multi-scale strategy is superior to that of the single-scale strategy (i.e., Black-Scholes and Leland) when there is low serial correlation in the return process for any hedging frequency[19]. If the return process has high serial correlation, under- or over-hedging can severely affect hedging performance given that more rebalancing activities occur at some pre-determined time intervals.

[19] Martellini (2000) uses the following performance measure to gauge the superiority of the multi-scale strategy: for a given level of expected transaction costs (expected tracking error), the superior strategy will produce smaller expected tracking error (expected transaction costs).

3.3 Move-Based Strategies

Move-based strategies are defined as strategies that rebalance the hedging portfolio according to the movement of the underlying asset price or the delta of the option. These strategies require continuous monitoring of market movements. The most common move-based strategies are described in the following sections.

3.3.1 Henrotte's Asset Tolerance Strategy

Instead of re-hedging a portfolio at fixed regular time intervals, Henrotte (1993) recommends a hedging strategy based on movement of the underlying asset price. The hedger monitors the market continuously and rebalances the hedging portfolio back to a perfectly hedged position when the percentage change in the price of the underlying asset exceeds a pre-determined amount. The perfectly hedged position is determined by the Black-Scholes model. The series of stopping times, ω_i, is recursively given by:

$$\omega_1 = t, \quad \omega_{i+1} = inf\left\{\omega_i < \omega < T: \left|\frac{S_\omega - S_{\omega_i}}{S_{\omega_i}}\right| > h\right\}, i = 1,2, \dots \qquad (3.13)$$

where h is a given constant percentage.

The choice of h depends on the hedger's risk aversion level. A risk-averse hedger would select a small h in order to obtain small hedging error at maturity but at large transaction costs. In contrast, a more risk-tolerant hedger is willing to face larger hedging error but enjoy lower transaction costs.

Compared to time-based strategies, this move-based strategy may improve the performance of the hedging portfolio. This strategy reacts to market movements, which can overcome the hedging slippage when the hedging time interval is large. Yet the relationship between stock price movement and the change in the hedging portfolio value is not monotonic. It is important for us to consider the sensitivity of the option delta (which represents the stock portion of the hedging portfolio) to the underlying asset price movement. Figure 5 below shows the relationship between the gamma of a call option (with strike price *X*) and the price of the underlying asset.

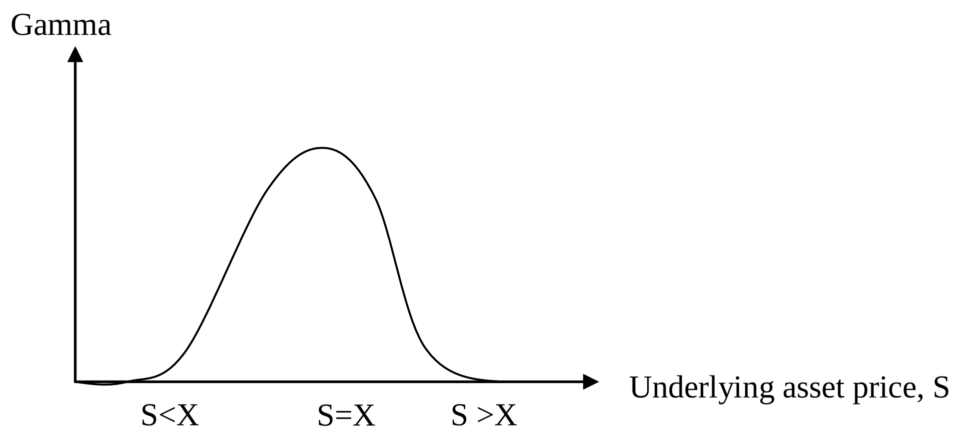

Figure 5 Long Call Option Gamma and Underlying Asset Price

Option gamma is largest when the option is at the money. Therefore, the implementation of an asset tolerance strategy may incur extra transaction costs for in-the-money and out-of-the-money options. This is because asset tolerance hedging criterion is based on the absolute percentage change in underlying asset price. Transaction costs are charged as long as the movement in the underlying asset price has breached the hedging bandwidth, even though the option gamma is low (or has low sensitivity to option delta).

3.3.2 Delta Tolerance Strategy

Another popular hedging strategy, introduced by Whalley and Wilmott (1993), is based on the movement of the option delta. With this hedging strategy, a hedger will rebalance a hedging portfolio to the Black-Scholes delta when the hedging ratio moves outside the tolerance level H. The series of stopping times, ω_i, is recursively given by:

$$\omega_1 = t, \quad \omega_{i+1} = inf\left\{\omega_i < \omega < T: \left|\Delta - \frac{\partial c}{\partial s}\right| > H\right\}, i = 1,2, \tag{3.14}$$

where $\frac{\partial c}{\partial s}$ is the Black-Scholes hedge, and H is a given constant tolerance level.

H is related to the desired hedging precision. A risk-averse hedger would choose a small H , while a more risk-tolerant hedger will choose a large H. This strategy improves the performance of the hedging portfolio by reducing the extra amount of transaction costs associated with time-based strategies and the asset tolerance strategy, given that rebalancing depends on the sensitivity of the change in portfolio value to the change in underlying asset price. Dewynne et al. (1994) point out that this hedging strategy is inappropriate for hedging options with large and positive gamma (i.e., long at-the-money call or put options). This is because the hedging portfolio value will increase in a very short time in the neighbourhood of the expiry date.

3.3.3 Utility-Based Hedging

Hodges and Neuberger (1989) derive an option hedging strategy explicitly taking into consideration the hedger's risk preference specified by a negative exponential utility function of the form

$$U(w) = -e^{-\gamma w}. \tag{3.15}$$

The utility-based approach in option pricing and hedging uses the concept of indifference in expected utility between final wealth with and without an option liability. The difference between the strategies with and without an option liability reflects the hedging action. The concept is illustrated in the following setup. This approach is considered as global in time in the hedging literature because it focuses on minimising the portfolio risk for the entire time horizon.

Assume that the hedger faces proportional transaction costs and maximises his expected utility of terminal wealth. The hedger has a finite horizon of $[t, T]$ and there is no transaction costs at terminal time T. The hedger has x_t unit of stocks and y_t in cash. Let $U(w_t)$ be the utility of wealth at time t. The value function of the hedger without an option liability is

$$J_0\big(t, x_t, y_t, S(t)\big) = \max E_t[U(\, x_T S(T) + y_T] \tag{3.16}$$

The value function of the hedger with an option liability is

$$J_w\big(t, x_t, y_t, S(t)\big) = \max E_t[U(\, x_T S(T) + y_T - \max\,(S_T - K, 0)] \tag{3.17}$$

The option value C and the optimal strategy are obtained by solving the following equation:

$$J_w\big(t, x_t, y_t + C, S(t)\big) = J_0(t, x_t, y_t, S(t)) \tag{3.18}$$

The above equation (3.18) implies that the hedger is willing to receive C in order to write an option such that his expected utility at terminal T is indifferent from not writing an option. The value function or indirect utility function is solved recursively backwards through time using the Hamilton–Jacobi–Bellman dynamic programming approach of stochastic optimisation:

$$C_t + rSC_S + \tfrac{1}{2}\sigma^2 S^2 C_{SS} - rC = F(S, C_{SS}) \tag{3.19}$$

The resulting hedging strategy involves a no-transaction region. Let us denote two boundaries Δ_l and Δ_u such that $\Delta_l < \Delta_u$. As long as the hedge ratio Δ_t lies within $\Delta_l \leq \Delta_t \leq \Delta_u$, the portfolio is not required to re-hedge. If Δ_t falls outside of the no-transaction region, then the hedger needs to re-hedge the portfolio back to the nearest boundary of the no-transaction region (or hedging bandwidth). For example, if $\Delta_t < \Delta_l$, the hedger should trade the underlying asset to bring the hedge ratio back to Δ_l and vice versa. Figure 6 below demonstrates the relationship between Black-Scholes delta and the optimal strategy. Hodges and Neuberger (1989), Clewlow and Hodges (1997) and Zakamouline (2006a) have documented that the middle of the optimal hedging strategy does not coincide with the Black-Scholes delta.

Although the resulting hedging strategy is optimal, it is rarely used as the strategy lacks a closed-form solution and is computationally demanding in implementation. Davis et al. (1993) provide a rigorous proof of the existence and uniqueness of the solutions. Constantinides and Zariphopoulou (1999) derive tighter closed-form upper and lower bounds of writing an option by maximising the expected utility. Clewlow and Hodges (1997) propose a numerical algorithm based on the binomial scheme in Cox et al. (1979) to compute the optimal hedging strategy while substantially reducing the computational burden in Hodges and Neuberger (1989). Through simulation, Clewlow and Hodges (1997) have also shown that the optimal strategy is superior to Leland's strategy, the Black-Scholes strategy and a heuristic strategy that centred the optimal control region on the Black-Scholes delta. Another method of numerical implementation is found in Davis and Pans (1994).

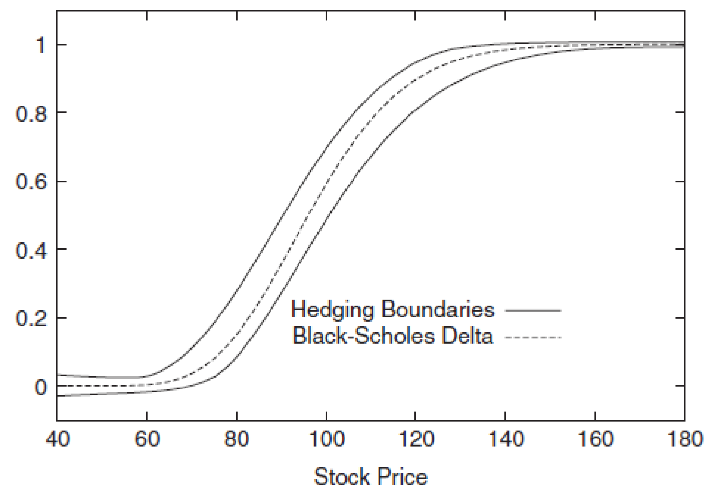

Figure 6 Black–Scholes and Optimal Strategy Reservation Short Call Option Prices
Source: Zakamouline (2006a).

There are two advantages of the assumed negative exponential utility function: the hedger's strategy is independent of the amount of his holdings in the cash account, and the utility function simplifies computation. The choice of the utility function is restrictive, and the result may differ if a different utility function was used. Davis et al. (1993) conjecture and Andersen and Damgaard (1999) show that the option price is insensitive to the choice of utility function, and instead the level of risk aversion plays an important role. One obvious advantage of utility-based hedging strategies over Leland's (1985) strategy is that it can be applied to a portfolio of long and short options with a simple modification to the payoff function. The disadvantage of the strategy is the necessity of prescribing the hedger's utility function.

3.3.3.1 Hedging to a Fixed Bandwidth around Delta

Given the computation burden in deriving the hedging strategy Hodges and Neuberger (1989) propose, a simplified version of the utility-based hedging strategy can be substituted. Following Martellini and Priaulet (2002), the boundary of no-transaction region for this simpler hedging strategy is as follows:

$$\Delta = \frac{\partial c}{\partial s} \pm H \tag{3.20}$$

where $\frac{\partial c}{\partial s}$ is the Black-Scholes hedge, and H is a given constant tolerance level.

As with previous case, H also represents the hedger's risk aversion level. For that reason, a risk-averse individual will choose a small no-transaction region while the relatively risk-tolerant agent will choose a relatively large no-transaction region. This strategy is closely related to the delta-tolerance hedging strategy. The major difference between this strategy and the delta-tolerance strategy is that the hedger will rebalance the hedge ratio to the nearest boundary of the hedging bandwidth. On the other hand, the delta-tolerance strategy will have a hedge ratio that is equal to the perfect Black-Scholes hedge position. As a result, the hedger is able to save the transaction costs for extra rebalancing between the perfect hedge position and the boundary.

This strategy is subject to the limitation that the hedging bandwidth is not varied according to the change in option gamma. It has been shown in the literature that gamma (as shown in Leland, 1985) plays an important role in option hedging. Gamma measures the sensitivity of the change of delta with respect to the stock price. Hence, it is often referred to as a measure of the level of transaction costs. If an option moves into a large gamma region, then the hedging portfolio is expected to re-hedge more often and therefore incur large transaction costs. A better strategy to avoid large transaction costs is to have a larger bandwidth when the option has large gamma; however, this strategy lacks the ability to self-adjust, so may still be subject to large transaction costs when the hedging ratio moves beyond the bandwidth in the high gamma region. For this reason, this hedging strategy may have poor performance for the at-the-money option with large gamma values.

3.3.3.2 Hedging to a Variable Bandwidth around Delta

Whalley and Wilmot (1997) propose another way to alleviate the computationally time-consuming problem of obtaining the optimal hedging strategy. They provide an asymptotic solution to Hodges and Neuberger's (1989) problem by assuming that transaction costs are small.

Davis et al. (1993) derive a transaction costs model expressed in PDE form:

$$C_t + rSC_S + \frac{1}{2}\sigma^2 S^2 C_{SS} - rC = \frac{\delta}{\gamma}\left(\frac{3k\gamma^2 S^4 \sigma^3}{8\delta^2}\right)^{\frac{2}{3}} \left(\left|C_{SS} - \frac{\delta(\mu - r)}{\gamma S^2 \sigma^2}\right|\right)^{\frac{4}{3}} \tag{3.21}$$

where γ is the index of risk aversion and $\delta = e^{-r(T-t)}$. Through asymptotic analysis, they show that the boundary of the no-transaction region is of the following form:

$$\Delta = \frac{\partial C}{\partial S} \pm \left(\frac{3}{2}\frac{e^{-r(T-t)}kS\Gamma^2}{\gamma}\right)^{\frac{1}{3}}, \quad (3.22)$$

where $\frac{\partial C}{\partial S}$ is the Black-Scholes hedge, Γ is the Black-Scholes gamma, γ is the hedger's absolute risk aversion and k is the proportional transaction costs. As k approaches zero, the optimal hedge will become the Black-Scholes hedge. The above formula can be further simplified as follows:

$$\Delta = \frac{\partial C}{\partial S} \pm h(e^{-r(T-t)}S\Gamma^2)^{\frac{1}{3}} \quad (3.23)$$

where h is a given constant tolerance level that reflects the hedger's risk aversion.

Instead of hedging to a fixed bandwidth around delta as in section 3.3.3.1, the hedger will rebalance the hedge ratio to the nearest boundary of the variable hedging bandwidth once the delta movement is beyond the non-hedging bandwidth. The bandwidth is determined by option gamma, spot underlying asset price, risk-free rate and time to maturity, and thus changes over time. As we expect there will be more re-hedging activities in regions with high gamma, the optimal control region is adjusted to be positively related to the gamma value so as to reduce the transaction costs that occur in frequent re-hedging. Another feature of the optimal strategy is that the no-transaction region becomes larger as the option close to maturity. This implies that a hedger tends to trade less towards the end of the option position and that most hedging transactions have been done in early stages. The role of the spot underlying asset price in determining the no-transaction region is linked to the fact that there is no one-to-one correspondence between the change in the underlying price and the change in the delta. The inclusion of the underlying asset price allows adjustment of the delta of the hedge only for the amount that is required to keep within the no-transaction region and to reduce some transaction costs.

Barles and Soner (1998) perform another version of asymptotic analysis of Hodges and Neuberger's (1989) model by assuming that both transaction costs and the hedger's risk tolerance are small. The optimal strategy has two key elements: a particular form of hedging bandwidth and volatility adjustment. This strategy is different from that of Whalley and Wilmott (1997) because of the additional term of volatility adjustment. The volatility adjustment is similar to the one in Leland (1985). For example, the hedger wants to hedge a short option position; the adjusted volatility is greater than the original volatility. As seen in equation (3.6), the volatility adjustment has a positive relationship with the transaction costs and option gamma. As with Hodges and Neuberger (1989) and Whalley and Wilmott (1997), the optimal hedging strategy is to keep the hedge ratio inside the no-transaction region defined as below:

$$\Delta = \frac{\partial C(\sigma_m)}{\partial S} \pm \frac{1}{k\gamma S} g(k^2 \gamma S^2 \Gamma), \tag{3.24}$$

where $\frac{\partial C(\sigma_m)}{\partial S}$ is the Black-Scholes delta with adjusted volatility given by

$$\sigma_m^2 = \sigma^2 \left(1 + f\left(e^{r(T-t)} k^2 \gamma S^2 \Gamma\right)\right). \tag{3.25}$$

The function $f(z)$ is the unique solution of the nonlinear initial problem

$$\frac{\partial f(z)}{\partial z} = \frac{f(z)+1}{2\sqrt{zf(z)} - z}, \; z \neq 0, \; f(0) = 0. \tag{3.26}$$

For $z > 0$, $f(z)$ is a concave increasing function. Both Whalley and Wilmott (1997) and Barles and Soner (1998) share the characteristics that the hedging bandwidth increases when the level of transaction costs, hedger's risk tolerance or option gamma increases.

Zakamouline (2006a) compares the performance of the exact numerical calculation of Hodges and Neuberger (1989) model with the asymptotic models and finds that the performance of the asymptotic models is worse than the exact strategy. The Barles and Soner (1998) model performs relatively better than Whalley and Wilmott's (1997) model due to the adjusted volatility, which reduces the sensitivity of the option delta to the underlying asset price movement. Zakamouline investigates the poor performance of the asymptotic strategies and discovers that the asymptotic analysis provides inaccurate solutions when the parameters are neither very large nor very small. In particular, the size of the hedging bandwidth and the volatility adjustments are overvalued. More importantly, the interrelationship between the size of the hedging bandwidth and the adjusted volatility is not sustainable. In Zakamouline's empirical testing, either undervaluation or overvaluation significantly affect the hedging performance; as a result, she proposes an approximation strategy to hedge a short European call option. Her proposed optimal hedging strategy has the general specification of

$$\Delta = \frac{\partial C(\sigma_m)}{\partial S} \pm (H_w + H_0), \tag{3.27}$$

where σ_m and H_w are functions which depend on option gamma, while H_0 is independent of option gamma. The term H_w is similar to those obtained in Whalley and Wilmott (1997) and Barles and Soner (1998). When an option becomes deep in-the-money or deep out-of-the-money, the gamma of the option approaches zero. This means that the hedging bandwidths suggested by Whalley and Wilmott (1997) and Barles and Soner (1998) approach zero. In fact, based on exact calculation, the numerical results show that the hedging bandwidth of the optimal strategy times the stock price converges to a constant value when gamma approaches zero, as seen in Figure 7 below.

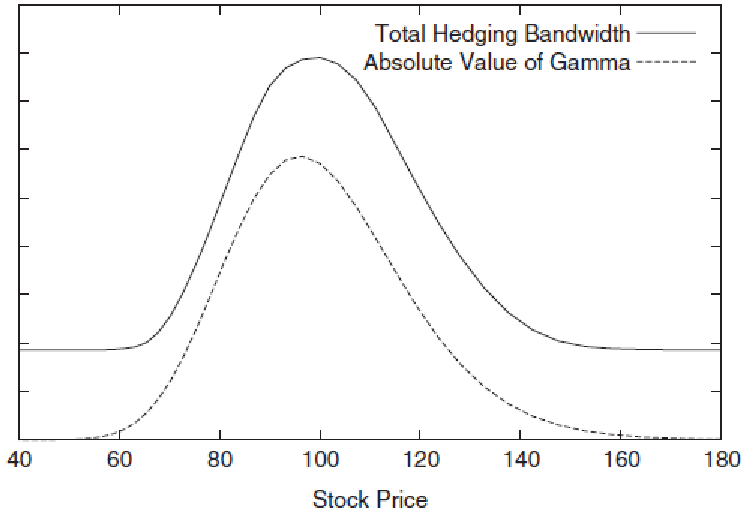

Figure 7 The Form of the Optimal Hedging Bandwidth versus the Option Gamma

Source: Zakamouline (2006a).

The term H_0 captures the constant value observed from the optimal hedging bandwidth using exact numerical calculation. The constant value turns out to be the half of the size of the no-transaction region in the optimal model without option liability. Based on Zakamouline's model calibration, the following approximate functions are obtained:

$$H_0 = \frac{k}{\gamma S \sigma^2 (T-t)}, \tag{3.28}$$

$$H_w = 1.12 k^{0.31} (T-t)^{0.05} \left(\frac{e^{-r(T-t)}}{\sigma}\right)^{0.25} \left(\frac{|\Gamma|}{\gamma}\right)^{0.5}, \tag{3.29}$$

$$\sigma_m^2 = \sigma^2 \left(1 + 4.76 \frac{k^{0.78}}{(T-t)^{0.02}} \left(\frac{e^{-r(T-t)}}{\sigma}\right)^{0.25} (\gamma S^2 |\Gamma|)^{0.15}\right). \tag{3.30}$$

However, this model is of limited practical use because the calibration requires a large dataset (and therefore it is very time-consuming to derive the optimal strategy) and the above estimations are only valid for a particular type of option. Nevertheless, this optimal strategy has been tested and its performance is close to the exact strategy derived from Hodges and Neuberger's model.

3.4 Other Hedging Strategies

3.4.1 Super Replication

Bensaid et al. (1992) and Edirsinghe et al. (1993) show that one can obtain a tighter upper bound on writing an option by replacing the goal of replicating the option payoff with the goal of dominating the payoff at maturity; that is, the terminal call option payoff becomes

$$C\big(T, S(T)\big) \geq \max(S(T) - X, 0). \tag{3.31}$$

This method is called super replication of an option payoff. The strategy derived from super replication is optimal since it satisfies the optimality criterion. At least two optimality criteria have been defined in the literature. One is defined in terms of maximising the expected utility at terminal date, which we have seen in section 3.3.3. Another is to minimise the initial cost of a strategy, which produces dominating payoff at maturity.

The advantage of using super replication is that the optimal strategy is independent of the hedger’s risk aversion. However, finding the minimum cost hedging strategy under super replication is a nonlinear problem. Bensaid et al. (1992) construct a dynamic programming algorithm to obtain the cost-minimising trading strategy. To overcome the computational burden, Edirsinghe et al. (1993) reformulate the nonlinear problem into either a linear programming model or a two-stage dynamic programming model under the assumption that the underlying asset price movement follows a binomial model. The difference between these two algorithms is that Edirsinghe et al. introduce current stock and bond position as the state variables while Bensaid et al. introduce the entire stock price path as the state variable. However, the derived optimal strategy does not have a general representation; instead, we only know about the characteristics of the optimal strategy. For a hedger to adopt such an optimal strategy, it is no longer necessary to trade in every period and it is optimal to establish a larger initial position and reduce the amount of trading in later periods. These characteristics can also be found in the utility-based hedging strategies.

A major disadvantage of the super replication approach is that the cheapest cost obtained depends on the number of trading periods. For a long position in a European call option, the price is an increasing function of the number of trading periods. Further, as noted earlier, Davis and Clark (1993) conjecture and Soner et al. (1995) prove that the cheapest super replication strategy is to purchase one share of the underlying asset initially and hold it until maturity. This leaves an unsatisfactory result of little economic interest for an option writer, for whom the option premium bound is the underlying asset price.

3.4.2 Stop-Loss Start-Gain Strategy

Consider a simple trading strategy in which the investor will hold no stocks or bonds when the option is at-the-money. The investor will only buy one unit of stock by using the borrowed fund every time the option becomes in-the-money. Conversely, the investor will sell one unit of stock and use the proceeds to repay the loan when the option becomes out-of-the money.

Seidenverg (1988) terms this strategy the stop-loss start-gain strategy and analyses it by modelling the stock price movement under a binomial framework. Assay and Edelsburg (1986) test the effectiveness of this strategy through a Monte Carlo simulation. A similar study by Dybvig (1988) examines the stop-loss strategy[20] in the asset allocation problem. Carr and Jarrow (1990) formalise the proof that this strategy is not self-financing. Both Assay and Edelsburg (1986) and Dybvig conclude that, even in the absence of transaction costs, this strategy requires more transactions and suffers from the large cost of inefficiency in implementation.

Further, the infinite crossing property of geometric Brownian motion implies infinite transaction costs may be incurred.

[20] This strategy is to invest in the stock until the portfolio value falls below a specified level. The investor is able to limit potential losses by switching his investment to bond.

3.4.3 Static Hedging

Carr, Ellis and Gupta (1998) propose static hedging for exotic options using standard options. Given the features of exotic options, which tend to be sensitive to volatility and have high gamma, static hedging is considered easier and cheaper than dynamic hedging using the underlying asset. They propose to use the European put-call symmetry to form the static hedge, that is, the method relies on the relationship between European put and call with different strike prices but they have the same maturity. Although the paper does not consider the impact of transaction costs on static hedging, this type of hedging strategy is expected to incur small transaction costs for trading in the standard options. Eventually, static hedging is like a buy-and-hold strategy and no intermediate trading during the life of the option.

3.5 Asset Allocation with Proportional Transaction Costs

A related strand of literature concerns asset allocation with transaction costs. Asset allocation refers to the decision of allocating wealth across different asset classes. In the literature, most authors consider a riskless asset and a risky asset in solving the problem. The objective of the investor is to maximise his or her expected terminal wealth. Therefore, the proposed solutions to the asset allocation problem are similar to those for the utility maximisation option hedging strategy as described in section 3.3.3.

Merton (1971) is the pioneer in solving the asset allocation problem. He assumes that there are only two assets in the economy. The riskless asset is earning at constant risk-free rate and the risky asset is log-normally distributed. An investor is assumed to have a power utility function, and will consume some of his wealth over time. At the same time, the investor will invest a fraction of $\alpha(t)$ of his wealth in the risky asset and the remaining fraction of $1-\alpha(t)$ in the riskless asset. The power utility function is of the form

$$U(W) = \frac{W^{\gamma}}{\gamma}, \gamma < 1. \tag{3.32}$$

In the absence of transaction costs, the optimal trading strategy for the investor is to invest a constant fraction $\alpha^*(t)$ of his wealth in the risky asset;

$$\alpha^*(t) = \frac{\mu - r}{(1-\gamma)\sigma^2}, \tag{3.33}$$

where μ is the expected return on the risky asset, r is the risk-free return and σ is the volatility of the risky asset. The optimal ratio is also known as the Merton ratio.

Kamin (1975), Magil and Constantinides (1976) and Constantinides (1979, 1986) consider the impact of transaction costs on the optimal choice of allocation of wealth between a riskless asset and a risky asset. The transaction costs are only deducted from the account holding the riskless asset. In addition, the investor's consumption is taken out from the riskless asset through time. In particular, Kamin (1975) and Magil and Constantinides (1976) show that there exists a no-transaction region for the fraction of risky assets. If a portfolio lies in the buy region, then the investor needs to buy the risky asset until the portfolio reaches the region's boundary. Conversely, if a portfolio lies in the sell region, then the investor needs to sell the risky asset until the portfolio reaches the sell region boundary. The portfolio space can be divided into three positive convex cones as shown in Figure 8, whereby X^0 and X^1 denote the amount of riskless and risky assets held in the portfolio. The Merton ratio, derived from an economy without transaction costs, stays inside the wedge of the no-transaction region.

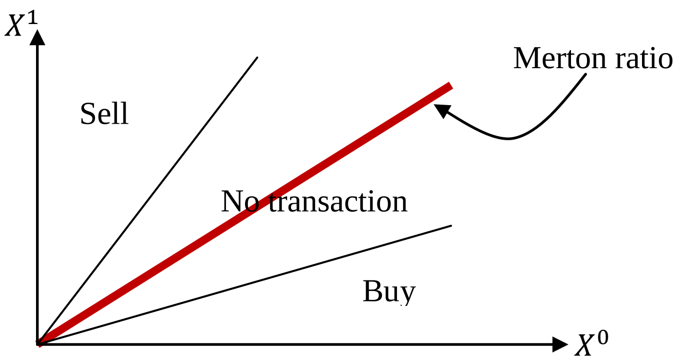

Figure 8 Portfolio Space and Trading Strategy

Constantinides (1979) addresses the situation for which one can simply reduce the multi-period consumption investment problem to a single-period framework, while Constantinides (1986) derives the approximate solution for the boundaries by assuming that the investor has a power utility function and restricting the consumption rate to a constant fraction of the holding in the riskless asset. Some important results documented by Constantinides (1986) are that:

- high transaction costs will widen the no-transaction region and shift the region towards more investment in the riskless asset;

- the width of the no-transaction region is not affected by the investor's risk aversion. However, an increase in the investor's risk aversion will shift the region towards the riskless asset; and
- the width of the no-transaction region is insensitive to the variance of the risky asset's rate of return (though an increase in the variance shifts the region towards the riskless asset).

Davis and Panas (1990) investigate the same problem, and show that the optimal buying and selling boundaries can be obtained by solving a free boundary problem of a nonlinear PDE without imposing restriction on the consumption process. They also provide an algorithm for numerical calculation. Shreve and Soner (1994) remove more of the restrictive assumptions in obtaining the optimal solution. For all these papers, the models assume that the investor has an infinite investment horizon and the investor maximises his or her discounted utility of intermediate consumption. On the other hand, Dumas and Luciano (1991) analyse the problem by modifying the assumption that the investor accumulates his wealth without consuming until a point in time at which he consumes all of his wealth; the investor's objective is to maximise his expected utility of terminal wealth. In their paper, they use the limiting argument that the finite time horizon becomes large. With this modification, the authors are able to obtain an analytical solution. Dumas and Luciano's (1991) findings are similar to those in Constantinides (1986), except that the investor has no preference for cash investment corresponding to an increase in his risk aversion level. This is due to the fact that there is no intertemporal consumption in their model.

