Natural leptogenesis and neutrino masses with two Higgs doublets

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(Received 14 June 2015; published 17 August 2015)

The minimal Type I seesaw model cannot explain the observed neutrino masses and the baryon asymmetry of the Universe via hierarchical thermal leptogenesis without ceding naturalness. We show that this conclusion can be avoided by adding a second Higgs doublet with $\tan\beta \gtrsim 4$. The models considered naturally accommodate a standard model-like Higgs boson and predict TeV-scale scalar states and low- to intermediate-scale hierarchical leptogenesis with $10^3 \text{ GeV} \lesssim M_{N_1} \lesssim 10^8 \text{ GeV}$.

I. INTRODUCTION

The discovery of a neutral Higgs boson at the LHC [1,2] has strengthened the case for the standard model (SM) paradigm of spontaneous electroweak symmetry breaking: a scalar doublet Φ gains a vacuum expectation value (vev) $\langle\Phi\rangle = v/\sqrt{2} \approx 174$ GeV by virtue of the potential

$$V_{\text{SM}} = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2, \tag{1}$$

with $\mu^2 < 0$. As well, the measurement of neutrino oscillations [3–8] suggests that the SM should be extended to incorporate neutrino masses. A straightforward way to achieve this is to include three right-handed neutrinos. Then gauge invariance allows two extra renormalizable terms to be added to the Yukawa Lagrangian,

$$-\Delta\mathcal{L}_Y = (y_\nu)_{ij}\bar{l}_L^i \tilde{\Phi} \nu_R^j + \frac{1}{2}M_i\overline{(\nu_R^i)^c}\nu_R^i + \text{H.c.}, \tag{2}$$

where $l_L = (\nu_L, e_L)^T$, $\tilde{\Phi} = i\tau_2\Phi^*$, and M_i are the right-handed neutrino masses. The SM extended in this way is what we refer to as the minimal Type I seesaw model [9–12].

If $y_\nu v \ll M_i$ then the minimal Type I seesaw provides an elegant explanation for the smallness of the neutrino masses. After electroweak symmetry breaking, the neutrino mass matrix is given by the seesaw formula

$$m_\nu = \frac{v^2}{2}y_\nu \mathcal{D}_M^{-1} y_\nu^T, \tag{3}$$

where $\mathcal{D}_M \equiv \text{diag}(M_1, M_2, M_3)$, suppressed by the presumably large right-handed neutrino mass scale.

The minimal Type I seesaw also provides a mechanism to reproduce the baryon asymmetry of the Universe (BAU). Fukugita–Yanagida hierarchical thermal leptogenesis [13] proceeds via the *CP*-violating out-of-equilibrium decays of the lightest right-handed neutrino N_1, creating a lepton asymmetry which is reprocessed into the baryon sector by the electroweak sphalerons. Successful hierarchical thermal leptogenesis is possible when the Davidson–Ibarra bound (ensuring enough *CP* asymmetry in the decays) is satisfied [14,15],

$$M_{N_1} \gtrsim 5 \times 10^8 \text{ GeV}\left(\frac{v}{246 \text{ GeV}}\right)^2, \tag{4}$$

where v is the vev that enters the seesaw of Eq. (3).

The ability of the minimal Type I seesaw model to simultaneously explain neutrino masses and the BAU is certainly intriguing. However, Vissani observed [16] that the model is incapable of doing so without generating a naturalness problem.[1] Equation (4) is simply incompatible with the conservative naturalness requirement that corrections to the electroweak μ^2 parameter of Eq. (1) not exceed 1 TeV^2. With three flavors of hierarchical right-handed neutrinos, this requires [17]

$$M_{N_1} \lesssim 3 \times 10^7 \text{ GeV}\left(\frac{v}{246 \text{ GeV}}\right)^{\frac{2}{3}}. \tag{5}$$

The incompatibility is exemplified in Fig. 1; nowhere at $v = 246$ GeV is it possible to simultaneously fulfill the Davidson–Ibarra and Vissani bounds.

A sensible question is then: in what minimal ways can this incompatibility be overcome? Figure 1 suggests three conspicuous (but not mutually exclusive) options: (1) Modify the correction to μ^2, e.g., by restoring supersymmetry or by partly cancelling the correction from the heavy fermion loop [18,19]; (2) lower the Davidson–Ibarra bound, e.g., by considering resonant leptogenesis [20], an alternative mechanism [21], or by introducing new fields which allow an increased *CP* asymmetry in the right-handed neutrino decay; (3) seek an extension of the canonical seesaw for neutrino mass, i.e., reduce the (possibly effective) v entering the seesaw Eq. (3).

In this paper we will consider the third option. Specifically we will examine alternative seesaw possibilities when the minimal Type I see-saw model is extended

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[1]Naturalness is admittedly an aesthetic requirement of a model, and the possibility remains that nature is just fine-tuned.
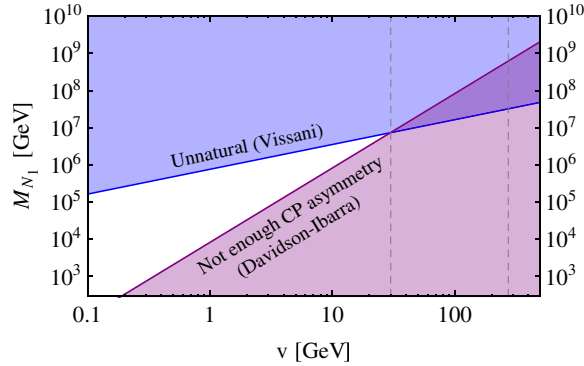
FIG. 1 (color online). Bounds on hierarchical thermal lepto-genesis as a function of v. Shown is the Davidson–Ibarra bound (purple) and the Vissani bound (blue). The dashed lines indicate $v = 246, 30$ GeV.

by a second Higgs doublet Φ_2. We are motivated by the following observation: if the seesaw neutrino mass of Eq. (3) is evaluated at $v \lesssim 30$ GeV, then Eqs. (4) and (5) become compatible, as is clear from Fig. 1. Thus, we expect that two-Higgs-doublet models with right-handed neutrinos (ν2HDMs) and $\tan\beta = v_1/v_2 \gtrsim 8$, where Φ_2 is responsible for a tree-level seesaw, can naturally accom-modate leptogenesis and neutrino masses. In fact, we find that $\tan\beta \gtrsim 4$ is possible, since the extra scalar states can be naturally TeV scale and the Vissani bound can be relaxed.

Upon examining the ν2HDM scenarios which will succeed, we rediscover the radiative Ma model [22] as the only possibility when $v_2 = 0$. Otherwise, in models without a significant radiative neutrino mass component, we require $0.3 \lesssim v_2/\text{GeV} \lesssim 60$. The potentially small vev is made natural by softly breaking a $U(1)$ or Z_2 symmetry, which also automatically results in one SM-like CP-even state. One advantage of this scenario is that it can work in all ν2HDM types, greatly increasing the opportunity for model building.

The paper is organized as follows. In Sec. II we build the ν2HDM models of interest, describe the scalar states, and briefly review the relevant experimental constraints. In Sec. III we pay particular attention to naturalness limits on the extra scalars; we verify that a natural ν2HDM of any type is still allowed by experiment. We discuss neutrino masses in Sec. IV and leptogenesis in Sec. V. The region of parameter space which naturally achieves hierarchical leptogenesis is identified. We conclude in Sec. VI.

II. ν2HDM MODEL

A. Lagrangian

The scalar content of the model contains two doublets $\Phi_{1,2}$ each with hypercharge $+1$. For simplicity we consider

TABLE I. The four models with no tree-level flavor-changing neutral currents and allowing for a GeV-scale vev to provide the seesaw while preserving perturbativity of y_t.

