

**SHAPE CLASSIFICATION:  
TOWARDS A MATHEMATICAL DESCRIPTION OF THE FACE**

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## ABSTRACT

Recent advances in biostereometric techniques have led to the quick and easy acquisition of 3D data for facial and other biological surfaces. This has led facial surgeons to express dissatisfaction with landmark-based methods for analysing the shape of the face which use only a small part of the data available, and to seek a method for analysing the face which maximizes the use of this extensive data set. Scientists working in the field of computer vision have developed a variety of methods for the analysis and description of 2D and 3D shape. These methods are reviewed and an approach, based on differential geometry, is selected for the description of facial shape.

For each data point, the Gaussian and mean curvatures of the surface are calculated. The performance of three algorithms for computing these curvatures are evaluated for mathematically generated standard 3D objects and for 3D data obtained from an optical surface scanner. Using the signs of these curvatures, the face is classified into eight "*fundamental surface types*" - each of which has an intuitive perceptual meaning. The robustness of the resulting *surface type description* to errors in the data is determined together with its repeatability.

Three methods for comparing two surface type descriptions are presented and illustrated for average male and average female faces. Thus a quantitative description of facial change, or differences between individual's faces, is achieved. The possible application of artificial intelligence techniques to automate this comparison is discussed. The sensitivity of the description to global and local changes to the data, made by mathematical functions, is investigated.

Examples are given of the application of this method for describing facial changes made by facial reconstructive surgery and implications for defining a basis for facial aesthetics using shape are discussed. It is also applied to investigate the role played by the shape of the surface in facial recognition.



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## **QUOTATION**

**"Had Cleopatra's nose been shorter, the whole face of the world would have been different."**

**Blaise PASCAL (1623-1662)**

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## INTRODUCTION

The face is a dynamic structure, unique to the individual and plays a very important role in recognising individuals and in attracting us emotionally, socially and sexually to an individual. It also plays an important role in portraying our emotions, signaling changes in mood and communicating feelings (Gorlin et al, 1975).

### Motivation for a description of facial shape

The study of the face is important to many different scientific and medical disciplines. Changes in facial appearance due to normal growth, abnormal growth, injury and surgery can and do have a profound effect on the person concerned. The psychosocial consequences of having to spend a significant portion of one's childhood with a major uncorrected facial malformation can be devastating (Cutting, 1989) whereas the surgical correction of facial deformity often gives a person more self-confidence and a less introverted personality. If facial surgery is to produce a consistent outcome it is necessary to have an objective means for describing facial change and to relate this to an assessment of the outcome. This requirement has led to many attempts being made over the years to measure and characterize changes in facial shape. The research described in this thesis arose from this need.

During the last decade computer systems have been increasingly used in the planning of facial surgery. A number of such systems have been developed including one at University College London (Arridge et al, 1985; Moss et al, 1988; Tan et al, 1988; Linney, 1992a; 1992b). These modern systems use computer graphics to provide the necessary three dimensional (3D) representations of the facial surface. They enable the face to be displayed as seen from any chosen viewpoint and allow the data to be manipulated to simulate the effect of surgery. The availability of computed, stored 3D data has opened up the possibility of a mathematical analysis and description of the facial surface.

A mathematical description of the face is also very important in research into facial recognition. In her book, "Recognising faces", Bruce reviews the research conducted by cognitive psychologists into this subject (Bruce, 1988). Much of the work in this field to date has been conducted on two dimensional images of the face. Bruce concluded that further understanding of the way in which we recognise faces, is dependant on treating the face as a 3D surface not a 2D pattern. This concept has provided a second motivation for this work.

The value of a mathematical description of the face will depend on firstly, its ability to be easily understood by surgeons and its relation to the way in which they perceive the structure of the face and secondly, its value for describing the ideas of cognitive psychologists about how we recognise faces. In this context, shape is a concept often

## ***Introduction***

used, although not well defined. I have therefore sought to produce a mathematical description of the face based on its shape.

Most of the mathematical methods which have previously been applied to facial analysis have been based on landmark points. These must first be accurately identified. Analyses have, for the most part, been limited to studies of the midline profile. Since most of the face consists of *surface in between landmarks* these methods of analysis represent a very high degree of abstraction. The application of landmark analyses to the characterization of facial change fails completely for cases in which the landmark points chosen have been moved only slightly or not at all but the perceived change in the facial shape is extremely marked.

## **What is shape?**

The question of *How should we describe shape?* is a question often dismissed as trivial but in fact, it is hard to answer precisely. Kendall (1989) has suggested one plausible definition of shape; that it is "what is left over" when the effects resulting from translations, changes of scale and rotations are filtered out. It is immediately apparent is that any objective description of shape (or form) must use only parameters for comparison which are *independent* of orientation (rotation), translation and linear scaling. Intuitively it seems that shape is to do with *curvature*.

The Oxford English Dictionary definition of shape is "the total effect produced by a thing's outlines". This is difficult to translate into an exact mathematical definition. In formulating a theory for shape, the properties associated with shape must be deduced. This is not an easy task. In 1967, Blum observed that the properties of shape have proved difficult to deduce primarily because "the number and variety of shapes are enormous" (Blum, 1967). A great deal of research has been done on the description of shape *per se* and associated computation methods (see chapters 1 and 2) but it is clear from the literature that the characterisation of facial shape and mathematical description of changes in facial shape have, apart from a few notable examples, been lacking. This is the subject of investigation in this work.

A practical definition of shape will undoubtedly transcend landmarks in its ability to produce a complete description of the face. Not only this, but shape also fits more closely our perceptual concepts of a face than the geometrical relationship between isolated surface points. Some recent work has indicated that a natural way of using shape as a descriptor, is to impose a hierarchical structure (Gauch et al, 1987; Pizer et al, 1987; Mokhtarian and Mackworth, 1986). In the case of the face, a hierarchical description would allow changes to be classified at varying levels of detail providing quantitative information.



**Purpose**

The purpose of this work has been to produce a *surface-based* description of the *shape* of face and to explore methods of characterizing changes in the shape of the face. Requirements are that the description is based on the entire 3D structure of the face, is invariant to the viewpoint from which the face is observed, robust against noise and repeatable for scans of the same face. It must also enable a *quantitative* comparison of the 3D changes in the face to be made that is explicable in terms of the perceived changes or differences. The description and the comparison of changes should be amenable to automation, so that they can be implemented for practical applications.

The development of such a method would address the requirements of facial reconstructive surgeons for a full quantitative description of the changes in facial appearance brought about by facial surgery. It is would also aid physical anthropologists in the classification of the face and to psychologists in their investigation of the mechanisms of facial recognition.

# CHAPTER 1

## SHAPE DESCRIPTION - LITERATURE REVIEW

In this first chapter, a brief review of the literature dealing with the description of shape and especially 3D shape is given. There has been an abundance of literature produced over the last two decades relating to shape description, much of which is in a similar vein. I have therefore been somewhat selective in referencing papers, seeking to point to authoritative texts describing a particular approach or idea or quoting an example of that approach, whilst endeavouring to still do justice to the ideas advanced by those scientists interested in this subject.

Cognitive psychologists have proposed a number of theories for describing the manner in which our visual system perceives shape and how our concepts of shape are formed (Bruce and Green, 1985). These have been intrinsically bound up with how we recognise objects (see chapter 9 for psychological literature pertaining to the recognition of faces). Apparently independent of this, scientists working in the computer vision field have produced various descriptions of shape which seem to be based on somewhat similar concepts. However, until the last few years there has been very little cross-referencing found in the literature. The computer vision approach to shape description uses ideas of image segmentation, feature extraction, artificial intelligence and differential geometry. The role of shape description in computer vision is to enable the recognition of objects, or scenes of objects, primarily for robotic applications. This role ties in well with one of the goals of cognitive psychologists, trying to understand how we recognize objects. In this chapter I will attempt to relate the approaches taken by these two fields. The material described here is of importance for the development of a method for describing facial shape.

Separate from these approaches has been the long history of interest in *biological shape* which stems, in modern times, from Darwin and his ideas about evolution. Early attempts to formulate biological development and evolution in terms of shape were made by D'Arcy Thompson in his classic work "On Growth and Form" of 1917 and emulated by others (eg. Richards (1955)). More recently Bookstein (1978a; 1978b) returned to Thompson's work and extended it. The aim of their work was the classification of *changes* in shapes, rather than the development of a description or concept of shape. Therefore this will be discussed in a separate chapter along with the statistical analysis of shape (chapter 2).

### **1.1 Human visual perception**

In order to describe shape adequately one first has to have a concept of what we mean by shape. The desire of cognitive psychologists to model the eye-brain system, ie. to

discover how the cognitive system works, quickly led them to form concepts of how the brain might describe shape. Considering a mathematical description of shape there is an intrinsic advantage claimed for using a conceptual model of shape based on the mechanisms of the brain's visual system, that is that the system is known to work! Let us now consider how shape may be represented in the brain.

### **1.1.1 Observations about the brains perception of shape**

It has been found that the perception of an object's shape enables us to identify that object and that this is true whether or not the object is rigid, since living things are just as easily recognisable as non-living ones (Attneave, 1967). So the perception of shape facilitates recognition.

Psychologists have made a number of basic observations about how we perceive shape. Psychophysical and neurophysiological experiments have found that *different areas of the brain respond to different shapes* and that shape outlines and the negation of a simple shape excite different areas of the visual cortex (eg. Perrett et al, 1988). Shape in the visual cortex seems to be mainly feature based and curvature along a curve or contour may be a key descriptor (Dobbins et al, 1987; Leymarie and Levine, 1988; 1989). This would explain how objects can be recognized from their silhouettes or outlines (Marr, 1977).

Regarding the perception of faces, clinical and experimental studies have shown that there are cells in the inferotemporal cortex of monkeys which *specifically* respond to faces (Tanaka et al, 1991). In man too there appear to be cells that respond specifically to faces and the neural mechanisms for face processing are predominantly, but not exclusively, located in the right cerebral hemisphere (Perrett et al, 1988).

Next, we can recognise objects when they are seen from *different viewpoints or in different states*. For example, a face can be recognised when seen from in front (anterior view), obliquely or in profile (lateral view) and with different facial expressions. It has been shown that faces are most easily recognisable as belonging to a particular individual when seen in three-quarter view and that profiles are less easy to recognise, unless the individual face is a very distinctive one (Bruce et al, 1987). It has been found that in the brain, *different* cells are excited by the same shape in different orientations or, in the case of a face, when portraying different emotions (Perrett et al, 1988).

Thirdly, objects can be identified as *belonging to a group of objects* according to their shape. Our ability to classify objects in this manner seems to be related to the functional use of the object (eg. objects perceived as tables may differ substantially from one another but are still recognised as tables). We can also name objects specifically, recognising our chair, John's shoe or Martina's face. This implies a strong

### ***Shape Literature***

interconnection with language area of the brain. We can also assign features to an object, or *segment an object into parts*. Facial features such as the nose can be recognised although it is contained within a continuous surface and varies enormously from individual to individual and race to race. Just *how* we can make such a distinction is still unknown.

The perceptual process is *reversible*. That is to say that we can somehow store the salient information about an object to enable us to recall it, and draw it from memory, at a later date. This recall ability appears to be influenced by our *familiarity* with the object, the importance of the object to the individual and the passage of time since its observation. It has been demonstrated that if one is asked to visualise an object, such as a cat, one visualises a *specific breed* of cat, which may vary from individual to individual (Attneave, 1967). These observations suggest perception is strongly dependent on learning. It also implies that information is abstracted from the retinal image and compressed for storage. Other observations suggest that incoherent information is discarded and only features of high informational value is retained (Attneave, 1954), as well as a preference for simple features over complex ones.

When we considering an image of an object, our interpretation of the image seems to be influenced by environmental cues such as vertical and horizontal orientation (eg. the distinction we make between a square and a rhombus), right angles, parallel lines leading to vanishing points and viewpoint. Highlights appear to reinforce the interpretation of surfaces (Blake and Bülhoff, 1991). Stevens (1981) showed that a localised highlight suggests an elliptic surface is being viewed whereas a linear, elongated highlight suggests a cylindrical one. The background surrounding an object appears to have little effect on how the object is perceived. However, if one reverses components of an object, great difficulty arises in identifying the object (eg. figure 1.1). This suggests that our perceptual sense is finely tuned.



Figure 1.1: Reversal of components of a face. (from Thompson 1980)  
*Reproduced with permission of the author and Pion London.*

If a percept is often used or is important, such as recognizing familiar faces, a special-purpose mechanism, entirely for that task is developed by the brain. Thus expert descriptions are formed differently from those of naive observers. Computer simulations of expert descriptions require the construction of more specialized models than those for naive observers. All adults are experts at recognizing faces (Diamond and Carey, 1986)



As long ago as the 17th century, people noticed that under certain circumstances, ambiguity exists in the way that the human visual system perceives shape. That is to say that some line drawings, or shaded line drawings, can be interpreted in two mutually exclusive ways. This phenomenon is termed "figure-ground reversal". Boring's figure (figure 1.2), which can be seen either as a mother-in-law or a wife, is a famous example of this and there are many others.

Figure 1.2: Boring's figure. An example of "figure-ground reversal", seen as either a young girl or an old woman.

The psychological experiments that have been conducted in order to investigate the perceptual organisation of the mind, often use the manner in which people "see" 2D contours or figures to examine how shape is perceived. These have not only provided some important observations about the perception of 2D outlines (which I shall call "contours") but have led to the formation of a concept of what the "same shape" is and, importantly, inferred that shape *is* amenable to mathematical description.

A definition of what we mean when we say that two objects have the same shape, termed "shape constancy", has been given as two objects that differ only in position, orientation, size and mirror-image reflection (Chen and Chen, 1987). This statement is profound since it implies that an object's shape can, and is, defined in terms of certain invariant parameters, such as the internal angular relationship between features, and therefore that the description of shape can be undertaken mathematically. The set of invariant transformations, which form the basis of shape constancy, is termed a similarity group (Selfridge and Neisser, 1963; Gibson, 1969). Thus mathematics, in the guise of group theory, was introduced into the investigation of human perception (Chen and Chen, 1987). Chen and Chen discovered that if the size of a figure was changed but its "sense" was unaltered, then the figure was recognised more quickly than vice versa. The sense of a figure is best explained by considering the direction of vectors connecting three points on the surface (figure 1.3). Since reversing the sense of the

### Shape Literature

figure made the figure less recognizable than changing its size they concluded that sense was more important as a recognition factor than size.

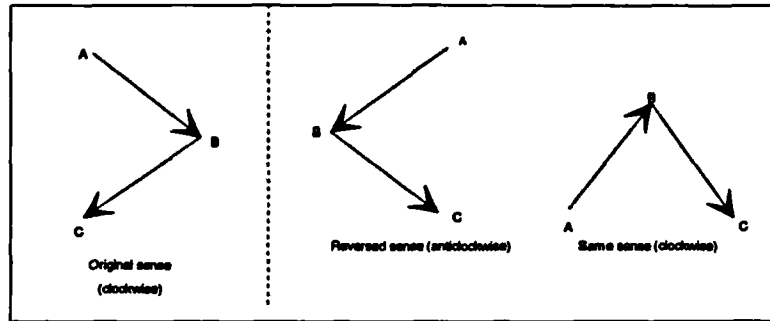


Figure 1.3: The "sense" of a figure. clockwise and anti-clockwise.

#### 1.1.2 Perceptual representation of shape

The observations described above tell us that the perception of an object's shape is bound up with its recognition. They have also provided some important pointers about how we perceive shape, helping us to develop concepts of what shape is. But how does the brain abstract this shape information from its visual input?

This has also been a subject of considerable interest, a review of research describing the perceptual representation of form with a view to defining mathematical metrics for describing shape was given as early as 1965 (Michels and Zusne, 1965). Two main theories have been advanced to explain how the shape information is encoded. In this section, I shall firstly describe these two theories and in the next section briefly review algorithms devised by computer vision scientists which have used these concepts.

##### a) Shape from edges

The first theory is rooted in the observation that artists will often make a preliminary sketch of a scene before painting it. This sketch is essentially a line drawing from which the scene can be recognised without any visual cues such as shading, colour or texture. The line drawing has been defined as the minimal representation of intensity discontinuities in a grey-level image that adequately conveys surface structure (Barrow and Tenenbaum, 1981). The identification of such lines with high frequency changes in intensity in the retinal image (see Bruce and Green, 1985) led to a theory which suggested that the human visual system works as a *high-pass filter*, capable of detecting *edges* and deducing shape information from them (Grimson and Pavlidis, 1985).

Edges in images can result from changes in intensity, extremal boundaries or discontinuities in boundaries. An extremal boundary is one where the surface turns smoothly away from the observer, where the line of sight is perpendicular to the surface normal and the tangent to the surface at that point. At a discontinuity boundary two or more surfaces intersect or terminate and the surface normal is orthogonal to the 3D

tangent in the plane of the space curve (Barrow and Tenenbaum, 1981).

It was Marr who suggested that because intensity discontinuities would often coincide with important boundaries in the visual scene, these edges are stored by the brain as a symbolic two dimensional representation of the object, a *primal sketch* (Marr, 1976; 1982). This representation is also supposed to record the turning points in curved edges (ie. changes of sign of the second order partial derivatives of curves) and the contrast, blur and local orientation of edges (by filtering the image with different size gaussian functions) (Marr and Hildreth, 1980; Marr and Poggio, 1979). A more complete *viewer-centred* representation, which Marr called a  $2\frac{1}{2} D$  *sketch*, was obtained by combining depth, motion and shading information with the primal sketch. This describes the layout of structures in the world from a particular viewpoint.

For natural scenes, the changes in light intensity associated with the edges of objects are embedded within the changes caused by surface texture, shape, shadows and arrangement of illuminating light sources (Watt, 1988). This makes any correspondence of intensity changes to the edges of objects difficult to ascertain.

Some psychophysical evidence has been obtained to suggest that the human visual system *does* use a kind of primal sketch (Watt, 1987a; 1987b). However, compared to Marr's primal sketch it is of a more dynamic and structured nature (Watt, 1987c). Marr and Hildreth's algorithm detects edges in an image scene by convolution with a range of different sized Gaussian filters and uses zero-crossings (ie. points of inflection) in the second directional derivative to locate edges. Pearson and Robinson (1985) found that for faces using the *peak* response of the filter produced a better edge description than using the zero-crossings. Watt and Morgan (1985) devised an algorithm called MIRAGE to simulate more closely the working of the human vision system and to allow any changes in image intensity to be described.

When assessing the usefulness of the concept of edge detection to us, we should consider that line drawings of objects do contain a lot of psychologically salient information (eg. figure 1.4) and a qualitative appreciation for the shape of a surface *can* be obtained from a line drawing. Some idea of its orientation is also relatively easy to judge, but its size is not (Stevens, 1981). Barrow and Tenenbaum (1981) have pointed out that the outline of a structure appears to influence the brains perspective and motion parallax cues (such as the Necker cube which is seen to reverse in depth, figure 1.5).

However, for a face, we would expect very few discontinuities and the boundary of the face against a background to have little meaning in terms of recognition, since profiles have been shown to be difficult to recognise (Bruce et al, 1987). Moreover, line

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drawings of the faces of individuals, have been shown to be inadequate for facial recognition (Bruce et al (in press<sup>a</sup>), see also chapter 9.1.1).



Figure 1.4: A line drawing of a face. Surface and boundary structure are perceived.

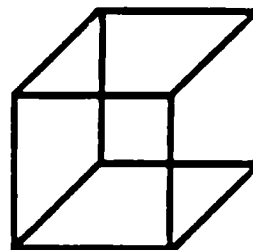


Figure 1.5: Necker's cube, which undergoes spontaneous reversals in depth.

Mathematically, this sort of edge-based description is inadequate because it is *not invariant* under monotonic transformations and it assumes that the general structure of an object is isotropic and smooth in order to obtain useful information. In the real-world this is often wrong. The well-documented figure-ground reversal observation provides evidence against the edge based theory. Consider Rubin's figure (first reported by Turton in 1819 referenced by Hoffman and Richards, 1985) which can be seen either as two faces or a wine goblet (figure 1.6). In 1915, Rubin found that if one sees the figure in one way, then later in the other way, he is no more likely to recognise the figure than if he had never seen it. Thus the relationship between the two drawings is in some sense "competitive". This suggests that object recognition is not *solely* do to with the contour properties, for if this were the case it should not matter which side of the contour the object is perceived to be.

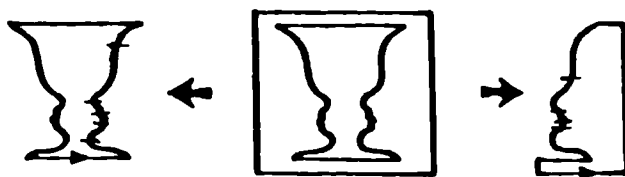


Figure 1.6: Rubin's figure, seen as left: a goblet, by defining part boundaries at curvature minima corresponding of the base, stem, bowl and lip or right: pair of facing facial profiles, by the curvature minima corresponding to nasion, nose base, lips and chin.

Although the description of 3D objects, and in particular the face, by edges and contour outlines is clearly not the whole story, this concept of shape representation has none the less led to some mathematical developments which are of importance. These will be discussed in section 1.2 after I have described another idea about how the visual system perceives shape which is also of importance for the description of shape.



**b) Shape as parts**

The second theory of how shape is represented in the visual system is complementary to the first. It is that the brain decomposes *shape into parts* in order to facilitate recognition. This idea helps to explain how objects can be recognised even though they may have some components missing or are seen from a number of viewpoints. It also takes account of motion. The question then arises as to how the brain segments an object into parts and whether the same strategy can be applied computationally. One consideration is that the parts must be invariant with time and viewing geometry (Marr and Nishihara, 1978).

In two dimensions, the points of curvature inflection of a contour have long provided a natural and useful method of segmentation. They also appear to have some psychological meaning. In 1954, Attneave (1954) estimated the points of highest curvature on a photograph of a cat. He drew a line drawing connecting these points from which he was still able to recognise the cat. He therefore surmised that these points must have a high information content.

Suggestions of how the visual system defines part boundaries have included segmentation at inflection points (Hollerbach, 1975; Marr, 1977), segmentation at points of maximum and minimum curvature (Duda and Hart, 1972) and segmentation at points with tangent and curvature discontinuities (Binford, 1981). Another hypothesis was that the visual system divides surfaces into parts using the loci of zero Gaussian curvature (ie. parabolic points) (Koenderink and van Doorn, 1982). This latter hypothesis was convincingly rebuked in theory by Hoffman and Richards (1985) and intuitively by considering the parabolic lines marked on a bust of Apollo Belvedere by the famous German mathematician Klein (Hilbert and Cohn-Vossen, 1952) where these lines seem to have no perceptual meaning.

Hoffman and Richards (1985) hypothesized that a rule from differential topology called "transversality regularity" is used by the visual system to segment a surface into parts. This rule is based on the observation that when two surfaces intersect they always meet in a contour of concave discontinuity in their tangent planes. Accordingly, surfaces are divided into parts along all contours of concave discontinuity of the tangent plane. Using differential geometric concepts, they show that *smooth* surfaces are divided by the loci of the negative minima of each principal curvature. The concepts of differential geometry are discussed further in chapter 5.

In contrast to earlier suggestions, Hoffman and Richards' theory has been shown to explain nicely some visual illusions, specifically that of figure-ground reversal. They showed that under figure-ground reversal, the minima of principal curvatures become

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maxima and vice versa. Therefore the part boundaries change and the figure is seen in a different way.

I tested this theory by conducting a small experiment. I asked 10 individuals to divide the curve shown in figure 1.7 into parts. All segmented the curve at points B,D,F,H, and J - the negative minima and not at points A,C,E,G and I - the positive maxima. When the figure was inverted all subjects segmented the curve at points A,C,E,G, and I. Thus this theory was verified.

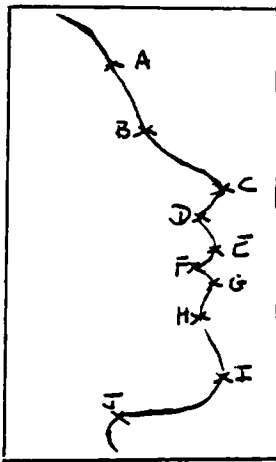


Figure 1.7: Segmentation of a profile at the points of negative minima in curvature.

For 2D curves, Hoffman and Richards (1985) derived six shape primitives, or "codons", terminated at points of negative minima and marking part boundaries (figure 1.8). There are only six possible ways in which a curve can vary between two negative minima. The segmentation of an object in this manner has been done intuitively by those seeking mathematical descriptions of curves and contours.

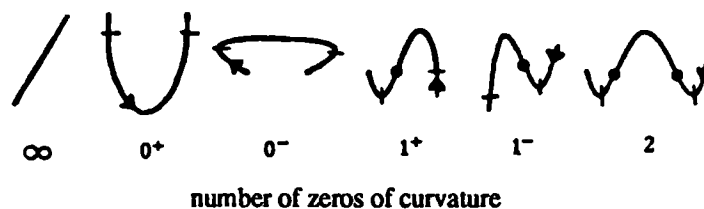


Figure 1.8: The "codons" of Hoffman and Richards. Zeros of curvature are indicated by dots, minima by slashes.

The full implication of this theory for surfaces is described later in this chapter. But first I shall discuss the methods which have been used to describe shape in two dimensions mathematically, since some useful concepts have resulted from these descriptions, which descriptions of 3D shape have built on.

## 1.2 Mathematical descriptions of 2D contours

In computer vision, the approaches adopted for describing contours or 2D shapes fall broadly into two classes, somewhat analogous to the two perceptual approaches of edges and parts described above. There are *global* methods, which describe the entire image, or object, in one go and are therefore not well suited to recognising partially complete objects, and *local* methods which describe parts of the object and then derive the connections between various parts. These are known as "top-down" and "bottom-

up" methods respectively. Local features have been demonstrated to play an important role in shape analysis (Rosenfeld and Johnson, 1970; Rosenfeld and Weszka, 1975), but for recognition purposes the whole shape must also be considered. In practice, a mixture of local and global methods often seem to give the best, fastest and most reliable descriptions. Multiple-scale, or hierarchical, representations have recently become an important part of any successful shape description because they allow the shape to be described both locally *and* globally.

For any representation of shape to be useful, it must be a shorter description (ie. have a smaller storage requirement) than the original shape and yet still contain the essential characteristics of the object's shape for it to be recognisable. Multi-level representations suppress surplus information to achieve this requirement (Hollerbach, 1975).

Throughout the literature pertaining to the mathematical description of 2D shape (outlines) the terms "contour" and "boundary" are liberally, and confusingly, interchanged. I shall use throughout the term "contour" to mean a closed (bounded) form and "boundary" to refer to the actual boundary of the form. The methods used for describing these 2D abstractions mathematically are now reviewed.

### **1.2.1 Description and classification of 2D contours**

The attempts made by scientists working in the computer vision field to find suitable descriptions of contours have been motivated by the desire to recognize such contours quickly and easily for a variety of tasks which range from industrial quality assurance to the recognition of aircraft outlines for defence purposes (Wallace et al, 1981). The first techniques that were developed for examining the shape of 2D contours subscribed to the part theory, describing the boundary in terms of segments. Syntactic methods could then be used to describe the boundary string. However, the computational expense of this approach prompted researchers to look for means of reducing the information content of the description. Decomposing the contour into parts (Shapiro, 1980) or deriving some sort of statistical measure was one way of doing this (see section a). Another approach was to extract properties of the contour, such as the medial axis or its representation in scale space (see section d), and use these to classify its shape.

#### **a) Description of the bounding contour**

The boundary of a contour can be extracted by standard edge detection algorithms (eg. Torre and Poggio, 1986). Once the boundary has been obtained a number of techniques may be used to segment it into appropriate parts. As discussed in the last section, points of curvature inflection have provided natural points at which to make a segmentation and these, together with other "critical points" such as end points, intersections or points of tangency (Freeman, 1978) are readily computable.

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The segmented boundary has been approximated by various functions to enable it to be quantified. Attneave (1954) used spline functions, but later iterative fitting methods were used to obtain a more accurate description. One of the first of these used circle segments (Shapiro and Lipkin, 1977) but this required many iterations to achieve a good match and produced many small segments. (Incidentally, these authors suggested that this technique could be extended to 3D using spherical segments but this would be an extremely expensive operation computationally and the subsequent matching of segments would be difficult.) A natural progression was to use polynomial functions to approximate the contour, iteratively refining the function until the error of fitting was lower than some threshold (Wallace et al, 1981).

A second description of the boundary used pattern recognition techniques to provide a syntactic description (Fu, 1974; Pavlidis, 1978). These can be quite complex and encounter the problem of closing incomplete boundaries, although this problem can be overcome by developing the syntactic description only up to a certain level (ie. using local properties) (Horowitz, 1975). Another difficulty was that noise in the data lead to the production of such complex syntactic strings that for some contours it becomes necessary to use a string containing the entire set of boundary points! Successful syntactic descriptions have been those which stress local aspects of the contour. However, these approaches enabled a language (or grammar) for describing shape to be defined, in terms of arcs, lines, protrusions etc., which is governed by semantic rules. The probabilistic or fuzzy character of the grammar often makes the governing rules appear to be unrealistic, but they contribute a flexibility to the approach which allows them to succeed. Hierarchical syntactic descriptors such as Pavlidis' (1979) have overcome some of these problems but are again computationally expensive.

Our ability to recognise an object from its silhouette, and hence its boundary, inspired Asada and Brady's (1986) *curvature primal sketch*. In this representation, the boundary was symbolically described using a set of curvature change primitives. Significant changes in curvature along the contour's boundary were represented in a similar manner to the intensity change representation of Marr (1976; 1982). A multi-level approach was adopted, in which the curvature changes are located at the finest level of detail at which they can still be detected. The greater the number of scales at which the curvature changes are detected, the more global a descriptor of shape the change is. This was extended to 3D by Ponce and Brady (1985) to describe significant surface changes (described later in this chapter on page 48).

The resulting representations of the boundary were often used to try to recognise individual contours. Additional information such as vector distances and angles was sometimes extracted from the contour and compared to a library of known contours using either a step-wise approach (Wallace et al, 1981) or a relaxation technique Davis

(1979). A statistical distance measure computed between the vectors facilitated recognition. Such representations have also been used to recognise objects from their partial boundaries (McKee and Aggarwal, 1977; Ansari and Delp, 1990). This requires an estimation of the closeness of match between the representation of a particular contour and the representations of a known contour stored in a "library". Leu (1989) observed that the recognition of objects from their boundary, becomes ineffective if the object is seen from a skewed angle, hence a normalization algorithm is needed to standardize the contour in order to facilitate recognition.

### **b) Mathematical morphology**

A very different approach for accurately extracting the boundary was devised by Matheron (1975) at the *Ecole des Mines de Paris*, and became known as mathematical morphology. The context in which it was developed was cellular automata and the connection between cellular arrays, retinal devices and image processing architectures have prompt considerable activity in this area (Skolnick, 1986).

In this method, the images being analysed are considered as sets of points and operations defined by set theory are used to describe the boundary (Serra, 1986). The method is different from other image processing techniques because it is based on the *logical* relations between points rather than arithmetic ones. It provides a means of decomposing global geometric measurements into sequences of local transformations. Different algorithms are specified by the different neighbourhoods of points defining these sequences of transformations. It begins with a "hit or miss" transform which defines whether a point on the boundary belongs to the enclosed object (the contour) or to the enclosing object outside of it (Serra, 1982; 1986) and proceeds with erosion/dilation transforms to build up a useful set of image processing algorithms such as skeletons and various filters (Skolnick, 1986). Mathematical morphology has been successfully applied to grey-scale images for the extraction of features (eg. Archibald and Sternberg, 1986). However, the algorithm is sensitive to lighting conditions for the scene under analysis. This method has also provided a decomposition of a contour into a union of simple components that is unique and invariant to translation, rotation and scale (Pitas and Venetsanopoulos, 1990).

### **c) Viewpoint invariant descriptions**

So far these methods have described the boundary of a contour. The computational expense of these methods drove researchers to seek methods of representing the contour in other ways. A number of statistical measures for describing contours were suggested such as area, perimeter, moments, coefficients of Fourier series or the distance between an enclosed pixel and its nearer boundary point (Hu, 1962, Zahn and Roskies, 1972; Shapiro, 1980; Danielsson, 1978). Whilst all of these maybe useful, some (moment invariants, introduced by Hu (1962), and Fourier descriptors (Zahn and Roskies, 1972;

## ***Shape Literature***

Persoon and Fu, 1977)) specifically address one requirement for the description of an object's shape. That is that the description is independent of object location, orientation, and viewpoint.

Moment invariants are so called because they are invariant with respect to rotation and scale change. The accuracy of the representation they produce depends on the number of moments used, but of course the greater the number of moments, the longer the computational time. They have been used to match scenes of objects (Wong and Hall, 1978). Originally applied to 2-D scenes, Sadjadi and Hall (1980) extended them to three dimensions, but these do not seem to have been used by others.

In the case of globally applied Fourier descriptors, no clear relationship between the representation and human perception is seen for the representation is unable to make the distinction between a square of side  $x$  and a circle of diameter  $x$  (Pavlidis, 1978). However, segments of boundaries have been described fairly successfully using Fourier descriptors calculated from chain-coding (Gorman et al, 1988). An advantage of this description is that the Fourier descriptors are independent of size, orientation, starting point and can be employed to recognise partial contours. They have also been used to recognize aircraft outlines (Wallace and Wintz, 1980; Arbter et al, 1990) and handwritten numerals (Persoon and Fu, 1977) via the extraction of skeletons, somewhat similar to medial axis representations (see section di). Other functions that have been used in a similar fashion include Walsh functions (Searle, 1970) and the rapid transform (RT) (Reitboeck and Brody, 1969).

A novel attempt at describing 2D convex shapes was the superimposition of a hexagonal 3 axes grid on the shape (Greene and Waksman, 1987). The hexagonal structure is based on the structure of the eye's visual receptor cells. The number of occluded grid points along each axes was used to measure the distance through the shape and the frequency of these occurrences compared with the distance through the grid gives a unique signature for the shape. The method reveals the difference between regular shapes such as squares, triangles and circles but appears to be totally unintelligible for irregular shapes. Another drawback is that it is not invariant to rotational changes. Interestingly, the authors claim that this *is* consistent with human perception as we perceive shapes differently when viewed from different angles, that is the same shape maybe seen as square or rhombus.

### **d) Extraction of information from the boundary**

#### **i) Skeletal representations**

Three separate but similar approaches have been proposed for the extraction of the essential shape information of 2D contours. These make use of symmetric properties of the bounding contour and produce a skeletal representation. The contour cannot,

however, be regenerated from these skeletal representations. But if the skeleton is unique, it may nonetheless be used for recognition purposes.

The three approaches are the smoothed local symmetries representation (SLS), the symmetric axis transform (SAT) and process-inferring symmetry analysis (PISA). The difference between them lies in their definitions which are closely related and illustrated geometrically in figure 1.9. The SAT is the locus of the centre  $O$ , SLS the locus of the midpoint  $P$  of the chord joining  $A$  and  $B$ , where  $A$  and  $B$  are the points where the maximal disk intersects the contour and PISA is the locus of the point  $Q$ , the midpoint of the arc  $AB$ . For a detailed comparison of these algorithms see Rosenfeld (1986).

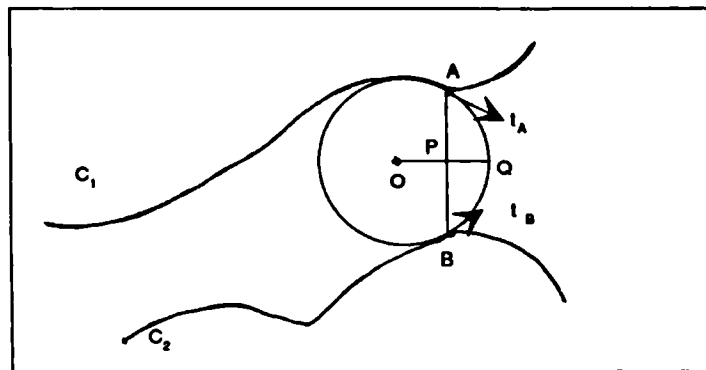


Figure 1.9: Geometric definitions of the SAT, SLS and PISA. Curves  $C_1$  and  $C_2$  have tangents vectors at  $A$  and  $B$ . The SAT-axis is the locus of circle centres  $O$ , The SLS the locus of points  $P$  and PISA-axis the locus of points  $Q$ .

### Symmetric axis transform (SAT)

The Symmetric Axis transform (SAT) is sometimes known as the *medial axis transform*, the term its inventor Blum used (Blum, 1967; 1973). Blum defined the medial axis of a contour by considering the brain's interpretation of wave fronts being propagated outwards from the centre of the shape and either exciting or inhibiting sensors placed in the field around it. These sensors could only be fired once and were unaffected by a second wave front passing through them. "Corners" occurred in the wave front contours at the minimum radius of curvatures and the locus of these corners defined the medial axis. A medial axis function was defined as the number of times a corner occurred on the medial axis.

Several alternative definitions of the medial axis exist: the medial axis can be generated by considering a contour being collapsed inward at a constant velocity in a direction perpendicular to the outline at all places, leaving a skeleton line drawing (figure 1.10). The medial points are the points where the outline meets itself and the medial axis is the locus of these points (which are equidistant from the outline). Yet another name for it is the "burning prairie algorithm" as the medial axis is what would be left over if one were to set fire to the entire boundary at the same instant in time. A *medial axis function* can also be defined as the locus of points when the occurrence of the medial points, or distance of the medial point from the perimeter, is included (de Souza and Houghton,

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1977). This function is a unique, and invertible, transform from the original form to the medial axis. Another way to define this representation is as the union of the centres of maximal disks that touch at least two points of the boundary of an object (figure 1.11). The symmetric axis is the locus of the maximal disks centres.

Blum and Nagel (1978) made a generalisation of the original form of the algorithm which reduced the effect of noise on the algorithm and the medial axis transform was more robustly defined for grey-level images by Moore and Seidl (1974).

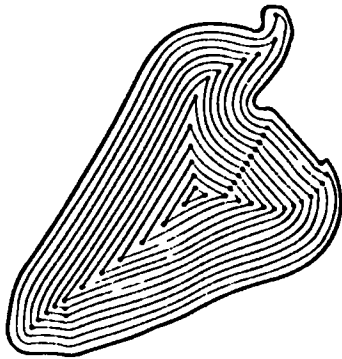


Figure 1.10: Formation of the medial axis by consideration of a contour being collapsed inwards at constant velocity (adapted from de Souza and Houghton, 1977).

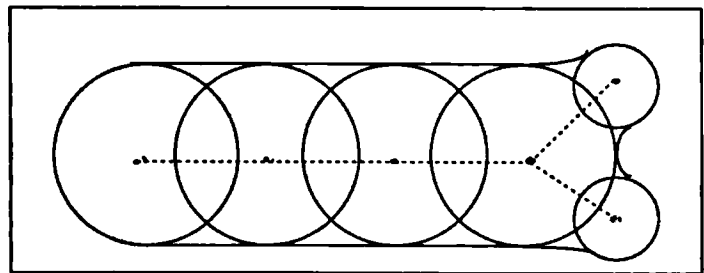


Figure 1.11: Formation of the medial axis by consideration of the locus of the maximal disks that touch the boundary at two points. (adapted from Brady, 1983).

Some advantages of the medial axis representation are that it is continuous and eliminates the need for the contour to be in a particular orientation when analysing its shape. The representation contains a local symmetry definition, is information preserving (Blum, 1973 p.216) and unique for a contour. The branching structure of the axis enables components of the shape to be defined and there is some evidence that human eye movements are related to the medial axis function of a line drawing (Richards and Kaufman, 1969).

It does, however, have a number of drawbacks; it is quite complicated to program and is computationally expensive (Pavlidis, 1982). It works best on smooth, curved objects such as biological shapes but does not provide the simplest representation for rectilinear figures. The SAT is very sensitive to small fluctuations in the boundary contour (Agin, 1974). Noise and fine detail in the bounding contour can produce extraneous axes that make the shape description more complex, but the use of multi-resolution descriptions can help overcome this (Gauch et al, 1987). One could also smooth the boundary *before* finding the medial axis.



Smoothed local symmetries (SLS)

The smoothed local symmetries representation (SLS), proposed by Brady and Asada (1984), represents a contour using the locus of the midpoint of the chord joining the points A and B on the bounding contour. The SLS is a more comprehensive descriptor than SAT. For instance, for an ellipse, the SAT finds only the major axis whereas SLS finds both the major and minor axes (Leyton, 1987b) (figure 1.12). The description produced is closely related to the generalised cylinders description for 3D objects, discussed later (Nevatia and Binford, 1977; Brooks, 1981).

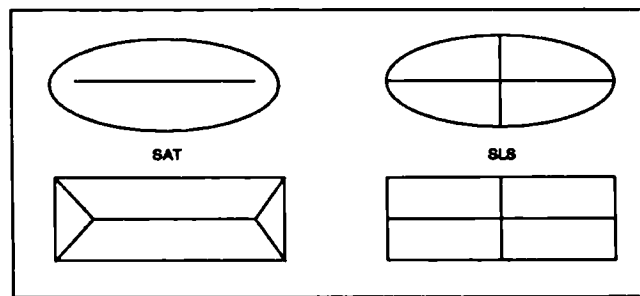


Figure 1.12: SAT and SLS representations of an ellipse and a rectangle.

Recently, Cho and Dunn (1991) have described a modification of the SLS called *Hierarchical local symmetries*, HLS, which excludes non-intrinsic and redundant local symmetries and enables a hierarchical decomposition of information based on local symmetry. The axis information is organised by the tangent difference between two boundary points forming a local symmetry. Rom and Medioni (1991) have proposed a similar hierarchical description of shape based on the SLS which allows the identification of parts of a contour.

Process-inferring symmetry analysis (PISA)

The third skeletal type representation was described by Leyton (1988). Termed process-inferring symmetry analysis (PISA) it considers shape to be the result of some historical process. Curvature extrema are obtained by using the "symmetric axis-curvature duality theory" (Leyton, 1987; Yuille and Leighton, 1987). This theorem states that "any segment of a smooth planar curve, bounded by two consecutive curvature extrema of same type has a unique differential symmetric axis which terminates at the curvature extrema of opposite type" (see figure 1.13). This idea is used, but not explicitly stated, in the SAT description.

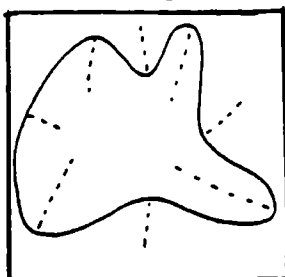


Figure 1.13: Symmetric axis-curvature duality (adapted from Leyton 1987).

The PISA description bears a close relation to the prototype description of 3D shape described by Pentland (1986) which is dealt with later in this chapter.

### ***Shape Literature***

Recently, a new method for obtaining the skeleton of a contour, based on active contours (or "snakes" (Kass et al, 1987)) has been proposed by Leymarie and Levine (1992). Leymarie and Levine argue that using the boundary information together with a contour's skeleton produces a richer, more powerful and efficient shape representation. Boundary information is included into the algorithm by the extraction of curvature extrema and arcs of constant curvature.

### **Multiresolution SAT description**

The characterization of changes between two contours requires an assessment of their overall similarity in shape. This involves measurement of the importance of each component of the contour. The multiresolution approach of Gauch, Pizer and their colleagues addressed this question (Gauch et al, 1987; Pizer et al, 1987; 1988). Here, 2D contours are obtained from a grey-scale image at different levels of intensity and the medial axis of each contour is extracted using the SAT. The importance of each branch of the medial axis is determined by its annihilation by, or of, an adjacent branch as one tracks the branch from low to high scales. Thus a hierarchical structure is imposed onto the axis branches allowing branches to be considered as sub-objects of one principal axis. In this hierarchy, termed an "axis pile", the width of the medial axis function and the axis of symmetry are used to characterize the contour.

This technique has been applied to analysis of the shape of jaws by simplifying the analysis of mandible outlines and was able to reveal differences between two types of jaw. However, a large storage capacity is required and the process is computationally expensive (Pizer has quoted times of half an hour to perform the necessary calculations for one representation).

Pizer also described another method called "vertex curves" in which vertices of one contour ("level curve") are followed to the next producing curves on the image surface. These curves supply boundaries to segment the image into "codon districts" and multiresolution analysis of these districts gives a representation which describes the spatial curvature properties of the image as a function of scale. Both this and Nackman's description give information about the shape of grey-scale images at various levels. Pizer and colleagues applied these techniques to the analysis of medical images (Pizer et al; 1988).

Information contained in a grey-scale image can be simultaneously represented at different scales using the approach of Crowley and Parker (1984) or Koenderink (1988). This approach implies using a hierarchy of scales to extract shape information from the image.

Multiresolution techniques such as these have been shown to improve the segmentation of an image over established techniques based on local pixel properties or edge strength (Lifshitz, 1987). The association of pixels to components of shape is crucial for this segmentation. Gauch et al (1987) postulated that if the multiresolution SAT could be used to impose pseudo-landmarks onto an image then a biorthogonal grid deformation analysis (Bookstein described later) might be used to describe morphological changes in soft-tissue surfaces where recognizable landmarks do not exist.

### **ii) Scale-space method**

Another method for describing contours by extracting information from the boundary was *Scale-space filtering* (Witkin, 1983). In this method, a signal or curve is successively filtered with a Gaussian mask of varying widths. This introduces a scale parameter. The curvature of the filtered signal is measured and the inflection points or "zero-crossings", where the curvature changes sign, determined at each scale. The zero-crossings are used to form a hierarchical description of the signal which is termed a "Scale-space Image" which contains information about the location and extent of features in the signal. An example for a facial profile is shown in figure 1.14. By applying a stability criteria, events which persist throughout the scale-space can be identified as major features of the curve.

The Scale-space image can be reduced to a simple tree structure by the relation of the contours, formed by the zero-crossings, to one another as parents and children. For these branches to be stable, a stability test is needed. Witkin developed a suitable stability test based on the correspondence between intervals of the signal and their perceptual salience (Witkin, 1983).

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Figure 1.14: Left: A profile smoothed with a scale parameter. Right above: A Scale-space image for a facial profile. The scale parameter ( $s$ ) is shown along the ordinate with the coarsest scale at the top and the path length along the profile ( $t$ ) as abscissa. The zero contours are plotted. The solid horizontal line shows the zero crossing of the profile at one particular scale. Right below: the facial (midline) profile.

### ***Shape Literature***

Witkin automatically determined a discrete set of scales which were useful symbolic descriptors of the curve and these were later interpreted in terms of primitive events by Asada and Brady (1986). Yuille and Poggio (1983a) found that the contours formed from zero-crossings of the second derivatives might have enough information content to enable their use in reconstructing the original signal to within a constant scale factor. They also found that the Gaussian convolution filter has the unique property of not introducing extraneous zero-crossings as one moves from fine to coarser scales (Yuille and Poggio, 1983b) and is, as such, "well-behaved" mathematically.

The Scale-space representation is sensitive to the amount of change made to the curve, except in the instance of a very small, very convex change being made. It is computationally efficient and not influenced by arbitrary choices made (such as starting point). Apart from for convex shapes, which have no zero crossings since the curvature is always positive, it is a unique representation for a curve. Mokhtarian and Mackworth used curvature, computed at various levels of detail by convolution of the path length parameters with a Gaussian kernel. They refined this method by reparameterizing each convolved curve by its normalized arc length parameter (Mackworth and Mokhtarian, 1988). This "renormalization" is equivalent to a continuous, non-linear horizontal shear of the scale-space image. It is more suitable for matching similar shaped curves in the presence of noise.

Rotem and Zeevi (1986) succeeded in recovering the original 2D signals from their zero-crossings thereby showing that no information is lost by working in scale-space. Bischof and Caelli (1988) maintained that scale-space is only useful in terms of what it can tell us about shape in conjunction with other methods, demonstrating how it can improve the shape-from-texture method (see section 1.4.1 for shape-from-texture).

Witkin's stability test extended to three dimensions produces a surface which splits and merges. Bischof and Caelli (1988) used a different stability test that assumes physical events are conceived of as boundaries, based on the observation that zero-crossings of region boundaries remain spatially stable over filter scale changes. If stable they will exist at multiple scales and the position of zero-crossings of the boundary edge will be unaffected by neighbour's boundaries. This edge detection technique is shown to have good noise resilience.

This method has been used to describe and match planar curves such as shorelines obtained from LANDSAT images by Mokhtarian and Mackworth (1986) or waveforms (Zhuang, 1988) and to extract points of maximum or minimum curvature (Degucii, 1988). It has recently been applied to the analysis of facial profiles (Campos et al, 1992; Campos et al, in press; Moss et al, in press), for the analysis of changes in the face before and after surgery.

The scale-space image is invariant to scaling, rotational and translational operations, which is a prerequisite for reliable matching and the matching that has been achieved by this method is very good.

**iii) Chord length distribution (CLD)**

Recently, a method for describing a contour, or class of contours, has been devised. In this method, called Chord length distribution, polygons are represented by the chord lengths between each pair of vertices (Cootes et al, 1991; Cooper et al, 1991). The correlation between pairs of chords is determined by calculating their covariance over a training set of objects. This allows a particular "class of shapes" to be defined by the principal eigenvalues of this covariance matrix. Thus the variability of the set of contours may be quantified from these eigenvectors.

**1.3 Shape in 3D**

Up to this point in this chapter, all the mathematical descriptions of shape described have ignored the fact that in real life a *surface* exists between the boundary lines or contours. To make further headway a new concept of shape, based on surface properties, is needed. The shape of an object is intrinsically bound up with its surface geometry, which can be visualised by the illumination of the surface. The mathematical language of surfaces is differential geometry, this is described in chapter 5.

Quite recently, the role played by shading and illumination in the human visual system has been investigated (Ramachandran, 1988). These studies have shown the following: firstly that the perception of symmetry is based on three dimensional shape rather than a distribution of dark and bright areas in the image, that the grouping of simple objects is based on 3D shape rather than luminance polarity and that the brain must compute 3D shape before it can perceive apparent motion. Thus the representation of 3D shape plays an important role in the human visual system.

**1.4 Mathematical description of 3D shape**

The concept of 3D shape representation has been realised by computer vision scientists seeking more accurate representations and interpretations of 3D scenes of objects for robotic applications. This has enabled some standards for 3D shape description to be laid down.

A good, useful, robust and rich description of a 3D object should exhibit at least some of the following characteristics: It should be viewpoint independent, able to deal with complex real objects, noise tolerant, computationally accurate and cost efficient and able to cope with novel objects or artifacts.

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Marr and Nishihara (1978) proposed three criteria for judging shape representations. These were that a representation should be computable relatively efficiently, have scope and have sensitivity - that is the information should be able to be represented at different scales. Brady (1983) added three other criteria: that the representation should also be rich (ie. information preserving), locally computed and be a local, extendible to a global, representation using frames that would eliminate other frames which they completely enclosed. These frames should be propagated between scales. He illustrated these criteria using the generalised cones and smoothed local symmetries representations described later in this section (Brady, 1983). These six criteria are also important as a basis for assessing our representation of facial shape (see Nackman and Pizer, 1985 for a further discussion of this).

Bearing these considerations in mind, two distinct approaches have been developed for describing object shape in terms of surface geometry. The first is based on measurement of the surface normal (section 1.4.1) and the second on measurement of depth (section 1.4.2).

#### **1.4.1 Surface normal based methods**

In intensity images, information about the surface geometry of an object is contained in the image intensity. Many different approaches have been used to derive the surface normals of an object from the image intensity, thus allowing the shape of the object to be extracted or represented. These methods are collectively known as "*shape-from-xxx*" methods and included shape from shading (Horn, 1975; 1977; Ikeuchi & Horn, 1981; Pentland, 1982; 1984a; Szeliski, 1991a), shape from stereo (Marr and Poggio, 1979; Grimson, 1981; Baker & Binford, 1982), shape from photometric stereo (Woodham, 1980; 1981), shape from contours (Brady and Yuille, 1984), shape from rotation (Szeliski, 1991b), shape from texture (Witkin, 1981) and shape from fractal geometry (Pentland, 1984b; Yokoya and Yamamoto, 1989; Chen et al, 1990).

The main difficulty of these methods is discerning the lighting geometry and taking account of physical properties of the surface, such as reflectivity and specularly, as well as characteristics of the light source (Wang et al, 1987; Wolf, 1983).

The representations derived from these methodologies, have been given many names including: 2½D sketch (Marr, 1976; 1982), needle maps and Gaussian image (Horn, 1984) but all are based on the calculation of the surface normal. Some issues have not been resolved in the literature, these are how the local surface normal is parameterized, whether the depth is explicit or computed by integration and whether second order quantities (such as the principal curvature, second fundamental form of the surface) are explicit or computed by differentiation (Brady and Yuille, 1984). These issues affect the accuracy of the representation and its susceptibility to noise.

Some of these representations (notably those derived from stereo and motion) describe the depth and orientation of a discrete set of points on the surface using certain points which show rapid change, such as those lying on the edges. A reason for this is that some theories about the specialized processes that occur in early human vision, such as stereopsis (Marr and Poggio, 1979), specify that explicit information about surfaces in a scene can only be inferred at "scattered" locations and that these locations correspond to significant changes in intensity in the retinal image (Terzopoulos, 1983). However, human perception is of *complete, smooth surfaces* and people have been found to be able to recognise objects from images where the illumination relationship is distorted, such as pseudocolour (Besl and Jain, 1985). This raises the question of whether the brain represents a surface by reconstruction from salient image points or encodes the surface in a more explicit fashion at an early stage of vision. Terzopoulos (1983) investigated the former question via multi-level reconstruction of a thin-plate surface. In this thesis, the latter possibility is investigated.

Of all the representations produced from intensity images, two are particularly interesting for this work. These are the *Topographic Primal Sketch* (TPS) (Haralick et al, 1983) and *Extended Gaussian Images* (EGI) (developed by Ikeuchi, Horn and Brou).

#### **a) Topographic primal sketch**

The Topographic Primal Sketch (TPS) groups together portions of a digitized image in terms of *intensity* regions. These regions are then classified using the directional derivatives functions of the surface (ie. the direction where the surface has the greatest rate of change) which are defined from differential geometry (see chapter 5). The surface is classified into seven topographical primitives namely; peak, pit, saddle, ridge, ravine, flat and hillside.

The accuracy of this description is dependent largely on the removal of noise from the image. This is achieved by modelling the surface with bivariate cubic splines across a certain window size. The directional derivatives are obtained from the fitted splines. An important consideration is, at which pixel in the window used should the classification be applied? Since each pixel represents an area of the image, using a large window size for the computation of the derivatives results in a less accurate classification because the cubic fit to the surface becomes less accurate. The window size used is therefore a function of noise and complexity of the image surface.

This representation is somewhat similar to the one I report in this thesis for the description of the facial surface. However, there is one very important difference, that is that the primitives used in the topographic primal sketch are derived from *intensity* images whereas I have used *range* images. Consequently, the TPS is dependent on the surface shading and is susceptible to the problem of shadowing. Moreover, the TPS

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representation does not interpret the surface in terms of its intrinsic geometry or take account of illumination or reflectance changes upon it and it is not invariant to viewpoint changes (Ponce and Brady, 1985) and its suitability for producing a multi-resolution description has not been demonstrated. Haralick and colleagues pointed out the importance of developing a technique that would group the topographically labelled pixels together and assemble them into primitive structures in order to facilitate higher level matching. Pong and colleagues (Pong et al, 1985) showed how the TPS representation could be used to estimate if a surface was either planar, developable, cylindrical, elliptical or hyperbolic in shape.

#### **b) Extended gaussian images**

Another interesting technique is *Extended Gaussian Images* (EGI's). In this representation, each point on an object's surface is mapped onto a corresponding point on a Gaussian sphere at which the orientation of the surface normal is the same. This mapping is reversible only if the object has positive Gaussian curvature everywhere, thus only convex objects have a unique representation. EGI's were introduced by Ikeuchi, Horn and Brou and are defined and discussed in depth by Horn (1984). They can be easily computed from depth map or needle map representations (produced from photometric stereo or from geometric object models). An advantage of this representation is that if the object's surface can be divided up into small patches, the equivalent patches on the Gaussian sphere can be computed without directly computing the Gaussian curvature of the entire surface. In a noisy image, information on the Gaussian sphere is spread out but noise in the data can be removed by convolution with a smoothing function. Some authors have argued for EGI's assuming that second order differential properties could not be reliably computed, however Brady and Yuille (1984) demonstrated that this is not the case. The EGI works well for convex objects without any occlusion but can't distinguish between some shapes (see Besl and Jain, 1985 figure 47).

#### **1.4.2 Depth based methods**

Depth or range based methods appear to have arisen in the last decade or so, directly as a result of the advances made in computers. Greater computing power has allowed more mathematically reliable data to be collected, in greater quantity and in less time. These systems have enabled *synthetic* and *analytic* methods to be developed to model and describe 3D objects.

Range images directly provide depth information which, unlike intensity information, is dependant only on the surface geometry. Therefore, shape information should therefore be easier to obtain using range images than intensity ones. The shape of a region in a range image bears a direct relationship to the 3D shape of visible object surface. Hence,



descriptions obtained from range images are more reliable than those obtained from intensity images.

**a) Synthetic models**

One approach to investigating an object's shape is attempt to synthesize the object, or scene of objects. This is a powerful technique because it provides the investigator with control over the object. The accuracy of synthesized models can be determined by comparison with object data.

The synthesization of object models was influenced by the computer vision requirement for object recognition. Models of complete objects produced for CAD/CAM required vast numbers of possible configurations of the object to be stored when viewed from different orientations. This quickly led to insoluble problems for recognizing objects, especially when the number of objects to be recognized was large or the object views were partial. A second approach was recognizing *parts* of objects and determine the spatial relationship between them. This had the advantage of incorporating the ability to learn new objects.

Object modelling uses mathematical functions to generate solid primitives that can be added or subtracted to produce an accurate description of the object surface. The first models for representing solid objects used cylinders with a straight line axis Marr (1976). This idea was extended by Binford (1981) to generalised cylinders and cones to represent any 3D volume. Generalized cones were concisely defined by Brady (1983) as the shape described by drawing a cross-section at a fixed angle along an arbitrary 3D "spine" curve and expanding the cross-section along the spine according to a "sweeping rule". The cross-section does not have to be circular (figure 1.15).

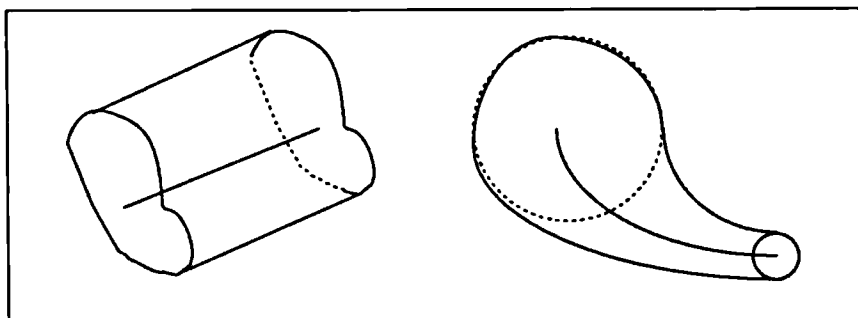


Figure 1.15: Generalised cylinders with left: a straight axis and b) a curved axis.

The axial basis of generalised cones means that they are suitable for describing objects which have a natural axis, including growing structures, such as animals, but unsuitable for objects that are essentially surfaces, especially smooth, featureless surfaces (Fan et al, 1986). Early attempts to use generalised cylinders to describe 3D objects were made by Agin and Binford (1973) and Nevatia and Binford (1977). Whilst they provided reasonable descriptions of elongated objects they do not cope with jointed or complex

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objects well. Generalized cones *can* provide a hierarchical description of shape but this is not a unique representation (Nackman, 1982 p.13) and constraints sufficient to bring about uniqueness-are unknown. Hence *ad hoc* rules have been implemented in order to obtain a single description.

The suggestion that geons (for "geometrical ions"), that is components of an object that can be modelled by generalized cones, play an important role in human perception (Marr and Nishihara, 1978; Biederman, 1987) has led to them being used by artists as a basis for facial structure (Dürer, 1582), see figure 1.16. However, they do not describe smooth featureless surfaces such as the face accurately enough for individual identification or assessment of the subtle changes that are brought about by reconstructive surgery.

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The production of a reliable model of an object requires that the parts chosen as primitives must be complex enough to be reliably recognized but simple enough to be used as building blocks (Koenderink and van Doorn, 1982). The solid modeller, "WINSOM", developed by Woodwark at the IBM (UK) scientific centre, uses primitives that may be thought of as sets of points that maybe operated on (eg. the union or difference of them) to form more

complex primitives such as cylinders, cones, torii, helixes or ellipsoids. This is a very adaptable system which has been used to model more complex objects. Some very complex shapes such as clouds and mountains have been represented by fractal-based models (Pentland, 1984b), but other structures such as trees, fire, hair or rapids have not yet been well described. Perhaps the recent theory of "chaos" (Gleick, 1989) will permit the description of some of these.

A more comprehensive parts set are the "*superquadratics*". These are mathematical solids defined by sweeping out cosine and sine functions in 3D space to produce a spatial surface (Barr, 1981). Like WINSOM, they are more powerful than the generalized cylinder representation, since they allow objects to be described in terms of a calculus formula. They were used by Pentland (1986) to develop a representation for 3D shape, which he claimed to be based on the human perception of shape and the way in which our perception organises the visual data received for the representation of natural form.

Specific objects, such as the face, can be generated from these components using the Boolean operators "and", "or", "not" etc. For the example shown in figure 1.17, 13 primitives have been used.

Superquadratics can be used to estimate 3D shape parameters from the image data. Their integral relationship to surface normals and surface shape means that they overconstrain this estimate and therefore have the potential to be extremely reliable modelling primitives. They produce two equations which are solved by linear regression to give the shape, orientation and shape parameter. This is important as it allows this representation to be robust and easily applied.