As opposed to the previous studies, Gennotte and Jung (1994) examine the effect of transaction costs on the optimal trading strategy by considering the investor has a finite horizon and he only consumes at the end of the horizon. They use a binomial approximation to numerically compute the optimal trading strategy. The results are consistent with Constantinides (1986). In addition, they also find that the width of the no-transaction region (i) becomes constant when the time to maturity is lengthened, and (ii) converges faster to a constant when the volatility of the risky asset increases, or transaction costs decrease, or risk aversion decreases. When the time horizon increases, the no-transaction region converges to the infinite horizon case.

Overall, the investor's utility is reduced compared to the no-transaction costs case. Balduzzi and Lynch (1999) use discrete approximations to solve a similar optimisation problem. Liu and Loewenstein (2002) introduce some jump events to resemble an investor's uncertain horizon by using an independent Poisson process. In contrast to Dumas and Luciano (1991), they find that it is not optimal for the investor to invest in stock subject to transaction costs if the expected horizon is short.

One common feature of the optimal trading strategies proposed is that, in order to reduce the impact of transaction costs, the investor will modify his or her trading strategy in terms of trading frequency and the size of their trades.

3.6 Empirical Studies

Toft (1996) analyses the cost and risk of hedging an option discretely under the MV framework. He provides closed-form solutions for expected hedging error, transaction costs and variance of the cash flow. His analysis indicates that the move-based strategy is superior to the time-based strategy when the volatility of the underlying asset is high, transaction costs are small and the hedger is less risk-averse.

Compared to the extensive list of theoretical models established over the years, there are relatively few empirical comparisons of different methods for option hedging with transaction costs. Only four published papers – Mohamed (1994), Martellini and Priaulet (2000), Zakamouline (2006a), and Zakamouline (2009) – systematically compare the performance of competing hedging strategies through Monte Carlo simulations. I discuss these papers from four perspectives: types of options, methods of comparison, hedging strategies covered, and results.

3.6.1 Type of Options

Mohamed (1994), Martellini and Priaulet (2002) and Zakamouline (2006a) focus on hedging plain vanilla short European call options, whereas Zakamouline (2009) examines the hedging performance of exotic options. The distinction between an exotic option and a plain vanilla option is in terms of its payoff function. The exotic option payoff function tends to contain jumps, discontinuities and barriers, and be path dependent. These options often have high gamma, meaning dynamic hedging is very costly. In addition, exotic options are highly sensitive to volatility. As a consequence, dynamic hedging may result in substantial hedging error due to volatility misspecification.

3.6.1.1 Methods of Comparison

Previous studies define the hedging error as the difference between the value of the replicating portfolio (excluding transaction costs) and the payoff on the option at maturity. The profit and loss (P&L) is then defined as the hedging error net of total transaction costs (TC),

$$P\&L = HE - TC. \quad (3.34)$$

The total transaction costs are the sum of transaction costs incurred during each rebalancing trade adjusted for the time value of money and so is expressed in present value terms. An alternative approach, and the one adopted in this thesis, is to adjust the value of the replicating portfolio at maturity by the cumulative effect of transaction costs incurred over the life of the hedging strategy. For this purpose, the net hedging error is the difference between the value of the replicating portfolio (after transaction costs) and the payoff on the option at maturity.

Following Toft (1996), Martellini and Priaulet (2002) and Zakamouline (2009), I compare the performance of the alternative hedging strategies under the MV framework. Varying parameters which serve as proxies for hedging frequency or bandwidth are used to calculate the expected net hedging error and the variance (or standard deviation) of the net hedging error. These values are then used to span the space of all possible strategies, from the most accurate hedging strategy to the least accurate one, in order to form an efficient frontier[21] in the MV plane for each type of strategy. The approach is described in more detail in section 4.3.1 below.

Instead of using variance as the measure of hedging risk, Mohamed (1994) uses the 95% risk of loss to gauge performance. The idea is similar to the Value at Risk (VaR), which is to estimate the hedging error for the 95th percentile of loss. In other

[21] The literature uses the terminology of 'efficient frontier' in describing the hedging errors in the MV space; arguably, MV curve is a more suitable term. This is because the efficient frontiers presented in the literature have inefficient outcomes that should not appear on an efficient frontier. Hence, in this thesis, I use the term MV curve instead of efficient frontier when forming the hedging performance comparison under the MV comparison framework.

words, we expect to observe the loss that exceeds this amount of hedging error with less than 5% probability. In Mohamed's study, the best hedging strategy is the one which has the lowest value of the 95% risk of loss.

Zakamouline (2006a) uses both standard deviation and 95% risk of loss as risk measures to compare the hedging performance under the MV framework.

All four studies listed above cover four popular hedging strategies proposed in the literature:

(1) re-hedging using Black-Scholes delta at fixed time intervals,
(2) re-hedging using Leland's delta at fixed time intervals,
(3) re-hedging using a delta-tolerance strategy, and
(4) re-hedging using variable bandwidth around delta based on Whalley and Wilmott's (1993) asymptotic solution.

In addition to the above strategies, Martellini and Priaulet (2002) test the multi-scale strategies proposed in Martellini (2000). Both Martellini and Priaulet (2002) and Zakamouline (2009) test the performance of Henrotte's (1993) strategy, which is based on the percentage change in the underlying asset price, with that of other strategies. In contrast, Zakamouline (2006a) mainly focuses on the performance of the optimal hedging strategies (which are based on utility maximisation) by using different closed-form solutions derived from the asymptotic analysis and approximation method.

3.6.1.2 Results

Mohamed (1994) and Martellini and Priaulet (2002) both show that a move-based strategy is superior to a time-based strategy for a plain vanilla short European call option. In particular, it is advantageous to adopt a move-based strategy when the drift of the underlying asset decreases and its volatility increases. However, the performance of the move-based strategy deteriorates following the introduction of stochastic volatility and a fixed transaction costs is incurred each trade. For exotic options, Zakamouline (2009) demonstrates that the ranking of the performance of the hedging strategies is mixed. The ranking depends on the composition and payoff function of the exotic option and the hedger's risk preference.

3.7 Selected Hedging Strategies

For comparative purposes with previous studies, I select six hedging strategies for empirical testing using simulated data in chapter 4 and actual market data in chapter 5. In addition to the MV rule used in the previous studies, I also use the SD rule to systematically compare the hedging strategies. The advantages of using the SD rule are stated in chapter 4. The specific hedging strategies examined in this thesis are:

Time-based strategies:

(1) Black–Scholes hedge at fixed time intervals (BS); and

(2) Leland's hedge (LS);

Move-based strategies:

(3) Henrotte's asset tolerance strategy (AT);

(4) delta tolerance strategy (DT);

(5) hedging to a fixed bandwidth around delta (FB); and

(6) hedging to a variable bandwidth around delta (after Whalley & Wilmott, 1997) (VB).

Chapter 4

4. Simulation Study

This chapter presents an examination of option hedging performance using Monte Carlo simulation. The simulated hedging performances of different hedging strategies serve as benchmarks for the subsequent empirical studies reported in chapter 5. In addition, simulation studies can solve some of the problems in empirical tests. For example, in the test of option models using historical data, researchers are often limited to examining a single set of data, and are unable to identify the true ex-ante price distribution. The simulations reported in this chapter allow me to assess how strategies perform in a controlled environment.

The objective of the study is to determine the optimal hedging strategy with a tradeoff between transaction costs and hedging precision - the smaller the hedging error the more precise the hedge. Theoretically, a hedging strategy with high rebalancing frequency will produce a small hedging error but involve high transaction costs. The existing literature focuses on comparing different classes of option hedging strategies using the MV analysis framework. Some weaknesses of the MV comparison method were identified in chapter 2. First, the MV framework only works well under the assumption of investor has a quadratic utility function or normally distributed random variables. Second, MV only focuses on the first and second moment of the distribution, rather than the whole distribution of the random variables.

In this study, I use the SD test to determine the existence of an optimal option hedging strategy in the presence of transaction costs. If a strategy stochastically dominates another strategy, then the dominant strategy maximises the expected utility of the hedger. As opposed to the MV analysis, SD considers the whole distribution of the net hedging errors. Further, I investigate whether MV and SD tests produce consistent or drastically different results.

As previously mentioned, the following six hedging strategies are examined:

(1) Black-Scholes hedge at fixed time intervals (BS),

(2) Leland's hedge (LS),
(3) Henrotte's asset tolerance strategy (AT),
(4) delta tolerance strategy (DT),
(5) hedging to a fixed bandwidth around delta (FB), and
(6) hedging to a variable bandwidth around delta (after Whalley & Wilmott, 1997) (VB).

For all hedging strategies except LS, the hedger wishes to optimise the tradeoff between transaction costs paid and hedging precision by allowing some hedging inaccuracy. The introduction of hedging inaccuracy allows the hedger to reduce the transaction costs. In BS, a reduction in transaction costs is achieved by rebalancing the hedging portfolio at discrete time points rather than continuously (as assumed in their model). However, regular periodic rebalancing is not necessarily optimal. For example, if the underlying asset price has not changed significantly since the last rebalancing point, then it may not be optimal to rebalance the portfolio. This gives rise to the idea of the AT strategy, which uses a hedging criterion based on the percentage change in the underlying asset price. For the AT strategy, the hedger will rebalance the stock position in the hedging portfolio back to the Black–Scholes delta when the rebalancing event is triggered. Further the relationship between a change in asset price and corresponding change in option delta is not one-to-one: a small change in the underlying price can lead to a large change in option delta and vice versa. Hence, the DT strategy is more closely related to the rebalancing event; that is, the portfolio is only rebalanced when the option delta changes significantly. Following the literature on optimal portfolio selection with transaction costs such as Constantinides (1986) and Davis and Norman (1990), it is more optimal to rebalance to the boundaries of the no-transaction region than to rebalance to the optimal portfolio holdings in the absence of transaction costs. Thus, one introduces a hedging bandwidth around the Black–Scholes delta with the aim of avoiding some intermediate but unnecessary trades. I therefore expect FB to outperform DT. Option gamma determines the change in option delta; the higher the gamma, the larger the change in delta. As a result, the total transaction costs incurred in DT and FB are proportional to option gamma since both hedging criterion are based on a change in delta. To further reduce the total amount of transaction costs paid, VB is introduced such that its hedging bandwidth increases when gamma is high. Hence, I expect VB will perform better than FB.

In contrast to the abovementioned hedging strategies, LS has a special mechanism that can reduce hedging volatility and may, at the same time, lower transaction costs. Given that LS is similar to the BS strategy but with modified volatility, I expect LS[22] to outperform BS when transaction costs are present.

4.1 Simulation Setup

Consider a market maker who has an imbalance in short and long option positions and so wishes to hedge a short call option. In this chapter, I consider hedging either a single call option or a portfolio of options on the same underlying asset, whereby all options have the same maturity. Evaluating a portfolio of options on a basket of underlying assets or the same options with different maturities are not within the scope of this study.

I take the viewpoint of a call option writer who has sold an European option on stock with exercise price X and maturity at time T. For a short call position, the option writer receives the option premium as compensation at time zero for the risk of a cash outflow at maturity. At time T, if the stock price is greater than X, then the option writer is obliged to pay the option buyer the difference between the stock price and X. On the other hand, if the stock price is less than X, then the option writer is not obliged to pay the option buyer. Given the potential cash outflow at maturity, the call option writer enters into a series of transactions in underlying stock and cash such that he is able to repay the potential cash outflow at time T. I term these transactions hedging a short call option or replicating a long call option. Therefore, the option writer also becomes a hedger.

In order to study the performance of alternative option hedging strategies, I first simulate the underlying stock price process. I define $S(t)$ as the spot price of the underlying stock at time t and $B(t)$ as the cash amount in money market account at time t. I assume $S(t)$ follows geometric Brownian motion, which means the spot price at time $t + dt$ is of the form:

$$S(t+dt) = \mu S(t)dt + \sigma S(t)dW(t) \quad (4.1)$$

[22] Leland's strategy assumes discrete time trading and proportional transaction costs in trading underlying assets.

where μ is the drift of the stock price process, σ is the volatility of the stock price process, dt is the time interval and $W(t)$ is a standard Brownian motion. A standard Brownian motion has a normal distribution with mean zero and volatility $\sqrt{t}$. Discretising the stock price process gives:

$$S(t+\delta t) = S(t)e^{\left(\mu-\frac{\sigma^2}{2}\right)\delta t+\sigma\sqrt{\delta t}Z}, \tag{4.2}$$

where Z is a standard normal distribution and δt is the time interval between each observable stock price. In my simulation study, δt is assumed to be $\frac{1}{250}$ which corresponds to a single trading day. Table 2 sets out the inputs used in simulating the stock price process. For consistency with my empirical study in chapter 5, I assume the underlying stock is the S&P 500 index and accordingly base the parameter inputs on the daily averages of the S&P 500 index for the period from January 2, 1996 to September 30, 2009. I simulate 200,000 stock price paths[23], each with a duration of six months[24].

Table 2 Inputs for Stock Price Simulation

Parameter	Simulation input[25]
Risk free rate (r)	4.60%
Volatility (σ)	16.09%
Drift (μ)	8.87%
Time interval (δt)	1/250
Time to maturity (T)	0.5 year

Let $B(t)$ be the amount of cash at time t. Similar to the Black–Scholes economy, I assume cash accumulates at a risk-free rate. Therefore, the process for $B(t)$ is of the form:

$$dB(t) = rB(t)dt. \tag{4.3}$$

[23] The antithetic variates method is used when generating the random variables in order to reduce the variance of the simulated stock price paths.

[24] Zakamouline's (2009) simulation method may have introduced inconsistent simulated stock price paths between time-based and move-based strategies. For time-based strategies, the stock price path is simulated based on the rebalancing interval δt as seen in (4.2). However, for move-based strategies, each stock price path consists of 250 equally spaced trading dates over the life of the option. Given that two different sets of random variables are used in the stock price path simulation, inconsistency is introduced to the Monte Carlo simulation that forms the basis of her net hedging error calculation.

[25] The standard deviation for the risk-free rate is 1.65. The volatility input is based on the average of VIX and its standard deviation is 4.39. The standard deviation for the drift rate is 0.17.

4.2 Calculation of Net Hedging Error

A hedging portfolio *V(t)* refers to the value of stock and cash held at time *t*. Further, I define *C(t)* as the option premium or price at time *t* and Δ_t to be the amount of stock holding at time *t*. At time zero, the hedger sets up a hedging portfolio consisting of Δ_0 units of stock and cash, by using an option premium received from the option buyer and borrowing. The option premium received by the hedger is assumed to be the Black–Scholes value of the option in my simulation study. Therefore, $V(0) = \Delta_0 S(0) + B(0) - transaction\,costs$ and *V(0)* = *C(0)*. This means that *B(0)* is the cash remaining after purchasing Δ_0 units of stock and transaction costs paid for purchasing Δ_0 units of stock. A negative *B(0)* represents borrowing. The hedging portfolio value will change over time, and rebalancing is required such that the hedger is able to deliver the cash flow at maturity if the option finishes in-the-money. As mentioned previously, I will test the performance of six hedging strategies, two of which belong to the time-based class and four to the move-based class. Each hedging strategy has different rebalancing criteria.

For the time-based strategies, BS and LS, I check at each δt if the hedging portfolio requires rebalancing. For example, at each $t+ \delta t$, a new delta is calculated. If delta at time $t+ \delta t$ is different from delta at time *t*, then the stock holding at time $t+ \delta t$ will be adjusted to the new delta at time $t+ \delta t$. When a rebalancing trade is performed, transaction costs are withdrawn from the cash account. Figure 9 shows that time-based strategies rebalance the hedging portfolio at regular intervals. For time-based strategies, a reduction of transaction costs can be achieved by adjusting the time interval between each rebalancing point. The main difference between BS and LS strategies is the calculation method for Δ_t, which determines the amount of stocks to purchase or sell at each rebalancing point (i.e. $\Delta_{t+\delta t}$ - Δ_t). For BS, Δ_t is the delta at time *t* derived from the Black-Scholes model; for LS, Δ_t is derived from the Black–Scholes model with Leland's modified volatility.

Move-based strategies (AT, DT, FB and VB) rebalance the portfolio whenever the hedging criteria are met. With these strategies, rebalancing occurs at non-constant time intervals. As suggested by its name, the hedging criteria for move-based strategies depend on changes in market-observable information. The threshold of the movement serves as the control for the transaction costs. Compared to time-based strategies, move-based strategies' hedging criteria have more variation. For AT and DT strategies, the amount of stocks to purchase or sell at each rebalancing point is equivalent to the difference between Δ_t and $\Delta_{last\ rebalancing\ time\ point}$. In contrast, FB and VB only rebalance their stock position to the nearest boundary of Δ_t. Detailed explanation of the hedging criteria for each strategy was provided in chapter 3.

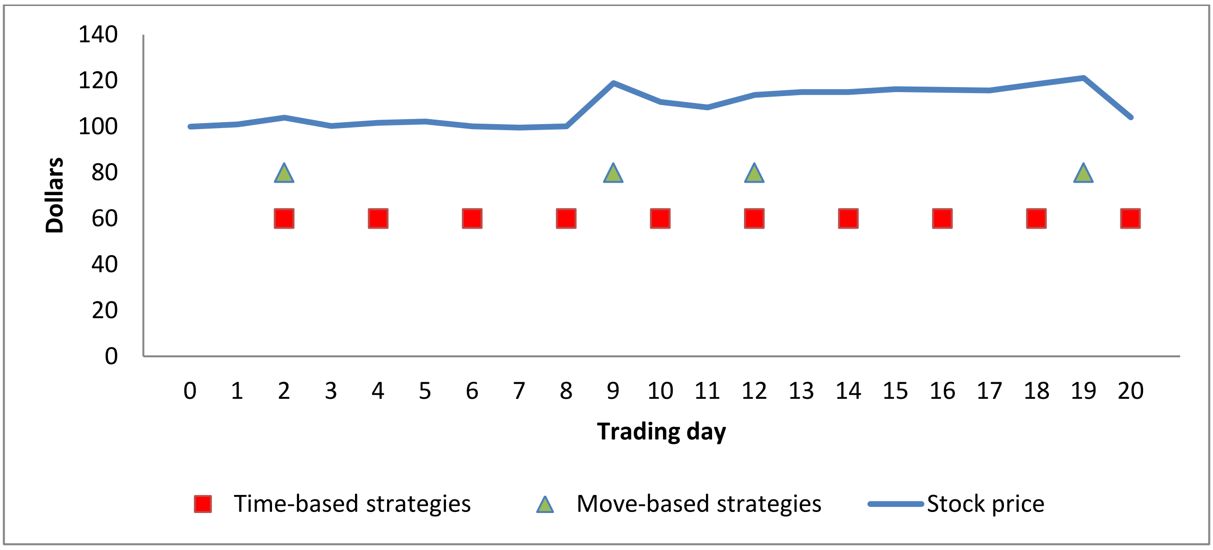

Figure 9 Time-based and Move-based Rebalancing Example

When rebalancing criteria for the hedging strategy are met, a series of rebalancing trades in undertaken. Next, I describe the change in portfolio value before and after rebalancing at time *t*.

Portfolio value before rebalancing:

$$V(t) = \Delta_{last\ rebalancing\ period} S(t) + B(t-1)e^{r\delta t} \tag{4.4}$$

Prior to rebalancing, the hedging portfolio value consists of the value of stock at time *t* based on the stock position carried over from the last rebalancing period and the cash value accumulated at a risk-free rate from the last period $t - \delta t$.

Portfolio value after rebalancing:

$$V(t) = \Delta_t S(t) + B(t) - \ transaction\ costs \tag{4.5}$$

The hedging portfolio value reflects the new stock position Δ_t at time *t* and new cash value *after deducting transaction costs*. It is noted that Δ_t is determined according to the choice of hedging strategy. Δ_t may not be the option delta derived from the Black–Scholes model if FB and VB are the choice of hedging strategy. I denote one-way transaction costs as k% of traded stock amount in dollars. Given that transaction costs are paid regardless of whether stock is bought or sold, the transaction costs incurred for a trade at time t is k times $\left|\Delta_t - \Delta_{last\ rebalancing\ period}\right| \times S(t)$, which is non-recoverable and will be deducted from the cash account.

At maturity time *T*, I compute the net hedging error as the value of the hedging portfolio *V(T)* minus the payoff of the option. The NHE for a call option is

$$NHE = V(T) - \max(S(T) - X, 0) \tag{4.6}$$

where *X* is the strike price of the call option. *NHE is inclusive of transaction costs deducted from the cash account throughout the life of the option when rebalancing is required.* If no transaction costs are incurred in trading stocks (i.e., k = 0%) then the difference between *V(T)* and option payoff at maturity represents the hedging error (HE) as commonly used in the literature. For each hedging strategy, 200,000 net hedging errors are generated - one for each of the 200,000 simulated stock price paths and then scaled by the initial option price calculated using the Black–Scholes formula.

4.2.1 Transaction Costs Assumption

In testing the performance of alternative hedging strategies, I chose the level of one-way proportional transaction costs k to be 50 basis points[26] of stock price at the time of trading. This assumes transaction costs are inclusive of one-half bid-ask spread and one-way trading fees[27]. This transaction costs assumption is consistent with Constantinides et al.'s (2008) study of mispricing of S&P 500 index options. Similar transaction costs are used in simulation studies by Mohamed (1994), Clewlow and Hodges (1997) and Zakamouline (2009). Further, a round-trip transaction costs of 1% is supported by the empirical findings in Hasbrouck (2009)[28]. In contrast, Bessembinders (2003a) shows that the level of proportional transaction costs can further reduce to

[26] The sensitivity of transaction costs assumption is tested and the results are reported in section 4.5.
[27] Trading fees refer to brokerage fees as a compensation for order processing costs. This assumption is consistent with the findings in my empirical study chapter. The empirical results show that the average bid-ask spread for the S&P 500 exchange traded fund is 40 basis points. Therefore, the assumption of 10 basis points of brokerage fees is reasonable.
[28] Hasbrouck (2009) finds that the effective trading costs are below 1% for the highest capitalisation quartiles of equities listed on AMEX and NYSE.

0.21% to 0.24% for a large investor trading in large and very liquid stocks. I assume there are no transaction costs in investing in cash or risk-free bonds.

4.3 Mean Variance Analysis

This section compares the performance of alternative hedging strategies using mean variance curves.

4.3.1 Construction of mean variance curves

In this section I describe how the simulated NHEs are used to construct one MV curve for each of the six hedging strategies. I illustrate the construction procedure in Figure 10 using the Black–Scholes hedging strategy. The starting point is to calculate the mean and standard deviation of the 200,000 simulated NHEs assuming Black–Scholes rebalancing every five days. This gives a *single point* in MV space as shown in the first panel of Figure 10. Under the MV framework, I use the mean NHE as the return measure and standard deviation of NHE as the risk measure. The process is repeated assuming Black-Scholes rebalancing every 10 days and again assuming Black-Scholes rebalancing every 20 days. This gives the second and third points in MV space in the first panel of Figure 10.

Extending the set of rebalancing frequencies to 2, 3, ..., 60 days and repeating the process gives the MV curve for the BS hedging strategy in the second panel of Figure 10. This MV curve represents the risk-return profile for the BS strategy for a range of different rebalancing frequencies.

A similar process is used to generate a MV curve for each of the other five strategies. For example, the MV curve for LS is presented in the third panel of Figure 10.

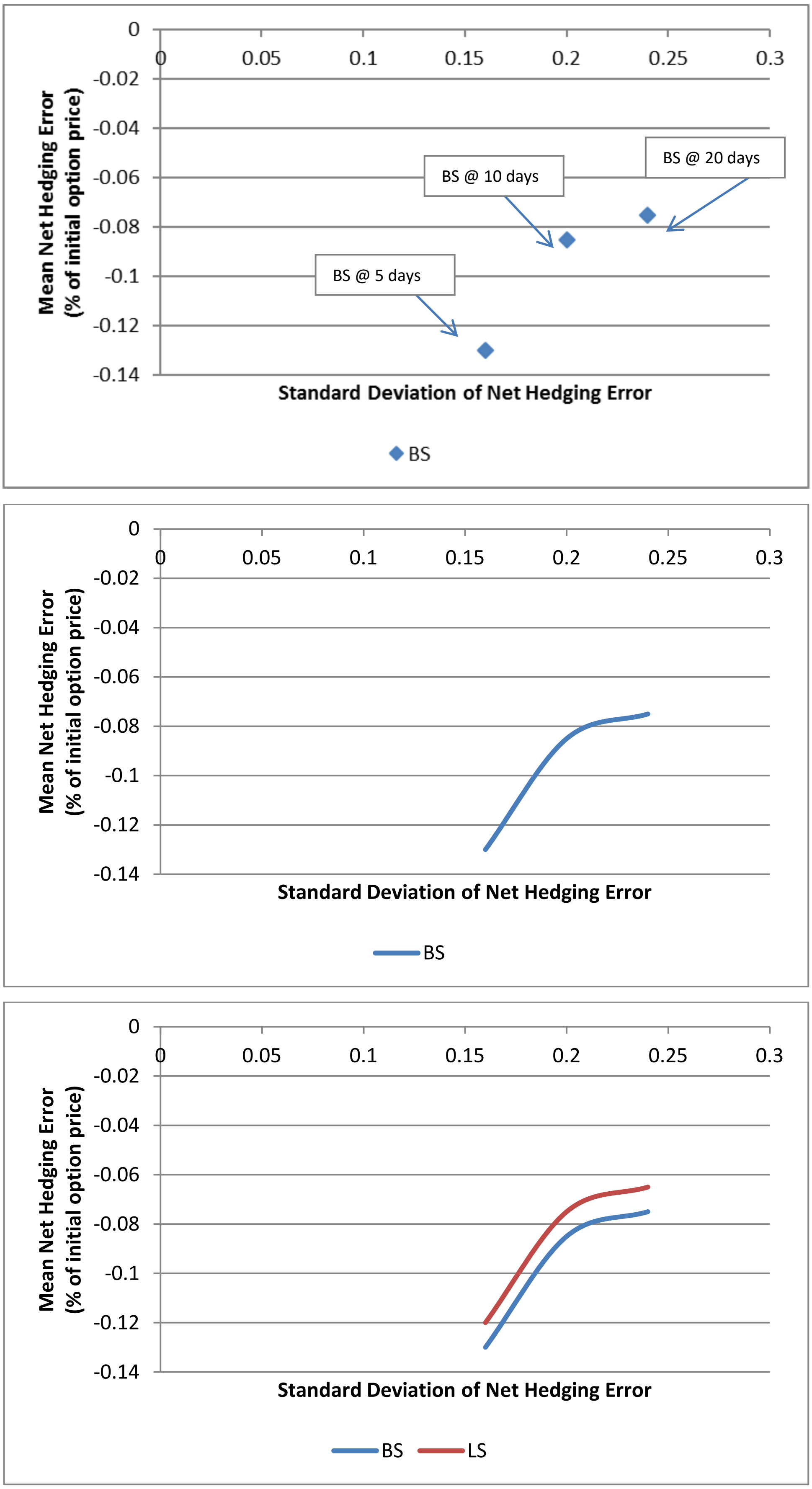

Figure 10 Example of Mean Variance Curve Construction

In the transaction costs literature, the degree of risk aversion has a direct impact on NHEs. A highly risk-averse hedger is unwilling to accept a large NHE at maturity as he prefers high precision in hedging outcome. Therefore, in the presence of transaction costs, a highly risk-averse hedger is willing to pay higher transaction costs in exchange for the likelihood of smaller NHEs. On the other hand, a less risk-averse hedger will prefer to pay lower transaction costs in exchange for the likelihood of larger NHEs (low precision in hedging outcome). As a result, there is a negative relationship between the precision of NHE and risk aversion and a positive relationship between total transaction costs of hedging and risk aversion. The desired level of hedging precision is incorporated by a specific parameter in each of the six hedging strategies. For example, a hedger who chooses to rebalance his portfolio every day by using the BS strategy has the choice of hedging parameter $\delta t = \frac{1}{252}$. Hedging every day allows the hedger to reduce his likely NHE compared to the case when he chooses to rebalance every week, e.g. $\delta t = \frac{5}{252}$. In another words, the more frequently a portfolio is rebalanced, the more risk-averse the hedger. On the flip-side, the hedger has to pay higher transaction costs throughout the life of the option. The six hedging parameters (and assumed range of possible values) for the six hedging strategies are described in Table 3.

In five of the strategies (BS, LS, AT, DT and FB), a higher parameter value corresponds to a less risk-averse hedger. In the VB strategy, however, a higher parameter value corresponds to a more risk-averse hedger, due to the mechanics of the exponential utility function.

The range of hedging parameter values in Table 3 is largely based on Zakamouline (2009)[29]. Note the choice of the hedging parameters ensures all MV curves lie on the same MV space. For BS and LS strategies, the upper bound of rebalancing every 60 days ensures the hedging portfolio is rebalanced at least once

[29] Zakamouline (2009) examines the hedging performance of exotic options versus a plain vanilla call option. In her study, time to maturity of the option is one year for the non-path-dependent option and six months for the path-dependent option. The simulation setup in my study is slightly different to Zakamouline's simulation setup.

during the assumed six-month life of the option. The parameters for the move-based strategies are chosen to match the MV curves resulting from the time-based strategies[30].

A hedging strategy is superior to all others if it offers the highest return on a hedging portfolio at a certain amount of risk. In another words, it has the highest mean NHE relative to other strategies for a given standard deviation of NHE.

