THE FLOOD HYDROLOGY OF URBAN CATCHMENTS IN GREATER LONDON

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ABSTRACT

William Stephen Eyre: The flood hydrology of urban catchments in Greater London.

The thesis investigated four south London catchments which drain northwards to the River Thames over superficial deposits which overlie clay in the North and chalk to the South. The catchments are densely urbanised ranging from 32.2 to 80.8 percent and range in area from 43.5 to 176.0 Km^2. After evaluating several deterministic sewer and flood routing models it was decided to analyse 96 storm events by the unit hydrograph method. Testing of alternative identification techniques using objective error functions indicated that matrix inversion of response runoff and effective rainfall calculated by the loss rate curve was the most consistently accurate. Analysis of the unit hydrographs indicated that those with high peak discharges and short times to peak were caused by short duration, high intensity storms on a dry catchment, whereas unit hydrographs with a small peak discharge and a long time to peak were caused by long duration, low intensity storms on a wet catchment. The unit hydrographs from the four catchments showed no significant change through time. The mean unit hydrographs of the four catchments were not related to the degree of urbanisation, but to the physical characteristics of the catchments. The unit hydrographs were analysed and split tested using four different models. A quasi-linear, straight line approximation of the unit hydrograph proved to be the most consistently accurate. A subsidiary analysis compared the performance of seventeen linear conceptual models but was not followed up. A sensitivity analysis was conducted on the thesis's findings and quantified the significant effect of rainfall separation and profile on peak discharge and spill volume. The effect of substituting a straight line approximation of the unit hydrograph was examined and found to have a minor effect on peak discharge estimates but a more significant effect on spill volume.

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Chapter 1

FLOODING IN URBAN CATCHMENTS: AN INTRODUCTION

The study of the flood hydrology of urban catchments has chiefly concentrated on measuring the effect of a given level of urbanisation on a dimensionless index of mean annual discharge. This permits the identification of different rates of increase of flood magnitude, with return period, for varying degrees of urbanisation. Attempts to synthesise the numerous studies of urban flood hydrology have been frustrated by the variable response of urban catchments. The current state of knowledge does not permit an accurate prediction of the effect of urbanisation on a catchment's response.

The hydrological response of an urban catchment, to a given storm event, is a combination of the effect of the topography, the urban area, the causative storm and the antecedent catchment wetness. Although several studies have defined relationships between the mean catchment response and the topography and the level of urbanisation, only a small number have investigated the range of hydrological response which may be expected from a single catchment.

This thesis, using data from four London catchments, has two objectives. First, to identify the causes of variation of catchment response for a given catchment and second, to identify the causes of variation of hydrological response between the four catchments.

This chapter defines the scope of the thesis and the relationship between the thesis and current research. The chapter has five sections. The first defines the role of the hydrologist and the hydraulic engineer in solving the problem of flooding in urban areas. Examples of the damage caused by flooding are presented together with a description of the constraints placed on the hydraulic engineer. The second section defines the role of flood design and flood forecasting in a flood alleviation scheme. The existing London flood warning system is described. The third and fourth sections present a review of the current research in urban hydrology and describe the search for an analytical technique to investigate urban flooding. The fifth section briefly describes the content of the eight chapters of the thesis.

1.1 Flooding in urban areas: The role of the hydrologist and the hydraulic engineer

Urbanisation has provided the hydraulic engineer with numerous problems. Urban development necessitates the provision of a water supply and a system to remove effluent. The change in the character of the land-use requires the provision of improved land drainage and protection from flood waves which are increased in both height and volume by urbanisation.

Flooding in urban areas is associated with three geographical factors, first location; second, the nature, and third, the expansion of urban areas. Most urban areas are sited on relatively flat land which facilitates easy expansion. For historical reasons of transport, communication and water supply many towns were located beside rivers. Urban areas create flooding by their very existence. Urbanisation increases the percentage runoff and increases the rapidity of response. Finally, expansion of urban areas has led to encroachment onto the natural flood plains, reducing channel storage and increasing the flood risk.

Urban areas have three types of drainage system. First, river channels which provide the main arterial conveyance of surface water and land drainage. Second, surface water sewers discharging into the main arterial river and third, minor watercourses, ditches and land drains which discharge into the main arterial river, either directly or via the sewer system. Flooding may be classified according to the origin of the flood water. Flooding from the arterial river is called primary flooding and that from the other two sources, secondary flooding. Examples of both types of flooding will be discussed to illustrate the problem and the damage which can result from a flood.

Severe primary flooding in the south London catchments of the Wandle and Beverley occurred as a result of a storm which lasted from 14th to 16th September 1968. The storm occurred after an exceptionally wet summer and was preceded by approximately 6mm of rain on 14th September, which ensured a high percentage runoff from normally non-contributory pervious areas. The storm started at approximately midnight on 14th September. "Hourly values of rainfall for the individual stations indicate that, after the first two hours, the maximum intensities fell from 11.4mm/hr to 4.3mm/hr, and then increased after 0900hrs to give a peak intensity of 19.6mm/hr at Worcester Park (TQ 229668) between 1200 and 1300hrs. Maximum intensities declined after 1500hrs but continuous heavy precipitation was recorded until 1800hrs" (Haggett, 1968). The maximum point rainfall for the 15th was 129.5mm at Bromley (TQ 399693). The mean 48 hour rainfall for the three southern catchments, the Ravensbourne, Wandle and Beverley was 149.1mm, 117.0mm and 91.6mm respectively.

Flooding was confined, on both the Wandle and Beverley, to the upper reaches, where considerable flooding reduced the peak discharge downstream to just below the design discharge. The discharge in the upper reaches was at least 30 percent higher than the design discharge, for example the flow recorder upstream of the Wandle Valley sewage works recorded a discharge approximately 30 percent greater than the design discharge of 20.81 cumecs, before being drowned and put out of action. In the Beverley catchment no flooding occurred downstream of the point where the Kingston by-pass crosses the river. There was no threat of flooding further downstream. The peak discharge at the Wimbledon Common gauge was 16.51 cumecs in a channel designed to hold 20.27 cumecs. Most of the flooding on both catchments was caused by overtopping of the river walls. The maximum depth of overbank flooding in the Wandle catchment was 1.2m and in the Beverley 0.6m. A 0.65Km^2 area west of Tooting in the Wandle catchment was the most severely affected. Sixteen roads were flooded together with several public properties, including the CEGB works, Wimbledon Stadium, Wandle Valley sewage works and Lambeth cemetery. Most of this flooding may be attributed to overtopping of a river wall at the CEGB works where water flowed from the river for five hours.

The flood caused £6.75m (1975 prices) of direct damage on the Wandle and was chiefly responsible for the decision to increase the degree of protection against floods on all London rivers from once in 30 years to once in 50 years.

Approximately 100 houses were flooded by the Kyd Brook, in the Ravensbourne catchment, as a result of a storm on 27th August 1977. The storm lasted for seven hours and produced a maximum fall of 69.8mm at Orpington (TQ 459652). The GLC's response to this was the allocation of a further £8,000 to improve and modify a flood warning system which was then in progress and which was initially costed at £50,000.

On the 23rd of June 1966 a maximum fall of 32.0mm was recorded at Addington Mills, Croydon (TQ 352643). The storm covered all the London catchments but only seven houses were flooded. Four of these were flooded as a result of a blocked culvert, this was solved by the subsequent installation of a grille upstream of the culvert. The remaining three properties were protected by improving the stream by removing trees and other obstructions and the construction of a levee along the footpath to increase the capacity of the channel.

The total damage caused by overbank flooding is far greater than the damage caused by secondary flooding. Further, the solutions to primary flooding are more expensive and involve many man-months in the design office and expensive contract work. In contrast secondary flooding may be called localised failure. The surface water sewers are able to deal with a flood event, except under exceptional circumstances, such as the 1975 Hampstead storm. The following example illustrates this.

On 31st of July 1978 a heavy fall from a 10 hour duration storm was recorded throughout London. The maximum fall of 55.0mm was recorded at Waddon, Croydon (TQ 314639). Five properties were flooded due to overland flow generated by the local surface water sewers being unable to accept and transmit the volume of rainwater.

Although this account has concentrated on describing the damage to property it should be pointed out that most flood events also inundate open spaces such as parks, golf links and car parks. These suffer minimal damage and attenuate the flood hydrograph, minimising the flood damage downstream. This inundation is part of most flood protection schemes and the areas inundated are called washlands. Basins built to temporarily hold flood water either on the river or the flood plain are called flood retention (or balancing) ponds.

To design against these two types of flooding requires close co-operation between the hydrologist and the hydraulic engineer. The hydrologist must predict the amount of water to be expected at a given point, for given conditions, for a given frequency and the hydraulic engineer uses this information to design the necessary structures. The engineer is not given free rein but is constrained by many factors, all of which, to some extent, affect the final engineering design. Several of these factors are discussed below.

Early engineering paid little heed to the impact it had on the environment. This was principally due to the absence of information and a poor understanding of natural processes. For example in the Act of Parliament which permitted the construction of two direct supply reservoirs on the upper Derwent, Derbyshire, it was stipulated that the minimum quantity of compensation water to be released by the reservoirs was to be one million gallons per day (4,550m^3/day). This is equivalent to a mean daily discharge of 0.053 cumecs, such a small discharge would cause severe modifications to the river channel as well as altering the floral and faunal communities. The current minimum compensation water is 1.075 cumecs, and reflects both an increase in awareness of the sensitivity of the environment to change and second, an increase in information about the hydrological system.

This environmental awareness has modified the role of the hydraulic engineer such that "... the traditional concerns of the hydraulic engineer, the structures and conveyances to supply water and control floods and storms, are not neglected, but the thrust of new research is toward the more sophisticated and challenging aspects of modern day problems. It no longer suffices to supply a stated amount of water to a given point in the cheapest possible manner. Today's hydraulic engineer may be part chemist, part biologist, part social scientist and planner to wrestle with problems of water quality, economics, ecology and aesthetics" (University of California, 1978, 1). For example, the solution to the problems of river flood control are not simple. Channel clearing can result in an increase in discharge velocity which will erode the banks. Widening the channel can reduce the velocity, causing siltation. Channel deepening will increase the discharge velocity. In addition to these hydraulic problems, pressures are now placed on the engineer to produce a design whose appearance is, where possible, pleasant. For example, the River Ravensbourne, south London, frequently caused flooding in Lewisham's town centre, the ideal solution would have been to widen the channel but this was impossible because of the extensive urban development and it would have been an eyesore. Instead a composite channel was built, which retained trees and grass within the channel. The scheme won a Civic Trust Commendation for the contribution it made to the local scene. In addition to aesthetics, Butters and Lane (1975,70) list six important influences, excepting engineering, which condition the hydraulic engineer's approach to the design of major flood alleviation works,

"(a) gradual erosion of the status of land drainage works as permitted development
(b) current trend towards public participation in the preparation of public works
(c) Ministry of Agriculture, Fisheries and Food's condition attaching to grants for land drainage
(d) numerous riparian owners affected by schemes in highly urbanised areas
(e) pressure on land use from many quarters
(f) conflicting demands for finance."

Finally, Acts of Parliament are an additional control on the engineer. For example, the Control of Pollution Act, 1974, Part II, December 12th and 13th 1978 which was implemented in 1979. This Act requires Water Authorities to disclose publicly, information about their discharges and the impact these have on the receiving streams. It allows Water Authorities to be prosecuted by members of the public or other bodies, in the event of their failure to comply with regulations.

1.2 Two solutions to the flood threat: Design and flood forecasting

To solve the problem of flooding it is necessary to understand the causes of flooding and to use this knowledge to predict flood hydrographs for given return periods, for given locations. Finally, these values are used by the hydraulic engineer to design the relevant structures within the constraints outlined above. Closer inspection of the problem indicates that there are two stages. The first is design, where a river channel is designed using the available hydrometric information. The second stage is concerned with flood forecasting and warning.

To predict the discharge for a given return period the GLC non-tidal rivers section uses seven different methods. These are based on the assumption that the catchment and the river channel exert no influence on the value of the peak discharge ordinate. Application of these seven methods to predict the 50 year return period discharge for the Beverley Brook at Wimbledon Common yielded a maximum estimate of 58.33 cumecs and a minimum of 16.42 cumecs (GLC, 1976). All these methods use arbitrary and subjective techniques to take account of increased urbanisation and the removal of flow retarding structures. Consequently, no single method may consistently be relied upon to produce the 'correct' estimate. The chosen method produced, in this instance, an estimate of 53.80 cumecs and reflects the propensity of the engineering profession to overdesign structures. One reason for overdesign is the lack of confidence which may be attached to the estimates of peak discharges for a given return period and consequently although London's rivers are nominally protected against the 50 year flood, overdesign has meant that the actual return period is much higher. It is argued that it is best to accept a higher estimate than a lower one and the risk of flood damage which it incurs. Overdesign is not necessarily expensive, "The cost of channels in urban areas is fairly insensitive to their capacity up to the point where they interfere with adjacent structures, sewers or large services. Analysis of a recent contract suggests that to increase the width of the channel from 2.75m to 3.25m would have increased the cost by about 4 percent and the capacity by 20 percent" (Osorio, 1978, 470). On the other hand, 4 percent of capital costs of at least £1m is not an insignificant amount.

Although such flood defences may prevent flooding, it is not economically viable to build defences which would contain a flood of any magnitude. Therefore a flood forecasting and warning system is desirable to forecast the peak and arrival time of a flood wave and to issue the relevant warnings. This constitutes the second stage of flood alleviation. Damage caused by a flood can be reduced if the warning can be issued sufficiently far in advance.

A completely different flood forecasting system is needed for the London catchments than for the large Water Authority catchments. On the latter flood warnings may be given days in advance of a flood. Flood warning is based on telemetry and observers who contact the regional office to report rainfall and river levels as they occur. An example of this is the Severn-Trent Water Authority's flood forecasting model for the River Trent (Lockyer, 1978).

Flood warning for the Greater London area depends on the London Weather Centre which contacts the GLC non-tidal rivers section in the event of a forecast of heavy rainfall. Small teams of workmen are then dispatched to known danger spots where they attempt to minimise flooding by the removal of debris from the rivers and keeping culverts and grilles clear. When the storm is in progress the intensity and total rainfall may be assessed by interrogation of the 19 telemetred rain-gauges and the discharge from the 12 telemetred flow gauges. In the event of severe flooding, information is exchanged between the GLC, the police, local boroughs and other emergency services.

This system does not fulfil a warning role since the information arrives as the flood is occurring and it is difficult for teams of workmen to reach the source of inundation because of the flooding. The solution to the problem is a detailed study of the response characteristics of the catchments to develop an on-line computer model, which can use telemetred information for input and for calibration. The spatial occurrence of flooding, together with the amount and timing, may then be predicted and teams may then be dispatched to take preventative action.

1.3 The state of urban hydrology in the late 1970s

The following section describes the significant developments in urban hydrology in the 1970s and points to those problems which require further investigation.

The hydrologist in the late 1970s was riding the wave of developments which started some fifteen years before. For example, in 1959 a general theory of the unit hydrograph was proposed (Dooge, 1959) and this was followed in 1968 by the exposition of a theory of modelling hydrological systems by uniform non-linearity (Dooge, 1968). These two papers laid the foundations for the use of conceptual models in hydrology. In 1962 the first results of continuous digital simulation were presented (Crawford and Linsley, 1962). Two objective solutions for the derivation of optimum unit hydrographs were proposed (Snyder, 1955; O'Donnell, 1960) and these marked the beginning of research into methods of deriving unit hydrographs. Such methods were superior to the subjective manual techniques since they permitted the analysis of long duration storms which hitherto were excluded from unit hydrograph analyses. Finally, the researcher was provided with easy access to fast, large core digital computers which meant that large quantities of data could be analysed quickly and accurately.

The development of scientific hydrology in the UK was slow before 1960. In 1962 the Department of Scientific and Industrial Research set up the Hydrological Research Unit which sought to investigate the effect of land use on the water balance within catchments. Previously hydrological research was conducted by universities on grants from the same Department. In 1968 the Institute of Hydrology was formed and was given the larger brief of developing the science of hydrology and to undertake research in the UK and overseas. The Institute represents the most important single body conducting hydrological research in the UK. Its large staff has developed methods and hardware which form the basis of current hydrological research in the UK.

The problems of urban hydrology in the UK have been investigated by five major projects, in four of these the Institute of Hydrology has taken or is taking a major part.

The first project resulted from the Institution of Civil Engineers Committee on Floods 1967 recommendation that a new investigation should examine all aspects of flood hydrology. This project was published as the five volume Flood Studies Report (NERC, 1975). It was the joint result of work undertaken principally by the Institute of Hydrology, the Hydraulics Research Station and the Meteorological Office. Meteorological records were studied to understand the causes of floods, to extend flood records, to assist in flood frequency analyses and to provide estimates of probable maximum precipitation. From a vast collection of discharge records a frequency analysis of flood peaks and volumes was carried out; regional analyses and correlations with catchment characteristics were used to improve single station frequency distributions and to estimate flood frequencies at ungauged sites. Unit hydrographs, soil infiltration characteristics and snowmelt were studied to derive rainfall-runoff models for use with the results of the meteorological studies. The Hydraulics Research Station reviewed, tested and improved flood routing techniques.

The Flood Studies Report (NERC, 1975) is one of the most important pieces of hydrological research conducted in the UK. Its thorough investigation of all the aspects of flood hydrology has significantly increased the quality of flood predictions. This thesis and other work (e.g. Wheater et al, 1978) may be said to be post-Flood Studies Report, because they draw from the findings of the Flood Studies Report (NERC, 1975). The methods the Report puts forward for urban catchments are unreliable but subsequent and continuing research has gone some way to rectifying this deficiency.

The second major project was the 1974 research colloquium held at Bristol under the auspices of the Construction Industry Research and Information Association (CIRIA). The findings of the colloquium were summarised by the following statement (CIRIA, 1974, 4).

> "The particular uncertainties that engineering designers of surface channels and sewer systems have to deal with at present for large (i.e. greater than 25Km^2) catchments which are entirely urban or being urbanised are believed to be:
>
> (1) response of the catchment to 'dynamic' rainfall (i.e. moving rainstorms that produce non-uniform precipitation in time and space)
>
> (2) effects of storage within the urban area
>
> (3) effects of growing development within catchments, particularly on downstream drainage
>
> (4) effects of variation of impermeability of natural surfaces.
>
> The underlying motives, of course, for all drainage system design is to reduce the incidence and cost of flooding".

These problems were not solved by the Flood Studies Report (NERC, 1975) because the Report was not concerned with small scale phenomena. In March 1974 the Working Party on the Hydraulic Design of Storm Sewers was established by the Department of the Environment. The terms of reference of the Working Party are, "To examine all aspects of the hydraulic design of systems for the conveyance of storm water from developed areas; to assess and co-ordinate research projects in progress; to promote any necessary new research both in the laboratory and in the field; and to publish guidance and produce a manual of good practice for the design of such systems" (National Water Council, 1977, 1). It was under the auspices of the Working Party that the effect of moving storms was studied (Shearman, 1977) and it was found that most storms move with greater speeds than those of general interest to the urban drainage engineer. The second and fourth points raised at the CIRIA colloquium are receiving attention from the Institute of Hydrology, through the use of data collected from instrumented urban catchments. The effect of urbanisation on the downstream flow regime may be investigated through the use of distributed models and flood-routing techniques (e.g. Packman, 1978).

The final project is nearing completion and is concerned with the design and management of river systems to cater for runoff from catchments with large-scale urbanisation. This recognises that the development of reliable methods will take a considerable amount of further research and hopes to provide an intermediate solution to enable engineers to calculate runoff from urban areas (CIRIA, 1978, 25).

The five research projects outlined above have been directed towards the development of a design method for ungauged basins. Workers in the UK have tended to ignore the variation of response for a single catchment which is essential for a reliable flood forecasting method. This thesis seeks to rectify this deficiency.

1.4 The search for an analytical methodology

In 1976 the Natural Environment Research Council awarded a three year research grant to the author to conduct an investigation of the response characteristics of the London catchments. A strong link was established between the GLC and University College London which contributed significantly to the success of the project.

The initial stages of the project consisted of identifying the most suitable analytical method to characterise the large, densely urbanised and heterogeneous London catchments. The complex nature of the catchments suggested the use of hydraulic continuous simulation models. These appeared to offer the advantage of defining the three components which control discharge in an urban area. First, the surface phase where the incoming rainfall is subjected to loss through depression storage and infiltration, and its subsequent transmission to the sewer system, which constitutes the second component. The third component is the flood routing phase which combines tributary and lateral inflows to produce a flood hydrograph. By using the channel cross-section and a stage-discharge relationship it is possible to estimate the spatial location of flooding and the volume of flood water. The first two components have been combined in sewer design and evaluation models. Four such models were obtained and implemented. First, the Transport and Road Research Laboratory model (Watkins, 1962); second, the Illinois Urban Drainage Area Simulator (Terstriep and Stall, 1974); third, the University of Cincinnati Urban Runoff model (Papadakis and Preul, 1972) and fourth, the Illinois Storm Sewer System simulation model (Sevuk, 1973). Several difficulties led to the abandonment of this approach. First, the quantity of data required to define a catchments sewer system was immense, for example, it was estimated that there were 70,000 individual pipe lengths in the Beverley Brook catchment. The minimum amount of data required to define each pipe length were the gradient, shape, diameter and roughness. To analyse a flow through this number of pipes would require a very large core requirement and excessive amounts of computer time. Second, the records of the sewer networks and pipe characteristics, held by the local boroughs, were incomplete. Frequently the size and gradient of the pipe were unknown, often the linkages within the system were inferred. In view of this it would have been counterproductive to use advanced models with data which were of doubtful accuracy. Third, to validate the results of the models, sewer gauges are required. These are few in number and are not located in surface water sewers. Their function is to measure the amount of effluent entering sewage treatment plants. Consequently, although a strongly distributed model was being used there was usually only one recorded outflow, and therefore there was no way of knowing where, in the catchment any errors in the prediction were being made. Further, with such a large system, errors of opposite sign could cancel each other out. A similar problem existed with the flood-routing down the river. A maximum of three gauges were present in a given catchment and no information was available on the amount of water being contributed as lateral inflows by the sewer network. Finally, the models were developed for design or evaluation, and consequently were unsuitable for the objectives of the project.

Parallel to these investigations, flood routing methods were compared. The most promising appeared to be the FLUCOMP suite of computer programs, developed at the Hydraulics Research Station

(Price, 1975; Samuels et al, 1976a, 1976b; Walton, 1974). These programs permit the prediction of discharges throughout the length of the river and calculate the associated stage levels. The influences of bridges, embankments and other obstructions are predicted. The programs require a vast amount of data, for example a river with 200 cross-sections requires a total of 25,000 items of data which, it has been estimated, would take four man-months to collect and codify. Price (1977b) advised the author that the programs were unsuitable for urban rivers and they were consequently rejected.

A second alternative sought to avoid some of these problems by considering a $4.37Km^2$ sewered catchment in Twickenham. The outfall of the catchment was monitored by a sewer gauge on a combined sewer. Two years of discontinuous discharge records were digitised at hourly intervals and attempts were made to correlate the parameters of the discharge hydrographs with storm and antecedent catchment conditions. This proved to be totally unsuccessful. This failure was attributed to an inadequate recording system and that the area was too large and consequently heterogeneous.

In view of these two investigations it was decided to use lumped parameter models, whose data requirements, an input and an output, could be met.

1.5 Description of the structure of the thesis

The following section outlines the content and structure of the thesis, which is a detailed application of unit hydrograph theory to four densely urbanised catchments.

Chapter one has defined the scope of the thesis and has described the reasons for adopting lumped-parameter techniques to analyse the rainfall-runoff relationship on the London catchments.

Chapter two commences by describing the topographical, geological and climatological characteristics of the London area. The small amount of published research on the flood hydrology of London's non-tidal rivers is reviewed and discussed. The absence of any analysis of the effect of storm and antecedent catchment characteristics on the flood hydrograph, together with simplistic assumptions of the effect of urbanisation on the flood hydrograph are identified as areas worthy of further investigation.

Chapter three consists of a comprehensive review of the development of methods to predict hydrograph characteristics for urban or urbanising catchments. Equal emphasis is placed on the methodology and the results, because the former represents the growth of hydrological theory whilst the latter serves as a guide to the effect of urbanisation on the hydrograph characteristics. The chapter points to the considerable variation in the effect of urbanisation on catchment response and emphasises that the correct choice of an analytical technique is essential if the hydrological characteristics of a catchment are to be suitably identified.

Chapter four commences by providing a thorough description of the unit hydrograph and goes on to describe the methods by which they may be derived. The two components of linear conceptual models, the linear reservoir and the linear channel, are described and the Nash cascade model is discussed as an example of this approach. The application of matrix inversion and harmonic analysis to the identification of a unit hydrograph are discussed and a new and superior method of removing oscillations in the unit hydrograph is presented. The assumptions and components of the suite of computer programs used to derive the unit hydrographs together with the effect of the choice of rainfall separation technique on unit hydrograph accuracy is discussed. Finally, the six objective error functions used throughout the thesis are described.

Chapter five describes the application of the techniques presented in chapter four, to the analysis of 62 storm events on the Beverley Brook. The chapter consists of three sections. The first section presents a comprehensive analysis using several methods of analysing unit hydrographs. First, the use of a new index to measure time; second, grouping the unit hydrographs and third, derivation of a mean unit hydrograph. The fourth method was the use of a dimensionless unit hydrograph and the fifth used a three parameter triangular approximation to the unit hydrograph, called the Parametric Triangular Unit Hydrograph (PTTUH). The last two methods assume that the unit hydrograph varies in shape between storms. The results of a comprehensive correlation analysis are presented. The second section consists of the analysis of the storm events using 17 linear conceptual models. Recommendations are made to improve the potential of such an analysis. The third section consists of a flood frequency simulation exercise which used the PTTUH developed in section one of this chapter and 250 historical rainfall profiles. This section commences with a detailed evaluation of the value of using historical rainfall profiles and concludes with a comprehensive description of the necessary improvements to the computer program which would increase the accuracy of the method. The method used in this section significantly improves existing work in the field of flood frequency simulation.

Chapter six presents the results of a further unit hydrograph analysis using 34 storm events from three catchments. The variation of response between catchments and between storm events on a given catchment is noted. Traditional methods of synthesis using the basin ratio and the centroid of the unit hydrograph are rejected in favour of a new method which appears to be more accurate.

Chapter seven analyses some of the assumptions and the results presented in chapters five and six. First, the effect of substituting a triangular approximation of the unit hydrograph is assessed both in terms of peak discharge estimate and volume. This is the first time that such an analysis has been conducted. Second, the effect of rainfall separation technique on flood frequency estimates were investigated and found to differ significantly from the results presented in the Flood Studies Report (NERC, 1975). Third, the relationship between hydrograph peak and the volume of water liable to cause flooding (spill volume) is investigated. It was found that although urbanisation may increase flood peaks, the spill volume increases at a lesser rate. The implications of this relationship, quantified for the first time, is discussed. Finally, the effect of rainfall profile on peak discharge and spill volume is investigated. The implications of the findings, again quantified for the first time, are discussed.

Chapter eight summarises the findings of the thesis. Namely, that the thesis developed several new methods which are superior to existing analytical techniques and presents several new findings. There are three findings of great significance. First, the storm characteristics and the antecedent catchment wetness are of major importance in controlling the catchment response. This relationship was quantified for one catchment and presented for a further three catchments. Second, on densely urbanised catchments, further urbanisation does not significantly affect the form of the hydrological response. Third, the form of the response on a densely urbanised catchment is not related to the level of urban development but to the topography of the catchment. It is suggested that, this thesis, on the basis of these three findings, makes a significant contribution to the understanding of the flood hydrology of densely urbanised catchments.

Chapter 2

A DESCRIPTION OF THE CHARACTERISTICS OF THE LONDON NON-TIDAL RIVER CATCHMENTS

This chapter describes the characteristics of the four catchments whose flood hydrology is analysed in this thesis.

After examining the effect of geology on the surface water hydrology, the seasonal characteristics of rainfall and soil moisture deficit are described. The morphology of the catchments is then described using several geomorphological indices. The sources of the discharge, precipitation and soil moisture deficit data are described and evaluated. Finally, studies conducted on the London catchments are reviewed.

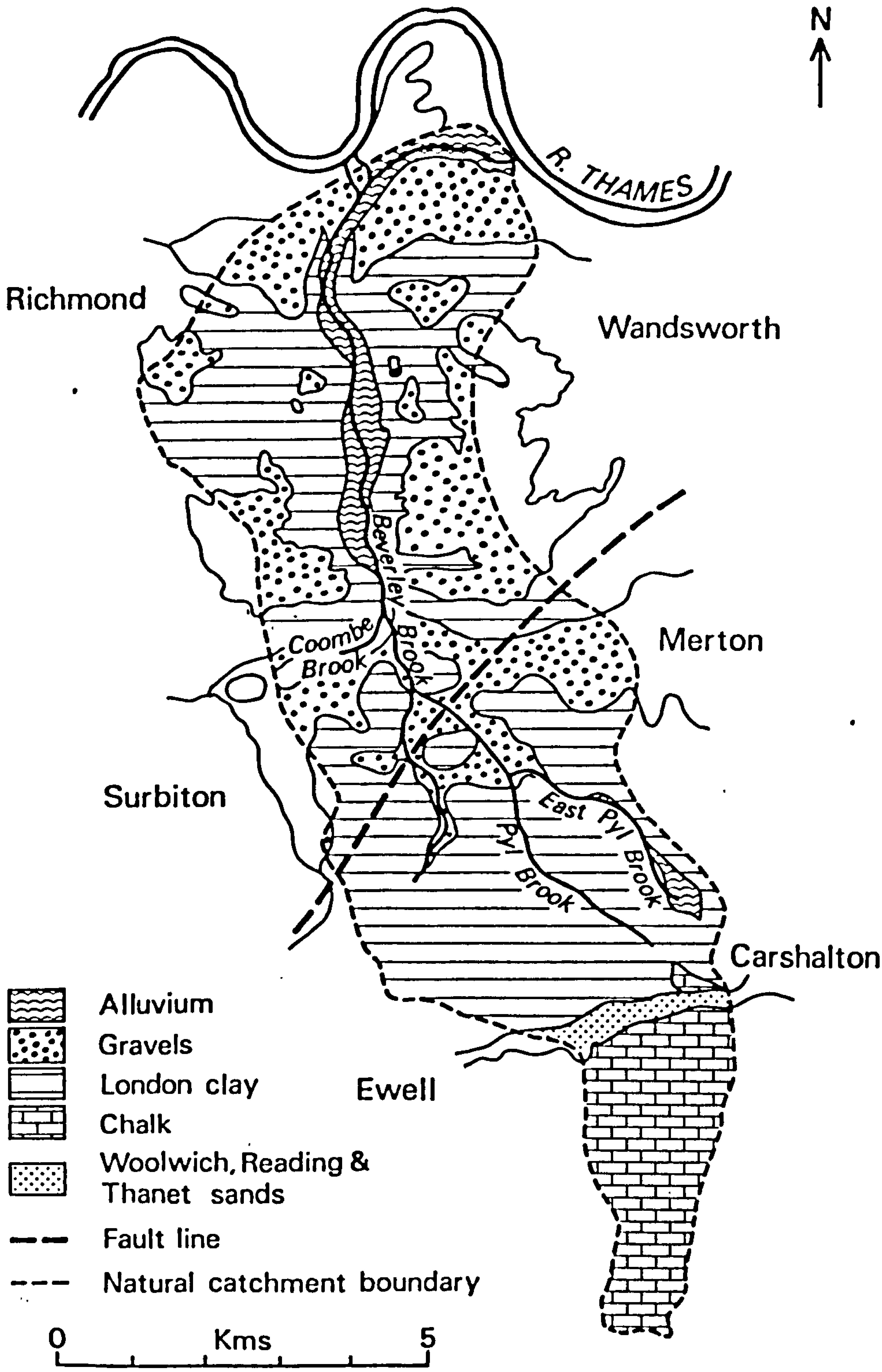
2.1 The geology of the London basin and its influence on the surface water hydrology

The London Basin rests on a platform of Palaeozoic rocks which has been buried by some 300m of younger Mesozoic and Tertiary deposits. The Mesozoic deposits consist of a 50m thick layer of Gault Clay which forms a waterproof layer for the overlying 4m of Upper Greensand and 200m of Chalk. The Upper Chalk, 70 to 100m thick, has well developed bedding planes, joints and fissures and consequently groundwater readily moves through it. Ground-water abstraction from the Chalk has caused dewatering of over 660Km^2 (Water Resources Board, 1972). The Mesozoic deposits are folded into a synclinal downfold with a west-east axis plunging eastward into the North Sea basin. In the centre of the syncline is a sequence of Tertiary clays and sands. The oldest of these, the Lower London Tertiaries, which rest directly on the Chalk, have been completely dewatered. The London Clay which underlies much of London has a high porosity but it lacks permeability and wells tapping this deposit yield water very slowly. The youngest in the sequence of solid rocks are fine sands with thin seams of clay and locally flint pebbles called the Bagshot and Claygate Beds. These outcrop extensively in south-west London and cap numerous isolated hills such as Harrow, Hampstead and Highgate. Springs emerge at the base of these deposits and are the source of minor tributaries to the Thames. This succession of solid rocks is masked almost everywhere by superficial deposits of clay, silt, sand and gravel ranging in thickness from a few centimetres to layers several metres thick. These deposits have been laid down over the past two million years as the Thames and its tributaries have cut into the hills and the valleys (Brown, 1978).

The present day configuration of these deposits is typified by a simplified geological map of the Beverley Brook, South London (Map 2.1). This river, and the three others considered in this thesis, have their origins in slope springs at the junction of the Chalk and the Tertiaries.

The abstraction of groundwater in the London basin has had two effects, first the reduction of spring flow and second the alteration of catchment response. The Upper Chalk represents the most important source of ground-water in the London basin. The Chalk rises in two limbs to outcrop in Hertfordshire to the north and in the North Downs to the south. The aggregate quantity abstracted has long exceeded replenishment from the two outcrops and a progressive fall of water levels in the Chalk under London, to a maximum of 75m, has taken place over the past 150 years. Analysis of records (Water Resources Board, 1972) suggests that springs in the London basin have either decreased in volume or have become ephemeral due to dewatering of the Chalk. For example, "The Ravensbourne originated as chalk springs, but today the flow is derived from the Lower London Tertiary outcrop, the Ravensbourne and Beck being spring-fed from the sandy gravel and the remaining tributaries originating as runoff from the London Clay" (Aston et al, 1979, 37). This implies that baseflow volumes will have decreased but in the absence of accurate, long records this cannot be verified.

The second effect, the alteration of catchment response, is difficult to quantify because the effect is only observable over long time periods and it is difficult to distinguish the effect of de-watering on catchment response from other factors. Nash (1959, 317) in a study of the River Wandle noted that, "Examination of the detailed map of the areas (south of) the Eocene-Chalk boundary indicates, by the absence of surface streams, that the contribution of these areas to storm runoff is negligible. Consequently, for the purposes of considering storm runoff the catchments may be considered terminated by the chalk line." This decision would seem to be justifiable in view of the high infiltration capacity of Chalk and the lowering of the water table. The major storm of the 14/15th September 1968 refuted these arguments (see section 1.1). Large amounts of antecedent precipitation decreased the soil moisture deficit to 4.06mm and the Chalk area, comprising approximately 70 percent of the total catchment area, made a significant contribution to the values of storm-water runoff. Had the Chalk area been included in the analysis, the under-



Map 2.1 Geology map of the Beverley Brook

design of the river channel may have been avoided. Therefore, although dewatering of the Chalk has decreased the hydrological significance of the Chalk springs, the Chalk area remains an integral part of the catchments which, under optimum conditions, can make a significant contribution to the volume of flood water.

2.2 The hydrological significance of precipitation and soil moisture

The characteristics of the distribution of precipitation in both time and space are important in a study of flood hydrology because they indicate the likely seasonal distribution of floods.

An analysis of mean annual precipitation for London, 1916 to 1950 (Chandler, 1965, 238) indicated two controls on areal distribution. First, a general decrease in rainfall totals from west to east and second, increased totals in the higher parts.

> "Less than 600mm falls near the Thames in a strip whose width is constricted from Richmond to Westminster between upstanding areas with higher falls, such as Hampstead to the north and the commons of Richmond, Wimbledon and Clapham to the south. The highest rainfall, over 860mm, falls on the crest of the North Downs north of Limpsfield and Oxted, southeast of London, and many of the surburban fringe areas of the conurbation in south and southeast London receive an annual average of more than 686mm. Regional trends and orographic influences are mainly responsible for the overall range of precipitation of 229 to 254mm over Greater London; this represents a very large percentage of the mean value"
> (Chandler, 1965, 238)

The spatial variation of rainfall, combined with variations in catchment morphology, is sufficient to produce different discharge regimes for each of the London catchments. Flood hydrograph characteristics for individual storm events also vary between catchments.

The incidence of a single flood event is related to the characteristics of the causative storm, namely, the intensity, the duration and the amount of rainfall (Table 2.1).

Rainfall amounts of greater than 50mm, irrespective of duration, tend to cause flooding in Greater London. Inspection of the rainfall record from 1865 to 1977 for the Greater London area produced 125 storms with over 50mm of rain (G.L.C., 1977). Assuming constant antecedent catchment conditions, it may be concluded from Table 2.1 that the flood season consists of the months June, July, August and September. The storms in these months are characterised by high intensity, short duration rainfall which is characteristic of a high degree of convective activity. The lack of spatial ordering of the maximum falls of these storms (Butters et al, 1977, 338) indicates that any part of Greater London is liable to receive a significant amount of rainfall.

Table 2.1 Monthly average of precipitation intensity and duration, Kew, 1927-1956 and monthly distribution

Sources: Chandler (1965, 221-222)
† G.L.C. (1977)

Table 2.2 Mean monthly precipitation at Kew (1916-1950) and mean monthly soil moisture deficit at Buckingham Palace (1941-1960)

Sources: Rainfall Chandler (1965, 220)
S.M.D. M.O. (1941-1960)

The incidence of a flood event is related to the causative storm characteristics and the catchment wetness. Table 2.2 indicates that the ground is near field capacity for the months of December to April, due to the low evaporation rate in these months and the heavier than average rainfall in August, October and November.

Flood events produced by relatively minor rainfall amounts show a greater predominance in the winter months of the year. This is a result of the wetter ground producing a response from light rains which would be smaller from drier ground (Table 2.3).

Table 2.3 Paired storms from the Beverley Brook illustrating the effect of catchment wetness on percentage response

Event Number	Rainfall			SMD (mm)	Percentage Response
	Duration (hours)	Total (mm)	Intensity (mm/hr)		
39	4.0	4.80	1.20	0.0	21.77
35	4.0	4.83	1.21	107.7	10.84
42	5.0	12.40	2.48	56.9	22.63
36	5.0	11.73	2.35	107.7	10.80

The potential of a rainstorm to cause a flood during the summer is related to the soil moisture deficit. High intensity storms on dry catchments produce significant flood hydrographs but a major proportion of the rainfall recharges the soil moisture deficit (Table 2.4).

Table 2.4 Beverley Brook: High intensity storms on a dry catchment

Event Number	Rainfall				SMD (mm)	Percentage Response
	Duration (hours)	Total (mm)	Intensity (mm/hr)	Effective (mm)		
13	0.5	25.91	51.82	4.241	70.9	16.37
50	0.5	5.90	11.80	0.764	21.2	12.95

Significant flood events are also produced by storms, with an intensity greater than the mean summer intensity (1.92 mm/hr), which fall on a wet catchment (Table 2.5).

Table 2.5 Beverley Brook: Moderate intensity storms on a wet catchment

Event Number	Rainfall				SMD (mm)	Percentage Response
	Duration (hours)	Total (mm)	Intensity (mm/hr)	Effective (mm)		
19	2.0	5.33	2.67	2.406	6.6	45.15
51	5.0	10.40	2.08	4.364	0.0	41.96

Event 13 (Table 2.4) and event 51 (Table 2.5) produce approximately the same amount of effective rainfall from dissimilar storm events. The principle cause of this is the different values of antecedent catchment wetness.

The results presented in these Tables (Tables 2.3, 2.4 and 2.5) point to the necessity of considering the causative storm characteristics and the antecedent catchment wetness in any study of catchment response.

2.3 Morphological description of the investigated catchments

There are six major catchments within the Greater London area, namely the Brent, Crane, Beverley Brook, Wandle, Ravensbourne and Marsh Dykes (Map 2.2). This thesis considered four south London catchments, the Beverley Brook, Wandle, Ravensbourne and the Pool (Table 2.6). The Pool is a subcatchment of the Ravensbourne.

The morphological indices are defined as follows.

The distance from the gauging station to the remotest part of the catchment (L_R) was defined as the radius of the circle centred on the gauging station where circumference just touched the most distant part of the catchment.

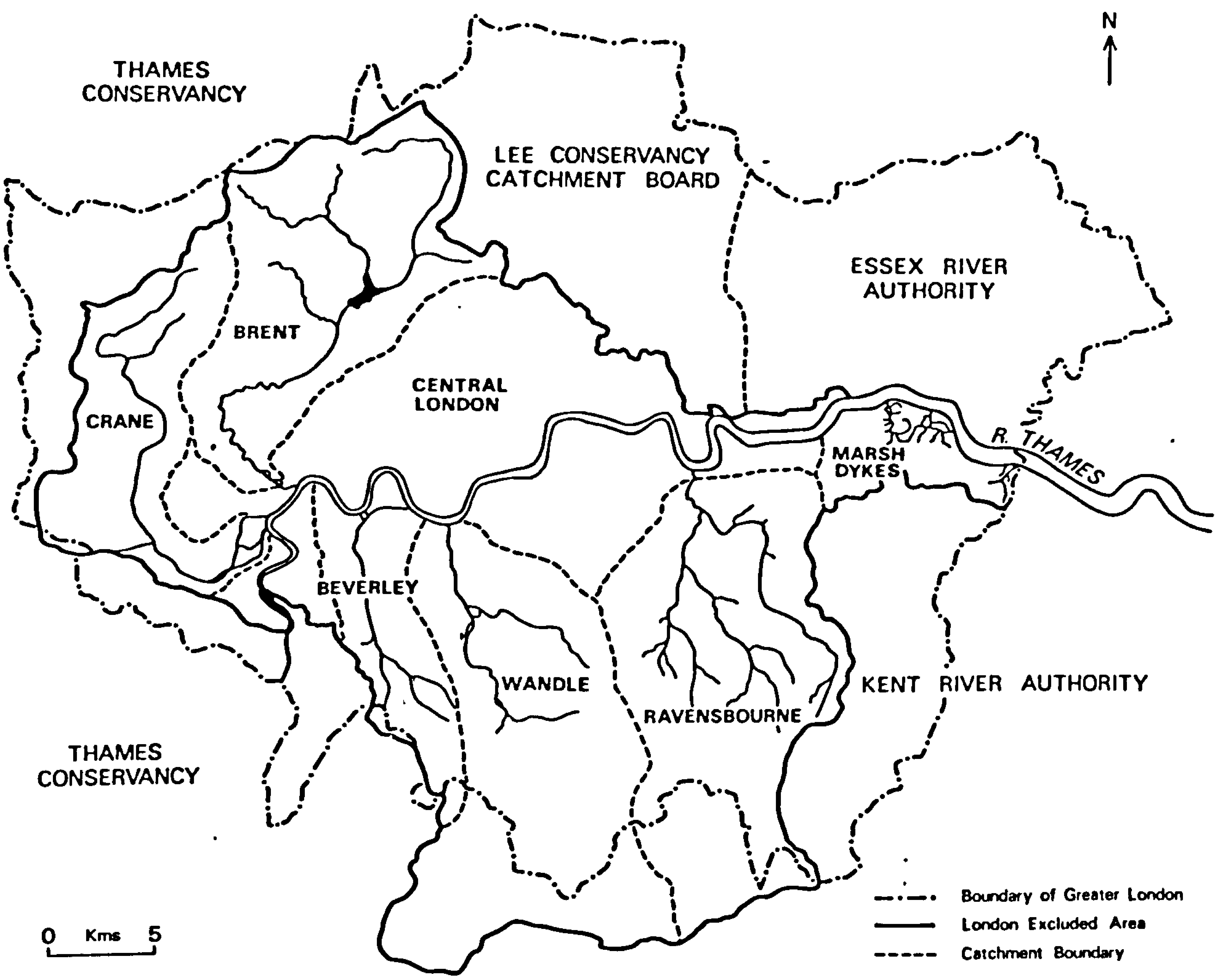
The main stream slope (S) was defined as the slope (m/km) of the main channel from 0.1L to 0.85L, where L is the main stream length (km), measured from a 1:50000 scale O.S. map.

The basin ratio (Z) was defined as

$$Z = L/\sqrt{S} \qquad (2.1)$$

where: L is the mainstream length (km)

S is the mainstream slope from 0.1L to 0.85L



Map 2.2 Map of the GLC catchments

Table 2.6 Morphology of the investigated catchments

Attribute	Catchment			
	Beverley	Wandle	Ravensbourne	Pool
Gauging Station	Wimbledon Common	Connollys Mill	Catford Hill	Winsford Road
Area (Km^2)	43.50	176.00	134.46	76.68
Mainstream length (km)	7.40	10.38	10.90	8.43
Distance from gauging station to remotest part of catchment (km)	12.86	20.02	17.93	18.81
Mainstream slope (m/km)	2.28	2.89	8.39	6.41
Basin ratio (Z)	4.90	6.11	3.76	3.33
Shape (K)	2.99	1.79	1.88	3.62
Altitude minimum (m)	11.00	10.27	14.49	16.95
maximum (m)	161.80	249.30	251.20	266.70
Width mean (km)	3.38	8.89	7.50	4.08
maximum (km)	4.55	15.77	7.60	5.76
Percentage urbanised	80.80	32.20	41.80	56.00

The shape factor (K) was the leminiscate loop, defined as

$$K = \frac{L_R^{2}\pi}{4A} \qquad (2.2)$$

where: L_R is the distance from the gauging station to the remotest part of the catchment (km).
A is the catchment area (km^2)

A K value of 1.0 indicates a circular catchment and increasing values of K indicate an increasing tendency to elongation of the catchment.

The maximum width of the catchment was defined as the maximum width perpendicular to the line drawn between the gauging station and the remotest part of the catchment. The mean width (W_{MEAN}) was defined as,

$$W_{MEAN} = A/L_R \qquad (2.3)$$

where A is the catchment area (km^2)
L_R is the distance from the gauging station to the remotest part of the catchment (km).

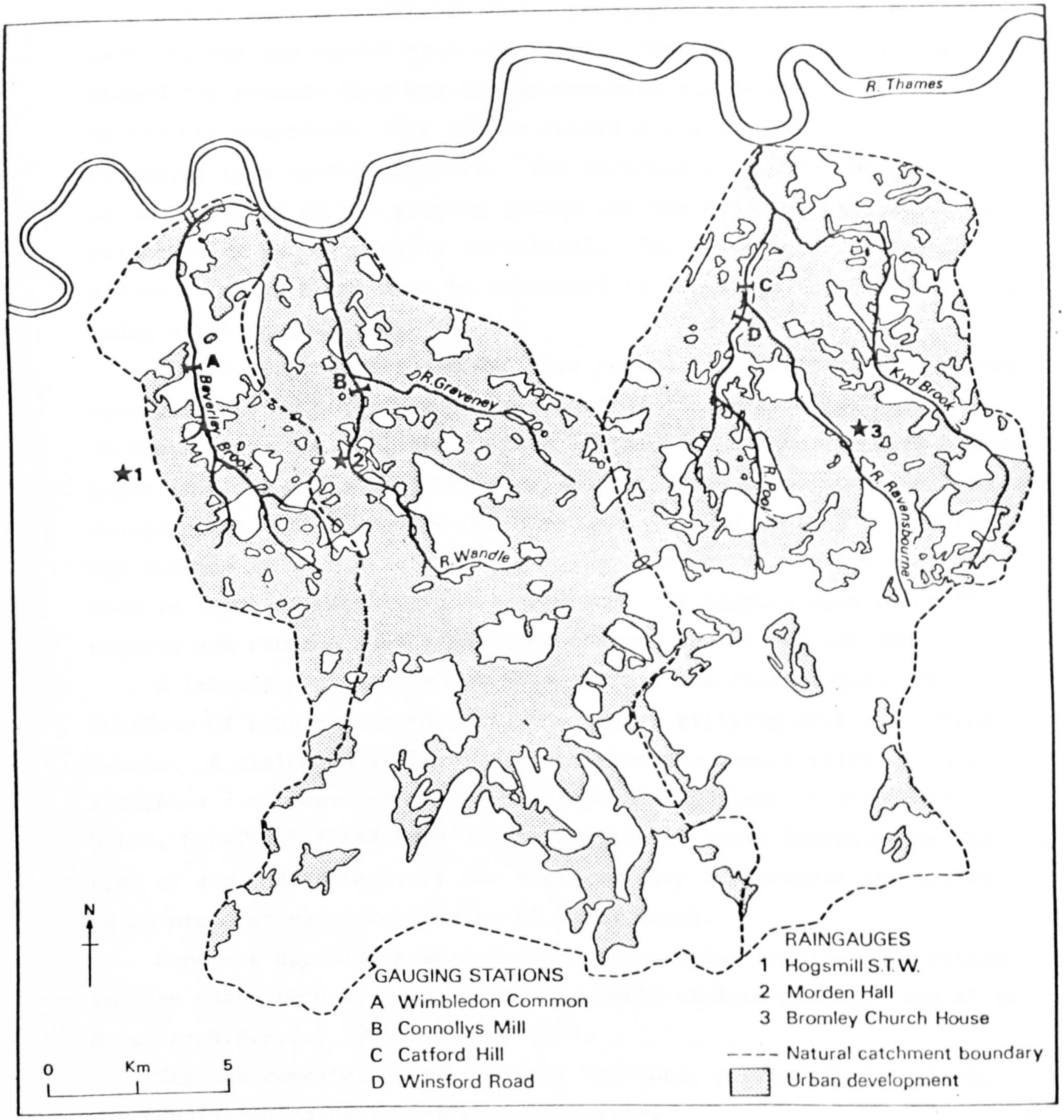
The percentage urbanised was defined by the orange area on a 1:50,000 O.S. map.

The spatial pattern of the urban development in each of the four catchments is different (Map 2.3). The southernmost parts of the catchments, with the exception of the Beverley Brook, display a lower density of urban development. All four catchments have large areas of parkland adjacent to the river which is used as washland for flood alleviation purposes. The spatial growth of urban development within the catchments, although not the concern of this thesis, shows a pattern of expansion along roads and later in-filling similar to that observed by Packman (1978) and Hall (1978a) in North London.

The four catchments are sufficiently diverse in both physical characteristics and the nature of urban development to ensure that a range of response types was observed.

2.4 Data Sources

Three types of hydrometric data were used in this thesis. First, discharge data measured continuously at fixed structures. Second, rainfall data measured both continuously and cumulatively. Third, the soil moisture deficit estimated from meteorological parameters.



Map 2.3 Location of the discharge and rainfall gauging stations and the extent of the urban development

The characteristics of the four discharge structures are presented in Table 2.7.

The primary recording devices, using pen and ink on paper charts, were Ott and Lea spiral drum recorders. The Lea recorders are being phased out because they are not as accurate and mechanically reliable as the Ott recorders. The former record a trace which is converted to discharge by a system of gears. The accuracy of the record depends on the accuracy of the gearing system and the accuracy with which the rating curve was originally calculated. The Ott recorder measures discharge stage which must be converted to discharge in the laboratory using a rating equation.

The Fisher and Porter punch tape recorder (PTR) forms a secondary recording system. These punch a record of discharge stage onto a 16-track paper and foil tape. This is translated into standard 8-track paper tape for use with a computer program which translates the discharge stage into discharge and derives daily and monthly statistics. The machine is more accurate than either the Lea or the Ott recorder both in terms of resolution and accuracy. It samples once every 15 minutes and records discharge stage changes to the nearest 1mm.

A telemetry system is installed at all the four gauges. This consists of sensors suspended in the gauges stilling well at varying levels. A rising water level may activate the sensor which in turn activates a microprocessor which telephones an alert to either the G.L.C. Non-Tidal Flood Room or the engineers' homes depending on the time of day. Alternatively the engineers may interrogate the system to obtain continuous monitoring of water level.

Separate appraisals were made of the gauging station, the rating and the chart record, resulting in a simple station grading from A1 to E (after N.E.R.C., 1975; Jones, 1975).

The instrumentation of the four stations, previously described, was considered to be reliable and accurate.

The stations were assessed by visits to the sites, discussions with the maintenance team and inspection of design drawings and flood records. The Wimbledon Common gauge was bypassed during the September 1968 flood. The banks were subsequently raised to prevent similar failures. None of the other three gauges have been bypassed because they have high walled flumes and channel approaches. All the stations are approached from a straight channel reach with a uniform, rivetted channel section. The Beverley Brook has been gauged at Wimbledon Common since 1935 but, following the re-organisation of local government, the weekly charts from 1935 to 1962 were lost.

Table 2.7 Gauging stations

		Catchment			
Name of Station		Wimbledon Common	Connollys Mill	Catford Hill	Winsford Road
Hydrometric No.		039005	039003	039056	039058
National Grid Reference (TQ)		216717	266706	372732	372726
Gauge Type		Critical depth trapezoidal	Standing wave	Venturi Critical depth flume	Venturi Critical depth flume
Recorder Type	OTT	X		X	X
	LEA		X		
	PTR	X	X	X	X
	Tele-metered	X	X	X	X
Record	From	8.8.1957	27.9.1962	2.12.1974	4.12.1974
	To	Present day	Present day	Present day	Present day
Classification After Jones 1975		A1	A1	A1	A1

Table 2.8 Rain gauges

		Rain gauges		
Site Name		Hogsmill	Morden Hall	Bromley Church House
National Grid Reference (TQ)		194682	261685	401691
Meteorological Office Number		02.286392	02.287909	03.288687
Instruments:				
Dines tilting syphon		X	X	X
Cassella tipping bucket		X		
Snowdon check gauge		X	X	X
Telemetered		X		
Record	From	1957	1960	1972
	To	Present day	Present day	Present day

The early record of the stations at Wimbledon Common and Connollys Mill are not continuous, but the installation of the P.T.R.s has meant that the records from the mid-1970s are virtually complete.

The rating curves for the Wimbledon Common, Catford Hill and Winsford Road stations were fixed by applying a theoretical formula to the design, this has been verified by numerous current meterings. The rating for the Lea and Ott recorders at Connollys Mill have been derived by current metering.

The stations are visited once a week to change the charts when any minor maintenance is carried out. Thorough checks are made once a month. This includes cleaning the stilling well and removal of debris from the channel upstream of the gauge. Bi-monthly current meterings are made at all stations and at times of high discharge as many stations as possible are visited to obtain current meterings. The evidence suggests that the discharge records are excellent and may be classified as A1 (after Jones, 1975).

The autographic raingauges were selected according to the following criteria. First, that the gauge should be located in the centre of the catchment. Second, that the gauge should be exposed according to Meteorological Office (M.O.) regulations. Third, that the gauge should be well maintained. Fourth, that the gauge should be available for the complete length of the discharge record for a given catchment (Table 2.8, Fig. 2.3).

The Hogsmill Valley gauge was used to provide the rainfall data for the Beverley Brook, and is one of the best autographic rainfall records in Greater London. The chart is changed at 0900 hours every day of the week and the M.O. exposure regulations are met. It is 1.5 km to the west of the Beverley Brook's western catchment boundary.

The Morden Hall gauge is in the Wandle catchment. It is slightly overshadowed by trees and buildings but is judged to be accurate. It is not changed at the weekends.

The Bromley Church House gauge provided rainfall records for both the Pool and the Ravensbourne and is located in the Ravensbourne catchment 0.5km east of the Pool-Ravensbourne catchment boundary. The gauge is changed every day but is overshadowed by young trees.

The values of daily precipitation, used to construct indices of antecedent precipitation, were obtained from the Snowdon check gauges associated with each autographic gauge.

The use of only one raingauge per catchment introduces errors, namely an inaccurate assessment of areal rainfall and no account is taken of storm movement. However, using the criteria outlined above, it was only possible to find the above gauges.

A deficit of soil moisture is defined as the equivalent depth of water which would be needed to restore a soil to field capacity. The measurement of soil moisture is one of the most intractible problems in hydrology. Areal assessments are complicated by the spatial heterogeneity of soils, consequently soil moisture is usually estimated for large areas from meteorological parameters.

To calculate daily values of soil moisture it is necessary to

(a) calculate the total potential evaporation

(b) calculate the average rainfall over the area

(c) using a land-use model of the area and models of potential to actual evaporation relationships, calculate a balance of rainfall, evaporation, soil moisture deficit and effective rainfall.

The values of soil moisture for this thesis were derived by the M.O. for the climatological station at Buckingham Palace, central London.

2.5 Hydrological investigations of the Greater London catchments

Hydrological investigations of the Greater London catchments fall into two groups, those that have been published in international journals and those published as internal memoranda by the G.L.C.

Nash (1959) predicted the effect of river improvement works on the flood frequency of the River Wandle, south London. It was assumed that the flood hydrology of the Beverley Brook and the Hogsmill, which are not obstructed by weirs and sluices, represented the 'post-works' condition of the River Wandle, provided that account was taken of differences in catchment physical characteristics. The 30 year design standard, adapted as a result of this investigation, proved to be insufficient to prevent serious flooding in 1968 which caused direct damage estimated at £6.75m (1975 prices). Nash's analysis was marred by several analytical and methodological errors. First, it was assumed that the parts of the catchment underlain by Chalk would not contribute to runoff. This was an incorrect assumption and has been commented on earlier in this chapter (section 2.1). Second; base flow was not subtracted from the hydrograph at Middle Mills (Nash, 1959, 328), this caused non-comparability of results between stations and is analytically incorrect. Third, too few events were analysed to derive an accurate estimate of catchment lag time (Table 2.9). The values of the standard deviation of lag time indicate that the value for Middle Mills should be treated with caution and that the values for Wimbledon Common and Wandle Park are acceptable. The reliability of lag time at Beddington and Grange Road cannot be assessed because only one instantaneous unit hydrograph (I.U.H.) was derived for each section. It is suggested that an insufficient number of IUHs were derived for all the catchments. The use of such a small sample prejudices the reliability of both the estimated lag time and the shape of the IUH. Current practice would suggest that at least 5 events should have been analysed for each catchment (Sutcliffe, 1978, 17). Fourth, it was assumed that channel improvement would reduce lag time but that the IUHs shape would be unchanged. Subsequent work has shown that the shape of the IUH changes from platykurtic to leptokurtic as a result of channel improvement. Consequently the increase in peak discharge, caused by channel improvement, was underestimated and this partially accounts for the under-design of the river channel. Fifth, Nash presents a graph of catchment shape versus area weighted lag time and observes that, "... the points lie roughly along a line ... with the Wimbledon Common points some distance below it. All this is in accordance with the expectation, except that the Grange Road point might have been lower" (Nash, 1959, 321). In fact, Grange Road,

Table 2.9 Analysis of IUH lag time from Nash's 1959 paper

Table removed due to third party copyright

Source: Nash (1959, Table 2, 321)

assumed to represent a post-works regime, plots on the same line as the pre-works stations. The assumption that post-works catchments have a different lag-time than the pre-works catchments, controlling for catchment area and slope, is therefore incorrect.

Nash's (1959) paper was partially handicapped by a lack of knowledge of the behaviour of urban rivers which today, with 20 years of subsequent research, is understood more fully. The application of a more rigorous scientific approach would have avoided some of the errors which the paper makes, namely the inconsistency in subtracting baseflow, the derivation of only one IUH each for two catchments and the use of a demonstrably incorrect hypothesis which proposed a difference in lag time between improved and unimproved ~~unimportant~~ river channels.

Hall (1974) suggested that the lag time of the unit hydrograph could be predicted from the basin ratio of the catchment, defined as the quotient of the main channel length and the square root of the main channel slope. Studies of the Silk Stream and the Dollis Brook in north London (Packman, 1974; Hall, 1977a) refuted this work. Substitution of a distributed model using a flood routing technique and the Flood Studies Report (NERC, 1975) rainfall-runoff model on the Silk Stream proved to be more accurate than the conventional 'lumped' catchment models.

These three studies (Packman, 1974, 1978; Hall, 1977a) are handicapped by the use of two gauging structures which are rated as class C (after Jones, 1975) because of errors in the rating curve. The Flood Studies Report (NERC, 1975) only used stations rated at A1, A2 and B for its unit hydrograph analysis and consequently it is suggested that the results of these studies should be treated with caution.

Hall's (1977a) analysis is marred by several errors. First, the iteration procedure to calculate the temporal distribution of the rainfall has a threshold of 0.02mm (Hall, 1977a, 148). This is a source of avoidable error since a computer can operate to a threshold of at least 0.0001mm, as was done in this thesis. Second, the analysis seeks to derive mean unit hydrographs for given time periods, however the unit hydrographs within these time periods are not consistent (Hall, 1977a, Fig. 2, 149). Hall ignored this variation which is attributable to differences in storm characteristics and antecedent catchment conditions. Further, negative ordinates on the recession limb have not been removed. These ordinates lack a physical interpretation and erroneously increase the value of the peak discharge. Third, the method of deriving a mean unit hydrograph (Hall, 1977a, 148) is incorrect because it does not align the unit hydrograph peaks before calculating the mean unit hydrograph (Sutcliffe, 1978, 17).

Although the paper is a useful contribution to urban hydrology these three errors reduce its authority. Further, the paper is one of many which investigate urbanising catchments and which ignore the effects of storm conditions and antecedent catchment conditions on catchment response. For example, Hall notes (Hall, 1977a, 149) "The standard deviation of the lag times of 1.48 hrs reflects the variability in the shape of the response functions", but offers no indication of the causes of the variability.

Two studies by the Greater London Council (Butters et al, 1975, 1977) used 250 rainfall profiles derived from historical profiles. These studies confirmed that the combination of storm duration and profile characteristics are important controls of the flood hydrograph shape. These studies, whilst using records from the Greater London catchments, have contributed to theoretical hydrology rather than to an understanding of the flood hydrology of the London catchments and are consequently not discussed here but in Section 5.4.2.

The non-tidal rivers section of the Greater London Council has examined the hydrology of the Brent, Beverley and the Wandle. These reports contain estimates of discharge to be expected, for given return periods, at different parts of the river system. These reports are not discussed here because they do not examine the flood hydrology of the catchments.

2.6 Conclusion

This chapter has described the characteristics of the four catchments which were analysed in the subsequent chapters of this thesis.

It was shown that, although dewatering of the Chalk has decreased the hydrological significance of the Chalk springs, the Chalk areas of the four catchments can make a significant contribution to the volume of flood water in periods of above average rainfall.

Evaluation of the data sources indicated that the data were of a high standard and were markedly superior to the recent studies of the Brent catchment (Hall, 1977a, Packman, 1974, 1978).

Investigation of several storms from the Beverley Brook indicated that there was a relationship between the catchment's response and the causative storm characteristics and the antecedent catchment wetness. The published studies of the flood hydrology of the London catchments have not investigated this variation which analyses, reported in Section 2.2, suggest is of great significance in controlling the volume of runoff.

Finally, as a result of a review of hydrological investigations of the London catchments it was concluded that little or no research had sought to identify first, the causes of variation of catchment response for a given catchment and second, the causes of variation of hydrological response between catchments. This thesis is directed towards rectifying these deficiencies.

Chapter 3

LITERATURE REVIEW

3.1 An introduction to the effects of urbanisation on the flood hydrograph

Chapter 1 has defined the research problem as the prediction of the discharge hydrograph to be expected from an urbanised catchment for a given storm. Chapter 2 has described the London catchments and reviewed the research conducted to date on these rivers. Chapter 3 consists of a description of the effects of urbanisation on the flood response of a catchment, describes the range of methods to measure this effect and presents the findings of several studies.

Urbanisation alters the rainfall-runoff process by replacing previously permeable surfaces with impervious surfaces. For a modern industrial society it has been estimated that 0.1 ha. of land is converted from rural to urban land use per head increase in population (Knapp, 1965). The two hydrological consequences of urbanisation are an increase in the percentage runoff and an increase in the rapidity of response. The first occurs because of the replacement of permeable surfaces with impervious surfaces and the associated reduction in surface roughness which reduces detention storage. The second is a result of the more rapid runoff from paved surfaces than from soil surfaces and the increased speed of subsurface transport by the sewer system laid down to remove the excess runoff. The principle effects of urbanisation have been known for some time (e.g. Savini and Kammerer, 1961), the most recent summary (Packman, 1977, 1979) forms the basis of this introduction.

Research in rural catchments has defined the sources of runoff as first, emergent interflow caused by the rainfall intensity being greater than the infiltration rate and when the interflow exceeds the capacity of the soil. Second, the most runoff results from source areas adjacent to the river and third, the spatial extension of the source areas during the storm produces a greater percentage runoff than the remainder of the catchment with the exception of the source areas. The magnitude of the effect of urbanisation on the flood response will depend on how it alters these sources of runoff.

First, the addition of impervious surfaces to a catchment should increase the percentage runoff. If the urbanisation is located on previous source areas then the effect will be less than if it were located in the non-source areas. Further, the spatial location of the urban development in the catchment will affect the timing and magnitude of the peak. Urbanisation at the upper end of the catchment may cause the rapid urban response to arrive at the outfall at the same time as the slower response from the rural area in the lower part of the catchment. Hence the flood magnitude is increased but the timing remains relatively unchanged. If the urban area is located in the lower part of the catchment then its response may pass the outfall before the arrival of the slower response from the upstream rural area. This would produce a double peaked hydrograph. Finally, the effect depends on the magnitude of the percentage runoff of the rural catchment. If the percentage is high then full urbanisation will have a smaller effect than if the percentage was originally low. Therefore it might be expected that for a similar increase in urbanisation, a clay catchment will show less change than a chalk catchment.

Second, the natural drainage system varies in length, density and spatial extent with the season and during a storm. The sewer system may be of greater length and density and cover a larger area, this would cause an increase in percentage runoff. Further the permanent nature of the sewer system as opposed to the time varient natural system results in a decrease in catchment lag time. The magnitude of this effect will depend on catchment size. For small catchments where the overland flow time represents a significant proportion of total flow time, the increased impervious area and sewer network may account for a large part of the increase in rapidity of response. On a large catchment where channel flow time is the most significant proportion of total flow time a large part of the increase in rapidity of response may be due to channel improvement. An increase in urban development will not increase the volume of runoff if there is insufficient drainage or if roads and railways block natural drainage courses. This situation contributes to anomalous findings of no increase or a decrease in the percentage runoff due to urbanisation.

Third, since urban response is more rapid than the corresponding rural response, urbanised catchments are able to respond more fully to shorter duration, higher intensity rainfall events than when in their rural condition. Consequently, since in the U.K. peakier storms are associated with summer convective events and since urbanisation reduces the effect of antecedent wetness on catchment response there is a tendency for the flood season to move to the summer or at the very least the summer season displays the most marked change in flood response.

Fourth, a given rainfall falling on a paved surface may experience less initial loss due to infiltration and surface storage than the same profile falling on a rural land use surface. Consequently, urbanisation may result in more evenly distributed losses during the storm than before. This will affect the shape of the hydrograph.

Lastly, several small effects may be significant depending on the catchment under consideration. Building activity disturbs the soil profile and destroys the subsurface flow paths, this will affect response time and percentage response. This effect will decrease in significance with time. Groundwater recharge by rainfall may be reduced, but the substantial leakage from the water supply and effluent disposal sewer systems, together with minor contributions from municipal and private garden watering may counteract this trend.

The variety of effects of urbanisation are a function of the increase in impervious area, the improvements to the drainage system and the location of the urban development within the catchment. The interaction of these three factors is complex as the above discussion has shown and explains why the results of different urban hydrology studies have yielded inconsistent results.

The literature review traces the historical development of methods to predict the peak discharge and the complete hydrograph. Emphasis has been placed on developments in methodology, which represent the growth of hydrological theory, and which have permitted the accurate prediction of these two properties of the hydrograph.

3.2 Empirical Methods

Two studies, conducted some 300 years ago, mark the beginning of scientific hydrology (Rodda, 1976, 258). In 1674 Perrault measured the rainfall and runoff for the 121.5Km^2 catchment upstream of Aignay le Duc, France. He found that for the three years commencing 1668 the discharge was about one-sixth of the rainfall. The second study considered the 60,356 Km^2 Seine basin above Paris. Mariotté (reported by de la Hire, 1686) found the rainfall to be about 400mm or 24x10^9 m^3 of water, while the runoff measured at Paris was of the order of 3.5x10^9 m^3.

These results showed that the rainfall was sufficient to produce runoff and therefore refuted the subterranean concept of the hydrological cycle which had endured since the time of the ancient Greeks.

Hydrology showed little progress in the next two hundred years. This may be attributed to the mutually dependent factors of scarcity of data and the lack of demand for reliable hydrological predictions. Runoff prediction was limited to peak discharge estimates. These were made by empirical formulas which were derived for particular cases and then applied to other cases on the assumption that conditions were similar enough for the predictions to be of the same order.

Chow (1962) has identified three groups of such formulas. First, those relating peak flow, of an unspecified frequency, to physical characteristics of the catchment.

$$q_d = aA^b \tag{3.1}$$

Where a, b are empirical factors
A is the catchment area
q_d is the design discharge.

Formulas of this type are a product of scarce hydrometric data and a poor understanding of the processes of the hydrological cycle. They are non-transferable because the empirical factors (a,b) are chosen by experience and have no physical relevance. Finally the frequency of the design discharge is unspecified and therefore the formula is of little use for planning purposes. An example of this group is an adaptation of a 1879 formula proposed by Jarvis (1926).

$$Q_m = 10000p\ A^{0.5} \tag{3.2}$$

Where Q_m is the maximum flood discharge (cusecs)
A is the catchment area (square miles)
p is a coefficient expressing the relationship of the estimated flood peak for a particular river to an assumed maximum of 10,000 cusecs for all rivers.

The second and third classes include a measure of frequency, this was only possible after the introduction of the necessary statistics in the 1930s (Pearson, 1930; Weibull, 1939).

The second group relates a peak discharge of specified frequency to the average annual flood for the catchment.

$$q_t = q_{ave} (1 + f(t)) \tag{3.3}$$

Where q_t is the design discharge of known frequency
q_{ave} is the average annual flood
t is the recurrence interval.

Such formulas are of little use if the catchment is ungauged, because q_{ave} must be estimated by some method. Alternatively, if q_{ave} can be defined from available data then a direct frequency analysis is a superior method.

The third group relates the peak discharge to the catchment characteristics and rainfall intensity.

$$q_p = f\ (ip, a, b \ldots) \tag{3.4}$$

The parameter ip is the rainfall intensity of probability p for a duration equal to the time of concentration of the catchment. These formulas assume that the probability of the discharge is equal to the probability of the rainfall. However, a given peak discharge may be produced by any number of combinations of intensity and duration. These methods fail when the storm duration is not equal to the time of concentration.

The most well known formula of this group is the rational method, presented to the Institution of Civil Engineers of Ireland by Mulvaney in 1851.

$$q_p = C.\ ip.\ A \tag{3.5}$$

Where C is a coefficient
A is the catchment area.

The runoff coefficient C takes account of the percentage of directly connected impermeable area. Standard values for different areas and surfaces are given in A.S.C.E. (1969). An alternative method is to use an empirical formula to relate C to the number of houses per acre (N) (Escritt, 1950).

$$C = \sqrt{N/10} \tag{3.6}$$

Whilst several workers (Kuichling, 1889; Bartlett, 1970) have presented tables of factors for different types of surface, it has been observed (e.g. Grubb, 1937) that the factor depends on the storm characteristics and the antecedent conditions. This results in errors in the estimated peak discharge. Watkins (1962) tested the method on 283 storms and found that the mean absolute percentage error for the peak discharge was 24.6, which is very poor.

Research work in urban hydrology using empirical formulas has been directed towards predicting the mean annual flood from the percentage of the catchment paved and various other catchment characteristics. They therefore represent an amalgamation of groups one and two. The method of analysis is called regionalisation. "To regionalise a model simply means to develop a scientific basis for predicting the model parameters on gauged watersheds from hydrologic and physiographic characteristics of that watershed. Regionalisation can be accomplished if there are enough bench mark watersheds with adequate storm rainfall and runoff data such that a statistical inference may be drawn" (Overton and Meadows, 1976, 246).

Several studies (Martens, 1968; Skelton, 1972; Johnson and Sayre, 1973a; Espey and Winslow, 1974) have found that the mean annual flood increases with an increase in the degree of imperviousness, but that the effect diminishes with increased flood return period. For example Johnson and Sayre (1973a) found that conversion of a rural catchment with no impervious area to one with 100 percent impervious increased the magnitude of the 2 year flood 9 times and the 50 year flood 5 times. The variability of the results from these and similar investigations may be attributed to spatial variation in response, the use of different analytical methods and the difficulty of fitting a flood frequency curve to a non-stationary record. It is significant that an attempt at synthesis (Hollis, 1975) should have fitted a curve to the published data by eye, when statistical methods would have indicated an insignificant relationship because of the variability of the results. Four examples of empirical formula will be discussed. The first relates the mean annual flood to catchment characteristics, the second relates a mean annual flood from a rural catchment to the flood to be expected from that catchment if it were urbanised. The third predicts the discharge to be expected from floods of varying recurrence intervals and the fourth predicts the peak discharge to be expected from a given storm event and catchment conditions.

Anderson (1970) defined the mean annual flood for the Washington area from the discharge records of 44 catchments ranging from 1 to 1476 Km^2 and with 0 to 30 percent of the catchment impervious.

$$\overline{Q} = 230K.A.^{0.82} \; T^{-0.48} \tag{3.7}$$

Where $\overline{Q}$ is the mean annual flood (cusecs)
K is the coefficient of imperviousness (1.0 + 0.015I)
I is the percentage of the catchment impervious
A is the catchment area (square miles)
T is the time lag (hours).

For ungauged catchments lag time is calculated from an equation according to the state of urban development within the catchment, for example:

a natural catchment: $$T = 4.64 \; (L/\sqrt{s})^{0.42} \tag{3.8}$$

a fully developed catchment: $$T = 0.56 \; (L/\sqrt{s})^{0.52} \tag{3.9}$$

Where L is the distance (miles) along the main channel from outlet to divide
S is the slope (feet/mile) of the main channel from 0.1L to 0.85L upstream of the outlet.

The mean annual flood is scaled to the T year flood using the urbanised growth curve. A growth curve is obtained by plotting return period (x-axis) against the quotient (growth factor) of the flood of a given return period and the mean annual flood (y-axis). The different rates of increase of flood magnitude with return period for varying degrees of urbanisation can clearly be seen using this dimensionless method of analysis. The urbanised growth curve was found by interpolating between the natural growth curve and the 100 percent urbanised growth curve, which was assumed to be equal to the rainfall growth curve.

$$R_i = \frac{R_n + 0.01I \; (2.5 \; R_{100} - Rn)}{1.0 + 0.015 \; I} \tag{3.10}$$

Where R_i is the flood ratio for a given I
I is the percentage of catchment impervious
R_{100} is the growth factor for a 100 percent impervious catchment
R_n is the growth factor for a natural basin.

This method of using a natural catchment as a benchmark was adopted by the Institute of Hydrology. Packman (1977) derived a scaling factor to increase a mean annual flood estimated for a rural catchment using the Flood Studies Report (NERC, 1975, I, 341) to take account of an increase in urbanisation.

$$\frac{\bar{Q}_u}{\bar{Q}_r} = (1 + \text{URBAN})^{2n} . \; \left(1 + \text{URBAN}\left(\frac{24}{PR_r} - 0.3\right)\right) \qquad (3.11)$$

Where $\bar{Q}$ is the mean annual flood (cumecs)
u is a subscript for urban
r is a subscript for rural
URBAN is the fraction of the grey area on a 1:63360 map
n is the rainfall continentality parameter defined from average annual rainfall (NERC, 1975, II, 26)
PR is the percentage response.

This equation, used in a simulation computer program, predicted the expected flattening out of the growth curve for less frequent floods.