The idea of using superquadratic functions and deforming them to fit the 3D data provided seems to have been first suggested by Barr (1984). Two different approaches were employed by Pentland (1986) and by Terzopoulos and his colleagues (Terzopoulos et al, 1987). Pentland proposed using parametric solid modeling primitives as deformable models. Here fitting was using the model primitives "inside-outside" fusion (Pentland, 1986; Solina and Bajcsy, 1990). Terzopoulos, on the other hand, employed a mesh-like surface model and fitting was achieved using a physically-

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motivated energy function. This function could be applied locally. Pentland's approach allows a unique, compact description to be produced that is suitable for recognition but may not have enough degrees of freedom to model fine surface details. Terzopoulos' approach allows accurate modelling of surface detail but the resulting descriptions are not unique or compact making recognition extremely difficult. Both are iterative methods and are therefore, computationally slow. These methods bear similarities to Bookstein's thin-plate splines (described in chapter 2) and to Biederman's geons (Biederman, 1987).

Recently, Pentland and Sclaroff (1991) have attempted to address these problems using a finite element method (FEM), which is a standard engineering method for simulating the dynamic behaviour of an object, and parametric solid modelling using implicit functions. They were able to obtain a unique 3D shape description but point out that a major weakness of their system is the need to segment the data into simple, approximately convex "blobs" in a stable viewpoint-invariant manner and to estimate the object orientation to within  $\pm 15$  degrees. Pentland and Sclaroff proposed that "snakes" or energy-minimizing curves (Kass et al, 1987) could be used to iteratively fit a required 2D curve and thus extract features of interest for an image. Similarly, in 3D, a surface model could be deformed in a dynamic and elastic fashion by fitting simulated rubbery sheets to obtain a good fit to the data (Terzopoulos et al, 1988). This approach provides considerable geometric flexibility and uses the idea that various physical forces (intrinsic and extrinsic) can be applied to produce the final object shape. By applying the deforming force over a small area of the object surface, local changes in the surface can be modelled as well as global ones. Naturally, this method requires powerful computers because of the iterative nature of the fitting algorithm. These methods have drawn wide interest from scientists working in the areas of machine vision and image analysis as a method for locating features of interest (eg. Cohen, 1991) and modeling 3D data.

### **b) Analytic descriptions**

The first analyses of range data used the range data to segment the object's surface into approximately planar regions. These regions were then matched for recognition purposes (Oshima and Shirai, 1983; Bhanu, 1984). Henderson (1983) added a region growing algorithm to achieve a cleaner segmentation of the data. Later, range data was used to find 3D boundaries (Hebert and Kanade, 1985; Parvin and Medioni, 1989). These boundaries included low frequency events, such as ridges and ravines as well as high frequency events, such as edges boundaries, allowing features to be extracted at a hierarchy of scale (Parvin and Medioni, 1989).

The idea of describing the shape of an object's surface in terms of curves and patches was expressed by Faugeras and Herbert (1986). The first to develop such a description

were Asda and Brady (1986), this was the *Curvature patches representation* which is analogous to smoothed local symmetries representation in two dimensions. This representation is a symbolic description of a 3D surface based on the theory that curves are perceived as part of a 3D surface. This theory is illustrated perceptually by example shown in figure 1.18. Most people interpret the lines as lines of curvature of the surface (Stevens, 1981).

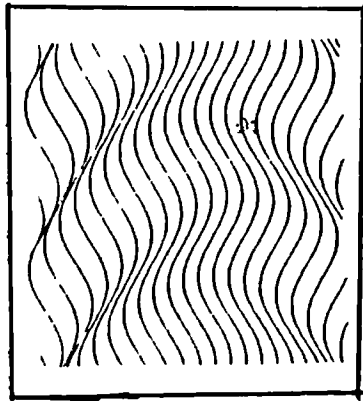


Figure 1.18: A series of lines which are readily interpreted as representing a surface.

In the curvature patches representation, the CAD definition of a parameterized surface patch is used and grid lines ( $du = 0$ ,  $dv = 0$ ) are set to the lines of curvature of the surface and the defining parameters  $u$  and  $v$  quantized. The surface is then interpolated from the grid lines using a blending function such as Bezier function or splines. By using the lines of curvature as "webbing" of the surface, flattening near patch corners is minimized and calculation of the principal curvatures is made easier.

A drawback of this method is that small changes in the observed surface produce large changes in the approximating curvature patches. Also a large number of patches are needed for this representation and the joins of these patches are not necessarily physically significant. Its main application has been in recognizing objects in a CAD database.

### **1.4.3 Characterization of changes in 3D shape**

A few researchers in computer vision have been specifically concerned with characterizing *changes* in 3D shape. These have either been for medical/anatomical purposes or for the automatic detection of defects in objects for industrial applications.

#### **a) Extension of SAT to 3D**

Nackman and Pizer's extended the SAT to 3D in order to characterize shape changes for medical applications (Nackman, 1984; Nackman and Pizer, 1985). In this method, the symmetric axis becomes the locus of maximal spheres instead of the locus of maximal circles in 2D. Nackman and Pizer decomposed the object using three kinds of primitives; width primitives based on radius, axis primitives based on simplified region curvatures and boundary primitives based on boundary surface curvatures. The sign of the mean and Gaussian curvatures were used to define curvature districts within these primitives.

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### **b) The surface primal sketch**

Ponce and Brady (1985) extended the curvature primal sketch to 3D, with the aim of detecting, localizing and symbolically describing surface changes from a depth map of the surface. The motivation for this development came from problems which had arisen with the algorithm used in the curvature primal sketch for computing lines of curvature on a surface and finding significant changes in curvature along the lines. Ponce and Brady devised a method for detecting changes in the height (depth) of a surface, this allowed them to classify the surface into some primitive surfaces (namely roofs, steps, smooth joins and shoulders). These surface classifications were obtained from the continuity of the surface, the continuity of the surface normal and the surface curvature, which can be distinguished between by their scale-space behaviour. The surface under investigation was segmented into smooth patches using surface intersections that are present at multiple scales. The surface patches were then matched to one of the primitives to generate a symbolic description (Asada and Brady, 1986).

The depth map of the surface can be smoothed using a gaussian function to remove noise if required, but this presents a problem with the boundary of the surface since the global application of the filter blurs the surface into the background. This was overcome by using a repeated averaging technique.

The surface primal sketch description has been applied to various objects including a mask of a face (figure 1.19). It enables the nose, eyes and mouth to be detected but lacks surface detail. Thus for relatively smooth featureless surfaces such as the face, the description produced is poor.

## **1.5 Summary**

In this chapter, I have brought together knowledge from two fields, that although seemingly independent, I have found to be related to the subject of this thesis. These are the theories formed by cognitive psychologists about how the brain perceives shape and the mathematical descriptions of shape that have been developed by computer vision scientists. I have tried to create a coherent whole from these diverse studies and to show how one reinforces the other.

First, observations of how the brain perceives shape were described. These revealed that the perception of an objects shape is strongly related to the recognition of that object. This implies that in the case of the face, the shape of the face affects how well the face is recognized. Thus, if a method can be developed for describing facial shape then this may also help with understanding how faces are recognized. In this thesis, the method for describing the shape of the face, developed in chapter 6, is used to examine the role played by the shape of the face in face recognition (chapter 9).

Next in this chapter, the question of how the brain abstracts shape information from its visual input was examined. Two complementary theories for this process were described: that shape is obtained from edges or from parts, ie. components. Examination of the mathematical descriptions produced for 2D shape (contours) showed two different approaches, somewhat analogous to these theories. These were global description methods, which describe the entire object in one go, and local methods which describe parts of objects and derive connections between various parts.

It was found that 2D contours had either been described by their boundary or by extracting information from the boundary. The latter case involved producing skeletal representations, such as the SAT, SLS, or PISA or using the scale-space method. If contours were to be compared, then producing these representations at a *hierarchy of scales* allowed the importance of each component branch of the contour to be assessed, and thus allow the similarity of shape to be assessed. A slightly different method of extracting information from the boundary was the chord length distribution.

These methods, and in particular the principle of describing shape changes by representing the shape at a hierarchy of scale provided useful tools for the shape description method and analysis of changes in shape described in chapters 6 & 7.

Next, we considered the fact that in the real world, *surfaces* are perceived and found, that the perception of 3D shape plays an important role in the human visual system (section 1.3). Mathematical descriptions of 3D shapes have not been as highly developed as those for 2D, but nevertheless some characteristics of a good shape descriptor have been realised (section 1.4). Again two different approaches have been

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developed for describing the an objects shape in terms of its surface geometry. The first is based on measuring the surface normal, which is extracted from intensity images (section 1.4.1). This encounters the problem of distinguishing between effects arising from physical surface properties and those arising from lighting conditions. The second approach, based on range data of the object which provides depth information, does not meet this problem.

The representations of 3D shape that have been produced have used the idea of an object as a collection of parts. Either the surface was segmented into primitive patches (eg. the TPS, section 1.4.1a or the curvature patch representation, section 1.4.2b), or objects were modelled synthetically and compared with acquired range data for identification (section 1.4.2a).

The characterization of *changes* in 3D shape has not been well addressed by the computer vision community. Only the extension of SAT to 3D to from "axes piles" and Ponce and Brady's surface primal sketch have attempted this. The description of *small scale* changes in shape has been addressed by a third field of research, that of biological shape description. Unlike most of the computer vision research, the emphasis in this field has been on defining *shape changes*.



## CHAPTER 2

### BIOLOGICAL AND STATISTICAL DESCRIPTIONS OF SHAPE (LITERATURE REVIEW)

Although a number of useful descriptions have been proposed by computer vision scientists for describing the shape of regular geometrical objects, finding an appropriate description of biological shape has proved very difficult. In 1967, Blum wrote that "despite more than 2 millenia of geometry, no formulation which appears natural for the biological problem has emerged" (Blum, 1967). The reason for this, he postulated, was that geometry had been born of surveying and grew in close collaboration with physical science and its mensuration problems. Clearly, description of the shape of biological forms requires a somewhat different framework.

In this chapter, a framework called morphometrics is discussed. Morphometrics seeks to describe *changes in shape* and therefore may be an appropriate framework for describing biological forms, which grow and evolve. Shape changes are described in terms of a deformation of the original form and quantified via the overlaying of cartesian grids (as per Thompson) or biorthogonal grids (as per Bookstein). An analogous method to the latter, called finite element analysis, is also briefly described. The main problem of these methods is their reliance on accurate landmark positioning. This is discussed in section 2.2.3. Other attempts at describing biological shapes in terms of bounding contours or a distortion of 3D space are discussed in sections 2.3 through 2.5. Finally, the multivariate approach to morphometrics made by statisticians, is described. The usefulness of these methods for describing the face is considered.

#### **2.1 Morphometrics**

A framework known as *morphometrics* began to emerge at the beginning of the century as biologists such as Thompson and Huxley tried to understand the interaction of size and shape in the evolution of species. Morphometrics was later defined as "the biometric study of effects upon form" (ie. biological shape) (Rohlf and Bookstein, 1990) and it evolved via two main strands of thinking. The first strand arose from geometric considerations with the relationship between one form and another measured as a "deformation". Interestingly, the artists Dürer and da Vinci were the first to consider the relationship between faces of different types as *deformations*. The second strand was provided by multivariate statistical analyses, whose first applications were in morphometrics. These analyses are concerned with the correlation of size measures with shape measures (termed *allometry*). A brief discussion of morphometrics is given in this chapter. For a more comprehensive treatment the reader is referred to Rohlf and Bookstein (1990).

### ***Biological Shape***

One of the driving forces in morphometrics has been Bookstein who has attempted to unify these two approaches, with some success (Bookstein, 1986; Rohlf and Bookstein, 1990). Bookstein has had a considerable influence on morphometrics and consequently this chapter is full of references to his ideas. Bookstein has made serious attempts to bring mathematics to bear on shape description and his biorthogonal grids have provided a conceptually useful description. His work has acted as a major source of simulation in this area.

Whilst Bookstein's ideas are highly regarded, there are a number of statisticians working in this area who have expressed mixed feelings about the robustness of his work (see the statistician's comments on Bookstein's 1986 paper). In particular, they have questioned some of the assumptions he has made in his analyses (eg. taking linear approximations of a Taylor series, which encourages zero covariance between the size and shape variables) (Campbell, 1986; Cressie, 1986), and the inability of his warp description of shape change (see section 2.4) to make the transition from 2-D space to 3-D space in practice (Cressie, 1986). Another comment has been that it is not made clear what is exact and what is approximate in the analysis (Campbell, 1986). However, Bookstein himself maintains that his approach to the statistical analysis of shape is analogous to the methods of other leading researchers in this area; the statisticians Goodall, Kendall and Mardia, and continues to answer his critics.

### **2.2 The deformation of form**

The idea behind deformation methods of morphology is that form can be described in terms of a *size* and *shape* variation of a *configuration of landmarks*. These methods were made possible by the foundation for the mathematical description and comparison of biological shape that was laid by the biologist D'Arcy Thompson in 1917. His method of Cartesian transformation grids is detailed in his famous work "On Growth and Form" which described the transformation of one contour shape into another via a smooth transition in accordance with biological homology (Thompson, 1917).

The importance of three dimensional analyses has been recognised by most scientists working with geometric methods. Specifically, the need for a description of the deformation of the 3D space *between landmarks* has been recognized, although none has yet been produced. The limitations of landmark based analyses are discussed later in this chapter (section 2.2 iv). Bookstein and Cutting (1988) have suggested a description based on differential geometry of the surface, and the ridge lines of the skull in particular. This thesis follows up this suggested approach and uses concepts of differential geometry to quantitatively describe the facial surface (see chapter 6).

**2.2.1 Thompson's cartesian grids**

Thompson's idea, regarding change between two forms, was that homologous points remained stationary while the Cartesian grids governing the change deformed, causing landmarks to appear to move. In this method, all points were simultaneously registered and therefore no arbitrarily chosen centre is needed. Thompson was concerned with evolution between species and he talked about a "biological force", of a postulated direction and magnitude, which acted on the original shape to bring about evolutionary change between species (Thompson, 1917, p.272). Natural selection is now widely believed to be responsible for these "forces" (Gould, 1971) This transformation between two species (what he called "forms") was assumed to be homogeneous and isotropic and to affect the entire form globally (Thompson, 1917, p.274). These two conditions depend on parts of the form evolving together, or at least not independently.

Thompson's application of this methodology to the study of the shapes of fishes as identical functions of different coordinate systems led him to the sweeping conclusion that the variation between the species had occurred along precise, orderly lines of growth. This idea was severely criticized by Hutchinson as a "floating mathematics for morphology, unanchored for the time being to physical science, but capable of valid generalization on its own level" (Hutchinson, 1948).

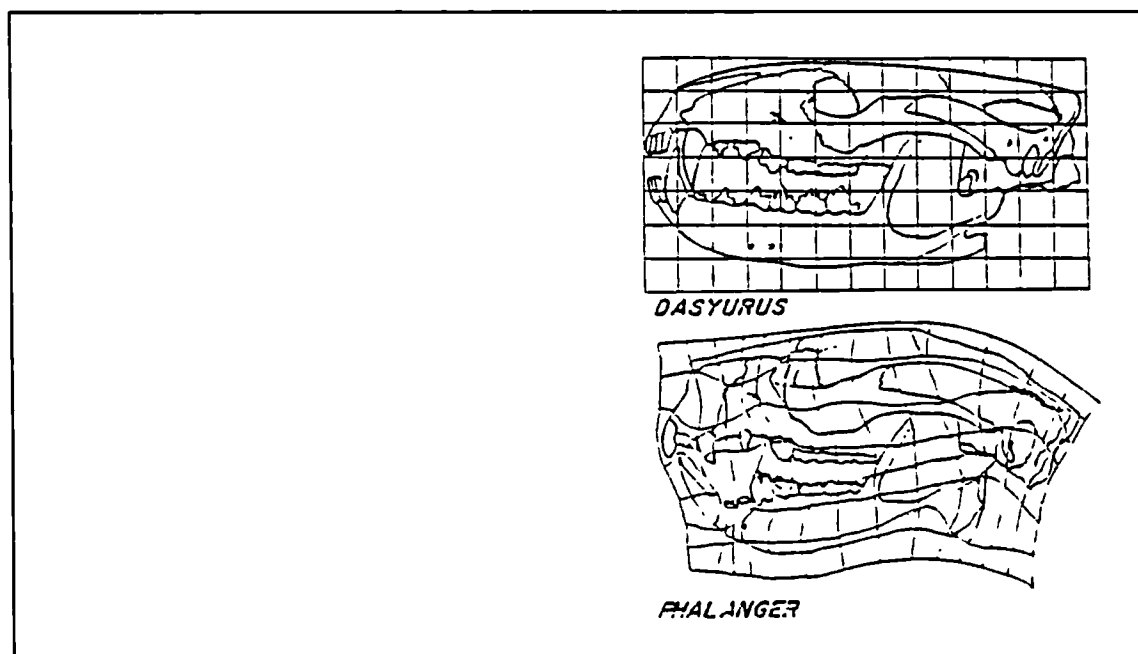


Figure 2.1: Cartesian transformation from Diodon to Orthogoriscus (from Thompson, 1917, p301).

Figure 2.2: A realistic complex deformation (after Sokal and Sneath, 1963).

Thompson illustrated his theory with a vast number of examples, the most famous one being a comparison of Diodon and Orthogoriscus (figure 2.1). Here the vertical coordinates of the first grid are deformed into concentric circles and horizontal coordinates become hyperbolas. Many researchers tried to repeat his method but none

### ***Biological Shape***

managed to obtain such nice, simple, mathematically describable functions as he did. Instead they obtained very complex deformations (such as figure 2.2) and all attempts at finding general laws failed. Sokal and Sneath (1963) defined this as a problem of feature enumeration: if there was more than one curvilinearity present, it became impossible to disintegrate multiple changes and produced constant shifting in relative grid line spacing and thus became visually highly complex. All compromises were so individually specialised that they failed to hold for more than a few examples.

A significant problem with the method was *how* to position the grid lines over a shape without any notable landmarks. Avery (1933) positioned a square grid over a tobacco leaf and photographed it during the leaf's growth. He then attempted to understand his results by mathematically analysing the grid. Later, Richards and Kavanagh (1943) looked at Thompson's technique and Avery's measurements and found that the lines of maximum and minimum linear growth of the tobacco leaf were orthogonal.

Thompson's work contained many weaknesses which were identified by Bookstein (1978a). For instance, Thompson's attempts at relating the skulls of humans to chimps and baboons in to order to demonstrate an evolutionary connection between them was not very successful and Thompson himself admitted that neither of these apes lie precisely in the same sequence as the other's connection to man. Another criticism was that although Thompson's draughtsmanship was good, there was no explanation of the way that individual grids were constructed. Bad fits were blamed on bad data leaving the methodology unquestioned (Bookstein, 1978a p.77 p.87).

No further headway was made in this area of morphometrics until Bookstein's seminal work of the late 1970's. It was his opinion that since Thompson until that time (1978) there had been "no improvement in morphometrics or without, no methodological advance ... that is comparable in stature with Thompson's original method." and that "six decades after its publication the method still resists quantification except in special cases. It remains ... much more difficult than it was supposed to be. ... Anyone trying to make new headway must begin to build, as I do, exactly where Thompson left off." (Bookstein, 1978a, p.89)

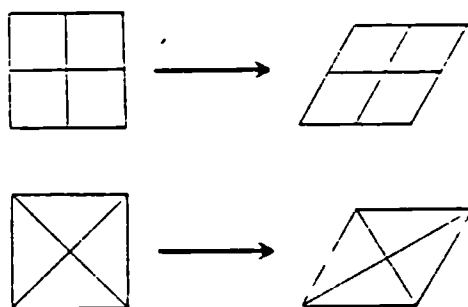
#### **2.2.2 Bookstein's biorthogonal grids**

Bookstein's re-analysis of Thompson's work and its mathematical elegance, together with the observation of orthogonal growth in tobacco leaves (Richards and Kavanagh, 1943) led him to develop a technique known as "biorthogonal grids". The concept of this method is based on the representation of an affine transform (one which maps parallel lines onto parallel lines). This idea is summarized below.

At this point it a word of caution regarding biorthogonal grid may be appropriate. Sampson (1986) pointed out that no one other than Bookstein has ever had the software to generate a biorthogonal grid so his claims for this method have not been fully tested and these grids have not been widely applied. A possible reason for this was suggested by Cheverud and colleagues (Cheverud et al, 1983). This was that although analytically correct, the graphical representations produced by the biorthogonal grids are visually confusing.

**a) Representation of an affine transform**

An affine transform can be described differently depending on the choice of starting grid. For example, the transformation of a square into a rhombus can be described in two ways: the angle between the diagonals can be altered and the length of the diagonals kept constant (as per Thompson), or one diagonal can shrink and the other expand keeping the angle constant (as per Bookstein) (see figure 2.3). The advantage of Bookstein's conception of the change is that the axis grid remains orthogonal for the transformation and allowing the changes to be described in terms of *dilatations*, or length multiplications, of the principal axes.



Any three points mapped onto any other three points can be expressed as an affine transform. So several affine transformations maybe performed and joined up to form a 'global' grid which may be used to describe the shape change.

Figure 2.3: Two ways of describing the same affine transformation from square to rhombus. Top: The axes lengths are constant and the angle between them alters. Below: The axes change in length, but the angle between them remains constant.

Bookstein (1984a) attempted to quantify the importance of changes in shape calculated via the biorthogonal grid technique. The mean size and shape changes occurring between one triangle of landmarks and a second were tested for statistical significance using a tensor description of shape change. Using an ancient theorem, a circle inscribed by the first triangle of landmarks is deformed into an ellipse in the second triangle of landmarks (see figure 2.4 and Hilbert and Cohn-Vossen, 1952 for a proof). The principal axes of the circle and ellipse lie at 90 degrees to each other both before and after the transformation but vary in length, or *dilate*. The product of these principal dilatations gives a measure of the ratio of change in the inscribed area and the quotient

## Biological Shape

of dilations a measure of the "anisotropy", or direction, of this size change (Bookstein, 1984a).

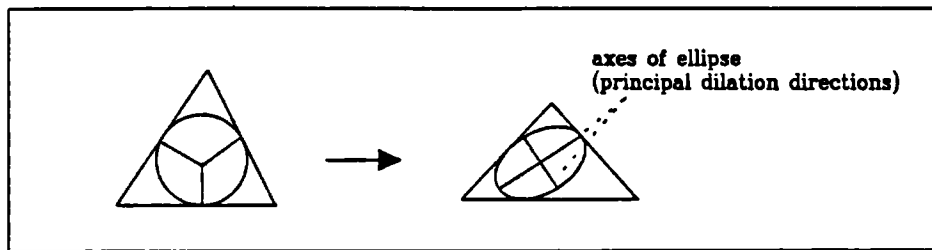


Figure 2.4: An ancient theorem tells us that an inscribed circle is transformed into an inscribed ellipse via the movement of the vertices of the triangle.

### b) Application to facial changes

Bookstein applied his technique to several problems including analysing the changes in hard tissue landmarks on the skull, which were obtained from cephalograms. The 2D changes in landmark locations were determined by consideration of the change in rotation and translation of a triangle of landmarks between two data sets (see figure 2.5).

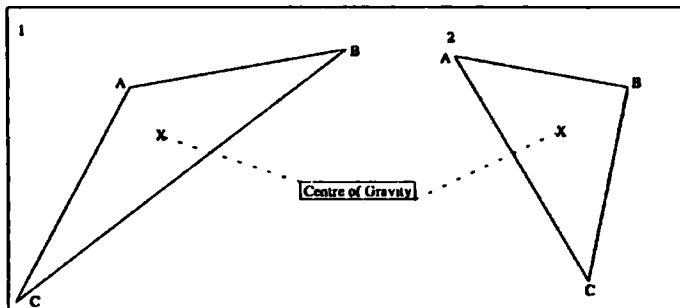


Figure 2.5: Illustration of the change in landmark location for a triangle of landmarks

located on a part of the face that is changed during surgery, the deformation of the triangle between the two cases may be obtained provided at least one landmark is kept invariant. This method provides a valuable but incomplete analysis of facial changes for small portions of the surface.

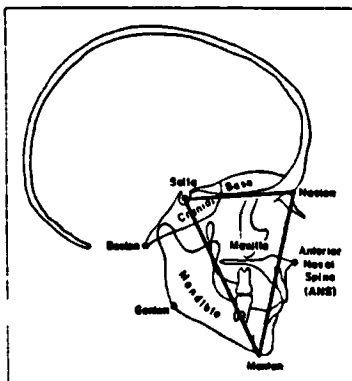


Figure 2.6: The sella-nasion-menton triangle

The choice of landmarks may be such that none are involved in the surgery, therefore allowing an accurate measure of the rotation and translation of the triangle to be obtained from case 1 to case 2. If one or two landmarks are

Bookstein (1984a) also used this method to analyse the changes that occurred during growth in the nasion-basion-menton triangle and sella-nasion-menton triangle. He found that at a 0.95 confidence level, between 66% and 88% of the changes were due to change in shape (his null hypothesis was no shape change had occurred). In a second paper, he applied this method to cases of Apert's syndrome and Crouzon's syndrome, which are deformities involving the cranium (Bookstein, 1984b).

In the study of craniofacial growth, the morphological differences between the head at different eras during a lifetime is important. These changes have traditionally been investigated using cephalometry. The problems of traditional X-ray cephalometry have been discussed at length by Moyers and Bookstein (1979) and Richtsmeier and Cheverud (1986). Geometrically, it is based on choosing a specific point as the centre of a registered system (often the sella) and an orientation line (often the sella-nasion line), to specify the direction of growth. The amount of growth which occurs at this "fixed point" and choice of different orientation lines have yielded incongruous results between studies (Bookstein, 1983). The biorthogonal method, as applied in cephalometrics, is intrinsically bound up with landmarks and fails to explain the curved form between them and its changes.

Both Moyers and Bookstein (1979) and Richtsmeier and Cheverud (1986) highlighted the need for a registration free method (in two and three dimensions) for investigating human growth and the necessity for presenting data about form at different levels of description. In traditional cephalometric studies, the changes in the angles formed between landmarks triplets do not reveal the cause of these changes. This is because the same magnitude of change maybe due to the movement of different landmarks (eg. figure 2.7).

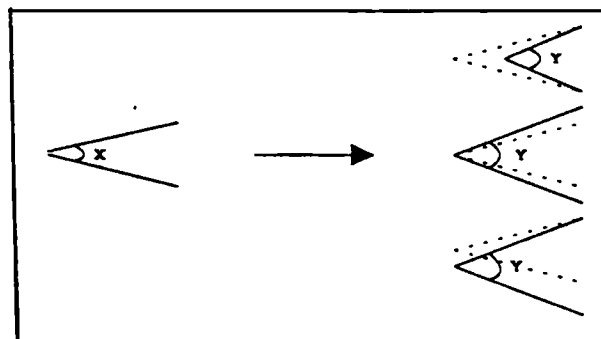


Figure 2.7: Three ways to produce the same magnitude of angle change, by movement of different landmarks.

### **c) Finite element analysis**

An analogous method to biorthogonal grids was the finite element scaling analysis (FEM) method of Richtsmeier and Cheverud (1986) which was developed in the field of continuum mechanics. Biological grids are essentially the simplest form of finite element analysis (Richtsmeier and Cheverud, 1986).

In finite element analysis, volumetric elements are constructed from landmarks and located in the changed object. A mapping function is calculated from the changes in landmark position, to enable comparison over the entire object. The smaller the elements used, the more accurate the description. The mapping can be used to obtain

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(size and form) strain tensors and deformation gradients, local to a landmark, and hence allow the difference in form to be quantified. Application of this method in two dimensions revealed that change in shape occurred at *all* landmarks across the face, with the least amount of change occurring over the neurocranium and the sella-nasion line. However this line, which has often been used as an invariant reference line for measuring the direction of growth, was shown to bend during growth! Finite element methods do not allow any more of the object surface to be described than any other analysis of landmark configuration.

### **2.2.3 Landmarks**

At the heart of all these deformation methods is the role that landmark points play on a surface and how they may be used to describe the shape of the surface and changes in shape. But what are landmarks?

A landmark, in the three dimensional sense, may be thought of as a specific point which is of necessity well-defined on the object and easily located. It is an anchor point that can be used to measure distances on the object or between objects. Most importantly, landmarks do not define the form but rather they serve as pointers to hold our conceptual place upon it (Moyers and Bookstein, 1979; Bookstein, 1978 p.17).

The position of an extremal landmark cannot be located until the orientation to be used for the analysis is fixed because they change with respect to orientation. That is, the orientation can be defined in terms of the location of these landmarks (Bookstein, 1978 p.13). It is therefore inappropriate to choose as landmarks points which have extreme values relative to the coordinate system. A sound landmark has an intrinsic definition in terms of anatomy or boundary curvature in its vicinity (Bookstein, 1984a). In Euclidean space, the most informative landmarks have been said to be vertices or points of very high Gaussian curvature (Attneave & Arnoult, 1956).

The importance of choosing good landmarks was acknowledged by Bookstein as essential for his analyses. Further, he has suggested three types of landmark, each with a different power for describing shape (Rohlf and Bookstein, 1990, p.216-222). In order of their power for the description of shape, these are: i) the confluence of three bony structures which is a recognisable point between regions of distinct histology ii) extrema of curvature and iii) extremal points such as endpoints lying at the end of a "medial axis" (Blum, 1973) or the most extreme point in some directions for the form. The third category of landmarks are explicitly not locally determined points whereas the other two kinds are.



According to Bookstein, landmarks are essential for the division a form into segments, although the segmentation must be meaningful in terms of the biological process acting on the form. They are the points at which one's explanations of biological processes are grounded (Rohlf and Bookstein, 1990), delimiting our explanations of effects upon form. The statistics of the landmark locations are also the statistics of all models of deformation driven by the landmarks (Bookstein 1986).

It is obvious that the use of landmarks for the analysis of 3D objects has severe limitations. Landmarks are of importance, but they not sufficient for shape analysis (Bookstein, 1978a p.14). For example, the three points a, b and c in figure 2.8 maybe joined in any of three ways.

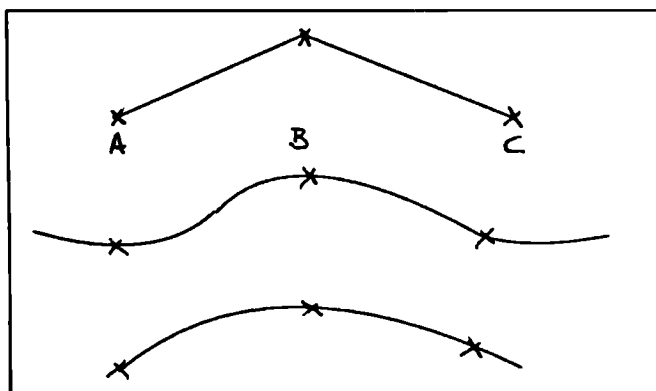


Figure 2.8: Illustration showing the different ways in which 3 landmarks maybe joined by a connecting surface.

In essence, the analysis of landmarks reduces to the study of triangles (Bookstein and Cutting, 1988). Today's technology allows the collection of a large amount of body data (eg. C.T. and optical surface scanners) and the use of such a small quantity of the

available information for the description of differences between data sets means that subtle anomalies in the data are not quantified and valuable information may be missed. It has been pointed out on many occasions over the last decade that the study of three-dimensional form must find a way of describing the curving form *in between* landmarks, independent of any scheme of conventional anatomical landmarks (eg. Bookstein and Cutting, 1988).

### **2.3 Contour descriptions**

Early attempts to describe the shape 2D outline contours found in biological shapes, were based on the number of lobes in the contour (Bowie, 1973) or parameterized the contour as a function of the perimeter and area (" $P^2/A$ ") (Bacus and Gose, 1972). However, these proved not to be robust, frequently leading to shapes of similar appearance being represented by wildly different " $P^2/A$ " values when evaluated using a discrete grid (Rosenfeld, 1973).

In 1974, Young and colleagues introduced another concept which defined the transformation between biological shapes in terms of *bending energy* (Young et al, 1974). Bending energy is defined as the amount of energy (or work) done in forming a

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biological shape from a linear, thin-shelled medium. Intuitively the shape in which a free elastic medium would minimize its stored energy in two dimensions is a circle and in three dimensions is a sphere. It follows that all other shapes, that can be formed from a continuous bounding contour, require work to be done to form them because the preferred shape will be the one which requires the least amount of energy. This work is termed the "bending energy" of the shape. Equivalent classes of shape require the same amount of bending energy to form. The bending energy can be calculated using a weighted sum of Fourier series coefficients.

Bowie and Young (1977a) showed that the bending energy of a contour is a better discriminator of complexity of the shape than the  $\text{perimeter}^2/\text{area}$  technique and that it relates well to human perception and assessment of complexity of shape. They also showed that the normalized mean absolute curvature of a shape was also a good discriminator of the complexity of a contour (Bowie and Young, 1977a). This was calculated by applying a convolution mask of different sizes to the contour. In a second paper, the segmentation of contours by applying a threshold to curvature at points of constriction around the contour was explored (Bowie and Young, 1977b). This also worked well.

#### **2.4 Thin-plate splines, principal and relative warps**

In 1989, a method *principal warps*, which are based the concept of bending energy, were applied for the description of more complex shape deformations (Bookstein, 1989). In this method, a shape deformation is modelled in terms of splines, which each have a certain amount of bending energy. The deformation is then modelled by an affine transformation and a number of principal warps. These warps are geometrically independent, affine-free deformations of progressively smaller geometrical scales. Bending energies of spline functions, describing the object at different scales, are inputted into a matrix and the principal warps are calculated from the eigenvectors of this matrix. *Relative warps*, eigenvectors of the variance-covariance matrix of landmark coordinates with respect to bending energy, are analogous to ordinary principal components (Rohlf and Bookstein, 1990, p.238).

The representation produced by this method is quite complex, although it allows the shape change to be decomposed into components. The addition of more landmarks along the curve being described, produces a better description but sometimes the construction of pseudo-landmarks is necessary to obtain a good fit! Bookstein maintains that this method can be extended to 3D relatively easily, but the mathematics gets tricky and this claim has not been demonstrated.

**2.5 Process history of shape change**

Yuille and Leyton's (1987) symmetry-curvature duality theory formed the basis not only for Leyton PISA description of shape but also for his *process history* concept of shape change (Leyton, 1988). In this concept, the symmetric (medial) axes of a contour are interpreted as the principal directions along which the imagined "shape forming processes" are most likely to act or have acted.

Leyton found that four types of curvature extrema occur on a contour. These are

$M^+$	local maximum, positive curvature	protrusion
$m^+$	local minima, positive curvature	indentation
$m^-$	local minima, negative curvature	squashing
$M^-$	local maxima, negative curvature	internal resistance

which can be interpreted as a protrusion, indentation, squashing or internal resistance respectively. Leyton viewed the shape of objects as a result of a historical process. Changes in shape from one contour to another were seen as a further example of this process. Thus, Leyton evaluated the shape of a bounding contour as its distortion from a circle (or sphere in 3D) - the shape which requires the minimum amount of energy to form. This distortion was described by a "process grammar" which consisted of six rules. These were derived by consideration of what happens to the four curvature extremum under firstly, continuation of the process which formed them and secondly, a bifurcation process (ie. the extremum splits into two copies of itself with an extremum of the opposite type formed in between the two). These rules represent the processes of: i) squashing until indentation, ii) internal resistance until protrusion, iii) a nodule becoming a lobe, iv) an inlet becoming a bay, v) a protrusion is introduced and vi) an indentation is introduced. Using these rules together with the asymmetry rule, a path between two bounding contours can be identified. The asymmetry rule says that the least deformed shape is one with no negative curvature. This process history concept of shape change allows a perceptually meaningful description of the change in shape to be made.

Leyton's process history represents a more developmental and growth based extension of Richards and Hoffman's (1985) "codons" description of 2D curves (described in chapter 1). Recall that "codons" are extrema triplets with minima endpoints. They have "duals" which are formed by exchanging extrema labels  $M^+ \leftrightarrow m^+$  and  $M^- \leftrightarrow m^-$ . This is a locally applied technique which lacks multi-resolution ability but although Leyton described his theory in two dimensions, the method nevertheless has the

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potential to be extended to 3D either by direct application to 3D lines or by development of 3D versions of the rules used.

### **2.6 Statistical analysis of shape change**

A second approach to morphometrics has been made by statisticians. If shape *can* be mathematically described then it will have a statistical theory associated with it. This introduces a whole new field which enhances the mathematical basis of shape description still further. A brief summary of recent ideas concerned with the statistics of shape description is given here.

The "multivariate approach" to morphometrics, that is the idea of using size and shape variables to describe biological forms, has been in the minds of biologists for a long time. Galileo was the first to draw attention to the effects of increasing the size of an animal without a corresponding change in shape, and the resulting inability to maintain the creature's same biological functions. This observation has been one of the foundation stones of evolution and zoology and is continually referred to by today's biologists (eg. Sir David Attenborough) as the reason why animals and plants of a specific species are limited in their size. The inter-relationship between size and shape was formally stated in Mosimann's theorem (Mosimann, 1970). It was, therefore, natural for investigators to try to analysis biological objects by their size and shape (eg. Sprent, 1972).

In 1977, Kendall introduced a theory of shape which defined shape in terms of a set of  $k$  points in  $m$  dimensions. In his 1989 review of the subject, he explains how "shape spaces" may be identified and how corresponding probabilities of real objects occupying them are assigned (Kendall, 1989). Points can then be mapped from real space,  $R^m$  to the shape space (an analogous idea to Extended Gaussian images). The shape spaces can be organised into 2D arrays, which are used to discover the discriminating global characteristics of shape space. The statistics of obtaining a set of points with various empirical characteristics can be determined. This approach enabled Kendall to deal with problems in archeology, astronomy, geography and physical chemistry such as the likelihood of a clusters of stones, such as Stonehenge in England, being laid down according to a plan, or the analysis of the position's in which quasars which appear to lie, along arcs of great circles on the celestial sphere, by mapping spherical triangles onto the shape space.

The application of Kendall's technique to morphometrics is explained by Bookstein (Comments on Kendall, 1989, p.99-105). The important questions here are not the colinearity of points, but rather differences of mean shape, factors underlying shape variation. One is specifically interested in small differences in shape. Bookstein

proposed a metric for shape change equivalent to the bending energy of a configuration of points, on an object (Bookstein 1989).

Kendall's technique is, essentially, the statistical analysis of the relative positions a set of points and therefore applicable to facial landmarks. Corrections are made for the effects of translation, rotation and scale and the resulting form is called the shape of the structure. Small (Comments on Kendall, 1989, p.107) points out that changes in the shape of the object can occur from two ways, either through the perturbation of individual points or through a global transform of the  $R^m$  space in which they lie. The lack of obvious landmarks on some structures hinders this. In the case of biological shapes, landmark identification very often proceeds on the basis of expert opinion in pin-pointing homologous points (Mardia's comments on Kendall, 1989, p.109). Mardia used an algorithm to calculate the local absolute curvature maxima to obtain landmarks on a contour. Additional information was provided by pseudolandmarks either side of a landmark.

Kendall explains the idea of "size-and-shape" of a set of points in Riemannian space and shape theoretic considerations of random Delaunay tessellations. Evaluation of the tessellations by the angles at the vertices is proposed by Stoyan (Kendall, 1989, p.115). Moss et al (1991) performed a similar sort of tessellation-based analysis using triangles constructed from facial landmark points to assess the change in area of triangles before and after facial surgery.

## **2.7 Summary**

Biological shapes are not static objects but organisms that grow and evolve. The methodology described here for describing their shape takes account of this fact. It might be thought that these methods would be the most useful for describing the face, which is after all part of a living organism. However, the deformation of form methods rely heavily on the accurate and consistent positioning of landmark points. This means that the analysis of shape changes produced by these methods is limited by the accuracy to which landmarks can be identified and restricted to small portions of the surface, failing to adequately describe the surface shape *in between* landmark points.

Other attempts at describing biological shapes have involved the calculation of a quantity called bending energy. This has been used to describe the shape of the outline of a contour (section 2.4) or to describe spatial warps containing the form (section 2.5). The use of bending energy to describe a contour, appears to give a useful measure of 2D shape and is used in chapter 7 of this thesis. The description of a shape in terms of spatial warps produces a complex representation of the shape change, which is not easy

### ***Biological Shape***

to interpret and difficult to extend to 3D. Thus does not appear to be a suitable method for quantifying the shape of the face.

Although not used in this work, Leyton's process history concept of shape change appears very promising and visually easy to interpret. It allows small scale shape changes to be described and may be extendible to 3D.

The mathematical basis of shape description has been enhanced by the introduction of statistical theory. At present, statistics has only been concerned with the analysis of a set of points (landmarks).

## CHAPTER 3

### 3D SURFACE DATA ACQUISITION SYSTEMS FOR THE FACE

The shape of the human body has been a topic of interest for centuries. Unlike regular solids, the complexity of its shape is hard to quantify exactly. In order to attempt a *mathematical* description of the body or the face, the three dimensional form must be first measured accurately.

The importance of acquiring 3D data for the body was realised by the ancient Egyptians, Greeks and Romans. Greek Sculptors would place boxes around people and measure the depth of the body at many points using rods. Stone would then be chiselled away to these depths at the corresponding points to form realistic figures. This was, of course, highly labour intensive and although beautifully proportioned figures were produced, they probably took many years to fashion. Even so this calliper approach was still in use up until about 15 years ago for making measurements of the body in radiotherapy because no other practical method existed.

The Renaissance era emphasised the idea that a work of art should faithfully represent nature. Artists developed the ideas of perspective and proportion in their pursuit of realistic representations. The principles of geometric representation were explained to sculptors by Alberti in his book *Della Statu* (c.1440). There he described an instrument called a "definer" which consisted of a disc fastened above a statue with a plumb-line suspended from it. The disc was rotated until the plumb-line made contact with a point of interest on the statue. This enabled the point to be defined in three dimensional space using a cylindrical coordinate system. In 1582, Albrecht Dürer explained the use of drawing frames and other devices for recording the size and shape of the human body in his four books on human proportion (Dürer, 1582).

In this chapter, methods for acquiring 3D measurements of the human form, and in particular the face, are reviewed. From these, one method was selected for collecting 3D data sets of the face for use in this work. The optical scanning system available at University College London was a natural choice of system and the data collected was of adequate resolution for my purpose. Optical scanners have certain advantages over other non-contact systems which are currently employed for the collection of 3D surface data, although some of these systems are also suitable. Other non-contact systems which have been reported include stereophotogrammetry, rasterstereogrammetry and moire fringes. A brief review of the capabilities of these systems is given by Gallup et al (1990). The principal advantages and disadvantages of each of these systems are briefly considered below.

### **3D Data Acquisition**

#### **3.1 Stereophotogrammetry**

The principle of stereophotogrammetry is analogous to that of depth perception in human vision, that is, it is based on the principle of looking at an object simultaneously from two slightly different viewpoints.

The mathematics of stereoscopy was first published in 1832 by Sir Charles Wheatstone and the first stereoscopic cameras were invented in the mid-1850's. One of the first practical uses for this new technique was the compilation of a photographic map of Grenoble by Aime Laussedat in the 1850's and it is in aerial surveys and latterly remote sensing where this method now has its primary application. Its potential for biostereometric measurement has also been explored. An excellent review of the history of body measurement using stereophotogrammetry is given in Herron (1972).

The potential of this method for recording the facial surface was first realised by Mansbach (1922) but it was Zeller (1939) who first demonstrated use of stereophotogrammetry to record the facial surface and he encouraged Thalmaan-Degan (1944) to apply the method clinically. She reported changes in facial morphology due to orthodontic treatment and illustrated and quantified them by means of contour plots. This method has since been used to study the face in many countries and by many authors. The most active of these were in Sweden, where researchers were particularly active in the 1940's, 50's and 60's, (eg. Bjorn et al, 1954), in Japan (eg. Haga et al, 1964), in England (eg. Burke and Beard, 1967; Dixon and Newton, 1972) , and in the United States (eg. Berkawitz and Cuzzi, 1977).

In stereophotogrammetry, an object is photographed simultaneously by two or more cameras, from different positions, and stereo analysis is used to obtain the 3D surface (eg. Burke and Beard, 1967; Herron, 1972; Berkawitz and Cuzzi, 1977; Frobin and Hierholzer, 1978; Savara et al, 1985). Three dimensional coordinates are obtained by identifying the same points on both images using either a pattern projected onto the objects surface or by placing physical markers on the surface. The main advantage of these systems are that the collection time is fast (a fraction of a second) and they therefore have the potential to be used for dynamic studies. They are also accurate in their measurement of the surface to sub-millimetre level. The disadvantages are that complex optical systems are involved and a skilled operator and complex computations are required to yield accurate, and consistently good, results. Although data can be acquired rapidly, considerable time is needed to process the film and digitize the image. This makes it unsuitable for clinical use. Recently attempts have been made to automate the analysis of stereophotographs and video frames and identify corresponding points on each photograph (Banda and Muller, 1991).



The accuracy to which this method records the facial surface was claimed by Savara to be 0.2mm (Savara, 1965a). However, further investigations suggested that the standard deviation error was 0.69mm (Burke, 1971; 1972) with errors of up to 0.8mm (Burke and Beard, 1967). The long processing time involved and its lack of automation together with recent advances in other data acquisition techniques have led to a reduction in the use of stereophotogrammetry. The expense and complexity of analysing images produced by stereophotogrammetric systems has led researchers to look for cheaper and simpler methods of making surface measurements amenable to direct computer acquisition and analysis.

### 3.2 Moiré topography

In 1970, Takasaki (1970) described a process called moiré Topography. In this technique, a light source is projected through a grid, casting a shadow onto the surface of the object to be measured. This shadow is then viewed through another grid and a set of fringe patterns are seen. These are called moiré fringes and they correspond to contours on the surface under investigation. Topographic information can be extracted from these fringe patterns but in practice this is a tricky and time consuming business. A permanent record of the moiré fringe pattern can be provided by photography.



Figure 3.1: Moiré fringes of a face.

Since then moiré fringes have been used by many researchers to try to measure parts of the human body, especially the back (eg. Takasaki, 1974; Groves et al, 1990) and are now being used to screen school children for signs of scoliosis (Burwell et al, 1990; Neugebauer and Windischbauer, 1990). Moiré fringes have also been used by a number of authors to measure the static face (Takasaki, 1970; 1974; Xenofos and Jones, 1979). Figure 3.1 shows an example of the moiré fringe pattern produced on a face. The fringe patterns have

been digitized by Tsuchiya et al (1985) and used to measure the movement of the facial surface forward, due to surgery, and regression.

Considerable debate has taken place in the last decade or so about the potential of this method with many techniques being devised to evaluate the fringe patterns. The fringe patterns are strongly dependent on the position and orientation of the patient (Hierholzer and Frobin, 1982; Turner-Smith, 1988) and considerable skill is needed to interpret them (Turner-Smith, 1988). The surface can only be measured from one viewpoint using this method. For 360 degree measurement, separate photographs taken from different viewpoints would be necessary as well a means of connecting them together, possibly by using surface markers. The characteristics of the fringe patterns

### ***3D Data Acquisition***

that make them difficult to analyze and quantify have been pointed out by Halioua and Liu (1986) and Elad and Einav (1990) and include ambiguity in the fringe order and sign, noise patterns which are generated by the grating, non-uniformity in sampling, the specific system geometry and the need for a large digital image processing capability to achieve some degree of automation. Recently, some advances have been made towards automatic analysis (Reid et al, 1986; Reid and Rixon, 1986; Boehnlein and Harding, 1986; Kawai et al, 1990).

On body surfaces, especially the facial surface, the complex patterns which are formed by moiré fringes are not well suited to automatic conversion into 3D coordinates (Arridge et al, 1985). The poor reflectivity of the facial surface produces blurring in the shadow of the projected grid, although this problem maybe surmountable by coating the face with a reflective powder. A more difficult problem is the need for precise positioning of the head since small changes in head position produce large changes in the fringe patterns (Kanazawa and Kamiishi, 1978).

### **3.3 Fourier transform method**

The Fourier Transform method was proposed by Takeda and Mutoh (1983) in order to overcome some of the difficulties associated with moiré fringe patterns. The Fourier transform was used to analyse the bands of light produced when a grating pattern was projected onto an object. This process avoids the need for determining the order of fringes, locating the centre of the fringe or interpolating between fringes since it provides the distribution of the object's height across the entire image. It is also sensitive to variation in height within fringes and can automatically distinguish between depressions and elevations in the shape of the object. However, despite these considerable advantages over traditional moiré fringe methods, a large computation capability is still required as well as very high resolution devices and problems remain when the slope of the surface is steep or contains step discontinuities (Halioua and Liu, 1986).

### **3.4 Projected-grid photography**

In the last two decades a number of enhancements to standard photogrammetry have been explored by researchers. Joel (1974) projected a plane of light onto the surface and photographed it obliquely. This method is sometimes known as "Lichtschnittsverfahren". He noted that distortions due to perspective arose and showed how they could be corrected for. Another method, termed "projected-grid photogrammetry" was devised by Lovesey (1973; 1974a; 1974b) specifically for recording the facial shape accurately. Lovesey required accurate facial data in order to improve the design of aircrew oxygen masks. A pattern of equally spaced parallel lines were projected on to one side of the face and the resulting distortion of these lines, lines of points of equal depth, producing a contour map of the surface were photographed

from the front of the face. The grid projected was composed of different coloured lines to make it easier to trace the path of an individual contour on the face. The major drawbacks of this arrangement were that the set-up allowed only one half of the face to be recorded at a time and that shadowing was not eliminated, thus parts of the face around the eye socket were not recorded. The system was reported to measure the surface to an accuracy of less than 0.7 mm and was cheap and relatively simple.

#### **3.5 Rasterstereogrammetry**

In 1980, the term "Rasterstereogrammetry" was introduced by Hierholzer and Frobin to describe their system which was designed specifically for the measurement of body surfaces (Hierholzer and Frobin, 1980). Rasterstereogrammetry is a hybrid of traditional stereophotogrammetry and moiré topography and its development reflects the concern of these authors about the difficulties of analysing moiré fringe patterns mathematically. In rasterstereogrammetry, one of the cameras used in traditional stereophotogrammetry set up is replaced by a projector which casts a structured light pattern (typically a grid) obliquely onto the object. A single camera placed at a different viewpoint, records the distortion of the grid on the surface yielding 3D information (see Frobin and Hierholzer, 1982b for the system and calibration details). Similar methods were described by Coray et al (1990a) and Lerch and Barish (1978).

Rasterstereogrammetry has been successfully employed to measure human back surfaces (Hierholzer and Frobin, 1980; Frobin and Hierholzer, 1981; 1982a; Hierholzer and Drerup, 1990; Elad and Einav, 1990) to a reported resolution in depth of 0.4mm (Frobin and Hierholzer, 1990). It has also been used to measure the facial surface to a precision of better than 2mm Coray et al (1990b).

This method had the drawback that a long time is spent in processing the information collected but recently the speed of data analysis has been improved by using a video camera to collect the data in place of the conventional camera, thereby automatically digitizing the data as it is collected ready for subsequent computer analysis (Frobin and Hierholzer, 1990). This advance has reduced the data collection and processing time to about seven minutes in total.

A similar system was produced by the Altschuler brothers and their colleagues in 1979 (Altschuler et al, 1979). In their system, dot patterns were projected by laser onto a surface which was then viewed from one or more suitable aspects. One application of this system has been to measure teeth. A resolution of 20 microns has now been achieved for a 1 cm<sup>3</sup> tooth (Altschuler, 1990). The field of view can be altered from 1 cm to several metres by changing lenses, making it suitable for a wide range of applications. This system has been used by NASA to design gloves for astronauts. The advantages of this particular system, apart from its accuracy, are that the data

### ***3D Data Acquisition***

acquisition time is fast (a fraction of a second) and the signal detected is clearly defined (as the peaks of laser light reflectance). The smaller versions are also portable!

#### **3.6 Sonic digitizers**

Sound and ultrasound measurements have been long used to measure the internal body structure but the almost total reflection of an incident ultrasound wave at the body surface, due to the very large change in acoustic impedance at the air/tissue interface, led researchers to investigate the possibility of using ultrasound to record the surface of the human body.

The feasibility of this technique was reported in 1974 by Short and colleagues (Short et al, 1974) and Lindstrom and his colleagues found that a resolution of between 1.0 - 0.1 mm in the axial direction was achievable depending upon the application (Lindstrom et al, 1982). A number of people have built these sort of systems one of the most recent to be reported was developed by Science Accessories Corporation, Connecticut (Gallup et al, 1990). In this particular system, a sonic emitter is located at a point on the object's surface and a sound pulse emitted which is detected by four microphones placed at different locations in the vicinity. The distance of the emitter from each sensor can be calculated from time elapsed between the transmission and detection. Thus "time-of-flight" measurements are made allowing the calculation of distance to the object's surface.

The precision in measurement of this method has been reported to be poor, 3mm error on a point (Gallup et al, 1990). The major errors in the measurements result from turbulence in the air traversed. The imaging procedure takes several minutes (6 minutes and 10 minutes have been quoted in the literature) which is nowadays regarded as being too long for clinical use. Its main advantages appears to be its portability and that it can be used to measure objects as large as the entire body surface on one go. It appears to be a very promising technique but clearly further development is needed to enable it to be used for clinical applications.

#### **3.7 Optical scanners**

Over the last decade or so a number of optical scanning devices have been developed by several groups of researchers in order to measure the surface of the human body (Arridge et al, 1985; Brunet, 1990; Cutting et al, 1986a; 1986b; Duffy and Yau, 1988; Halioua and Liu, 1986; 1989; Halioua et al 1990a; 1990b; 1990c; Livingstone and Rioux, 1986; Maldague et al, 1986; Moss et al, 1989; Turner-Smith, 1988; Turner-Smith et al 1988). A forerunner of these was the laser spot scanner, developed by Ishida and colleagues, used to measure cross-sections of the trunk for the detection of scoliosis (Ishida et al, 1982). Quicker and more efficient methods soon followed.

In optical surface scanners a single line of light is generally projected onto the object's surface and its distortion recorded by a camera viewing it obliquely. The surface geometry along the line is computed from its distortion. A combination of camera, line source and subject movement are used in order to cover the desired surface. This is termed "scanning".

Optical scanners have been made with between two and five degrees of freedom. Those with three or more degrees of freedom (Koch, 1990; Laser Design Inc, of Minneapolis' system) allow data to be collected from very complex and occluded objects. However, these are too slow at data collection for clinical use (Sadler et al, 1990). For most human applications those with two degrees of freedom have proved to be adequate, one exception might be the foot.

These systems have the great advantage that they are driven by computers and so the data collected is immediately available for analysis. For example, the data may be electronically archived and transmitted with measurements or 3D images produced for validation and these may all be done extremely rapidly. In clinical applications, where rapid collection and processing times are demanded, this is a considerable advantage over the other data collection systems described above. Additionally, operators require little training. The main disadvantage is that the collection time is not instantaneous, typically of the order of 5 to 30 seconds. However, this has not proved to be a handicap for the majority of clinical applications. The only exception, in our experience, are children under 4 years of age who have difficulty in keeping still.

There are several commercially available systems based on this principle. These are most notably, the ISIS system produced by Oxford Metrics which measures the back surface (Turner-Smith, 1988). CYBERWARE Laboratories Inc. of Monterey, California, Dimensional Measurement Systems Inc. of New York (Halioua and Liu, 1986; Halioua et al, 1990a; 1990b) and Vision 3D system of France (Brunet, 1990) all manufacture a range of laser scanners for collection of 3D data from various parts of the body. Loughborough University of Technology have produced a non-commercial system for scanning the whole body (called "LASS") which is being used by clothing manufactures and the Armed Services (Jones et al, 1989). 3D Scanners Ltd now market the system for scanning the face and head designed at U.C.L. (Moss et al, 1989).

### **3.8 Holography and phase-measuring techniques**

The use of a high-powered laser to construct a hologram of the face was described by Ansley (1970). The accuracy of this technique was assessed by Cobb (1971) using a dummy head. He found that an accuracy of 2mm could be achieved. The technique requires a burst of energy of 1-3 Joules for a duration of 30 nanoseconds. Holographic techniques have not been used for facial measurement partly because of concern at the

### **3D Data Acquisition**

dangers of tissue damage from these high energies but also because the accuracy is not considered good enough.

In the late 1980's, some very ingenious and versatile systems were developed by Halioua and Liu for industrial and body measurement applications (Halioua and Liu, 1986; 1989; Halioua et al 1990a; 1990b; 1990c). Three separate measurement techniques using the same computer hardware but with different optical arrangements were devised. In "Phase-measuring profilometry" a sinusoidal grating pattern is projected onto an object, seated on a turn-table. The distorted pattern is viewed obliquely by a CCD camera which collects the data (Halioua and Liu, 1989). This represents a cross between optical scanning and rasterstereogrammetry. A second technique "Phase-measuring polarization interferometry" was designed to measure 3D mirror surfaces. Here a laser beam is projected through a crystal onto the surface. The incident and reflected waves are caused to interfere with each other and CCD camera captures the resulting interference patterns. The third technique is "Phase-measuring holographic interferometry". Here two states of a surface, one at rest and one under stress, are caused to interfere with each other using a double-exposure technique.

All these techniques have a fast data acquisition time (under 1 second) and processing time (30 seconds). They have very good precision, achieving a RMS error of 0.65mm on a back surface. They can scan over 360 degrees and are low cost. Specific systems have been devised for applications as small as teeth (measured to 5  $\mu\text{m}$ ) and as large as whole bodies. Profiles and other measurements can be readily extracted and the data visualized either as a contour map or as a (shaded) wire mesh model.

Halioua and Liu have applied these techniques to a wide range of industrial and body measurement applications (eg. to the back, the breasts and the face) producing very good quality images.

#### **3.2 The UCL optical surface scanner**

In this work, the facial surface data used was obtained from the optical scanning system at University College London (UCL). This system has been fully described elsewhere (Arridge et al, 1985; Moss et al, 1987, 1989) so only a short resume of the scanner and its precision is given here.

The UCL optical surface scanner was originally designed for the purpose of following changes in the facial surface brought about by facial surgery and to provide a database for the planning and simulation of surgery (Arridge et al, 1985).

A laser beam is fanned out into a vertical line using a cylindrical lens. This line is then projected onto the subject's face, or surface to be scanned, and two mirrors reflect the

light along disparate paths into separate halves of the field of view of a video camera (figures 3.2 and 3.3). This is equivalent to two cameras viewing the laser line from either side. The reason for this unusual optical arrangement is the need to avoid occlusion of parts of the face adjacent to the prominence of the nose. The video signal is digitized and superfluous signals suppressed using a digital comparator to leave only the signal due to the images of the line. To record data over the entire face, the subject is rotated under computer control and the distortion of the laser line as it illuminates the face is used to compute the facial geometry. To avoid loss of data, caused by the high angle of incidence under the chin, the head is tilted back slightly.

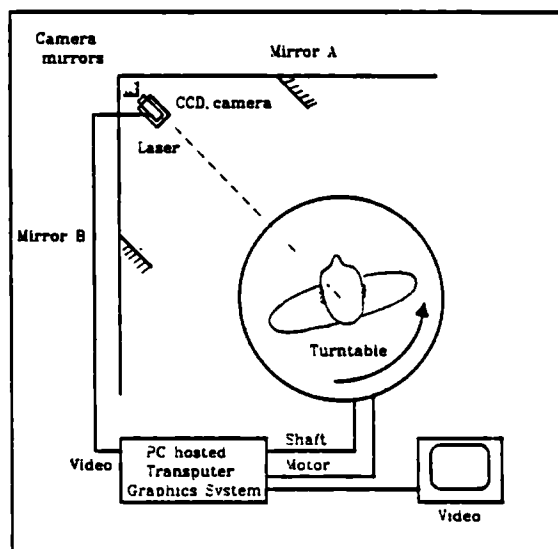


Figure 3.2: Schematic diagram of the optical surface scanner. (from Moss et al, 1990)

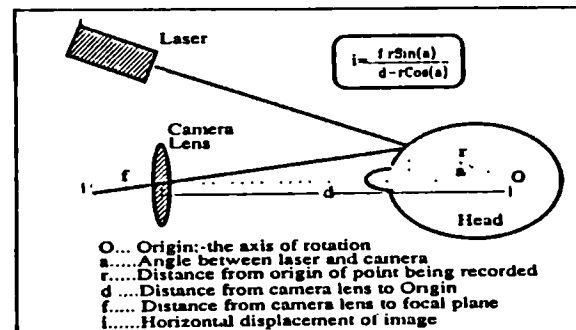


Figure 3.3: Geometry of the viewing system. Only one laser beam is shown (from Moss et al, 1990).

The vertical extent of the profiles is limited by the slope of the forehead since the intensity of light reaching the camera from this region falls rapidly due to the angle of incidence. The scanning interval is programmable. In this work, the laser line is recorded every 2.8 degrees of rotation, except over the central portion of the face, between the two inner canthii, where it is recorded at every 1.4 degrees. The smaller interval across the central portion of the face is used because of the greater facial detail in this region. Individual profiles are recorded with a radial spatial resolution that is better than 0.5mm. The vertical resolution, within a profile, is better than 1mm. Assuming a radial distance of 100mm, the sampling interval used gives resolutions around the head of 2.4mm over the central regions and 4.8mm elsewhere.

Hair does not reflect laser light very well and can produce artifacts. These can be avoided by covering the hair with a stocking. This was done for the "normal" subjects used in this work. These subjects were scanned over 360 degrees, enabled the shape of the whole of the head to be recorded. This also had the effect of removing the hairstyle of the person, which was important for our collaborative work, in investigating facial recognition, carried out with psychologists at Nottingham University. The facial surgery

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patients were scanned from just behind one ear to just behind the other (a 220 degree scan) with the hair held back from the face by clips to avoid spurious data points from hair overhanging the face.

The acquired data are stored in computer memory. Approximately 20 000 3D coordinates are derived for the facial surface and approximately 40 000 for a whole head. These are filed to disk for storage.

The stability of the optical surface scanner has been investigated by Hammond (1987), who measured a calibration line positioned at 70mm from the axis of rotation at different times of the day, over several weeks. He found that the mean value of this line was 69.96mm with a standard deviation of 0.11mm. Hammond also scanned a Roman bust on different days and at different times during the day and determined the accuracy of a profile through 3 points marked on the surface. He found differences of less than 1mm, with a mean difference of less than 0.3mm. The calibration and validation of the system have been described by Moss et al (1990).

#### **3.10 Visualization of the data**

An important requirement for this work is a means of visualizing this enormous data set. A graphics systems based on Transputers has been designed for this purpose (Moss et al, 1987; 1989; Tan et al, 1988; Linney et al, 1989; 1991 and Linney, 1992b).