Table 3 Hedging Parameters for Simulation

This table shows the hedging parameters for each hedging strategy used in the simulation study. The first column represents the choice of hedging strategy, the second column represents the hedge parameter in symbol format and the last column specifies the range of parameter values[31].

Hedging strategy	Hedging parameter	Range of parameter values
Black–Scholes hedge at fixed time intervals (BS)	δt (in days)	$[\frac{1}{252}, \frac{60}{252}]$
Leland's hedge (LS)	δt (in days)	$[\frac{1}{252}, \frac{60}{252}]$
Henrotte asset tolerance (AT)	H (percentage change in underlying asset price)	$[0.001, 0.24]$
Delta tolerance (DT)	H (option delta)	$[0.001, 0.30]$
Hedging to a fixed bandwidth around delta (FB)	H (option delta)	$[0.001, 0.30]$
Hedging to a variable bandwidth around delta (VB)	γ (risk aversion)	$[0.0001, 100]$

4.3.2 Results

The MV curves for the six hedging strategies tested on the simulated price data are shown in Figure 11. There are two points to note to assist with their interpretation.

[30] Note my choice of VB hedging parameter is larger than Zakamouline's, which has a range between 0.1 and 2. With Zakamouline's VB parameters assumption, my MV curve for VB will start slightly later than the starting point of MV curves for the time-based hedging strategies, Hence, there is a lack of comparison points for the TB strategy rebalancing every one and two days. Further, with my parameter assumption, the VB MV curve appears to be smoother because the MV points obtained from risk aversion parameter values between 2 and 100 are heavily clustered at the starting points of the curve.

[31] For each hedging strategy, there are 60 hedging parameters within the listed range in Table 3. In another words, these 60 points will form an MV curve for each parameter value of the six hedging strategies.

First, in accordance with expectations, the hedger's level of risk aversion increases as we move from right to left. Second, the negative mean NHEs are explained as follows. The more frequent the rebalancing activity, the greater the transaction costs incurred. Hence, it has greater negative impact on the mean NHE. Further, the initial hedging portfolios, for each of the six strategies, are constructed assuming the amount of option premium received is equivalent to the Black–Scholes value of the option. However, the option premium derived by transaction costs models are higher for a short option position. For example, Boyle and Vorst (1992) show that, in the presence of transaction costs, there exists a bound for option price. Cho and Engle (1999) also demonstrate that option hedging activity explains the bid-ask spreads in an option market. In another words, the option premium derived from the Black–Scholes model that assumes a frictionless market is the value obtained using the cheapest and most efficient way to hedge the option payoff at maturity using stocks and bonds. When more restrictive assumptions are relaxed, option bounds become wider and centred around the Black–Scholes value. As a result, the hedging portfolio value is always in deficit in my simulation because of insufficient funds used to set up the initial hedging portfolio.

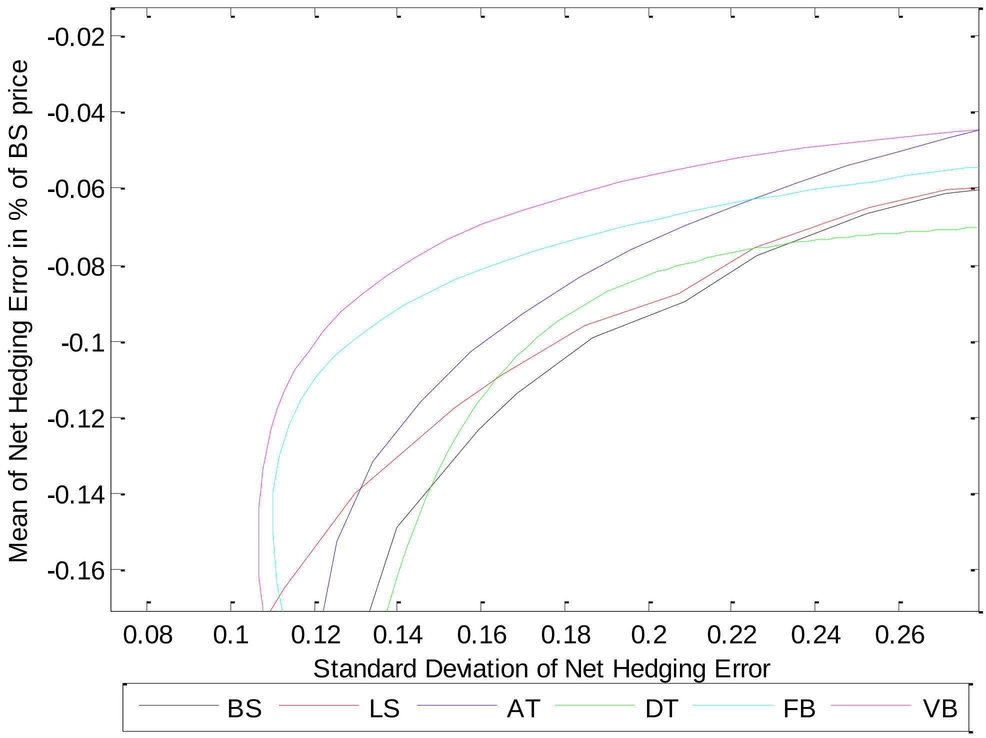

Figure 11 Comparison of Alternative Hedging Strategies in the Mean-Standard Deviation Framework

To determine the optimal hedging strategy in Figure 11, I choose a fixed level of hedging precision (standard deviation of NHE) on the horizontal axis and draw a vertical line to find the strategy that produces the least negative mean NHE. The most striking feature of Figure 11 is that the variable bandwidth VB strategy clearly outperforms all other strategies at all levels of hedging precision[32]. The result is intuitive, as VB is derived by maximising the hedger's utility. Transaction costs increase the volatility of the NHE of the hedging portfolio due to its random nature[33]. However, the mechanism used in VB is able to reduce the volatility by adjusting the hedging bandwidth, that is, the hedging bandwidth is a function of option gamma and reducing trading activity when the option is close to maturity. The fixed bandwidth FB strategy appears to be the next best strategy, except when hedging precision is low. Another important feature of Figure 11 is that the LS strategy is always superior to the BS strategy, because the Leland strategy adjusts the number of stocks traded in each rebalancing point by a modified volatility, which take into consideration the magnitude of transaction costs per trade. The special mechanism in LS allows the hedger to reduce expected NHE and pay less transaction costs at the same time.

The performance ranking of the hedging strategies depends on the level of precision. Given that the hedging strategies are examined in an incomplete market, the hedger is concerned about hedging risk due to tradeoff between transaction costs and hedging precision. In the following performance rankings, high, moderate and low hedging precision correspond to the hedger's high, moderate and low risk aversion level. When the hedger's risk aversion is moderate (the middle section of the MV curve), the ranking of the strategies is

$$VB \succ FB \succ AT \succ DT \succ LS \succ BS.$$

As the hedger's risk aversion increases (left-hand side of the MV curve), the LS performs better than the DT strategy. The ranking becomes

$$VB \succ FB \succ LS \succ AT \succ BS \succ DT.$$

[32] The hedging precision is determined using standard deviation of NHE.

[33] There are two random components that are path dependent: (1) the amount of transaction costs paid depends on the rebalancing amount of underlying index (2) move-based strategies have random rebalancing points which are linked to the hedging criterion. These two components depend on the realisation of the index price path in the future.

On the other hand, as the hedger becomes more risk-tolerant (right-hand side of the MV curve), the FB strategy is inferior to the AT strategy. The ranking becomes

$$VB \succ AT \succ FB \succ LS \succ BS \succ DT.$$

It is also observed that the performance gap among the strategies declines when the hedger becomes less risk-averse. As mentioned previously, the hedger faces a continuous dilemma between hedging precision and transaction costs. Apart from hedging precision, I can also assess the performance of hedging strategies using total transaction costs paid, i.e. a strategy is preferred to another if the total transaction costs paid is lower for a given level of the volatility of the NHE. However, the ranking may be different from the results obtained above. This is because transaction costs only explain part of the mean NHE.

4.3.2.1 Benefits of Switching Existing Hedging Strategies

Given the performance of the six hedging strategies in Figure 11, I further analyse the level of benefits of switching from one hedging strategy to another. The ideal result is to obtain an economically significant reduction in NHE when the hedger switches from an inferior strategy to a superior one. In this analysis, the performance of the hedging strategies is divided into different levels of hedging precision. I first fit the mean variance curves in Figure 11 by using a power function[34] of the following form:

$$Y = aX^b + c$$

where Y is the mean NHE and X is the standard deviation of NHE. a, b and c are fitted parameters[35].

I define three levels of hedging precision – high, medium and low – that correspond to the standard deviation of the NHE obtained from the BS strategies with hedging frequencies of 2 days[36], 10 days and 20 days. With this definition, I am able to

[34] The reason I fit the MV curves into a power function is because there are 60 mean-standard deviation points for each hedging strategy in Figure 11. However, there is no guarantee that the discrete MV points for each hedging strategy refer to the same standard deviation. Hence, fitting the MV curve into a function allows me to compare the mean NHE of the six hedging strategies based on the same amount of standard deviation of NHE.

[35] The adjusted R-squared is high for most of the strategies. They are 0.9854, 0.9808, 0.9645, 0.9529 and 0.7643 corresponding to LS, AT, DT, FB and VB strategies, respectively.

[36] Instead of using the daily rebalancing as a reference, the BS strategy with 2 days rebalancing frequency is treated as a reference to the high risk aversion level. The reason for not using the one day frequency is that all move-based strategies with zero bandwidth are degenerated into the BS strategy with daily rebalancing in my simulation design.

compare the mean NHE for each hedging strategy across all classes while fixing the level of hedging precision.

Table 4 shows the reduction in mean NHE if the hedger switches from one strategy to another. (Note I use the term "reduction" to indicate an improvement in hedging outcome based on the change in the magnitude of the NHE.) The results are presented in heatmap format, in which green implies that it is worthwhile to switch from a strategy on the left-hand side (listed in rows) to a strategy on the right-hand side (listed in columns) whilst red indicates it is not worthwhile to switch. To illustrate the results, consider the following example. A highly risk-averse hedger (a hedger who prefers high hedging precision) employing the BS hedging strategy would be able to reduce his portfolio mean NHE by 20% if he switched to the AT strategy whilst being exposed to the same level of risk. Alternatively, he would have reduced the mean NHE by 57% if he switched to the VB strategy. Table 4 also shows that there are benefits for a hedger irrespective of his level of hedging precision to switch from a time-based to a move-based strategy (except the DT strategy). However, the percentage of mean NHE reduction decreases when hedging precision is lower, that is, the hedger becomes more risk-tolerant. When the hedger prefers low hedging precision, switching to the AT or VB strategy is equally beneficial. The DT strategy appears to be an inferior performing strategy. In particular, DT has the worst performance when the hedger prefers high hedging precision. This result is partially due to assuming the option to be hedged is at-the-money. The gamma of an at-the-money option is at its peak compared to other moneyness levels. High option gamma leads to more frequent change in option delta, which increases the transaction costs of the hedging activities. As a result, the gains from DT are less than from the others. It is noted that the reduction in mean NHE can be attributed to a reduction in transaction costs and (or) reduction in hedging error. It is possible that the mean hedging error[37] of a strategy increases due to the hedging mechanism, which reduces transaction costs substantially and then leads to a reduction in mean NHE. My results show that, at the three levels of hedging precision, over 70% of the reduction in mean arising from a switch in hedging strategy is due to a reduction in total transaction costs paid for a new hedging strategy that performs better than the existing strategy.

[37] Recall, hedging error is the difference between hedging portfolio value and option payoff at maturity under the condition that there are no transaction costs in trading the underlying index.

Table 4 Comparative Mean of Net Hedging Error Reduction for Switching Strategies

The table shows the percentage of mean NHE reduction when the hedger switches from one strategy to another. The leftmost column is the hedger's current strategy. The rest of the columns are the strategies that the hedger chooses to switch to. I use the standard deviation of NHE to determine the level of hedging precision. The BS strategies with hedging frequency of 2 days, 10 days and 20 days represent the hedger who prefers high, moderate and low hedging precision respectively. I then determine the mean NHE for LS, AT, DT, FB and VB strategies using the same scale of standard deviation obtained from the BS strategy. A negative percentage means that there is a reduction in the mean NHE by switching from strategy A to strategy B. A positive percentage means that there is an increase in the mean NHE by switching from strategy A to strategy B. The colour scale at the bottom illustrates the scale of the mean NHE reduction.

Panel A: High hedging precision

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-31%	-20%	11%	-47%	-57%
	LS	46%		16%	61%	-22%	-37%
	AT	26%	-14%		39%	-33%	-45%
	DT	-10%	-38%	-28%		-52%	-61%
	FB	87%	29%	49%	107%		-19%
	VB	131%	58%	83%	155%	23%	

Panel B: Moderate hedging precision

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-6%	-22%	-13%	-27%	-40%
	LS	7%		-16%	-7%	-22%	-36%
	AT	28%	20%		12%	-7%	-24%
	DT	14%	7%	-11%		-17%	-32%
	FB	37%	29%	8%	20%		-18%
	VB	68%	57%	31%	47%	22%	

Panel C: Low hedging precision

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-7%	-37%	-1%	-23%	-38%
	LS	8%		-32%	7%	-17%	-33%
	AT	59%	48%		58%	22%	-1%
	DT	1%	-6%	-37%		-22%	-37%
	FB	30%	21%	-18%	29%		-19%
	VB	60%	48%	1%	58%	23%	

Next, I inspect the relative expensiveness of the strategies. I perform similar power function fitting[38] to determine the relationship between total transaction costs paid and the standard deviation of NHE. Table 5 shows the comparative transaction costs gains when the hedger switches his current strategy to a new one. Again, similar results pattern to comparative mean NHE are observed. A hedger who prefers high hedging precision would have saved almost half of the total transaction costs paid for the BS strategy by simply switching to the VB strategy. For most cases, switching from a time-based strategy to a move-based strategy will reduce the total transaction costs by an average of 15%. It is noted that total transaction costs savings decreases when the hedger becomes more risk-tolerant. This is not a surprising result, as a hedger benefits from paying lower transaction costs when he is willing to accept higher hedging risk or lower hedging precision. My results for the DT strategy are also consistent with the findings in Table 4. The DT strategy involves larger transaction costs, and is not recommended for use when the option is at-the-money as the change in delta is sensitive to the change in underlying stock price. In particular, higher transaction costs for the DT strategy occur when the hedger is highly risk-averse. This is because the hedging bandwidth is easily breached and more stock is required to rebalance to the new delta position.

Table 5 Comparative Transaction Costs Gains for Switching Strategy

The table shows the percentage of transaction costs reduction when the hedger switches from one strategy to another. The leftmost column is the hedger's current strategy. The rest of the columns are the strategies that the hedger chooses to switch to. I use the standard deviation of NHE to determine the level of hedging precision. The BS strategies with hedging frequency of 2 days, 10 days and 20 days represent the hedger who prefers high, moderate and low hedging precision respectively. I then determine the total transaction costs of LS, AT, DT, FB and VB strategies using the same scale of standard deviation of NHE obtained from the BS strategy. A negative percentage means a reduction in total transaction costs by switching from strategy A to

[38] The adjusted R-squared is high for most of the strategies. They are 0.987, 0.9956, 0.9664, 0.9587 and 0.7659 corresponding to LS, AT, DT, FB and VB strategies.

strategy B. A positive percentage means an increase in the total transaction costs by switching from strategy A to strategy B. The colour scale at the bottom illustrates the scale of the transaction costs saving.

Panel A: High risk aversion

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-26%	-31%	7%	-45%	-45%
	LS	35%		-6%	45%	-26%	-26%
	AT	44%	7%		55%	-21%	-21%
	DT	-7%	-31%	-35%		-49%	-49%
	FB	81%	34%	26%	95%		-1%
	VB	83%	35%	27%	96%	1%	

Panel B: Moderate risk aversion

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-5%	-11%	-12%	-23%	-26%
	LS	6%		-6%	-7%	-19%	-22%
	AT	13%	7%		-1%	-13%	-16%
	DT	14%	8%	1%		-13%	-16%
	FB	30%	23%	16%	15%		-3%
	VB	35%	28%	20%	19%	4%	

Panel C: Low risk aversion

		Switching to					
		BS	LS	AT	DT	FB	VB
Switching from	BS		-6%	-14%	-8%	-21%	-21%
	LS	6%		-8%	-2%	-16%	-16%
	AT	16%	9%		7%	-9%	-8%
	DT	8%	2%	-6%		-14%	-14%
	FB	26%	19%	9%	17%		0%
	VB	26%	19%	9%	17%	0%	

Average total transaction costs paid

Worse off — Better off

4.3.2.2 Different Risk Measures

Previous studies use mean or root mean squared hedging error to evaluate the performance of different option pricing and hedging models. Conversely, the hedging literature employs the variance of the hedging error as the main estimation method. In addition, there are other risk measures such as VaR and expected shortfall (ES) which focus on the left tail of the distribution. VaR, stated in Jorion (2001), measures the magnitude of the maximum loss at a certain confidence level. Mohamed (1994) uses VaR as a risk measure to determine the best-performing strategies; however, VaR does not indicate the loss that exceeds VaR and is not a coherent risk measure[39]. VaR breaches the sub-additivity property, which means that the VaR of a portfolio can be greater than the sum of the VaR of individual components. On the other hand, ES[40] measures the average loss below the VaR (see Acerbi & Tasche, 2002); therefore, it is more sensitive to the loss distribution in the tail and unlike VaR is a coherent risk measure.

Given that standard deviation is a quadratic risk function, it penalises both profits and losses. I assess the robustness of the MV results by changing the risk measure to VaR and ES, which focus on the left tail of the NHE distribution. Figure 12 and Figure 13 illustrate the performance of alternative strategies based on one-sided 95% VaR and ES. The ranking of the hedging strategies for both VaR and ES risk measures remain the same as the one obtained using an MV framework. Note, all the MV curves in Figures 10 and 11 have an "efficient" part and an "inefficient" part in accordance with well-known portfolio theory concepts. This means that for each hedging strategy a better outcome may be possible by changing the desired level of hedging precision. For example, in the BS model it may be possible to reduce the mean NHE but maintain the same level of risk (VaR of NHE) by rebalancing the hedge portfolio less frequently.

[39] A coherent risk measure is a function that satisfies properties of monotonicity, sub-additivity, homogeneity and translational invariance.
[40] Expected shortfall is also termed expected tail loss, conditional value at risk or average value of risk.

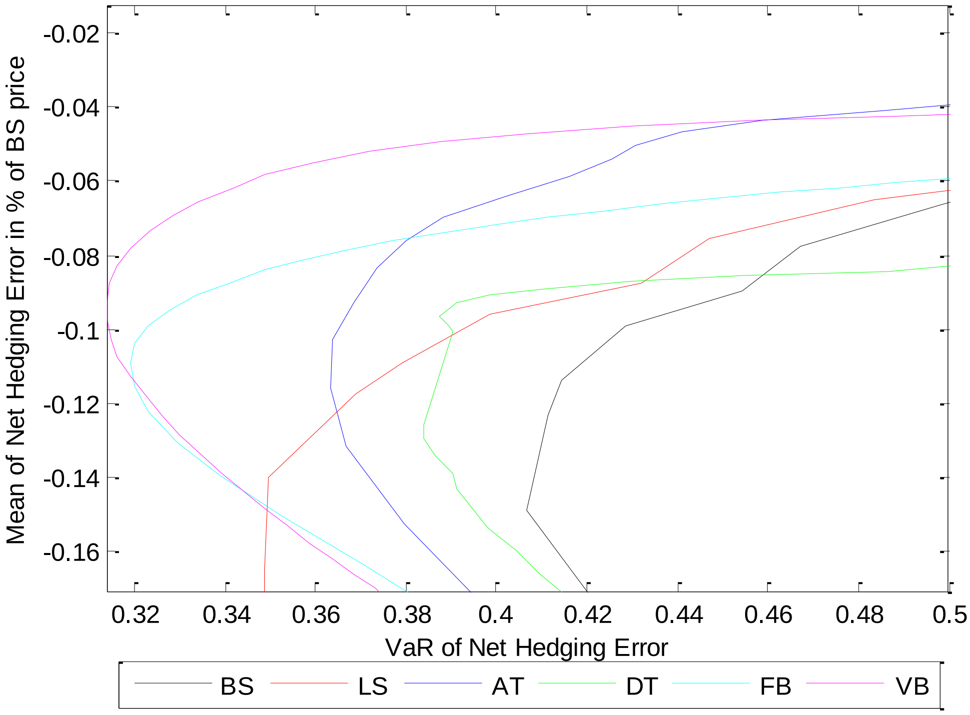

Figure 12 Comparison of Alternative Hedging Strategies in the Mean-VaR Framework

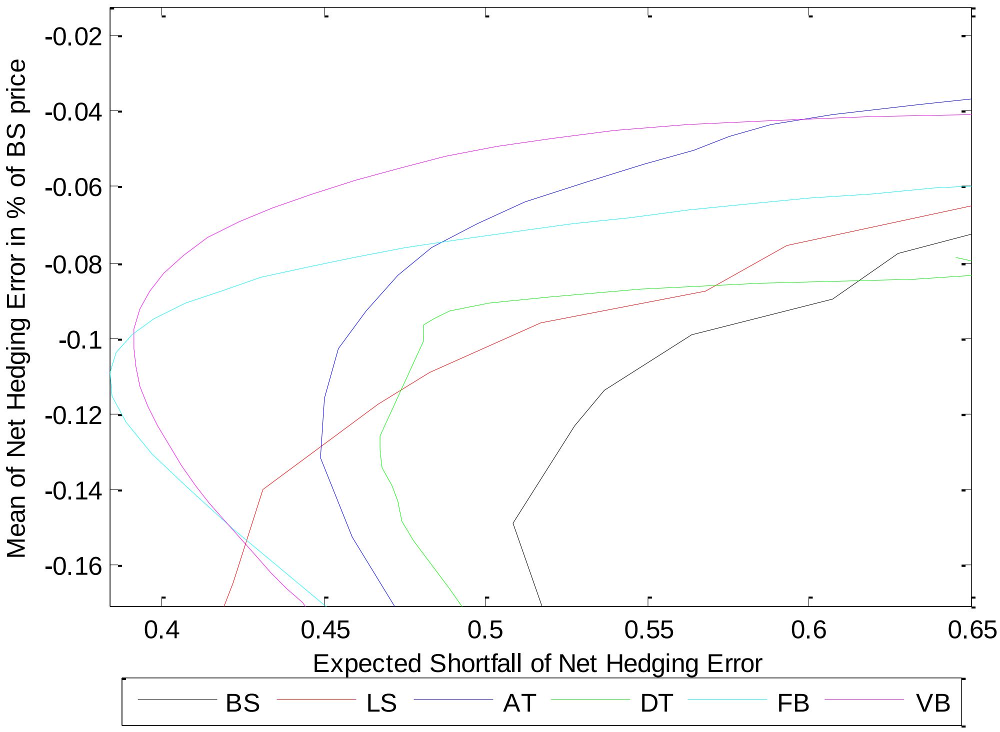

Figure 13 Comparison of Alternative Hedging Strategies in the Mean-Expected Shortfall Framework

4.4 Stochastic Dominance Analysis

The SD rules and the MV rules use partial information about the investor's preferences. There are several advantages of using the SD rules over the MV rules. First, the MV rules assume quadratic utility or normally distributed random variables. Second, MV only focuses on the first and second moment of the distributions, rather than the whole distribution of the random variables. In the following subsections, I present the SD rules and explain how to implement them in an empirical setting and then compare and contrast the SD results with the MV results.

4.4.1 Stochastic Dominance Rule

I denote U_i for i=1, 2 and 3 as the class of utility function where U_1 includes all utility functions such that $U' \geq 0$, U_2 includes all utility functions such that $U' \geq 0$ and $U'' \leq 0$, and U_3 includes all utility functions such that $U' \geq 0$, $U'' \leq 0$ and $U''' \geq 0$. In what follows I assume the investor maximises the von Neumann–Morgenstern expected utility and I state the SD criteria.

Theorem 4.1 First-order stochastic dominance test

Let [a, b] be the support of each distribution function. Let $F_X(w)$ and $F_Y(w)$ be the cumulative distribution of the net hedging error of strategies X and Y, respectively. X dominates Y in the first order if $F_Y(w) - F_X(w) \geq 0, \forall w \in [a, b]$.

For the FSD test, the only requirement for the choice of the utility function *U(x)* is that U is an increasing monotonic function of x, that is, $U'(x) \geq 0$. Economically, this assumes that an investor prefers more to less. In my context, this means that a hedger would prefer a positive NHE to a negative NHE. Positive NHE means that there is a positive cash flow from the hedging position. In other words, the hedging portfolio value (after taking account of transaction costs) is greater than the liability or option payoff at maturity.

Theorem 4.2 Second-order stochastic dominance test

Let [a, b] be the support of each distribution function. Let $F_X(w)$ and $F_Y(w)$ be the cumulative distribution of the net hedging error of strategies X and Y, respectively. X dominates Y in the second order if $\int_0^w F_x(t)dt \leq \int_0^w F_Y(t)dt \ \forall w \in [a, b]$.

For the SSD test, the requirement for the choice of utility function *U(x)* is that U is monotonically increasing and it is concave, that is, $U'(x) \geq 0$ and $U''(x) \leq 0$. Economically, this assumes that an hedger prefers more to less and he is risk-averse. Note that the MV analysis is a special case of SSD.

For proof of FSD and SSD, see Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970).

In addition, Whitmore (1970) introduces third-order stochastic dominance (TSD) by adding the condition that the utility function has a non-negative third derivative in addition to a non-negative first derivative and non-positive second derivative. Economically, the non-negative third derivative assumes that the hedger has decreasing absolute risk aversion.

According to Levy (2006), there is a unique relationship among stochastic dominance at different orders. Formally, if a strategy X stochastically dominates a strategy Y at the n^{th} order, then strategy X also dominates strategy Y at any order higher than n. Therefore, we can infer that

X FSD Y => X SSD Y=> X TSD Y.

4.4.2 Application of the SD Rules to Empirical Data

Early studies propose different algorithms for testing SD (see Porter et al., 1973; Levy & Kroll, 1979; Levy, 1998; Levy et al., 2004). Most SD tests are designed to test two uncertain choices, assuming that we know the true probability distribution of the two choices. However, in empirical studies, it is often the case that we do not know the true probability distribution of NHE of a strategy. Given that we have to rely on the empirical distribution function, *which is subject to sampling error*, more recent studies of SD rules have focused on econometric issues.

Anderson (1996) and Davidson and Duclos (2003) consider the problem of making statistical inferences for various forms of SD; however, their method of comparison is undesirable and introduces the possibility of test inconsistency. This is because statistical inferences are made by comparing objects at a fixed number of arbitrarily chosen points. A desirable test compares the objects at all points of the supports. Accordingly, Barrett and Donald (2003) propose a consistent test for SD based on the Kolmogorov–Smirnov test, which compares the objects at all points in the support of the object. We employ Barret and Donald's method to test the existence of an SD relationship among the hedging strategies. Note that the test proposed by Barret and Donald assumes the situation in which the samples are independent with possibly different sample sizes. This assumption matches my testing samples, for which each hedging strategy is independent of each other. Depending on the required characteristics of the utility function, different orders of SD rules can be tested. All levels of SD orders assume the hedger has von Neumann–Morgenstern utility function and would like to maximise his expected utility.

The general hypotheses for testing SD of order s can be written as follows:

$$H_0^s: D_G^s(x) \le D_F^s(x) \text{ for all } x \in [0, \bar{x}],$$

$$H_1^s: D_G^s(x) > D_F^s(x) \text{ for all } x \in [0, \bar{x}],$$

where $F(x)$ and $G(x)$ are the cumulative distribution functions for strategy F and G, which have the common support $[0, \bar{x}]$ where $\bar{x} < \infty$. $D_F^{(s)}(x)$ is the function that integrates F to order s-1. For example,

$$D_F^{(1)}(x) = F(x), \tag{4.7}$$

$$D_F^{(2)}(x) = \int_0^x F(t)dt, \tag{4.8}$$

$$D_F^{(3)}(x) = \int_0^x \int_0^t F(s)dsdt = \int_0^x D_F^{(2)}(t)dt, \tag{4.9}$$

If the null hypothesis is not rejected, then G stochastically dominates F at order j. The above test focuses on identifying the pairwise dominance relationship. The test is used when two strategies are compared; hence it is useful to determine the ranking of the strategies at the same level of hedging precision. Since this SD test is designed for pairwise comparison, the test must be conducted twice in order to determine the existence of a dominance relationship. For example, to compare strategies A and B, the first step is to check if strategy A dominates strategy B. If there is no dominance relationship, then we need to check if strategy B dominates strategy A before making a conclusion. If there is no dominance relationship, then strategies A and B are equally acceptable to the hedger.

