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Magnetic Field Generation in Stars

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Abstract Enormous progress has been made on observing stellar magnetism in stars from the main sequence (particularly thanks to the MiMeS, MAGORI and BOB surveys) through to compact objects. Recent data have thrown into sharper relief the vexed question of the origin of stellar magnetic fields, which remains one of the main unanswered questions in astrophysics. In this chapter we review recent work in this area of research. In particular, we look at the fossil field hypothesis which links magnetism in compact stars to magnetism in main sequence and pre-main sequence stars and we consider why its feasibility has now been questioned particularly in the context of highly magnetic white dwarfs. We also review the fossil versus dynamo debate in the context of neutron stars and the roles played by key physical processes such as buoyancy, helicity, and superfluid turbulence, in the generation and stability of neutron star fields.

Independent information on the internal magnetic field of neutron stars will come from future gravitational wave detections. Coherent searches for the Crab pulsar with the Laser Interferometer Gravitational Wave Observatory (LIGO) have already constrained its gravitational wave luminosity to be $\lesssim 2\%$ of the observed spin-down luminosity, thus placing a limit of $\lesssim 10^{16}\,$G on the internal field. Indirect spin-down limits inferred from recycled pulsars also

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yield interesting gravitational-wave-related constraints. Thus we may be at the dawn of a new era of exciting discoveries in compact star magnetism driven by the opening of a new, non-electromagnetic observational window.

We also review recent advances in the theory and computation of magnetohydrodynamic turbulence as it applies to stellar magnetism and dynamo theory. These advances offer insight into the action of stellar dynamos as well as processes which control the diffusive magnetic flux transport in stars.

Keywords Magnetic fields · Main sequence stars · white dwarfs · neutron stars

1 Introduction

It was Hale (1908), one of the greatest astrophysicist of the twentieth century, who built the first spectroheliograph and used it to establish that sunspots are magnetic and grouped in pairs of opposite polarities. Further pioneering work on solar magnetism conducted by Hale and collaborators in 1919 revealed an East-West direction of polarity in the sunspots' magnetic fields exhibiting a mirror symmetry with respect to the solar equator. Such polarity was observed to undergo inversion according to the 11 years solar cycle. This is commonly known as the "Hale-Nicholson law".

The first detection of a magnetic field in a star other than our own Sun, 78 Vir, was obtained in 1947 by Babcock. In 1958, Babcock published the first catalogue of magnetic stars and in 1960 he discovered what is still today the most magnetic main sequence star known, HD 215441 ($\sim 3.4 \times 10^4$ G). This became known as "Babcock's star". Surface fields discovered in more recent years that rival in strength that of Babcock's star are those of HD 154708 ($\sim 2.45 \times 10^4$ G; Hubrig et al. 2005), HD 137509 ($\sim 2.9 \times 10^4$ G; Mathys 1995; Kochukhov 2006) and HD 75049 ($\sim 3 \times 10^4$ G; Freyhammer et al. 2008; Elkin et al. 2010).

Following the detections of strong fields in main sequence stars, Blackett (1947) suggested that if magnetic flux is conserved, some white dwarfs should exhibit magnetic fields of up to $10^7 - 10^8$ G. However, spectroscopic surveys aimed at detecting magnetic fields in white dwarfs yielded negative results (Preston 1970). Kemp (1970) argued that electrons spiralling in a magnetic field would emit linearly and circularly polarised radiation that should be detectable in the continuum of strongly magnetic white dwarfs. A spectropolarimetric survey of white dwarfs led to the discovery of strong circular polarisation in the continuum of the white dwarf Grw+70°8247 (Kemp et al. 1970).

Baade and Zwicky (1934) first suggested that some stars could be made up of neutrons and that a supernova could be the result of a rapid transition of a normal star into a neutron star. Baade (1942) and Minkowski (1942) found unusual emission arising from the central parts of the Crab Nebula. Later on radio pulsations in this nebula were discovered by Staelin and Reifenstein

(1968). Such radio emission had been predicted by Shklovsky (1953) as caused by relativistic electrons spiralling around magnetic field lines.

Woltjer (1964) first proposed that under magnetic flux conservation, if a star contracts to the density expected in a neutron star, then the surface field strength could be amplified to values of up to $10^{14} - 10^{16}$ G. The first pulsar, a highly magnetised rapidly spinning neutron star (Pacini 1968), was discovered in 1967 by Jocelyn Bell (Hewish et al. 1968).

In this chapter we review progress made on the origin of magnetic fields in stars. The origin of magnetic fields is still a major unresolved problem in astrophysics. The "fossil field" hypothesis is often invoked to link magnetism in compact stars to magnetism on the main sequence (see Sect. 3.1 and Sect. 4.1). In the fossil scenario, some fraction of the magnetic flux of the progenitor star is conserved during the collapse process, because the stellar plasma is highly conducting and hence, by Faraday's law, the magnetic field is 'frozen in' to the fluid (Woltjer 1964; Ruderman 1972; Braithwaite and Spruit 2004a; Ferrario and Wickramasinghe 2006a, 2007, 2008a). Under these circumstances, the field strength B scales with radius R of the star as $B \propto R^{-2}$. A main sequence B star with radius $5R_{\odot}$ and field $\sim 1,000$ G compresses to give a neutron star with a field of $\sim 10^{14}$ G; a main sequence A star with radius $1.6R_{\odot}$ and field $\sim 1,000$ G compresses to give a magnetic white dwarf with a field of $\sim 10^7$ G. The fossil field scenario is indeed quite attractive and can explain the existence of magnetic fields even in the strongest magnets in the universe, the so-called "magnetars". However, the feasibility of the fossil field hypothesis does have its problems and has been recently questioned, in the context of highly magnetic white dwarfs, on the basis of recent observational results related to their binarity (see Sect. 3.2). The fossil versus dynamo-generated fields debate for neutron stars is analysed in Sect. 4.1.1. Alternative explanations for the origin of fields in stars are presented in Sect. 2, 3.2 and 4.2.

In this chapter we also review recent progress in magnetohydrodynamics (MHD) turbulence theory and computation as it applies to stellar magnetism (see Sect. 5), which underpins the operation of stellar dynamos, and controls diffusive magnetic flux transport in stars.

2 Magnetism in non-degenerate stars

Magnetic fields in main sequence stars have been measured mainly via spectropolarimetry. Different techniques measure different field properties, e.g., line-of-sight component, volume-averaged field strength, or dipole moment. A thorough review on magnetic field measurements of non-degenerate stars across the Hertzprung-Russell diagram can be found in this book in the chapter by Linsky & Schöller.

Direct measurements of magnetic fields in the chemically peculiar main sequence Ap and Bp stars, which form about 10% of stars in the $1.6 - 5\,M_{\odot}$ range, have revealed the existence of strong ($\sim 3\times10^2 - 3\times10^4$ G) large scale fields (e.g. Aurière et al. 2007; Donati and Landstreet 2009). Landstreet et al. (2007a) found that the majority of magnetic objects have an average longitudinal field (line of sight component of the surface field) $B > 0.25$ kG in every $1\,M_{\odot}$ mass bin between $2M_{\odot}$ and $9M_{\odot}$ (Bagnulo et al. 2006a; Landstreet et al. 2007a). The lack of fields below 300 G, value that corresponds to the strength at which the magnetic field is in equipartition with the gas pressure in the stellar photosphere, is not a detection threshold effect since Aurière et al. (2010) have set a 3σ upper limit of longitudinal fields down to $1 - 10\,$G depending on stellar brightness. The absence of magnetic stars below this cutoff has been referred to as "the Ap/Bp magnetic desert". This indicates that Ap and Bp stars are not simply the high field tail of a continuous field distribution. Landstreet et al. (2008) also find that the field strengths seem to show some decline with stellar age but the field incidence does not. Such large scale fields are observed throughout the main sequence phase and, in more recent years, have also been observed in a small number of stars on the red giant branch (Aurière et al. 2008).

The recent detections of sub-gauss fields in Vega (Lignières et al. 2009), and possibly in the Am stars Sirius A (Petit et al. 2011) and β UMa and θ Leo (Blazère et al. 2014) have unveiled a new class of magnetic stars which is at the 1 gauss end of the magnetic desert, thus potentially suggesting a dichotomy between strong and ultra-weak magnetic fields among intermediate-mass stars (Lignières et al. 2014). This intriguing observational result has prompted Braithwaite and Cantiello (2013) to investigate the origin of these low fields. They argued that such fields could be the remnants of fields already present or formed during or immediately after the star formation stage. Hence, these fields would still be evolving on a timescale that is comparable to the age of the star. According to these studies, all intermediate and high mass stars lacking strong fields should display sub-gauss field strengths that would slowly decline over their main sequence lifetime. There has been some recent effort to test this prediction by Wade et al. (2014a), Neiner et al. (2014b) and Neiner et al. (2014a) but without any clear conclusion.

Alecian et al. (2008) conducted a spectropolarimetric study of Herbig Ae/Be (HAeBe) objects (Herbig 1960). HAeBe stars are pre-main sequence stars of $2 - 15\,M_{\odot}$ which are still embedded in their protostellar gas-dust envelope and exhibit emission lines in spectra of type A/B. The observations of Alecian et al. (2008), Alecian et al. (2009), Hubrig et al. (2009) and more recently Alecian et al. (2013a) and Hubrig et al. (2013) have revealed that about 7% of HAeBe objects are magnetic. These studies have also found that they display large scale dipolar fields of strength comparable to that of Ap and Bp stars, under the assumption of conservation of magnetic flux. Interestingly, although some magnetic HAeBe stars are in the totally radiative evolutionary phase while others have already developed convective cores, they seem to share the same magnetic field strength and structure thus indicating that the convective core is not responsible for generation or destruction of fields (Alecian et al. 2013b).

The Magnetism in Massive Stars (MiMeS, Wade et al. 2011) project, the MAGORI (Hubrig et al. 2011; Schöller et al. 2011) project and the B fields in OB star (BOB) Collaboration (Hubrig et al. 2014) have conducted large surveys of bright and massive Galactic stars of spectral types B and O. It is now clear that about 10% of all stars with radiative envelopes in the mass range $1.5-50\,\mathrm{M}\odot$ possess large scale mostly dipolar magnetic fields (Grunhut and Wade 2013). Even more interestingly, the MiMes project has revealed that all of the basic field characteristics do not vary significantly from the coolest spectral types F0 ($\sim 1.5\,\mathrm{M}\odot$) to the hottest spectral types O4 ($\sim 50\,\mathrm{M}\odot$) stars thus supporting a common formative scenario over a very large range of stellar masses (Wade et al. 2014b). Furthermore, the upper dipolar field limits placed on the undetected magnetic O stars sample studied by the MiMeS collaboration are 40 G at 50% confidence, and 130 G at 80% confidence (Wade and the MiMeS Collaboration 2014). This result seems to indicate that the field distribution of massive stars may also have a magnetic desert, similar to that observed in intermediate-mass stars. However, the recent work of Fossati et al. (2014), who detected weak longitudinal fields of $\lesssim 30\,\mathrm{G}$ in the early B-type main sequence stars β CMa and ϵ CMa, appears to support a more continuous distribution of fields in massive stars. They also claim that weak fields in massive stars could be more widespread than currently observed because of the numerous observational biases associated with the detection of weak fields in massive stars using current instrumentation and techniques.