Johnson and Sayre (1973b) found that the peak discharge of a T year flood could be estimated for the Houston area using equation (3.12)

$$Q_T = aA^b \, I^c \qquad (3.12)$$

Where Q_T is the peak discharge of a flood with a return period T (cusecs)
A is the catchment area (square miles)
I is the percent of impervious catchment
a,b,c are constants for the Houston region, Table 3.1

Table 3.1 Regional relations for the T year flood, Houston region (Johnson and Sayre, 1973b)

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The decrease of the c exponent with increasing return period simulates the observed diminishing effect of urbanisation on flood return period with higher return periods.

Finally, Narayana et al (1970) analysed 200 events from 50 rural catchments and 193 events from 20 urban catchments and found that the peak discharge (Q_p) was best predicted by:

$$Q_p = 0.777\ W_1 S_1 U_1 \tag{3.13}$$

Where W_1 is the watershed parameter:

$$W_1' = \frac{A^{0.738}\ S^{0.206}}{L^{0.042}} \tag{3.14}$$

S_1 is the storm parameter:

$$S_1 = \frac{P^{1.016}\ P_{30}^{\ 0.179}}{D^{0.26}} \tag{3.15}$$

U_1 is the urbanisation parameter:

$$U_1 = \frac{1}{\phi^{1.28}\quad cf^{0.797}} \tag{3.16}$$

- A is the catchment area (acres)
- S is the main channel slope (percent)
- L is the main channel length (miles)
- P is the total storm rainfall (inches)
- P_{30} is the maximum 30 minute rainfall (inches)
- D is the storm duration (hours)
- ϕ is the Espey and Winslow (1969) factor
- cf is the watershed imperviousness factor ($1.0 - R_i$)
- R_i is the ratio of impervious catchment to pervious catchment.

This is a relatively complex empirical formula which requires a considerable amount of data. It could be argued that a better use of such data would be to calculate an analytical expression of catchment response, preferably one which gave the complete hydrograph. However the method probably represents a good first approximation of the expected peak discharge, due to the large number of storms (393) on which the formula was based.

These empirical formulas only give a value for peak discharge and are therefore of use at the planning stage to give an indication of the expected changes due to urbanisation. They are unable to take account of spatial variations in development and cannot give the response hydrograph and are therefore unable to help in the more complex design problems such as for flood balancing ponds.

Modifications to the rational method took place in two areas, first the introduction of a rainfall profile with varying intensity and second consideration of catchment shape. The first of these improvements used the time-area diagram and the second was based on the 'tangent method'. These modifications to the rational method have been summarised as Table 3.2 (Dooge, 1973, 194).

Table 3.2 Modifications of the rational method

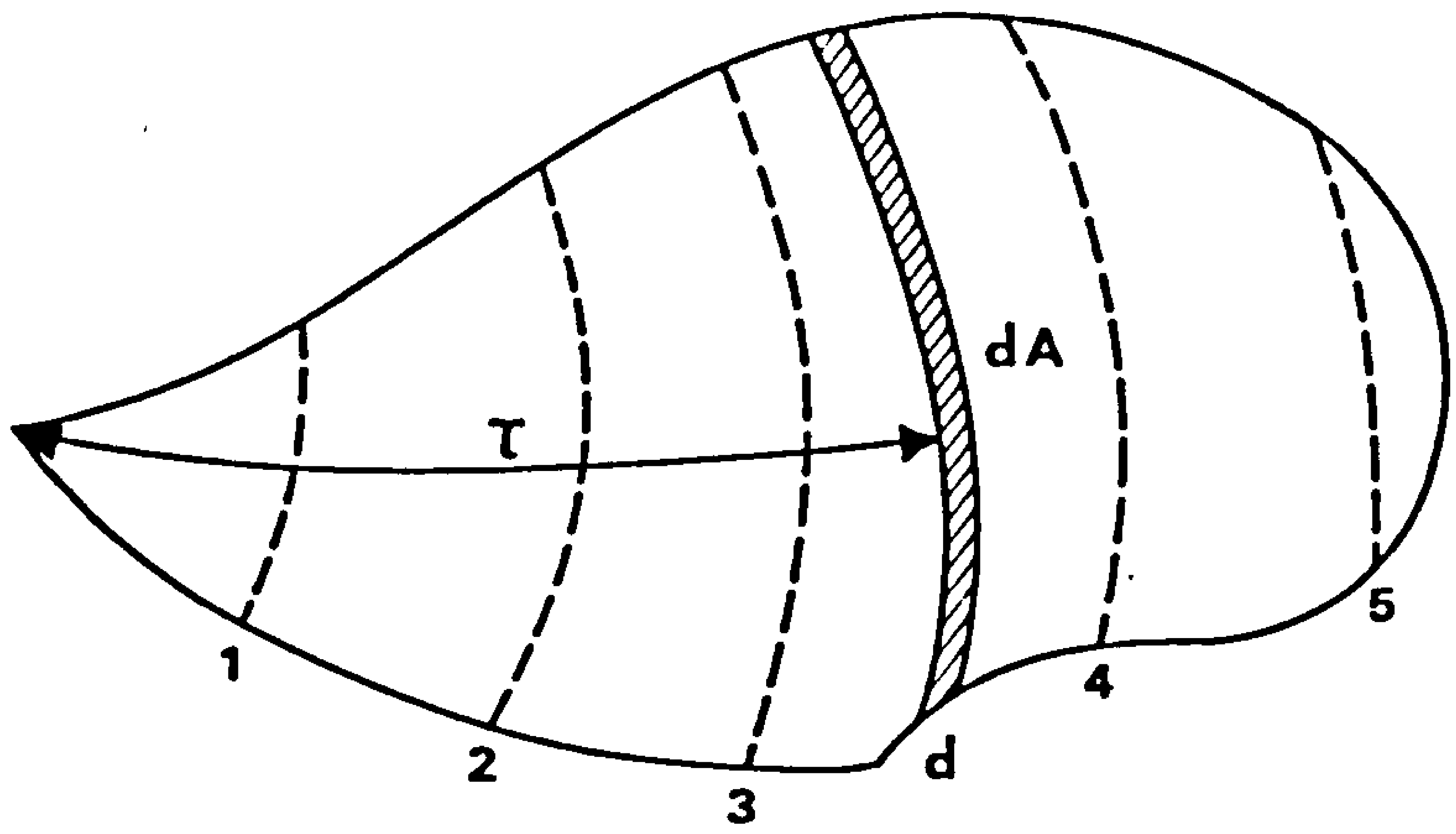
	Rainfall Assumption		
Type	Area	Intensity	Authors
Classical rational method	Full	Uniform	Mulvaney (1851)
			Kuichling (1889)
			Chamier (1897)
			Lloyd-Davies (1906)
Time-area	Full	Hypothetical	Ross (1921)
			Rousculp (1927)
			Ormsby (1933)
		Critical	Hawken and Ross (1921)
			Judson (1932)
		Typical	Coleman and Johnson (1931)
			Jens (1948)
'Tangent' methods	Partial	Uniform	Reid (1926)
			Riley (1931)
			Escritt (1950)
			Munro (1956)

A time-area diagram is constructed by drawing isochrones of equal travel time on a map of the catchment. The position of the isochrones can be calculated by estimating the time required for the overland flow to reach the river, and then calculating the time of flow in the river by Mannings formula. The area of the catchment with a time of travel less than or equal to t is plotted against that value of t to produce the time-area curve (Fig. 3.1).

Ross (1921) proposed a modification to the time-area diagram and constructed the time-area-concentration diagram (Fig. 3.2), this corresponds to the instantaneous unit hydrograph in the unit hydrograph method (Dooge, 1973, 82). Combining this modified diagram with a hypothetical storm permitted the production of the total hydrograph. This method is the graphical equivalent of the numerical procedure called convolution. Therefore, 11 years before the introduction of the unit hydrograph, engineers were using synthetic unit hydrographs, derived by using Mannings formula to estimate the time-area-concentration diagram. Further improvements to the method included the maximising of the peak discharge by shuffling the rainfall intensity pattern (Hawken and Ross, 1921) which inevitably led to overdesign. Coleman and Johnson (1931) recommended that rainfall profiles typical of the area under investigation should be used, a far-sighted recommendation, and one which only recently has begun to gain acceptance.

Application of the time-area methods to urban catchments were unsuccessful because "It is possible for a greater runoff to be produced from only part of a catchment under the effect of the rainfall applicable to the time of concentration of that part than would come from the whole catchment as a result of the storm applicable to the total time of concentration. The tangent methods are a means of finding what part of the catchment and what time of concentration produce the greatest runoff" (Escritt, 1964, 49). The tangent method derives its name from the method of calculation which consists of drawing a tangent to the time-area curve. Watkins (1962) investigated the use of the tangent method for 50 storms from 5 sewer catchments. Although reasonable results were obtained on symmetrical catchments, inconsistent results were obtained on non-symmetrical catchments. It was concluded that "...only in the few cases where the rational formula underestimates the rate of runoff will the tangent methods reduce the error and, since it is impossible during design calculation to forecast when this will occur, it is considered that there is no justification for using the tangent method" (Watkins, 1962, 71).

The improvements to the rational method did little to improve the accuracy of the peak discharge estimates, but it did provide the total hydrograph. The chief source of error of methods based on the time-area curve was that no account was taken of storage effects and therefore when combined with 'real', non-uniform rainfall profiles,



Isochrones of travel time

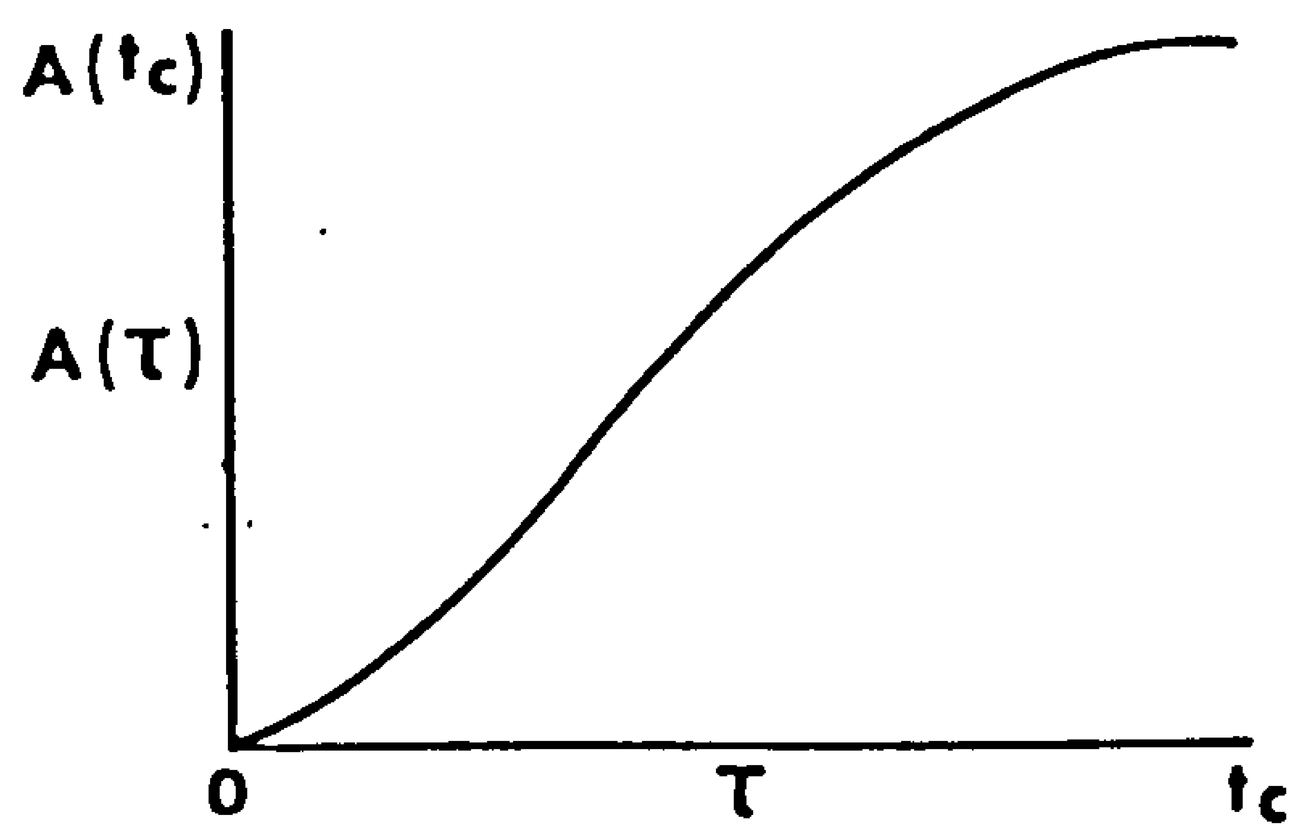


Fig. 3.1 Time-area curve

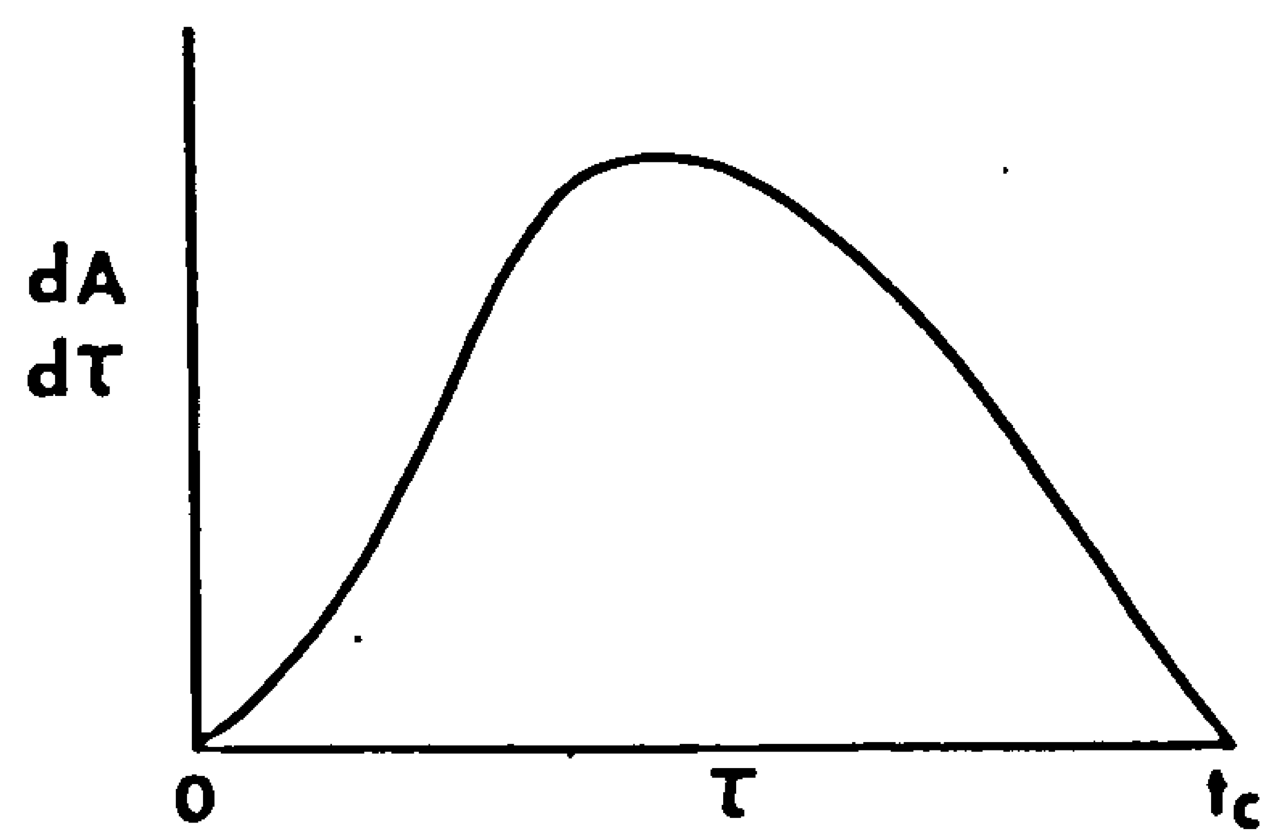


Fig. 3.2 Time-area-concentration diagram

overprediction was common. Escritt (1964) argued that the use of a uniform profile tended to underpredict the peak and therefore, taken together, peak discharge estimates should be of the correct order. This situation was unsatisfactory and a superior solution to the problem was found in the unit hydrograph method, first proposed in 1932.

3.3 The Unit Hydrograph

The unit hydrograph, proposed by Sherman (1932a), and its subsequent development, represent one of the most significant developments in twentieth century hydrology. The T hour unit hydrograph (TUH) is defined as the hydrograph of response runoff resulting from one inch of effective rainfall falling uniformly over the catchment, at a uniform rate for a unit duration of T hours. A detailed description of the TUH is presented in section 4.2. Dooge (1973, 84-89) has pointed out that the unit hydrograph method was used for 25 years without recognition of the assumptions of the model, namely that the rainfall-runoff relationship is assumed to be linear and time-invarient. When this was realised the powerful techniques of systems analysis and linear mathematics were applied and achieved considerable advances.

"The unit hydrograph method has been used quite extensively in urbanising catchments because it affords a method of separating the increase in rapidity of response being evidenced by a shortening of the time base and a corresponding heightening of the peak of the unit hydrograph" (Packman, 1977, 4).

One of the earliest applications of the TUH to urban hydrology for design was by Horner and Flynt (1936) in St. Louis, U.S.A., who investigated 3 sewer catchments each of less than 2ha. The runoff was calculated by multiplying the rainfall increment by the TUH ordinates (expressed as a percentage of the volume of the TUH) and the impermeable area. This was claimed to yield predictions in "good agreement" with the observed hydrographs.

The TUH has also been used in many analytical studies, however the wide variation of results produced by such studies precludes any precise conclusion other than an increase in urbanisation tends to increase the TUH peak. The results of three studies are presented to indicate the range of response to be expected.

Crippen (1965) analysed a 99.15ha. catchment in California which between 1959 and 1963 saw urbanisation rise from 0 to 45.3 percent. The TUH peak increased by 1.4 times (5.1 to 7.0 cumecs). Crippen points to the problems caused by an irrigated golf course just above the mouth of the catchment and concludes "...it is difficult to separate the components and thus ascribe a specific part of change to a specific detail of basin alteration" (Crippen, 1965, D198). This problem is common to all lumped parameter analyses and is only soluble by using distributed models.

Seaburn (1969) using records from 1956 to 1962 for a 80.29 Km^2 catchment on Long Island, New York, U.S.A., analysed the effect of extending the area connected by sewers to the river by 530 percent. The TUH peak increased during this time by 2.5 times (8.86 to 21.97 cumecs).

Hollis (1974) investigated a 21.4 Km^2 clay catchment at Harlow, Essex, U.K., using records from 1950 to 1968. The TUH peak increased during this period by 4.6 times (1.73 to 7.95 cumecs) for an increase of 16.6 percent of the basin being paved.

Whilst such studies as these indicate the effect of urbanisation on the response of a catchment, the findings are applicable only to that catchment. In order to progress towards the goal of being able to predict the discharge hydrograph for an ungauged catchment it is necessary to derive relationships between the TUH characteristics and catchment characteristics, that is, to regionalise the relationship.

The first synthetic hydrograph was proposed by Sherman (1932b). The method consisted of using a derived TUH from a catchment with similar characteristics to the ungauged catchment, but with the time axis adjusted in proportion to the square root of the ratio of the two areas.

There are a considerable number of morphometric variables available for the hydrologist, Newson (1975, 3-6) lists 44 which were considered for the U.K. Flood Studies Report (N.E.R.C., 1975). A literature search (Newson, 1975, 17) indicated that the most popular morphometric variables used by hydrologists were catchment area, stream channel slope and length of the main stream. The TUH parameters used in a catchment correlation study may be the peak discharge, a measure of the delay imposed on the excess rainfall by the catchment, the recession coefficient and the time base.

Five examples of synthetic unit hydrographs which were used for urban catchments will be discussed. First, the Flood Studies Report

(N.E.R.C., 1975) TUH is based on a triangular approximation of the unit hydrograph. This is constructed from a single parameter, time to peak, Tp (Packman, 1977, 5)

$$Tp \;=\; 46.6L^{0.14}\, S^{-0.38}\, RSMD^{-0.42}\, (1 + URBAN)^{-1.99} \qquad (3.17)$$

where L is the main stream length (Km.)
S is the mainstream slope from 0.1L to 0.85L upstream
RSMD is the one day, once a year effective rainfall (rainfall SMD) which is related to average annual rainfall (NERC, 1975, I, 461).
URBAN is the fraction of the 'grey' area of the catchment as defined on a 1:63360 O.S. map.

The peak discharge, Qp, of a 10mm. 100 Km^2 1 hour TUH is given by:

$$Qp \;=\; 220/Tp \qquad (3.18)$$

and the time base, TB, by:

$$TB \;=\; 2.52.Tp \qquad (3.19)$$

This procedure ensures that the unit volume of the TUH is maintained. Subsequent work at the Institute of Hydrology has defined the losses relating to a given total rainfall (Stoneham and Kidd, 1977) and the distribution of these losses through time (Kidd, 1978b). The equation to predict the volume of runoff is:

$$TOTPQ = -33.6 + 0.924\ PIMP + 53.4\ SOIL + 0.0649\ UCWI \qquad (3.20)$$

where TOTPQ is the percentage runoff (runoff expressed as mm. over the total catchment area)
PIMP is the percentage impervious
SOIL is the soil index (NERC, 1975, I, Section 4.2.3)
UCWI is the urban catchment wetness index (125 + 8API5 - SMD).

The equation was derived from 368 storms from 14 catchments. The equation indicates that antecedent conditions exert a strong control on runoff volume. None of the urban simulation models make allowance for antecedent wetness, which is an area for further improvement. The constant proportional loss model was adopted after comparative tests using 188 storm events from 16 urban catchments. It has the added advantage of being relatively simple and is popular among the relevant literature.

The Flood Studies Report (NERC, 1975) critics have pointed out that the method should be limited to catchments where urban development does not exceed 25 percent and that the method is unable to predict the effects of increasing urbanisation (Hamlin and O'Donnell, 1977). The errors incurred by assuming a symmetrical rainfall profile have been quantified (Kelway, 1977). The continuing research programme (Institute of Hydrology, 1979, 80) to collect rainfall and runoff data from sub-catchments in Bracknell, Southampton, Stevenage and Wallingford will be used to extend the preliminary results and go some way to answering the first two criticisms. The use of symmetrical storm profiles is a product of the requirement to produce a nationally applicable model and is largely unavoidable. Any subsequent refinement is dependent upon the availability of local data. Inspite of these criticisms the work completed by the Institute of Hydrology is the most important research into urban hydrology in the U.K. because a nationally applicable method has been derived.

An alternative synthetic unit hydrograph method is to make the TUH dimensionless in terms of lag time, T_L (Hall, 1974, 1977a, Packman 1974). The functional form of the curve being

$$U_t . T_L = f\ (t/T_L) \tag{3.21}$$

where U_t is the ordinate of the actual TUH at time t. Following the work of Carter (1961) and Anderson (1970) lag time was correlated with basin ratio (Z) for different levels of urbanisation (Hall, 1974)

$$Z = L/\sqrt{S} \tag{3.22}$$

where L is the main stream length (Km.)

S is the main stream slope from 0.1L to 0.85L.

The equations to predict T_L were derived from storm events from 7 catchments. For rural areas with less than 4 percent impervious,

$$T_L = 3.61\ Z^{0.42} \tag{3.23}$$

For urban areas with 25 percent impervious,

$$T_L = 1.1Z^{0.5} \tag{3.24}$$

Hall (1977a) tested the accuracy of the equations on two North London catchments and found that they were unreliable. One catchment exhibited a T_L three times the predicted value for its state of development. It was concluded that simple indices are unable to account for the effect of urbanisation which is characterised by inter-catchment variations in storage potential (lakes and sewers) and by the spatial distribution of the urban development.

The third synthetic unit hydrograph method was derived to study the effects of urbanisation on the hydrological characteristics of 33 Texan catchments (Espey, 1965; Espey and Morgan, 1964; Espey et al, 1969). The TUH parameters of peak discharge and time of rise were predicted from area, length of main channel, slope, percentage impervious and an empirical factor, phi. In the initial analysis phi was "...an urbanisation factor...defined to consider the efficiency of the conveyance system" (Espey et al, 1969, 218). A subsequent analysis using 50 catchments led to the phi index being, "...redefined to reflect changes in channel roughness as a result of seasonal variations in vegetation" (Espey et al, 1969, 223).

This study points to the dangers of a non-scientific approach. First, the need to change the definition of phi indicates that it is merely an empirical factor, which being less than unity serves to reduce the time of rise for urbanised streams. Second, the value of phi is selected by the operator and consequently is unreproducable. Further, because the latter study uses an additive form ($\phi_1 + \phi_2 = \phi$) it is clearly impossible for the final phi value to possess any physical relevance.

The final two studies include an index of the causative storm characteristics. These methods may be used to model individual storm events, rather than the preceding three studies which attempt to predict the hydrograph of an unspecified frequency.

Wheater et al (1978) analysed 112 storm events from eight catchments in the Gloucester region, U.K., and developed a 6 parameter, trapezoidal synthetic unit hydrograph. The value of the soil moisture deficit was included in the analysis in an attempt to explain the observed variability of the TUHs for a given catchment. The omission of storm duration, a significant parameter (e.g. Rao et al, 1972) led to an over-complex method which is unable to accurately predict the TUH for a given storm. The six regression equations to predict the TUH parameters explain between 34.46 and 80.46 percent of the variance in the parameters and consequently are unreliable. It is presumably for this reason that no results of testing the equations on catchments not included in the analysis are presented.

Schulz and Lopez (1974) analysed storm events from nine urban catchments in the Denver metropolitan area. The TUH parameters were correlated with storm and catchment parameters. The changes in the TUHs were related to the decrease in catchment lag time (T_{LC}). Two equations were used:

$$T_{LC} = 210.233 \frac{T_{10}^{0.031} \; E_{RF}^{0.323} \; H_R^{0.80}}{U^{0.342}} \qquad (R^2=0.613) \quad (3.25)$$

$$\frac{Q_p}{A} = \frac{32.223}{T_{LC}^{0.867} \; V_{RF}^{0.036}} \qquad (R^2=0.687) \quad (3.26)$$

where T_{LC} is the lag time (hours)
Q_p is the peak discharge (cusecs)
T_{10} is the duration of the total storm rainfall (minutes)
E_{RF} is the volume of rainfall excess (inches)
H_R is the hydrologic radius (A/catchment perimeter)
A is the catchment area (square miles)
U is the index of imperviousness $(1 + (A_I/A))$
A_I is the impervious area (square miles)
V_{RF} is the total volume of rainfall (inches)

The study did not consider any antecedent condition index, in the light of other work (Stoneham and Kidd, 1977) its inclusion would probably have increased the already high predictive power of the equations.

Research which uses synthetic unit hydrographs is moving towards models which consider the causative storm characteristics. This is a result of work (e.g. Minshall, 1960) which showed the considerable effect of the storm characteristics on the derived unit hydrographs.

3.4 Conceptual Models

The three methodologies discussed so far belong to two groups. The first group, consisting of the rational method and time-area diagrams, are based on the assumption that each catchment has a unique TUH. The second group consists of synthetic unit hydrograph methods and is based on the assumption that a catchments TUH may be obtained through the use of a single curve (the dimensionless TUH, e.g. Hall, 1974) or a set of equations (e.g. NERC, 1975). Dooge (1973, 190-208) has shown that by 1958, the two groups had coalesced into the single group called conceptual models. Conceptual models are based on the assumption that the translation and storage elements in a catchment may be separated and represented by linear channels and linear reservoirs (for further details see section 4.3).

The first group merged into the conceptual models through two papers. First, Clark (1945) suggested that an instantaneous unit hydrograph (IUH) could be derived by routing the time-area-concentration curve through a single linear reservoir. This method rectified the error introduced by the time-area methods which only considered time of travel to the exit and ignored storage effects. O'Kelly (1955) found that there was no loss of accuracy if the time-area-concentration curve was replaced by an isosceles triangle. Inter-catchment variation in TUH shape was accounted for by different storage coefficients of the linear reservoir. Therefore a two-parameter model was developed using first, the base of the triangle (T) and second, the reservoir storage coefficient (k). The method became a synthetic unit hydrograph when correlations between T and k and the catchment characteristics were developed.

The second group had already become synthetic unit hydrographs, the remaining requirement was the substitution of an equation or shape to define the TUH. This was first achieved in Japan (Sato and Mikawa, 1956; Sugawara and Maruyam, 1957) where one or two linear reservoirs were used to simulate a catchment. Nash (1958) proposed the use of the two-parameter gamma distribution because its shape resembles an IUH. The method may be considered as a cascade of linear reservoirs.

In 1959 Dooge (1959) proposed a general theory of the unit hydrograph based on combinations of linear channels and linear reservoirs. This theory included the Japanese and Nash's work as special cases and therefore represents a milestone in scientific hydrology, comparable in importance to Sherman's (1932a) paper proposing the unit hydrograph.

Conceptual models are applied by correlating the models parameters with catchment and/or storm characteristics. This was first demonstrated by Nash (1960) who analysed 90 hydrographs and their causative rainfall from British catchments. The first two moments (see section 4.3) of the IUHs derived from the 90 events were correlated with catchment characteristics:

$$m_1 = 27.6\ A^{0.3}\ OLS^{-0.3} \tag{3.27}$$

$$m_2 = 0.41\ L^{-0.1} \tag{3.28}$$

where m_1 is the first moment of the IUH

m_2 is the second moment of the IUH

A is the catchment area (square miles)

OLS is a measure of overland slope (parts per 10,000)

L is the distance from the gauging station to the catchment boundary via the main channel (miles)

The parameters of the Nash model, n and k, were derived from;

$$n = 1/m_2 \tag{3.29}$$

$$nk = m_1 \tag{3.30}$$

which are substituted in equation (3.31) to yield the catchment IUH

$$U(o,t) = \frac{1}{k\,\Gamma(n)} e^{-t/k(t/k)^{n-1}} \tag{3.31}$$

where Γ is the gamma function.

A catchment IUH may be derived for an ungauged site by calculating the values of A, OLS and L from a map of the catchment.

This method was used by Gray (1961) and Wu (1963) with the exception that the parameters n and k were calculated directly from the catchment data removing the necessity for equations (3.29) and (3.30).

The application of conceptual models to urban areas has been associated with comparisons of the suitability of these models to simulate the urban rainfall-runoff process (e.g. Rao et al, 1972; Sarma et al, 1973; Kidd, 1978b). Rao et al (1972) presented the results of analyses based on 131 storm events from eight urbanised and four rural catchments in Indiana and Texas. It was found that for catchments greater than $13Km^2$ the Nash model yielded consistently better results than the other 4 models considered. The parameters n and k varied between storms on a given catchment and to solve this, the model was linearised by including storm characteristics in the regression equations to predict n and k.

$$nk = 0.831 \frac{A^{0.458} \; T_R^{\;0.371}}{(1+U)^{1.622} \; P_E^{\;0.267}} \tag{3.32}$$

$$k = 0.575 \frac{A^{0.389} \; T_R^{\;0.222}}{(1+U)^{0.622} \; P_E^{\;1.06}} \tag{3.33}$$

where A is the catchment area (square miles)

T_r is the duration of the effective rainfall (hours)

U is the percentage impervious

P_E is the volume of effective rainfall (inches)

Using simulation techniques with these equations it is possible to predict the effect of urbanisation on peak discharge and lag time. For example, an increase of impervious area from 0 to 40 percent would halve the time to peak and increase the peak discharge value by 90 percent.

It should be noticed that there is no difference in the methods of synthesis between this study and the unit hydrograph study by Schultz and Lopez (1974). The choice of model will vary according to the problem, conceptual models do have significant advantages over unit hydrograph methods (see section 4.3), but both, when used correctly, will yield results of similar accuracy.

3.5 Developments since 1960

The 1960s saw advances in four fields. First, a proliferation of analytical techniques. Second, the development and refinement of digital simulation, this was a result of the increasing power and size of digital computers. Third, the urban hydrologist instrumented catchments in order to define relationships which hitherto were only inferred. Fourth, the late 1960s saw the rise of distributed models which were necessary to take account of spatial variations in urban growth.

Three major advances were made in analytical techniques, first, model derivation; second, objective error functions and third, optimisation.

Sherman (1932a) derived TUHs by selecting storms with near uniform intensity rainfall, of a duration T_2 and then adjusting this to time T using the S-curve (see section 4.2). The method was unsatisfactory because, first, the necessity to use uniform intensity rainfall events introduced a bias to the analysis and second, the S-curve method introduced oscillations in the recession limb of the TUH. Collins (1939) iterative method removed the first criticism but the method concentrates the error in the TUH before the maximum rainfall increment. A major improvement occurred in 1955 when Snyder (1955) introduced a method based on matrix inversion (see section 4.4.2).

This method was based on a linear algebraic representation of the rainfall-runoff process and is the first of many objective analytical techniques which may be classified into three groups (Neuman and Marsily, 1976, 253). The first group involve transformation of the input and output data into another domain. The transform of the impulse response is then found by dividing the transform of the output by the transform of the input. To determine the impulse response as a function of time involves the numerical inversion of the transform of the impulse response. Examples include harmonic analysis (O'Donnell, 1960, 1966), Fourier transforms (Blank et al 1971; Navet, 1972; Evans et al, 1972) and the Z transform (Delleur and Rao, 1971a). The second group involves the solution of the linear algebraic equations of the rainfall and runoff data by matrix inversion when the system is square (Sage and Melsa, 1971), by least squares (Snyder, 1955; Newton and Vinyard, 1967) or by linear programing (Deininger, 1968; Diskin and Boneh, 1973, 1974). The third group involves projection of the response function onto a space characterised by a set of given coordinate functions and a number of unknown parameters which can be determined by various optimisation techniques. The coordinate functions may be chosen as straight line segments (Gwyn, 1969), Laguerre functions (Dooge, 1965) or Walsh functions (Emsellem and Marsily, 1971). Alternatively they may be taken as the solution of a linear differential equation with constant coefficients (Lobert, 1969) or the Wiener-Hopf equations (Eagleson et al, 1966).

One of the chief reasons for the proliferation of these analytical techniques is the instability in the derived kernal function. Although this instability is partly due to the assumption of a linear system, of equal importance are the data errors in the rainfall and runoff data (Delleur and Rao, 1971). Different methods display different abilities to manipulate this error pervaded data (Laurenson and O'Donnell, 1969) and each method is an attempt to derive a kernal function with the minimum instability. All these methods permit the derivation of a TUH from long duration storms (greater than T hours) and ensure that the TUH is optimum for a given storm. They therefore represent a considerable advance over manual techniques which can achieve neither of these ends.

The second technique is objective error functions. These are statistical or numerical indices which measure the quality of fit of the observed and the predicted either for a single variable or for a complete hydrograph. Hydrological analysis abounds with models which are justified by a single graph which shows the 'close' fit of the predicted with the observed. This characteristic shows no signs of abatement (e.g. Mander, 1979) and does not merit the name scientific hydrology. Six error functions are discussed in Section 4.5. Objective error functions are essential if the full value of the third development, automatic optimization techniques, is to be realised.

The output predicted by a model varies according to the value of each of the parameters in the model. If the efficiency of the model is assessed by an objective error function, then the optimum values of the models parameters are those which minimise the error function. Optimisation is indispensable for multi-parametric models. The problems of defining optimum parameter values has been investigated with the object of improving the design of these models. An example of the problem is that multiparametric models frequently have different sets of parameters which fit the data equally well, this becomes more pronounced as the data set length decreases (Ibbitt, 1972, 72). In this situation the relationships derived between model parameters and catchment characteristics would be imprecise. Therefore the hydrologist cannot be confident that by changing a particular parameter value he can claim that this is representing the effect of a change in land use or whatever (Mein and Brown, 1978). Rosenbrocks (1960) systematic search technique has been found to be the most efficient (Ibbitt and O'Donnell, 1971).

The second major area of advance has been digital simulation. The first use of a digital computer by hydrologists was in 1958 (Lawler and Drumel, 1958) when it was used to simulate a continuous discharge record. The computer has meant that it has been possible to incorporate thresholds into the computer program, this makes such models non-linear. Whilst these models do not use large amounts of computer time they make considerable demands in their data requirements and operator time and experience.

There are a considerable number of digital simulation models for urban areas, virtually all these models have been developed to design or evaluate sewer systems, and are consequently outside the scope of this literature review. Only one model may be used to illustrate the effect of urbanisation on a catchment, this is the Stanford Watershed Model (Crawford and Linsley, 1966). James (1972) presents an example of its usage for urban runoff prediction. A continuous synthetic hydrograph was developed for 1905 to 1963, using the model which was calibrated for the urban conditions prevailing between 1959 and 1963. The synthesised record was analysed to yield quantitative information on the effect of urbanisation on the catchment with reference to a rural base period. This showed that the runoff volume increased by 2.29 times and surface runoff increased by 5 times in wet years but 125 times in dry years.

Proponents of continuous simulation models have argued that they are the answer to the urban hydrologists problems of the frequency of peak discharges and the frequency of discharge volumes. This, it is claimed, may be achieved by conversion of a long rainfall record into a discharge record using a calibrated model. However, there is a problem of defining what length of record is necessary in order to calibrate the model. This is complicated on urbanising catchments where the response is non-stationary. Finally a record of 100 years of rainfall is insufficient to determine either the volume characteristics of the discharge (Wallis and O'Connell, 1973) or the probability of peak flows (Wallis et al, 1974). Storm event models based on conceptual models are now being used to simulate both of these properties (Butters et al, 1977) and they are considerably cheaper to use than continuous simulation models. An alternative use of these models is to calibrate the model on a rural catchment which is subsequently urbanised. The model is calibrated on the rural catchment and then run using the rainfall data from the period of urbanisation. The divergence in discharge record can then be interpreted as indicating the effect of urbanisation (Hollis, 1970). Again similar results can be obtained by using lumped parameter models and examining the progressive change of the model parameters. This later method is cheaper and does not depend on an accurate calibration in the rural stage.

Simulation models are unsuitable to predict the effects of increasing urbanisation on a catchment. Conceptual models are able to produce the same results at less expense. A further reason for this decision is that the only available model has 23 parameters, far too many for practical purposes.

The movement of the hydrologist to field work was the result of the need to obtain sufficient data from which to derive relationships. A considerable amount of the research has been directed towards the definition of the inlet hydrograph. This is the hydrograph which is propagated by the gutter upstream of a sewer inlet. Watkins (1962) derived this inlet hydrograph by assuming that impervious areas contribute 100 percent of the rainfall and pervious areas 0 percent.

The time of entry, the time taken for the runoff to enter the inlet, was taken to be between 2 and 4 minutes. The method produced tolerable results inspite of these tenuous assumptions and in the absence of the requisite data it was not possible to refine the method.

Two field studies produced equations to derive an inlet hydrograph for a small catchment, however the techniques they developed are important for the contribution they made to hydrology.

The inlet method was developed from work undertaken by the John Hopkins University (Boch, 1958; Kaltenback, 1963; Miller et al, 1972; Viessman, 1968; Viessman et al, 1962, 1970). The method is based on calculating an inlet hydrograph for individual sub-catchments draining to a sewer inlet. The hydrographs are summed for each pipe length and the result is attenuated by an attenuation formula. The inlet hydrograph is based on a 1 minute unit hydrograph represented by: (Viessman, 1968)

$$Q_{max} = (1 - e^{-1/K}) \tag{3.34}$$

$$Q_r = Q_{max} e^{-r/K} \tag{3.35}$$

where I is the inflow to the reservoir (effective rainfall)
Q_{max} is the peak outflow rate occurring at $t = 1$ minute
r is the time variable equal to $(t - 1)$
t is the time from the beginning of effective rainfall

The factor K is derived from,

$$K = 0.0015 \; (nL_2/0.015 \sqrt{S} \;)^{0.66} \tag{3.36}$$

where K is the storage coefficient
n is the Mannings roughness coefficient
L_2 is the maximum gutter flow distance
S is the mean gutter slope (feet/feet).

The second example is a simple method based on a dimensionless hydrograph for designing storm sewers draining motorway cuttings (Swinnerton et al, 1972, 1973a, 1973b). The dimensionless hydrograph is defined by three parameters, first, the peak rate of runoff (R_p), second, the time of rise (T_R) and third, the time of recession (T_F). The hydrograph is scaled using two polynomial equations, one for the rising and one for the falling limb of the hydrograph. The independent parameter is the quotient of the time from the start of the hydrograph and T_R, and the quotient of the time after the peak and T_F respectively. The mean absolute percentage error of the peak was 22, compared to 80 by the rational method and 83 by the unit hydrograph method. The success of the method points to the importance of allowing for variations in hydrograph shape as a result of storm characteristics.

Studies such as the two outlined above were based on the assumption that it was possible to define an inlet hydrograph in terms of catchment characteristics. With the development of the technology to measure the inlet hydrograph (Blyth and Kidd, 1977) it was possible to initiate a rigorous field study of urban runoff. Three European countries are prominent in this field, first the United Kingdom. The research programme has been co-ordinated by the Institute of Hydrology and the data is derived from 7 catchments used by the Road Research Laboratory and a further 7 which are continuing to yield results (Helliwell et al, 1976, 85). The results of the work has led to the development of a model which will predict an inlet hydrograph from a given rainfall on a given catchment (Helliwell et al, 1976; Kidd, 1976, 1978a; Kidd et al, 1977; Stoneham et al, 1977). The equation to predict the volume of runoff has been discussed (equation 3.20). The model is based on a non-linear reservoir, defined by,

$$S = kQ^n \quad \text{(dynamic equation)} \tag{3.37}$$

$$\frac{dS}{dt} = I - Q \quad \text{(continuity equation)} \tag{3.38}$$

where S is the storage
Q is the discharge
I is the inflow to the reservoir
d is a time interval
k, n are coefficients

The values of k and n are found by performing a Newton-Raphson iteration on a finite difference formulation of equations (3.37) and (3.38) combined. The non-linear reservoir proved to be more accurate than either the linear reservoir or the time of entry model. Sensitivity analysis suggested that a satisfactory fit is possible with any number of pairs of k and n. Application of the Chezy equation in a situation where the depth of flow is small in comparison to the width (i.e. a gutter) yielded an n value of 0.67. Further analysis fixed the k value as; (Kidd, 1978b, 182)

$$k = 0.0166\text{AREA}^{0.358} \tag{3.39}$$

where AREA is the catchment area (m^2).

The Swedes initiated an urban research project in 1972 with a laboratory model (Gottschalk et al, 1976). This led to the development of a gully meter and the establishment of nine instrumented sub-catchments in the Klostergarden suburb of Lund (Arnell et al, 1977; Falk et al, 1976, 1978; Lindh, 1976). The data was analysed by a non-linear reservoir with time lag (τ),

$$S_{t-\tau} = kQ_t^{\,n} \qquad \text{(storage equation)} \tag{3.40}$$

$$\frac{ds}{dt} = I-Q \qquad \text{(continuity equation)} \tag{3.41}$$

This is a three parameter model (k, n, τ). The parameter τ was found by trial and error to be equal to one minute. Re-writing equations (3.40) and (3.41) and including a further parameter S_d, the depression storage, assumed to be the amount of surface storage needed before any runoff will be produced, yielded,

$$Q_t = k\,(S_{t-1}-S_D)^n \tag{3.42}$$

The value of S_d was fixed as:
S_d = 1.0 mm for $S_w < 1\%$
S_d = 0.5 mm for $1\% < S_w < 3\%$
S_d = 0.3 mm for $S_w > 3\%$
where S_w is the average weighted slope.

Due to the strong internal correlation between k and n, only one parameter, k, was optimised. The parameter n was fixed at 1.5. Parameter k was predicted by,

$$k = 6.7 + 5.51\,S_w \tag{3.43}$$

The model may be used predictively from a knowledge of average weighted slope and surface area.

The third research project is based at Lelystad, one of the new towns in the polders in the Lake Ijassel, Netherlands. A housing area (2.0ha.), a parking lot (0.7 ha.) and a shopping centre (2.5 ha.) have been instrumented (Berg 1974, 1976, 1978; Berg et al 1977a, 1977b; Kloet et al, 1977). The surface response was modelled by the Nash model (Nash, 1960). Whilst this research has not produced predictive equations it has produced results which suggest that an increase of the percentage of flat roofed area tends to reduce and delay the peak flow. Further, flat roofs have a larger storage potential than streets and therefore result in a greater loss at the beginning of the rainfall.

None of the models used in these three projects are directly related to reality and therefore advances can only be made through the collection of a large number of storm events from a variety of catchments. On the assumption that the relationships are independent of climate, an International Workshop on rainfall-runoff processes over urban surfaces was held at the Institute of Hydrology in April 1978 (Kidd, 1978b). Data from the U.K., Sweden and the Netherlands produced a data base of 188 storm events from 16 catchments. Exhaustive analyses showed that a models parameters could be estimated from catchment slope and overland flow length, the former explaining a greater percentage of the variance. In terms of overall fit the linear reservoir, the non-linear reservoir and the Muskingum models were superior to the others. In terms of peak estimation, there was nothing to choose between any of the models. The unit hydrograph model performed poorly compared to the other models in all respects.

The implications of this Workshop are that, given reliable data, it should be possible to define the effect of urbanisation on the hydrological response of a catchment. Earlier sections of this review have shown that although many regionalisation studies have internal consistency there is no consistency between studies. It is significant that the workshop should conclude that "...the choice of surface routing model is less critical than the manner in which it is utilised" (Kidd, 1978b, 58). It is inevitable that no universal relationship will be derived if un-scientific analytical techniques are used. Major advances in this field are possible, the Workshop is a step in the right direction.

Several projects to investigate the effect of catchments undergoing urbanisation are in progress in the U.K. The Institute of Hydrology is studying two catchments in Milton Keynes. One of which is to remain rural for several years and the other to be completely urbanised. The Scottish Development Department is gauging the 26 Km^2 Calder Water catchment at Stonehouse to study the effects of a new town on runoff quantity and quality. The University of Newcastle and the Northumbrian Water Authority are studying runoff from a developed industrial estate at Cramlington. None of these projects has produced any published results. One project to produce published results is based on a 0.26 Km^2 catchment on the north-east margin of Exeter and is maintained by Exeter University. The catchment has been monitored continuously since 1968, urbanisation which accounted for 12.25 percent in 1974 commenced in 1971. Analysis of 659 storm events (Gregory, 1974) indicated that urbanisation increased peak discharge by 2 times, decreased lag time to half its rural value and increased runoff between 1.1 and 3 times its rural value. These results do not take account of variations in storm and antecedent wetness, consequently more information could be obtained from this data if some analytical model were applied, for example, unit hydrograph or conceptual models.

The final example is of a study with unrealised potential (Williams, 1976). A 11.74 Km^2 catchment of the Wairu Creek, Auckland, New Zealand was transformed from bush and scrub vegetation in 1959 to 75 percent urban in 1975. The study derived an equation to relate the urban development to peak discharge.

$$y = 0.4797 - 0.0244x \tag{3.44}$$

where y is the peak discharge (cumecs)
x is the percent urban
($R = 0.7665$, $R^2 = 0.5876$)

This equation is meaningless because Williams used the observed peak discharge which is a function of the total rainfall, the storm duration and the antecedent wetness as well as the urban development. Unless studies use the appropriate analytical techniques and use them correctly the conclusions they draw are invalid.

The rise of distributed models is associated with the effect of the location of urban development within a catchment, which has two effects. First, urban development in previously non-contributing areas of the catchment will increase percentage runoff more than urban development in previously contributing areas. Second, urban development in the upper part of a catchment may generate a hydrograph which coincides with the hydrograph generated by the rural lower part of the basin producing an enlarged single peaked hydrograph. Alternatively urban development in the lower part of the basin may cause the urban response to pass before the rural response, producing a double peaked hydrograph.

Lumped parameter models, used analytically, can identify the variations in contributary area. Packman (1978, 688) derived unit hydrographs for the Silk Stream in North London and found that a flashy response associated with a low percentage response derived mainly from the downstream urban area whereas the longer response, associated with a high percentage response occurred when the upstream undeveloped area was contributing significantly. It is not possible to define the unit hydrograph for a given urban configuration and a given storm duration and amount, therefore the method is unsuitable for design and prediction.

The solution to the problem is to consider the catchment as temporally and spatially varying in its ability to generate runoff. Lumped parameter models are one-dimensional, the catchment is assumed to be uniform in its reponse to a uniform input. Two-dimensional models allow runoff source areas to vary either temporally or spatially and the three-dimensional models allow for spatial and temporal variations in runoff source areas. As the number of dimensions increase so the data requirements and the computer time increases. Three-dimensional models, for example Freeze's (1972a, 1972b) physics based model of the rainfall-runoff process, are only suitable as research tools for studying individual storm events on a sub-catchment. A model of this type, SHE (Systeme Hydrologique Europeen) has been under joint development by the Danish Hydraulic Institute, SOGREAH of Grenoble and the Institute of Hydrology, U.K. since 1976. This distributed model will be based on physical principles and is designed to predict the hydrological consequences of land-use changes whether or not streamflow records exist for a basin (Institute of Hydrology, 1979, 10).

Two-dimensional models are suitable for most design situations where only a rudimentary amount of information is available. Examples of both the temporal and spatial variable models will be discussed.

Several authors have recognised the different characteristics of paved and unpaved areas to convert rainfall to runoff. Terstriep and Stall (1974) extended the Transport and Road Research Laboratory (TRRL) model (Watkins, 1962). The TRRL model assumes that the runoff hydrograph is produced by the convolution of the effective rainfall profile and a time-area-concentration curve for the impervious part of the catchment. Terstriep and Stall (1974) used two effective rainfall profiles and two time-area-concentration curves, one each for the paved and unpaved parts of the catchment. The total runoff hydrograph is obtained by summing the runoff hydrographs from the paved and unpaved areas. Similar methods of deriving two hyetographs for use with urban runoff models have been used by Stubchaer (1975), Watt et al, (1975), Wittenberg (1975) and Diskin et al (1978).

Diskin et al (1978) used the Nash (1960) model to route the two rainfall hyetographs and tested the model on 11 storms from a 9.1 Km^2 catchment in Tucson, Arizona. It was concluded that there was a linear relationship between the volume of runoff and the degree of urbanisation but that the relationship for the peak discharge is non-linear because of the difference in the time to peak of the two hydrographs from the paved and unpaved areas. The success of this technique depends on the ability of the temporal variations to account for both the temporal variations in the amount of runoff generated and the spatial variations in the location of the urban development within the catchment. The method simulates the effect of differences in response from the paved and unpaved and for the relative amounts of each but it cannot account for differences in distribution. Consequently an optimisation scheme on the routing model parameters is necessary to take account of the spatial distribution of urban development. It is significant that these optimised parameter values were not used on other catchments, errors would have inevitably occurred because of differences in spatial distributions of urban development.

A more successful method has been to use unit hydrographs representing subcatchment response and to route these to give total catchment response. This method is able to take account of variations in the percentage response and differences in response form caused by subcatchment characteristics.

Two American models HEC-1 and TR-20 have been developed by the U.S. Army Corps of Engineers (1971) and the U.S.D.A. Soil Conservation Service (1972) respectively. The effect of urban development and flood alleviation works may be estimated by using previously published results to adjust the unit hydrograph shape, the percentage response and the temporal distribution of that loss.

Packman (1978) presents the results of a study which combined the unit hydrograph rainfall-runoff model (NERC, 1975) with a variable parameter Muskingum-Cunge channel routing method called FLOUT (Price, 1977a). The results obtained using the distributed model were superior to those obtained by the lumped unit hydrograph model, though Packman (1978) does not provide any quantitative evidence other than simple graphs. It was observed that the distributed method tended to overpredict the beginning of the hydrograph and underpredict the recession. This suggests that some improvement in the effective rainfall separation method is required and points to the problem that the methods presented in the Flood Studies Report (NERC, 1975) were for lumped catchments and consequently are inappropriate for a universal application to subcatchments consisting of entirely non-standard topography.

The distributed catchment model represents an area with considerable potential for improvements, not least of which is an investigation of the effect of a catchment type on the rainfall loss and the temporal distribution of that loss.

3.6 Conclusion

This chapter has traced the historical development of linear mathematical methods in hydrology with special reference to the unit hydrograph.

The unit hydrograph method, traditionally dated as 1932 (Sherman, 1932a), was shown to have originated in 1921 (Ross, 1921), when a graphical equivalent of the numerical procedure of convolution was proposed.

The unit hydrograph method was used for 25 years without recognition of the assumptions of the method, namely that the rainfall-runoff relationship is assumed to be linear and time-invarient. When this was realised, systems analysis and linear mathematics were applied and achieved considerable advances. This, combined with the introduction of the digital computer led to the establishment of objective procedures for TUH derivation which produced a rational science, such that it was possible for a second hydrologist to use another hydrologist's data and reach exactly the same conclusion. In 1959, a general theory of the TUH was presented (Dooge, 1959). This paper, included all previous published research as special cases, and is the fundamental paper for hydrological analyses using conceptual models, in the same way as Sherman's (1932a) paper forms the basis for empirical analyses.

The TUH method, in both its empirical and conceptual forms, represents a fundamental element of modern analytical hydrology. The unit hydrograph is the basis of national design methods in the USA (USDA, SCS, 1972) and the UK (NERC, 1975). It has been used to trace the effect of the growth of urbanisation on hydrological response (e.g. Hollis, 1974), as a component of distributed models (e.g. Packman, 1978) and to examine the effect of storm characteristics on catchment response (e.g. Schulz and Lopez, 1974).

In situations with minimal data resources and in the continuing absence of a thorough understanding of the hydrological system, the TUH continues to be an important method for the analytical hydrologist.

The following chapter presents a thorough description of the theory of the TUH and methods of TUH identification. Chapters 5 and 6 present the results of analysing 96 storm events by the TUH method.

Chapter 4

THE MATHEMATICAL ANALYSIS OF LINEAR HYDROLOGICAL SYSTEMS

4.1 Definitions and an introduction to modelling the hydrological system

A hydrological system may be defined as the inter-relating of an input and an output in some time reference (Dooge, 1973, 4). A system comprises of first, variables which assume different values when measured at different times (e.g. daily rainfall) and second, parameters which characterise a hydrological system and are time-invariant (e.g. catchment area) (Clarke, 1973, 1).

A linear system may be defined mathematically by a linear differential equation. The principle of superimposition applies and the response of the system is only a function of the system itself. An example of a linear system representation is the unit hydrograph. A non-linear system may be defined mathematically by a non-linear differential equation, the system's response depends on the system itself and the input intensity. An example of a non-linear system representation is the equation of gradually varied open channel flow. Although most systems are non-linear, the lack of sufficient information to define them has led to the substitution of linear systems.

A time-invariant system is one in which the input-output relation is not dependent upon the time at which the input is applied to the system. Most hydrological systems are time-variant, for example, there are seasonal variations in hydrological variables throughout the year. The mathematical advantages of assuming a system to be time-invariant are such that these variations are usually ignored.

Finally, the input and output variables and parameters may be either lumped or distributed. A lumped system is one in which the spatial variations either do not exist or have been ignored. For example, the input is said to be lumped if the rainfall input to a model is considered to be spatially uniform. The behaviour of lumped systems is governed by ordinary differential equations with time as the independent variable. The behaviour of distributed systems is governed by partial differential equations.

A model is a simplified representation of a complex system. Hydrological models may be either first, physical models which are a scaled down facsimile of the full scale prototype, second, analogue or third, mathematical, in which the behaviour of the system is represented by a set of equations. Physical models are completely unsuitable for modelling runoff from urban catchments and analogue models have been superseded by mathematical models. This thesis used linear mathematical models exclusively. These may be classified according to whether they are either conceptual or empirical. Each type may be subclassified according to the degree of linearity and lumping.

The distinction between conceptual and empirical hydrological models is according to whether or not the model was suggested by consideration of the physical processes acting on the input to produce the output. The distinction is artificial since, "... models originally formulated without reference to physical processes may have parameters for which some physical interpretation can be found, whilst models apparently firmly based in physics may contain obviously empirical components" (Clarke, 1973, 7). In the literature the term 'empirical' corresponds to 'analytical' and 'black-box', whereas 'conceptual' corresponds to 'synthetic'.

The unit hydrograph, a linear mathematical model, was selected (Section 1.4) to analyse the flood hydrology of the four urbanised south London catchments.

> "The operation of the whole watershed system in converting precipitation excess to direct storm runoff is summarised in the form of the unit hydrograph. We are not concerned with arguments about whether there is, or is not, such a thing as interflow, nor with arguments as to whether overland flow actually occurs; and if it does, what the friction factor is. We may overlook our ignorance of the physical laws actually determining the process in various parts of the hydrological cycle. We may ignore the problem of trying to describe the complex watershed with which we are dealing; we do not have to survey the whole watershed by taking cross-sections on every stream as we would have to do if we wanted to solve the problems by classical hydraulics. Instead we assume that all the complex geometry in the watershed and all the complex physics in the hydrologic cycle is described for that particular watershed by the unit hydrograph" (Dooge, 1973, 6).

This chapter presents a thorough description of the analytical methods which were used to derive the unit hydrographs. Following a presentation of the theory of the unit hydrograph, the methods of deriving unit hydrographs by matrix inversion and harmonic analysis are described. The conversion of the total rainfall into effective rainfall and the separation of the discharge hydrograph into slow and rapid response runoff are described together with several indices used throughout the thesis. Finally the results of an evaluation of the best method of deriving the unit hydrograph are presented.

The methods described in this chapter were used to analyse 96 storm events from four catchments. The results of this analysis are presented in Chapters 5 and 6.

4.2 The unit hydrograph

The unit hydrograph concept states that for a given land use, an initial catchment moisture condition and a unit depth of excess rainfall, uniformly spread in both time and space, a discharge hydrograph of a certain shape will be produced corresponding to the particular duration of excess rainfall that caused the discharge.

This may be defined mathematically as follows. For a given input of unit depth of excess rainfall p(t), for a duration T, the output discharge q(t) may be represented by the catchment response function (or kernal) u (T,t). This function is the T-hour unit hydrograph (TUH). Thus, if p(t) represents, as a function of time t, an input to a system and q(t) the corresponding output from the system, then the linear, time-invarient system may be defined by

$$A_n \cdot \frac{d^n p}{d t^n} + A_{n-1} \frac{d^{n-1} p}{dt^{n-1}} + \ldots\ldots . A_1 \cdot \frac{d p}{d t} + A_o p = q \qquad (4.1)$$

Equation 4.1 possesses the property known as the principle of superimposition common to all linear systems. If the output q_1 is caused by the input p_1, and q_2 is caused by p_2, then the output q_3 is given by p_3 or the sum of $(p_1 + p_2)$. Thus to obtain the catchment response function U(T,t) equation 4.1 must be solved for the values of p(t) and q(t). By maintaining the boundary conditions the solution will be identical for each event considered if the system being described by P and Q is linear.

The unit hydrographs derivation and synthesis can be formulated in terms of black-box analysis and synthesis because of the assumptions of its linear, time-invarient operation (Dooge 1959, 1973, 1977).

If the record of effective precipitation is represented by a histogram, then each element of the input may be represented by a pulse function, P_T, defined by

$$P_T\,(t-sT) = \frac{1}{T} \quad \text{where } sT < t < (s+1)\,T \qquad (4.2a)$$

$$P_T\,(t-sT) = 0 \quad \text{elsewhere} \qquad (4.2b)$$

where T is the standard interval and s is the number of intervals elapsed before the beginning of the interval in question.

If the area of effective precipitation for each time interval is given by $P(sD)$ then the volume of input may be represented by

$$p(t) = \sum_{s=0}^{\infty} P(sT) \cdot P_T\,(t-st) \qquad (4.3)$$

The unit hydrograph $U_T(t)$ has been defined as the response to a unit volume of rain falling uniformly in a unit period T, and hence must be the output corresponding to the input $P_T(t)$.

Assuming time-invariance, the output corresponding to $P_T(t-sT)$ is $U_T(t-sT)$, and assuming linearity the output due to $\beta(sT).P_T(t-sT)$ is $\beta(sT).U_T(t-sT)$. The relationship between the input, defined by equation 4.3, and the output may be represented by

$$q(t) = \sum_{s=0}^{\infty} P(sT).\ U_T(t-sT) \tag{4.4}$$

Equation 4.4 is a continuous function and since both the input and output are obtained at discrete time intervals, equation 4.4 may be re-written as

$$q(T,t) = \sum_{s=0}^{\infty} P(sT).\ U_T(tT-sT) \tag{4.5a}$$

$$q(t) = \sum_{s=0}^{\infty} P(s).\ U_T(t-s) \tag{4.5b}$$

Equation 4.5 is known as the convolution integral, and means that the output at any given sampling point is found by the convolution of the input and the kernal function corresponding to the particular time interval.

To derive the kernal function, 4.5b is re-written as

$$q_i = \sum_{j=0}^{\infty} P_j\, U_{i-j} \tag{4.6}$$

Differentiating equation 4.6 yields a matrix form of the relationship

$$Q = PU \tag{4.7}$$

where Q is the vector of outputs (discharge), U is the vector of the unknown unit hydrograph ordinates and P is the matrix of inputs (rainfall).

Equations 4.3 to 4.6 suggest that the set of linear equations to be solved is infinite however by assuming that no output can occur before the corresponding input and that the input is isolated, equation 4.6 may be written as

$$q_i = \sum_{j=0}^{j=i} P_j\, U_{i-j} \tag{4.8}$$

in which the number of equations to be solved is finite.

A reduction of the input duration T, whilst maintaining a constant input volume causes the input rate to increase. This produces a TUH with an earlier and higher peak, Fig. 4.1.

As the duration decreases the TUH shape reaches a limiting form which at T = 0 is called the instantaneous unit hydrograph (IUH), represented by U(0,t). The IUH has been called the idical response to the unit impulse and the impulse response of the system.

The IUH may be derived graphically by the S-hydrograph or S-curve. The S-curve is defined as the hydrograph of surface runoff produced by a continuous effective rainfall at a constant rate. The S-curve can be used to convert a TUH of one duration to another. An S-curve is produced from a TUH of any duration T_1 by an accumulation procedure. A TUH of a new unit period T_2 is derived by displacing the S-curve by the required amount, subtracting the ordinates of the two S-curves and adjusting the volume. This procedure may be represented by

$$U_{T_2}(t) = \frac{S(t) - s(t - T_1)}{T_1} \tag{4.9}$$

The limit of equation 4.9, when T = 0, may be represented by

$$U_T(t) = \frac{d}{dt}\left[S(t)\right] \tag{4.10}$$

Equation 4.10 is the equation of the IUH, which may be expressed in continuous form by

$$q(t) = \int_0^\infty P(\tau).\ U(t - \tau).d\tau \tag{4.11}$$

in which the output is the continuous convolution of the input and the kernal function (IUH). A flood hydrograph may be predicted using an IUH and a storm with a given duration T and a given volume of excess rainfall by first deriving a TUH and then convolving the TUH with the effective rainfall. The TUH is derived by

$$U(T,t) = \frac{1}{T}\int_{t-T}^{t} U(0,t).dt \tag{4.12}$$

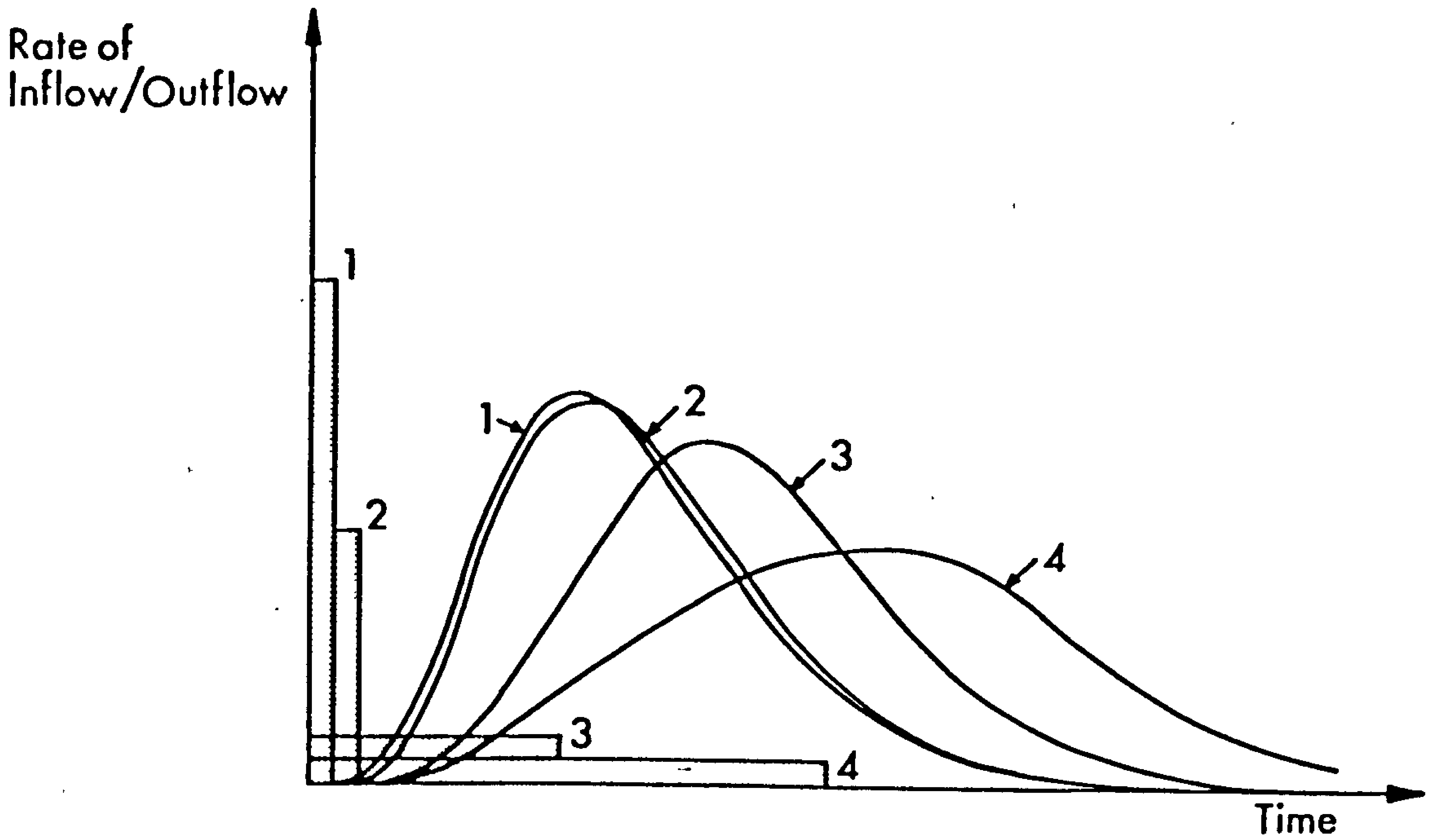


Fig. 4.1 Shape of the TUH for various values of T, maintaining a constant input volume (after Leur, 1966, 32)

4.3 Conceptual models

Conceptual models are mathematical representations of a system where the system's structure is described a priori. These models are based on the assumption that the translation and storage elements in a catchment may be separated and represented by linear channels and linear reservoirs respectively. A review of some of the results obtained using these models has been presented in section 3.4. There are two advantages of using these models to represent the unit hydrograph.

First, the conceptual model provides automatically a number of constraints on the form of the derived unit hydrograph. Unit hydrographs derived by conceptual models consist of positive ordinates and a smooth shape. This avoids the need for subjective smoothing and adjustment which is necessary for unit hydrographs derived by inversion methods (matrix inversion and harmonic analysis).

Second, "...by using a conceptual model with a small enough number of parameters, it is possible to concentrate the information content of the data into this small number of parameters and thus increase the chances of a reliable correlation with catchment characteristics" (Dooge, 1977, 86).

This section continues with a description of the linear reservoir and the linear channel and concludes with a description of the Nash (1960) cascade model as an example of the use of conceptual models.

A linear reservoir may be defined by

$$S = KQ \tag{4.13}$$

where the storage (S) is directly proportional to the outflow (Q) and a storage coefficient (K). The rate of change of storage is a function of the inflow and the outflow in a given time interval, this may be represented by a continuity equation

$$I-Q = \frac{dS}{dt} \tag{4.14}$$

Substituting 4.13 in 4.14 and setting both Q and t to zero yields

$$Q = I\ (1-e^{-t/K}) \tag{4.15}$$

If t is set to infinity, equation 4.15 becomes Q = I, which is the equilibrium condition. If the inflow terminates at time t_n since the outflow began, then equation 4.16 represents the outflow at t in terms of discharge Q_n at t_n

$$Q = Q_n e^{-\tau/K} \tag{4.16}$$

where $\tau = t-t_n$, which is equal to the time since inflow terminated.

For an instantaneous inflow which fills a reservoir of storage S, equation 4.14 yields Q = S/K, and since 4.16 gives the outflow, the outflow from an instantaneous inflow may be represented by

$$Q = \frac{S}{K} e^{-t/K} \tag{4.17}$$

For a unit input of S = 1, the IUH of a linear reservoir may be represented by

$$U(t) = \frac{1}{K} e^{-t/K} \tag{4.18}$$

A linear channel is a channel in which the time T required to translate a given discharge Q, of any magnitude, through the channel of length x is constant. Therefore the outflow function is identical in shape to the inflow function (Fig. 4.2).

If a segment of inflow of duration ΔT and volume S is routed through a linear channel the outflow Q may be represented by

$$Q = S\,\delta(t, \Delta t) \tag{4.19}$$

where $$\delta(t, \Delta t) = \frac{1}{\Delta t} \tag{4.20}$$

for $0 \leq \tau \leq \Delta t$ and $t = \tau + T$, otherwise it is zero.

τ is the time measured from the beginning of the segment.

When Δt approaches zero equation 4.20 becomes the impulse function $\delta(t)$ or the Dirac-delta function, which represents the IUH of a linear channel.

The precise shape of the IUH is difficult to obtain analytically from discrete interval data, however Nash (1960) demonstrated that it was possible to derive fundamental properties of the IUH shape using the moments of its area. The moment relationships involve no approximation and are a property of time-invariant, linear systems. The moments of an IUH can be found directly from the recorded discharge and precipitation using equations 4.21 and 4.23.

The first moments about the origin of the impulse response (U'_1), the input (p'_1) and the output (q'_1) of a system are related by

$$U'_1 = q'_1 - p'_1 \tag{4.21}$$

In continuous form this may be represented as

$$U'_R(f) = \int_0^\infty f(t) \cdot t^R \cdot dt \tag{4.22}$$

where R is the value of the moment.

The first moment of the IUH is equal to the distance between the centre of the input and the centre of the output, which is the catchment lag. The remaining moments are dimensionless.

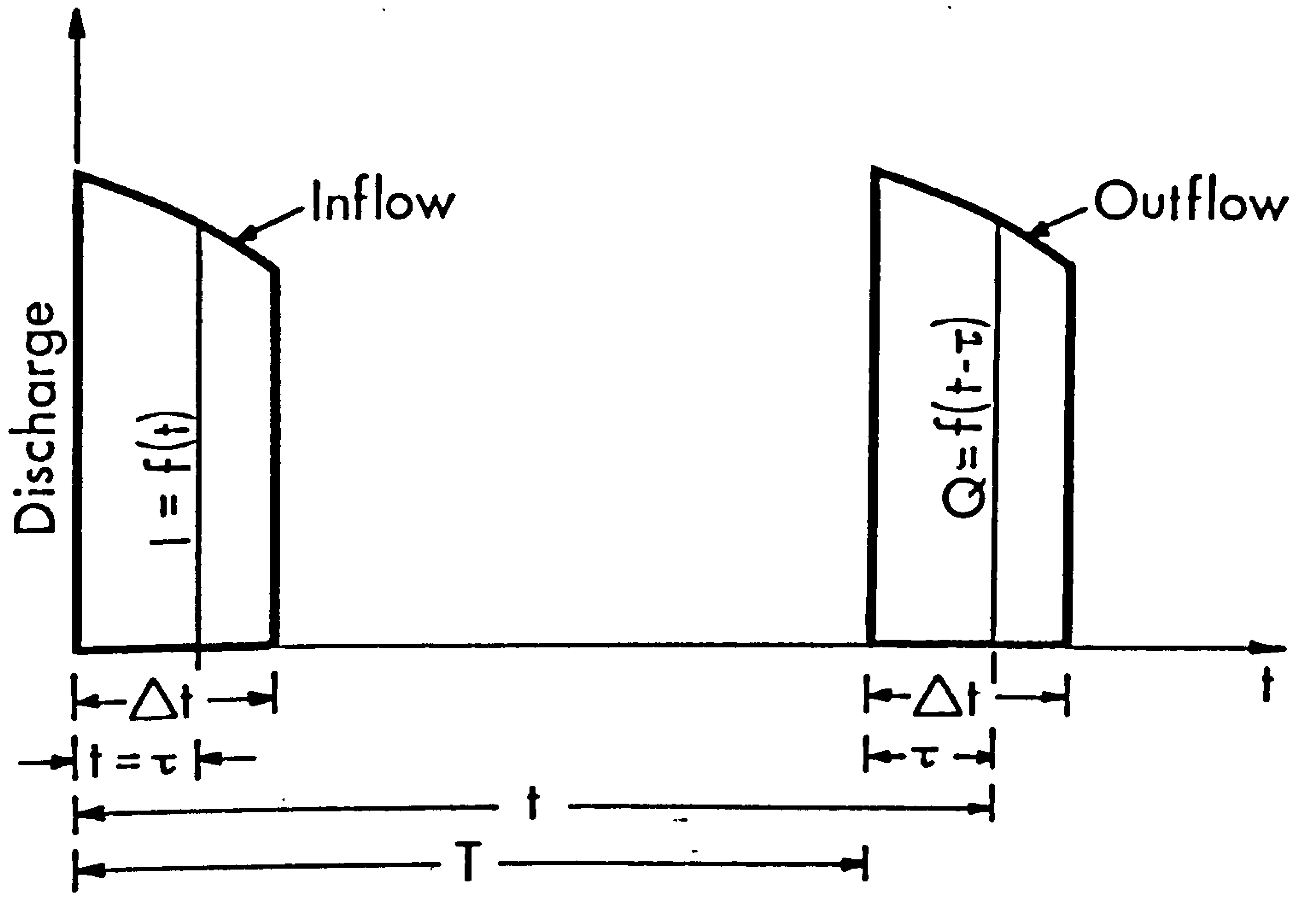


Fig. 4.2 Routing a segment of a hydrograph through a linear channel (after Chow, 1964, 28)

The second moments about the centroid U_2, p_2 and q_2 are related by

$$U_2 = q_2 - p_2 \tag{4.23}$$

Which in continuous form may be represented as

$$U_R(f) = \int_0^{\infty} f(t) \cdot (t - U'_1)^R \, dt \tag{4.24}$$

Nash used the concept of routing effective rainfall through a series of identical linear reservoirs. The IUH of such a system is a gamma density function of the form

$$U(0,t) = \frac{1}{K\,\Gamma(N)} \cdot \left[\frac{t}{K}\right]^{N-1} \cdot e^{-t/K} \tag{4.25}$$

In which $\Gamma(.)$ is the gamma function.

In equation 4.25, N is the number of linear reservoirs and K is the storage factor. For equation 4.25, the values of N and K are derived from the observed moments of the precipitation and discharge data

$$N = \frac{(U'_1)^2}{U_2} \tag{4.26a}$$

$$K = \frac{U_2}{U'_1} \tag{4.26b}$$

The observed values of N and K vary from storm to storm. To use the model as a predictive tool it is necessary to determine which storm and catchment parameters affect the conceptual model parameters. The standard method is a multiple regression analysis which will produce equations of the following form (after Rao et al, 1972, 1213).

$$NK = 0.831 \; \frac{A^{0.458} \; T^{0.371}}{(1+U)^{1.622} \; P^{0.267}} \tag{4.27a}$$

$$K = 0.575 \; \frac{A^{0.389} \; T^{0.222}}{(1+U)^{0.622} \; P^{1.06}} \tag{4.27b}$$

where A is the catchment area, T the duration of rainfall, (1+U) is the ratio of paved area to the total area and P is the volume of rainfall excess.

Although the above description has been concerned with the Nash model the principles of relating the moments of the discharge and the precipitation to the parameters of a model, which are in turn related to catchment and storm variables, holds for all linear conceptual models.

The fitting of conceptual models is made easier by deriving shape factors which are dimensionless moments and are defined as,

$$S_R = \frac{U_R}{(U'_1)^R} \qquad (4.28)$$

where U_R is the Rth moment about the centre and U'_1 is the first moment about the origin.