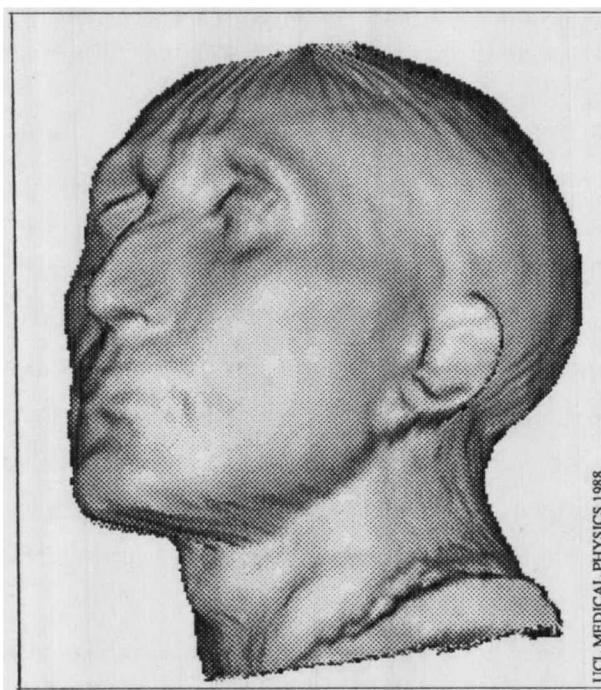


Figure 3.4: A rendered surface image of a bust of a Roman general.

The technique used for visualizing the data creates a patchwork of triangles (termed "facets") from the acquired data, taking advantage of its structured format. These facets



are then shaded using the Gouraud surface-rendering technique (Gouraud, 1971), simulating their illumination by a notional light source. This light source may be positioned at any angle. The 3D graphic image of the data is then displayed on a monitor (eg. figure 3.4). The data may be transformed to produce an image displayed from any chosen viewpoint and scaled to any size.

#### **3.11 Summary**

A number of different non-contact systems for acquiring 3D data on the facial surface have been described along with their principal advantages and disadvantages. From these one method, optical surface scanning, was selected for acquiring 3D data sets of a number of faces, in order to facilitate this work. A short description of the scanner used, its operation and precision was given in section 3.9 and the visualization of the acquired data was described in section 3.10.

The method developed in chapter 6 for describing the shape of the face, operates on the data sets acquired from optically scanning the face.

## CHAPTER 4

### FACIAL MEASUREMENT AND ANALYSIS: (LITERATURE REVIEW)

In the previous chapter, modern methods of acquiring 3D data for the face and other body surfaces were reviewed. The availability of this data allows measurements, in 3 dimensions, to be made with relative ease between points on the facial surface (Moss et al, 1989). However, there has been a long history of measurement of the face which extends as far back as the ancient Greeks and beyond - to the development of classical geometry.

The face has been measured for various reasons and with different aims in mind. Orthodontists, anthropologists and police researchers have all been motivated to produce a description of the face simply because they needed one, however poor, in order to accomplish what they were trying to do (ie. to plan and assess facial surgery, describe differences between peoples or to identify a criminal). These descriptions have been fairly *ad hoc* and pragmatic. Others have sought to produce more theoretical descriptions using a more disciplined mathematical approach, treating the face as a special case of a surface. A few, such as Bookstein, have had both of these motivations.

In this chapter, the various measurements and observations about the face and differences between faces that have been made by researchers from these different areas are described. This is to set the context for the work described in later chapters. Frequently the progress of research on facial description, measurement and analysis has been limited by the scarcity of suitable data. The rapid measurement optical scanning systems now available overcome this hindrance and the use of such a system has allowed us to proceed with some of these.

#### **4.1 Facial measurement and analyses of facial changes**

Facial anomalies can be very noticeable and their presence was observed as long ago as 2000 B.C. by the Chaldeans (Ballantyne, 1984). Many attempts have been made to describe these anomalies, the differences between two faces, and the change in a face observed at two different times. In this section, I will briefly describe the methods used by orthodontists and craniofacial biologists to study changes in the face that have been induced either by surgical correction, normal or abnormal facial growth.

These scientists have long been interested in three dimensional measurement, aware that facial anomalies occur in 3D space and change in shape with time, yet they have generally lacked the necessary equipment to make sufficient 3D measurements or the computational facilities to make adequate 3D analyses. One of the first to realise that the face should be measured in three dimensions was da Vinci. His diagram, shown in

figure 4.1 (Clark, 1968), suggests that the midline profile should be measured by projection.



Figure 4.1: da Vinci's diagram.

3D measurement in these fields was begun by the anatomists and anthropologists of the 19th century who measured distances between points on dried skulls. This allowed anatomical landmark points and reference planes, (such as the Frankfort plane) to be defined and used as a basis for comparisons. These points and planes were believed to be subject to the smallest amount of individual variation. Likewise, certain landmarks were defined on the soft tissue surface and distances between them measured using callipers. The definitions of these landmarks have served as a basis for nearly all subsequent analyses.

Röntgen's discovery of X-rays in 1895 enabled machines to be built to record bone and soft tissue profiles together of a living head. These techniques were first published in 1931 by two researchers working independently; Hofrath of Düsseldorf (Germany) and Broadbent of Cleveland (USA). Initially two X-rays were taken orthogonally and concurrently allowing three-dimensional information obtained using geometry, but difficulties and imprecision in this method (see section 4.1.1) limited its use and restricted analyses to two dimensional studies of the midline profile.

The two dimensional analyses that have been made using this technique have concentrated on performing isolated measurements between points that were easy to measure. Whilst this has allowed progress to be made in comparing two faces, one effect of this framework has been to divert attention away from the surface *in between*

### ***Face Measurement***

the landmarks points. I believe this surface to play an important role in describing the shape of the face. These measurements have, however, led to various standards for midline profile being laid down, such as the Ricketts (1981) and Downs (1948) indices, and has allowed some sort of definition of "normality". This is a practical requirement in orthodontics since it allows the establishment of degree of abnormality.

Later, analysis of the entire facial surface was attempted by producing contour plots from stereophotogrammetry (section 4.1.3). Whilst these have proved helpful there have been difficulties in using them to assess the shape of the face at more than just a few points and in particular in using them to predict how a face would change due to surgery. In 1986, a leading American orthodontist, Court Cutting, expressed the opinion that the major requirement for the advancement of computer-aided planning and evaluation of facial surgery was the development of a surface-based, facial shape analysis methodology. To my knowledge, no such full 3D analysis of facial shape and shape changes has ever been made.

#### **4.1.1 Cephalometric analysis**

The traditional method used by orthodontists for measuring changes in the face was to take lateral skull X-rays (cephalograms) of the patient to reveal the position of the facial bones and soft tissue outline (Broadbent, 1931). This naturally focussed attention, and analyses of the outcome of facial surgery or growth, on changes in the midline profile (eg. Subtelny, 1959; Brodie, 1949). Although this is of value, it does not give a complete picture of the change in shape nor change in movement of the soft tissues and, by itself, it is of little value for evaluating changes in a patient with conditions such as unilateral craniofacial microsomia because of the degree of asymmetry involved (Grayson et al, 1983a).

The characterization of shape by cephalometry was made entirely in terms of distances between landmarks and angles between pairs of lines drawn through landmarks. It was assumed that landmarks could be located accurately and repeatedly so it must have been a bit of a surprise when Miller and colleagues investigated error in marked points on the maxilla and found that although the error due to measuring 3D distances was 0.1mm  $\pm$  0.05mm, the error caused by variability in the operator landmark location was 5 times that amount (Miller et al, 1965)! In fact, the location of landmarks on the facial surface is highly subjective in nature and may not be consistent for independent observers. The accuracy to which soft tissue points can be measured in a mobile area like the lips has been estimated to be  $\pm$  1 to 1.5 mm. (Wisth and Boe, 1975; Hillesund et al, 1978).

In 1979, a strong attack on the lack of theory involved in the cephalometric method was delivered by Moyers and Bookstein (1979). They pointed out that because of the need to provide a standardized representation, cephalometry was based on the properties of

dry skulls not on craniofacial growth. Further, they said that these analyses provide no information about the shape of the surface (the curved form) between the measured landmark points, despite its high clinical value. Better methods, they suggested, would involve measuring tangents and curvatures, extracting medial axes (Blum, 1967) or using biorthogonal grids (Bookstein, 1978a; 1978b). These were described earlier in this thesis.

#### **4.1.2 The prediction of soft tissue changes**

The production of this measurement technique, led to attempts at *predicting* the outcome of facial surgery. The first attempts involved cutting life-size photographs along the plane where a surgical incision would be made and repositioning the pieces according to the amount of bone movement undertaken. In the 1970's, this practice was superseded when a foundation stone for morphanalysis was laid by Rabey (1971; 1977). Rabey superimposed two dimensional radiographs (cephalograms) enabling the study of the magnitude and direction of growth in the midline profile. This led to many studies of the midline profile, using lateral cephalograms (eg. Freihofer, 1977; Holdaway, 1983; 1984; Bishara et al, 1985). These allowed the movement of the soft tissue landmarks to be related to the movement of the underlying hard tissue landmarks. For instance, Hershey and Smith (1974) found that in prognathic mandibular surgery, a movement of 1mm at the Pogonion produced a movement of about 0.6mm at the lower lip, about 0.2mm over the upper lip and 0.9mm at the overlying soft tissue point. A summary of how profiles, and their relationship to various skeletal planes, have been used in the planning of facial surgery can be found in Powell and Rayson (1974).

The use of cephalometric studies to try and *predict* how the soft tissues of the midline profile would move with surgery has been relatively unsuccessful. The main reason for this is that the ratio of movement between the soft tissues and the underlying hard tissues is not well known. Some authors have found a 1:1 correlation (Suckiel and Kohn, 1978; Robinson et al, 1972) while others have found better correlations in some areas than others (eg. a 90% correlation over the chin and lower lip but less over the upper lip area, Hershey and Smith, 1974). Willmot (1981) found that the soft tissues tend to lag behind the hard tissues by a small extent. In yet another study, more variability in soft tissue movement was found in the vertical direction than the horizontal (Kajikawa, 1979). In cases of growth, the changes in soft tissues of the midline have *not* been found to be analogous to changes in the skeletal structure (Subtelny, 1959). The accuracy to which soft tissue changes at selected, corresponding, hard-tissue landmarks can be predicted has been found to be around 1mm (Denis and Speidel, 1987). This is also the accuracy to which commercially available cephalostats can reposition a head (Newton, 1974).

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A three dimensional understanding of the head was sought by some orthodontists, such as Broadbent (1931) and Sassouni (1958), who studied lateral and posteroanterior views. However, these two views were not incorporated into a coherent 3D analysis until Savara (1965a) reminded the community how three dimensional measurements could be extracted from a concurrently taken pair of cephalograms (see also Baumrind et al, 1983a; 1983b and Grayson, et al, 1983b). But even then, the two cephalograms were often not taken concurrently and problems existed in registering them and extracting the correct 3D coordinates for landmarks (Marsh and Vannier, 1983; Vannier et al, 1984; Moss et al 1987; 1988; Cutting et al, 1989) as well producing reliable 3D measurements (Baumrind et al, 1983a). The last decade has seen the advent of 3D methods for planning surgery, firstly from cephalograms (Bhatia and Sowray, 1984) and then using C.T. and M.R.I. scans to reconstruct the skeletal structure and simulating surgical procedures using computer graphics (eg. Cutting et al, 1986a; 1986b; 1987; Moss et al, 1988). This has focussed attention on the lack of methodology for predicting corresponding soft-tissue changes and describing the facial surface in general.

### **4.1.3 Analysis of facial contours and Moiré fringes**

Another technique that has been widely used for facial measurement is the use of contour plots or photographs (eg. figure 4.2). These are obtainable from stereophotogrammetry (Burke, 1972) or by projecting a calibrated radial grid onto the face and photographing the face together with the distorted grid pattern that is produced. This method was invented in 1953 by Sassouni. Amongst other studies, contour plots have been used to measure facial change with respect to time (ie. growth) (Burke, 1974; 1983; Burke et al, 1978) and the effect on the face of a congenital disease (eg. pulmonary stenosis commonly called "moon face", Ainsworth et al, 1978).

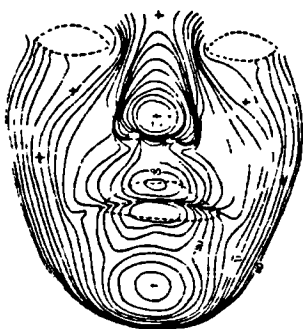


Figure 4.2: Contour plot of a face.

Only a relatively small number of measurements, between landmarks and of profiles, can be made by this method and the analysis of these measurements is labour intensive. They do not lend themselves easily to automatically extracting facial surface coordinates (Arridge et al, 1985). The accuracy of these measurements is better than 1 mm but their reproducibility on a second contour photograph of the same person has been found to vary by as much

as 2.0-2.3 mm (Leivesley, 1983). Leivesley assigned this difference to involuntary changes in facial expression. The reliability of photogrammetry of the face was reported by Farkes and colleagues (Farkas et al, 1980).

Moiré fringe patterns have been used in a similar manner to make similar measurements (Takasaki, 1970; Xenofos and Jones, 1979) but because of the difficulties discussed in chapter 3, small facial changes due perhaps to growth or retention of a surgical case are not recorded with sufficient accuracy.

Surgeons and biometricians generally feel that these forms of analysis are not sufficient for a realistic assessment of the shape of the face to be made. This belief combined with the fact that the elastic soft-tissues do not move in the same way as the supporting skeletal form makes prediction of the outcome of surgery to the face extremely difficult, especially away from landmark points. Currently prediction of change in shape of the face from these measurements is thought to be impossible.

As late as 1986, it was realised that in order to analyse a surface in three dimensions a large number of points on the facial surface would have to be measured (Segner, 1986). Segner calculated that using 1 point per mm<sup>2</sup> of surface over 10 000 points on the facial surface would need to be measured and asserted that this was "impractical in clinical use" (Segner, 1986). Thankfully, the introduction of optical scanning technology and high powered computers means this number of measurements can be made rapidly.

### **4.1.4 New profile analysis methods**

The need for a better analysis of the soft tissue changes has been articulated by many facial surgeons (eg. Cutting et al, 1986a). Attempts to use the movements of hard tissue (bony) surfaces, obtained from standard cephalograms, to predict movements of the soft tissues due to surgery have failed (Park and Burstone, 1986). This is because of the variation in soft-tissue thicknesses between individuals, the convexity/concavity of the individual face and the lack of documented data on these depths. The susceptibility of the soft tissues to change and relapse after the operation has increased the desire of surgeons to obtain a better understanding of the morphology and morphological changes of the face (Bhantia et al, 1985).

Recently, some new methods for analysing the shape of facial profiles have been proposed. These have been developed by mathematicians and computer vision scientists. Fourier analysis has been applied to profiles taken through the cranial base by Lestrel and Roche (Lestrel, 1978; Lestrel and Roche, 1986), to profiles of the mandible by Halazonetis and colleagues (Halazonetis et al, 1991) and to fronto-facial sagittal profiles of modern man (*Homo sapiens sapiens*) and *Australopithecus africanus* by Pesce-Delfino and colleagues (Pesce-Delfino et al, 1987). In this method, a curve-fitting method is used to approximate the shape of the profile and obtain an accurate quantification of the profile. The approximating curve is then described using the Fourier method. However, it is difficult to relate the values of the Fourier coefficients to

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the shape of parts of the profile because of the global effect of each Fourier waveform on the fitted curve.

A second method for describing facial profiles is the pattern recognition technique of Scale-space (Witkin, 1983; Mokhtarian and Mackworth, 1986; Mackworth and Mokhtarian, 1988) (see chapter 1). In this method, the curve is segmented at either the maxima or minima and the curvature of the segments is calculated. These segments can then be compared. The advantage of using the inflection points to segment the curve is that this leads to a repeatable, objective segmentation, free from observer error. Dudek (1991) and Campos, Moss and Linney (Campos et al, in press; Moss et al, in press) applied this method to facial profiles.

## **4.2 Distinguishing between faces**

So far in this chapter, I have discussed methods used by orthodontists to measure the face. Anthropologists and psychologists as well as orthodontists have made important observations on the differences between faces and these are reported in this section. The interest of anthropologists has been to identify and study racial, tribal or familial differences between faces. They have measured the face either using i) callipers on live or dry skulls or photographs, ii) x-rays or iii) stereophotogrammetry to measure a number of points on the face and compute distances between them (eg. Domokos and Kismartoni, 1974). Psychologists have ~~used~~ investigated such topics as what makes a face recognisable, male or female, familiar or unfamiliar using experiments with photographs, computer generated images and live people. These topics are discussed further in chapter 9.

### **4.2.1 Male and female faces**

There are (usually) noticeable differences between the faces of men and women but these are hard to describe and quantify. In fact, surprisingly few anthropological measurements have been made comparing male and female faces. However, a qualitative assessment of these differences has been published.

In his book about the human face Liggett (1974) summarized the differences between the male and female face as follows: women tend to have smaller noses which are more concave and wider at the alar base than men. Liggett observed that women's noses appear similar to children's noses and postulated that some connection may exist between femininity of a face and age. Women have small mouths and a smaller upper lip height than men and less pronounced jawlines and brow ridges. They also have larger eyes and a darker surround to the eye. Men tend to have squarer jaw, thicker eyebrows, and a larger overall size of face. Women have less mobile faces due to smaller muscles, hidden beneath more fatty tissues, which makes the texture smoother.



The orthodontist Enlow (1975) observed that the male nose appears larger at the nasion than the female and his eyes therefore appear to be deeper set and cheekbones less prominent. The female appears to have a less prominent nose, larger eyes and more prominent cheekbones than the male. He also suggested that the shape of the nose differs between the sexes with the tip of the nose being more pointed and tipped downwards in the male and more rounded and tipped upwards in the female. His observations have led psychologists to investigate the importance of the nose region in sex judgment (see chapters 7 and 9).

In a study of 10 faces using stereophotogrammetric data, Haga and colleagues (Haga et al, 1964) found that the volumes of the buccal regions were greater in females than males. Generally speaking the volume of the cheek were also greater in females than males but, given the limited number in their study, there was an equal probability of which side of the face was larger for a given individual. Shepherd (1989) noticed that women appear to have fuller cheeks than men.

Psychologists have addressed the question of how we distinguish between male and female faces. O'Toole and her colleagues have used principal component analysis (described in chapter 9) to distinguish between male and female faces with 74% accuracy (O'Toole et al, 1991). Burton and colleagues attempted to find a discriminate function between male and female faces based on the measurement a large number of distances, in 2 and 3 dimensions, and various ratios and angles computed from them (Burton et al, in press). They found that the sex of the faces could be classified correctly using 12 2D distances on full face photographs with 85% accuracy (compared to 95% human ability). Using 3D measures the same accuracy could be achieved with only 6 variables. Combining 16 3D and "picture plane" variables gave a performance close to human accuracy (94%). Interestingly, those misclassified by the analysis are *not* those misclassified by humans. They noted that 3D information is likely to play an important role in our ability to distinguish between male and female faces.

In a companion paper to Burton and colleagues, Bruce and colleagues (Bruce et al, in press<sup>b</sup>) investigated the perceptual basis for sex judgement and showed that humans use a combination of local cues, relationships between key positions and 3D information (see chapter 9). They concluded that knowledge of 3D shape aids interpretation in the picture plane.

### **4.2.2 Facial asymmetry**

It was discovered long ago that the two sides of the skull are not equal in size and shape. This led to some debate about whether or not this phenomenon is related to the left-handedness or right-handedness of the individual. Studies do not seem to have supported this idea (eg. McManus, 1982). Despite the asymmetry typically present in

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skulls, the "normal" face has been found to be surprisingly symmetric in morphology, size and area and although it may exhibit some asymmetry of facial expression these differences are not correlated to any differences in size asymmetry (Sackeim et al, 1984). On the contrary, the "abnormal" face may be markedly asymmetric in form, for instance in cases of hemifacial microsomia.

Facial asymmetry has been measured using the techniques of stereophotogrammetric (Burke, 1971; 1974; 1983; Burke and Beard, 1967), morphanalysis (Rabey, 1977) and from computer analysis of photographs (Coghlan et al, 1987).

#### **4.2.3 Racial differences**

Many studies of the size and shape differences between races and tribes have been undertaken. Race is a very difficult thing to define and there have been many disagreements about how to define it. Some have used skin colour, others shape and proportions of the head and even hair or eye colour. There seems to be no consensus of opinion. Unfortunately, few measurements of the face have resulted and these have been limited to a comparison of facial proportions; heights or widths of heads, noses etc. Although, Marcellino et al (1978) discovered that shape was three times as important as size in accounting for intertribal variation amongst south American Indian tribes.

Investigation of Jewish populations in different geographical locations have shown that they are much closer to one another in body proportions than their non-Jewish neighbours (Kobyliansky and Livshits, 1985)

O'Toole and her colleagues have used principal component analysis (described in chapter 9, section 4v) to distinguish between Caucasian and Japanese faces with 88.6% accuracy (O'Toole et al, 1991).

#### **4.2.4 Family resemblance**

A number of anthropometric studies have indicated there is a strong genetic component in the variability of facial dimensions. Comparisons of craniofacial measurements for a sample of Indian families showed some correlation of these measurements between families with a very strong correlation for twins (Byard et al, 1985). Sharma and Sharma (1984) found that for Indian families there was strong genetic determination for gross head size and that a paternal inheritance route for this was likely.

These studies have mainly been based on calliper measurements and the results have been difficult to compare because of differences in the statistical analysis performed, variation in choice of subject material (live subjects, dry skulls or X-rays), age and sex of subjects. In 1985, a view was expressed that progress in providing a better

description of the genetically determined facial structure was dependant on the introduction of new measuring techniques that would enable a more accurate description of the facial features (Hauspie et al, 1985).

#### **4.3 Landmark analyses**

To date the methods proposed for the analysis of facial shape have almost always been firmly based on the movement of homologous landmarks (Bookstein, 1978a; 1978b; 1984a; 1984b; 1986; 1988; Siegel and Benson, 1982; Thompson, 1917). As already discussed in chapter 2, the weakness of such an approach is that the landmark points must be readily, and unambiguously, identifiable. On gently curving surfaces such as the human back, or most of the face, this is not the case. Bookstein (1978a) has pointed out the importance of examining "the curving surface bearing the points (landmarks) and not the points (landmarks) *per se*" and this view has been reiterated by facial surgeons who feel that they are not adequate to describe the intricacies of deformity (Cutting et al, 1986a). Bookstein has recently proposed using "thin-plate splines" to interpolate the edges between two dimensional landmarks (Bookstein, 1989). However, the small number of homologous points on two curves is still a constraint on the accuracy of the interpolation, and pseudo-landmarks need to be created. This method has not yet been demonstrated in 3D although the potential for this may exist.

#### **4.4 Possible implications of shape analysis for surgeons**

The need for a comprehensive system comparing all aspects of the facial surface to allow the formation of a normative data set has been expressed by facial surgeons (Bookstein and Cutting, 1988; Udupa, 1986). Many facial conditions are defined using qualitative terms. For instance, hemifacial microsomia involves a "flattening of the maxilla" or a "narrowing of the maxilla" and a "displacement of the chin point", thus a quantitative analysis of the face should *also* be easily interpretable in linguistically definable terms. A more rigorous analysis of facial shape would help these conditions to be defined quantitatively and the results of surgical correction to be assessed.

The question of aesthetic appearance has hardly been treated at all analytically, although the functionality of the occlusal bite has been studied by the analysis of electro-myographic signals (which indicate the balance of muscular function as the patient bites). An analytic treatment would allow the surgeon some objective method of evaluating the outcome of surgery, to determine its conformity to some plan and to pinpoint discrepancies between the planned and actual outcome. Again, Cutting has said that a truly *scientific* investigation of aesthetic surgery is dependent on a comprehensive method of surface deformation analysis (Cutting et al, 1986a).

It is anticipated that the formation of a database for quantifying the changes brought about by surgical procedures and the development of quantitative norms (Savara et al,

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1985) will eventually help in the planning and prediction of surgery. A 3D analysis may also have a significant influence on the accuracy of craniofacial diagnosis and allow the establishment of a biologically sound treatment plan (Christiansen, 1978).

Additionally, an analysis and characterization of facial shape may well throw some light on dysmorphobia. In this condition, the patient may complain of flat cheeks, the jaw being out of proportion to the rest of his face, or some aspect of asymmetry in the shape of his face, which others have difficulty perceiving. It maybe possible to quantify the problems of which the patient complains, or, in some cases, convince the patient that their problem is of a purely psychological nature. Moreover, slow changes in the face, either perceived or real, could be investigated and quantified. This maybe particularly important in the case of developing facial asymmetry or in the slow growth of tumours.

Other studies where a quantification of surface shape changes is needed include categorizing facial morphological types and assessing their relationships to developmental anomalies and in the study of growth and development of the face in general. The quantification of changes may help to explain the effect on the developing face of surgical intervention and discover the effects of racial and sexual differences on the outcome of the surgery.

### **4.5 Summary**

The history of the measurement of the face that have been undertaken by orthodontists, anthropologists and others has been described here. Measurements made by orthodontists have largely been limited to two dimensional analyses, such as cephalometry and profile analysis, although the production contour plots and Moire fringe patterns have allowed some assessment of the 3D structure of the face to be made.

Anthropologists have made some 3D measurements of face using callipers, but the measures used by individual researchers have not been standardized, limiting the usefulness of some of the comparisons. Attempts made by anthropologists at measuring facial asymmetry, racial differences and family resemblance are described in section 4.2 along with a qualitative assessment of the differences between male and female faces. The differences between male and female faces are discussed further in chapters 7 and 9.

The limitations of methods to date for describing facial shape, which have been based on the location of landmark points, are mentioned in section 4.3. Methods for describing facial shape have also been limited by the lack of availability of 3D facial

data. The data acquisition methods described in chapter 3 have now overcome this obstacle.

The possible implications for facial surgeons of the production of a method for analysing facial shape is described in section 4.4. In 1986, one surgeon Cutting, predicted that differential geometry would provide the appropriate tools for the analysis of facial surface deformities (Cutting et al, 1986). This thesis follows this suggestion and shows how differential geometry may be used to describe the 3D changes in shape of the face which occur during surgery qualitatively or quantitatively. In the next chapter, the criteria for a successful shape description method are outlined and the potential of a method based differential geometry to meet these is discussed.

## CHAPTER 5

### CHOICE OF METHODOLOGY

In the search for an algorithm that could be used to provide a mathematical description of the face, and to distinguish differences between faces, a number of requirements were noted. In this chapter, these requirements are set out and it is shown how the approach that was selected, a method based on the mathematics of differential geometry, meets them. The general concepts of differential geometry are reviewed and previous work which has used certain aspects of differential geometry for object and face description is described. The hardware and software that was used to implement the chosen method is described. The actual method is described fully in chapter 6.

#### **5.1 General requirements**

The first consideration is the nature of the object to be described. The face is a smooth, continuous *surface*, therefore a surface-based method should be used as opposed to a volumetric method. Since the face may be viewed, and recognized, from many angles, the encoding of the facial surface should be *independent of viewpoint*.

In the analysis of medical images, it is most often small departures from the norm that are more significant than the gross structure (Trivedi, 1986). Bearing in mind that the aim is produce a method for describing changes in the face, the method chosen to describe the face should be able to encode changes in the *small scale structure* of the face. Ideally, the approach should also be simple and visual, allowing a qualitative appreciation of the face. A hierarchical approach would provide flexibility allowing both small and large scale changes to be described in a quantitative manner.

The 3D data sets which are produced from optically scanning the face are very detailed. If all this data is used, the computational requirements are likely to be large. This factor influenced the choice of hardware (see section 5.4). The chosen method needed to be robust against noise because of its potential application in facial reconstructive surgery. However, for surgery accuracy is of the utmost importance, this limits the amount to which the acquired data can be smoothed in order to eliminate noise since smoothing removes not only noise but fine surface structure.

Many of the techniques for shape description which have previously been reported have been concerned with modelling 3D objects. These techniques have arisen from the use of computer graphics for CAD/CAM for applications such as building descriptions of a car for the design of a new car-line. I sought a means of reversing these models to generate a description of a specific object, from acquired data, in terms of parts or primitives.

## **5.2 A neural network for the face?**

Bearing these requirements in mind, I first considered using a neural network to describe the face, since they have been applied to face recognition questions (see Chapter 9). Neural networks are based on the observation that the human brain consists of millions of cells, or neurons, which communicate with each other in parallel. A multi-layered network of computational "cells" are built which respond, in a binary manner, to stimuli and pass on information to subsequent "cells". The network can be trained, by repeated presentation, to recognize patterns correctly. These include distorted or shifted patterns (Fukushima and Miyake, 1982), partial or incomplete patterns (Fukushima, 1988). However, the complexity of the system quickly grows with the number and complexity of the pattern presented because more layers of cells are required for recognition. To recognise a 3D object, the network would be need to be very large. Although new patterns can be recognised, a large amount of time is needed to train the network in the first place. It may be possible to "train" the network to recognise a face or facial feature such as the nose and hence reject any face or feature that had changed "too much" from that target face or feature, however it is not clear how one would assess how much change is "too much" nor how a quantitative assessment of that change could be made.

Neural networks do not provide an explicit description of the face and their output is isoteric in nature. This means that a neural network can not output *how* its conclusion was reached, merely that conclusion. In a medical context this is a serious drawback (Price, 1989). It is also doubtful whether neural nets would have the required sensitivity for the description of small scale facial changes, indeed questions concerning their use for describing faces have been raised (Bruce, 1988). Another consideration is the expense of construction. This rises rapidly with the complexity of the network and the number of objects it is required to recognise. For all these reasons, this approach was rejected.

## **5.3 Differential geometry**

A more promising approach, and the one I eventually adopted, was to describe the (facial) surface using the mathematics of differential geometry.

In the last two decades, methods for describing the surfaces visualized in range images have been sought and a number of descriptions have been obtained. These have included; parametric polynomial surfaces, tensor splines, contours showing maxima and minima extrema, maps of Gaussian and mean curvatures, principal curvatures, lines of curvatures and geodesics (Beck et al, 1986; Brady et al, 1985). All these description have been based on various aspects of differential geometry.

## Methodology

The theory of differential geometry is applicable to *smooth differentiable* surfaces, which are readily obtainable from range data. This requirement means that differential geometry is ideal for studying a surface with few discontinuities such as the face. Importantly, the surface curvatures defined by differential geometry are *locally invariant* to rotation and translation, allowing the surface to be described in a fashion that is independent of viewpoint. They are also invariant to scale. Thus, it was in this branch of mathematics that a method suitable for implementation and further development was found that could be used to describe a face. Provided the resulting description was stable enough to noise, changes in the facial surface could also be quantified.

### 5.3.1 Principles of differential geometry

Differential geometry is a well documented mathematical science but a brief review of its concepts is given here for completeness and to aid the reader unfamiliar with this topic. Further explanations can be found from any standard text (eg. Lipschultz, 1969) and good summaries are containing in Beck et al (1986) and Hilbert and Cohn-Vossen (1952).

Consider a surface in parametric form  $\mathbf{r} = \mathbf{r}(u,v)$  that may be differentiated with respect to  $u$  and  $v$ . The first derivatives are denoted by  $\mathbf{r}_u$ ,  $\mathbf{r}_v$ , and the second derivatives by  $\mathbf{r}_{uu}$ ,  $\mathbf{r}_{vv}$ ,  $\mathbf{r}_{uv}$ . These derivatives allow intrinsic characteristics of the surface, independent of the parameterization, to be derived. For instance the surface normal  $\mathbf{n}$  is defined

$$\mathbf{n} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{\text{mod}[\mathbf{r}_u \times \mathbf{r}_v]} \quad \text{-- (5.1)}$$

In Euclidean space, Pythagorus' Theorem gives the shortest distance between two points (a straight line). On a curved surface, the infinitesimal distance element between two neighbouring points  $(u,v)$  and  $(u + du, v + dv)$  is given by:

$$ds^2 = \mathbf{r}_u \cdot \mathbf{r}_u du^2 + 2\mathbf{r}_u \cdot \mathbf{r}_v dudv + \mathbf{r}_v \cdot \mathbf{r}_v dv^2 \quad \text{-- (5.2)}$$

Integrating  $ds$  along a specified path  $u = u(t)$ ,  $v = v(t)$  on the surface gives the path that is the shortest distance between the two points (a geodesic curve). Equation (5.2) is known as the first fundamental form of the surface and can also be written as

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2 \quad \text{-- (5.3)}$$

where  $E(u,v) = \mathbf{r}_u \cdot \mathbf{r}_u$ ,  $F(u,v) = \mathbf{r}_u \cdot \mathbf{r}_v$ ,  $G(u,v) = \mathbf{r}_v \cdot \mathbf{r}_v$ . It gives the distance between two points to first order in  $du$  and  $dv$ . Since the distance  $ds$  lies in the tangent plane at



the point (u,v), it does not tell us how the surface *curves away* from the tangent plane at that point. The curvature comes from the second order equation, known as the second fundamental form of the surface. The component of displacement between points (u,v) and (u + du,v + dv) perpendicular to the tangent plane is one half of

$$dh^2 = Ldu^2 + 2Mdudv + Ndv^2 \quad \text{-- (5.4)}$$

where  $L(u,v) = \mathbf{n} \cdot \mathbf{r}_{uu}$ ,  $M(u,v) = \mathbf{n} \cdot \mathbf{r}_{uv}$ ,  $N(u,v) = \mathbf{n} \cdot \mathbf{r}_{vv}$

If u and v can be expressed as a function of a single parameter t ( $u = u(t)$ ,  $v = v(t)$ ), then, the normal curvature of the surface at a point in the direction (u,v), where the dot indicates differentiation with respect to t, is given by:

$$k = - \frac{L\dot{u}^2 + 2M\dot{u}\dot{v} + N\dot{v}^2}{E\dot{u}^2 + 2F\dot{u}\dot{v} + G\dot{v}^2} \quad \text{-- (5.5)}$$

and the radius of curvature is  $1/k$ . A sign convention is used to define a convex surface as one having positive curvature and a concave surface as one having negative curvature. It can be demonstrated that the first fundamental form and second fundamental form of a surface uniquely determine the local surface shape, in terms of its curvature, torsion and speed for a 3D space curve (Lipschultz, 1969) and that these are invariant to rotation, translation and changes in the parameterization.

In equation (5.5), a curvature k is associated with each direction (u,v) on a surface. The directions in which the normal curvature k attains extremum values occur when  $dk/du = 0$  and  $dk/dv = 0$ . This occurs when

$$(L + kE)\dot{u} + (M + kF)\dot{v} = 0 \quad \text{-- (5.6)}$$

and  $(M + kF)\dot{u} + (N + kG)\dot{v} = 0$

For a consistent solution,

$$k^2 - 2Hk + K = 0 \quad \text{-- (5.7)}$$

must be satisfied where K is the *Gaussian curvature* of the surface and H the *mean curvature* of the surface and are defined as

$$K = \frac{LN - M^2}{EG - F^2} \quad \text{and} \quad H = \frac{2FM - (EN + GL)}{2(EG - F^2)} \quad \text{-- (5.8)}$$

### Methodology

In 1760, Euler discovered that there is always a direction on a surface in which the surface curves least and another direction, orthogonal to the first, in which the surface curves most. These directions are called the principal directions and the curvatures associated with them, the principal curvatures. The solutions of the quadratic equation (5.7) are the *principal curvatures* ( $k_{\max}$  and  $k_{\min}$ ). These can be written in terms of the Gaussian and mean curvature of the surface:

$$\begin{aligned} k_{\max} &= H + (H^2 - K)^{1/2} & \text{-- (5.9)} \\ \text{and} \quad k_{\min} &= H - (H^2 - K)^{1/2} \end{aligned}$$

From equation (5.9) it can be seen that  $k_{\max} = k_{\min}$  when  $H^2 = K$ . Such a point is termed *umbilic* or *spherical* as the curvature  $k$ , is independent of direction and the surface is locally spherical.

The Gaussian and mean curvatures can also be expressed in terms of the principal curvatures. The Gaussian,  $K$  is the product of the principal curvatures and the mean,  $H$  is the arithmetic mean of them ie.

$$K = k_{\max} k_{\min} \quad \text{and} \quad H = \frac{(k_{\max} + k_{\min})}{2} \quad \text{-- (5.10)}$$

If the two principal curvatures have the same sign, the Gaussian curvature will be positive and the surface is called *elliptic*. Whereas if the two principal curvatures have the opposite sign the Gaussian curvature will be negative and the surface is called *hyperbolic*. A point at which one of the principal curvatures is zero is called a *parabolic point* (Hilbert and Cohn-Vossen, 1952). When either one of the principal curvatures is zero, the product of the principal curvatures, and therefore the Gaussian curvature, becomes zero. The surface is then termed *developable* (ie. it can be rolled out onto a plane). However, this condition is not sufficiently constraining to uniquely define a surface (Barrow and Tenenbaum 1981).

The Gaussian curvature,  $K$ , is an intrinsic property of the surface and therefore it remains invariant when the surface is bent (without stretching or tearing) (Peet and Sahota, 1985). The mean curvature is an extrinsic property and changes as the surface is bent. Collectively, they are known as the *surface curvatures*. Their properties are more comprehensively explained by Besl and Jain (1986).

A surface curve ( $u = u(t)$ ,  $v = v(t)$ ), whose derivatives  $(u,v)$  satisfy either of the two equations in (5.6) is a tangent to a principal direction at every point. This is termed a *line of curvature* and shows the directional flow of the maximum and minimum curvature across a surface. Lines of curvature can be used to define "principal patches" (Brady and Yuille, 1984). An *asymptotic curve* follows the direction of zero normal

curvature,  $k$ , on the surface - the antithesis of a line of curvature which follows the direction of extremal normal curvature. The direction of a surface's asymptotes can be found from Euler's theorem if the principal curvatures are opposite in sign, ie. Gaussian curvature is negative.

### 5.3.2 Analysis of surfaces using differential geometry

The first recorded attempt to use the mathematics of differential geometry to describe the facial shape was made by the renowned German mathematician Klein around 1926. Klein marked out parabolic curves (ie. the smoothed loci of parabolic points) on a bust of the classical statue Apollo Belvedere in an attempt to put facial aesthetics onto a mathematical foundation (Hilbert and Cohn-Vossen, 1952). However, these curves are not simple, do not appear to correspond to the facial features and are not suitable for producing a robust description of the face (Brady et al, 1985). Parabolic lines have recently been automatically marked on computer generated models of the face (Gordon, 1991b). Figure 5.1 shows the parabolic lines marked by Klein and Gordon.

Early differential geometric analyses of surfaces were limited by the enormity of computational requirement and, until the last few years, many authors employed point-wise descriptions of surfaces rather than extracting characteristic features. This was probably also due to the lack of availability of sufficiently large sets of 3D data. But with the advent of range data from optical scanning, together with more powerful computers, descriptions of the entire surface have become possible.

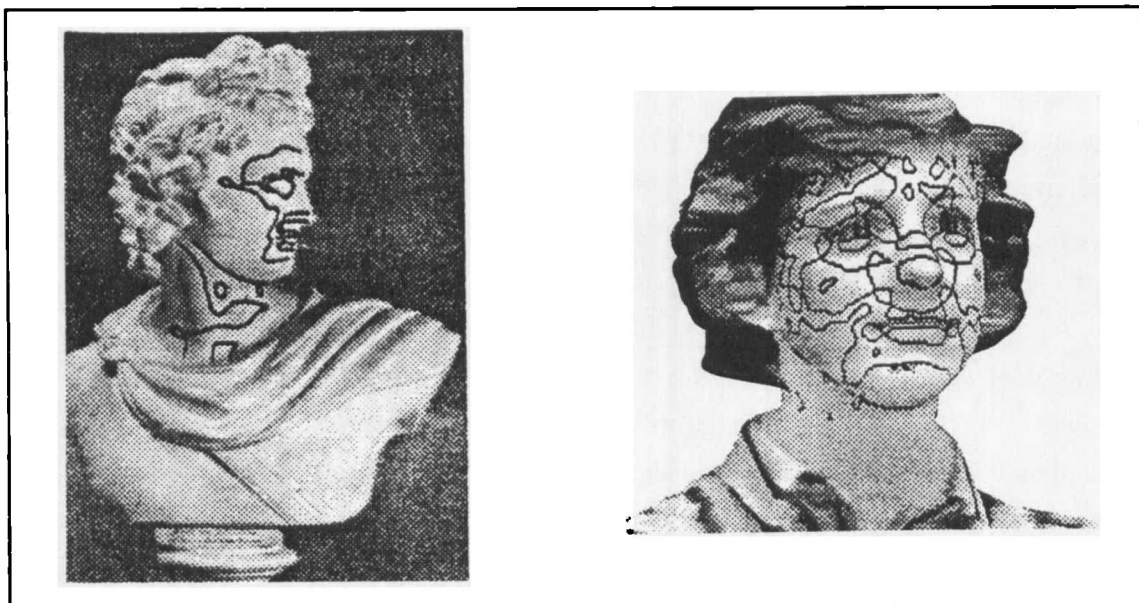


Figure 5.1: Left: The parabolic lines marked by Klein on Apollo Belvedere. Right: automatically generated parabolic lines superimposed onto a 3D model of the face (from Gordon, 1991b. *Reproduced with permission*).

A number of different approaches have been made using various curvature measures to describe the shape of a 3D object and a number of different *segmentations* for range

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images based on these have been proposed. Besl (1988) and Fan (1990) describe in depth some of these approaches. These approaches have all considered, in some way, the *curvature* of the surface and very recently, some of these have been applied to facial range images. For instance, the potential for using the sign of Gaussian curvature to describe a surface for recognition purposes was noted by Marr and Nishihara (1978) and Stevens (1981). Its usefulness is due to the fact that it allows an object-centred description to be obtained and thus the description is complete for any chosen, continuous surface, is local and additionally is readily computable. The segmentations that have been produced fall broadly into two categories: segmentation into primitive patches and segmentation by lines of curvature. These are discussed in turn.

### **a) Segmentation into primitive patches:**

In the 1980's, several attempts were made to segment a surface into geometrically describable patches. Faugeras and colleagues (Faugeras et al, 1983) proposed a segmentation into planar and quadratic patches, by fitting the surface with a quadratic least squares fit algorithm. Medioni and Nevatia (1984) used the Gaussian curvature and the principal curvatures to obtain elliptic, hyperbolic, parabolic and planar patch primitives. The zero-crossing of Gaussian curvature and the maximum principal curvature were used to detect surface discontinuities.

Ittner and Jain (1985) investigated the power of six different curvature measures for the identification of four surface primitives: sphere, plane, cylinder and cone. These six measures were: the average curvature, minimum curvature, maximum curvature, Gaussian curvature, mean curvature and ratio curvature. All six measures were found to be robust. The number of points in the surface patch and size of the neighbourhood used to calculate the curvature affected the accuracy of the measures in the presence of different levels of noise. These findings provide confidence that if a curvature based method for describing the facial surface is developed, it will prove suitable and sufficiently robust, for surgical applications.

Vemuri et al (1986) computed the principal curvatures by fitting smooth patches to the object surface using spline functions. Maximal regions were then formed by coalescing patches with similar intrinsic curvature-based properties and surface points were then classified accordingly.

Hoffman and Jain (1987) proposed a segmentation into planar, concave and convex regions. Saddle points were classified as concave or convex according to the direction in which the magnitude of the curvature was larger. This method was based on clustering (or region growing) techniques using nonparametric statistical tests. This results in a large number of moderately sized patches.

In the late 1980's, Besl and Jain (1986; 1988) used the signs of the Gaussian and mean curvatures: positive, negative or zero, to segment surfaces into eight "surface types". The importance of their segmentation was, they claimed, that these eight surface type could be used to describe *any* smooth and differentiable surface. Thus they were *fundamental surface primitives*. It was this segmentation method that was applied to the 3D data set, in order to describe the face (see chapter 6).

Medioni and Nevatia's (1984) surface type labels are in fact, a subset of Besl and Jain's primitives. The difference between Besl and Jain's surface type labels and those used for the Topographic Primal Sketch (TPS, see chapter 1), for intensity images, is explained in detail by Besl and Jain (1986). They described, in lengthy papers, the application of differential geometry to recognizing 3D objects and the properties of the Gaussian and mean curvature.

Bhanu and Nuttall (1989) devised a description based on the magnitude and orientation of the principal curvatures, which allowed them to describe objects using a "continuum of surface types". This method of description arose from their observation that conical surfaces were not represented satisfactorily using Besl and Jain's 8 surface types. They showed that certain regular objects, such as spheres, cones and cubes formed well-defined clusters at specific points on a graph whose axes were the principal curvatures. The goal of this work was object recognition.

Recently, Wienshall (1991) has used knowledge of the sign of the Gaussian curvature, computed directly from motion disparities, to classify a surface into elliptic, hyperbolic, cylindrical and planar regions.

### b) Segmentation by curvature lines:

Additional, complementary information for the segmentation of a surface may be found from considering the surface discontinuities and extremal values of curvature. Amongst these methods, Brady and colleagues (Brady et al, 1985) discussed the idea of using the *lines of curvature* to parameterize the surface (Brady and Yuille, 1984). They showed that this was *not* a good basis for describing many surfaces (Brady et al, 1985) and proposed that *asymptotes* might prove to be a better descriptor. Brady also investigated the use of surface intersections, planar and spherical surface patches and bounding contours (Brady et al, 1985).

Calculation of the surface curvatures allows certain properties of the surface to be extracted. "Jump boundaries", where a surface discontinuity occurs and "folds" where a surface normal discontinuity occurs can be extracted from zero-crossings (ie. change of sign) in curvature. "Ridge lines", the local extrema of curvature can be obtained from the extremal values (Fan et al, 1986). Fan and colleagues calculated the surface

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curvature in four different directions and obtained the curvature extrema and zero-crossings for each of these one dimensional curves (Fan et al, 1986). These were used to identify surface and depth discontinuities that might then be used to describe or recognize an object.

Haralick (1983) proposed identifying ridges and valleys on intensity images from zero-crossings of the first derivative, taken in a direction which maximizes or minimizes the second directional derivative. He demonstrated how this could be applied to a facial image. However, the ridges he identified corresponded to highlights in the image and the valleys corresponded to shadows illustrating the difficulty of using intensity images to derive surface properties.

Gordon (1991a; 1991b) also proposed the use of "ridge" and "valley" lines to segment a facial range image. She defined "ridge lines" as the local maxima in principal curvature  $k_{\max}$  along the line of maximum curvature and "valley lines" as local minima in  $k_{\min}$  along the line of minimum curvature. Using  $k_{\max}$  greater than a certain threshold and  $k_{\min}$  less than a different threshold, these produced a much clearer, and more stable pattern (see figure 5.2 for an example). Gordon's work will be discussed further later in this section.

Jain and Hoffman (1988) combined surface patch information with jump edge information to produce a shape representation of a 3D object. The object could then be recognised using an evidence rulebase. A better segmentation of the image has been sought by Haddon and Boyce (1990) by unifying region and boundary information.

c) Application to body surfaces:

In the early 1980's some pioneering work was reported on the use of the Gaussian and mean curvatures for the analysis of human back shape (Hierholzer and Frobin, 1980; Frobin et al, 1982; Frobin and Hierholzer, 1982a). Frobin and Hierholzer used photogrammetric measurements of the back, sampled at a number of points and interpolated between points, to produce maps of the Gaussian and mean curvatures of the back surface. They used the sign of these to classify the surface into planes, cylinders, spheres and saddle surfaces. The importance of this method for analysing biological surfaces is that the need for a unique, body-fixed, coordinate system based on landmarks is avoided and the description produced allows the major features of the back to be identified in an objective way. Later, Schwartz and Merker (1986) also used the Gaussian and mean curvatures to describe the cortex of a monkey's eye.

Besl and Jain's demonstration of how the signs of Gaussian and mean curvatures could be used to segment a surface into eight surface type primitives, that could be used to completely describe *any* surface (Besl and Jain, 1985; 1986), prompted Hidson of Defense Research Establishment, Ottawa (Hidson et al, 1990) and myself (Coombes et al, 1990), working independently, to show how the facial surface could be described in this manner. This work was reported in 1988 at the 5th International Conference on Surface Tomography and Body Deformity held in Vienna. Since then, Gordon at Harvard in her thesis (1991b) and I, Coombes (1991a; 1991b; 1992) have independently pursued the use of these techniques for facial description and recognition. In chapter 6, Besl and Jain's surface type description is described and used for analysing facial shape.

Lee and Milios (1991) used Besl and Jain's method to calculate the surface curvatures by convolution (using a window size 7x7) and segmented range images of faces into the 8 surface types. In order to match faces for recognition purposes they then represented each convex region by its Extended Gaussian Image (EGI - see chapter 1), which maps points of the region to corresponding points on a unit sphere. The reason for using the EGI representation is that it is invariant to changes in scale and orientation. A similarity measure based on the correlation features in the EGI was used to match the data. They observed that features such as the nose, chin, cheek and eyebrow correspond to convex regions of the face and that these convex regions appear to be subject to less change between facial expression than concave regions. They encountered difficulties in EGI interpolation due to the inadequate sampling of areas of high curvature (eg. the nose).

In this work, the need for using the EGI representation to achieve invariance in scale and orientation is removed by using a different method for calculating the surface curvatures. This method is described in chapter 6 (section 6.2.4).

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The use of the negative minima of the principal curvature,  $k_{\min}$ , to segment a surface into regions corresponding to features was suggested by Hoffman and Richards (1985). Gordon (1991a) implemented this idea for facial surfaces. Although the segmentation produced by this method was not complete, application of dilation and shrinking operators enabled her to extract the eyes, nose regions effectively. To extract the face from hair region, Gordon used the fact that the face is smooth compared to the hair. On smooth areas, both principal curvatures are low, therefore Gaussian curvature will be less than some threshold. Gordon found an even better criteria for detecting smooth areas was  $k_{\min}^2 + k_{\max}^2 < \text{threshold}$ . This function has a steeper slope at high curvature boundaries.

Gordon aimed to identify facial features through segmentation of the facial surface. In her method she made use of the protrubance of the nose and its typical ridge characteristic to identify the nose bridge from depth values and thereby define the plane of symmetry of the face. This enabled her to locate other facial features such as the eyes, by searching for concave (or convex) surfaces of a certain size and distance from the line of symmetry. By applying a further constraint, that two regions be found, symmetrical about the midline, a more accurate location of these features was obtained. She considered extracting facial features to be an iterative process, as features are extracted information is provided that can be used to refine, or correct earlier feature definitions. Gordon noted the effect that facial expression would have on her analysis and therefore she deliberately did not use such changeable regions as the mouth.

### **d) Sensitivity of surface curvatures to noise.**

It has been pointed out by a number of authors that the calculation of the surface curvatures is likely to be sensitive to noise because they contain second order partial derivatives. However, this need not create a prohibitively large problem (Fan et al, 1986). Smoothing of the data with a Gaussian filter has been used by a number of authors to obtain a reliable description (Fan et al, 1986; Gordon, 1991b). Gordon analysed the effect of smoothing on the curvature values obtained and concluded that smoothing the whole image with the highest level of smoothing required would severely modify the curvature values and location of features in highly curved areas. She proposed that an adaptive smoothing method, based on local estimates of curvature, should be used in order to obtain accurate and reliable measurements (Gordon, 1991b).

For the purpose of object recognition, tolerance of small differences can be achieved by the setting of a threshold below which differences in the surface description are ignored. Abdelmalek (1990) reported an algebraic analysis of the effect of noise on the segmentation of range images into Besl and Jain's surface types. He measured the noise standard deviation for a sample range image and found that both the Gaussian and the mean curvatures were more susceptible to noise when they have small values, ie. on



nearly planar surfaces, and that the Gaussian curvature is more susceptible to noise than the mean curvature.

For our application, the assessment of small changes to the facial shape, it is these fine scale differences that we are interested in and wish to preserve. For facial surface changes, the difference between two individual faces and facial expressions are all small changes. Therefore the stability of the segmentation with respect to noise in the data is important. This is discussed in chapter 6.

#### **5.4 Implementation**

Before the production of a description for the face based on its Gaussian and mean curvatures is discussed, it is appropriate to mention how the programmes were written and implemented. The amount of 3D data and the iterative nature of some of the procedures made the anticipated computational requirements large. These computational requirements, together with the available resources limited the choice of hardware and programming language. Only high powered machines were suitable. Another possibility was using parallel processors such as Transputers.

When this work was begun, the Medical Graphics and Imaging group were in the process of converting their programmes to run on P.C. hosted Transputer networks, in order to market a computer graphics workstation that would simulate facial surgery. Thus it seemed logical to write programmes in a compatible format that could eventually be included with this system. Transputers were chosen over other alternatives providing graphics because of their cost effectiveness in producing graphical display. All the routines and algorithms described in this thesis were written in OCCAM2 and implemented on a P.C. hosting Transputers.

OCCAM2 is a language based on communicating sequential processes (the second version was released in 1987). It is concurrent, that is to say it has potential for executing in parallel. OCCAM2 was written especially to enable the Transputer to function in its most efficient manner. (Both OCCAM and the Transputer were designed by INMOS.) The Transputer is a programmable VLSI (very large scale integration) device containing communication links for point-to-point inter-transputer connections.

In this work, only one Transputer was used to execute the OCCAM code and a second Transputer executed the graphics display commands concurrently.

#### **5.5 Summary**

The general requirements for a good shape descriptor for the face were discussed in section 5.1. It has been shown that application of differential geometry techniques to describing surfaces inherently meets some of these requirements. For instance, the

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*surface* geometry is described, independent of the viewpoint from which the surface is seen. The recent availability of large amounts of 3D body data, from optical surface scanning, has allowed entire surfaces to be described using this methodology. This methodology allows the surface to be segmented into regions or primitive patches and either by modelling the surface using fundamental surfaces or by using extrema curvature values to define valleys and ridges. Gordon, Hidson and I have pioneered the application of these techniques to the facial surface. In order to make full use of the available data, I have implemented these techniques on a Transputer-based PC.

The suitability of differential geometry for meeting the other requirements of a good facial shape descriptor has not been demonstrated in this chapter and some concern has been expressed regarding the tolerance of the surface curvatures to noise in the data. However, in the next chapter, differential geometry is applied to the face producing a *hierarchical* description of facial shape, which is also easy to interpret visually. It is also shown that the facial description is reasonably robust against noise in the data and the ability of the description to represent small scale changes in the facial structure is examined.

## CHAPTER 6.

### DESCRIPTION OF THE FACIAL SURFACE

#### **6.1 Strategy**

In the previous chapter, the selection of a methodology based on differential geometry and implemented on a Transputer-based PC was discussed. In this chapter, the adaptation and implementation of these methods for the description of the facial surface is described.

Three separate algorithms for calculating the surface curvatures are described, two of which have previously been reported and the third, a least squares surface fitting algorithm for irregularly sampled surfaces, is new. The information carried by the surface curvatures is represented graphically on a KH-map according to the distribution of the curvature values. The *signs* of the surface curvatures, positive, negative or zero, are used to encode the surface into eight *fundamental surface types*. By colour coding image pixels to represent these different surface types, a *surface type image* (STI) can be displayed. Alteration of the thresholds on the surface curvatures which delineates between zero curvatures and the other two signs, allows a series of STI's to be produced which are *hierarchical* in terms of curvature.

The performance of all three surface curvature algorithms for describing both regular test objects and the facial surface is assessed. These are important and new evaluations for all three algorithms. The assessment shows the greater accuracy of the least squares algorithm over the other two algorithms.

An assessment is made of how robust the least squares algorithm is to artifacts that are produced by the optical scanning process and random and quantization noise in the acquired data. The stability and repeatability of the resulting facial description is demonstrated. These are important considerations for the practical application of this method in describing faces or changed faces.

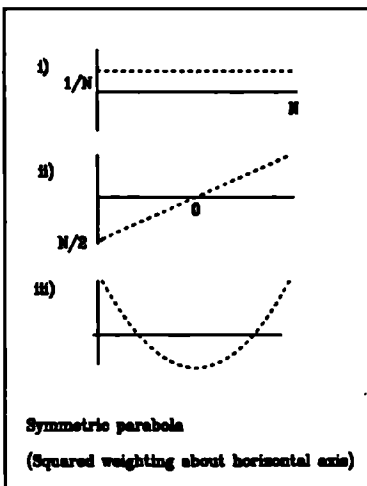
#### **6.2 Calculation of surface curvatures:**

In the computer vision literature, two methods have been reported for calculating the Gaussian and mean curvatures of a point on a continuous surface. The first uses convolution windows (Besl and Jain, 1987, 1988) and the second uses selective local fitting of windows to the surface (Yokoya and Levine, 1989). In addition to these, I report here another method for calculating these curvatures based on fitting a least squares quadratic surface patch around each point for an irregularly sampled set of points (such as optical surface scan data). I will describe each of these methods in turn.

### Facial Description

The first two methods begin by representing the surface as a discrete depth-map (z-buffer), regularly sampled in x & y, from a single viewpoint. This is simply a range image representing the surface from a single viewpoint by the depth value (z) at a point (x,y) on the 2D projection, which is given by a single-valued function  $z(x,y)$ . The depth map is generated from the irregularly sampled 3D source data from the chosen viewpoint using a standard interpolation algorithm (Newman and Sproull, 1981). The third method is *not* calculated from a single viewpoint and performs calculations using the actual x, y and z coordinates of the measured data points.

#### 6.2.1 The method of Besl and Jain (1988)



In this method, the surface curvatures are computed for each pixel of the depth map image, by passing a local neighbourhood operator over the depth map (Besl and Jain, 1988). The size of the convolution window can be varied in order to optimize the surface sample (see section 6.2.3). The column vectors for the convolution windows are calculated in the following manner:

For an  $N \times N$  window of border  $M = (N-1)/2$ , the 3 vector operators of the convolution filter are given by:

Figure 6.1: Convolution filters for the Besl and Jain algorithm i)  $d_0$  ii)  $d_1$  and iii)  $d_2$ .

- i)  $d_0 = 1/N$  for all  $u$ , where  $-M \leq u \leq M$   
eg. for a  $7 \times 7$  window,  $d_0 = 1/7 (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1)$
- ii)  $d_1 = 3 u / ( M ( M + 1 ) ( 2M + 1 ) )$   
eg. for a  $7 \times 7$  window,  $M = 3$   $d_1 = 1/28 (-3 \ -2 \ -1 \ 0 \ 1 \ 2 \ 3)$
- iii)  $d_2 = 1/P(M) * ( u^2 - ( M(M+1) / 3 ) )$   
where  $P(M) = 8/45 M^5 + 4/9 M^4 + 2/9 M^3 - 1/9 M^2 - 1/15 M$   
eg. for a  $7 \times 7$  window,  $d_2 = 1/84 ( 5 \ 0 \ -3 \ -4 \ -3 \ 0 \ 5)$

These functions are graphically depicted in figure 6.1.

The convolution matrices, which are equally weighted least-squares derivative estimation window operators, are given by the following combinations of these column vectors:

## Facial Description

$$\begin{aligned} [G_u] &= d_0 d_1^T & [G_v] &= d_1 d_0^T & [G_{uu}] &= d_0 d_2^T \\ [G_{vv}] &= d_2 d_0^T & [G_{uv}] &= d_1 d_1^T & & \end{aligned} \quad \text{-- (6.1)}$$

The convoluted image data are stored in 5 matrices of the same dimensions as the image data (GV, GU, GUU, GVV, GUV) and these are then used to calculate the Gaussian (K) and mean (H) curvatures at each point in the image.

$$\begin{aligned} H(x,y) &= \frac{(1 + GV^2(x,y))*GUU(x,y) + (1 + GU^2(x,y))*GVV(x,y) - 2*GU(x,y)*GV(x,y)*GUV(x,y)}{2(1 + GU^2(x,y) + GV^2(x,y))^{3/2}} \\ K(x,y) &= \frac{GUU(x,y)*GVV(x,y) - GUV^2(x,y)}{(1 + GU^2(x,y) + GV^2(x,y))} \end{aligned} \quad \text{-- (6.2)}$$

### **6.2.2 The method of Yokoya and Levine (1989)**

In this method, the curvatures are calculated from the selective local fitting of bi-quadratic functions to the surface. The method is based on the computation of the first and second partial derivatives of the surface by locally approximating the object surface using bi-quadratic polynomials. The Gaussian and mean curvatures are computed from these partial derivatives. The local surface fit used is claimed to improve the curvature estimates compared to the windowing technique. The partial derivatives can also be employed to calculate two edge-based segmentations of the surface, a "jump-edge" segmentation which detects discontinuities in depth and a "roof-edge" segmentation which detects discontinuities in surface normals (see section 6.7). These segmentations can be combined with the surface type segmentation of the surface to give more information about the surface. I will return to these two edge descriptors later in this chapter.

The selection of an appropriate window size is discussed in section 6.2.3. The best fit function for the window, centred at each point (x,y), is determined. Then for each point, the window is chosen from all the windows which overlap the point that provides a minimum fitting error.

At each point in the depth-map, a continuous differentiable function is fitted to the surface. The form of this bi-quadratic function is:

$$z(x,y) = ax^2 + by^2 + cxy + dx + ey + f \quad \text{--(6.3)}$$

Higher order equations attempt to model the surface more closely. However, this means that they also include the undesirable effect of fitting the noise in the data better. They would also yield many more surface types giving a more complex description. The local bi-quadratic surface is fitted within a (2m + 1) by (2m + 1) window centred on point

**Facial Description**

(x,y) using a standard least-squares method. The coefficients a through f, associated with the point (x,y), are obtained by mask operators. The fitting error of the window W(x,y) is calculated as the sum of the squared fitting error, E<sup>2</sup>(x,y).

$$E^2(x,y) = \sum_{i=-m}^m \sum_{j=-m}^m \{ai^2 + bj^2 + cij + di + ej + f - z(x + i, y + j)\}^2 \quad --(6.4)$$

It is assumed that the presence of a discontinuity in the data, within the window area used for calculation, leads to inaccuracies in the estimation of the partial derivatives. This is because differential geometry is applicable only to smooth continuous surfaces. This approach seeks to minimize errors introduced by surfaces discontinuities by fitting a best-fit window to each data point. This window is the one which provides the minimum fitting error. For each point (x,y) the best fit window W(x - u, y - v) (where u and v are the offset of the window from the point (x,y)) amongst (2m + 1) \* (2m + 1) windows minimizes:

$$E^2(x - u, y - v) = \min \{E^2(k,l):(k,l) \in W(x,y)\} \quad -- (6.5)$$

Once the best fit window has been determined, the surface fitted at the point (x,y) is represented using the set of coefficients a(x - u, y - v) through f(x - u, y - v), The fitted depth value at point (x,y) is determined by

$$z(x,y) = a(x - u,y - v)u^2 + b(x - u,y - v)v^2 + c(x - u,y - v)uv + d(x - u,y - v)u + e(x - u,y - v)v + f(x - u,y - v) \quad --(6.6)$$

This is the selective local surface fit. This is compared with the actual surface values to check the accuracy of the fit. If required, this equation can be used to derive a smoothing operator for reducing the noise effects in the data whilst preserving the edges in the data (see Yokoya and Levine, 1989 for details). In this work, an alternative algorithm was used for smoothing the data which is described section 6.2.5.