The vexed question on the origin of fields in main sequence stars is still unanswered. The two main hypothesis are the fossil field, according to which magnetism in stars is a relic of the interstellar field from which the star was born (e.g. Borra et al. 1982; Moss 2001), and dynamo action taking place in the rotating cores of main sequence stars. Both theories are unable to explain why only a small subset of main sequence stars is magnetic, although the fossil field theory does assume that differences such as initial density and magnetic field strength in interstellar clouds could be responsible for this. However, it is peculiar that observations have never revealed the presence of magnetism in both components of a binary system. The BinaMIcS (Binarity and Magnetic Interactions in various classes of Stars) project (Alecian et al. 2014; Neiner and Alecian 2013) is a collaborative endeavour that has been recently setup to investigate the phenomenon of magnetism in close (orbital periods shorter than 20 d) binary systems. The study of the magnetic properties of these systems, at a detection limit of 150 G, will give us crucial information on the origin of fields in magnetic main sequence stars which are well known to be rare in binaries (Carrier et al. 2002). Since the components of binaries share the same history, the study of the two stars will help us unravel the importance of their physical parameters and their studies should enable us to distinguish effects caused by initial conditions at formation from those caused later on by evolutionary processes. Preliminary studies of 314 massive stars in close binaries have revealed the presence of fields in 6 systems yielding an incidence of magnetism of $\lesssim 2\%$ which is up to 5 times lower than what is observed in single stars. This confirms the results of Carrier et al. (2002) which were based on a smaller sample of A-type stars.

In order to explain the rarity of short period binaries containing magnetic main sequence stars and also the fact that magnetism is only present in a small percentage of single main sequence stars, Ferrario et al. (2009) proposed that magnetic fields in main sequence stars could form when two proto-stellar objects merge late on their approach to the main sequence and when at least one of them has already acquired a radiative envelope. The N-body simulations of pre-main sequence evolution of Railton et al. (2014) have indicated that in high initial cluster densities the number of collisions between stars is twice as high as that of stars already on the main sequence. Furthermore, they find that massive stars generally form through the merging of lower mass stars. Thus, the expectation is that the incidence of magnetism should increase with mass, which seems to be demonstrated by the observations of Power et al. (2007) for a volume limited sample of A and B type stars with $M \leq 4\,\mathrm{M}_{\odot}$. Observations also seem to be in general agreement with the predictions of Ferrario et al. (2009) that the Ap and Bp stars should have pre-main sequence progenitors with similar magnetic field flux and structure.

Another merger scenario has been proposed by Tutukov and Fedorova (2010). They propose that the coalescence of the two components of a close binary system with masses in the range $0.7 - 1.5\,M_{\odot}$, which are expected to have convective envelopes and strong magnetic fields, would be responsible for the formation of Ap and Bp stars with $M \lesssim 3\,\mathrm{M}_{\odot}$. In this picture, the magnetic Ap and Bp stars with radiative envelopes owe their strong magnetic fields to progenitors with convective envelopes. Tutukov and Fedorova (2010) speculate that systems such as the W UMa-type contact binaries could represent the precursory phase of these merging events.

However all of these predictions are so far only of a very qualitative nature and need to be supported by detailed quantitative calculations in the future.

3 Magnetic white dwarfs

Magnetism in white dwarfs is revealed either through Zeeman splitting and circular polarisation of spectral lines or, at very high field strengths, as continuum polarisation. The magnetic white dwarfs, which represent about $8 - 13\%$ of the total white dwarf population (Liebert et al. 2003; Kawka et al. 2007) and exhibit polar field strengths of $\sim 10^3 - 10^9\,\mathrm{G}$, have been discovered mainly in optical sky surveys. At spectral resolutions of about $10\,\text{Å}$, the Zeeman triplet in the lower members of the Balmer and Lyman series can be easily recognised at fields of $\sim 10^6 - 10^7\,\mathrm{G}$ when the splitting in intensity spectra is smaller than other broadening factors, such as pressure broadening. At higher spectral resolutions, it is possible to detect fields down to $\sim 10^5\,\mathrm{G}$. However, circular spectropolarimetry is a much more sensitive observational technique that is used to measure even lower ($< 10^5\,\mathrm{G}$) fields, since it can independently measure the σ^+ and σ^- oppositely polarised components of the Zeeman split absorption features. On the other hand, due to the faintness of white dwarfs, fields below $\sim 3 \times 10^4$ G have only been recently detected. Spectropolarimetric observations on the ESO VLT by Aznar Cuadrado et al. (2004); Jordan et al. (2007) revealed fields down to $\sim 10^3$ G in about 10% of the surveyed white dwarfs.Landstreet et al. (2012) have confirmed this finding by conducting a survey of objectsrandomly drawn from a list of nearby cool ($T_{\rm eff} \lesssim 14,000$ K) WDs.

Observations seem to indicate that there is a paucity of white dwarfs with fields in the intermediate field range $10^5 - 10^6$ G (Kawka and Vennes 2012), reminiscent of the magnetic desert of Ap and Bp stars. However, this finding has not been fully corroborated by observations and future surveys may find objects also in this field range. In this context we need to stress that while magnetic fields in bright main sequence and pre main sequence stars have been mainly found via circular polarisation observations, only a small percentage of magnetic white dwarfs has been discovered using this method. This non-systematic approach has created, over the past 30 years, a sample of stars whose properties are difficult to investigate since the observational biases are not fully understood and thus are difficult to remove.

Another interesting characteristic of strongly magnetic white dwarfs is that their mean mass ($0.85 \pm 0.04\,{\rm M}_\odot$) is higher than that of non-magnetic or weakly magnetic white dwarfs ($0.593 \pm 0.002\,{\rm M}_\odot$, see the chapter on magnetic white dwarfs in this book). This indicate that their progenitors are more massive than those of ordinary white dwarfs. Interestingly, the observations of Vennes (1999) of the EUVE/Soft X-ray selected ultra-massive white dwarfs ($M > 1.2\,{\rm M}_\odot$) have found that $\sim 20\,\%$ of them are strongly magnetic. Thus it appears that the incidence of magnetism in the high field group increases with white dwarf mass and hence with progenitor mass, unless they result from a merger (e.g. EUVE J0317-853 Barstow et al. 1995; Ferrario et al. 1997).

3.1 Origin of fields in highly magnetic white dwarfs: the fossil hypothesis

According to the "strong" fossil field hypothesis, magnetic stars originate in gas clouds whose fields are at the upper end of the field distribution observed in the interstellar medium (10^{-6} and 10^{-4} G, Heiles 1997). One of the very first articles on star formation and flux freezing is that of Mestel (1966). Briefly, in the simple model proposed by Tout et al. (2004), a star of mass M that collapses from the interstellar medium entraps a magnetic flux $\Phi \propto B_{\rm ISM} M^{2/3}$ Mx where $B_{\rm ISM}$ is the interstellar (primordial) field. The variations expected in the interstellar magnetic field will determine the distribution of magnetic fluxes in the protostars. The magnetic flux lines would freeze in the radiative upper layers of the emerging star that will then evolve towards the main sequence. Assuming perfect magnetic flux conservation, a contraction by a 10^7 factor could in principle create fields of the order of 10^8 G in a main sequence star. However during the cloud's collapse most of the initial magnetic flux would be lost via ohmic dissipation or ambipolar diffusion, so that much lower fields than this upper limit can be realised. The survival of significant large-scale magnetic flux through the pre-main sequence evolution has been addressed by Moss (2003).

According to the "weak" fossil field hypothesis, stellar fields could just be the remnants of those generated by the dynamos of active pre-main sequence stars or by late mergers of protostars with at least one of the two having a radiative envelope (see Sect. 2 and Ferrario et al. 2009). The magnetic field flux would then be conserved during the subsequent evolution from main sequence to the white dwarf stage.

Under the fossil field hypothesis, the magnetic Ap and Bp stars in the mass range $1.5-8\,\mathrm{M}_\odot$ would be the progenitors of the highly magnetic white dwarfs ($B \geq 10^6\,\mathrm{G}$). The paucity of white dwarfs with fields in the range $10^5-10^6\,\mathrm{G}$ could be interpreted as being related to the lower field cut off of Ap and Bp stars.

The main issue is whether magnetic fields can survive complex phases of evolution when a star develops convective and radiative zones that contract and expand in size with time. Tout et al. (2004) point out that a dynamo-generated field in the convective regions of a star is transient and has no large-scale component. A star can conserve its primordial fossil field as long as this star possesses a radiative zone throughout its life, because the field would be pumped out of any developing convective region into an adjacent radiative region. The conclusions of Tout et al. (2004) is that a poloidal fossil field that is present in a main sequence star can appear in the compact star phase with a similar field structure and with its magnetic flux nearly conserved.

There have been many semi-analytical and numerical calculations that have addressed the issues concerning the evolution of a magnetic field during the main sequence phase. Braithwaite and Spruit (2004b) and Braithwaite and Nordlund (2006a) have shown that stable magnetic fields with roughly equal poloidal and toroidal field strengths can exist in the radiative interior of a star and their exterior appearance would be that of a dipole with minor contributions from higher multipoles (see also Mestel and Moss 2010). Interestingly, they find that Ohmic dissipation diffuses the field outward over time so that the field strength at the surface of the star increases while the field structure in the stellar interior would switch from being mainly toroidal to poloidal. Hence they predict an increase in the surface field strength with the age of the star. In order to test these theoretical findings, it is necessary to study a sample of Ap and Bp stars with known age and fraction of the main sequence evolution completed. However, it is quite difficult to estimate age-related quantities for magnetic stars (see Bagnulo et al. 2006b). The evolution of magnetic fields from the zero-age main sequence (ZAMS) to the terminal-age main sequence (TAMS) has been investigated observationally by Bagnulo et al. (2006b) and Landstreet et al. (2007b) through the study of Ap stars in clusters, since the age of a cluster can generally be established to better than about $\pm 0.2\,\mathrm{dex}$. They find that magnetic fields exist at the ZAMS phase for stellar masses $2-5\,\mathrm{M}_\odot$ when stars have fractional ages (that is, fraction of the main sequence evolution completed) below about 0.05 and for fractional ages of less than about $\lesssim 0.10$ for masses up to $9\,\mathrm{M}_\odot$. This is consistent with the existence of fields in the late pre-main sequence stars Herbig AeBe (see section 2). However, the study of Landstreet et al. (2007b) also reveals that the evolutionary time of magnetic fields is less straightforward to interpret. For stars with $M \gtrsim 3\,\mathrm{M}_\odot$ they find that the field strength decreases on a timescale of about 23×10^7 yr, in agreement with Kochukhov (2006). However it is not clear whether the magnetic flux really decreases or whether it is the conservation of magnetic flux that causes the surface field to decrease as the star expands. Interestingly, for stars with $M \lesssim, 3\,\mathrm{M}_\odot$ they find no conclusive evidence of field strength or magnetic flux reduction on a time scale of a few 10^8 yr.

Support for the fossil field hypothesis also comes from spectropolarimetric studies of fifty red giants by Auriere et al. (2013). This work has revealed the existence of four magnetic giants that have been identified as probable descendants of Ap/Bp stars.