The first shape factor measures lag time, the second a coefficient of the variation of lag time and the third a coefficient of skew.

The conceptual models used in this thesis were fitted using shape factors (Appendix 4).

4.4.1 Methods of unit hydrograph derivation

Prior to 1939 all TUHs had been derived by selecting a storm event where the rainfall was uniform in time and space and the associated discharge hydrograph was comparatively smooth. The ordinates of the response hydrograph were divided by the excess rainfall which yielded a TUH with T equal to the duration of the rainfall. The TUH was then adjusted to any desired duration using the S-curve method (see section 4.2 for further details). The method is subjective unless a criterion of acceptable fit, between the observed and simulated hydrographs, is objectively defined.

Collins's (1939) iterative method was a distinct improvement on this approach. Wilson (1969, 140) has described the method as follows,

> " The ...method requires the initial selection of a set of coefficients...of the unitgraph. This distribution graph is then applied to the various rain periods, excepting the largest one, and the resulting discharge subtracted from the actual discharge to obtain a set of 'residuals'. These residuals should represent the discharge of the unitgraph applied to the largest rain. If correspondence is poor, the initial coefficients are altered and another trail is made. By a series of converging approximations, the residual graph is made to correspond with the assumed distribution graph".

Computerised versions of the Collins method have been developed (e.g. Body, 1959). However, if the unit hydrograph is constrained to be causative (no negative ordinates) then Collins's method concentrates any error in the matching of the runoff hydrograph into the portion of the hydrograph due to rainfall before the maximum rainfall increment.

The rapid expansion in the availability of digital computers in the 1960s stimulated the development of numerous methods to derive the unit hydrograph (see Section 3.5). Two methods, each of a different analytical type, were used in this thesis. The first, matrix inversion, consists of the solution of the linear algebraic equations of the rainfall and runoff data. The second, harmonic analysis, involves the transformation of the input and output data into another domain. The transform of the impulse function is then found by dividing the transform of the output by the transform of the input. To determine the impulse response as a function of time involves the numerical inversion of the transform of the impulse response.

The conceptual models parameters were optimised by equating the shape factors of the conceptual model to the estimated shape factors of the unit hydrograph as derived from the rainfall and runoff data. This method has been explained in the previous section with reference to the Nash (1960) model.

The remainder of this section describes the mathematics used to derive a unit hydrograph by matrix inversion and harmonic analysis.

4.4.2 The use of matrix algebra to derive a unit hydrograph

Given a response runoff hydrograph and an excess rainfall hyetogram the number of ordinates of the unit hydrograph which will be produced from these data sets is given by

$$j = n-p + 1$$

where j is the number of unit hydrograph ordinates, n is the number of response runoff ordinates and p the number of excess rainfall ordinates.

A set of equations (4.29) may be written to define the relationship between the excess rainfall ordinates p(i), the unit hydrograph ordinates U(i) and the discharge ordinates q(i),

$$P_1U_1 = q_1 \quad (4.29a)$$
$$P_2U_1 + P_1U_2 = q_2 \quad (4.29b)$$
$$P_3U_1 + P_2U_2 + P_1U_3 = q_3 \quad (4.29c)$$
$$P_3U_2 + P_2U_3 + P_1U_4 = q_4 \quad (4.29d)$$
$$P_3U_3 + P_2U_4 = q_5 \quad (4.29e)$$
$$P_3U_4 = q_6 \quad (4.29f)$$

The problem is that, given the values of p and q, solve the equations (4.29) for the values of U. This may be achieved by forward substitution. By solving the first equation for U_1; substituting this value in the second equation and solving for U_2, substituting for the value of U_1 and U_2 in the third equation and solving U_3, and so on until all the unknown values of U are determined. Errors in p and q will cause incorrect values of U, which, with repeated substitution can cause significant errors in the later ordinates of the unit hydrograph. Barnes (1959), noting that a calculation which is unstable in one direction is usually stable if taken in the reverse direction, suggested that the estimated effective rainfall should be adjusted until the unit hydrograph obtained in forward and reverse directions was substantially the same.

A superior method to either of these is to solve the equations simultaneously and to ensure that the unit hydrograph is optimum according to a given objective error function. Snyder (1955) developed such a method which became known as the least squares unit hydrograph method. The method was used in this thesis and is described below.

To solve equations 4.29 it is necessary to re-write them into matrix notation. Matrix P is the excess rainfall matrix, consisting of n rows and j columns. Matrix U is a vector of j rows, consisting of the unknown unit hydrograph ordinates.

$$P = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_1 & 0 & 0 \\ P_3 & P_2 & P_1 & 0 \\ 0 & P_3 & P_2 & P_1 \\ 0 & 0 & P_3 & P_2 \\ 0 & 0 & 0 & P_3 \end{bmatrix} \qquad U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

The product of P and U is equivalent to the vector Q, of n rows, containing the rapid response runoff ordinates

$$P \cdot U = Q \tag{4.30}$$

or

$$\begin{bmatrix} P_1U_1 & 0 & 0 & 0 \\ P_2U_1 & P_1U_2 & 0 & 0 \\ P_3U_1 & P_2U_2 & P_1U_3 & 0 \\ 0 & P_3U_2 & P_2U_3 & P_1U_4 \\ 0 & 0 & P_3U_3 & P_2U_4 \\ 0 & 0 & 0 & P_3U_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$

To solve equation 4.30 for matrix U requires an inverse matrix $[P^{-1}]$ for the excess rainfall matrix. For a matrix to have an inverse it must be both square and possess a non-zero determinant. To achieve this matrix P is multiplied by its transpose, P^T. The transpose is constructed by interchanging the columns and rows. Substituting in equation 4.30 yields

$$P^T . P . U = P^T . Q \tag{4.31}$$

Equation 4.31 generates the simultaneous normal equations for a least squares solution. The product $P^T.P$ produces the sums of squares and cross products of the independent variables. $P^T.Q$ forms the Σxy terms

$$\hat{Q} = U_1P_1 + U_2P_2 + \ldots\ldots\ldots\ldots U_jP_j \tag{4.32}$$

Equation 4.31 is solved by Guassian elimination, the inverse matrix of $P^T.P$ is added to both sides

$$(P^T. P)^{-1} \ (P^T. P) \ U = (P^T. P)^{-1} \ P^T. Q \tag{4.33}$$

This solves the equation explicitly for U

$$U = (P^T. P)^{-1} . P^T. Q \tag{4.34}$$

For a given set of excess rainfall values and response runoff ordinates the derived unit hydrograph ordinates are the 'best' in the least squares sense, that is the sum $\sum_{i=1}^{i=n} (Q_i - \hat{Q}_i)^2$ is minimised.

Snyder (1955) proposed a method to produce 'improved' estimates of the loss rate through time. The objective error function is defined as the difference between the observed response runoff and the simulated response runoff using the TUH, derived from equation 4.34, and the excess rainfall. The 'improved' rainfall data ~~is~~ are used to produce another estimate of the TUH and the process is repeated. Although this method will regenerate the observed storm to a high degree of accuracy, it is only of use in cases where the rainfall and discharge data ~~is~~ are available. In the design situation a technique for estimating rainfall excess is still required and there is considerable strength in the argument that consistency in technique must be maintained in the development and subsequent design stages.

The correction procedure uses the same types of matrix procedures as those used to derive the TUH.

Each value of rainfall excess is assumed to be inaccurate, and a correction term, e, is therefore added to each increment.

$$P(t) = P(t) + e \tag{4.35}$$

Vector E consists of the correction terms. By multiplying matrices P and E by the first estimate of U, from 4.34, the response runoff hydrograph vector Q is generated.

$$U.P + U.E = Q \tag{4.36}$$

Substituting $\hat{Q}$ for U.P in 4.36 produces

$$\hat{Q} + U.E = Q \tag{4.37a}$$

$$\text{or} \quad U.E = Q - \hat{Q} \tag{4.37b}$$

Equation 4.37b now has a form similar to 4.30 and the method of solution is the same. To solve 4.37b for E an inverse matrix of U is required, U^{-1}.

Premultiplying 4.37b by U^T produces a square matrix which can be inverted

$$E = (U^T.U)^{-1} . U^T. \quad (Q-\hat{Q}) \tag{4.38}$$

Equation 4.38 provides the corrections which should be made to vector P. The corrections are based on the ordinate by ordinate fitting error, $Q-\hat{Q}$.

4.4.3 <u>The use of Harmonic Analysis to derive a unit hydrograph</u>

A function x(t), can for all values of t, be represented by the sum of an infinite series of other functions (O'Donnell, 1966, 79):

$$x(t) = \sum_{m=0}^{\infty} c_m.f_m(t) \tag{4.39}$$

in which the coefficients, c_m, are constants. An example of this is a polynomial function of the nth degree (Wilson and Kirkby, 1975, 76):

$$y = a_n x^n + a_{n-1} x^{n-1} + \dots a_o \tag{4.40}$$

$$= \sum_{m=0}^{n} a_m x^m \tag{4.41}$$

Therefore it is possible to represent the input (p), the output (q) and the kernal function (u) of a time-invariant linear hydrologic system as: (O'Donnell, 1966, 80)

$$p(t) = \sum_{m=0}^{\infty} (c_p)_m \cdot f_m(t) \tag{4.42}$$

$$q(t) = \sum_{m=0}^{\infty} (c_q)_m \cdot f_m(t) \tag{4.43}$$

$$u(t) = \sum_{m=0}^{\infty} (c_u)_m \cdot f_m(t) \tag{4.44}$$

The problem is that given either p(t) and q(t) or $(cp)_m$ and $(cq)_m$ is it possible to derive either u(t) or $(c_u)_m$ respectively. Several types of orthogonal functions are available to solve this problem namely Fourier and Laplace Transforms, and Laguerre functions. These solutions are based on the derivation of algebraic relationships between the input and output functions and the unknown kernal function. The kernal function derived by continuous orthogonal functions is the instantaneous unit hydrograph. When continuous methods are adapted for use with discrete data the kernal function is a TUH, where T is equal to the sampling interval. This thesis used Harmonic analysis, which is the discrete solution of the continuous Fourier transform.

A direct runoff hydrograph (q_i, i=1,2,...n) with ordinates at equal intervals of t hours can be represented exactly at every point by the Fourier series:

$$q(t) = a_o + \sum_{r=1}^{p} \left[a_r \cdot \cos r \frac{2\pi t}{n} + b_r \sin r \frac{2\pi t}{n} \right] \tag{4.45}$$

Equation (4.45) indicates that the direct runoff hydrograph is the sum of a sine and cosine curve of a given frequency, both of which are in phase with the effective rainfall. "If, then, one were to break down rainfall excess and runoff curves for a given catchment into their harmonic sine and cosine components of various frequencies, all based on a common fundamental frequency, one might expect that the four components of any one frequency (two from the rainfall curve and two from the runoff hydrograph) would be linked in some way" (O'Donnell, 1960, 548).

The coefficients of equation (4.45) are given by:

$$\left.\begin{aligned} a_r &= \frac{2}{n}\sum_{k=1}^{n} q_K \cdot \cos K \cdot \frac{2\pi r}{n} \quad \underline{\text{but}} \quad a_o = \frac{1}{n}\sum_{k=1}^{n} q_K \\ b_r &= \frac{2}{n}\sum_{k=1}^{n} q_K \cdot \sin k \cdot \frac{2\pi r}{n} \end{aligned}\right] \qquad (4.46)$$

Using coefficients (a,b) for p(t), (A,B) for q(t) and (c,d) for the TUH, O'Donnell (1960, 549) has shown that substitution into the convolution equation (Eq. 4.47)

$$q(t) = \sum_{s=0}^{\infty} p(s) \cdot U_T\,(t-s) \qquad (4.47)$$

yields the following relationships between the coefficients:

$$\left.\begin{aligned} c_r &= \frac{2}{n} \cdot \frac{a_r A_r + b_r B_r}{a_r^2 + b_r^2} \quad \underline{\text{but}} \quad c_o = \frac{1}{n} \cdot \frac{A_o}{a_o} \\ d_r &= \frac{2}{n} \cdot \frac{a_r B_r - b_r A_r}{a_r^2 + b_r^2} \end{aligned}\right] \qquad (4.48)$$

The n TUH ordinates are obtained by substitution of the coefficients (c,d) in equation (4.49)

$$U(j) = c_o + \sum_{j=1}^{p} \left[c_j \cdot \cos j \cdot \frac{2\pi i}{n} + d_j \sin j \cdot \frac{2\pi i}{n} \right] \qquad (4.49)$$

The maximum number of harmonic coefficients is equal to n, however the use of the full number of coefficients leads to high frequency oscillations in the TUH. There are two solutions to this. First, an arbitrary restriction is placed on the number of coefficients, in this thesis this was achieved by:

$$P = (\text{NSFL} - \text{NHRF} + 1)/2 \qquad (4.50)$$

where P is the number of coefficients and NSFL and NHRF are the number of direct runoff and effective rainfall ordinates respectively. The advantage of a reduction in computer time is offset by the inflexibility of the method. The number of harmonics required to obtain a smooth TUH varies from storm to storm, this leads to the second method. Early work (Hall, 1974; Packman, 1974) was based on an iterative procedure of truncation of the series down to a minimum of three harmonics subject to the condition that on reconvolution the peak discharge ordinate was within 5% of the observed peak discharge. This wasted a considerable amount of computer time and further work by Hall (1977b) derived a relationship to define the exact number of harmonics necessary to obtain an optimum solution.

Spurious oscillations near the recession of the TUH, caused by truncation of the infinite series, may be displaced away from that region to later ordinates by the addition of extra data, in the form of zeros, at the end of the real runoff data. This procedure ensures an accurate computation of the recession limb and the oscillations at the extreme portion of the recession limb of the TUH may be ignored. Ten zero value ordinates were added to each set of runoff data by the unit hydrograph derivation program LPROG6 (Appendix 3).

4.5 Objective error functions

Objective error functions are statistical or numerical indices which measure for either a single variable or for a complete hydrograph the quality of fit of the observed and the modelled. The error functions used in this thesis are among the most commonly used in the literature (e.g. Kidd, 1978b) and permit first, an objective measure of the efficiency of a particular model and second the comparison of a models performance with others in the literature.

Six error functions were used.

First, the Integral Square Error (ISE). This is a version of the least squares function scaled to a dimensionless measure of fit of the observed and modelled hydrograph. An ISE value of 0.0 percent indicates a perfect fit and higher values indicate a worsening fit. ISE is defined as,

$$\mathrm{ISE}(\%) = \left[\sqrt{\Sigma(Q_o-Q_m)^2} \,/\, \Sigma Q_o\right] \times 100 \tag{4.51}$$

where Q_o is the observed discharge

Q_m is the discharge predicted by the model.

Second, to attach more significance to the goodness of fit around the peak the ISE function was modified to,

$$\mathrm{ISE2}(\%) = \left[\sqrt{\Sigma(Q_o^2 - Q_m^2)} \,/\, \Sigma Q_o\right] \times 100 \tag{4.52}$$

The ISE2 function is identical in interpretation to the ISE function.

Third, the Partial Integral Square Error (PISE) was defined as,

$$\text{PISE}(\%) = \left[\sqrt{\Sigma(Q_o - Q_m)^2} \,/\, \Sigma Q_o\right] \times 100 \tag{4.53}$$

$$\text{for } Q_o \geq P_o/2$$

where P_o is the observed peak discharge.

This error function is identical in form and interpretation to the ISE function except that it assesses the quality of fit around the peak. A comparison of the values of the ISE and PISE functions for a given event will indicate whether or not the errors are uniformly distributed over the hydrograph. If the value of PISE is greater than that of ISE then the largest errors occur in the region of the peak.

Fourth, the Root Mean Square (RMS) error function is defined as,

$$\text{RMS} = \sqrt{\left[\Sigma(Q_m - Q_o)^2 / N\right]} \tag{4.54}$$

where N is the number of hydrograph ordinates.

This function is identical in interpretation to the ISE function. The RMS value is dimensional and consequently was more difficult to interpret than either ISE, ISE2 or PISE.

Fifth, the peak discharge error (QPE) is defined as,

$$\text{QPE } (\%) = \left[QP_m - QP_o/QP_o\right] \times 100 \tag{4.55}$$

where QP_m is the modelled peak discharge.
QP_o is the observed peak discharge.

This function indicated the error in the prediction of the peak. A negative percentage indicated an underestimate and a positive percentage an overestimate.

Finally, the time to peak error (TPE) was defined as,

$$\text{TPE (Hours)} = (TP_m - TP_o) \times \text{TINT} \tag{4.56}$$

where TP_m is the modelled time to peak (hours)
TP_o is the observed time to peak (hours)
TINT is the data time interval (hours).

This error function measured the error in time to peak in hours. A negative percentage indicated an underestimate and a positive percentage an overestimate.

A subroutine was written to calculate ISE, ISE2, PISE, QPE and TPE and was included, where necessary, in the computer programs used in this thesis.

The RMS error function was used in the original versions of the computer programs LPROG6 (Appendix 3) and PICOMO (Appendix 4).

4.6 Smoothing and adjustment of the unit hydrograph

Unit hydrographs derived by analytical methods may possess oscillations, negative ordinates and a non-zero origin. When the oscillations are mild their effect on the accuracy of the prediction of the discharge hydrograph is small. Using a perturbation analysis Blank et al (1971) found that an error in the unit hydrograph is reduced by a factor of between 0.167 and 0.04 in the resulting output.

There are four sources of error which can contribute to an unstable unit hydrograph.

First, errors may be generated by computational limitations, principally truncation and rounding. The unit hydrographs used in this thesis were derived using a CDC 7600 computer which has a single precision word length of 15 bits. It may therefore be assumed that the error introduced by computational limitations is very small.

Second, the input and output data may be subject to error. This can be due to errors in recording the stage, an inaccurate rating curve and errors associated with the baseflow separation method. Similarly the rainfall may be incorrectly measured, both spatially and temporally, and the method of calculating rainfall excess may be inappropriate. The discharge data used in this thesis was reasonably error free compared to the rainfall data, this is a source of error which could not be avoided.

Third, the catchment response may be excessively non-linear. Non-linear response can be modelled by linear models with different degrees of success depending on which analytical method is used. The temptation to use non-linear analytical models should be resisted because catchment response is not uniformly non-linear.

Fourth, errors may be increased by the inability of the analytical method to filter out errors in the data. "For...error-free data, the choice between methods is merely one of the ease of computation, and there is no reason to go beyond the direct solution of the simultaneous equations involved in the method of forward substitution. If however, there are errors in the data, or if the system is not truly linear, then the values of the ordinates obtained for the optimum linear response may vary according to the method used. The choice between methods depends not only on the convenience of computation but also on the manner in which the various methods handle errors in the data and linearise any non-linear properties of the system under identification" (Dooge, 1973, 142).

Dooge (1977, 90) presents the results of a study of the effect of a 10 percent error in the input and output data on the unit hydrograph (Table 4.1).

Table 4.1 The effect of a 10% error in data on the unit hydrograph

Unit hydrograph derivation method	Mean absolute error as percent of peak		
	Error free	Systematic error	Random error
Least Squares	.00029	6.6	21.5
Harmonic Analysis	3.4	5.3	7.8
Routed Triangle	6.8	8.1	7.7
Cascade of reservoirs	2.8	6.0	5.2

Of the algebraic methods (least squares and harmonic analysis) the harmonic analysis method proved to be the most consistently reliable. The least squares method was the best of all methods when operating on error free data, the very small error is due to computer rounding and truncation. The two conceptual models (routed triangle and cascade of reservoirs) are reasonably consistent for all types of error. The difference in performance of these two models emphasises the importance of selecting the most appropriate model for a given catchment. The ability of the four methods to identify an accurate unit hydrograph from data subject to systematic and random errors is relatively uniform, with the exception of the least squares method. This method, unlike the other three, is not constrained to produce a unit hydrograph with a realistic shape.

An unstable unit hydrograph may be adjusted to a more realistic shape by smoothing of either the unit hydrograph or the input and output data. Smoothing of the input and output data was rejected because it produces a loss of information at the beginning and end of a data set. When the data set is large (e.g. discharge) the loss is less important than when a small data set (e.g. rainfall) is smoothed. Smoothing causes an alteration in volume which must be adjusted by multiplying the ordinates by the quotient of the volume of the original data set and the volume of the smoothed data set. Smoothing of a unit hydrograph using a moving average filter can reduce the peak discharge ordinate by up to 20 percent (N.E.R.C., 1975, I, 396). Such a reduction can produce considerable errors in subsequent predictions of a discharge hydrograph.

The moving average filter used in the Flood Studies Report (N.E.R.C., 1975) was tested on 44 oscillatory unit hydrographs derived from data from the Beverley Brook. The filter was defined as,

$$Q_t = \frac{Qt}{3} + \frac{2}{9}(Q_{t-1} + Q_{t+1}) + \frac{1}{9}(Q_{t-2} + Q_{t+2}) \qquad (4.57)$$

where Q_t is the discharge at time t.

The objective error functions (section 4.5) were applied to the results (Table 4.2). Visual assessment of the smoothed unit hydrographs indicated that the oscillations had not been removed but that their amplitude had been reduced. Further, negative ordinates had not been removed and therefore the unit hydrographs required further subjective, manual adjustment.

An alternative scheme was developed which was completely automatic (Appendix 6). This consisted of fitting a polynomial function to the unit hydrograph which required smoothing. The number of degrees of the polynomial was incremented from one to ten until the sum of squares reached a minimum. Any negative ordinates were then removed to produce a single pulse unit hydrograph, which was then rescaled to the unit volume. This method produced smoothed unit hydrographs which were more accurate than those derived by using the moving average filter (Table 4.2). It is suggested that this method represents a significant advance in the smoothing and adjustment of unit hydrograph estimates.

In several cases the discharge at the origin of the unit hydrograph was not zero. This may be due to errors in the relative timings of the rainfall and discharge data. Since this is unproven, any subsequent adjustment introduces unwarranted subjectivity into the analysis. It is believed that the non-zero origin was caused by the use of too coarse a time interval (30 minutes). The lack of sufficient resolution of the discharge record prohibited any reduction in the

Table 4.2 Comparison of the effect of polynomial smoothing and a moving average filter on regeneration error functions

Error Function	Polynomial		Moving Average	
	Mean	Standard Deviation	Mean	Standard Deviation
ISE %	16.694	11.540	17.641	9.129
PISE %	11.507	8.036	12.171	6.209
QPE %	-2.911	8.711	-7.316	6.936
Absolute QPE %	4.963	7.662	7.334	6.916
TPE (hours)	-0.206	0.398	-.147	0.424
Absolute TPE (hours)	0.324	0.303	0.324	0.303

time interval. Inspection of published work indicated that a non-zero origin is common (e.g. Delleur and Rao, 1971b, Fig.5; Hall, 1977b, Fig.4).

4.7 Programming a computer to derive unit hydrographs

The unit hydrographs were derived by two computer programs written by Dr. M.J. Lowing of the Institute of Hydrology for the Flood Studies Report (N.E.R.C., 1975). The first program (LPROG5, Appendix 2) separates the rapid response runoff from the slow response runoff. The volume of the response runoff is converted to an equivalent depth of response runoff. Effective rainfall hyetograms are calculated by distributing this depth by four different methods. The second program (LPROG6, Appendix 3) uses the response runoff and the effective rainfall to derive unit hydrographs by matrix inversion and harmonic analysis. The two programs are described in the Flood Studies Report (N.E.R.C., 1975, I, Chapter 6). A complete description of the input data together with sample data sets is presented in Appendix 1.

The supplied programs were written in FORTRAN IV and were developed and run on an ICL 1900 computer. The ICL installation specific features (e.g. switches) and ICL FORTRAN extensions were removed and the programs will now compile on the UCLCC IBM 360 WATFIV and FORTRAN G Compilers and the ULCC CDC 7600 Minnisota Fortran compiler. There were several errors in the supplied programs.

First, the equations to calculate $API5_d$ (Eq.4.61) and $API5_t$ (Eq.4.63) did not correspond to those published in the Flood Studies Report (N.E.R.C., 1975).

Second, the algorithm to calculate $API5_d$ was incorrect if more than 5 days of antecedent rainfall were read into the program.

Third, when a rainfall or discharge data start time was half-past the hour the program rounded it up to the next hour. This affected calculations of lag-time and affected the shape of the unit hydrograph.

Fourth, in certain circumstances the number of simultaneous equations to be solved for matrix inversion was incorrectly calculated and consequently the program failed.

Fifth, the iterative algorithm for the calculation of the loss curve had an accuracy of 0.01mm, whereas the other rainfall separation methods had an accuracy of 0.0001mm. Consequently there were differences in the volume of effective rainfall calculated by the loss curve and the other three methods.

Minor errors included a faulty 24 hour clock algorithm and a rainfall smoothing algorithm which was inoperable.

All these errors were rectified and the programs were further improved by the addition of first, a subroutine to calculate objective error functions which compared the observed with the modelled discharge. Second, the option to read in data in imperial units which were converted to S.I. units by the program. Third, the addition of the phi method of rainfall separation to the existing three methods. Fourth, the supplied program offered the user the choice of using one of the three rainfall separation methods. Therefore to obtain effective rainfall hyetograms for each of the three methods meant that three separate production runs were necessary. The improved program produced effective rainfall hyetograms by the four methods from a single production run.

In addition to all these improvements, the layout of the print-out was improved.

Three aspects of the programs will be discussed, first the method of separating the rapid and slow response runoff. Second, the four indices used to estimate catchment wetness and third the four rainfall separation techniques.

The characteristics of rapid and slow response runoff differ and must be treated separately in problems involving short term events.

In the absence of water quality data it is impossible to differentiate between the two types of discharge and therefore an arbitrary technique must be adopted for hydrograph analysis.

Several workers have used a straight line to separate the slow response runoff from the rapid response runoff. The line starts at the time of rise and terminates at some point on the recession limb of the hydrograph. The methods to define this point are numerous, for example Sarma et al (1973, 331) defined the point (Q_B) as,

$$Q_B = (Q_p - Q_A/100) + Q_A \tag{4.58}$$

where Q_A is the discharge at the time of rise
Q_p is the peak discharge.

Nash (1960, 251) defined the point (B), "...such that the time elpased between the end of effective rainfall and the point B is equal to three times the time lag between the centres of area of the graphs of effective rainfall and storm runoff". Finally, by plotting the recession limb on semi-log paper, the point is defined where a linear recession commences (e.g. Packman, 1974, 19). This method is equivalent to fitting a single linear reservoir to the data. Some workers have advocated the use of a master recession curve (e.g. Wilson, 1974, 124) but the shape of the recession curve depends on storm characteristics (Dooge, 1973, 89) and percentage runoff of an event (Lowing, 1979), consequently this method is incorrect.

The use of a straight line introduces errors, because the continued decrease of the slow response runoff at the beginning of the event is ignored. The method used by the Flood Studies Report (N.E.R.C., 1975, I, 389) and which was adopted by this thesis avoids this error (Fig. 4.3).

The recession limbs of the preceding event and the selected event are extended using a linear, time variant reservoir, which is defined as,

$$S = K(t).Q \tag{4.59}$$

where S is the storage
K is the storage coefficient, which is a function of time (t)
Q is the outflow

The recession limb of the preceding event is extended using equation (4.59) only if two conditions are met. First, that there are six or more ordinates in the recession limb and second that the decay

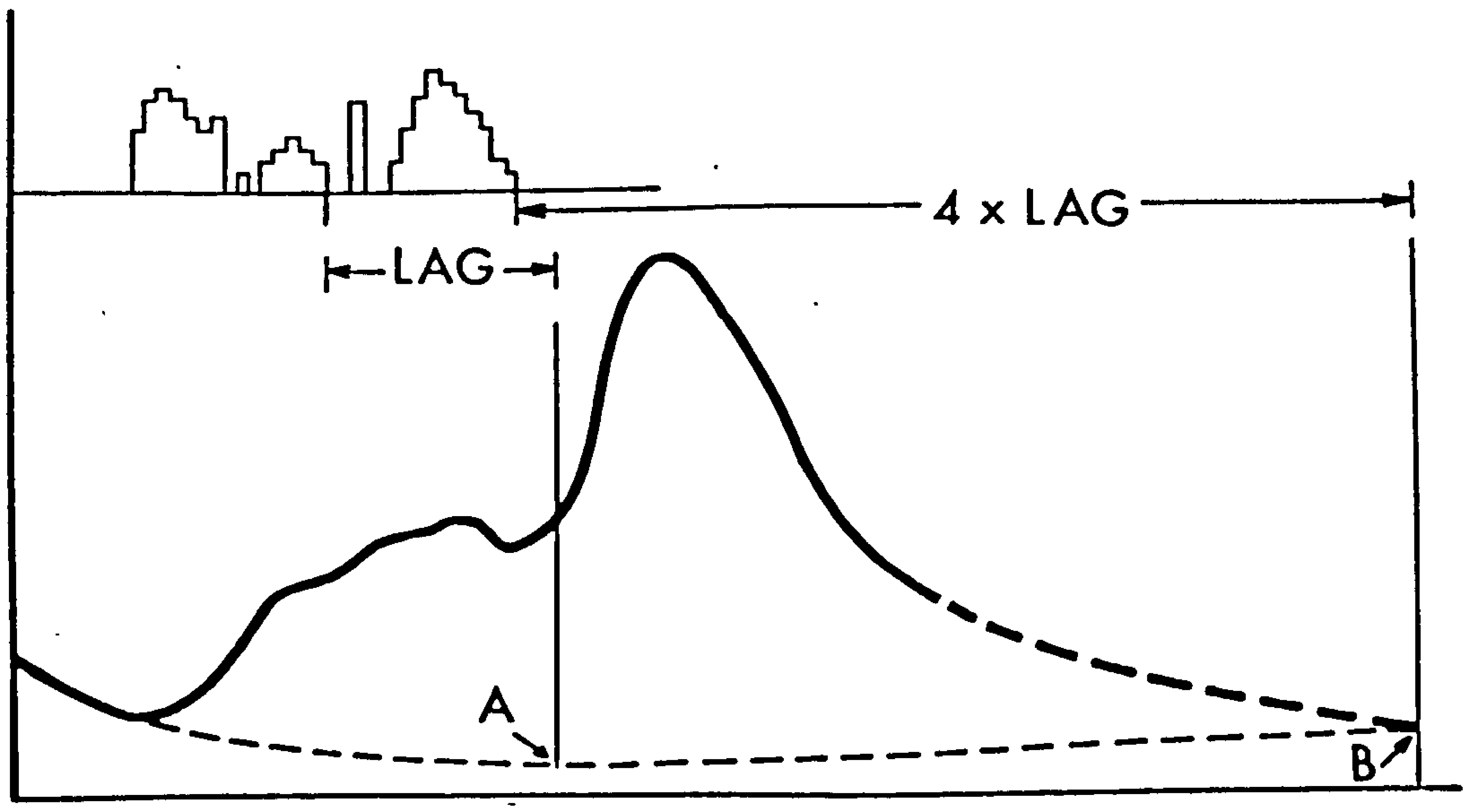


Fig. 4.3 Definition of rapid response runoff (after NERC, 1975, I, 389)

factor of the curve is less than RL which is defined as,

$$RL = 0.995^{TINT} \tag{4.60}$$

where TINT is the time interval of the data (hours). All the events were sampled at half-hourly intervals and therefore the critical value of RL was 0.99749. The first test was a safeguard against the extension of a recession limb consisting of a small number of ordinates which may be unrepresentative of a full recession limb. The second test saved computer time by recognising that if the value of RL exceeds 0.99749, the substitution of a horizontal line in place of a curvi-linear recession will produce an insignificant loss of accuracy. The preceding recession limb is extended to the time interval corresponding to either the centroid of the peaks for a multi-peaked hydrograph or the peak for a single peaked hydrograph (Point A). The end point of the rapid response runoff is defined as four times the lag time from the centroid of the peaks, where lag time is defined as the centroid of the peaks minus the centroid of the total rainfall (Point B). A straight line is drawn from the intersection of the preceding recession with the centroid of the peaks to the endpoint of the rapid response runoff. If the value of the discharge at the endpoint is less than the discharge at the time of rise, the endpoint is moved back until it corresponds to the discharge at the time of rise.

Three indices of antecedent precipitation (API) were used.

First, the five day antecedent precipitation index ($API5_d$) which is defined as (N.E.R.C., 1975, I, 390, Eq.6.3),

$$API5_d = 0.5^{0.5}(P_{d-1} + 0.5P_{d-2} + (0.5)^2P_{d-3} + (0.5)^3\,P_{d-4} + (0.5)^4\,P_{d-5}) \tag{4.61}$$

where P_d is the total rainfall on day d. The constant, 0.5, outside the bracket ensures that the assumption of a uniform distribution of rain through the day is consistent with the value of $API5_d$ at the end of the day. The value of $API5_d$ is used to calculate the catchment wetness index (Eq.4.64).

Second, the 28 day antecedent precipitation index ($API28_d$) is defined as (N.E.R.C., 1975, I, 390, Eq.6.2),

$$API28_d = P_{d-1} + (API28_{d-1} \cdot 0.9) \quad (4.62)$$

The index is calculated by iterating P from d-1 to d-28.

The third index ($API5_t$) records the change in $API5_d$ throughout a storm event. This is defined as (N.E.R.C., 1975, I, 476),

$$API5_t = P_{t-1} \cdot 0.5^{T/48} + API5_{t-1} \cdot 0.5^{T/24} \quad (4.63)$$

where $API5_t$ is the value of API5 at time t
P_t is the total rainfall in the time interval t
T is the data interval.

The value of API5, calculated by 4.63, using 24 individual hourly calculations will be the same as the value of API5 calculated by 4.61 in a single calculation. This third index was combined with the soil moisture deficit to calculate the catchment wetness index (CWI) at time t during a storm event.

The catchment wetness index (CWI) represents the wetness of the catchment, and is defined as (N.E.R.C., 1975, I, 390, Eq.6.4),

$$CWI = 125 + API5 - SMD \quad (4.64)$$

The constant, 125, ensures that the value of CWI is always positive.

The four rainfall separation methods used in this thesis were first the constant loss percentage, second the loss rate curve, third the constant loss percentage varying with catchment wetness and fourth the phi index.

The first method, the constant loss percentage is defined by,

$$ERF1_t = (PRO.HRF_t)/100.0 \quad (4.65)$$

where $ERF1_t$ is the effective rainfall at time t
HRF is the total rainfall at time t
PRO is the percentage runoff and is defined by

$$PRO = \frac{VOL}{SR} \quad (4.66)$$

where VOL is the equivalent depth of runoff
SR is the total rainfall which fell during the storm event.

This simple method is suitable for solution on a pocket calculator.

The second method, the loss rate curve, requires the use of a digital computer because of the iterative method of its calculation. The curve is based on the reciprocal of the catchment wetness index for each rainfall increment. The method allows the loss rate to recover during drier periods within a storm and to have higher initial values and decay faster in dry antecedent conditions.

The loss for each rainfall increment at time t is defined by,

$$ERF2_t = HRF_t - \frac{1}{CWI_t} \cdot \frac{BLOSS}{SACC} \tag{4.67}$$

where ERF2 is the effective rainfall at time t

HRF is the total rainfall at time t

CWI is the catchment wetness index at time t

BLOSS is (1) on the first iteration, the depth of rainfall loss (ALOSS).

(2) on the second and subsequent iterations,

$$BLOSS = ALOSS - \sum_{t=1}^{t=NHRF} ERF2_t \tag{4.68}$$

where NHRF is the number of rainfall ordinates.

SACC is (1) on the first iteration,

$$SACC = \sum_{t=1}^{t=NHRF+1} \frac{1}{CWI_t} \tag{4.69}$$

(2) on the second and subsequent iterations,

$$SACC = \sum_{t=1}^{t=NHRF} \frac{1}{CWI_t} \quad \text{where } HRF_t > ERF2_t \tag{4.70}$$

The coefficient, BLOSS/SACC is the proportion by which the ordinates of 1/CWI curve should be adjusted such that the volume of effective rainfall equals the volume of response runoff.

To ensure that the effective rainfall always started before or at the same time as the discharge hydrograph, it was assumed that 100 percent of 1 percent of the catchment contributed to the response runoff. The remaining 99 percent of the total rainfall in each increment was then eligible for reduction by equation (4.67).

An iteration technique was necessary because the estimated loss in a given increment may exceed the amount of total rainfall in that increment. The iteration ceased when BLOSS, defined by equation (4.68), was less than 0.0001. This difference between the sum of the estimated effective rainfall and the equivalent depth of runoff is called the residual loss. This limit ensured that the volume of effective rainfall calculated by this method was equal to the equivalent depth of runoff calculated from the response runoff hydrograph.

The third method is based on the assumption that the percentage runoff increases through the storm according to the catchment wetness index. This method does not involve any iteration, but eight separate computations are required for each rainfall increment which makes the method more amenable to solution on a computer than a pocket calculator. The change in catchment wetness through a storm is measured by calculating the value of API5 and SMD for each rainfall increment. Table 4.3 contains an example of calculating effective rainfall by this method. The values in columns C to J are calculated as follows,

$$C = API5_{t-1} \cdot 0.5^{T/24}$$

$$D = P_{t-1} \cdot 0.5^{T/48}$$

$$E = C + D$$

$$F = SMD_{t-1} - P_{t-1}$$

$$G = 125 - SMD + API5$$

$$H = \text{Total rainfall} \times CWI$$

$$I = CWI \times PF$$

$$J = \text{Runoff coefficient} \times \text{total rainfall}$$

$$PF = \frac{\Sigma P}{\Sigma K} = \frac{0.618}{80.12} = 0.0077159$$

Table 4.3 Method of computation of the constant loss percentage varying with catchment wetness

A	B	C	D	E	F	G	H	I	J
Time (t)	Total Rainfall (P) (mm)	Adjusted $API5_{t-1}$	Adjusted P_{t-1}	API5	SMD	CWI	K	Runoff coefficient	Net Rain (mm)
0.0	0.254	0.0	0.0	0.045	105.16	19.88	5.05	0.153	0.039
0.5	0.254	0.044	0.252	0.297	104.91	20.39	5.18	0.157	0.040
1.0	0.254	0.292	0.252	0.545	104.65	20.89	5.31	0.161	0.041
1.5	0.508	0.537	0.252	0.789	104.40	21.39	10.87	0.165	0.084
2.0	1.524	0.778	0.504	1.282	103.89	22.39	34.12	0.173	0.263
2.5	0.254	1.264	1.513	2.777	102.37	25.41	6.45	0.196	0.050
3.0	0.508	2.737	0.252	2.989	102.11	25.88	13.14	0.200	0.101

The final method, the phi index, assumes a constant rainfall loss throughout the storm. This method is iterative because the calculated phi index may exceed the total rainfall in a given increment and consequently the residual loss must be distributed throughout the remaining rainfall increments. Effective rainfall, calculated by this method, is defined by,

$$ERF4_t = HRF_t - PHI \qquad (4.71)$$

where ERF4 is the effective rainfall at time t
HRF is the total rainfall at time t
PHI is the loss constant

The initial estimate of PHI is defined by,

$$PHI = \frac{SR - VOL}{DUR.TINT} \qquad (4.72)$$

where SR is the total rainfall which fell during the storm event
VOL is the equivalent depth of runoff
DUR is the duration of the storm event (hours)
TINT is the time interval of the data (hours).

The iterations ceased when the residual loss was less than 0.0001. This method did not ensure that the effective rainfall either started before or coincided with the start of the discharge hydrograph, consequently the method proved to be unsuitable to derive unit hydrographs. The method of allowing a fixed percentage of each rainfall increment to runoff would have avoided this problem.

The application of the four methods to a single rainfall profile will produce four different effective rainfall profiles (Fig.4.4, 4.5, 4.6 and 4.7, Table 4.4).

Table 4.4 A comparison of the four rainfall separation methods

Increment	Total rainfall (mm)	Effective rainfall (mm) Rainfall separation method			
		1	2	3	4
1	0.2540	0.0786	0.0025	0.0773	0.0000
2	2.0320	0.6287	1.2776	0.6195	1.2916
3	0.7620	0.2358	0.0186	0.2357	0.0216
4	0.7620	0.2358	0.0222	0.2369	0.0216
5	0.7620	0.2358	0.0257	0.2380	0.0216
6	1.2700	0.3930	0.5371	0.3985	0.5296
7	0.2540	0.0786	0.0025	0.0804	0.0000

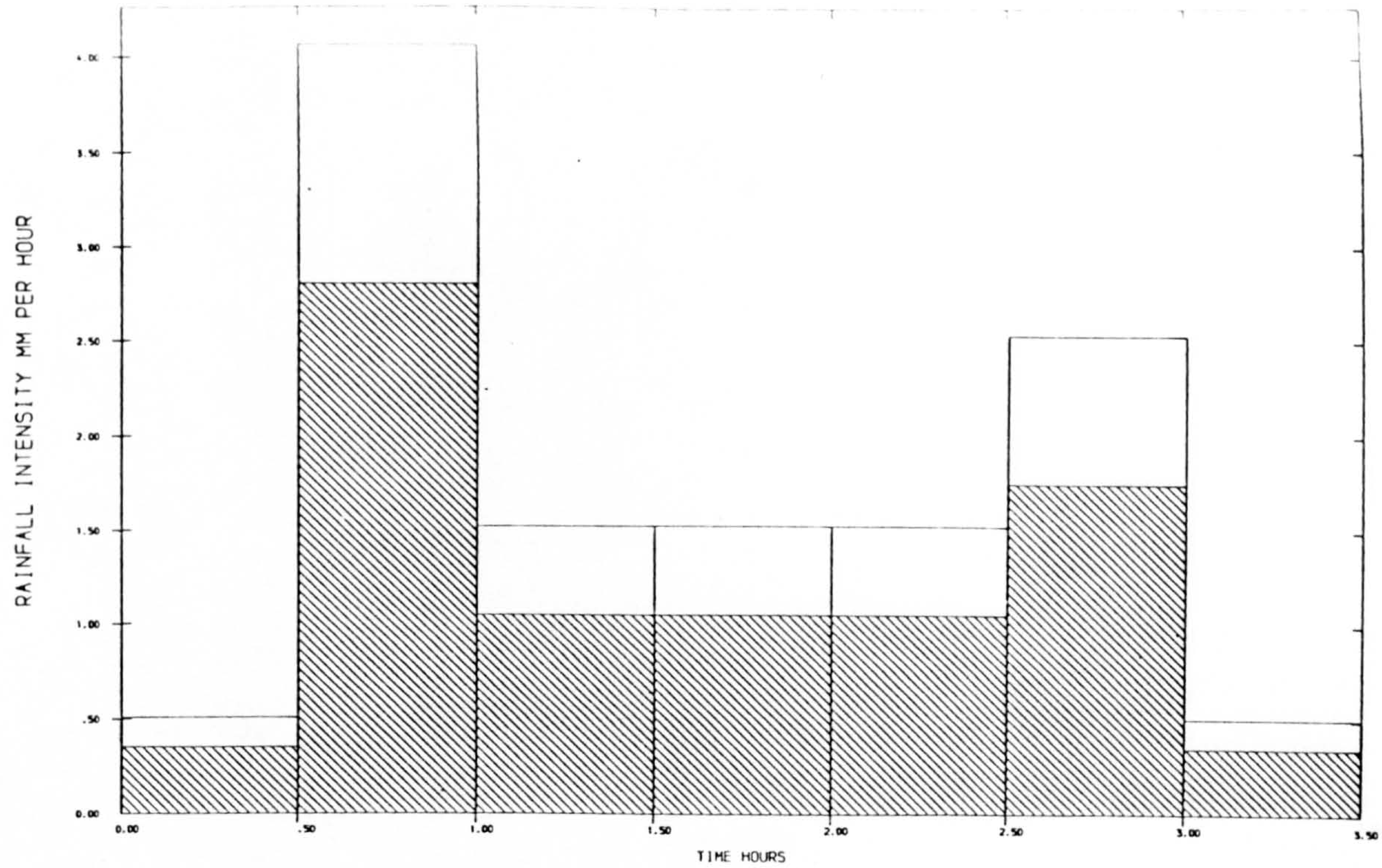


Fig. 4.4 Rainfall separation method 1: The constant loss percentage. Shaded area equals rainfall loss

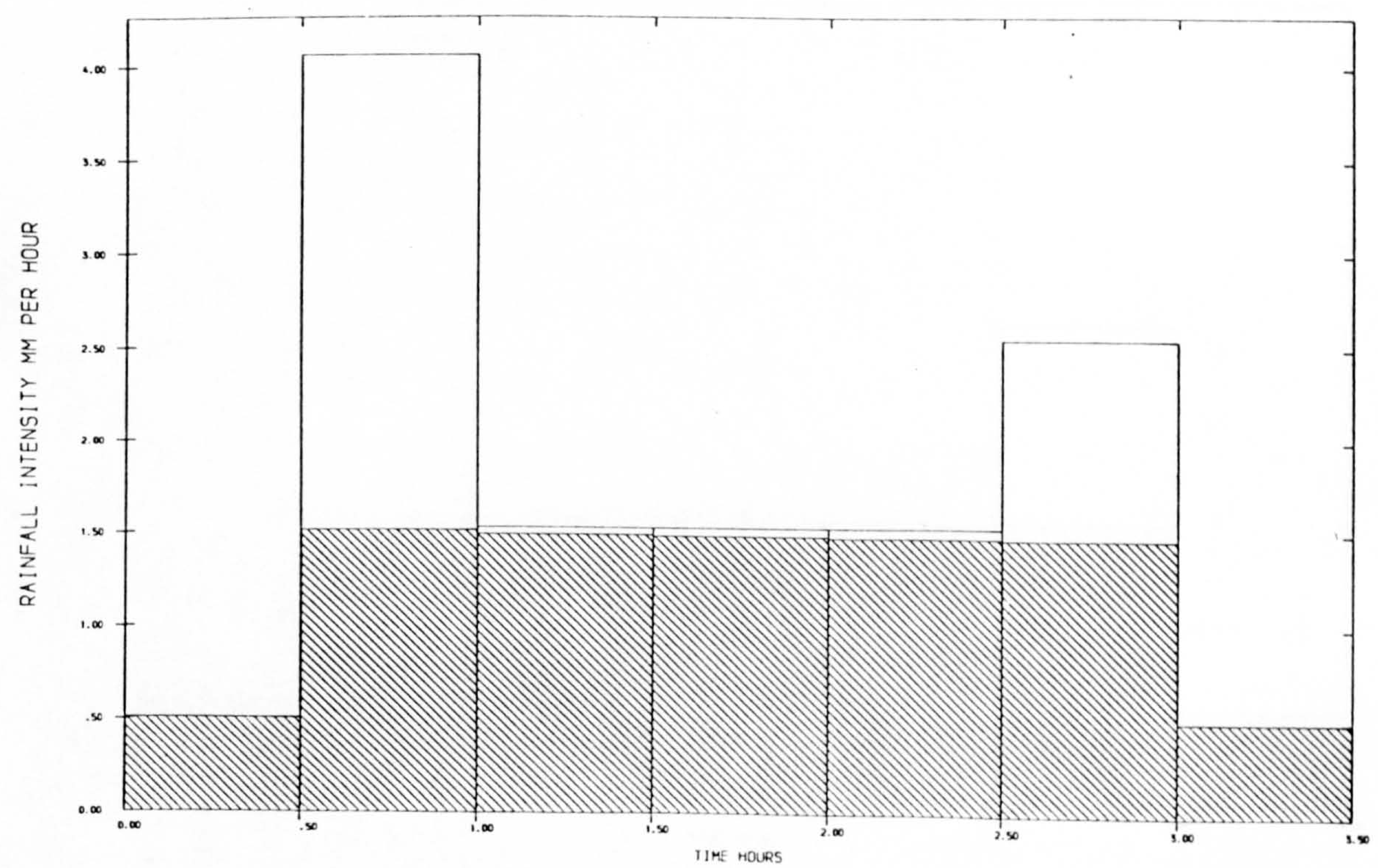


Fig. 4.5 Rainfall separation method 2: The loss rate curve. Shaded area equals rainfall loss

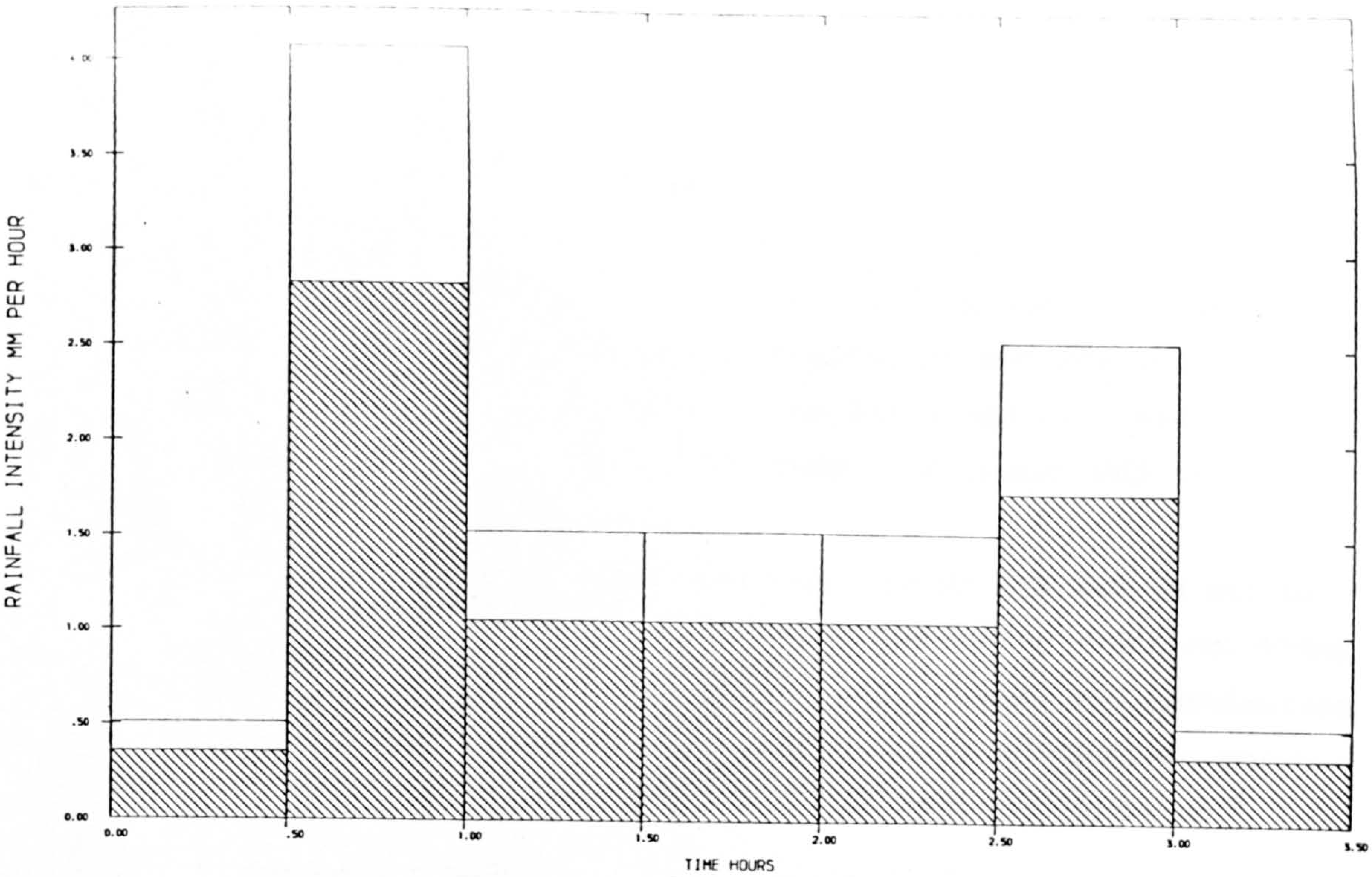


Fig. 4.6 Rainfall separation method 3: The constant loss percentage varying with the catchment wetness index
Shaded area equals rainfall loss

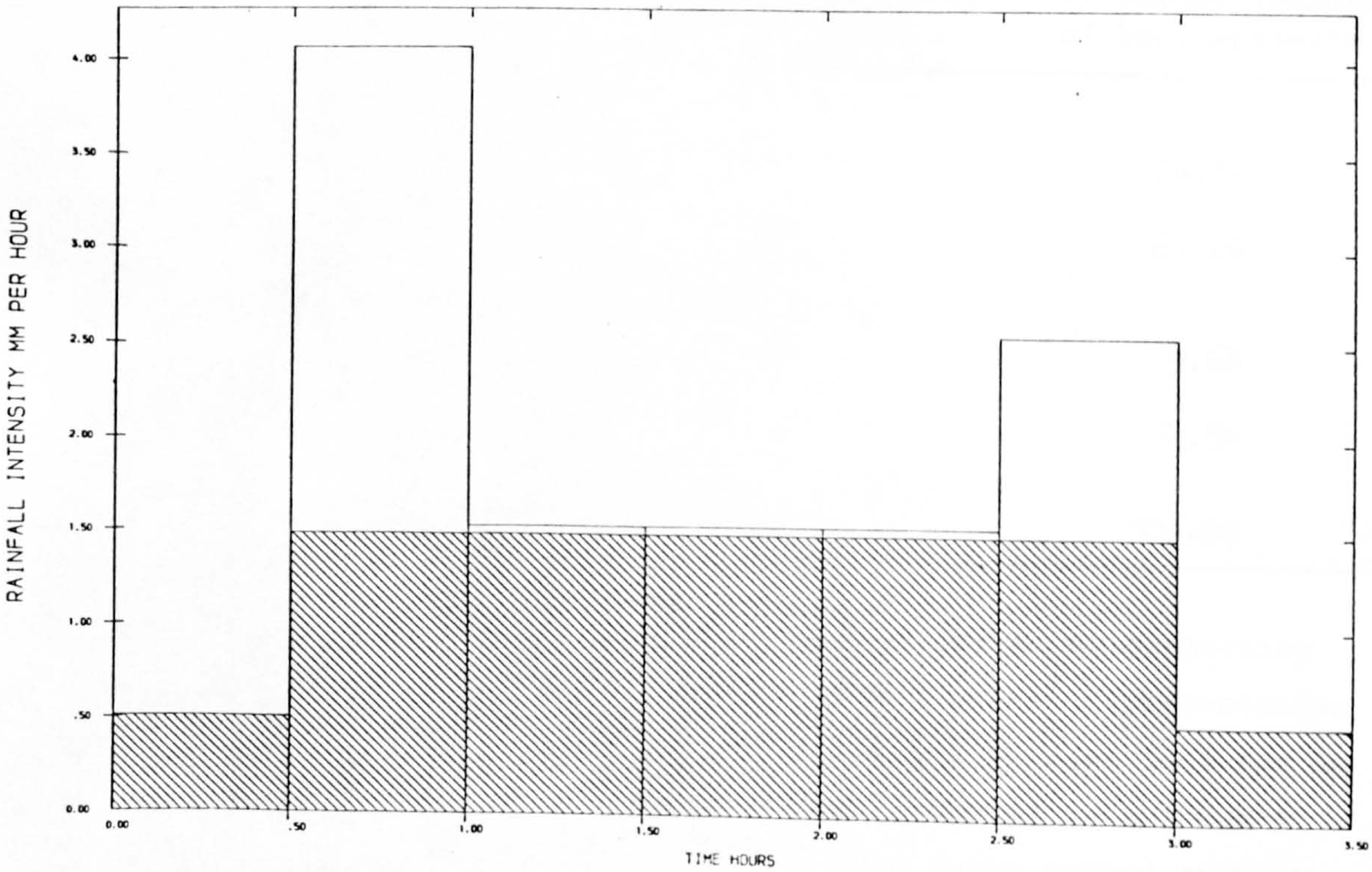


Fig. 4.7 Rainfall separation method 4: The phi index.
Shaded area equals rainfall loss

Consequently, to decide which combination of unit hydrograph derivation method and rainfall separation method produced the optimum unit hydrographs an analysis was conducted on 62 storm events from the Beverley Brook, south London. For each event 16 unit hydrographs were derived from a combination of the four rainfall separation methods and the two unit hydrographs methods using the unsmoothed and smoothed options. For each event the combination which produced the lowest RMS error (Eq. 4.54) on regeneration was noted.

This analysis indicated first, for every event the matrix method was superior to the harmonic method. Second, for every event smoothing of the unit hydrograph increased the RMS error. Third, the combination of matrix inversion with effective rainfall derived by the loss curve method consistently produced the most accurate unit hydrographs (Table 4.5).

Table 4.5 Four rainfall separation methods and matrix inversion: Results

Rainfall separation method	Absolute frequency of best estimate	Percentage frequency of best estimate
Constant loss Percentage	15	24.19
Loss rate curve	38	61.29
Constant loss percentage varying with CWI	6	9.68
Phi index	3	4.84
TOTAL	62	100.00

The results of this analysis cannot be interpreted as supporting the argument that the loss rate curve is a good physical representation of the infiltration process. The analysis simply indicates that for an analytical study which assumes a linear rainfall-runoff relationship the combination of matrix inversion and the loss curve method produce a unit hydrograph which in most cases is an accurate representation of a linear catchment response function.

The 34 storm events derived for the other south London catchments were analysed using this combination.

4.8 Conclusion

This chapter has presented a review and evaluation of the techniques by which linear hydrological systems may be analysed using the unit hydrograph method.

The mathematical description of the unit hydrograph derived the convolution integral (Eq. 4.5) and the equation to derive a TUH from an IUH (Eq. 4.9). These equations were included in the computer programs used in the analysis.

The linear conceptual models used in the following chapter (section 5.3) were based on combinations of the linear reservoir (Eq. 4.17) and the linear channel (Eq. 4.19). The conceptual models were fitted by matching the shape factors (Eq. 4.28) of the discharge and precipitation data to the parameters of the model.

The methods of deriving a unit hydrograph by matrix inversion (Section 4.4.2) and by harmonic analysis (Section 4.4.3) were evaluated for their accuracy and consistency in deriving unit hydrographs. This indicated that the matrix inversion method was superior to the harmonic analysis method. The effect of rainfall separation method on the accuracy of the derived TUH was also evaluated. This indicated that matrix inversion of the rapid response runoff and effective rainfall derived by the loss curve method produced the most consistently accurate TUHs. This method of deriving TUHs was used in chapters 5 and 6.

A new automatic method of removing oscillations and negative ordinates in a derived TUH was devised and found to be superior to the moving average filter used by the Flood Studies Report (NERC, 1975). It is suggested that this new method represents a significant advance in the smoothing and adjustment of TUH estimates.

Finally, some of the components of the suite of computer programs used to derive the unit hydrographs were discussed. The description of the method of separation of the hydrograph and the four rainfall separation methods are among the most detailed in the literature.

This chapter has presented an evaluation of the assumptions on which the analysis was based and provides the requisite information from which future work can proceed.

Chapter 5

AN ANALYSIS OF THE FLOOD HYDROLOGY OF THE BEVERLEY BROOK

5.1 Introduction

This chapter presents the results of a comprehensive analysis of 62 storm events from the Beverley Brook. The objective was to find which particular model of the unit hydrograph was best suited to analyse and predict the flood hydrographs.

The Beverley Brook is a 43.5 Km^2 catchment in south London (see section 2.3 for further details). The 62 storm events were selected from the period 1963 to 1976. The data for each event consisted of half-hourly discharge and precipitation data, daily totals of precipitation and daily values of soil moisture deficit (see section 2.4 for further details). The characteristics of the 62 events are presented in Table 5.1.

Table 5.1 Statistical attributes of the 62 storm events from the Beverley Brook

	Peak Discharge Cumecs	Total Rainfall mm.	Mean Rainfall intensity mm/hr	Peak Rainfall intensity mm/hr
Mean	6.610	12.048	3.338	7.018
Minimum	1.376	2.54	0.464	0.510
Maximum	15.105	36.45	51.820	51.820
Standard Deviation	3.373	8.729	6.555	7.424

The mean discharge (6.61 cumecs) is 45.34 percent of the mean annual discharge (14.58 cumecs). The largest flood event (15.11 cumecs) has a return period of 3.3 years. Consequently the results of this analysis refer to frequent events.

The data set for each storm event was analysed by the computer program LPROG5 (Appendix 2, see section 4.7 for further details). This provided the rapid response runoff and the effective rainfall which was used by all the models of the unit hydrograph described in this chapter. In addition to this, information was also provided for each storm, namely the duration of the rainfall, the total rainfall and the peak and mean intensity of the rainfall. The API5, API28 and the CWI at the beginning and the end of the storm were also calculated. Other information included the percentage runoff during the event and the initial and final rainfall loss rates. These variables provided a detailed description of the characteristics of each storm event and were used in the chapter to examine the relationship between the parameters of a model of the TUH and the causative storm and antecedent catchment characteristics.

The analysis proceeded by deriving unit hydrographs by matrix inversion (section 4.4.2) of the effective rainfall (calculated by the loss curve method) and the rapid response runoff. Oscillatory TUHs were smoothed by the polynomial method described in section 4.6. To explain the considerable variation in the shape of the unit hydrographs a statistical analysis was made to relate the TUH parameters to the statistical attributes of the storms. This indicated that the antecedent conditions of the catchment and the characteristics of the causative storm had a significant effect on the shape of the TUH. Three models of the unit hydrograph were evaluated to determine which one analysed 36 events and predicted 26 events with the minimum error. These models were first, the time invariant TUH; second, the dimensionless TUH and third, a straight-line approximation of the TUH called the Parametric Triangular unit hydrograph (PTTUH). This last model proved to be the most consistently accurate of the three models.

The second stage consisted of the analysis of the 62 events by 17 conceptual models. Analysis of the error functions on regeneration indicated that the convective diffusion reach conceptual model was the best of all and was superior to the matrix inversion method of identification. The parameters of the model displayed a variability similar to that of the TUH parameters but it proved to be impossible to derive significant equations to relate the parameters of the conceptual model to the storm and catchment characteristics. Suggestions for future analyses are presented which it is believed would rectify the deficiencies of the analysis reported in this section.

In the final section the PTTUH was used together with a set of historical rainfall profiles to simulate a flood frequency curve. The method was not successful and several recommendations are put forward to improve the accuracy of the method.

The results presented in this chapter represent the most comprehensive investigation of the flood hydrology of any of the London catchments.

5.2.1 Unit hydrograph analysis

Unit hydrographs were derived for the 62 storm events from the Beverley Brook. Oscillatory TUHs were smoothed by the polynomial method described in section 4.6. To ensure that the unit hydrographs were an accurate representation of the catchment response for a given event, inclusion criteria, based on the regeneration error functions, were defined as,

(1) that ISE should be less than 40 percent;

(2) that QPE should be less than 33 percent;

(3) that the absolute TPE should be less than or equal to one hour;

(4) that the discharge ordinate at time zero should be less than 50 percent of the peak discharge.

In addition to these objective criteria, unit hydrographs were also rejected if they were oscillatory after smoothing.

Application of these five criteria resulted in the rejection of 26 (41.94%) TUHs and the inclusion of 36 (58.06%) TUHs. Most of the rejected TUHs were rejected by two or more criteria (Table 5.2).

Table 5.2 The distribution of the rejected events by inclusion criteria

Inclusion Criteria	ISE >40.0%	QPE >33.3%	Y>QP/2 for x=0.0	Oscillatory after smoothing	TPE >1.0 hours
Percent of the 26 events rejected by each inclusion criterion	73.08	30.77	19.23	38.46	15.38

An analysis of the characteristics of the rejected and included TUHs (Table 5.3) indicated that the rejected TUHs causative storms were typically, long duration with moderately high rainfall totals. On average, storms of greater than 9 hours duration yielded an ISE value greater than 40 percent and would therefore cause the TUH to be rejected (see section 6.2 for further details). The rejection of the 26 TUHs reduced the scope of the analysis, but the inclusion of all the 62 TUHs in the subsequent analysis would have prejudiced the accuracy of any findings. The subsequent analyses were based on the 36 included TUHs and the 26 rejected TUHs were used to test the derived relationships.

Table 5.3 Characteristics of the rejected and analysed events

Variable	Rejected Events		Analysed Events	
	Mean	SD	Mean	SD
Storm Duration (hrs)	6.865	3.445	4.014	2.055
Total Rainfall (mm)	16.585	10.398	8.771	5.407
Peak Rainfall Intensity (mm/hr)	7.585	5.515	6.609	8.597
Mean Rainfall Intensity (mm/hr)	2.507	1.360	3.938	8.526
API5 (mm)	3.787	5.409	4.844	10.175
API28 (mm)	18.032	10.535	21.566	22.192
SMD (mm)	56.250	44.026	48.450	42.663
Percent Response	16.872	6.898	20.255	8.521
Number of Events	26	-	36	-

The characteristics of the selected TUHs are presented in Table 5.4. This table indicates that the TUH peak varied from 44.17 to 134.44 cumecs and the time to peak from 1.0 to 4.5 hours. The shape of the TUH was measured by the curvature around the peak, defined as (Price, 1973, 85),

$$\text{Curvature} = \frac{(Qp_{t+1} + Qp_{t-1}) - 2.Qp_t}{TINT^2} \qquad (5.1)$$

where Qp is the TUH peak discharge, cumecs
TINT is the data interval, hours
t is the time interval.

A TUH with a low absolute curvature is 'blunt' whereas a TUH with a high absolute curvature is 'sharp'. The curvature of the TUHs used in this analysis ranged from -16.0 to -466.36.

A graph (Fig.5.1) of the 36 TUHs indicates the wide range of catchment response.

Table 5.4 Results of the TUH analysis for the Beverley Brook

	TUH Parameters				Error Functions			
IEN	Qp	Tp	TB	CURV	ISE	PISE	QPE	TPE
1	103.19350	4.5	9.0	-110.6614	0.023	0.022	-0.0168	0.0
2	77.70310	3.5	14.0	-45.0493	19.535	14.005	-7.9170	0.0
3	88.26679	2.5	9.5	-82.9243	7.066	4.502	0.3918	0.0
5	73.06878	2.0	10.5	-62.2097	38.566	26.844	-1.6981	0.5
6	127.40164	2.0	9.5	-249.9211	0.337	0.091	0.0410	0.0
7	74.10626	4.0	10.5	-27.2107	9.973	9.197	2.4541	-0.5
8	88.73317	3.0	15.0	-103.8014	0.019	0.018	-0.0177	0.0
9	67.60489	3.0	11.5	-23.9962	32.647	14.775	-0.4168	0.0
10	55.79662	3.0	15.0	-15.9976	12.238	9.788	0.0314	0.0
11	83.09225	2.0	11.0	-89.7914	1.967	0.983	0.4267	0.0
13	79.25877	3.5	14.5	-53.3014	0.003	0.003	-0.0009	0.0
14	75.20418	1.5	18.0	-52.2860	33.972	25.919	-31.8920	-0.5
15	127.69356	1.0	5.0	-287.0890	0.359	0.127	-0.1090	0.0
19	44.17286	3.0	19.0	-16.7480	14.210	9.021	-6.1272	0.5
20	83.61784	2.0	11.0	-62.2402	11.907	10.860	-1.2317	-0.5
21	134.43515	1.0	6.0	-466.3569	0.816	0.395	-0.3066	0.0
22	124.43213	1.5	7.5	-428.8147	0.006	0.005	0.0028	0.0
26	68.15772	1.5	8.5	-39.5390	18.849	8.991	3.7419	0.0
27	54.54952	2.5	13.5	-16.3122	5.858	4.155	-2.1208	0.0
30	114.78821	1.0	5.5	-184.8546	1.843	0.532	0.5833	0.0

Table 5.4 (cont.)

	TUH Parameters				Error Functions			
IEN	Qp	Tp	TB	CURV	ISE	PISE	QPE	TPE
36	89.85403	2.0	12.5	-109.6242	1.037	0.437	-0.2640	0.0
38	88.98547	2.0	8.5	-64.8332	5.113	3.458	0.1315	-0.5
39	78.67450	1.5	11.0	-58.6202	6.490	4.693	-2.8202	0.0
40	53.82806	2.0	13.5	-20.0808	6.008	5.222	5.1444	-1.0
41	54.21298	2.0	15.0	-20.8876	12.165	7.495	2.3845	-0.5
42	58.93164	3.5	14.5	-18.8138	36.527	28.061	-12.7048	-0.5
43	59.42129	3.0	14.5	-27.0092	22.568	14.513	-7.6842	0.0
50	104.61229	2.5	10.5	-151.6371	0.007	0.006	-0.0046	0.0
52	111.46709	2.0	11.5	-239.6457	2.779	1.094	0.6537	0.0
54	111.37129	1.5	8.5	-187.3096	3.586	2.085	1.1869	0.0
55	76.95395	1.5	10.5	-64.4036	15.738	6.120	3.3475	0.0
56	64.98646	2.0	15.5	-25.4604	12.677	8.628	3.1652	-0.5
58	113.61966	2.0	8.5	-148.4157	0.211	0.196	0.0870	0.0
59	116.11508	1.5	7.5	-148.1374	0.004	0.004	0.0019	0.0
60	123.69212	4.5	14.5	-312.0107	0.011	0.009	-0.0081	0.0
61	94.06414	1.0	6.5	-90.1384	7.193	2.824	-0.1923	0.0

Error functions refer to the regenerated response runoff compared to the observed response runoff

Notation:

IEN - Event number
Qp - TUH peak, cumecs per $100Km^2$
Tp - TUH time to peak, hours
TB - TUH time base, hours

CURV - Curvature around the peak of the TUH
ISE - Integral square error, percent
PISE - Partial integral square error, percent
QPE - Peak discharge error, percent
TPE - Time to peak error, hours

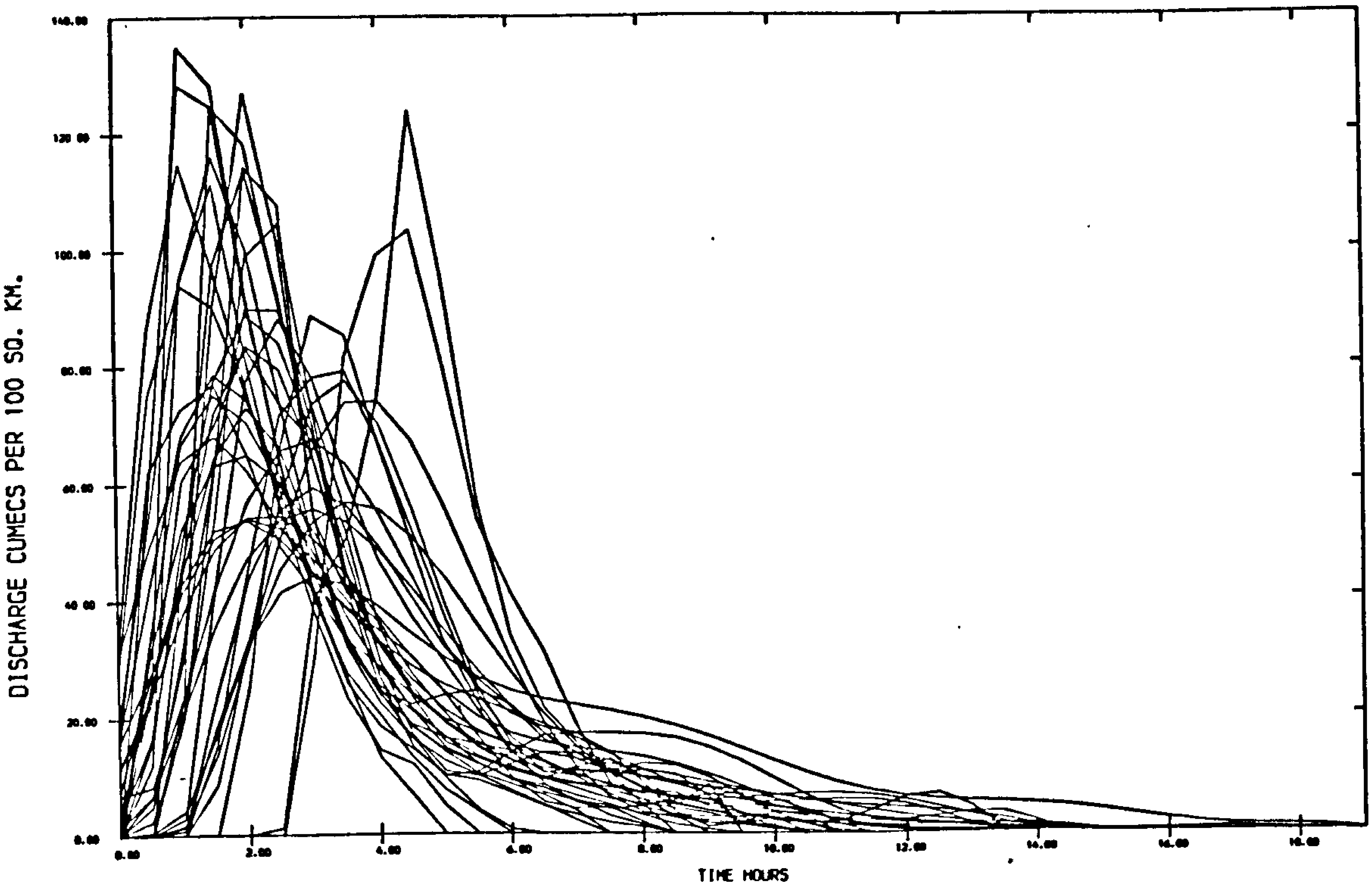


Fig. 5.1 Unit hydrographs for the Beverley Brook

Investigations of urbanising catchments have analysed the TUH parameters, peak discharge (Qp) and time to peak (Tp), through time in an attempt to show the effect of urbanisation and flood alleviation schemes on the catchment's response. Conventionally arbitrary time periods are selected (e.g. Hall, 1977a) within which the TUHs are averaged, this method may be criticised because of the subjective selection of the time periods. A new method was used in this thesis which consisted of correlating the time of each storm event, expressed as days elapsed since January 1st 1900, with the TUH Qp and Tp. The algorithm for time was an adaption of a method by Organick and Meissner (1976, 220). This method was objective and involved no loss of information through the averaging of the TUHs. The correlations were insignificant indicating that there had been no significant change in the catchment response during the 14 years which were analysed. Hall's (1974, 1977a) and Packman's (1974) findings that infilling of the urban area has no measurable effect on the flood hydrograph are probably applicable to the Beverley Brook.

To assist the interpretation of the variable catchment response the 36 TUHs were divided into four groups. The two TUHs in Group 4 had a time to peak greater than the rest of the TUHs. The class boundaries of the remaining three groups were fixed to ensure a uniform number of TUHs in each group (Table 5.5, Figures 5.2, 5.3, 5.4 and 5.5).

Table 5.5 TUH group classification

Group Number	Minimum Qp (cumecs)	Maximum Qp (cumecs)	Number of Events
1	0.0	70.0	10
2	70.1	90.0	12
3	90.1	135.0	12
4	-	-	2

An analysis of the characteristics of the TUHs and the boundary conditions for each of the four groups was made (Table 5.6).