Differentiating equation (6.3) and using the selective local surface fit, the first and second partial derivative estimates at point(x,y) are

$$\frac{\partial z(x,y)}{\partial x} = 2a(x - u,y - v)u + c(x - u,y - v)v + d(x - u,y - v)$$

$$\frac{\partial z(x,y)}{\partial y} = 2b(x - u,y - v)v + c(x - u,y - v)u + e(x - u,y - v)$$

$$\frac{\partial^2 z(x,y)}{\partial x^2} = 2a(x - u,y - v)$$

$$\frac{\partial^2 z(x,y)}{\partial y^2} = 2b(x - u,y - v)$$

$$\frac{\partial^2 z(x,y)}{\partial x \partial y} = \frac{\partial^2 z(x,y)}{\partial x \partial y} = c(x - u, y - v) \quad \text{-- (6.7)}$$

Substituting into these equations for the coefficients of the best-fit window gives a set of partial derivative estimates at point (x,y). The Gaussian (K) and mean (H) curvatures for each point are computed from these as follows:

$$K = \frac{\frac{\partial^2 z}{\partial x^2} * \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2}{\left(1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)^2}$$

$$H = \frac{\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} * \left(\frac{\partial z}{\partial y}\right)^2 + \frac{\partial^2 z}{\partial y^2} * \left(\frac{\partial z}{\partial x}\right)^2 - 2 * \frac{\partial z}{\partial x} * \frac{\partial z}{\partial y} * \frac{\partial^2 z}{\partial x \partial y}}{2 \left(1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)^{3/2}} \quad \text{-- (6.8)}$$

### 6.2.3 Optimizing the surface sample

For these two algorithms, the surface sample was optimized by varying the size of the window used for the convolution (in the Besl and Jain algorithm) or to fit the bi-quadratic functions (in the Yokoya and Levine algorithm). If the window size is too small, the effect of noise becomes more significant. On the other hand, if the window size is too large, then the values obtained for the surface curvatures may not adequately reflected the 3D structure of the surface (figure 6.2). This latter case, has a similar effect to smoothing the surface data before performing the surface curvature calculation.

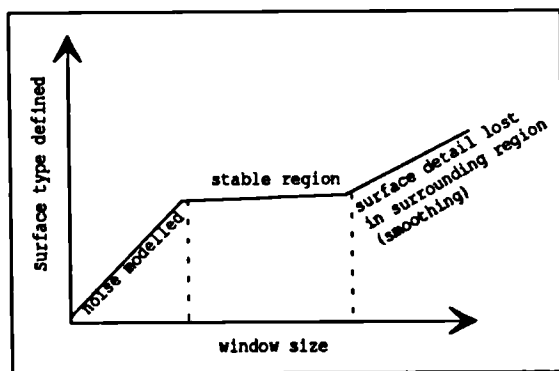


Figure 6.2: Effect on defined surface type with increasing window size.

A number of different window sizes were implemented for both algorithms, these ranged from 3 x 3 through 13 x 13. The general formulation, as well as some explicit formula, for the convolution matrices or the quadratic functions for these windows sizes are given in Besl and Jain (1988) and Yokoya and Levine (1989) respectively.

It was decided that a size of 7 x 7 was the smallest window size that gave rise to a description of the surface that was stable to noise in the data. Therefore, a window size of 7 x 7 was used to evaluate the performance of these algorithms on regular test objects and the face (see section 6.4).

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### **6.2.4 Least squares fitting of surface patches**

The first two methods that were used to calculate the surface curvatures were based on the extraction of a depth map of the surface from a particular viewpoint. The depth map approximation is not truly independent of surface orientation since the areas of the surface represented on the depth map by the image pixels, and therefore used by the convolution windows and surface fit algorithms, are *not constant* but depend on the slope of the surface as seen from the chosen viewpoint. If the slope appears great then the image pixels will represent larger areas of the surface than if the slope is shallow. For the face, seen from the anterior view, this means that it is quite likely that large errors will occur at the edges of the facial image and on the sides of the nose. Conversely, the method now presented of least squares fitting is conducted on the *actual* surface data and does not require the extraction of a depth map from a particular viewpoint. The curvatures of the surface are calculated directly from the acquired 3D data and so this method *is* fully independent of the orientation of the data.

In this method, a least squares quadratic surface is fitted about each point in the data. The coefficients of this fitted surface are used, as in the Yokoya-Levine algorithm, to calculate the partial derivatives and hence the Gaussian and mean curvatures.

However, before the least squares surface is fitted a number of steps are taken to ensure a good fit to the data. Firstly for each point, which will subsequently be referred to as the "current point", all the surrounding points within a given radius are found (limits on the maximum number of profiles and scanlines along the profile, either side of the current point are set). The surrounding points are used to fit a least squares surface patch to the current point, I will call these the "patch points". Next the transformation matrix,  $M$ , which orientates the surface normal of the current point outwards along the  $z$  axis (the radial distance) is found. All the patch points are then transformed by the matrix  $M$ , so that the orientation of the patch is now centred along the  $z$  axis. This is to standardize the direction from which the patch is fitted and the curvatures calculated. The derivation of the transformation matrix is based on geometrical consideration's and is detailed below.

**Lemma: Calculation of transformation matrix to align the surface normal of the current point along the  $z$  axis.**

If we consider the surface normal at a point on the surface  $(x,y,z)$  to be pointing in some direction in space with components  $(x_n, y_n, z_n)$ , its orientation to the  $z$  axis  $(0,0,1)$  can be achieved by two rotations. First a rotation about the  $y$  axis by an angle  $\delta$ , followed by a rotation about the  $x$  axis by an angle  $(90 - \beta)$ , where  $\beta$  is the angle made by the surface normal vector to the  $y$  axis.



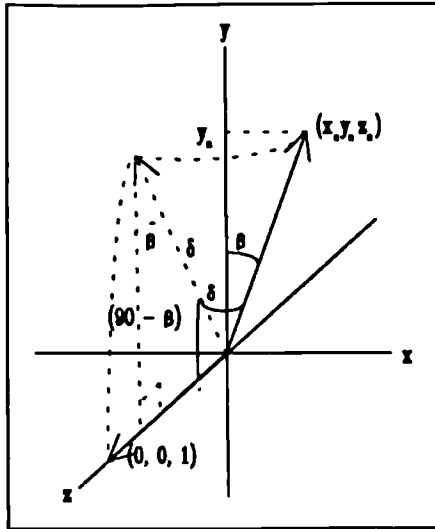


Figure 6.3: Alignment of a surface normal vector  $(x_n, y_n, z_n)$  to  $(0,0,1)$  by a rotation of  $\delta$  about the  $y$  axis and  $(90 - \beta)$  about the  $x$  axis. Clockwise is a positive rotation.

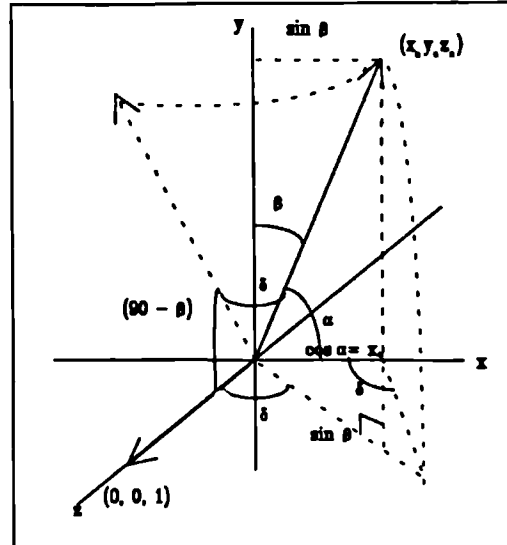


Figure 6.4: Illustration showing how the angle  $\delta$  may be defined.

Now from trigonometry,

$$\begin{aligned} \cos \beta &= y_n \text{ and } \sin \beta = \sqrt{(1-y_n^2)} \\ \cos (90 - \beta) &= \sqrt{(1-y_n^2)} \text{ and } \sin (90 - \beta) = y_n \end{aligned} \quad \text{-- (6.9)}$$

Also,

$$\sin \delta = \cos \alpha / \sin \beta,$$

where  $\alpha$  is the angle the surface normal vector makes with the  $x$  axis.

therefore,

$$\sin \delta = x_n / \sqrt{(1-y_n^2)}$$

and using the relation,  $z_n^2 + y_n^2 + x_n^2 = 1$

$$\cos \delta = z_n / \sqrt{(1-y_n^2)} \quad \text{-- (6.10)}$$

Matrices  $R_x$  and  $R_y$  are standard matrices which rotate a vector about the  $x$  and  $y$  axis respectively. They are

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(90-\beta) & -\sin(90-\beta) \\ 0 & \sin(90-\beta) & \cos(90-\beta) \end{pmatrix} \quad R_y = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix}$$

Multiplying these together gives the combined rotation matrix  $M$ ;

$$\begin{pmatrix} \cos\delta & 0 & \sin\delta \\ \sin(90-\beta)\sin\delta & \cos(90-\beta) & -\sin(90-\beta)\cos\delta \\ -\cos(90-\beta)\sin\delta & \sin(90-\beta) & \cos(90-\beta)\cos\delta \end{pmatrix} \quad \text{-- (6.11)}$$

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Substituting for these in terms of the surface normal components;

$$M = \begin{pmatrix} z_n/\sqrt{(1-y_n^2)} & 0 & x_n/\sqrt{(1-y_n^2)} \\ y_n x_n/\sqrt{(1-y_n^2)} & \sqrt{(1-y_n^2)} & -y_n z_n/\sqrt{(1-y_n^2)} \\ -x_n & y_n & z_n \end{pmatrix} \quad \text{-- (6.12)}$$

which is the form of the matrix required.

Verification of this matrix can be obtained from checking the result of

$$M \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{-- (6.13)}$$

It was also verified by plotting the components along each of the axes for a test patch of face data. The appearance of the patch of data viewed along the z axis should be a roughly circular patch and along the x and y axis, where the transformed data is seen edge-on, a relatively linear patch or elongated ellipse should be seen. Thus the matrix M is used to transform all the patch points, and their surface normals thus

$$M \begin{pmatrix} x_{\text{data}} \\ y_{\text{data}} \\ z_{\text{data}} \end{pmatrix} = \begin{pmatrix} x'_{\text{data}} \\ y'_{\text{data}} \\ z'_{\text{data}} \end{pmatrix} \quad \text{-- (6.14)}$$

so that the patch is orientated along the z-axis.

#### **a) Fitting a least squares quadratic surface patch**

The transformed patch points are now fitted by a least squares quadratic surface. The mathematical detail of this is given in Lancaster and Salkaukas (1986, pp147-151). A quadratic surface cannot be well fitted to a discontinuous surface. Fortunately, the facial surface is a largely continuous surface and providing the size of surface patch fitted is not too large, little surface detail is lost. Fitting a least squares surface does not always yield a unique solution (see Lancaster and Salkaukas for details), but this is not a problem for this application.

Each of the set of  $(x_i, y_i)$  data points in the patch, where  $i = 0, 1, \dots, N$  have a value "zdata". It is assumed that the surface data is approximated by the quadratic function z of the form:

$$z(x,y) = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \quad \text{-- (6.15)}$$

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for some choice of coefficients  $a_0$  through  $a_5$ . At each point in the data the difference between the surface elevation,  $z$ , and actual surface elevation,  $zdata$ , is  $z(x_i, y_i) - zdata_i$ . For a least squares solution, function  $z$  is adjusted to minimize:

$$E(z) = \sum_{i=0}^N (z(x_i, y_i) - zdata_i)^2 \quad -- (6.16)$$

This is achieved by altering the choice of values for the coefficients  $a_0$  through  $a_5$ . The accuracy of the surface patch fitted to the data was calculated by comparing the patch fit at the current point with both i) the current points actual depth value and ii) the mean fitting error for the patch.

The partial derivatives of the equation can then be derived allowing the Gaussian and mean curvatures at the current point to be calculated. This entire process is repeated for all the points in the data. For a test image of 18 profiles, this algorithm took just over 3 minutes to run on one Transputer. A typical facial data set contains approximately 210 profiles therefore, the projected running time for one face was approximately 40 minutes, which is very slow.

#### **b) Using a variable patch size.**

The accuracy of the surface fit may be improved if the size of the patch fitted around each point is allowed to vary. This forces larger patches to be fitted over shallowly curved surfaces, such as the cheeks, and smaller patches to be fitted at more highly curved portions of the face, such as the nose. Thus programme was iterated with the size of the patch being decreased from a maximum radial size (eg. 12 mm), removing the most distant point each iteration, until the mean error of the fitted patch to the patch points was within a predefined limit (eg. 0.5 mm). To ensure that the patch did not grow too small during this process a minimum patch size was also defined (eg. 2 mm). If the programme reached the minimum size and no fit was found within the error allowed, the curvatures were assigned a dummy value to indicate this and no surface type was assigned to that point, which was displayed as black on the surface type image. The orientation of the points' surface normals were checked to ensure that no points were included in the patch that were located on a backward facing surface. Without this check, the quadratic surface fit function would treat these points as if they were located on the same surface. On the face, this could lead to possibly quite significant errors around the nose bridge region.

For this variable-patch size algorithm, the calculation time would obviously be a lot longer than before because of the iterative procedure, which fits several patches for each one fitted in the fixed-patch algorithm. In view of this anticipated long run time, consideration was given to what could reasonably be done to shorten it. The options

### ***Facial Description***

were as follows: i) to further optimize the code or ii) to implement the code on more than one Transputer ie. in parallel. If the run time could not be quickly shortened then the option to run the programme overnight remained. The first option could obviously be done fairly quickly and would have significant effect on the run time, although the run time would still remain long. The second would require further programming and strategy as well as the purchase (or loan) a number of Transputers. Approximately 4 to 8 Transputers would be required to bring the run time down to around 10-20 minutes. Since the cost of Transputers is approximately £1500 each and the necessary funds were not available and neither at this advanced stage in the work, was the time required for the programming, this course of action was not embarked upon. The code was optimized and the programme run overnight.

The following optimization of the code was carried out. The distances of the patch points from the current point were calculated. These were then sorted by order of distance from the current point using a standard heap-sort type algorithm (Press et al, 1986, pp.231-232). This allowed the number of points included in the patch to be decreased by one (or more) at a time as the patch retreated inwards, without having to recalculate these distances every iteration. One could simply read the first  $n$  points from an array and use them. Everything that could be removed from inside the iterative loop was removed. When this was completed, the computation for one head (typically 210 profiles) still took some 3 hours to run.

The minimum and maximum patch sizes, as well as the fitting error of the patch were selected by experimentation on a test image. The test image was actually a portion of a real laser scan of the face (figure 6.5). The criteria used for selection were that the surface type representation produced (see section 6.3.2) would contain as few as possible single pixel patches and the patches would not appear too "blocky" in nature. Most importantly the description should be stable to small variations of these initial conditions. I found that the following sizes produce a good description. A minimum patch size of 2.0 mm, containing some 12 points, and maximum patch 20.0 mm, containing some 200 points. The mean error between the patch of data and the fitted surface was either 0.5mm or 1.0mm.

Comparisons were made between the fixed patch size method and the variable patch size method for the test image (figure 6.5). These showed that the variable patch method was more noise tolerant than the fixed patch method resulting in tidier surface type patches at a smaller error.

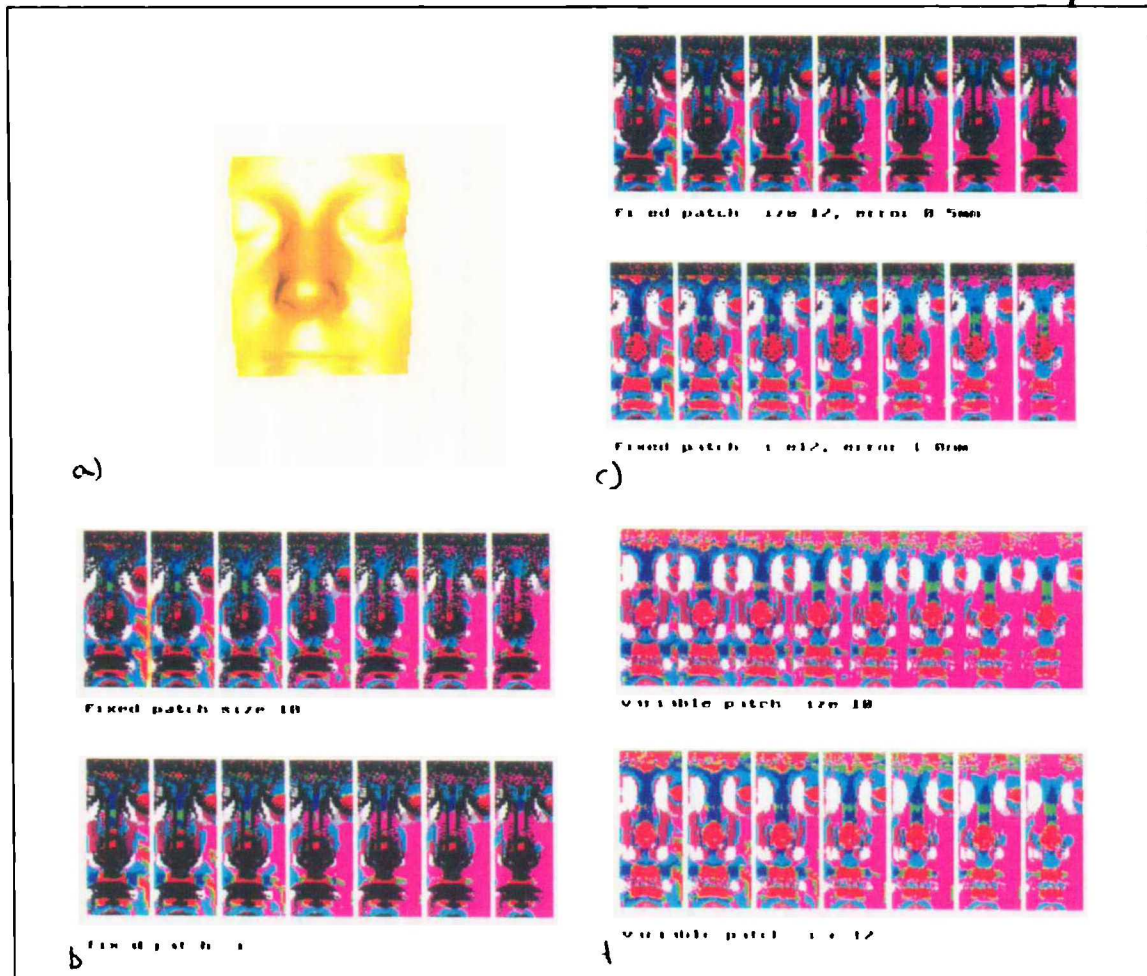


Figure 6.5: a) Test optical surface scan image used and surface type descriptions of the test image with increasing thresholds from left to right for b) fixed patches of size 10mm and 12mm to a fit error of 0.5mm c) fixed patch of 12mm to a fit of 0.5mm and 1.0mm and d) variable patches of size 10mm and 12mm.

### 6.2.5 Smoothing the data

Altering the tolerance on the allowable error for the fitted patch is similar in effect to smoothing the data. Additional smoothing can be achieved by pre processing the data using a low pass filter. Some smoothing pre-processing was used for the older, more noisy scans. The algorithm used to smooth the data averaged the data values at a point's neighbours in the manner shown in figure 6.6.

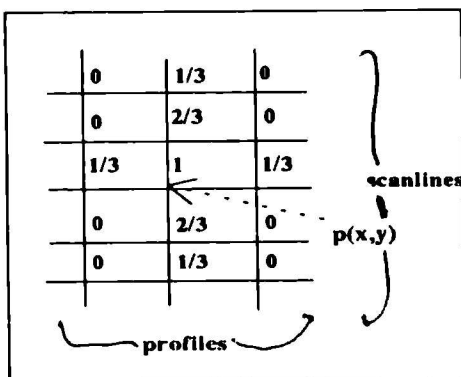


Figure 6.6: Smoothing of data using a low pass filter showing the contribution of adjacent point to the point  $p(x,y)$

The effect of smoothing the data was assessed by smoothing one laser scan a number of times. The effect on the surface type description can be seen in figure 6.7. Some of the smaller surface type patches were removed (such as the vertical green, ridge lines on the cheeks) and some smaller patches were amalgamated together (eg. the blue



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saddle ridge patches corresponding to the philtrum of the upper lip). The shape of the remaining patches were smoother, but well preserved - even after smoothing 10 times. An evaluation was made of the amounts of each surface type on the face, at 11 chosen threshold levels and for the levels of smoothing shown in figure 6.7. This revealed only very small changes in the amount of surface type between smoothing levels. The *greatest difference* between the original unsmoothed representation and the 10 times smoothed one was for the flat surfaces which increased by 2%. The least abundant surface type, the valleys, changed by only 0.4%.

This study suggests that smoothing the data and hence the removal of data noise, *does not* effect the surface type representation drastically. Indeed, if a large amount of smoothing is made then the gross facial structure should be more easily extracted by the surface encoding scheme.

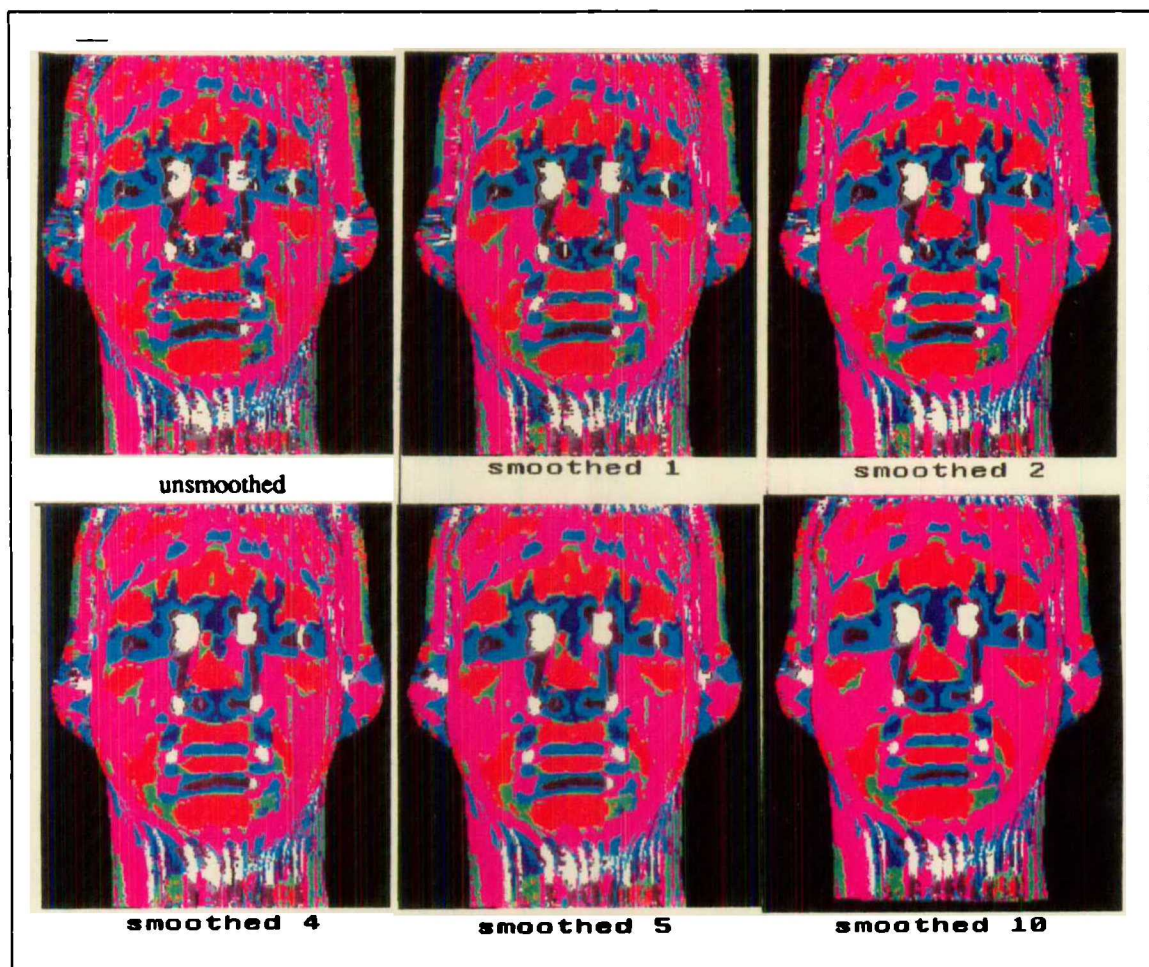


Figure 6.7: The effect of smoothing the data on the surface type description of an individual. Amount of smoothing indicated.

### **6.3 Representation of the curvatures values**

Once the curvature values for each point on the surface have been calculated a method is required for visualizing this information in the simplest possible sense. Two methods are presented for graphically representing the data.

### 6.3.1 The KH-map

The magnitude and distribution of the values of surface curvatures together with the frequency with which they occur on a surface, can be illustrated by plotting a graph of the Gaussian curvature,  $K$ , (as ordinate) against the mean curvature,  $H$ , (as abscissa). A form of this graph was first drawn up by Yokoya and Levine (1989). Eight *surface types* can be determined by the signs of the two curvatures and correspond to certain regions on this graph (figure 6.8). A forbidden region exists above the curve  $K = H^2$ , where it is physically impossible to obtain such a surface. In my version of these graphs (the KH-map), the axis scale for each surface curvature was defined as  $\pm 2$  standard deviations of the most frequently occurring curvature value. 50 bins were allotted to the four standard deviations and the frequency of occurrence of values of Gaussian and mean curvature within the bins are displayed as one of six intensities, with darker shades indicating more frequently occurring values. An end bin on each axis was used to indicate values outside the axis scale range. These are the bottom and right hand bins respectively. Figure 6.9 shows an example for the surface curvature values calculated for one face.

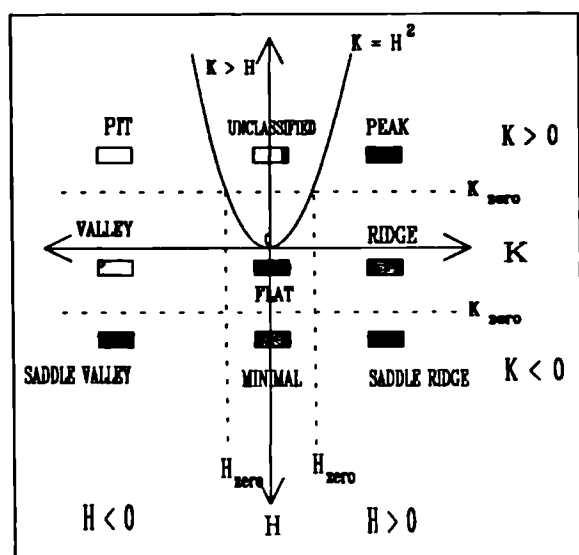


Figure 6.8: KH-map. This plot of mean curvature ( $H$ ) vs Gaussian curvature ( $K$ ) demonstrates how thresholds are applied to segment into regions of each surface type.

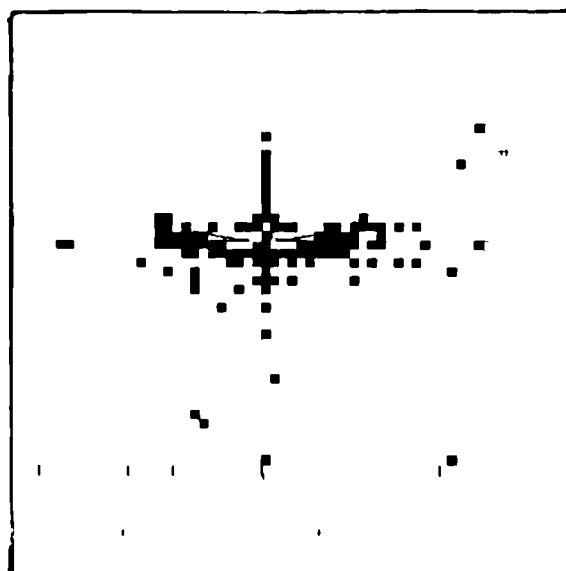


Figure 6.9: KH-map for a face. Blue squares indicate single points, grey squares of increasing density indicate multiple occurrences of points.

### 6.3.2 Surface type description

As mentioned above, the computed values for Gaussian and mean curvatures of the surface can be used to define the "surface type" at each image pixel by consideration of the sign of both curvatures (Besl and Jain, 1988) (figure 6.10). The sign of the curvature is calculated by comparing the curvature value with a pre-selected threshold value. For example for the mean curvature,  $H$ :



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$$\begin{array}{ll}
 H > \text{threshold value} & \text{signH} = +1 \\
 |H| < \text{threshold value} & \text{signH} = 0 \\
 H < -\text{threshold value} & \text{signH} = -1
 \end{array}
 \quad \text{-- (6.17)}$$

and similarly for the Gaussian curvature, K. The surface type at the pixel, T(x,y) is calculated from these signs:

$$T(x,y) = 1 + 3(1 + \text{signH}(H(x,y)) + (1 - \text{signK}(K(x,y)))) \quad \text{-- (6.18)}$$

By colour-coding the pixel according to its surface type a "surface type image" (or STI) may be produced. This is a useful, readily understandable way of displaying the information. These surface types together with the colour coding that I have used throughout this work to represent them are shown in figure 6.10.

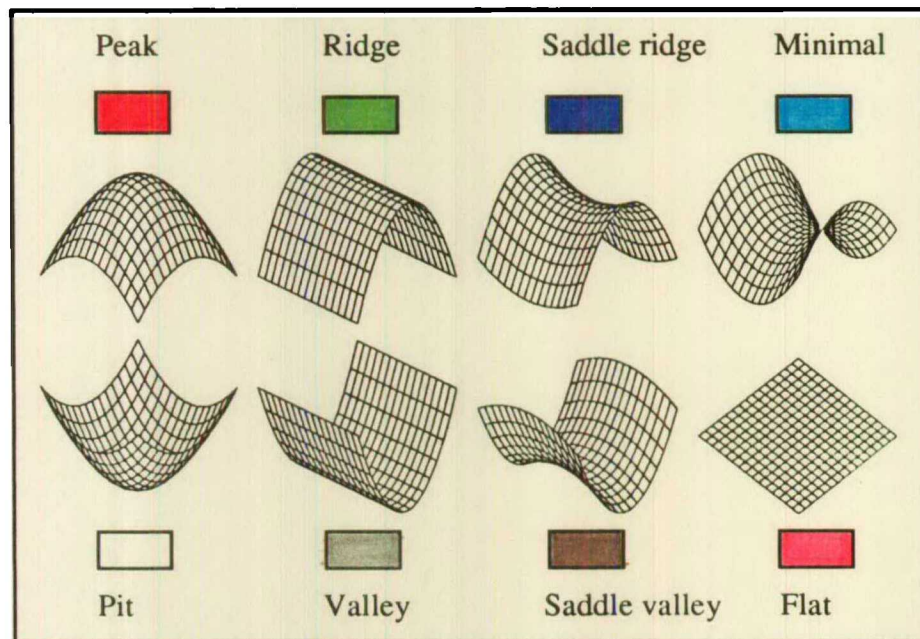


Figure 6.10: The eight fundamental surface types.

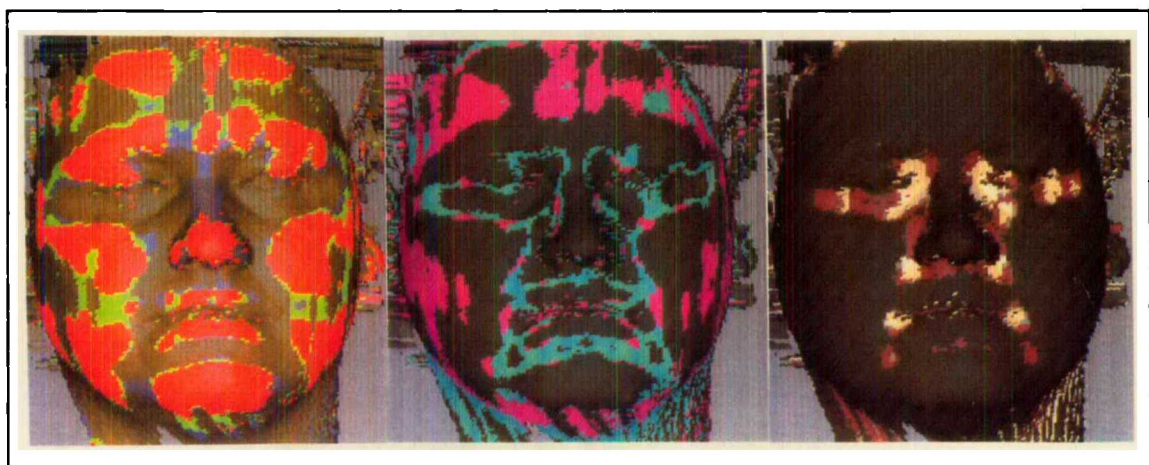


Figure 6.11: Left to right, surfaces of positive, zero and negative mean curvature superimposed onto the optical surface scan of the face



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Figure 6.11 shows separately for one face the surfaces of positive mean curvature (ie. peak, ridge, saddle ridge), zero mean curvature (flat and minimal) and negative mean curvature (pit, valley, saddle valley). The least squares fit method was used to calculate the surface curvatures.

#### **6.3.3 Encoding the face (threshold settings and susceptibility to noise)**

For the face, the data have some random variation from calibration errors or noise (see section 6.6). Therefore it is necessary to set thresholds on the curvatures,  $K_{zero}$  and  $H_{zero}$ , below which the surface is classified as flat. The surface type description is stable over a range of thresholds. Varying the thresholds also allows the surface to be described at a hierarchy of levels according to the magnitude of curvature of interest. The manner in which the threshold should be altered is still a matter for debate but the condition,  $K_{zero} \leq H_{zero}^2$ , must hold or some points will lie in the forbidden zone, which is geometrically impossible. The rule  $K_{zero} = H_{zero}^2$  was used to alter the thresholds, ie. along the parabola bounding the forbidden zone. Besl and Jain (1986) used the maxima of both curvatures and set the threshold values according to the amount of noise estimated to be in the image data. This estimation was obtained from fitting a equally-weighted least-squares planar surface to the surface. Yokoya and Levine (1989) did not address the problem specifically but pointed out that the optimal values for these thresholds is thought to depend on the level of noise in the data. They suggested scanning a reference flat surface and finding the minimum thresholds on K and H necessary to classify the surface as flat without falling into the forbidden zone. Besl and Jain (1988) attempted to eliminate noise from their data using a iterative region growing procedure, based on variable order bivariate surface fitting.

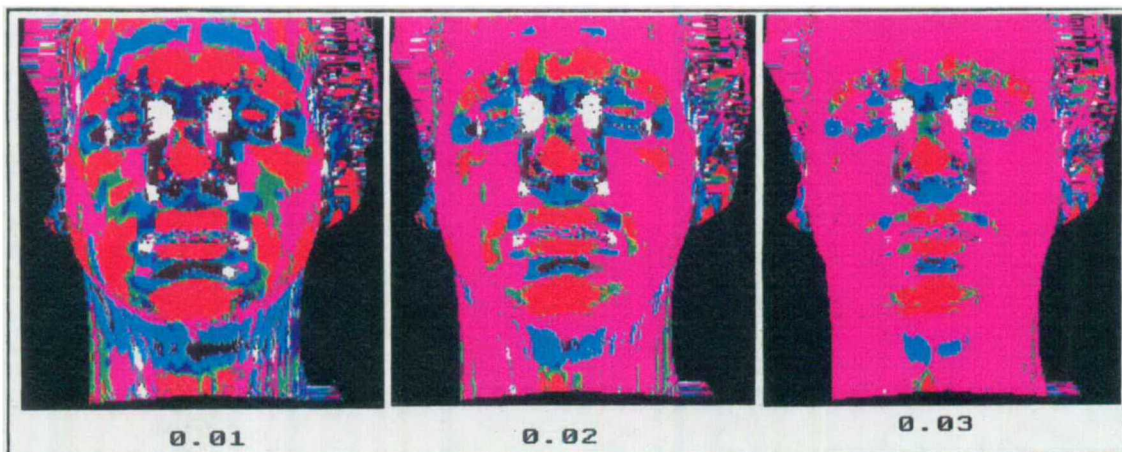


Figure 6.12: A face encoded into surface types at three different threshold levels. Left to right: low, medium and high thresholds (the threshold on H is indicated).

Applying this description to the face, when the surface curvature thresholds are set high (eg. figure 6.12 right), the most curved parts of the face are easily seen; represented as peaks, pits, saddle ridges and saddle valleys. These major facial features persist as the

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thresholds are lowered and less curved features are detected (see figure 6.12). The patches of each surface type that are produced by the description are perceptually meaningful and it is easy to identify such parts of the face as the chin, eye orbits, nose, lips and inner canthi of the eyes from visual inspection of the surface type image. So the segmentation of the face achieved by this approach matches the human interpretation. One way of summarizing the information contained by a series of these hierarchical STI's is by the use of graphs. In figure 6.13, the overall amount of each surface type present, as a percentage of the entire surface is shown for increasing thresholds on the curvatures for the face represented in figure 6.12. At high threshold levels, ie. very curved surfaces, a large amount of the face is classified as flat, only small regions of the face, which correspond to the major facial features are described by other surface types. At lower threshold levels, corresponding to less curved surface, smaller amounts of the surface are represented as flat and large amount by other surface types.

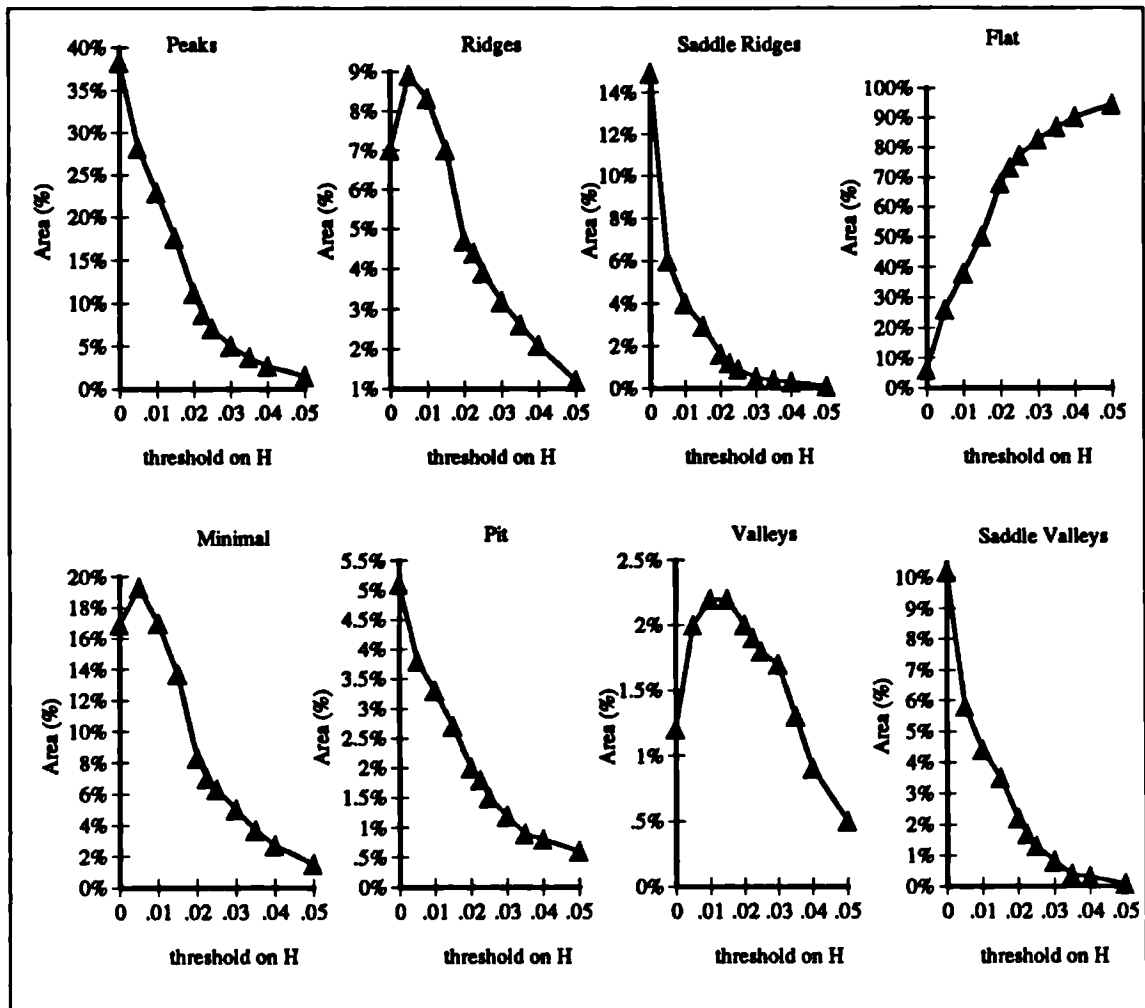


Figure 6.13: Variation of the total area of each surface type on one individual's face with increasing curvature thresholds.

## **6.4 Evaluation of curvature algorithms**

Two of the three algorithms for calculating surface curvatures described in this chapter have previously been reported, but to date no one has reported an assessment of their relative performance on either standardized, geometric surfaces or on anatomical surfaces such as the face. In this section, I assess the performances of all three algorithms described in section 6.2 for geometric surfaces and the face, using the KH-map and surface type representations. Where appropriate a window size of 7 x 7 was chosen for these tests, as stated in section 6.2.3.

### **6.4.1 Geometric objects**

The geometric test objects used to evaluate the performance of the curvature algorithms were spheres, cylinders and saddles. Data for these objects were obtained from either optical surface scanning the object, or calculated by one of two different ways (see appendix A for the equations of these solids and the code for generating this data).

In the first calculation, a test depth map of the object was written directly and then described by both the Besl and Jain and Yokoya and Levine algorithms (section a). These two algorithms rely on a depth map being extracted from the surface data, so the simulation of the depth map enabled an assessment to be made of whether the process of extracting a depth map from the surface data influenced the performance of these algorithms.

A second test object was mathematically generated as a series of profiles, simulating optical surface scan data that is free from noise or calibration errors. Descriptions of this test object were produced from all three algorithms (section b). Comparison of the descriptions produced from these two forms of the test object allowed some assessment to be made of the effect of extracting a depth map.

Finally, optical surface scans of geometric objects were taken and again all three algorithms used to describe them (section c). Comparison with the simulated test object (section b) allowed some indication of the robustness of the algorithms to noise in the acquired data to be made. The effect of data noise is evaluated in section 6.5.

Figure 6.14 shows an example of the KH-map representation and surface type description for the simulated cylindrical and saddle surface respectively produced using the Besl and Jain algorithm. Note the clustering of curvature values in different parts of the KH-map. One would expect that for a given geometric surface, the curvatures would cluster in a small region on the KH-map corresponding to the surface type that one associates with that surface (eg. a point in the peak section of the KH-map for a sphere). Thus the ability of the algorithm to produce a small distribution of points is a



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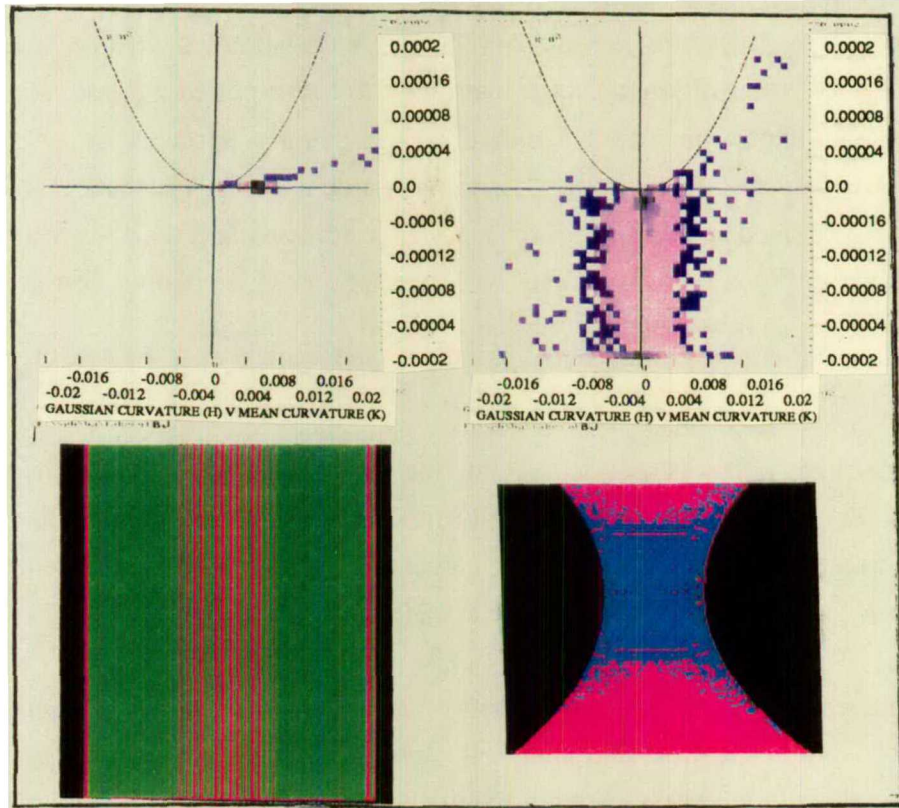


Figure 6.14: Above, KH-map representations and below, surface type descriptions for left, a cylinder and right, a saddle using the Besl and Jain algorithm.

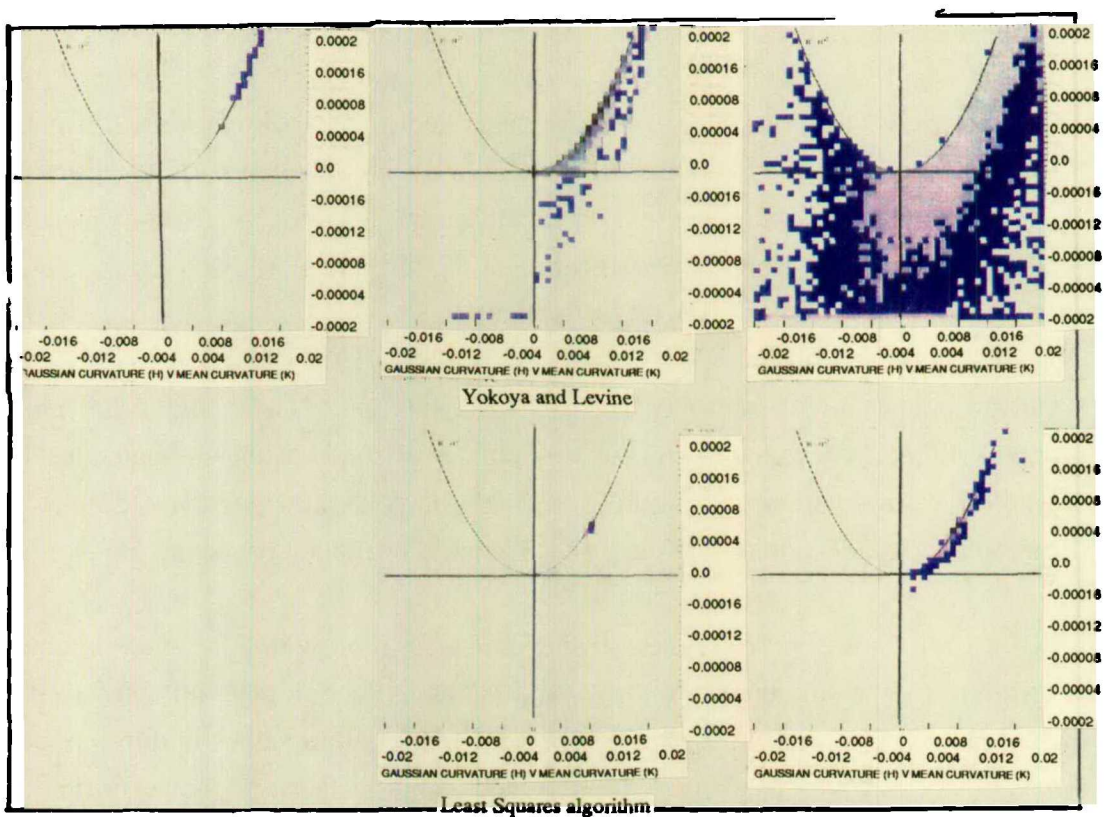


Figure 6.15: Comparison of the performance of the Yokoya and Levine algorithm (above) with the least squares algorithm (below) for spheres. Left, depth map generated sphere, centre, simulated sphere and right, optical surface scan of a sphere.

indication of its accuracy. For the cylinder, the cluster on the Gaussian curvature axis corresponds to the ridge surface and for the saddle surface the cluster around the mean curvature axis corresponds to the minimal surface.

a) Depth map generated surfaces

As mentioned previously, depth maps were directly written for the test objects and used to calculate the mean and Gaussian curvatures using the Besl and Jain or the Yokoya and Levine algorithm. The value of the calculated curvatures was determined for a small number of centrally located points on the surface. These were compared with the expected values of the mean and Gaussian curvatures for the test object (the sum and product of the principal curvatures respectively) to establish the accuracy of the algorithm. Very good agreement was found for both algorithms. In the case of the sphere, the algorithms were accurate to 0.5% for the mean curvature and 0.1% for the Gaussian curvature.

Further evaluation showed that there is a marked difference in the *distribution* of the values of the curvatures calculated by the algorithms. Considering the spherical test objects, whilst the curvatures calculated at the centre of a spherical surface are very accurate, towards to edge of the "field of view", the values obtained for the curvatures become much less accurate. This is because at the edges, a larger area of the surface is being represented by the image pixel, from which the curvature is calculated. These edge effects are visible as surface misclassification in the surface type images produced using the Besl and Jain algorithm (eg. see figure 6.16). The Yokoya and Levine algorithm's surface approximation method removes these edge effects.

The curvature values obtained using the Yokoya and Levine algorithm were more scattered on the KH-map than those obtained using the Besl and Jain algorithm.

b) Simulated scanned surfaces

The test objects were simulated in optical surface scan format by writing a series of profiles. For the Besl & Jain and Yokoya & Levine algorithms, a depth map was extracted from this data in order to calculate the surface curvatures. The least squares algorithm used the simulated data directly.

Interestingly, both depth-map based algorithms obtained *less accurate* curvature values for the simulated laser scanned sphere than the directly written depth-map (mean to 0.7%, Gaussian to 1.5%). The KH-map representations produced using these algorithms show that extracting a depth map from the laser scan data introduces a greater variation in the surface curvatures that are calculated using these algorithms (see figure 6.15 (top left and centre) for an example using the Yokoya and Levine algorithm).

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Figure 6.16 (above) shows a comparison of the surface type descriptions, at one threshold level, for the simulated sphere obtained from all three algorithms. The strong edge effects produced by the Besl and Jain algorithm can be clearly seen together with horizontal striations in both the Besl and Jain and the Yokoya and Levine algorithms. The least squares algorithm represents every pixel as peak surface. The close clustering of points on the KH-map produced by the least squares algorithm compared to the other algorithms (see figure 6.15 centre) demonstrates its greater accuracy over the other algorithms.

### **c) Optically scanned surfaces**

Three geometric objects were scanned; a sphere of 125mm radius, a cylinder and a saddle surface. The greater accuracy of the least squares algorithm compared with the other algorithms was demonstrated for these objects. In figure 6.15 (right), the much smaller variation of points on the KH-map for the least squares compared to the Yokoya and Levine algorithm is demonstrated. Figure 6.16 (below) shows the surface type descriptions produced by the three algorithms for the optically scanning sphere.

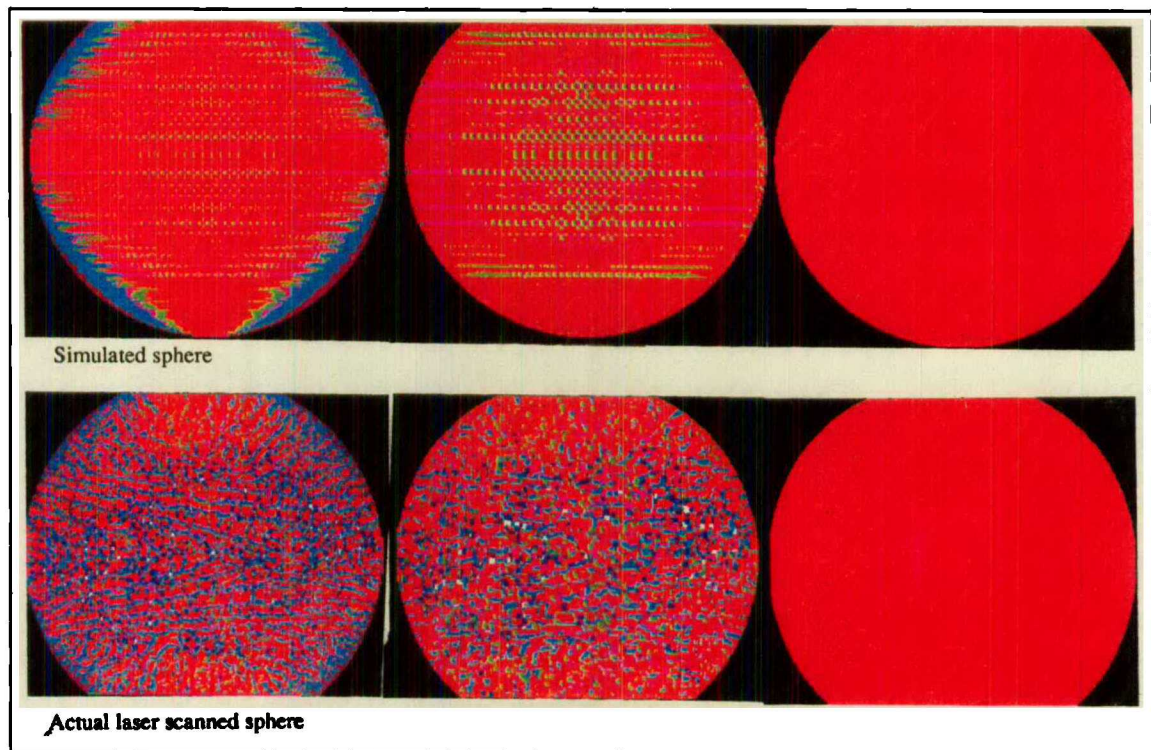
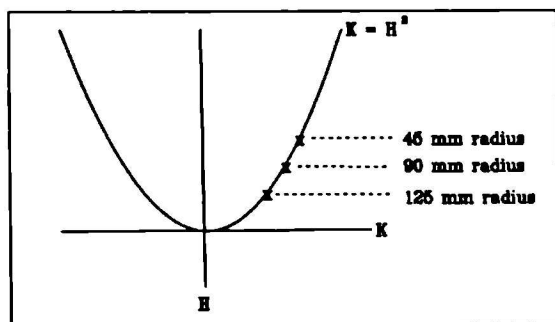


Figure 6.16: Comparison of the surface type descriptions produced by the Besl and Jain (left), Yokoya and Levine (centre) and irregularly sampled least squares (right) algorithms respectively. Above, a simulated sphere. Below, an optical surface scan.

Comparison of the optically scanned sphere (section c) with the simulated sphere (section b) revealed the imperfections in the laser scanned data, due to system noise. These differences, although detectable via the curvature values, do not greatly effect the surface type description produced when the least squares algorithm is used.



**d) Scale**



The effect of scale was investigated using three sizes of spheres with radii of 45mm, 90mm and 125mm respectively. KH-maps and surface type description's were generated for these spheres and it was shown that as the size of sphere increased, the position of the most frequently occurring curvature values moves down the

Figure 6.17: The effect of size of sphere on the distribution of curvature values.

parabola  $K = H^2$  towards zero (see figure 6.17). This was predicted as the surface represented by an image pixel is less curved for spheres of larger radii.

**e) Additional testing of the Yokoya-Levine algorithm**

In the Yokoya-Levine algorithm, the matrices given for the coefficients a through f and the calculated fitted depth were tested using test objects by considering the expected values for them. The objects used were a test plane (inclined in various directions), a test curve and the two test spheres, described above. The minimum error ( $E_{min}$ ) for these objects was also calculated giving an indication of the limitation of the algorithm in fitting these surfaces. The partial derivatives and values of the mean and Gaussian curvatures were also obtained. These allowed an assessment of the accuracy on the mean and Gaussian curvature values that will be obtained from this algorithm. The algorithm calculated values of the coefficients and depth were in close agreement with the expected values.

**6.4.2 The face**

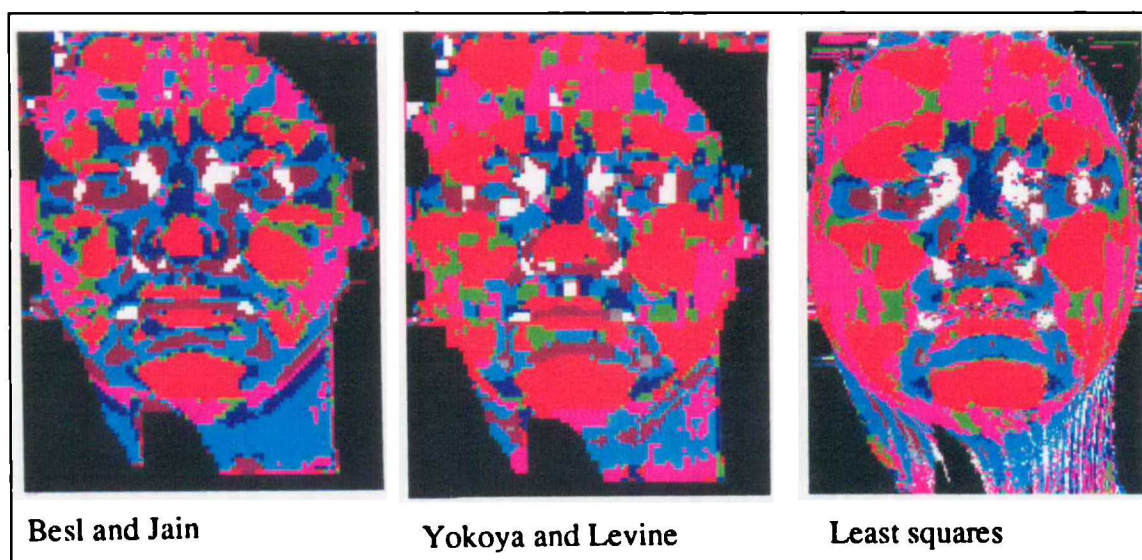


Figure 6.18 Comparison of the surface type descriptions produced for the same face using the three algorithms

### ***Facial Description***

The performance of the algorithms for the facial surface are shown in figure 6.18. Whilst all three algorithms represent the same portions of the face with the same or similar surfaces, the least squares representation demonstrates a clearer correspondence to the perceived facial surface structure and the patches of each surface type are less "blocky".

### **6.5 Sensitivity of the algorithms to noise**

The next step was to consider the effect on the representation of possible distortions in the optically scanned data. The errors involved in the collection of a data set for a person's face, from the UCL optical surface scanner, has been estimated to be 1mm in the vertical direction (ie. along a profile) and 0.6 mm between profiles (Moss et al, 1989). These errors could arise from noise which could be either random noise, and from several sources, or quantization noise due to the radial resolution of the optical scanner or from systematic problems to do with the measuring techniques employed.

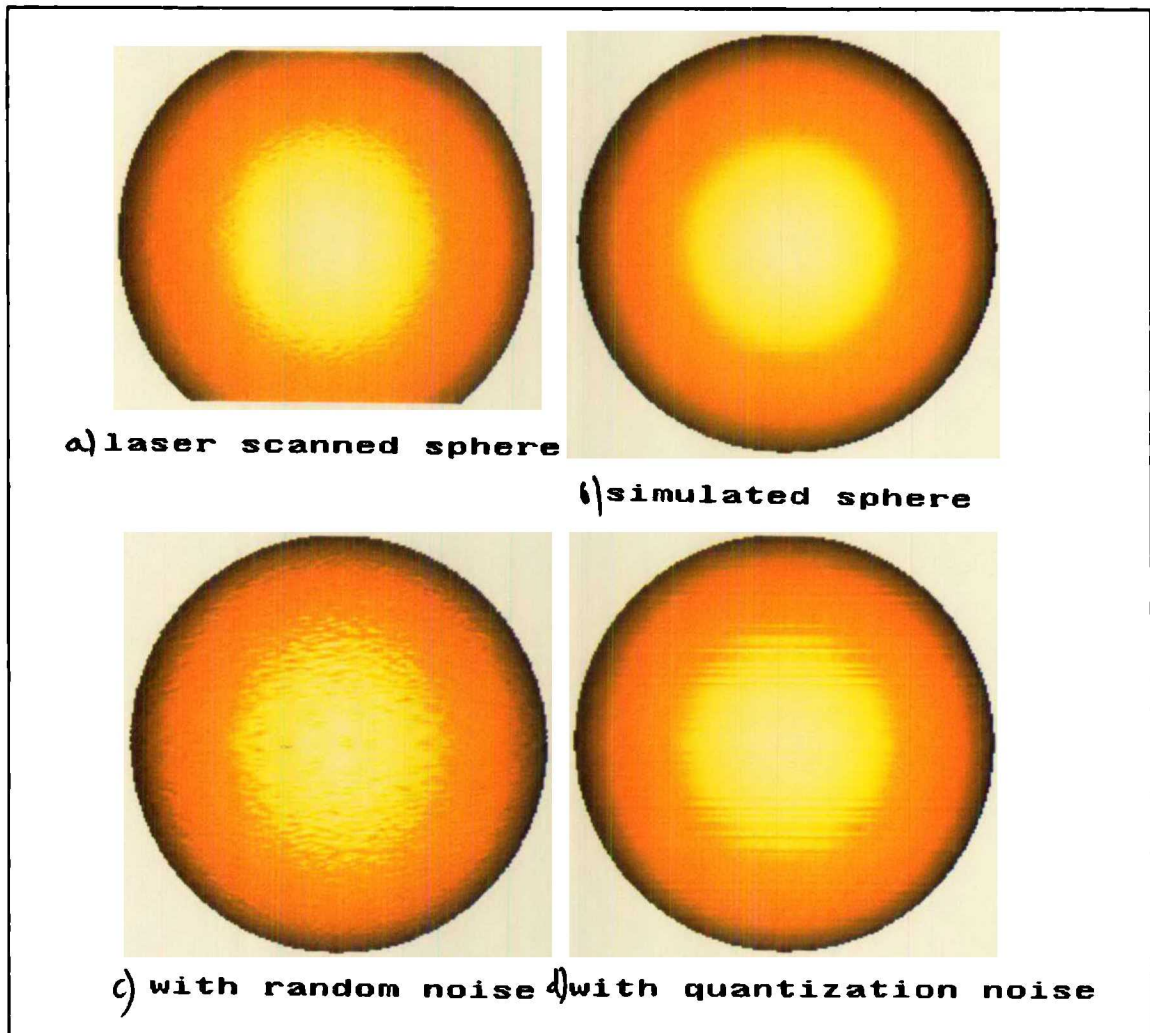


Figure 6.19: Comparison of an optically scanned sphere with a simulated sphere with noise added

For the UCL scanner, one systematic problem is known. This is due to the head not being correctly positioned above the axis of rotation of the chair on which the subject is



### *Facial Description*

seated. The effect of this is to alter the spatial resolution at which the surface is sampled and the presence of this error is noticeable in the distortion from horizontal of the quantization noise bands on the optically scanned sphere (see figure 6.16 below). This problem was noted but no full assessment of this systematic problem was made. The effect of noise is now described.