Let N and M be the sample size of strategy A and B. The empirical distributions used to construct the statistical tests are respectively

$$\widehat{D}_A^{(1)}(x) = \frac{1}{N}\sum_{i=1}^{N} 1(X_A \leq z) \text{ and } \widehat{D}_B^{(1)}(z) = \frac{1}{M}\sum_{i=1}^{M} 1(X_B \leq z). \quad (4.10)$$

For higher order:

$$\widehat{D}_A^{(s)}(x) = \int_{-\infty}^{x} \widehat{D}_A^{(s-1)}(t)dt \text{ for s = 2,3} \quad (4.11)$$

$$\widehat{D}_B^{(s)}(x) = \int_{-\infty}^{x} \widehat{D}_B^{(s-1)}(t)dt \text{ for s = 2,3} \quad (4.12)$$

For the null hypothesis that strategy A dominates B at first and sth order where s = {2; 3}, the test statistics can be written in the form

$$T_{NM}^{(1)} = (\frac{\text{NM}}{\text{N+M}})^{1/2} sup_{z\in X}\left[\widehat{D}_A^{(1)}(x) - \widehat{D}_B^{(1)}(x)\right] \quad (4.13)$$

and

$$T_{NM}^{(s)} = (\frac{\text{NM}}{\text{N+M}})^{1/2} sup_{z\in X}\left[\widehat{D}_A^{(s)}(x) - \widehat{D}_B^{(s)}(x)\right] \text{ for s = 2,3.} \quad (4.14)$$

In searching for the optimal hedging strategy among all possible strategies, I use a consistent testing method proposed by Linton et al. (2005). Consider there are K hedging strategies, $X_1, \ldots, X_K$. Denote N as the full sample size of the net hedging error observations of X_k for k = 1, 2,...,K. Let $F_k(x)$ be the cumulative distribution function of strategy k. The null and alternative hypotheses for testing the dominance relationship for a particular hedging strategy are:

H_0: *Strategy k stochastically dominates all other strategies at order s,*
H_1: *Strategy k does not stochastically dominates all other strategies at order s.*

Let N denote the full sample size of hedging error observations of strategy X_{ki} for k = 1,..., K and i = 1,..,N. The test statistic $T_N^{(s)}(k)$ for the full sample is computed as below:

$$T_N^{(s)}(k) = \max_{l:k \neq l} sup_{x \in \chi} \sqrt{N} \left[\widehat{D}_k^{(s)}(x) - \widehat{D}_l^{(s)}(x) \right] \text{ for s} \geq 1, \qquad (4.15)$$

where $\widehat{D}_k^{(1)}(x) = \widehat{F}_{kN}(x) = \frac{1}{N}\sum_{i=1}^{N} 1(X_{ki} \leq x)$ and $\widehat{D}_k^{(s)}(x) = \int_{-\infty}^{x} \widehat{D}_k^{(s-1)}(t)dt$ for s ≥ 2. Let χ denotes the union of supports of all cumulative distributions of the hedging strategy, $\widehat{F}_{jN}$ for j = 1,…,K. To test the null hypothesis, Linton et al. (2005) suggest taking the maximum over all l with $k \neq l$. I apply this test to the hedging strategies across different classes.

For both pairwise and all strategies' SD tests, the critical value is obtained through bootstrapping simulation by using the observed NHE as the population. To determine whether a null hypothesis is rejected or not, the simulated p-value is calculated and compared at 1%, 5% and 10% levels of significance. The procedure of obtaining the test statistics and simulated p-value are summarised in three steps (the details are stated in Appendix A).

In section 4.4.4, I use the SD tests for the selected six hedging strategies at three different levels of hedging precision[41]. For each strategy *k*, I test whether it dominates all other strategies by FSD, SSD and TSD at the selected level of hedging precision. This gives three *p* values per strategy with each corresponding to FSD, SSD and TSD. The significance levels of 1%, 5% and 10% are used to decide whether or not I can reject the null hypothesis. Further, I determine for each strategy the highest SD order for which the null hypothesis of SD cannot be rejected. A strategy is FSD if we cannot reject the null hypothesis that the strategy dominances all other strategies. An FSD strategy also implies that the strategy is SSD and TSD. For an SSD strategy, the null hypothesis of the strategy dominates all other strategies at SSD but is rejected at FSD. Lastly, a TSD strategy means that the null hypothesis of the strategy dominates all other strategies at TSD but rejected at FSD and SSD.

[41] Unlike the hedging precision in the MV test, which is derived from a fitted power function using the same level of standard deviation, the hedging precision in the SD test is based on a list of selected hedging parameters.

In order to determine the ranking of the hedging performance of each hedging strategy, the pairwise SD test (as described previously in this section) is used. A strategy *k* is ranked higher than strategy *l* if the null hypothesis that strategy *k* dominates strategy *l* cannot be rejected but the null hypothesis that strategy *l* dominates strategy *k* is rejected. On the other hand, if both null hypotheses are rejected, then the strategy are equally ranked. The FSD, SSD and TSD tests are also performed at high, moderate and low hedging precision levels.

4.4.3 SD Tests Setup

I carry out the SD tests by considering three different levels of hedging precision for each class of hedging strategy. The selected hedging parameters for the corresponding hedging precision level are presented in Table 6 below.

Table 6 Hedging Parameters for Stochastic Dominance Test

The table shows the hedging parameters selected for each class of hedging strategy. These parameters represent three different levels of hedging precision: high, moderate and low. The first column presents the name of the hedging strategy, the second column indicates the measurement unit of the hedging parameters and the remaining columns indicate the parameter value for its corresponding level of hedging precision. The bracket term in second column specifies the meaning of the parameter symbol.

Hedging Strategy	Hedging Parameters	High	Moderate	Low
Black–Scholes hedge at fixed time intervals (BS)	δt (in days)	2	10	20
Leland's hedge (LS)	δt (in days)	2	10	20
Henrotte asset tolerance (AT)	H (percentage change in underlying asset price)	2%	4%	6%
Delta tolerance (DT)	H (option delta)	0.05	0.1	0.2
Hedging to a fixed bandwidth around delta (FB)	H (option delta)	0.05	0.1	0.2

Hedging to a variable bandwidth around delta (VB)	γ (risk aversion)	5	0.1	0.001

In my simulation setting, the rebalancing frequency for a move-based strategy with zero hedging bandwidth will be degenerated into a BS hedging strategy with daily rebalancing frequency. As a result, I choose the BS and LS strategies rebalance every 2 business days for time-based strategies with expected high hedging precision. The 10 business days and 20 business days represent bi-weekly and monthly rebalancing strategies, which have relatively less hedging precision than the 2 days strategy. For move-based strategies, a wider hedging bandwidth implies that the hedger is less risk-averse and willing to accept a less precise hedging outcome. Although the choice of the move-based strategies' hedging parameter is somewhat arbitrary, these parameter values will generate a three-point MV curve for hedging strategy, which spans the same MV space, for consistency comparison between MV and SD tests. The choice of the hedging parameters for the AT strategy is based on historical daily index return. For the period of January 2, 1996 to September 30, 2009, I observe that the average daily index return is 0.9% and the maximum return is 11%. As a result, I set 2% as the high hedging precision strategy and 6% as low hedging precision strategy. For the delta move-based strategy, the hedging parameter is chosen such that rebalancing does not occur only when the option has changed from in-the-money position to the out-of-the-money position. An in-the-money option always has delta greater than 0.5 and an out-of-the-money option delta is always less than 0.5. Therefore, the low hedging precision hedging parameter is set at 0.2, and the high hedging precision strategy will rebalance the hedging portfolio when delta changes by 0.05.

The hedging parameter for the VB bandwidth strategy is also chosen arbitrarily. It is chosen on the basis that the MV curve lies within all other MV curves. The greater the risk aversion parameter, the more risk-averse the hedger. In addition, the choice of these hedging parameters produces a performance ranking consistent with the results obtained in section 4.3. It is possible that different choices of hedging parameter values for each hedging strategy can yield different SD test results; this poses a limitation to my study. The derivation of a consistent comparable set of hedging parameters will be investigated in a future study.

4.4.4 Results

4.4.4.1 Simulated net hedging error distribution

Prior to presenting the stochastic dominance test results, I study the characteristics of NHE using the simulated results in section 4.3. For each hedging strategy, probability density functions (PDF) and EDFs corresponding to the hedging parameter are plotted. The PDF and EDF are constructed using the simulated NHE. Each hedging strategy has 200,000 simulated NHEs for its corresponding hedging parameters. As a result, each simulated net hedging error is assigned a probability of 1/200,000. A kernel distribution function in Matlab is used to plot the PDF. A kernel distribution is a non-parametric representation of the PDF of a random variable. Let $\{x_1, \dots, x_n\}$ be independent and identically distributed random variables with common density function. The kernel density estimator is of the following equation:

$$\widehat{f_h}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right);\ -\infty < X < \infty, \tag{4.16}$$

where *n* is the sample size, *K(.)* is the kernel smoothing function and *h* is the bandwidth. The bandwidth value controls the smoothness of the resulting density curve. The kernel smoothing function is a non-negative function that integrates to one and has mean zero. The resulting kernel distribution is similar to a histogram (which places sample values into discrete bins) except that it is a smooth and continuous probability curve. The kernel distribution allows study of the characteristics of the NHE distribution without making assumptions about the distribution of the data.

The EDF is defined as

$$\widehat{F_n}(t) = \frac{1}{n}\sum_{i=1}^{n} 1\{x_i \le t\} = \frac{number\ of\ elements\ in\ the\ sample \le t}{n}, \tag{4.17}$$

where 1{A} is the indicator of event A. It is the CDF associated with the empirical measure of the sample value. In my study, the empirical measure of each net hedging error is 1/200,000.

Figures 13–15 display the NHE for each hedging strategy at different hedging precision levels. As explained previously, the level of hedging precision is represented by the hedging parameter value. The PDF plots show that the distributions of NHE for all hedging strategies are skewed to the left. This is because total transaction costs paid during the hedging period are non-recoverable and therefore constitute permanent losses to the replicating portfolio. It is noted that the distributions of each strategy are shifted slightly to the right when hedging precision becomes less precise (or the hedger is willing to accept greater amount of NHE and therefore lower transaction costs). As suggested by theorem 4.1, a first-order dominance strategy's EDF will stay below that of the rest of the hedging strategies; this also means that there will be no crossing with the EDF of other hedging strategies. If a strategy is dominating at second order, then there will be one crossing between the EDF of two strategies. Graphically, the results in Figure 14 imply that there is a first-order SD relationship for the FB strategy at a high level of hedging precision. In addition, the DT strategy is potentially dominated by all other strategies at second or higher order[42], given that Figure 15 shows that the DT strategy's EDF is generally the highest of all strategies and crosses other strategies' curves at least once. To confirm the preliminary visual results, I present the results of statistical testing in the next section.

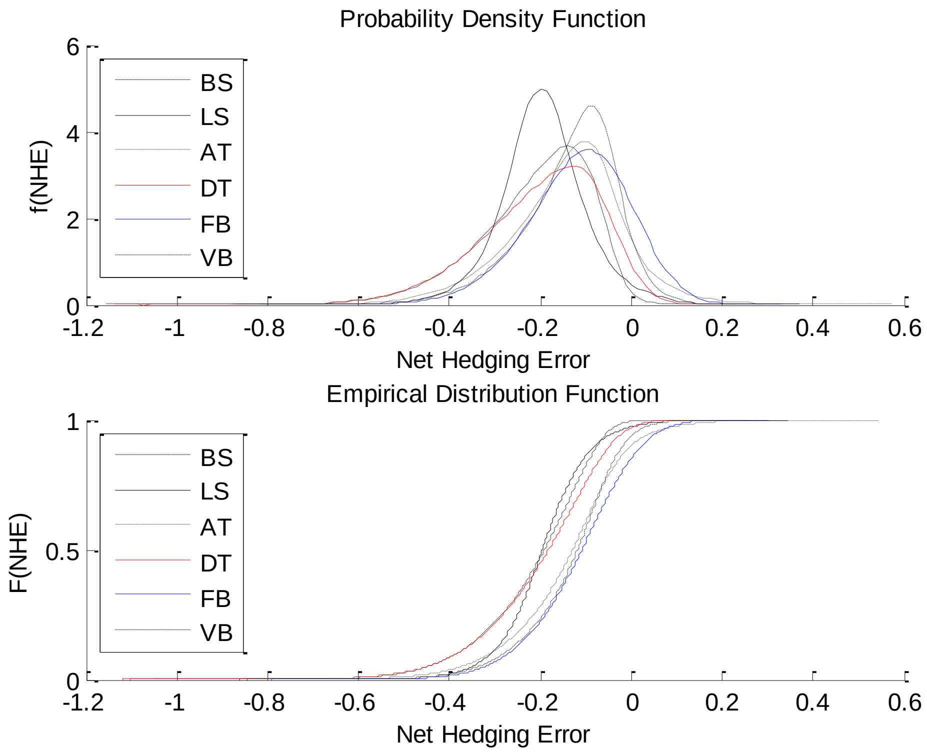

Figure 14 Probability Density Function and Empirical Distribution Function of Net Hedging Error Across Strategies – High Hedging Precision

[42] This could be due to the choice of the parameters in Table 6. Another possibility is that the DT strategy can be quite costly to a hedger. One can imagine that the hedger will need to rebalance the underlying index position to the perfect BS delta position even though the only change is time and all else has not changed. An option with high gamma may require the hedger to rebalance the portfolio daily and hence hedging becomes very costly.

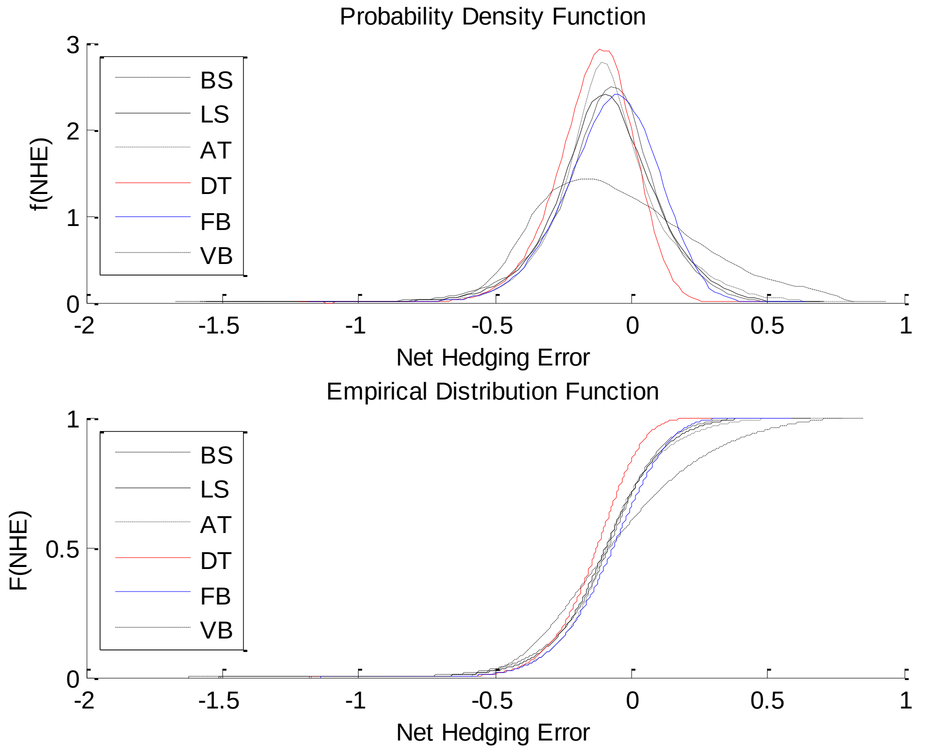

Figure 15 Probability Density Function and Empirical Distribution Function of Net Hedging Error Across Strategies – Moderate Hedging Precision

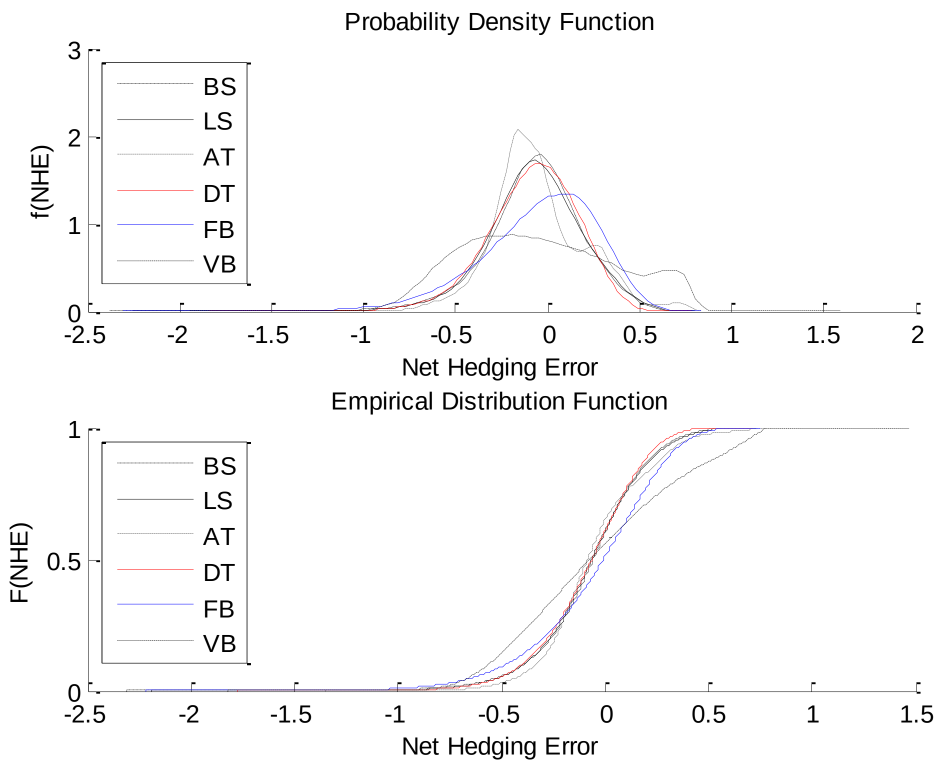

Figure 16 Probability Density Function and Empirical Distribution Function of Net Hedging Error Across Strategies – Low Hedging Precision

4.4.4.2 SD Test Results

I present the test results of the null hypothesis that a particular hedging strategy dominates all other strategies at three levels of hedging precision (high, moderate and low). The results in Table 7 show that FB dominates all other strategies at first order at high hedging precision. This implies that FB is the optimal strategy for a hedger who prefers a highly precise hedging outcome. As I decrease the level of hedging precision, there is no dominating strategy when the hedging precision becomes moderate. Last but not least, AT is the preferred strategy when the hedger prefers low hedging precision in my testing. It is noted, at low hedging precision level, AT dominates all other hedging strategies at second order. Given that the SD results presented in this section are based on hedging parameters defined in Table 6, I re-run the mean variance test for the six hedging strategies using those hedging parameters defined in Table 6, generating a new set of six different MV curves[43]. This is to ensure that both MV and SD results are compared consistently.

The untabulated results[44] show that the ranking of the hedging strategies are slightly different from those presented in section 4.3.2. The reason behind this is that the comparison in the previous section is based on calibrating NHE to a power distribution such that mean NHE is derived using BS NHE standard deviation as an input. In contrast, the untabulated MV results are obtained without any calibration, that is, the mean and standard deviation of NHE are directly derived from the hedging parameters in Table 6. In addition, the MV comparison is based on the location of the particular strategy on the MV curve; hence, this comparison method is slightly different from the one introduced in the previous section. These SD results are consistent with the MV results for high and low hedging precision. However, the contrasting result is that there is no conclusive dominating strategy at moderate hedging precision given that VB, FB and AT strategies dominate all other strategies at third order. Unlike the pairwise test, the all-strategy test does not indicate these three strategies are regarded equally. The

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[43] Each MV curve is constructed using three hedging parameters defined in Table 6.

[44] For a high level of hedging precision, the ranking becomes

$$FB \succ VB \succ AT \succ LS \succ DT \succ BS.$$

For a moderate level of hedging precision, the ranking becomes

$$VB \succ AT \sim FB \succ LS \succ BS \succ DT.$$

For a low level of hedging precision, the ranking becomes

$$VB \succ FB \succ AT \succ LS \succ BS \succ DT.$$

results are inconclusive, so the consistency of the MV and SD test results is not confirmed.

Table 7 Stochastic Dominance Test for All Other Strategies

The table shows SD test results obtained from a simulation of 200,000 NHEs for each of the six hedging strategies based on the hedging parameters listed in Table 6. One-way proportional transaction costs of 50 basis point are assumed. Under the null hypothesis of the SD tests, a particular hedging strategy dominates all other hedging strategies at the selected hedging precision level. If the null hypothesis is rejected, no dominance relation is present. The test statistics are calculated using equation (4.14). The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. FSD implies that we cannot reject the null hypothesis that the strategy dominates all other strategies at the selected hedging precision level by first-order. Similarly, SSD means that we can reject the null hypothesis that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by second-order. Finally, if a strategy is TSD, we can reject the hypothesis that the strategy dominates all other strategies by FSD and SSD, but not that it dominates all other strategies in the set by third-order. No SD means that we reject the null of SD by any order. $^{\circ}$,$^{\circ\circ}$,$^{\circ\circ\circ}$ denote the significance level of the hypothesis test for SD identifications, i.e. 10%, 5%, 1% level respectively.

	Hedging Strategy					
Hedging Precision	BS	LS	AT	DT	FB	VB
High	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	**FSD**	No SD $^{\circ\circ\circ}$
Moderate	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	**TSD**	No SD $^{\circ\circ\circ}$	**TSD**	**TSD**
Low	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	**SSD**	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$	No SD $^{\circ\circ\circ}$

Pairwise SD tests using equation (4.13) and (4.14) are performed in order to determine the optimal ranking of the hedging strategies. I obtain the following ranking at different levels of hedging precision. When the hedging precision is moderate, the ranking of the strategies is

$$FB \sim AT \succ VB \succ LS \succ BS \succ DT.$$

As the hedging precision increases (i.e., high hedging precision level), the ranking becomes

$$FB \succ VB \succ AT \succ LS \succ DT \succ BS.$$

On the other hand, as the hedging precision decreases (i.e., low hedging precision level), some of the move-based strategies are regarded equally to the time-based strategy. The ranking becomes

$$AT \succ FB \succ VB \sim LS \sim BS \sim DT.$$

These dominance relationships are presented at first-, third- and second-order with respect to high, moderate and low hedging precision levels. In terms of hedging performance, the dominating strategy obtained from pairwise results are consistent with the all-strategies SD results. In comparison, the ranking of high hedging precision strategies coincides with the MV results. For moderate and low hedging precision, both MV and SD correctly identify the top and bottom three strategies, but the ranking is different in each test.

It is important to note that the probability distribution of the hedging strategies is dissimilar to the normal distribution; these distributions are negatively skewed and most have long tails due to transaction costs. As a result, it is no surprise that the ranking of the strategies under the SD test is different from the one under the MV test, for which the accuracy of the test depends on the normality assumption.

4.5 Sensitivity Tests

To access the robustness of the hedging performance, I perform sensitivity tests on the choice of optimal hedging strategy to different factors such as option moneyness, time to maturity, transaction costs, volatility, drift and interest rate. I analyse the impact of each factor through three levels of hedging precision which are high, moderate and low. The corresponding hedging parameters have been defined in Table 6.

4.5.1 Option Moneyness

I consider different levels of moneyness for which the strike price of the option ranges from 900 to 1100 compared to the starting value of the index of 1000. We find that the total transaction costs paid in dollars is the highest for the at-the-money option. This result is consistent with the fact that the at-the-money option has the highest gamma, and therefore more frequent rebalancing is involved. The performance of the hedging strategies is not robust with the change in option moneyness when using MV analysis. Instead, the ranking of the strategy depends on moneyness and hedging precision level. For all levels of hedging precision, the move-based strategies consistently perform better than the time-based strategies, even though the rankings of the strategies are not always consistent when option becomes out-of-the-money for all hedging precision levels. In general, VB is the preferred strategy at all hedging precision levels. However, FB becomes the preferred strategy as the option becomes deeper out-of-the-money at moderate and low hedging precision levels. We also observe that, for moderate and low hedging precision levels, there is increasing preference for the time-based strategy when an option becomes in-the-money.

The contrasting result in my study is that, although the ranking of the hedging strategies using the SD test is different from the ranking using MV analysis, the performance of the hedging strategies are robust with the change in option moneyness at high and moderate levels of hedging precision at third-order of pairwise SD. In particular, all strategies' SD test results are consistent across all moneyness in identifying the dominant hedging strategy. SD results demonstrate that FB is preferred at high and moderate hedging precision levels, whereas AT is preferred at low hedging precision level.

4.5.2 Time to Maturity

I implement the hedging strategy by varying time to maturity of the call option from 30 days to 250 days. For all strategies, the total transaction costs increase with time to maturity because of more rebalancing transactions through time. I also find that the standard deviation of NHE increases with the length of maturity, but the mean is relatively unaffected. This reflects the fact that hedging outcomes are increasingly uncertain over longer periods, given that underlying stocks prices can fluctuate with larger magnitude over longer periods of time. My results show that the MV and SD test results conform with each other when hedging precision is high or moderate. Hence, time to maturity does not play an important role in determining the relative performance of hedging strategies.

4.5.3 Transaction costs

The proportional transaction costs increase the transaction costs paid by the hedger. I observe that the NHE becomes more negative in value when transaction costs increase. The influence of proportional transaction costs on strategy ranking depends on hedging precision level. I consider the situation in which the proportional transaction costs increases from 1 to 100 basis points. Under the MV test, move-based strategies (except the DT strategy) perform better than time-based strategies across different levels of hedging precision. Although the ranking of the strategies is slightly altered, VB appears to be the optimal hedging strategy consistently at high and moderate hedging precision. As with all other parameters considered in the sensitivity test, the MV test correctly identifies the top three and bottom three performing strategies consistently. Under the SD test at third-order, the base case results are robust when transaction costs are non-trivial at all levels of hedging precision.

In practice, a transaction will involve both fixed and proportional transaction costs. I therefore consider the impact of fixed transaction costs on hedging performance. I assume each transaction will incur 40 cents of fixed transaction costs and 50 basis points of proportional transaction costs. The ranking of the strategies under the MV test alters significantly, such that time-based strategies are within the top three best-performing strategies. The reason is that move-based strategies involve an infinite number of trades near the hedging bandwidth and become very costly when fixed costs are included.

4.5.4 Volatility

When the underlying asset becomes more volatile, there is greater risk involved in the hedging process. More transactions must be undertaken in order to attain certain level of accuracy of a hedging position. Consequently, higher transaction costs are expected when volatility increases. I test the impact by varying levels of volatility from 10% to 50% and find that the base case result is robust to the change in volatility under both MV and SD tests at all levels of hedging precision.

The previous analysis assumes the underlying stock price process follows a geometric Brownian motion, so the instantaneous relative volatility of the stock price is constant. Studies such as Hull and White (1987a), Bakshi et al. (1997) and Dumas et al. (1998) demonstrate that we should always hedge for volatility risk. As a result, I analyse the impact of stochastic volatility on hedging performance. I implement the same set of hedging strategies on the assumption that the underlying stock return exhibits stochastic volatility. I assume that the stock price process follows Heston's

(1993) model. The model assumes that the stock price $S(t)$ follows the stochastic process:

$$dS(t) = \mu S(t)dt + \sqrt{v(t)}S(t)dW^S(t), \tag{4.18}$$

where $v(t)$ is the instantaneous variance which follows the Cox, Ingersoll and Ross (1985) process:

$$dv(t) = k(\theta - v(t))dt + \sqrt{v(t)}\varepsilon dW(t), \tag{4.19}$$

and $dW^S(t)$ and $dW(t)$ are Wiener processes with correlation ρ. I denote μ as the rate of return of the stock price, θ as the long-run average price for volatility, k as the speed of mean reversion to the long-run average and ε as the volatility of volatility.

For simulation, I set μ=0.039, θ=0.0457, k= 5.07 and ε=0.48 based on the estimates in Table 6 from Aït-Sahalia and Kimmel (2007)[45] and simulate 200,000 stock price paths. I then use the Heston option pricing model to compute the option price, which includes the price of volatility risk[46]. I rebalance the hedging portfolio according to the proposed hedging strategies. It should be noted that the implementation of the hedging strategy is inconsistent with the Black–Scholes assumption, which is based on constant volatility. I find that the hedging performance deteriorates, partly due to the volatility risk that is not captured by the constant volatility model. This exercise allows us to access the robustness of the result with respect to stochastic volatility. For both MV and SD test results, I consistently find that FB is the best hedging strategy across all levels of hedging precision.

4.5.5 Drift

The value of the drift varies from 5% to 20%. In general, I expect a higher drift value will reduce the standard deviation of NHE. This is because the drift rate is the average increase in underlying price per unit of time, and hence a higher drift will drive the option in-the-money sooner compared to a relatively lower drift rate. For example, a call option will reach a further in-the-money position when drift rate increases and hence rebalancing transactions may reduce the number of stock transactions, which leads to lower transaction costs. In my sensitivity test, I find that there is a decrease in

[45] These estimates are obtained from the daily data of S&P 500 index and the Chicago Board Options Exchange Volatility Index from January 2, 1990 to September 30, 2003.
[46] Note that the Black–Scholes price is no longer valid for use as a benchmark for option price as before. This is because the underlying stock return has stochastic volatility.

the standard deviation of NHE when the drift rate increases; however, the magnitude of decrement is trivial. Overall, the ranks of the strategies are robust to the base case results obtained from both tests.

4.5.6 Interest rate

The present value of expected transaction costs decreases when the interest rate increases due to the effect of the time value of money. Given that the base case option is an at-the-money option that has high gamma, it therefore involves a relatively large number of trading activities. The effect of interest rate is mixed, and depends on when most trades happen. If trades happen to occur in the early time period, then the present value of transaction costs is greater and vice versa. Hedging performance deteriorates as more transaction costs are incurred.