Population synthesis calculations carried out by Wickramasinghe and Ferrario (2005) have shown that starting with a distribution of magnetic fields on the main sequence, as observed for the Ap and Bp stars and under the assumption of magnetic flux conservation, the predicted magnetic field and mass distribution in white dwarfs are in general agreement with observations, provided that a modified initial to final mass relation is employed for magnetic white dwarfs (their "scenario A"). That is, in order to fit the mass distribution, one needs to assume that mass loss mechanisms are partly inhibited by the presence of strong fields. However, such models can at most yield an incidence of magnetism of $\sim 5\%$ as compared to the observed $\sim 8-12\%$ (see also Kawka et al. 2007). Wickramasinghe and Ferrario (2005) have shown that it is possible to achieve a better agreement with observations if one assumes that in addition to the actually observed fields in Ap and Bp stars, about 40% of stars more massive than $4.5\,\mathrm{M}_\odot$ (early B-type stars) have fields of $10-100$ G and evolve into high fields magnetic white dwarfs (see Figure 1). This is their "scenario B" which would only be viable if magnetic B-type stars do not have a "magnetic desert", which seems to be, albeit not conclusively, supported by the work of Fossati et al. (2014).

Larger and more sensitive spectropolarimetric surveys of white dwarfs may confirm the classification of white dwarfs within either the very low ($\lesssim 10^4$ G) or the very high field group ($\gtrsim 10^6$ G), hence forming a bimodal field distribution. This would point to two different channels for field formation. It is interesting to note that the weak magnetic fields which have recently been detected in Vega and Sirius (Lignières et al. 2009; Petit et al. 2011), would scale up to a surface field of a few $\sim 10^4$ G at the white dwarf stage under magnetic flux conservation. Such weakly magnetised white dwarfs would belong to the low field component of the bimodal white dwarf field distribution.

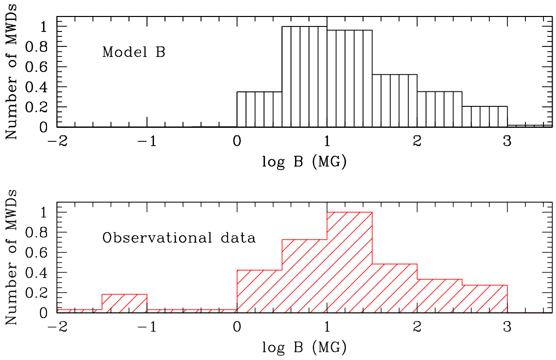

Fig. 1 Population synthesis calculations of Wickramasinghe and Ferrario (2005) (their scenario B) for the field distributions of magnetic white dwarfs based on the fossil field hypothesis compared with observations

3.2 Origin of fields in highly magnetic white dwarfs: the stellar merger hypothesis

We have plotted in Fig. 2 the ratio of poloidal magnetic flux to mass of the highest field magnetic white dwarfs and of the most magnetic main sequence stars. This diagram shows that the two groups of stars share similar characteristics indicating that their magnetic fields may have a common origin. However, the possible evolutionary link between the two populations, and thus the viability of the fossil field hypothesis, has been questioned by Tout et al. (2008) and Wickramasinghe et al. (2014). The main argument hinges on the fact, first highlighted by Liebert et al. (2005), that there is not a single example of a high field magnetic white dwarf ($B \gtrsim$ a few $10^6\,$G) with a non-degenerate companion star (generally an M dwarf, but see Ferrario 2012) in a non-interacting binary. Further searches on a much larger sample of objects conducted by Liebert et al. (2015) have confirmed the hypothesis that magnetic field in white dwarfs and binarity (with M or K dwarfs) are independent at a $9\,\sigma$ level. However, we cannot invoke some as yet unknown physical mechanism that could inhibit the formation of a strong magnetic field in a white dwarf when a companion star is present, because such a mechanism would also prevent the formation of Magnetic Cataclysmic Variables (MCVs) which are interacting binaries consisting of a magnetic white dwarfs with a low-mass red dwarf companion. This curious lack of detached white dwarf - non-degenerate star systems indicates that there are no known progenitors of MCVs (for more details see also the chapter on magnetic white dwarfs in this book).

An merger scenario for the generation of fields has also been proposed by Nordhaus et al. (2011). Here a low-mass star would be tidally disrupted by its proto-white dwarf companion during common envelope evolution to form an accretion disc. In this disc seed fields would be amplified through turbulence and shear and then advected on to the object that will become an

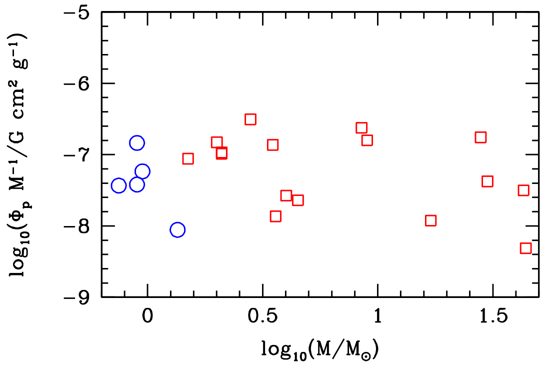

Fig. 2 The ratio of magnetic flux to mass ϕ_p/M for the most magnetic main sequence stars (squares) and white dwarfs (circles) (Wickramasinghe et al. 2014)

isolated highly magnetic white dwarf. García-Berro et al. (2012) conducted three-dimensional hydrodynamic simulations of merging double degenerates and argued that a hot and differentially rotating convective corona would form around the more massive star. Their population synthesis calculations of double degenerate mergers are in general agreement with observations. Similar population synthesis calculations conducted for a wide range of merging astrophysical objects have also been carried out by Bogomazov and Tutukov (2009).

Following the work of Tout et al. (2008), Wickramasinghe et al. (2014) proposed that the fields are generated by an Ω dynamo within the common envelope of a binary system where a weak seed poloidal field would wind up by differential rotation. The closer the cores of the two stars are drawn before the envelope is ejected, the stronger the final field of the star emerging from common envelope will be. Therefore the strongest fields occur when the two stars merge, forming an isolated highly magnetic white dwarf. If the two stars do not coalesce but emerge from the common envelope when they are about to transfer mass, they become the MCVs (Tout et al. 2008).

Observations indicate that highly magnetic stars are typified by (see Fig. 2)

$$10^{-8.5} < \frac{\Phi_{\rm p}/M}{\rm G\,cm^2\,g^{-1}} < 10^{-6.5}, \tag{1}$$

where $\Phi_{\rm p} = R^2 B_{\rm p}$ is poloidal magnetic flux and M and R are the total mass and radius of the star respectively.

If the dynamo mechanism generates a magnetic field from differential rotation $\Delta\Omega$, then we have

$$0 \le \Delta\Omega \le \Omega_{\rm crit} = \frac{1}{\tau_{\rm dyn}} = \sqrt{\frac{GM}{R^3}}. \tag{2}$$

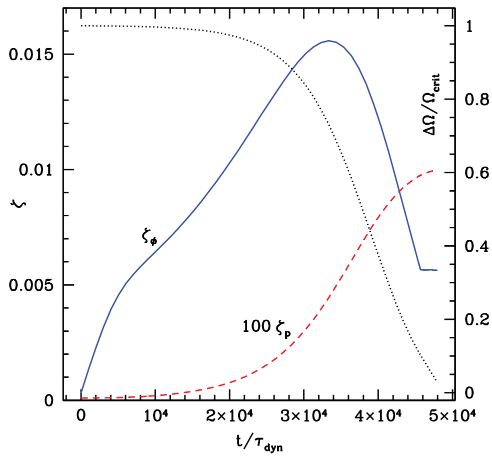

Fig. 3 The evolution of the magnetic field components $B_{\rm p}$ and B_ϕ ($\zeta = \sqrt{\eta} \approx B$) following a stellar merger which produces a differential rotation equal to the break up spin of the merged object. The decay of differential rotation $\Delta\Omega$ follows the right hand axis while the fields follow the left hand axis. Toroidal field decays unless $\eta_{\rm p} > a\eta^2$ and this determines the final ratio of toroidal to poloidal field (Wickramasinghe et al. 2014)

Toroidal and poloidal fields are unstable on their own (Braithwaite 2009). The process that gives rise to the decay of toroidal field leads to the generation of poloidal field with one component stabilising the other and thus limiting field growth until a stable configuration is reached. The results of Fig 2 have been interpreted by Wickramasinghe et al. (2014) as follows. If $E_{\rm p}$, E_ϕ and $E = E_{\rm p} + E_\phi$ are the poloidal, toroidal and total magnetic energies respectively and $\eta = E/U$, $\eta_{\rm p} = E_{\rm p}/U$ and $\eta_\phi = E_\phi/U$ are, respectively, the ratios of poloidal and toroidal magnetic to thermal energy U, then by scaling to the observed maximum poloidal flux, Wickramasinghe et al. (2014) find that

$$\eta_{\rm p} = \frac{10^{-8}}{\lambda} \left(\frac{\hat{\Phi}_{\rm p}}{10^{-6.5}\,{\rm G\,cm^2\,g^{-1}}} \right)^2 , \tag{3}$$

where $\hat{\Phi}_{\rm p}$ is the ratio of magnetic flux to stellar mass. The observational data in Fig. 2 show that the maximum $\eta_{\rm p}$ is independent of the mass and type of star. In non-rotating stars, a stable poloidal-toroidal configuration must satisfy (Braithwaite 2009)

$$a\eta^2 < \eta_{\rm p} < 0.8\eta, \tag{4}$$

where $a \approx 1$ is a buoyancy factor for main sequence stars. The left hand side inequality is caused by the stabilising effect of a poloidal field on the Taylor instability in purely toroidal fields. A lower limit to the poloidal field is determined by the relative importance of magnetic to gravitational–thermal energy through buoyancy effects. The upper limit is due to the stability of poloidal fields which requires they be not significantly larger than the toroidal field. Braithwaite (2009) argued that the same inequalities are also likely to hold for stable fields in rotating stars.

Fig. 3, where $\zeta = \sqrt{\eta} \propto B$, shows how $B_{\rm p}$ and B_ϕ evolve as $\Delta\Omega$ decreases from its maximum at $\Delta\Omega = \Omega_{\rm crit}$. Their single model parameter is chosen to give the observed maximum $\eta_{\rm p} \approx 10^{-8}$, so that $\zeta_{\rm p} \approx 10^{-4}$. Toroidal field is initially generated by the winding up of the seed poloidal field through differential rotation. As soon as the toroidal field is large enough, the poloidal field starts to grow. As long as the differential rotation is sufficiently large, the toroidal field continues to grow up to a maximum value. Then it decreases until it reaches equilibrium with the poloidal field and $a\eta^2 = \eta_{\rm p}$. The magnetic torque extinguishes $\Delta\Omega$ to yield a final object that rotates as a solid body. Wickramasinghe et al. (2014) find that the dynamical timescale for a $2\,M_\odot$ star is of the order of 40 min so this evolution is completed in about 3.7 yr which is a lot smaller than the corresponding Kelvin-Helmholtz timescale (2.3×10^6 yr for a $2\,M_\odot$ star). The conclusion of Wickramasinghe et al. (2014) is that the final poloidal field is stable and proportional to the initial quantity of differential rotation, but is independent of the size of small initial seed field and.

More sophisticated $\alpha - \Omega$ dynamo models are described in Nordhaus et al. (2007), although these calculations are aimed at understanding the different problem related to the shaping mechanisms in planetary nebulae.

In any case, the work of Nordhaus et al. (2011) highlights that for the dynamo mechanism of Wickramasinghe et al. (2014) to be viable, the transport of a strong field generated in the envelope to the proto-white dwarf's surface would require a diffusion coefficient $\geq 10^{21} - 10^{22}\,{\rm g\,cm^{-3}}$ which is unphysical. Hence Nordhaus et al. (2011) propose an alternative scenario whereby a companion star, of sufficiently low mass to avoid a premature ejection of the envelope, would be tidally destroyed by the gravitational field of the primary star and would form a disc extending all the way to the surface of the proto-white dwarf. Accretion would then transport the strong fields that are formed in the disc towards the proto-white dwarf where they would become anchored to its surface.