In order to make an overall assessment of the amount of noise present in the acquired data, the optical surface scan of a sphere, 125 mm in radius (figure 6.19a) was compared with a mathematically generated one of the same size (figure 6.19b). Various amounts of noise were added to the simulated sphere, and comparison was made with the optically scanned sphere until the KH-map and surface type descriptions corresponded well.

#### **6.5.1 The effect of random noise addition**

Two sorts of noise were added to the sphere to simulate the possible noise present in the acquired laser scans of facial surfaces. These were normally distributed (Gaussian) random noise and quantization noise. The random noise was calculated from a standard algorithm (Groeneveld p.108, 1979) where the noise added to the radial position of the surface,  $x$  is given by

$$x = x + sd * err$$

where  $sd$  is the standard deviation of noise to be added in mm and  $err$  is

$$err = \sqrt{-2 * \log(random1)} * \cos(2\pi * random2)$$

$random1$  and  $random2$  being random numbers between 0 and 1. This formula was tested by calculating the value of  $err$  100 000 times using different random numbers and checking the number of times each value of  $err$  occurred. This was indeed found to be a normal distribution (figure 6.20).

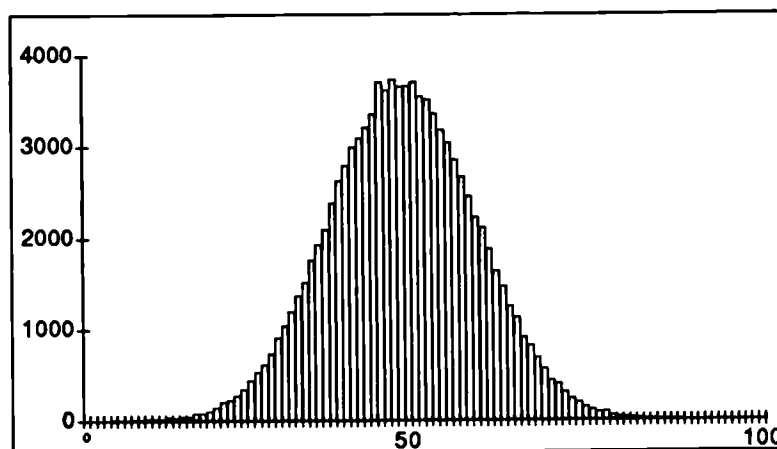


Figure 6.20: Distribution of 100 000 randomly generated numbers, normalised to represent a Gaussian distribution of noise

### ***Facial Description***

Various amounts of random noise were added to the simulated sphere (figure 6.19c) and KH-maps and surface type images were produced for each noisy sphere using the least squares algorithm (figure 6.21). Comparison of these representations with those produced for the optically scanned sphere (figures 6.15 bottom right and 6.16 bottom right) shows that not more than 0.2 mm of normally distributed random noise is present in the laser scans, if this noise is the sole cause of error in the surface curvature values.

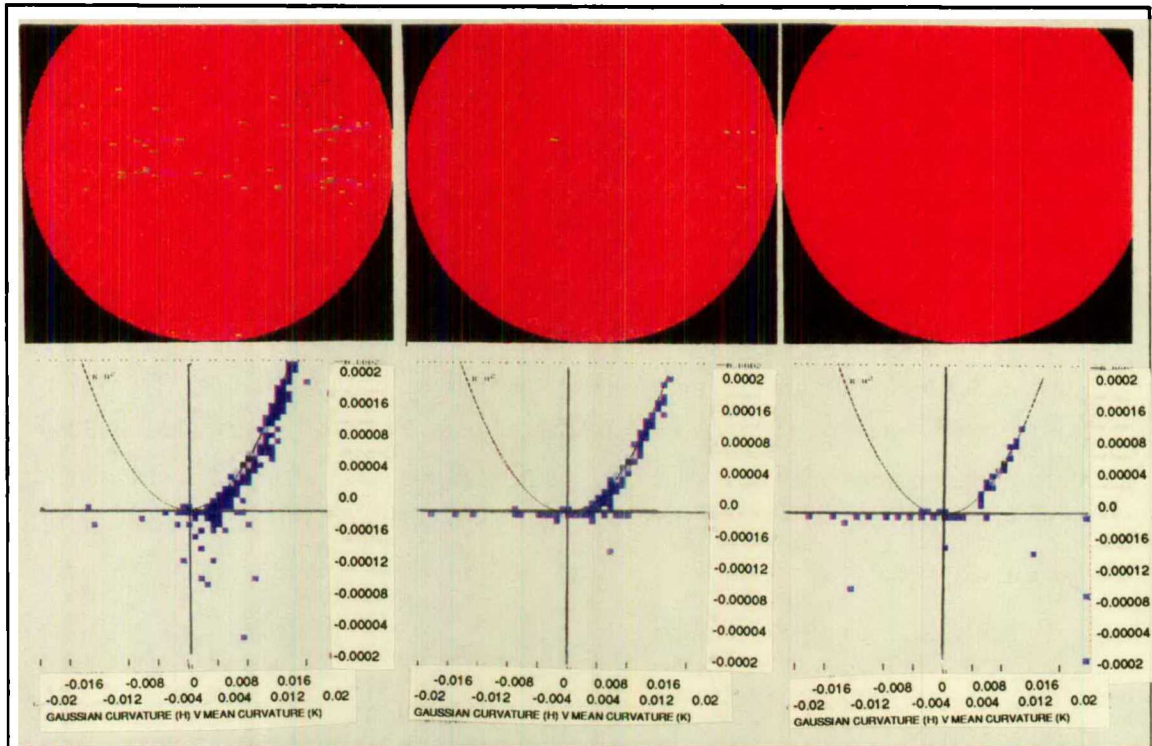


Figure 6.21: Effect of adding random noise to a sphere. Left, 0.5mm noise. Centre, 0.2mm noise. Right 0.1mm noise.

### **6.5.2 The effect of quantization noise**

In optical surface scans, quantization noise can arise due to the fact that points along the acquired profiles are digitized to the nearest pixel by the CCD camera. Hence a smooth spherical surface will be encoded with small horizontal bands running across it (see figure 6.19d). To simulate the effect of quantization noise, the real numbers values for the radial position of the surface, were rounded to the nearest 1mm, 0.5mm or 0.25 mm. A simulated sphere was again used to evaluate the effect of this kind of noise and three noisy spheres were produced with the above amounts of noise. KH-map and surface type representations were produced using the least squares method (figure 6.22) and comparisons were made with those obtained for the optically scanned sphere. These comparisons show that less than 0.25mm of quantization noise is present in the optically scanned sphere.

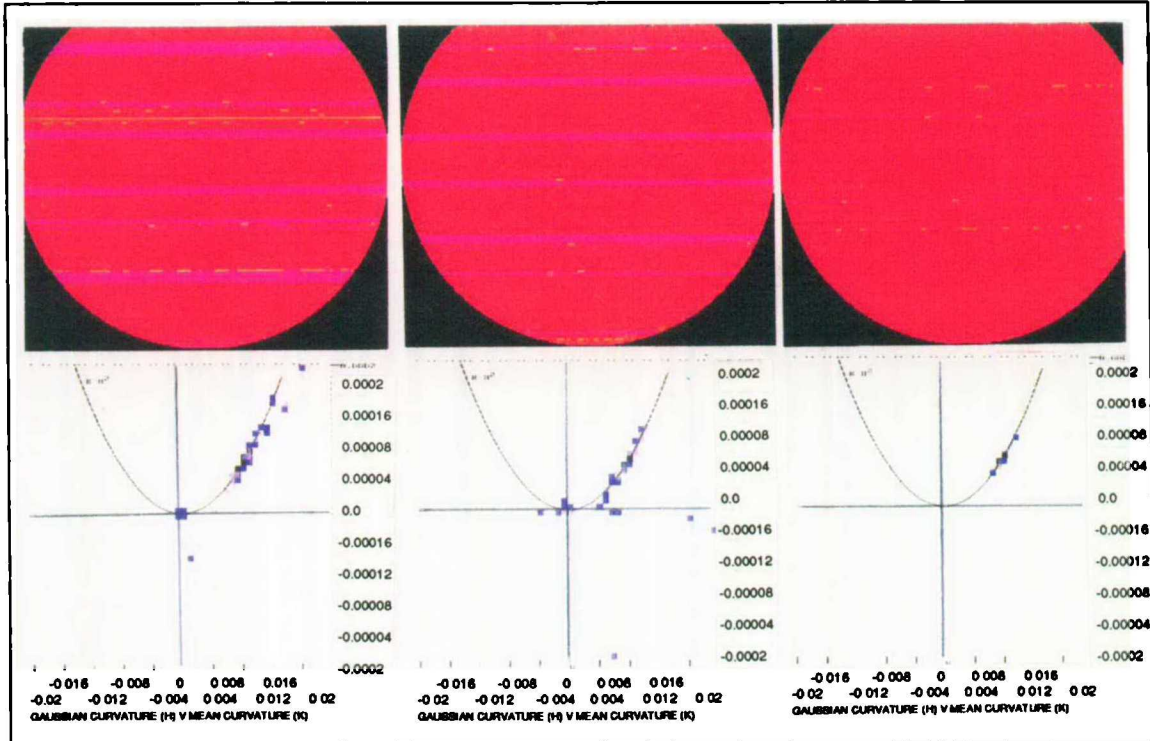


Figure 6.22: Effect of adding quantization noise to a sphere. Left, 1.0mm noise. Centre, 0.5mm noise. Right 0.25mm noise.

**6.5.3 Overall assessment of noise in laser scans**

The combined effect of random and quantization noise errors was assessed by generating a number of simulated spheres with various amounts of both types of noise. Again, the KH-maps and surface type images for these spheres were compared with those for the optically scanned sphere.

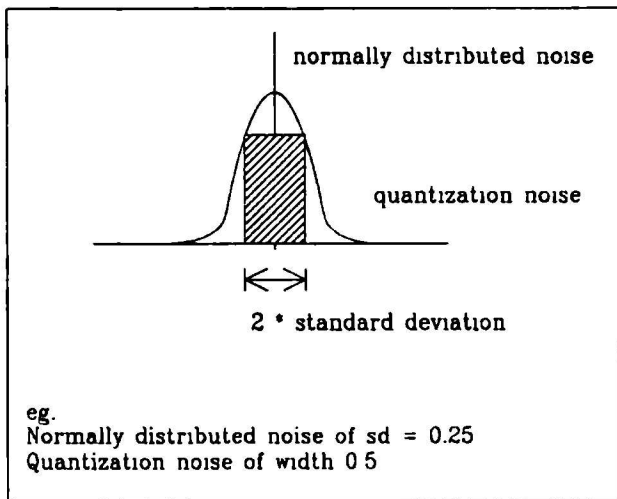


Figure 6.23: The relative widths of normally distributed and quantization noise.

Figure 6.23 shows that the normally distributed random noise should be half the quantization noise. Figure 6.24 shows that 0.25mm quantization noise and between 0.1mm and 0.15mm of normally distributed noise would give a good explanation of the KH-map and surface type image of the laser scanned sphere. Thus relative amounts of these two types of noise is consistent with theory.



## Facial Description

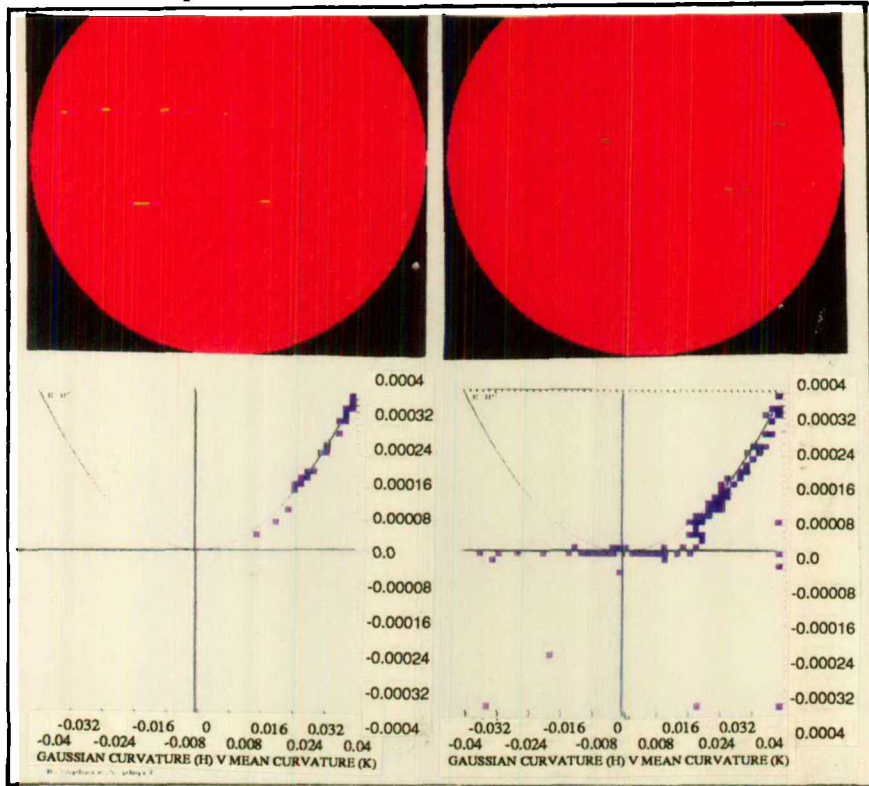


Figure 6.24: Above, surface type images and below, KH-maps for two combinations of normally distributed and quantization noise added to a simulated sphere.

0.25mm quantization noise &  
0.1mm random noise

0.25mm quantization noise &  
0.15mm random noise

## 6.6 Stability and repeatability of the facial description

### 6.6.1 Errors in facial surface scans

The optically scanned sphere, used to investigate the effect of data noise above, has been measured to be accurate to within 0.1mm at a point and is a rigid object. However, for a face, the amount of noise present could reasonably be expected to be greater than this value due to the difficulty in holding one's head absolutely still during the scan. Also, it is well known that a person grows shorter during the day due to gravity. Other factors which may possibly affect facial appearance are facial activity such as eating and talking and skin blemishes such as spots or changes of mood. These effects have not been measured. However, the optical surface scanner has been shown to provide accurate and repeatable scans (the stability of the optical surface was discussed in chapter 5).

### 6.6.2 Repeatability of the description for an individual

To estimate the difference between facial scans incorporating these differences in facial appearance, several scans of the same person were taken over a two week period. These were then registered together, ie. the orientation of the heads were matched using the technique described in chapter 7. The surface curvatures and surface type at every point were calculated and differences in the resulting surface type descriptions compared. Four of these scans and the corresponding surface type descriptions, at the same medium threshold level, are shown in figure 6.25.

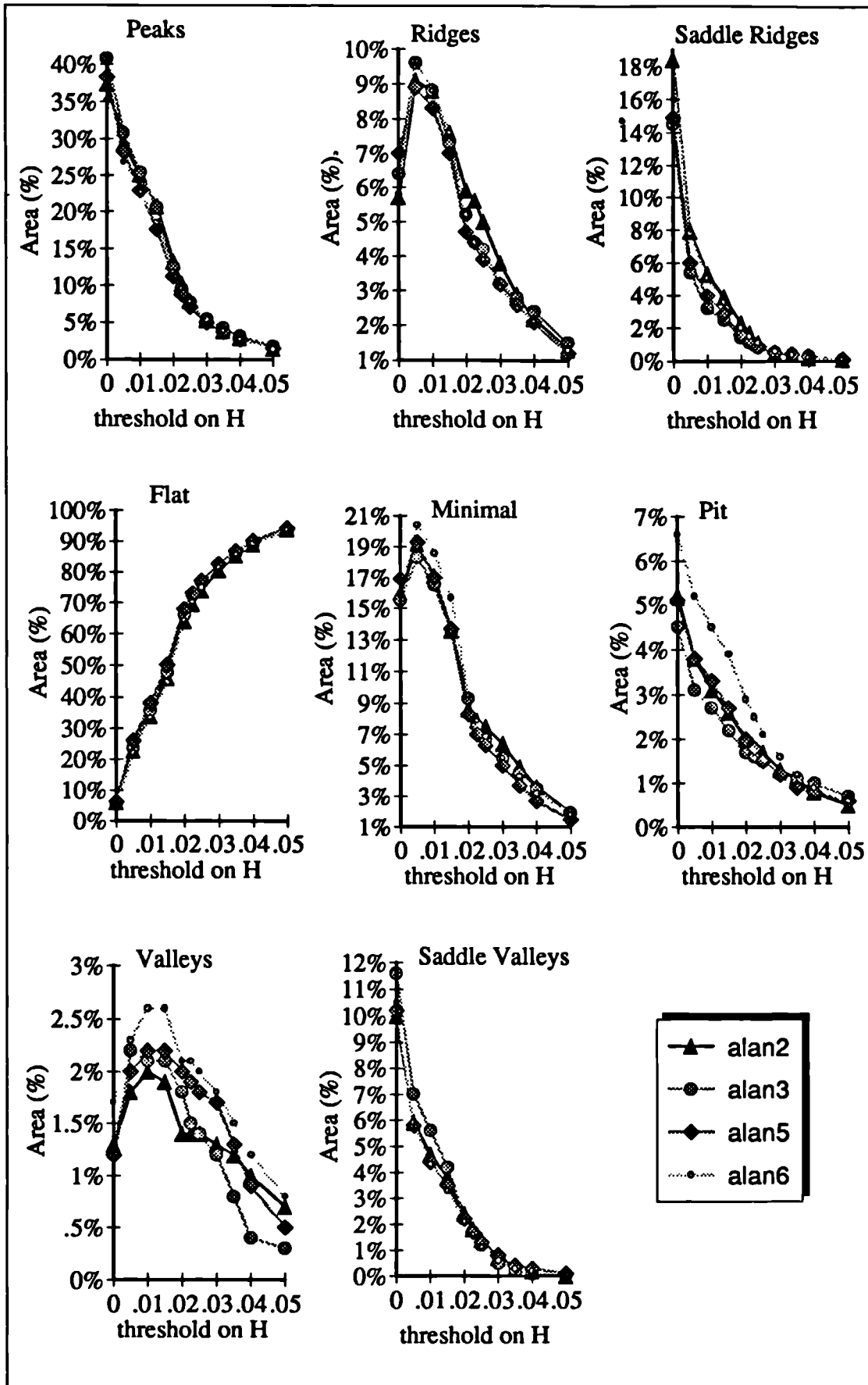


Figure 6.26: Variation of each surface type over the face for four scans of one individual.

## ***Facial Description***

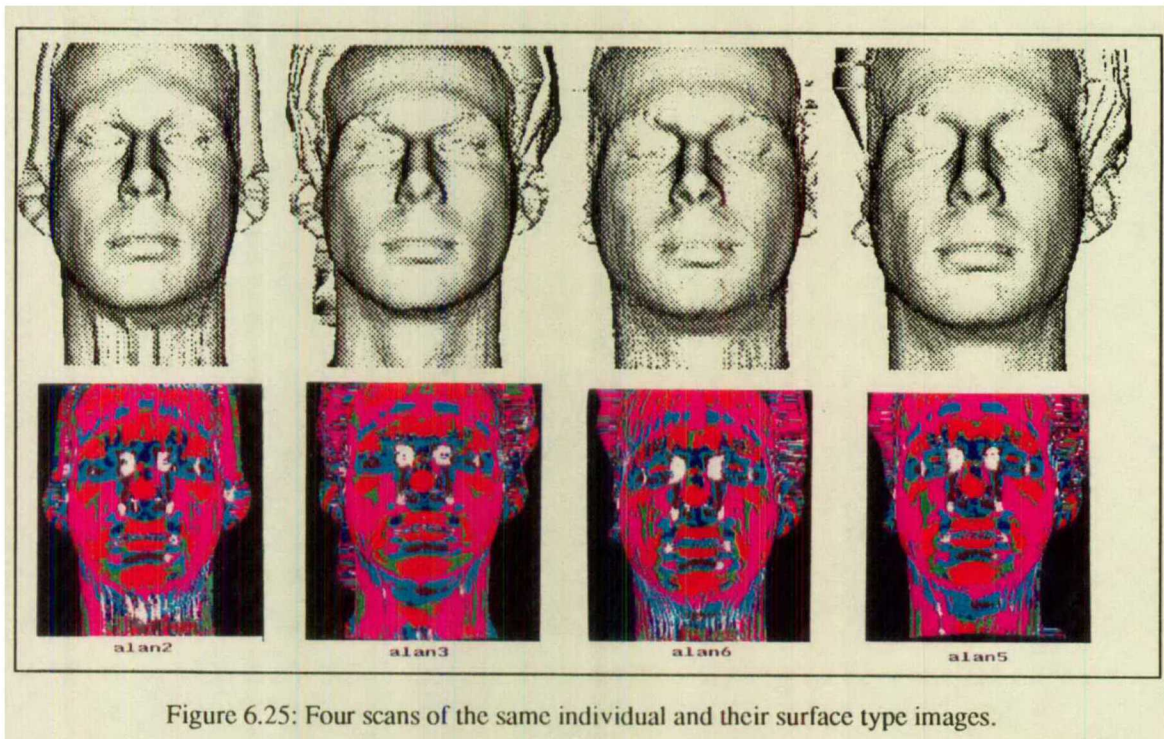


Figure 6.25: Four scans of the same individual and their surface type images.

Some small differences are visible especially in the more mobile parts of the face, such as green ridge lines on the cheeks and the eyes. Note also that in the left most scan the subject had a large spot between his brows that was not present when the other scans were taken. No smoothing of the data was made.

Figure 6.26 compares of the amount of each surface type on the face with increasing thresholds, for the four surface type descriptions shown in figure 6.25. Variations in the amount of each surface type between scans are 1% of the entire surface or better. This is more significant for some surface types others, depending on the proportion of the surface which they cover. Thus for the valleys this difference is more significant that for the peaks. In general, good agreement is shown between the four analyses.

### **6.6.3 Effect of facial expression**

The mobility of the soft tissues allows the face to assume a range of different facial expressions and yet these changes have little effect on our recognition of a face. The role played by facial expressions in communication, has led researchers to investigate how they are produced, and latter to attempt to simulate them.

Drawings of how different emotions are portrayed in a face were made in 1806 by Sir Charles Bell. His work was aided in the late nineteenth century by the advent of photography (Ekman, 1973). Duchenne investigated the muscle forces which produce facial expressions. Working at the time of the French Revolution, he applied electrical impulses to freshly guillotined heads to produce different expressions. The soft tissue movements that correspond to a particular expression have been documented. For



### ***Facial Description***

instance, a smile widens the mouth and nostrils, decreases the height of the upper lip, raises the eyebrows, partially closes the eyes, opens the mouth and decreases the distance between the mouth and base of the nose (Bledsoe, 1966). Ekman and Friesen (1971) discovered that some expressions panculturally convey the same meaning. This led them to develop a method for counting the activity of facial muscles (Ekman and Friesen, 1976).

In the last decade or so, the quantification of the soft tissue movements produced by an expression and the computer simulation of facial expressions have been undertaken. Pilowsky and colleagues (Pilowsky et al, 1985) used a mathematical model developed by Thornton and Pilowsky (1982) to manipulate facial muscles in order to simulate expressions and to quantify facial measurements relative to facial expression. This method was photography based. A 3D model based on muscle-action has been developed by Walters and Terzopoulos (Walters, 1987; 1989; Walters and Terzopoulos, 1990; 1991). This model has been used to simulate different facial expressions and, recently, speech.

The effect of facial expression on the surface type description is demonstrated in figure 6.27 and the variation, across the facial surface, of the amount of each surface type with increasing curvature threshold levels is shown in figure 6.28.

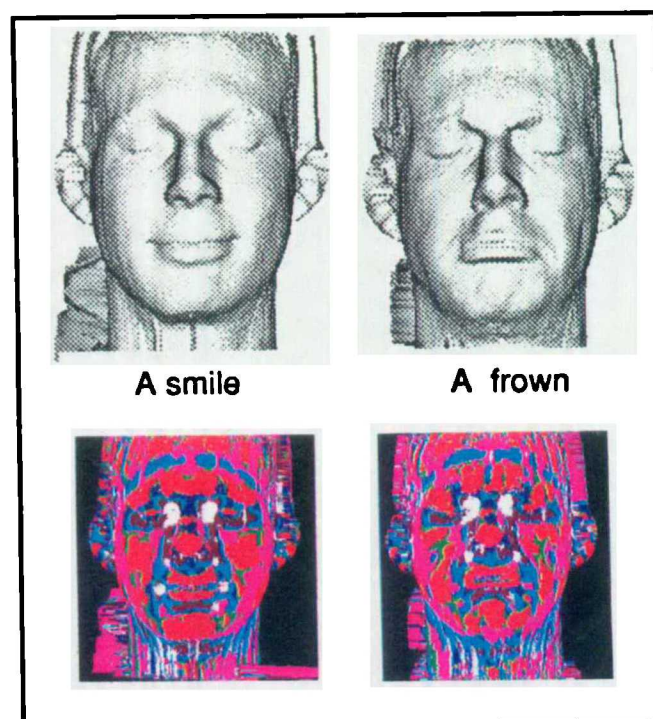


Figure 6.27 - The effect of facial expression on the surface type description

**Facial Description**

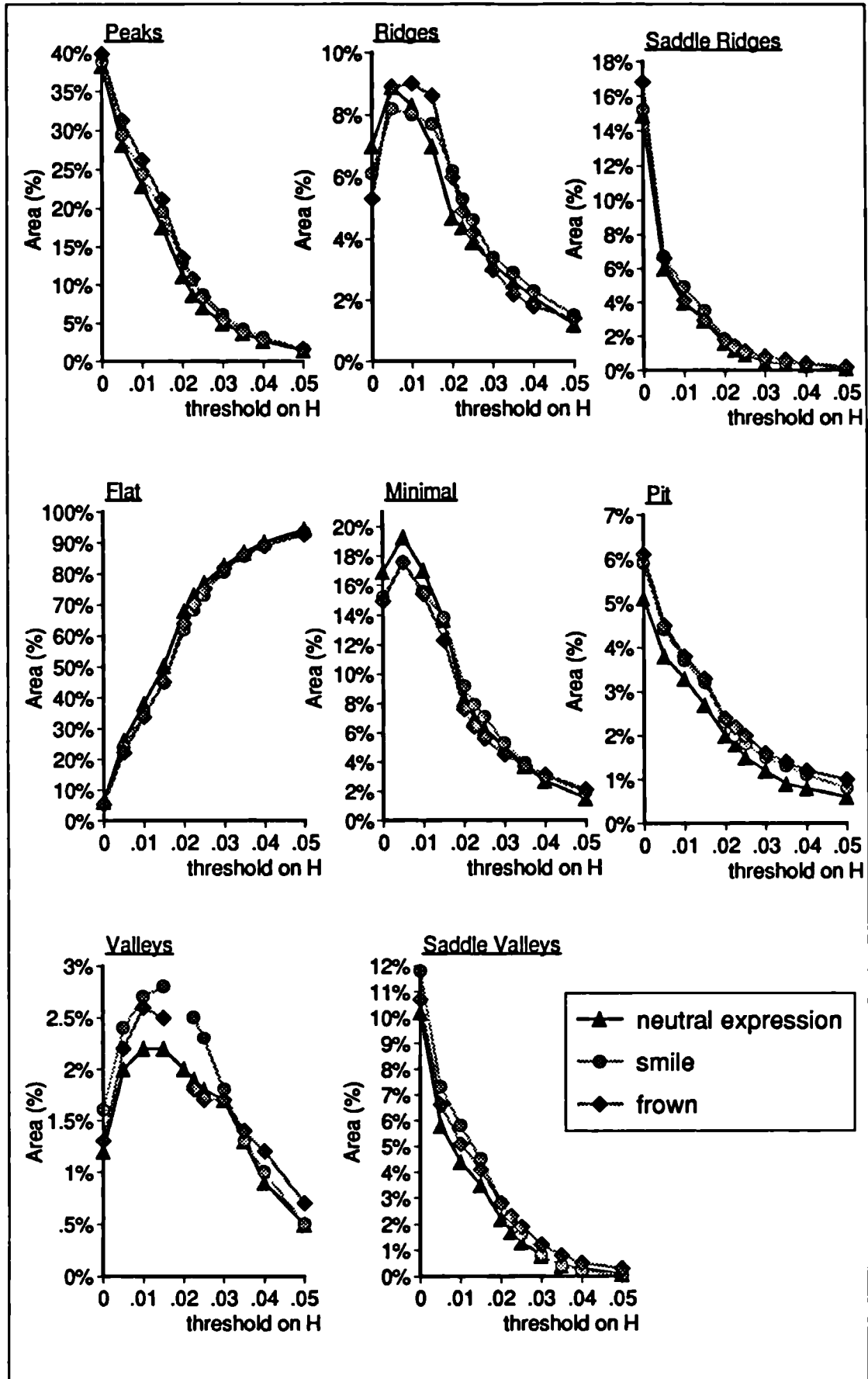


Figure 6.28: The effect of facial expression on the amount of each surface type across the entire facial surface.



### **6.7 Summary and enhanced segmentation**

The method described in this chapter allows the segmentation of the facial surface into patches of different types (shapes) of surface. A hierarchical description of the face, based on its curvature is produced, and this description has proved to be perceptually meaningful and allows a large amount of data to be readily appreciated.

Three algorithms for calculating the surface curvatures were described and their ability to describe geometrically regular objects and the facial surface has been described. The advantage of the novel least squares algorithm, in terms of the accuracy of its representation has been demonstrated. The comparative long run time of this algorithm is a concern, but is a problem that is summountable by parallelization of the computer code.

An assessment of the effect of random and quantization noise on the representation has been made. Noise in data was estimated to be between 0.1mm and 0.15mm of random noise and 0.25mm of quantization noise. This amount of noise will only effect the surface type description at low threshold level. Smoothing the data was shown to have very little effect on the surface type description. This establishes confidence that noise in the data will not serious effect the representation produced. and that fairly small scale changes can be described with confidence.

The surface type description has been shown to be reproducible for a face with a neutral expression. The effect of facial expression on the description has been demonstrated for two instances. It is important to establish the degree of reproducibility of the description for an individual face, so that a level of confidence can be placed on any comparison made between two faces. I have shown here, that a relatively high degree of confidence can be placed in the surface type description produced for a face.

Combining the surface type description with discontinuity maps of the surface ("jump edges") and surface normal ("roof edges") has been suggested for enhancing the segmentation of objects (Yokoya and Levine, 1989). For the face, very few surface discontinuities exist. However, the production of discontinuity maps, suitably thresholded, may prove of value.

## CHAPTER 7.

### ANALYSIS OF DIFFERENCES AND CHANGES IN THE FACE

In this chapter, I discuss how the differences in shape between individual faces, or the changes in shape made to one face may be described using the surface type representation. Three methods of analysis are described (section 7.3) and some ideas on how an *automatic* comparison of facial changes or differences might be made in future work are then discussed (section 7.4). This is novel work. However, before the surface type descriptions can be analysed satisfactorily, the two surfaces in question must be accurately registered together so that the comparison made is meaningful. It is also useful for a standard or "average" face to be defined. These requirements are described in the next two sections.

Finally, in section 7.5, an assessment is made of the sensitivity of the surface type description to global changes made to the facial data (such as caricaturing) and to local alterations, made by application of a mathematical function across a small region of the face. The resilience of the surface type description against these changes to the data set will allow the determination of the magnitude of change in facial shape that is required in order for the change to be perceptible. This innovation could be useful in assessing the outcome of facial surgery and for the investigation of the role of various parts of the face in facial recognition.

#### **7.1 Registration of 3D surfaces**

In order for a valid comparison to be made between two optical surface scans, a method must first be found to accurately register the two surfaces together. A suitable method for registration has been derived by Fright and Linney (in press). This technique permits the comparison of two or more head surfaces without the need for a common coordinate system by enabling surface measurements to be registered, or normalised, with respect to spatial position, orientation and scale in three dimensions. This method is briefly described here.

Firstly, an operator identifies a set of landmark points on the surfaces of both heads. These are marked interactively by moving a cursor across a shaded image of the facial surface, which is displayed on the graphics system. The accurate location of the landmarks is assisted by the display of vertical and horizontal profiles across the face through the cursor point concurrently with the shaded image. This guides the landmark selection to points of maximum convexity or concavity. The 3D position of the landmark is computed from the two dimensional screen position, the z buffer value at that point and the matrix transformation from screen space to object space.

The landmarks used for the registration should be located, as far as possible, across the whole facial surface so that the coordinates are well distributed in all three planes of space, thereby facilitating an accurate registration. The landmarks used for the registration of non-patients were: the inner canthi, the outer canthi, soft tissue nasion, the left and right alar base, subnasale, soft tissue B point, menton and otobasion inferius (figure 7.1).

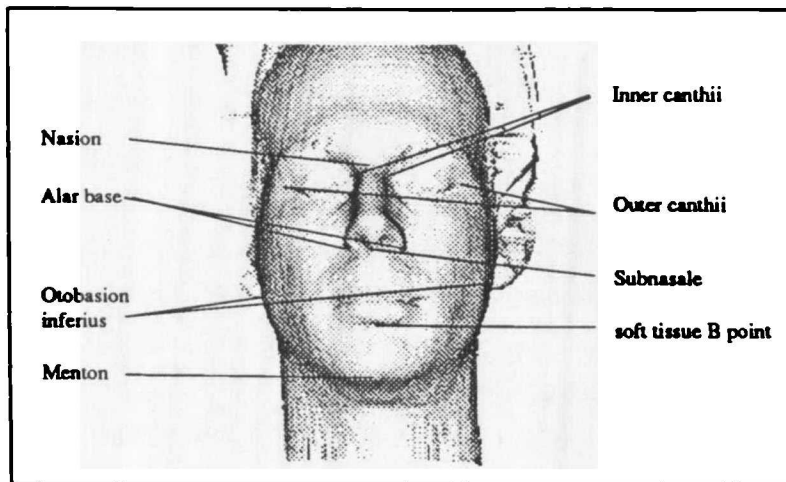


Figure 7.1 Landmarks used for the registration of facial surfaces

It is a requirement of this technique that the landmarks used must be located on regions of the surface that have remained *unchanged* between the two scans. For the facial surgery patients studied in chapter 8, much of the lower two-thirds of the face was moved during their surgery. Therefore, in these cases the landmarks chosen were confined to the forehead and eye regions (figure 7.2). These were the left and right, inner and outer canthi and the nasion together with five points on the forehead. The forehead points were defined by mathematical construction as follows. A best fit line was constructed through the five marked points and the face was orientated with the mid saggital plane at 90 degrees to this line. The first forehead point was marked a distance of 30mm from the nasion along a perpendicular projection from the constructed line. Two pairs of points were marked either side of this point at 15mm intervals, perpendicular to the mid saggital plane.

Once appropriate landmarks have been selected, the spatial locations of their 3D coordinates are averaged together to effect the registration. The error in registration of the landmarks is minimised using a least squares technique. The accuracy of the registration increases with the number of landmark points chosen and with their wider distribution across the facial surface. This ensures that the effects of individual errors, incurred by the measuring system and the operator, are minimised. Once the facial surfaces are registered together, a comparison of surface changes or differences can be made with confidence.

## ***Analysis of Changes***

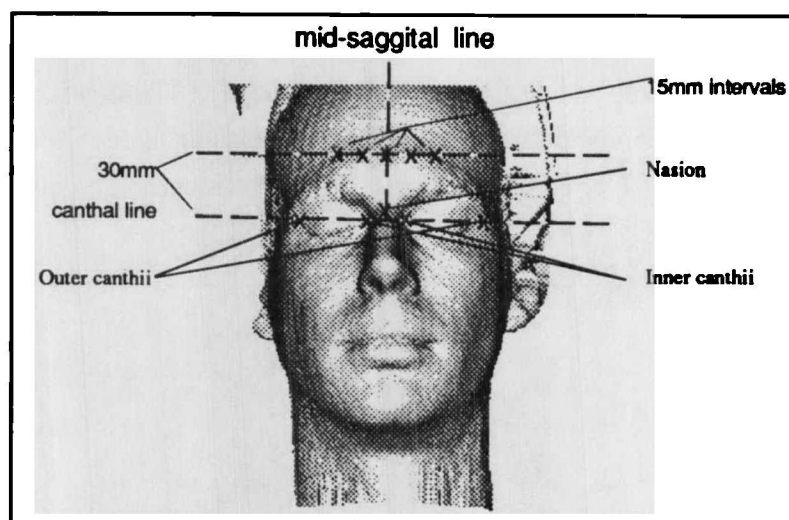


Figure 7.2 Landmarks used for the registration of facial surfaces for patients

### **7.2 Production of an average face.**

The derivation of an average face provides a useful standard against which different faces can be compared, thus allowing abnormal facial conditions to be described and in some slight cases, detected. Eventually, it may also be of help for defining a comparative basis for facial aesthetics (see chapter 8). The availability of an "average" face allows an assessment to be made of the result of an operation to correct facial deformity revealing the areas in which the face has become less extreme in appearance (moved closer to the "average") or become more extreme (moved further away from the "average").

Orthodontists have long been interested in the production of average or standard faces and different types of faces. Angle (1900), Downs (1948) and Ricketts (1981) all developed facial indices based on the classification of the facial profile into groups. These indices were designed to describe the types of facial abnormality that are congenital in nature and to tell the facial surgeon how to move the facial bones in order to correct them.

Galton (1878) was interested in the production of average faces for another reason. He superimposed photographs of two or more faces using multiple exposures, in an attempt to define facial characteristics of health, disease and criminality. A requirement of Galton's technique is that the original images must be of the same size and pose, have features of similar proportions and the pupils of the eyes must be exactly aligned. Interest in this superposition method is still being shown today. Langlois and Roggman (1990) have recently tried to improve it by adjusting the facial images to be equal in size and orientation and vertically scaling and averaging the image intensity. Benson and Perrett (1991b; 1992) have also manipulated photographic images prior to

producing a composite face. In their composites, the problem of blurred edges is avoided by using some 200 coordinates marking the outlines of various facial features. They have used this method to produce average male, average female and average person (androgynous) faces (see figure 7.3). They have also produced "hyper-male" and "hyper-female" faces by doubling the differences between each feature point in the average male and average female faces.



Figure 7.3 Benson and Perrett's Photographic averages, Left to right, a "hyper-female" face (this accentuates the differences between the average female and male); average female (16 faces); androgynous face (average of 16 female and 16 males); average male (16 faces) and a "hyper-male" face. *Reproduced with permission.*

A technique for averaging 3D optical surface scan data has recently been derived by Fright and Linney (submitted). For this method, the individual surfaces must first be registered using the technique described in section 7.1. Differences can be expected between the size of individual heads and in their position and orientation during scanning. Thus before an average head surface can be produced a number of factors must be normalised. Firstly, a transformation matrix, which minimises the error between the individual landmarks and the reference in a least squares sense is derived. This matrix then maps the landmarks and the rest of the data for an individual head onto the average. The surfaces are then resampled on a regular cylindrical grid in the common coordinate system. The new radial measurements of the surfaces are averaged to produce an average head surface. Thus the head surface acquires a standard (the average) position and orientation in space and a standard size and proportion.

Optical surface scans of 13 females and 14 males were used to produce an average female head and an average male head via this technique (figure 7.4). Surface type descriptions were then computed for these. Figure 7.4 shows these surface type descriptions at a medium threshold level.

### **7.3 Analysis of surface type descriptions**

In this section, I describe three methods for analysing the differences between two surface type descriptions. These analyses are illustrated using the surface type descriptions computed for the average male face and the average female face (figure 7.4).

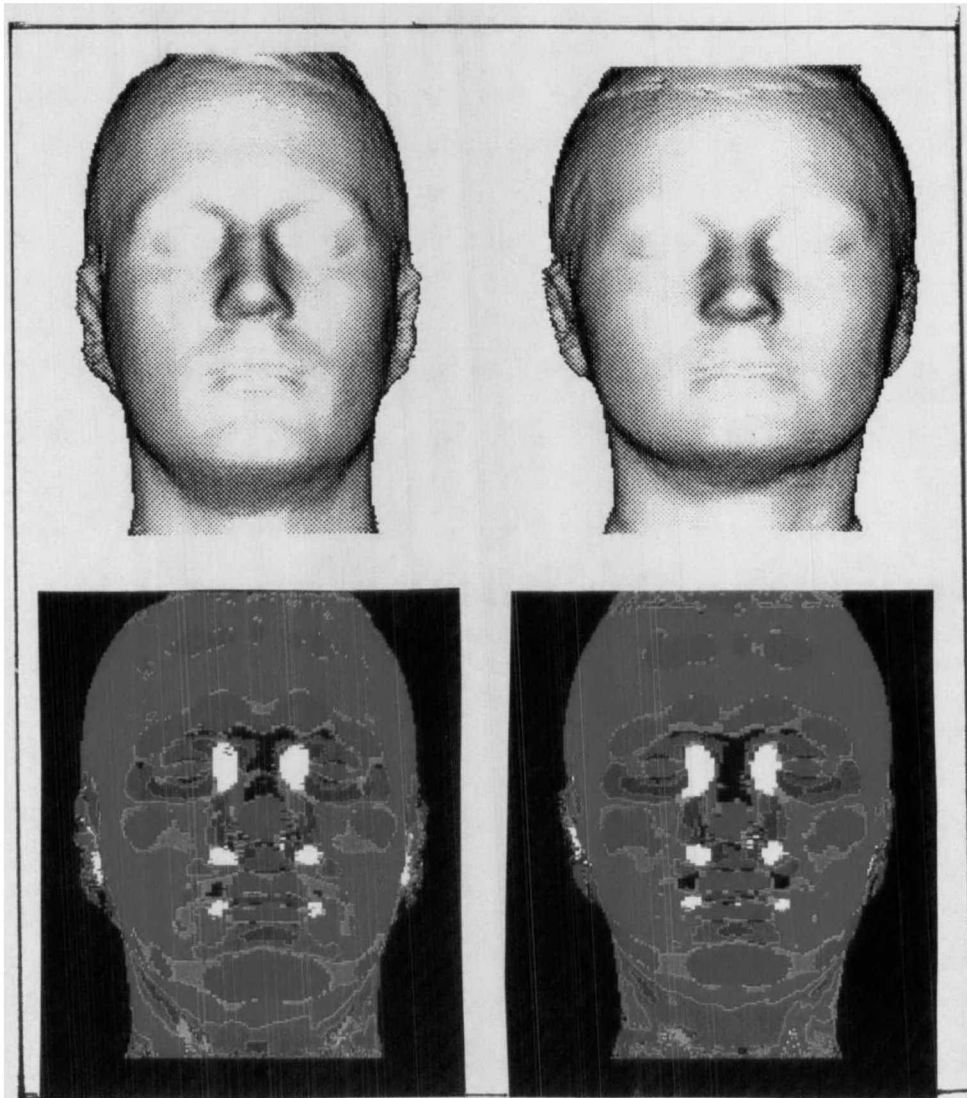


Figure 7.4 Above, the average male head (left) and average female head (right). Below, the corresponding surface type descriptions.

### **7.3.1 Qualitative analysis**

The first method of analysis is a qualitative description of the surface types, in which the relative position and size of certain surface type patches are assessed by visual observation of the surface type images.

A qualitative consideration of the surface type encodings for the average male and average female heads reveals some notable differences: compared to the female head, the average male head has larger areas of peak surfaces at the inner corners of the eyebrows and at the chin but smaller areas of peak surfaces across the cheeks. The average male also has areas of ridge surfaces extending from the upper lip downwards, parallel to the lip corners which are absent in the female, and a larger area of minimal surface between the lower lip and the chin centred on the soft tissue B point.

Some of these differences may be related to the relative protuberance of facial features on male and female faces. Bruce, Richards and I have recently demonstrated, using simple quantitative comparisons between the average male and average female, that in

the male face the brows, chin and nose are more protuberant than the female whereas the female has slightly more protuberant cheeks (Bruce et al, submitted). This could explain the differences in amount of peak surfaces. However it does not account for the ridges extending from the upper lip.

This analysis of the 3D average male and female heads broadly concurs with Benson and Perrett's findings for their photographic average faces. Benson and Perrett (1991b, 1992) found that the male faces tend to be longer with more protuberant nostrils, thicker eyebrows and squarer jaws with subtle differences in the shape of the cheeks. Psychological tests on the rated masculinity and femininity of Benson and Perrett's average faces have shown that the sex of these faces are correctly perceived, implying that the shape of the internal facial features are sufficient for judging the sex of a face.

### **7.3.2 Regional analysis**

A second method of analysing the surface type descriptions is to divide the face into regions and compute the amount of each surface type within each region. The region, or feature, of interest was defined interactively by using a cursor to mark a box around it on a Mercator projection of the surface type data (see figure 7.5).

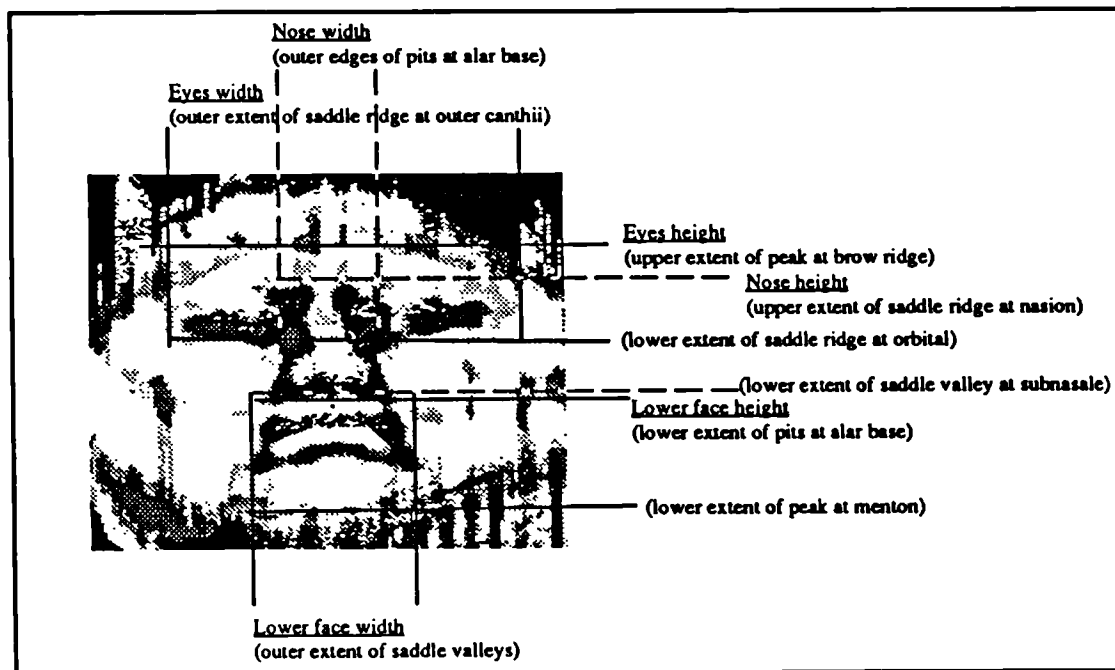


Figure 7.5 Definitions of eyes, nose and lower face regions, as marked from a low threshold level surface type image, shown here as a Mercator projection.

The consistent placing of this enclosing box was ensured by using the location of certain "landmark" surface type patches, at the lowest threshold level computed. Whilst, it may be argued that the human visual system does *not* use such artificial linear constructions to define regions, the consistent marking of the region from face to face at least allows us to make a reliable comparison of the surface encoding and to begin to

### ***Analysis of Changes***

explore whether or not this sort of surface encoding carries any useful information. In this work, three regions of the face have been considered; the nose, the eyes and the lower face. These were defined as shown in figure 7.5.

Once a feature or region has been defined, the percentage of each component surface type contained within the region can be computed, yielding a quantitative analysis of the amount of each surface type in the region. This can be carried out for different curvature thresholds and represented graphically as a function of increasing thresholds on the curvatures (eg. figure 7.6).

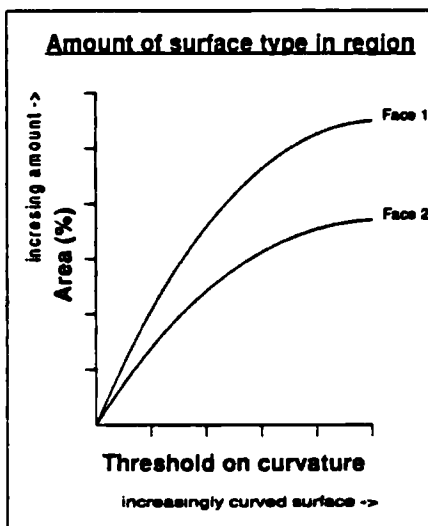


Figure 7.6: Graphical comparison of the amount of surface type against threshold level.

A summary of this analysis for the average male and female heads is shown in figure 7.7. If we assume that the surface types that can be associated with prominence are those of positive mean curvature (ie. peaks, ridges and saddle ridges) and those associated with retardation have negative mean curvature (ie. pit, valley, saddle valley), then we would expect to find more peaks, ridges and saddle ridges on the male brow, chin and nose and more flat and minimal surfaces (of zero mean curvature) for the female.

In the eye region, which includes the brows, there is a greater amount of peak and ridge surfaces for the average male compared to the average female, this could correspond to the relative prominence of the surface. The male also has more ridges at high threshold levels, implying that the brows are more curved. The average female has more flat, less curved surfaces.

At the chin, the average male has more ridge surfaces at low thresholds (ie. a shallowly curved surface) and also more flat surfaces. These maybe indicative of a squarer mandible. The female head has more saddle ridge, minimal and peak surfaces.

At the nose, the average male has more ridge surfaces at high thresholds (ie. a highly curved surface) but more flat and minimal surfaces too perhaps indicating a square shape. The average female has more saddle ridge and pit surfaces.

Overall, this analysis of the amount of each surface type in the eyes, nose and lower face regions for the average male and female heads, tends to support the notion that the



prominence or retrusion of the facial surface is reflected by the surface type amounts, but this is not particularly clear.

surface type	eyes	nose	lower face
peaks	m > f (all)	m / f similar	f > m (high)
ridges	m >> f (high)	m > f (high)	m >> f (low)
saddle ridges	m / f similar	f > m (all)	f >> m (low)
flat	f >> m (all)	m > f (all)	m >> f (high)
minimal	m > f (high)	m > f (all)	f >> m (all)
pit	f > m (low), m > f (high)	f > m (all, low)	f > m (low), m > f (high)
valley	m slightly > f (all)	m / f similar	f slightly > m (low)
saddle valley	m > f (all)	m slightly > f (all)	m slightly > f (all)

Figure 7.7: Summary of a comparison of the amount of each surface type over low to high threshold level for the average male and average female. m = male, f = female, in three regions of the face. Brackets indicate threshold range over which the comparison is valid, if not all thresholds.

### 7.3.3 Analysis of Surface Type Patches

A far better method for analysing the surface type description would be to use various statistical measures to quantify the surface type patches produced by the description. For instance, one could calculate, for each surface type patch within a region of interest, the area, orientation, length and width of each patch (see figure 7.9). The interfacial variation could then be assessed directly from the variability of these patches. Although the comparison between two sets of such parameters is straight-forward, a method for comparing two surface type descriptions by their patches has not yet been fully implemented since its robust implementation relies on developing a fully automatic method for *identifying* the surface type patches. This is by no means easy and is not attempted here, but is the subject of on-going research. However, by manually identifying of a small number of patches, such a detailed comparison between two faces is illustrated.

#### a) Forming patches

Adjacent pixels of the same surface type can be linked together into *patches*. These patches appear to correspond to parts of the facial surface that are perceptually meaningful, such as the nose tip, chin or eye orbitals. The boundaries of the patches are found as follows.

For each point in the data set, the surface type, T, is noted and keeping the position along the profile constant, an algorithm steps back through the profiles to find the point at which the surface changes type. This point, bp, lies on the boundary of the patch. A boundary search algorithm then starts with the boundary point (bp) and, using a eight-way connected search algorithm, searches its neighbouring points to find the next point of the boundary. This second boundary point becomes the starting point for the next

### Analysis of Changes

search and so on (Press et al, 1986) (see figure 7.8). Once, all the boundary points have been found, a "flood-fill" algorithm is used to fill the patch with a dummy number (Pavlidis, 1982) thereby ensuring that the patch pixels are not found again when this process is repeated for the next patch. It is straightforward to extract various parameters of the patch at this stage, such as the area or centre of gravity. Some salient parameters are discussed below.

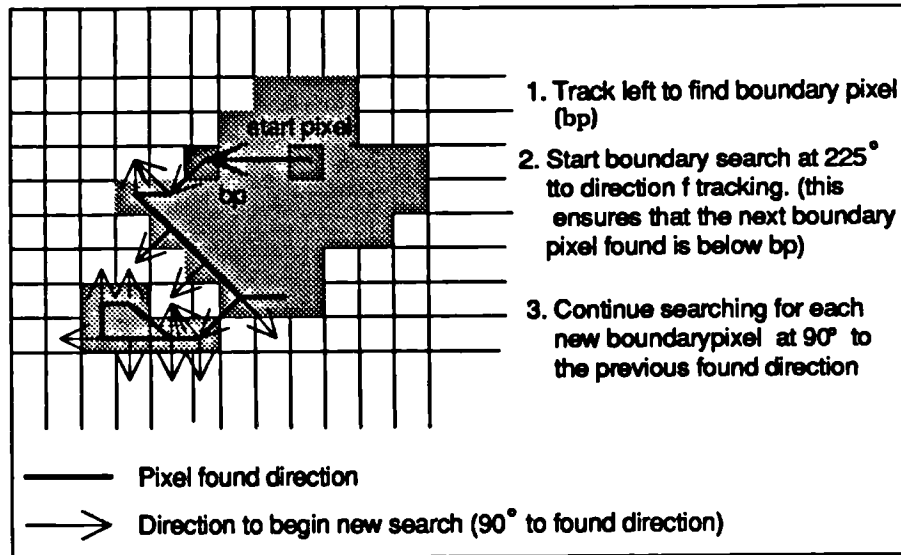


Figure 7.8: The boundary search algorithm. Starting with the boundary point (bp), neighbouring points are examined in turn to find the next boundary pixel. The start direction is at  $90^\circ$  to the direction of the previous search.

### b) Patch Parameters

In the computer vision literature, a number of parameters that provide meaningful and stable measures of the shape of bounding contours have been suggested. A review of these parameters is given in Brady and Horn (1983). Some of these measures were selected and used to describe the shape of the surface type patches. These measures are illustrated in figures 7.9 and 7.10 and described below.

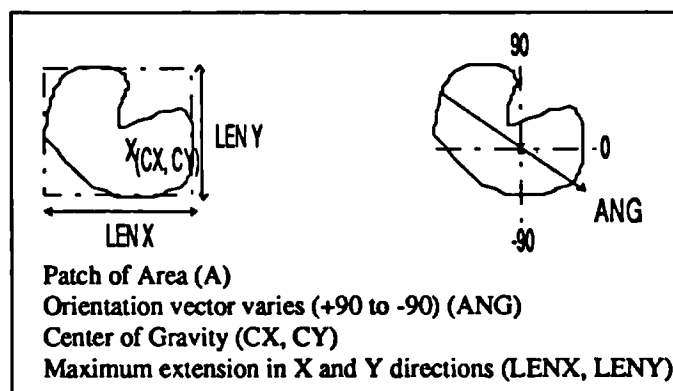


Figure 7.9: Some measures of surface type patches.

**aa) Geometric measures:**

- i) **Area:** The number of pixels in the patch.
- ii) **Surface Type:** The fundamental surface type of the patch.
- iii) **Maximum size in the x and y directions:** The maximum and minimum extension of the patch across the acquired profiles (x) and scanlines (y).
- iv) **Centre:** The geometric centre of the patch (centre of gravity).
- v) **Orientation:** The orientation of the principal eigenvector of the patch. (The direction is inverted if the vector lies outside -90 degrees to +90 degrees range).

**bb) Measures estimating the complexity of the contour shape:**

i) **Bending Energy:** The bending energy of a contour was defined by Bowie and Young (1977a) and provides an estimate of the complexity of a contour in terms of the energy needed to deform a circle into the contour. Thus a contour with 2 "lobes" would have lower bending energy than one with 3 "lobes". It has also been called the curve of least energy (Horn 1983).

ii) **P2A:** P2A is a well-known measure estimating the complexity of an enclosing contour. It is defined as:

$$P2A = \text{Perimeter}^2 / \text{Area}$$

P2A has been used with some success for interpreting irregularly shaped objects (Witkin, 1981; Brady and Yuille, 1984) and is commonly used in pattern recognition and industrial vision systems. Brady and Yuille (1984) showed that this measure is fairly insensitive to noise and stable with respect to the viewpoint, provided the eccentricity is not large. However, Pavlidis (1979) criticized it for giving the same value for widely different shapes ie. not being information preserving!

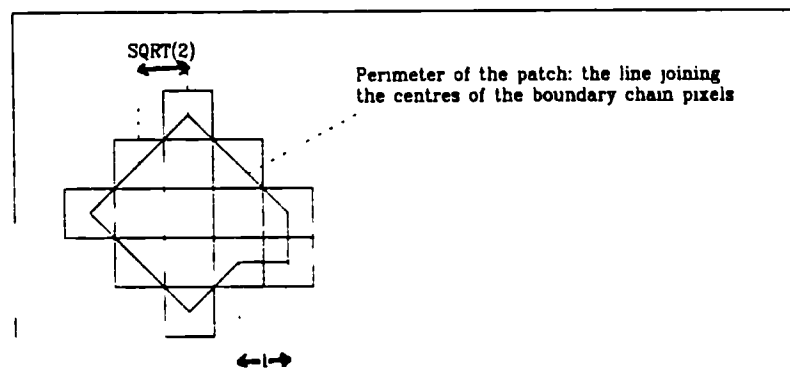


Figure 7.10: The patch perimeter is defined as the line joining the centres of the boundary chain pixels. The area of the patch, calculated as the sum of the pixels in the patch, is therefore overestimated by the enclosed perimeter and hence a correction has to be applied.

### Analysis of Changes

For the surface type patches, a patch is defined as the number of pixels of the same surface type which are adjacent to each other. Thus to calculate P2A for a patch, the perimeter of the patch must be calculated using the distance from the *centre* of each boundary pixel to the *centre* of the adjoining boundary pixel (this may be either 1 or  $\sqrt{2}$ ) (see figure 7.10). This distance is recorded in the patch boundary's chain code. When the area of the patch is calculated, account has to be taken of the parts of a boundary pixel which are outside this perimeter. A good approximation to the area *enclosed* by the perimeter is:

$$\text{Area} = (\text{number of pixels in patch}) - ((\text{chain code length} / 2) + 1)$$

In addition, a normalization factor of  $4\pi$  is needed, in order to set the value of P2A for a circular patch to be zero (since  $\text{circumference/area} = (2\pi r)^2 / \pi r^2$ ).

iii) Smoothness of a space curve: Another measure that was considered was the smoothness of the curve, which was defined by Barrow and Tenenbaum (1981) as:

$$\int \frac{(dk)^2}{(ds)}$$

Where  $k$  is the differential curvature (defined as the reciprocal of the radius of the osculating circle at each point on the curve) and  $s$  is position on the curve. However, this parameter is very dependent on high order derivatives of the curve and consequently on small scale behaviour. Hence small wiggles in the bounding contour contribute a disproportionately large amount to the integral for the contour as a whole. This is obviously not desirable as it will be very noise sensitive. The measure is minimized by  $dk/ds$ , which is zero for straight lines and so a bias towards linearities is introduced. For these reasons this measure was not used.

Patch	Type	cx	cy	area	length x	length y	orient. angle	P2A
upper lip	peak	85	172	143	19	11	-4.06	5
nose tip	peak	89	142	283	29	18	-87.81	14
nose bridge	saddle ridge	88	97	412	61	31	-85.84	119
inner canthus (right)	pit	74	115	199	33	14	-80.89	369
right ala	pit	76	157	56	12	7	78.32	1943
inner canthus (left)	pit	97	116	127	32	20	74.28	1342
left ala	pit	99	157	40	14	6	-79.69	1277
nose side (right)	saddle valley	78	132	189	32	14	-82.91	607
nose side (left)	saddle valley	96	136	322	66	35	-81.09	1159

Figure 7.11: List of parameters describing surface type patches on a face. Cx, cy is the centre of the patch; length x, length y the extend of the patch in the x direction (across the profiles) and y direction (across the scanlines); orient.angle is the orientation of the principal axis.

Algorithms were written to automatically calculate these parameters for each patch on the image. A listing of some of these parameters, describing nine sample patches on one face is given in figure 7.11.