Two additional properties of the TUH were defined (Meynink and Cordery, 1976, 1211). First, peakedness, was defined as,

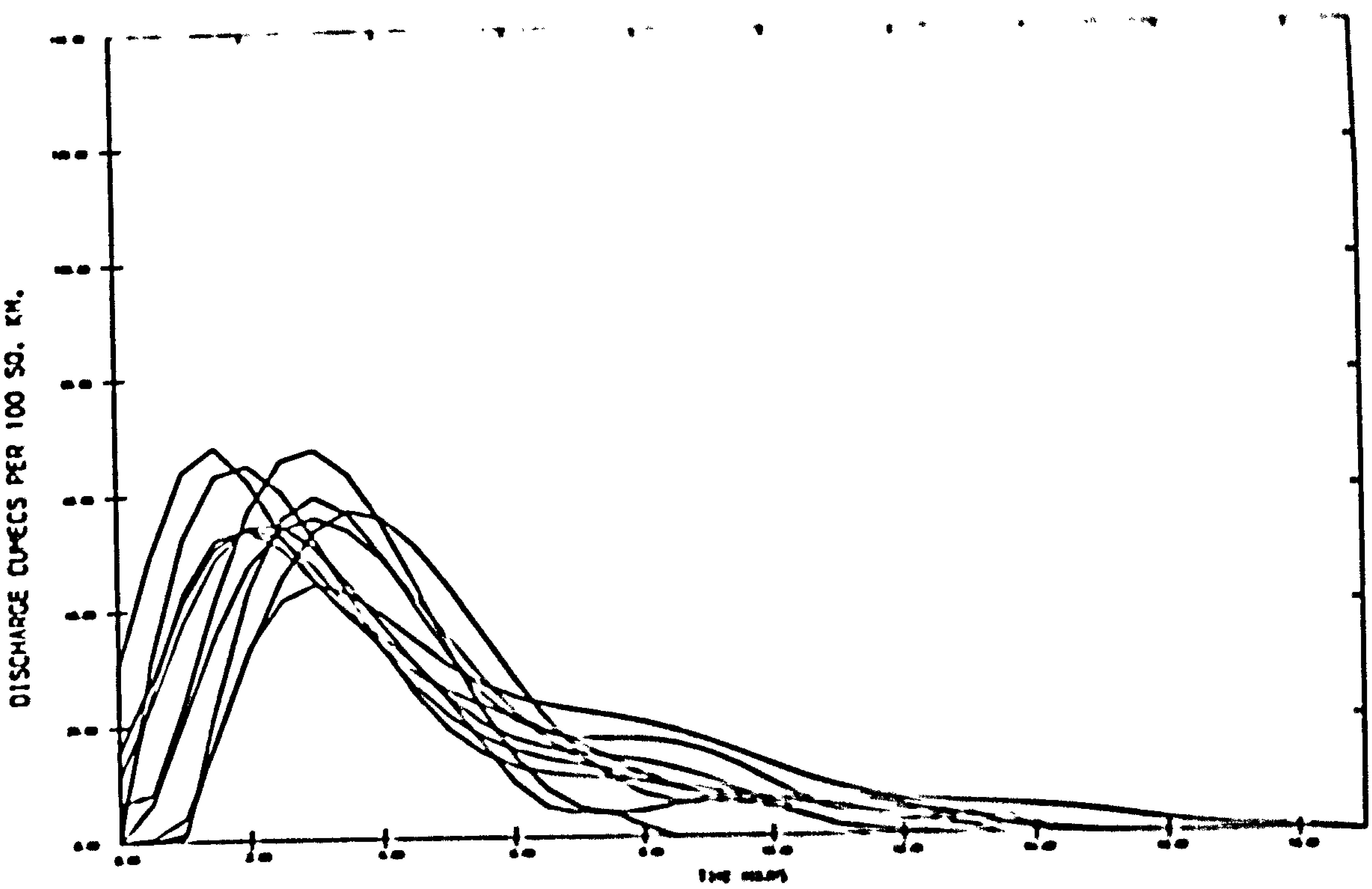


Fig. 5.2 Unit hydrographs for the Beverley Brook, Group 1

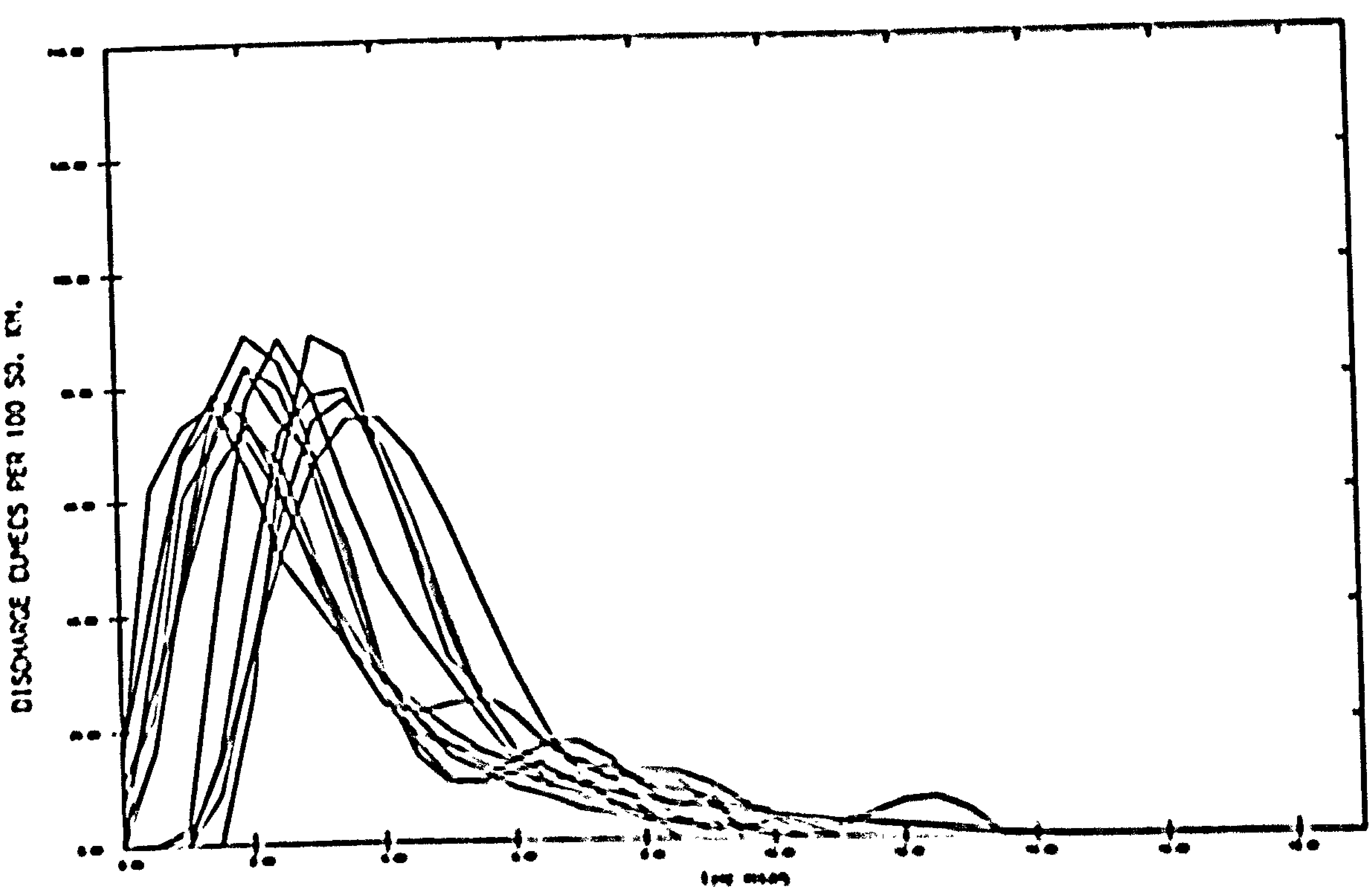


Fig. 5.3 Unit hydrographs for the Beverley Brook, Group 2

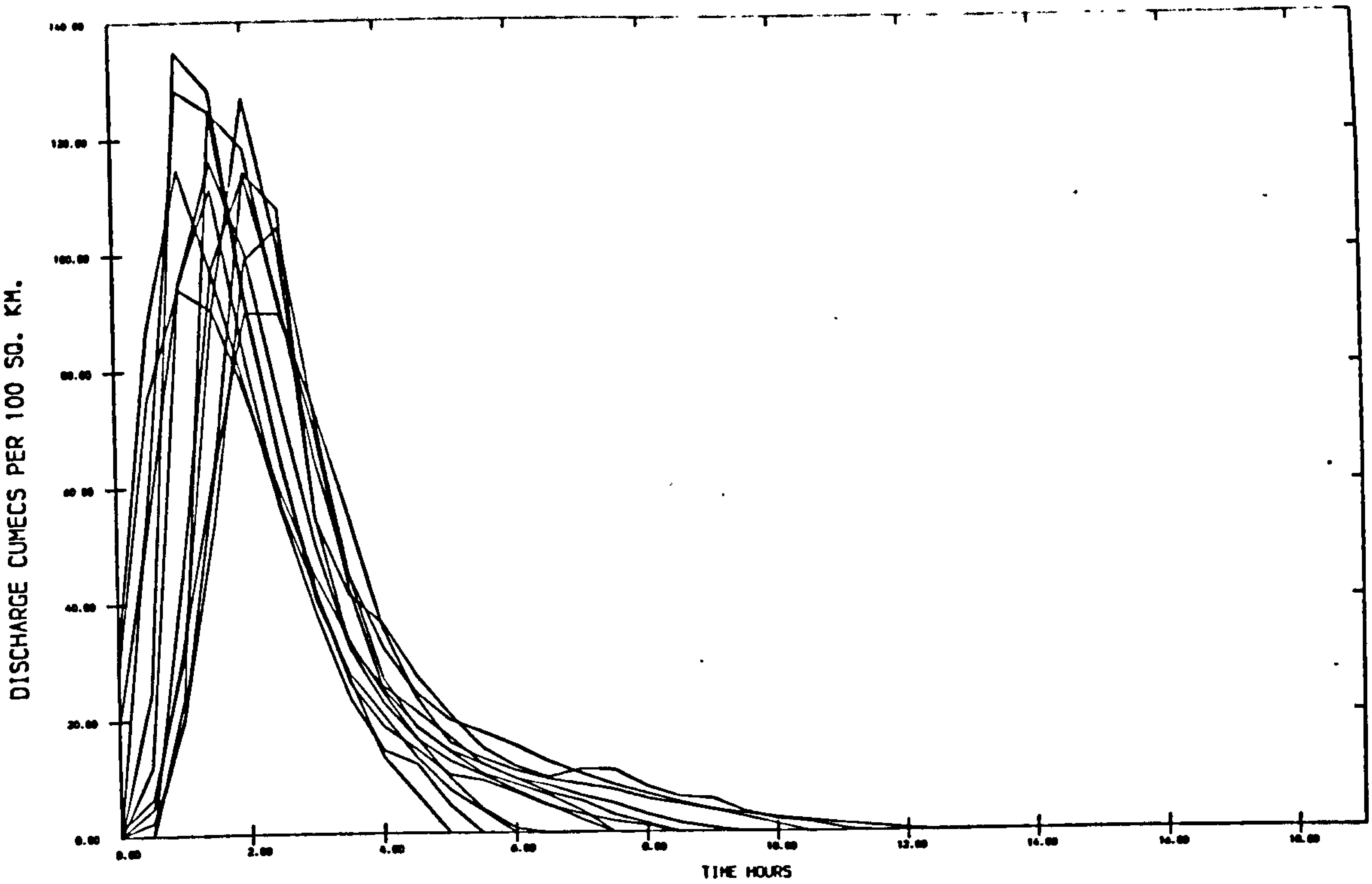


Fig. 5.4 Unit hydrographs for the Beverley Brook, Group 3

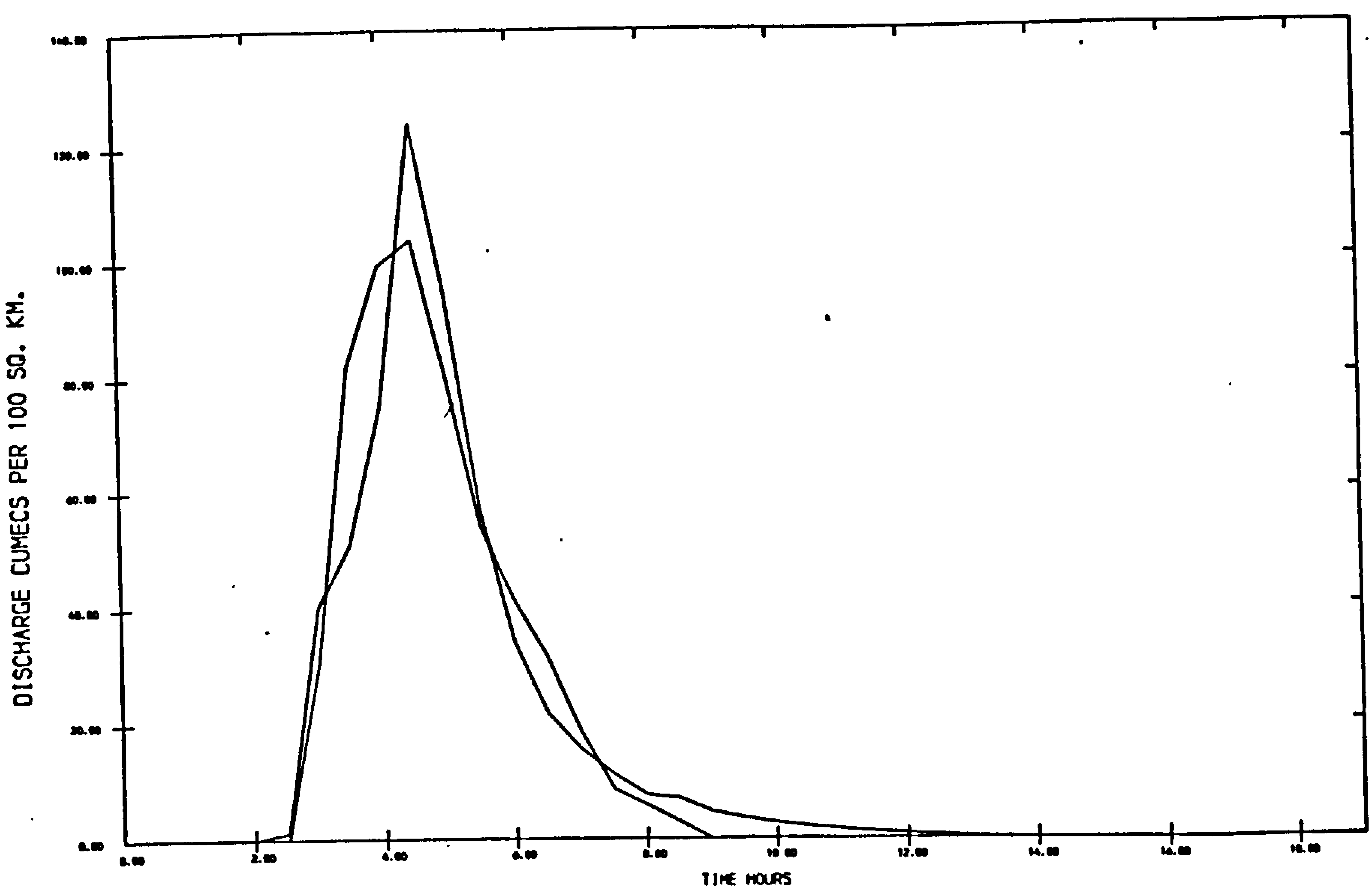


Fig. 5.5 Unit hydrographs for the Beverley Brook, Group 4

Table 5.6 Characteristics of TUHs and boundary conditions for each TUH group

	Variable	Group 1		Group 2		Group 3		Group 4	
		Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
TUH Parameters	Peak (cumecs)	58.166	7.353	80.639	5.766	114.155	13.418	113.443	14.495
	Time to Peak (hours)	2.550	0.643	2.417	0.875	1.583	0.515	4.5	0.0
	Peakedness	2.427	0.864	6.974	6.294	7.284	4.299	21.624	4.624
	Shape	0.630	0.125	0.673	0.153	0.629	0.087	0.923	0.029
	Curvature	-22.484	7.129	-63.889	20.467	-224.329	119.151	-211.181	142.594
Regenerative Error Parameters	ISE (%)	17.375	10.417	12.529	12.614	1.515	2.136	0.017	0.008
	PISE (%)	11.065	6.873	8.884	9.228	0.650	0.912	0.016	0.009
	QPE (%)	-1.459	5.750	-3.236	9.458	0.140	0.443	-0.012	0.006
	TPE (hours)	-0.200	0.422	-0.125	0.311	0.0	0.0	0.0	0.0
Storm Characteristics and Boundary Conditions	Storm Duration (hours)	5.350	0.492	4.583	2.009	2.583	1.663	2.500	1.414
	Total Rainfall (mm)	10.818	4.542	9.408	7.017	7.294	3.919	3.580	0.028
	Mean Rainfall Intensity (mm/hr)	2.086	0.786	5.739	14.526	4.052	3.481	1.709	0.978
	Time of Rise (hours)	5.050	1.165	3.375	1.110	2.208	1.033	2.500	0.0
	S.M.D. (mm)	23.900	38.417	42.592	38.941	64.642	39.014	109.200	5.657
	Number of events	10	-	12	-	12	-	2	-

$$\text{Peakedness} = \frac{Qp \cdot t_L}{A \cdot R} \tag{5.2}$$

where Qp is the TUH peak discharge, cumecs
t_L is the time lag, defined as the centroid of the peaks minus the centroid of the total rainfall, hours
A is the catchment area, Sq.Km.
R is the depth of runoff, mm.

Second, shape which defined whether the hydrograph was early, middle or late peaked was defined as,

$$\text{Shape} = tp/t_L \tag{5.3}$$

where tp is the TUH time to peak, hours
t_L is the lag time, defined as the centroid of the peaks minus the centroid of the total rainfall, hours.

The variation between the groups was attributed to the causative storm and the antecedent catchment conditions. Group 1 TUHs were caused by long duration, low intensity storms on a wet catchment. Groups 2 and 3 were the result of relatively short duration, high intensity storms on a dry catchment. To provide a quantitative assessment of the relationship of the TUH Qp and Tp for each of the groups, to the storm and catchment conditions a correlation analysis was undertaken (Tables 5.7 and 5.8). Correlation coefficients were not calculated for Group 4 because it contained only two TUHs. The results confirmed the findings of N.E.R.C. (1975) and Stoneham and Kidd (1977) that the variation of the TUHs parameters cannot be successfully 'explained' by simple correlations of the TUH parameters with the storm and boundary conditions.

It was considerably more difficult to 'explain' time to peak than peak discharge. The correlation coefficients varied markedly between each of the first three groups preventing the development of an objective method of classifying a storm event into one of the four groups. The variation may indicate the non-linear behaviour of the catchment. The simple correlation of one variable onto either TUH Qp or Tp may be misleading because many variables influence the magnitude of the TUH parameters and therefore a multiple regression analysis was made.

Table 5.7 Correlation coefficients:
TUH time to peak versus boundary conditions

Variable	Group 1		Group 2		Group 3		All 36 events	
	R	R^2	R	R^2	R	R^2	R	R^2
Storm Duration	-.3676	.1351	-.5387	.2902	0.1946	.0379	-.0825	.0068
Total Rainfall	.2350	.0552	.1940	.0376	.2250	.0506	.1119	.0125
Peak Rainfall Intensity	.4763	.2269	.3760	.1414	.3055	.0933	.1578	.0249
S.M.D.	.0058	-	.3411	.1163	-.0925	.0086	.1176	.0138
C.W.I.	.0701	.0049	-.3580	.1282	.0309	.0010	-.0578	.0033
Mean Rainfall Intensity	.5255	.2762	.3929	.1544	.5603	.3139	.1953	.0381

Table 5.8 Correlation coefficients:
TUH peak discharge versus boundary conditions

Variable	Group 1		Group 2		Group 3		All 36 events	
	R	R^2	R	R^2	R	R^2	R	R^2
Storm Duration	.7063	.4989	-.3039	.0924	-.3922	.1538	-.5872	.3448
Total Rainfall	.3308	.1094	-.2304	.0531	-.3604	.1299	-.3378	.1141
Peak Rainfall Intensity	.2100	.0441	-.1028	.0106	-.0899	.0081	.0732	.0054
S.M.D.	.3721	.1385	-.0127	.0002	-.3216	.1034	.4042	.1634
C.W.I.	-.3353	.1124	.0277	.0008	.4008	.1606	-.3657	.1337
Mean Rainfall Intensity	-.2760	.0762	-.0769	.0059	-.0610	.0037	.0285	.0008

It has been found that the time of rise of the hydrograph has a significant effect on the TUH Qp and Tp (Schultz and Lopez, 1974). This finding was incorporated in the analysis by first predicting the time of rise and then using the predicted value in equations to predict the TUH Qp and Tp.

Two alternative forms of the multiple regression equation were tested. First the additive form ($Y = M + A_1X_1 + A_2X_2 + \ldots.$) and second the multiplicative form ($Y = M.X_1^{A_1} . X_2^{A_2} . \ldots.$). The former is linear and the latter non-linear (Table 5.9).

Table 5.9 Comparison of linear and non-linear forms of scaling equations

Dependent Variable	Linear Equation R^2	Non-Linear Equation R^2
Catchment Response Time	0.56590	0.55574
TUH Qp	0.66389	0.70469
TUH Tp	0.27908	0.35386

All equations significant at 1%

The linear form was used to predict the catchment response time and the non-linear form to predict TUH Qp and Tp (Table 5.10). The equation to predict TUH Tp had the lowest R square value of the three equations (0.354). The error in prediction was compounded by the necessity to round the predicted value to the nearest half hour so that it was suitable for use with half-hourly rainfall data.

The three equations (Table 5.10) were used to scale the dimensionless unit hydrograph (section 5.2.3) and the parametric triangular unit hydrograph (section 5.2.4).

The following three sections describe the evaluation of three models of the unit hydrograph, first the time-invariant TUH, second the dimensionless unit hydrograph and third the parametric triangular unit hydrograph.

Table 5.10 Regression equations to predict TUH peak discharge and TUH time to peak

(1) Equation to predict catchment response time

$$\text{Time of Rise (Hours)} = 1.6124 + 0.2828\ \text{DUR} + 0.1714\ \text{TRF} + 0.1005\ \text{MIN} + 0.0172\ \text{API5} - 0.0049\text{SMD} - 0.1650\ \text{PIN} \qquad (5.4)$$

R = 0.75226 R^2 = 0.56590 DF = (6.29)

Critical F = 3.50 F = 6.30072 Equation significant at 1%

(2) Equation to predict TUH peak

$$\text{TUH peak (cumecs)} = 116.6310 \cdot \frac{\text{SMD}^{0.0264} \cdot \text{PIN}^{0.2947}}{\text{TOR}^{0.3202} \cdot \text{TRF}^{0.1892} \cdot \text{MIN}^{0.1652}} \qquad (5.5)$$

R = 0.83946 R^2 = 0.70469 DF = (5.30)

Critical F = 3.70 F = 14.31747 Equation significant at 1%

(3) Equation to predict TUH time to peak

$$\text{Time to peak (hours)} = 1.2285 \cdot \frac{\text{TOR}^{0.5965} \cdot \text{API5}^{0.0504} \cdot \text{SMD}^{0.0479}}{\text{DUR}^{0.2429}} \qquad (5.6)$$

R = 0.59486 R^2 = 0.35386 DF = (4.31)

Critical F = 4.02 F = 4.24432 Equation significant at 1%

Notation:

DUR - Storm duration, hours

TRF - Total rainfall, mm

PIN - Peak rainfall intensity, mm/hr

MIN - Mean rainfall intensity, mm/hr

TOR - Time of rise, hours

API5 - API5, mm

SMD - SMD, mm

5.2.2 The time-invariant unit hydrograph

Hydrological analyses conventionally use a single time-invariant TUH which assumes that the response characteristics of a catchment are constant.

A mean TUH was calculated by aligning the peaks of the TUHs and calculating mean values for the ordinates on either side. The mean TUH was positioned on the time (x) axis such that the time to peak was equal to the mean time to peak. The area under the curve is then rescaled to the unit volume to be expected from the unit storm (10mm) over the unit catchment area (100 Km^2) for the unit time period (0.5 hours). A computer program was written to calculate a mean TUH from several TUHs (Appendix 8).

The mean TUH of the 36 events was calculated (Fig. 5.6) and examined for its performance in regeneration and prediction (Table 5.11). The results indicate the futility of using a single TUH. A mean TUH was calculated for each of the four groups (Fig. 5.7) and the accuracy of the regeneration of the hydrograph was examined (Table 5.12). These TUHs yielded a better fit than the use of a single TUH. The use of four TUHs improved the accuracy of the peak discharge estimate more than the time to peak estimate. The weighted mean absolute QPE showed a reduction of 8.958 percent (or an improvement of 57.48 percent) whereas the weighted mean absolute TPE showed a reduction of 0.542 percent (or an improvement of 31.61 percent). The results of using four TUHs points to the advantages of a quasi-linear TUH, that is, a TUH which is linear during storms but non-linear between storms. The results of two quasi-linear models, the dimensionless unit hydrographs and the parametric unit hydrograph are discussed in the following two sections.

5.2.3 The dimensionless unit hydrograph

The mean TUH for the Beverley Brook (section 5.2.2) was converted to a dimensionless unit hydrograph (DUH) by dividing the discharge and time for each ordinate by the peak discharge and the time to peak of the TUH, respectively. This produced a dimensionless unit hydrograph with a peak discharge and time to peak of unity. A TUH was calculated by rescaling the DUH using multiple regression equations to calculate Qp and Tp (Table 5.10). A computer program was written to perform the necessary calculations (Appendix 9).

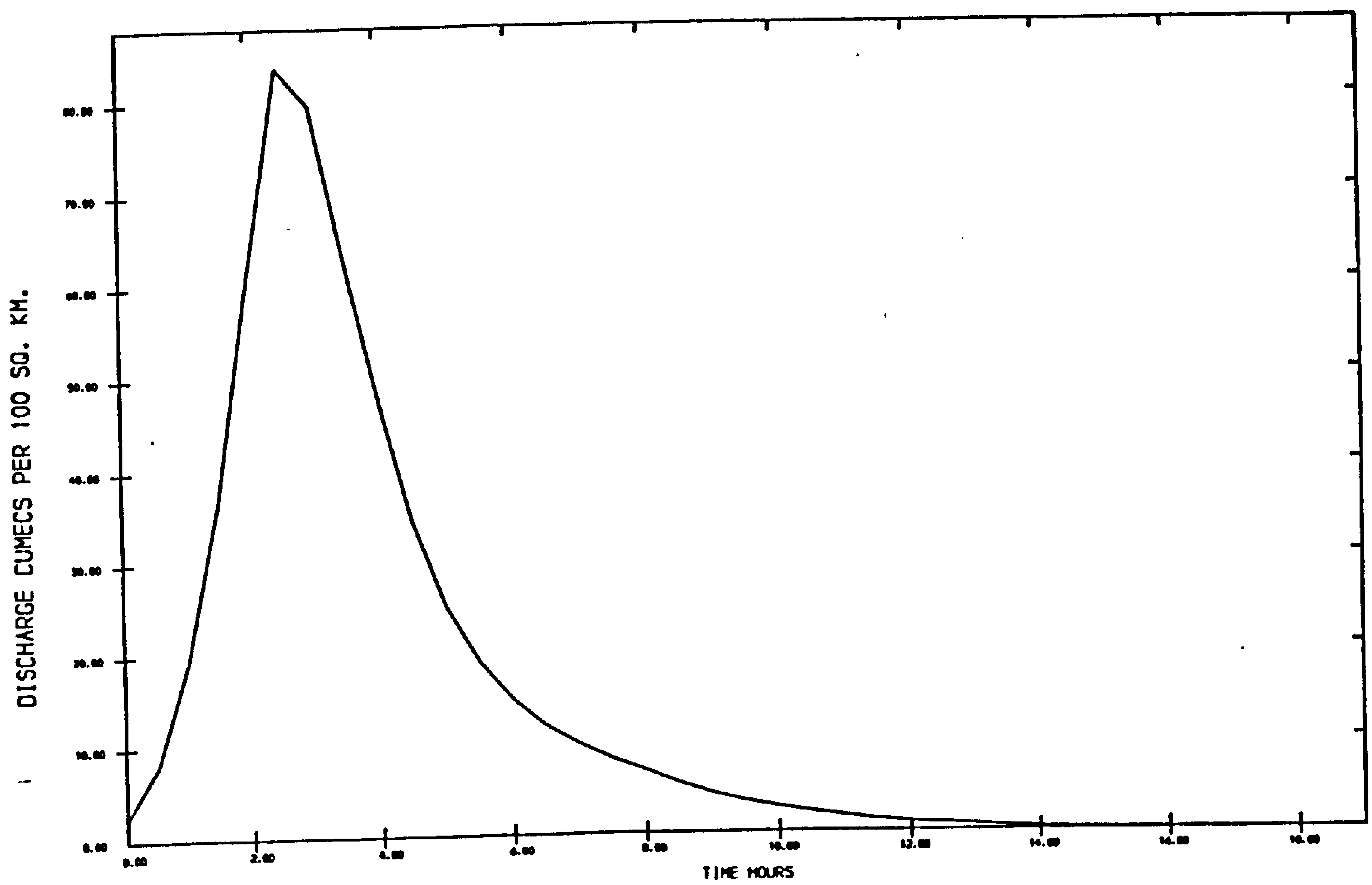


Fig. 5.6 Mean unit hydrograph for the Beverley Brook

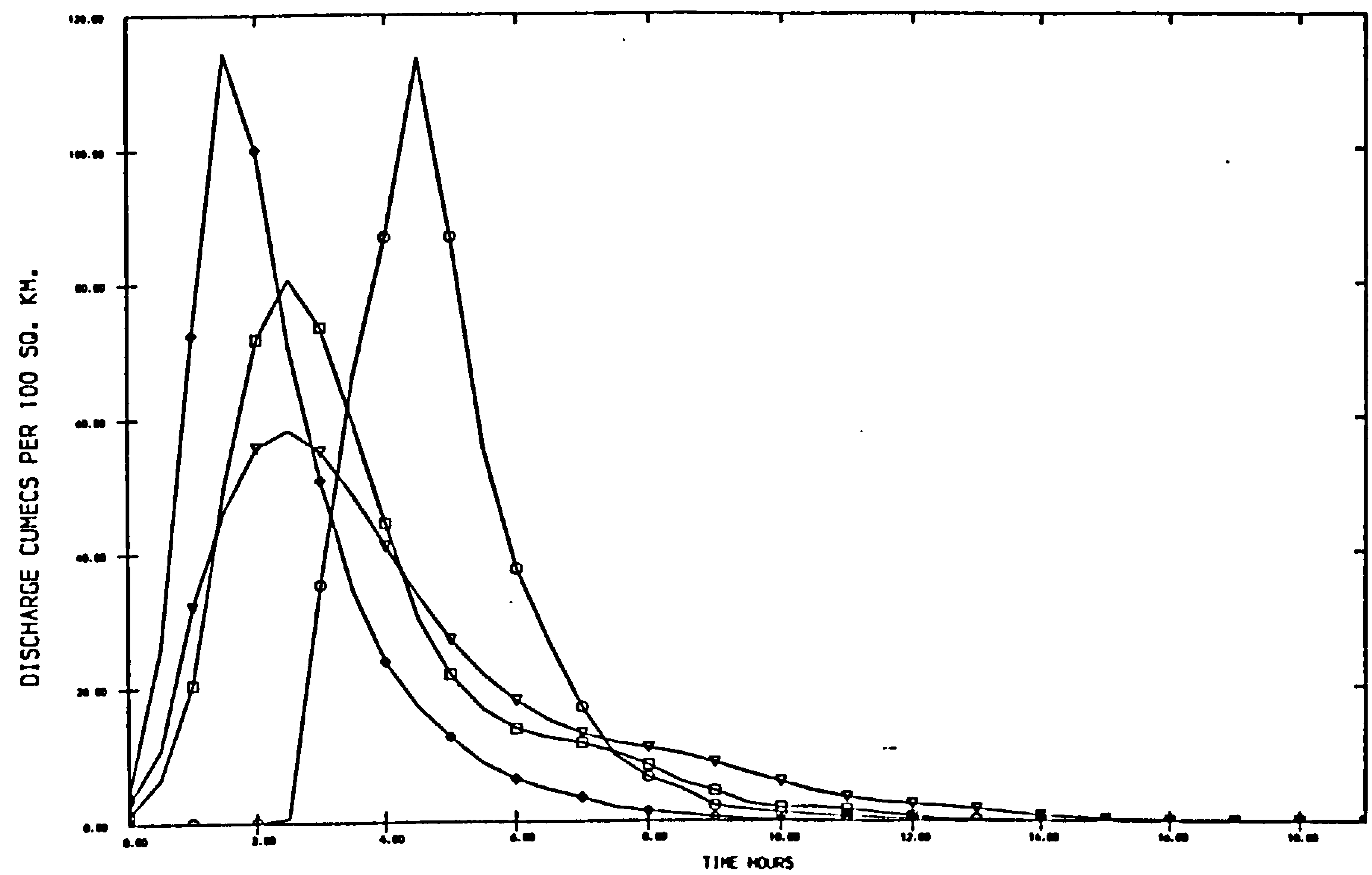


Fig. 5.7 Mean unit hydrographs of the four groups for the Beverley Brook

Group:

1 - ▽

2 - □

3 - ◇

4 - ○

Table 5.11 Performance of a time invariant TUH

Error Function	Regeneration Mean	Regeneration Standard Deviation	Prediction Mean	Prediction Standard Deviation
ISE %	80.749	34.413	170.413	61.047
PISE %	53.912	21.176	102.761	34.196
QPE %	-0.768	25.577	13.175	35.579
Absolute QPE %	21.069	14.078	26.128	27.137
TPE (hours)	0.208	1.003	1.788	1.872
Absolute TPE (hours)	0.792	0.637	2.250	1.251

Table 5.12 Performance of a time invariant TUH

Error Function	Group 1		Group 2		Group 3		Group 4	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
ISE %	52.317	25.691	64.225	32.800	57.185	18.088	16.591	0.153
PISE %	40.613	24.377	47.054	24.880	44.518	15.639	15.450	0.155
QPE %	0.309	12.693	-3.901	10.236	1.309	12.044	0.492	12.383
Absolute QPE %	10.468	6.282	7.606	7.634	9.084	7.543	8.756	0.695
TPE (hours)	-.250	0.635	-.083	0.949	-.167	0.615	0.0	0.0
Absolute TPE (hours)	0.450	0.497	0.750	0.544	0.500	0.369	0.0	0.0

The DUH method was used to predict the hydrographs for the 26 rejected events and the regenerative performance of the four TUH groups was assessed both individually and together (Tables 5.13 and 5.14).

The DUH method was inferior to the mean TUH method used in the preceding section (section 5.2.2) because of the problems of rescaling the DUH to a TUH. When the time to peak predicted by the multiple regression equations (Table 5.10) was less than the time to peak of the mean TUH, the ordinates of the TUH had to be increased to maintain the unit volume. This resulted in overprediction of the peak discharge. The peak discharge was underestimated when the predicted time to peak was greater than the time to peak of the mean TUH.

In spite of the introduction of quasi-linearity and a considerable increase in the number of calculations the DUH method was inferior to the time invariant TUH method. This led to the final model of the TUH which abandoned the formulation of the TUH as a set of ordinates (as in the case of the DUH) and replaced them with a straight line approximation. This method was the Parametric triangular unit hydrograph and is discussed in the next section (section 5.2.4).

5.2.4 The parametric triangular unit hydrograph

The parametric triangular unit hydrograph (PTTUH) is defined by three parameters, the peak discharge (Qp), the time to peak (Tp) and the time base (TB) (Fig. 5.8). Qp and Tp are predicted from the equations in Table 5.10 and TB is calculated to maintain a unit volume, TB=555.5/Qp. Since Tp and TB have to be rounded to the nearest half hour, the whole of the PTTUH is rescaled after its initial construction. It was assumed that the TUH starts at zero hours with a discharge of zero cumecs. In some cases (events 1 and 61) it was found that the recession limb was shorter than the rising limb. In these cases an isosceles triangle was substituted, with the peak at the predicted time to peak. The procedures to derive a PTTUH are contained in a computer program "PATRUH" (Appendix 5).

The reduction of the TUH curve to two straight lines has little effect on the accuracy of the peak discharge estimate, producing a mean absolute error of 3.322 percent. The absolute error in calculating spill volume using the PTTUH depends on the value of the spill volume threshold and can range from 4.935 to 14.323 percent (see section 7.5 for further details). In a design situation more error will be introduced by estimating the percentage response than the substitution of a straight line approximation of the TUH.

Table 5.13 Performance of a dimensionless unit hydrograph

Error Function	Regeneration		Prediction	
	Mean	Standard Deviation	Mean	Standard Deviation
ISE %	84.133	37.021	167.340	63.385
PISE %	65.502	32.042	105.544	37.719
QPE %	28.017	37.860	23.721	39.996
Absolute QPE %	36.486	29.513	33.254	32.168
TPE (hours)	-.357	0.982	1.692	1.817
Absolute TPE (hours)	0.729	0.741	2.077	1.339

Table 5.14 Performance of a dimensionless unit hydrograph

Error Function	Group 1		Group 2		Group 3		Group 4	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
ISE %	105.605	33.435	71.649	36.364	69.423	28.473	133.699	23.699
PISE %	89.585	28.028	58.596	32.789	46.955	20.057	94.349	23.319
QPE %	63.438	27.909	28.336	34.894	5.339	24.558	-14.777	23.074
Absolute QPE %	63.438	27.909	34.210	28.521	19.475	14.848	16.316	20.898
TPE (hours)	-.700	0.675	-.409	0.917	0.292	0.782	-2.250	0.354
Absolute TPE (hours)	0.700	0.675	0.591	0.801	0.625	0.528	2.250	0.354

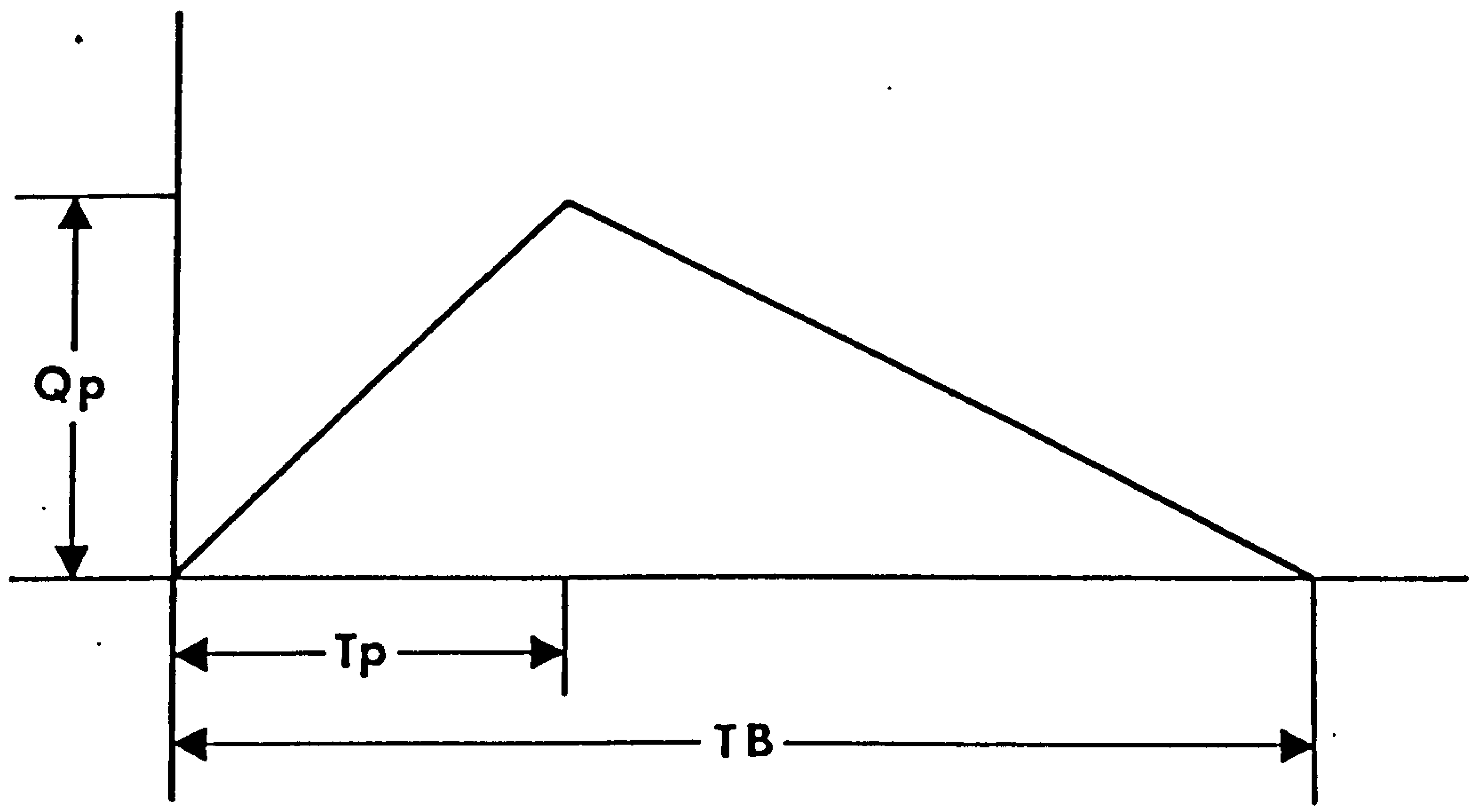


Fig. 5.8 The parametric triangular unit hydrograph

Table 5.15 Performance of a quasi-linear unit hydrograph (PTTUH)

Error Function	Regeneration		Prediction	
	Mean	Standard Deviation	Mean	Standard Deviation
ISE %	74.293	38.082	155.314	62.500
PISE %	56.323	30.795	95.696	29.814
QPE %	-2.333	19.631	0.304	22.456
Absolute QPE %	16.235	10.940	17.234	13.981
TPE (hours)	-.286	0.926	1.615	1.779
Absolute TPE (hours)	0.686	0.676	2.000	1.311

Table 5.16 Performance of a quasi-linear unit hydrograph (PTTUH)

Error Function	Group 1		Group 2		Group 3		Group 4	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
ISE %	65.404	39.709	58.500	21.180	79.916	26.276	171.863	9.139
PISE %	48.983	36.939	45.423	18.413	60.408	19.313	128.461	3.954
QPE %	6.099	21.645	-9.745	16.084	-4.576	20.494	9.733	3.617
Absolute QPE %	16.235	14.730	15.861	9.297	17.661	10.128	9.733	3.617
TPE (hours)	-.450	0.643	-.227	0.876	0.125	0.829	-2.250	0.354
Absolute TPE (hours)	0.550	0.550	0.500	0.742	0.708	0.396	2.250	0.354

The results of using the PTTUH are presented in Tables 5.15 and 5.16. The predictive performance of the PTTUH was superior to both the time invarient TUH and the DUH, all the objective error functions were at their lowest for this model. However the fit of the observed and the predicted is still very poor and the time to peak is clearly of little use with a mean absolute error of two hours. Only the peak discharge estimate is moderately reliable, the difference of the mean absolute error between the regeneration and prediction was a mere 0.999% which indicated a consistently accurate model.

5.3.1 An analysis of the catchment response using conceptual models

This section presents the results of an analysis using 17 conceptual models and 62 storm events derived from the Beverley Brook. The principles of deriving these models was described in section 4.3 and the results of analyses using conceptual models have been reviewed in section 3.4.

A conceptual models parameters may be calculated by two alternative methods. First, by equating the moments of the conceptual model to the estimated moments of the unit hydrograph as derived from the rainfall and runoff data. This method is confined to linear models. Second, by direct solution of the equations of the models. For linear models, with a unique solution, an analytical solution is adopted. For non-linear models a numerical scheme is required, for example by performing a Newton-Raphson iteration on a finite-difference formulation of the models equations (e.g. Kidd, 1978b, 27). The moment matching scheme was adopted because of the availability of the computer program 'PICOMO', Program for the Identification of Conceptual Models (Appendix 4) which was written by Dooge and O'Kane (1977). The program was modified for the catchments under consideration and by the addition of an objective error function subroutine. The numbering of the models (Table 5.17) is consistent with Dooge (1977) to facilitate ease of comparison.

Table 5.17 Conceptual models used in the analysis

One Parameter Models:

Model No.	Model.
1	Single linear channel
2	Rectangle
4	Scalene triangle, peak at $\frac{1}{3}$ of the base
5	One linear reservoir
6	Two equal linear reservoirs
9	Diffusion reach

Two Parameter Models:

Model No.	Model.
11	Isosceles triangle
12	Channel and reservoir
13	Routed rectangle
14	Routed triangle
16	N equal linear reservoirs
17	Cascade of two unequal reservoirs
19	Cascade of two equal linear reservoirs
20	Convective diffusion reach

Three Parameter Models:

Model No.	Model.
22	Lagged cascade of N equal linear reservoirs
23	Cascade of three reservoirs
24	Two reservoirs with lateral inflow

The 62 storm events, derived from the Beverley Brook, which were used in this section, were the same 62 events used in the unit hydrograph analysis (section 5.1). Each data set, consisting of effective rainfall and rapid response runoff, was processed by the computer program PICOMO (Appendix 4) to calculate the parameters of each of the 17 conceptual models. The models were fitted by matching the shape factors (equation 4.28) of the conceptual model to those of the IUH derived from the effective rainfall and rapid response runoff. Six events (18, 25, 31, 44, 48, 57) were rejected from subsequent analysis because the shape factors of the IUH were negative and consequently had no physical interpretation. This occurred for those events where the response time was short, 0.5 hours, and where the rainfall distribution was strongly negatively skewed. These six events were also rejected from the TUH analysis by two or more rejection criteria. It was not always possible to calculate model parameters for the remaining 56 events because the shape factors were sometimes out of range. The extent to which parameters could be found was largely due to the number of parameters in the model (Table 5.18). The greater the number of parameters, the larger the number of events which were rejected. An increase in the number of parameters in a model tended to increase the accuracy of the model and therefore the choice of model was dependent on both the consistency of deriving a model and on the accuracy of the model.

The choice of the best model was based on the value of the standard objective error functions (section 4.5) which measured the fit between the observed and the modelled hydrograph. This indicated that the routed triangle (model 14), the Nash model (model 16) and the convective diffusion reach (model 20) were the most consistently accurate models. These three models were compared using a set of 38 storm events (Table 5.19). This indicated that model 16 was inferior to models 14 and 20 and that model 14 was slightly superior to model 20. The advantage of model 20 was that it had been derived for all the 56 events, whereas model 14 had been unable to derive an IUH for 15 (26.79%) of the events. Rather than reject one model in favour of the other, both models were used in the subsequent analyses.

Table 5.18 Percentage of events for which an IUH could not be derived for each conceptual model

Model Number	Number of Parameters	Percentage of Events Lost
1	1	0.0
2	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0
9	1	0.0
11	2	57.14
12	2	21.43
13	2	28.57
14	2	26.79
16	2	3.57
17	2	23.21
19	2	21.43
20	2	0.0
22	3	69.64
23	3	100.0
24	3	69.64

Number of Events = 56

Table 5.19 Regeneration performance of three conceptual models

Routed triangle

	Mean Error		Absolute Error			
	QPE	TPE	QPE	TPE	ISE	PISE
Mean	8.420	-0.276	17.299	0.566	7.019	5.687
S.D.	19.121	0.685	11.716	0.475	2.652	2.410

N equal linear reservoirs

	Mean Error		Absolute Error			
	QPE	TPE	QPE	TPE	ISE	PISE
Mean	-19.749	-1.118	32.757	1.145	16.600	10.505
S.D.	36.088	0.729	24.888	0.687	9.573	5.644

Convective diffusion reach

	Mean Error		Absolute Error			
	QPE	TPE	QPE	TPE	ISE	PISE
Mean	8.128	-0.382	19.195	0.539	7.362	6.176
S.D.	22.138	0.590	13.701	0.450	2.740	2.514

Notation:

QPE - Peak discharge error, percent

TPE - Time to peak error, percent

ISE - Integral square error, percent

PISE - Partial integral square error, percent

Table 5.20 Results of an analysis of 62 events from the Beverley Brook

Derivation technique	ISE percentage on regeneration
Matrix inversion	37.238
Convective diffusion reach (model 20)	11.509

A comparison of the performance of model 20 with the matrix inversion method (Table 5.20) indicates that the former method is superior in terms of accuracy. In addition to this, the TUHs derived by model 20 consisted of positive ordinates and required no smoothing. This result points to the considerable advantages of using conceptual models.

Although the parameters of the models were optimal in terms of the moments they were not necessarily the most accurate because the values of the higher moments may be distorted by a long recession limb (Dooge, 1973, 213). To increase the accuracy of the parameters an optimisation scheme was adopted where one parameter was held constant and the other iterated. The parameter with the smallest standard deviation was held constant. For model 14, the base of the triangle was iterated and the delay time was held constant. For model 20, the 'a' parameter was iterated and the 'b' parameter was held constant. The choice of optimisation technique was made on the grounds of ease of programming (Appendix 10). The improvement in the quality of the estimate was assessed by three objective error functions, ISE, PISE, and QPE. When two of these functions improved by less than 0.001%, the iterative process was stopped. The iterative step was 0.0001.

The optimised model parameters exhibited a great range which was assumed to be due to variations in the storm characteristics and antecedent catchment conditions. This was tested following the procedure described in section 5.2.1, however the multiple regression equations were statistically insignificant (Table 5.21) and the analysis was terminated.

Table 5.21 Results of the multiple regression analysis for conceptual model parameters

Model No.	Dependent variable	Variance explained R	R^2
14	Delay time	.61374	.37668
	Base of triangle	.25483	.06494
20	Parameter A	.50412	.25413
	Parameter B	.36351	.13214

5.3.2 Conceptual models: suggestions for future research

The results of this section indicate that conceptual models are superior to inversion methods for deriving consistent and accurate TUHs. The subsequent failure to capitalise on this finding was due to the lack of experience on the part of the author. Retrospectively it was possible to identify several improvements to the analysis.

First, the two disadvantages of using the 'PICOMO' program were that it was not possible to use non-linear conceptual models and that model parameters were not estimated for each storm event. Direct analytical and numerical solutions would rectify both these omissions. It would seem profitable to test the performance of the two methods by estimating the model parameters using both methods and using these estimates to predict the discharge hydrograph for a separate set of storm events. Subsequent analyses should include non-linear models because these have been found to predict marginally better than linear models (Kidd, 1978b).

Second, the use of three objective error functions in the optimisation program is non-standard, conventional analyses use one. The adopted method measured three properties of the difference between the observed and predicted hydrographs therefore the hydrograph would not be optimum for one at the expense of the other two. A better method is to decide which property of the hydrograph is the most important and to use an error function which measures that property. For example, discharge peak analyses should use QPE, whereas a flood volume analysis should use ISE. If it is felt necessary to use three error functions then a weighting system should be devised which incorporates a ranking of the relative importance of the three hydrograph properties.

Third, the computer program 'OPTMIZ', which optimised the parameters, was inefficient. Frequently 200 iterations were performed without reaching the optimum parameter value. It is suggested that subsequent analyses should use the Rosenbrock (1960) method of parameter optimisation. This recursive method consists of a search in n-dimensional space for the best set of n model parameters formed by n-orthogonal parameters and bounded by limits set on the parameters. Clarke's (1973) computer program would be suitable for such an analysis.

Fourth, the selection of the fixed parameter was subjective and should be replaced by an objective error surface mapping computer program. This consists of a plot of error function values on two axes relating to the specified parameters. A parameters sensitivity is defined by the slope of the error surface in the region of the optimum, in the direction of the relevant parameter axis. Part of the failure of the analysis may have been due to optimising the least sensitive parameter.

Fifth, whilst a given parameter was held constant for each storm event, the value of that parameter varied between events. This was the most serious error of the analysis. "Where there is more than one parameter, a high degree of inter-correlation is generally observed which makes optimisation of more than one parameter difficult; and, in this case, it is unlikely that realistic relationships between model parameters and catchment characteristics could be obtained for more than one parameter of a given model" (Kidd, 1978b, 20). The solution is to fix the least sensitive parameter(s) with a value which is constant for <u>all</u> storms, the one free parameter is then optimised. This increases the information content of the free parameter and allows a more reliable correlation with storm and catchment characteristics. The value of the fixed parameter is the mean optimum value derived from the error surface mapping analysis.

5.4.1 Methods of estimating flood frequency

This section uses the findings of section 5.2 to derive a flood frequency curve.

A flood frequency curve will tell the engineer the discharge which may be expected to occur for a given return period. The return period to which a structure is designed is a function of the degree of risk to be provided for, the life of the structure and economic constraints.

There are two alternative, though complementary, methods of flood frequency estimation. The first is a statistical method which provides an estimate of the peak discharge for a given return period. The second is a convolution method which, in addition to providing peak discharge return periods, provides an estimate of the return period of a given flood volume. The first method has received a detailed description in the Flood Studies Report (NERC, 1975) and was not considered here. The second method is based on convolving a TUH with a rainfall profile and was the method chosen to evaluate the PTTUH as a model to predict flood frequency curves. The success of the method is dependent on an accurate definition of both the TUH and the rainfall profile. Recently research has been directed towards definition of the rainfall profile because of the realisation of its importance in defining the shape of the response hydrograph. This chapter considers the advantages of using historical profiles over synthetic design profiles and describes a flood frequency simulation program and its application to the Beverley Brook.

5.4.2 Rainfall profiles: historical versus design profiles

A rainfall profile may be defined by, first, frequency of occurrence; second, duration; third, peakedness; fourth, volume and fifth, location. The last factor, location, is important because rainfall profiles are spatially varient. There are two alternative sources of rainfall profile, either synthetic design profiles or those based on observed historical profiles. The latter method is usually dismissed for two reasons. First, the time and expense involved in collecting and analysing the data may be considered inappropriate for design work. In the absence of long rainfall records, statistical methods developed from large data sets (e.g. NERC, 1975) may be preferable. Second, it is arguable whether precision in the selection of the profile is necessary when the hydrologist is beset with problems of greater importance, for example the estimation of rainfall loss and the temporal distribution of that loss.

The arguments for the use of observed profiles are based on the observed deficiencies of design storms. First, section 5.2.1 indicated that there was a relationship between hydrograph characteristics and the characteristics of the causative storm. A design storm profile is unrepresentative of the observed range of profiles and therefore flood frequency curves calculated using design storms are liable to be inaccurate. Second, Howard (1978, 3) has shown that the effect of profile shape varies with the severity of the storm and the size of the catchment. Response to frequent events depends on the catchment while response to infrequent events depends on the storm profile. Spatial variations in rainfall profile on large catchments have a small effect on the shape of the discharge hydrograph because a large catchment damps the response characteristics. Points one and two indicate the importance of considering the meteorological conditions for a given catchment rather than adopting a national or regionalised mean profile. Third, it is difficult to define the frequency of design storms. Conventionally a 1:1 relationship is assumed between the rainfall and discharge. The Flood Studies Report (NERC, 1975, I, 464) presents an alternative relationship, but one which is still fixed (Table 5.22).

Table 5.22 Recommended storm return period to yield flood peak of required return period (NERC, 1975)

Flood Peak Return Period	2.33	5	10	20	30	50	100	250	500	1000
Rainfall Return Period	2	8	17	35	50	81	140	300	520	1000

This assumption was refuted by Butters et al (1977, 349) who found that, "...never did a 30 year return flood result from rainfall having the same frequency but always from those having frequencies varying from 3 to 12 years". This problem is removed by considering an historical record if the record is of sufficient length to define rainfall return periods. Fourth, Kelway (1977, 264) has shown that the profile varies in both volume and shape for a given storm across a single catchment. Conventional design methods largely ignore this, for example the design profile volume for use in the TRRL method (HMSO, 1976, 22) is calculated by multiplying a mean rainfall profile with a coefficient derived from a map which is assumed to take account of the spatial variation of rainfall volume for a given frequency. Fifth, design profiles are adequate for the purposes of assessing peak discharges in sewers and rivers but are inadequate for the assessment of flood volumes and the design of balancing ponds. This is because design storms ignore the temporal structure of a storm which has a strong influence on the peak discharge and the shape of the hydrograph. In the absence of hydrometric data the use of design methods (e.g. Hall et al, 1978) which incorporate design storms will provide over-estimates of flood volumes. The use of historical records would enable a reduction in capital costs by accurately estimating flood volumes but would cause an increase in design costs.

In spite of the overwhelming advantages associated with the use of historical storms, the U.K., in both research and application, lags behind the rest of the world in using these methods. This may be attributed to the strangle-hold exerted on the British engineer by the TRRL method and the Flood Studies Report (NERC, 1975) both of which use design profiles. Work by Kelway (1977) and workers at the Greater London Council (G.L.C.) (Butters, 1968; Butters et al 1975, 1977) has gone some way to rectifying this deficiency in U.K. hydrological research.

This thesis estimated flood frequency by combining the PTTUH method (section 5.2.4) with profiles derived from historical records.

5.4.3 The derivation of rainfall profiles from a historical record

This section expands the brief, published (Butters et al, 1977) description of the method of deriving rainfall profiles from historical records used by the G.L.C. and this thesis.

Thirty-three years of point rainfall from autographic gauges in the London area were examined for storms with more than 25 mm of rain and of less than 36 hours duration. This yielded 106 storm events. The profile, measured at half-hourly intervals, was standardised by setting the peak intensity to 100 units and expressing the remainder of the profile as percentages of the peak intensity. The 106 standardised profiles were divided into ten groups, each with an arbitrary characteristic distribution. The characteristics of these groups are shown in Figure 5.9. To derive a mean profile for a given group, the time taken for a given percentage of the total rain to fall was averaged. This controlled for the range (the time constraints) of durations within a given group. Earlier work (Butters, 1968) found that on average, 2½ significant rainfall events, of 25 mm or more occurred each year over the London area. Therefore, to represent 100 years of significant rainfall events, 250 rainfall profiles were synthesised. Each of the 250 profiles was generated by the following iterative technique. One of the ten profiles was selected at random. For the chosen profile the time constraints were noted and a storm duration within these constraints was selected together with a rainfall frequency, both being selected at random. Using this information, the volume of rainfall was read from a frequency-duration-volume matrix stored in the computer. The matrix was supplied by the Meteorological Office and was calculated specifically for the London area. The mean group profile was then scaled, by a factor, to the required frequency-duration-volume relationship. The factor is the quotient of the volume and the sum of the peak intensities for the selected duration for the given profile.

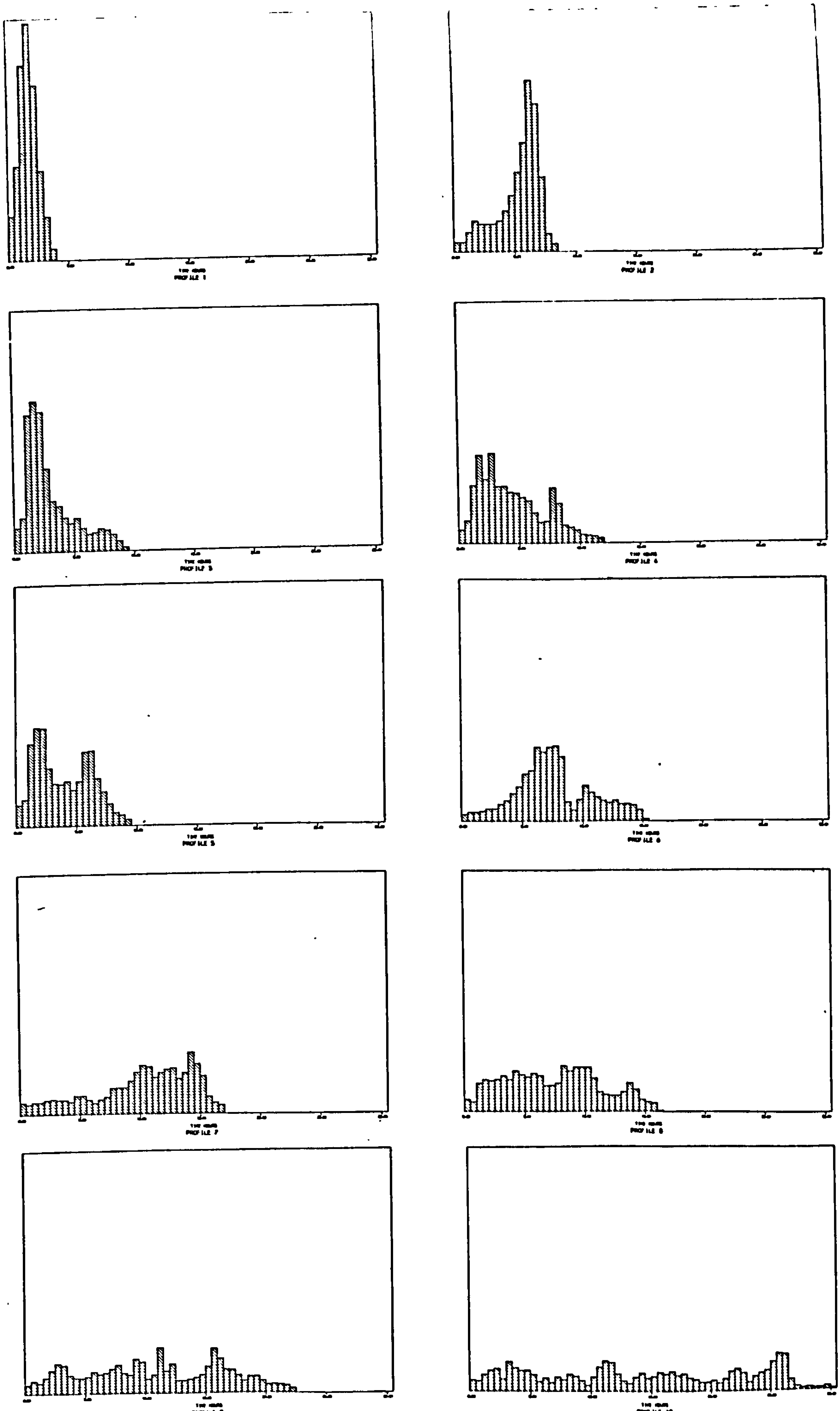


Fig. 5.9 Rainfall profile groups (after Butters and Vairavamoorthy, 1977, 359)

To test the validity of the derived profiles a frequency-duration-volume table was calculated using the 250 profiles. The mean absolute error for all frequencies, when compared to the Meteorological Office table was of the order of 5mm (Vairavamoothy, 1979). Although this is significant for short duration storms it is insignificant for long duration storms.

The original program convolved a given TUH with the 250 profiles and calculated the frequency of the peak flows and spill volumes from the generated hydrographs. Spill volume is defined as the volume of water in a hydrograph in excess of a given threshold,

$$Q_s = \sum_{t=1}^{t=n} Q(t) - q_s \cdot \frac{3600}{T} \tag{5.7}$$

where Q_s is the spill volume (cubic metres)
Q is the discharge (cumecs)
q_s is the spill volume threshold (cumecs)
t_1, t_n define the duration of the flood hydrograph (hours)
t is a time interval
T is the data interval (hours)

The computer program FLOODS (Appendix 7) was extended by writing three new subroutines. The first allowed the user to distribute a given rainfall loss by one of four methods. Either, constant loss percentage, loss rate, constant loss percentage varying with CWI or the phi index (see section 4.7 for further details). The second subroutine analysed the characteristics of the effective rainfall profiles. The characteristics were total rainfall, maximum intensity, mean intensity, the variance and the skewness. This subroutine, in combination with the ranking subroutine (supplied), provided information on the effect of rainfall separation technique on profile shape. The third subroutine calculated a PTTUH from given input parameters. The PTTUH could be used in place of an input TUH.

5.4.4 Flood frequency simulation

The computer program 'FLOODS' was used to estimate a flood frequency curve for the Beverley Brook. The input data consisted of the 250 profiles (described above) and the mean percentage response, CWI and API5 for the 62 storm events analysed from the Beverley Brook (Table 5.23). The effective rainfall was calculated using the loss rate separation method.

Table 5.23 Comparison of the observed and simulated flood frequency curve for the Beverley Brook

	Discharge return period (years)			
	2.33	10	50	100
Simulated (cumecs)	19.45	24.00	28.50	30.40
Observed (cumecs)	14.58	16.42	19.11	20.13
Error (%)	33.40	46.16	49.14	51.02

Curves fitted by the Gumbel distribution
Error = (Simulated-Observed)/Observed x 100

The method is not reliable, the divergence from the observed flood frequency curve is very large. There are three reasons for this. First, the observed flood frequency curve was based on annual peak discharges for 12 years (1963 to 1975), consequently it could be argued that the extrapolation of the curve to 50 and 100 years is very dangerous. Further, in 1968 a large storm generated a peak discharge which bypassed the recorder but which was estimated as 30.6 cumecs. This has a return period of over 1000 years by the observed curve or 100 years by the simulated curve, either way the flood is so significantly different from the remaining 11 annual floods that it distorts the observed flood frequency curve. Second, the observed flood frequency curve was based on an annual series, but the simulated curve was based on a partial duration analysis. This has a minor effect above a return period of 2.33 years (Haan, 1977, 149) but is an unnecessary problem. Its removal would require an increase of the number of storms from 250 to 300 and to assume that each year was represented by three events. The peak 'annual' discharge could then be selected for 100 'years' to produce a curve derived in a similar way to the observed curve. Third, the method assumes that the rainfall profile falls uniformly over the catchment, an areal reduction factor would correct this assumption.

From the evidence of one catchment it would appear that the method is in need of improvement.

5.4.5 Suggested improvements to the flood frequency computer program

The current version of the computer program assumes that both the percentage runoff and the antecedent catchment conditions are constant. Removal of these unrealistic constraints would substantially improve the program. A further improvement would be to replace the constraintless random profile selection method with one which selected profiles according to some observed seasonal distribution.

The first step in such an analysis would be to define the time interval into which the year should be divided. The decision to use either months or quarters would depend on the statistical significance of the variation of the parameter values between the selected time intervals. Within the chosen time interval an analysis would then be made of the distribution of the 10 profile groups, CWI, API5 and percentage runoff. A relationship should then be developed to predict the percentage runoff from CWI and API5. The number of storm profiles would have to be increased to at least 1000 to maintain statistical validity. Each profile would be generated by the method outlined in section 5.4.3, except that the random selection would use a weighted method to take account of the frequency of occurrence of the profiles within the chosen time interval. Similarly, CWI and API5, being largely independent of each other, would be randomly selected, within the observed range for the time interval. The percentage runoff would then be predicted from the values of CWI and API5. The profile would be reduced to an effective profile by using the value of percentage runoff. The effective rainfall profile would be convolved with the PTTUH calculated from the characteristics of the profile and the values of CWI and API5.

Whilst this procedure would be more realistic it would involve a considerable amount of extra analysis to derive the seasonal distributions and would increase the core requirement and the execution time of the program enormously. In view of this, it is doubtful if the increased accuracy of the estimates would be justified by the extensive modifications to the program and the large increase in computing costs.

5.5 Conclusion

This chapter has presented the results of a comprehensive investigation of the flood hydrology of the Beverley Brook.

The chapter consisted of three sections, the first was an analysis of 36 unit hydrographs. The principle conclusions of the section were, first, the response characteristics of the catchment, measured by the unit hydrograph, were not uniform and the form of the response was a function of the causative storm and antecedent catchment wetness. Second, densely urbanised catchments, subject to subsequent urban development do not exhibit a marked change in hydrological response. Third, after exhaustive tests a three parameter quasi-linear model called the Parametric Triangular Unit Hydrograph (PTTUH) proved to be the most successful model of the unit hydrograph. The three parameters were peak discharge, time to peak and time base. The first two parameters were predicted from the storm and antecedent catchment characteristics. The third was calculated from a simple formula and maintained a unit volume.

The second section consisted of the analysis of 62 storm events by 17 conceptual models. Analysis of the error functions on regeneration indicated that the convective diffusion reach conceptual model was the best of all and was superior to the matrix method of identification. The two parameters of the model displayed a high degree of inter-correlation which made it impossible to derive significant equations to relate the parameters of the conceptual model to the storm and antecedent catchment characteristics. Several suggestions were made which it is believed would rectify the deficiencies of the analyses reported in this section.

The final section described a method of simulating a flood frequency curve by convolving historical rainfall profiles with a quasi-linear unit hydrograph. Comparison of the observed and simulated flood frequency curve indicated that the method was in need of improvement and required further evaluation. Three improvements were suggested. First, an increase in the accuracy with which the PTTUH was calculated from storm and catchment conditions. Second, the introduction of seasonal variation in the frequency of occurrence of the storm profiles and the catchment conditions. Finally, the method required a long discharge record to establish accurate estimates of the discharge return periods.

The results presented in this chapter represent, first, a significant advance in the understanding of the variability of catchment response and second, a comprehensive evaluation of methods with which this variability may be analysed.

Chapter 6

THE RELATIONSHIP BETWEEN CATCHMENT CHARACTERISTICS AND THE UNIT HYDROGRAPHS FOR THE BEVERLEY, POOL, RAVENSBOURNE AND WANDLE

6.1 Introduction

This chapter commences by presenting the results of a unit hydrograph analysis of three south London catchments; the Wandle at Beddington, the Pool at Winsford Road and the Ravensbourne at Catford Hill. The characteristics of these catchments together with details of the rainfall and discharge gauges have been presented in sections 2.3 and 2.4. The characteristics of the unit hydrographs from these three catchments, together with those of the Beverley, are discussed and a relationship is established between the catchment characteristics and the parameters of the unit hydrograph.

6.2 Analysis

It was observed from the analysis of the 62 storm events from the Beverley that it became increasingly difficult to identify a stable TUH as the storm duration increased. To avoid non-productive analysis a regression analysis was performed to relate the integral square error (ISE) function to the storm duration (DURN),

$$ISE = 0.03196 DURN^{3.2897} \tag{6.1}$$

$$R = 0.79227$$

$$R^2 = 0.62769$$

Equation (6.1) indicated that storms of greater than 9 hours duration would yield an ISE of greater than 40 percent and would therefore cause the TUH to be rejected because it exceeded the critical value (section 5.2.1). Storms of greater than 9 hours duration were not used for the analysis of the three catchments. Although this introduced a sampling bias, such storms would have been rejected because the low regenerative performances of the TUHs would have indicated that they were poor identifications of the catchment response. The increase in efficiency was significant, 58.06 percent of the events selected from the Beverley yielded stable TUHs, whereas the exclusion of long duration storms for the other three catchments increased the success rate to 70.59 percent.

Thirty-four storm events were chosen for the three catchments (Table 6.1). The characteristics of the TUHs are presented in Tables 6.2, 6.3 and 6.4 and Figures 6.1, 6.2 and 6.3.

Table 6.1 Number of Storms and TUHs used in the analysis by catchment

River	Hydrometric Number	Number of Storms Analysed	Number of Stable TUHs	Percent Success
Wandle	39003	14	9	64.29
Beverley	39005	62	36	58.06
Ravensbourne	39056	10	7	70.00
Pool	39058	10	8	80.00

Table 6.2 Results of the TUH analysis of the Wandle

	TUH Parameters				Error Functions			
IEN	Qp	Tp	TB	CURV	ISE	PISE	QPE	TPE
1	32.35121	3.0	16.5	-6.5339	7.772	6.210	2.1600	0.0
2	28.57930	10.5	22.5	-1.1899	4.088	3.296	-0.9402	0.0
3	27.80936	5.5	22.5	-1.8328	7.741	5.393	0.9580	0.0
4	43.12053	2.0	19.0	-10.5640	20.608	6.772	-2.6664	0.0
5	89.56057	3.0	8.5	-108.0399	0.806	0.249	-0.1302	0.0
6	56.08219	3.0	12.5	-20.4050	11.106	3.304	-1.2869	0.0
8	47.94051	4.0	11.5	-41.5228	6.848	2.265	1.3575	0.0
9	28.20828	6.5	25.0	-3.1536	15.788	6.821	0.3180	0.0
10	77.60567	2.0	9.0	-34.3022	0.009	0.009	-0.0037	0.0

Table 6.3 Results of the TUH analysis of the Ravensbourne

	TUH Parameters				Error Functions			
IEN	Qp	Tp	TB	CURV	ISE	PISE	QPE	TPE
1	119.50692	2.0	9.0	-266.4704	0.022	0.010	-0.0059	0.0
2	154.11021	1.0	5.0	-680.4406	0.091	0.053	-0.0433	0.0
3	123.87988	1.0	6.5	-312.4505	5.239	2.177	-1.8488	0.0
4	126.29695	1.5	9.0	-462.2452	1.909	0.558	-0.4012	0.0
5	80.51446	2.0	9.5	-159.5039	0.015	0.014	0.0053	0.0
6	150.11705	2.0	8.5	-663.2895	0.241	0.105	-0.0732	0.0
7	101.58818	1.5	6.5	-82.6023	0.523	0.178	-0.0945	0.0

Error functions refer to the regenerated response runoff compared to the observed response runoff

Notation:

IEN - Event number
Qp - TUH peak, cumecs per 100Km^2
Tp - TUH time to peak, hours
TB - TUH time base, hours
CURV - Curvature around the peak of the TUH
ISE - Integral square error, percent
PISE - Partial integral square error, percent
QPE - Peak discharge error, percent
TPE - Time to peak error, hours

Table 6.4 Results of the TUH analysis of the Pool

	TUH Parameters				Error Functions			
IEN	Qp	Tp	TB	CURV	ISE	PISE	QPE	TPE
1	122.07731	1.5	7.0	-279.2382	0.160	0.069	-0.0696	0.0
2	227.86370	0.5	3.0	-1205.4267	0.132	0.048	0.0561	0.0
4	110.50759	1.5	10.0	-398.2026	1.078	0.283	-0.2990	0.0
5	76.75625	2.0	9.5	-47.1788	27.915	22.687	8.1956	0.0
6	123.95042	1.5	6.5	-236.5598	22.792	15.873	-17.3745	0.5
7	198.02321	1.0	4.0	-1066.9781	7.261	5.264	-2.3725	0.0
9	72.56644	1.5	11.0	-35.0383	20.519	11.527	-3.3949	-0.5
10	80.64569	1.0	7.5	-32.7341	3.603	2.864	-2.4097	0.0

Error functions refer to the regenerated response runoff compared to the observed response runoff.