The population synthesis calculations carried out by Briggs et al. (2015) have given further support to the stellar merging hypothesis for the origin of fields in the highly magnetic white dwarfs. Briggs et al. (2015) find that the mass distribution and the fraction of stars that merge during a common envelope phase are in good agreement with the observations of magnetic white dwarfs for a wide range of common envelope efficiency parameter α. Fig. 4 shows the contributions from the various merging pre-common envelope progenitor pairs for $\alpha = 0.1$. They find a theoretical incidence of magnetism among white dwarfs, for $\alpha = 0.1 - 0.3$, of about 13-19 %, which is consistent with observational results (e.g. Kawka et al. 2007).

For the sake of completeness, we remark that similar population synthesis calculations, but related to the viability of the core-degenerate scenario for Type Ia supernovae, have been conducted by Ilkov and Soker (2013) (and references therein).

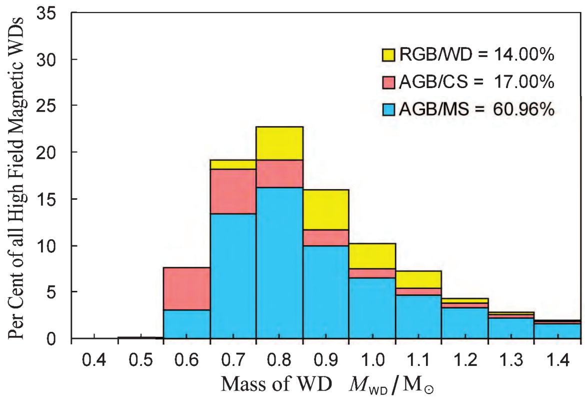

Fig. 4 Theoretical mass distribution of magnetic white dwarfs for $\alpha = 0.1$. The contributions are separated according to their pre-CE progenitors. AGB= Asymptotic Giant Branch; MS= Main Sequence; RGB= Red Giant Branch; CS= Convective Star and WD=White Dwarf. Other paths also contribute but are less important than those shown. The Galactic disc age is chosen to be 9.5 Gyr

Finally, we would like to bring the attention of the reader to the calculations of Duez et al. (2010b); Duez and Mathis (2010); Duez et al. (2010a) (and references therein) on the stability of non-force-free magnetic equilibria in radiative stellar regions (cores of Sun-like stars, envelopes of intermediate and high mass stars, and compact stars). Through the use of semi-analytic techniques they constructed and tested an axisymmetric non-force-free magnetostatic equilibrium. Their numerical calculations recovered the instabilities which are characteristic to purely poloidal and toroidal magnetic fields and proved computationally, and for the first time, that only a mixed configuration is stable under all types of perturbations. Their important results can be applied to describe the magnetic equilibrium topology in stellar radiative regions (instead of choosing the initial field configuration arbitrarily) and to provide initial conditions for magneto-rotational transport in state-of-the-art stellar evolution codes (e.g., see the models of Maeder and Meynet 2014, for giants and supergiant stars). More general applications of these results to multi-dimensional magnetohydrodynamic computations are also envisaged.

4 Neutron stars

Magnetic fields of neutron stars at birth are estimated to be in the range $\sim 10^{11} - 10^{15}$ G. However, measurements of the birth magnetic fields of neutron stars are always indirect. Sometimes the field or its dipole moment is measured today, e.g., from neutron star spin-down or Zeeman spectropolarimetry of suitable progenitors, and then extrapolated backwards or forwards in time respectively in the context of an evolutionary model. Otherwise, the birth field is inferred indirectly from some relic of the birth event, e.g., calorimetry of the supernova remnant. If the internal magnetic field evolves slowly, so that its strength today approximately equals its strength at birth, then gravitational wave upper limits also provide constraints. We discuss each of these approaches briefly below. More information can be found in other chapters in this volume.

Population synthesis models have a long history of being used to infer the birth fields of neutron stars (Gunn and Ostriker 1970; Hartman et al. 1997; Faucher-Giguère and Kaspi 2006; Kiel et al. 2008). Radio timing measurements of the spins, fields, and radio luminosities of the current neutron star population are combined with prescriptions for binary evolution, source kinematics in the Galactic gravitational potential, radio emission properties (e.g., beaming), supernova kicks, and observational selection effects to constrain the spin and field distributions at birth. Population synthesis models have benefited recently from the discovery of many new radio pulsars, both isolated and recycled, in large-sale radio multi-beam surveys (Manchester et al. 2001; Morris et al. 2002; Kramer et al. 2003; Faulkner et al. 2004; Hobbs et al. 2004), an updated Galactic electron density map (Cordes and Lazio 2002), and proper motions from very-long-baseline interferometry. The results are that the birth spins are inferred to be normally distributed, with average birth period $\langle P_0 \rangle = 0.30\,\mathrm{s}$ and standard deviation $\sigma_{P_0} = 0.15\,\mathrm{s}$, while the birth fields B_0 are log-normally distributed, with $\langle \log B_0 \rangle = 12.65$ and $\sigma_{\log B_0} = 0.55$ (B_0 in gauss) (Faucher-Giguère and Kaspi 2006; Kiel et al. 2008). The latter studies find no evidence for magnetic field decay over $\lesssim 0.1\,\mathrm{Gyr}$, assuming that the radio luminosity scales roughly as the square root of the spin-down luminosity. The no-decay conclusion sits in partial tension with the inference of field decay over $\sim 10^4\,\mathrm{yr}$ in a different sub-population of neutron stars, e.g., the magnetars and the thermal X-ray sources (Pons et al. 2012). Faucher-Giguère and Kaspi (2006) also find no evidence for bimodality in the distributions of P_0 and space velocity.

The energetics of the birth event, which are related to B_0 and P_0, leave their imprint on the supernova remnant. From X-ray measurements of the remnant's radius, temperature, and emission measure, one can infer the total blast energy assuming Sedov expansion (Reynolds and Chevalier 1981; Reynolds 2008). The blast wave is powered by the core-collapse event, which depends weakly on B_0 and P_0, and the relativistic wind emitted by the newly born pulsar, whose luminosity scales $\propto B_0^2 P_0^{-4}$. Drawing on X-ray Multi-Mirror Mission Newton (XMM-Newton) data, Vink and Kuiper (2006) showed that the pulsar wind played an insignificant role powering the blast wave in two anomalous X-ray pulsars (AXPs) and one soft-gamma-ray repeater (SGR), implying $P_0 \gtrsim 5\,\mathrm{ms}$ and hence no significant protoneutron star dynamo in these three objects at least. Steep density gradients in the interstellar medium can modify this conclusion, especially if the gradient correlates with the interstellar magnetic field (Vigelius et al. 2007).

Gravitational wave upper limits and future detections furnish independent information on the *internal* magnetic field of a neutron star. At the time of writing, coherent searches for the Crab pulsar with the Laser Interferometer Gravitational Wave Observatory (LIGO) constrain its gravitational wave luminosity to be $\lesssim 2\%$ of the observed spin-down luminosity, thereby limiting the internal field to $B_0 \lesssim 10^{16}$ G (Abbott et al. 2010; Aasi et al. 2014). An analogous result has been obtained for the Vela pulsar (Abadie et al. 2011; Aasi et al. 2014).

The gravitational wave strain scales as the square of the spin frequency, so the best constraints come from rapid rotators. Two classes of object are especially interesting in this regard. First, the large deformation of a newly born, fast-spinning ($P_0 \sim 1$ ms) magnetar caused by a super-strong internal magnetic field ($> 10^{16}$ G) radiates powerful gravitational waves, which should be detectable with Advanced-LIGO-class detectors up to the distance of the Virgo cluster (Stella et al. 2005; Dall'Osso and Stella 2007; Dall'Osso et al. 2009; Mastrano et al. 2011; Melatos and Priymak 2014). A future gravitational-wave detection of a millisecond magnetar would provide a direct measurement of B_0, if the distance is known (e.g., from a counterpart whose redshift is measured electromagnetically). Second, recycled millisecond pulsars have low external magnetic dipole moments, but their internal fields may be much stronger, if the external dipole is reduced by accretion-driven diamagnetic screening (Payne and Melatos 2004), which leaves the birth field in the interior untouched. Combining this scenario with rapid rotation and tight electromagnetic spin-down limits (ellipticity $\epsilon \lesssim 10^{-8}$ in some cases), recycled millisecond pulsars already yield some of the best constraints on the internal B_0 by any method, ruling out $B_0 \gtrsim 10^{13}$ G in some objects (Mastrano and Melatos 2012), depending on the poloidal-toroidal flux ratio and whether the core is superconducting or not.

The above conclusions change, if the internal field evolves significantly in ordinary pulsars (as opposed to magnetars), cf. Faucher-Giguère and Kaspi (2006). Radio pulse evolution over long time-scales may be a signature of magnetic polar wandering or plate tectonics, where the field wanders inside the star (Macy 1974; Ruderman 1991; Melatos 2012). The recent discovery of changes in the flux ratio (0.1 per century) and component separation (0.6 degrees per century) of the Crab's radio pulses (Lyne et al. 2013) is noteworthy in this regard, although other explanations like radiative precession are also possible (Melatos 2000; Barsukov et al. 2013, 2014). Stairs et al. (2000) discovered switching between two discrete magnetospheric states (and hence two pulse shapes and spin-down rates) in PSR B1828-11.

The central, enduring debate regarding the origin of neutron star magnetic fields revolves, as it does in magnetic white dwarfs (see Sect. 3), around whether the field is a fossilised relic of the progenitor's field or is generated afresh by dynamo action in the protoneutron star in the first ~ 10 s after core collapse. Below, we summarise briefly the pros and cons of each scenario, drawing heavily on an excellent review by Spruit (2009). The reader is directed to the latter reference for more discussion.

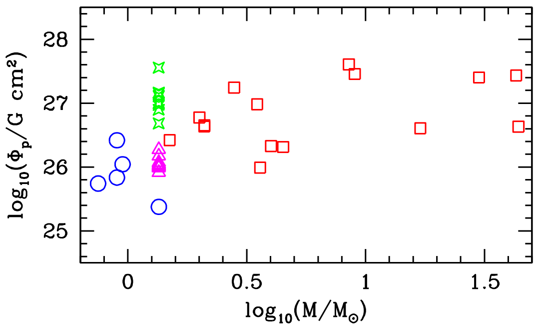

Fig. 5 The magnetic flux ϕ_p for the most magnetic main sequence stars (squares), white dwarfs (circles), high field radio pulsars (triangles) and magnetars (stars)

4.1 Origin of fields in neutron stars: the fossil hypothesis

Although the surface magnetic fields in pre-main sequence stars, Ap/Bp, OB stars, magnetic white dwarfs and neutron stars cover a wide range of magnetic field strength values (from a few 10^2 to 10^{15} Gauss), the magnetic fluxes near the upper limit of the observed field ranges are all a few $10^{27}\,\mathrm{G\,cm^2}$. We show in Fig. 5 the magnetic fluxes of the most magnetic compact stars and non-degenerate stars with radiative envelopes. This finding has been used in support of the fossil field hypothesis for the origin of fields.

The fossil field hypothesis for the origin of fields in neutron stars was first proposed by Woltjer (1964). Ruderman (1972) also remarked that the similarity of magnetic fluxes in magnetic white dwarfs and neutron stars could be explained through flux conservation during the evolution from main sequence to compact star stage.