#### **7.4 Towards An Automatic Description of Facial Changes**

In this section, a possible approach, based on artificial intelligence concepts, is outlined for the identification of these surface type patches in a surface type image. The implementation of a robust method for automatically identifying which patches on one surface type image correspond to the same patches on another surface type image, would enable a fully automatic method for describing changes in facial shape to be achieved.

##### **7.4.1 Assignment of patches to features**

As already mentioned, the surface type patches carry a perceptually meaningful interpretation corresponding to facial features, or portions of facial features, such as the nose tip, eyebrow, cheeks etc. Labelling some or all of the patches with their perceptual interpretation would provide a linguistic description of the face and hence facial change. However, the number of patches to be identified, and their liability to change in size and shape, as well as location makes the computer recognition of these patches very difficult.

In the last decade or so, methods for object recognition have incorporated expert knowledge about the object and applied a series of "rules" based on this knowledge in order to classify the object. This has led to more reliable object identification and allowed a level of confidence to be attached to the recognition of an object. The application of techniques from the field of artificial intelligence and mathematics including expert knowledge, rule-based methods (eg. Xu and Wan, 1988; Feldman and Yakimovsky, 1974) and fuzzy logic (Zadeh, 1983; 1988) may give assistance to the recognition of surface type patches.

A first stage, would be to subdivide the surface type patches into major facial features (eye, nose, mouth, cheek etc.), thereby reducing the patch recognition problem. The frame-based techniques of Yager (1984a; 1984b) may facilitate this stage. The principles of this technique are outlined below.

##### **a) Linguistic Frame representation.**

The linguistic frame method (Yager, 1984a; 1984b) represents information about an image and its interpretation. It provides a neat way for assessing the degree of matching between image data and a predescribed object and is based on the way in which an "expert" would apply his knowledge.

### ***Analysis of Changes***

In this method, a frame is defined as a "data structure for representing stereotyped situations in artificial intelligence", a structured representation of an object or class of objects. If one wishes to determine whether or not a particular frame can be applied to all, or part, of an image then knowing some elements composing the frame, one can predict the rest of the elements (termed "slots"). These slots have allowable values containing facts about the object we are searching for in the data and are tested to see if they are fulfilled by the data. An example would be:

Frame: Family (Coombes)		
Slots	Potential value	(A possible match)
Father	Male	(Richard)
Mother	Female	(Esther)
No. of children	A non negative integer	(2)

Figure 7.12: Example of a linguistic frame representation.

It is possible to use "fuzzy values", from the theory of approximate reasoning, (summarized in Zadeh, 1988b) for these slots. These are imprecise prepositions used to describe a value. For instance, the value "height" maybe described as "tall", "fairly tall", "average", "rather short" etc.

The linguistic frame representation operates on a rule-basis of interdependence containing knowledge about the object model to which the data is to be matched, ie. If (condition X is true) then (condition Y is also true). For example, If *location* is *west town* then *prices* are *high*. Hence conditional probability values can be used to assess whether a statement is likely to be true and the slot condition fulfilled. The truth of a statement may also be fuzzy; eg. at least half, more than 5 etc. Confidence levels can be attached to a statement via probability. Hence if X is the most likely interpretation and Y is the second most likely interpretation then the confidence of the assignment (Conf.) is:  $Conf. = X/Y$ . The frame can be evaluated using these inference rules, ie. if slot 1 and slot 2 and either slot 3 or slot 4 are satisfied (or mostly satisfied) then frame is true.

A mechanism for determining the effects of applying a rule and for establishing the order in which the rule-base is searched can be incorporated into the system. When a "fireable" rule is found, the data is searched for information. If the information is found the rule is "triggered" and the description updated. An iterative process can be established until no new information about the data is found until or the frames are completed.

#### **b) Facial feature frames**

Applying this approach to the surface type patches, the knowledge which humans (as experts) have about the constitution of the human face should be utilized. That is that

most faces have one nose, two eyes, two cheeks, a mouth, a forehead and a chin, and that these have a specific relationship to one another (eg. the nose is above the mouth). These "major features" would consist of the frames. The slots would consist of the surface type patches. For the "nose" frame, the slots would include the patches corresponding to the nose bridge, nose tip, nasion, nares and the sides - where the nose joins the cheeks.

Thus the rule-base could be formed by the feature structure and used to assign patches to frames and interpret them in terms of the constituents of the facial features. Probability analysis could be used to attach a level of confidence to the interpretation of each patch and patches with a high level of confidence could be used as "seeds" for placement of the feature frames. A standard orientation of the face would need to be assumed. Figures 7.13 and 7.14 give an example of how the nose frame might look.

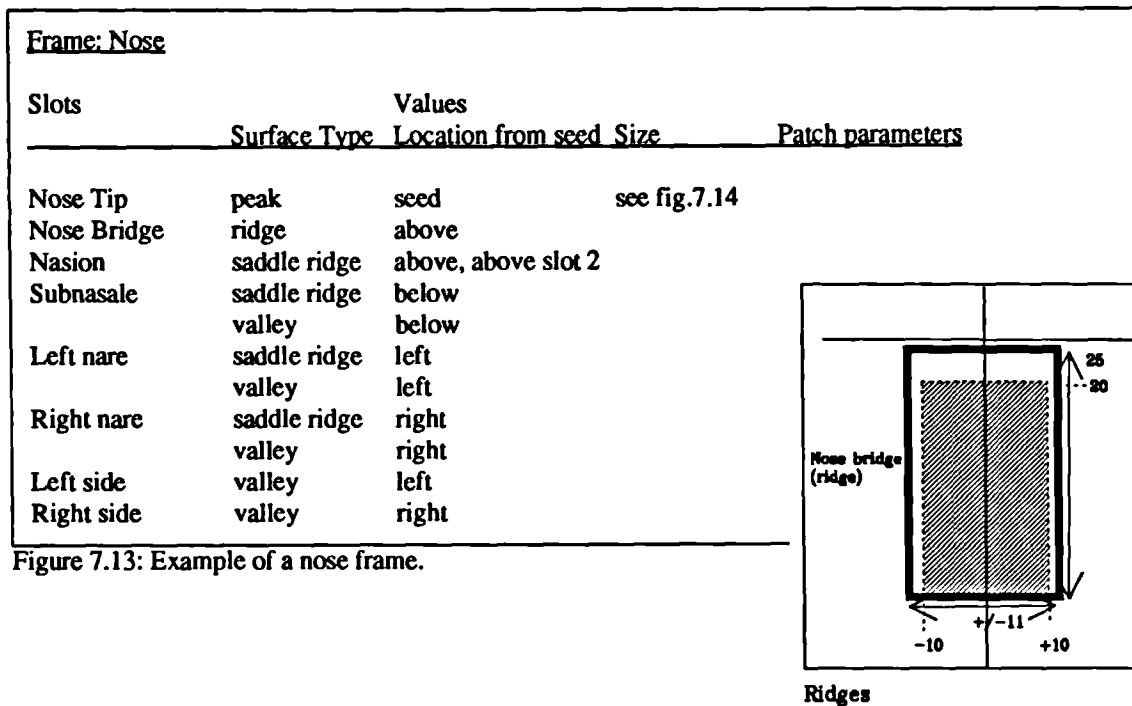


Figure 7.13: Example of a nose frame.

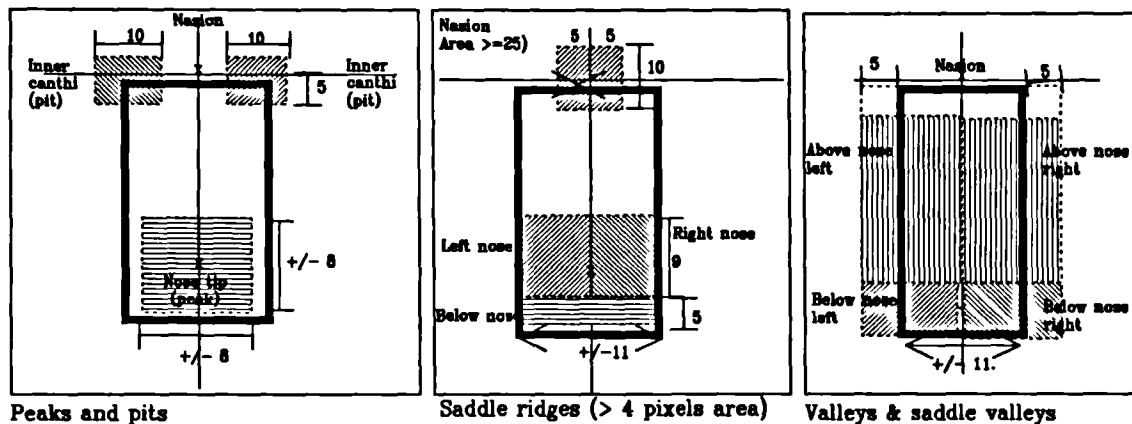


Figure 7.14: Patch locations in the nose frame.

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Once a methodology for identifying the surface type patches has been established, the surface type patches found on the average face could be used to evaluate which patches are essential for the recognition of a feature. An evaluation of the reliability of patch assignment and correct interpretation should also be made.

#### **7.4.2 Comparison between two faces.**

The identification of surface type patches with their perceptual interpretation provides a powerful tool for the analysis of changes in facial shape, brought about by growth or surgery or for describing the differences between individual faces in terms of shape. Two or more surface type images can be compared by comparing patches with the same interpretation (ie. considered to be the same feature). Such patches may be termed "*Changed features*" to distinguish them from patches that are only found in the first (eg. pre-surgery) image ("*Disappearing features*") and those found only in the second (eg. post-surgery) image ("*New features*").

The parameters of the surface type patches described in section 7.3.3b) can easily be compared using algorithms, based on simple geometry and measured as a percentage of the first image patch.

Although an automated method for *identifying* the surface type patches has not yet been achieved, I have illustrated the potential of this method for describing the changes in facial shape produced by reconstructive surgery (Coombes et al, 1991a). This involves identifying and selecting several patches from a list of all patches for a surface type image and making the following comparisons between them.

- i) Change in area.
- ii) Change in lengths of the principal and secondary axes.
- iii) The two components of shifts in the location of the centre of gravity.
- iv) Change in Bending Energy.
- v) Change in P2A (perimeter<sup>2</sup>/area)

#### **a) Change in the central location of the patch**

The change in the central location of the patch is measured in terms of two components parallel to the principal and secondary axis of the patch in the first image. A negative sign indicates movement *towards* the centre and a positive sign indicates movement *away* from the centre. Thus the change is measured with respect to the patch's initial position is not dependent on the coordinate system. The magnitude of the change, *vec* is:

$$\text{vec}^2 = \delta x^2 + \delta y^2$$



If  $\alpha$  is the orientation of the principal axis in the first image, the orientation of the principal axis in the second image  $\beta$  is,  $\beta = \arctan(\delta x / \delta y)$ .

The vector joining the first centre to the second centre is rotated through an angle  $(\beta - \alpha)$ , to remove the difference between the two centres due to the orientation of the principal axis in the first image (figure 7.15).

Thus the shift in the direction of the principal axis is:

$$\delta.\text{prin} = \sin(\beta - \alpha) * \text{vec}$$

and the shift in the direction of the secondary axis is:

$$\delta.\text{sec} = \cos(\beta - \alpha) * \text{vec}$$

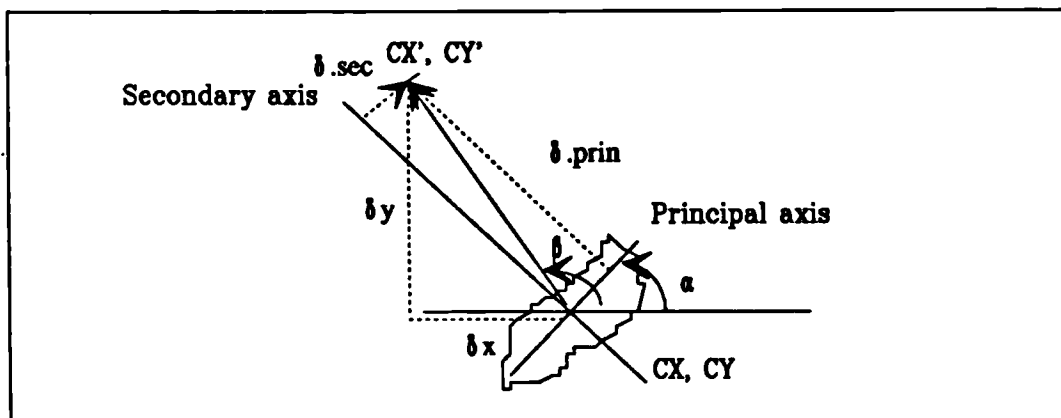


Figure 7.15: Diagram showing the transformation of the patch centre from CX,CY in the first image to CX',CY' in the second image. The components of the transformation are  $\delta.\text{prin}$  perpendicular to the principal axis of the patch in the first image and  $\delta.\text{sec}$  perpendicular to the secondary axis.  $\alpha$  is the orientation of the principal axis in the first image, and  $\beta$  the orientation of the patch in the second image.

**b) Change in the angle of the principal axis**

The change in angle of the principal axis is a measure of the change in the orientation of the patch. A negative sign indicates a clockwise change and a positive sign indicates an anti-clockwise change (figure 7.16)

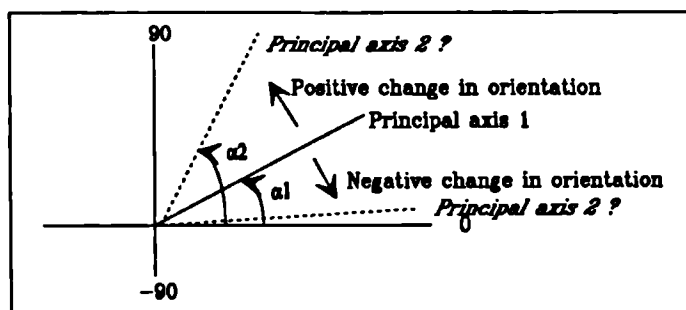


Figure 7.16: Change in the orientation of the principal axis.

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For the first orientation angle  $\alpha_1$  and second orientation angle  $\alpha_2$ :

```
IF
  a1 > a2
    angle.change := -(a1 - a2)
  a1 < a2
    angle.change := (a2 - a1)
TRUE
  angle.change := 0.00
```

## **7.5 Sensitivity of surface type description to global and local changes**

### **7.5.1 Caricature**

*"Caricature is the tribute that mediocrity pays to genius"*  
Oscar Wilde (1856-1900)

One method of globally altering the shape of the face is to caricature it. This technique is popular with cartoonists since it conveys identity rapidly through an outline form and shading. The technique of caricaturing is to exaggerate the more prominent features of the face compared to less prominent ones. Therefore, in regard to face recognition, it has been expected that caricaturing the face would *improve* recognition performance. However, for artist-drawn caricatures this has not been found to be the case (Hagen and Perkins, 1983), suggesting that face recognition is *not* based merely on the most prominent features (which are assumed to be the most distinctive).

A pre-requisite for computerised caricaturing of the face is the generation<sup>of</sup> an "average" face. For line-drawn caricatures, an average face may be produced from a set of coordinates marked on a number of faces (Brennan 1982; 1985; Dewdney, 1986). A line-drawn caricature of an individual can then be produced by comparing the coordinates of various features for the individual with the average coordinates and exaggerating those features which differ from the average the most. Brennan (1982; 1985) found that if such line-drawn caricatures are exaggerated enough, they lose their human qualities and degenerate into a chaotic state that she termed "facelessness". These caricatures were *not* found to be any more recognisable than realistic line drawings, although highly exaggerated caricatures were recognized significantly *faster* than the true line drawing (approximately twice as fast). This is only true for full-face views.

Benson and Perrett (1990) extended Brennan's method to produce "continuous-tone" caricatures. A photograph of individual was divided into triangular portions using landmark coordinates. The caricature was formed by exaggerating the triangular grid and "texture mapping" the photographic portions onto this caricatured grid. This method was applied to investigate face recognition questions. Benson and Perrett found that the recognition of a photographic caricature depended on the familiarity of the face to the observer, the distinctiveness of the features, the features that are exaggerated and

degree of their exaggeration (Benson and Perrett, 1990). They also produced "anticaricatures", these are images in which the distinctiveness of the face is reduced by decreasing its deviation from the average face (Benson and Perrett, 1991a) (see figure 7.17). Benson and Perrett (1991a) found that the most recognisable images occurred for a small positive exaggeration (4.4%).

Another method of caricaturing the face was proposed by Kirby and Sirovich (1990). Digitized photographs were standardized using the interocular distance and then averaged. The test face was then subtracted from the mean face and the sum of the eigenpictures used to give the degree of caricature, using the Karhunen-Loeve expansion (also called principal component analysis (PCA) or the Hotelling transform).

The production of 3D average heads from optical surface scan data (section 7.2) enabled Fright to exaggerate the face of an individual with respect to that average to produce a *three dimensional caricature* of that individual. This involves a global alteration of the facial surface. The alteration is made radially with respect to an axis passing through the tip of the chin and the crown of the head. The extent of exaggeration, with respect to the average male or female, can be varied allowing a whole series of progressively more caricatured faces or less caricatured faces to be produced (figure 7.17).

The effect on the facial shape achieved by caricaturing the face in this manner has been assessed by calculating surface type descriptions for each of a series of caricatures of an individual (figure 7.18). Qualitative examination of the surface type descriptions show that the shape of the face has been changed considerably by the caricaturing process, especially at high levels of exaggeration.

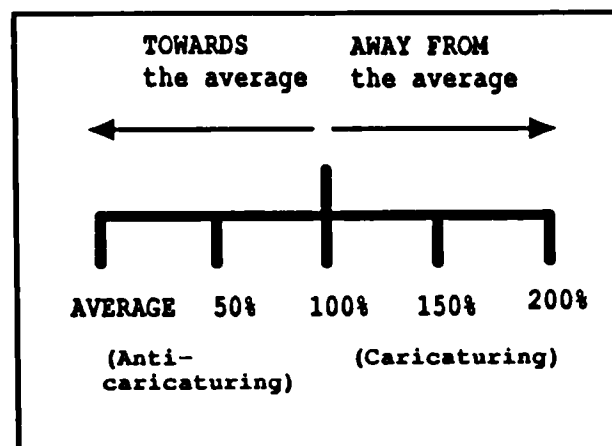


Figure 7.17: Production of caricatures

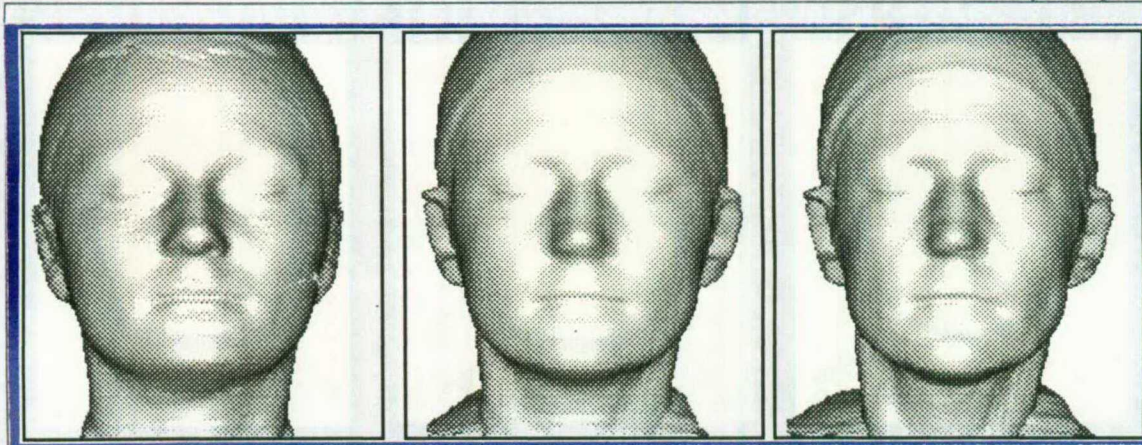
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The recognisability of images of these 3D caricatures has been assessed by psychological experiments (Bruce et al, 1992). Bruce and colleagues found that the original facial surface images were preferred over both caricatures and anticaricatures of 50%, 75%, 125%, 150% and 200% degrees of caricature. This indicates that either this particular method of caricaturing is *not* equivalent to two dimensional methods, or that the degree of caricaturing used was too great to aid recognition. The surface type description shows that the face has been considerably changed in shape, providing a possible explanation as to why these caricatures are badly recognised.

### **Next page:**

Figure 7.18: A series of 3D caricatures of an individual (above) and corresponding surface type images (below). Above, left to right, the average woman, 50% caricature, 100% caricature (the original scan). Below, left to right, 100% caricature, 150% caricature and 200% caricature.

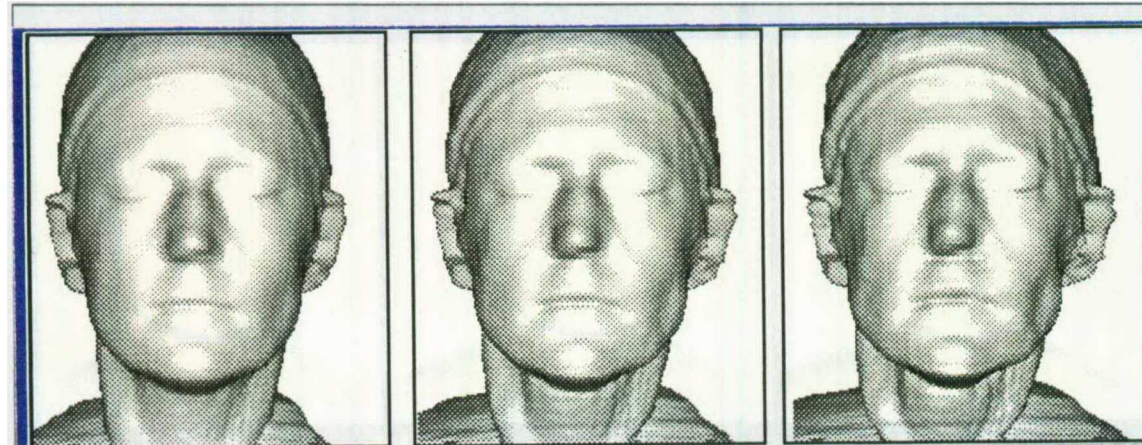




Average woman

50% caricature

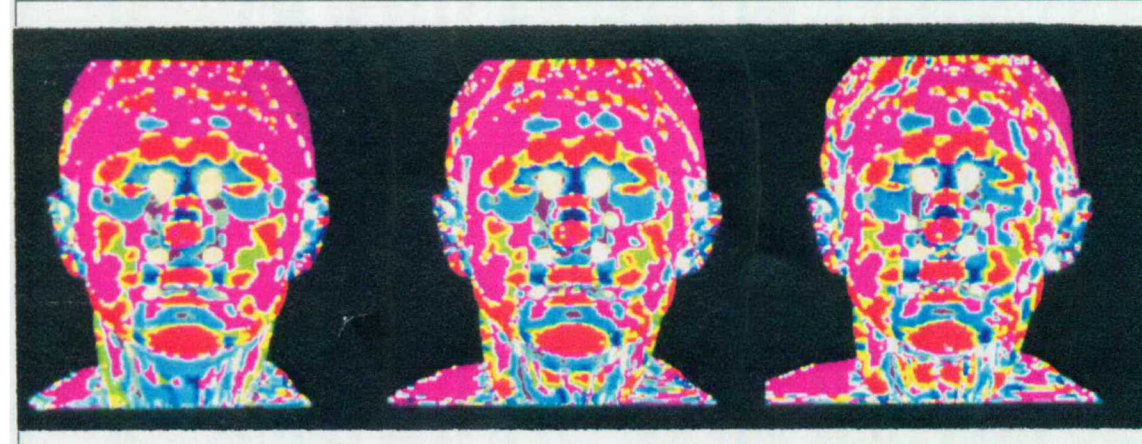
individual (100% caricature)



Individual

150% caricature

200% caricature



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### **7.5.2 Local Spline Alteration**

The facial surface data can also be manipulated in a locally defined region. One way of doing this is to apply a two-dimensional B-spline algorithm between two surface points and modify the enclosed surface gradient (Hanna and Bruce, 1992). Hanna has implemented this algorithm to make the nose and chin more convex or concave in shape. The gradient of the recorded profiles were altered by a certain percentage in either direction.

For the nose, the length of the section of profile to be changed was determined by selecting, interactively, the points on the profile that most closely corresponded to the nasion, the nose tip and the mid-point between the brows. The adjustment to the nose gradient was applied to the "midline" profile - passing through the centre of the nose ridge, and to three profiles to either side. This procedure resulted in a smooth altered surface (figure 7.20).

Changes to chin were produced in a similar manner, though to produce changes which were as visible as those to the nose, larger percentage alterations to the gradient were needed (because chins are naturally flatter). The chin changes were applied to 17 profiles - the midline profile and eight on either side. The control points were selected to be those which corresponded most closely to the menton (tip of the chin), the soft tissue B point (the point of inflection between the lower lip and menton) and a point approximately half-way between (see figure 7.19).

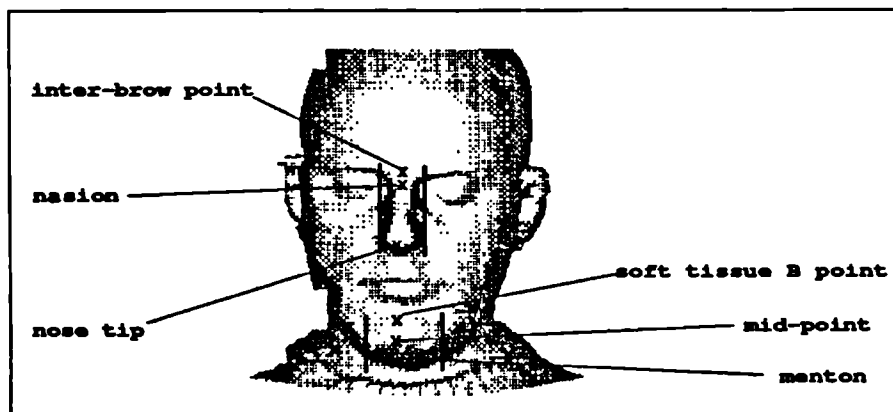


Figure 7.19: Alteration of the nose and chin using a b-spline. Area of application between the solid lines and anchor points shown.

Figure 7.20 shows an example, where the nose has been moved outwards (ie. made more convex) or inwards (more concave) by 60% of the original nose gradient. The effects of these alterations can be quantified by calculating the surface type image for the distorted surface and comparing it with the original unchanged surface. In the example shown, the outward movement has resulted in the enlargement of the mid-bridge peak and the peaks at the inner brows and the pit at the right inner canthus is



reduced. The inward movement has removed the mid-bridge peak, replacing it with saddle ridge surface.

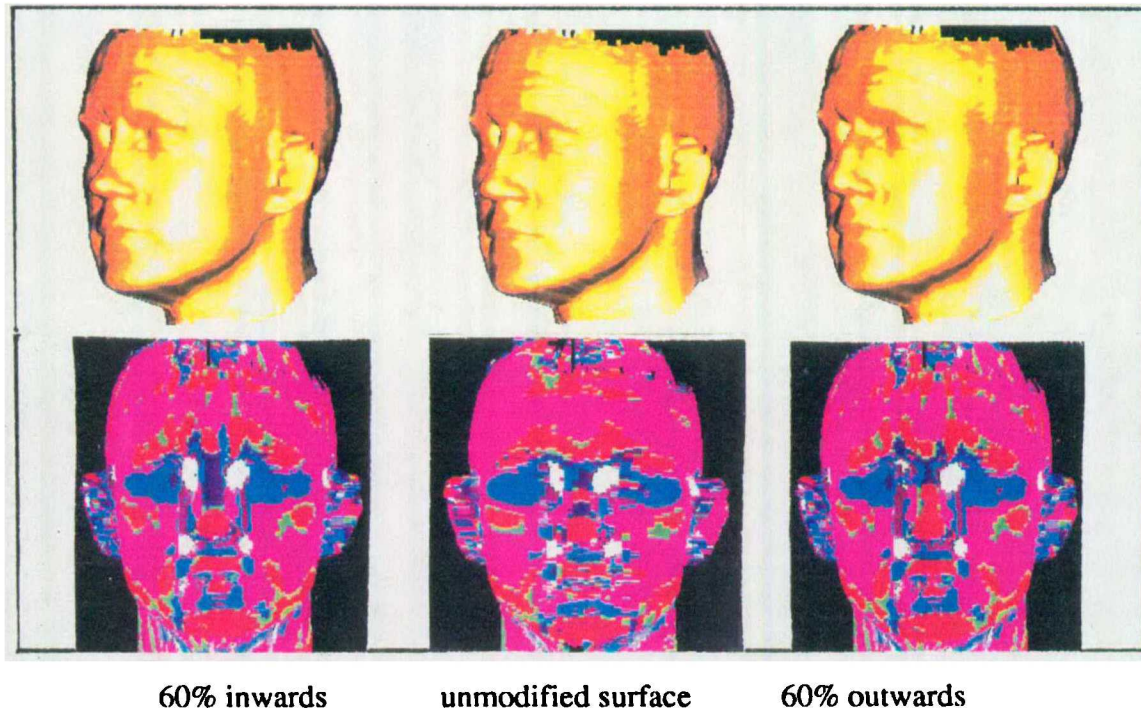


Figure 7.20: Alteration of the nose using a b-spline technique. Above facial surface images. Below the corresponding surface type descriptions.

This technique was used to alter the shape of the nose in order to investigate its effect on the perceived masculinity or femininity of the resulting surface image (Bruce et al, in press b). This investigation found that more convex noses were described as more masculine and more concave noses were described as more feminine in appearance for both male and female heads.

A second investigation considered the effect on the amounts of the surface types in the nose region (section 7.3.2) caused by altering the nose gradient, by 20%, 40% and 60% in an inward and an outward direction. Two faces were used, one male and one female, both of which appeared to us to have unremarkable nose shapes. A much greater effect in terms of the proportions of surface types was seen for the female face than the male in both inward and outward directions. This greater impact of the nose changes for the female face compared with the male is clearly shown for the peaks, saddle ridges, minimal surfaces and pits and, to a lesser extent for the saddle valleys. For the flat surfaces, there are no obvious differences between the two heads. Only for the ridges and the valleys, does the male head seem to give a somewhat greater difference in amount of surface type. These two surfaces have zero Gaussian curvature. One would expect from this analysis, that the nose changes would be more easily detectable in the female head. This was indeed the case. For the female head, both inward and outward

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changes reduce the amount of pit surface. This is due to a reduction in the pits at the inner canthi. This reason for this effect may be due to the spline being applied over too great a range of profiles, since the nose bridge is very narrow between the inner canthi for this woman.

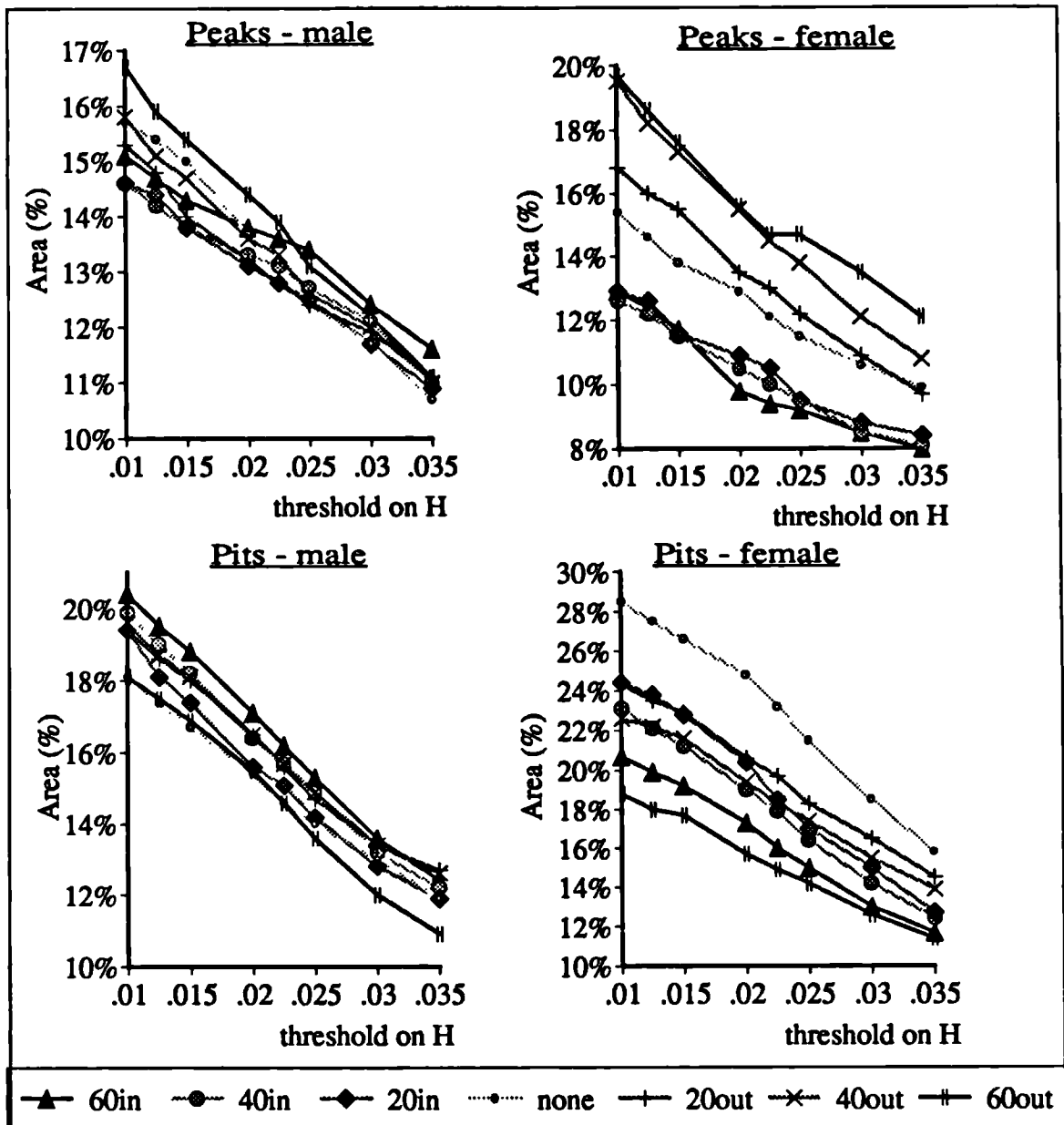


Figure 7.21: Graphs illustrating the variation of surface types with percentage of nose gradient alteration for a female face and a male face. (continued overleaf)

In a more general investigation into the detectability of these nose changes on the facial surface image and the corresponding surface type image, psychological experiments were conducted in order to discover whether these changes were noticeable. We found that for both images an alteration of the nose gradient by 20% was perceived as *not* noticeable, whereas an alteration of 40% was *just* noticeable and a 60% alteration was *definitely* noticeable (Bruce et al, submitted).



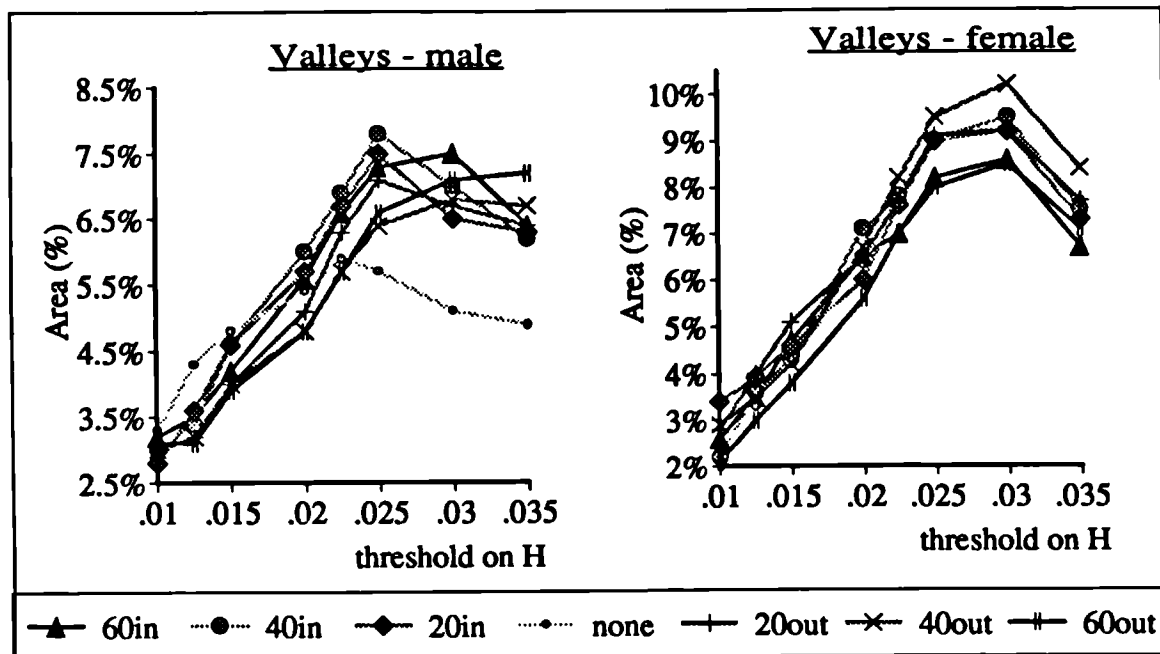


Figure 7.21(cont.): Graphs illustrating the variation of surface types with percentage of nose gradient alteration for a female face and a male face.

### 7.5.3 Addition and Subtraction of Surface Type Patches

The b spline technique described in the previous section enabled the shape of the nose and the chin to be altered. In order to alter other parts of the face a new method, based on the surface type patches was sought. In this method, the 3D data (optical surface scan) corresponding to the surface type patch (or an amalgamation of surface type patches) was moved outwards or inwards. Zero movement was applied to the patch boundary, a maximum movement to a central axis of the patch and the movement of the rest of the patch was blended between the two. Thus the three stages involved in this method are i) to smooth the patch boundary, ii) to find the central axis of the patch and iii) to blending the movement across the patch data using a function that alters the data in a smooth manner. The resulting smooth alteration of the data set, by a known amount, will enable controlled experiments to be conducted to investigate which parts of the face are important for various face recognition tasks and to simulate the results of surgical operations.

Since the surface type patches typically have somewhat ragged edges, the first step was to smooth the edges using an erosion/dilation algorithm. This algorithm and its effect on a typical patch is illustrated in figures 7.22 and 7.23. It was found that dilating then eroding three times gave a reasonably smooth boundary whilst preserving the overall shape of the patch. The patch was dilated first in order to prevent patches such as the one in figure 7.24 splitting in two.

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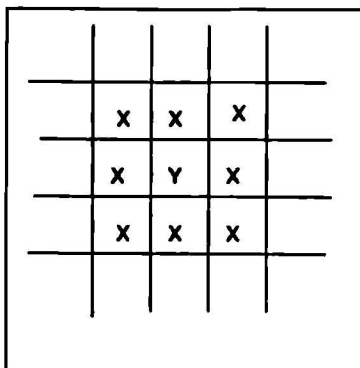


Figure 7.22: Erosion and dilation of a patch. Dilation algorithm. For each patch pixel Y, examine locations X. If location X is not part of the patch then include in the patch.

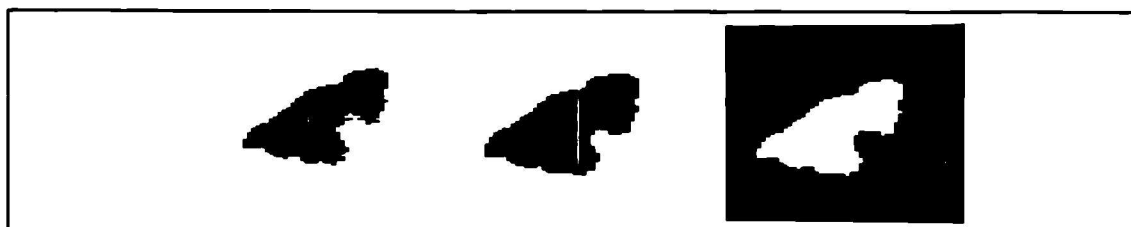


Figure 7.23: Erosion and dilation of a patch. Left, unaltered patch. Centre, patch after 1 dilatation. Right, patch after 1 dilatation followed by erosion.

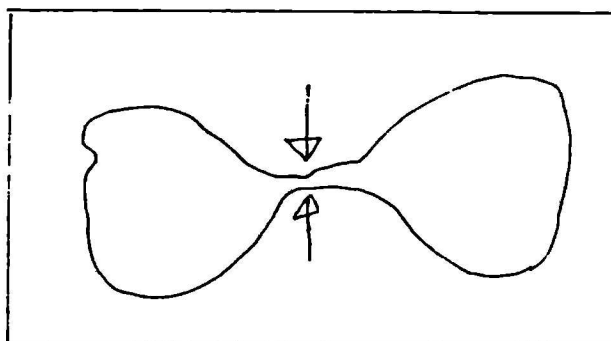


Figure 7.24: Example of a patch where eroding first would split the patch

Once a relatively smooth patch boundary was achieved, a method was sought for determining the central axis of the patch, that would correspond to the maximum movement. Suitable methods considered were finding the medial axis of the patch (eg. Pavlidis, 1982; Zhang and Suen, 1984) (figure 7.25) or eroding the patch boundary to produce contours (figure 7.26). Smoothing the patch boundary, frees the medial axis from erroneous branches and produces smoother contours.

Either of these techniques seemed to be suitable. However, for the next stage, producing a uniform blending between the central axis and the boundary, the contouring technique was simpler to implement. Therefore, this method was chosen.



Figure 7.25: The medial axis of the patch.

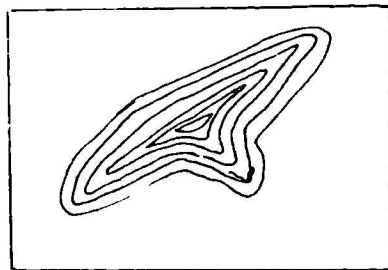


Figure 7.26: Erosion of the boundary to produce contours.

Several functions were implemented to blend the 3D data corresponding to the surface type patch, between zero movement at the boundary and one at the central axis. These functions included a Gaussian function and a cosine function, raise to a power. The chosen function was sampled according to the number of contours,  $M$ , produced when the patch was eroded. These functions are illustrated in figure 7.27.

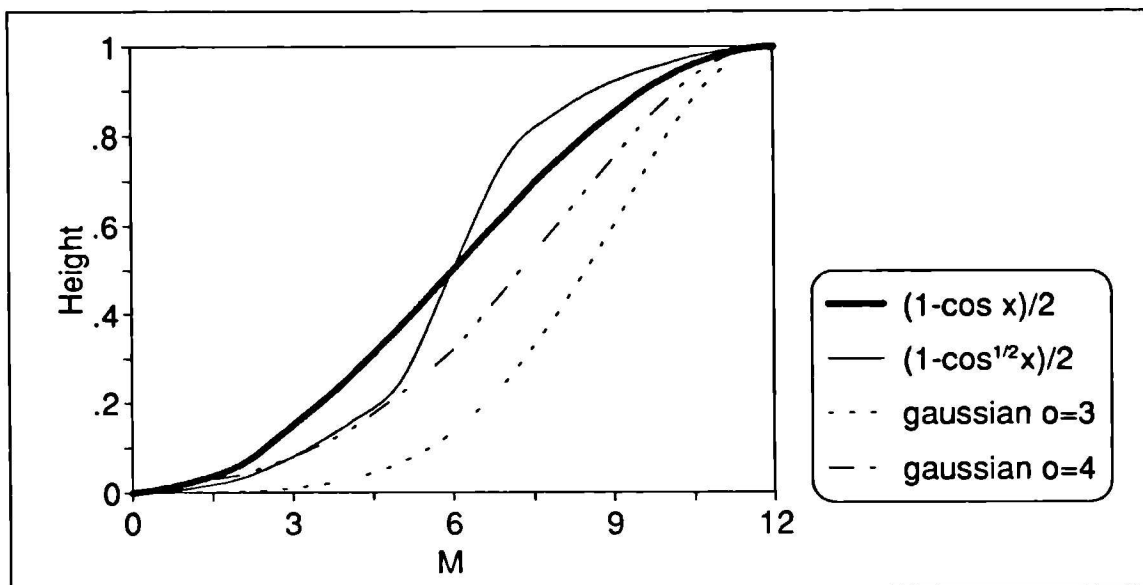


Figure 7.27: Possible functions used to blend the patch movement.

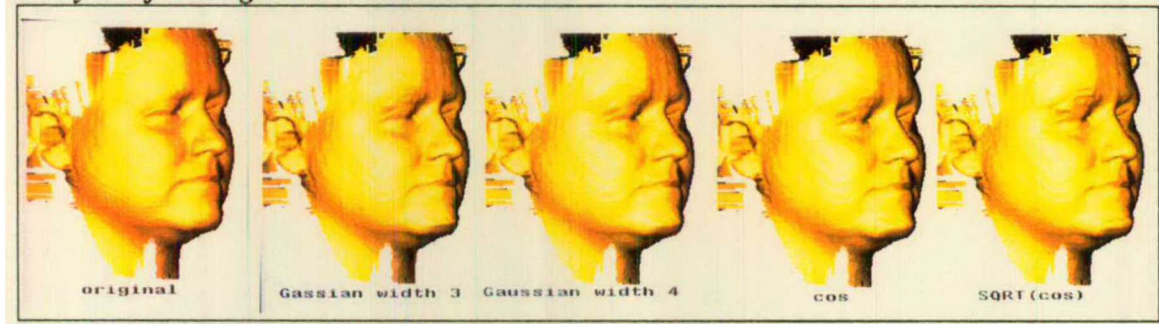


Figure 7.28: Movement of the portion of the optical surface scan image corresponding to the right eyebrow peak and chin peak using different blending functions.

The movement  $f(x)$  was defined as follows;

For a Gaussian function:

$$f(x) = \sum_{x=-M}^0 \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \text{ where } \sigma \text{ is the width of the Gaussian}$$

A correction of  $\sigma\sqrt{2\pi}$  is applied so that  $f(x)$  lies in the range  $0 \rightarrow 1$ .

For a cosine function raised to a power,  $p$ :

$$f(x) = \sum_{x=0}^M \frac{1 - \cos^p\left(\frac{\pi x}{M}\right)}{2}$$

$f(x)$  lies in the range  $0 \rightarrow 1$ .

The amount of movement required is simply

$$\text{patch.movement} := f(x) * \text{maximum.movement}$$

The results of applying these functions to the patch data are shown in figure 7.28. The function  $\cos^p$  appears to produce the best blending, however, the movement made to the patch is still noticeable and very localised. A better method, might be to blend the change over a larger area and set half the change to occur at the patch boundary. This would mean that most of the change would occur across the patch and a small change would be made, in order to blend to zero, outside the patch. This idea has not been implemented yet.

## **7.6 Summary**

In this chapter, comparison of two surface type description by three different methods have been presented and illustrated using the "average" male and "average" female heads. In the first method, a qualitative comparison is made which allows differences in shape to be described in terms of facial features. For the average heads, this comparison suggests that the differences in shape maybe due, in part, to the relative prominence of the facial surface (or underlying skull). It would be interesting to investigate this relationship more closely. This could be done for patients who have undergone reconstructive surgery by comparing the differences in facial shape, quantified by the surface type method to the differences in movement of the soft tissues and skeletal form, which can be described using a radial distance metric, developed by Fright and used by McCance in his thesis (McCance, 1992).

A quantitative method for describing differences in various *regions* of the surface type description was reported. Differences in the eyes, nose and lower face region were described. This method allows us to begin to explore whether the surface type encoding carries any useful information and is used for the examples discussed in chapters 8 and 9.

A better method is to quantify the differences in the patches of each surface type. Suitable parameters for comparison were discussed. The implementation of such a comparison for a large number of faces relies on developing a fully automated method for *identifying* these patches on the surface type image. A suitable method for this maybe to define a frame for each feature, and use a rule-based method to assign patches to appropriate frames. This is an area for further work.

The accuracy of these comparisons relies on a good registration between the two data sets. The registration method that has been used could be improved by increasing the accuracy with which the landmark points are identified and or perhaps by using different landmark points. The production of an average head has been extremely useful for making comparisons between faces. The differences between the 3D average male head and average female head appear to correspond well to the 2D photographic averages that Benson and Perrett (1991b, 1992) produced.

The response of the surface type description to changes in the face was demonstrated for a global alteration, caricaturing, and for local alterations, made with spline functions to the nose and chin. Considerable alteration to the description was produced by caricaturing. Contrary to the expected outcome, the recognizability of the facial surface images was found not to be aided by the caricaturing process. Local alterations produced local changes to the surface type description. This technique was used to

### ***Analysis of Changes***

relate the shape change produced to the perceptibility of that change. This has implications for ideas about facial recognition and maybe of use in assessing the impact of facial surgery.

Finally, local alterations to the data set were made, over a specific surface type patch. The performance of a number of different blending functions were evaluated, in order to obtain a smooth, undetectable change for visual presentation. Of those tested, the cosine function appeared to give the most gradual change. The ability to modify facial surface data in a principled mathematically fashion, based on the shape of the surface will be of benefit for the simulation of facial surgery and experiments into facial recognition. However, it is unlikely to be possible to apply one function to all modifications, since the perceptibility of the alteration will depend on the shape of the area to which the function is applied and the curvature of that area.

## CHAPTER 8

### APPLICATIONS I: CLINICAL

*"The psychosocial consequences of having to spend a significant portion of one's childhood with a major uncorrected facial malformation can be devastating."*

Cutting (1989)

#### **8.1 Facial surgery**

The ability to change the shape of the face through surgery is not as new as one might think. The first published work, showing diagrams of operations, was *De Chirurgia Curtorum* by Tagliacozzi which appeared in 1597, but a form of surgery was practised at least a thousand years ago in India (Liggett, 1974). Operations have gradually become more sophisticated in nature and recently the planning of reconstructive surgery using computer-graphics based techniques has been made possible (Cutting et al, 1986a; Moss et al, 1988). Today, facial surgeons believe that further advances in the computer-aided planning of surgery are dependent on a better understanding of soft tissue changes (Cutting et al, 1986a).

The significance of the advances made in facial reconstructive surgery is underlined by the profound psychological effects of facial deformity. Some individuals can become quite neurotic about their facial appearance, although their face appears quite normal to independent observers. The perceived imperfection in one's face has been shown to strongly influence the decision of a prospective patient on whether or not to undergo surgery (Bell et al, 1985) and has long been recognised by orthodontists (eg. Simon in 1923 cited by Wood, 1969). Gayed (1988) investigated the psychological effect of facial shape in the assessment he made of dysmorphic patients. It has also been shown that people have a tendency to avoid contact with a person who appears in some way to be unusual or abnormal. In a study conducted on the Strathclyde Underground, Bull and Houston (1990) found that people actually moved away from a person who had a port wine stain.

In chapter 4, the desire of facial surgeons for a mathematical methodology that could analyse the complex soft tissue changes which result from surgery to the face was described. In this chapter, the method of surface type analysis, developed in chapters 6 and 7, is applied to some clinical cases. These examples include cases of congenital malformation, facial asymmetry and growth. This application allows its potential for meeting this challenge, as well its limitations, to be assessed. In addition, the semi-automatic extraction of (3D) landmark points from the surface type description is described. This is a new development which may be of clinical value. Finally, the potential of the surface type method for advancing our concepts of facial aesthetics is indicated.

## ***Clinical Applications***

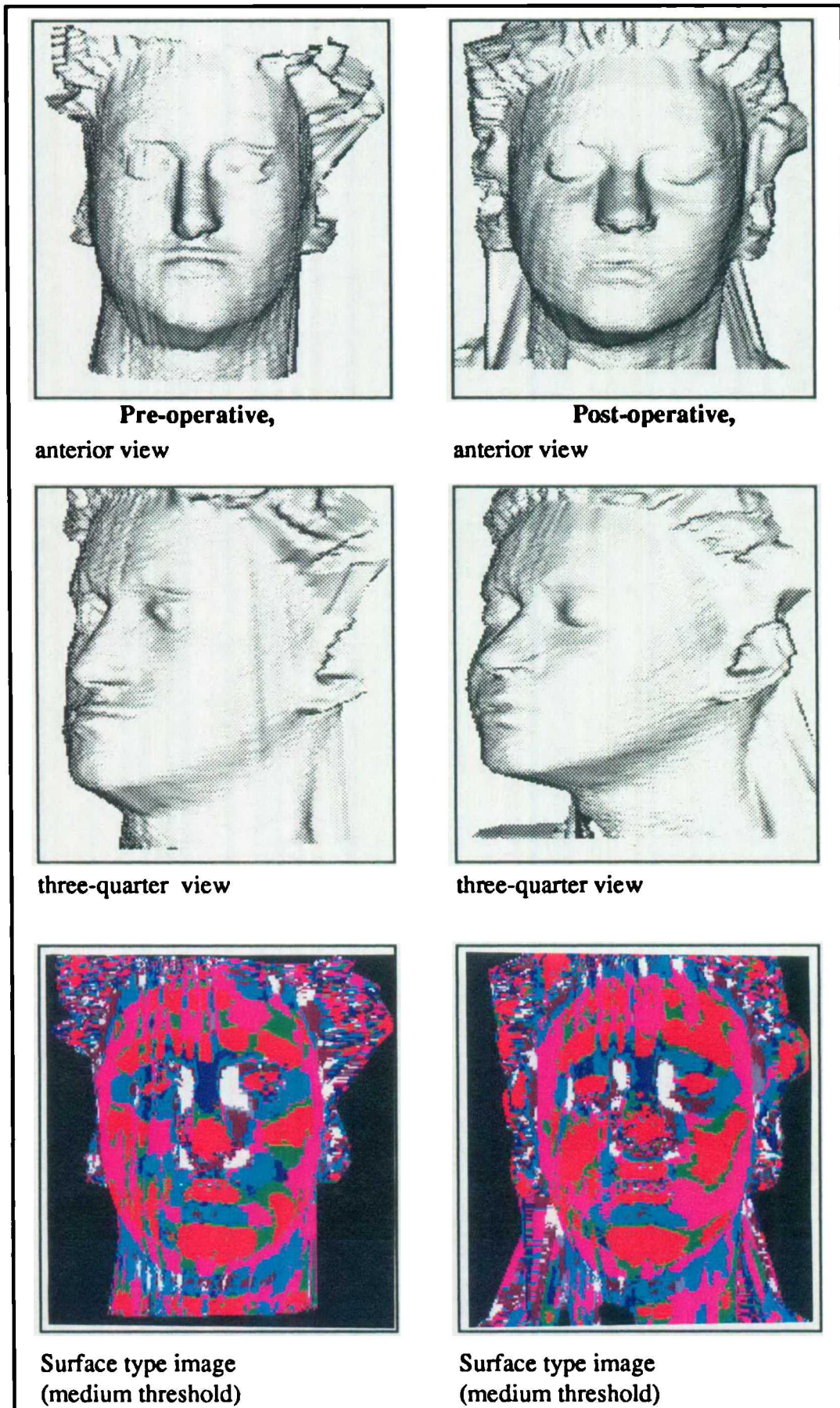
### **8.1.1 Cleft palate patient**

Cleft palate is the commonest congenital facial deformity with an incidence of between 3.6 per 1000 births for Indians, 0.5 per 1000 for Negroes and 1 per 1000 for Caucasians (Houston and Tulley, 1986). Clefts may involve the primary and/or the secondary palates and may be incomplete (not involving the lip), unilateral or bilateral. Usually, clefts of the lip are repaired shortly after birth and clefts of the palate at around 6 to 12 months. Many patients treated in the Western world develop midface hypoplasia and lack of development in the midface. There is controversy as to why this happens, some surgeons believe it to be because of an intrinsic deficiency of tissue, others feel it is caused by the surgery itself with the scarring inhibiting growth. People with cleft palate have a sunken appearance in their mid-face due to lack of growth in this region. This lack of growth is apparent in three dimensions: antero-posteriorly, laterally and vertically. It is most commonly corrected by a Le Fort I osteotomy operation that moves the maxilla forward (see Albery et al, 1986 for further details of this and other facial operations). The cut lines for this operation are illustrated in figure 8.5 (on page 168). Sometimes it is necessary to move the mandible as well during the operation either due to deformity of the mandible or in order to obtain good occlusion.

Figure 8.1 shows facial surface images of a cleft palate patient, aged 14, before (left) and after (right) an operation to correct the lack of growth to the midface. It also shows surface type images produced from the optical surface scan data. These are shown at a medium threshold level. During the operation the maxilla was advanced and the tip of the nose turned upwards. The effect on the appearance of the face was dramatic. Indeed, some of the facial surgeons involved thought that she appeared quite beautiful after the operation and the patient herself was delighted and more out-going in temperament.

The surface type analysis reveals how the facial shape has been changed by the operation. Moreover, it seems that these changes can be intuitively understood in terms of the operation performed. Before the operation, the peak surface (red) corresponding to the upper lip is just separated from the peak surface corresponding to the lower lip. Post-operatively, this peak is much larger and entirely separate, due to the advancement of this region. Also before the operation, the areas extending from the alar base to the corners of the mouth were largely represented by minimal surfaces (light blue). These minimal surfaces were considerably reduced post-operatively, and replaced by ridge and peak surfaces. Thus the sunken form over the maxilla has been turned into protuberant form by the operation. Other notable changes include, a reduction in the saddle valley (brown) along the left side of the nose and an increase in saddle valley surfaces over the lower eye orbits.