To access the impact of interest rate level on hedging performance, I implement the test with different interest rate values ranging from 0% to 10%. The NHE distribution skews to the right for all levels of hedging precision when interest rates increase. This observation can be explained by the fact that the present value of total transaction costs is reduced when the interest rate increases. The ranking of hedging strategies is consistent with the base case results under the MV and SD tests across different risk-free rates; however, these comparison methods do not yield the same ranking results. This means that hedging performance is insensitive to the change in interest rate but the choice of comparison method will change the optimal hedging strategy. For example, VB is the optimal strategy under the MV test but FB is optimal when the strategies are compared using the SD test.

4.6 Conclusion

In this chapter I examine the performance of alternative option hedging strategies in the presence of transaction costs in a Monte Carlo simulation setting. The objective of the comparison was to determine the existence of an optimal hedging strategy among the six proposed hedging strategies. The presence of transaction costs introduces a tradeoff between hedging precision and the amount of transaction costs paid. In general, there are two main classifications of hedging strategies in the literature: time-based strategies and move-based strategies.

The existing literature uses MV rules to assess the performance of the competing hedging strategies. However, the use of the MV test has to satisfy the assumption that either the hedger has a quadratic utility function or the NHE is normally distributed. These assumptions are hardly satisfied, questioning the validity of the results of previous studies. To overcome the deficiency, I propose a systematic comparison of alternative hedging strategies using the SD test. In contrast to the MV decision rule, which is based on the first and second moment of NHE distribution, the SD test considers the entire distribution of NHE. The SD rule is also consistent with the von Neumann–Morgenstern expected utility maximisation framework.

When I test the performance of alternative hedging strategies using the MV test, six MV curves based on the mean and standard deviation of the NHE of each hedging strategy, with varying hedging parameters, form the basis of comparison. I find that, with proportional transaction costs, the optimal hedging strategy is to rebalance the hedging portfolio to the nearest boundary of the hedging bandwidth (VB) under the MV test. I also find that move-based strategies are superior to time-based strategies when drift decreases and volatility increases. This suggests that a hedger should switch from a time-based strategy to a move-based strategy, which allows the hedger to save substantial transaction costs for the same level of hedging precision. However, the move-based strategies' performance is weakened when fixed transaction costs are introduced. Indeed, the performance of time-based strategies improve and they enter the top three preferred strategies. This is because hedging at a fixed time point will reduce transactions incurred due to movement close to hedging bandwidth in move-based strategies.

Another consistent performance measure I use in testing the alternative of hedging strategies is the SD test. Given the intensity of the calculation, I simplify the comparison by limiting three hedging parameters for each hedging strategy in order to reflect high, moderate and low hedging precision. I use the pairwise SD test (introduced by Barret & Donald, 2003) and an all-strategies SD test (introduced by Linton et al., 2005) to compare the performance of six hedging strategies at three different levels of hedging precision. The SD test results show that FB is the optimal hedging strategy when a hedger requests high hedging precision and AT is optimal when a hedger requests low hedging precision in the base case scenario. However, I cannot determine the optimal strategy when hedging precision is moderate, due to the absence of dominance of a single strategy. I also find that the SD results are robust to all types of sensitivity tests.

For consistency comparison between MV and SD tests, I also construct a reduced version of the MV test based on three hedging parameters for each hedging strategy. These hedging parameters are the same as those used in the SD test. Although both test's resulting rankings do not match most of the time, they consistently identify the same set of top three and bottom three hedging strategies in terms of hedging performance. Given the robustness of SD test results at high and low hedging precision levels, I conclude that SD is a better method for assessing the performance of hedging strategies. The superiority of the SD test is largely due to the fact that it considers the full distribution of NHE and has less restrictive assumptions than the MV test.

Chapter 5

5. Empirical Study: The S&P 500 Index Option

The simulation study in chapter 4 compares the performance of alternative hedging strategies under a controlled environment. Although simulation results provide a performance benchmark for the approximated NHE of alternative hedging strategies, an empirical study allows me to gauge the effectiveness of hedging strategies in practice. In real life, practitioners tend to use move-based strategies to hedge their option exposure because of their intuitive appeal in theory and superior hedging results obtained from Monte Carlo simulations based on the mean variance rule. However, it is also noted that there is no standard way of implementing a hedging strategy. For example, a market maker who aims to maintain a delta neutral position on a daily basis (either through a covered position or dynamic hedging) can also implement a move-based strategy within the day. The hedging practice in real life is largely driven by the risk appetite of the market maker (or his/her firm)[47]. This chapter offers better guidance for market participants in choosing an appropriate hedging strategy.

In this chapter, I investigate the empirical performance of alternative option hedging strategies using S&P 500 index options data. As listed in chapter 3, the six hedging strategies that I examine in this chapter are:

(1) Black-Scholes hedging at fixed time intervals (BS),
(2) Leland's hedge (LS),
(3) Henrotte's asset tolerance strategy (AT),
(4) delta tolerance strategy (DT),
(5) hedging to a fixed bandwidth around delta (FB), and
(6) hedging to a variable bandwidth around delta (Whalley & Wilmott, 1997) (VB).

[47] As stated in chapter 4, I would like to highlight our assumption of holding the position until maturity (or liquidation period) is for consistent comparison purposes but it is not necessarily how hedging is being done most of the time in real life.

Recall, the first two strategies belong to the class of time-based strategies and the remaining four belong to the class of move-based strategies.

The chapter is organised as follows. Section 5.1 and 5.2 explain the data and methodology for the empirical study. Section 5.3 summarises the findings under both MV and SD frameworks. Section 5.4 examines the performance of hedging strategies when a delta-vega-neutral hedge is formed. Section 5.5 concludes the chapter.

5.1 Data

S&P 500 index options are the focus of much option pricing research, including Bakshi, Cao and Chen (1997, 2000), Dumas et al. (1998), Rubinstein (1994) and Constantinides et al. (2009). Consistent with the reasons previous studies provide, I use S&P 500 index options traded on the Chicago Board of Exchange (CBOE) to examine the performance of alternative hedging strategies. There are two reasons why I use S&P 500 index options as the market data. First, S&P 500 index options satisfy most of the conditions required by the Black–Scholes formula. Second, options written on this index are the most actively traded European-style index option contract. Thus, S&P 500 index options offer an ideal test for most of the option hedging models.

For all hedging exercises conducted in this study, I require two types of hedging instrument: the underlying asset that closely resemblances the S&P 500 index[48], and a cash account. Following Bakshi et al. (1997), the spot S&P 500 index[49] rather than the

[48] I acknowledge a suggestion from an examiner that a Contract for Difference (CFD) can be a potential hedging instrument. From United States investors perspective, CFD is not permitted to trade under the Dodd-Frank Act which is implemented by the Securities and Exchange Commission (SEC). Although CFD is traded in Australia, the exchange traded version of CFD only starts from November 2007 which is towards the end of our sample period. This poses data availability issue to my study. It is noted that CFD has been traded exclusively over-the-counter (OTC) prior to November 2007. Further, CFD is a margined product and similar to a futures contract except it does not have a specified maturity date. Reasons for not choosing futures as the underlying asset used in this study are discussed above.

[49] It is known that the S&P 500 index is non-tradeable. Another financial instrument that closely tracks the performance of the S&P 500 index is its exchange traded fund, called Standard and Poor's Depository Receipts (SPDR). Although the return on SPDR almost replicates the S&P 500 return, several factors affect SPDR returns apart from the S&P 500 index return. First, SPDR charges management expenses of 11 basis points. Second, the dividends received in SPDR are invested in a non-interest bearing account and the accumulated dividends are distributed quarterly. Ackert and Tian (1998) show that the non-interest bearing component has resulted in the underperformance of the SPDR relative to the S&P 500 index by approximately 10 basis points. Third, the SPDR closing price may differ from its net asset value, although the amount is very small. Fourth, transaction costs incurred in replicating the index, such as

S&P 500 index futures contract is used in place of the spot asset in each hedge. The decision was made based on the following considerations. First, the spot S&P 500 and the S&P 500 futures contract with immediate expiration month generally have a correlation coefficient close to one. As a result, whether a spot index or a futures contract is used in a hedge, the conclusion most likely will be the same. Second, the hedging ratio drawn from the literature has to be modified if a futures contract is used as a hedging instrument. This would introduce a misalignment of the hedging strategy derived from the option hedging model, and hence bias the results. Third, futures contracts require margin account maintenance throughout the life of the futures contract. This will involve cash in- and out-flow which has further implications for the cash account in my hedging process. Last, I also need to consider the liquidity of the futures contract when using it as a hedge instrument. Given that the S&P 500 futures contract matures quarterly but the S&P 500 index options contract has monthly maturity, I need to roll over the futures contract to the next most liquid futures contract (if the maturity month of the futures contract does not coincide with the maturity of the options contact) so that hedging can be continued. The rollover effect adds another layer of complication to the hedging performance. In addition, the contract size rounding effect would introduce basis risk if an actual tradable instrument was used as the hedging instrument.

5.1.1 Option Metrics Database

The data on S&P 500 index options are obtained from Option Metrics. The S&P 500 index options are standard European options on the spot index listed on the CBOE. Daily closing quotes on each option contract (bid and ask) and their corresponding contract specifications such as strike price and time-to-maturity are recorded in this database. The dataset also includes a unique option contract identifier to facilitate the tracking of an option contract over time. Option Metrics also supply the underlying index level at close, interest rate curve and projected dividend yield. The advantage of the database is that option data are directly linked with the underlying issue data to ensure consistency of the historical time series. My hedging exercises are based on the ask option price quotes, given that the objective is to hedge short position (or replicate a long call position) in European call options. The difference between mid-price and ask

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portfolio components, change when S&P 500 index components change. Based on the above factors, I do not use SPDR as the underlying asset even though it is a highly liquid tradeable asset.

price is the market friction costs inclusive of market perceived transaction costs paid for hedging[50].

Option Metrics collects different types of S&P 500 index options in the database. These include the S&P 500 index option (which has an expiry date of the Saturday following the third Friday of the expiration month), the quarterly S&P 500 index option (which maturity is the last business day of the quarter) and the S&P 500 Long-term Equity AnticiPation Securities (LEAP) option. The first type of option has maturity spanning from 30 days to 12 months, while the LEAP option has maturity greater than 12 months. The quarterly index options were listed on July 10, 2006 and trading commenced on February 21, 2007. Given the sample period stated in the following section, and consistent with previous literature, I exclude the quarterly S&P 500 index call and put options from my sample, which consist of 4.36% and 5.33% of the original sample respectively. Similar to Bakshi et al. (1997), I only report the results based on the calls.

5.1.2 Other Data

Daily Treasury bill data are collected from the Federal Reserve Bank of St. Louis Economics Research Database (FRED®). There are four different maturities for Treasury bills: 28 days, 91 days, 182 days and 264 days. In order to obtain continuously compounded interest rates that match the options' maturities, I linearly interpolate or extrapolate interest rates from these four rates[51]. Daily S&P 500 index prices are obtained from the Option Metrics database.

5.1.3 Sample Period

The sample period is from January 2, 1996 to September 30, 2009. This period covers four major financial crises: the long-term capital management (LTCM) crisis in 1998, the tech bubble in the year 2000, the subprime mortgage crisis in 2007 and the subsequent GFC in 2008. The LTCM crisis began with the devaluation of the Russian

[50] If the mid-price is used, then I might underestimate the market-perceived transaction costs in hedging the option. In the option hedging with transaction costs literature, such as Boyle and Vorst (1992), the presence of transaction costs introduces an option price bound.
[51] There is at least one Treasury bill rate available for each day in my sample period. Therefore I can interpolate or extrapolate when other Treasury rates are missing.

Ruble and default of Russian bonds on August 17, 1998 and ended on November 17, 1998[52]. The database includes 66 trading days within the LTCM crisis period. The tech bubble began on March 31, 2000 and ended on March 31, 2001[53]. The subprime mortgage crisis began on July 1, 2007, but there is no clear date for its end[54] because it was followed by the more severe GFC.

Although the sample period ends in September 2009, I downloaded option prices up to September 2010. The reason is that I need to allow for extra observations to extract implied volatility from the option data beyond September 2009 that is used in the hedging estimation process. For example, if I examine the hedging performance of an option which initiates in July 2009 and has maturity of six months, daily implied volatilities up to December 2009 are required for the dynamic hedging process. Figure 17 below illustrates why extra option prices are needed.

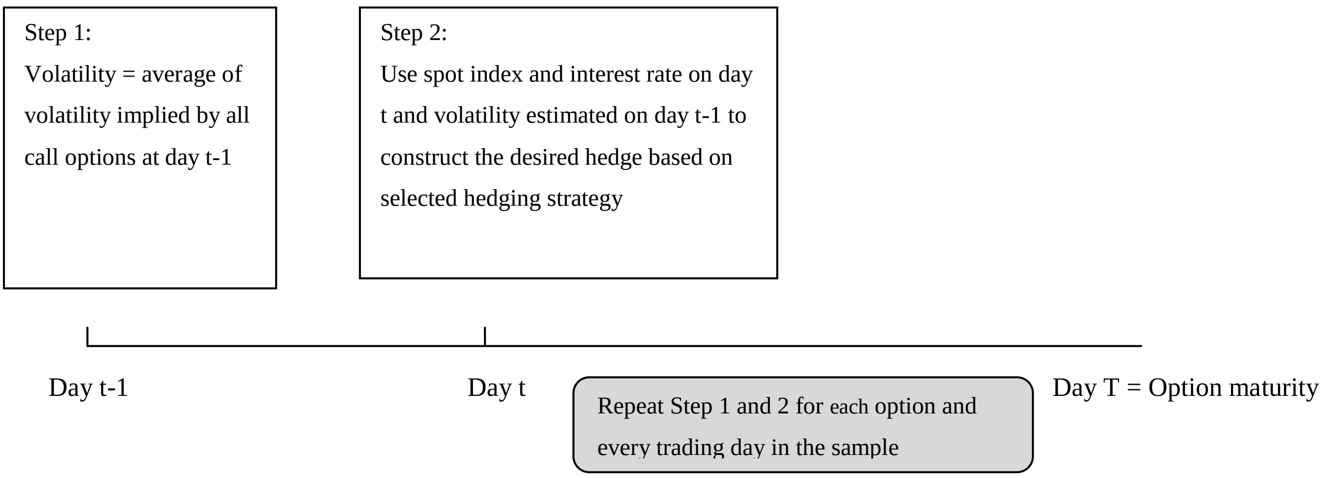

Figure 17 Option Data Requirement for Dynamic Hedging Process

Given the construction of the dynamic hedging process, call options on first trade day in the sample are excluded because these options are used as inputs for implied volatilities (this serves as criterion E1 in Figure 18 as described later). The

52 Cai (2002) and Adrian (2007) illustrate the timeline for the start and end of the long-term capital management crisis.

53 According to Griffin, Harris, Shu and Topaloglu (2011), the run-up period of the tech bubble was January 2, 1997 to March 27, 2000. The bursting of the tech bubble was the crisis period that lasted for a year from March 31, 2000. The technology index lost most of its gains (-72.9%) during this one-year period.

54 The choice of the starting date of the subprime mortgage crisis is arbitrary but consistent with the market consensus.

average of the implied volatilities on day 1 is then used as a volatility input for the option hedging process on day 2. In addition, I restrict the data to option samples with maturity no later than October 1, 2010 (this serves as criterion E2 in Figure 18) due to data availability at the time of download from Option Metrics. This restriction is due to the last day of implied volatility estimation being September 30, 2010.

5.1.4 Data Cleaning

Since S&P 500 index options are European-style contracts, the spot index level must be adjusted for dividends. For each option with τ periods to expiration from time t, the ex-dividend index level becomes

$$\overline{S(t)} \ = S(t)e^{-d\tau}, \tag{5.1}$$

where $S(t)$ is the dividend-inclusive S&P 500 index level and d is the dividend yield. As mentioned in section 5.1.1, dividend yield information is provided in the Options Metrics database. This procedure is repeated for all option maturities and for each day in my sample.

I apply various filters to the raw database in order to construct the option data. I exclude observations with the following properties:

1. options with less than six days to expiration, to avoid any liquidity related biases[55];
2. option price quotes lower than \$3/8, to mitigate the impact of price discreteness on option valuation[56];
3. duplicated options contracts[57]; and
4. options that violate the (simple static) arbitrage restriction

$$Call: \qquad \max\ (S(t) - X, S(t)e^{-d\tau} - Xe^{-r\tau}, 0) \le C(t,\tau) \le S(t). \tag{5.2}$$

where X is the exercise price and r is the risk-free rate.

Based on the data cleaning procedure illustrated in Figure 18, 676,358 call options are included in the final sample, which is about 46% of the original sample.

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[55] This filter follows Bakshi et al.'s (1997) data cleaning procedure.
[56] This filter follows Bakshi et al.'s (1997) data cleaning procedure.
[57] There are data duplication problems in the Option Metrics database from 2005 to 2009. There are 191 days, 251 days, 251 days, 253 days and 182 days of duplicated data in 2005, 2006, 2007, 2008 and 2009 respectively.

Most elimination is due to bid quotes of less than \$3/8 and duplicated data. The eliminated calls are mostly deep out-of-the-money.

I divide the option data into several categories according to either option moneyness or maturity. A call option is said to be in-the-money (ITM) if its $S/X \geq 1.03$; at-the-money (ATM) if its $S/X \in (0.97, 1.03)$; and out-of-the-money (OTM) if its $S/X \leq 0.97$. For ITM and OTM options, I further divide observations into deep ITM (if $S/X \geq 1.06$) and deep OTM (if $S/X < 0.94$). An option is said to be short-term if it has less than 60 days to expiration; medium term if it has between 60 and 180 days to expiration; and long-term otherwise. This creates 18 categories of options, which are analysed and the results presented in subsequent sections.

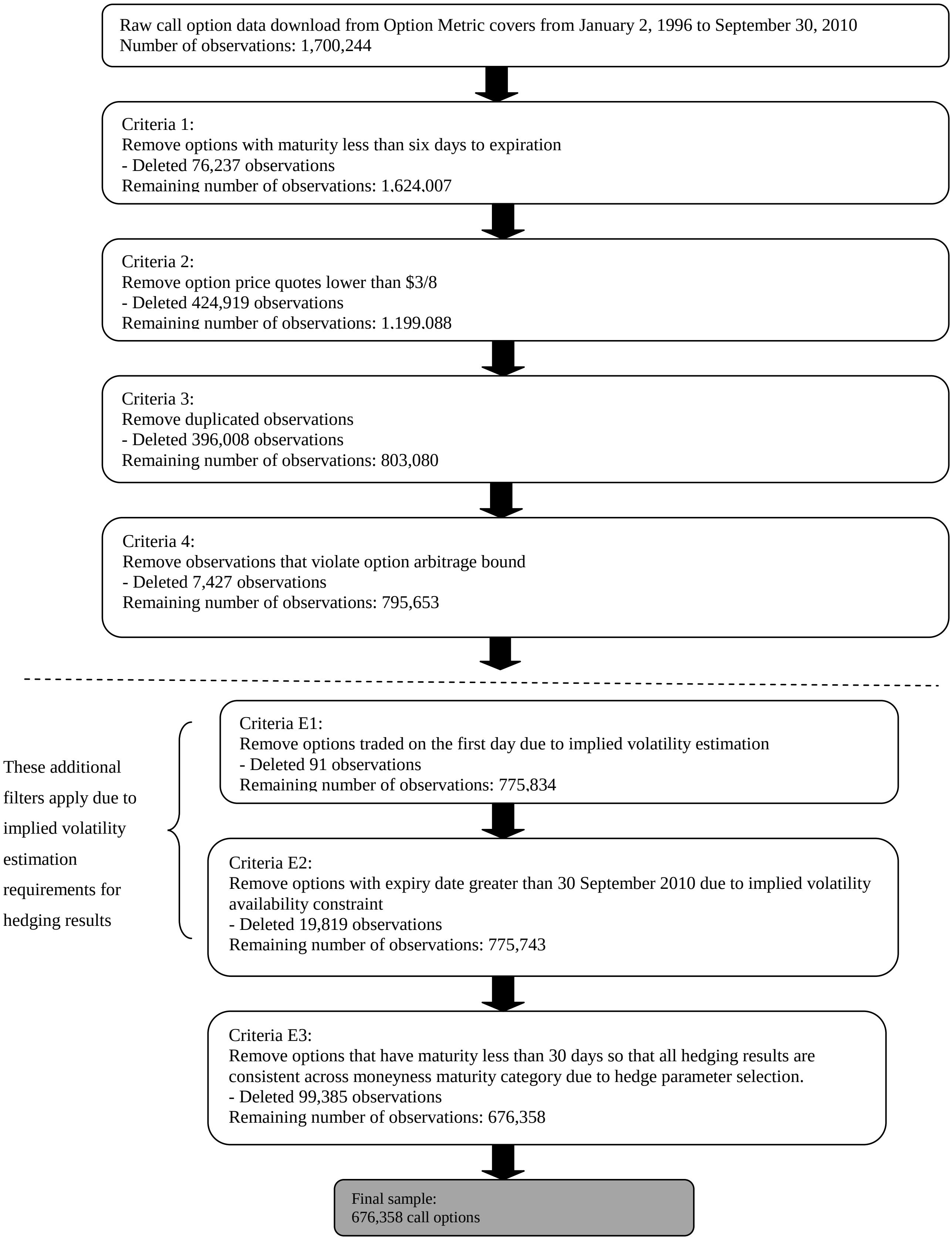

Figure 18 Options Data Cleaning Procedure

Table 8 Summary Statistics of the S&P 500 Option Sample

This table reports (i) the average quoted bid-ask mid-point option price, (ii) the average effective bid-ask spread (ask price minus bid-ask mid-point) in brackets, and (iii) the total number of observations in braces for each moneyness-maturity category. The sample period is from January 2, 1996 to September 30, 2009. Daily end-of-day call option quotes of each contract is used in this table. *S* denotes the spot S&P 500 index level and *X* is the exercise price. ITM, ATM and OTM denote in-the-money, at-the-money and out-of-the-money options respectively. Short, medium and long term-to-expiration options represent options with maturity less than 60 days, between 60 and 180 days, and greater than 180 days.

	Moneyness	Term-to-Expiration			
		Short	Medium	Long	
	S/X	<60	60-180	≥ 180	Total
Deep OTM	< 0.94	$ 505	$ 953	$ 3,221	
		(54.73)	(65.00)	(108.03)	
		{18,529}	{50,132}	{115,826}	{184,487}
OTM	0.94-0.97	$ 1,010	$ 2,475	$ 7,976	
		(57.65)	(85.43)	(130.14)	
		{14,365}	{18,492}	{18,656}	{51,513}
ATM	0.97-1.00	$ 2,229	$ 3,917	$ 9,962	
		(81.45)	(97.38)	(131.42)	
		{16,382}	{20,500}	{19,229}	{56,111}
ATM	1.00-1.03	$ 4,210	$ 5,918	$ 11,988	
		(98.43)	(102.65)	(132.61)	
		{15,356}	{19,100}	{18,376}	{52,832}
ITM	1.03-1.06	$ 6,653	$ 8,295	$ 14,035	
		(102.79)	(106.07)	(132.37)	
		{12,648}	{16,121}	{17,152}	{45,921}
Deep ITM	≥ 1.06	$ 19,627	$ 22,620	$ 29,745	
		(113.67)	(109.85)	(135.95)	
		{52,353}	{91,035}	{142,106}	{285,494}
	Subtotal	{129,633}	{215,380}	{331,345}	{676,358}

Table 8 summarises the properties of the S&P 500 call price sample. The reported summary statistics include average bid-ask mid-point price, average effective spread[58] and total number of observations for each moneyness-maturity category. There are 676,358 call options in the sample, with deep ITM and deep OTM call options respectively taking an average of 42% and 27% of the sample for different maturity categories. The average call price ranges from $505 for short-term deep OTM options to $29,745 for long-term deep ITM options. The effective spread is smallest for OTM options ($54.73) and largest for ITM options ($135.95).

5.2 Methodology

This section explains how to estimate the NHE of each hedging strategy. In particular, I focus on forming a delta-neutral hedge by using the spot S&P 500 index. My choice of hedging instrument is consistent with Bakshi et al. (1997), for the reasons given in section 5.1.

5.2.1 Net Hedging Error Estimation

Consider a situation in which an option writer intends to hedge a short position in a European call option with strike price X and τ periods to expiration. I refer to this call option as the target call option for the remaining of the chapter. It is worth mentioning that the objective of hedging a short call is equivalent to replicating a long call option. I first examine hedges in which only a single instrument (i.e., the underlying asset) can be employed. This constraint implies that the uncertainties that are affecting option value, but are uncorrelated with the underlying asset, are not hedged. Therefore, hedge results are expected to be not as good as the case when I control for more dimensions of uncertainty.

As mentioned previously, I will test the performance of six hedging strategies, the first two belonging to the time-based class and the rest to the move-based class. Each strategy has different rebalancing criteria, and hence Δ_t depends on the choice of the hedging strategy. I assume no liquidation of the underlying asset at maturity and hence no transaction costs will be paid at maturity. This assumption is consistent with

[58] Effective spread is ask price minus bid-ask mid-price.

most of the theoretical assumptions in the option pricing and hedging models with transaction costs.[59]

The calculation of the NHE is as previously described in section 4.2. In short, NHE is the difference between the hedging portfolio value and the option payoff at maturity. There are some differences between the simulation and empirical settings. First, the market-observed S&P 500 index option price is used when setting up the hedging portfolio initially. Second, there is only one realised S&P 500 index path for each option sample in empirical testing. Third, market conditions are fed into the hedging portfolio through implied volatility (as explained in the following paragraph) input in the option delta calculation. Fourth, the NHE is standardised since the empirical test is performed on a set of option samples with varying option moneyness and time to maturity.

To obtain the empirical hedging results presented in section 5.3, I follow three steps. First, estimate the volatility values implied by all call options of day *t-1*. Next, on day *t*, use the volatility estimate and current day's spot index and interest rates to construct the desired hedge using Δ_t unit of underlying index and *B(t)* amount of cash. Finally, rebalance the hedge by repeating step 1 and 2 whenever there is a rebalancing need throughout the life of the option. At maturity, I calculate the NHE. Figure 17 in section 5.1.3 shows the parameters estimation procedure and how the calculation is applied to my empirical study. This three-step procedure is applied to every option in my sample. Figure 19 shows how the option samples are used in my empirical study.

[59] To assess the impact of trading at the ending period, transaction costs can be added easily. The transaction costs may increase significantly depending on how deep in moneyness the option is at maturity.

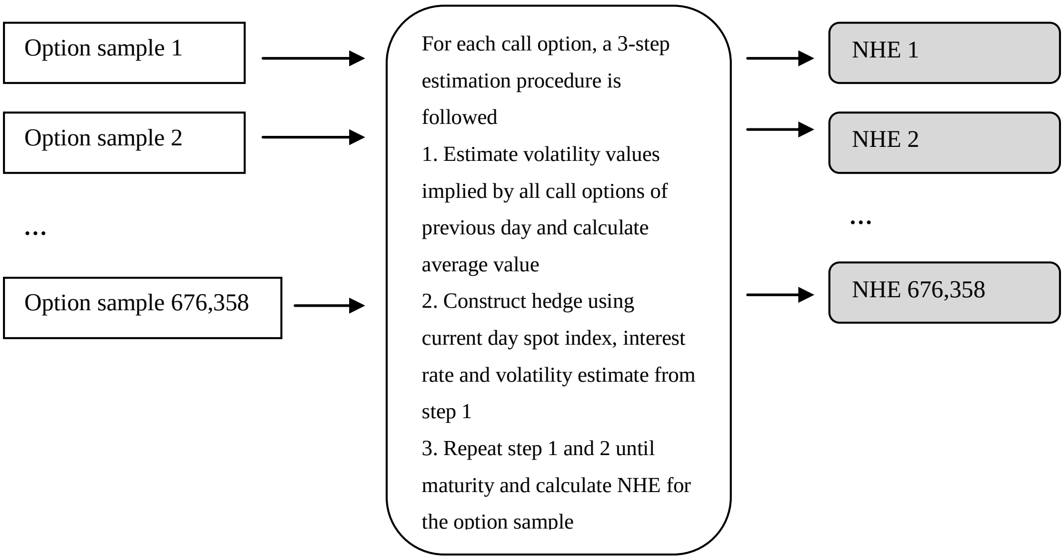

Figure 19 Option Hedging Estimation Process

Following Carr and Wu (2014), I normalise the NHE as a percentage of the underlying index value at the starting date of the hedging exercise for consistent comparison of NHE obtained from the option sample on different trade days. Given that each option sample has different maturity, the normalised NHE is then standardised by the number of days to maturity so that the comparison among different option samples will be consistent.

My method of hedging portfolio construction assumes that market participants are aware of the presence of transaction costs, and hence the discounted expected transaction costs of hedging the option are incorporated into the market price of option[60]. In contrast, the simulation study in chapter 4 expects a deficit at maturity. This is because I used the frictionless option price to construct the hedging portfolio[61]. My

[60] Note there is no longer a single option price in the presence of transaction costs; instead, there is a bid-ask spread. The market bid and ask option price include not only the transaction costs of hedging the position but incorporate the liquidity of the option itself as well as other market frictions.

[61] In the simulation, I am unable to incorporate the present value of the expected transaction costs in setting up the hedging portfolio. This is because the amount of expected transaction costs depends on which transaction costs model is used. Given the objective of comparing the hedging strategies consistently, I want to avoid bias towards any particular transaction costs model in setting up the hedging portfolio.

empirical study measures the effectiveness of the six types of hedging strategies in practice by using the actual market prices.