Ferrario and Wickramasinghe (2006b) have investigated the effects of the fossil field hypothesis for the origin of magnetic fields in neutron stars by carrying out population synthesis calculations for different assumptions on the distribution of the magnetic flux of massive ($\geq 8\,M_\odot$) main sequence stars and on the dependence of the initial birth period of neutron stars. They used the observed properties of the population of isolated radio pulsars in the 1374 MHz Parkes Multi-Beam Survey (Manchester et al. 2001) to constrain model parameters. These were then used to deduce the required magnetic properties of their progenitor stars. Their conclusion is that the fossil field hypothesis, which does not allow for magnetic flux loss in the post-main sequence evolution, requires a very specific distribution of magnetic fields for massive main sequence stars as shown in Fig. 6 for an assumed dipolar field structure. In this picture the field distribution in massive stars would be continuous and all massive stars would be magnetic. This distribution is predicted to have a peak near 46 G with low- and high-field wings covering a field range from 1 to 10^4 G. About 8 per cent of main sequence stars would require to have fields in excess

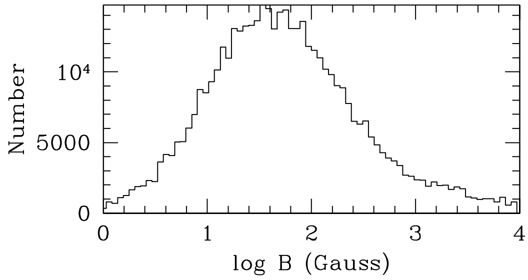

Fig. 6 Predicted magnetic field distribution of massive stars ($8 - 45\,M_\odot$), according to the population synthesis modelling of Ferrario and Wickramasinghe (2006b) under the assumption of fossil fields and magnetic flux conservation

of 10^3 G and these would the progenitors of the highest-field neutron stars. This main sequence distribution of B0-O type stars is an observable quantity and thus a prediction of the fossil field model.

Further population synthesis calculations of the observed properties of magnetars were carried out by Ferrario and Wickramasinghe (2008b) under the assumption that magnetars originate from main sequence stars that are much more massive than those giving rise to normal radio pulsars. These studies were prompted by (i) the observation that most magnetars have been linked to progenitors in the mass range $\sim 20 - 45\,M_\odot$ (Gaensler et al. 2005; Muno et al. 2006) and (ii) the discovery of strong fields in massive O-type stars (see Sect. 2). Thus fossil magnetic fluxes similar to those observed in the magnetars may already be present in stellar cores prior to their collapse to neutron star and would explain the presence of fields of up to $\sim 10^{15}$ G in magnetars. According to Ferrario and Wickramasinghe (2008b), the anomalous X-ray emission of magnetars would be caused by the decay of the toroidal field that does not contribute to the spin down of the neutron star. Ferrario and Wickramasinghe (2008b) predict a number of active magnetars (see Fig. 7) that is consistent with the number of sources detected by ROSAT and a Galactic birth rate compatible to that inferred by Kouveliotou et al. (1998).

Since stars of spectral types B0 to O are the progenitors of neutron stars, it remains to be seen whether there is a sufficiently large range of magnetic fluxes in these stars to explain the entire range of magnetic fluxes in neutron stars. So far, observations of magnetism in massive stars by the MiMeS, MAGORI and BOB collaborations have led to the discovery of a large number of new objects. The paper by Petit et al. (2013) lists the physical, rotational and magnetic properties of about 60 highly magnetic massive OB stars. Most of the detected fields are in the $> 10^3$ G regime with masses $> 8\,M_\odot$ and radii $> 5\,R_\odot$. Under magnetic flux conservation, they would be the progenitors of the highest magnetic field neutron stars ($> 10^{13}$ G).

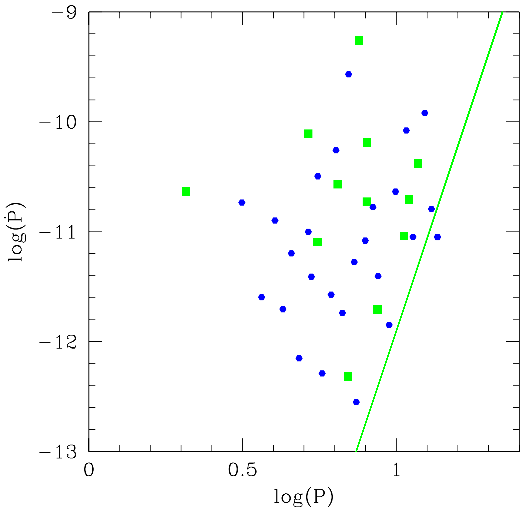

Fig. 7 Filled squares: observed magnetars; filled circles: magnetars as derived from the modelling of Ferrario and Wickramasinghe (2008b). The solid line is an empirically determined boundary (a "magnetar death line") given by $\log(\dot{P}) = 8.4\log(P) - 20$ (Zhang 2003)

4.1.1 Problems with the fossil hypothesis

Taken at face value, the fossil field arguments seem to be reasonable. Certainly, there is no trouble accounting for the magnetar end of the distribution, since typical surface fields measured in Ap and Bp stars compress to give fields $\gtrsim 10^{13}$ G in neutron stars. If anything, there is a problem at the lower end, where it is hard to produce neutron stars with $B \sim 10^{10} - 10^{11}$ G without some other accretion-related physics. In fact, a main sequence star with a surface field $B \sim 300$ G and a radius $R \sim 2R_\odot$ compresses to give $B \sim 6 \times 10^{12}$ G. There are three problems with the fossil picture. First, the innermost $\sim 1.4\,M_\odot$ of the progenitor, which collapses to form a neutron star, occupies $\sim 2\%$ of the progenitor's cross-subsectional area, reducing the maximum compressed field to $B \sim 10^{14}$ G. One can raise the limit somewhat, if the progenitor's field is centrally concentrated, but the three-dimensional MHD simulations of Braithwaite and Nordlund (2006b) (which assume polytropic, spherically symmetric density and temperature profiles and neglect core convection) do not predict much central concentration. Second, the magnetar birth rate is comparable to the ordinary neutron star birth rate (Keane and Kramer 2008; Woods 2008), yet, as already mentioned in Sect. 4.1, there may be too few known progenitors with $B_0 \gtrsim 10$ kG to create magnetars in the numbers observed. Furthermore, the relatively low incidence of intermediate-strength fields (10–100 G) (Wade and the MiMeS Collaboration 2014) may also represent a problem for the fossil field hypothesis, unless future, more sensitive spectropolarimetric surveys of massive stars discover that intermediate fields (e.g., see Fossati et al. 2014) are indeed much more common than current observations show. Third, the core of the progenitor couples magnetically to the giant envelope. The coupling is 'frictional' — the field lines are disrupted into a tangle by instabilities as they shear, behaving like a turbulent magnetic viscosity rather than 'bungee cords' — but it is strong nonetheless at high fields, scaling as B_0^2. Consequently, the core decelerates too much to explain the distribution of neutron star spin periods observed today. However, note that this is a problem for any neutron star with a magnetised progenitor, even if the neutron star's field is generated ultimately in a dynamo; it is not an argument against fossil fields specifically. Indeed, a protoneutron star dynamo may never take root, if the progenitor core rotates too slowly. Core-envelope coupling has stimulated the suggestion that neutron star spins are imparted by the birth kick during supernova core collapse (Spruit and Phinney 1998). Alignment is therefore expected between the spin axis and proper motion, for which some evidence exists in X-ray images of supernova remnants and pulsar radio polarimetry (Wang et al. 2006).

4.2 Origin of fields in neutron stars: dynamo generated fields

In this model, the seed field is amplified by a a dynamo in the proto-neutron star (Ruderman and Sutherland 1973; Duncan and Thompson 1992; Thompson and Duncan 1993; Bonanno et al. 2005). There are two main kinds of protoneutron star dynamos: those driven by convection and those driven by differential rotation. Convection-driven dynamos were first analysed in the neutron star context by Thompson and Duncan (1993). A thermal luminosity L is transported outwards through the star by turbulent convection with characteristic eddy velocity $V \approx (L/4\pi R^2\rho)^{1/3}$, where ρ is the mean stellar density, and this generates a magnetic field of strength $B_{\rm PNS} \approx (4\pi\rho V^2)^{1/2}$, if the mechanical and magnetic stresses are in equipartition. For a typical photon luminosity, we obtain $B_{\rm PNS} \approx 10^9$ G $(L/10^{38}$ erg s$^{-1})^{1/3}(R/10^8$ cm$)^{7/6}$. For a typical neutrino luminosity, $B_{\rm PNS}$ is $\sim 10^2$ times higher ($L \sim 10^{44}$ erg s^{-1}). When the protoneutron star collapses from $R = 10^8$ cm to 10^6 cm, the field is compresses further by flux conservation to give $B \approx 10^4 B_{\rm PNS}$.

A convective dynamo faces several obstacles to explain all neutron star magnetic fields, although it may certainly explain a subset. The most serious obstacle is the assumption of equipartition. Numerical simulations reveal time and again that self-sustaining dynamos do not reach equipartition (e.g., Cook et al. 2003) because then the magnetic stresses would paradoxically self-quench the driving shear. In a variety of contexts, and for various numerical set-ups, the magnetic stresses saturate at $\lesssim 5\%$ of the mechanical stresses (Cook et al. 2003; Braithwaite 2006). These theoretical results are consistent with observations of the Sun, whose dipole amounts to $\approx 0.3\%$ of equipartition (Charbonneau 2014). A second obstacle arises when stratification quenches convection ~ 10 s after the dynamo begins. Does the field retreat or leave behind a permanent dipole moment? The danger here is that pockets ('domains') of tilted polarity are left behind, if the convective eddies turn over faster than they are quenched, producing a dipole moment lower than one inferred in magnetars (e.g., from spin down Kouveliotou et al. 1998), even if the higher-order multipoles are substantial. The consequences for magnetic structure of forming convective pockets (which 'insulate') and radiative pockets (which 'conduct') has been explored by Tout et al. (2004).

A second type of dynamo is driven by differential rotation. It is sometimes presumed that a sheared, magnetised fluid develops a toroidal field component B_ϕ, which reacts back to shut off the shear completely. In practice, the back reaction can only ever be partial in a continuously driven system; if the fluid velocity gradient were to vanish momentarily, B_ϕ generation would cease, and the gradient would eventually reassert itself in response to the driver (Spruit 2009). Microphysically, quenching is avoided because toroidal wind-up is limited to $B_\phi \lesssim 10B_p$ by instabilities, where B_p is the poloidal field component (Spruit 2009). Hence it is necessary to grow B_ϕ and B_p together in a differentially driven dynamo. The dynamo loop can be closed by Tayler (Braithwaite 2006) or magneto-rotational (Moiseenko et al. 2006) instabilities, for example. The growth window is open for ~ 10 s, before the star stratifies stably, and lasts for a time $\approx R/v_A$ or $(\Delta\Omega)^{-1}$ in the latter scenarios respectively, where v_A is the Alfvén speed and $\Delta\Omega$ is the angular velocity shear. Differential rotation can also be driven by stellar mergers; the reader is referred to section 3.2 and Wickramasinghe et al. (2014) for details.