**Figure 8.1:** Cleft palate patient.

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**Figure 8.2: Cleft Palate patient. Comparison of the amount of each surface type in mid-face (left) and lower face regions (right).**

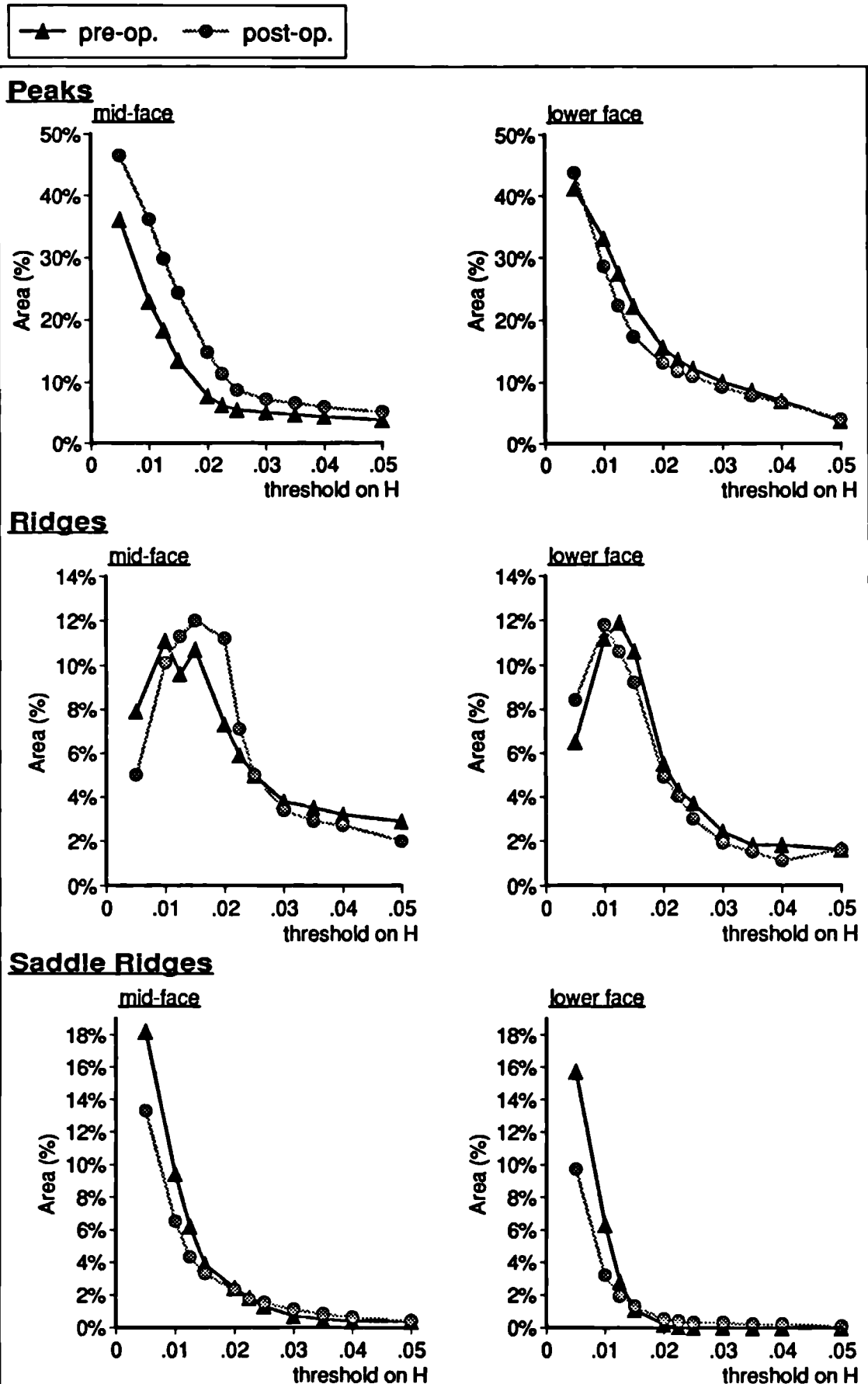
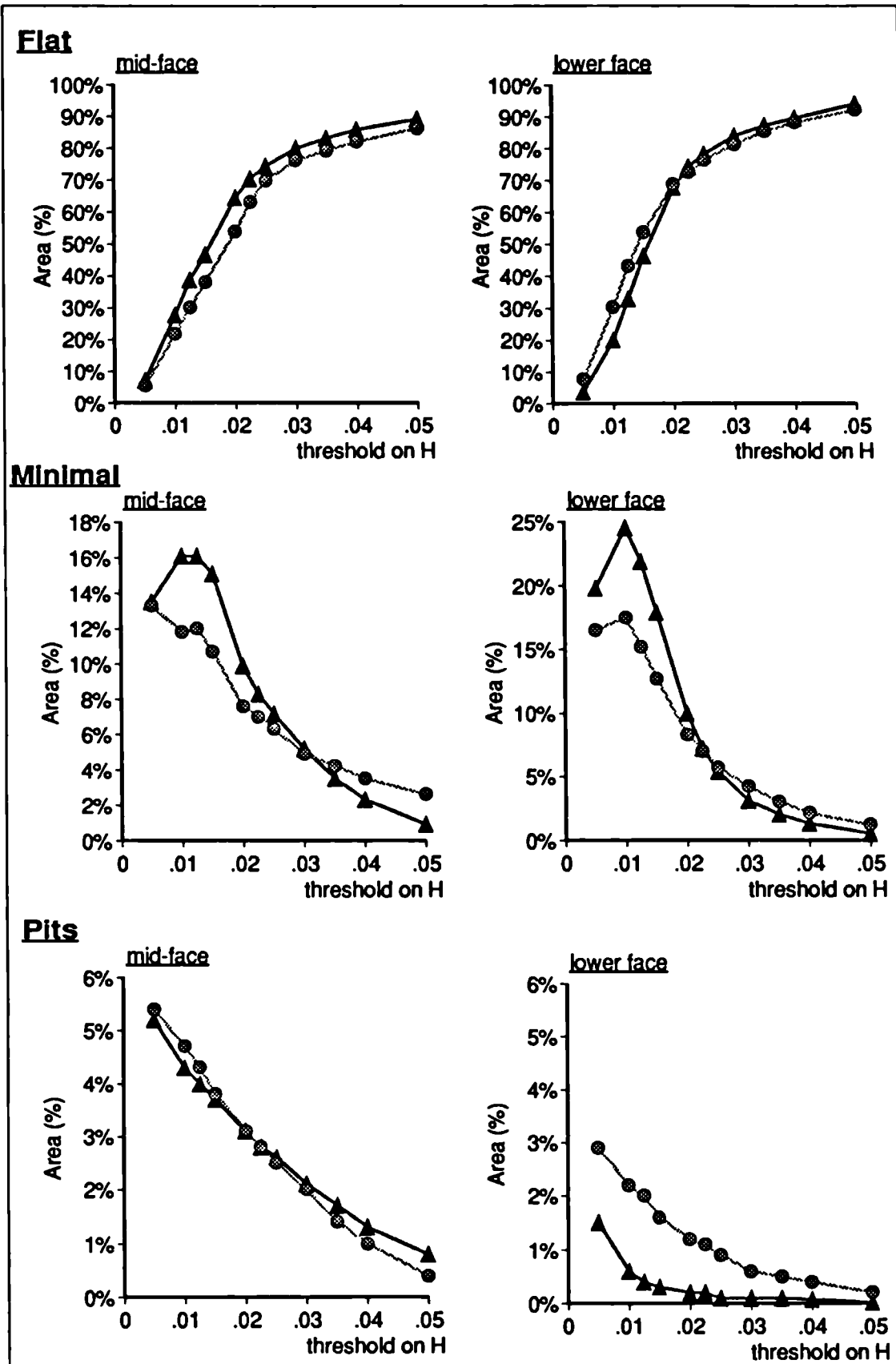


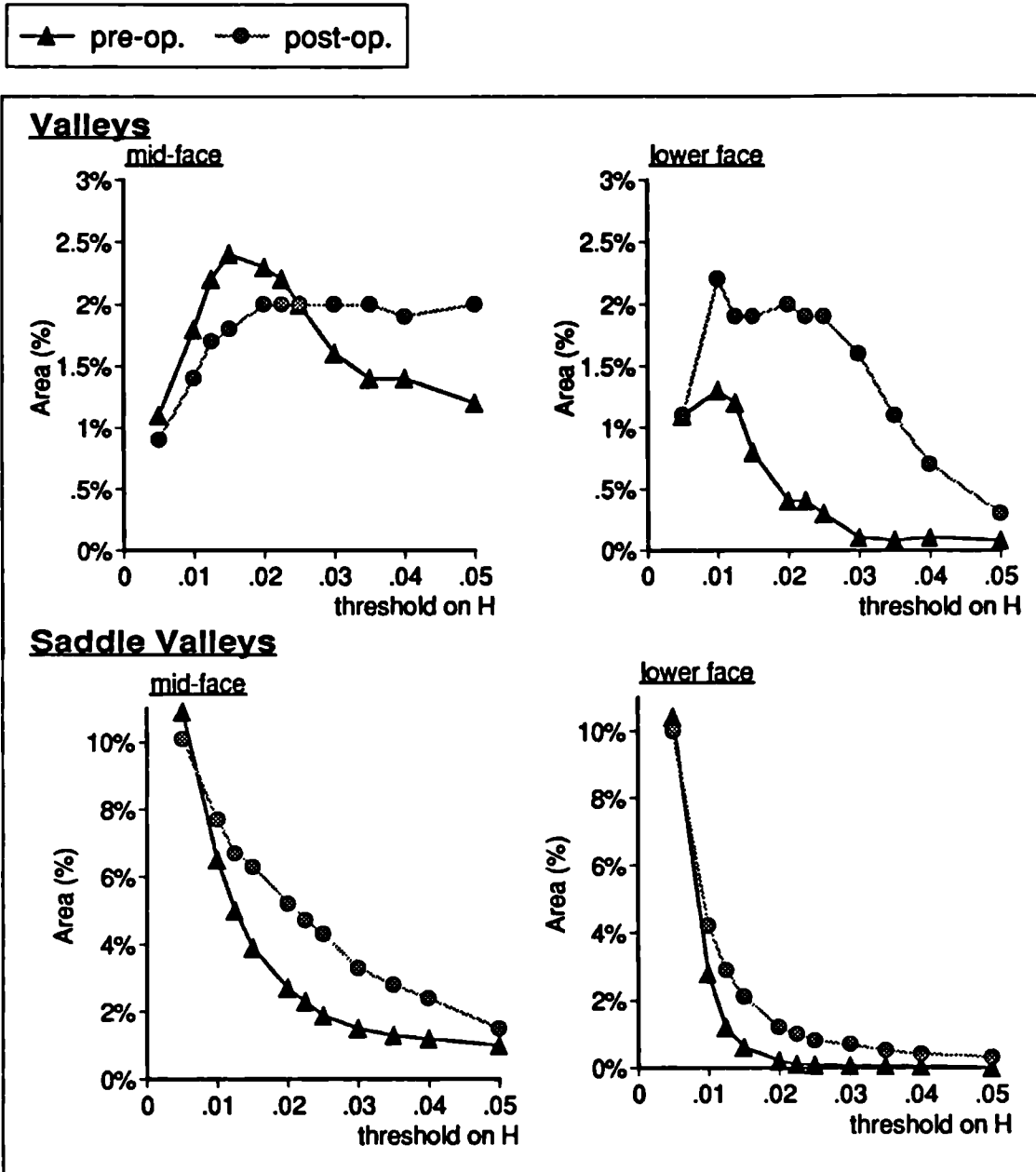
Figure 8.2(cont.): Cleft Palate patient.

▲ pre-op. ● post-op.



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**Figure 8.2(cont.): Cleft Palate patient.**



Further analysis of the surface type description can be made for regions of the face (see figure 7.5) Analysis of the mid-face and lower-face regions is presented in figure 8.2. In the mid-face region, this shows an increase in peak, ridge and saddle valley surfaces with threshold level (ie. curvature), together with a reduction in the minimal surfaces. In the lower face region, little change is shown except for a reduction in minimal surfaces

**8.1.2 Skeletal Class II patient**

This condition is defined by the antero-posterior relationship of the maxilla and the mandible to each other and is such that the maxilla is prognathic relative to the mandible or the mandible is retrognathic relative to the maxilla or a combination of the two (see figure 8.3b). Patients of this class typically exhibit a small receding lower jaw.

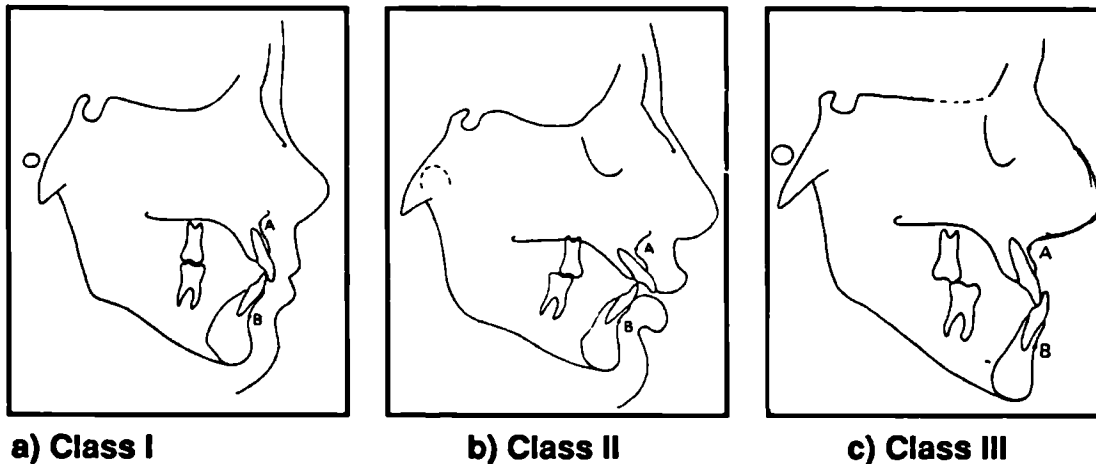
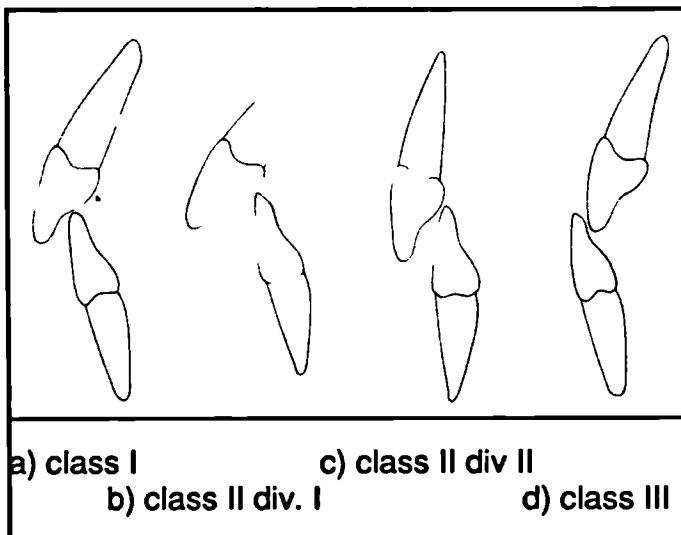


Figure 8.3: Skeletal classification. a) class I - the facial profile is well balanced with the mandible and maxilla in correct alignment. b) class II - the mandible is retrude with respect to the maxilla. c) class III - the mandible is prominent with respect to the maxilla.



This class is divided into two divisions which may be defined in terms of the position of the incisors (see figure 8.4). In division I, the incisors point forwards giving the face a "goofy" appearance. In division II, the incisors are relatively upright and there is typically a deep bite and prominent malars. The profile often appears quite good in division II, and in fact a

Figure 8.4: Incisor classification.

lot of top models have a class II, division II face. Corrective surgery is usually bimaxillary with the mandible moved forward during surgery.

In the example of a skeletal II, division I patient shown in figure 8.6, we see optical surface scan and surface type images pre-operatively, post-operatively and approximately one year post-operative (after a second operation). In the first operation the patient had: a bimaxillary Le Fort I osteotomy (see figure 8.5) - this moved the maxilla retrusively and the mandible forward via an inverted "L" mandibular osteotomy

### ***Clinical Applications***

approximately one year post-operative (after a second operation). In the first operation the patient had: a bimaxillary Le Fort I osteotomy (see figure 8.5) - this moved the maxilla retrusively and the mandible forward via an inverted "L" mandibular osteotomy

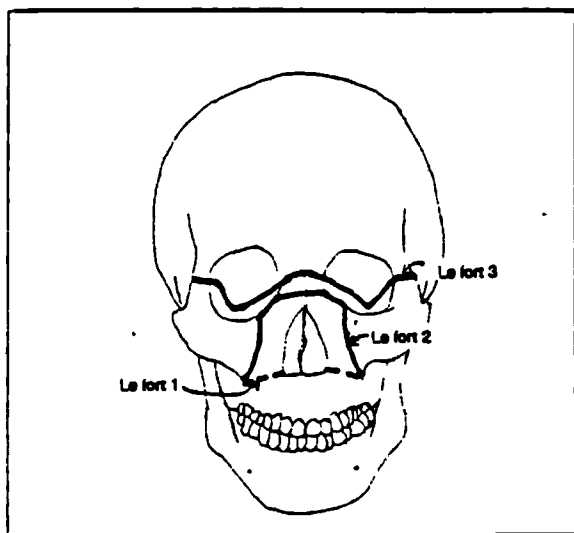


Figure 8.5: Le fort lines of facial skeletal weakness.

and costochondral graft on her right hand side. In the costochondral graft, a portion of a rib adjacent to the sternum is removed and inserted into the jaw. In the second operation almost one year later, she had a genioplasty, which moved the chin forward and a dermofat graft, in which fat was taken from her inner thigh and placed over the left hand side of the face to "pad it out". One and a half years after this she had a third operation, a rhinoplasty, this is not shown or discussed here.

The relative advancement of the mandible compared to the maxilla is seen best from the three-quarter viewpoint and can be seen after the first operation. The effect of the dermofat graft is most easily seen in the anterior view on the right side of the laser scan image.

Considering the first operation, the surface type images show an increase in peak (red) and ridge (green) surfaces around the jawline, related to the forward movement of the mandible relative to the maxilla. The reduction in the peak surface corresponding to the upper lip, the reduction in the pits (white) at the alar base and in the minimal surfaces (light blue) over the area extending from alar base to corners of mouth, indicate that the whole of the upper lip has, in general, become less curved. New pits at the corners of the mouth and ridge surface below the corners of the mouth show that the curvature at the corners of the mouth has been increased.

Considering the second operation, the surface type images show small changes, the most visible of which is the replacement of some peak surfaces on the lower left cheek with flat (pink) surfaces. Thus the main effect of the dermofat operation was to flatten the lower left cheek. The peak at the chin has become a separate region from the cheek peaks implying that the chin has been further advanced.

Analysis of the changes in the amount of each surface type in the mid-face and lower face regions are shown in figure 8.7. In the mid-face region, the first operation



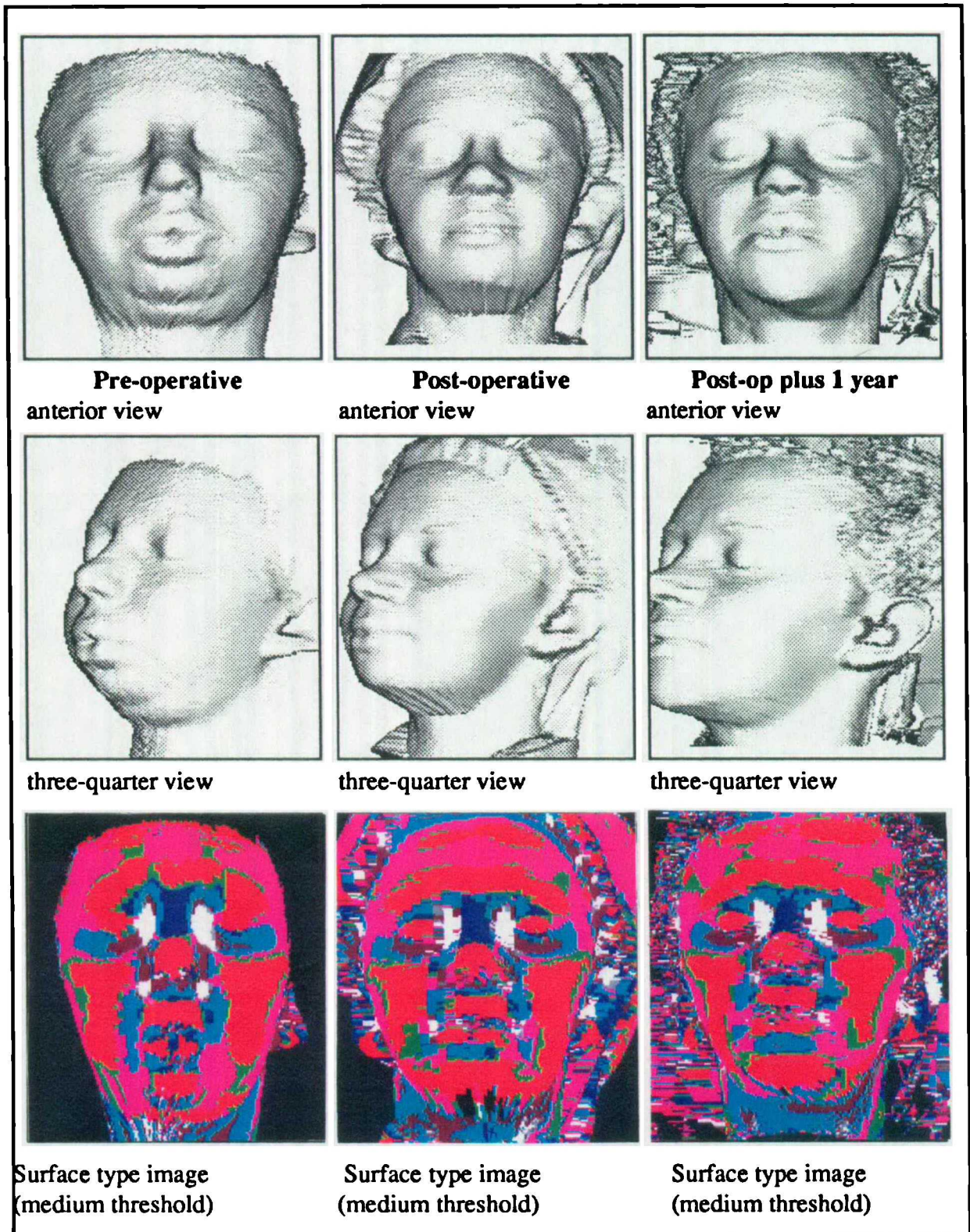


Figure 8.6: Skeletal Class II patient

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**Figure 8.7: Skeletal Class II patient. Comparison of the amount of each surface type in mid-face (left) and lower face regions (right).**

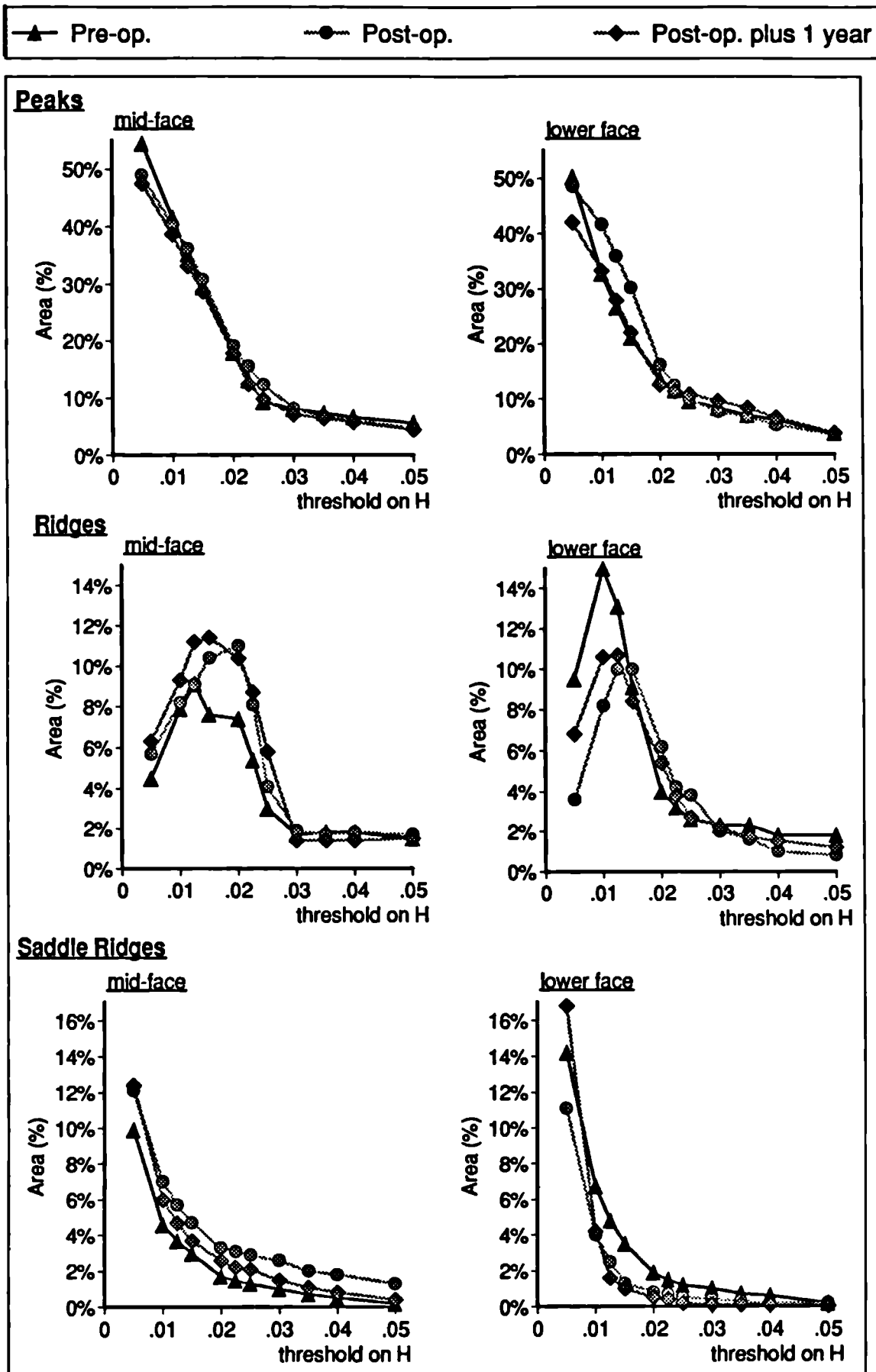
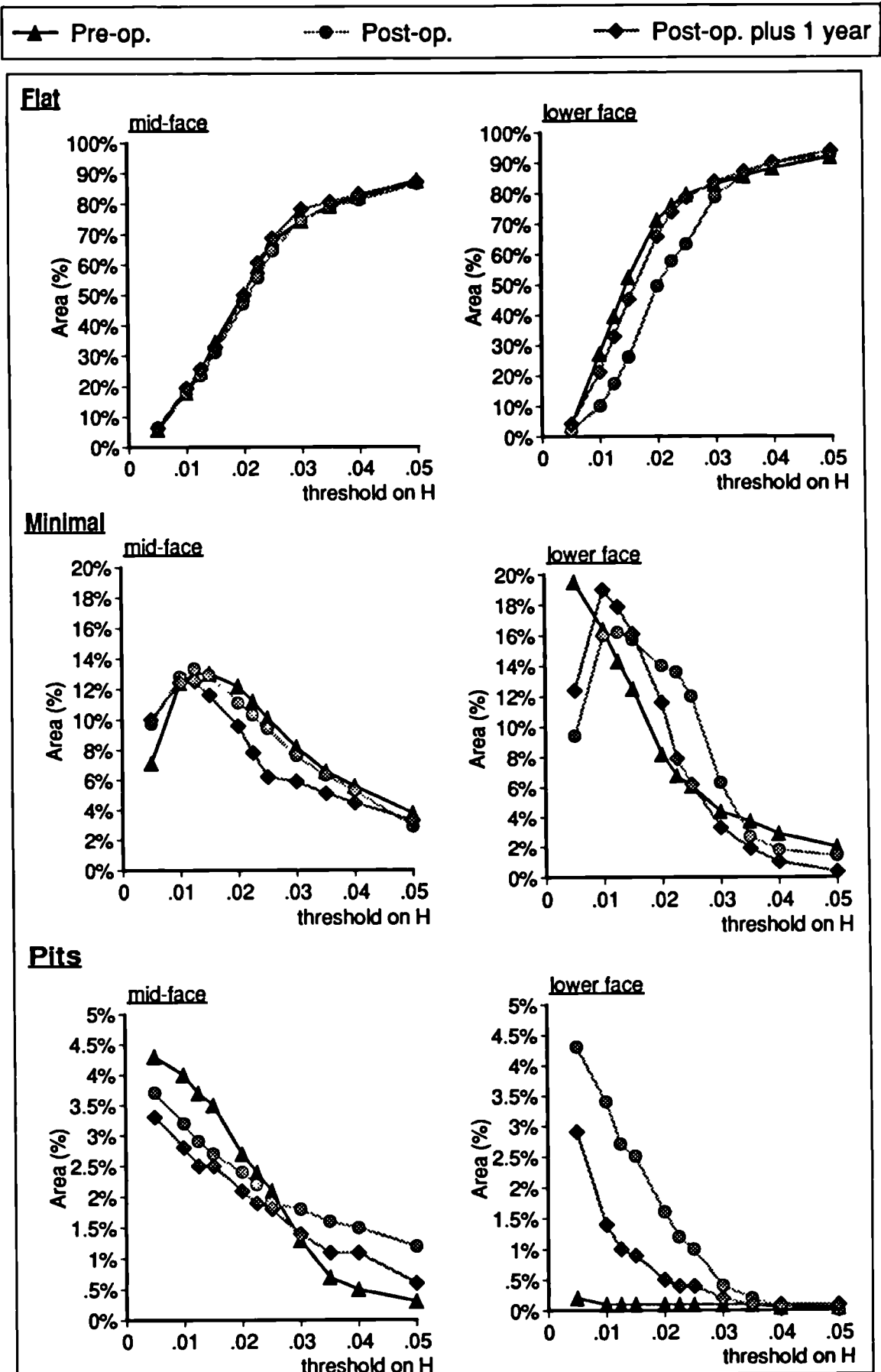


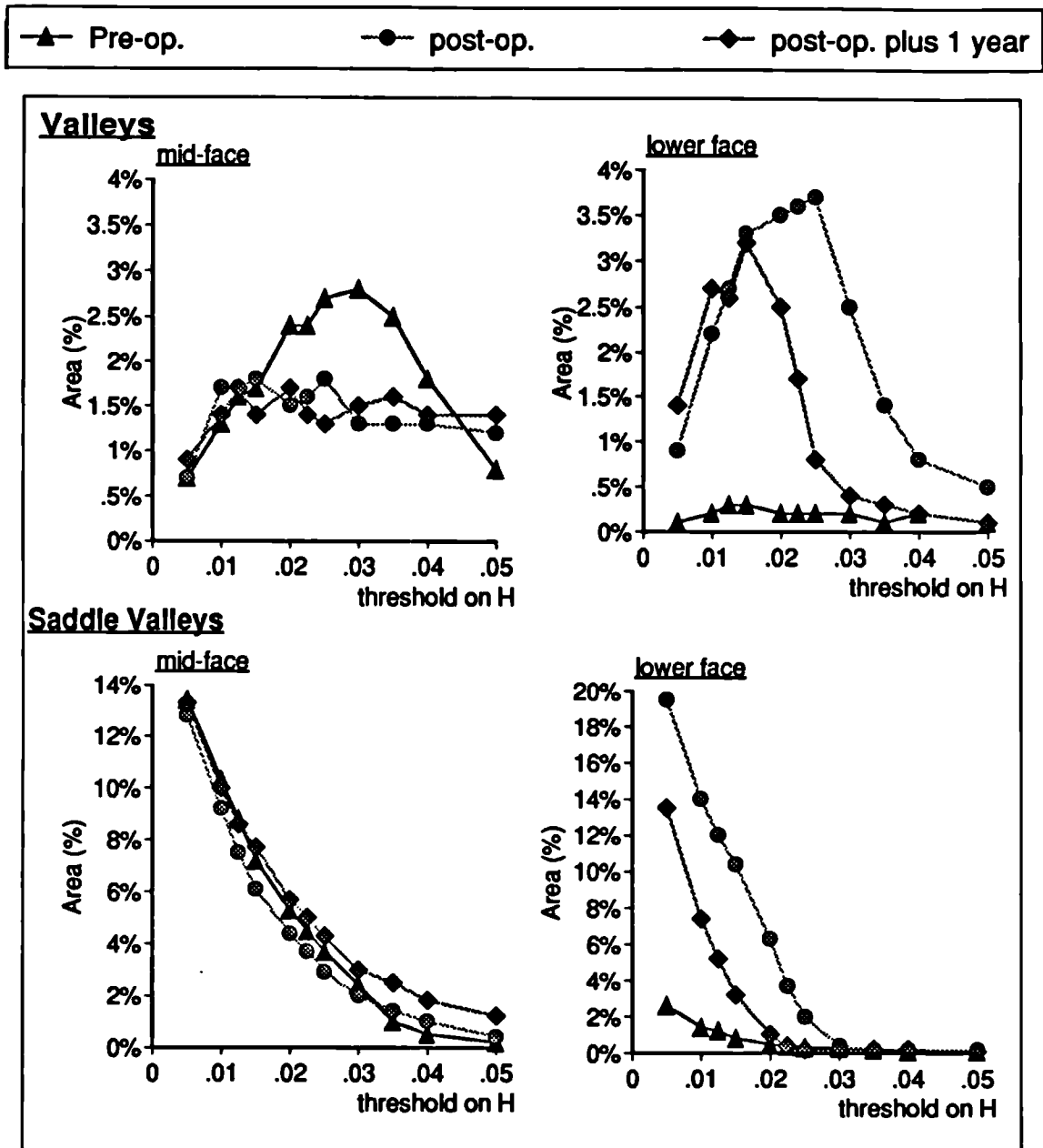


Figure 8.7(cont.): Skeletal Class II patient.



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**Figure 8.7(cont.): Skeletal Class II patient.**



the first operation and reduced medium level minimal surfaces. Across both operations, there are small reductions in peak surfaces, little change in the flat surfaces, a reduction at medium levels of the valley surfaces and small changes in the saddle valleys.

In the lower face region, the first operation, caused the following changes to shallowly curved surfaces: the ridges and saddle ridges were reduced and the pits and saddle

there were considerable reductions in pit and saddle valley surfaces, whilst at high curvatures, the valleys surfaces were considerably reduced.

### **8.1.3 Skeletal Class III patient**

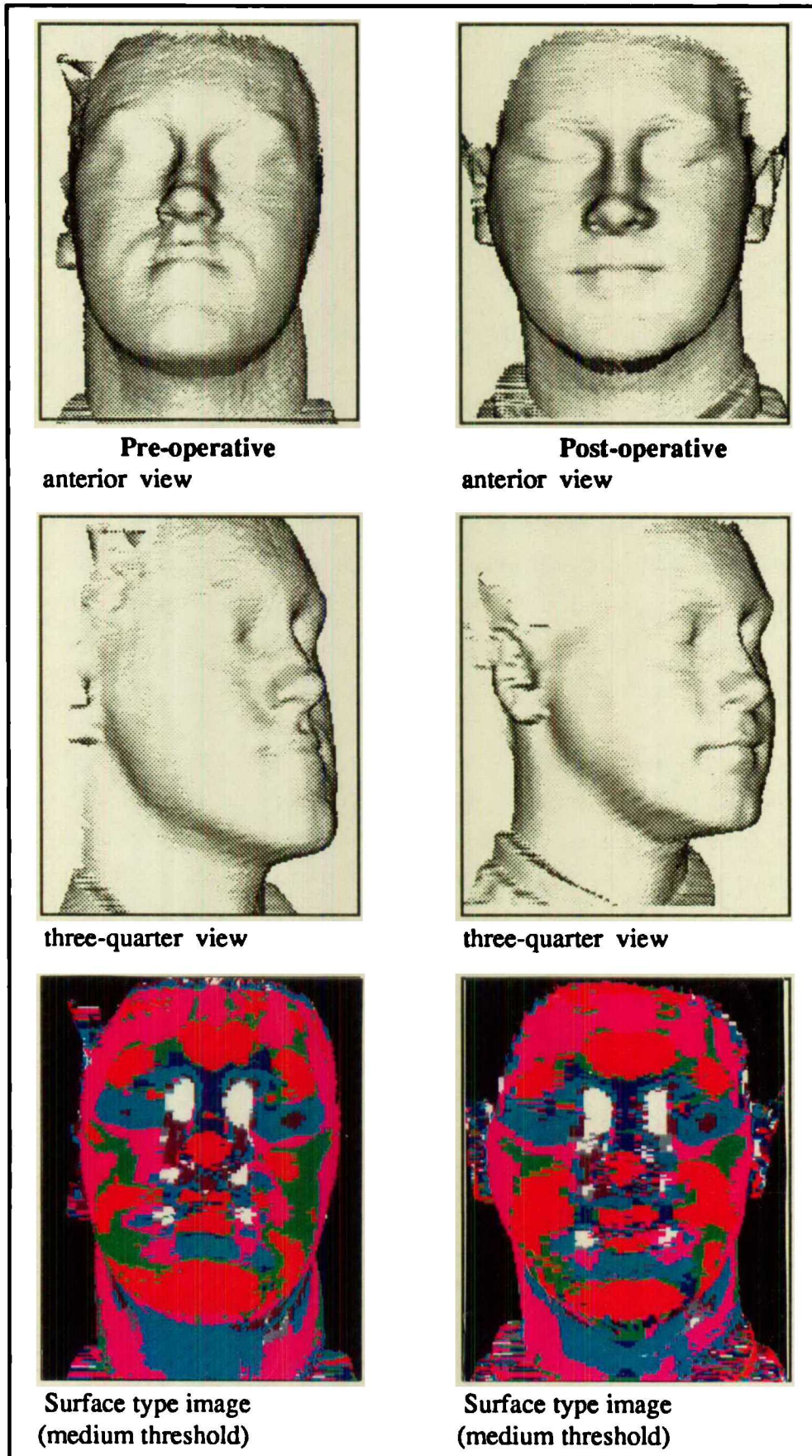
This condition is also defined by the antero-posterior relationship of the maxilla and the mandible to each other. In this case, the maxilla is retrognathic relative to the mandible or the mandible is prognathic relative to the maxilla or a combination of the two (see figure 8.3c). In this condition the chin appears prominent and long with a large mandibular facial height. Again, corrective surgery is often bimaxillary.

Figure 8.8 shows an example of a skeletal III patient before and after corrective surgery. He had Le Fort I maxillary osteotomy to advance the maxilla and a mandibular osteotomy with a subsigmoid pushback to decrease the prominence of the lower face. The resulting change in facial shape is revealed by the surface type images. There is a large reduction in the amount of peak surface across the chin and lower cheeks, which are replaced by mostly minimal surfaces. The peak surfaces over both cheeks are enlarged at the expense of ridge surfaces, the lower lip peak is reduced, and the upper lip peak enlarged.

An analysis of changes in the amount of each surface type in the mid-face and lower face regions is shown in figure 8.9. In the mid-face, increases in peak and valley surfaces at low threshold levels, are balanced by a reduction in ridge surfaces. At high thresholds, corresponding to greater curvatures, there is a small increase in flat surfaces. Across all thresholds, slight reductions occur in minimal, pit and saddle valley surfaces.

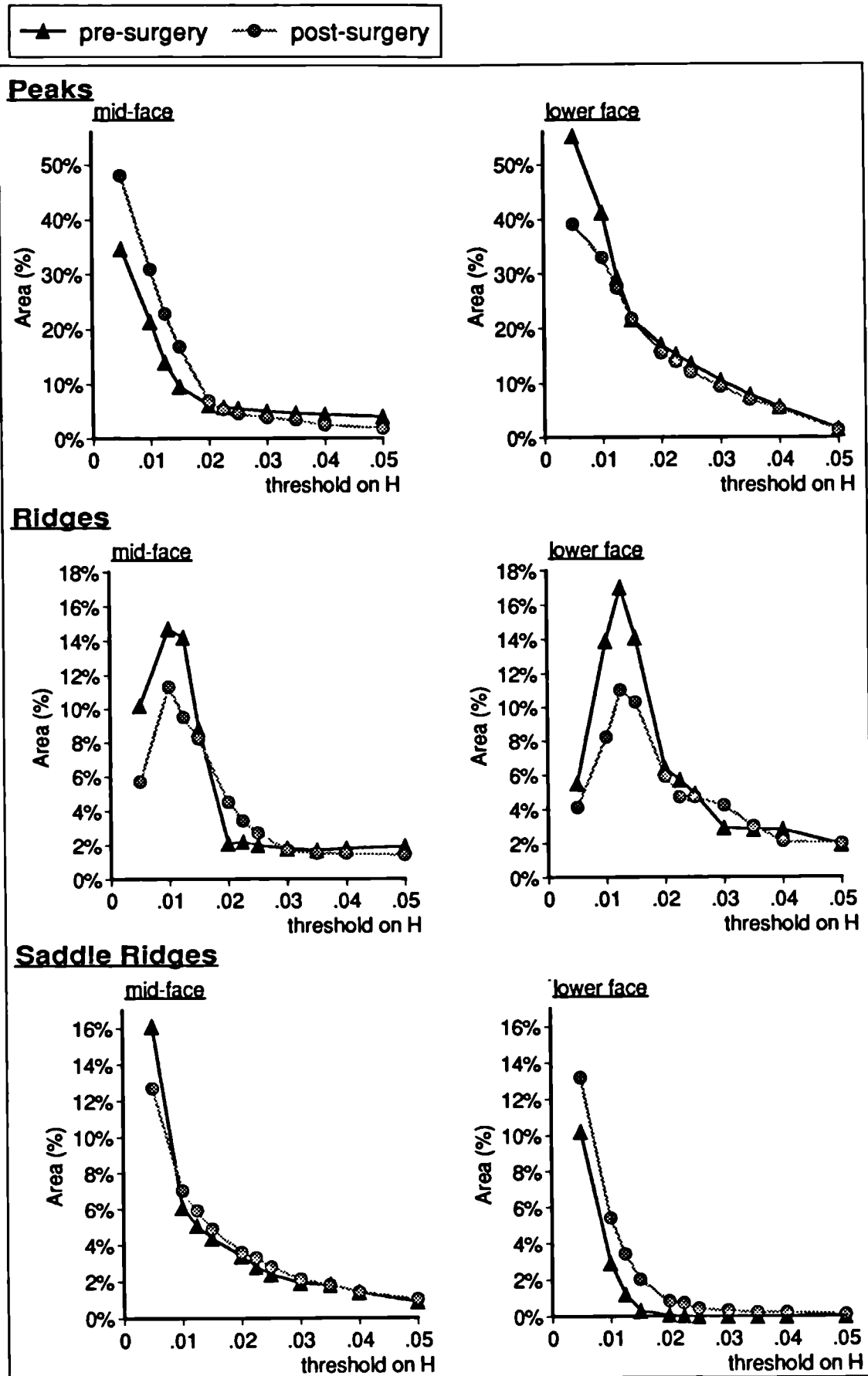
In the lower face, the most change occurs at low threshold levels, indicating that the changes in shape are small. A large reduction in peak and ridge surface is offset by sharp increases in minimal, pit, saddle valley and valley surfaces.

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**Figure 8.8:** Skeletal Class III patient

Figure 8.9: Skeletal Class III patient. Comparison of the amount of each surface type in mid-face (left) and lower face regions (right).



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**Figure 8.9 (cont.): Skeletal Class III patient.**

▲ pre-surgery    ● post-surgery

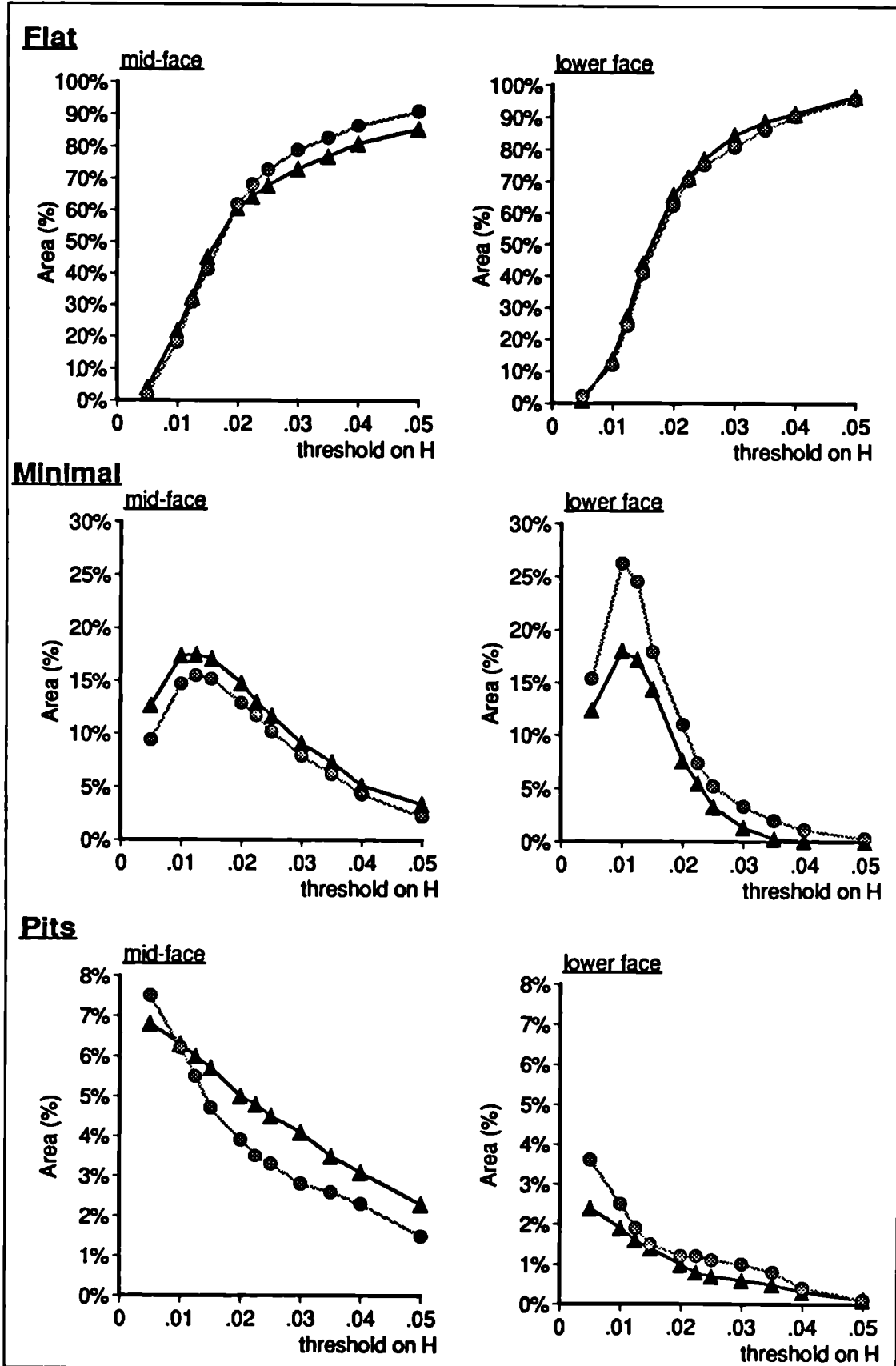
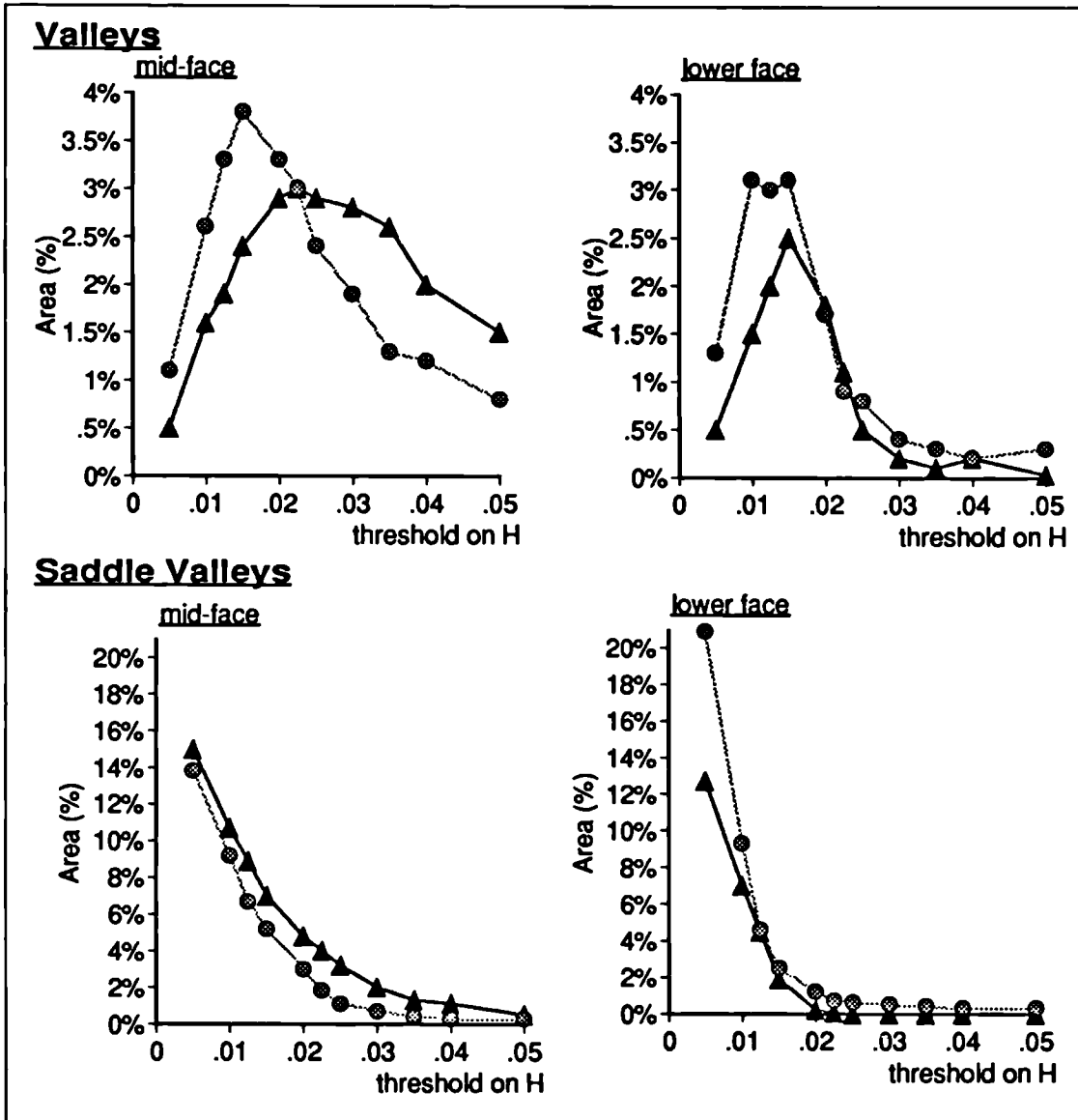


Figure 8.9(cont.): Skeletal Class III patient.



## ***Clinical Applications***

### **8.1.4 Asymmetry patient**

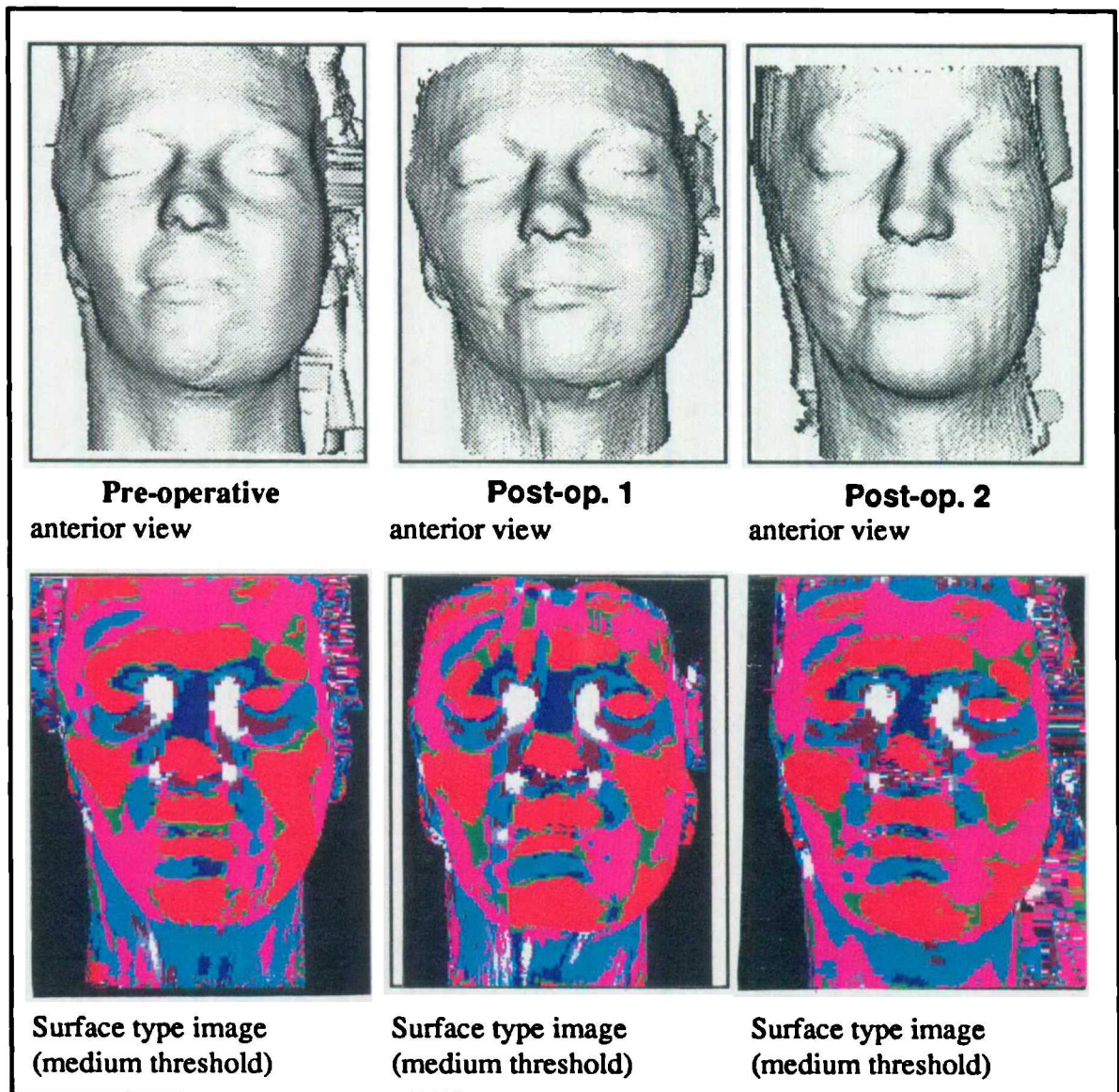
Gross congenital asymmetries of the face arise from deformations of the first and second branchial arches of the fetus, and involve the eyes, ears, mid-face and mandible. The most common is Hemifacial microsomia (also called craniofacial microsomia) which occurs in about 1 in 3500 births. This condition can be extremely mild or very severe. During growth, the "normal" side of the face will often grow to compensate for the asymmetry and tends to overgrow, leading to increased facial asymmetry. The timing of an operation is very difficult and every time surgery is undertaken there is inevitably scar tissue which inhibits normal growth. All surgery has a morbidity and this surgery can be life threatening.

An example of a hemifacial microsomia patient is shown in figure 8.10. The patient has had three operations to date. The first operation was a Le Fort I bimaxillary which moved the maxilla 2-3mm to the left, and involved the insertion of a rib graft in the mandible and a myotomy (repositioning muscle tissue). The centre illustrations show the face after this operation. The second operation was a dermofat graft to the right side, together with a reduction of the masseter muscle on the left. The right-hand illustrations show the face after this operation. She has also had a third operation, a genioplasty within the last twelve months. This last operation is not discussed here.

The surface type image for the pre-operative scan shows a large peak surface on the left hand side of the lower face, compared to flat surfaces on the right hand side. The changes in shape shown by the surface type images for the first operation, are a reduction in this peak area and movements to the right of the lower lip peak and the minimal surface that lies between the lower lip and chin. In the second operation, the lower lip peak and this minimal surface is moved leftwards and a further small reduction in the peak on lower left cheek occurs.

Figure 8.11 shows an analysis of the amount of each surface type in the *left* and the *right* hand sides of the faces for the three images shown in figure 8.10. The division between left and right sides was made by the mid-sagittal line, passing through the nasion and centre of the nose tip. Considering the first operation, surface types which show a reduction in asymmetry are the shallowly curved minimal surfaces and flat surfaces of a medium threshold. Interestingly, the amount of valley surfaces on the right hand side has been considerably increased whilst those on the left side considerably decreased. However, only a very small amount of the facial surface is represented by valley surfaces. The analysis shows that the second operation has considerably improved the facial asymmetry with all surface types showing an improved balance between the left and right sides.





**Figure 8.10** Asymmetry patient

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**Figure 8.11: Asymmetry patient. Comparison of amounts of surface types on the left and right hand sides of the face.**

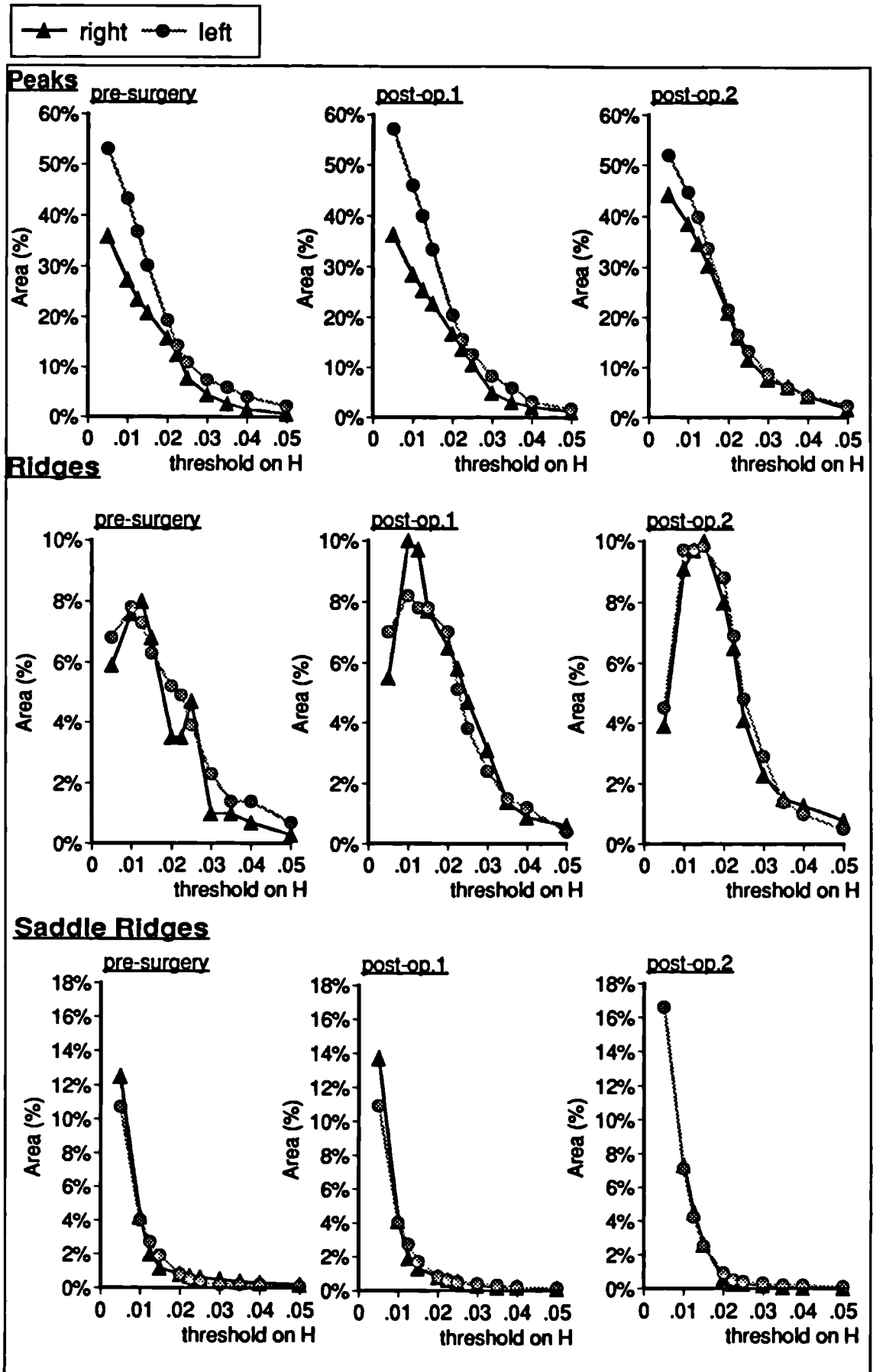
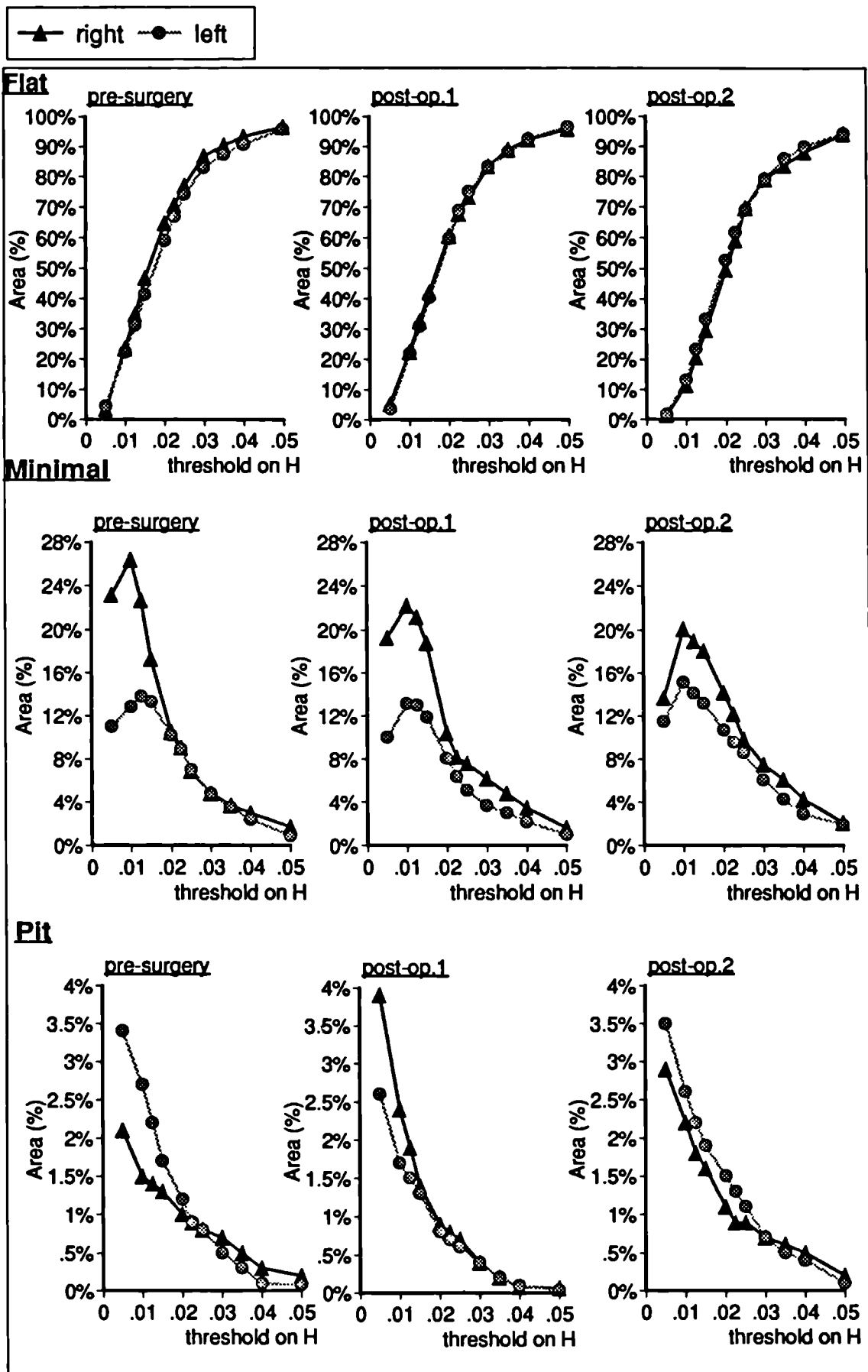
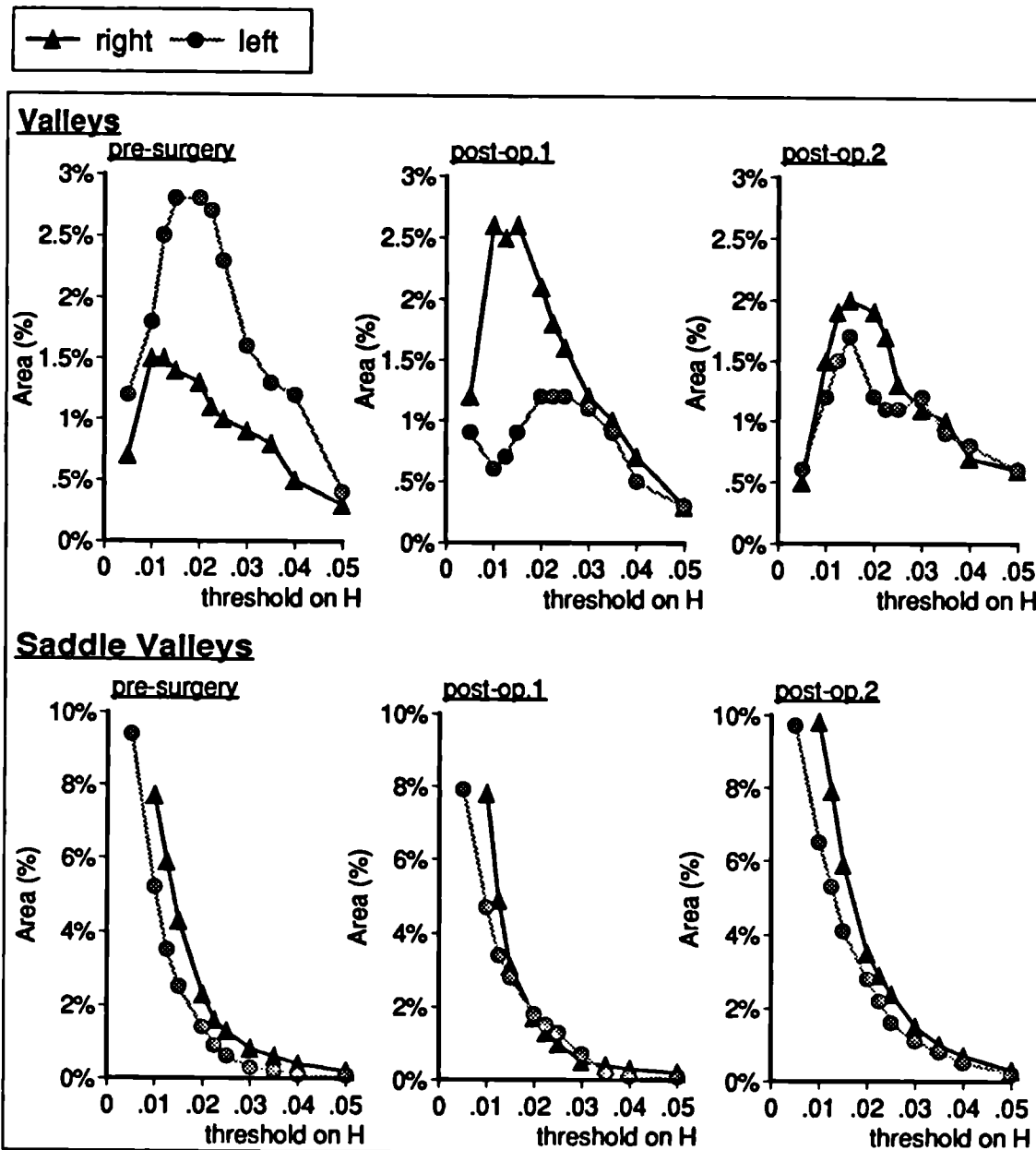


Figure 8.I(cont.): Asymmetry patient.