Model	u_R^i	d_R^i	e_R^i	ν_R^i
Type I	Φ_1	Φ_1	Φ_1	Φ_2
Type II	Φ_1	Φ_2	Φ_2	Φ_2
LS	Φ_1	Φ_1	Φ_2	Φ_2
Flipped	Φ_1	Φ_2	Φ_1	Φ_2

the softly broken, CP-conserving, Z_2-symmetric potential (see, e.g., Ref. [23])

$$
\begin{aligned}
V_{2\text{HDM}} = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\
& + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 \\
& + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2],
\end{aligned} \tag{6}
$$

where all the parameters are real. To explain observations, at least one of these doublets must obtain a nonzero vev. We consider $m_{11}^2 < 0$ and a CP-conserving vacuum,

$$
\langle\Phi_1\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{7}
$$

where $v_1 > 0$, $v_2 \geq 0$, and $v_1^2 + v_2^2 = v^2 \approx (246 \text{ GeV})^2$.

A general 2HDM will have flavor-changing neutral currents at tree level. These can be avoided if right-handed fermions of a given type (u_R^i, d_R^i, e_R^i) couple to only one of the doublets [24,25]. Although not strictly necessary, we will assume that this is realized and adopt the convention that only Φ_1 couples to the u_R^i. In a ν2HDM, if we assume this also applies for the ν_R^i, then there are eight possibilities. As mentioned in the Introduction, the seesaw constraint Eq. (3) can be made consistent with naturalness and leptogenesis if the vev contributing to the seesaw is sufficiently small. Since we would like our model to remain perturbative, and already $y_t \approx 1$ for $v \approx 246$ GeV, we anticipate that Φ_2 obtains the small vev, and thus we couple it to the ν_R^i. Remaining are four possible ν2HDMs which we refer to by their conventional types as listed in Table I.[2] The Yukawa Lagrangian is then given by

$$
\begin{aligned}
-\mathcal{L}_Y = {} & + y_u \overline{q_L} \tilde{\Phi}_1 u_R + y_d \overline{q_L} \Phi_I d_R \\
& + y_e \overline{l_L} \Phi_J e_R + y_\nu \overline{l_L} \tilde{\Phi}_2 \nu_R \\
& + \frac{1}{2} M_N \overline{(\nu_R)^c} \nu_R + \text{H.c.}, \tag{8}
\end{aligned}
$$

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[2]Type I ν2HDMs with $v_2 \sim$ eV were considered in Refs. [26–29]. We will end up considering v_2 of $\mathcal{O}(0.1 - 10)$ GeV.

where I, J depend on the model type, and family indices are implied.

B. Scalar masses and mixings

Consistency with experiments requires the extra scalar states to have masses at least $\gtrsim 80$ GeV. To construct models with potentially TeV-scale scalars with a naturally small v_2, we will consider $m_{22}^2 > 0$ and $m_{12}^2/m_{22}^2 \ll 1$ [26]. This is technically natural, since in the limit $m_{12}^2/m_{22}^2 \to 0$ a $U(1)$ or Z_2 symmetry is restored if $\lambda_5 = 0$ or $\lambda_5 \neq 0$, respectively.

For $m_{11}^2 < 0$, the vevs are given by

$$v_1 \approx \sqrt{\frac{-2m_{11}^2}{\lambda_1}}, \tag{9}$$

$$v_2 \approx \frac{1}{1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345}} \frac{m_{12}^2}{m_{22}^2} v_1, \tag{10}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. These relations become exact when $m_{12}^2 = 0$ and when $\lambda_2 = 0$, respectively. For $m_{22}^2 \gg \lambda_{345} v_1^2$, $\tan\beta \equiv v_1/v_2 \approx m_{22}^2/m_{12}^2$.

There is a useful constraint on m_{22}^2 which is derived as follows. The minimization conditions give

$$m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 = \tan^2\beta\left(m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2\right), \tag{11}$$

where $\lambda_1 v_1^2 \approx m_h^2 \approx (125\text{ GeV})^2$ (see below). In the limit $m_{22}^2 \gg \lambda_2 v_2^2/2$, it can be seen that m_{22}^2 is bounded above by

$$m_{22}^2 \lesssim \frac{1}{2}m_h^2\tan^2\beta \tag{12}$$

if m_{11}^2 is to remain negative. Figure 2 illustrates how m_{11}^2 deviates from its standard value of $-(88\text{ GeV})^2$ as m_{22}^2 approaches this bound. For m_{22}^2 above this bound, m_{11}^2 very quickly grows to values $> v^2$, and $v \approx 246$ GeV is only
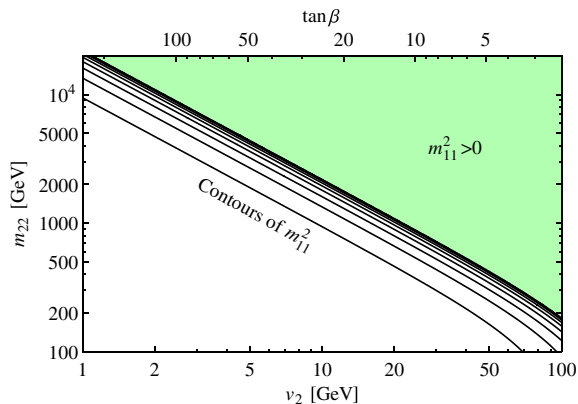

FIG. 2 (color online). Contours of $m_{11}^2/\text{GeV}^2 = -80^2, -70^2$, and so on. The shaded region has no solution for $m_{11}^2 < 0$.

explained by a miraculous balance of m_{11}^2 against $m_{22}^2/\tan^2\beta$, which constitutes a fine-tuning. Thus, we adopt Eq. (12) as a consistency condition.

The charged scalar and pseudoscalar (neutral scalar) mass-squared matrices are diagonalized by a mixing angle β (α). The neutral mass eigenstates are

$$h = \rho_1 \cos\alpha + \rho_2 \sin\alpha,$$
$$H = \rho_2 \cos\alpha - \rho_1 \sin\alpha,$$
$$A = \eta_2 \sin\beta - \eta_1 \cos\beta, \tag{13}$$

where $\rho_i = \sqrt{2}\text{Re}(\Phi_i^0) - v_i$ and $\eta_i = \sqrt{2}\text{Im}(\Phi_i^0)$. The masses are given by

$$m_h^2 = \lambda_1 v_1^2 + \mathcal{O}\left(\frac{m_{12}^4}{m_{22}^4}v_1^2\right),$$

$$m_H^2 = m_{22}^2 + \frac{1}{2}\lambda_{345}v_1^2 + \mathcal{O}\left(\frac{m_{12}^4}{m_{22}^4}m_{22}^2\right),$$

$$m_A^2 = m_{22}^2 + \frac{1}{2}(\lambda_{345} - 2\lambda_5)v_1^2 + \mathcal{O}\left(\frac{m_{12}^4}{m_{22}^4}m_{22}^2\right),$$

$$m_{H^\pm}^2 = m_{22}^2 + \frac{1}{2}\lambda_3 v_1^2 + \mathcal{O}\left(\frac{m_{12}^4}{m_{22}^4}m_{22}^2\right), \tag{14}$$

i.e., the same as in the inert doublet model [23] up to corrections proportional to m_{12}^4/m_{22}^4, which we provide in Appendix A. Clearly, if $m_{22}^2 \gg v^2$, the mass scale of extra scalar states is $\approx m_{22}$.