Braithwaite (2006) showed that a shear-driven dynamo can even persist in a stably stratified star. Fluid is displaced parallel to equipotential surfaces in the Tayler instability. Hence, the condition for dynamo operation reduces to requiring that the maximum hydromagnetic mode wavelength allowed by buoyancy exceeds the minimum wavelength set by ohmic dissipation — a condition which is easily satisfied in a highly conducting neutron star. Braithwaite (2006) showed explicitly through simulations that a Tayler dynamo is indeed self-sustaining in the presence of strong stratification and that it saturates with $B_\phi \sim 5B_P$ and $B_\phi^2/8\pi \approx 10^{-2}\rho V^2$, i.e., at 1% of equipartition. However, Zahn et al. (2007) have strongly contested this result, finding that the Tayler instability cannot sustain a dynamo even at high magnetic Reynolds number.

A mean-field variant of the shear-driven dynamo was analysed in the neutron star context by Mastrano and Melatos (2011). In a mean-field dynamo, the turbulent electromotive force $\langle \mathbf{v} \times \mathbf{B} \rangle$ in Faraday's law is the sum of two terms: the 'α' term proportional to $\mathbf{B}$, which arises from turbulent vorticity, and the so-called '$\mathbf{\Omega} \times \mathbf{J}$' term, which is proportional to $\eta_{ij}(\nabla \times \mathbf{B})_j$ and arises from off-diagonal entries in the resistivity tensor η_{ij} (Moffatt 1978; Biskamp 1997). A natural seat for a mean-field dynamo is a tachocline, where the angular velocity jumps sharply (Ott et al. 2006; Burrows et al. 2007); in a protoneutron star, this may occur at the boundary between the convection and neutron finger zones (Bonanno et al. 2003, 2005). Mastrano and Melatos (2012) proved that a tachocline mean-field dynamo operates self-sustainingly in a protoneutron star, if η_{ij} is anisotropic and the jumps in 'α' (vorticity) and angular velocity are coincident, and also (less realistically) if η_{ij} is isotropic but the jumps are displaced.

4.2.1 Generation physics

The ultimate fate of a dynamo-generated magnetic field depends on buoyancy — does the field float out of the star or remain anchored in the core? — and helicity — does the field maintain its large-scale integrity against the action of hydromagnetic instabilities, or is it shredded?

Buoyancy exists in a neutron star because, in β equilibrium, the ratio of charged to neutral species densities increases with depth (Reisenegger and Goldreich 1992). Magnetic flux tubes ('light fluid') rise to the surface if the Alfvén speed satisfies $v_A \gtrsim HN$, where H is the hydrostatic scale height, and N is the Brunt-Väisälä frequency (Reisenegger and Goldreich 1992; Spruit 2009). The field rises most easily when the protoneutron star is young and neutrinos help to lower N to $\lesssim$ 10 rad s^{-1}. By contrast, in a normal neutron star, theory predicts $N \gg 10^2\,\mathrm{rad\,s}^{-1}$ (Reisenegger and Goldreich 1992). In this way, magnetic loops inside the star penetrate the surface and add up to form a permanent dipole moment, as long as their footpoints remain anchored in the cooling crust or core and do not float away entirely.

Assuming that the field does remain anchored, is it stable? Many analyses have shown that a purely poloidal magnetic field in a highly conducting plasma is always unstable (Markey and Tayler 1973; Wright 1973; Flowers and Ruderman 1977). If we regard the star's eastern and western hemispheres as two bar magnets, it is energetically favourable for the magnets to flip and counteralign, destroying the dipole moment (Flowers and Ruderman 1977; Marchant et al. 2011); in practice, the instability proceeds by forming Rayleigh-Taylor 'mushrooms' in the toroidal direction. Similarly, a purely toroidal magnetic field is always unstable (Tayler 1973; Akgün et al. 2013). However, linked poloidal-toroidal fields appear to be stable under a range of conditions, both in analytic calculations (Prendergast 1956; Tayler 1973; Akgün et al. 2013) and numerical simulations (Braithwaite and Spruit 2004a; Braithwaite and Nordlund 2006b; Braithwaite 2009).

Linked poloidal-toroidal fields are stabilised, because their helicity is conserved approximately in a highly conducting plasma. Helicity is defined as $H = \int \mathrm{d}^3\mathbf{x} \quad \mathbf{A}\cdot\mathbf{B}$, where $\mathbf{A}$ is the magnetic vector potential; it is a gauge-dependent quantity which describes the number of times one flux tubes winds around another (Biskamp 1997). In a weakly resistive plasma, H is conserved globally, even when H is scrambled locally on individual flux tubes. Hence, if a dynamo-generated field has $H \neq 0$ initially, the twist can never be undone completely, leading to $B \neq 0$ in the final state. If magnetic energy is minimised during the field's evolution, it tends to a force-free configuration, cf. laboratory spheromaks (Broderick and Narayan 2008; Mastrano and Melatos 2008).

Numerous papers have been written recently reporting stability calculations on poloidal-toroidal fields, e.g., (Braithwaite and Spruit 2004a; Braithwaite and Nordlund 2006b; Akgün et al. 2013; Armaza et al. 2013; Ciolfi and Rezzolla 2013; Gourgouliatos et al.

2013; Lander and Jones 2012; Mitchell et al. 2013; Passamonti and Lander 2013). A comprehensive review lies outside the scope of this chapter; we simply mention a few highlights. In a barotropic star, it has become apparent that stability depends in subtle ways on the boundary conditions and the entropy distribution within the star (Ciolfi et al. 2011; Lander and Jones 2012). In a non-barotropic star, magnetic field configurations with an equatorial 'torus' are generically stable, when the poloidal-toroidal flux ratio is 'modest'. Akgün et al. (2013) obtained the stability condition $0.25 < B_\phi^2/B_p^2 < 0.5(\gamma_{\rm ad}/\gamma - 1)|E_g|/(B_p^2/8\pi)$, where $\gamma_{\rm ad}$ and γ are the adiabatic and non-barotropic specific heat ratios respectively, and $|E_g|$ is the total gravitational potential energy. Non-barotropic solutions also match self-consistently to an external dipole (or any arbitrary multipole), extending the power of future observational tests involving gravitational waves (Mastrano et al. 2011; Mastrano and Melatos 2012; Mastrano et al. 2013). Finally, in a superconducting star, the stability problem changes character fundamentally for two reasons: the Lorentz force is much stronger ($|\mathbf{F}_B| \propto H_{\rm c1}B \sim 10^3 B^2$, where $H_{\rm c1}$ is the critical field in a type II superconductor), and its vectorial structure is more complicated $[\mathbf{F}_B \propto \mathbf{B} \times (\nabla \times \mathbf{H}_{\rm c1}) + \rho\nabla(|\mathbf{B}|\partial|\mathbf{H}_{\rm c1}|/\partial\rho)]$. The equilibrium magnetic structure and its stability are then controlled by $H_{\rm c1}/\langle B\rangle$ at the crust-core boundary (Glampedakis et al. 2012; Lander et al. 2012; Lander 2013, 2014). In fact, the internal field configuration now depends very strongly on field strength, suggesting significant differences between the interior fields of pulsars and those of magnetars (Lander 2014).

4.2.2 Superfluid turbulence

Does magnetic field evolution in a neutron star conclude ~ 10 s after the protoneutron star is born and stably stratifies? The traditional consensus has been in the affirmative, except for the slow ohmic and Hall evolution described elsewhere in this volume and in Geppert et al. (2012); Pons et al. (2012); Viganò et al. (2012). However, this picture may need modification in light of recent work suggesting that the neutron superfluid and charged components to which it is coupled are turbulent. In this scenario, internal magnetic activity may be ongoing, even though the magnetic flux thereby produced struggles to rise buoyantly to the surface. Hints of abrupt magnetospheric changes observed on ~ 1 yr time-scales (see Sect. 4) — much longer than the dynamical time of the magnetosphere, but much shorter than the ohmic and Hall time-scales — are relevant in this context.

In the standard picture, the vorticity field inside a neutron star is uniform and quasi-static. Macroscopically, this means that there is zero meridional circulation. Microscopically, it means that the superfluid is threaded by a rectilinear array of vortices, each carrying a quantum of circulation $\kappa = h/(2m_n)$, where h is Planck's constant and m_n is the mass of the neutron. As the star brakes electromagnetically, an angular velocity lag $\Delta\Omega$ builds up between the crust and superfluid, whose value is set by the balance between the Magnus and pinning forces. One finds $\rho\kappa R\Delta\Omega \approx E_{\rm pin}(\xi_{\rm coh}\xi_{\rm pin})^{-1}$ and hence $\Delta\Omega \approx 1$ rad s^{-1} independent of Ω and $\dot{\Omega}$, where $E_{\rm pin}$ is the pinning energy, $\xi_{\rm coh}$ is the superfluid coherence length, and $\xi_{\rm pin}$ is the pinning site separation (Warszawski and Melatos 2013). Glitches reset $\Delta\Omega$ but only partially, because there is no observed correlation between glitch sizes and waiting times in most pulsars (Melatos et al. 2008).

The lag $\Delta\Omega$ changes the flow structure completely. It is a well-known result of fluid mechanics that a shear flow in a sphere cannot be purely toroidal. The boundary conditions induce Ekman circulation, wherein interior fluid is drawn into a viscous boundary layer, spun down through contact with the crust, then recycled back into the interior (Abney and Epstein 1996; van Eysden and Melatos 2010). This occurs with or without a rigid core. Furthermore, Ekman circulation is known to be unstable at high Reynolds numbers (see Sect. 5.2) $\mathrm{Re} \gtrsim 10^3$, forming unsteady, nonaxisymmetric flow patterns like herringbone waves ($\mathrm{Re} \approx 6 \times 10^3$) and Taylor-Gortler vortices ($\mathrm{Re} \approx 10^4$) (Nakabayashi 1983). In a neutron star, where the effective Reynolds number is much greater [$\mathrm{Re} \gtrsim 10^7$ for e^--e^- shear viscosity modified by Landau damping by transverse plasmons; see Shternin and Yakovlev (2008)], wave modes resembling fully developed turbulence are expected (Melatos and Peralta 2007).

At the microscopic level, the vorticity field is disrupted by vortex-line instabilities. One example is the Donnelly-Glaberson instability, familiar from laboratory experiments with liquid helium and Bose-Einstein condensates (Glaberson et al. 1974). If there is a superfluid counterflow directed along the vortices, Kelvin waves are excited and amplified by mutual friction to produce a reconnecting, self-sustaining vortex tangle (Tsubota et al. 2003, 2013). This process has a low threshold (counterflow $\approx$ mm s^{-1}, easily exceeded during Ekman pumping) and a fast growth time ($\lesssim$ one spin period) (Peralta et al. 2006). Recently, a related Kelvin-wave instability has been discovered by Link (2012a; 2012b), which does not rely on a counterflow and arises from imperfect pinning, which creates a lag to amplify the Kelvin waves [cf. Glampedakis et al. (2009)]. This instability grows over $\sim$ days, which is still fast.

Turbulent, tangled vorticity is an important ingredient in the story of magnetic field generation. The turbulent neutron condensate couples to charged components in the star through mutual friction and entrainment (Mendell 1998; Prix et al. 2002; Haskell et al. 2009, 2012; Chamel 2013), as well as through fluxoid-vortex interactions (Srinivasan et al. 1990; Jahan-Miri 2010). Consequently, the charged components circulate too (Peralta et al. 2005). Simulations that incorporate the magnetic field dynamics explicitly are required to understand the behaviour of the coupled fluids in detail. There is reason to be hopeful that such simulations will be conducted in the next few years, motivated by the prospect of gravitational wave experiments.