For each option in the sample, I assume the option is held until its maturity. Without this assumption, I would not be able to calculate the total transaction costs of an option. Further, consistent comparison among the six hedging strategies is not possible because move-based strategies have random rebalancing periods. I do not track an option contract's price change since its inception; rather, I assume an option contract is commenced on its trade date[62]. At each time point *t*, I calculate option delta Δ_t and gamma Γ_t according to the corresponding index and interest rate movement at that time and its remaining time to maturity. I also use this set of information to rebalance the hedging portfolio. The option delta and gamma are expressed in the following forms:

$$\Delta_t = \frac{\partial C(t,S(t))}{\partial S(t)}, \tag{5.3}$$

$$\Gamma_t = \frac{\partial^2 C(t,S(t))}{\partial S(t)^2}. \tag{5.4}$$

Note that the volatility input for both delta and gamma[63] calculations is the average implied volatility by all call options on day *t-1*.

At maturity, the normalised NHE for each observation in each monyness-maturity category is calculated. The average values for each category are then reported in the results section. In my testing, the market value of the option contract will not affect the value of the hedging portfolio throughout the rebalancing process. There is only one exception (to be explained later): when I use the market option price to construct the hedging portfolio. With such a procedure, I am able to calculate the total transaction costs spent on hedging the option. This also allows me to compare across different hedging strategies in a consistent way.

It is noted that my computation method differs from Bakshi et al.'s (1997) and Carr and Wu's (2014), because those studies test time-based hedging strategies without accounting for transaction costs. For example, Bakshi et al. (1997) compute the hedging error at time *t* as the difference between the market value of an option contract and the

[62] Note that the database provides the option contract identification number. However, the scope of this study is not on the daily changes in the market contract value.
[63] The option delta and gamma are derived from the Black–Scholes model.

value of a hedging portfolio constructed at $t - \delta t$. Their hedging portfolio is reconstructed at every rebalancing time interval δt. Figure 20 below demonstrates the difference between my NHE and Bakshi et al.'s (1997) hedging error calculation method.

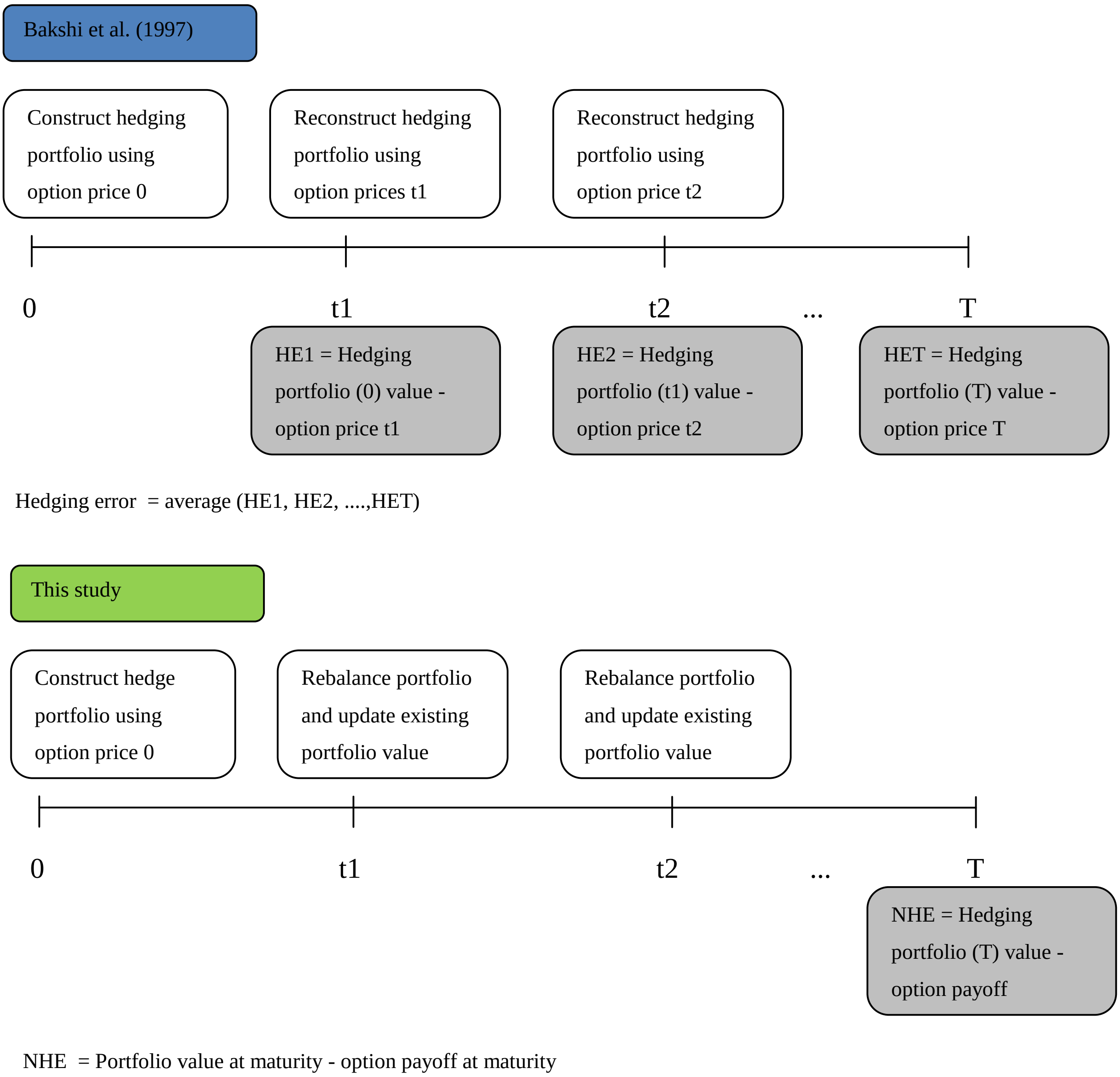

Figure 20 Illustration of Difference in Net Hedging Error Calculations

Bakshi et al.'s (1997) calculation method is not applicable in my testing for the following reason. There is a major difference between time-based strategies and move-based strategies in terms of NHE calculation. For time-based strategies, the rebalancing frequency must be decided when a hedging portfolio is established. Once the rebalancing frequency is selected, the hedger is not required to constantly monitor the movement of the underlying asset. Therefore, the hedging error[64] is calculated only on the day when rebalancing takes place. For example, if I choose to rebalance a hedging portfolio every five days, then I calculate the hedging error on day *t+5* when I set up the portfolio on day *t*. I reconstruct the hedging portfolio on day t+5 and repeat the hedging error calculation on day *t+10*, and so on. The average hedging error is the average of the recorded hedging errors at each rebalancing interval. Hence, the average hedging error is a function of rebalancing frequency. Bakshi et al. (1997) and others employ this method to compute the hedging error, since they only consider time-based hedging strategies and transaction costs are ignored.

On the other hand, move-based strategies require constant monitoring of the movement of the underlying asset price. The hedging portfolio is rebalanced whenever the percentage change of the underlying asset price or change in delta breaches the selected hedging bandwidth. As a result, the time interval for calculating the hedging error is random. In order to compare the performance of the hedging strategies consistently, my method differs from those in the existing literature as I take account of transaction costs. Therefore, I use the terminology of *net* hedging error in order to distinguish my hedging error calculation from Bakshi et al.'s hedging error calculation without transaction costs. It is noted, however, that the NHE presented in this chapter may still be larger than the actual amount. This is due to the fact that a market maker will seldom hold an option position until maturity; instead, he will close his position (by entering into an offsetting position) as soon as possible in order to eliminate or reduce his risk exposure. Therefore, the NHE reported in my study is the most that a market maker could have lost on average if he holds his short position until maturity.

5.2.2 Transaction Costs Assumptions

I assume constant proportional one-way transaction costs are incurred when trading the underlying index throughout the sample period. Following Constantinides et al. (2008), I assume 50 basis points of the index value as the one-way transaction costs.

[64] The term hedging error refers to the difference between hedging portfolio value and option payoff when there is no transaction costs. In Bakshi et al. (1997), the option payoff is replaced by the market option price at the end of each rebalancing period.

The assumed transaction costs are inclusive of one-half bid-ask spread and one-way trading fees[65]. Similar amount of transaction costs are used in simulation studies by Mohamed (1994), Clewlow and Hodges (1997) and Zakamouline (2009). Empirically, a round-trip transaction costs of 1% is supported by the empirical findings in Hasbrouck (2009). To assess the sensitivity of the transaction costs assumption, I perform the same hedging exercise using 25 basis points of the index value as the one-way transaction costs. The findings are similar to those achieved with the 50 basis points assumption, except the mean and standard deviation of NHE are smaller. I assume no costs are involved in investing cash. In practice, transaction costs vary according to market conditions. The impact of time-varying transaction costs are beyond the scope of this study but are useful topics for future research.

5.2.3 Assessment of Hedging Performance

In the transaction costs literature, the degree of hedging precision has a direct impact on the NHE of a hedging strategy. The hedging precision is also related to the hedger's risk aversion. A highly risk-averse hedger is unwilling to accept large NHE at maturity. Therefore, in the presence of transaction costs, this highly risk-averse hedger is willing to pay more transaction costs in exchange for small NHE. On the other hand, a less risk-averse hedger pays lower transaction costs for willing to bear relatively larger NHE. As a result, there is a negative relationship between NHE and risk aversion. In contrast, there is a positive relationship between total transaction costs paid for hedging and risk aversion. For example, a highly risk-averse hedger who prefers a precise hedging outcome will rebalance a hedging portfolio more frequently to minimise NHE. Frequent rebalancing will incur higher transaction costs during the hedging period. The choice of hedging parameter in the selected six hedging strategies reflects how frequently a hedging portfolio is rebalanced or the precision of the hedging outcome. Attaining a more precise hedging outcome requires the hedger to rebalance the hedging portfolio more frequently than a less precise hedging outcome in order to control for the risk of having large NHE at maturity. Therefore, a hedging parameter serves as a proxy for the desired level of hedging precision.

[65] Trading fees refer to brokerage fees as a compensation for order processing's costs.

In Table 10, I have listed the range of hedging parameters for each hedging strategy tested in the empirical study. The specific parameters values used in the empirical test in this chapter are mostly the same as those used in the simulation test in chapter 4. The sole exception is for the BS and LS strategies where the maximum rebalancing frequency has been reduced from 60 to 30 days. The reason for reducing the frequency is to avoid further reduction in the sample size of options with short maturity (i.e., maturity less than 60 days[66]). For all hedging parameters except γ, the greater the parameter value, the less risk-averse the hedger.

Table 9 Hedging Parameters for Empirical Study

This table shows the hedging parameters for each hedging strategy used in my empirical study. The first column represents the choice of hedging strategy, the second column represents the hedging parameter in symbol format, and the last column specifies the range of parameter values.

Hedging strategy	Hedging parameter	Range of parameter values
Black–Scholes hedge at fixed time intervals (BS)	δt (in days)	$[\frac{1}{252}, \frac{30}{252}]$
Leland's hedge (LS)	δt (in days)	$[\frac{1}{252}, \frac{30}{252}]$
Henrotte asset tolerance (AT)	H (percentage change in underlying asset price)	$[0.001, 0.24]$
Delta tolerance (DT)	H (option delta)	$[0.0001, 0.30]$
Hedging to a fixed bandwidth around delta (FB)	H (option delta)	$[0.001, 0.30]$
Hedging to a variable bandwidth around delta (VB)	γ (risk aversion)	$[0.0001, 100]$

5.2.3.1 Mean Variance Test

Since an option cannot be hedged perfectly in the presence of transaction costs by trading the underlying asset, the hedging strategy chosen by a hedger is highly dependent on his risk aversion level. A hedger faces a tradeoff between hedging

[66] This is to ensure the testing of BS and LS hedging performance is consistent across their corresponding range of hedging parameters (i.e., the number of option samples for testing BS rebalances every day is the same as the number of option samples for testing BS rebalances every 30 days).

accuracy and transaction costs. Ultimately, a hedger would like to seek a hedging strategy that gives reasonable accuracy and low transaction costs. The objective of my study is to determine the best hedging strategy considering both hedging accuracy and transaction costs.

An MV test is often used in the transaction costs literature to compare the performance of different hedging strategies. In my context, this test requires the construction of an MV curve for each hedging strategy. For each hedging strategy, I fix a value for a hedging parameter chosen from the range listed in Table 9 and calculate the standardised NHE for each option sample using the methodology described in section 5.2.1. The average and standard deviation of the standardised NHEs for all option samples belonging to each moneyness-maturity category are then reported for each hedging strategy. Under the MV framework, mean and standard deviation of standardised NHE are proxied as return and risk respectively. For each hedging strategy and its associated hedging parameter within the range in Table 9, I calculate 30 different combinations of risk and return, which form an MV curve of the hedging strategy. An example of MV curve construction is provided in section 4.3.1. Note that each hedging strategy has its own MV curve as each has a different risk and return profile. In total, I have six MV curves that allow me to compare hedging performance consistently. Under the MV framework, a rational hedger will always prefer a hedging strategy that minimises risk for a given level of return or maximises return for a given level of risk. As a result, a hedging strategy is optimal if it offers the highest hedge return that is, highest mean NHE relative to the other strategies for a given level of hedge risk or standard deviation of NHE[67].

For ease of comparison, I will present the best hedging strategy under the MV framework at three levels of hedging precision: high, moderate and low for each moneyness-maturity category. To present the results at different level of hedging precision for each moneyness-maturity category, I fit each MV curve by using a quadratic function of the following form:

$$Y = aX^2 + bX + c$$

[67] Ideally, a hedging strategy performs best if the average NHE is close to zero at minimal hedging risk. When I compare six hedging strategies simultaneously, a strategy performs better if it has less deficit (or more surplus) in the difference between hedging portfolio value and final payoff for the same level of hedge risk.

where Y is the mean standardised NHE and X is the standard deviation of standardised NHE. a, b and c are fitted parameters. I performed similar fitting in chapter 4[68]. With the fitted functions, the mean standardised NHE is a function of the standard deviation of standardised NHE. The high, moderate and low hedging precision levels correspond to the standard deviation of standardised NHE obtained from the Black–Scholes strategy with hedging frequency of 2 days, 10 days and 20 days. With this definition, I am able to compare the mean standardised NHE for each hedging strategy at the same level of hedging precision. The resulting optimal strategy will be reported in Table 11 in section 5.3.1.

In addition, I examine the relative expensiveness of the hedging strategies. I perform similar quadratic function fitting to determine the relationship between total transaction costs and the standard deviation of standardised NHE. Results in Table 11 show the best- and worst-performing hedging strategies along with the amount of transaction costs saved if the hedger switched from the worst to the best strategy.

5.2.3.2 Stochastic Dominance Test

I use two types of SD test in empirical testing. The first test determines if one hedging strategy stochastically dominates the remaining five strategies; if a dominance relationship exists, then that hedging strategy is optimal. I refer to the first test as the all-strategies SD test. The second test compares one strategy to another, pairwise; hence I refer to the second test as the pairwise SD test. Given that the true distributions of standardised NHE are unknown in practice, SD tests have to rely on an EDF, which is subject to sampling error. Hence I employ Barret and Donald (2003) for the pairwise SD test and Linton et al. (2005) for the all-strategies SD test. The details of the test hypotheses and their associated test statistics are set out in section 4.4.2.

For both pairwise and all-strategies SD tests, the procedure of obtaining the test statistics can be summarised in three steps (detailed in Appendix A). Note that this procedure is applied to each moneyness-maturity category. For example, for the ATM option (strike to underlying index ratio between 0.97 and 1.00) with medium-term maturity, there are 20,500 standardised NHEs for one hedging strategy with selected

[68] Power function tends to have poor fitting for the empirical data. Hence a quadratic function is used here.

hedging parameters. These 20,500 observations form an EDF which is then used in both SD tests.

Given the extensiveness of the SD test, I simplify the results presentation to three levels of hedging precision: high, moderate and low. The selected hedging parameters for each corresponding hedging precision level are presented in Table 10 below. The choice of hedging parameters for the SD test is the same as for the test in Table 5 in chapter 4.

Table 10 Hedging Parameters for Stochastic Dominance Test

The table shows the hedging parameters selected for each class of hedging strategy. These parameters represent three different levels of hedging precision: high, moderate and low. The first column presents the name of the hedging strategy, the second column indicates the measurement unit of the hedging parameters, and the remaining columns indicate the parameter value for its corresponding level of hedging precision. The bracket term in the second column specifies the meaning of the parameter symbol.

Hedging Strategy	Hedging Parameters	High	Moderate	Low
Black–Scholes hedge at fixed time intervals (BS)	δt (in days)	2	10	20
Leland's hedge (LS)	δt (in days)	2	10	20
Henrotte asset tolerance (AT)	H (percentage change in underlying asset price)	2%	4%	6%
Delta tolerance (DT)	H (option delta)	0.05	0.1	0.2
Hedging to a fixed bandwidth around delta (FB)	H (option delta)	0.05	0.1	0.2
Hedging to a variable bandwidth around delta (VB)	γ (risk aversion)	5	0.1	0.001

5.3 Results

5.3.1 Mean Variance Test Results

For each moneyness-maturity category, a set of MV curves similar to the form in Figure 21 is obtained. To interpret the magnitude of the mean NHE in Figure 21, I use the index level of 2160 and 30 days to maturity as an example. For move-based strategies, a mean NHE of 0.00001 is equivalent to $0.26. For time-based strategies, a mean NHE of 0.00015 is equivalent to $3.86. If I compare the amount of mean NHE obtained from the simulation study, -0.1 of mean NHE is equivalent to -$42.90. As a result, the mean NHE is smaller in the empirical study than in the simulation study.

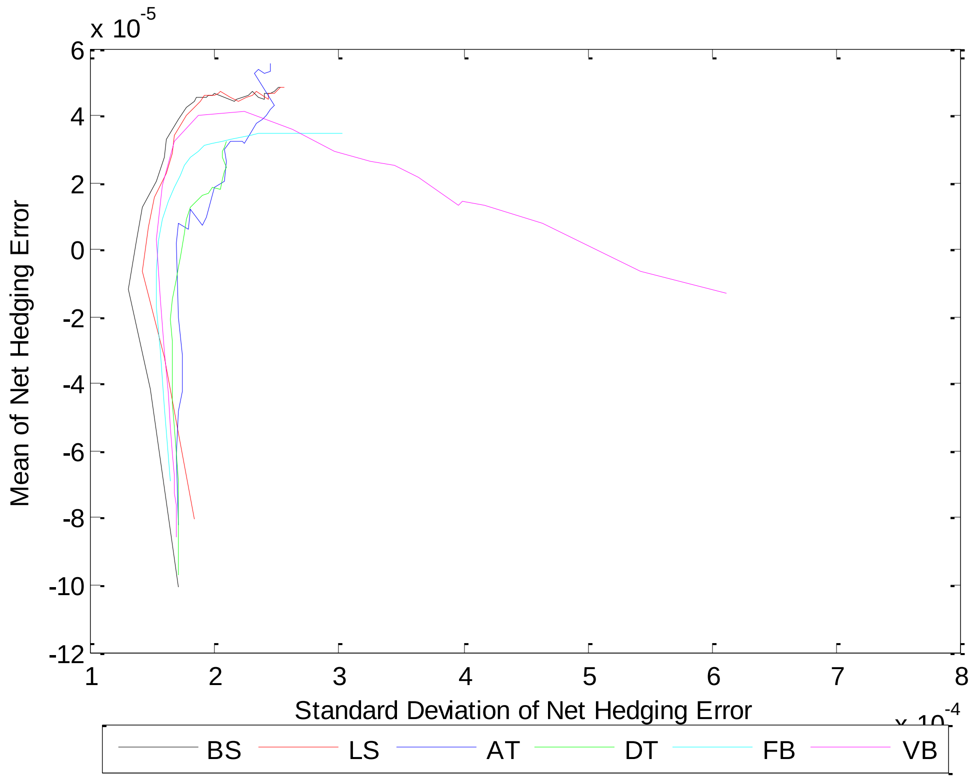

Figure 21 Mean Variance Curves of the Performance of Alternative Hedging Strategies under Delta-Neutral Hedge

The MV curves in Figure 21 based on actual market data are, not surprisingly, less smooth than the MV curves based on simulated data in Figures 10-12. Notwithstanding, the basis of comparison of the alternative hedging strategies remains the same in both the controlled setting in chapter 4 and the actual market setting here. Under the MV framework, the best hedging strategy is the one with an MV curve in the upper left-hand corner. This means that the strategy has the highest positive (or smallest negative) mean NHE at the smallest amount of hedging risk (i.e., the standard deviation of NHE). An important finding in the figure is that both time-based strategies are in the upper left-hand corner in the MV space when they are compared with the move-based strategies. This result is opposed to my findings in the simulation study; indeed, a discrete time Black–Scholes hedging strategy is the optimal hedging strategy. At first, this result may be counter-intuitive, since the ex ante result is that VB should be the optimal hedging strategy, but is likely due to the dynamic volatility update in my hedging exercise. The volatility input used in the rebalancing activity has effectively fed market information into the hedging portfolio. On the other hand, the move-based strategies are derived from the framework that assumes constant volatility input since inception of the hedging portfolio. A potential explanation as to why VB underperforms the time-based strategy is related to the formation of its hedging bandwidth, which is negatively related to the option's gamma. The volatility input in my hedging exercise is the average implied volatility from call options on the previous day. If implied volatility is greater than realised volatility, then VB's hedging bandwidth becomes narrower than it should be. This may result in over-hedging and therefore more transactions in the underlying asset. Consequently, this leads to more uncertainty in hedging outcome and greater variation in transaction costs. Another observation is that the theoretically superior LS underperforms BS. LS is derived by incorporating discrete time hedging and non-zero transaction costs for the underlying asset. The LS hedging mechanism is to make systematic gains over the course of option life in order to offset the expected transaction costs of hedging. In my empirical study, the volatility input is non-constant and hence the expected systematic gain may be insufficient to cover the transaction costs over the hedging period. Nonetheless, the gap between BS and LS decreases when the standard deviation of NHE increases (or required hedging precision decreases). This finding is similar to my simulation results in chapter 4. In terms of the hedging performance of move-based strategies, VB has better performance than AT, DT and FB, except when the hedging precision is low (or the standard deviation of NHE is large).

Table 11 presents the best and the worst delta-neutral hedging strategies for across the eighteen moneyness-maturity categories. The table shows that time-based strategies are the best hedging strategies most of the time for different moneyness-maturity groups at different levels of hedging precision. For moderate hedging precision, BS is the optimal hedging strategy across seventeen moneyness-maturity groups. On the other hand, LS is optimal for ATM options regardless of the length of option maturity when hedging precision is high. It is also often the case that DT and VB are the worst hedging strategies[69]. The transaction costs saved by switching from the worst to the best hedging strategy are also reported in the same table. The average transaction costs saving is 46%, and this amount is approximately constant at high, moderate and low hedging precision level. Most of the time, the average transaction costs saving is greater when the maturity of the option is longer; however, there are a few exceptions. These exceptions may be due to two reasons: (1) each moneyness maturity group has a different number of observations, and (2) the market price of options contains the cost of other frictions such as liquidity of option price. The transaction costs saving in the empirical study is about 31% higher than that obtained in the simulation study. The detailed hedging performance ranking results for the six hedging strategies is set out in Table 1A of Appendix B.

Table 11 Best and Worst Delta-Neutral Hedging Strategy Under Mean Variance Test

In this table, the S&P 500 index is used as the hedging instrument. Parameters and spot volatility implied by all call options of the previous day are used to establish the current day's hedges. The hedging portfolio is rebalanced according to the rule of the selected hedging strategy until option maturity. For each target call option, its NHE is, as of maturity day, the difference between the hedging portfolio value and its payoff normalised by initial option ask price and number of days to maturity. The best and the worst hedging strategy tested under the MV framework are presented in each panel at different level of hedging precision. The standard deviation of the standardised NHE of the BS strategy is used as a proxy for the level of hedging precision. The BS strategies with hedging frequency of 2 days, 10 days and 20 days represent the hedger who prefers high, moderate and low hedging precision correspondingly. The mean NHE for LS, AT, DT, FB and VB strategies are derived from the standard deviation obtained from the BS strategy through a fitted quadratic function for each strategy. The sample period is from January 2, 1996 to September 30, 2009. There are 676,358 observations distributed across eighteen moneyness-maturity categories. In each moneyness-maturity category,

[69] The finding in Table 11 that VB appears to be the worst hedging strategy seems to contradict the MV curves in Figure 21. The difference however results from using a quadratic function for fitting purposes in Figure 21.

the first row represents the best-performing hedging strategy and the second row (in *italic*) represents the worst-performing hedging strategy within time-based or move-based strategies. The third row in the category represents the total transaction costs saving by switching from the worst to the best hedging strategy. A negative value means saving and a positive value means paying more transaction costs (as a percentage of total transaction costs paid when adopting the worst hedging strategy).

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS	LS	FB
		LS	*DT*	*LS*
		-27%	-30%	-84%
	0.94-0.97	FB	LS	FB
		AT	*DT*	*VB*
		-25%	-46%	-50%
ATM	0.97-1.00	LS	LS	LS
		DT	*DT*	*BS*
		-40%	-48%	-43%
	1.00-1.03	LS	LS	LS
		DT	*AT*	*BS*
		-55%	-33%	-52%
ITM	1.03-1.06	LS	LS	LS
		DT	*AT*	*BS*
		-71%	-28%	-56%
	≥ 1.06	FB	AT	LS
		DT	*DT*	*BS*
		-53%	-93%	-48%

Panel B: Moderate Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS	BS	BS
		VB	*VB*	*LS*
		-33%	-47%	-91%

	0.94-0.97	BS *VB* -23%	BS *VB* -43%	BS *VB* -54%
ATM	0.97-1.00	BS *VB* -10%	BS *VB* -38%	BS *VB* -52%
	1.00-1.03	BS *VB* -24%	BS *VB* -36%	BS *VB* -50%
ITM	1.03-1.06	BS *VB* -22%	BS *VB* -33%	BS *VB* -50%
	≥ 1.06	FB *DT* -20%	BS *DT* -86%	BS *DT* -46%

Panel C: Low Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS *VB* -29%	BS *VB* -37%	BS *VB* -70%
	0.94-0.97	LS *DT* -4%	BS *VB* -38%	LS *VB* -60%
ATM	0.97-1.00	LS *DT* -63%	BS *VB* -34%	LS *VB* -59%
	1.00-1.03	LS *DT* -71%	DT *VB* -59%	LS *VB* -59%
ITM	1.03-1.06	LS *DT* -71%	DT *VB* -65%	BS *VB* -56%

≥ 1.06	FB	BS	LS
	DT	*DT*	*VB*
	-71%	-92%	41%

5.3.2 Stochastic Dominance Test Results

Using Linton et al.'s (2005) SD test method, the all-strategies SD results in Table 12 show that there is no SD relationship for deep ITM options and deep OTM options for all maturities and all hedging precision levels. I examine the undocumented empirical distributions of NHE of each hedging strategy for deep ITM options and deep OTM options and compare them with options at other moneyness levels. I find that these options tend to have empirical distributions that are clustered together and hence no strong preference is concluded from the SD test. Deep ITM options and deep OTM options have smaller gamma compared to other moneyness options. This means that the hedging portfolios tend to have less rebalancing transactions and the stickiness of delta may produce hedging results that have minimal difference among the six hedging strategies. The results also apply to long maturity options across all option moneyness. This means that a hedger does not prefer any particular hedging strategy, since all hedging strategies are equal under the SD test. As opposed to the MV results, move-based FB and VB strategies are no worse than either time-based strategy. Indeed, VB is the optimal hedging strategy for ATM options with short maturity (when hedging precision is high) and ATM options with medium maturity (when hedging precision is moderate). The reason is the distribution of NHE for VB has more positive NHEs (long right tail) and larger standard deviation than the remaining strategies. The MV test fails to consider the impact of thick tail distribution on hedging performance. Using Barrett et al.'s (2003) methodology, I also perform a pairwise SD test, however, no meaningful results are obtained. The results are grouped similarly to those obtained from the all-strategies SD test. Hence, the pairwise results are not presented.

Table 12 All-Strategies Stochastic Dominance Test for Delta-Neutral Hedge

In this table, the S&P 500 underlying index is used as the hedging instrument. Parameters and spot volatility implied by all call options of the previous day are used to establish the current day's hedges. The hedging portfolio is rebalanced according to the rule of selected hedging strategy until option maturity. For each target call option, its standardised NHE is, as of maturity day, the difference between the hedging portfolio value and its payoff. The dominating hedging strategy tested under the all-strategies SD framework is presented in each panel at a different level of hedging precision. Under the null hypothesis of the SD tests, a particular hedging strategy dominates all other hedging strategies at the selected hedging precision level. If the null hypothesis is rejected, no dominance relation exists. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. FSD implies that we cannot reject the null hypothesis that the strategy dominates all other strategies at selected risk aversion levels by first-order. Similarly, SSD means that we can reject the null hypothesis that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by second-order. Finally, if a strategy is TSD, we can reject the null hypothesis that the strategy dominates all other strategies by FSD and SSD, but not that it dominates all other strategies in the set by third-order. No SD means that we reject the null hypothesis of SD by any other. $^{\circ}$,$^{\circ\circ}$,$^{\circ\circ\circ}$ denote the significance level of the null hypothesis test for SD identifications, i.e. 10%, 5%, 1% level respectively. If there is more than one strategy in the moneyness-maturity category, then these strategies are ~equal. The sample period is from January 2, 1996 to September 30, 2009. There are 676,358 observations distributed across eighteen moneyness-maturity categories.