In the alignment limit $\cos(\alpha - \beta) \to 0$, the couplings of h to SM particles become SM-like. We calculate

$$\cos^2(\alpha - \beta)$$
$$\approx \frac{m_{12}^4}{m_{22}^4}\frac{v_1^4}{m_{22}^2}\frac{(\lambda_1 - \lambda_{345})^2}{\left(1 - \frac{v_1^2}{2m_{22}^2}(2\lambda_1 - \lambda_{345})\right)^2\left(1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345}\right)^2}, \tag{15}$$

suppressed by the approximate $U(1)$ or Z_2 symmetry ($m_{12}^2/m_{22}^2 \ll 1$) as well as the usual decoupling limit suppression ($v_1^2/m_{22}^2 \ll 1$) [30]. Thus, the model naturally accommodates a SM-like neutral scalar state.

C. Constraints

With $M_N > m_{22}$ the constraints (and search strategies) for a ν2HDM of a given type are largely identical to those for a 2HDM of the same type, for which there is extensive literature (see references henceforth). The 2HDM potential Eq. (6) is subject to a few standard theoretical constraints [23]. The necessary and sufficient conditions for positivity of the potential in all directions are [31–33]

$$\lambda_{1,2} \geq 0,$$
$$\lambda_3 \geq -\sqrt{\lambda_1 \lambda_2},$$
$$\lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}. \tag{16}$$

Vacuum stability of the potential minimum is more difficult to evaluate. An inequality which ensures a global minimum, missing possible metastable vacua, is presented in Ref. [34]. Tree-level perturbative unitarity of scalar-scalar scattering is ensured by bounding the eigenvalues of the scattering matrix [23,35–37]. Perturbativity of the λ_i can also be demanded [38]. At the very least, these bounds should be implemented at the mass scale of the scalar states. In addition they may be demanded up to some high scale under the renormalization group evolution, which results in nontrivial constraints on the parameter space (see, e.g., Refs. [39–41][3]). Type II, lepton-specific (LS), and flipped 2HDMs are particularly susceptible to exclusion by such a demand at large $\tan\beta$; at one loop, their Yukawa couplings hit a Landau pole before $M_{\text{Pl}} \sim 10^{18}$ GeV when $v_2 \lesssim 3.6, 2.3, 3.3$ GeV ($\tan\beta \gtrsim 68$, 107, 75) respectively [42]. These Landau poles merely indicate the breakdown of perturbativy.

The scalar boson discovered at the LHC is to be identified with the mass eigenstate h. Its couplings have been measured to be SM-like, which constrains the ν2HDM to lie in the alignment limit $\cos(\alpha - \beta) \approx 0$, particularly at large $\tan\beta$, and for Type II and flipped 2HDMs.[4] As is evident from Eq. (15), the alignment limit is automatically preferred in our model due to the approximate Z_2 or $U(1)$ symmetry. Thus, we limit the following discussion on additional experimental limits to those that constrain moderate to large $\tan\beta$ models very close to the alignment limit.

In Type II and flipped 2HDMs, the Φ_2 coupling to down-type quarks is $\tan\beta$ enhanced. The $H^{\pm}$ state then contributes significantly to radiative $B \to X_s\gamma$ decay; the experimental measurement [49] combined with a recent next-to-next-to-leading-order SM calculation [50] bounds $m_{H^{\pm}} \gtrsim 480$ GeV at 95% C.L. for $\tan\beta \gtrsim 2$. This bound along with the consistency condition Eq. (12) implies $v_2 \lesssim 45$ GeV ($\tan\beta \gtrsim 5.4$) for these ν2HDMs. In the Type II 2HDM, the Φ_2 coupling to e_R^i is also $\tan\beta$ enhanced, and the bound on $m_{H^{\pm}}$ from $B \to \tau\nu$ decays exceeds the radiative bound when $\tan\beta \gtrsim 60$ [23].

Direct searches at Large Electron–Positron Collider constrain $m_{H^{\pm}} \gtrsim 80$ GeV assuming decay to SM particles [51]. At the LHC, searches for $H/A \to \tau\tau$ [52,53] are particularly constraining in the Type II 2HDM. The 95% C.L. limit rises approximately linearly from $m_A \gtrsim 300$ GeV at $\tan\beta = 10$ to $m_A \gtrsim 1000$ GeV at $\tan\beta = 60$. Such searches can also be mildly constraining for the LS 2HDM at moderate $\tan\beta$. Searches for $H^{\pm} \to \tau\nu$ [54,55] cannot compete with $B \to X_s\gamma$ for Type II/flipped 2HDMs or with $H/A \to \tau\tau$ for the LS 2HDM. However, for $m_{H^{\pm}} < 160$ GeV, significant parameter space is ruled out in Type I 2HDMs with moderate $\tan\beta$.

The $(y_\nu)_{ij}\overline{l_L^i}\tilde{\Phi}_2\nu_R^j$ Yukawa term related to the neutrino masses can induce lepton flavor violating decays; these are suppressed by the small y_ν and the right-handed neutrino mass scale $M_N > m_{22}$. The processes of interest are $l_\alpha \to l_\beta\gamma$, $l_\alpha \to 3l_\beta$, and $\mu \to e$ conversion in nuclei (see Ref. [56] for expressions). As well, $b \to sl_\alpha\bar{l}_\beta$ decays are induced in Type II and flipped ν2HDMs. In practice, lepton flavor violating measurements constrain linear combinations of $(y_\nu)_{ij}$ bi- and trilinears as well as the M_{N_i}.

In summary, for moderate to large $\tan\beta$ and $\cos(\alpha - \beta) \approx 0$, experiments are most constraining for the Type II and flipped 2HDMs, with $m_{22} \gtrsim 480$ GeV necessary (implying $v_2 \lesssim 45$ GeV). For Type I and LS 2HDMs, even additional scalars with masses down to 80 GeV may still have evaded detection.

III. NATURALNESS

In the SM, the renormalization group equation (RGE) for the electroweak μ parameter [as in Eq. (1)] is dominated by the top quark Yukawa,

$$\frac{d\mu^2}{d\ln\mu_R} \approx \frac{1}{(4\pi)^2}6y_t^2\mu^2, \qquad (17)$$

where μ_R is the renormalization scale. At low energy we measure $\mu^2 \approx -(88 \text{ GeV})^2$, and under SM running it is apparent that $|\mu|$ remains ~ 100 GeV even up to the Planck scale $M_{\text{Pl}} \sim 10^{18}$ GeV. Thus, there is no *measurable* naturalness problem in the SM alone; there is no fine-tuning of any measurable parameter at a high scale, only the cancellation of an unmeasurable bare parameter against an unphysical cutoff scale, which should be assigned no physical significance. With this understood, it is clear that a measurable naturalness problem can only arise when $d\mu^2/d\ln\mu_R \gtrsim \mu^2$. Indeed, this is exactly how the Vissani bound in the Type I seesaw model can be interpreted [16,17]. Let us now examine when the ν2HDM encounters such a problem.

In practice, the naturalness considerations can be divided into two distinct calculations: the influence of m_{22} on m_{11} and the influence of M_N on m_{22}. These influences will be considered in turn.[5]

A. Corrections to m_{11}^2

If $m_{22}^2 \ll m_h^2\tan^2\beta/2$, then m_{11}^2 sets the mass of the observed SM-like Higgs via Eqs. (9) and (14). The one-loop RGE for the m_{11}^2 parameter is [23] (see Ref. [41] for a recent two-loop calculation)

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[3]Note that some of the bounds derived in these papers do not apply to the softly broken Z_2-symmetric case and also do not apply to the Z_2-symmetric case when one of the vevs vanishes.

[4]We refer the reader to Refs. [41,43–48] for allowed parameter space as a function of $\cos(\alpha - \beta)$ and $\tan\beta$ in all 2HDM types.