Can superfluid turbulence be quenched by stratification, i.e., by the buoyancy force produced by the charge-neutral ratio increasing with depth (see Sect. 4.2.1)? Counterintuitively, perhaps, the situation is borderline. Stratified turbulence is characterised by two dimensionless quantities: the Froude number, Fr, which is related to the buoyancy force, and the Reynolds number, Re (Brethouwer et al. 2007). Even for $\mathrm{Fr}^{-1} \gtrsim 10^2$, the turbulence is not quenched, if Re is sufficiently large. Turbulence persists for Re $\gtrsim$ Fr^{-2} or equivalently $R^2(\Delta\Omega)^3 \gtrsim \nu N^2$, where ν is the kinematic viscosity (Iida et al. 2009; Lasky et al. 2013). Neutron stars lie near the boundary defined by the above condition in the Fr^{-1}–Re plane. In the regime Re $\gtrsim$ Fr^{-2} and Fr^{-1} $\gtrsim 10^2$, the flow is modified strongly away from Kolmogorov isotropy, even though it remains circulatory; the Earth's atmosphere and oceans also occupy this regime (Brethouwer et al. 2007; Chung and Matheou 2012).

Another possibility is that superfluid turbulence is quenched by magnetic stresses. Again, though, the case is unclear. At the microscopic level, the imperfect pinning instability which drives a vortex tangle (Link 2013) persists even when a magnetic field is present; in fact, Link's (2013) analysis assumes magnetic locking of the viscous proton-electron plasma to the crust. Does the B_ϕ wound up by flux freezing erase the shear? More simulations are needed, but for now it seems unlikely: the magnetic back reaction saturates at $\lesssim 1\%$ of the mechanical (shear) stress in simulations attempted to date (Cook et al. 2003; Braithwaite 2006), and the Tayler instability self-organizes such that $\mathbf{B}\cdot\nabla\Omega = 0$, i.e., such that wind-up stops, before B_ϕ quenches the process (Duez et al. 2006; Rüdiger et al. 2009). Magnetised Ekman pumping acts on the rapid time-scale $\approx (\Omega R/v_A)^{2/3}\Omega^{-1}$ (where V_A is the Alfvén speed) in simple magnetic geometries, e.g., uniform magnetisation (Easson 1979b). However, it is known to operate much more slowly in complicated geometries and stratified conditions (Goedbloed and Poedts 2004; Melatos 2012). Finally, Easson showed that many — perhaps most — realistic magnetic topologies (e.g., those with closed loops) cannot enforce corotation (Easson 1979a); see also Melatos (2012). Physically, this occurs because the toroidal component of the magnetic field perturbation created by spin down satisfies a nonconservative equation of motion in a superconductor with Fermi liquid forces (Easson 1979a). Again, large-scale numerical simulations are needed to determine if the class of magnetic topologies that permit corotation is restricted or generic.

5 Status of MHD turbulence theory and computation

The basic physics underlying stellar magnetic fields lies in the theory of magnetised fluids, or magnetohydrodynamics (MHD). Of particular importance is the manifestation of magnetohydrodynamic turbulence because its presence underpins the operation of stellar dynamos, and controls diffusive magnetic flux transport in stars. This section is intended to give a high-level overview of recent progress in MHD turbulence theory and computation as it applies to stellar magnetism.

5.1 Equations of motion

The equations of compressible MHD can be written as

$$\frac{\partial\rho}{\partial t} = \nabla\cdot(\rho\mathbf{u}) \tag{5}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \nabla p + \rho\nu\nabla^2 \mathbf{u} \qquad (6)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta\nabla^2 \mathbf{B} \qquad (7)$$

where in Equation 6 $\mathbf{u}$ is the fluid velocity, ρ is the mass density, and ν is the kinematic viscosity. Equation 7 is Faraday's law together with the phenomenological Ohm's law $\mathbf{E} = -\mathbf{u}\times\mathbf{B}+\eta\mathbf{J}$. In general an equation for the transport of energy must also be supplied along with an equation of state relating the gas pressure to its temperature and density. However, if incompressibility is assumed, as it is in most analytical treatments of turbulent dynamo, the energy equation is superfluous.

5.2 Dimensionless numbers

If L and u represent characteristic length and velocity magnitudes of the system in question, then the ratio $\mathrm{Re} = \mathrm{uL}/\nu$ is referred to as the Reynolds number. It represents the relative importance of inertial and viscous forces. It can also be seen as the ratio of the viscous time $t_{visc} = L^2/\nu$ to the advective time $t_{adv} = L/u$. Flows having large Re are in general susceptible to turbulence as their dynamics are dominated by the non-linear coupling in Equation 6. Due to the enormous length scales involved in astrophysical flows, it is commonly appropriate to drop the viscous term altogether and just work with the Euler equation.

Analogously, one can define the *magnetic* Reynolds number $\mathrm{Rm} = \mathrm{uL}/\eta$ which measures, correspondingly the relative importance of magnetic induction and diffusion.It is also the ratio t_{res}/t_{adv} where $t_{res} = L^2/\eta$ is the damping time scale for magnetic fluctuations of scale L. The limiting case of infinite Rm corresponds to the case of a perfectly conducting fluid. This limit has the special property that the magnetic field is "frozen" into the fluid. Flux-freezing is a conservation law for the which the quantity $\int_S \mathbf{B} \cdot d\mathbf{A}$ is a constant of the motion, when S represents a surface which is deformed along with the flow. The limit of vanishing viscosity and resistivity is referred to as ideal MHD.

One can also define the magnetic Prandtl number $\mathrm{Pm} = \mathrm{Rm}/\mathrm{Re}$ which characterises the relative importance of viscous and resistive dissipation. In turbulent flows, it also corresponds to the ratio of length scales where the velocity and magnetic fields become smooth under the action of their respective dissipation mechanisms. Thus, for flows having large Pm, the velocity field is smoothed out by viscosity over scales where the magnetic field is still frozen into the fluid.

As an example, let us estimate the magnetic Prandtl number for various parts of the sun. The resistively of a fully ionised plasma is given by Priest (2014)

$$\eta = \frac{m_e}{\mu_0 n_e e^2 \tau_e} \qquad (8)$$

where m_e is the electron mass, n_e is electron number density, and τ_e is the electron mean free path. In the solar photosphere this gives $\eta \sim 10^4$ m^{2}s^{-1}. Thus vortices on the solar surface of size $\sim$ 500 km rotating with typical speed of $1\,\mathrm{km\,s^{-1}}$ (Simon and Weiss 1997) have magnetic Reynolds number $\mathrm{Rm} \sim 5 \times 10^4$. By comparison, the Reynolds number is of order 10^2 making the magnetic Prandtl number of order 10^2. A similar estimate for the solar convection zone gives $\mathrm{Pm} \sim 10^{-2}$. The magnetic Prandtl number of diffuse astrophysical plasmas is vastly larger, for example it is $\sim 10^{15}$ Maron and Cowley (2001) in the Galaxy and as large as 10^{22} (Schekochihin et al. 2002b) in the intergalactic medium. These values reflect their extraordinarily low collisionality.

Other dimensionless numbers of interest include the Lundquist number $S = Lv_A/\eta$ where $v_A = B/\sqrt{4\pi\rho}$ is the speed of Alfvén wave propagation. The sonic and Alfvén Mach numbers are flow speed measured in units of their corresponding wave propagation speeds. The sonic Mach number $\mathcal{M}$ is also roughly the square-root of the ratio of kinetic to thermal energy densities, and similarly with the Alfvén Mach number $\mathcal{M}_A$ except with the magnetic energy instead of the thermal energy.

5.3 Conserved quantities

Conserved quantities in MHD belong to one of three types (Bekenstein 1987). The first kind, referred to as type A, is where the closed line integral of a vector field is a constant of the motion, where the path of circulation is carried along with the local flow. The canonical example is that of Kelvin's circulation theorem, which states that in a perfect fluid the circulation $\Gamma = \oint_C \mathbf{u} \cdot \mathbf{ds}$ is a constant of the motion, where the closed path C is advected and deformed along with the flow. Kelvin's circulation theorem has been generalised to relativistic hydrodynamics by Taub (1959), and to non-relativistic and relativistic MHD by Bekenstein and Oron (2000). Another example is the Alfvén law of magnetic flux conservation, which states that $\Phi = \oint_C \mathbf{A} \cdot \mathbf{ds}$ is a constant of the motion. This law can also be stated as the constancy of magnetic flux through the surface whose boundary is the closed contour C. It formally applies in ideal MHD, where the viscosity and resistivity are vanishing. This law, commonly referred to as "flux freezing", is generally considered a good approximation in astrophysical plasmas, where the microphysical diffusion time of magnetic fields is far longer than other dynamical times. However, for turbulent flows in which the velocity field stays chaotic down to scales below those of interest (referred to as "rough" velocity fields) turbulent diffusivity effectively violates the perfect fluid assumptions. The violation of Kelvin's circulation theorem in high Reynolds number hydrodynamic flows has been analysed by Chen et al. (2006), and similarly for the Alfvén theorem by Eyink (2011).

The second type of conservation law (type B) is where a local fluid quantity is a constant of the motion along streamlines. Bernoulli's theorem is the canonical example of type B conservation laws.

Type C conservation laws apply to the global conservation of scalar or vector quantities whose transport is strictly local. These are like Gauss's law in that they relate the conserved quantity's time rate of change in a finite volume to its flux through the volume's bounding surface. They can thus be stated in either differential or integral form, the latter of which forms the basis for modern numerical methods for solving the hydrodynamic and MHD equations. The prototypical type C conservation law is the continuity equation $\dot{\rho} + \nabla \cdot \rho \mathbf{u} = 0$. In the absence of viscous and resistive effects, momentum and magnetic flux are also conserved in this way, where $\rho\mathbf{u}$ is replaced with a generic flux function for that conserved quantity. Similarly, in the absence of heat transport by conduction, the evolution of total energy can also be written as a continuity equation.

5.4 Consequences of helicity conservation

A particularly important conserved quantity is the magnetic helicity $H_M = \int d^3x \mathbf{A} \cdot \mathbf{B}$. H_M is a global topological characterisation of the magnetic field, which indicates the degree to which magnetic field loops are mutually linked. It is analogous to the kinetic helicity $H_K = \int d^3x \mathbf{u} \cdot \omega$ (where $\omega = \nabla \times \mathbf{u}$ is the vorticity) which represents the degree to which the vortex lines are mutually linked. In a perfect fluid, both helicity measures are a constant of the motion.

Magnetic helicity is important because it tends to establish magnetic fields well above the turbulence integral scale. This consequence of magnetic helicity conservation was first investigated by Frisch et al. (1975), where it was predicted that the magnetic energy would peak around the scale H_M/E_B (where E_B is the total magnetic energy), even if the injection of magnetic energy took place at much smaller scales. This process is referred to as inverse cascading because the magnetic energy cascades from small scales toward large scales, unlike the direct cascade of energy observed in hydrodynamic turbulence. It is the underlying mechanism for the formation of large-scale dynamos which may be responsible for establishing stellar magnetic fields.

5.5 Inverse cascade of magnetic energy

Magnetohydrodynamic turbulence exhibits an unusual phenomenon where, at least in its freely decaying state, magnetic field correlations arise over distances much longer than the original eddy size. This *inverse cascading* effect is reminiscent of what is seen in two-dimensional (but not three-dimensional) hydrodynamic turbulence. Although this effect may be related to the conservation of magnetic helicity (Frisch et al. 1975), until recently it was not appreciated that it happens even when the field is initially helicity-free. That is, when it is topologically equivalent to the uniform (or zero) magnetic field.