Notation:

IEN - Event number
Qp - TUH peak, cumecs per $100Km^2$
Tp - TUH time to peak, hours
TB - TUH time base, hours
CURV - Curvature around the peak of the TUH
ISE - Integral square error, percent
PISE - Partial integral square error, percent
QPE - Peak discharge error, percent
TPE - Time to peak error, hours

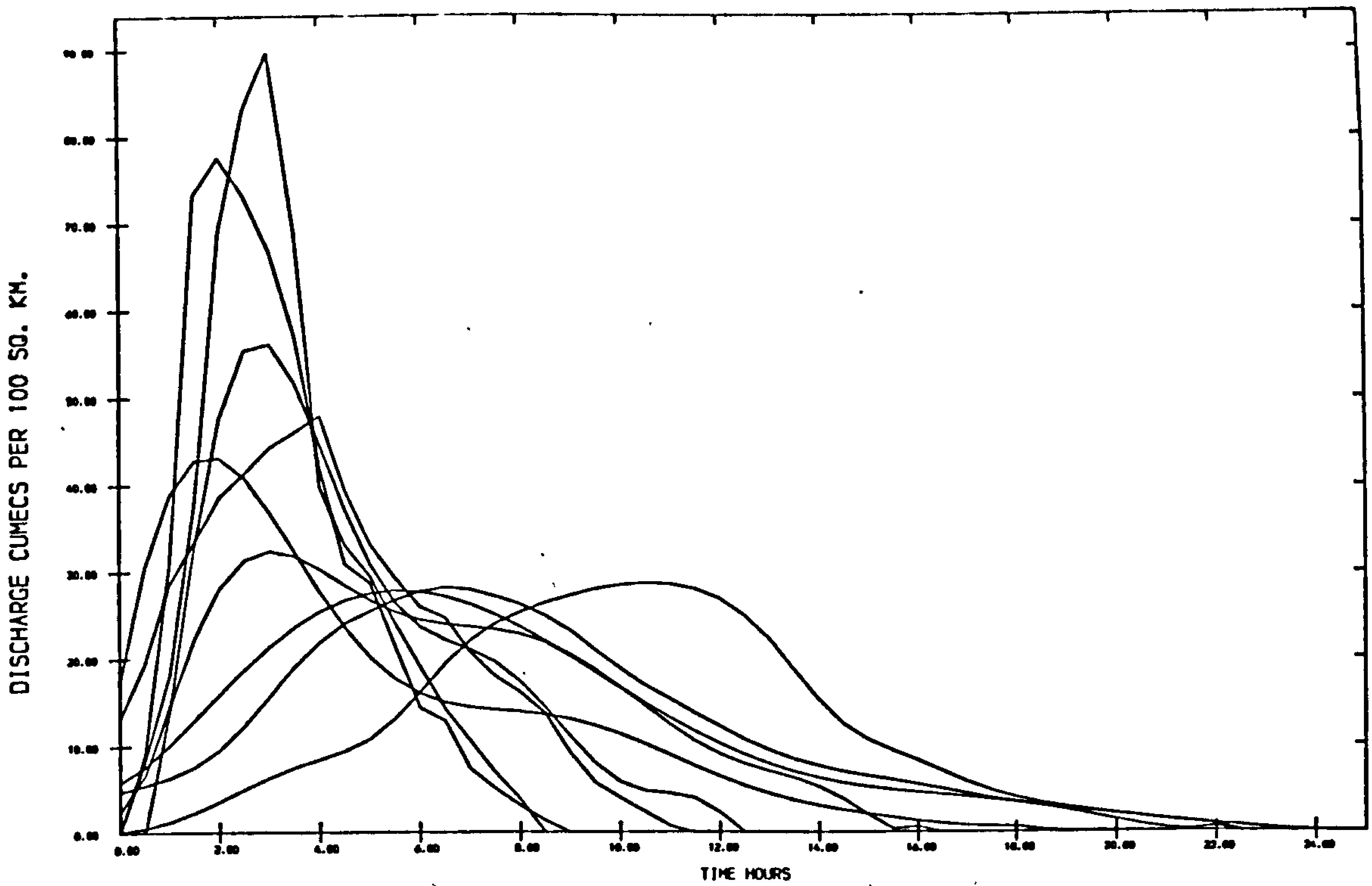


Fig. 6.1 Unit hydrographs for the Wandle

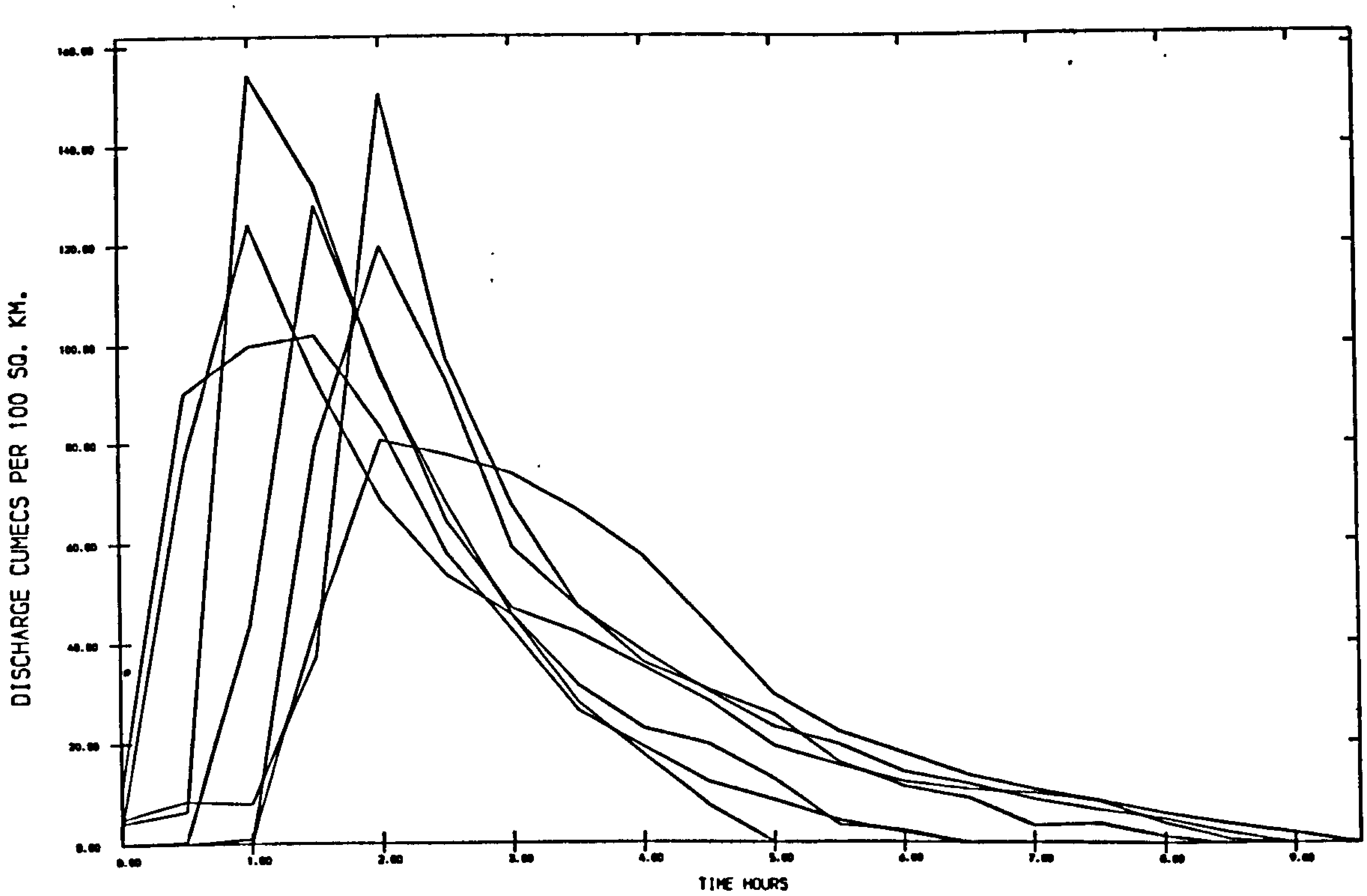


Fig. 6.2 Unit hydrographs for the Ravensbourne

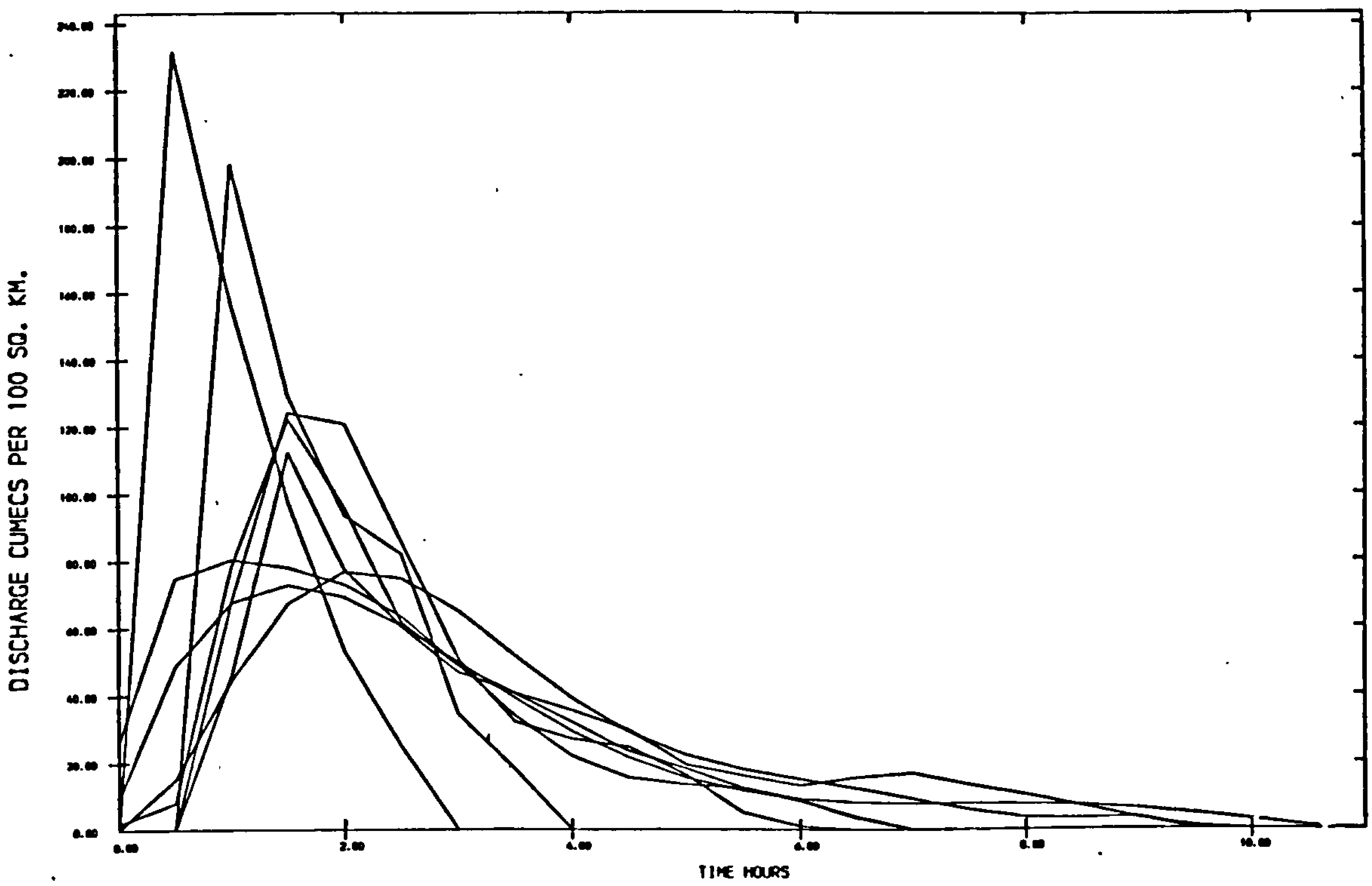


Fig. 6.3 Unit hydrographs for the Pool

The method of deriving TUHs by matrix inversion (section 4.4.2) was uniformly successful for three of the catchments and extremely successful on one (Table 6.5).

Table 6.5 Comparison of the mean regenerative ISE of the accepted TUHs for the four catchments

CATCHMENT NAME	MEAN ISE (%)
Wandle	8.307
Beverley	9.509
Ravensbourne	1.149
Pool	10.433

The weighted mean ISE of the TUHs for the four catchments was 8.477 percent, which indicated that the TUHs were accurate representations of the catchments response. The variation of the value of ISE between the catchments was attributed to three sources. First, a constant, very small percentage of the error was due to the computational limitations of the computer programs. Second, the rainfall and discharge data varied in quality between events and between catchments. Third, catchments differ in the degree of non-linearity and in the extent to which this non-linearity varies between storms. The success of the unit hydrograph method lay in its ability to identify a TUH from noisy data derived from non-linear catchments.

The characteristics of the TUHs derived from the Beverley catchment have been discussed in depth in section 5.2.1. The catchment exhibited a wide range of response which was attributed to storm and antecedent catchment characteristics. The Wandle catchment exhibited a wider range of response which was attenuated in comparison to the Beverley and was probably due to the larger size of the Wandle catchment. The TUHs had lower peaks, slower time to peaks and longer recession limbs. The Pool and the Ravensbourne exhibited a smaller variation in response. The range of the time to peak of the TUHs was small, one hour for the Ravensbourne and 1.5 hours for the Pool, compared to 3.5 hours and 8.5 hours for the Beverley and the Wandle respectively. The Pool produced a more flashy runoff than the Ravensbourne.

Mean TUHs were derived for the catchments following the method described in section 5.2.2 (Table 6.6, Fig. 6.4). The lag time was defined as the time from the origin of the TUH to its centroid, minus one time interval, and was expressed in hours.

Table 6.6 Characteristics of the Mean TUHs (10mm, 0.5 hour, 100 Sq.Km)

Catchment Name	Peak (Qp) Cumecs	Time to Peak (Tp)hours	Curvature around the Peak	Lag Time (TL)hours	Time base (TB)hours
Wandle	48.94982	4.5	-25.82735	6.05	23.00
Beverley	83.69503	2.5	-108.38564	3.68	19.00
Ravensbourne	122.43926	1.5	-375.75126	2.55	9.00
Pool	126.54883	1.5	-412.66958	2.56	11.00

The mean TUHs for the Ravensbourne and the Pool are virtually identical. The Pool's peak discharge and lag time are 3.35 percent and 0.39 percent greater than those of the Ravensbourne respectively. The slow response of the Wandle produced a curve which was almost isosceles in shape, quite the reverse to the shape which is intuitively expected from a catchment with 32.2 percent urbanised. The Beverley plots at an intermediate position between the Wandle and the Ravensbourne and the Pool mean TUHs. The mean TUHs (Fig. 6.4) were smooth which may suggest that they were accurate representations of the mean catchment response, but inspection of the individual TUHs which made up the mean TUH suggested otherwise. The TUHs for the Ravensbourne and the Pool (Fig. 6.2 and 6.3 respectively) were reasonably consistent and therefore averaging did not introduce much error. The TUHs for the Wandle and Beverley (Fig. 6.1 and 5.1 respectively) showed such a large variation that the accuracy of the mean TUH was in doubt. A valid comparison of mean TUHs can only be made if the range of storms and antecedent conditions from which the TUHs were derived is broad and if the relative proportions of storm type and antecedent conditions are equal for all catchments. This has received no attention in the literature and is no longer justifiable because of the known effect of storm characteristics and the antecedent catchment conditions on the shape of the TUH (section 5.2.1). The mean TUHs used in this study were derived without considering this effect because an insufficient number of storms with comparable characteristics were analysed for each catchment.

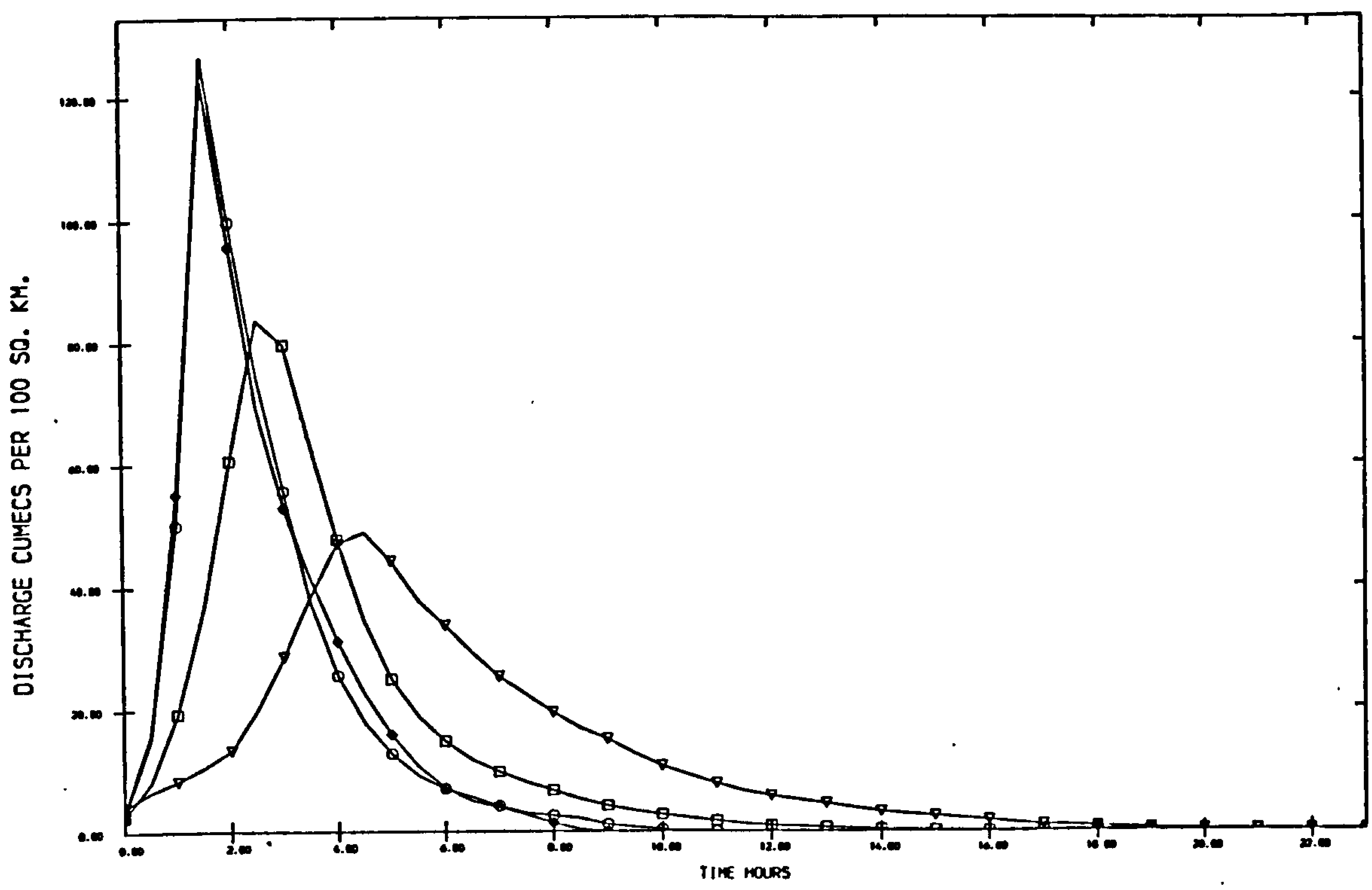


Fig. 6.4 Mean unit hydrographs for the four catchments

Catchment:

Wandle - ▽

Beverley - □

Ravensbourne - ◇

Pool - ○

6.3 Regionalisation of the results

"To regionalise a model simply means to develop a scientific basis for predicting the model parameters on ungauged watersheds from hydrologic and physiographic characteristics of that watershed. Regionalisation can be accomplished if there are enough bench mark watersheds with adequate storm rainfall and runoff data such that a statistical inference may be drawn" (Overton and Meadows, 1976, 246). Four catchments are an insufficient number from which to derive reliable equations. A further disadvantage was the imbalance in the number of TUHs derived for each catchment, 60 percent of all the TUHs were derived from the Beverley. Ideally at least ten catchments with at least ten TUHs for each should be used if a successful regionalisation analysis is to be made (e.g. Rao et al, 1972; Schultz and Lopez, 1974). The following analyses were based on the mean TUHs derived from the four catchments. Although statistical significance tests were applied to each of the findings, it should be stressed that an analysis based on only four catchments is fraught with uncertainty.

The first analysis considered the relationship between the percentage urbanised and the parameters of the mean TUHs (Table 6.7). The parameters of the TUH were analysed and found to be stationary through time for each catchment and showed no significant correlation with the percentage urbanised. This indicated that densely urbanised catchments, subject to subsequent urban development do not exhibit a marked change in hydrological response.

Table 6.7 The relationship between percentage urbanised and the mean TUH parameters

Catchment Name	Percentage Urbanised	Mean TUH Parameters		Period from which storm events were selected
		Qp (cumecs)	Tp (hours)	
Wandle	32.2	48.95	4.5	1963-1977
Ravensbourne	41.8	122.44	1.5	1974-1977
Pool	56.0	126.55	1.5	1974-1977
Beverley	80.8	83.70	2.5	1963-1976

The second analysis consisted of evaluating methods to predict a TUH from catchment characteristics (Table 6.8).

Table 6.8 Physical characteristics of the catchments

Catchment River	Hydrometric Number	Area Km^2	Mainstream length (Km)	Mainstream slope (10/85) (m/Km)	Basin ratio
Wandle	39003	176.00	10.38	2.89	6.11
Beverley	39005	43.50	7.40	2.28	4.90
Ravensbourne	39056	134.46	10.90	8.39	3.76
Pool	39058	76.68	8.43	6.41	3.33

The dimensionless TUH method was rejected because the mean TUHs showed quite a large variation in skewness between catchments, and as a result no single dimensionless TUH could fit all the catchments.

The basin ratio was the second method to be examined. The basin ratio (Z) has been used by Carter (1961), Anderson (1970) and Hall (1974) and is defined as,

$$Z = L / \sqrt{S} \tag{6.2}$$

where L is the mainstream length, Km

S is the slope between 0.1L and 0.85L, m/Km.

Using the basin ratio in conjuncion with a TUH derived for a constant catchment area ($100Km^2$ in this thesis) includes the most significant morphometric variables to predict flood parameters namely, area, length of mainstream and stream channel slope (Newson, 1975, 17).

The basin ratio and the percentage urbanised is assumed to control the lag time of a catchments TUH. The lag time (T_L) is defined as the time from the origin of the TUH to its centroid, minus one time interval, and is expressed in hours. The calculated value of T_L is used to rescale a semi-dimensionless TUH. The semi-dimensionless TUH is calculated by,

$$U_t \cdot T_{LM} = f\ (t\ /\ T_{LM}) \tag{6.3}$$

where U_t is the ordinate of the actual TUH at time t

T_{LM} is the lag time of the actual TUH.

The relationship between basin ratio, percentage urbanised and lag time may be represented graphically (Fig. 6.5) and by an equation (6.5).

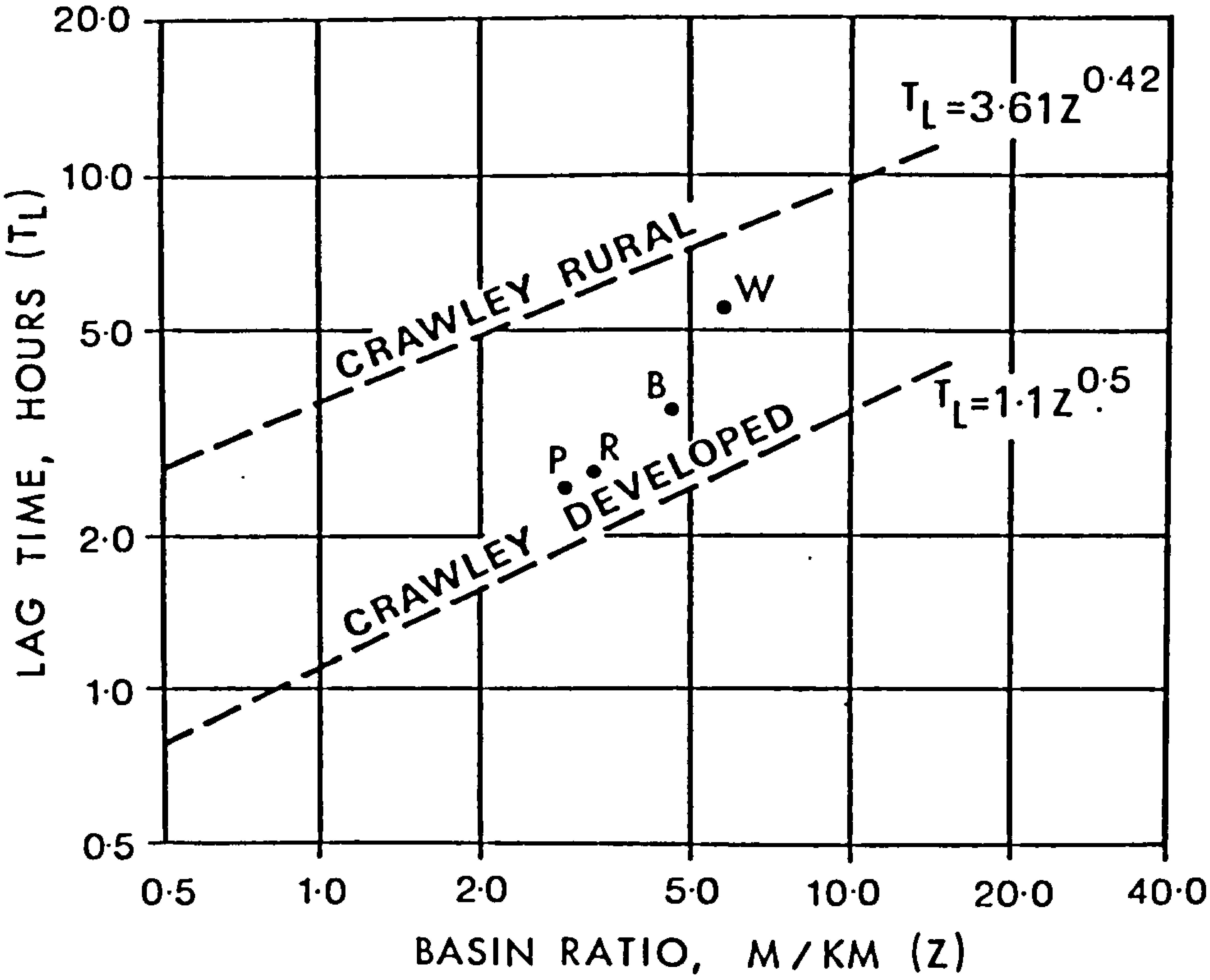


Fig. 6.5 Graph of basin ratio and lag time for the four catchments

Catchment:

Wandle - W

Beverley - B

Ravensbourne - R

Pool - P

A graph (Fig. 6.5) of the relationship produces a straight line of the Beverley, Ravensbourne and Pool catchments. The Wandle, which has the least urbanisation (32.2 percent) of the four catchments, plots near the 'rural' line, a line supposedly for catchments with less than 4 percent urbanised. Further, although the other three catchments are in a straight line, the percentage urbanised varies from 41.8 to 80.8. If the hypothesis was correct they would plot at different levels in the Y-plane and all would be below the 'Crawley Developed' line. Clearly, the hypothesis is incorrect, and these results confirm earlier work (Hall, 1974) that the use of percentage paved is an inappropriate index of the effect of urbanisation on the hydrological response of a catchment.

Hall (1974) derived an equation to predict T_L from basin ratio for catchments with 25 percent urbanisation,

$$T_L = 1.1Z^{0.5} \tag{6.4}$$

Testing this equation on two north London catchments (Hall, 1977a) showed that it consistently underpredicted. Hall's (1974) equation was derived for 1.0 hour TUHs with a different unit volume and unit area than the TUHs derived in this thesis. To test the equation (6.4), the mean TUHs, derived for the four catchments, were converted from 0.5 to 1.0 hour TUHs, using a computer program. The unit volume and unit area were not altered because these do not affect the values of T_L. These new values of T_L were plotted in Fig. 6.5 and used to evaluate Hall's (1974) equation (Table 6.9).

Table 6.9 Analysis of Hall's (1974) equation to predict T_L

Catchment name	T_L (Hours)		Error (percentage)
	Observed	Predicted	
Wandle	6.30	2.72	-56.825
Beverley	3.93	2.43	-38.168
Ravensbourne	2.81	2.13	-24.199
Pool	2.82	2.01	-28.723

Error = (Predicted - Observed)/Observed x 100

The large mean error (-36.979 percent) and the consistent underprediction, both on the four catchments and the two north London catchments, suggests that the equation is unsuitable for the London catchments.

An equation was derived for the four catchments,

$$T_L = 0.4037Z^{1.4523} \tag{6.5}$$

$$R = 0.96722$$

$$R^2 = 0.93551 \qquad \text{Significant at 1\%}$$

Equation (6.5) is little more than an academic exercise because it is based on only four catchments, further the failure of the method (Table 6.9) would suggest that an alternative method should be used.

It was noted that there was a linear relationship between the unit hydrograph parameters (Qp and Tp) and the basin ratio. The two TUH parameters can be used to calculate a triangular unit hydrograph following the same procedure as the parametric triangular unit hydrograph (section 5.2.4). This method has the advantage that the TUH is defined directly, rather than the preceding method which requires the derivation of a semi-dimensionless unit hydrograph. Regression equations were derived to calculate the parameters by two alternative sets of equations. First, the two parameters were predicted independently,

$$Qp = 932.4Z^{-1.5836} \tag{6.6}$$

$$R = 0.97341$$

$$R^2 = 0.94753$$

$$Tp = 0.1395Z^{1.8745} \tag{6.7}$$

$$R = 0.97455$$

$$R^2 = 0.94975$$

$$TB = 555.5/Qp \tag{6.8}$$

where TB is the time base of the TUH.

The second set uses equation (6.6) and calculates Tp from equations (6.9) and (6.10).

$$\frac{Tp.Qp}{2} = VRL = 225.5Qp^{-0.1808} \tag{6.9}$$

$$R = 0.94777$$

$$R^2 = 0.89827$$

where VRL is the volume of the rising limb.

$$Tp = \frac{VRL.2}{Qp} \tag{6.10}$$

The time base (TB) is calculated by equation (6.8).

It is suggested that the second set of equations should be used because the error, measured by the sum of squares, is significantly less using equation (6.9) than equation (6.7). The value of this alternative method could not be assessed with only four catchments, however, it is suggested that, although urbanisation affects catchment response, the shape of the mean TUH is strongly dependent upon the physical properties of the catchment, as measured by the basin ratio.

The principle behind the use of the Hall (1974) method and the new method is to predict the mean TUH for an ungauged catchment. The shape of the TUH varies markedly between storms and therefore a mean TUH is only an elementary planning tool. The first method has been found to be inappropriate to predict the scaling parameter (T_L) of the TUH and it would seem profitable to abandon the method and concentrate on developing alternative methods. The method proposed in this chapter, although tentative, would seem to offer such an opportunity.

6.4 Conclusion

This chapter has presented the results of a synthesis of 60 TUHs from four catchments. The traditional method of deriving a mean TUH by using a scaling parameter (T_L) derived from basin ratio and percentage urbanised was shown to be inaccurate. An alternative method was proposed based on the finding that the catchment's physical characteristics are the dominant controls on TUH shape in spite of varying levels of urbanisation. It is stressed that this finding should be tested by analysing storm events from other catchments.

Chapter 7

A SENSITIVITY ANALYSIS OF SOME OF THE IMPLICIT ANALYTICAL ASSUMPTIONS OF THE UNIT HYDROGRAPH METHOD

7.1 Introduction

A sensitivity analysis is an integral part of any modelling analysis, answering questions concerning the relative importance of certain model components. The Flood Studies Report (NERC, 1975, I, 449-454) conducted sensitivity analyses to determine the effect of catchment wetness, storm duration, rainfall profile, unit hydrograph shape and rainfall separation method on peak discharge. The analyses used the Flood Studies Report (NERC, 1975) rainfall-runoff model and symmetrical design rainfall profiles. The findings published in the Flood Studies Report (NERC, 1975) have been complemented and extended by the results of the sensitivity analyses described in this chapter. Four analyses were made, first, the effect of rainfall separation method on flood frequency estimates. Second, the effect of the TUH on spill volumes; third, the effect of rainfall profile on peak discharge and spill volume and fourth, the effect of substituting a triangular approximation of a TUH for a curved TUH.

The sensitivity of a variable, for example peak discharge, is assessed by a series of simulations where only one variable, for example rainfall profile, is changed. When a given variable is found to be sensitive to another variable the design procedure must be improved to take account of the observed sensitivity. For example, if peak discharge is found to be sensitive to rainfall profile then several profiles should be used in place of a single profile. Failure to make this substitution will result in errors in the predicted peak discharge. Errors can only be recognised and removed if either more accurate analytical methods or better quality data are available. The decision to remove errors depends on the required level of accuracy of the analysis.

7.2 The effect of rainfall separation method on peak discharges and flood frequency estimates

The Flood Studies Report (NERC, 1975, I, 413, 454) compared four different methods of rainfall separation; first, the constant loss percentage; second, the loss rate curve; third, the constant loss percentage varying with CWI and fourth, the phi index. Symmetrical storm profiles were used in the analysis. The results indicated that the second and fourth methods yielded synthesised peak discharges 5 to 10 percent higher than those produced by the percentage loss methods (methods one and three). The analysis undertaken by the Flood Studies Report (NERC, 1975) was extended by using historical rainfall profiles. Peak discharges and flood frequency curves were predicted by convolving the mean TUH for the Beverley Brook (Figure 5.6) with the 250 historical rainfall profiles (section 5.4.3) using the computer program FLOODS (section 5.4.3 and Appendix 7). Four separate analyses were made, each separated the 250 total rainfall profiles by one of the four rainfall separation methods and convolved the effective rainfall profiles with the mean TUH for the Beverley Brook. The mean catchment conditions for the Beverley Brook (Table 5.1) were used to determine the percentage response and the shape of the loss curve. The phi index algorithm contained in FLOODS (Appendix 7) was unstable and this separation method was therefore excluded from further analysis. Comparison of the peak discharge predicted using the effective rainfall profiles produced by the percentage loss method and the loss curve method showed that the latter's produced peak discharges 26.94 percent greater than those by the former method. This is considerably greater than the difference observed by the Flood Studies Report (NERC, 1975) and indicates first, the significance of the rainfall separation method and second, that the shape of the rainfall profile has a significant effect on the peak discharge of the hydrograph. The flood frequency simulation displayed a similar relationship, though the difference between the loss curve and the percentage techniques decreased with increasing return period (Table 7.1).

Table 7.1 The effect of rainfall separation method on flood frequency estimates

Figures in parenthesis indicate percentage difference from separation method no.2.

RAINFALL SEPARATION METHOD	PEAK DISCHARGE (CUMECS) RETURN PERIOD (YEARS)			
	2.33	10	100	1000
Constant loss percentage (1)	31.27 (30.00)	44.48 (24.93)	63.04 (20.93)	81.27 (18.61)
Loss rate curve (2)	44.67	59.25	79.73	99.85
Constant loss percentage varying with CWI (3)	32.33 (27.62)	45.77 (22.75)	64.66 (18.90)	83.21 (16.66)

The significance of the findings may be illustrated with reference to Kelway's (1977, 263) finding that the 30 year flood predicted by the Flood Studies Report (NERC, 1975) rainfall-runoff model underpredicted by approximately 30 percent. The sensitivity analysis described above (section 7.2) suggested that the underprediction was due to first, the use of different rainfall separation methods for analysis and design. Analysis was "...based on loss rate separation but the finally recommended design method and prediction equations are based on percentage runoff" (NERC, 1975, I, 394). These two rainfall separation methods were found to predict peak discharges which were substantially different. Second, the symmetrical design storm profile may not be typical of the area under consideration and since rainfall profile was found to be an important control on peak discharge this also introduces error. The significance of profile shape is discussed further in section 7.4.

7.3 The effect of unit hydrograph characteristics on spill volume

The damage caused by a given flood hydrograph is proportional to the volume of water, spill volume, in excess of the channel capacity. Spill volume is defined as the volume of water in a hydrograph in excess of a given threshold (equation 7.1)

$$Q_s = \sum_{t=1}^{t=n} Q(t) - q_s \cdot \frac{3600}{T} \qquad (7.1)$$

where Q_s is the spill volume, cubic metres

Q is the discharge, cumecs

q_s is the spill volume threshold, cumecs

t_1, t_n define the duration of the flood hydrograph, hours

t is a time interval, hours

T is the data interval, hours.

To assess the effect of the TUH on spill volume, and therefore flood damage, a simulation analysis was made using the computer program FLOODS (Appendix 7) and the 36 TUHs derived from the Beverley Brook. The spill volumes were analysed at the 50 years recurrence interval because this is the design standard for the GLC non-tidal rivers.

Spill volumes were found to be dependent upon three characteristics of the TUH. First, the curvature around the peak; second, the peak ordinate and third, the shape of the TUH.

A graph of TUH peak and volume spilt over a threshold of 20 cumecs (Fig. 7.1) displays a positive curvi-linear relationship. The scatter of points in the Y-plane is due to the curvature of the TUH around the peak. Curvature was measured by equation 7.2 (see section 5.2.1 for further details)

$$\text{CURVATURE} = \frac{(Qp_{t+1} + Qp_{t-1}) - 2 \cdot Qp}{\text{TINT}^2} \qquad (7.2)$$

where Qp is the TUH peak discharge, cumecs
TINT is the data interval, hours
t is the time interval, hours.

For a given TUH peak, the TUH with the smallest curvature (a blunt peak) will produce the greatest spill volume. The envelope of points (Fig. 7.1) defined by the TUHs derived from the Beverley Brook indicate that differences in curvature can produce differences in spill volume of 10,000 cubic metres for a given TUH peak. This has implications for those design methods which use straight line triangular approximations of the TUH, this was investigated and reported in section 7.5.

Recognising that the spill volume threshold controls the spill volume, unique relationships between TUH peak and spill volume for the thresholds of 10, 20, 30 and 40 cumecs were derived by fitting a polynomial to the data (Fig. 7.2). This indicated that first, spill volume was positively correlated to TUH peak and second, that there was a non-linear relationship between the spill volume and spill volume threshold for a given TUH peak. The relationship described in Fig. 7.2 can be used by the hydraulic engineer to estimate the volume required for off channel detention ponds. First, the capacity of the channel at a given point is estimated and a spill volume curve is extrapolated between those curves plotted in Fig. 7.2. Second, the TUH peak is predicted using the equations developed in section 5.2.1 (Equations 5.4, 5.5 and 5.6). Third, the spill volume resulting from a given TUH peak is calculated using the extrapolated curve on Fig. 7.2. Chapter 5 indicated that the TUHs with the highest peak discharges were caused by short duration, high intensity storms on a dry catchment. The engineer should conduct a series of analyses to produce the TUH with the highest peak, within the hydrological constraints of the catchment, which will yield an upper estimate of spill volume to be expected at the given point.

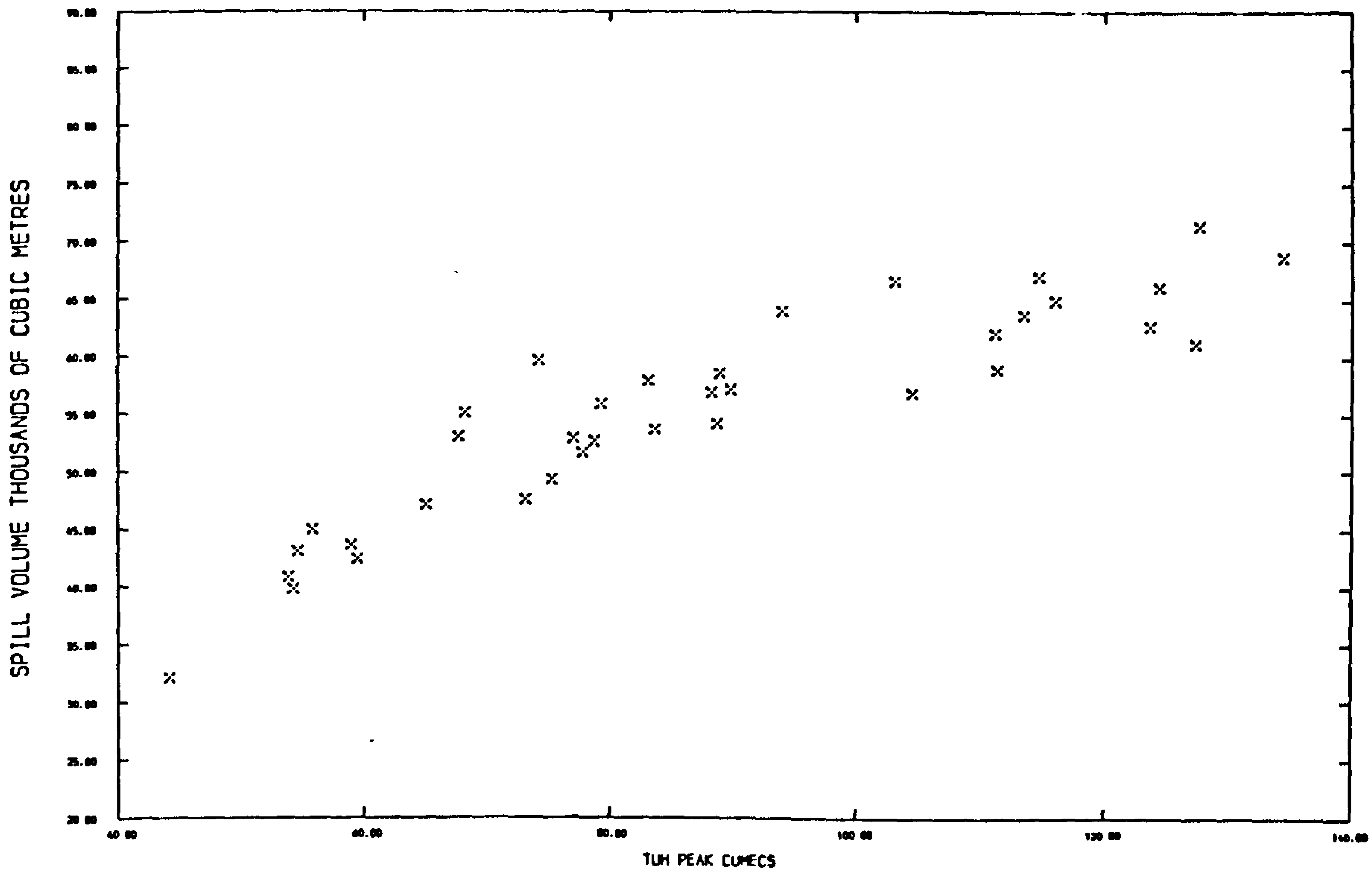


Fig. 7.1 TUH peak versus spill volume for a spill volume threshold of 20 cumecs

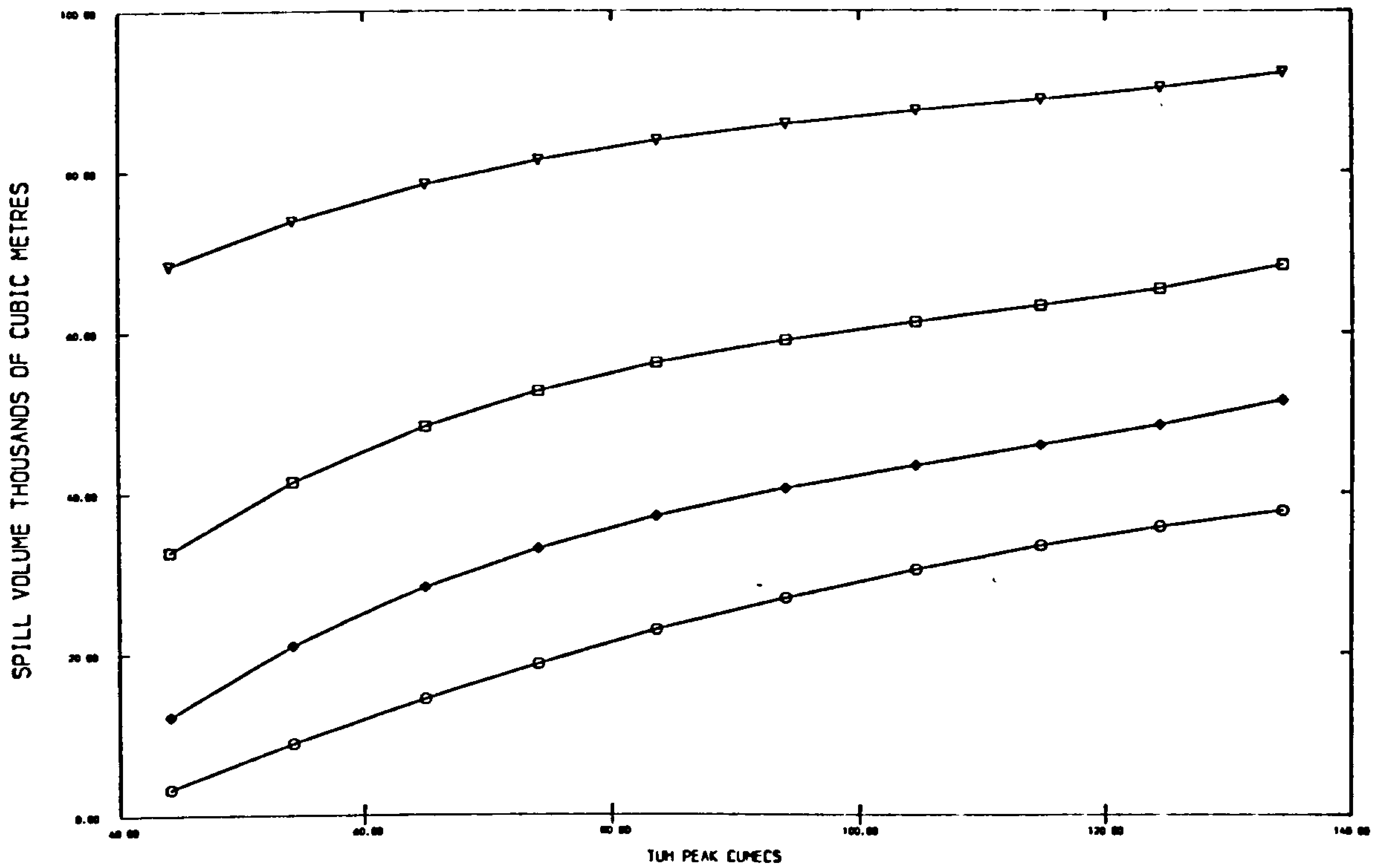


Fig. 7.2 TUH peak versus spill volume for spill volume thresholds of 10, 20, 30 and 40 cumecs

Spill volume threshold:

10 cumecs - ▽

20 cumecs - □

30 cumecs - ◇

40 cumecs - ○

The third control on spill volume was the shape of the TUH. The curves in Fig. 7.2 exhibit a decrease in gradient as the TUH peak increases, this is caused by the decrease in significance of the recession limb on TUHs with large peak discharges. This means that for a given spill volume threshold, the spill volume produced by high peaked TUHs is proportionally less than that generated by low peaked TUHs. Conventional TUH analyses (e.g. Hollis, 1974; Hall, 1974) have provided an index of the effect of urbanisation on floods but have not considered the actual increase in flood potential, as measured by the spill volume. Therefore, assuming that the river channel is first, unaltered in the transition from a rural to an urban catchment and second, that the heightened flood hydrograph is contained within the channel, then it is suggested that the increase in flood potential due to urbanisation is not as great as the literature suggests. Attempts to demonstrate this using published material were frustrated because published unit hydrographs were too small to digitise, however Fig. 7.2 provides irrefutable evidence in support of the hypothesis.

7.4 The effect of rainfall profile on peak discharge and spill volume

The Flood Studies Report (NERC, 1975, I, 453, 454, 455) recommends that the 75 percent winter profile should be used for peak discharge estimates because "...it is the profile which typically produces a peak of the same size as the average from sampling across the full set of summer and winter profiles". It was observed that the rainfall profile became important on responsive catchments and when the peakedness, defined as the ratio duration/time to peak, was greater than 2. Neither of these were considered to be important enough to merit alteration of the symmetrical profile. The structure of the 'worst profile' varies spatially (Sutcliffe, 1978, 31) however the Meteorological Office has no plans to investigate storm structure (Folland and Colgate, 1978, 68) and consequently it is argued that the symmetrical profile is adequate and is the only viable profile for peak discharge estimates. It is acknowledged that for the assessment of flood volumes an analysis of storm structure is essential.

These views have been criticised by several research workers. First, Butters et al (1977, 349) found that "...never did a 30 year return flood result from rainfall having the same frequency but always from those having frequencies varying from 3 to 12 years". Second, Kelway (1977) argued that the Flood Studies Report (NERC, 1975) rainfall-runoff model is invalid if it is used in an area where the 75 percent winter profile is not typical of the meteorological conditions. He demonstrated this by using six different profiles with constant duration and rainfall depth which yielded peak discharges ranging from 5.7 to 9.3 cumecs (Kelway, 1977, 265-266).

To investigate the effect of profile structure on peak discharge and spill volume a simulation exercise was undertaken using the mean TUH for the Beverley Brook (Fig. 5.6) and four different profiles, consisting of 40mm in 4.5 hours (Fig. 7.3 and 7.4, Table 7.2). The rectangular profile (P2) produced a peak discharge 16.51 percent less than the isosceles triangle profile (P1). Spill volume variations correspond to the peak discharge variations. The value of the peak discharge is not correlated to the maximum rainfall intensity as the Rational Method (section 3.2) would suggest. Profiles P3 and P4 have identical peak rainfall intensities but differ in the value of the predicted peak discharge. The cause of variations in the value of the peak discharge ordinate is the distribution of the rainfall increments within the profile. The Flood Studies Report (NERC, 1975, I, 453) found that, "...the hyetograph shape that maximises the peak from any given unit hydrograph is one with equal and opposite skewness to the unit hydrograph, provided that no portion of the hyetograph is of greater return period than the whole". Historical profiles, such as those used in this thesis, fall outside this relationship and little attention has been paid to the effect of non-symmetrical profiles on peak discharge and spill volumes. The remainder of this section attempts to rectify this deficiency.

The ten profiles (Fig. 5.9), which were used in section 5.4, were non-dimensionalised such that each profile consisted of 100 units. Each profile was rescaled twenty-five times at increments of 10mm, starting at 10mm and rising to 250mm. This produced 250 profiles, consisting of 25 profiles for each of the ten profiles. This meant that the volume of rainfall within a profile was controlled for and the differences between profiles, for a given rainfall amount, lay in the duration and the structure.

To calculate the peak discharge and the spill volume, the mean TUH of the Beverley Brook (Fig. 5.6) was convolved with each of the 250 profiles using the computer program FLOODS (Appendix 7). The profiles were separated using the loss curve method and the mean catchment conditions for the Beverley Brook (Table 5.1) were used to determine the percentage response and the shape of the loss curve.

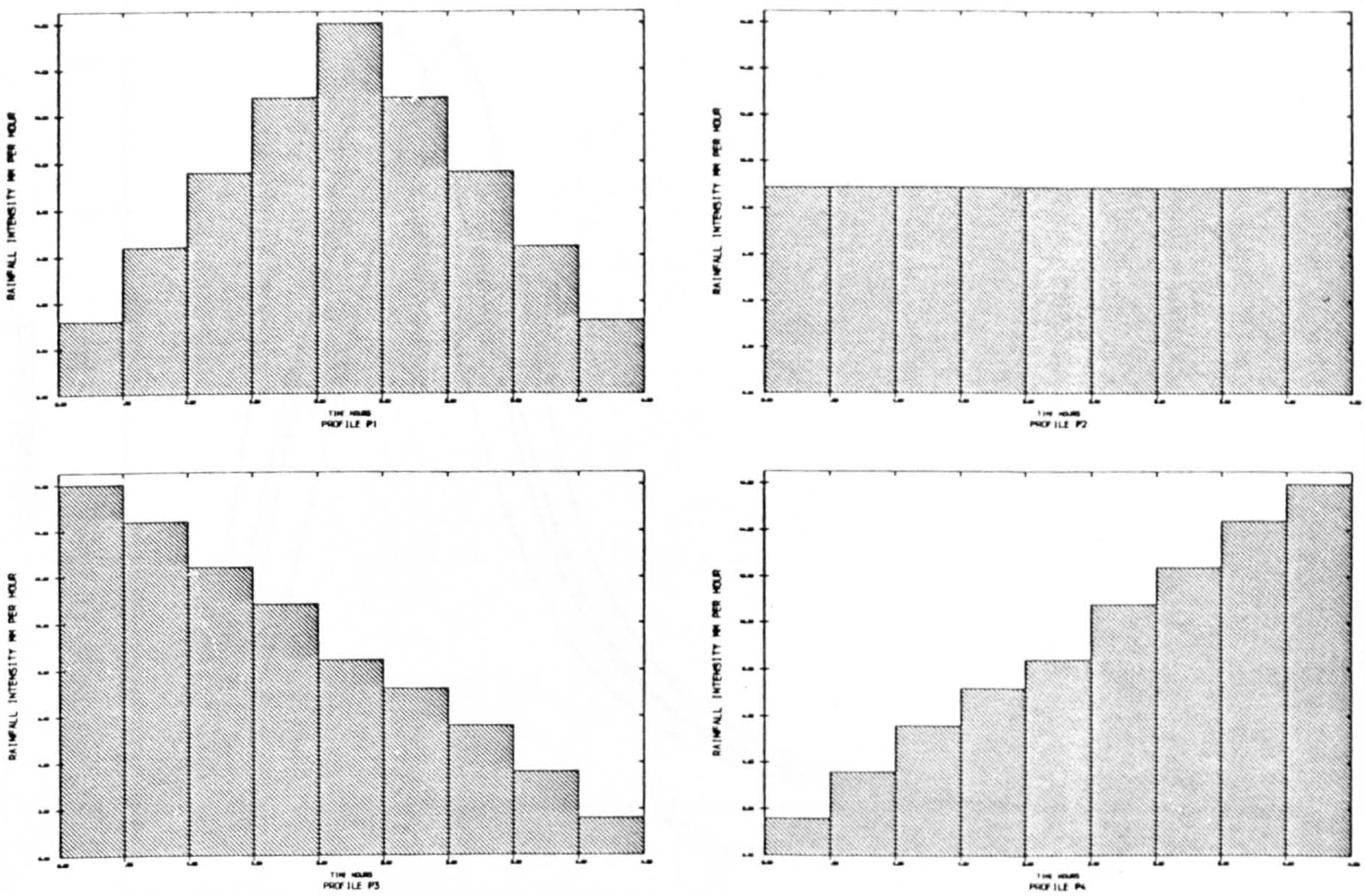


Fig. 7.3 Four synthetic rainfall profiles

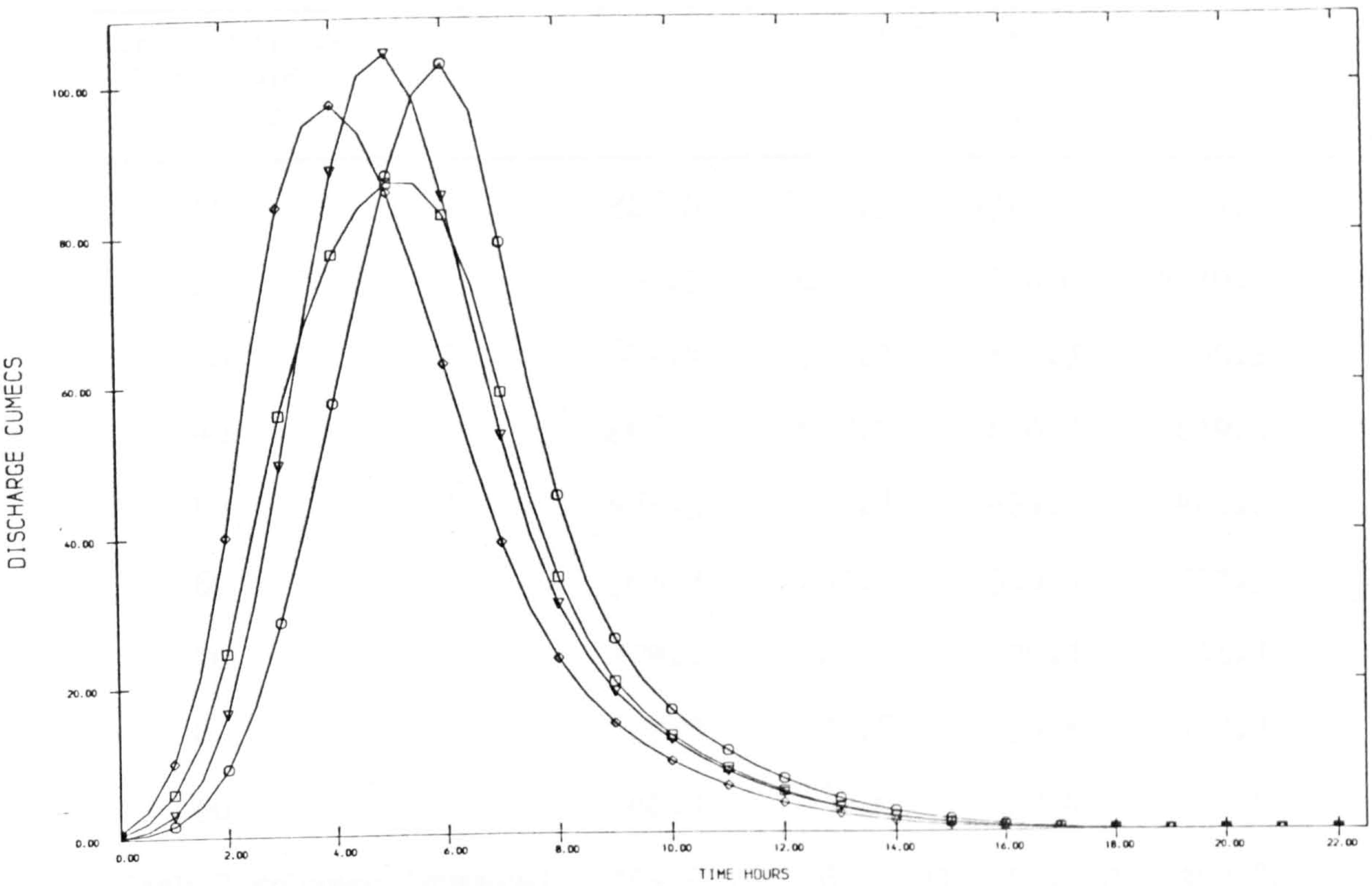


Fig. 7.4 The effect of rainfall profile on the flood hydrograph

Rainfall profile:

P1 - ▽

P2 - □

P3 - ◇

P4 - ○

Table 7.2 The effect of rainfall profile on spill volume and peak discharge

Spill Volume Threshold cumecs	Profile Number P1	P2	P3	P4
10	1327.8	1311.3	1327.0	1314.4
20	1052.3	1018.9	1048.0	1030.4
30	829.4	783.2	823.3	805.8
40	646.8	577.5	626.8	619.0
50	485.3	404.8	464.7	461.6
60	353.3	251.5	320.1	321.9
70	229.0	128.5	199.1	213.1
80	139.0	36.8	99.8	118.8
90	60.1	0.0	28.4	50.5
Peak Discharge (cumecs)	104.2076	87.0023	97.4521	102.8224

Spill volume in thousands of cubic metres

P1: Isosceles triangle

P2: Rectangle

P3: Positive skewed right angled triangle

P4: Negative skewed right angled triangle

The peak discharges produced by the 10 profiles (Fig. 5.9) for the total rainfalls of 50, 100, 150, 200 and 250mm were expressed as a percentage of the discharge produced by profile 1 and averaged (Table 7.3). The ability of the profile to generate a peak discharge was evaluated in terms of the duration and the skewness of the profile (Table 7.3) and indicated that first, longer duration storms tend to produce lower peaks and second, the skewness of the profile had no consistent relationship with the magnitude of the peak discharge. Comparison of profiles 3 and 5 indicated the importance of storm structure. Both profiles had the same duration, 9.5 hours, and the same amount of rainfall (for this study), only the structure, measured by the skewness, varied (0.27327 and 0.04514 respectively). This structural variation produced peaks 95.12 and 57.46 percent those of profile 1 (Fig. 5.9). This analysis emphasised the importance of selecting a profile which corresponds to those observed in the area under consideration.

The structure of a rainfall profile controls the shape of the synthesised hydrograph and therefore the use of a symmetrical profile to design balancing ponds can introduce large errors in the estimated spill volume, if the observed profiles do not correspond to the design profiles. Recent project work (e.g. Hall et al; Davis and Woods, 1979) has used the Flood Studies Report (NERC, 1975) symmetrical profiles because of the absence of the necessary hydrometric data. To rectify the deficiency of published work on the relationship between profile and spill volume, a sensitivity analysis was made (Table 7.4 and Fig. 7.5). Profile 1 produced the maximum spill volume at all thresholds, but the variation between profiles was greater than that observed with the peak discharge, the effect being greatest at the higher spill volume thresholds. The variation was measured by the relative range index, which was defined as the difference between the maximum and minimum spill volume for a given threshold divided by the maximum spill volume. The index was expressed in percent (Table 7.4). The index indicated the extreme importance of rainfall profile and how profile became increasingly important at the higher spill volume thresholds. Although profile 1 will produce an upper estimate of spill volume it may be uneconomical to design to this if that type of storm does not occur with sufficient frequency to justify the expenditure to protect against such a flood. For example, if the river channel has a capacity of 45 cumecs and a flood balancing pond is designed, using profile 1, to accommodate the overbank floodwater (93,000 cu. m.), when profile 10 is the locally important profile, then the balancing pond will be overdesigned to the order of five times its necessary size.

Table 7.3 Characteristics of the rainfall profiles and their effect on peak discharge

Profile Number	Profile Characteristics Duration (hours)	Skewness	Peak Q as a percentage of Peak Q produced by Profile 1
1	4.0	0.00993	100.00
2	8.5	-0.03326	98.24
3	9.5	0.27327	95.12
4	12.0	0.10944	54.76
5	9.5	0.04514	57.46
6	15.5	0.01631	81.67
7	17.0	-0.04262	60.51
8	16.5	0.02081	58.39
9	22.5	0.00254	43.58
10	30.5	0.00121	44.86

Table 7.4 The effect of rainfall profile on spill volume

Profile Number	Spill Volume Threshold (cumecs) 0	5	10	15	20	25	30	35	40	45
1	205	183	166	151	139	128	118	109	101	93
2	205	181	164	149	137	126	116	107	99	91
3	205	182	165	150	137	125	116	107	98	90
4	205	177	156	137	120	104	90	76	62	50
5	205	178	156	138	121	105	91	77	64	52
6	205	180	162	146	132	120	109	99	90	81
7	205	175	154	136	120	106	92	80	68	58
8	205	173	149	127	107	90	74	64	55	47
9	205	168	141	117	95	75	57	42	30	21
10	205	154	117	86	65	49	38	30	24	19
Relative range %	0.00	15.85	29.52	43.05	53.24	61.72	67.80	72.48	76.24	79.57

Spill volume in thousands of cubic metres

Relative range = ((Profile 1 - Profile 10)/Profile 1) x 100

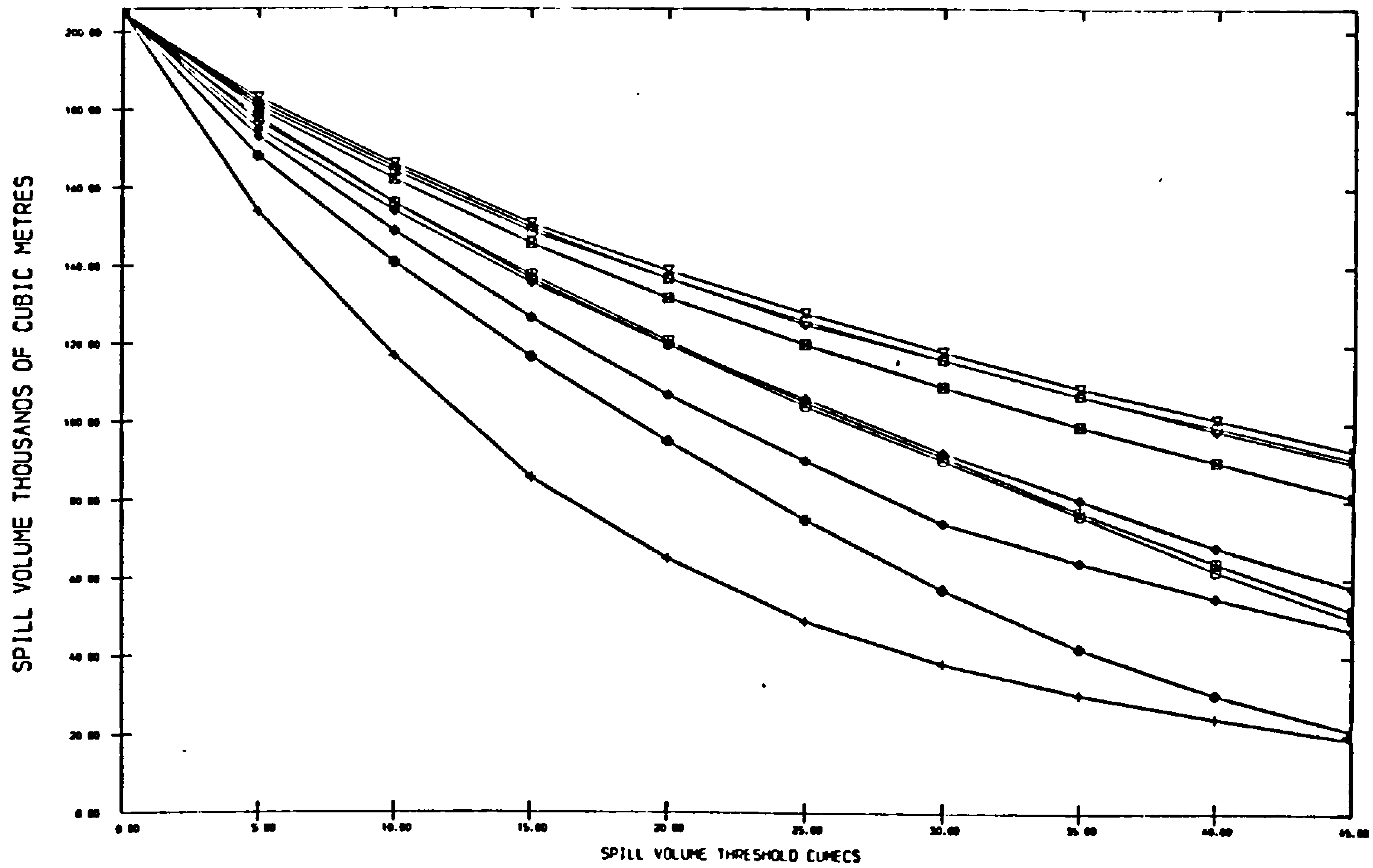


Fig. 7.5 The effect of rainfall profile on spill volume

Profile Number:

1 - ▽	6 - ⊠
2 - □	7 - ⊕
3 - ◇	8 - ⊗
4 - ○	9 -
5 - ⊞	10 -

This section has quantified the significance of rainfall profile on peak discharge and spill volume and has pointed to the necessity of analysing the characteristics of the rainfall profiles for the area under consideration rather than adopting national symmetrical design profiles (e.g. NERC, 1975).

7.5 The effect of substituting a parametric triangular unit hydrograph for a curved TUH

Section 5.2.4 described the construction of a triangular approximation of the TUH called the Parametric Triangular Unit hydrograph (PTTUH). The PTTUH was constructed using three parameters, the peak discharge (Qp), the time to peak (Tp) and the time base (TB). Qp and Tp were predicted from equations and TB was calculated to maintain a unit volume. Tp and TB were rounded to the nearest half hour and the whole PTTUH was rescaled after its initial construction.

The first sensitivity analysis investigated the effect on peak discharge of substituting a PTTUH for a curved TUH. The analysis was based on the 36 TUHs for the Beverley Brook. The calculations for this and the second sensitivity analysis were performed by the computer program FLOODS (Appendix 7) and a separate program to calculate the ordinates of the PTTUHs. For each event the Qp and Tp were the same values for both the TUH and the PTTUH. The sensitivity analysis therefore measured the effect of assuming that the PTTUH starts at zero hours, with a discharge of zero cumecs, and the effect of substituting two straight lines for the TUH (Table 7.5). The effect of substituting a PTTUH had a very small effect on peak discharge estimates, the mean absolute error was 3.322 percent.

Table 7.5 Absolute error of substituting a PTTUH for a curved TUH: Peak Discharge

	Peak Discharge Return Period (Years)			
	2.33	10	50	100
Error %	3.309	3.314	3.325	3.341

The second sensitivity analysis compared the spill volume predicted by the TUH and the PTTUH (Table 7.6). The errors were larger than those in predicting peak discharge and were inevitable because the PTTUH, although preserving the parameters Qp and Tp, ignored the shape of the TUH which is largely responsible for the value of the spill volume.

Table 7.6 Absolute Error of Substituting a PTTUH for a Curved TUH: Spill Volume

	Spill Volume Threshold (Cumecs)				
	5	15	25	35	45
Error %	4.935	10.768	13.438	14.525	14.323

It may be concluded that the use of the PTTUH will introduce little error in predicting peak discharge but will produce significant errors in predicting spill volume. This conclusion corresponds to current engineering practice which advocates the use of symmetrical profiles for peak discharge estimates of both sewer and river flows but the use of historical profiles to design balancing ponds.

7.6 Conclusion

Four sensitivity analyses were conducted to test the relative importance of some of the assumptions made in this thesis.

First, the choice of rainfall separation method was found to be a significant control on the magnitude of the synthesised peak discharge.

Second, spill volume was related to three parameters of the TUH, namely, the curvature around the peak, the peak ordinate and the shape of the TUH. Assuming that urbanisation increases peak discharges, then the actual flood danger, measured by the spill volume, although increasing, does so at a lesser rate than the peak discharge.

Third, the peak discharge and the spill volume produced by a rainfall profile is related to the duration and the structure of the profile. The results indicated the necessity of analysing the characteristics of the rainfall profiles for the area under investigation rather than adopting national symmetrical design profiles.

Finally, the effect of substituting a PTTUH for a curved TUH was found to be insignificant for peak discharges but important for estimates of spill volume.

Chapter 8

SUMMARY AND CONCLUSIONS

This thesis has presented the results of a study of the flood hydrology of four urbanised catchments in Greater London. This chapter considers three aspects of the thesis. First, the problem to be solved. Second, the use of computer based methods to analyse the data and third, the results of the analysis.

8.1 The analysis of the flood hydrology of four London catchments

Urban areas are subject to two separate flood threats, namely, primary and secondary flooding. The former is caused by the overtopping of river channel banks and the latter by sewers being unable to remove the surface water quickly enough. Secondary flooding causes relatively little damage and its solution is adequate pipe sizing and a rigorous maintenance scheme to remove obstructions and blockages from culverts and grilles. Primary flooding causes the greatest amount of damage and is avoided or mitigated through a combination of design and flood forecasting. Flood alleviation through design requires the prediction of the peak discharge to be expected at a given point, for a given frequency. In the absence of a scientifically derived method to predict design discharges, subjective techniques are employed which tend to lead to overdesign. Flood forecasting requires the construction of a model which predicts the complete hydrograph and is operated in real-time, using telemetered catchment data. This is more complicated than the design stage because the model attempts to simulate a given storm event rather than the mean response. Although a comprehensive telemetry network has been installed in Greater London, there is no operational flood-forecasting system because no reliable catchment model has been developed.

Chapter two indicated that the four catchments were significantly different from previous studies because of their large size and the large proportion of the catchments covered by urban development. A survey of published work on the London catchments found that the variation of response between storms for a given catchment has been ignored. Further, conventional methods of synthesis were found to be unreliable.

Preliminary investigations rejected deterministic models because of their inappropriateness to the thesis's objective and their excessive demands on data. This thesis, using lumped parameter models, sought first, to remedy the absence of a design and flood-forecasting model for the London catchments and second, to make a significant contribution to the understanding of the variation of catchment response.

The detailed survey of the literature presented in chapter three traced the historical development of methods to predict the peak discharge and the complete hydrograph. Emphasis was placed on developments in methodology because these represent the growth of hydrological theory and have permitted the accurate prediction of the hydrograph. The methodological developments in the 1960s provided hydrologists with the opportunity to produce more accurate predictions than ever before, but unless studies use the appropriate analytical techniques, and use them correctly, the conclusions they draw will be invalid.

8.2 Analytical methods

The computer was the most important single tool used in the thesis, it permitted the fast and accurate analysis of large quantities of data. The statistical linear regression analyses used the Statistical Package for Social Scientists (SPSS) implemented on the University of London Computer Centre's (ULCC) CDC 6600. All the FORTRAN programs were run on the ULCC CDC 7600, using the MNF compiler. The computer graphics were produced using the ULCC DIMFILM library. Both the CDC machines have a 15 bit word length and parallel runs on the University College IBM 360, which has a 7 bit word length, indicated that the increase in accuracy gained by using the CDC machines was significant, especially for matrix inversion and harmonic analysis. The CDC machines provided results of the required level of accuracy for this project. Frequently the accuracy of the hardware is not assessed in studies using computers.

The computer programs presented in the appendices are the first readily available suite of programs to derive unit hydrographs from rainfall and runoff data and which permit the smoothing of unit hydrographs, the derivation of mean unit hydrographs and dimensionless unit hydrographs. The first two programs, LPROG5 and LPROG6, are based on the programs written by Dr. M.J. Lowing for the Flood Studies Report (NERC, 1975). They have been extensively modified by the removal of inefficient programming, incorrect algorithms and ICL FORTRAN extensions to provide a comprehensive range of analyses.

In chapter five, three unit hydrograph models were tested by comparing the observed and predicted hydrographs. This represents three important aspects of the thesis. First, the fit of the observed and predicted hydrographs was measured by six objective error functions. These avoided subjectiveness and produced an objective guide to the accuracy of the model. The use of standard objective error functions permits the comparison of models developed by different researchers. Such comparisons, based on objective criteria, are a prerequisite for scientific hydrology. Second, the models were split-tested, that is, they were tested on events which were not used, at any stage, in the design and calibration of the model. This is an important feature of the thesis and was considered essential if the conclusions of the thesis were to be valid. Frequently hydrological models are proposed and either never tested or tested using data used to derive the model. Third, maximum values, forming rejection criteria, were set for each of the objective error functions and applied to each unit hydrograph. This ensured that only those unit hydrographs which were accurate representations of catchment response were used in the analysis. This represents a significant advance over several unit hydrograph studies which have not considered the accuracy of the derived unit hydrographs. Unless the unit hydrographs used in an analysis are accurate, the results of the study will be subject to doubt.

8.3 Summary of the results

The analysis was directed towards first, the derivation of unit hydrographs; second, the investigation of the causes of variation of the unit hydrographs and third, studies using the derived relationships.

The analysis was based on 96 storm events selected from four catchments.

Sixty-two storm events were analysed to find the most accurate method of deriving unit hydrographs. It was found that first, matrix inversion was superior to harmonic analysis. Second, smoothing of the unit hydrograph by a moving average filter decreased the accuracy of the unit hydrograph. Third, an analysis of the effect of rainfall separation technique on the accuracy of the unit hydrograph was made using the phi index, the loss curve and two versions of the percentage loss method. The loss curve method produced, on average, the most accurate unit hydrograph. A comparison was made of the relative efficiency of smoothing the unit hydrograph by a moving average filter and a new method, fitting a polynomial function. The new method proved to be superior. An analysis of regeneration using objective error functions obtained for parallel analyses using a moving average filter and a polynomial function indicated that first, the overall fit was superior using a polynomial function. Second, the polynomial function reduced the absolute error of the peak discharge, partially due to smoothing, from 7.334 percent to 4.963 percent. Third, the polynomial function was more efficient at removing severe oscillations than the moving average filter. Reduction of the error caused by smoothing is important if an accurate synthesis is to be made. All the unit hydrographs used in the analysis were derived by matrix inversion of an effective rainfall, calculated by the loss curve method, and the response runoff. Smoothing, where necessary, was performed by fitting a polynomial function. This yielded 60 unit hydrographs with a mean integral square error (ISE) of 8.477 percent which indicated that the unit hydrographs were, on average, very accurate representations of catchment response. The accuracy of the unit hydrograph, measured by ISE was related, approximately, to the cube of the storm duration. Using this relationship it was found that unit hydrographs could not be derived for storms of greater than 8.5 hours duration. This appears to be the first published test of the efficiency of matrix inversion in deriving unit hydrographs, namely that the maximum storm length should be 17T, where T is the elemental time of the unit hydrograph.

Analysis of the unit hydrographs produced the following findings. First, traditional analyses of urbanising catchments have derived mean unit hydrographs for groups of unit hydrographs, where each group represents specific catchment characteristics, for example, pre- and post-urbanisation. This is potentially inaccurate because of the subjectivity in the selection of the time span for each group. The new method correlated the time of each event, expressed as the number of days elapsed since 1st January 1900, with the peak discharge and time to peak of the TUH derived for that event. No significant relationship was found for the 14 year record of the Beverley Brook. This was attributed to the hydrological effect of increased urbanisation being masked by the nature of urban growth, namely infilling. It is suggested that the new method has great potential and should be used in place of traditional grouping techniques.

Second, the variation in the shape of a unit hydrograph is due to storm characteristics and antecedent catchment wetness. Unit hydrographs with high peak discharges and a short time to peak were caused by short duration, high intensity storms on a dry catchment. Unit hydrographs with low peak discharges and a long time to peak were caused by long duration, low intensity storms on a wet catchment. Early unit hydrograph studies did not consider the effect of changes in antecedent catchment wetness and causative storm characteristics on the shape of the unit hydrograph. Research conducted to date on urban catchments has assumed that the effect of the storm and antecedent catchment wetness on catchment response is minimised by the urban development and have sought to explain the changes in catchment response solely in terms of the degree of urbanisation. Several American studies have refuted this assumption (e.g. Rao et al, 1972; Schultz and Lopez, 1974) and the results of this study indicate the large range of response which may be expected due to variations in storm and antecedent catchment wetness conditions. Part of the success in identifying this effect is due to the derivation of a large number of unit hydrographs (36) from one catchment. Frequently a small sample is used which is unable to identify a consistent relationship.

Third, split-testing of the three unit hydrograph models indicated that first, the use of a mean unit hydrograph to predict the hydrograph for a range of storm events was very inaccurate. The absolute error in peak discharge and time to peak were 26.128 percent and 2.25 hours, respectively. Second, a two parameter, dimensionless unit hydrograph, scaled by two equations produced worse results. The absolute error in peak discharge and time to peak were 33.254 percent and 2.077 hours, respectively. Finally a three parameter, parametric triangular unit hydrograph (PTTUH), scaled by two equations, whose independent variables were the storm and antecedent catchment characteristics produced the best results with an absolute error in peak discharge and time to peak of 17.234 percent and 2.0 hours, respectively. Further research is necessary to reduce the inaccuracies, especially in the prediction of the time to peak where the error is so great that it makes the method useless for flood forecasting. The method is tolerable for flood peak prediction.

Fourth, analysis showed that the peak discharge and the time to peak of the mean unit hydrographs of the four catchments were not related to the percentage of the catchment urbanised. This, together with the failure of a traditional method of synthesis utilising the basin ratio, indicated that the response of large, heavily urbanised catchments is, contrary to much of the published literature, not controlled by the percentage urbanised. Rather, a new method indicated that the peak of the unit hydrograph was related to the topography (measured by the basin ratio) and therefore it was concluded that, although urbanisation will affect the percentage response, the shape of the catchment response is a function of the shape and physical characteristics of the catchment. This conclusion requires further investigation because it is based on only four catchments.