[5]In the following we ignore the influence of the small y_ν Yukawas on m_{11}^2, and hence those results also hold in a general 2HDM.

$$\frac{dm_{11}^2}{d\ln\mu_R} = \frac{1}{(4\pi)^2}[(4\lambda_3 + 2\lambda_4)m_{22}^2 + \mathcal{O}(m_{11}^2)]. \quad (18)$$

The $\mathcal{O}(m_{11}^2)$ term contains gauge, λ_1, and Yukawa contributions, which, as in the SM case, do not induce a naturalness problem. However, if $\lambda_{3,4}$ are nonzero, then a naturalness problem is induced for sufficiently large m_{22}^2; we are interested in when this generically occurs. Even if $\lambda_{3,4} = 0$ at some scale, they will quickly be reintroduced by gauge interactions at one loop. Their one-loop RGEs are given by

$$\frac{d\lambda_3}{d\ln\mu_R} = \frac{1}{(4\pi)^2}\left[\frac{3}{4}(g_Y^4 - 2g_Y^2 g_2^2 + 3g_2^4) + \cdots\right],$$
$$\frac{d\lambda_4}{d\ln\mu_R} = \frac{1}{(4\pi)^2}[3g_Y^2 g_2^2 + \cdots], \quad (19)$$

where $g_2^2(m_Z) \approx 0.43$ and $g_Y^2(m_Z) \approx 0.13$ are the gauge couplings and the ellipses contain terms multiplicative in $\lambda_{3,4}$, terms proportional to λ_5^2, and terms related to the Yukawas. Let us ignore those effects for now and return to them later. Note that ignoring the contribution from λ_5^2 is equivalent to assuming $\lambda_5 \lesssim 0.2$, so that its contribution is subdominant to the gauge couplings. Typically, one would expect

$$|\lambda_3(\mu_R)| \gtrsim \frac{1}{(4\pi)^2}\frac{3}{4}(g_Y^4 - 2g_Y^2 g_2^2 + 3g_2^4),$$
$$|\lambda_4(\mu_R)| \gtrsim \frac{1}{(4\pi)^2}3g_Y^2 g_2^2, \quad (20)$$

and thus

$$\left|\frac{dm_{11}^2}{d\ln\mu_R}\right| \gtrsim \frac{1}{(4\pi)^4}(3g_Y^4 + 9g_2^4)m_{22}^2. \quad (21)$$

This lower bound is of the same order as the two-loop pure gauge contribution [41].

Equation (21) represents a conservative bound on the running of the m_{11}^2 parameter above the scale $\sim m_{22}$. Naturalness demands that this running not be significantly larger than the value measured at a low scale, $|m_{11}| \approx 88$ GeV. A very conservative naturalness bound is therefore

$$\frac{1}{(4\pi)^4}(3g_Y^4 + 9g_2^4)m_{22}^2 < 1 \text{ TeV}^2, \quad (22)$$

$$\Rightarrow m_{22} \lesssim 1 \times 10^5 \text{ GeV}. \quad (23)$$

Alternatively, we can try to bound a quantity which measures the fine-tuning in m_{11}^2 at some high scale Λ_h. A typical quantity is [57,58]

$$\Delta(\Lambda_h) = \left|\frac{m_{11}^2(\Lambda_h)}{m_{11}^2(0)}\frac{\partial m_{11}^2(0)}{\partial m_{11}^2(\Lambda_h)}\right|, \quad (24)$$

which compares percentage variations of two (in principle) measurable parameters. Let us now estimate how such a bound might constrain m_{22}.

For simplicity, and anticipating that the m_{22} scale is not far above the electroweak scale, we will evolve the dimensionless parameters using the (ν)2HDM RGEs from the m_Z scale. First, the one-loop gauge coupling RGEs [23,59] can be solved analytically. Upon substitution into the $\lambda_{3,4}$ RGEs [Eqs. (19)], and considering only the pure gauge contribution, the $\lambda_{3,4}$ running can be solved for given initial conditions. For simplicity we take $\lambda_3(m_{22}) = \lambda_4(m_{22}) \equiv \lambda_{3,4}(m_{22})$ and consider it a free parameter. Next we solve Eq. (18) for $m_{11}^2(\mu_R)$ with the initial condition $m_{11}^2(m_{22}) = -(88 \text{ GeV})^2$ (neglecting any RGE evolution of m_{22}^2). With these simplifications $\partial m_{11}^2(0)/\partial m_{11}^2(\Lambda_h) = 1$, and the fine-tuning measure is given simply by $\Delta(\Lambda_h) = |m_{11}^2(\Lambda_h)/(88 \text{ GeV})^2|$.

Note that in setting the initial condition $m_{11}^2(m_{22}) = -(88 \text{ GeV})^2$ we have implicitly assumed that $m_{22}^2 \ll m_h^2 \tan^2\beta/2$ [see Eq. (12) and Fig. 2]. This is conservative for negative m_{11}^2, since $|m_{11}^2(m_{22})|$ shrinks as $m_{22}^2/\tan^2\beta \to m_h^2/2$ and the naturalness constraint would become more stringent. In some circumstances we will obtain naturalness bounds on m_{22}^2 which exceed $m_h^2 \tan^2\beta/2$, which just indicates that the naturalness constraint is weaker than the consistency condition Eq. (12).

In Fig. 3 we show $\Delta = 10$ and $\Delta = 100$ contours as a function of Λ_h and $\lambda_{3,4}(m_{22})$. These represent naturalness upper bounds on m_{22}. The cusplike structures of apparently low fine-tuning in m_{11}^2 occur when m_{11}^2 runs negative before turning and passing through $m_{11}^2 = 0$, which just corresponds to a fine-tuning in $(\lambda_{3,4}, \Lambda_h)$. A stringent naturalness constraint is obtained by demanding $\Delta < 10$ at $\Lambda_h = M_{Pl}$; from Fig. 3 it is clear that this implies

$$m_{22} \lesssim \text{few} \times 10^3 \text{ GeV}. \quad (25)$$

If any new physics comes in below M_{Pl}, then the running of m_{11}^2 could change, and these bounds do not apply. If that is the case, then it is more appropriate to consider Λ_h at the scale of the new physics, which weakens the bound, as is clear from Fig. 3. In the ν2HDM this new physics scale is the right-handed neutrino scale M_N, after which the right-handed neutrinos can contribute to the running of m_{11}^2 through m_{22}^2 at one loop.