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	FB~VB (FSD)	No SD$^{\circ\circ\circ}$	No SD$^{\circ\circ\circ}$
	0.94-0.97	LS~FB~VB (FSD)	LS~AT~FB~VB (FSD)	No SD$^{\circ\circ\circ}$
ATM	0.97-1.00	VB (FSD)	No SD$^{\circ\circ\circ}$	No SD$^{\circ\circ\circ}$
	1.00-1.03	VB (FSD)	No SD$^{\circ\circ\circ}$	No SD$^{\circ\circ\circ}$
ITM	1.03-1.06	FB~VB (FSD)	BS~LS~FB~VB (FSD)	No SD$^{\circ\circ\circ}$
	≥ 1.06	No SD$^{\circ\circ\circ}$	No SD$^{\circ\circ\circ}$	No SD$^{\circ\circ\circ}$

Panel B: Moderate Hedging Precision

	Moneyness S/X	Term-to-Expiration Short	Medium	Long
OTM	< 0.94	No SD°°°	No SD°°°	No SD°°°
	0.94-0.97	BS~LS~AT~FB~VB (FSD)	BS~LS~AT~FB~VB (FSD)	No SD°°°
ATM	0.97-1.00	LS~VB (FSD)	VB (FSD)	No SD°°°
	1.00-1.03	BS~LS~VB (FSD)	VB (FSD)	No SD°°°
ITM	1.03-1.06	BS~LS~VB (FSD)	LS~VB (FSD)	No SD°°°
	≥ 1.06	No SD°°°	No SD°°°	No SD°°°

Panel C: Low Hedging Precision

	Moneyness S/X	Term-to-Expiration Short	Medium	Long
OTM	< 0.94	No SD°°°	No SD°°°	No SD°°°
	0.94-0.97	BS~LS~AT~FB~VB (FSD)	BS~LS~AT~FB~VB (FSD)	No SD°°°
ATM	0.97-1.00	BS~LS~VB (FSD)	BS~LS~AT~FB~VB (FSD)	No SD°°°
	1.00-1.03	BS~LS~VB (FSD)	BS~LS~AT~FB~VB (FSD)	No SD°°°
ITM	1.03-1.06	BS~LS~VB (FSD)	BS~AT~FB~VB (FSD)	No SD°°°
	≥ 1.06	No SD°°°	No SD°°°	No SD°°°

5.4 Delta-Vega-Neutral Hedge

Empirical evidence provided by Bakshi et al. (1997) and Dumas et al. (1998) shows that, in terms of hedging performance, the ad-hoc Black–Scholes model performs no worse than more sophisticated models, especially when one takes account of stochastic volatility risk. In addition, Hull and White (1987) use simulated and actual foreign currency data from the Philadelphia Exchange to demonstrate that we should always hedge for volatility risk if we are given a choice to hedge between gamma and vega. Much research has also demonstrated that it is important to control for volatility risk. For example, Bakshi et al. show that a simple delta-vega-neutral hedge would have improved the hedging performance significantly; the results of the Black–Scholes delta-plus vega-neutral strategy and the delta-neutral strategies for the other models that considered stochastic volatility are indistinguishable. A simple delta-vega-neutral hedge is formed by using a combination of underlying stock and vanilla call options to neutralise the sensitivity of the hedge to underlying price risk *and volatility risk*. One should note that jump risk would also have an impact on hedging effectiveness. However, Bates (1996), Cox and Ross (1976) and Merton (1976) find that it is difficult to create a perfect hedge in the presence of stochastic jump risk. As a result, the hedge effectiveness provided in this study is not controlled for jump risk.

The formation of a delta-vega-neutral hedge is similar to the previously described delta-neutral hedge in section 5.2. However, the methodology is slightly different due to data limitations. In this section, I first describe the selection of a hedge instrument, then demonstrate the procedure of computing the NHEs resulting from the delta-vega-neutral hedge, and finally present the results.

5.4.1 Formation of Delta-Vega-Neutral Hedge

Suppose again that the target option is a short European call option with strike price X and τ periods to expiration from time *t*. The hedger will need a position in (i) $n_{S,t}$ shares of underlying spot index (to control for price risk), (ii) $B(t)$ amount of cash and (iii) $n_{H,t}$ units of another call option with the same maturity but different strike price $\bar{X}$ (to control for volatility risk). The time *t* value of this hedging portfolio is $B(t) + n_{S,t}S(t) + n_{H,t}\bar{C}(t, S(t))$, where $\bar{C}(t, S(t))$ is the (actual market) price of the hedge instrumental option with strike price $\bar{X}$ at time *t*. Deriving the dynamics for the hedging portfolio with those of target call option $C\big(t, S(t)\big)$, I present the following solution:

$$n_{H,t} = \frac{\mathrm{K}_{\mathrm{t,target}}}{\mathrm{K}_{\mathrm{t,hedge}}}, \tag{5.5}$$

$$n_{S,t} = \Delta_{t,target} - n_{H,t}\Delta_{t,hedge}, \quad (5.6)$$

$$B_t = C(t, S(t)) - n_{S,t}S(t) - n_{H,t}\bar{C}(t, S(t)), \quad (5.7)$$

where $\Delta_{t,target}$ and $\Delta_{t,hedge}$ are the delta of the target and hedge instrumental option as determined in equation (5.3); and $\mathrm{K}_{t,target}$ and $\mathrm{K}_{t,hedge}$ are the vega of the target and hedge instrumental option as demonstrated in equation (5.8) below:

$$\mathrm{K}_t = \frac{\partial C(t,S(t))}{\partial \sigma}, \quad (5.8)$$

and σ denotes the volatility of the corresponding option.

To examine the hedging effectiveness, I set up a hedging portfolio as described above. At each time interval, I rebalance the portfolio in a way similar to the delta-neutral hedge in section 5.2[70]. In the case of the delta-vega-neutral hedge, the hedging portfolio has added an additional option to control for volatility risk. As a result, the residual cash amount invested after transaction costs is of the following form:

$$B(t+\delta t) = e^{r\delta t}B(t) - (n_{s,t+\delta t} - n_{s,t}) \times S(t) - k \times |n_{s,t+\delta t} - n_{s,t}| \times S(t)$$
$$-(n_{H,t+\delta t} - n_{H,t})\bar{C}(t, S(t)) - k_H \times |n_{H,t+\delta t} - n_{H,t}| \times \bar{C}(t, S(t)) - const, \quad (5.9)$$

where k_H is the proportional one-way transaction costs incurred when trading the hedge instrumental option. Note that δt is not necessarily a fixed interval. Other than in BS and LS strategies, δt is determined by the hedging rule that indicates rebalancing is required when conditions are met. The above hedging exercise also means that the hedging portfolio will be rebalanced to a delta-vega-neutral hedge position at each rebalancing interval.

5.4.2 Net Hedging Error Estimation

To derive a hedge effectiveness measure for the delta-vega-neutral hedge, I construct the desired hedge as described and rebalance the hedging portfolio according to the selected hedging strategy rule. I rebalance the hedging portfolio (whenever it is necessary) until the option matures. At maturity, I compute the NHE, being the difference between the hedging portfolio value and the option payoff at maturity. The computed NHE is of the following form:

$$NHE = n_{S,T-\delta t}S(T) + n_{H,T-\delta t}\bar{C}(T, S(T)) + B(T) - \max(\mathrm{S(T)} - \mathrm{X}, 0) \quad (5.10)$$

[70] I use a delta-neutral hedge rebalancing rule to rebalance the delta-vega-neutral hedge portfolio.

Given that the selected hedge instrumental option and the target option have the same maturity but different strike price, the payoff of the hedge instrumental option is therefore equivalent to max $(S(T) - \bar{X}, 0)$.

Based on the hedge equation (5.9), the data requirement for the empirical test is stricter than the previous delta-neutral hedging in section 5.2. The delta-vega-neutral hedge requires (i) the availability of prices for the hedge instrumental option at each rebalancing point: $\bar{C}(t, S(t))$ and (ii) the computation of Δ_t and K_t for both the target and hedge instrumental option. One further complication in my testing is that the option prices are only available up to six days before maturity due to the data cleaning process[71]. Hence, I need to revise the NHE computation to

$$NHE' = n_{S,t'-\delta t}S(t') + n_{H,t'-\delta t}\bar{C}(t', S(t')) + B_{t'} - \mathrm{C}(\mathrm{t}', \mathrm{S}(\mathrm{t}')) \qquad (5.11)$$

where t' is the last day where both target and hedge instrumental option prices are available in my dataset. $C(t', S(t'))$ and $\bar{C}(t', S(t'))$ are the market prices for both target option and hedge instrumental option on t'. In Bakshi et al. (1997), the delta-vega-neutral test does not suffer from an early liquidation problem because their estimation of hedging error uses the option prices at the end of each rebalancing interval. A new hedging portfolio is reconstructed and new hedging error calculated at the end of next rebalancing interval. Given the lack of transaction costs term in their hedging error computation, their method does not face the same issue I do. In my revised NHE estimation, I have indirectly assumed that the hedger liquidates his position prior to maturity. This exercise results in lower transaction costs paid throughout the hedging process due to early exit of the position. However, it is not an uncommon practice for practitioners to liquidate their position before option maturity. In any event, the relative performance comparison is consistent since all strategies are using the same set of information to form the delta-vega-neutral hedge and exit the position at the same time[72].

There are some occasions when multiple hedge instrumental options (options with same maturity but strike price different to the target option specification) match the target option after applying the matching criteria (i) and (ii). Under this situation, I

[71] Following Bakshi et al. (1997), options with less than six days to expiration are excluded in order to avoid any liquidity-related bias.
[72] I have also performed the delta-neutral hedge in which the hedging portfolio is liquidated six days prior to maturity. The results show that a reduction of 20% in total transaction costs due to the early liquidation of the hedging portfolio.

select the option that is the furthest away from the ATM position as the matched hedge instrumental option. The rationale is that the ATM option has the highest option gamma and therefore it will trigger more rebalancing activities due to the hedge instrumental option instead of the target option. I aim to reduce the gamma impact as much as possible through the selection of a hedge instrumental option. Moreover, if there is a choice between ITM and OTM options, I select the OTM option as the hedge instrumental option because it is cheaper in terms of transaction costs. Based on the requirements above, the remaining sample for this hedging exercise contains 249,111 matched pairs. As before, I use the current day's spot index and interest rate, but the volatility implied by all of the previous day call options to determine the current hedging position for the target call. The hedge is rebalanced according to the rule of the selected hedging strategy. At time t', I calculate NHE using equation (5.11). NHE is then normalised by the underlying index value at the initial hedging portfolio setup date and number of days to maturity. This procedure is repeated for each target call option in my new sample pool.

5.4.3 Results

5.4.3.1 Mean Variance Test

The MV curves for each hedging strategy generated from the delta-vega-neutral hedge are shown in Figure 22. The most striking feature is the zigzag patterns which arise from (1) relatively lengthy unhedged periods for the time-based strategies, and (2) actual option prices containing frictions other than transaction costs. These results are similar across eighteen moneyness-maturity categories. A second key distinction from the delta-neutral hedge results in Figure 21 is that the DT and FB MV curves only span a small area in the MV space based on the defined range of hedging parameters. This is because the DT and FB results are relatively insensitive to the change in hedging parameter value. As a result, the best and worst hedging strategies and the transaction costs saving in Table 13 for DT and FB are most likely extrapolated at moderate and low hedging precision level. I also observe that the mean and standard deviation of NHE in the delta-vega-neutral hedge are larger than those in the delta-neutral hedge, although the translated dollar value is still small. This finding is different from the results in the empirical option hedging literature, such as Bakshi et al. (1997), Nandi (1998) and Dumas et al. (1998), whereby ad hoc Black–Scholes hedging will improve hedging performance after controlling for volatility risk. My finding may be due to the fact that previous research has not taken account of transaction costs paid in the hedging process and the hedging error is calculated based on liquidation of the option at each rebalancing interval. As discussed previously, my NHE calculation traces the option position until the option matures (for the delta-neutral hedge) or is liquidated early (for the delta-vega-neutral hedge). Given the randomness of transaction costs, hedging an extra risk dimension may have increased NHE risk through the adjustment of the delta position due to the use of a hedge instrumental call option. Nonetheless, my delta-vega-neutral position is formed only when the rebalancing criterion is triggered. Another observation in Figure 22 is that LS performs better than BS since its MV curve always sits on the left-hand side of the BS MV curve.

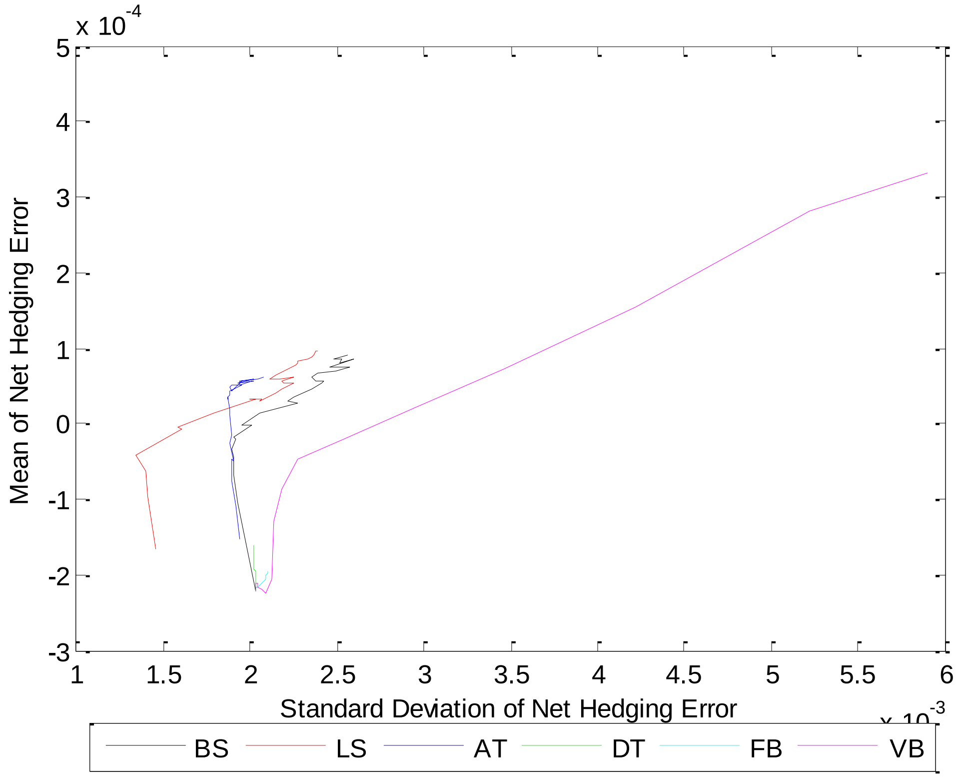

Figure 22 Mean Variance Curves of the Performance of Alternative Hedging Strategies under Delta-Vega-Neutral Hedge

Based on the hedging results presented in Table 13, I find that although there is no single strategy that is outstanding in terms of hedging optimality, the MV results are relatively consistent across three levels of hedging precision for all options except deep ITM and deep OTM options. The detailed of hedging performance ranking results for the six hedging strategies are set out in Table 1B in Appendix C. The average transaction costs saving in the delta-vega-neutral hedge is 86% of the total transaction costs of the worst hedging strategy. Note that the 46% saving in the delta-neutral hedge and 86% savings in the delta-vega-neutral hedge cannot be compared directly. This is because the amount of savings is presented on a relative comparison among alternative hedging strategies under the same hedge type, that is, using either a delta-neutral or delta-vega-neutral hedge. As a result, I compare the total amount of transaction costs in dollar value in order to assess the reduction in total transaction costs paid when changing from a delta-neutral hedge to a delta-vega-neutral hedge. Relative to the total transaction costs paid in the delta-neutral hedge, my unreported results demonstrate that there is an average reduction of 29%, 21% and 19% in total transaction costs with respect to high, moderate and low hedging precision when a delta-vega-neutral hedge is applied. The amount of reduction in total transaction costs is partially due to early liquidation of the hedging portfolio in the delta-vega-neutral hedge[73]. The total transaction costs in the delta-vega-neutral hedge do not capture the full amount paid at the maturity of the option. In addition, the saving decreases when option maturity becomes longer and option moneyness increases.

Table 13 Best and Worst Delta-Vega-Neutral Hedging Strategy under Mean Variance Test

In this table, the S&P 500 index and a call option with the same maturity as the target call option but different strike price are used as the hedging instruments. Parameters and spot volatility implied by all call options of the previous day are used to establish the current day's hedges. The hedging portfolio is rebalanced according to the rule of selected hedging strategy until six days prior to maturity. For each target call option, its standardised NHE is, as of liquidation day, the difference between the hedging portfolio value and its market price. The best and the worst hedging strategy tested under the MV framework are presented in each panel at different levels of hedging precision. The sample period is from January 2, 1996 to September 30, 2009. The standard deviation of the standardised NHE of the BS strategy is used as a proxy for the level of hedging

[73] I have checked the percentage of transaction cost reduction due to the introduction of early liquidation. Early liquidation reduces total transaction costs by an average of 67% for rebalancing the portfolio until option maturity in my sample. For consistency, the comparison of transaction costs reduction between delta-neutral hedge and delta-vega-neutral hedge portfolio is based on the early liquidation samples described in section 5.4.2. For example, if we are paying $100 worth of transaction costs for rebalancing the portfolio until option maturity, then we are paying only $33 when we liquidate the hedging portfolio early due to short hedging time period. This means that implementation of delta-vega-neutral hedge still allows the hedger to save some total transaction costs.

precision. The BS strategies with hedging frequency of 2, 10 and 20 days represent the hedger prefers high, moderate and low hedging precision respectively. The mean standardised NHEs for LS, AT, DT, FB and VB strategies are derived from the standard deviation obtained from the BS strategy through a fitted quadratic function for each strategy. There are 249,111 observations distributed across eighteen moneyness-maturity categories. In each moneyness-maturity category, the first row represents the best-performing hedging strategy and the second row (in *italic*) represents the worst-performing hedging strategy within time-based or move-based strategies. The third row in the category represents the amount of total transaction costs saving by switching from the worst to the best hedging strategy. A negative value means saving and a positive value means paying higher transaction costs (as a percentage of total transaction costs paid when adopting the worst hedging strategy).

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	AT	DT	AT
		DT	*VB*	*DT*
		-100%	-100%	-100%
	0.94-0.97	DT	DT	LS
		BS	*FB*	*DT*
		-100%	-100%	-100%
ATM	0.97-1.00	LS	DT	LS
		DT	*FB*	*DT*
		-67%	-100%	-99%
	1.00-1.03	LS	VB	LS
		DT	*FB*	*DT*
		-76%	-100%	-95%
ITM	1.03-1.06	LS	AT	FB
		VB	*DT*	*DT*
		-61%	-66%	-100%
	≥ 1.06	LS	FB	LS
		VB	*DT*	*DT*
		-32%	-63%	-90%

Panel B: Moderate Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	FB	LS	AT
		DT	*FB*	*DT*
		-100%	-73%	-83%
	0.94-0.97	DT	DT	LS
		AT	*FB*	*DT*
		-100%	-100%	-100%
ATM	0.97-1.00	AT	DT	LS
		DT	*FB*	*DT*
		-75%	-100%	-99%
	1.00-1.03	LS	VB	AT
		DT	*FB*	*DT*
		-79%	-100%	-97%
ITM	1.03-1.06	LS	BS	FB
		DT	*DT*	*DT*
		-62%	-67%	-100%
	≥ 1.06	LS	FB	AT
		VB	*DT*	*DT*
		-39%	-69%	-89%

Panel C: Low Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	FB	DT	AT
		DT	*FB*	*DT*
		-100%	-100%	-99%
	0.94-0.97	DT	DT	LS
		AT	*FB*	*DT*
		-100%	-100%	-100%
ATM	0.97-1.00	LS	DT	LS
		DT	*FB*	*DT*
		-85%	-100%	-99%

	1.00-1.03	LS	VB	LS
		DT	*FB*	*DT*
		-81%	-100%	-95%
ITM	1.03-1.06	LS	BS	FB
		DT	*DT*	*DT*
		-64%	-73%	-100%
	≥ 1.06	LS	BS	BS
		AT	*DT*	*DT*
		-24%	-75%	-77%

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5.4.3.2 Stochastic Dominance Test

The results from the all-strategies SD test are presented in Table 14. I find that there is no SD relationship, that is, all hedging strategies are equal, for the following: (1) deep ITM options at high hedging precision, (2) deep OTM options at moderate hedging precision and (3) deep ITM and OTM options at low precision. I observe that some dominating relationships exist at different level of hedging precision. At a high level of hedging precision, AT and (or) LS appear to be the preferred strategy. This relationship can be seen from the MV curves, such that LS and AT are sitting at the left-hand side of the MV curves of other hedging strategies. At moderate hedging precision, VB is preferred when options have medium to long maturity. At low hedging precision, all strategies are equal except DT and FB. The DT and FB results have to be interpreted with caution, as the MV curves for these strategies span very little of the MV space. This means that their results at moderate and low hedging precisions are extrapolated based on the fitted curve. Nonetheless, the observations do not mean DT and FB are statistically dominated by other strategies under the all-strategies test. Last but not least, it is not surprising that there is not much difference in hedging performance when the hedging precision level is low. This is because the relaxation in achieving accurate hedging outcomes leads to less frequent trades, which may avoid some unnecessary and costly intermediate trades.

I also perform a pairwise SD test; the results are not documented here, since the exercise produces no meaningful ranking. Although there is no obvious pairwise ranking for the six hedging strategies across moneyness-maturity categories, I consistently find that the move-based strategies DT and FB do not dominate the remaining strategies.

Table 14 All Strategies Stochastic Dominance Test for Delta-Vega-Neutral Hedge

In this table, the S&P 500 index and a call option with the same maturity as the target call option but a different strike price are used as the hedging instruments. Parameters and spot volatility implied by all call options of the previous day are used to establish the current day's hedges. The hedging portfolio is rebalanced according to the rule of the selected hedging strategy until six days prior to maturity. For each target call option, its standardised NHE is, as of liquidation day, the difference between the hedging portfolio value and its market price. The dominating hedging strategy tested under the all-strategies SD framework is presented in each panel at a different level of hedging precision. Under the null hypothesis of the SD tests, a particular hedging strategy dominates all other hedging strategies at the selected hedging precision level. If the null hypothesis is rejected, no dominance relation exists. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. FSD implies that we cannot reject the null hypothesis that the strategy dominates the strategies at all other hedging precision levels by first-order. Similarly, SSD means that we can reject the null hypothesis that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by second-order. Finally, if a strategy is TSD, we can reject the hypothesis that the strategy dominates all other strategies by FSD and SSD, but not that it dominates all other strategies in the set by third-order. No SD means that we reject the null of SD by any other. °,°°,°°° denote the significance level of the hypothesis test for SD identifications, i.e. 10%, 5%, 1% level respectively. If there is more than one strategy in the moneyness-maturity category, then these strategies are ~equal. The sample period is from January 2, 1996 to September 30, 2009. There are 249,111 observations distributed across eighteen moneyness-maturity categories.

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	No SD°°°	No SD°°°	No SD°°°
	0.94-0.97	BS~LS~AT (FSD)	LS~AT (FSD)	BS~LS~AT (FSD)

ATM	0.97-1.00	AT (FSD)	AT (FSD)	LS~AT (FSD)
	1.00-1.03	AT (FSD)	BS~LS~AT (FSD)	LS~AT (FSD)
ITM	1.03-1.06	LS~AT (FSD)	LS~AT (FSD)	LS~AT (FSD)
	≥ 1.06	LS~AT (FSD)	LS~AT (FSD)	LS~AT (FSD)

Panel B: Moderate Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	LS~AT (FSD)	LS~AT (FSD)	LS~AT (FSD)
	0.94-0.97	BS~LS~AT~VB (FSD)	AT~VB (FSD)	VB (FSD)
ATM	0.97-1.00	LS~AT~VB (FSD)	VB (FSD)	VB (FSD)
	1.00-1.03	BS~LS~AT~VB (FSD)	AT~VB (FSD)	VB (FSD)
ITM	1.03-1.06	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)	AT~VB (FSD)
	≥ 1.06	No SD°°°	No SD°°°	No SD°°°

Panel C: Low Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	No SD°°°	No SD°°°	No SD°°°
	0.94-0.97	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)
ATM	0.97-1.00	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)	AT~VB (FSD)

	1.00-1.03	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)	AT~VB (FSD)
ITM	1.03-1.06	BS~LS~AT~VB (FSD)	BS~LS~AT~VB (FSD)	BS~AT~VB (FSD)
	≥ 1.06	No SD°°°	No SD°°°	No SD°°°

Given that move-based strategies may have controlled for volatility risk indirectly through their hedging criteria, which avoid intermediate transaction costs paid due to transient market movements, I also use the samples of options that are liquidated early (as described in section 5.4.2) to compare the hedging performance of time-based strategies using a delta-vega-neutral hedge with the performance of move-based strategies using a delta-neutral hedge. Although this type of comparison is inconsistent with the hedging setup, such treatment may in some sense give the time-based strategies a fairer chance. Further, the poor performance of move-based strategies in the delta-vega-neutral hedge may be due to overly hedging the volatility risk. There is also a potential of introducing more volatility risk instead of controlling the hedging risk. To examine whether time-based strategies' hedging performance under delta-vega-neutral hedge is comparable to move-based strategies' hedging performance under delta-neutral hedge, I re-calculate the NHE of move-based strategies using a delta-vega-neutral hedge sample. It is worth mentioning that this delta-neutral hedge exercise is different from the one in section 5.2.1. For consistency with data limitations in the delta-vega-neutral hedge, I trace the hedging portfolio until the target option's liquidation day instead of the target option maturity date. The NHE is therefore the difference between hedging portfolio value and option price on the liquidation day. I consistently obtain the set of MV curves of the form in Figure 23 across all eighteen moneyness-maturity categories. We can see that move-based strategies are located in the northwest location, whereas time-based strategies are located in the inefficient region. The results show that the hedging performance of time-based strategies after controlling for volatility risk is worse than the performance of move-based hedging strategies implemented through a delta-neutral hedge at all levels of hedging precision.

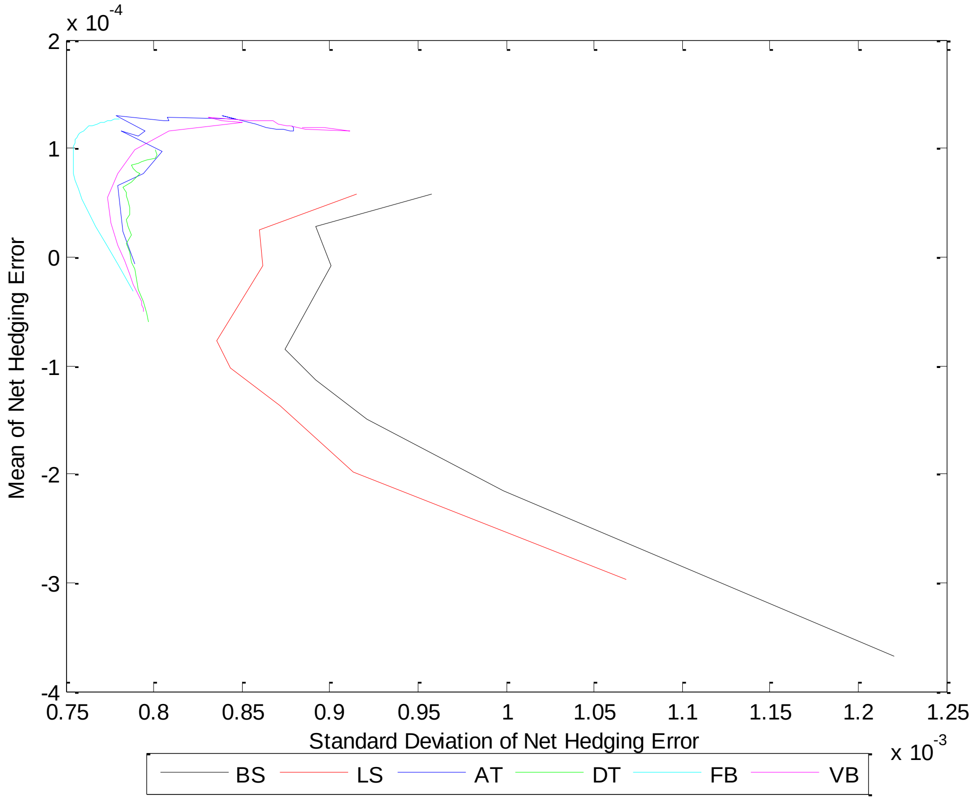

Figure 23 Mean Variance Curves of Time-Based Strategies under Delta-Vega-Neutral Hedge versus Move-based Strategies under Delta-Neutral Hedge

Based on the findings above, I investigate two more questions: (i) will the introduction of fixed transaction costs affect the results? (ii) does the hedging performance vary between crisis and tranquil markets?[74] To answer the first question, I introduce a fixed transaction costs of 40 cents[75] for each transaction in the underlying index for both delta-neutral and delta-vega-neutral hedges. The findings do not change, except there is an increase in total transaction costs paid. To answer the second question, I divide the samples into crisis and non-crisis periods according to the periods defined in section 5.1.3. For both delta-neutral and delta-vega-neutral hedge, I find that the ranking of hedging performance does not vary much between crisis periods (which

[74] I acknowledge an examiner's comment on the importance of how time series variation of illiquidity of the underlying stock can potentially affect hedging performance. A classical hedge ratio (which is used in my study) may no longer be valid when considering the feedback effect of illiquidity. Further the illiquidity problem is also closely linked to time varying transaction costs. This suggests an interesting area for future research is to examine how each hedging strategy accommodates the problem of high transaction costs during liquidity dries up period.