Unit hydrographs may be derived synthetically by the use of conceptual models. The full value of such an analysis was not realised through the lack of experience on the part of the author. These models have been used extensively by American hydrologists but they have yet to be used on large urban catchments in the UK. Consequently, a full list of recommendations were provided to help hydrologists maximise the benefits derived from such models.

An existing method of simulating a flood frequency curve was improved and tested. It was based on a statistical simulation of rainfall, a deterministic conversion of rainfall to discharge and a probabilistic analysis of the resulting discharge. The existing method convolved a total rainfall profile with a unit hydrograph adjusted to take account of catchment land use. The new method convolved an effective rainfall profile (derived by the loss curve separation method) with a PTTUH. Although the method proved to be more accurate than the existing method, several improvements are necessary. First, an improvement in the accuracy with which the PTTUH is predicted; second, the introduction of seasonal variations in the profile type and catchment conditions and finally a thorough testing of the method using a long discharge record. Although the method is still in need of improvement, the thesis made some significant advances in the techniques of flood frequency simulation by using a time variant unit hydrograph and effective rainfall profiles.

The final four results were derived from studies to measure the significance of some of the findings presented earlier in the thesis.

First, the effect of substituting a straight line approximation of the unit hydrograph (PTTUH) in place of a curved unit hydrograph was investigated using the flood frequency and spill volume computer program. This is the first study of the effect. The absolute error in predicting the peak discharge for return periods of 2.33 to 100 years was 3.222 percent, which is negligible. The error in predicting spill volume was greater, the maximum absolute error was 14.525 percent, for discharges in excess of 35 cumecs. This indicates the level of inaccuracy which can result from using triangular unit hydrographs to design balancing ponds and other structures which require an estimate of the volume of flood water. The triangular unit hydrograph consistently over-predicts the spill volume and therefore leads to overdesign.

The second study investigated the effect of rainfall separation method on peak discharge and flood frequency estimates. Comparison of the peak discharge predicted using the effective rainfall profiles produced by the percentage loss method and the loss curve method showed that the latter produced peaks 26.94 percent greater than those by the former method. This is some four times greater than the difference observed by the Flood Studies Report (NERC, 1975, I, 413). A similar difference was observed with a flood frequency simulation, though the difference between the loss curve and the percentage loss method decreased with increasing return period. This stressed the importance of the correct selection of rainfall separation method and the use of historical rather than design rainfall profiles.

The third study considered the relationship between the peak of the unit hydrograph and the spill volume. It was found that the spill volume produced by high peaked unit hydrographs was proportionally less than that generated by low peaked unit hydrographs. Consequently conventional unit hydrograph analyses, although providing an index of the effect of urbanisation on floods, have not considered the actual increase in flood damage as measured by the spill volume. It is suggested therefore that the increase in flood damage, due to urbanisation, may not be as great as the literature suggests. This should be tested by correlating spill volume and peak discharge on a catchment which has experienced urbanisation.

Finally, the effect of rainfall profile on peak discharge and spill volume was investigated. The effect of rainfall profile on peak discharge was noted and it was recommended that the hydrologist should take account of both the frequency and the seasonal occurrence of profiles because this will affect the significance of a profile as a flood producer. Similarly, the choice of profile to simulate spill volumes is dependent on first, the capacity of the channel for a given design return period. Second, although a given profile may produce the maximum estimate of spill volume it may be uneconomical to design to this, if that type of storm does not occur with sufficient frequency to justify expenditure to protect against such a flood. Studies of the effect of rainfall profile on the runoff hydrograph are important for the efficient design of flood alleviation structures, these results are the first to investigate the relationship.

8.4 Conclusion and suggestions for future research

This thesis represents a significant contribution to the study of the effect of storm and antecedent conditions on the response of urban catchments. The techniques, computer programs, results and suggestions provide a basis from which further research into urban hydrology may continue.

The variable response of urban catchments, identified by the unit hydrograph model, is caused by variations in the storm profile and antecedent catchment wetness. This variability handicaps attempts to define a relationship between the urban fraction of a catchment and the increase in mean annual discharge caused by urbanisation. To define this relationship it is necessary to understand the interaction between the catchment response, measured by the TUH, and the storm profile and antecedent catchment wetness. This thesis has presented a model to define such a relationship on one catchment. Further studies are required, for a range of levels of urbanisation, so that the relationship between the urban fraction of a catchment and the increase in mean annual discharge caused by urbanisation, for different combinations of storm and antecedent catchment wetness, may be defined. This relationship may then be used to design flood alleviation works for urbanising catchments, using local meteorological and hydrological information, rather than standard national conditions (e.g. NERC, 1975) which may not be typical of the area under consideration.

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APPENDICES

Appendix 1: User documentation for LPROG5 and LPROG6

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*****************************************************************************
THE INSTITUTE OF HYDROLOGY UNIT HYDROGRAPH DERIVATION PROGRAMS.
*****************************************************************************

THE INSTITUTE OF HYDROLOGY, WALLINGFORD, RETAINS OWNERSHIP OF ALL THE
PROPERTY RIGHTS ON BEHALF OF THE CROWN AND RESERVES CROWN COPYRIGHT.
MODIFICATIONS, DOCUMENTATION AND UCL IMPLEMENTATION BY W.S. EYRE.

*****************************************************************************

INTRODUCTION.
*************
THE UNIT HYDROGRAPH (UH) AND ITS DERIVATIVES CONTINUE TO BE WIDELY USED
BY HYDROLOGISTS BOTH IN A DESIGN AND ANALYSIS MODE ON UNGAUGED AND
GAUGED CATCHMENTS.  THE DEPARTMENT OF GEOGRAPHY, UCL, POSSESSES TWO
COMPUTER PROGRAMS WHICH PROVIDE THE OPPORTUNITY TO DERIVE A UH FASTER
AND WITH GREATER PRECISION THAN MANUAL METHODS (E.G. COLLINS, 1939).
THE USE OF MATHEMATICAL TECHNIQUES FACILITATE THE ANALYSIS OF RAINFALL
AND DISCHARGE RECORDS OF COMPLEX FLOODS.  THESE TECHNIQUES AVOID THE BIAS
INTRODUCED BY MANUAL GRAPHICAL TECHNIQUES WHICH CAN ONLY STUDY ISOLATED
STORMS, WITH NEAR UNIT DURATION, SUCCESSFULLY.
THE TWO PROGRAMS WERE WRITTEN BY M.J.LOWING OF THE INSTITUTE OF HYDROLOGY
FOR THE FLOOD STUDIES PROGRAMME (1970-1975).  THE FIRST PROGRAM (LPROG5)
SEPARATES THE RAPID RESPONSE RUNOFF FROM THE SLOW RESPONSE RUNOFF BY
EXTENSION OF THE RECESSION CURVE USING A TIME VARIENT LINEAR RESERVOIR.
THE VOLUME OF RUNOFF IS CONVERTED TO AN EQIVALENT DEPTH OF EFFECTIVE
RAINFALL.  THE EFFECTIVE RAINFALL IS SEPARATED FROM THE TOTAL RAINFALL BY
FOUR METHODS.  THE SECOND PROGRAM (LPROG6) USES THE RAPID RESPONSE RUNOFF
AND EFFECTIVE RAINFALL TO DERIVE UNIT HYDROGRAPHS BY MATRIX INVERSION
(SNYDER(1955),BODY(1959),NEWTON AND VINYARD(1967)) AND HARMONIC ANALYSIS
(O'DONNELL(1960)).  THE TWO PARAMETERS NECCESARY TO OBTAIN THE IUH OF THE
NASH(1960) MODEL ARE ALSO PROVIDED.
FURTHER DETAILS OF THE PROGRAMS MAY BE FOUND IN THE FLOOD STUDIES REPORT
(NERC, 1975), VOL. 1, SECTION 6. A GUIDE TO DATA COLLECTION IS PRESENTED
IN FSR, VOL.4, SECTION 3.
THE UNIT HYDROGRAPHS DERIVED BY THE PROGRAMS MAY EXHIBIT OSCILLATIONS,
NEGATIVE ORDINATES AND POSSESS A NON-ZERO ORIGIN.  ANY SUBSEQUENT
SUBJECTIVE ADJUSTMENT TO THE UH MUST ENSURE THAT THE ORDINATES OF THE NEW
UH CORRESPOND TO THE VOLUME OF RUNOFF TO BE EXPECTED FROM THE UNIT
RAINFALL OVER THE UNIT AREA.
THE TWO PROGRAMS ARE STORED IN UCMODLIB, THE REQUISTE JCL FOR RUNNING THE
MODULES IS PRESENTED TOGETHER WITH SAMPLE INPUT FILES.

METHODS OF RAINFALL SEPARATION.
*******************************
(1) CONSTANT LOSS PERCENTAGE.

(2) LOSS RATE CURVE.

(3) CONSTANT LOSS PERCENTAGE VARYING WITH CATCHMENT WETNESS INDEX (CWI).

(4) UNIFORM LOSS RATE - THE PHI METHOD.

    CWI = 125 + API5 + SMD
    WHERE: API5 IS THE FIVE DAY ANTECEDANT PRECIPITATION INDEX (MM).
           SMD  IS THE SOIL MOISTURE DEFICIT (MM).


DESCRIPTION OF THE INPUT DATA.
******************************
FOR EACH VARIABLE, PARAMETER, ARRAY OR SWITCH FOUR TYPES OF INFORMATION
ARE PRESENTED:
           COLUMN 1: THE NAME.
           COLUMN 2: THE FORMAT AND TYPE.
           COLUMN 3: THE LOCATION ON AN 80-COLUMN CARD.
           COLUMN 4: DESCRIPTION.

CARD FORMAT FOR LPROG5.
***********************

(1) SYSTEM CONTROL CARD.
************************

NAME    FORMAT   COLUMNS   DESCRIPTION
----    ------   -------   -----------
NDT     I1       1         NUMBER OF DATA TYPES (CARDS 3 TO 7)
ANH     F5.2     2-6       CORRECTION TO THE RELATIVE TIMING OF THE
                           DISCHARGE AND THE RAINFALL.
                           ANH IS THE NUMBER OF HOURS THAT THE
                           RAINFALL SHOULD BE ADJUSTED TO ENSURE
                           ASSUMED SYNCHRONISATION.
                           ANH IS EXPRESSED AS A DECIMAL OF AN HOUR AND MAY
                           BE EITHER -VE OR +VE.
ICAPL1  I2       7-8       THIS PARAMETER REFERS EXCLUSIVELY TO THE LOSS RATE
                           CURVE RAINFALL SEPARATION METHOD.
                           ICAPL1% OF EACH TOTAL RAINFALL INCREMENT
                           CONTRIBUTES TO RUNOFF, THE REMAINDER
                           (100-ICAPL1)% IS ELIGIBLE FOR REDUCTION
                           BY THE LOSS RATE CURVE.
                           ICAPL1 IS SET TO 1% AS A DEFAULT VALUE.
ISKIP   I1       11        TYPE OF RUN:
                           0 = FULL ANALYSIS CONDUCTED.
                           1 = READ DATA AND CHECK FOR CORRECT FORMATTING
                               PRODUCES NO OUTPUT, USEFUL IF THE 'PFILE'
                               WAS PRODUCED BY HAND.
OSMRF   I1       13        RAINFALL SMOOTHING:
                           0 = NO SMOOTHING.
                           1 = SMOOTHING USING A MOVING AVERAGE OF 3.

(2) DATA DEFINITION CARD.
*************************

NAME    FORMAT   COLUMNS   DESCRIPTION
----    ------   -------   -----------
ICN     I5       1-5       CATCHMENT NUMBER.
IEN     I5       6-10      EVENT NUMBER.
ITOD    I5       11-15     DATA TYPE:
                           '1' - DISCHARGE.
                           '2' - RAINFALL
                           '3' - SOIL MOISTURE DEFICIT.
                           '4' - ANTECEDANT PRECIPITATION.
                           '5' - COMMENT CARD.


CARD 2 SHOULD PRECEDE EACH SET OF DATA TYPE CARDS.
DATA TYPES 1,2,3 AND 4 ARE MANDATORY.
DATA TYPE 5 IS OPTIONAL.


(3) DISCHARGE HYDROGRAPH (DATA TYPE 1).
***************************************

NAME    FORMAT   COLUMNS   DESCRIPTION
----    ------   -------   -----------
AREA    F8.2     1-8       CATCHMENT AREA, SQ. KM.
NTFL    I5       9-13      NUMBER OF DATA VALUES.
N(3)    I2       16-17     DAY.
N(2)    I2       18-19     MONTH.
N(1)    I2       20-21     YEAR, THE LAST TWO DIGITS.
TINT    F5.2     22-26     TIME INTERVAL OF THE DATA (HOURS, DECIMAL).
HSFL    F5.2     27-31     START TIME, DECIMAL.
```

NAME	FORMAT	COLUMNS	DESCRIPTION
IWSG1	I1	70	INPUT DATA UNIT OF MEASUREMENT. 0 = SI, 1 = IMPERIAL.
TFL(I)	F10.4	------	DATA VALUES (CUMECS OR CUSECS). 8 TEN COLUMN FIELDS. TO ENSURE AN ACCURATE BASEFLOW SEPARATION IT IS RECOMMENDED THAT: (1) SIX OR MORE ORDINATES FROM THE PRECEEDING RECESSION SHOULD BE INCLUDED. (2) THE NUMBER OF ORDINATES ON THE RECESSION LIMB SHOULD BE EQUAL TO FOUR TIMES THE LAG TIME. WHERE LAG TIME IS DEFINED AS THE TIME FROM THE CENTROID OF TOTAL RAINFALL TO PEAK FLOW. DO NOT END THE DISCHARGE HYDROGRAPH WITH A RISING LIMB. A RECESSION LIMB OF AT LEAST 4 ORDINATES IS REQUIRED.

(4) RAINFALL DATA (DATA TYPE 2).
****

NAME	FORMAT	COLUMNS	DESCRIPTION
AREA2	F8.2	1-8	CATCHMENT AREA, SQ. KM.
NHRF	I5	9-13	NUMBER OF DATA VALUES.
N(7)	I2	16-17	DAY.
N(6)	I2	18-19	MONTH.
N(5)	I2	20-21	YEAR, THE LAST TWO DIGITS.
TINT2	F5.2	22-26	TIME INTERVAL OF THE DATA (HOURS, DECIMAL).
HSRF	F5.2	27-31	START TIME, DECIMAL.
RPOBS	F10.4	32-41	RAINFALL AFTER 0900 HRS., BUT BEFORE THE START OF THE STORM (MM OR INCHES).
IWSG2	I1	70	INPUT DATA UNIT OF MEASUREMENT. 0 = SI, 1 = IMPERIAL.
HRF(I)	F10.4	------	DATA VALUES (MM OR INCHES). 8 TEN COLUMN FIELDS.

(5) SOIL MOISTURE DEFICIT DATA (DATA TYPE 3).
****

NAME	FORMAT	COLUMNS	DESCRIPTION
AREA3	F8.2	1-8	CATCHMENT AREA, SQ. KM.
IDS	I2	16-17	DAY.
IMS	I2	18-19	MONTH.
IYS	I2	20-21	YEAR, THE LAST TWO DIGITS.
NOFSTN	I3	22-24	NUMBER OF MET. OFFICE SMD STATIONS USED TO OBTAIN SMD VALUE.
SMD	F6.2	25-30	SMD VALUE (MM OR INCHES). WHEN NO DATA ARE AVAILABLE INSERT A VALUE OF 0.0
NUMSTN	I10	31-40	)
NUMSTN	I10	41-50	) STATION NUMBERS OF SMD STATIONS USED.
NUMSTN	I10	51-60	)
IWSG3	I1	70	INPUT DATA UNIT OF MEASUREMENT. 0 = SI, 1 = IMPERIAL.

(6) ANTECEDANT PRECIPITATION DATA (DATA TYPE 4).
****

NAME	FORMAT	COLUMNS	DESCRIPTION
AREA4	F8.2	1-8	CATCHMENT AREA, SQ. KM.
NARD	I5	9-13	NUMBER OF DAYS FOR WHICH AP IS AVAILABLE.
IDA	I2	16-17	DAY.
IMA	I2	18-19	MONTH.
IYA	I2	20-21	YEAR, THE LAST TWO DIGITS.
IWSG4	I1	70	INPUT DATA UNIT OF MEASUREMENT. 0 = SI, 1 = IMPERIAL.
ANTE	F10.4	------	DAILY TOTALS OF PRECIPITATION (MM OR INCHES). 8 TEN COLUMN FIELDS, IN CHRONOLOGICAL ORDER. IT IS RECOMMENDED THAT 28 DAYS OF AP ARE INSERTED. 5 DAYS OF AP ARE MANDATORY. WHEN NO DATA ARE AVAILABLE INSERT 5 CONSECUTIVE VALUES OF 0.0

(7) COMMENT CARD(S) (DATA TYPE 5).
****

NAME	FORMAT	COLUMNS	DESCRIPTION
NOWOC	I5	1-5	NUMBER OF 80 COLUMN CARDS USED. NOWOC SHOULD BE LESS THAN OR EQUAL TO 4.
SDCOM	20A4	1-80	COMMENT CARD, USE FOR LABELING.

NOTES.

(1) START TIME (HSFL, HSRF) IS EXPRESSED AS A DECIMAL ON A 24 HOUR CLOCK.
    E.G. HALF PAST THREE IN THE AFTERNOON IS CODED AS 15.50
         QUARTER TO FOUR IN THE AFTERNOON IS CODED AS 15.75
(2) TIME INTERVAL OF THE DATA (TINT) AND 'ANH' ARE EXPRESSED AS A DECIMAL OF AN
    E.G. HALF-HOURLY DATA ARE CODED AS 0.5
         QUARTER-HOURLY DATA ARE CODED AS 0.25

HOW TO RUN LPROG5 ON THE UCL IBM 360.
****

```
//-------- JOB (100,WSGE,0.10,7C),'LPROG5',MSGLEVEL=(2,0)
// EXEC LINKANGO
//L.SYSIN DD *
  INCLUDE MODLIB(UCFAG651)
//G.SYSIN DD *
( DATA )
//G.FT08F001 ETC, SEE LIST OF AVAILABLE OPTIONS (BELOW
/*
//
```

THE OUTPUT OF LPROG5 IS CALLED THE SEPFILE AND FORMS PART OF THE INPUT FOR LPROG6. THE SEPFILE IS WRITTEN ON CHANNEL 8. THE PERIPHERAL DEVICE THAT SHOULD BE INVOKED SHOULD BE RELATED TO THE USERS NEEDS. THREE OPTIONS ARE LISTED BELOW, OTHERS INCLUDE A MAGNETIC TAPE, 4C002 DISK OR THE GEOGRAPHY DEPARTMENTS DISK.

OPTIONS.
****

(1) TO LIST THE SEPFILE.

```
//G.FT08F001 DD SYSOUT=A
```

(2) TO PLACE THE SEPFILE IN A GUTS FILE.

```
//G.FT08F001 DD UNIT=SCR14,SPACE=(TRK,(5,2)),DSN=&&SEPF,
//              DISP=(NEW,PASS),DCB=(LRECL=80,BLKSIZE=80,RECFM=FB)
// EXEC GUTSGEN,NAME=--------
//U.SYSIN DD DSN=&&SEPF,DISP=(OLD,DELETE)
```

(3) TO OBTAIN A CARD DECK OF THE SEPFILE.
  (A) REPLACE THE HASP PARAMETERS ON THE JOB CARD SUCH THAT:
      (100,WSGE,0.10,7C,50)
      WHERE 50 IS THE NUMBER OF CARDS TO BE PUNCHED.
  (B)

```
//G.FT08F001 DD UNIT=SCR14,SPACE=(TRK,(5,2)),DSN=&&SEPF,
//              DISP=(NEW,PASS),DCB=(LRECL=80,BLKSIZE=80,RECFM=FB)
// EXEC CARDPCH
```

```
//U.SYSIN  DD  DSN=&&SEPF,DISP=(OLD,DELETE)
```

THIS OPTION WILL ALSO PROVIDE A LISTING OF THE SEPFILE.
THE CARDS PUNCHED BY THE IBM 360 COMPUTER ARE LIABLE TO BE OFF-CENTRED.

SAMPLE INPUT FILE FOR LPROG5.
****

```
< J.C.L. >
4
39005    1    1
   43.50   18  240963 0.5 02.00                                          1
   52.6      51.3      51.3      52.6      81.0     128.2     144.4     148.5
  126.9     102.6      90.4      81.0      68.8      59.4      56.7      54.0
   51.3      48.6
39005    1    2
   43.50    7  230963 0.5 23.00                                          1
    0.01      0.01      0.01      0.02      0.06      0.01      0.02
39005    1    3
   43.50       230963  1  4.14     246263                                1
39005    1    4
   43.50   28  230963                                                    1
    0.07      0.17      0.0       0.0       0.08      0.26      0.41      0.06
    0.04      0.46      0.53      0.0       0.01      0.04      0.0       0.02
    0.06      0.0       0.0       0.0       0.0       0.0       0.0       0.0
    0.0       0.01      0.0       0.0
< J.C.L. >
```

****

CARD FORMAT FOR LPROG6.
****

LPROG6 METHOD CONTROL CARD.
****

NAME	FORMAT	COLUMNS	DESCRIPTION
IS1	I1	1	HARMONIC ANALYSIS TECHNIQUE ONLY.
IS2	I1	3	MATRIX INVERSION TECHNIQUE ONLY.
IS3	I1	5	HARMONIC ANALYSIS AND MATRIX INVERSION TECHNIQUES.
IS4	I1	7	0 = NO OUTPUT. 1 = PROVIDES A REPRODUCED RESPONSE HYDROGRAPH TO COMPARE WITH THE INPUT.
IS5	I1	9	0 = NO OUTPUT. 1 = PROVIDES THE PARAMETERS FOR THE NASH CASCADE MODEL.
IS6	I1	11	0 = NO OUTPUT. 1 = LINEPRINTER PLOT OF TUH.
ISW1	I1	13	OUTPUT FOR HARMONIC ANALYSIS TECHNIQUE. 0 = SMOOTHED TUH ONLY. 1 = UNSMOOTHED TUH ONLY. 2 = SMOOTHED AND UNSMOOTHED TUH'S.
ISW2	I1	15	OUTPUT FOR MATRIX INVERSION TECHNIQUE. 0 = SMOOTHED TUH ONLY. 1 = UNSMOOTHED TUH ONLY. 2 = SMOOTHED AND UNSMOOTHED TUH'S.
IRFT(1)	I1	17	)
IRFT(2)	I1	19	) REFERENCE NUMBER OF THE EFFECTIVE RAINFALL
IRFT(3)	I1	21	) PROFILE FOR WHICH THE TUH SHOULD BE DERIVED.
IRFT(4)	I1	23	)

NOTES:

1. IS1, IS2 AND IS3 SHOULD BE SET BY PLACING A '0' FOR 'NO' AND A '1' FOR 'YES'.

2. LPROG6 SUPPLIES THE TWO PARAMETERS NECCESARY TO OBTAIN THE IUH OF THE NASH MODEL. THIS IS A CONCEPTUAL MODEL CONSISTING OF A CASCADE OF 'N' EQUAL LINEAR RESERVOIRS, EACH WITH A STORAGE COEFFICIENT 'K'. IT SHOULD BE NOTED THAT SEVERAL WORKERS HAVE FOUND THAT THE VALUES OF 'N' AND 'K' VARY FROM STORM TO STORM.

3. THE DATA FOR LPROG6 CONSISTS OF THE SEPFILE FROM LPROG5 TOGETHER WITH A METHOD CONTROL CARD WHICH TELLS THE PROGRAM THE METHOD(S) BY WHICH THE UNIT HYDROGRAPHS SHOULD BE DERIVED. ONE METHOD CONTROL CARD IS REQUIRED FOR EACH SEPFILE, IT IS PLACED AFTER THE FIRST CARD IMAGE IN THE SEPFILE.

HOW TO RUN LPROG6 ON THE UCL IBM 360.
****

```
//-------- JOB (100,WSGE,0.20,9C),'LPROG6',MSGLEVEL=(2,0)
// EXEC  LINKANGO
//L.SYSIN  DD  *
  INCLUDE MODLIB(UCFAG652)
//G.SYSIN  DD  *
 < DATA >
/*
//
```

SAMPLE INPUT FILE FOR LPROG6.
****

```
< J.C.L. >
39005    1
0 0 1 1 1 1 1 1 1 2 3 4
  43.50   0.50
 24
   0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
   0.0000    0.0368    0.8410    2.1776    2.8363    2.7524    2.1407    1.4527
   1.1072    0.8410    0.4955    0.2294    0.1529    0.0765    0.0000    0.0000
 7
   0.0442    0.0442    0.0442    0.0883    0.2649    0.0442    0.0883
 7
   0.0025    0.0025    0.0025    0.0051    0.5979    0.0025    0.0051
 7
   0.0390    0.0400    0.0409    0.0838    0.2633    0.0498    0.1014
 7
   0.0000    0.0000    0.0000    0.0000    0.8182    0.0000    0.0000
< J.C.L. >
```

****

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****

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Appendix 2: Source listing of LPROG5

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C     PROGRAM LPROG5(INPUT,OUTPUT,SEPFILE,
C                    TAPE5=INPUT,TAPE6=OUTPUT,TAPE8=SEPFILE)
C
C   LPROG5: THE INSTITUTE OF HYDROLOGY, WALLINGFORD, U.K., RETAINS
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C           UCL, LONDON (JANUARY 1978).
C
      INTEGER OSMRF
      COMMON/A/NHRF,HRF(150),SR,TFL(400),NTFL,BDR,NRL,SFL(400),VOL,NSFL,
     *     EPR(400),ANH,TINT2,PIRF
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/C/OSMRF,ERF1(150),ERF2(150),ERF3(150),TINT,CAP,AREA,AEX,
     *ISKIP,ERF4(150)
      COMMON/D/EC,CRO,CRT,NORT,NORTA,RL,QR(50),NEND
      COMMON/H/SDCOM(80),NOWOC
      COMMON/JADE/ NX(4),HSFL,HSRF,PQ4A,PQ4B
      AEX=1.0
      ISM=125
   10 BDR=0.0
      DO 20 I=1,400
      TFL(I)=0.0
      SFL(I)=0.0
   20 EPR(I)=0.0
      CALL READIN
   30 SR=0.0
      DO 40 I=1,NHRF
   40 SR=SR+HRF(I)
      DUR=NHRF*TINT
      XMINT=SR/DUR
      WRITE     (6,50) ICN,IEN
   50 FORMAT (1H1,//,T110,2I5)
      WRITE     (6,60) ICN,IEN,N(7),N(6),N(5),AREA,DUR,TINT,SR,
     *    PIRF,XMINT,HSRF,N(7),N(6),N(5)
   60 FORMAT(/,T26,' HYDROGRAPH ANALYSIS FOR CATCHMENT NUMBER ',
     *I7,'.  EVENT ',I3,' (',I2,'/',I2,'/',I2,').'/,
     *26X,10('*******'),'**',///,
     *10X,'CATCHMENT AREA =',F7.2,
     *   ' SQ. KM.',T71,'DURATION OF STORM =',F6.2,' HOURS'/
     *    10X,'DATA INTERVAL  =',F7.2,' HOURS',T78,'TOTAL RAIN =',F8.2,
     *' MM'//10X,'PEAK RAINFALL INTENSITY =  ',F8.2,' MM PER HOUR',//10X,
     *'MEAN RAINFALL INTENSITY =  ',F8.2,' MM PER HOUR',
     *                              //10X,'FIRST RAIN VALUE APPLIES TO INTERV
     *AL BEGINNING',F6.2,' HOURS ON ',I2,'/',I2,'/',I2)
      ANH1=ANH
      CALL TIMEFI
      IF(ABS(ANH-500.0).LT.0.005)GO TO 10
      IF(ABS(ANH1).LT.0.005)GO TO 100
      IF(ANH.GE.0.0)GO TO 80
      WRITE     (6,70) ANH1
   70 FORMAT (/10X,'RELATIVE TIMING ERROR SUSPECTED - RUNOFF',
     *  ' ADVANCED(MOVED LEFT) BY',F6.2,' HOURS')
      GO TO 100
   80 WRITE     (6,90) ANH1
   90 FORMAT (/10X,'RELATIVE TIMING ERROR SUSPECTED - RAINFALL',
     *  ' DELAYED(MOVED RIGHT) BY',F6.2,' HOURS')
  100 CALL HYGRAP
      CALL ANTECE
      CALL RAINSP
      NH=ANH
      IF(BDR) 10,110,110
  110 IF(NOWOC.EQ.0)GO TO 150
      WRITE     (6,120)
  120 FORMAT(/10X,'THE FOLLOWING COMMENT RELATES TO THE DATA EXTRACTED',
     *' FOR THIS EVENT:-')
      DO 130 J=1,NOWOC
  130 WRITE     (6,140) (SDCOM(I),I=1,20)
  140 FORMAT(20X,20A4)
C
C     ADJUST DATA FOR PRINTING
C
  150 WRITE     (6,160)ICN,IEN
  160 FORMAT (//T110,2I5)
      NH9=NEND
      NH8=NHRF
      IF(NH) 200,230,170
  170 DO 180 I=1,NHRF
      IJ=NHRF-I+1
      IJH=IJ+NH
      HRF(IJH)=HRF(IJ)
      ERF1(IJH)=ERF1(IJ)
      ERF2(IJH)=ERF2(IJ)
      ERF3(IJH)=ERF3(IJ)
  180 ERF4(IJH)=ERF4(IJ)
      DO 190 I=1,NH
      ERF1(I)=-1.0
      ERF2(I)=-1.0
      ERF3(I)=-1.0
      ERF4(I)=-1.0
  190 HRF(I)=-1.0
      NH8=NHRF+NH
      GO TO 230
  200 DO 210 I=1,NEND
      IJ=NEND-I+1
      IJH=IJ-NH
      TFL(IJH)=TFL(IJ)
      EPR(IJH)=EPR(IJ)
  210 SFL(IJH)=SFL(IJ)
      IAB=-NH
      DO 220 I=1,IAB
      EPR(I)=-1.0
      SFL(I)=-1.0
  220 TFL(I)=-1.0
      NH9=NEND-NH
  230 WRITE     (6,50)ICN,IEN
      WRITE     (6,240)
  240 FORMAT(////6X,'TIME AT START',4X,'TOTAL RAIN',10X,'EFFECTIVE RAINF
     *ALL (MM).',14X,'TOTAL FLOW',5X,'PREVIOUS FLOW',4X,'SEPARATED'/
     *7X,'OF INTERVAL',60X,'EXTENDED',9X,'FLOW'/
     *8X,'(HOURS)',11X,'(MMS)',8X,'(1).',7X,'(2).',7X,'(3).',7X,'(4).',7
     *X,'(CUMECS)',7X,'(CUMECS)',7X,'(CUMECS)'//)
      TIME=PQ4A-TINT
      DO 310 I=1,NH9
      TIME=TIME+TINT
      TIME=AMOD(TIME,24.0)
  250 WRITE     (6,260)TIME
  260 FORMAT (7X,F6.2)
      IF(I.GT.NH8)GO TO 280
      IF(HRF(I).LT.0.0)GO TO 280
      WRITE     (6,270) HRF(I),ERF1(I),ERF2(I),ERF3(I),ERF4(I)
  270 FORMAT (1H+,20X,F10.4,1X,4(F11.4))
  280 IF(TFL(I).LT.0.0)GO TO 310
      WRITE     (6,290) TFL(I),EPR(I)
  290 FORMAT (1H+,80X,F10.4,5X,F10.4)
      IF(SFL(I).LT.0.0)GO TO 310
      WRITE     (6,300)SFL(I)
  300 FORMAT (1H+,110X,F10.4)
  310 CONTINUE
      WRITE     (6,320)
  320 FORMAT(//10X,'RAINFALL SEPARATION METHOD:-',
     *//10X,'(1) CONSTANT LOSS PERCENTAGE.'
```

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     */10X,'(2) LOSS RATE CURVE.'
     */10X,'(3) CONSTANT LOSS PERCENTAGE VARYING WITH CWI.'
     */10X,'(4) UNIFORM LOSS RATE, THE PHI INDEX.')
      WRITE      (6,180) ICN,IEN
C
C     ADJUST DATA AGAIN FOR TRANSFER
C
      NTR=NRL+NH
      IF(NH.LE.0)GO TO 350
      NH7=NH
      IF(NTR.LT.NH) NH7=NTR
      NH8=NH8-NH7
      DO 330 I=1,NH8
      ERF1(I)=ERF1(I+NH7)
      ERF2(I)=ERF2(I+NH7)
      ERF3(I)=ERF3(I+NH7)
  330 ERF4(I)=ERF4(I+NH7)
      NH9=NH9-NH7
      DO 340 I=1,NH9
  340 SFL(I)=SFL(I+NH7)
  350 CONTINUE
      DO 360 I=1,NH8
      IF(ERF1(I).LT.0.0)  ERF1(I)=0.0
      IF(ERF2(I).LT.0.0)  ERF2(I)=0.0
      IF(ERF3(I).LT.0.0)  ERF3(I)=0.0
  360 IF(ERF4(I).LT.0.0)  ERF4(I)=0.0
      DO 370 I=1,NH9
  370 IF(SFL(I).LT.0.0)  SFL(I)=0.0
      WRITE      (6,380)  ICN,IEN
  380 FORMAT  (2I5,2X)
      WRITE      (6,390)  AREA,TINT
  390 FORMAT  (F8.2,F7.2)
      WRITE      (6,400)  NH9,(SFL(I),I=1,NH9)
      WRITE      (6,400)  NH8,(ERF1(I),I=1,NH8)
      WRITE      (6,400)  NH8,(ERF2(I),I=1,NH8)
      WRITE      (6,400)  NH8,(ERF3(I),I=1,NH8)
      WRITE      (6,400)  NH8,(ERF4(I),I=1,NH8)
  400 FORMAT  (I3/(8F10.4))
      GO TO 10
      END
      SUBROUTINE READIN
C
C     TO READ IN A COMPLETE SET OF DATA FOR ONE EVENT.
C
      INTEGER OSMRF
      COMMON/A/NHRF,HRF(150),SR,TFL(400),NTFL,BOR,NRL,SFL(400),VOL,
     *NSFL,EPR(400),ANH,TINT2,PIRF
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/C/OSMRF,ERF1(150),ERF2(150),ERF3(150),TINT,CAP,AREA,AEX,
     *ISKIP,ERF4(150)
      COMMON/H/ SDCOM(80),NOWOC
      COMMON/JADE/ NX(4),HSFL,HSRF,PO4A,PO4B
      COMMON/K/NARD,IDS,IMS,IYS,NOFSTN,NUMSTN(3),SMD
      DIMENSION IDT(5)
      NSTV(X)=INT(X+SIGN(0.5,X))
  410 READ       (5,420,END=840)NDT,ANH,ICAPL1,ISKIP,OSMRF
  420 FORMAT(I1,F5.1,I2,2X,I1,1X,I1)
      IF(ICAPL1.EQ.0) ICAPL1=1
      CAP=ICAPL1
  430 NOWOC=0
  440 IC=1
      DO 450 I=1,5
  450 IDT(I)=0
      IENP=0
      ICNP=0
  460 READ       (5,470) ICN,IEN,ITOD
  470 FORMAT(3I5)
      IF(ISKIP.EQ.1.AND.IC.EQ.1)WRITE      (6,480)ICN,IEN
  480 FORMAT(/' CATCH.',I6,' EVENT',I4,' READ BUT NOT ANALYSED')
      IF(IC.EQ.1)GO TO 490
      IF(ICNP.NE.ICN.OR.IENP.NE.IEN)GO TO 820
  490 ICNP=ICN
      IENP=IEN
      IDT(ITOD)=1
      GO TO (500,540,590,620,660),ITOD
  500 READ       (5,510)AREA,NTFL,N(3),N(2),N(1),TINT,HSFL,IWSG1,
     *      (TFL(I),I=1,NTFL)
  510 FORMAT(F8.2,I5,2X,3I2,2F5.1,38X,I1/(8F10.0))
      IF(IWSG1.EQ.0)GO TO 530
      DO 520 I=1,NTFL
  520 TFL(I)=TFL(I)*0.0283168
  530 CONTINUE
      N(4)=NSTV(HSFL)
      GO TO 700
  540 READ       (5,550)AREA2,NHRF,N(7),N(6),N(5),TINT2,HSRF,RP9BS,IWSG2,
     *      (HRF(I),I=1,NHRF)
  550 FORMAT(F8.2,I5,2X,3I2,2F5.1,F10.4,28X,I1/(8F10.4))
      IF(IWSG2.EQ.0)GO TO 570
      DO 560 I=1,NHRF
  560 HRF(I)=HRF(I)*25.4
      IF(RP9BS.GT.0.0) RP9BS=RP9BS*25.4
  570 CONTINUE
      N(8)=NSTV(HSRF)
      PIRF=0.0
      DO 580 I=1,NHRF
  580 IF(HRF(I).GT.PIRF) PIRF=HRF(I)
      PIRF=PIRF*(1.0/TINT2)
      GO TO 700
  590 READ       (5,600)AREA3,IDS,IMS,IYS,NOFSTN,SMD,(NUMSTN(I),I=1,3),IWS
     *G3
  600 FORMAT(F8.2,7X,3I2,I3,F8.2,3I10,9X,I1)
      IF(IWSG3.EQ.0)GO TO 610
      SMD=SMD*25.4
  610 CONTINUE
      GO TO 700
  620 READ       (5,630)AREA4,NARD,IDA,IMA,IYA,IWSG4,(ANTE(I),I=1,NARD)
  630 FORMAT(F8.2,I5,2X,3I2,46X,I1,/(8F10.4))
      IF(IWSG4.EQ.0)GO TO 650
      DO 640 I=1,NARD
  640 ANTE(I)=ANTE(I)*25.4
  650 CONTINUE
      GO TO 700
  660 READ       (5,670) NOWOC
  670 FORMAT(I5)
      DO 680 J=1,NOWOC
  680 READ       (5,690) (SDCOM(I),I=1,20)
  690 FORMAT(20A4)
  700 IF(IC.EQ.NDT)GO TO 710
      IC=IC+1
      GO TO 460
  710 DO 730 I=1,5
      IF(IDT(I).EQ.1)GO TO 730
      IF(I.EQ.5)GO TO 730
      ISKIP=1
      WRITE      (6,720)I,IEN
  720 FORMAT(//' VITAL DATA TYPE ',I1,' MISSING. EVENT ',I2,' NOT ANALYS
     *ED')
  730 CONTINUE
      IF(ISKIP.EQ.1)GO TO 410
      IF(TINT2.EQ.TINT) RETURN
      IF(TINT2.GT.TINT)GO TO 790
      WRITE      (6,740)
```

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  740 FORMAT(/9X,' RAIN DATA INTERVAL INCREASED TO MATCH WITH FLOW',
     * ' DATA')
      IRAT=TINT/TINT2
      NHRF1=NHRF/IRAT
      IF(NHRF1*IRAT.EQ.NHRF)GO TO 760
      NHRF1=NHRF1+1
      IAC=NHRF+1
      IAD=NHRF1*IRAT
      DO 750 I=IAC,IAD
  750 HRF(I)=0.0
  760 IS=1
      NHRF=NHRF1*IRAT
      DO 780 I=1,NHRF,IRAT
      SUMHRF=0.0
      IAE=IRAT+IS-1
      DO 770 J=IS,IAE
  770 SUMHRF=SUMHRF+HRF(J)
      IIRA=(I+IRAT-1)/IRAT
      HRF(IIRA)=SUMHRF
      IS=IS+IRAT
  780 CONTINUE
      NHRF=IIRA
      RETURN
  790 IRAT=TINT2/TINT
      NHRF1=IRAT*NHRF
      WRITE      (6,800)
  800 FORMAT(/9X,' RAIN DATA INTERVAL REDUCED TO MATCH WITH FLOW DATA')
      NHRF=NHRF1
      DO 810 I=1,NHRF,IRAT
      IJ=NHRF-I+1
      IJIRAT=IJ/IRAT
      HRT=HRF(IJIRAT)
      DO 810 J=1,IRAT
      IJOHN=IJ-J+1
  810 HRF(IJ-J+1)=HRT/IRAT
      NHRF=IJOHN
      RETURN
  820 WRITE      (6,830)
  830 FORMAT(' CATCHMENT NUMBER OR EVENT NUMBER QUERY. PROGRAM HALTED')
      STOP
  840 WRITE (6,850)
  850 FORMAT(////,' END OF THE JOB.')
      STOP
      END
      SUBROUTINE TIMEFI
C
C     TO CALCULATE NO. OF HOURS BETWEEN FIRST ITEMS OF RAINFALL
C     AND STREAMFLOW. DATA.
C
      REAL NH,NH1
      COMMON/A/NHRF,HRF(150),SR,TFL(400),NTFL,BDR,NRL,SFL(400),VOL,NSFL,
     *          EPR(400),NH,TINT2,PIRF
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/JADE/ NX(4),HSFL,HSRF,PQ4A,PQ4B
      IRF1=0
      PQ4A=HSFL
      PQ4B=HSRF
      DO 860 I=1,4
  860 NX(I)=N(I)
      DO 870 I=1,3
      IF  (N(I)-N(I+4)) 920,870,900
  870 CONTINUE
      IF(HSFL-HSRF) 920,880,900
  880 WRITE      (6,890)IEN
  890 FORMAT(/10X,'IDENTICAL START TIMES IN EVENT ',I2)
      RETURN
  900 DO 910 I=1,4
      NI=N(I)
      N(I)=N(I+4)
  910 N(I+4)=NI
      IRF1=1
      TEMP=PQ4A
      PQ4A=PQ4B
      PQ4B=TEMP
  920 IF(N(5)-N(1)-1) 940,930,1010
  930 IF(N(6).NE.1.OR.N(2).NE.12)GO TO 1010
      N(6)=13
  940 IF(N(6)-N(2)-1) 1000,950,1010
  950 IF(MOD(N(1),4).EQ.0.AND.N(2).EQ.2) N(2)=14
      N2=N(2)
      GO TO (960,990,960,970,960,970,960,960,970,960,970,960,960,980),N2
  960 N(7)=N(7)+1
  970 N(7)=N(7)+1
  980 N(7)=N(7)+1
  990 N(7)=N(7)+28
 1000 ND=N(7)-N(3)
      NH1=(FLOAT(ND*24))+PQ4B-PQ4A
      IF(IRF1.EQ.1)NH1=-NH1
      NH=NH1+NH
      RETURN
 1010 WRITE      (6,1020)IEN
 1020 FORMAT(' RAINFALL AND/OR RUNOFF START TIMES ARE SUSPECT. EVENT NO.
     * ',I2,' NOT ANALYSED')
      NH=500.0
      RETURN
      END
      SUBROUTINE HYGRAP
C
C     FIND LAG BETWEEN CENTRE OF MASS OF TOTAL RAINFALL AND PEAK OF
C     FLOW HYDROGRAPH. ASSUME QUICK RESPONSE RUNOFF ENDS AT 4 TIMES
C     THIS LAG AFTER LAST RAINFALL AND SEPARATE IT FROM PRECEDING
C     FLOW AND THE SLOW RESPONSE. CALCULATE VOLUME OF RUNOFF.
C
      REAL NH,NPF
      DIMENSION EER(400),FLOWDI(20),FLOWPE(20),KA(20),KB(20)
      COMMON/A/NHRF,HRF(150),SR,TFL(400),NTFL,BOR,NRL,SFL(400),VOL,NSFL,
     *          EPR(400),NH,TINT2,PIRF
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/C/OSMRF,ERF1(150),ERF2(150),ERF3(150),TINT,CAP,AREA,AEX,
     *ISKIP,ERF4(150)
      COMMON/D/EC,CRO,CRT,NORT,NORTA,RL,OR(50),NEND
      COMMON/JADE/ NX(4),HSFL,HSRF,PQ4A,PQ4B
      NSTV(X)=INT(X+SIGN(0.5,X))
C
C     FIND CENTRE OF MASS OF TOTAL RAINFALL HYETOGRAM
C
      NESL=0
      NH=NH*(1/TINT)
      VOL=0.0
      SRT=0.0
      DO 1030 I=1,NHRF
 1030 SRT=SRT+(HRF(I)*I)
      HCMRF=SRT/SR
C
C     ANALYSE BURST SEQUENCE WITHIN RAINFALL
C
      WRITE      (6,1040)
 1040 FORMAT (/10X,'BURST ANALYSIS - X MMS IN Y HOURS FOLLOWED',
     * ' BY Z HOURS WITH NOTHING',T81,'X',4X,'Y',5X,'Z')
      HSC=0.0
      HZC=0.0
      RRA=0.0
```

```
      ITR=0
      DO 1100 I=1,NHRF
      HR=HRF(I)
 1050 IF(ITR.NE.1)GO TO 1080
      IF(HR.NE.0.0)GO TO 1060
      HZC=HZC+TINT
      GO TO 1100
 1060 WRITE     (6,1070)RRA,HSC,HZC
 1070 FORMAT (T78,F5.1,F6.2,F6.2)
      ITR=0
      RRA=0.0
      HZC=0.0
      HSC=0.0
 1080 IF(HR.NE.0.0)GO TO 1090
      HZC=HZC+TINT
      ITR=1
      GO TO 1100
 1090 RRA=RRA+HR
      HSC=HSC+TINT
 1100 CONTINUE
      IF(ITR.EQ.0)WRITE      (6,1110)RRA,HSC
 1110 FORMAT (T78,F5.1,F5.1)
      IF(ITR.EQ.1)WRITE      (6,1070)RRA,HSC,HZC
C
C     FIND PEAKS AND TROUGHS OF TOTAL FLOW HYDROGRAPH.
C
      K1=1
      SUM1=0.0
      SUM=0.0
      I=1
      TFLMAX=TFL(I)
 1120 THJ=TFL(I)
      IAF=I+1
      DO 1130 K=IAF,NTFL
      KJ=K
      IF(TFL(K).GT.THJ)GO TO 1140
 1130 THJ=TFL(K)
 1140 I=KJ
      IF(I.EQ.NTFL)GO TO 1180
      FLOWDI(K1)=THJ
      KA(K1)=KJ-1
      THJ=TFL(I)
      IAG=I+1
      DO 1150 K=IAG,NTFL
      KJ=K
      IF(TFL(K).LT.THJ)GO TO 1160
 1150 THJ=TFL(K)
 1160 I=KJ
      IF(I.EQ.NTFL)GO TO 1180
      FLOWPE(K1)=THJ
      IF(THJ.LE.TFLMAX)GO TO 1170
      TFLMAX=THJ
      NOPF=KJ-2
 1170 KB(K1)=KJ-1
      K1=K1+1
      GO TO 1120
 1180 K1=K1-1
      WRITE     (6,1190) (FLOWDI(I),KA(I),FLOWPE(I),KB(I),I=1,K1)
 1190 FORMAT (///16X,'TROUGH',21X,'PEAK',/14X,'FLOW',6X,'INTS',12X,
     *     'FLOW',6X,'INTS',/(9X,F10.4,6X,I3,7X,F10.4,6X,I3))
C
C     RAIN BEFORE MAIN PEAK.
C
      SUMRF=0.0
      DO 1200 I=1,NHRF
      I3=I
      IF(HRF(I).NE.0.0) I2=0
      IF(HRF(I).EQ.0.0.AND.I2.EQ.0) I2=I3
      SUMRF=SUMRF+HRF(I)
      IF(I.EQ.(NOPF-NH))GO TO 1210
 1200 CONTINUE
 1210 IF(I2.GT.0)I3=I2
      DURBP=I3*TINT
C
C     FIND SIGNIFICANT POINT OF RISE AND CENTROID OF PEAKS.
C
      TFLMIN=TFLMAX
      DO 1220 I=1,K1
 1220 IF(FLOWDI(I).LT.TFLMIN)TFLMIN=FLOWDI(I)
      RHTL=TFLMAX-TFLMIN
      DO 1230 I=1,K1
      IK=I
      IF((FLOWPE(I)-FLOWDI(I)).GT.0.005*RHTL)GO TO 1240
 1230 CONTINUE
 1240 NTR=KA(IK)-1
      PFL=TFL(NTR+1)
      DO 1250 I=IK,K1
      SUM1=SUM1+FLOWPE(I)*KB(I)
 1250 SUM=SUM+FLOWPE(I)
      SNPF=SUM1/SUM
      WRITE     (6,1260)SNPF
 1260 FORMAT (/,9X,' CENTROID OF PEAKS AT ',F8.2,' (INTERVALS)',/)
      WRITE     (6,1270)TFLMAX,SUMRF,DURBP
 1270 FORMAT (/10X,'BEFORE THE PEAK FLOW OF',F8.4,
     *  ' CUMECS, THERE WERE ',F5.1,' MMS. IN',F8.2,' HOURS')
C
C     CHECK THAT RISE DOES NOT OCCUR BEFORE RAIN.
C
      NRL=NTR-NH
      IF(NRL.LT.0)WRITE      (6,1280)
 1280 FORMAT (/,' *****'/' **WARNING-SELECTED RISE IS BEFORE RAIN',
     *    /' *****')
C
C     CALCULATE LAG AND END POINT.
C
      FCT=SNPF*TINT+PQ4A-TINT
      RCT=HCMRF*TINT+PQ4A-TINT
      IF(NH.GT.0) RCT=RCT+NH*TINT
      IF(NH.LT.0) FCT=FCT-NH*TINT
      IF(FCT.GE.24.0) FCT=FCT-24.0
      IF(RCT.GE.24.0) RCT=RCT-24.0
      HLAG=SNPF-NH-HCMRF
 1290 IF(HLAG.GE.0.0)GO TO 1310
      WRITE      (6,1300)
 1300 FORMAT (' *****'/' **WARNING - LAG WAS NEGATIVE :',
     * 'NOW INCREASED TO 1 INTERVAL - MUST BE RERUN'/'*****')
      HLAG=1.0
 1310 IF(HLAG.GT.37.0)WRITE      (6,1320)
 1320 FORMAT (' *****'/' **WARNING - LAG IS TOO LARGE'/
     *  ' *****')
      HLAGH=HLAG*TINT
      NEND=NH+NHRF+NSTV(4*HLAG)+1
C
C     LIMITING VALUE OF RECESSION FACTOR FOR DATA INTERVAL
C
      RL=0.995**TINT
      WRITE      (6,1330)HLAGH,RCT,FCT,HSFL,NX(3),NX(2),NX(1),NTR,NRL,PF
     *L
 1330 FORMAT (/10X,
     *'LAG TIME 1 (LT1) =',F8.2,' HOURS',/10X,
     *    '(TOTAL RAIN CENTROID AT',F8.2,' HOURS: CENTROID OF PEAKS AT',
     *    F8.2,' HOURS)'
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```
     *//10X,'TOTAL FLOW HYDROGRAPH IS AVAILABLE FROM ',F5.2,' HOURS ON '
     *,I2,'/',I2,'/',I2,'.  IT BEGAN TO RISE ',I2,' INTERVALS '/10X,
     *   'LATER WHICH IS',I3,' INTERVALS AFTER FIRST RAIN.  TOTAL',
     *   ' FLOW AT START OF RISE = ',F8.4,' CUMECS')
C
C     CHECK PRECEDING RECESSION STEEPNESS IN
C     RELATION TO EVENT SIZE AND FOR FLATNESS.
C
      PRORF=1.0
      IF(NTR.LE.1.OR.TFL(1).EQ.0.0)GO TO 1340
      PRORF=(PFL/TFL(1))**(1.0/(NTR-1))
 1340 WRITE      (6,1350)PRORF
 1350 FORMAT (/,9X,' AVERAGE PRECEDING RECESSION FACTOR =',F9.6)
      IF(PRORF.GE.RL)NESL=1
      IF(PFL*(1-PRORF**(SNPF-NTR)).GE.(TFLMAX-PFL)/10.0)WRITE      (6,13
     *60)
 1360 FORMAT (' *****'/' **WARNING-PREVIOUS RECESSION RATHER STEEP',
     *    ' IN RELATION TO SIZE OF THIS EVENT'/' *****')
      IF(NTR.LT.5.OR.NESL.EQ.1)GO TO 1390
C
C     EXTEND PRECEDING RECESSION IF THERE ARE MORE THAN 6 TERMS
C
      IAH=NTR+1
      DO 1370 I=1,IAH
 1370 OR(I)=TFL(I)
      NORTA=NTR+1
      NORT=NEND
      CALL RECEX1 (EPR)
      WRITE      (6,1380) NTR,EC
 1380 FORMAT (/10X,'PREVIOUS FLOW HAS BEEN EXTENDED: ',I2,' VALUES',
     *    ' WERE USED (EC =',F9.6,')')
      INPF=NSTV(SNPF)
      EPROP=EPR(INPF)
      GO TO 1420
 1390 WRITE      (6,1400)
 1400 FORMAT (/10X,'PREVIOUS FLOW HAS BEEN ASSUMED CONSTANT')
C
C     PRECEDING RECESSION EXTENDED AS HORIZONTAL STRAIGHT LINE
C     IF TOO FEW TERMS AVAILABLE OR IF IT IS ALMOST FLAT.
C
      DO 1410 I=1,NEND
 1410 EPR(I)=PFL
 1420 IF(NTFL.GE.NEND)GO TO 1480
C
C     FIND HOW MUCH RECESSION AT END OF EVENT.
C
      I1=0
      DO 1430 I=1,50
      IF((TFL(NTFL-I)/TFL(NTFL-I+1)).LT.1.0)GO TO 1440
      I1=I
 1430 CONTINUE
 1440 NERT=I1
      DO 1450 I=1,NERT
 1450 OR(I)=TFL(NTFL-NERT+I)
      NORTA=NERT
      NORT=NEND-NTFL+NERT
      CALL RECEX1(EER)
      IAI=NEND-NTFL
      DO 1460 I=1,IAI
 1460 TFL(NTFL+I)=EER(I+NERT)
      NEX=NEND-NTFL
      WRITE      (6,1470) NEX,NERT,EC
 1470 FORMAT (/10X,'EVENT RECESSION EXTENDED BY',I3,' INTERVALS.  ',
     *    I2,' VALUES USED FOR FITTING (EC =',F9.6,')')
      GO TO 1500
 1480 NRED=NTFL-NEND
      WRITE      (6,1490) NRED
 1490 FORMAT (/10X,I2,' INTERVALS IGNORED ON EVENT RECESSION')
C
C     SEPARATE TOTAL FLOW FROM PREVIOUS FLOW. IF EVENT RECESSION
C     FALLS BELOW PREVIOUS FLOW EXTENSION, END POINT BROUGHT FORWARD.
C
 1500 IAJ=NTR+1
      DO 1510 I=IAJ,NEND
      IN=I
      SFL(I)=TFL(I)-EPR(I)
      IF(SFL(I).LT.0.0.AND.I.GT.SNPF)GO TO 1520
 1510 CONTINUE
      GO TO 1540
 1520 NEND=IN
      SFL(NEND)=0.0
      NEX=NEND-NTFL
      WRITE      (6,1530) NEX
 1530 FORMAT (10X,'BUT ONLY ',I3,' INTERVALS EXTRA AS RECESSIONS MEET')
 1540  IF(NTR.EQ.0) NTR=1
      IF(NTR.EQ.1)GO TO 1560
      DO 1550 I=1,NTR
 1550 SFL(I)=-1.0
 1560 SLORPK=SFL(NEND)
      WRITE      (6,1570) SLORPK
 1570 FORMAT (/10X,'EVENT FLOW MINUS EXTENDED PREVIOUS FLOW AT END =',
     *     F10.4,' CUMECS')
      IF(SLORPK.LE.0.0)GO TO 1590
C
C     SEPARATED FLOW ADJUSTED BY FURTHER REDUCTION FROM
C     CENTROID OF PEAKS ONWARDS SO THAT IT IS ZERO AT END POINT.
C
      SFLN=SFL(NEND)
      IAK=SNPF+1
      DO 1580 I=IAK,NEND
      SUBFL=(I-SNPF-1)/(NEND-SNPF-1)*SFLN
 1580 SFL(I)=SFL(I)-SUBFL
C
C     VOLUME OF SEPARATED FLOW EXPRESSED
C     IN TERMS OF MMS OVER CATCHMENT.
C
 1590 TVOL=0.0
      IAL=NTR+1
      DO 1600 I=IAL,NEND
      TVOL=TVOL+TFL(I)
 1600 VOL=VOL+SFL(I)
      ANRFL=TVOL-VOL
      AVERNS=ANRFL/(NEND-NTR)
      EYEY1=100.0*(ANRFL/TVOL)
      EYEY2=100.0*(VOL/TVOL)
      XTINT=TINT*3600.0
      TVOL1=TVOL*XTINT
      VOL1=VOL*XTINT
      ANRFL1=ANRFL*XTINT
      WRITE      (6,1610)  TVOL1,VOL1,ANRFL1,AVERNS,EYEY1,EYEY2
 1610 FORMAT(/10X,'TOTAL FLOW(TFL), RESPONSE FLOW(SFL) AND NON-RESPONSE
     *FLOW(NRFL) OVER THE RESPONSE PERIOD.',
     */10X,'VOLUME OF TFL    =',F14.4,' CUBIC METRES',
     */10X,'VOLUME OF SFL    =',F14.4,' CUBIC METRES',
     */10X,'VOLUME OF NRFL   =',F14.4,' CUBIC METRES',
     */10X,'MEAN NRFL        =',F10.4,' CUMECS',
     */10X,'VOL NRFL/VOL TFL=',F6.2,' %',
     */10X,'VOL SFL/VOL TFL =',F6.2,' %')
C
C     CALCULATE LAG2: CENTROID TOTAL RF. TO CENTROID TOTAL FLOW.
C
      EYEY3=0.0
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```
      EYEY4=0.0
      DO 1620 I=1,NEND
      EYEY3=EYEY3+TFL(I)
 1620 EYEY4=EYEY4+(TFL(I)*I)
      SNPF2=EYEY4/EYEY3
      EYEY5=(SNPF2*TINT)+PQ    INT
      IF(NH.LT.0) EYEY5=EYE    H*TINT
      IF(EYEY5.GE.24.0) EYEY5=EYEY5-24.0
      EYEY6=(SNPF2-NH-HCMRF)*TINT
      WRITE    (6,1630) EYEY6,RCT,EYEY5
 1630 FORMAT(/10X,'LAG TIME 2 (LT2) =',F6.2,' HOURS',
     */10X,'(TOTAL RAIN CENTROID AT',F6.2,' HOURS; CENTROID OF TOTAL',
     *' FLOW AT',F6.2,' HOURS)')
      VOL=VOL*TINT*3.6/AREA
      NSFL=NEND
      RETURN
      END
      SUBROUTINE RECEX1 (QRP)
C
C     ANALYSES AND EXTENDS A RECESSION
C     CURVE TIME VARIANT LINEAR RESERVIOR.
C
      DIMENSION QRP(400)
      COMMON/D/EC,CRO,CRT,NORT,NORTA,RL,QR(50),NEND
      COMMON/INNER/NC,T(250),Y(250)
 1640 NC=NORTA-1
      HMF=1.0/3.0
      IAM=NC-1
      DO 1660 I=2,IAM
      R=(QR(I+2)/QR(I-1))**(HMF)
 1650 Y(I)=RL-R
 1660 CONTINUE
      Y(1)=RL-(QR(2)*QR(3)/QR(1)/QR(1))**HMF
      Y(NC)=RL-(QR(NC)*QR(NC)/QR(NC-1)/QR(NC-2))**HMF
      CALL REMOM(CY,A)
      EC=EXP(-CY)
 1670 CRO=A
 1680 IF(EC.LT.1.0)GO TO 1690
      EC=1.0
      CRO=RL-(QR(NORTA)/QR(1))**(1.0/NC)
 1690 RT=RL-CRO
      QRP(1)=QR(1)
      DO 1700 I=2,NORTA
      QRP(I)=QRP(I-1)*RT
 1700 RT=RL-(RL-RT)*EC
      QRP(NORTA)=QR(NORTA)
      IAN=NORTA+1
      DO 1710 I=IAN,NORT
      QRP(I)=QRP(I-1)*RT
 1710 RT=(RL-(RL-RT)*EC)
      RETURN
      END
      SUBROUTINE REMOM(CY,A)
C
C     FOR ESTIMATION BY MOMENTS OF PARAMETERS IN Y=A*EXP(-CY*T)
C
      COMMON/INNER/NC,T(250),Y(250)
      SY=0.0
      SYT=0.0
      SYS=0.0
      DO 1720 I=1,NC
      SY=SY+Y(I)
      SYT=SYT+Y(I)*(I-1)
 1720 SYS=SYS+Y(I)*Y(I)
      AMO=SY-(Y(1)+Y(NC))/2.0
      AM1Y=SYT+Y(1)/8-Y(NC)*((NC-1)/2+0.125)
      C1Y=AMO/AM1Y
      CY=C1Y
      IF(CY.GE.0.0)GO TO 1730
      I=0
      GO TO 1750
 1730 DO 1740 I=1,50
      PCY=CY
      CYNC=CY*(NC-1)
      EX=EXP(-CYNC)
      CY=C1Y*(1-EX*(1+CYNC))/(1-EX)
      IF(ABS(PCY-CY).LT.0.001)GO TO 1750
 1740 CONTINUE
 1750 A=CY*AMO/(1-EXP(-CY*(NC-1)))
      WRITE     (6,1760) C1Y,CY,I
 1760 FORMAT (/,9X,' RECESSION FIT:-'/
     *9X,' FIRST ESTIMATE OF DECAY CONSTANT = ',F10.6/
     *9X,' FINAL ESTIMATE OF DECAY CONSTANT = ',F10.6/
     *9X,' ITERATIONS REQUIRED:- ',I2)
      RETURN
      END
      SUBROUTINE ANTECE .
C
C     CALCULATES CWI AT START OF RAIN.
C
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/K/NARD,IDS,IMS,IYS,NOFSTN,NUMSTN(3),SMD
      COMMON/JADE/ NX(4),HSFL,HSRF,PQ4A,PQ4B
      API5=0.0
C
C     API5  AT 0900 HRS. ON DAY RAIN STARTS
C     AND API5S AT START OF STORM.
C
      API5=0.5**0.5*((ANTE(NARD)+0.5*ANTE(NARD-1))+(0.5**2*ANTE(NARD-2))
     *       +(0.5**3*ANTE(NARD-3))+(0.5**4*ANTE(NARD-4)))
C
C     AVERAGE SMD VALUES AT NEAREST GAUGES ARE CORRECTED
C     FOR RAINFALL AND EVAPORATION OCCURING AFTER 9.00 AM.
C
C     SMDS IS THE SMD AT START OF STORM
C
      WRITE     (6,1770) SMD
 1770 FORMAT(/10X,'SMD AT 0900 ON EVENT DATE =',F6.1,' MMS.')
      IF(NOFSTN.EQ.0)GO TO 1790
      WRITE     (6,1780)  (NUMSTN(I),I=1,NOFSTN)
 1780 FORMAT(10X,'(STATION NO.',I10,')'/)
 1790 CONTINUE
      SMDA=SMD-RP9BS
      GO TO  (1850,1850,1820,1820,1800,1800,1800,1800,1820,1820,1850,1850
     *),IMS
 1800 TEVAP=HSRF-9
      IF(TEVAP.GE.9)GO TO 1810
      SMDS=SMDA+(TEVAP*0.4)
      GO TO 1860
 1810 SMDS=SMDA+(9*0.4)
      GO TO 1860
 1820 TEVAP=HSRF-10
      IF(TEVAP.GE.6)GO TO 1840
      IF(TEVAP.LE.0)GO TO 1830
      SMDS=SMDA+(TEVAP*0.2)
      GO TO 1860
 1830 SMDS=SMDA
      GO TO 1860
 1840 SMDS=SMDA+(6*0.2)
 1850 SMDS=SMDA
 1860 CONTINUE
      IF(SMDS.LT.0.0) SMDS=0.0
```

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      API5S=((API5+RP9BS)*0.5**((HSRF-9.0)/24.0))*SQRT(0.5)
C
C     CATCHMENT WETNESS INDEX AT START OF STORM.
C
      CWI=ISM-SMDS+API5S
      IF(CWI.LE.5.0) CWI=5.0
      WRITE     (6,1870) CWI,SMDS,API5S
 1870 FORMAT(/10X,'CATCHMENT WETNESS INDEX AT START OF RAIN = ',F8.2,
     *  '  (SMD =',F7.2,' MMS; API5 =',F6.2,')')
C
C     NARD DAY API.
C
      API28=0.0
      DO 1880 I=1,NARD
      IN=NARD-I+1
 1880 API28=API28*0.9+ANTE(IN)
      WRITE (6,1890) NARD,API28
 1890 FORMAT(/,10X,I2,' DAY API  (DECAY CONSTANT OF 0.9) =',F6.2)
      RETURN
      END
      SUBROUTINE RAINSP
C
C     EFFECTIVE RAINFALL SEPARATION:
C     (A) CONSTANT LOSS PERCENTAGE  (ERF1).
C     (B) LOSS RATE CURVE  (ERF2).
C     (C) CONSTANT LOSS PERCENTAGE VARYING WITH CWI  (ERF3).
C     (D) UNIFORM LOSS RATE, THE PHI INDEX  (ERF4).
C
      INTEGER OSMRF
      DIMENSION SHRF(150),K(150),ALT(150),ACC(150)
      COMMON/A/NHRF,HRF(150),SR,TFL(400),NTFL,BDR,NRL,SFL(400),VOL,NSFL,
     *          EPR(400),NH,TINT2,PIRF
      COMMON/B/SOIL(60),CWI,SMDS,API5S,RP9BS,ANTE(30),N(8),IEN,ICN,ISM
      COMMON/C/OSMRF,ERF1(150),ERF2(150),ERF3(150),TINT,CAP,AREA,AEX,
     *ISKIP,ERF4(150)
C
C     FIND TOTAL LOSS.
C
      ALOSS=SR-VOL
      IF(ALOSS) 1900,1900,1920
 1900 WRITE      (6,1910)
 1910 FORMAT('*****'/'**WARNING - RAINFALL DOES NOT EXCEED RUNOFF'
     *  /'*****')
      BDR=-1.0
      RETURN
 1920 PRO=VOL/SR*100.0
C
C     SMOOTH RAINFALL VALUES IF REQUIRED.
C
      IF(OSMRF.EQ.0)GO TO 1960
      SHRF(1)=(2*HRF(1)+HRF(2))/3.0
      SHRF(NHRF)=(2*HRF(NHRF)+HRF(NHRF-1))/3.0
      IAO=NHRF-1
      DO 1930 I=2,IAO
 1930 SHRF(I)=(HRF(I-1)+HRF(I)+HRF(I+1))/3.0
      DO 1940 I=1,NHRF
 1940 HRF(I)=SHRF(I)
      WRITE      (6,1950)
 1950 FORMAT(/10X,'RAINFALL VALUES HAVE BEEN SMOOTHED'/)
 1960 CONTINUE
      DO 1970 I=1,NHRF
 1970 ERF1(I)=PRO*HRF(I)/100.0
C
C     DETERMINE SHAPE OF LOSS CURVE
C
 1980 AK=0.5**(TINT/24.0)
      AK2=0.5**(TINT/48.0)
      IF(AEX.LT.0.0) AEX=(250.0-CWI)/50.0
      SACC=1.0/CWI**AEX
      ACC(1)=SACC
      S=SMDS
      AA=API5S
      SUMRO=0.0
      HRF(NHRF+1)=0.
      DO 1990 I=1,NHRF
      S=S-HRF(I)
      IF(S.LT.0)S=0.0
      AA=(HRF(I)*AK2)+(AA*AK)
      CWIR=ISM-S+AA
      IF(CWIR.LE.5.0)CWIR=5.0
      ACC(I+1)=1.0/(CWIR)**AEX
      SUMRO=SUMRO+HRF(I+1)/ACC(I+1)
 1990 SACC=SACC+ACC(I+1)
      PF=VOL/(SUMRO+HRF(1)*CWI)
      CWIEND=ISM-S+AA
C
C     ADJUST INCREMENTAL LOSSES.
C
      BLOSS=ALOSS
      RLC1=((BLOSS/SACC)**(1.0/AEX))/TINT
      SALT=0.0
      ITERS=0
      DO 2000 I=1,NHRF
      ALT(I)=0.0
 2000 K(I)=0
 2010 DO 2020 I=1,NHRF
      ARF=(100-CAP)/100.0*HRF(I)
      IF(K(I).EQ.1)GO TO 2020
      ALT(I)=ACC(I)*BLOSS/SACC+ALT(I)
      IF(ARF.GT.ALT(I))GO TO 2020
      ALT(I)=ARF
      K(I)=1
 2020 SALT=SALT+ALT(I)
      RESID=ALOSS-SALT
      IF(RESID.LT.0.0001)GO TO 2060
      IF(ITERS.LE.100)GO TO 2040
      WRITE      (6,2030)
 2030 FORMAT(/10X,'100 ITERATIONS IN LOSS CURVE - PROGRAM STOPPED')
      STOP
 2040 BLOSS=RESID
      SACC=0.0
      SALT=0.0
      DO 2050 I=1,NHRF
      IF(K(I).EQ.1)GO TO 2030
      SACC=SACC+ACC(I)
 2050 CONTINUE
      ITERS=ITERS+1
      GO TO 2010
 2060 DO 2070 I=1,NHRF
 2070 ERF2(I)=HRF(I)-ALT(I)
      DO 2080 I=1,NHRF
      IF(K(I).NE.1)GO TO 2090
 2080 CONTINUE
 2090 RLC2=((ALT(I)/ACC(I))**(1.0/AEX))/TINT
      ROL1=RLC2/CWI
      ROL2=RLC2/CWIEND
 2100 WRITE      (6,2110)VOL,ALOSS,PRO,CWIEND
 2110 FORMAT(/10X,'VOLUME OF RESPONSE RUNOFF IS THE EQUIVALENT OF ',
     *     F6.2,' MMS OVER THE CATCHMENT'/14X,'LOSS = ',F6.2,' MMS'/
     *10X,'RESPONSE = ',F6.2,' %'//10X,'CATCHMENT WETNESS INDEX AT',
     *     ' END OF RAIN = ',F6.2)
      WRITE      (6,2120) RLC1,RLC2,ROL1,ROL2
```