We have so far ignored the RGE contributions from possibly large Yukawas. There are two situations in which the Yukawas play a significant role. The first is in Type II, LS, and flipped ν2HDMs with $\tan\beta$ large enough such that an early Landau pole is induced (see Sec. II C), and the second is in Type II and flipped ν2HDMs with
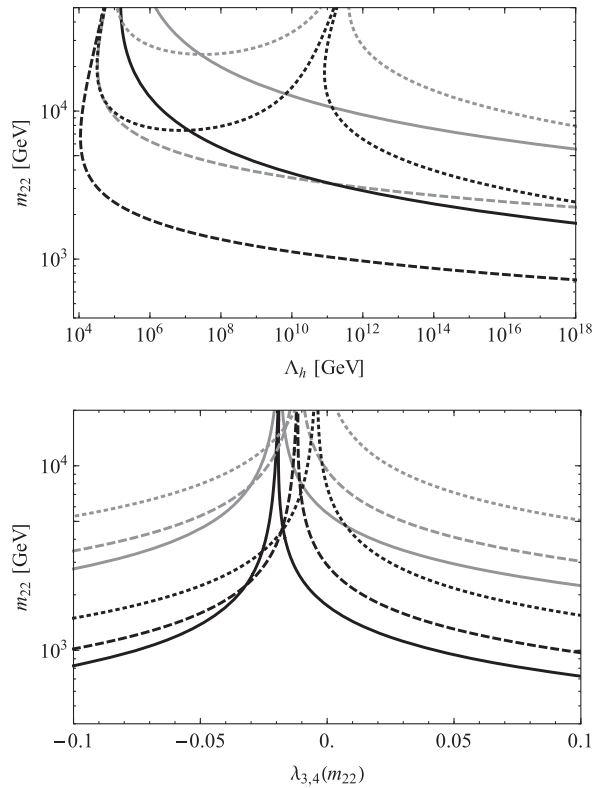
FIG. 3.   Contours of the fine-tuning measure $\Delta(\Lambda_h) = 10(100)$ in black (gray) as a function of (top) Λ_h for $\lambda_{3,4}(m_{22}) = 0.0, 0.1, -0.01$ (solid, dashed, dotted) and (bottom) $\lambda_{3,4}(m_{22})$ for $\Lambda_h = 10^{18}, 10^{12}, 10^7$ GeV (solid, dashed, dotted). See the text for the assumptions that accompany this plot.
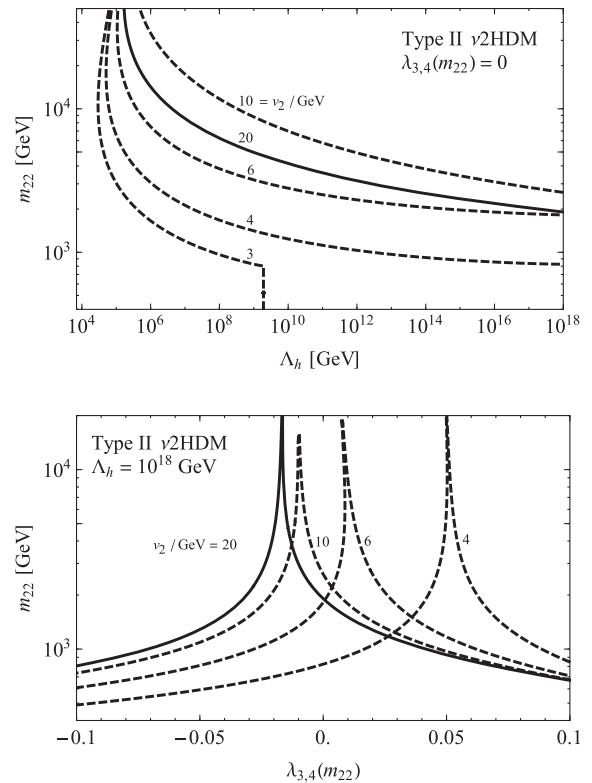
FIG. 4.   Contours of the fine-tuning measure $\Delta(\Lambda_h) = 10$ in an illustrative Type II ν2HDM (see the text) for different values of v_2. The solid lines for $v_2 = 20$ GeV match approximately onto the solid lines in Fig. 3.

moderate to large $\tan\beta$ when the pure Yukawa term $\pm\frac{1}{(4\pi)^2}12y_b^2y_t^2$ induced by a quark box diagram contributes significantly to the $\lambda_{3,4}$ RGEs [Eqs. (19)]. In Fig. 4 we show how the $\Delta = 10$ contours change as a function of v_2 in an example Type II ν2HDM. For this figure we have numerically solved the full set of one-loop RGEs [41] including the top/bottom/tau Yukawas, taking the following values at the scale M_Z: $g_s^2 = 1.48$, $\lambda_1 = \lambda_2 = 0.26$, and $y_t = 0.96/\sin\beta$, $y_b = 0.017/\cos\beta$, $y_\tau = 0.010/\cos\beta$. Comparing to Fig. 3 it can be seen that the pure Yukawa term has a noticeable effect when $v_2 \lesssim 20$ GeV. It is also apparent from Fig. 4 that nearing $v_2 \approx 3.6$ GeV (below which a Landau pole is induced before M_{Pl}) can act to degrade *or* improve the naturalness bound. The $v_2 = 3$ GeV bound in Fig. 4 shows the effect of hitting the Landau pole at $\sim 10^9$ GeV. We note that this only signals the breakdown of perturbation theory, and of our one-loop RGEs; we cannot calculate $m_{22}(\mu_R)$ above this scale, though it is perfectly possible that the theory remains natural.

In a repeated full one-loop RGE analysis, we found that the flipped ν2HDM gave essentially the same results as the Type II ν2HDM in Fig. 4, and there was no noticeable Yukawa effect in the LS ν2HDM until the Landau pole was

reached. Thus, we found that the stringent naturalness bounds of Eq. (25) and Fig. 3 are applicable at all times in the Type I ν2HDM, for $v_2 \gtrsim 2$ GeV in the LS ν2HDM, and for $v_2 \gtrsim 20$ GeV in the Type II and flipped ν2HDMs. Otherwise, Yukawa effects must be taken into account. Either way, the important point is now clear: a TeV-scale m_{22} can be both completely natural and, as was discussed in the previous subsection, is experimentally allowed in all ν2HDM types.

B. Corrections to m_{22}^2

Let us now consider the influence of the right-handed neutrinos. The one-loop RGE for m_{22}^2 is [17,60]

$$\frac{dm_{22}^2}{d\ln\mu_R} = \frac{1}{(4\pi)^2}[-4\mathrm{Tr}[y_\nu\mathcal{D}_M^2 y_\nu^\dagger] + \mathcal{O}(m_{22}^2)]. \quad (26)$$

A conservative naturalness bound is obtained by bounding the running as we did in Eq. (23),

$$\frac{1}{4\pi^2}\mathrm{Tr}[y_\nu\mathcal{D}_M^2 y_\nu^\dagger] < \Lambda_{\mathrm{bound}}^2, \quad (27)$$

where taking $\Lambda_{\mathrm{bound}} = 1$ TeV gives the Vissani bound on M_{N_1} of Eq. (5) [17]. However, now we are bounding corrections to m_{22}^2 rather than m_{11}^2, which may be TeV
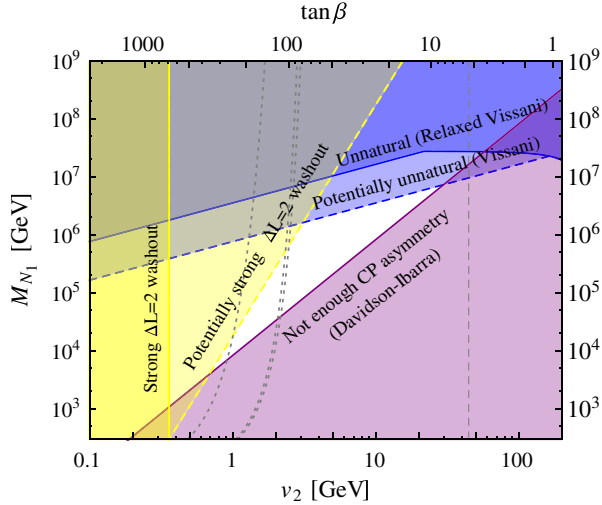
FIG. 5 (color online). Bounds on the ν2HDM as a function of v_2. Shown (as labelled) are the Davidson–Ibarra bound, the Vissani and relaxed Vissani naturalness bounds, and the areas of parameter space with strong $\Delta L = 2$ scattering washout. The Type II and flipped ν2HDMs are excluded by $B \to X_s \gamma$ for values of v_2 greater than indicated by the gray dashed line (see Sec. II C). The gray dotted lines indicate the v_2 below which the Yukawas hit a Landau pole before M_{N_1} in the Type II, flipped, and LS ν2HDMs right to left.

scale. Thus, depending on the mass of the extra scalars, it is possible that we can sensibly take $\Lambda_{\text{bound}} > 1$ TeV, in which case the the naturalness bound is somewhat relaxed; in Fig. 5 we show a relaxed Vissani bound for $\Lambda_{\text{bound}} = \min(10 \text{ TeV}, 10\sqrt{m_h^2 \tan^2\beta/2})$, where we have kept in mind the consistency condition Eq. (12). The Vissani bound still represents the unnatural area of parameter space if m_{22} is closer to 100 GeV.