Inverse cascading of magnetic energy was first seen in numerical simulations by Meneguzzi et al. (1981). It has since been confirmed by many others using direct numerical simulations (Christensson et al. 2001; Banerjee and Jedamzik 2004; Kahniashvili et al. 2010; Tevzadze et al. 2012), quasi-analytical techniques such as the "Eddy damped quasi normal mode" approximation (Son 1999) and also turbulence "shell models" (Kalelkar and Pandit 2004). Such inverse cascading is a process of magnetic field self-assembly, whereby magnetic energy provided by random motions at very small scales, may over time establish dynamically important magnetic fields over scales that are much larger. For example, the sun's convective cells, which set the turbulence integral scale, are far smaller than the solar diameter. The ordered magnetic fields that extend over the whole surface of the sun (and many other stars) may come from such a self-assembly process. Dynamos capable of establishing magnetic fields on scales far larger than the turbulence integral scale are generically referred to as large scale dynamos.

The literature to date is still conflicted on whether non-zero magnetic helicity is a necessary condition for inverse cascading to occur. It was shown by Olesen (1997) and Shiromizu (1998) that inverse cascading could be expected even for non-helical magnetic field configurations, as a consequence of rescaling symmetries native to the Navier-Stokes equations. But no inverse cascading was seen in numerical studies based on EDQNM theory (Son 1999) or direct numerical simulations with relatively low resolution (Christensson et al. 2001; Banerjee and Jedamzik 2004).

Very recently, three numerical studies have appeared which confirm the existence of inverse cascading in non-helical three-dimensional MHD turbulence. Brandenburg et al. (2014) carried out high resolution direct numerical simulations of compressible MHD turbulence with magnetic energy injected at very small scales. The results confirm the growth of magnetic energy spectrum $P_M(k,t)$ at low wavenumbers. Zrake (2014) found similar results in high resolution simulations of freely decaying relativistic MHD turbulence. In the latter study, the evolution of $P_M(k,t)$ was parametrised around a self-similar ansatz which may be used to evaluate the time required for magnetic fields to be established over scales arbitrarily larger than the energy injection scale. Berera and Linkmann (2014) also found inverse cascades in decaying MHD turbulence. That study utilised a novel technique, whereby the system was evolved hydrodynamically with a passive magnetic field. The resulting energy spectrum also exhibits growth of magnetic field fluctuations at large scales, suggesting that inverse cascading of passively advected vector fields is a generic property of hydrodynamic turbulence.

5.6 Small-scale turbulent dynamo

In general, the term "dynamo" refers to the conversion of kinetic energy into magnetic energy. In magnetohydrodynamic theory, a number of distinct processes can facilitate this conversion. The only thing they all have in common is the presence of turbulence. Many excellent reviews have been written on this the connection between MHD turbulence and dynamo processes:

Brandenburg and Subramanian (2005); Lazarian and Cho (2005); Kulsrud and Zweibel (2008); Tobias et al. (2011). Here we will present a very brief overview of a process known as small-scale turbulent dynamo.

Small-scale turbulent dynamo operates in highly conducting turbulent fluids, even when the turbulence is isotropic and non-helical. Conceptually it is quite simple. Magnetic field lines carried along by the turbulent flow become increasingly chaotic and distorted over time, leading to enhancement of their energy. When the field is very weak, i.e. $v_A \ll u$, its back-reaction on the fluid can be ignored, and the field evolves as a passively advected vector field in otherwise hydrodynamic turbulent flow. This approximation is referred to as *kinematic* small-scale dynamo, and was first studied by Kazantsev (1968), but more modern analytic treatments exist such as that of Vincenzi (2001). The original formulation, known as the Kraichnan-Kazantsev dynamo starts with the assumption of an arbitrary spectrum of velocity fluctuations that are delta-correlated in time. Despite its simplicity, this model makes accurate predictions regarding the time rate of change and spectral energy distribution $P_M(k,t)$ of the magnetic field, where k is the inverse length scale. In particular, solutions for $P_M(k,t)$ that are exponentially growing in time exist when the microphysical diffusion coefficient η is small relative to the turbulent diffusivity, a condition which is met when the Reynolds number of the flow is sufficiently large.

The Kraichnan-Kazantsev dynamo predicts for the unstable growing solutions, (1) that $P_M(k,t) \propto k^{3/2}$ and peaks at the resistive cutoff, and (2) that the magnetic energy exponentiates at the fastest eddy turnover time of the turbulent hydrodynamic cascade. The first numerical experiments to confirm the basic features of the Kraichnan-Kazantsev dynamo were by Meneguzzi et al. (1981). At the time, direct numerical simulations were exceedingly expensive, and those simulations were limited to grid sizes of 32^3 and 64^3. Since then, many groups have continued this investigation utilising high performance parallel computing architectures, and modern numerical schemes for obtaining solutions to the MHD equation.

Although the original work by Kazantsev predicted that the magnetic Reynolds number Rm had to be "sufficiently large" for small scale dynamo action to occur, the exact value of the critical Rm needed to be found through numerical experiments. For the case of $\mathrm{Pm} = 1$, Haugen et al. (2004b) found that the critical magnetic Reynolds number $\mathrm{Rm}_{\mathrm{crit}}$ for dynamo action was about 35. For smaller Pm, it was found that $\mathrm{Rm}_{\mathrm{crit}}$ increases as Pm decreases. This may be partially understood as having to do with the flow properties in the resistive interval. When the Pm is very large, the resistive scale lies deep below the viscous cutoff, so that magnetic energy peaks where the flow is very smooth. However, when Pm is smaller than one, the resistive scale moves into the turbulent inertial range of the flow, and the velocity field is *rough* at the resistive scale. Haugen et al. (2004c) found that $\mathrm{Rm}_{\mathrm{crit}}$ increased to 70 when the flow is supersonic, even when $\mathrm{Pm} = 1$. Federrath et al. (2011) found that for compressible flows the dynamo growth rate depends upon how the turbulence is forced; vortically driven flows produced more efficient dynamo than did dilatational forcing. It was argued analytically by Boldyrev and Cattaneo (2004) that in the small Pm regime, the critical magnetic Reynolds number should be a strong function of the *roughness* exponent α of the velocity field, $v_\ell \propto \ell^\alpha$, such that smaller α (a rougher field) requires a larger Rm to produce small-scale kinematic dynamo.

A central question around small-scale dynamo is the nature of its saturation. The exponentially growing solution of the Kazentsev model holds only as long as the dynamo is in the kinematic regime, so that magnetic back-reaction on the flow is negligible. Once the field attains a strength high enough to compete with the fluid inertia, the kinematic assumption breaks down, and some sort of saturation behaviour is expected. This saturation is a highly nonlinear problem, and thus very difficult to study analytically. Schekochihin et al. (2002a) argued that in the non-linear stage, at least for very large Pm, magnetic energy would remain concentrated in the sub-viscous range, moving toward the inertial range on a resistive rather than dynamical time. Were this to be the case, equipartition of the turbulent kinetic and magnetic energy densities at the largest scales would require an asymptotically long time to be established. However, this scenario has not been observed in the numerical literature. Instead, the emerging consensus (other differences aside) (Cho and Vishniac 2000; Brandenburg et al. 2003; Haugen et al. 2004a; Schleicher et al. 2013; Schekochihin et al. 2004; Maron and Cowley 2001; Zrake and MacFadyen 2011, 2013) is that magnetic energy shifts into the inertial range regardless of the magnetic Prandtl number. Once there, the field attains coherence at increasing scales until finally reaching the energy-containing scale of turbulence, thus establishing scale-by-scale equipartition. Both the kinetic and magnetic energy follow a Kolmogorov spectrum, with the magnetic energy per unit wavelength exceeding that of the kinetic energy by a factor of around 2.

The time-scale for the flow to fully establish scale-by-scale equipartition is still not completely clear. It was argued by Beresnyak (2012) that non-linear small-scale dynamo exhibits universality in the sense that a constant fraction C_E of the work done by turbulent pumping accumulates in magnetic energy. In this scenario, scale-by-scale equipartition should be attained after a time $1/C_E$. Careful modelling of the approach to saturation by Zrake and MacFadyen (2013) finds that indeed scale-by-scale equipartition is reached after a number of dynamical times that is independent of the Reynolds number.

6 Conclusions

In this chapter we have reviewed progress made on the origin of magnetic fields in intermediate and massive main sequence stars and in compact stars.

Observations of magnetic fields in white dwarfs may indicate that the distribution is bi-modal, exhibiting a paucity of objects between $10^5 - 10^6$ G, although further sensitive spectropolarimetric surveys conducted on 8 m class telescopes are needed in order to confirm this finding. Should this bimodality be confirmed, then two different channels for field formation and evolution could be at play. In particular, one could identify weakly magnetic main sequence stars such as Vega and Sirius as the progenitors of weakly magnetic white dwarfs belonging to the low field component. This would lend some support to the fossil field hypothesis for the origin of fields in compact stars.

However, the viability of the fossil field hypothesis has been recently questioned because of the absence of magnetic white dwarfs paired with non-degenerate stars. These observations mirror the dearth of magnetic non-degenerate A, B and O stars in short-period binary systems. It is not clear whether there is any connection or analogy to be made between fields on the main sequence and compact star phases, but a merger scenario for field formation has been suggested to overcome some of the problems raised by recent observations.

Here we stress that the pairing properties of magnetic white dwarfs do not necessarily invalidate the viability of the fossil scenario since more than one formation channel could be at work. However, the complete lack, rather than just a dearth, of magnetic white dwarfs paired with non-degenerate companions in non-interacting systems, raises strong doubts on whether the fossil route can be the main channel for the formation of magnetic white dwarfs. If magnetic white dwarfs were the descendants of magnetic main sequence stars, then magnetic white dwarfs should be commonly found in binary systems to reflect the incidence of binarity observed in (magnetic) stars of F to late B spectral types. However, there is *not one single example* of a magnetic white dwarf paired with a non-degenerate companion in a non-interacting binary.

In order to explain this curious finding, it has been proposed that fields in isolated highly magnetic white dwarfs could be generated during common envelope evolution when the two components of a binary system merge.

The observational situation surrounding birth magnetic fields of neutron stars remains frustratingly uncertain, because what measurements exist are indirect. Population synthesis models point, on balance, to moderate birth fields $\sim 10^{12.65\pm0.55}$ G. Zeeman spectropolarimetry has revealed strongly magnetised (i.e., a few thousand gauss) progenitors across a range of spectral classes, but even so it seems that the numbers may be insufficient to account for the magnetar birth rate under the fossil field scenario. At least for magnetars, therefore, a proto-neutron star dynamo is favoured, driven by differential rotation and the Tayler or magneto-rotational instabilities rather than thermal convection (Spruit 2009). Emergence (through buoyancy), stability (through helicity), and ongoing activity (through superfluid turbulence) play roles in determining the strength of the (observable) surface field. Future gravitational wave observations offer the best prospects for direct internal field measurements. Until these observations become available we can only say that, as for magnetic white dwarfs, it is possible that both fossil and dynamo processes are at work in different neutron stars.

We have also reviewed the theory of magnetohydrodynamic turbulence because its presence underpins the operation of stellar dynamos and controls diffusive magnetic flux transport in stars. But MHD turbulence poses a great theoretical challenge due to its inherent complexities, and many features of its operation are yet to be satisfactorily explained. Advances in numerical algorithms and computing resources have yielded considerable gains in the empirical grounding of various phenomenological pictures.

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