```
2120 FORMAT(/10X,'INITIAL LOSS CURVE MULTIPLYING FACTOR =',F10.3,
    *    /10X,'FINAL LOSS CURVE MULTIPLYING FACTOR   =',F10.3
    *    /10X,'STARTING LOSS RATE = ',F10.3,' MMS/HOUR',
    *    /10X,'   FINAL LOSS RATE = ',F10.3,' MMS/HOUR')
2130 ERF3(1)=PF*CWI*HRF(1)
     DO 2140 I=2,NHRF
2140 ERF3(I)=PF*HRF(I)/ACC(I)
     PF=PF*100
     WRITE     (6,2150) PF
2150 FORMAT(/10X,'METT RAIN (3) DETERMINED ON BASIS OF % RUNOFF AND',
    *' CWI TIMES ',F7.4)
C
C     CALCULATE THE 'PHI' INDEX.
C
     DO 2160 I=1,NHRF
2160 ERF4(I)=HRF(I)
     JEYE=0
     DO 2170 I=1,NHRF
2170 IF(HRF(I).EQ.0)JEYE=JEYE+1
     EYE3=ALOSS/(NHRF-JEYE)
2180 EYE1=0.0
     DO 2200 I=1,NHRF
     IF(ERF4(I).LE.0)GO TO 2200
     IF(EYE3-ERF4(I)) 2200,2200,2190
2190 EYE1=EYE1+(EYE3-ERF4(I))
     ERF4(I)=-1.0
     JEYE=JEYE+1
2200 CONTINUE
     EYE2=EYE1/(NHRF-JEYE)
     PHI=EYE2+EYE3
     EYE3=PHI
     IF(EYE1.GT.0.0001)GO TO 2180
     DO 2210 I=1,NHRF
     IF(HRF(I).GT.PHI) ERF4(I)=HRF(I)-PHI
2210 IF(HRF(I).LE.PHI) ERF4(I)=0.0
     WRITE     (6,2220) PHI
2220 FORMAT(/10X,'VALUE OF PHI INDEX = ',F7.4,' MM.')
     RETURN
     END
```

Appendix 3: Source listing of LPROG6

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C
C   LPROG6: THE INSTITUTE OF HYDROLOGY, WALLINGFORD, U.K., RETAINS
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C
      COMMON/A/NHRF,ERF(375),NSFL,SFL(375),UT(150),UTA(150),VOL,NUT3,
     *          TINT,AREA,L
      DIMENSION IRFT(4)
   10 READ (5,20,END=250) ICN,IEN
   20 FORMAT(2I5)
      DO 30 I=1,375
   30 SFL(I)=0.0
      IOPT=3
      READ (5,40) IS1,IS2,IS3,IS4,IS5,IS6,ISW1,ISW2,(IRFT(I),I=1,4)
   40 FORMAT(12(I1,1X))
      IF(IS1.EQ.1) IOPT=1
      IF(IS2.EQ.1) IOPT=2
      IF(IS3.EQ.1) IOPT=3
      READ (5,50) AREA,TINT
   50 FORMAT(F8.2,F7.2)
      READ (5,60) NSFL
   60 FORMAT(I3)
      NSFL5=NSFL+5
      READ (5,70) (SFL(I),I=6,NSFL5)
   70 FORMAT(8F10.4)
      NSFL=NSFL+5
      IAB=NSFL+1
      IAC=NSFL+11
      DO 80 I=IAB,IAC
   80 SFL(I)=0.0
      NSFL=NSFL+10
      VOL=100.0/(0.36*TINT)
      NIRFT=0
      DO 90 I=1,4
   90 IF(IRFT(I).GT.0) NIRFT=NIRFT+1
      DO 240 LI=1,NIRFT
      READ (5,100) NHRF,(ERF(I),I=1,NHRF)
  100 FORMAT(I3/(8F10.4))
      ISTEVE=IRFT(LI)
      NUT2=2*NHRF
      NUT3=0
      IF(NSFL.GE.NUT2)GO TO 110
      NUT3=NUT2-NSFL+1
      L=NSFL-NHRF+NUT3
      GO TO 120
  110 L=NSFL-NHRF+1
  120 CONTINUE
      IF(L.GT.148)GO TO 240
      IF(IS5.EQ.0)GO TO 140
      WRITE (6,130) ICN,IEN
  130 FORMAT(1H1,//,' NASH CASCADE MODEL FOR CATCHMENT NUMBER',I6,', EVE
     *NT NUMBER',I4,'.',/,32('—'))
      CALL NASH(TINT,NHRF,ERF,NSFL,SFL,VOL,AREA,ISTEVE)
  140 IF(IOPT.EQ.2)GO TO 200
      CALL UHDHAR
      IF(ISW1.EQ.0)GO TO 170
      WRITE (6,150) ICN,IEN
  150 FORMAT(1H1,//,' UNIT HYDROGRAPH DERIVATION FOR CATCHMENT NUMBER',
     *I6,', EVENT',I4,'.',/,13('———'))
      WRITE (6,160)
  160 FORMAT(//,' TUH DERIVED BY HARMONIC ANALYSIS WITHOUT SMOOTHING.')
      CALL CONVOL(UTA,NHRF,ERF,NSFL,SFL,NUT3,IS4,IS6,TINT,AREA,L,ISTEVE)
  170 IF(ISW1.EQ.1)GO TO 190
      WRITE (6,150) ICN,IEN
      WRITE (6,180)
  180 FORMAT(//,' TUH DERIVED BY HARMONIC ANALYSIS SMOOTHED BY A MOVING
     *AVERAGE FILTER.')
      CALL CONVOL(UT,NHRF,ERF,NSFL,SFL,NUT3,IS4,IS6,TINT,AREA,L,ISTEVE)
  190 IF(IOPT.EQ.1)GO TO 240
  200 CALL UHDMAT
      IF(ISW2.EQ.0)GO TO 220
      WRITE (6,150) ICN,IEN
      WRITE (6,210)
  210 FORMAT(//,' TUH DERIVED BY MATRIX INVERSION WITHOUT SMOOTHING.')
      CALL CONVOL(UTA,NHRF,ERF,NSFL,SFL,NUT3,IS4,IS6,TINT,AREA,L,ISTEVE)
  220 IF(ISW2.EQ.1)GO TO 240
      WRITE (6,150) ICN,IEN
      WRITE (6,230)
  230 FORMAT(//,' TUH DERIVED BY MATRIX INVERSION SMOOTHED BY A MOVING A
     *VERAGE FILTER.')
      CALL CONVOL(UT,NHRF,ERF,NSFL,SFL,NUT3,IS4,IS6,TINT,AREA,L,ISTEVE)
  240 CONTINUE
      GO TO 10
  250 WRITE (6,260)
  260 FORMAT(///////,' END OF THE JOB.')
      STOP
      END.
      SUBROUTINE UHDHAR
C
C         DERIVES UNIT HYDROGRAPH BY HARMONIC ANALYSIS. THE NUMBER OF
C         PAIRS OF COEFFICIENTS CONSIDERED IS RESTRICTED WITH THE
C         AIM OF BUILDING IN A SMOOTHING EFFECT.
C
      COMMON/A/NHRF,ERF(375),NSFL,SFL(375),UT(150),UTA(150),VOL,NUT3,
     *          TINT,AREA,L
      DIMENSION CC1(150),CC2(150),SC1(150),SC2(150),CUT(150),SUT(150),
     *          C(375),S(375)
      NCS=(NSFL-NHRF+1)/2
      N3=1.25*NSFL
      IF(N3/2*2.NE.N3) N3=N3+1
  270 PI=3.1415926536
      EEE=2.0*PI/N3
      ARG=0.0
      DO 280 J=1,N3
      C(J)=COS(ARG)
      S(J)=SIN(ARG)
  280 ARG=ARG+EEE
      CALL HARM(ERF,NHRF,CC1,SC1,NCS,C,N3,S)
      CALL HARM(SFL,NSFL,CC2,SC2,NCS,C,N3,S)
      EEE=2.0/N3
      CUT(1)=EEE*CC2(1)/(2.0*CC1(1))
      SUT(1)=0.0
      DO 290 I=2,NCS
      G=EEE/(CC1(I)*CC1(I)+SC1(I)*SC1(I))
      CUT(I)=G*(CC1(I)*CC2(I)+SC1(I)*SC2(I))
  290 SUT(I)=G*(CC1(I)*SC2(I)-SC1(I)*CC2(I))
      UTSUM=0.0
      DO 310 I=1,L
      UTA(I)=CUT(1)
      J=I
      DO 300 K=2,NCS
      UTA(I)=UTA(I)+CUT(K)*C(J)+SUT(K)*S(J)
      J=J+I-1
      IF(NSFL.GE.J)GO TO 300
      J=J-NSFL
  300 CONTINUE
      UTSUM=UTSUM+UTA(I)
  310 CONTINUE
      DO 320 I=1,5
  320 UTSUM=UTSUM-UTA(I)
```

```
      COR=VOL/UTSUM
      DO 330 I=1,L
  330 UTA(I)=UTA(I)*COR
C
C     SMOOTHING OF THE U.H.
C
      UT(1)=(6*UTA(1)+2*UTA(2)+UTA(3))/9.0
      UT(2)=(3*UTA(1)+3*UTA(2)+2*UTA(3)+UTA(4))/9.0
      UT(L-1)=(3*UTA(L)+3*UTA(L-1)+2*UTA(L-2)+UTA(L-3))/9.0
      UT(L)=(6*UTA(L)+2*UTA(L-1)+UTA(L-2))/9.0
      IAG=L-2
      DO 340 I=3,IAG
  340 UT(I)=(UTA(I-2)+2*UTA(I-1)+3*UTA(I)+2*UTA(I+1)+UTA(I+2))/9.0
      SS2=0.0
      DO 350 I=6,L
  350 SS2=SS2+UT(I)
      COR2=VOL/SS2
      DO 360 I=1,L
  360 UT(I)=UT(I)*COR2
      RETURN
      END
      SUBROUTINE HARM(DATA,M,CC,SC,NCS,C,N3,S)
C
C        EVALUATES THE FIRST 'NCS' PAIRS OF HARMONIC COEFFICIENTS.
C
      DIMENSION DATA(375),CC(100),SC(100),C(375),S(375)
      DO 380 I=1,NCS
      SSUM=0.0
      CSUM=0.0
      K=1
      DO 370 J=1,M
      CSUM=CSUM+DATA(J)*C(K)
      SSUM=SSUM+DATA(J)*S(K)
      K=K+I-1
      IF(K.LE.N3)GO TO 370
      K=K-N3
  370 CONTINUE
      CC(I)=CSUM*2.0/N3
  380 SC(I)=SSUM*2.0/N3
      CC(1)=0.5*CC(1)
      SC(1)=0.0
      RETURN
      END
      SUBROUTINE UHDMAT
C
C        DERIVES UNIT HYDROGRAPH BY MATRIX INVERSION. NET RAINFALL
C        INCREMENTS ARE FORMED INTO A SYMMETRIC POSITIVE MATRIX WHICH
C        IS THE LHS OF THE NORMAL EQUATIONS IN THE LEAST SQUARES
C        SOLUTION OF THE CONVOLUTED SUMMATION. THE MATRIX IS, FOR THE
C        SAKE OF EFFICIENCY, STORED IN A COLUMN VECTOR. SO IS THE RHS
C        (INPUT/OUTPUT CROSS PRODUCTS) OF THE NORMAL EQUATIONS.
C        SOLUTION IS BY DECOMPOSITION OF THE UPPER TRIANGLE AND A
C        FORWARD AND BACKWARD PROCEDURE. THE UH IS SMOOTHED BY TWO
C        PASSES OF A SIMPLE MOVING AVERAGE FILTER.
C
      COMMON/A/NHRF,ERF(375),NSFL,SFL(375),S2DUH(150),DUH(150),VOL,
     *      NUT3,TINT,AREA,L
      DIMENSION AR(11000)
C
C        PREPARE LHS AND RHS.
C
      DO 390 I=1,11000
  390 AR(I)=0.0
      KS=1
      DO 420 I=1,NHRF
      AI=0.0
      IAD=NHRF-I+1
      DO 400 J=1,IAD
      JI=J+I-1
  400 AI=AI+ERF(J)*ERF(JI)
      KI=0
      IAE=L-I+1
      DO 410 J=1,IAE
      JK=KS+KI
      AR(JK)=AI
  410 KI=KI+I+J
  420 KS=KS+I
      DO 440 I=1,L
      AI=0.0
      DO 430 J=1,NHRF
      JI=J+I-1
  430 AI=AI+ERF(J)*SFL(JI)
  440 DUH(I)=AI
C
C        DECOMPOSE UPPER TRIANGLE.
C
      II=1
      DO 520 I=1,L
      AII=AR(II)
      K1=II-I+1
      K4=II-1
      IF(K4.LT.K1)GO TO 480
      DO 450 KI=K1,K4
      AKI=AR(KI)
  450 AII=AII-AKI*AKI
  460 IF(AII.GE.0.0)GO TO 480
      WRITE  (6,470)
  470 FORMAT(/' MATRIX DECOMPOSITION FAILURE')
      RETURN
  480 AII=SQRT(AII)
      AR(II)=AII
      IJ=II+I
      IAF=I+1
      DO 510 J=IAF,L
      AIJ=AR(IJ)
      KJ=IJ-I+1
      IF(K4.LT.K1)GO TO 500
      DO 490 KI=K1,K4
      AIJ=AIJ-AR(KI)*AR(KJ)
  490 KJ=KJ+1
  500 AR(IJ)=AIJ/AII
  510 IJ=IJ+J
  520 II=II+I+1
C
C        FORWARD SOLUTION.
C
      II=1
      DO 550 I=1,L
      SUM=DUH(I)
      I1=I-1
      IF(I1.LT.1)GO TO 540
      DO 530 J=1,I1
  530 SUM=SUM-DUH(I1-J+1)*AR(II-J)
  540 DUH(I)=SUM/AR(II)
  550 II=II+I+1
C
C        BACK SUBSTITUTION.
C
      K=II-1
      II=II-L-1
      DO 580 I=1,L
      I3=L-I+1
```

```
      SUM=DUH(I3)
      I2=I3+1
      IF(L.LT.I2)GO TO 570
      DO 560 J=I2,L
      SUM=SUM-DUH(J)*AR(K)
  560 K=K+J
  570 DUH(I3)=SUM/AR(II)
      K=II-1
  580 II=II-I3
C
C        VOLUME ADJUSTMENT AND SMOOTHING OF U.H.
C
      S2DUH(1)=(6*DUH(1)+2*DUH(2)+DUH(3))/9.0
      S2DUH(2)=(3*DUH(1)+3*DUH(2)+2*DUH(3)+DUH(4))/9.0
      S2DUH(L-1)=(3*DUH(L)+3*DUH(L-1)+2*DUH(L-2)+DUH(L-3))/9.0
      S2DUH(L)=(6*DUH(L)+2*DUH(L-1)+DUH(L-2))/9.0
      IAG=L-2
      DO 590 I=3,IAG
  590 S2DUH(I)=(DUH(I-2)+2*DUH(I-1)+3*DUH(I)+2*DUH(I+1)+DUH(I+2))/9.0
      SS=0.0
      SS2=0.0
      DO 600 I=6,L
      SS=SS+DUH(I)
  600 SS2=SS2+S2DUH(I)
      COR=VOL/SS
      COR2=VOL/SS2
      DO 610 I=1,L
      DUH(I)=DUH(I)*COR
  610 S2DUH(I)=S2DUH(I)*COR2
      RETURN
      END
      SUBROUTINE CONVOL(UNIT,NHRF,ERF,NSFL,SFL,NUT3,IS4,IS6,TINT,AREA,L,
     *ISTEVE)
C
C        CONVOLUTES A UNIT HYDROGRAPH WITH A NET RAINFALL
C        SEQUENCE AND COMPARES WITH ORIGINAL NET HYDROGRAPH.
C
      DIMENSION UNIT(150),RSFL(375),JRR(3),KRR(10),ERF(375),SFL(375)
      INTEGER LINE(112),MARK,SPACE,CROSS
      DATA MARK/'*'/,SPACE/' '/,CROSS/'+'/
      NSTV(X)=INT(X+SIGN(0.5,X))
      NSFL2=NSFL+NUT3
      WRITE (6,620) TINT,(UNIT(I),I=1,L)
  620 FORMAT(///,' 10 MM. ',F4.2,' HOUR 100 SQ KM TUH:',/,5F12.5,/,(10F1
     *2.5))
      UHM=UNIT(6)
      IJM=6
      DO 630 IJ=7,L
      IF(UNIT(IJ).LE.UHM)GO TO 630
      UHM=UNIT(IJ)
      IJM=IJ
  630 CONTINUE
      TP=(IJM-6)*TINT
      WRITE (6,640) UHM,TP
  640 FORMAT(//,' TUH QP =',F10.5,' CUMECS',/,' TUH TP =',F7.2,4X,'HOURS
     *')
      IF(IJM.EQ.6)GO TO 660
      CURV=(UNIT(IJM+1)+UNIT(IJM-1)-2.0*UHM)/TINT**2
      WRITE (6,650) CURV
  650 FORMAT(' CURVATURE AROUND PEAK OF TUH =',F10.4)
  660 CONTINUE
      WRITE (6,670) ISTEVE
  670 FORMAT(/,' RAINFALL SEPARATION BY METHOD (',I1,').')
      DO 680 I=1,375
  680 RSFL(I)=0.0
      DO 690 I=1,NHRF
      DO 690 J=6,L
      IJ=I+J-1
  690 RSFL(IJ)=RSFL(IJ)+ERF(I)*UNIT(J)*AREA/1000.0
      IF(IS4.EQ.0)GO TO 710
      WRITE (6,700) (RSFL(I),I=6,NSFL2)
  700 FORMAT(//,' PREDICTED RESPONSE RUNOFF:',/,(10F12.4))
  710 CALL OBERFN(NSFL,SFL,RSFL)
      DSQ=0.0
      DO 720 I=6,NSFL2
      D=RSFL(I)-SFL(I)
  720 DSQ=DSQ+D*D
      DSQ=SQRT(DSQ/NSFL)
      WRITE (6,730) DSQ
  730 FORMAT(' 6. RMS ERROR =',F11.7)
      IF(IS6.EQ.0)GO TO 830
      IRAN=500
      DO 740 I=1,L
      IF(UNIT(I).GT.8.0.AND.IRAN.EQ.500) IRAN=100
      IF(UNIT(I).GT.43.0.AND.IRAN.EQ.100) IRAN=50
      IF(UNIT(I).GT.86.0.AND.IRAN.GE.50) IRAN=25
  740 CONTINUE
      IF -(IRAN.EQ.100)GO TO 760
      DO 750 I=1,L
  750 UNIT(I)=UNIT(I)*IRAN/100.0
  760 WRITE  (6,770) IRAN
  770 FORMAT(//,55X,' CUMECS PER',I4,' SQ. KM.')
      WRITE  (6,780)
  780 FORMAT(1X,
     *   '     -10.0      -5.0       0.0       5.0      10.0',
     *   '      15.0      20.0      25.0      30.0      35.0',
     *   '      40.0      45.0')
      DO 790 I=1,112
  790 LINE(I)=SPACE
      DO 820 I=1,L
      LINE(20)=CROSS
      TIME=(I-6)*TINT
      J=NSTV(2*UNIT(I))+20
      IF(J.GT.0.AND.J.LE.110) LINE(J)=MARK
      WRITE  (6,800) TIME,LINE
  800 FORMAT(1X,F6.2,'-  +',112A1)
      IF(TIME.EQ.0.0)WRITE (6,810)
  810 FORMAT(1H+,19X,'+',19X,9('+',9X))
      IF(J.GT.0.AND.J.LE.110) LINE(J)=SPACE
  820 CONTINUE
  830 CONTINUE
      RETURN
      END
      SUBROUTINE NASH (NUHT,NHRF,ERF,NSFL,SFL,VOL,AREA,ISTEVE)
C
C        FINDS THE PARAMETERS OF A NASH 10MM NUHT-HR, UH.
C
      DIMENSION SFL(375),ERF(375)
      REAL NUHT
      S1=0.0
      S2=0.0
      S3=0.0
      DO 840 I=1,NSFL
  840 S1=S1+SFL(I)*(I-1)
      DO 850 I=1,NHRF
      S3=S3+ERF(I)
  850 S2=S2+ERF(I)*(I-0.5)
      ERF1=S2/S3
      VOLR=S3*VOL*AREA/1000.0
      SFL1=S1/VOLR
      U1=SFL1-ERF1-5
      S1=0.0
```

```
      S2=0.0
      DO 860 I=1,NSFL
      I2=I-1-SFL1
  860 S1=S1+SFL(I)*I2*I2
      SFL2=S1/VOLR
      DO 870 I=1,NHRF
      I2=I-0.5-ERF1
  870 S2=S2+ERF(I)*I2*I2
      ERF2=S2/S3
      U2=SFL2-ERF2
      AN=U1*U1/U2
      AN1=AN-1
      AK=U1/AN
      XARG=NUHT/(AK*AN1)
      TTP=NUHT/(1.-EXP(-XARG))
      WRITE  (6,880) SFL1,ERF1,U1,SFL2,ERF2,U2,AN,AK,NUHT,TTP,ISTEVE
  880 FORMAT(/////' FIRST MOMENTS ABOUT ORIGIN:',
     * /'      RESPONSE RUNOFF',F13.4,
     * /'      NET RAIN       ',F13.4,
     * /'      DIFFERENCE     ',F13.4,
     * //' SECOND MOMENTS ABOUT CENTROID :',
     * /'      RESPONSE RUNOFF',F13.4,
     * /'      NET RAIN       ',F13.4,
     * /'      DIFFERENCE     ',F13.4,
     * //' NASH SEQUENTIAL RESERVOIRS MODEL :',
     * /'     N =',F12.4,'      K =',F12.4,
     * //' TIME TO PEAK OF NASH  ',F4.2,' HOUR UH =',F6.2,' HOURS',
     *//' RAINFALL SEPARATION BY METHOD (',I1,').')
      RETURN
      END
      SUBROUTINE OBERFN(NY,Y,PY)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(375),PY(375)
      EOO=0.0
      DO 890 I=6,NY
  890 EOO=EOO+Y(I)
      PKY=0.0
      PKPY=0.0
      DO 910 I=6,NY
      IF(PY(I).LT.PKPY)GO TO 900
      PKPY=PY(I)
      TPPY=(I-6)*0.5
  900 IF(Y(I).LT.PKY)GO TO 910
      PKY=Y(I)
      TPY=(I-6)*0.5
  910 CONTINUE
      X=0.0
      DO 920 I=6,NY
  920 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EOO))*100.0
      X=0.0
      DO 930 I=6,NY
  930 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EOO))*100.0
      HPY=PKPY/2.0
      X=0.0
      DO 940 I=6,NY
      IF(PY(I).LT.HPY)GO TO 940
      X=X+((Y(I)-PY(I))**2)
  940 CONTINUE
      Z3=(SQRT(X/EOO))*100.0
      Z4=((PKPY-PKY)/PKY)*100.0
      Z5=TPPY-TPY
      WRITE (6,950) Z1,Z2,Z3,PKY,PKPY,Z4,TPY,TPPY,Z5
  950 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS.',
     *//,' 1. ISE  = ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'    OBSERVED  = ',F10.4,' CUMECS',
     */,'    PREDICTED = ',F10.4,' CUMECS',
     */,'    ERROR     = ',F10.4,' %',
     */,' 5. TIME_TO PEAK:',
     */,'    OBSERVED  = ',F5.2,' HOURS',
     */,'    PREDICTED = ',F5.2,' HOURS',
     */,'    ERROR     =',F6.2,' HOURS')
      RETURN
      END
```

Appendix 4: Source listing of PICOMO

```
C-----------------------------------------------------------------------
C
C   XXX  XXX  XXX   XX   X   X   XX        A PROGRAM FOR THE
C   X  X  X  X     X  X  XX XX  X  X       IDENTIFICATION OF
C   X  X  X  X    X     X X X X     X      CONCEPTUAL MODELS
C   XXX   X  X    X     X X X X     X
C   X     X  X     X  X  X   X  X  X       COPYRIGHT:
C   X    XXX  XXX   XX   X   X   XX        DOOGE AND O'KANE 1974.
C
C-----------------------------------------------------------------------
C
C                              NOTATION
C         ICN    = CATCHMENT NUMBER.
C         IEN    = EVENT NUMBER.
C         NX     = NUMBER OF ACTIVE RAINFALL ORDINATES.
C         NY     = NUMBER OF RUNOFF ORDINATES.
C         DUR    = DURATION OF THE TIME INTERVAL BETWEEN SUCCESSIVE
C                  RAINFALL/RUNOFF ORDINATES.
C         NDELT  = NUMBER OF SUBDIVISIONS OF THE UNIT TIME INTERVAL
C                  TO BE USED IN NUMERICAL INTEGRATION.
C         NTNT   = NY*NDELT THE NUMBER OF ORDINATES USED IN NUMERICAL
C                  INTEGRATION.
C         DT     = DUR/NDELT LENGTH OF SUB-INTERVALS IN NUMERICAL
C                  INTEGRATION.
C         X      = THE VECTOR OF (NORMALIZED ACTIVE) RAINFALL ORDINATES.
C         Y      = THE VECTOR OF (NORMALIZED) RUNOFF ORDINATES.
C         ORDER  = THE VECTOR OF INTERNAL MODEL NUMBERS IN THE ORDER IN
C                  WHICH THEY ARE TO BE EXECUTED.
C         REFNUM = THE REFERENCE NUMBERS TO BE ASSOCIATED WITH EACH
C                  MODEL ON OUTPUT.
C         DUH    = ORDINATES OF THE UNIT PULSE RESPONSE OF A MODEL AS
C                  VOLUMES IN SUCCESSIVE UNIT INTERVALS OF TIME.
C         PY     = NORMALIZED OUTFLOW FROM A MODEL. THE PREDICTED
C                  RUNOFF AS VOLUMES IN SUCCESSIVE UNIT INTERVALS OF
C                  TIME.
C         S      = VALUES OF THE S-CURVE FOR A MODEL SAMPLED AT INTERVAL
C                  OF DT.
C         SDUH   = VALUES OF THE UNIT PULSE RESPONSE FOR A MODEL
C                  SAMPLED AT INTERVALS OF DT.
C
C-----------------------------------------------------------------------
C
C    THIS EXTENDED VERSION BY W.S. EYRE, JULY 1978.
C    DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C
C-----------------------------------------------------------------------
C
      IMPLICIT REAL(A-Z)
      INTEGER I,J,NX,NY,NDELT,NTNT,L,K,MAXO,MINO,MODEL,SKIP,ORDER,
     *        REFNUM,N,RN,MOD1
      REAL DUH,PY,P,X,Y,DUR
      DIMENSION DUH(150),P(5,30),X(150),SKIP(30),ORDER(17),REFNUM(17),
     *          Y(150),PY(150)
      DATA P/150*0.0/,X/150*0.0/,Y/150*0.0/,SKIP/30*0/
      DATA ORDER/4,13,2,1,3,14,12,7,5,6,9,10,8,15,11,20,21/
      DATA REFNUM/1,2,4,5,6,9,11,12,13,14,16,17,19,20,22,23,24/
C
C***********************************************************************
C     ACTIVITY 1     READ IN, ECHO AND PREPARE DATA.
C***********************************************************************
C
   10 WRITE  (6,20)
   20 FORMAT(1H1)
      READ   (5,30,END=710) ICN,IEN
   30 FORMAT(2I5)
      WRITE  (6,40)ICN,IEN
   40 FORMAT(' CONCEPTUAL MODELS FOR CATCHMENT NUMBER ',I5,', EVENT NUMB
     *ER ',I5,'.',//)
      READ   (5,50)NDELT,DUR
   50 FORMAT(I5,F5.2)
      READ   (5,60) NX,(X(I),I=1,NX)
      READ   (5,60) NY,(Y(I),I=1,NY)
   60 FORMAT(I3/(8F10.4))
      J=NX+1
      DO 70 I=J,NY
   70 X(I)=0.0
      WRITE  (6,80)NX,NY,NDELT,DUR
   80 FORMAT(1H ,'NUMBER OF ACTIVE RAIN ORDINATES =',I4//' NUMBER OF RUN
     *OFF ORDINATES =',I4//' NUMBER OF SUBDIVISIONS OF TIME INTERVAL =',
     *I4//' TIME INTERVAL = ',F5.2)
      WRITE  (6,90) (X(I),Y(I),I=1,NY)
   90 FORMAT(1H0,'  RAINFALL AND RUNOFF DATA',/(F10.4,2X,F10.4))
C
C     NORMALISE DATA.
C
      VX=0.0
      VY=0.0
      DO 100 I=1,NY
      VX=VX+X(I)
  100 VY=VY+Y(I)
      DO 110 I=1,NX
  110 X(I)=X(I)/VX
      DO 120 I=1,NY
  120 Y(I)=Y(I)/VY
      WRITE  (6,130)
  130 FORMAT(1H0,7X,'NORMALISED')
      WRITE  (6,140) (X(I),Y(I),I=1,NY)
  140 FORMAT(1H0,'  RAINFALL AND RUNOFF DATA',/(F10.4,2X,F10.4))
      WRITE  (6,150)VX,VY
  150 FORMAT(1H0,' NORMALISING CONSTANTS:  VX =',F14.4,'    VY =',F14.4)
      SSOD=0.0
      DO 160 I=1,NY
      DIF=Y(I)-X(I)
  160 SSOD=SSOD+DIF*DIF
      RMS=(SSOD/NY)**0.5
      WRITE  (6,170)RMS
  170 FORMAT(1H0,'.RMS DIFFERENCE BETWEEN X AND Y =',F10.4)
C
C     CALCULATE MOMENTS OF RAINFALL AND RUNOFF.
C
      UP1X=0.0
      UP2X=0.0
      UP3X=0.0
      DO 180 I=1,NX
      XM1=I*DUR
      UP1X=UP1X+XM1*X(I)
      XM2=XM1*(I*DUR)
      UP2X=UP2X+XM2*X(I)
      XM3=XM2*(I*DUR)
  180 UP3X=UP3X+XM3*X(I)
      U2X=UP2X-UP1X**2
      U3X=UP3X-3.0*UP1X*UP2X+2.0*UP1X**3
      WRITE  (6,190) U3X,U2X,UP1X
  190 FORMAT(///,' ACTIVE RAINFALL MOMENTS',//,' U3X =',F10.4,4X,'U2X ='
     *,F10.4,4X,'UP1X =',F10.4)
      UP1Y=0.0
      UP2Y=0.0
      UP3Y=0.0
      DO 200 I=1,NY
      YM1=I*DUR
      UP1Y=UP1Y+YM1*Y(I)
      YM2=YM1*(I*DUR)
```

```
      UP2Y=UP2Y+YM2*Y(I)
      YM3=YM2*(I*DUR)
  200 UP3Y=UP3Y+YM3*Y(I)
      U3Y=UP3Y-3.0*UP1Y*UP2Y+2.0*UP1Y**3
      U2Y=UP2Y-UP1Y**2
      WRITE  (6,210) U3Y,U2Y,UP1Y
  210 FORMAT(///,' RUNOFF MOMENTS',//,' U3Y =',F10.4,4X,'U2Y =',F10.4,4X
     *,'UP1Y =',F10.4)
C
C     MOMENTS OF IUH:  DUR CORRECTIONS ASSUMING A TRUELY PULSED
C                      INPUT AND A SAMPLED OUTPUT.
C
      UP1H=UP1Y-UP1X-DUR/2.0
      U2H=U2Y-U2X-(DUR*DUR)/12.0
      U3H=U3Y-U3X
C
C     SHAPE FACTORS OF IUH.
C
      S1=1.0
      S2=U2H/(UP1H**2)
      S3=U3H/(UP1H**3)
      WRITE  (6,220)UP1H,U2H,U3H,S1,S2,S3
  220 FORMAT(///,' MOMENTS OF THE IUH',//,' UP1H =',F12.4,2X,'U2H =',F12
     *.4,2X,'U3H =',F12.4,///' SHAPE FACTORS OF THE IUH',//' S1 =',F12.4
     *,2X,' S2 =',F12.4,2X,'S3 =',F12.4)
      IF(UP1H.GT.0.0.AND.U2H.GT.0.0.AND.U3H.GT.0.0)GO TO 240
      WRITE  (6,230)
  230 FORMAT(///,' ***',/,' *** ANALYSIS TERMINATED DUE TO NEGATIVE MOME
     *NTS OF THE IUH.',/,' ***')
      GO TO 10
C
C*********************************************************************
C     ACTIVITY 2     FIT MODELS OF IUH BY MOMENT MATCHING.
C*********************************************************************
C
  240 N=17
      WRITE  (6,20)
C
C     SWITCH 1.     BEGIN.
C
      DO 630 I=1,17
      MODEL=ORDER(I)
      RN=REFNUM(I)
      IF(MODEL.GT.10)GO TO 250
      GO TO (280,300,320,340,360,380,400,420,440,460),MODEL
  250 MOD1=MODEL-10
      IF(MOD1.GT.5)GO TO 260
      GO TO (480,500,520,540,560),MOD1
  260 MOD1=MOD1-8
      IF(MOD1.GT.2)GO TO 270
      GO TO (580,600),MOD1
  270 N=N-1
      GO TO 620
C
C     SWITCH 1.     END.
C
C     1. ONE LINEAR RESERVOIR.
C
  280 LRK=UP1H
      LRS2=1.0
      DLRS2=S2-LRS2
      WRITE  (6,290)RN,LRK,LRS2,DLRS2
      P(1,1)=LRK
  290 FORMAT(////,I3,'. ONE LINEAR RESERVOIR.',
     *//,' STORAGE DELAY FACTOR =',F10.4,
     *//,' MODEL SHAPE FACTOR S2 =',F10.4,
     *//,' DIFFERENCE IN S2: DATA-MODEL =',F10.4)
      GO TO 620
C
C     2. SCALENE TRIANGLE (1:3).
C
  300 BTR=(9.0*UP1H)/4.0
      TRS2=7.0/32.0
      DTRS2=S2-TRS2
      WRITE  (6,310)RN,BTR,TRS2,DTRS2
      P(1,2)=BTR
  310 FORMAT(////,I3,'. 1:3 TRIANGLE.',
     *//,' BASE OF TRIANGLE =',F10.4,
     *//,' MODEL SHAPE FACTOR S2 =',F10.4,
     *//,' DIFFERENCE IN S2: DATA-MODEL =',F10.4)
      GO TO 620
C
C     3. TWO EQUAL LINEAR RESERVOIRS.
C
  320 LR2K=UP1H/2.0
      LR2S2=0.5
      DLR2S2=S2-LR2S2
      WRITE  (6,330)RN,LR2K,LR2S2,DLR2S2
      P(1,3)=LR2K
  330 FORMAT(////,I3,'. TWO EQUAL LINEAR RESERVOIRS.',
     *//,' STORAGE DELAY TIME =',F10.4,
     *//,' MODEL SHAPE FACTOR S2 =',F10.4,
     *//,' DIFFERENCE IN S2: DATA-MODEL =',F10.4)
      GO TO 620
C
C     4. SINGLE CHANNEL.
C
  340 SCT=UP1H
      SCS2=0.0
      DSCS2=S2-SCS2
      WRITE  (6,350)RN,SCT,SCS2,DSCS2
      DT=DUR/MDELT
      P(1,4)=SCT
      P(2,4)=DT
  350 FORMAT(////,I3,'. SINGLE CHANNEL.',
     *//,' DELAY TIME FOR THE CHANNEL =',F10.4,
     *//,' MODEL SHAPE FACTOR S2 =',F10.4,
     *//,' DIFFERENCE IN S2: DATA-MODEL =',F10.4)
      GO TO 620
C
C     5. ROUTED RECTANGLE.
C
  360 A=1.0/3.0
      B=-UP1H
      C=UP1H*UP1H-U2H
      CALL QUAD(A,B,C,RRT,SKIP(5))
      RRK=UP1H-RRT/2.0
      IF(RRK.LE.0.0)SKIP(5)=1
      RRS3=(2.0*RRK**3)/((RRT/2.0+RRK)**3)
      DRRS3=S3-RRS3
      WRITE  (6,370)RN,RRT,RRK,RRS3,DRRS3
      P(1,5)=RRT
      P(2,5)=RRK
  370 FORMAT(////,I3,'. ROUTED RECTANGLE.',
     *//,' BASE OF RECTANGLE =',F10.4,
     *//,' STORAGE DELAY TIME =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     6. ROUTED TRIANGLE.
C
```

```
  380 A=7.0/24.0
      B=-UP1H
      C=UP1H*UP1H-U2H
      CALL QUAD (A,B,C,RTT,SKIP(6))
      RTK=UP1H-RTT/2.0
      IF(RTK.LT.0.0)SKIP(6)=1
      RTS3=(2.0*RTK**3)/((RTT/2.0+RTK)**3)
      DRTS3=S3-RTS3
      WRITE  (6,390)RN,RTT,RTK,RTS3,DRTS3
      P(1,6)=RTT
      P(2,6)=RTK
  390 FORMAT(////,I3,'. ROUTED TRIANGLE.',
     *//,' BASE OF TRIANGLE =',F10.4,
     *//,' STORAGE DELAY TIME =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     7. CHANNEL AND RESERVOIR (LAG AND ROUTE).
C
  400 A=1.0
      B=-2.0*UP1H
      C=UP1H*UP1H-U2H
      CALL QUAD(A,B,C,CRT,SKIP(7))
      CRK=UP1H-CRT
      IF(CRK.LT.0.0)SKIP(7)=1
      CRS3=(2.0*CRK**3)/((CRT+CRK)**3)
      DCRS3=S3-CRS3
      WRITE  (6,410)RN,CRT,CRK,CRS3,DCRS3
      P(1,7)=CRT
      P(2,7)=CRK
  410 FORMAT(////,I3,'. CHANNEL AND SINGLE LINEAR RESERVOIR.',
     *//,' DELAY TIME FOR THE CHANNEL =',F10.4,
     *//,' STORAGE DELAY TIME =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     8. CASCADE TWO E.L.R.S. WITH LATERAL INFLOW.
C
  420 A=2.0
      B=-4.0*UP1H
      C=U2H+UP1H*UP1H
      CALL QUAD(A,B,C,CLIK,SKIP(8))
      BETA=(UP1H-CLIK)/CLIK
      CLIS3=2.0*(2.0-(1.0-BETA)**3)/(1.0+BETA)**3
      DCLIS3=S3-CLIS3
      WRITE  (6,430)RN,CLIK,BETA,CLIS3,DCLIS3
      P(1,8)=CLIK
      P(2,8)=BETA
  430 FORMAT(////,I3,'. CASCADE 2 E.L.R.S. WITH LATERAL INFLOW.',
     *//,' DELAY TIME FOR THE RESERVOIRS =',F10.4,
     *//,' FRACTION OF INFLOW INTO FIRST RESERVOIR =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     9. N EQUAL LINEAR RESERVOIRS.
C
  440 LRNK=U2H/UP1H
      LRNN=(UP1H*UP1H)/U2H
      LRNS3=2.0/(LRNN*LRNN)
      DLRNS3=S3-LRNS3
      IF(LRNK.LE.0.0.OR.LRNN.LE.0.0) SKIP(9)=1
      WRITE  (6,450) RN,LRNN,LRNK,LRNS3,DLRNS3
      P(1,9)=LRNK
      P(2,9)=LRNN
  450 FORMAT(////,I3,'. N EQUAL LINEAR RESERVOIRS.',
     *//,' NUMBER OF RESERVOIRS IN CASCADE =',F10.4,
     *//,' STORAGE DELAY TIME FOR EACH RESERVOIR =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     10. CASCADE OF TWO UNEQUAL RESERVOIRS.
C
  460 A=2.0
      B=-2.0*UP1H-U2H
      C=UP1H*UP1H-U2H
      CALL QUAD(A,B,C,C2RK2,SKIP(10))
      C2RK1=UP1H-C2RK2
      IF(C2RK1.LT.0.0) SKIP(10)=1
      IF(C2RK1.EQ.C2RK2)SKIP(10)=1
      C2RS3=(2.0*C2RK1**3+2.0*C2RK2**3)/((C2RK1+C2RK2)**3)
      DC2RS3=S3-C2RS3
      P(1,10)=C2RK1
      P(2,10)=C2RK2
      WRITE  (6,470)RN,C2RK1,C2RK2,C2RS3,DC2RS3
  470 FORMAT(////,I3,'. CASCADE OF TWO UNEQUAL RESERVOIRS.',
     *//,' DELAY TIME FOR RESERVOIR 1 =',F10.4,
     *//,' DELAY TIME FOR RESERVOIR 2 =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     11. LAGGED CASCADE OF N EQUAL LINEAR RESERVOIRS.
C
  480 LCNK=U3H/(2.0*U2H)
      IF(LCNK.LT.0.0)SKIP(11)=1
      LCNN=(4.0*U2H**3)/(U3H**2)
      IF(LCNN.LT.0.0)SKIP(11)=1
      LCNT=UP1H-(2.0*U2H**2)/U3H
      IF(LCNT.LT.0.0)SKIP(11)=1
      P(1,11)=LCNK
      P(2,11)=LCNN
      P(3,11)=LCNT
      WRITE  (6,490)RN,LCNK,LCNN,LCNT
  490 FORMAT(////,I3,'. LAGGED CASCADE OF N EQUAL RESERVOIRS.',
     *//,' STORAGE DELAY TIME FOR EACH RESERVOIR =',F10.4,
     *//,' NUMBER OF RESERVOIRS IN CASCADE =',F10.4,
     *//,' DELAY TIME BEFORE THE CASCADE =',F10.4)
      GO TO 620
C
C     12. ISOSCELES TRIANGLE.
C
  500 A=2.0*S2-1.0
      B=4.0*S2+1.0
      C=2.0*S2-1.0
      CALL QUAD(A,B,C,AB,SKIP(12))
      BTR=3.0*UP1H/(1.0+AB)
      SUB=9.0*AB/(1.0+AB)**2
      ITS3=(2.0-SUB)/10.0
      DITS3=S3-ITS3
      WRITE  (6,510)RN,AB,BTR,ITS3,DITS3
      P(1,12)=AB
      P(2,12)=BTR
  510 FORMAT(////,I3,'. ISOSCELES TRIANGLE.',
     *//,' PEAK TIME AS A FRACTION OF THE BASE =',F10.4,
     *//,' BASE OF TRIANGLE                    =',F10.4,
     *//,' MODEL SHAPE FACTOR S3               =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL        =',F10.4)
      GO TO 620
```

```
C
C     13. RECTANGLE.
C
  520 BR=2.0*UP1H
      RS2=1.0/3.0
      DRS2=S2-RS2
      WRITE  (6,530)RN,BR,RS2,DRS2
      P(1,13)=BR
  530 FORMAT(////,I3,'. RECTANGLE.',
     *//,' BASE OF RECTANGLE',F10.4,
     *//,' MODEL SHAPE FACTOR S2 =',F10.4,
     *//,' DIFFERENCE IN S2: DATA-MODEL =',F10.4)
      GO TO 620
C
C     14. DIFFUSION REACH.
C
  540 A=(UP1H**3/(2.0*U2H))**0.5
      WRITE  (6,550)RN,A
      P(1,14)=A
  550 FORMAT(////,I3,'. DIFFUSION REACH.',
     *//,' PARAMETER A =',F10.4)
      GO TO 620
C
C     15. CONVECTIVE DIFFUSION REACH.
C
  560 A=(UP1H**3/(2.0*U2H))**0.5
      B=(UP1H/(2.0*U2H))**0.5
      CDS3=0.75/(A*A*B*B)
      DCDS3=S3-CDS3
      WRITE  (6,570)RN,A,B,CDS3,DCDS3
      P(1,15)=A
      P(2,15)=B
  570 FORMAT(////,I3,'. CONVECTIVE DIFFUSION REACH.',
     *//,' PARAMETER A =',F10.4,
     *//,' PARAMETER B =',F10.4,
     *//,' MODEL SHAPE FACTOR S3 =',F10.4,
     *//,' DIFFERENCE IN S3: DATA-MODEL =',F10.4)
      GO TO 620
C
C     20. CASCADE OF THREE RESERVOIRS.
C
  580 D=-(UP1H**3-3.0*UP1H*U2H+U3H)/6
      C=(UP1H**2-U2H)/2.0
      B=-UP1H
      A=1.0
      CALL CUBIC(A,B,C,D,K1,K2,K3,SKIP(20))
      IF(K1.LT.0.0.OR.K2.LT.0.0.OR.K3.LT.0.0) SKIP(20)=1
      WRITE  (6,590)RN,K1,K2,K3
      P(1,20)=K1
      P(2,20)=K2
      P(3,20)=K3
  590 FORMAT(////,I3,'. CASCADE OF 3 RESERVOIRS.',
     *//,' DELAY TIME FOR RESERVOIR 1 =',F10.4,
     *//,' DELAY TIME FOR RESERVOIR 2 =',F10.4,
     *//,' DELAY TIME FOR RESERVOIR 3 =',F10.4)
      GO TO 620
C
C     21. TWO RESERVOIRS WITH LATERAL INFLOW.
C
  600 UP2H=U2H+UP1H*UP1H
      UP3H=U3H+3.0*UP1H*U2H+UP1H**3
      A=UP1H**2-UP2H/2.0
      B=UP3H/6.0-UP1H*UP2H/2.0
      C=(UP2H/2.0)**2-UP1H*UP3H/6.0
      CALL QUAD(A,B,C,K2,SKIP(21))
      K1=C
      IF(K1.LT.0.0.OR.K2.LT.0.0)SKIP(21)=1
      BETA=(UP1H-K2)/K1
      P(1,21)=BETA
      P(2,21)=K1
      P(3,21)=K2
      WRITE  (6,610)RN,K1,K2,BETA
  610 FORMAT(////,I3,'. TWO RESERVOIRS WITH LATERAL INFLOW.',
     *//,' DELAY TIME FOR RESERVOIR 1 =',F10.4,
     *//,' DELAY TIME FOR RESERVOIR 2 =',F10.4,
     *//,' FRACTION OF INFLOW INTO FIRST RESERVOIR =',F10.4)
  620 CONTINUE
  630 CONTINUE
C
C*****************************************************************************
C     ACTIVITY 3    RECONSTITUTE OUTFLOW USING EACH MODEL.
C*****************************************************************************
      WRITE  (6,20)
C
C     SWITCH 2      BEGIN.
C
      DO 700 J=1,N
      MODEL=ORDER(J)
      RN=REFNUM(J)
C
C     SWITCH 2      END.
C
      IF(SKIP(MODEL).EQ.1)GO TO 680
      CALL RECON(NX,NY,DUR,NDELT,X,MODEL,P,DUH,PY)
      WRITE  (6,640) RN
  640 FORMAT(////,1H0,' RAINFALL   DUH-MODEL ',I2,' PREDICTED Y  RUNOFF(
     *Y)   ERROR    TIME',/)
      SSQD=0.0
      TOTDUH=0.0
      DO 660 I=1,NY
      DIF=Y(I)-PY(I)
      WRITE  (6,650)X(I),DUH(I),PY(I),Y(I),DIF,I
  650 FORMAT(1H ,5(F10.7,2X),I3)
      TOTDUH=TOTDUH+DUH(I)
  660 SSQD=SSQD+DIF*DIF
      RMS=(SSQD/NY)**0.5
      WRITE  (6,670)TOTDUH,RMS
  670 FORMAT(1H0,4X,'DUH TOTAL =',F9.7,7X,'RMS ERROR =',F10.7)
      CALL OBERFN(NY,RN,VY,Y,PY,DUR)
      GO TO 700
  680 SKIP(MODEL)=0
      WRITE  (6,690)RN
  690 FORMAT(////,10X,'THE',I3,' TH MODEL HAS BEEN SKIPPED',
     */,10X,'DUE TO ILLEGAL PARAMETER VALUES.',////)
  700 CONTINUE
      GO TO 10
  710 WRITE  (6,720)
  720 FORMAT(/////,'  END OF THE JOB.')
      STOP
      END
      SUBROUTINE QUAD(A,B,C,P,SKIP)
C
C     SOLVES A QUADRATIC EQUATION EXPLICITLY.
C
      INTEGER SKIP
      SKIP=0
      DELSQ=B*B-4.0*A*C
      IF(DELSQ.LT.0.0)GO TO 760
      DEL=DELSQ**0.5
      R1=(-B+DEL)/(2.0*A)
      R2=(-B-DEL)/(2.0*A)
      IF(R1.LT.0.0)GO TO 730
```

```
      IF(R2.LT.0.0)GO TO 740
      P=AMIN1(R1,R2)
      C=AMAX1(R1,R2)
      RETURN
  730 IF(R2.LT.0.0)GO TO 750
      P=R2
      RETURN
  740 P=R1
      RETURN
  750 SKIP=1
      P=0.0002
      RETURN
  760 SKIP=1
      P=0.000001
      RETURN
      END
      SUBROUTINE CUBIC(A,B,C,D,X1,X2,X3,SKIP)
C
C     SOLVES A CUBIC EQUATION EXPLICITLY.
C
      INTEGER SKIP
      SKIP=0
      P=(3.0*A*C-B*B)/(9.0*A*A)
      Q=B**3/(27.0*A**3)-B*C/(6.0*A*A)+D/(2.0*A)
      IF(Q) 770,790,780
  770 EPS=1.0
      GO TO 800
  780 EPS=-1.0
      GO TO 800
  790 IF(P.LT.0.0)GO TO 820
      Y1=0.0
      Y2=(3.0*P)**0.5
      Y3=-Y2
      GO TO 810
  800 IF(P.GT.0.0)GO TO 820
      DISC=P**3+Q**2
      IF(DISC.GT.0.0)GO TO 820
      R=EPS*(-P)**0.5
      A1=Q/R**3
      PHI=ARCOS(A1)
      A2=PHI/3.0
      A3=(3.1415926536-PHI)/3.0
      A4=(3.1415926536+PHI)/3.0
      Y1=-2.0*R*COS(A2)
      Y2= 2.0*R*COS(A3)
      Y3= 2.0*R*COS(A4)
  810 COR=B/(3.0*A)
      X1=Y1-COR
      X2=Y2-COR
      X3=Y3-COR
      RETURN
  820 SKIP=1
      X1=0.0
      X2=0.0
      X3=0.0
      RETURN
      END
      SUBROUTINE RECON(NX,NY,DUR,NDELT,X,MODEL,P,DUH,PY)
C
C     CALCULATES THE ORDINATES OF A MODELS S-CURVE AT THE SPECIFIED
C     FREQUENCY OF POINTS ON THE TIME INTERVAL BETWEEN SUCCESSIVE
C     ORDINATES OF RAINFALL AND RUNOFF BY INTEGRATING NUMERICALLY THE
C     ANALYTICAL EXPRESSION FOR THE MODELS IUH PROVIDED BY THE FUNCTION
C     IUH.  THE S-CURVE IS DIFFERENCED TO GIVE THE ORDINATES OF THE UNIT
C     PULSE RESPONSE (UPR).  THE UPR IS CONVERTED TO VOLUMES OF RUNOFF
C     IN SUCCESIVE INTERVALS OF TIME IN CORRESPONDENCE WITH THE RAINFALL
C     AND RUNOFF DATA TO YIELD THE MODEL DUH.  PREDICTED OUTFLOW IS
C     CALCULATED BY CONVOLUTING THE MODEL DUH WITH THE NORMALIZED
C     ACTIVE RAINFALL.
C
      IMPLICIT REAL(A-Z)
      REAL X(150),S(1000),SDUH(1000),DUH(150),PY(150),P(5,30)
      INTEGER I,J,NX,NY,NDELT,NTNT,L,K,MAXO,MINO,MODEL
C
C     CALCULATE S-CURVE BY TRAPEZOIDAL RULE.
C
      NTNT=NY*NDELT
      DT=DUR/NDELT
      S(1)=0.0
      F1=IUH(0.0,MODEL,P)
      DO 840 I=1,NTNT
      T=I*DT
      F2=IUH(T,MODEL,P)
  830 S(I+1)=S(I)+DT*(F1+F2)/2.0
  840 F1=F2
C
C     FIND UNIT PULSE RESPONSE FROM S-CURVE AT INTERVALS OF DT.
C
      L=NTNT-NDELT+1
      DO 850 I=1,L
      J=NTNT-I+2
  850 SDUH(J)=(S(J)-S(J-NDELT))/DUR
      DO 860 I=1,NDELT
  860 SDUH(I)=S(I)
C
C     DUH SAMPLED AT INTERVALS OF DUR.
C
      DO 870 I=1,NY
      J=NDELT*(I-1)+1
      DUH(I)=SDUH(J)
  870 CONTINUE
C
C     FIND DUH AT INTERVALS OF DUR BY TRAPEZOIDAL RULE.
C
      J=0
      L=1
      DUH(L)=0.0
      DO 890 I=1,NTNT
      IF(J.NE.NDELT)GO TO 880
      J=0
      L=L+1
      DUH(L)=0.0
  880 J=J+1
  890 DUH(L)=DUH(L)+DT*(SDUH(I)+SDUH(I+1))/2.0
C
C     CONVOLUTE X WITH DUH TO RECONSTRUCT Y AS PY.
C
      DO 900 I=1,NY
      PY(I)=0.0
      K=MINO(I,NX)
      DO 900 J=1,K
  900 PY(I)=PY(I)+DUH(I-J+1)*X(J)
      RETURN
      END
      SUBROUTINE OBERFN(NY,MN,VY,Y,PY,DUR)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(150),PY(150)
C-----FIND SUM OF OBSERVED (Y) Q ORDINATES.
      EOB=0.0
      DO 910 I=1,NY
```

```
  910 EQO=EQO+Y(I)
C-----FIND PEAK DISCHARGE AND TIME TO PEAK.
      PKY=0.0
      PKPY=0.0
      DO 930 I=1,NY
      IF(PY(I).LT.PKPY)GO TO 920
      PKPY=PY(I)
      TPPY=(I-1)*DUR
  920 IF(Y(I).LT.PKY)GO TO 930
      PKY=Y(I)
      TPY=(I-1)*DUR
  930 CONTINUE
      PKPY2=PKPY*VY
      PKY2=PKY*VY
C-----1. INTEGRAL SQUARES ERROR (ISE).
      X=0.0
      DO 940 I=1,NY
  940 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EQO))*100.0
C-----2. BIASED INTEGRAL SQUARES ERROR (ISE2).
      X=0.0
      DO 950 I=1,NY
  950 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EQO))*100.0
C-----3. PARTIAL INTEGRAL SQUARE ERROR (PISE).
      HPY=PKPY/2.0
      X=0.0
      DO 960 I=1,NY
      IF(PY(I).LT.HPY)GO TO 960
      X=X+((Y(I)-PY(I))**2)
  960 CONTINUE
      IF(X.LT.0.0)X=ABS(X)
      Z3=(SQRT(X/EQO))*100.0
C-----4. PEAK DISCHARGE.
      Z4=((PKPY-PKY)/PKY)*100.0
C-----5. TIME TO PEAK.
      Z5=TPPY-TPY
      WRITE (6,970) MN,Z1,Z2,Z3,PKY,PKY2,PKPY,PKPY2,Z4,TPY,TPPY,Z5
  970 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS FOR DUH-MODEL ',I2,'.',
     *//,' 1. ISE  = ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'    OBSERVED  = ',F9.7,'  OR  ',F10.4,
     */,'    PREDICTED = ',F9.7,'  OR  ',F10.4,
     */,'    ERROR     = ',F8.3,' %',
     */,' 5. TIME TO PEAK:',
     */,'    OBSERVED  = ',F5.2,
     */,'    PREDICTED = ',F5.2,
     */,'    ERROR     = ',F5.2)
      RETURN
      END
      REAL FUNCTION IUH(T,MODEL,P)
C
C     IUH CONTAINS THE ANALYTICAL EXPRESSION FOR THE IUH OF ALL THE
C     MODELS CONSIDERED.  IUH RETURNS TO RECON THE VALUE OF THE IUH
C     AT A SPECIFIED POINT IN TIME (T) FOR A GIVEN MODEL.
C
      IMPLICIT REAL (A-Z)
      INTEGER MODEL,MOD1
      REAL P(5,30)
      IF(MODEL.GT.15)GO TO 980
      GO TO (990,1000,1020,1030,1040,1060,1090,1100,1110,1120,1130,1140,
     *1160,1170,1180),MODEL
  980 MOD1=MODEL-19
      GO TO (1190,1200),MOD1
C-----1. ONE LINEAR RESERVOIR.
  990 LRK=P(1,1)
      IUH=(1.0/LRK)*EXP(-T/LRK)
      RETURN
C-----2. SCALENE TRIANGLE 1:3.
 1000 BTR=P(1,2)
      IUH=0.0
      IF(T.GE.BTR)RETURN
      IF(T.LE.BTR/3.0)GO TO 1010
      IUH=-3.0*(T-BTR)/(BTR*BTR)
      RETURN
 1010 IUH=6.0*T/(BTR*BTR)
      RETURN
C-----3. TWO EQUAL LINEAR RESERVOIRS.
 1020 LR2K=P(1,3)
      IUH=(T/LR2K)*EXP(-T/LR2K)/LR2K
      RETURN
C-----4. SINGLE CHANNEL.
 1030 SCT=P(1,4)
      DT=P(2,4)
      H1=SCT-DT/2.0
      H2=SCT+DT/2.0
      IUH=0.0
      IF(T.GE.H1.AND.T.LE.H2)IUH=1.0/DT
      RETURN
C-----5. ROUTED RECTANGLE.
 1040 RRT=P(1,5)
      RRK=P(2,5)
      H=EXP(-T/RRK)
      IF(T.GT.RRT)GO TO 1050
      IUH=(1.0-H)/RRT
      RETURN
 1050 H1=EXP((RRT-T)/RRK)
      IUH=(H1-H)/RRT
      RETURN
C-----6. ROUTED TRIANGLE.
 1060 RTT=P(1,6)
      RTK=P(2,6)
      H=4.0/(RTT*RTT)
      H1=RTK*EXP(-T/RTK)
      IF(T.GT.RTT/2.0)GO TO 1070
      IUH=H*(T-RTK+H1)
      RETURN
 1070 H2=EXP(RTT/(2.0*RTK))
      IF(T.GT.RTT)GO TO 1080
      IUH=H*(RTK-T+RTT-2.0*H1*H2+H1)
      RETURN
 1080 H3=EXP(RTT/RTK)
      IUH=H*(1.0-2.0/H2+1.0/H3)*H3*H1
      RETURN
C-----7. CHANNEL AND RESERVOIR (LAG AND ROUTE).
 1090 CRT=P(1,7)
      CRK=P(2,7)
      IUH=0.0
      IF(T.LT.CRT)RETURN
      IUH=(1.0/CRK)*EXP((CRT-T)/CRK)
      RETURN
C-----8. CASCADE 2 E.L.R.S. WITH LATERAL INFLOW.
 1100 CLIK=P(1,8)
      BETA=P(2,8)
      H=EXP(-T/CLIK)
      IUH=H*(BETA*T/(CLIK*CLIK)+(1.0-BETA)/CLIK)
      RETURN
C-----9. N EQUAL LINEAR RESERVOIRS.
 1110 LRNK=P(1,9)
```

```
      LRNN=P(2,9)
      H=T/LRNK
      IUH=(H**(LRNN-1.0))*EXP(-H)/(LRNK*GAMMA(LRNN))
      RETURN
C-----10. CASCADE OF TWO UNEQUAL RESERVOIRS.
 1120 C2RK1=P(1,10)
      C2RK2=P(2,10)
      IUH=EXP(-T/C2RK1)-EXP(-T/C2RK2)
      IUH=IUH/(C2RK1-C2RK2)
      RETURN
C-----11. LAGGED CASCADE OF N EQUAL LINEAR RESERVOIRS.
 1130 LCNK=P(1,11)
      LCNN=P(2,11)
      LCNT=P(3,11)
      IUH=0.0
      IF(T.LT.LCNT)RETURN
      H=(T-LCNT)/LCNK
      IUH=(H**(LCNN-1.0))*EXP(-H)/(LCNK*GAMMA(LCNN))
      RETURN
C-----12. ISOSCELES TRIANGLE.
 1140 BTR=P(2,12)
      A=P(1,12)
      IUH=0.0
      IF(T.GE.BTR)RETURN
      IF(T.LE.A*BTR)GO TO 1150
      IUH=-2.0*(T-BTR)/(BTR*BTR*(1.0-A))
      RETURN
 1150 IUH=2.0*T/(A*BTR*BTR)
      RETURN
C-----13. RECTANGLE
 1160 BR=P(1,13)
      IUH=0.0
      IF(T.GT.BR)RETURN
      IUH=1.0/BR
      RETURN
C-----14. DIFFUSION REACH.
 1170 A=P(1,14)
      IUH=0.0
      IF(T.EQ.0.0)RETURN
      IUH=A/(3.1415926536*T**3)**0.5
      IUH=IUH*EXP(-A**2/T)
      RETURN
C-----15. CONVECTIVE DIFFUSION REACH.
 1180 A=P(1,15)
      B=P(2,15)
      IUH=0.0
      IF(T.EQ.0.0)RETURN
      IUH=A/(3.1415926536*T**3)**0.5
      IUH=IUH*EXP(-(A-B*T)**2/T)
      RETURN
C-----20. CASCADE OF THREE RESERVOIRS.
 1190 K1=P(1,20)
      K2=P(2,20)
      K3=P(3,20)
      P(4,20)=4.0
      H1=K1*EXP(-T/K1)
      H2=K2*EXP(-T/K2)
      H3=K3*EXP(-T/K3)
      IUH=H1/((K1-K2)*(K1-K3))+H2/((K2-K1)*(K2-K3))+H3/((K3-K1)*(K3-K2))
      RETURN
C-----21. TWO RESERVOIRS WITH LATERAL INFLOW.
 1200 BETA=P(1,21)
      K1=P(2,21)
      K2=P(3,21)
      H1=EXP(-T/K1)
      H2=EXP(-T/K2)
      IUH=(1.0-BETA)*H2/K2+BETA*(H1-H2)/(K1-K2)
      RETURN
      END
      REAL FUNCTION GAMMA(X)
C
C     COMPUTES THE GAMMA FUNCTION FOR A GIVEN ARGUMENT.
C
      IF(X-57.0) 1220,1220,1210
 1210 GAMMA=1.E75
      RETURN
 1220 GX=1.0
      ERR=1.0E-6
      IF(X-2.0) 1250,1250,1240
 1230 IF(X-2.0) 1310,1310,1240
 1240 X=X-1.0
      GX=GX*X
      GO TO 1230
 1250 IF(X-1.0) 1260,1320,1310
 1260 IF(X-ERR) 1270,1270,1300
 1270 Y=FLOAT(INT(X))-X
      IF(ABS(Y)-ERR) 1330,1330,1280
 1280 IF(1.0-Y-ERR) 1330,1330,1290
 1290 IF(X-1.0) 1300,1300,1310
 1300 GX=GX/X
      X=X+1.0
      GO TO 1290
 1310 Y=X-1.0
      GY=1.0+Y*(-0.57710166+Y*(+0.98585399+Y*(-0.87642182+Y*(+0.8328212+
     *Y*(-0.5684729+Y*(+0.25482049+Y*(-0.05149930)))))))
      GAMMA=GX*GY
 1320 RETURN
 1330 GAMMA=GX
      RETURN
      END
```

Appendix 5: Source listing of PATRUH

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C-----------------------------------------------------------------
C
C     PATRUH: CALCULATES A PARAMETRIC TRIANGULAR UNIT HYDROGRAPH
C             FROM SCALING EQUATIONS AND CONVOLUTES THE PTTUH WITH
C             EFFECTIVE RAINFALL.
C             WRITTEN BY W.S. EYRE, JULY 1978.
C             DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C
C-----------------------------------------------------------------
C
C                     - NOTATION -
C
C     AREA  - CATCHMENT AREA, SQ. KM.
C     TINT  - ORDINATE TIME INTERVAL, HOURS.
C     ICN   - CATCHMENT NUMBER.
C     IEN   - EVENT NUMBER.
C     VAR05 - STORM DURATION, HOURS.
C     VAR06 - TOTAL RAINFALL, MM.
C     VAR07 - PEAK RAINFALL INTENSITY, MM/HR.
C     VAR25 - API5, MM.
C     VAR27 - SMD, MM.
C     QPG   - CALCULATED VALUE OF THE TUH QP, CUMECS (OPTIONAL).
C     TPG   - CALCULATED VALUE OF TUH TP, HOURS (OPTIONAL).
C     NHRF  - NUMBER OF EFFECTIVE RAINFALL ORDINATES.
C     ERF   - ARRAY OF EFFECTIVE RAINFALL ORDINATES.
C     NSFL  - NUMBER OF RESPONSE RUNOFF ORDINATES.
C     SFL   - ARRAY OF RESPONSE RUNOFF ORDINATES.
C
C-----------------------------------------------------------------
C
      DIMENSION TUH(100),TUHI(100),OBER(100,6)
      DATA OBER/600*0.0/
      REAL NSTV,NST1
      NSTV(X)=INT(X+SIGN(0.5,X))
      NST1(X)=(NSTV(X*2.0))/2.0
   10 FORMAT(/,26('-----'))
      ITOT=0
   20 READ (5,30,END=260) AREA,TINT
   30 FORMAT(F10.2,F5.2)
      ITOT=ITOT+1
      DO 40 I=1,100
      TUH(I)=0.0
   40 TUHI(I)=0.0
      L5=0
      ISOC=0
      READ (5,50) ICN,IEN,VAR05,VAR06,VAR07,VAR25,VAR27,QPG,TPG
   50 FORMAT(2I5,5F10.3,F10.4,F10.2)
      WRITE (6,60) ICN,IEN
   60 FORMAT(1H1,//,' DERIVATION AND CONVOLUTION OF A PARAMETRIC TUH FOR
     * ',I5,', EVENT',I3,'.',/,67('-'))
C-----CALCULATE QP AND TP FROM REGRESSION EQUATIONS.
C-----EQUATIONS APPLY TO THE BEVERLEY BROOK ONLY.
      VARA01=VAR06/VAR05
      VAR24=1.6124163+(VAR05*0.28278249)+(VAR06*0.17141147)+(-0.164975830
     **VAR07)+(VAR25*0.017188065)+(-0.0046975845*VAR27)+(VARA01*0.0171880
     *065)
      IF(VAR24.GT.0.0)GO TO 80
      WRITE (6,70) VAR24
   70 FORMAT(/,' CALCULATED VALUE OF VAR24 WAS',F10.4,
     *', ANALYSIS OF THIS EVENT WAS ABANDONED.')
      L5=1
      ITOT=ITOT-1
      GO TO 250
   80 CONTINUE
      TP=1.22850781*(VAR24**0.59646148)*(VAR25**0.050380014)*
     *(VAR27**0.047913909)*(VAR05**(-0.24289357))
      QP=116.63100019*(VAR27**0.026351351)*(VAR07**0.29474203)*
     *(VAR24**(-0.32020119))*(VAR06**(-0.18928325))*
     *(VARA01**(-0.16524179))
      WRITE (6,90) QP,TP
   90 FORMAT(//,' INITIAL ESTIMATES:  QP =',F9.4,'     TP =',F6.2)
      IF(QPG.GT.0.0)WRITE (6,100) QPG,TPG
  100 FORMAT(1H+,70X,'CALCULATED VALUES:  QP =',F9.4,'     TP =',F6.2)
      X1=1.0/TINT
      TP=NST1(TP)
C-----CALCULATE THE TIME BASE OF THE PTTUH.
      VOL=100.0/0.36
      TB=NST1(VOL/QP)
      D1=TB-TP
      IF(D1.LE.TP)GO TO 130
C-----INTERPOLATE VALUES OF TUH(I) AT 'TINT' INTERVALS FROM 0.0 TO TB.
      N=TP*X1+1.0
      X=0.0
      GRAD1=QP/TP
      DO 110 I=1,N
      TUH(I)=GRAD1*X
      X=X+TINT
  110 CONTINUE
      N2=N+1
      N3=TB*X1+1.0
      GRAD2=QP/(TP-TB)
      DO 120 I=N2,N3
      T4=X-TP
      TUH(I)=(GRAD2*T4)+QP
      X=X+TINT
  120 CONTINUE
      GO TO 170
C-----ISOCELES TRIANGLE DEFAULT.
  130 B2=NST1(VOL/QP)
      TR=TP-B2
      TPI=B2
      TBI=2.0*B2
      N=TPI*X1+1.0
      X=0.0
      GRAD3=QP/TPI
      DO 140 I=1,N
      TUHI(I)=GRAD3*X
      X=X+TINT
  140 CONTINUE
      N2=N+1
      N3=TBI*X1+1.0
      GRAD4=QP/(TPI-TBI)
      DO 150 I=N2,N3
      T4=X-TPI
      TUHI(I)=(GRAD4*T4)+QP
      X=X+TINT
  150 CONTINUE
      IQ=TR*X1
      DO 160 I=1,N3
      J=IQ+I
  160 TUH(J)=TUHI(I)
      ISOC=1
      N3=N3+IQ
  170 CONTINUE
C-----RESCALE AND WRITE OUT THE PTTUH.
      VOL=100.0/(0.36*TINT)
      VOLTUH=0.0
      DO 180 I=1,N3
  180 VOLTUH=VOLTUH+TUH(I)
      COR=VOL/VOLTUH
      DO 190 I=1,N3
  190 TUH(I)=TUH(I)*COR
```

```
      WRITE (6,200)
  200 FORMAT(/////,' PARAMETRIC TRIANGULAR TUH:')
      IF(ISOC.EQ.1)WRITE (6,210)
  210 FORMAT(1H+,29X,'ISOCELES TRIANGLE SUBSTITUTED')
      C1=0.0
      C2=0.0
      DO 220 I=1,N3
      IF(TUH(I).LT.C1)GO TO 220
      C1=TUH(I)
      C2=(I-1)*TINT
  220 CONTINUE
      C3=(N3-1)*TINT
      WRITE (6,230) C1,C2,C3
  230 FORMAT(//,2X,'PTTUH PEAK',8X,'TP',7X,'TB',/F12.5,F8.1,F8.1)
      WRITE (6,10)
      WRITE (6,240)(I,TUH(I),I=1,N3)
  240 FORMAT(6(I3,',',F11.5,6X))
      WRITE (6,10)
  250 CALL CONVOL(TUH,N3,AREA,L5,ITOT,OBER)
      GO TO 20
  260 CALL OBERST(OBER,ITOT)
      WRITE (6,270)
  270 FORMAT(///////,' END OF THE JOB.')
      STOP
      END
      SUBROUTINE CONVOL(EUH3,MUH2,AREA,L5,ITOT,OBER)
      DIMENSION EUH3(50),ERF(50),SFL(100),RSFL(150),OBER(100,6)
      READ (5,280) ICN,IEN
  280 FORMAT(2I5)
      READ (5,290) NHRF,(ERF(I),I=1,NHRF)
      READ (5,290) NSFL,(SFL(I),I=1,NSFL)
  290 FORMAT(I3/(8F10.4))
      IF(L5.EQ.1) RETURN
      WRITE (6,300)
  300 FORMAT(/////,' CONVOLUTION OF PTTUH AND EFFECTIVE RAINFALL')
      DO 310 I=1,150
  310 RSFL(I)=0.0
      DO 320 I=1,NHRF
      DO 320 J=1,MUH2
      IJ=I+J-1
  320 RSFL(IJ)=RSFL(IJ)+ERF(I)*EUH3(J)*AREA/1000.0
      CALL OBERFN(NSFL,SFL,RSFL,ICN,IEN,ITOT,OBER)
      WRITE (6,330)
  330 FORMAT(///3(8X,'OBSERVED',5X,'PREDICTED',10X))
      WRITE (6,340)(I,SFL(I),RSFL(I),I=1,NSFL)
  340 FORMAT(3(I4,',',F11.4,3X,F11.4,10X))
      WRITE (6,350)
  350 FORMAT(/,26('————'))
      RETURN
      END
      SUBROUTINE OBERFN(NY,Y,PY,ICN,IEN,ITOT,OBER)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(100),PY(150),OBER(100,6)
      EQO=0.0
      DO 360 I=1,NY
  360 EQO=EQO+Y(I)
      PKY=0.0
      PKPY=0.0
      DO 380 I=1,NY
      IF(PY(I).LT.PKPY)GO TO 370
      PKPY=PY(I)
      TPPY=(I-1)*0.5
  370 IF(Y(I).LT.PKY)GO TO 380
      PKY=Y(I)
      TPY=(I-1)*0.5
  380 CONTINUE
      X=0.0
      DO 390 I=1,NY
  390 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EQO))*100.0
      X=0.0
      DO 400 I=1,NY
  400 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EQO))*100.0
      HPY=PKPY/2.0
      X=0.0
      DO 410 I=1,NY
      IF(PY(I).LT.HPY)GO TO 410
      X=X+((Y(I)-PY(I))**2)
  410 CONTINUE
      Z3=(SQRT(X/EQO))*100.0
      Z4=((PKPY-PKY)/PKY)*100.0
      Z5=TPPY-TPY
      OBER(ITOT,1)=Z4
      OBER(ITOT,3)=Z5
      OBER(ITOT,5)=Z1
      OBER(ITOT,6)=Z3
      WRITE (6,420) ICN,IEN,Z1,Z2,Z3,PKY,PKPY,Z4,TPY,TPPY,Z5
  420 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS FOR ICN =',I6,' AND IEN =',
     *I3,//,' 1. ISE  = ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'      OBSERVED   = ',F10.4,' CUMECS',
     */,'      PREDICTED  = ',F10.4,' CUMECS',
     */,'      ERROR      = ',F10.4,' %',
     */,' 5. TIME TO PEAK:',
     */,'      OBSERVED   = ',F4.1,' HOURS',
     */,'      PREDICTED  = ',F4.1,' HOURS',
     */,'      ERROR      = ',F4.1,' HOURS')
      RETURN
      END
      SUBROUTINE OBERST(OBER,ITOT)
C
C     CALCULATES THE MEAN AND STANDARD DEVIATION OF THE OBJECTIVE
C     ERROR FUNCTIONS.
C
      DIMENSION OBER(100,6),SUM(6),AV(6),S(6),SD(6)
      M=6
      DO 430 I=1,ITOT
      OBER(I,2)=ABS(OBER(I,1))
  430 OBER(I,4)=ABS(OBER(I,3))
      DO 440 I=1,M
      S(I)=0.0
  440 SUM(I)=0.0
      DO 450 I=1,M
      DO 450 J=1,ITOT
  450 SUM(I)=SUM(I)+OBER(J,I)
      DO 460 I=1,M
  460 AV(I)=SUM(I)/ITOT
      DO 470 I=1,M
      DO 470 J=1,ITOT
  470 S(I)=S(I)+((OBER(J,I)-AV(I))**2)
      DO 480 I=1,M
  480 SD(I)=SQRT(S(I)/ITOT)
      WRITE (6,490)(AV(I),I=1,M),(SD(I),I=1,M),ITOT
  490 FORMAT(1H1,//,' SUMMARY OF THE OBJECTIVE ERROR FUNCTIONS:',////,16X
     *,'OPE %',3X,'A OPE %',4X,'TPE HR',2X,'A TPE HR',5X,'ISE %',4X,'PIS
     *E %',//,' MEAN',6X,6F10.3,//,' S.D.',6X,6F10.3,////,' NUMBER OF CAS
```

```
*ES =',I3)
 RETURN
 END
```

Appendix 6: Source listing of SMOOTH

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C-----------------------------------------------------------------------
C
C     SMOOTH:  SMOOTHING OF AN ESTIMATED TUH BY POLYNOMIAL REGRESSION.
C              POLYNOMIAL REGRESSION ROUTINE FROM FORTRAN SSP.
C              WRITTEN BY W.S. EYRE, MAY 1978.
C              DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C
C-----------------------------------------------------------------------
C
C                       - NOTATION -
C
C     ICN  - CATCHMENT NUMBER.
C     IEN  - EVENT NUMBER.
C     AREA - CATCHMENT AREA, SQ. KM.
C     TINT - ORDINATE TIME INTERVAL, HOURS.
C     NHRF - NUMBER OF EFFECTIVE RAINFALL ORDINATES.
C     ERF  - ARRAY OF EFFECTIVE RAINFALL ORDINATES.
C     NSFL - NUMBER OF RAPID RESPONSE RUNOFF ORDINATES.
C     SFL  - ARRAY OF RAPID RESPONSE RUNOFF ORDINATES.
C     N    - NUMBER OF TUH ORDINATES.
C     X    - ARRAY OF TUH ORDINATES (OUTPUT FROM LPROG6).
C
C-----------------------------------------------------------------------
C
      DIMENSION X(1100),DI(100),D(66),B(10),E(10),SB(10),T(10),XBAR(11),
     *          STD(11),COE(11),SUMSQ(11),ISAVE(11),ANS(10),ERF(40),SFL
     *          (80),EUH(80),UH(80),RSFL(150),P(300)
      INTEGER LINE(112),MARK,SPACE,CROSS
      DATA MARK/'*'/,SPACE/' '/,X/1100*0.0/,CROSS/'+'/
      NSTV(X)=INT(X+SIGN(0.5,X))
C
C-----------------------------------------------------------------------
C     ACTIVITY 1.  READ IN THE DATA.
C-----------------------------------------------------------------------
C
   10 READ (5,20,END=620) ICN,IEN
   20 FORMAT(2I5)
      READ (5,30) AREA,TINT
   30 FORMAT(F8.2,F7.2)
      READ (5,40) NHRF,(ERF(I),I=1,NHRF)
      READ (5,40) NSFL,(SFL(I),I=1,NSFL)
   40 FORMAT(I3/(8F10.4))
      READ (5,50) N
   50 FORMAT(I3)
      M=10
      X(1)=0.0
      DO 60 I=2,N
      J=I-1
   60 X(I)=X(J)+TINT
      L=N*M
      L1=L+1
      L2=L+N
      READ (5,70) (X(I),I=L1,L2)
   70 FORMAT(8F10.5)
      WRITE (6,80)
   80 FORMAT(1H1)
      WRITE (6,90) ICN,IEN
   90 FORMAT(////,' SMOOTHING OF THE TUH ESTIMATE FOR CATCHMENT NUMBER '
     *,I5,' EVENT NUMBER',I5,'.',////)
C
C-----------------------------------------------------------------------
C     ACTIVITY 2:  LEAST SQUARES POLYNOMIAL REGRESSION.
C-----------------------------------------------------------------------
C
      CALL GDATA(N,M,X,XBAR,STD,D,SUMSQ)
      MM=M+1
      SUM=0.0
      NT=N-1
      DO 600 IA=1,M
      ISAVE(IA)=IA
      CALL ORDER(MM,D,MM,IA,ISAVE,DI,E)
      CALL MINV(DI,IA,DET,B,T)
      CALL MULTR(N,IA,XBAR,STD,SUMSQ,DI,E,ISAVE,B,SB,T,ANS)
      WRITE (6,100) IA
  100 FORMAT(/////,' POLYNOMIAL REGRESSION OF DEGREE ',I3)
      IF(ANS(7)) 120,110,110
  110 SUMIP=ANS(4)-SUM
      IF(SUMIP) 120,120,140
  120 WRITE (6,130)
  130 FORMAT(' NO IMPROVEMENT.')
      GO TO 610
  140 WRITE (6,150) ANS(1)
  150 FORMAT(' INTERCEPT',F20.7)
      WRITE (6,160) (B(J),J=1,IA)
  160 FORMAT(' REGRESSION COEFFICIENTS',/(6F20.7))
      WRITE (6,170) IA
  170 FORMAT(/24X,' ANALYSIS OF VARIANCE FOR',I4,'  DEGREE POLYNOMIAL',/)
      WRITE (6,180)
  180 FORMAT(1H0,5X,19HSOURCE OF VARIATION,7X,9HDEGREE OF ,7X,6HSUM OF,9
     *X,4HMEAN,10X,1HF,9X,20HIMPROVEMENT IN TERMS/33X,7HFREEDOM,8X,7HSQU
     *ARES,7X,6HSQUARE,7X,5HVALUE,8X,17HOF SUM OF SQUARES)
      SUM=ANS(4)
      WRITE (6,190) IA,ANS(4),ANS(6),ANS(10),SUMIP
  190 FORMAT(' DUE TO REGRESSION',14X,I6,F17.5,F14.5,F13.5,F20.5)
      NI=ANS(8)
      WRITE (6,200) NI,ANS(7),ANS(9)
  200 FORMAT(' DEVIATION ABOUT REGRESSION',5X,I6,F17.5,F14.5)
      WRITE (6,210) NT,SUMSQ(MM)
  210 FORMAT(8X,'TOTAL',19X,I6,F17.5///)
      COE(1)=ANS(1)
      DO 220 J=1,IA
  220 COE(J+1)=B(J)
      LA=IA
      IF(LA.LE.3)GO TO 600
      NP3=N+N
      DO 230 I=1,N
      NP3=NP3+1
      P(NP3)=COE(1)
      L=I
      DO 230 J=1,LA
      P(NP3)=P(NP3)+X(L)*COE(J+1)
  230 L=L+N
      N2=N
      L=N*M
      DO 240 I=1,N
      P(I)=X(I)
      N2=N2+1
      L=L+1
  240 P(N2)=X(L)
      WRITE (6,250)
  250 FORMAT(/////27X,'TABLE OF RESIDUALS',//' OBSERVATION NO.',5X,
     *'X VALUE',7X,'Y VALUE',7X,'Y ESTIMATE',7X,'RESIDUAL',/)
      NP2=N
      NP3=N+N
      DO 260 I=1,N
      NP2=NP2+1
      NP3=NP3+1
      RESID=P(NP2)-P(NP3)
      EUH(I)=P(NP3)
  260 WRITE (6,270) I,P(I),P(NP2),P(NP3),RESID
  270 FORMAT(4X,I6,F18.5,F14.5,F17.5,F15.5)
C
```

```
C-----------------------------------------------------------------------
C     ACTIVITY 3:  RESCALE TUH, CONVOLUTE, CALCULATE ERROR PARAMETERS.
C-----------------------------------------------------------------------
C
      NUHF=N
      VOL=100.0/(0.36*TINT)
C-----CALCULATE SCALING FACTOR AND RESCALE ESTIMATED TUH.
      DO 280 I=1,NUHF
      UH(I)=EUH(I)
  280 IF(UH(I).LT.0.0) UH(I)=0.0
      K=0
      XYR=0.0
      DO 290 I=1,NUHF
      IF(UH(I).LT.XYR)GO TO 290
      XYR=UH(I)
      IXY=I
  290 CONTINUE
      DO 300 J=1,IXY
  300 IF(UH(J).EQ.0.0) K=J
      IF(K.EQ.0)GO TO 320
      DO 310 J=1,K
  310 UH(J)=0.0
  320 CONTINUE
      K=0
      DO 330 J=IXY,NUHF
  330 IF(UH(J).EQ.0.0)GO TO 340
      GO TO 360
  340 K=J+1
      DO 350 J=K,NUHF
  350 UH(J)=0.0
  360 CONTINUE
      VOLUH=0.0
      DO 370 I=1,NUHF
  370 VOLUH=VOLUH+UH(I)
      COR=VOL/VOLUH
      DO 380 I=1,NUHF
  380 UH(I)=UH(I)*COR
      IF(UH(NUHF).EQ.0.0)GO TO 390
      NUHF=NUHF+1
      UH(NUHF)=0.0
      EUH(NHRF)=0.0
  390 CONTINUE
C-----CALCULATE DIMENSIONLESS CENTROID.
      SRT=0.0
      DO 400 I=1,NUHF
  400 SRT=SRT+(UH(I)*I)
      DCENT=((SRT/VOL)/NUHF)*100.0
C-----CALCULATE CURVATURE AROUND THE PEAK OF THE TUH.
      UHM=UH(1)
      IJM=1
      CURV1=0.0
      DO 410 IJ=2,NUHF
      IF(UH(IJ).LE.UHM)GO TO 410
      UHM=UH(IJ)
      IJM=IJ
  410 CONTINUE
      IF(IJM.GT.1)GO TO 420
      CURV1=-1.0
      GO TO 430
  420 CURV=(UH(IJM+1)+UH(IJM-1)-2.0*UHM)/TINT**2
  430 CONTINUE
C-----CALCULATE THE SURFACE RUNOFF HYDROGRAPH.
      DO 440 I=1,60
  440 RSFL(I)=0.0
      DO 450 I=1,NHRF
      DO 450 J=1,NUHF
      IJ=I+J-1
  450 RSFL(IJ)=RSFL(IJ)+ERF(I)*UH(J)*AREA/1000.0
C-----CALCULATE THE ERROR FUNCTION - RMS.
      DSQ=0.0
      DO 460 I=1,NSFL
      V=RSFL(I)-SFL(I)
  460 DSQ=DSQ+(V*V)
      DSQ=SQRT(DSQ/NSFL)
      WRITE (6,470) (UH(I),I=1,NUHF)
  470 FORMAT(/' ADJUSTED TUH:',/(10F12.5))
      WRITE (6,480) DCENT
  480 FORMAT(/' DIMENSIONLESS CENTROID OF ADJUSTED TUH =',F9.4)
      IF(CURV1.EQ.0.0)WRITE (6,490) CURV
  490 FORMAT(/' CURVATURE AROUND PEAK OF THE TUH       =',F9.4)
C-----PLOT THE ADJUSTED TUH.
      IRAN=500
      DO 500 I=1,NUHF
      IF(UH(I).GT.8.0.AND.IRAN.EQ.500) IRAN=100
      IF(UH(I).GT.43.0.AND.IRAN.EQ.100)IRAN=50
      IF(UH(I).GT.86.0.AND.IRAN.GE.50) IRAN=25
  500 CONTINUE
      IF(IRAN.EQ.100)GO TO 520
      DO 510 I=1,NUHF
  510 UH(I)=UH(I)*IRAN/100.0
  520 WRITE (6,530) IRAN
  530 FORMAT(//55X,' CUMECS PER',I4,' SQ.KMS')
      WRITE (6,540)
  540 FORMAT(1X,
     *   '      -10.0      -5.0       0.0       5.0      10.0',
     *   '       15.0      20.0      25.0      30.0      35.0',
     *   '       40.0      45.0')
      DO 550 I=1,112
  550 LINE(I)=SPACE
      DO 570 I=1,NUHF
      LINE(20)=CROSS
      TIME=(I-1)*TINT
      J=NSTV(2*UH(I))+20
      IF(J.GT.0.AND.J.LE.110) LINE(J)=MARK
      WRITE (6,560) TIME,LINE
  560 FORMAT(' ',F6.2,'-  +',112A1)
      IF(J.GT.0.AND.J.LE.110) LINE(J)=SPACE
  570 CONTINUE
      WRITE (6,580) (I,SFL(I),RSFL(I),I=1,NSFL)
  580 FORMAT(/10X,'INCREMENT',7X,'OBSERVED',6X,'SIMULATED',
     */(15X,I2,7X,F10.4,5X,F10.4))
      WRITE (6,590) DSQ
  590 FORMAT(/10X,'ERROR FUNCTION (RMS) =',F8.6)
      CALL OBERFN(NSFL,SFL,RSFL)
  600 CONTINUE
  610 CONTINUE
      GO TO 10
  620 WRITE (6,630)
  630 FORMAT(///,' END OF THE JOB.')
      STOP
      END
      SUBROUTINE OBERFN(NY,Y,PY)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(80),PY(150)
      EQO=0.0
      DO 640 I=1,NY
  640 EQO=EQO+Y(I)
      PKY=0.0
      PKPY=0.0
      DO 660 I=1,NY
```

```
      IF(PY(I).LT.PKPY)GO TO 650
      PKPY=PY(I)
      TPPY=(I-1)*0.5
  650 IF(Y(I).LT.PKY)GO TO 660
      PKY=Y(I)
      TPY=(I-1)*0.5
  660 CONTINUE
      X=0.0
      DO 670 I=1,NY
  670 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EQQ))*100.0
      X=0.0
      DO 680 I=1,NY
  680 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EQQ))*100.0
      HPY=PKPY/2.0
      X=0.0
      DO 690 I=1,NY
      IF(PY(I).LT.HPY)GO TO 690
      X=X+((Y(I)-PY(I))**2)
  690 CONTINUE
      Z3=(SQRT(X/EQQ))*100.0
      Z4=((PKPY-PKY)/PKY)*100.0
      Z5=TPPY-TPY
      WRITE (6,700) Z1,Z2,Z3,PKY,PKPY,Z4,TPY,TPPY,Z5
  700 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS.',
     *//,' 1. ISE  = ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'    OBSERVED  = ',F10.4,
     */,'    PREDICTED = ',F10.4,
     */,'    ERROR     = ',F10.4,' %',
     */,' 5. TIME TO PEAK:',
     */,'    OBSERVED  = ',F4.1,' HOURS',
     */,'    PREDICTED = ',F4.1,' HOURS',
     */,'    ERROR     = ',F4.1,' HOURS')
      RETURN
      END
```