As before, we could instead bound a quantity which measures the fine-tuning in m_{22}^2 at some high scale Λ_h. In this case, the fine-tuning measure of Eq. (24) is

$$\Delta(\Lambda_h) = 1 + \frac{1}{4\pi^2} \frac{\sum_{i,j}(y_\nu)_{ij} M_j^2 (y_\nu^\dagger)_{ji} \ln(M_j/\Lambda_h)}{m_{22}^2(M_j)}. \quad (28)$$

Taking $m_{22}(M_j) \sim 1$ TeV and demanding $\Delta(M_{\text{Pl}}) < 10$ gives a similar bound to Vissani [Eq. (27) with $\Lambda_{\text{bound}} = 1$ TeV]. Note that there is no naturalness bound on M_N in the $y_\nu \to 0$ limit. This is the technically natural limit corresponding to an enhanced Poincaré symmetry in which ν_R decouples from the theory [61].

In summary, there are up to three scales in the ν2HDM: v, m_{22}, and M_N. We have described the conditions under which v^2 (or m_{11}^2) is protected from m_{22}^2, and m_{22}^2 from M_N^2. Under such conditions it follows that m_{11}^2 is also protected from $M_{N_1}^2$ and the model is entirely natural.

IV. NEUTRINO MASSES

If $m_{12}^2 = 0$ and $\lambda_5 = 0$, then a $U(1)$ lepton number symmetry can be defined, and neutrinos remain massless. Let us now consider turning each nonzero in turn.

A. $m_{12}^2 > 0$, $\lambda_5 = 0$

If $m_{12}^2 > 0$ then the $U(1)$ lepton number symmetry is softly broken; i.e., the breaking does not force us to insert a nonzero λ_5 term in order to introduce a divergent counter-term, and it is consistent to consider $m_{12}^2 > 0$, $\lambda_5 = 0$.

In this situation the neutrino mass matrix is given by the seesaw formula,

$$m_\nu = \frac{v_2^2}{2} y_\nu \mathcal{D}_M^{-1} y_\nu^T \approx \frac{1}{\tan^2\beta} \frac{v^2}{2} y_\nu \mathcal{D}_M^{-1} y_\nu^T, \quad (29)$$

where $\tan\beta \approx m_{22}^2/m_{12}^2$ for $m_{22}^2 \gg \lambda_{345} v_1^2$ [see Eq. (10)].

The analogous Davidson–Ibarra and Vissani bounds are given by the standard Eqs. (4) and (5) with the replacement $v \to v_2$. These bounds are depicted in Fig. 5. If $v_2 \lesssim 30$ GeV ($\tan\beta \gtrsim 8$), then both bounds are satisfied. As well, as discussed in Sec. III, if m_{22} is TeV scale, the Vissani bound can be relaxed, and the required CP asymmetry needed to reproduce the BAU via leptogenesis may be naturally achieved for $v_2 \lesssim 60$ GeV ($\tan\beta \gtrsim 4$). In the Type I ν2HDM, v_2 can be naturally $\ll$ GeV. Otherwise, requiring a perturbative theory up to M_{N_1} restricts $v_2 \gtrsim 1$ GeV ($v_2 \gtrsim 2$ GeV) for the LS (Type II/ flipped) ν2HDM in the parameter space region of interest, as depicted in Fig. 5.

B. $m_{12}^2 = 0$, $\lambda_5 \neq 0$

In this situation $v_2 = 0$, and a Z_2 symmetry remains unbroken; this is the scenario of Ma [22]. The model yields a radiative neutrino mass and a dark matter candidate. This is only possible in the Type I ν2HDM, since in any other type the unbroken Z_2 forbids a Dirac mass term for any charged fermion coupling to Φ_2. Note that the limit $\lambda_5 \to 0$ is technically natural, since in that limit the $U(1)$ lepton number symmetry is reinstated.

If $M_N^2 \gg m_{22}^2, v^2$ the radiatively induced neutrino mass matrix is

$$(m_\nu)_{ij} \approx \frac{v^2}{2} \frac{(y_\nu)_{ik}(y_\nu^T)_{kj}}{M_k} \frac{\lambda_5}{8\pi^2} \left(\ln\left[\frac{2M_k^2}{(m_H^2 + m_A^2)}\right] - 1 \right). \quad (30)$$

The analogous Davidson–Ibarra and Vissani bounds are given by the standard Eqs. (4) and (5) with the intuitive replacement $v^2 \to v^2 \frac{\lambda_5}{8\pi^2} (\ln[2M_{N_1}^2/(m_H^2 + m_A^2)] - 1)$. This assumes that there is no fine-tuning in the complex y_ν parameters to reproduce the observed neutrino masses (see Appendix B for details). These bounds are depicted in Fig. 6, where the Davidson–Ibarra bound has been
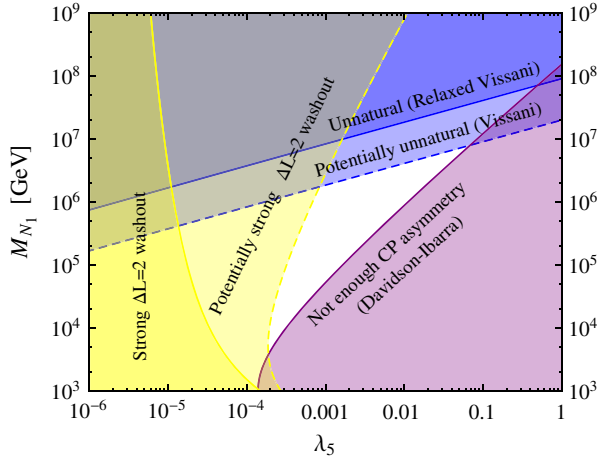
FIG. 6 (color online). As in Fig. 1 but for the Ma model. The Vissani and relaxed Vissani bounds are evaluated at $m_{22} = 100, 1000$ GeV respectively. The Davidson–Ibarra bound and strong $\Delta L = 2$ washout region are shown for $m_{22} = 500$ GeV, though they are only mildly sensitive to m_{22}.

evaluated at $m_{22} = 500$ GeV as an illustrative example (the bound is only mildly sensitive to m_{22}). We find that the Ma model with $\lambda_5 \lesssim 0.5$ can naturally achieve the required CP asymmetry to reproduce the BAU via hierarchical leptogenesis.[6]

C. $m_{12}^2 > 0$, $\lambda_5 \neq 0$

In this case both the tree-level seesaw and the radiative mechanism will contribute to the neutrino mass. Both contributions are calculable, and either might dominate. Note that it is still technically natural to take $\lambda_5 \to 0$ in this case, since it restores a softly broken $U(1)$ symmetry. In other words, the λ_5 RGEs to all orders will be multiplicative in λ_5, indicative of the fact that the soft-breaking term can only generate finite $U(1)$-breaking corrections.

V. LEPTOGENESIS

The observed BAU is achieved analogously to standard hierarchical thermal leptogenesis [13]; the out-of-equilibrium CP-violating decays of the lightest right-handed neutrino $N_1 \to l\Phi_2$ create a lepton asymmetry which is transferred to the baryons by the electroweak sphalerons above $T \sim 100$ GeV.