[75] The amount of 40 cents refers to the trading fees paid by the market maker in the CBOE and Chicago Mercantile Exchange.

include the tech bubble and GFC) and non-crisis periods. However, the mean and standard deviation of NHE are elevated during crisis periods. In addition, VB's standard deviation of NHE increases for the same range of hedging parameters, and therefore its MV curve spans further than the remaining hedging strategies. On the other hand, the MV curves formed during the LTCM crisis are different from those in the non-crisis period. The MV curves formed during the LTCM crisis period (albeit a short one) cluster together, and the ranking of hedging performance has no clear pattern.

5.5 Conclusion

The performances of six different hedging strategies are examined using S&P 500 index options obtained from Option Metrics. The sample period covers January 2, 1996 to September 30, 2009. The empirical tests are performed with reference to Bakshi et al. (1997), including investigating various option pricing models such as Black and Scholes' (1973) model as well as more complicated models that allow volatility, interest rates and jumps to be stochastic. However, my empirical testing methodology does not fully replicate Bakshi et al.'s approach. This is because transaction costs are introduced to my empirical test. In order to study the impact of transaction costs, the hedger (who hedges a short European call option or replicates a long European call option) is assumed to rebalance the hedging portfolio whenever required using a selected hedging strategy until the option matures. I assess the hedging performance by using two performance comparison frameworks: MV and SD tests.

I first study the delta-neutral hedging performance of alternative hedging strategies. The MV results show that time-based strategies are preferred to move-based strategies, contradicting the findings of my simulation study. In particular, the BS hedging strategy is the optimal hedging strategy. The average saving from switching from the worst to the best hedging strategy is 46% of the total transaction costs paid for worst-performing strategy. Based on SD test results, I am only able to draw the conclusion that move-based strategies of delta tolerance and hedging with fixed bandwidth are no worse than the time-based strategies.

Given the widely documented importance of controlling volatility risk in option hedging process, I test the performance of the delta-vega-neutral hedge using the same set of hedging strategies. The test is designed to control volatility risk in the hedging portfolio by adding a European call option (with same maturity as the target option but different strike price) position into the existing hedging portfolio, which consists of the underlying index and cash. It is assumed that the hedging portfolio is delta-vega-neutral at each rebalancing point. In another words, volatility risk is tilted back to a neutral position only when the delta hedge is triggered. Given the characteristics of my sample, the delta-vega-neutral hedging portfolio will be liquidated before the target option matures. Therefore, total transaction costs paid for hedging using this testing approach will be less than for the hedging activity that maintains delta-vega-neutral throughout the life of the option. My empirical evidence based on two hedging performance measures indicates that, after controlling for volatility risk, neither time-based nor move-based hedging strategies are consistently optimal. Indeed, the performance of the delta-vega-neutral hedge is worse than that of the delta-neutral hedge. Note that my simple implementation of move-based strategies introduced some volatility to the hedging outcomes. Hence, I also compare the hedging performance of time-based strategies implemented using a delta-vega-neutral hedge and move-based strategies implemented using a delta-neutral hedge.

Overall, my results support the contention that time-based strategies, in particular, the Black–Scholes hedge at fixed time interval (or the so-called ‘ad hoc Black–Scholes hedge’ – Bakshi et al., 1997) is the optimal hedging strategy when transaction costs are taken into consideration. A delta-neutral hedge is sufficient for a hedger to control total transaction costs paid while attaining the optimal hedging outcome for hedging a short European call option. My empirical evidence also shows that MV and SD tests conform, so I arrive at the conclusion that time-based hedging strategies are superior to move-based hedging strategies when the fat tails of NHE distributions are not large. However, the SD test does not enable me to draw a conclusion on the exact hedging performance ranking of the six hedging strategies.

Chapter 6

6. Conclusion and Future Research

Consider the situation where a trader has written a European call option. For some reason, the trader is unable to offset (or reduce) his position. Therefore, this sold European call option position represents his net exposure to the market. The trader faces the problem of how to mitigate his risk exposure, and becomes a hedger. In a Black and Scholes' (1973) economy, he is able to create a riskless hedge by continuously rebalancing a portfolio of underlying assets and risk-free bonds throughout the life of the option. However, if transaction costs are paid for trading the underlying asset, it is no longer viable to engage in the previous hedging strategy. This is because a continuously rebalancing hedging strategy will generate infinite transaction costs. On the other hand, he has been presented with choices to hedge his position in order to maintain a riskless hedge position and minimise transaction costs paid during the hedging process.

The literature proposes two major types of hedging strategy: time-based strategies and move-based strategies. A hedger who has chosen a time-based hedging strategy will rebalance the hedging portfolio regularly. On the other hand, a move-based hedging strategy requires the hedger to rebalance the hedging portfolio according to the movement of a pre-defined parameter such as change in stock price or change in option Greeks. In this thesis, I examine two time-based strategies, being Black and Scholes' (1973) rebalance at fixed time intervals and Leland's (1985) hedging strategy, and four move-based strategies, being Henrotte's (1993) asset tolerance strategy, Whalley and Wilmott's (1993) delta tolerance strategy, Hodges and Neuberger's (1989) hedging to a fixed bandwidth around delta strategy, and Whalley and Wilmott's (1997) hedging to a variable bandwidth around delta strategy. I sought to identify which of these six strategies allows the hedger to incur the lowest transaction costs and yet achieve the desired hedging precision.

The objective of my research is to fill two important gaps in the existing literature. First, despite an extensive list of articles describing theoretical models of the pricing and hedging of options in the presence of transaction costs, only four published papers compare the performance of competing hedging strategies in a Monte Carlo simulation setting. There is also a lack of empirical testing of the performances of alternative hedging strategies based on actual market data. Further, research has clearly demonstrated that the volatility of the return on an underlying asset changes over time, and therefore a hedging portfolio is subject to volatility risk. To this author’s knowledge, no published empirical study focuses on delta-vega-neutral hedge performance in the presence of transaction costs. Second, existing comparisons of hedging strategies are largely based on the MV rule. The MV rule is subject to a restrictive set of assumptions, namely that the hedger has a quadratic utility function and /or the returns on asset prices are normally distributed. Alternatively, SD rules maximise the investor's expected utility and utilise information about the entire probability function of the asset returns. As a result, my key original contributions to the existing literature are (1) to examine the performance of alternative hedging strategies using actual market data, and (2) to compare the results using both mean variance and stochastic dominance rules. The findings of my research allow a hedger to make better decisions when actual market conditions are taken into account, and to understand better how hedging results may differ when different performance assessment methods are used.

Unlike Bakshi et al.’s (1997) study and other studies of the performance of time-based hedging strategies, my hedging performance estimation method involves estimating the hedging error and cumulative transaction costs at maturity of the option. Bakshi et al. compute the hedging error as the difference between the market value of an option contract and the value of the hedging portfolio constructed at fixed time intervals. However, there is a major difference between time-based strategies and move-based strategies. For time-based strategies, a hedging error can be calculated on the day rebalancing takes place. On the other hand, move-based strategies require constant monitoring of movement of the underlying asset price so that the hedging portfolio is rebalanced whenever a change in the underlying asset return or delta breaches the predefined hedging bandwidth. Hence, the time interval to calculate a hedging error is random. To distinguish my work from the hedging literature without transaction costs, I term the difference between option payoff at maturity and hedging portfolio value after transaction costs as NHE instead of hedging error.

With reference to the existing literature, I use the MV rule as a starting point for consistent comparison of the performance of the six alternative hedging strategies. The MV rule determines that one strategy is superior to another if it produces a greater mean NHE given the same level of variance (or standard deviation) of NHE or vice versa. The simulation results show that move-based strategies outperform time-based strategies in a delta-neutral hedge. In particular, VB has the best performance under the MV framework. In contrast, my empirical study demonstrates that time-based strategies are better than move-based strategies. Based on my analysis of S&P 500 index option historical prices, a Black–Scholes hedge at a fixed time interval is the optimal hedging strategy. In my testing, I also use an SD rule to determine the superiority of hedging performance at first, second and third order. SD rules require fewer assumptions about NHE distribution, which gives the results greater explanatory power. To carry out an SD test, the distributions of NHEs are obtained. The SD test is based on (i) Barrett and Donald's (2003) Kolmogorov–Smirnov type test, which is able to test SD consistently by assuming each strategy is independent from each other, and (ii) Linton et al.'s (2005) test for whether a single strategy dominates the remaining five hedging strategies. I find that the MV results are consistent with the SD results in identifying the outperforming class of hedging strategies (e.g. time-based strategies outperform move-based strategies in a delta-neutral hedge) when the fat tails of the NHE distribution are not large. However, the SD test does not permit me to draw a conclusion about the hedging performance ranking of the six hedging strategies in my empirical test, due to the absence of a strong dominance relationship. Another important result obtained in my simulation is that, if a hedger is willing to switch to a better hedging strategy, he can save an average of 15% of total transaction costs paid for the inferior strategy. The average transaction costs saving decreases when the required hedging outcome is less precise. More interestingly, the estimated cost saving was as high as 46% in my empirical test, and the magnitude of saving is relatively insensitive to high, moderate and low hedging precision.

In empirical testing it is possible to obtain a positive average NHE for a hedging strategy. This may be because the market data are inclusive of different types of market frictions, such as transaction costs, in trading the underlying asset and option bid-ask spread. Both methods tend to prefer a hedging strategy with the largest positive NHE due to their assumption that the hedger prefers more to less. In contrast, the existing literature on empirical testing of the performance of alternative option hedging in the presence of transaction costs does not report positive NHE results; this is because those studies compare hedging performances in simulation settings. The frictionless Black–Scholes price is often used as the premium received when forming the hedging portfolio initially. Given that the Black–Scholes price does not consider transaction costs, average NHEs in simulations are always negative. Therefore, the resulting optimal hedging strategy is that with the least negative average NHE, which coincides with the strategy that closely replicates the short option position.

I supplement my findings by extending my examination beyond the delta-neutral hedge in empirical tests. Given that volatility risk in hedging has been widely documented in empirical studies, I study how hedging performance can vary when the hedger maintains a delta-vega-neutral hedge. To control for volatility risk throughout the hedging period, I include a European call option position (either long or short) to neutralise the vega of the hedging portfolio that consists of underlying asset and cash. It is important to note that my testing methodology assumes that the hedging portfolio will have zero delta and zero vega whenever the portfolio is rebalanced according to the existing set of hedging strategies. In another words, volatility risk is controlled when a delta hedge is triggered. As opposed to delta-neutral hedge results, I find neither time-based nor move-based strategies are consistently preferred. My ad hoc implementation of a delta-vega-neutral hedge may over-hedge volatility risk when using move-based strategies. Therefore, I also perform a simple hedging performance comparison between time-based strategies using a delta-vega-neutral hedge and move-based strategies using a delta-neutral hedge. I find that the Black–Scholes hedge at a fixed time interval is the optimal hedging strategy, and a delta-neutral hedge is sufficient in achieving optimal result for the hedger.

In contrast to the simulation study, in the empirical study the theoretically optimal hedging strategy, VB, appears to be the worst under the MV test. I show that switching from the VB hedging strategy to one of the time-based strategies proposed in my study allows a hedger to save an average of 37% of the transaction costs paid for the VB strategy at moderate and low hedging precision. This suggests that a hedger should try to avoid using a VB strategy in order to achieve an optimal tradeoff between hedging precision and transaction costs. In addition, my research shows that a delta-neutral hedge achieves a better hedging outcome than the ad hoc delta-vega-neutral hedge when time-based strategies are the preferred choices. My study provides useful information for the hedger about the magnitude of transaction costs that can be saved and the best options to choose when he considers switching hedging strategy. For a hedger who has restrictive access to other derivatives when forming a hedging portfolio, my research shows that the hedger is able to achieve a favourable hedging outcome without using another derivative hedge instrument as long as he is adopting a Black–Scholes hedge at a fixed time interval and forming a delta-neutral hedge for his short position. From a practicality perspective, if a hedger is unable to change an existing strategy due to contractual obligations, my study allows him to analyse how much money to set aside in order to cover the potential loss due to non-optimal choice of hedging strategy.

Although this research was prepared and conducted carefully, it has some limitations. First, I assume the hedger will hold the option position until maturity. NHEs obtained from a delta-neutral hedge are likely to be larger than the realised amount. This is because a market maker will seldom hold an option position until maturity. Instead, a hedger will close his position (by entering into an offsetting position) as soon as possible in order to eliminate or reduce his risk exposure. Therefore, the NHE reported in my study may serve as, on average, the most a market maker could have lost if he holds the position until maturity. Second, I assume that the hedger will only use one strategy to hedge the option. This assumption eliminates the possibility that he changes strategy when he discovers non-optimal hedge results. If option maturity is not long dated, it is likely that a single strategy will better serve the hedging objective. Adoption of a single hedging strategy can also be a true reflection of market practice, whereby hedging activities can be restricted by a firm’s policy or portfolio mandate. Third, transaction costs are a fixed proportion of the underlying asset price and the fixed amount does not change over time. In fact, transaction costs are time-varying, especially during a market downturn. In addition, the amount of transaction costs paid by a hedger depends on his market power. The assumption of 50 basis points would be too high for a hedger who transacts frequently and in large amounts. Finally, for the delta-vega-neutral hedge, my results are limited to the assumption that the hedger has an existing option to hedge against volatility risk. In addition, the vega of the hedging portfolio is only neutralised according to the hedging criteria derived from the selected strategies. It is noted that my selected strategies are mostly based on the formation of a delta-neutral position. Therefore, my results demonstrate how a simple adjustment can reduce the volatility risk of a hedge position rather than identify the optimal delta-vega-neutral hedge strategy.

Throughout the thesis, I identify various avenues of research that require further investigation.

First, time-varying transaction costs are observed in market data. In particular, the magnitude of transaction costs during tranquil and crisis periods can be very different. Market participants are less willing to transact during crisis periods, and therefore may request a premium in order to transact. Consequently, transaction costs are expected to increase during crises. A useful future study would involve examining whether the optimal hedging strategy varies with the change in transaction costs over time. For example, is it sufficient for a hedger to adopt one strategy when market conditions have changed significantly since hedging inception? Or, is the choice of the hedging strategy dependent on market conditions? It would also be valuable to understand if hedging portfolio performance changes significantly under different market conditions.

Second, the delta-vega-neutral hedge studied in this thesis is based on the same hedging criteria derived from the delta-neutral hedge. In contrast, Gondzio et al. (2003) propose a stochastic optimisation model for hedging options that takes account of stochastic volatility, transaction costs and trading restrictions. In their model, hedging errors are only minimised at the first few trading dates rather than every trading date up to option maturity. The formation of a delta-vega-neutral hedge model consists of underlying stock, cash and different options to hedge at different trading dates. Gondzio et al.'s simulation study finds that a delta-vega-neutral hedge model can improve hedging performance considerably compared to traditional hedging strategies. To overcome the limitations of my own study, an extension of my empirical study with respect to the optimal delta-vega-neutral hedge rebalancing rule, along the lines of the model introduced by Gondzio et al., will provide insights about the optimal hedging approach.

Third, the hedging parameters I use in my SD test were chosen arbitrarily to a certain extent. Future research on how to derive the hedging parameters for each hedging strategy at the same level of hedging precision, without relying on the standard deviation of NHE, will allow the results to be applied on a more general basis.

My research has filled important gaps in the literature through addressing unanswered research questions, finding that: (1) the superiority of time-based strategies to move-based strategies is strongly evident in historical S&P 500 index option data, and (2) MV and SD results conform, to arrive at the ultimate conclusion that time-based strategies have stronger hedging performance than move-based strategies. Unlike the MV test, the SD test is able to provide a precise performance ranking for hedging strategies only when a strong a dominance relationship is present. This research allows a hedger to understand that the choice of hedging strategy has a significant impact on the amount of total transaction costs paid for the hedging process, and that this ultimately affects the performance of a hedging position. It also strengthens confidence in using the MV rule as a consistent performance measure in assessing hedging outcome in the presence of transaction costs.

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Glossary Table

Term	Acronym	Description
Hedging error	HE	The difference between hedging portfolio value (exclusive of transaction costs) and option payoff at maturity.
Net hedging error	NHE	For a delta-neutral hedge, the difference between hedging portfolio value (inclusive of transaction costs) and option payoff at maturity. For a delta-vega-neutral hedge, the difference between hedging portfolio value (inclusive of transaction costs) and hedge instrumental option price on liquidation date.
Target option	N/A	The short European call option to be hedged or long European call option to be replicated.
Hedge instrumental option	N/A	The long European call option that has the same maturity as the target option but at a different strike price.
Mean variance curve	N/A	A set of means and standard deviations of NHE corresponding to each hedge parameter of a hedging strategy.
Empirical distribution function	EDF	The distribution function associated with the empirical measure of the sample. It is a step function that jumps up by 1/N at each of the N data points. The function will converge with probability one.
Probability density function	PDF	A function that describes the relative likelihood for the NHE to take on a given value.
Mean variance test	MV	Test rule based on the mean and variance (or standard deviation = square root of variance). A strategy has the best performance if it has the highest mean NHE (greatest positive value or lowest negative value) for the same level of standard deviation of NHE or vice versa.
Stochastic dominance test	SD	Test rule based on the whole distribution of NHE. Comparison is formed by constructing an EDF and testing at different order using the EDF and its derivatives.

Appendix A Stochastic Dominance Test Procedure

Let N and M be the sample size of strategy A and B. The empirical distribution functions used to construct the statistical tests are respectively,

$$\widehat{D}_A^{(1)}(x) = \frac{1}{N}\sum_{i=1}^{N} 1(X_A \le z) \text{ and } \widehat{D}_B^{(1)}(z) = \frac{1}{M}\sum_{i=1}^{M} 1(X_B \le z). \quad \text{(A1)}$$

For higher order,

$$\widehat{D}_A^{(s)}(x) = \int_{-\infty}^{x} \widehat{D}_A^{(s-1)}(t)dt \text{ for s = 2, 3} \quad \text{(A2)}$$

$$\widehat{D}_B^{(s)}(x) = \int_{-\infty}^{x} \widehat{D}_B^{(s-1)}(t)dt \text{ for s = 2, 3.} \quad \text{(A3)}$$

<u>Pairwise stochastic dominance test</u>

The hypothesis for testing the dominance relationship is

H_0: *Strategy A stochastically dominantes strategy B at order s,*

H_1: *Strategy A does not stochastically dominantes strategy B at order s,*

where s = 1,2 and 3.

The test statistics for first and s^th order of SD can be written in the following form:

$$T_{NM}^{(1)} = (\frac{\text{NM}}{\text{N+M}})^{1/2} sup_{z\in X}\left[\widehat{D}_A^{(1)}(x) - \widehat{D}_B^{(1)}(x)\right] \quad \text{(A4)}$$

and

$$T_{NM}^{(s)} = (\frac{\text{NM}}{\text{N+M}})^{1/2} sup_{z\in X}\left[\widehat{D}_A^{(s)}(x) - \widehat{D}_B^{(s)}(x)\right] \text{ for s = 2, 3.} \quad \text{(A5)}$$

<u>All strategies stochastic dominance test</u>

The hypothesis for testing the dominance relationship for a particular hedging strategy is

H_0: *Strategy k stochastically dominates all other strategies at order s,*

H_1: *Strategy k does not stochastically dominate all other strategies at order s,*

where s = 1,2 and 3.

Let N denotes the full sample size of normalised NHE observations of strategy X_{ki} for k = 1,..., K and i = 1,..,N. The test statistic $T_N^{(s)}(k)$ for the full sample is computed below:

$$T_N^{(s)}(k) = \max_{l:k\neq l} sup_{x\in\chi}\sqrt{N}\left[\widehat{D}_k^{(s)}(x) - \widehat{D}_l^{(s)}(x)\right] \text{ for s = 1,2 and 3,} \quad \text{(A6)}$$

where $\widehat{D}_k^{(1)}(x) = \widehat{F}_{kN}(x) = \frac{1}{N}\sum_{i=1}^{N} 1(X_{ki}\leq x)$ and $\widehat{D}_k^{(s)}(x) = \int_{-\infty}^{x}\widehat{D}_k^{(s-1)}(t)dt$ for s =2,3. Let χ denote the union of supports of all cumulative distributions of the hedging strategy, $\widehat{F}_{jN}$ for j = 1,…,K.

The three-step procedure below is used to obtain the test statistics for both pairwise and all-strategies SD tests.

Step1:

Compute the test statistics $T_{NM}^{(s)}$ and $T_N^{(s)}(k)$ for $s = 1,2,3$ using equations defined in (A1)- (A6).

Step 2:

Use a bootstrapping method to recompute the test statistics as detailed in the following.

I define N as the full sample size of NHE observations for each hedging strategy and $\omega = \{X_{1,1}, X_{1,2}, \dots, X_{K,N}\}$ as the sample pool of observations where $X_{k,i}$ is i^{th} observation of NHE of hedging strategy k where k = 1, 2, ..., K and i = 1, 2, ..., N. In my simulation study, N is 200,000 and K is 6. For each bootstrapping iteration, a random subsample $\omega^*$of b observations of NHE of each hedging strategy is selected from ω, for example, $\omega^* = \{W_i, \dots, W_{i+b-1}\}$ where $W_i = \{X_{ki}: k = 1, \dots K\}$ and i=N-b+1. Kläver (2005) recommends a circular blocking method to recompute the test statistics so that observations at the beginning and the end of the distribution are equally considered, as well as the observations in the middle of the full distribution. Hence, the test statistics in (A1)-(A6) are computed for N-b+1 different subsamples $\{W_i, \dots, W_{i+b-1}\}$ where $i = N - b + 1$ and additionally for the subsamples $\{W_i, \dots, W_N, W_1, W_{i+b-N-1}\}$ where i=N-b+2,...,N. The idea is that each subsample ω^* represents a sample of the true sampling distribution. For each ω^*, I calculate the test statistics $t_{NM}^{(s)}$ and $t_N^{(s)}(k)$. Note that the test statistics are multiplied by the square root of the subsample size b. The resulting distribution of the subsampling statistics will approximate the sampling distribution of the full sample statistics obtained from Step 1. Following Kläver (2005), the subsample size in my computation is $b(N) = 10\sqrt{N}$.

Step 3:

Compute the rejection rate of the null hypothesis

H_0^s: $Strategy\ G\ dominates\ strategy\ F$ for pairwise SD test

and

H_0^s: $Strategy\ k\ stochastically\ dominances\ all\ other\ strategies\ at\ order\ s$ for all strategies SD test. The rejection rate is denoted as $\hat{P}_{NM}^{(s)} = \frac{1}{b}\sum_{i=1}^{b} 1(t_{NM}^{(s)} > T_{NM}^{(s)}$ and $\hat{P}_N^{(s)}(k) = \frac{1}{b}\sum_{i=1}^{b} 1(t_N^{(s)}(k) > T_N^{(s)}(k)$ for s = 1 ,2 and 3. 1(.) is an indicator function. The rejection rate can be interpreted as the number of occurrences when sub-sampling statistics $t_{NM}^{(s)}$ and $t_N^{(s)}(k)$ are greater than the full-sample test statistics $T_{NM}^{(s)}$ and $T_N^{(s)}(k)$.

Appendix B Delta-Neutral Hedge Additional Results

<u>Mean variance results</u>

Table 1A shows the pairwise ranking of the six hedging strategies at each level of hedging precision based on the MV test.

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS>FB>VB>AT>DT>LS	LS>FB>BS>VB>AT>DT	FB>BS>AT>DT>VB>LS
	0.94-0.97	FB>BS>LS>VB>DT>AT	LS>FB>BS>VB>AT>DT	FB>DT>LS>AT>BS>VB
ATM	0.97-1.00	LS>FB>BS>VB>AT>DT	LS>FB>BS>VB>AT>DT	LS>FB>DT>AT>VB>BS
	1.00-1.03	LS>FB>BS>VB>AT>DT	LS>FB>BS>DT>VB>AT	LS>FB>DT>AT>VB>BS
ITM	1.03-1.06	LS>FB>BS>VB>AT>DT	LS>DT>FB>BS>VB>AT	LS>FB>DT>AT>VB>BS
	≥ 1.06	FB>LS>AT>VB>BS>DT	AT>LS>FB>VB>BS>DT	LS>AT>DT>FB>VB>BS

Panel B: Moderate Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS>FB>LS>AT>DT>VB	BS>LS>FB>DT>AT>VB	BS>FB>AT>DT>VB>LS
	0.94-0.97	BS>FB>LS>DT>AT>VB	BS>LS>FB>DT>AT>VB	BS>FB>LS>DT>AT>VB
ATM	0.97-1.00	BS>FB>LS>DT>AT>VB	BS>LS>FB>DT>AT>VB	BS>DT>FB>LS>AT>VB
	1.00-1.03	BS>FB>LS>DT>AT>VB	BS>LS>FB>AT>DT>VB	BS>DT>FB>LS>AT>VB
ITM	1.03-1.06	BS>LS>FB>DT>AT>VB	BS>LS>FB>AT>DT>VB	BS>DT>LS>FB>AT>VB
	≥ 1.06	FB>BS>LS>AT>VB>DT	BS>AT>LS>FB>VB>DT	BS>LS>AT>FB>VB>DT

Panel C: Low Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	BS>LS>FB>DT>AT>VB	BS>LS>FB>DT>AT>VB	BS>LS>FB>AT>DT>VB
	0.94-0.97	LS>BS>FB>AT>VB>DT	BS>LS>AT>FB>DT>VB	LS>BS>FB>AT>DT>VB
ATM	0.97-1.00	LS>VB>BS>FB>AT>DT	BS>LS>AT>FB>DT>VB	LS>BS>FB>AT>DT>VB
	1.00-1.03	LS>BS>AT>FB>VB>DT	DT>LS>BS>AT>FB>VB	LS>BS>FB>AT>DT>VB
ITM	1.03-1.06	LS>BS>AT>FB>VB>DT	DT>AT>BS>LS>FB>VB	BS>LS>FB>DT>AT>VB
	≥ 1.06	FB>LS>BS>AT>VB>DT	BS>LS>FB>VB>AT>DT	LS>BS>AT>DT>FB>VB

Appendix C Delta-Vega Neutral Hedge Additional Results

<u>Mean variance results</u>

Table 1B shows the pairwise ranking of the six hedging strategies at each level of hedging precision based on the MV test.

Panel A: High Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	AT>BS>VB>LS>FB>DT	DT>LS>AT>BS>FB>VB	AT>LS>VB>FB>BS>DT
	0.94-0.97	DT>FB>AT>LS>VB>BS	DT>LS>AT>BS>VB>FB	LS>AT>BS>FB>VB>DT
ATM	0.97-1.00	LS>AT>BS>FB>VB>DT	DT>VB>AT>LS>BS>FB	LS>AT>BS>VB>FB>DT
	1.00-1.03	LS>AT>BS>VB>FB>DT	VB>LS>AT>BS>DT>FB	LS>AT>BS>VB>FB>DT
ITM	1.03-1.06	LS>AT>DT>FB>BS>VB	AT>LS>FB>BS>VB>DT	FB>LS>AT>BS>VB>DT
	≥ 1.06	LS>FB>AT>DT>BS>VB	FB>LS>AT>BS>VB>DT	LS>AT>BS>VB>FB>DT

Panel B: Moderate Hedging Precision

	Moneyness S/X	Term-to-Expiration: Short	Medium	Long
OTM	< 0.94	FB>VB>BS>LS>AT>DT	LS>BS>DT>AT>VB>FB	AT>LS>BS>FB>VB>DT
	0.94-0.97	DT>FB>VB>BS>LS>AT	DT>AT>LS>BS>VB>FB	LS>BS>AT>VB>FB>DT
ATM	0.97-1.00	AT>LS>BS>VB>FB>DT	DT>VB>AT>LS>BS>FB	LS>AT>BS>VB>FB>DT
	1.00-1.03	LS>AT>BS>VB>FB>DT	VB>LS>BS>AT>DT>FB	AT>LS>BS>VB>FB>DT
ITM	1.03-1.06	LS>AT>BS>FB>VB>DT	BS>AT>LS>FB>VB>DT	FB>AT>LS>BS>VB>DT
	≥ 1.06	LS>FB>BS>DT>AT>VB	FB>BS>LS>AT>VB>DT	AT>LS>BS>VB>FB>DT

Panel C: Low Hedging Precision

	Moneyness	Term-to-Expiration		
	S/X	Short	Medium	Long
OTM	< 0.94	FB>VB>BS>LS>AT>DT	DT>LS>BS>AT>VB>FB	AT>FB>LS>BS>VB>DT
	0.94-0.97	DT>FB>VB>LS>BS>AT	DT>AT>LS>BS>VB>FB	LS>BS>AT>FB>VB>DT
ATM	0.97-1.00	LS>BS>AT>VB>FB>DT	DT>VB>AT>LS>BS>FB	LS>AT>BS>VB>FB>DT
	1.00-1.03	LS>AT>BS>VB>FB>DT	VB>LS>BS>AT>DT>FB	LS>BS>AT>VB>FB>DT
ITM	1.03-1.06	LS>BS>AT>FB>VB>DT	BS>AT>LS>FB>VB>DT	FB>LS>BS>AT>VB>DT
	≥ 1.06	LS>FB>BS>DT>VB>AT	BS>LS>AT>FB>VB>DT	BS>LS>AT>VB>FB>DT