Appendix 7: Source listing of FLOODS

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C-----------------------------------------------------------------------
C
C  FLOODS: A FLOOD FREQUENCY SIMULATION PROGRAM.
C          DEVELOPED BY A. VAIRAVAMOORTHY.
C          DEPARTMENT OF PUBLIC HEALTH ENGINEERING, G.L.C.,
C          THIS EXTENDED VERSION BY W.S. EYRE.
C          DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C          AUGUST 1978.
C
C-----------------------------------------------------------------------
C
C                          - NOTATION -
C
C  TITLE - GENERAL LABEL APPLYING TO THE SERIES OF ANALYSES
C          BEING PERFORMED.
C  IRUN  - 1. 250 TOTAL RAINFALL PROFILES CONVOLUTED WITH A
C             GIVEN TUH.
C          2. 250 EFFECTIVE RAINFALL PROFILES CONVOLUTED WITH
C             A GIVEN TUH
C          3. 250 TOTAL RAINFALL PROFILES CONVOLUTED WITH A
C             PTTUH DERIVED FOR EACH PROFILE.
C          4. 250 EFFECTIVE RAINFALL PROFILES CONVOLUTED WITH A
C             PTTUH DERIVED AUTOMATICALLY FOR EACH PROFILE.
C  IRFSEP - METHOD OF EFFECTIVE RAINFALL SEPARATION:
C          1. CONSTANT LOSS PERCENTAGE
C          2. LOSS RATE CURVE
C          3. CONSTANT LOSS PERCENTAGE VARYING WITH CWI
C          4. UNIFORM LOSS, PHI INDEX
C  LIST  - LISTS THE 250 TOTAL RAINFALL PROFILES.
C  LIST1 - LISTS THE TOTAL RAINFALL FOR EACH PROFILE.
C  LIST2 - LISTS THE TOTAL EFFECTIVE RAINFALL FOR EACH PROFILE.
C  LIST3 - LISTS THE 250 EFFECTIVE RAINFALL PROFILES.
C  LIST4 - LISTS THE CHARACTERISTICS OF THE 250 PROFILES USED
C          IN THE CONVOLUTION PROCESS.
C  PRO   - PERCENTAGE RESPONSE.
C  SMD   - SOIL MOISTURE DEFICIT.
C  API5  - 5 DAY ANTECEDENT PRECIPITATION INDEX (MM)
C  AREA2 - CATCHMENT AREA (SQ. KM.)
C  RAINF - AN ARRAY OF 72 RAINFALL ORDINATES.  THIS REPRESENTS ONE
C          RAINFALL PROFILE.  THE PROFILES ARE READ FROM A
C          MAGNETIC TAPE OR A PERMANENT FILE.
C  HEAD  - LABEL FOR EACH TUH OR 250 PTTUHS.
C  NAREA - NUMBER OF TUH ORDINATES.
C  AREA  - TUH ORDINATE ARRAY (CUMECS).
C
C-----------------------------------------------------------------------
C
      PROGRAM FLOODS(INPUT,OUTPUT,TAPE15,PROFILE,
     * TAPE5=INPUT,TAPE6=OUTPUT,TAPE18=PROFILE)
      COMMON/UNIT/ KT,LP
      DIMENSION TITLE(20),AREA(100),RAINF(72),SUM(250,11),HEAD(20),
     * HYD(120),ARGES(250),T(250),TT(11),X(11),IX(11),JUMP(250,16),
     * TERF(250),AB(4,11),B1(11,11),B2(11,11),IMAX(16),VAROGA(250),
     * TPRSFL(250)
      DATA KR,LP,NRAIN,KT/5,6,72,15/,
     *  TT/2.,5.,10.,20.,30.,50.,100.,200.,500.,1000.,2.33/,
     *  IMAX/1,2,4,6,8,10,12,16,20,24,28,32,36,40,60,72/,
     * TERF/250*0.0/,VAROGA/250*0.0/,IRUN/0/,IRFSEP/0/
      READ   (KR,10) TITLE
   10 FORMAT (20A4)
      READ (KR,20) IRUN
   20 FORMAT(I1)
      IF(IRUN.GE.1.AND.IRUN.LE.4)GO TO 40
      WRITE (LP,30)
   30 FORMAT(///,10X,'ANALYSIS TERMINATED DUE TO FAILURE TO SPECIFY RUN
     *TYPE.')
      STOP
   40 CONTINUE
      IPAGE=1
      WRITE  (LP,50)  TITLE,IPAGE
   50 FORMAT(1H1,/,1X,20A4,T110,'PAGE ',I4)
      NN=250
      WRITE (LP,60)
   60 FORMAT(//,' FLOOD FREQUENCY SIMULATION BASED ON 250 STORM PROFILES
     * AND A CATCHMENT KERNAL FUNCTION.')
      K=14
      NK=0
C
C     READ IN THE 250 STORM PROFILES.
C
   70 READ  (KR,80,END=630)LIST,LIST1,LIST2,LIST3,LIST4,PRO,SMD,API5,
     *AREA2,IRFSEP
   80 FORMAT(5(I1,1X),2X,4F10.4,1X,I1)
      REWIND 18
      REWIND KT
      IF(IRUN.GT.1)WRITE (LP,90) PRO,SMD,API5,IRUN,IRFSEP
   90 FORMAT(//,10X,'BOUNDARY CONDITIONS:  PRO =',F6.2,5X,'SMD =',F7.2,5
     *X,'API5 =',F6.2,/,32X,'RUN TYPE =',I2,4X,'RAINFALL SEPARATION METH
     *OD =',I2)
      DO 170 KQ=1,NN
      READ  (18,100)(RAINF(I),I=1,72)
  100 FORMAT(18F5.1)
      DO 110 I=1,72
  110 VAROGA(KQ)=VAROGA(KQ)+RAINF(I)
      IF(LIST.EQ.1)GO TO 150
      IF(NK.LT.K)GO TO 130
      IPAGE=IPAGE+1
      WRITE  (LP,120)  IPAGE
  120 FORMAT(1H1,T110,'PAGE ',I4)
      K=K+16
  130 WRITE  (LP,140)  KQ,RAINF
  140 FORMAT(/2X,I3,2X,24F5.1,/(7X,24F5.1))
      NK=NK+1
  150 IF(IRUN.EQ.1.OR.IRUN.EQ.3)GO TO 170
      CALL RFSEPN(RAINF,PRO,SMD,API5,IRFSEP)
      DO 160  I=1,72
  160 TERF(KQ)=TERF(KQ)+RAINF(I)
  170 WRITE  (KT)  RAINF
      IF(LIST1.EQ.1)GO TO 190
      WRITE  (LP,180)(I,VAROGA(I),I=1,250)
  180 FORMAT(//,' TOTAL RAINFALL FOR EACH STORM PROFILE  (MM).',
     *//,8(5X,I3,F8.1))
  190 IF(LIST3.EQ.1)GO TO 260
      IPAGE=IPAGE+1
      WRITE  (LP,200)  IPAGE
  200 FORMAT(1H1,' 250 EFFECTIVE RAINFALL PROFILES.',T110,'PAGE',I4)
      REWIND KT
      INDEX=0
      DO 240  IK=1,NN
      READ  (KT)  RAINF
      DO 210  I=1,72
  210 IF(RAINF(I).GT.0.0)  K3B=I
      DO 220  I=1,72
      IF(RAINF(I).GT.0.0)GO TO 230
  220 CONTINUE
  230 K3A=I
      K3B1=K3B+1
      NLINES=((K3B1-K3A)/17)+2
      INDEX=INDEX+NLINES
      IF(INDEX.LE.56)GO TO 240
      INDEX=0
      IPAGE=IPAGE+1
```

```
      WRITE (LP,200) IPAGE
  240 WRITE (LP,250) IK,(RAINF(I),I=K3A,K3B)
  250 FORMAT(/,I4,2X,17F7.3/(6X,17F7.3))
  260 IF(IRUN.EQ.1.OR.IRUN.EQ.3.AND.LIST2.EQ.1)GO TO 280
      IPAGE=IPAGE+1
      WRITE (LP,50) TITLE,IPAGE
      WRITE (LP,270) (I,TERF(I),I=1,250)
  270 FORMAT(//,' TOTAL EFFECTIVE RAINFALL FOR EACH STORM PROFILE   (MM).
     *',//,8(5X,I3,F8.3))
  280 CONTINUE
      IF(LIST4.EQ.0) CALL RFSTAT(TITLE,IPAGE)
C
C     CONVOLUTE RAINFALL WITH THE UNIT HYDROGRAPH.
C
      NAREA=0
  290 IPAGE=IPAGE+1
      REWIND KT
      READ (KR,10,END=630) HEAD
      IF(IRUN.GT.2)GO TO 310
      READ  (KR,300) NAREA,(AREA(I),I=1,NAREA)
  300 FORMAT(5X,I5/(8F10.5))
  310 CONTINUE
      DO 320 I=1,11
      IX(I)=(I-1)*5
  320 X(I)=FLOAT(IX(I))
      DO 400 II=1,NN
      READ  (KT) RAINF
      IF(IRUN.LT.3)GO TO 330
      VAROG=VAROGA(II)
      CALL PTTUH(RAINF,AREA,APIS,SMD,VAROG,NAREA)
  330 CONTINUE
      DO 340 IL=1,11
  340 SUM(II,IL)=0.0
      ILIM = NAREA+NRAIN-1
      DO 350 K=1,ILIM
  350 HYD(K)=0.0
      DO 360 J=1,NAREA
      DO 360 I=1,NRAIN
      K=J+I-1
  360 HYD(K)=HYD(K)+RAINF(I)*AREA(J)*AREA2/1000.0
      DO 390 K=1,ILIM
      IF(HYD(K).LT.SUM(II,1))GO TO 370
      SUM(II,1)=HYD(K)
      TPRSFL(II)=(K-1)*0.5
  370 IL=1
  380 IL=IL+1
      IY=IL-1
      IF(HYD(K).LE.X(IY))GO TO 390
      SUM(II,IL)=SUM(II,IL)+(HYD(K)-X(IY))*.18
      IF(IL.GE.11)GO TO 390
      GO TO 380
  390 CONTINUE
  400 CONTINUE
C
C     STATISTICAL ANALYSIS OF PEAK FLOWS.
C
      WRITE (LP,410) HEAD,IPAGE
  410 FORMAT(1H1,2X,20A4,T110,'PAGE ',I4/)
      WRITE  (LP,420) (J,SUM (J,1),J=1,250)
  420 FORMAT(/5X,' MAXIMUM HYDROGRAPH VALUE   (CUMECS) ',//,8(5X,I3,F8.2))
      IPAGE=IPAGE+1
      WRITE (LP,410) HEAD,IPAGE
      WRITE (LP,430) (J,TPRSFL(J),J=1,250)
  430 FORMAT(/5X,' TIME TO PEAK OF PREDICTED RESPONSE RUNOFF (HOURS) ',
     *//,8(5X,I3,F8.1))
      DO 440 JJ=1,11
      DO 440 II=1,250
  440 JUMP(II,JJ)=II
      CALL RANK(SUM,JUMP,250,11)
      DO 450 I=1,100
  450 T(I)=ALOG10(101./FLOAT(I))
      IX1=0
      IX2=0
      DO 490 III=1,5
      IX1=IX2+1
      IX2=IX2+50
      IPAGE=IPAGE+1
      WRITE (LP,410) HEAD,IPAGE
      WRITE (LP,460)  (IX(I),I=1,10)
  460 FORMAT(16X,'FREQUENCY ANALYSIS - PARTIAL DURATION SERIES - PEAK FL
     *OWS - SPILL VOLUMES RANKED IN DESCENDING ORDER'/48X,'(ORIGINAL POS
     *ITION IN PARANTHESES)'//4X,'ORDER   PEAK FLOW',1X,10(2X,'V',I2,'*1
     *000',1X))
      DO 470 I=IX1,IX2
  470 WRITE (LP,480) I,(SUM(I,LL),JUMP(I,LL),LL=1,11)
  480 FORMAT(5X,I4,1X,11(F6.0,'(',I3,')'))
  490 CONTINUE
      IPAGE=IPAGE+1
      WRITE (LP,410) HEAD,IPAGE
      DO 510 LL=1,11
      DO 500  I=1,100
  500 ARGES(I)=SUM(I,LL)
      CALL LSTSQ(T,ARGES,100,A,B)
      AB(1,LL)=A
      AB(2,LL)=B
      CALL GUMB(ARGES,100,X4,X5)
      AB(3,LL)=X4
      AB(4,LL)=X5
  510 CONTINUE
      WRITE (LP,520)(IX(I),I=1,10)
  520 FORMAT(//,8X,'PEAK FLOW ',10(2X,'V',I2,'*1000',1X))
      WRITE (LP,530)  ((AB(LL,I),I=1,11),LL=1,4)
  530 FORMAT(5X,'A',11F11.3/5X,'B',11F11.3/4X,'X4',11F11.3/4X,'X5',
     *11F11.3,/////)
      DO 550 IL=1,11
      DO 540 IK=1,11
      Y=-ALOG(-ALOG(1-1/TT(IK)))
      Q=Y*AB(3,IL)+AB(4,IL)
      B1(IL,IK)=Q
      P=AB(1,IL)+ALOG10(TT(IK))*AB(2,IL)
      B2(IL,IK)=P
  540 CONTINUE
  550 CONTINUE
      WRITE (LP,560)(TT(I),I=1,10)
  560 FORMAT(/' CHARTS USING GUMBELS DISTRIBUTION '//9X,T50,'RETURN PERI
     *OD  (YEARS) '/10X,10F12.2)
      WRITE (LP,570)  (B1(1,KJ),KJ=1,10)
  570 FORMAT(5X,'QPEAK',10F12.2)
      DO 580  LL=2,11
  580 WRITE (LP,600)  IX(LL-1),(B1(LL,KJ),KJ=1,10)
      WRITE (LP,590) B1(1,11)
  590 FORMAT(/,5X,'MEAN ANNUAL FLOOD (2.33 YEARS) =',F6.2,' CUMECS')
  600 FORMAT(5X,'V',I4,10F12.2)
      WRITE (LP,610)(TT(I),I=1,10)
  610 FORMAT(/////' CHARTS USING LEAST SQUARES'//9X,T50,'RETURN PERIOD
     *(YEARS) ',/10X,10F12.2)
      WRITE (LP,570)(B2(1,KJ),KJ=1,10)
      DO 620 LL=2,11
  620 WRITE (LP,600)  IX(LL-1),(B2(LL,KJ),KJ=1,10)
      WRITE (LP,590) B2(1,11)
      GO TO (290,70,70,70),IRUN
  630 WRITE (LP,640)
```

```
  640 FORMAT(//////////,' END OF THE JOB.')
      STOP
      END
      SUBROUTINE RFSEPN(HRF,PRO,SMDS,API5S,IRFSEP)
C
C     EFFECTIVE RAINFALL SEPARATION FROM LPROGS (UCL VERSION).
C
      DIMENSION HRF(80),ACC(80),ALT(80),K(80),ERF(80)
      IF(IRFSEP.GT.1)GO TO 660
      DO 650 I=1,72
  650 HRF(I)=PRO*HRF(I)/100.0
      RETURN
  660 ISM=125
      AEX=1.0
      CAP=1.0
      CWI=125.0-SMDS+API5S
      NHRF=72
      SR=0.0
      TINT=0.5
      DO 670 I=1,NHRF
  670 SR=SR+HRF(I)
      VOL=(PRO*SR)/100.0
      ALOSS=SR-VOL
      IF(IRFSEP.EQ.4)GO TO 800
  680 AK=0.5**(TINT/24.0)
      SACC=1.0/CWI**AEX
      ACC(1)=SACC
      S=SMDS
      AA=API5S
      SUMRO=0.0
      HRF(NHRF+1)=0.0
      DO 690 I=1,NHRF
      S=S-HRF(I)
      IF(S.LT.0) S=0.0
      AA=AA*AK+HRF(I)
      CWIR=ISM-S+AA
      IF(CWIR.LE.5.0) CWIR=5.0
      ACC(I+1)=1.0/(CWIR)**AEX
      SUMRO=SUMRO+HRF(I+1)/ACC(I+1)
  690 SACC=SACC+ACC(I+1)
      IF(IRFSEP.EQ.3)GO TO 780
      BLOSS=ALOSS
      SALT=0.0
      DO 700 I=1,NHRF
      ALT(I)=0.0
  700 K(I)=0
  710 DO 720 I=1,NHRF
      ARF=(100-CAP)/100.0*HRF(I)
      IF(K(I).EQ.1)GO TO 720
      ALT(I)=ACC(I)*BLOSS/SACC+ALT(I)
      IF(ARF.GT.ALT(I))GO TO 720
      ALT(I)=ARF
      K(I)=1
  720 SALT=SALT+ALT(I)
      RESID=ALOSS-SALT
      IF(RESID.LT.0.01)GO TO 750
  730 BLOSS=RESID
      SACC=0.0
      SALT=0.0
      DO 740 I=1,NHRF
      IF(K(I).EQ.1)GO TO 740
      SACC=SACC+ACC(I)
  740 CONTINUE
      GO TO 710
  750 DO 760 I=1,NHRF
  760 ERF(I)=HRF(I)-ALT(I)
      DO 770 I=1,NHRF
  770 HRF(I)=ERF(I)
      RETURN
  780 PF=VOL/(SUMRO+HRF(1)*CWI)
      HRF(1)=PF*CWI*HRF(1)
      DO 790 I=2,NHRF
  790 HRF(I)=PF*HRF(I)/ACC(I)
      RETURN
  800 DO 810 I=1,NHRF
  810 ERF(I)=HRF(I)
      JEYE=0
      DO 820 I=1,NHRF
  820 IF(HRF(I).EQ.0) JEYE=JEYE+1
      EYE3=ALOSS/(NHRF-JEYE)
  830 EYE1=0.0
      DO 850 I=1,NHRF
      IF(ERF(I).LE.0)GO TO 850
      IF(EYE3-ERF(I)) 850,850,840
  840 EYE1=EYE1+(EYE3-ERF(I))
      ERF(I)=-1.0
      JEYE=JEYE+1
  850 CONTINUE
      EYE2=EYE1/(NHRF-JEYE)
      PHI=EYE2+EYE3
      EYE3=PHI
      IF(EYE1.GT.0.0001)GO TO 830
      DO 860 I=1,NHRF
      IF(HRF(I).GT.PHI) HRF(I)=HRF(I)-PHI
  860 IF(HRF(I).LE.PHI) HRF(I)=0.0
      RETURN
      END
      SUBROUTINE RFSTAT(TITLE,IPAGE)
C
C     CALCULATES THE PROPERTIES OF THE RAINFALL PROFILES.
C
      COMMON/UNIT/ KT,LP
      DIMENSION RAINF(72),TITLE(20),KJUMP(250,4),RFSTA(250,4),X(72)
      REWIND KT
      DO 870 I=1,4
      DO 870 J=1,250
  870 RFSTA(J,I)=0.0
      IPAGE=IPAGE+1
      WRITE (LP,880) TITLE,IPAGE
  880 FORMAT(1H1,2X,20A4,T110,'PAGE',I4,/)
      WRITE (LP,890)
  890 FORMAT(///,15X,'CHARACTERISTICS OF THE RAINFALL PROFILES USED IN T
     *HIS ANALYSIS.',//,15X,'PROFILE',7X,'DURATION',7X,'TOTAL RF',7X,'PE
     *AK INTENSITY',7X,'MEAN INTENSITY',/,16X,'NUMBER',9X,'HOURS',11X,'M
     *M',15X,'MM/HR',16X,'MM/HR',11X,'S2',12X,'S3',/)
      INDEX=0
      DO 970 II=1,250
      INDEX=INDEX+1
      READ (KT) RAINF
      VAR06=0.0
      VAR07=0.0
      DO 900 I=1,72
      IF(RAINF(I).GT.VAR07) VAR07=RAINF(I)
      IF(RAINF(I).GT.0.0) K4A=I+1
  900 VAR06=VAR06+RAINF(I)
      DO 910 I=1,72
      IF(RAINF(I).GT.0.0)GO TO 920
  910 CONTINUE
  920 K3A=I
      VAR05=(K4A-K3A)*0.5
      VAR07=VAR07*2.0
      VARA01=VAR06/VAR05
```

```
      DUR=0.5
      DO 930 I=1,72
      UP1X=0.0
  930 X(I)=RAINF(I)/VAR06
      UP2X=0.0
      UP3X=0.0
      DO 940 I=1,72
      XM1=I*DUR
      UP1X=UP1X+XM1*X(I)
      XM2=XM1*(I*DUR)
      UP2X=UP2X+XM2*X(I)
      XM3=XM2*(I*DUR)
  940 UP3X=UP3X+XM3*X(I)
      U2X=UP2X-UP1X**2
      U3X=UP3X-3.0*UP1X*UP2X+2.0*UP1X**3
      S2=U2X/(UP1X**2)
      S3=U3X/(UP1X**3)
      IF(INDEX.LE.50)GO TO 950
      IPAGE=IPAGE+1
      INDEX=1
      WRITE (LP,880) TITLE,IPAGE
      WRITE (LP,890)
  950 WRITE (LP,960) II,VAR05,VAR06,VAR07,VARA01,S2,S3
  960 FORMAT(17X,I3,11X,F4.1,9X,F7.3,12X,F6.3,15X,F6.3,4X,2(5X,F9.5))
      RFSTA(II,1)=VAR05
      RFSTA(II,2)=VAR06
      RFSTA(II,3)=VAR07
      RFSTA(II,4)=VARA01
  970 CONTINUE
      DO 980 JJ=1,4
      DO 980 II=1,250
  980 KJUMP(II,JJ)=II
      CALL RANK(RFSTA,KJUMP,250,4)
      IX1=0
      IX2=0
      DO 1020 III=1,5
      IX1=IX2+1
      IX2=IX2+50
      IPAGE=IPAGE+1
      WRITE (LP,880) TITLE,IPAGE
      WRITE (LP,990)
  990 FORMAT(//,15X,'CHARACTERISTICS OF THE RAINFALL PROFILES USED IN TH
     *IS ANALYSIS.',/15X,'RANKED IN DESCENDING ORDER, ORIGINAL POSITION
     *IN PARENTHESIS.',//,29X,'DURATION',7X,'TOTAL RF',7X,'PEAK INTENSIT
     *Y',7X,'MEAN INTENSITY',/,17X,'ORDER',9X,'HOURS',11X,'MM',15X,'MM/H
     *R',16X,'MM/HR',/)
      DO 1000 I=IX1,IX2
 1000 WRITE (LP,1010)I,(RFSTA(I,LL),KJUMP(I,LL),LL=1,4)
 1010 FORMAT(18X,I3,7X,F4.1,'(',I3,')',5X,F7.3,'(',I3,')',
     *7X,F6.3,'(',I3,')',10X,F6.3,'(',I3,')')
 1020 CONTINUE
      RETURN
      END
      SUBROUTINE PTTUH(RAINF,TUH,VAR25,VAR27,VAR06,N3)
C
C     DERIVATION OF A PARAMETRIC TRIANGULAR UNIT HYDROGRAPH.
C
      DIMENSION RAINF(72),TUH(100),TUHI(100)
      REAL NSTV,NST1
      NSTV(X)=INT(X+SIGN(0.5,X))
      NST1(X)=(NSTV(X*2.0))/2.0
      TINT=0.5
      VAR07=0.0
      DO 1030 I=1,72
      IF(RAINF(I).GT.VAR07) VAR07=RAINF(I)
      IF(RAINF(I).GT.0.0) K4A=I+1
 1030 CONTINUE
      DO 1040 I=1,72
      IF(RAINF(I).GT.0.0)GO TO 1050
 1040 CONTINUE
 1050 K3A=I
      VAR05=(K4A-K3A)*TINT
      VAR07=VAR07*2.0
C-----CALCULATE QP AND TP FROM REGRESSION EQUATIONS.
C-----EQUATIONS FOR BEVERLEY BROOK, DERIVED FROM 36 STORM EVENTS.
      VARA01=VAR06/VAR05
      VAR24=1.8124183+(VAR05*0.28278249)+(VAR06*0.17141147)+(-0.16497563
     **VAR07)+(VAR25*0.017188065)+(-0.0048975845*VAR27)+(VARA01*0.017188
     *065)
      IF(VAR24.LT.0.0) VAR24=0.0001
      TP=1.22850781*(VAR24**0.59646148)*(VAR25**0.050380014)*
     *(VAR27**0.047913909)*(VAR05**(-0.24289357))
      QP=116.83100019*(VAR27**0.026351351)*(VAR07**0.29474203)*
     *(VAR24**(-0.32020119))*(VAR06**(-0.18928325))*
     *(VARA01**(-0.16524179))
      X1=1.0/TINT
      TP=NST1(TP)
      VOL=100.0/(0.36*TINT)
C-----CALCULATE THE TIME BASE OF THE PTTUH.
      TB=NST1(VOL/QP)
      D1=TB-TP
      IF(D1.LT.TP)GO TO 1080
C-----INTERPOLATE VALUES OF TUH(I) AT 'TINT' INTERVALS FROM 0.0 TO TB.
      N=TP*X1+1.0
      X=0.0
      GRAD1=QP/TP
      DO 1060 I=1,N
      TUH(I)=GRAD1*X
      X=X+TINT
 1060 CONTINUE
      N2=N+1
      N3=TB*X1+1.0
      GRAD2=QP/(TP-TB)
      DO 1070 I=N2,N3
      T4=X-TP
      TUH(I)=(GRAD2*T4)+QP
      X=X+TINT
 1070 CONTINUE
      GO TO 1120
C-----ISOCELES TRIANGLE DEFAULT.
 1080 B2=NST1(VOL/QP)
      TR=TP-B2
      TPI=B2
      TBI=2.0*B2
      N=TPI*X1+1.0
      X=0.0
      GRAD3=QP/TPI
      DO 1090 I=1,N
      TUHI(I)=GRAD3*X
      X=X+TINT
 1090 CONTINUE
      N2=N+1
      N3=TBI*X1+1.0
      GRAD4=QP/(TPI-TBI)
      DO 1100 I=N2,N3
      T4=X-TPI
      TUHI(I)=(GRAD4*T4)+QP
      X=X+TINT
 1100 CONTINUE
      IQ=TR*X1
      DO 1110 I=1,N3
      J=IQ+I
```

```
 1110 TUH(J)=TUHI(I)
      N3=N3+IQ
 1120 CONTINUE
C-----RESCALE THE PTTUH.
      VOLTUH=0.0
      DO 1130 I=1,N3
 1130 VOLTUH=VOLTUH+TUH(I)
      COR=VOL/VOLTUH
      DO 1140 I=1,N3
 1140 TUH(I)=TUH(I)*COR
      RETURN
      END
      SUBROUTINE RANK(PRAIN,KJUMP,N,M)
C
C     RANKS AN ARRAY (PRAIN) IN DESCENDING ORDER AND RECORDS THE
C     ORIGINAL POSITION OF EACH ELEMENT (KJUMP).
C
      DIMENSION PRAIN(N,M),KJUMP(N,M)
      DO 1190 J=1,M
      DO 1180 KK=1,N
      JJ=N-KK
      ITSW=0
      DO 1170 I=1,JJ
      IF(PRAIN(I,J)-PRAIN(I+1,J)) 1150,1160,1160
 1150 ALPHA=PRAIN(I,J)
      PRAIN(I,J)=PRAIN(I+1,J)
      PRAIN(I+1,J)=ALPHA
      IT=KJUMP(I,J)
      KJUMP(I,J)=KJUMP(I+1,J)
      KJUMP(I+1,J)=IT
      ITSW=1
 1160 CONTINUE
 1170 CONTINUE
      IF(ITSW) 1190,1190,1180
 1180 CONTINUE
 1190 CONTINUE
      RETURN
      END
      SUBROUTINE LSTSQ(T,ARGES,NN,A,B)
C
C     COMPUTES A REGRESSION EQUATION BY THE METHOD OF LEAST SQUARES.
C
      DIMENSION T(NN),ARGES(NN)
      SX=0.0
      SY=0.0
      SXY=0.0
      SX2=0.0
      DO 1200 I=1,NN
      SX=SX+T(I)
      SY=SY+ARGES(I)
      SXY=SXY+T(I)*ARGES(I)
      SX2=SX2+T(I)*T(I)
      XN=NN
 1200 CONTINUE
      B=(XN*SXY-SX*SY)/(XN*SX2-SX*SX)
      A=(SY-B*SX)/XN
      RETURN
      END
      SUBROUTINE GUMB(ARGES,NUM,X4,X5)
C
C     COMPUTES THE PARAMETERS OF THE GUMBELL DISTRIBUTION.
C
      DIMENSION ARGES(NUM)
      X1=0.
      X2=0.
      DO 1210 IK=1,NUM
      X1=X1+ARGES(IK)
 1210 X2=X2+ARGES(IK)**2
      X1=X1/NUM
      X2=X2/NUM
      A=NUM/(NUM-1)
      X3=(A*(X2-X1*X1)**0.5)
      X4=0.780*X3
      X5=X1-0.577*X4
      RETURN
      END
```

Appendix 8: Source listing of DEMETU

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C-----------------------------------------------------------------------
C
C     DEMETU: DERIVES A MEAN TUH BY CENTERING THE PEAK DISCHARGE
C             ORDINATES OF SEVERAL TUHS.  THE TUH IS CONVOLVED
C             WITH EFFECTIVE RAINFALL.
C             WRITTEN BY W.S. EYRE, JULY 1978.
C             DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C
C-----------------------------------------------------------------------
C
C                           - NOTATION -
C
C     ICONV - 0 = NO ACTION.
C             1 = CONVOLVE MEAN TUH WITH EFFECTIVE RAINFALL.
C     AREA  - CATCHMENT AREA, SQ. KM.
C     NIENS - NUMBER OF TUHS FROM WHICH A MEAN TUH IS TO BE CALCULATED.
C     ITUH  - 0 = NO ACTION.
C             1 = MEAN TUH SUPPLIED BY USER.
C     IEN   - EVENT NUMBER.
C     NUH   - NUMBER OF TUH ORDINATES.
C     UH    - ARRAY OF TUH ORDINATES.
C     NUH2  - NUMBER OF ORDINATES OF SUPPLIED MEAN TUH.
C     EUH3  - ARRAY OF SUPPLIED MEAN TUH ORDINATES.
C     ICN   - CATCHMENT NUMBER.
C     NHRF  - NUMBER OF EFFECTIVE RAINFALL ORDINATES.
C     ERF   - ARRAY OF EFFECTIVE RAINFALL ORDINATES.
C     NSFL  - NUMBER OF RESPONSE RUNOFF ORDINATES.
C     SFL   - ARRAY OF RESPONSE RUNOFF ORDINATES.
C
C-----------------------------------------------------------------------
C
      DIMENSION UH(100),EUH1(50,100),EUH2(100),EUH3(100),DTUHQ(100),DTUH
     *T(100),OBER(100,6)
      INTEGER TP,P,P1,P3
      DATA EUH1/5000*0.0/,EUH2/100*0.0/,EUH3/100*0.0/,OBER/600*0.0/
      NSTV(X)=INT(X+SIGN(0.5,X))
      SUMTP=0.0
      N=0
      READ (5,10) ICONV,AREA,NIENS,ITUH
   10 FORMAT(I1,F9.2,2I5)
      IF(ITUH.EQ.1)GO TO 300
      WRITE (6,20)
   20 FORMAT(1H1,//,' DERIVATION OF A MEAN TUH BY CENTERING THE PEAK DIS
     *CHARGE ORDINATE.',////)
      DO 100 L=1,NIENS
      READ (5,30) IEN,NUH
   30 FORMAT(2I5)
      READ (5,40) (UH(I),I=1,NUH)
   40 FORMAT(8F10.5)
      NUHA=NUH+1
      DO 50 I=NUHA,100
   50 UH(I)=0.0
      N=N+1
      CALL STATS(UH,NUH,PK,TP,TB,CU)
      SUMTP=SUMTP+TP
      TPX=(TP-1)*0.5
      WRITE (6,60) IEN,PK,TPX,TB,CU
   60 FORMAT(//,' EVENT NUMBER ',I3,//,
     *2X,'TUH PEAK',6X,'TP',7X,'TB',5X,'CURVATURE',/,
     *F10.5,F8.1,F9.1,F14.4,/,24('---'))
      WRITE (6,70) (I,UH(I),I=1,NUH)
   70 FORMAT(6(I3,'.',F11.5,6X))
      WRITE (6,80)
   80 FORMAT(24('---'))
      P=30-TP
      P3=100-P
      DO 90 I=1,P3
      J=P+I
   90 EUH1(N,J)=UH(I)
  100 CONTINUE
      WRITE (6,110)
  110 FORMAT(1H1,//,' FIRST ESTIMATE OF TUH',
     *//,' ORDINATE',10X,'MEAN   STAND. DEVN.',8X,'MAXIMUM',8X,'MINIMUM'
     *,10X,'RANGE',/)
      SUM=0.0
      DO 160 J=1,100
      DO 120 K=1,N
  120 SUM=SUM+EUH1(K,J)
      AVA=SUM/N
      SA=0.0
      DO 130 I=1,N
  130 SA=SA+((EUH1(I,J)-AVA)**2)
      SDA=SQRT(SA/N)
      AMIN=1000.0
      AMAX=0.0
      DO 140 I=1,N
      IF(EUH1(I,J).LT.AMIN) AMIN=EUH1(I,J)
  140 IF(EUH1(I,J).GT.AMAX) AMAX=EUH1(I,J)
      ARAN=AMAX-AMIN
      WRITE (6,150) J,AVA,SDA,AMAX,AMIN,ARAN
  150 FORMAT(5X,I3,5(F15.5))
      EUH2(J)=AVA
      SUM=0.0
  160 CONTINUE
      TP2=SUMTP/N
      P=NSTV(30.0-TP2)
      P1=100-P
      DO 170 I=1,P1
  170 EUH3(I)=EUH2(P+I)
      AMAX2=0.0
      IPEAK=0
      DO 180 I=1,100
      IF(EUH3(I).LT.AMAX2)GO TO 180
      IPEAK=I
      AMAX2=EUH3(I)
  180 CONTINUE
      K=0
      DO 190 J=IPEAK,100
      IF(EUH3(J).EQ.0.0)GO TO 200
  190 CONTINUE
      K=101
      GO TO 220
  200 K=J+1
      DO 210 J=K,100
  210 EUH3(J)=0.0
  220 CONTINUE
      NUH2=K-1
      CALL STATS(EUH3,NUH2,PK,TP,TB,CU)
      TPX=(TP-1)*0.5
      WRITE (6,230)
  230 FORMAT(1H1,//,' FINAL ESTIMATE OF TUH.',/)
      WRITE (6,70) (I,EUH3(I),I=1,NUH2)
      WRITE (6,240) PK,TPX,TB,CU
  240 FORMAT(///,' PEAK =',F12.5,' CUMECS',
     */,' TP   =',F12.5,' HOURS',
     */,' TB   =',F12.5,' HOURS',
     */,' CURV =',F12.5)
      WRITE (6,80)
      DO 260 I=1,NUH2
      IF(EUH3(I).EQ.0.0)GO TO 250
      DTUHQ(I)=EUH3(I)/PK
      GO TO 260
```

```
250 DTUHQ(I)=0.0
260 CONTINUE
    DTUHT(1)=0.0
    DO 270 I=2,NUH2
    J=I-1
270 DTUHT(I)=(J/2.0)/TPX
    WRITE (6,280)
280 FORMAT(//////////,' DIMENSIONLESS TUH:  (Q/QP AND T/TP).',
   *//,3(11X,'TIME',8X,'DISCHARGE',10X))
    WRITE (6,290) (I,DTUHT(I),DTUHQ(I),I=1,NUH2)
290 FORMAT(3(I3,'.',F11.7,5X,F13.8,10X))
    WRITE (6,80)
    GO TO 350
300 IF(ICONV.EQ.1)GO TO 320
    WRITE (6,310)
310 FORMAT(///,' EXECUTION TERMINATED. PROGRAM CONTROL PARAMETERS ARE
   * INCORRECTLY SET.')
    STOP
320 READ (5,330) NUH2,(EUH3(I),I=1,NUH2)
330 FORMAT(I5/(8F10.5))
    CALL STATS(EUH3,NUH2,PK,TP,TB,CU)
    WRITE (6,340)
340 FORMAT(1H1,///,' CONVOLUTION OF A GIVEN MEAN TUH WITH EFFECTIVE RA
   *INFALL.',////,' MEAN TUH:',/)
    WRITE (6,70) (I,EUH3(I),I=1,NUH2)
350 IF(ICONV.EQ.1) CALL CONVOL(EUH3,NUH2,AREA)
    WRITE (6,360)
360 FORMAT(///////////,'  END OF THE JOB.')
    STOP
    END
    SUBROUTINE STATS(UH,NUH,PK,TP,TB,CU)
    DIMENSION UH(50)
    INTEGER TP
    VOL=100.0/(0.36*0.5)
    VOLUH=0.0
    DO 370 I=1,NUH
370 VOLUH=VOLUH+UH(I)
    COR=VOL/VOLUH
    DO 380 I=1,NUH
380 UH(I)=UH(I)*COR
    IF(UH(NUH).EQ.0.0)GO TO 390
    NUH=NUH+1
    UH(NUH)=0.0
390 CONTINUE
    UHM=UH(1)
    IJM=1
    DO 400 IJ=2,NUH
    IF(UH(IJ).LE.UHM)GO TO 400
    UHM=UH(IJ)
    IJM=IJ
400 CONTINUE
    IF(IJM.GT.1)GO TO 410
    CU=0.0
    GO TO 420
410 CU=(UH(IJM+1)+UH(IJM-1)-2.0*UHM)/0.5**2
420 CONTINUE
    PK=0.0
    TP=0
    DO 430 I=1,NUH
    IF(UH(I).LT.PK)GO TO 430
    PK=UH(I)
    TP=I
430 CONTINUE
    TB=(NUH-1)*0.5
    RETURN
    END
```

```
    SUBROUTINE CONVOL(EUH3,NUH2,AREA)
    DIMENSION EUH3(100),ERF(100),SFL(100),RSFL(150),OBER(100,8)
    ITOT=0
    WRITE (6,440)
440 FORMAT(1H1,//,' REGENERATION PERFORMANCE USING FINAL ESTIMATE OF T
   *UH.',////)
450 READ (5,460,END=530) ICN,IEN
460 FORMAT(2I5)
    ITOT=ITOT+1
    READ (5,470) NHRF,(ERF(I),I=1,NHRF)
    READ (5,470) NSFL,(SFL(I),I=1,NSFL)
470 FORMAT(I3/(8F10.4))
    DO 480 I=1,150
480 RSFL(I)=0.0
    DO 490 I=1,NHRF
    DO 490 J=1,NUH2
    IJ=I+J-1
490 RSFL(IJ)=RSFL(IJ)+ERF(I)*EUH3(J)*AREA/1000.0
    CALL OBERFN(NSFL,SFL,RSFL,ICN,IEN,ITOT,OBER)
    WRITE (6,500)
500 FORMAT(///3(8X,'OBSERVED',5X,'PREDICTED',10X))
    WRITE (6,510) (I,SFL(I),RSFL(I),I=1,NSFL)
510 FORMAT(3(I4,'.',F11.4,3X,F11.4,10X))
    WRITE (6,520)
520 FORMAT(/,26('-----'))
    GO TO 450
530 CALL OBERST(OBER,ITOT)
    RETURN
    END
    SUBROUTINE OBERFN(NY,Y,PY,ICN,IEN,ITOT,OBER)
C
C   OBJECTIVE ERROR FUNCTIONS.
C
    DIMENSION Y(100),PY(150),OBER(100,8)
    EQO=0.0
    DO 540 I=1,NY
540 EQO=EQO+Y(I)
    PKY=0.0
    PKPY=0.0
    DO 560 I=1,NY
    IF(PY(I).LT.PKPY)GO TO 550
    PKPY=PY(I)
    TPPY=(I-1)*0.5
550 IF(Y(I).LT.PKY)GO TO 560
    PKY=Y(I)
    TPY=(I-1)*0.5
560 CONTINUE
    X=0.0
    DO 570 I=1,NY
570 X=X+((Y(I)-PY(I))**2)
    Z1=(SQRT(X/EQO))*100.0
    X=0.0
    DO 580 I=1,NY
580 X=X+((Y(I)**2)-(PY(I)**2))
    IF(X.LT.0.0) X=ABS(X)
    Z2=(SQRT(X/EQO))*100.0
    HPY=PKPY/2.0
    X=0.0
    DO 590 I=1,NY
    IF(PY(I).LT.HPY)GO TO 590
    X=X+((Y(I)-PY(I))**2)
590 CONTINUE
    Z3=(SQRT(X/EQO))*100.0
    Z4=((PKPY-PKY)/PKY)*100.0
    Z5=TPPY-TPY
    OBER(ITOT,1)=Z4
```

```
      OBER(ITOT,3)=Z5
      OBER(ITOT,5)=Z1
      OBER(ITOT,6)=Z3
      WRITE (6,600) ICN,IEN,Z1,Z2,Z3,PKY,PKPY,Z4,TPY,TPPY,Z5
  600 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS FOR ICN =',I6,' AND IEN =',
     *I3,//,' 1. ISE  =  ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'     OBSERVED  =  ',F10.4,' CUMECS',
     */,'     PREDICTED =  ',F10.4,' CUMECS',
     */,'     ERROR     =  ',F10.4,' %',
     */,' 5. TIME TO PEAK:',
     */,'     OBSERVED  =  ',F4.1,' HOURS',
     */,'     PREDICTED =  ',F4.1,' HOURS',
     */,'     ERROR     =  ',F4.1,' HOURS')
      RETURN
      END
      SUBROUTINE OBERST(OBER,ITOT)
C
C     CALCULATES THE MEAN AND STANDARD DEVIATION OF THE OBJECTIVE
C     ERROR FUNCTIONS.
C
      DIMENSION OBER(100,6),SUM(6),AV(6),S(6),SD(6)
      M=6
      DO 610 I=1,ITOT
      OBER(I,2)=ABS(OBER(I,1))
  610 OBER(I,4)=ABS(OBER(I,3))
      DO 620 I=1,M
      S(I)=0.0
  620 SUM(I)=0.0
      DO 630 I=1,M
      DO 630 J=1,ITOT
  630 SUM(I)=SUM(I)+OBER(J,I)
      DO 640 I=1,M
  640 AV(I)=SUM(I)/ITOT
      DO 650 I=1,M
      DO 650 J=1,ITOT
  650 S(I)=S(I)+((OBER(J,I)-AV(I))**2)
      DO 660 I=1,M
  660 SD(I)=SQRT(S(I)/ITOT)
      WRITE (6,670) (AV(I),I=1,M),(SD(I),I=1,M),ITOT
  670 FORMAT(1H1,//,' SUMMARY OF THE OBJECTIVE ERROR FUNCTIONS:',///,16X
     *,'OPE %',3X,'A OPE %',4X,'TPE HR',2X,'A TPE HR',5X,'ISE %',4X,'PIS
     *E %',//,' MEAN',6X,6F10.3,//,' S.D.',6X,6F10.3,///,' NUMBER OF CAS
     *ES =',I3)
      RETURN
      END
```

Appendix 9: Source listing of DUHCON

```
C-----------------------------------------------------------------------
C
C     DUHCON:  DERIVES A DUH FROM A TUH AND CONVOLUTES THE DUH
C              WITH EFFECTIVE RAINFALL.
C              WRITTEN BY W.S. EYRE, AUGUST 1978.
C              DEPARTMENT OF GEOGRAPHY, U.C.L., LONDON, WC1E 6BT.
C
C-----------------------------------------------------------------------
C
C                          - NOTATION -
C
C     NFLOW - NUMBER OF TUH ORDINATES.
C     FLOW1 - ARRAY OF TUH ORDINATES.
C     AREA  - CATCHMENT AREA, SQ. KM.
C     TINT  - ORDINATE TIME INTERVAL, HOURS.
C     ICN   - CATCHMENT NUMBER.
C     IEN   - EVENT NUMBER.
C     VAR05 - STORM DURATION, HOURS.
C     VAR06 - TOTAL RAINFALL, MM.
C     VAR07 - PEAK RAINFALL INTENSITY, MM/HR.
C     VAR25 - API5, MM
C     VAR27 - SMD, MM.
C     QPG   - CALCULATED VALUE OF TUH QP, CUMECS (OPTIONAL).
C     TPG   - CALCULATED VALUE OF TUH TP, HOURS (OPTIONAL).
C     NHRF  - NUMBER OF EFFECTIVE RAINFALL ORDINATES.
C     ERF   - ARRAY OF EFFECTIVE RAINFALL ORDINATES.
C     NSFL  - NUMBER OF RESPONSE RUNOFF ORDINATES.
C     SFL   - ARRAY OF RESPONSE RUNOFF ORDINATES.
C
C-----------------------------------------------------------------------
C
      DIMENSION FLOW1(100),FLOW2(100),TIME1(100),TIME2(100),DUHQ(100),
     *          DUHT(100),TUH(100),OBER(100,6)
      DATA OBER/600*0.0/
      TINT=0.5
      X1=1.0/TINT
      NTB=0
      ITOT=0
      READ (5,10) NFLOW,(FLOW1(I),I=1,NFLOW)
   10 FORMAT(I5/(8F10.5))
      NTIME=NFLOW
      TIME1(1)=0.0
      DO 20 I=2,NTIME
      J=I-1
   20 TIME1(I)=TIME1(J)+TINT
      CALL STATS(TINT,NFLOW,FLOW1,QP1,TP1)
      DO 30 I=1,NFLOW
      DUHQ(I)=FLOW1(I)/QP1
   30 DUHT(I)=TIME1(I)/TP1
   40 READ (5,50,END=160)-AREA,TINT,ICN,IEN
   50 FORMAT(F10.2,F5.2,2I5)
      L5=0
      ITOT=ITOT+1
      WRITE (6,60) ICN,IEN
   60 FORMAT(1H1,//,' DERIVATION AND CONVOLUTION OF A DUH FOR ',
     *I5,', EVENT',I3,'.',/,57('-'))
      CALL SCALEF(QPP,TPP,L5)
      IF(L5.EQ.0)GO TO 70
      ITOT=ITOT-1
      GO TO 150
   70 DO 80 I=1,NFLOW
      FLOW2(I)=DUHQ(I)*QPP
   80 TIME2(I)=DUHT(I)*TPP
      WRITE (6,90)
   90 FORMAT(//,' INCREMENT',4X,'TIME',7X,'DISCHARGE',11X,'TIME/TP',4X,
     *'DISCHARGE/QP',9X,'TIME',7X,'DISCHARGE',/)
      WRITE (6,100) (I,TIME1(I),FLOW1(I),DUHT(I),DUHQ(I),TIME2(I),FLOW2(I
     *), I=1,NFLOW)
  100 FORMAT(4X,I3,7X,F4.1,7X,F9.5,10X,F8.5,7X,F7.5,10X,F5.2,7X,F9.5)
      CALL TUHEST(TUH,NTB,TIME2,FLOW2,NFLOW,TINT,X1)
      CALL REKAL(TINT,TUH,NTB)
      CALL STATS(TINT,NTB,TUH,QP2,TP2)
      WRITE (6,110)
  110 FORMAT(1H1,///,' ESTIMATED TUH:')
      C3=(NTB-1)*TINT
      WRITE (6,120) QP2,TP2,C3
  120 FORMAT(//,8X,'PEAK',6X,'TP',7X,'TB',/F12.5,F8.1,F9.1)
      WRITE (6,130)
  130 FORMAT(/,26('-----'))
      WRITE (6,140) (I,TUH(I),I=1,NTB)
  140 FORMAT(6(I3,',',F11.5,6X))
      WRITE (6,130)
  150 CALL CONVOL(TUH,NTB,AREA,L5,ITOT,OBER)
      GO TO 40
  160 CALL OBERST(OBER,ITOT)
      WRITE (6,170)
  170 FORMAT(////,' END OF THE JOB.')
      STOP
      END
      SUBROUTINE STATS(TINT,N,ARRAY,QP,TP)
C
C     CALCULATES THE PEAK DISCHARGE AND TIME TO PEAK.
C
      DIMENSION ARRAY(100)
      QP=0.0
      TP=0.0
      DO 180 I=1,N
      IF(ARRAY(I).LT.QP)GO TO 180
      QP=ARRAY(I)
      TP=(I-1)*TINT
  180 CONTINUE
      RETURN
      END
      SUBROUTINE SCALEF(QP,TP,L5)
C
C     CALCULATES PEAK DISCHARGE AND TIME TO PEAK FROM
C     GIVEN STORM AND CATCHMENT CONDITIONS.
C     REGRESSION EQUATIONS FOR THE BEVERLEY BROOK ONLY.
C
      READ (5,190) ICN,IEN,VAR05,VAR06,VAR07,VAR25,VAR27,QPG,TPG
  190 FORMAT(2I5,5F10.3,F10.4,F10.2)
      VARA01=VAR06/VAR05
      VAR24=1.6124163+(VAR05*0.28278249)+(VAR06*0.17141147)+(-0.16497563
     **VAR07)+(VAR25*0.017108065)+(-0.0046975845*VAR27)+(VARA01*0.017168
     *065)
      IF(VAR24.GT.0.0)GO TO 210
      WRITE (6,200) VAR24
  200 FORMAT(/,' CALCULATED VALUE OF VAR24 WAS',F10.4,
     *', ANALYSIS OF THIS EVENT WAS ABANDONED.')
      L5=1
      RETURN
  210 CONTINUE
      TP=1.22650781*(VAR24**0.59646146)*(VAR25**0.050380014)*
     *(VAR27**0.047913909)*(VAR05**(-0.24289357))
      QP=116.63100019*(VAR27**0.026351351)*(VAR07**0.29474203)*
     *(VAR24**(-0.32020119))*(VAR06**(-0.18928325))*
     *(VARA01**(-0.16524179))
      WRITE (6,220) QP,TP
  220 FORMAT(//,' INITIAL ESTIMATES:  QP =',F9.4,'      TP =',F8.2)
      IF(QPG.GT.0.0)WRITE (6,230) QPG,TPG
  230 FORMAT(1H+,70X,'CALCULATED VALUES:  QP =',F9.4,'      TP =',F8.2)
      RETURN
```

```
      END
      SUBROUTINE TUHEST(TUH,NTB,TIME2,FLOW2,NFLOW,TINT,X1)
C
C     INTERPOLATES VALUES OF DISCHARGE ORDINATES FOR THE
C     TUH AT TIME INCREMENTS OF 'TINT'.
C
      DIMENSION TUH(100),TIME2(100),FLOW2(100)
      SST1(X)=INT(X+SIGN(0.5,X))
      SST2(X)=(SST1(X*2.0))/2.0
      NTB=(SST2(TIME2(NFLOW)))*X1+1.0
      TUH(1)=FLOW2(1)
      X=TINT
      DO 280 M=2,NTB
      DO 240 I=1,NFLOW
      IF(TIME2(I)-X) 240,250,260
  240 CONTINUE
  250 TUH(M)=FLOW2(I)
      GO TO 270
  260 J=I-1
      A=TIME2(J)
      B=FLOW2(J)
      C=TIME2(I)
      D=FLOW2(I)
      E=X-A
      TUH(M)=B+(((D-B)/(C-A))*E)
  270 X=X+TINT
  280 CONTINUE
      RETURN
      END
      SUBROUTINE REKAL(TINT,ARRAY,N)
C
C     RESCALES A KERNAL FUNCTION TO A UNIT VOLUME.
C
      DIMENSION ARRAY(100)
      VOL=100.0/(0.36*TINT)
      VOLA=0.0
      DO 290 I=1,N
  290 VOLA=VOLA+ARRAY(I)
      COR=VOL/VOLA
      DO 300 I=1,N
  300 ARRAY(I)=ARRAY(I)*COR
      A=(COR-1.0)*100.0
      WRITE (6,310) A
  310 FORMAT(////,' ALL ORDINATES WERE SUBSEQUENTLY ADJUSTED BY',F7.2,'
     *% TO MAINTAIN A UNIT VOLUME.')
      RETURN
      END
      SUBROUTINE CONVOL(EUH3,NUH2,AREA,L5,ITOT,OBER)
C
C     CONVOLUTES EFFECTIVE RAINFALL WITH A TUH.
C
      DIMENSION EUH3(50),ERF(50),SFL(100),RSFL(150),OBER(100,6)
      READ (5,320) ICN,IEN
  320 FORMAT(2I5)
      READ (5,330) NHRF,(ERF(I),I=1,NHRF)
      READ (5,330) NSFL,(SFL(I),I=1,NSFL)
  330 FORMAT(I3/(8F10.4))
      IF(L5.EQ.1) RETURN
      WRITE (6,340)
  340 FORMAT(/////,' CONVOLUTION OF TUH AND EFFECTIVE RAINFALL')
      DO 350 I=1,150
  350 RSFL(I)=0.0
      DO 360 I=1,NHRF
      DO 360 J=1,NUH2
      IJ=I+J-1
  360 RSFL(IJ)=RSFL(IJ)+ERF(I)*EUH3(J)*AREA/1000.0
      CALL OBERFN(NSFL,SFL,RSFL,ICN,IEN,ITOT,OBER)
      WRITE (6,370)
  370 FORMAT(///3(8X,'OBSERVED',5X,'PREDICTED',10X))
      WRITE (6,380) (I,SFL(I),RSFL(I),I=1,NSFL)
  380 FORMAT(3(I4,'.',F11.4,3X,F11.4,10X))
      WRITE (6,390)
  390 FORMAT(/,26('-----'))
      RETURN
      END
      SUBROUTINE OBERFN(NY,Y,PY,ICN,IEN,ITOT,OBER)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(100),PY(150),OBER(100,6)
      EOO=0.0
      DO 400 I=1,NY
  400 EOO=EOO+Y(I)
      PKY=0.0
      PKPY=0.0
      DO 420 I=1,NY
      IF(PY(I).LT.PKPY)GO TO 410
      PKPY=PY(I)
      TPPY=(I-1)*0.5
  410 IF(Y(I).LT.PKY)GO TO 420
      PKY=Y(I)
      TPY=(I-1)*0.5
  420 CONTINUE
      X=0.0
      DO 430 I=1,NY
  430 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EOO))*100.0
      X=0.0
      DO 440 I=1,NY
  440 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EOO))*100.0
      HPY=PKPY/2.0
      X=0.0
      DO 450 I=1,NY
      IF(PY(I).LT.HPY)GO TO 450
      X=X+((Y(I)-PY(I))**2)
  450 CONTINUE
      Z3=(SQRT(X/EOO))*100.0
      Z4=((PKPY-PKY)/PKY)*100.0
      Z5=TPPY-TPY
      OBER(ITOT,1)=Z4
      OBER(ITOT,3)=Z5
      OBER(ITOT,5)=Z1
      OBER(ITOT,6)=Z3
      WRITE (6,460) ICN,IEN,Z1,Z2,Z3,PKY,PKPY,Z4,TPY,TPPY,Z5
  460 FORMAT(///,' OBJECTIVE ERROR FUNCTIONS FOR ICN =',I6,' AND IEN =',
     *I3,//,' 1. ISE  = ',F7.3,' %',
     */,' 2. ISE2 = ',F7.3,' %',
     */,' 3. PISE = ',F7.3,' %',
     */,' 4. PEAKS:',
     */,'      OBSERVED  = ',F10.4,' CUMECS',
     */,'      PREDICTED = ',F10.4,' CUMECS',
     */,'      ERROR     = ',F10.4,' %',
     */,' 5. TIME TO PEAK:',
     */,'      OBSERVED  = ',F4.1,' HOURS',
     */,'      PREDICTED = ',F4.1,' HOURS',
     */,'      ERROR     = ',F4.1,' HOURS')
      RETURN
      END
      SUBROUTINE OBERST(OBER,ITOT)
C
```

```
C     CALCULATES THE MEAN AND STANDARD DEVIATION OF THE OBJECTIVE
C     ERROR FUNCTIONS.
C
      DIMENSION OBER(100,6),SUM(6),AV(6),S(6),SD(6)
      M=6
      DO 470 I=1,ITOT
      OBER(I,2)=ABS(OBER(I,1))
  470 OBER(I,4)=ABS(OBER(I,3))
      DO 480 I=1,M
      S(I)=0.0
  480 SUM(I)=0.0
      DO 490 I=1,M
      DO 490 J=1,ITOT
  490 SUM(I)=SUM(I)+OBER(J,I)
      DO 500 I=1,M
  500 AV(I)=SUM(I)/ITOT
      DO 510 I=1,M
      DO 510 J=1,ITOT
  510 S(I)=S(I)+((OBER(J,I)-AV(I))**2)
      DO 520 I=1,M
  520 SD(I)=SQRT(S(I)/ITOT)
      WRITE (6,530) (AV(I),I=1,M),(SD(I),I=1,M),ITOT
  530 FORMAT(1H1,//,' SUMMARY OF THE OBJECTIVE ERROR FUNCTIONS:',///,16X
     *,'OPE %',3X,'A OPE %',4X,'TPE HR',2X,'A TPE HR',5X,'ISE %',4X,'PIS
     *E %',//,' MEAN',6X,6F10.3,//,' S.D.',6X,6F10.3,///,' NUMBER OF CAS
     *ES =',I3)
      RETURN
      END
```

Appendix 10: Source listing of OPTMIZ

```
      IMPLICIT REAL(A-Z)
      INTEGER I,J,NX,NY,NDELT,NTNT,L,K,MAXO,MINO,MODEL,SKIP,N,MOD1,
     *TP1,TP2,IOPT,IC,ICN,IEN
      REAL DUH,PY,P,X,Y,DUR
      DIMENSION DUH(150),X(150),SKIP(30),Y(150),PY(150)
      DATA X/150*0.0/,Y/150*0.0/,DUR/1.0/,NDELT/8/
      NSTV(X)=INT(X+SIGN(0.5,X))
      READ (5,10) MODEL
   10 FORMAT(I2)
   20 WRITE (6,30)
   30 FORMAT(1H1)
      READ (5,40,END=250) ICN,IEN
   40 FORMAT(2I5)
      WRITE (6,50) MODEL,ICN,IEN
   50 FORMAT(' PARAMETER OPTIMISATION OF CONCEPTUAL MODEL',I3,' FOR CATC
     *HMENT NUMBER ',I5,', EVENT NUMBER ',I5,'.',//)
      READ (5,60) NX,(X(I),I=1,NX)
      READ (5,60) NY,(Y(I),I=1,NY)
   60 FORMAT(I3/(8F10.4))
      J=NX+1
      DO 70 I=J,NY
   70 X(I)=0.0
      VX=0.0
      VY=0.0
      DO 80 I=1,NY
      VX=VX+X(I)
   80 VY=VY+Y(I)
      DO 90 I=1,NX
   90 X(I)=X(I)/VX
      DO 100 I=1,NY
  100 Y(I)=Y(I)/VY
      READ (5,110) PARM1,PARM2
  110 FORMAT(2F10.4)
C-----CALCULATE ERROR FOR PARAMETERS AS CALCULATED BY IUH MOMENTS.
      CALL RECON(NX,NY,DUR,NDELT,X,PARM1,PARM2,DUH,PY,MODEL)
      CALL OBERFN(NY,VY,Y,PY,Z1,Z2,Z3,Z4,Z5)
      ISE1 =(NSTV(ABS(Z1)*1000))/1000
      PISE1=(NSTV(ABS(Z3)*1000))/1000
      PEAK1=(NSTV(ABS(Z4)*1000))/1000
      TP1=Z5
      WRITE (6,120)
  120 FORMAT(///,' ITERATION',13X,'ISE',16X,'ISE2',16X,'PISE',16X,'PEAK'
     *,9X,'TP',/)
      J=0
      WRITE (6,130) J,Z1,Z2,Z3,Z4,Z5
  130 FORMAT(5X,I5,4(5X,F15.10),3X,F4.1)
C-----DECIDE IF OPT SHOULD BE ADDED OR SUBTRACTED.
      IC=0
      OPT=0.0001
  140 PARM2A=PARM2+OPT
      CALL RECON(NX,NY,DUR,NDELT,X,PARM1,PARM2A,DUH,PY)
      CALL OBERFN(NY,VY,Y,PY,Z1,Z2,Z3,Z4,Z5)
      ISE2 =(NSTV(ABS(Z1)*1000))/1000
      PISE2=(NSTV(ABS(Z3)*1000))/1000
      PEAK2=(NSTV(ABS(Z4)*1000))/1000
      TP2=Z5
      ICHECK=0
      IF(PEAK2.LT.PEAK1) ICHECK=1
      IF(TP2.LE.TP1) ICHECK=ICHECK+1
      IF(ISE2.LT.ISE1) ICHECK=ICHECK+1
      IF(PISE2.LT.PISE1) ICHECK=ICHECK+1
      IF(ICHECK.GE.3)GO TO 170
      IF(IC.GT.0)GO TO 150
      OPT=-0.0001
      IC=1
      GO TO 140
  150 WRITE (6,160) PARM1,PARM2
  160 FORMAT(///,' OPTIMUM SOLUTION:',/,' PARAMETER 1 (FIXED) =',F10.4,/
     *,' PARAMETER 2 (OPT.)  =',F10.4)
      GO TO 20
  170 CONTINUE
      J=1
      WRITE (6,130) J,Z1,Z2,Z3,Z4,Z5
      PEAK1=PEAK2
      TP1=TP2
      ISE1=ISE2
      PISE1=PISE2
      PARM2=PARM2A
C-----ENTER ITERATION LOOP.
      DO 230 I=2,100
      PARM2B=PARM2
      PARM2=PARM2+OPT
      CALL RECON(NX,NY,DUR,NDELT,X,PARM1,PARM2,DUH,PY)
      CALL OBERFN(NY,VY,Y,PY,Z1,Z2,Z3,Z4,Z5)
      ISE2 =(NSTV(ABS(Z1)*1000))/1000
      PISE2=(NSTV(ABS(Z3)*1000))/1000
      PEAK2=(NSTV(ABS(Z4)*1000))/1000
      TP2=Z5
      ICHECK=0
      IF(PEAK2.LT.PEAK1) ICHECK=1
      IF(TP2.LE.TP1) ICHECK=ICHECK+1
      IF(ISE2.LT.ISE1) ICHECK=ICHECK+1
      IF(PISE2.LT.PISE1) ICHECK=ICHECK+1
      IF(ICHECK.GE.3)GO TO 220
      WRITE (6,160) PARM1,PARM2B
      CALL RECON(NX,NY,DUR,NDELT,X,PARM1,PARM2B,DUH,PY)
      WRITE (6,180)
  180 FORMAT(////,' TIME  RAINFALL',5X,'   MODEL',4X,'PREDICTED',5X,'OBS
     *ERVED',8X,'ERROR',6X,'PREDICTED',4X,'OBSERVED',7X,'ERROR'/)
      SSOD=0.0
      TOTDUH=0.0
      DO 200 J=1,NY
      AY=Y(J)*VY
      BY=PY(J)*VY
      CY=AY-BY
      DIF=Y(J)-PY(J)
      WRITE (6,190) J,X(J),DUH(J),PY(J),Y(J),DIF,BY,AY,CY
  190 FORMAT(1H ,I4,5(F10.7,3X),3(F12.4))
      TOTDUH=TOTDUH+DUH(J)
  200 SSOD=SSOD+DIF*DIF
      RMS=(SSOD/NY)**0.5
      WRITE (6,210) TOTDUH,RMS
  210 FORMAT(1H0,4X,'DUH TOTAL =',F9.7,7X,'RMS ERROR =',F10.7)
      GO TO 20
  220 PEAK1=PEAK2
      TP1=TP2
      ISE1=ISE2
      PISE1=PISE2
      WRITE (6,130) I,Z1,Z2,Z3,Z4,Z5
  230 CONTINUE
      WRITE (6,240) PARM1,PARM2
  240 FORMAT(///,' AFTER 100 ITERATIONS AN OPTIMUM SOLUTION WAS NOT FOUN
     *D.',//,' PARAMETER VALUES AFTER 100 ITERATIONS:',/,' PARAMETER 1 (
     *FIXED) =',F10.4,/,' PARAMETER 2 (OPT.)  =',F10.4)
      GO TO 20
  250 WRITE (6,260)
  260 FORMAT(///,' END OF THE JOB.',///)
      STOP
      END
      SUBROUTINE RECON(NX,NY,DUR,NDELT,X,PARM1,PARM2,DUH,PY,MODEL)
C
C     CALCULATES THE ORDINATES OF A MODELS S-CURVE AT THE SPECIFIED
```

```
C     FREQUENCY OF POINTS ON THE TIME INTERVAL BETWEEN SUCCESSIVE
C     ORDINATES OF RAINFALL AND RUNOFF BY INTEGRATING NUMERICALLY THE
C     ANALYTICAL EXPRESSION FOR THE MODELS IUH PROVIDED BY THE FUNCTION
C     IUH.  THE S-CURVE IS DIFFERENCED TO GIVE THE ORDINATES OF THE UNIT
C     PULSE RESPONSE (UPR).  THE UPR IS CONVERTED TO VOLUMES OF RUNOFF
C     IN SUCCESIVE INTERVALS OF TIME IN CORRESPONDENCE WITH THE RAINFALL
C     AND RUNOFF DATA TO YIELD THE MODEL DUH.  PREDICTED OUTFLOW IS
C     CALCULATED BY CONVOLUTING THE MODEL DUH WITH THE NORMALIZED
C     ACTIVE RAINFALL.
C
      IMPLICIT REAL (A-Z)
      REAL X(150),S(1000),SDUH(1000),DUH(150),PY(150),PARM1,PARM2
      INTEGER I,J,NX,NY,NDELT,NTNT,L,K,MAXO,MINO,MODEL
C
C     CALCULATE S-CURVE BY TRAPEZOIDAL RULE.
C
      NTNT=NY*NDELT
      DT=DUR/NDELT
      S(1)=0.0
      F1=IUH(0.0,PARM1,PARM2)
      DO 280 I=1,NTNT
      T=I*DT
      F2=IUH(T,PARM1,PARM2)
  270 S(I+1)=S(I)+DT*(F1+F2)/2.0
  280 F1=F2
C
C     FIND UNIT PULSE RESPONSE FROM S-CURVE AT INTERVALS OF DT.
C
      L=NTNT-NDELT+1
      DO 290 I=1,L
      J=NTNT-I+2
  290 SDUH(J)=(S(J)-S(J-NDELT))/DUR
      DO 300 I=1,NDELT
  300 SDUH(I)=S(I)
C
C     DUH SAMPLED AT INTERVALS OF DUR.
C
      DO 310 I=1,NY
      J=NDELT*(I-1)+1
      DUH(I)=SDUH(J)
  310 CONTINUE
C
C     FIND DUH AT INTERVALS OF DUR BY TRAPEZOIDAL RULE.
C
      J=0
      L=1
      DUH(L)=0.0
      DO 330 I=1,NTNT
      IF(J.NE.NDELT)GO TO 320
      J=0
      L=L+1
      DUH(L)=0.0
  320 J=J+1
  330 DUH(L)=DUH(L)+DT*(SDUH(I)+SDUH(I+1))/2.0
C
C     CONVOLUTE X WITH DUH TO RECONSTRUCT Y AS PY.
C
      DO 340 I=1,NY
      PY(I)=0.0
      K=MINO(I,NX)
      DO 340 J=1,K
  340 PY(I)=PY(I)+DUH(I-J+1)*X(J)
      RETURN
      END
      SUBROUTINE OBERFN(NY,VY,Y,PY,Z1,Z2,Z3,Z4,Z5)
C
C     OBJECTIVE ERROR FUNCTIONS.
C
      DIMENSION Y(150),PY(150)
C-----FIND SUM OF OBSERVED (Y) Q ORDINATES.
      EOO=0.0
      DO 350 I=1,NY
  350 EOO=EOO+Y(I)
C-----FIND PEAK DISCHARGE AND TIME TO PEAK.
      PKY=0.0
      PKPY=0.0
      DO 370 I=1,NY
      IF(PY(I).LT.PKPY)GO TO 360
      PKPY=PY(I)
      TPPY=(I-1)*0.5
  360 IF(Y(I).LT.PKY)GO TO 370
      PKY=Y(I)
      TPY=(I-1)*0.5
  370 CONTINUE
      PKPY2=PKPY*VY
      PKY2=PKY*VY
C-----1. INTEGRAL SQUARES ERROR (ISE).
      X=0.0
      DO 380 I=1,NY
  380 X=X+((Y(I)-PY(I))**2)
      Z1=(SQRT(X/EOO))*100.0
C-----2.
      X=0.0
      DO 390 I=1,NY
  390 X=X+((Y(I)**2)-(PY(I)**2))
      IF(X.LT.0.0) X=ABS(X)
      Z2=(SQRT(X/EOO))*100.0
C-----3. PARTIAL INTEGRAL SQUARE ERROR (PISE).
      HPY=PKPY/2.0
      X=0.0
      DO 400 I=1,NY
      IF(PY(I).LT.HPY)GO TO 400
      X=X+((Y(I)-PY(I))**2)
  400 CONTINUE
      Z3=(SQRT(X/EOO))*100.0
C-----4. PEAK DISCHARGE.
      Z4=((PKPY-PKY)/PKY)*100.0
C-----5. TIME TO PEAK.
      Z5=TPPY-TPY
      RETURN
      END
      REAL FUNCTION IUH(T,MODEL,A,B)
      IF(MODEL.EQ.20)GO TO 430
C-----ROUTED TRIANGLE.
      H=4.0/(A*A)
      H1=B*EXP(-T/B)
      IF(T.GT.A/2.0)GO TO 410
      IUH=H*(T-B+H1)
      RETURN
  410 H2=EXP(A/(2.0*B))
      IF(T.GT.A)GO TO 420
      IUH=H*(B-T+A-2.0*H1*H2+H1)
      RETURN
  420 H3=EXP(A/B)
      IUH=H*(1.0-2.0/H2+1.0/H3)*H3+H1
      RETURN
C-----CONVECTIVE DIFFUSION REACH.
      IUH=0.0
      IF(T.EQ.0.0) RETURN
      IUH=A/(3.1415926536*T**3)**0.5
      IUH=IUH*EXP(-(A-B*T)**2/T)
      RETURN
```