The details of the leptogenesis are largely defined by the decay parameter

$$K = \frac{\Gamma_D}{H|_{T=M_1}} = \frac{\tilde{m}_1}{m_*}, \qquad (31)$$

---

[6]A similar observation was made in a recent paper [62].

comparing the rate for decays and inverse decays to the expansion rate at the time of departure from thermal equilibrium. Here, the rates

$$\Gamma_D = \frac{1}{8\pi}(y_\nu^\dagger y_\nu)_{11} M_1, \qquad (32)$$

$$H \approx \frac{17T^2}{M_{\text{Pl}}}, \qquad (33)$$

are typically rescaled and expressed in terms of an effective neutrino mass $\tilde{m}_1$ and an equilibrium neutrino mass m_*,

$$\tilde{m}_1 = \frac{(y_\nu^\dagger y_\nu)_{11} v^2}{2M_1},$$

$$m_* \approx 1.1 \times 10^{-3}\, \text{eV} \left(\frac{v}{246\,\text{GeV}}\right)^2, \qquad (34)$$

where v is the vev that enters the seesaw Eq. (2). In the ν2HDM with $\lambda_5 = 0$ (with $m_{12}^2 = 0$, $\lambda_5 \neq 0$), the analogous definitions make the replacement $v^2 \to v_2^2$ ($v^2 \to v^2 \frac{\lambda_5}{8\pi^2}(\ln[2M_{N_1}^2/(m_H^2 + m_A^2)] - 1)$). Note that for the scenarios we are interested in (e.g., $v_2 \ll v$) m_* is smaller than its usual value in standard leptogenesis.

When only decays and inverse decays are considered, leptogenesis for given K proceeds exactly as in standard hierarchical thermal leptogenesis (see, e.g., Ref. [63] for a review). In the weak washout regime $K \ll 1$, the baryon asymmetry strongly depends on the initial asymmetry and the initial N_1 abundance, with N_1 decays occurring at $T \ll M_1$. The strong washout regime $K \gg 1$ is independent of the initial conditions, and the asymmetry is generated as the N_1 fall out of thermal equilibrium.

The $2 \leftrightarrow 2$ scatterings with $\Delta L = 1$ (see, e.g., Ref. [64]) provide a correction to the simple decays plus inverse decays picture; they act to increase N_1 production at $T > M_1$ and contribute to washout at $T < M_1$. In standard hierarchical thermal leptogenesis, the scattering contributions involving the top quark and the gauge bosons are roughly equal. In the present model, the gauge boson contribution is the same as in the standard scenario. However, by construction, the Φ_2 involved here in leptogenesis does not couple directly to the top quark, and thus the usual s-channel ($Nl \leftrightarrow tq$) and t-channel ($Nt \leftrightarrow lq$, $Nq \leftrightarrow lt$) scattering contributions do not occur. Instead, at large $\tan\beta$ they can be replaced by the analogous contribution from other charged fermions, i.e., the bottom quark in Type II and flipped ν2HDMs and/or the tau lepton in Type II and LS ν2HDMs. A large tau lepton Yukawa will also introduce new s-channel ($N\Phi_2 \leftrightarrow \tau\Phi_2$) and t-channel ($N\Phi_2 \leftrightarrow \tau\Phi_2$, $\tau N \leftrightarrow \Phi_2\bar{\Phi}_2$) scattering contributions. All of these processes are proportional to $(y_\nu^\dagger y_\nu)_{11}$ and hence $M_1\tilde{m}_1/v^2$, with the appropriate ν2HDM replacement for v^2. Therefore, they scale with the decays and inverse

decays so that they represent only a minor (but obviously important) departure from the standard leptogenesis scenario.

The $2 \leftrightarrow 2$ scatterings with $\Delta L = 2$ mediated by the right-handed neutrinos ($\Phi_2 l \leftrightarrow \bar{\Phi}_2 \bar{l}$, $\Phi_2 \Phi_2 \leftrightarrow ll$) occur as they do in the standard scenario. These processes are proportional to $\text{Tr}[(y_\nu y_\nu^T)(y_\nu y_\nu^T)^\dagger]$ and hence $M_1^2 \bar{m}^2/v^4$ where $\bar{m}^2 = \sum m_i^2$ is the neutrino mass scale $\gtrsim 0.05$ eV. Comparing this rate to the decay/scattering rates $\propto M_1 \tilde{m}_1/v^2$, it is easy to see that after making the appropriate ν2HDM replacement for v^2, e.g., $v^2 \to v_2^2 \ll v^2$, these scatterings will become comparatively more important than in the standard case. For $T \lesssim M_1/3$ the thermally averaged $\Delta L = 2$ scattering rate is well approximated by [63]

$$
\begin{aligned}
\frac{\Gamma_{\Delta L=2}}{H} &\approx \frac{T}{2.2 \times 10^{13} \text{ GeV}} \left(\frac{246 \text{ GeV}}{v}\right)^4 \\
&\quad \times \left(\frac{\bar{m}}{0.05 \text{ eV}}\right)^2,
\end{aligned}
\tag{35}
$$

where the previously described ν2HDM replacements for v^2 hold (see Appendix B). In Fig. 5 (Fig. 6), we show the region in the $\lambda_5 = 0$ ($m_{12}^2 = 0$, $\lambda_5 \neq 0$) ν2HDM where these scatterings are still in equilibrium at $T \lesssim M_1/3$.[7] This is the region where strong $\Delta L = 2$ scatterings can potentially wash out the generated asymmetry, depending on the details of the leptogenesis (e.g., in a weak washout scenario with N_1 decays at $T \ll M_1$, this washout may be avoided). Demanding that the scatterings fall out of equilibrium before sphaleron freeze-out at $T \sim 100$ GeV provides a lower bound $v_2 \gtrsim 0.3$ or $\lambda_5 \gtrsim 10^{-5}$; this is represented by the strong $\Delta L = 2$ scattering washout regions in Figs. 5 and 6, respectively. We note that this calculation has been performed in the context of a perturbative theory. This is reliable for the Type I ν2HDM but not for Type II, LS, or flipped ν2HDMs with sufficiently small v_2, when perturbativity breaks down.

Putting this all together, we can now read off from Figs. 5 and 6 the regions of parameter space which can achieve natural hierarchical thermal leptogenesis. For ν2HDMs with $m_{12}^2 > 0$ and $\lambda_5 = 0$, we find 10^3 GeV $\lesssim M_{N_1} \lesssim$ few $\times 10^7$ GeV is viable for Type I ν2HDMs, and 10^4 GeV $\lesssim M_{N_1} \lesssim$ few $\times 10^7$ GeV for all other types if they are to remain perturbative. For the Ma model with

$m_{12}^2 = 0$ and $\lambda_5 \neq 0$, we find viable parameter space for 10^3 GeV $\lesssim M_{N_1} \lesssim 10^8$ GeV and $10^{-5} \lesssim \lambda_5 \lesssim 0.5$.

Lastly we note that the lightest scalar state in the Ma model is stable.[8] It is therefore possible that this state, if it is neutral, constitutes some or all of the observed dark matter. During the leptogenesis epoch, Φ_2 is produced in abundance in N_1 decays. Overproduction of dark matter is of no concern as long as Φ_2 efficiently thermalizes at or below the temperatures when N_1 decays occur, which suggests $m_{22} \ll M_{N_1}$. In this case the lightest state is a thermal relic dark matter candidate.

VI. CONCLUSION

The minimal Type I seesaw model is unable to explain neutrino masses and the BAU via hierarchical thermal leptogenesis without ceding naturalness. The main conclusion of this paper is the observation that a second Higgs doublet can avoid this problem. These ν2HDM models provide a natural solution by reducing the (possibly effective) vev entering the seesaw formula. This can be done radiatively, or by having the second Higgs doublet provide a tree-level seesaw with a small vev v_2, kept natural by softly breaking a $U(1)$ or Z_2 symmetry.

The models naturally accommodate a SM-like Higgs and predict the existence of approximately TeV-scale extra scalar states in order to remain natural. We rediscovered the radiative Ma model as the only possibility when $v_2 = 0$; in that case we found 10^3GeV $\lesssim M_{N_1} \lesssim 10^8$GeV and $10^{-5} \lesssim \lambda_5 \lesssim 0.5$ could simultaneously explain neutrino masses and the BAU via leptogenesis while remaining natural. The $v_2 > 0$ models require $\tan\beta \gtrsim 4$; we found 10^3 GeV $\lesssim M_{N_1} \lesssim$ few $\times 10^7$ GeV was viable for Type I ν2HDMs, and 10^4 GeV $\lesssim M_{N_1} \lesssim$ few $\times 10^7$ GeV for all other types if they are to remain perturbative. The interesting areas of parameter space are well summarized in Figs. 5 and 6.

APPENDIX A: SQUARED SCALAR MASSES AT $\mathcal{O}(m_{12}^4/m_{22}^4)$

To order m_{12}^4/m_{22}^4, the scalar masses Eqs. (14) are given by

---

[7]A similar plot to Fig. 5 appears in Ref. [65] in the context of the Type I ν2HDM with $v_2 > 0$. We are not aware of any plot similar to Fig. 6 in the literature, though see Refs. [66–70] for leptogenesis studies at points in the Ma model parameter space.

[8]The lifetimes of the heavier scalar states are governed by mass splittings Δ via $\Gamma \sim G_F^2 \Delta^5/(10^2 \pi^3)$. In the parameter space of interest, one can check that Δ is typically already large enough at tree level so that lifetimes remain well below $\mathcal{O}(1 \text{ s})$ and therefore do not disturb big bang nucleosynthesis.

$$m_h^2 \approx v_1^2 \left[\lambda_1 + \frac{m_{12}^4}{m_{22}^4} \frac{2\lambda_{345} - \lambda_1 - \frac{v_1^2}{2m_{22}^2}\lambda_1\lambda_{345}}{(1 + \frac{v_1^2}{2m_{22}^2}(\lambda_{345} - 2\lambda_1))(1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345})^2} \right],$$

$$m_H^2 \approx m_{22}^2 \left[1 + \lambda_{345}\frac{v_1^2}{2m_{22}^2} + \frac{m_{12}^4}{m_{22}^4} \frac{1 - \frac{v_1^2}{2m_{22}^2}(2\lambda_{345} - 3\lambda_2) + \frac{v_1^4}{4m_{22}^4}(\lambda_{345}^2 + 3\lambda_2\lambda_{345} - 6\lambda_1\lambda_2)}{(1 + \frac{v_1^2}{2m_{22}^2}(\lambda_{345} - 2\lambda_1))(1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345})^2} \right],$$

$$m_A^2 \approx m_{22}^2 \left[1 + (\lambda_{345} - 2\lambda_5)\frac{v_1^2}{2m_{22}^2} + \frac{m_{12}^4}{m_{22}^4} \frac{1 + \frac{v_1^2}{2m_{22}^2}(\lambda_{345} + \lambda_2 - 2\lambda_5)}{(1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345})^2} \right],$$

$$m_{H^\pm}^2 \approx m_{22}^2 \left[1 + \lambda_3 \frac{v_1^2}{2m_{22}^2} + \frac{m_{12}^4}{m_{22}^4} \frac{1 + \frac{v_1^2}{2m_{22}^2}(\lambda_2 + \lambda_3)}{(1 + \frac{v_1^2}{2m_{22}^2}\lambda_{345})^2} \right]. \tag{A1}$$

APPENDIX B: BOUNDS FOR MORE GENERAL m_ν

We consider the seesaw Lagrangian as in Eq. (2) and a neutrino mass matrix of the form

$$m_\nu = \frac{v^2}{2} y_\nu \mathcal{D}_M^{-1} \mathcal{D}_{f(M)} y_\nu^T, \tag{B1}$$

where $\mathcal{D}_x \equiv \mathrm{diag}(x_1, x_2, x_3)$. Note that in the ν2HDM with $\lambda_5 = 0$ we have $f(M_j) = v_2^2/v^2$, and in the Ma model with $M_N \gg m_{22}$, we have

$$f(M_j) = \frac{\lambda_5}{8\pi^2} \left(\ln\left[\frac{M_j^2}{(m_H^2 + m_A^2)/2}\right] - 1 \right). \tag{B2}$$

Following Casas–Ibarra [71], it is possible to write

$$y_\nu = \frac{\sqrt{2}}{v} U^\dagger \mathcal{D}_m^{\frac{1}{2}} R \mathcal{D}_M^{\frac{1}{2}} \mathcal{D}_{f(M)}^{-\frac{1}{2}}, \tag{B3}$$

where R is a (possibly complex) orthogonal $(RR^T = R^T R = \mathbb{1})$ matrix. The Vissani bound on each right-handed neutrino mass becomes [17]

$$\frac{1}{4\pi^2}\frac{2}{v^2}\frac{M_j^3}{f(M_j)}\sum_i m_i |R_{ij}|^2 < 1 \text{ TeV}^2$$

$$\Rightarrow M_{N_1} \lesssim 3 \times 10^7 \text{ GeV} \times f(M_{N_1})^{\frac{1}{3}}. \tag{B4}$$

The *CP* asymmetry for hierarchical neutrinos [14] becomes

$$|\epsilon_1| = \frac{6}{8\pi}\frac{M_1}{v^2}\frac{\mathrm{Im}[(R^\dagger \mathcal{D}_m (R\mathcal{D}_{f(M)}^{-1}R^T)\mathcal{D}_m R^*)_{11}]}{(R^\dagger \mathcal{D}_m R)_{11}}$$

$$\lesssim \frac{6}{8\pi}\frac{m_3 M_{N_1}}{v^2}\frac{1}{\min[f(M_j)]}, \tag{B5}$$

where the approximate inequality holds for $\max(|R_{ij}|) \leq 1$. For larger $\max(|R_{ij}|)$ the inequality can be exceeded, but this corresponds to a fine-tuning (see Ref. [17]). With this caveat the Davidson–Ibarra bound for $\min[f(M_i)] = f(M_{N_1})$ therefore becomes

$$M_{N_1} \gtrsim 5 \times 10^8 \text{ GeV} \times f(M_{N_1}). \tag{B6}$$

The $\Delta L = 2$ scatterings are proportional to [63,72]

$$\sum_{i,j} \mathrm{Re}[(y_\nu^\dagger y_\nu)_{ij}(y_\nu^\dagger y_\nu)_{ij}]\frac{1}{M_i M_j}$$

$$= \mathrm{Tr}[(y_\nu \mathcal{D}_M^{-1} y_\nu^T)(y_\nu \mathcal{D}_M^{-1} y_\nu^T)^\dagger]$$

$$= \frac{4}{v^4} \mathrm{Tr}[\mathcal{D}_m (R\mathcal{D}_{f(M)}^{-1}R^T)\mathcal{D}_m (R\mathcal{D}_{f(M)}^{-1}R^T)^\dagger]$$

$$\lesssim \frac{4}{v^4}\frac{\bar{m}^2}{\min[f(M_j)]^2}, \tag{B7}$$

where the last line is an exact equality for $f(M_1) = f(M_2) = f(M_3)$, an exact inequality when R is real, and an approximate inequality (as indicated) if R is complex with $\max(|R_{ij}|) \leq 1$. Again, for larger $\max(|R_{ij}|)$ the inequality can be exceeded. Then for $\min[f(M_i)] = f(M_{N_1})$ the $\Delta L = 2$ scattering Eq. (35) becomes

$$\frac{\Gamma_{\Delta L=2}}{H} \lesssim \frac{T}{2.2 \times 10^{13} \text{ GeV}}\frac{1}{f(M_{N_1})^2}\left(\frac{\bar{m}}{0.05 \text{ eV}}\right)^2. \tag{B8}$$

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Title:
Natural leptogenesis and neutrino masses with two Higgs doublets

Date:
2015-08-17

Citation:
Clarke, J. D., Foot, R. & Volkas, R. R. (2015). Natural leptogenesis and neutrino masses with two Higgs doublets. PHYSICAL REVIEW D, 92 (3), https://doi.org/10.1103/PhysRevD.92.033006.

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