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**Figure 81(cont.): Asymmetry patient.**



## **8.2 Facial growth**

Change of shape is implicit in the term growth implying that facial growth proceeds through a change of shape. The advancement of studies of growth is thus dependent on the development of appropriate mathematical tools for analyzing size and shape and these have played a key role in the progression of growth studies.

Many investigations of facial growth have been made and for many purposes. In the last century, an abundance of research papers have been published in journals of anthropology, genetics, orthodontistry, plastic surgery, radiology and biometrics to name but a few. Three examples of these which show this wide interest are; facial growth before and during puberty (Baughan et al, 1979), the growth of the cranium and palate in Downs Syndrome children (Barden, 1983) and the differences in the cranial base shape described by Fourier analysis (Lestrel and Roche, 1986). In this latter investigation, the strongest changes found were due to the puberty growth spurt and showed stronger changes, of longer duration, for males compared to females. No apparent shape change was found during adulthood. It is not proposed to review these many studies in this work, the reader is referred to the reference list of Sadler and colleagues for a broad sample of this work (Sadler et al, 1990a).

It is, however, important to note that these studies have all been based on charting changes in the position of various landmark points with time. In this sense, the literature is very complete but all these measurements reveal very little about changes in the *shape* of the face during growth. It is this knowledge which is vital in testing models of facial growth (such as the spline model for growth of the nose used by Sadler et al, 1990a) and for predicting the growth of a face.

Whilst surgeons have concentrated on abnormal growth due to deformity or tumours, psychologists, anthropologists and orthodontists have become interested in "normal" growth. Early this century, the founder of the British orthodontic society, George Northcroft, charted the growth of his son William's face by taking plaster casts every year from the age of six until twenty-one. In 1987 this son, aged 72 then, agreed to visit University College to have the procedure repeated and also to have an optical surface scan. As a result, it was shown that the weight of the plaster distorted the facial surface by around 10mm (Moss, 1989). Orthodontists have tended to assume that once a person has reached 21 years, the amount of facial change due to growth is negligible (Bjork, 1966, Baer and Harris, 1969, Enlow, 1975). In contrast to this opinion, the Northcroft study showed that between the ages of 21 and 72 years, the chin had grown outwards and some (smaller) change had occurred to the nose.

### ***Clinical Applications***

year from the age of six until twenty-one. In 1987 this son, aged 72 then, agreed to visit University College to have the procedure repeated and also to have an optical surface scan. As a result, it was shown that the weight of the plaster distorted the facial surface by around 10mm (Moss, 1989). Orthodontists have tended to assume that once a person has reached 21 years, the amount of facial change due to growth is negligible (Bjork, 1966, Baer and Harris, 1969, Enlow, 1975). In contrast to this opinion, the Northcroft study showed that between the ages of 21 and 72 years, the chin had grown outwards and some (smaller) change had occurred to the nose.

In order to see how the shape of the face changed due to growth, the surface type method was applied to facial data of an adolescent boy, who had four optical surface scans taken over a two year period. Figure 8.12 shows anterior facial surface images for these four scans together with medium threshold surface type images. Whilst many surface type regions remain in the same place, small changes to these regions are visible over this period.

Analyses of the changes in the amount of each surface type for these four scans, over the entire face and in the eyes, nose and lower face regions are presented in figure 8.13. Disappointingly, no clear trends, such as an increase in the amount of peak surface with growth, are revealed by the analysis.



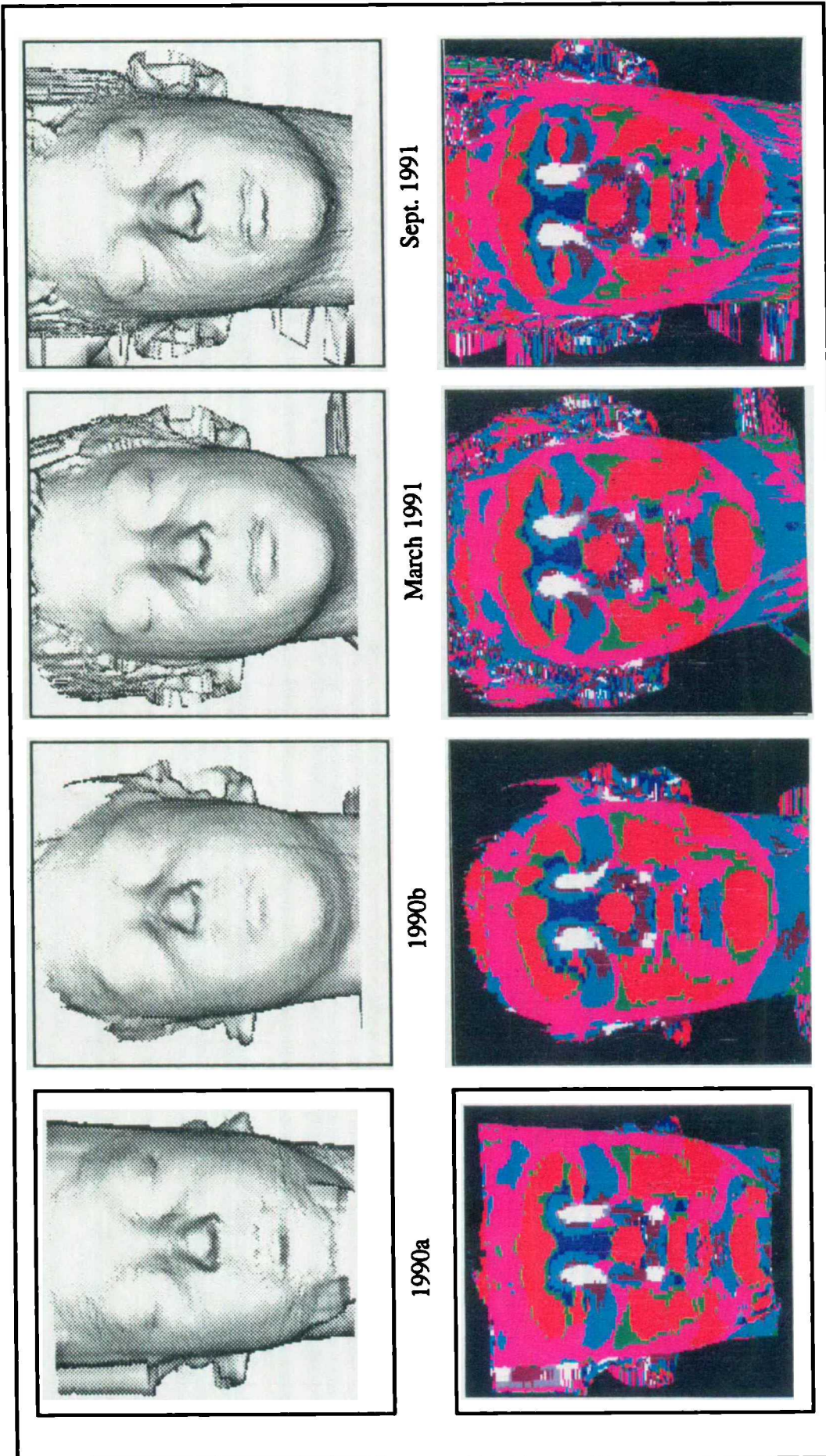


Figure 8.12 Facial Growth of an adolescent boy over 2 years. Anterior view and corresponding surface type images.

Figure 8.13 Facial growth of an adolescent boy. Comparison of the amount of each surface type across the entire facial surface, in the eyes, nose and lower face regions.

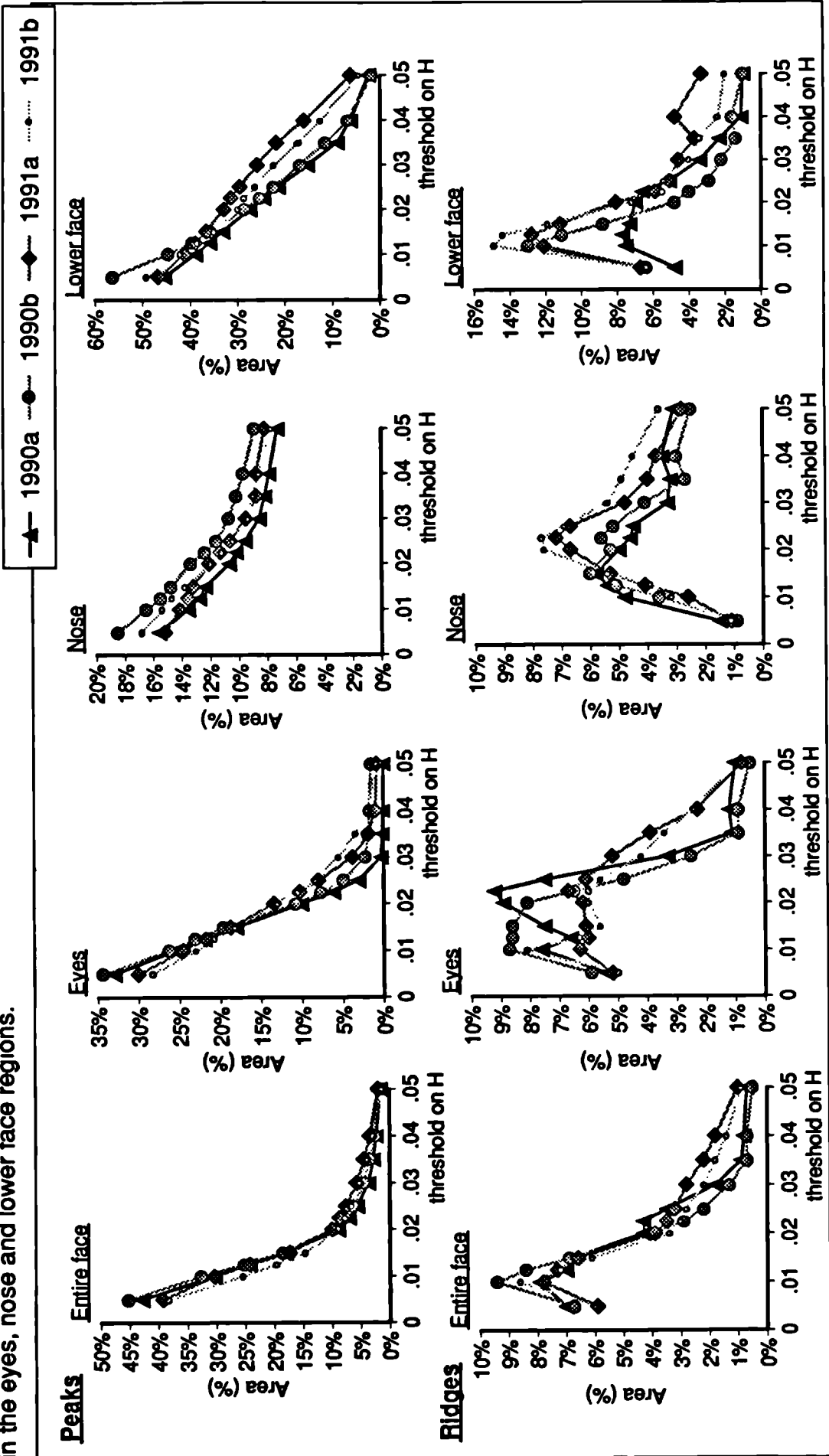




Figure 8.13(cont.) Facial growth of an adolescent boy.

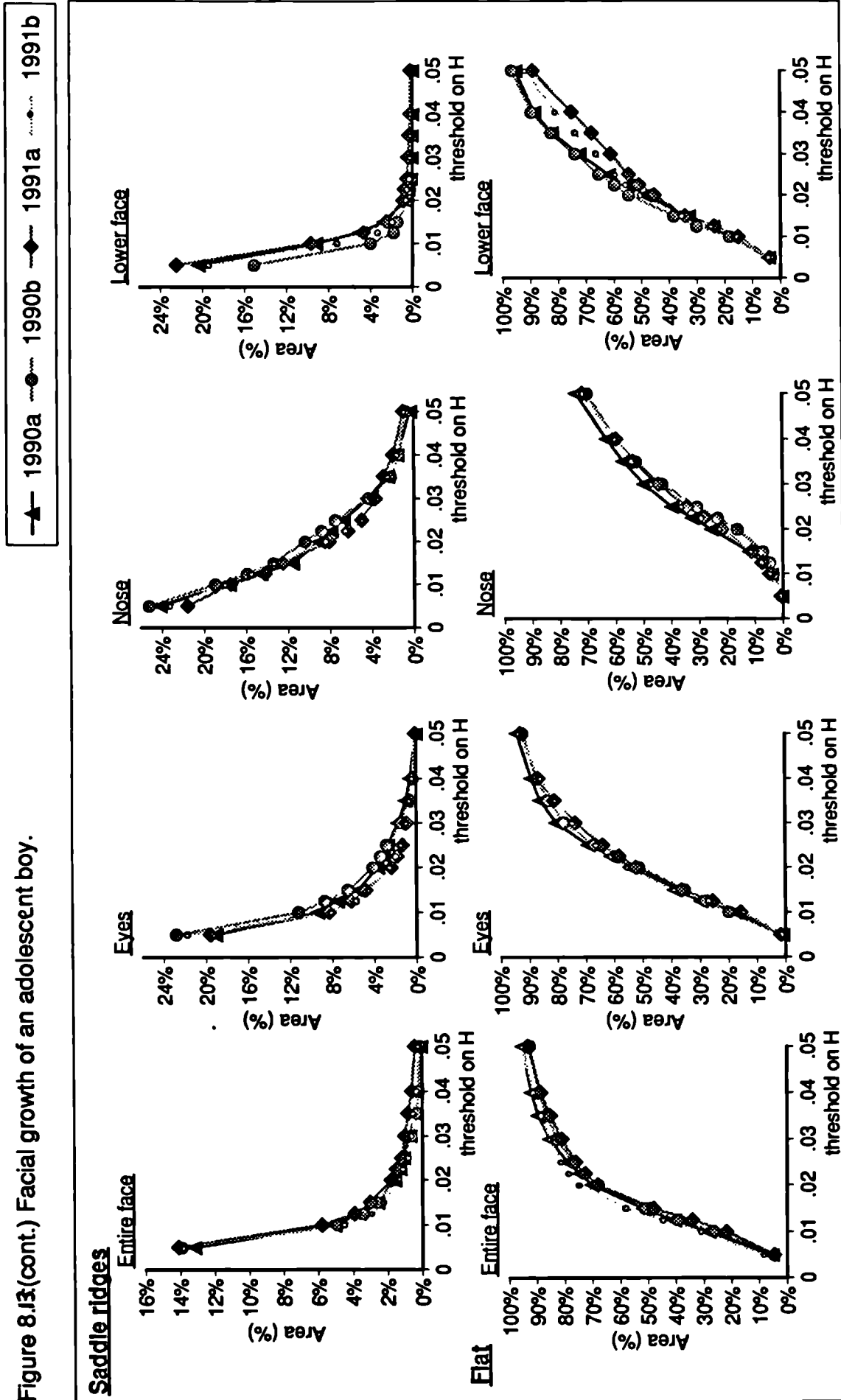


Figure 8.13(cont.): Facial growth of an adolescent boy.

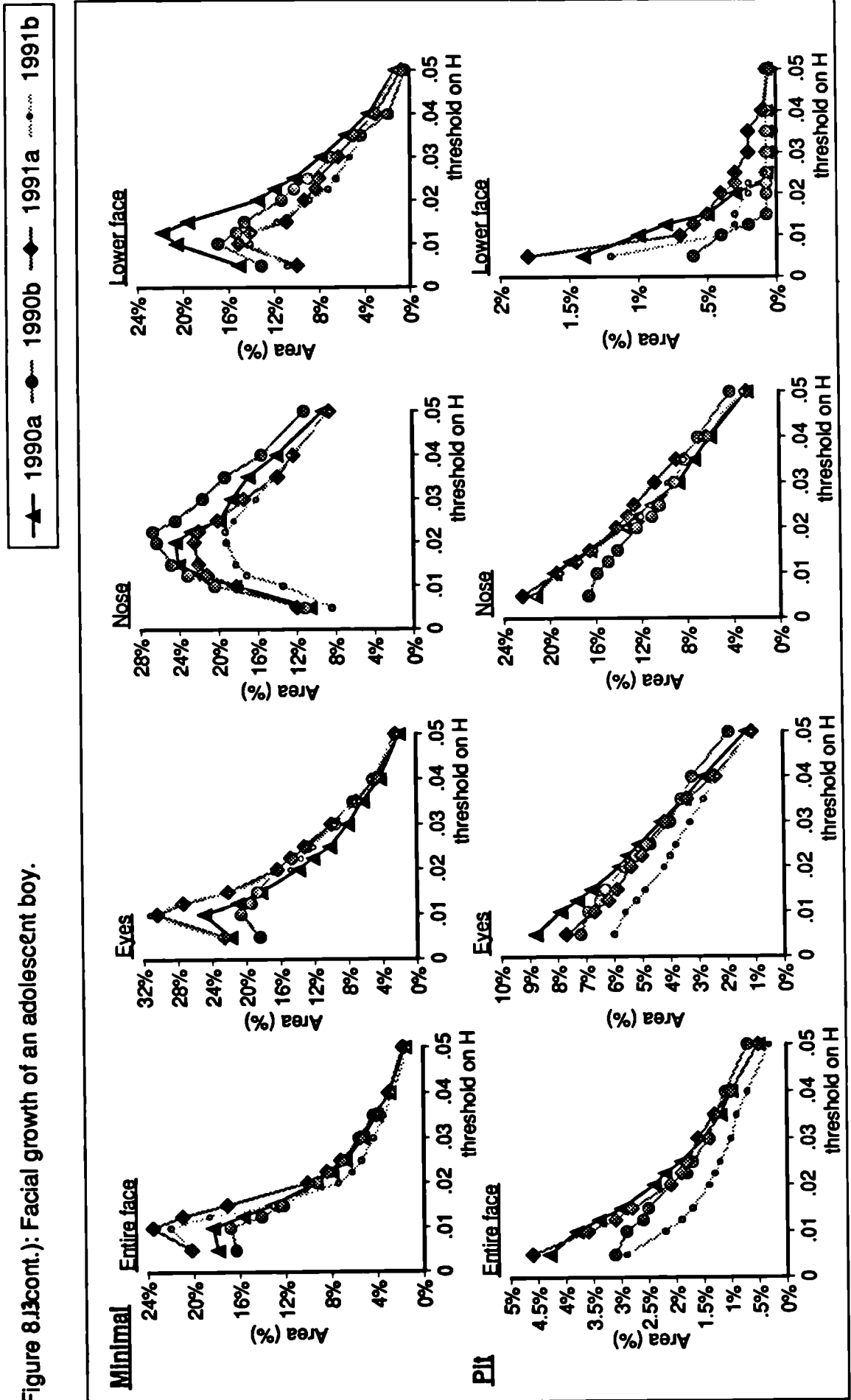
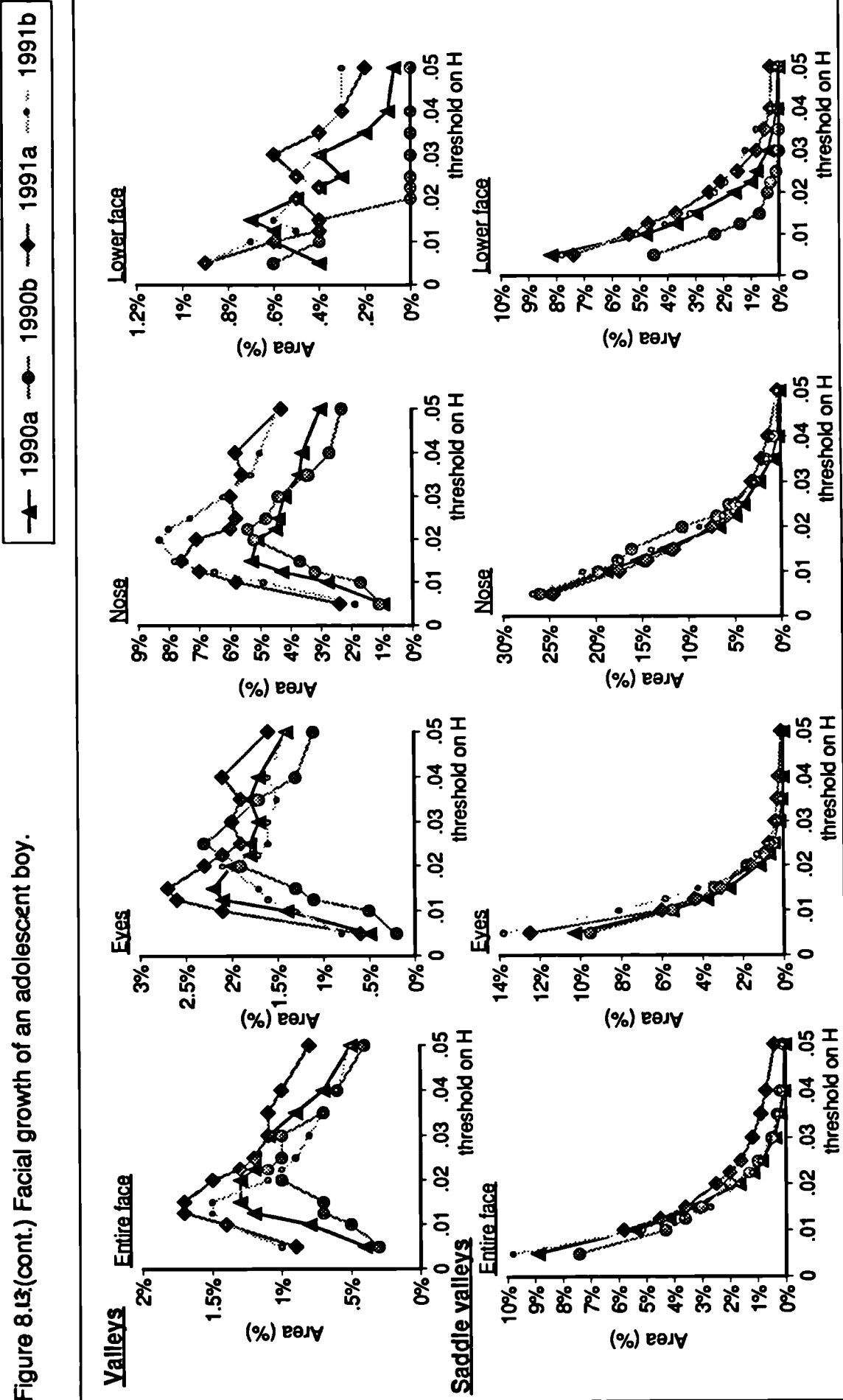


Figure 8.13(cont.) Facial growth of an adolescent boy.



## Clinical Applications

### 8.3 Curvature based definition of landmarks

Another possible clinical application of the surface type method is to use the regions of each surface type to *automatically* define landmark points on the facial surface. These landmarks may then be used to register two scans together, or to determine relative change between pairs of landmarks.

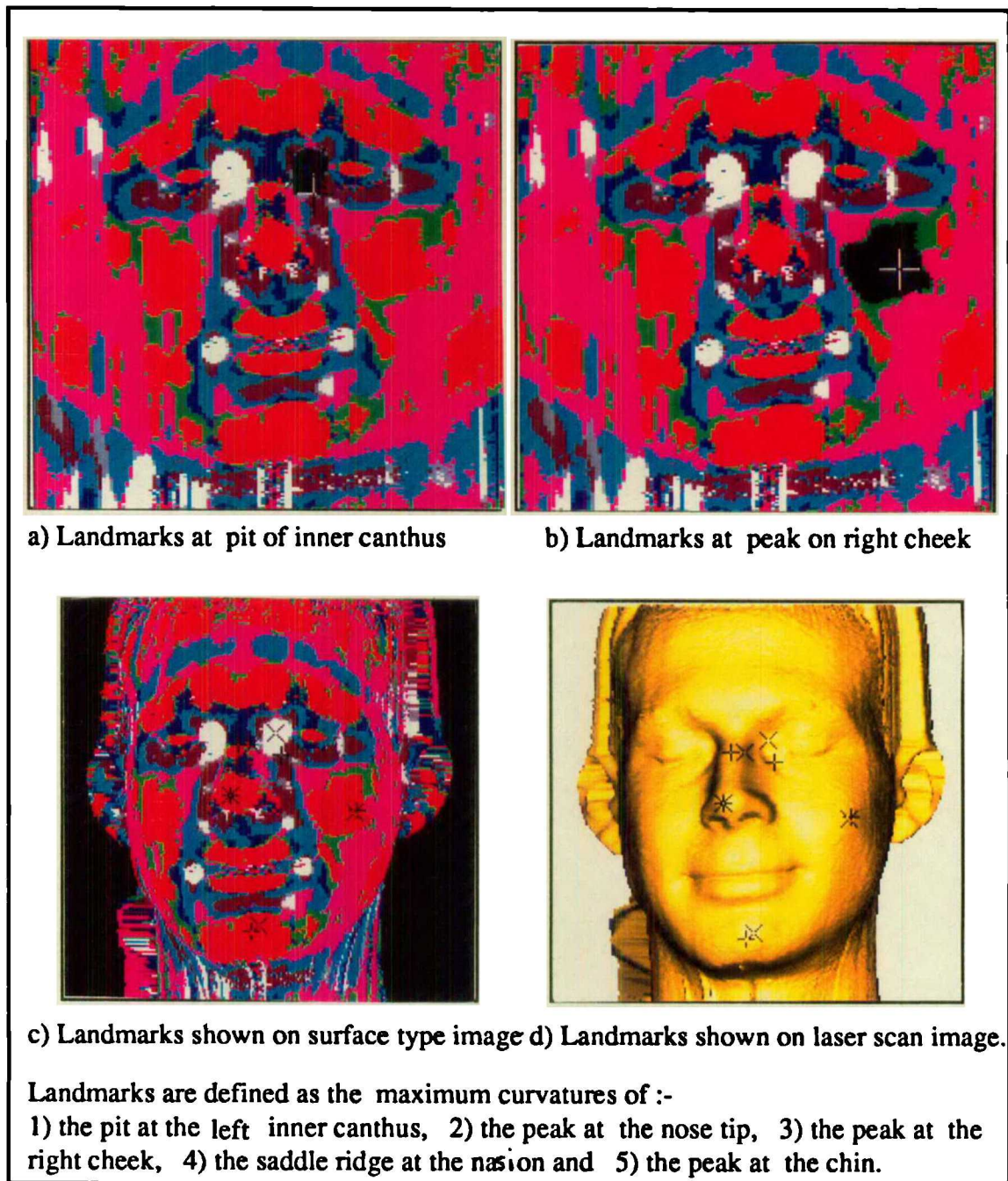


Figure 8.14: The definition of landmark points using the maximum curvature of each region. Maximum mean curvature (H) shown as a "+" and maximum Gaussian curvature (K) shown as a "x".

Figure 8.14 shows an example of this application. A cursor is moved interactively to select a region of interest. Two landmarks are then automatically derived for each region; these are the (absolute) maximum curvature of the mean curvature, and

(absolute) maximum of the Gaussian curvature. These are sometimes coincident (eg. figure 8.14b) and sometimes not (eg. figure 8.14a). This procedure can be repeated to select as many landmark points as required. Once all landmark points have been derived, they can automatically be displayed over the facial surface image and saved to file. Figure 8.14d shows the location of five pairs of landmarks, found from the surface type image (figure 8.14c). Good correspondence is seen with the nose tip, nasion, left inner canthi, left cheek and chin. Either or both landmarks from a region could be used for registration.

The advantage of using this approach to obtain landmarks is that the landmarks can be computed mathematically from the curvature data, thereby enabling the *consistent* positioning of the landmark and removing observer error from its location. In practice, for facial landmarks such as the outer canthi, observer error can be quite large. This approach also allows landmarks to be reliably located on shallowly curving areas of the facial surface such as the cheeks which would not normally be selected as good landmark points due to large observer errors. Hopefully, this method will lead to a more robust registration between surface scans but this is the subject of on-going work.

#### **8.4 Facial aesthetics**

*"There is nothing so rare as perfect beauty in women"*

Raphael (cited by Angle, 1900, p.16)

In this last section, the concept of facial aesthetics is discussed, including a brief summary of how ideas concerning an aesthetically pleasing face have varied through the centuries. The ways in which the surface type description method may help with achieving an aesthetically pleasing outcome of surgery is discussed and the possibility of using it as a basis for putting facial aesthetics on to a stronger mathematical foundation is mentioned.

The shape of the face is undoubtedly important for the aesthetic impact of the face. The appearance of the face is extremely important to a person for their self-confidence and self-love and is often, if not most commonly, the reason why a person seeks corrective surgery. One of the main criteria used to assess the success of a facial operation is how aesthetically pleasing the result of surgery is to the patient, the surgeon and others. At first sight therefore, it is somewhat surprising, given its surgical importance, that so little work has been undertaken on establishing a basis for facial aesthetics. This maybe because the language employed to describe an aesthetically pleasing face is highly nebulous. Before discussing the implications of this thesis for facial aesthetics, I must firstly describe the various ways in which aesthetics has been defined and mention the different concepts of what constitutes an aesthetically pleasing face.

### ***Clinical Applications***

Aesthetics is defined in the Chambers concise dictionary as "relating to perception by the senses; generally relating to, possessing ... a sense of beauty" and beauty, according to the artist Dürer "was the reverse of deformity" (Angle, 1900, p.16). However, a lot of terms are used to describe an aesthetically pleasing face. Some commonly used ones are "pretty", "handsome", "attractive", "harmonious", "balanced". These are all qualities which are hard to quantify but it is generally agreed that these terms involve an intuitive sense of *proportion*.

The idea of proportion is evident in the earliest attempts at defining aesthetics. These date back to the Greek philosophers, Plato and Aristotle. It was Plato's opinion that "the qualities of measure and proportion invariably... constitute beauty and excellence" (cited by Peck and Peck, 1970). Aristotle concurred. "True beauty is necessarily displayed by harmony" he said (cited by Peck and Peck, 1970). And so Greek sculptors defined "classic" or "golden" proportions for the face to use as standards. These golden proportions were rooted in the Pythagorean theory of numbers and defined using the Fibonacci sequence. Yet the Greeks believed that standardization of all faces into only one ideal face was impossible and many statues of the same beauty display considerable individuality.

Another idea, put forward by the philosopher Kant, was that beauty was intricately bound up with *simplicity*. Psychologists of the Gestalt school concurred, laying emphasis on the importance of the *whole form*, not just one a particular feature.

Other writers have suggested that the beauty of a face is connected to the "sexual ability" suggested by its form (notably, Frumkin, the American sociologist). Others strongly disagree, notably Simone de Beauvoir and Schopenhauer. Given this idea, it is interesting to observe that has been popular over the centuries to exaggerate the masculinity or femininity of a face. The *symmetry* of a face may also play a role but its importance can be overstated since no face is entirely symmetric and, as Francis Bacon observed, a wholly symmetric face can be very boring.

However, *any* standards used for judging facial aesthetics must be treated with care because of differences of opinion between cultures and epoches as well as individual taste (Pepper, 1974). The old adage, "Beauty is in the eye of the beholder" (Margaret Hungerford, cited by Wood, 1969) is very true. Anthropologists have found that ideas of beauty vary from people group to people group and that these ideas can be very localised. Aesthetic judgements have also been demonstrated to vary with age (Eysenck, 1940). Art, and more recently the media, has played an important role in influencing our ideas of beauty and our familiarity with particular types of faces. In the developed world today, the standards for aesthetics are a result of the ideas and



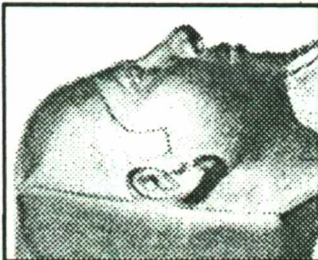


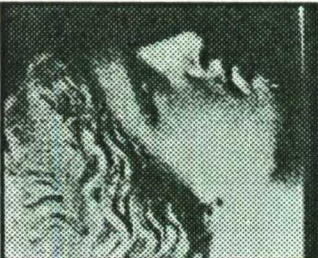
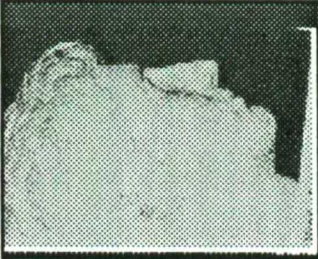



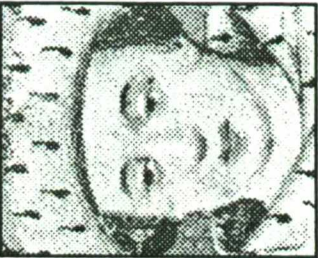


MEN		WOMEN	
	a) Egyptian King Mycerinus ca. 2580 B.C.		g) Egyptian Queen Nefertiti ca. 1350 B.C.
	b) Greek Apollo Beveledere 4th century B.C. (Roman copy)		h) Greek Aphrodite 4th century B.C.
	c) Roman Emperor Augustus 1st century A.D.		j) Medieval Mona Lisa by da Vinci 1560 A.D.
	d) Renaissance David by Michelangelo		k) early 20th century Marlene Dietrich 1934
	e) Middle Ages King Edward IV 15th century		l) later 20th century Marilyn Monroe 1960's
	f) Modern Cary Grant		

Figure 8.15: Examples of faces from different ages that were considered to be beautiful by people of that era..

### ***Clinical Applications***

influence of the artists, sculptors and philosophers of western civilisation and modern media preferences and movie makers.

#### **8.4.1 The ideal face through the ages**

Ideas of facial aesthetics have changed over the ages and thus it is interesting to consider examples of great beauties from various ages. Figure 8.15 shows some examples of beautiful and/or typical faces from different eras.

The ancient Egyptians liked faces which were a mixture of Negroid and Caucasian features. Queen Nefertiti was contemporarily renowned for her beauty (figure 8.15 a). The Greeks admired faces with "ideal" proportions. The Greek ideal had a "short, finely curved and prominent upper lip, a full round but less prominent lower lip with a marked depression at the base, giving roundness and character to the chin" (Angle, 1900, p.16) as well as a longish straight nose. Some of the best examples of this ideal are the statues Apollo Belvedere (figure 8.15b) and Venus de Milo and the works of the sculptors Phidias and Praxiteles. The Romans used the standards set down by the Greeks as guide-lines, but tended to favour more lifelike representations such as the statue of Emperor Augustus shown in figure 8.15c. European Medieval culture believed the perfect face was divided into sevenths and not thirds as the Greeks thought. A number of "canons", to describe the way a face should be divided, were expounded by Renaissance artists such as da Vinci, Dürer and Martinez (Farkes et al, 1985). Leonardo da Vinci believed that the nose had a great effect on a face and he and Dürer drew sketches of different sized noses to illustrate just how dramatically the facial profile is altered by the nose. Medieval society idealised a small, thin mouth and pale skin. Leonardo's Mona Lisa typifies this. English Victorian society (1827-1901) also liked a pale skin. In modern times, a face based on a full, slightly protrusive dentition is preferred.

Some renowned beauties display quite marked orthodontic abnormalities. For instance, Queen Nefertiti the Beautiful (28th century B.C.) was severely skeletal II (Wood, 1969) and many other examples can be found amongst works of art. Even as recently as the 1960's some of the beauty queens had noticeable class II, division I malocclusions (Wood, 1969). On the other hand some beauties such as Queen Nefertiti, David by Michelangelo (figure 8.15d), the statue Apollo Belvedere and the 1967 Peruvian Miss World display features and proportions that are considered orthodontically normal.

In modern times, interest has been shown in the facial structure of declared beauty queens. Investigations have involved the measurement of various angles and ratio relationships. Holdaway used lateral cephalograms to investigate the appearance of a Miss America (Holdaway, 1983; 1984). He found that measurements of various lengths



and soft tissue depths showed good proportions of the face and that the angles between some landmarks (eg. facial angle, nasion to menton) were approximately 90 degrees. Peck and Peck (1970) carried out a similar analysis of beauty queens and discovered that certain relationships between soft-tissue landmarks were preferred.

#### **8.4.2 The search for an aesthetic standard.**

It has been widely recognized that faces fall into a number of classes or types and therefore, that one aesthetic standard is not feasible. One of the earliest facial classifications was made by Dürer who, recognising the variability of faces, suggested variations on the canons of absolute beauty which he and other Renaissance artists laid down. He describe what is now termed a normal Class II, division I face and a normal Class II face. Others have added more categories to this classification (Angle, Downs, Ricketts etc). These are all based on the relationship of the skeletal bones to each other or the incisors.

In the subject of orthodontics, a great interest has naturally been taken in facial aesthetics. The first orthodontist to write on the subject of aesthetics was Angle (1900) and his ideas have been influential. In his early writings, Angle seemed to subscribe to the view that the statues of the Greek god Apollo should be used as a standard of beauty (Riedel, 1950) but later, according to Riedel after he met a certain Dr. Wuerpel, he revised his position and assumed that placing the teeth in normal occlusion would yield the most harmonious face possible for an individual. Angle stated that a profile was in perfect harmony when a straight line joined three specific points together, these were the most prominent points of the frontal and mental eminences and the middle of the ala of the nose. This "line of harmony" was taken from a statue of Apollo. However, Angle felt formulae for describing facial appearance should not be rigidly applied, but used instead as guide-lines for each facial type. He sensed the conflict between good occlusion and a pleasing facial appearance which sometimes exists.

In their work, orthodontists have largely tried to standardize facial appearance to some norm, although some such as Brodie (1944 cited by Riedel, 1950 and Wood, 1969) maintained that orthodontists were not qualified offer opinions on the beauty of a face, since all were unique and individual. Brodie appealed for the entire concept of "the norm" to be abandoned and for orthodontists instead to be guided only by the individual.

In the search for an aesthetic standard, the basis for aesthetic judgement needs to be known. Some studies have investigated whether a common basis of judgment exists regarding what is considered an aesthetically pleasing face. In 1960, Iliffe (1960) asked the readership of a national UK newspaper to order, in terms of preference, a set of

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photographs of young women. He found that a common basis of judgement existed, between all social classes, geographical location and both sexes. There was however, a slightly lower correlation between the oldest (over 55 years) group and the youngest, which led him to suggest that culturally accepted norms maybe transmitted through the educational process and that these maybe subject to slow change.

Faces of different ethnic origins, such as Nordic, Assyrian or Greek, are widely different in appearance. However, Martin (1964) found that the standards for judging the aesthetic nature of faces from a particular racial origin were the same for people both from the *same* racial origin and from other origins. In 1939, the orthodontist Hellman wrote that when "differences in development become abnormal they present the same aspect in all racial types and become distinguishable as malocclusal types" (cited by Riedel, 1950). Thus standards for aesthetics as well as the recognition of abnormal or deformed faces spans racial origin.

The aesthetic preference of people seems to be for typical or non-extreme faces. Carello and colleagues (Carello et al, 1989) suggested that the attractiveness of a facial profile was dependent on its distance from an archetype (standard profile type), which confirmed the ideas of de Smit and Dermaut (1984) who conducted a similar experiment for faces with various sorts of deformities. They found that people tend to prefer less extreme faces (class I types with deep set faces compared to open faces and class II compared with class III). Some orthodontists have conducted some studies to determine whether fellow orthodontists have a common opinion as to the aesthetic nature of a face. Riedel (1950) reported the judgments of United States Midwest orthodontists on profiles traced from people with normal and abnormal occlusion, and found that profiles with their skeletal parts arranged in a straight line with little dental protrusion were preferred. Fromm and Lundberg (1970) conducted an evaluation of the success of an operation (to correct for a prognathic mandible) using a panel of medical assessors judging pre- and post-surgical cephalograms.

Whilst studies using profiles found preferences for certain types of profiles, the use of other viewpoints has given inconsistent results. In one investigation, orthodontic surgeons were asked to match profiles with anterior and three-quarter views of the same face (Powell and Rayson, 1974). This was done very badly. In addition, if silhouettes of the profiles were used, the sex of the face could not even be reliably judged. This illustrates the unreliability of assessments from 2D outlines. Another part of Powell and Rayson's investigation showed that whilst the profile may be deemed to be satisfactory, the anterior view of the same face might be assessed differently or vice versa. They note that three-quarter views provided more information than anterior or profile views but were commonly used because of the difficulty in standardizing them.

Earlier in this section, various methods of dividing the face into ratios or canons were discussed. A fascinating paper by Ricketts (1982), investigated whether the "Golden Section" defined by the Greeks occurs in the body and, in particular, in the face. The Golden section originated in the Pythagorean theory of chosen numbers and was taken up by Plato as a mathematical relationship expressing universal harmony. It is a proportion of  $0.618 (\sqrt{5}+1)/2$  whose reciprocal 1.618) which is derivable from the Fibonacci sequence (0,1,1,2,3,5,8,13,21...). Ricketts related this to the concept of beauty (which he defined as something which arouses the senses to an emotional level of pleasure) and showed how by standardizing with the trichion (the point on the forehead where the skull-cap begins) and the nose tip - tragus line, many occurrences of these ratios, both vertically and horizontally, can be found in people considered by others to be beautiful (such as film stars or beauty queens). He also said that a beautiful face will also have *rhythm* since rhythm occurs when a proportion recurs uniformly and demonstrated how the mandible grows along a logarithmic spiral (which can be formed from "golden" triangles). It is intriguing to see that form and beauty can<sup>be</sup> described using such mathematics but the debate over whether the "golden section" is aesthetically preferable to divisions of unity is still current today. One recent paper reported that no such preference was found although the authors only considered the division of geometric figures such as squares and rectangles and not natural objects such as faces (Davis and Jahnke, 1991)!

#### **8.4.3 Implications from the surface type methodology**

From observation of the faces selected through the ages as being beautiful (figure 8.15), it appears that these faces have two factors in common. Firstly, they are of smooth appearance, with no small lumps or bumps and smooth texture. Secondly, the facial features are not extreme and no single feature dominates the face. There is individual variation, and this variation appears to make the face particularly interesting but the variation is small and adds rather than detracts from the aesthetic appearance. This indicates that an aesthetically pleasing face should be free from sudden changes in curvature on surfaces such as the cheeks or forehead which are otherwise smoothly curving in form.

It would be interesting to describe some faces considered to be particularly beautiful and some not so aesthetically pleasing using the surface type methodology. This has not been attempted here, but it is likely that an investigation of this sort would reveal which types of surfaces should be present at specific locations on the face. For instance, a nose may consist of a peak at the tip, pits at the inner canthi, a saddle ridge on bridge and saddle valleys at the side. Some noses (those which are convex in shape) also have a small peak on the bridge. On a female face this may not be aesthetically pleasing since a preference for concave shaped noses (Bruce et al, 1992), which do not have this peak

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has been noted. It may be that a range of aesthetically pleasing surface types at certain locations may be specified and any deviation from that range may not be aesthetically pleasing. Hence, a target face, composed of certain surface types may be defined that could act as an aesthetic guide to the surgeon planning an operation. However, the application of such as aesthetic guide to surgical planning, relies on the correspondence between bone movement and soft tissue movement becoming better known.

### **8.5 Summary**

The application of the surface type description and analyses, described in chapter 6 and 7, to the clinical cases studied here reveals the potential of this method for analysing the complex changes in shape which occur in the soft tissues, when a face is altered by reconstructive surgery.

For the four patients studied, the *areas* of shape change are fairly easily grasped by studying the two surface type description produced for the pre- and post-surgery cases. The *amount* of change can be obtained from the graphical analysis of the amount of each surface type, in the relevant region, before and after surgery. The *magnitude* of the change can also be interpreted from these graphs, since larger changes will effect higher threshold levels. Thus this method, and analysis does appear to provide useful information about how the shape of the face has been altered by reconstructive surgery. The hierarchical nature of the analysis, incorporated into the graphs on the threshold axis, allows an assessment of the *importance* of the change to be made.

However for the facial growth case, the graphical analyses do not give as clear a picture. It is difficult to draw any definite conclusions from them. One possible explanation for this may lie in the poor quality of the first scan (1990a) in the chin area. It may be that when scans, more widely spaced in age, become available the trends will become clearer.

A second clinical application of the surface type description was demonstrated in section 8.3. The automatic location of landmark points on the facial surface may help to improve the accuracy of registration, since landmarks can be positioned consistently, as well as to speed up this procedure. This will also make the relative change between pairs of landmarks easy to determine.

Finally, the concept of facial aesthetics was examined and the changing ideas of an aesthetically pleasing face through the ages was described. One of the concerns in the planning of orthodontic treatment and reconstructive surgery, is the aesthetic impact of the alteration to the face. Thus some form of standard, or guidelines, for aesthetic beauty has been sought. My hypothesis that the faces that have been, and are today,

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considered as aesthetically pleasing should be smoothly curving in form could be tested using a set of faces, rated by independent observers to be of different aesthetic levels and encoding them using this curvature-based method. This is area of future research.

## CHAPTER 9

### APPLICATIONS II: 3D SHAPE AND FACIAL RECOGNITION

*"He had that sort of face that, once seen, is never remembered"*

Oscar Wilde (1856-1900)

The mechanisms by which we recognise faces have been a subject of great interest throughout the second half of this century. The aim of this chapter, is to place the work reported in this thesis, and in particular the analysis techniques described in chapter 7, into its appropriate context as it pertains to previous face recognition studies. Over the last three years, Bruce and her colleagues at the university of Nottingham (now at Stirling), in collaboration with myself and my UCL colleagues, have examined the role which maybe played by 3D shape in facial recognition (Bruce et al, 1989; Bruce et al, 1991b; 1991c; submitted). Our investigations have been facilitated by the surface-type methodology reported in this thesis.

Faces and their attributes are extremely recognisable to us. We recognise objects as faces and discriminate between male and female faces with an extremely high degree of confidence. Other tasks we can perform with faces are to say how unusual they are (ie. whether or not the face is *distinctive*), whether a face is *familiar* or not and assess its *aesthetic impact* (eg. rating it in terms of "attractiveness"). All these tasks can be performed with great speed. However, it is interesting to note that people are really not very good at verbally describing faces (Davies et al, 1978; Laughery and Fowler, 1980) and are susceptible to confusion when asked to recognize faces that they are not very familiar with. Familiar faces can be identified as belonging to specific individuals; although sometimes our memory will fail to assign a name to a face. This memory failure suggests that name information is stored in a separate part of the brain from the visual processing information (Bruce et al, 1991b). Our understanding of the mechanisms involved in the visual processing procedures is still largely unknown (Wu and Huang, 1990).

Cognitive psychologists have put forward several theories to explain the relationships between different aspects of facial recognition (see A. Ellis, 1991 for a review and Bruce, et al 1991b for an example of current thinking) but it is the understanding of the *visual* information that is encoded from faces, to enable these tasks to be performed, that remains very unclear (Bruce et al, submitted). In the last few years, some psychologists have expressed the opinion that 3D shape information may play an important role in these processes (Bruce, 1988; Bruce and Burton, 1989; 1991d; Bruce et al, 1991b; 1991c; Bruce et al, submitted; Burton et al, in press). Other face-psychologists such as Watt, Perrett and Craw, insist that 3D shape is *not* an important

~~psychologists such as Watt, Perrett and Crow, insist that 3D shape is not an important factor and emphasise how much might be achieved without any explicit description of 3D shape.~~

In parallel with this psychological research, an increasing amount of work has been done by computer vision scientists on devising systems to *automatically* recognise faces. These systems are aimed at application in security and forensics (Hawkes, 1989; Sherman, 1990; Nixon and Jia, private communication). Attempts at producing automated face recognition systems have analysed facial images, inferring information about 3D shape from shading. Shading is dependent on illumination conditions, the reflectivity of the surface (albedo) as well as the surface geometry, information about obtained about the shape of the surface from this method is restricted. For instance, whilst it is certainly true that faces *are* recognized from photographs, it is nevertheless the case that if a full-face photograph is studied, one can tell very little about the appearance of the face in say profile.

In this chapter, the factors which are believed to be important for recognition are described and, where appropriate, the manner in which a surface analysis could be of benefit for this research is indicated. These factors fall into two groups, the facial features themselves (section 9.1) and their configural arrangement (section 9.2). This is followed by a description of the psychological evidence which suggests 3D shape *does* play a role in our ability to recognise faces (section 9.3). In section 9.4, our preliminary investigations of the role that may be played by the facial surface in sex judgements and other face recognition tasks are presented.

In addition to considering the role of shape in facial recognition, a brief review of systems that have been devised in order to automatically recognise faces is made in section 9.5. Advantages of using a depth based method are described. Finally, the implications for forensic systems of a role played by 3D shape in face recognition are discussed (section 9.6).

## **9.1 The interrelation of facial features**

It seems clear that faces are encoded by reference to a general face prototype (Valentine and Bruce, 1986; Valentine and Ferrara, 1991) which contains information about both the *facial features* and their *configuration*. In this section, the studies which have indicated the relative importance of the features for facial recognition are reviewed. The configural aspects are dealt with in the following section (9.2). Some 22 facial features and accessories as well as the facial proportions have been reported which may be used for recognition (Laughery and Fowler, 1980). These are: eyes, nose, mouth, lips, ears, forehead, cheeks and cheek bones, jaw and jawline, chin, hair, hairline, eyebrows, sideburns, moustache, beard, face shape, glasses, eye colour,

## ***Face Recognition***

Some of these features could be identified, and characterized using the surface type description.

The relative importance of these various features in recognition has been of great interest to cognitive psychologists. Studies of the eye movements of observers viewing facial photographs have shown that there is concentration on certain areas of the face which correspond to various features (eg. Walker-Smith et al, 1977) but the correlation of these fixations with the amount of information taken in by the observer is not yet known (Haig, 1985). It is important to note that most, if not all, of the investigations into the relative importance of facial features have been carried out on full-face views. It is, therefore, not surprising that features such as the nose, whose shape is best observed from other viewpoints, have proved unhelpful. This was acknowledged by Davies and his colleagues (Davies et al, 1977).

It is thought that different facial features hold different importance for different recognition tasks. Certain features may be important for recognising different individuals (Haig, 1986a), such as the eyebrows for Denis Healey, and for distinguishing between the sexes (Bruce et al, 1991a). It has been postulated that a major hindrance to these experiments may be the way a feature is defined (Haig, 1986a).

Surface type analysis could be of benefit for these investigations, since it allows features to be defined in a *mathematically consistent* manner. Moreover, the importance of each facial feature, or even components of a facial feature, in facial recognition could be assessed by altering the portion of the facial surface which corresponds to that feature. This could be achieved by changing the shape of the portion of the surface defined by a surface type patch (or patches), using the technique described in chapter 7 (section 5.3). The importance of a feature could be investigated by relating the detectability of changes in the facial surface, as perceived by impartial observers, to the shape change produced. In our preliminary investigations, the effect of altering the shape of nose and chin, using a B spline technique (section 7.5.2), was shown to have a significant effect on the perceived masculinity and femininity of the face (Bruce et al, submitted).

### **2.1.1 Familiarity and sex judgment**

In the brain, different facial features appear to be used for the recognition of familiar and unfamiliar faces. Familiar faces are recognised more quickly from their internal features (ie. the eyes, nose, mouth etc.) than unfamiliar faces but there is no difference in the speed with which familiar and unfamiliar faces are recognised from the external features (ie. hair and facial surround) (Young et al, 1985).



An effective method for hindering recognition is the use of disguise (Patterson and Baddeley, 1977). Thus, one method that has been used for investigating the importance of various features in recognition tasks is to hide selected features. Another method used is to move features within the facial image (Hosie et al, 1988). Roberts and Bruce (1988) found that masks that concealed the eyes, but not the brows, slowed familiarity decisions the most. This was consistent with earlier work which noticed the importance of eye region for these decisions (Shepherd et al 1974; Haig 1986a).

Masks that concealed the nose, including the nasion area, slowed sex decisions the most but had no noticeable effect on familiarity decisions. If noses were presented in isolation from the rest of the face, the sex judgement task was reduced to the level of chance. This led Roberts and Bruce (1988) to suggest that it was the *interaction* of the size and shape of nose with the rest of the face that was important for sex judgement. This indicated that strong roles maybe played by 3D shape information and the configuration of features in sex decisions.

In a later report which used unfamiliar faces, Bruce et al (in press<sup>b</sup>) found that masking noses seems to have a greater effect on male faces and masking eyes had a greater effect on female faces. Whilst this is somewhat at odds with Roberts and Bruce's results, it is generally believed that the nasion area is important for sex judgement.

### **2.1.2 Unfamiliar faces**

A number of researchers have investigated the relative saliency of features in the recognition of unfamiliar faces. These studies are of importance for evaluating the reliability of eye-witness identifications. Amongst these studies, Ellis and colleagues found that the most noticeable feature was the hair followed by the eyes, mouth, nose and chin (Ellis et al, 1975a). Others have placed slightly different emphases on the importance of these features. Davies and colleagues suggested an order of forehead, eyes, mouth, chin and nose (Davies et al, 1977) and Haig the eyes and eyebrows, hairline above temples, mouth and upper lip area, lateral hairline beside the temples (Haig, 1985), though in an earlier paper Haig had considered the head outline to be the dominant influence (Haig, 1984).

Eye movement studies have shown that the most noticed facial features are the hair, eyes, eyebrows, nose and the shape of the face (ie. whether its round, lean and oval faces or square) (Davies et al, 1979a) and the least noticed features are the ears, forehead and chin (Ellis et al, 1989). This is interesting since ears and chins can be very distinctive features (Farkes and Munro, 1986). Several other studies have indicated that most attention is paid to the upper facial features rather than the lower ones (eg. Laughery and Fowler, 1980).

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Haig (1985) conducted an experiment which concealed various portions of the face by a distributed aperture technique. He found that there was a great deal of disagreement between observers on *how* the target faces differed and he argued that we remember most clearly what we *perceive* to be the most prominent features and that this perception may vary between individuals.

#### **2.1.3 Different types of human faces**

All of the studies mentioned so far in this chapter appear to have been conducted using Caucasian faces viewed by Caucasian observers, although this is not made very clear in the literature. The well-known difficulty that Caucasian observers experience in recognising unfamiliar Negroid/Mongoloid faces (and vice versa) may be due perhaps to them placing a different importance on the facial features. Ellis (1975) hypothesized that different features were used by different races for discriminating between faces. Later that year, Ellis and colleagues found that black Zimbabweans more often used eye size, eyebrows and ears to describe a face whereas white Scots used hair texture and eye colour (Ellis et al, 1975b). The white subjects seemed to observe the target face more widely, using more features to describe it but were considerably less accurate at identifying black faces than black subjects. Negroid subjects have also been found to be worse at recognising white faces than white subjects (Shepherd et al, 1974). Later, the basis used for judging African and European faces was shown to be different and moreover, that these bases were used by observers from both races (Shepherd and Deregowski, 1981).

Exposure to a variety of faces from another race has been demonstrated to improve the performance of white subjects in recognising 'Negroid' Oriental faces (Malpass et al, 1973; Elliot et al, 1973). This may be because they begin to make use of the more subtle changes in features which make better discriminators for these faces (Ellis, 1975). Studies that have been conducted in more multi-racial areas, such as California using American and Japanese observers, show little difference in discrimination ability. This is thought to be due to an increased familiarity with faces of different ethnic origins.

If a database of Negroid/Mongoloid faces were collected it may be possible to use the surface type description to assess differences between racially different human faces.

#### **2.2 Disruption of facial recognition**

Our ability to recognize faces can be disrupted in several ways. These disruptions are briefly stated below because they provide important evidence of the role played by the face as a *whole entity* in recognition, compared with the role of each feature in a collection of parts, described above.

The idea that the configural relationship between the facial features is more important than any individual feature was first expressed by Galton in the late 19th century (see Young et al, 1987 for a summary of Galton's ideas). Nowadays, the available evidence suggests that faces have both component *and* configural properties which give rise the different processing strategies which can occur simultaneously and interact with each other (Sergent, 1984; Rhodes, 1988).

In 1990, Kirby and Sirovich (1990) drew attention to evidence that the brain uses parallel pathways to process information (Anderson and Hinton, 1981) and suggested that recognition processes may occur in parallel. In the case of the face, the eyes, mouth, nose etc. could be recognized simultaneously, which could explain how an individual may be perceived as having "her eyes" or "his nose". However, Haig felt that a combination of parallel and serial processes was more likely since some responses would need to be evaluated individually (Haig, 1986b).

### **2.2.1 Feature displacement and configural arrangement**

The movement of facial features within the face inevitably affects its configuration (Sergent, 1984). This may have serious consequences for recognition if, as some research has suggested, the face is treated more as a whole than as a sum of individual parts. Evidence to support this view include the findings that the perception of one feature can be influenced by surrounding features (Haig, 1986a) and that recognition is disrupted by viewing the top half of one face and the bottom half of another face simultaneously (Young et al, 1987).

The effect of displacing facial features on recognition has been investigated by using images stored on computer, isolating and slightly displacing facial features to produce Photo-fit type pictures (Haig, 1984; 1986a; Bruce et al, 1991a). Haig discovered that people are very sensitive to vertical mouth movements, vertical eye and nose movements and to movements of the eyes inward but not to outward eye movements. He suggested that this may be to do with the relative position of features, their apparent size and saliency and, especially, their interaction with the surrounding areas of the face. This ability to remember, and subsequently recognize, subtle configurations of the face was confirmed by Bruce and colleagues using faces generated by "Mac-A-Mug" software for the Macintosh computer (Bruce et al, 1991a). This research has serious consequences for forensic kits such as Photo-fit which treat the face<sup>as</sup> a sum of individual parts which can be independently retrieved by eye-witnesses.

### **2.2.2 Inversion and photographic negation**

Two other disruptions to recognition are to "invert" (turn upside-down) or "negate" (take the photographic negative of) the face. The inversion and negation effects are additive, ie. subjects are worse at identifying displacements of facial features when the

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face is both inverted and negated (Kemp et al, 1990). This implies that they affect independent processes. Both inversion and negation have been found to be disruptive for sex classification (Bruce et al, in press<sup>b</sup>). People have been shown to be less sensitive to changes in facial features when a face is either inverted (Yin, 1969; Sergent, 1984; Valentine, 1988) or negated (Galper and Hochberg, 1971).

It has been known for a long time that people find upside-down faces difficult to recognize. The same effect has been reported for inverted words, but inverted faces have been found to be disproportionately difficult to recognize (Yin, 1969). It was first postulated that this was due to a disruption of facial expression cues and, indeed Michelangelo and other artists noticed that facial expression is extremely difficult to portray on inverted faces. A profound effect on expression has been found by inverting features within an upright face. This has become known as the "Thatcher illusion" after the face it was first demonstrated on (Thompson, 1980, see figure 1.1). When the compound face is viewed from upside down, a normal appearance is perceived but when it is viewed upright, a grotesque expression is seen. Further experiments have shown that the presence of nearby facial features can influence the interpretation of an inverted feature when they are viewed in isolation from the rest of the face (Parks et al, 1985). Conversely, if the positions of the eyes, nose and mouth are altered within a face, the same sort of disruption to facial expression is seen *only* when the face is inverted (Valentine and Bruce, 1985). These studies have provided evidence for a uprightly orientated "frame of reference" for faces.

In fact, faces are not *uniquely* vulnerable to inversion. The same difficulties arise when dog-breeders try to recognize inverted dogs (Diamond and Carey, 1986). Thus it appears that inversion only affects "experts" badly, and all adults are expert at face recognition. Children do not suffer the same difficulties with inverted faces. This is attributed to them being less expert at face recognition than adults (Diamond and Carey, 1986). It has been suggested that in adults inversion interferes with the encoding of the facial configuration (Carey and Diamond, 1977). This idea has been supported by investigations conducted by Sergent (1984), Young and colleagues (Young et al, 1987) and Valentine (1988). If this is correct then it follows that our perception should not be affected by inversion if we are *only using* local features in a given face processing task.

Faces displayed in photographic negative are also extremely difficult to recognise. As already mentioned it has been suggested that negation may disrupt cues of shape obtained from shading and shadows. Phillips (1972) and Hayes and colleagues found that it was only the low frequencies of the image that are affected by negation (Hayes et al, 1986). But negating a photographic image with a wide range of grey-levels creates a complex representation of the original scene with a variety of factors that will influence its interpretation by the brain. It has been proposed that the brain assumes an overhead

lighting model when interpreting 3D scenes, probably because of the overhead Sun (Ramachandran, 1988). One method of investigating the effects of negation, which may allow a separation of the factors involved, would be to study the effects of reversing the depths of surface under different conditions of illumination. It is apparent that negative photographs of faces do not look like faces turned inside out, as one would expect if this processing were entirely due to shading information. In fact, we choose to see such real masks as faces (Gregory hollow face illusion) and there seems to be a strong bias to see convex rather than concave surfaces in general and faces in particular.

An investigation into this effect should take account of the three dimensional nature of the surface. The experimental material required to investigate these phenomena can be produced using the techniques we have developed at UCL for displaying facial surfaces. Such an investigation could separate the shading and depth cues. It would be possible to position the source of illumination at any required angle. Different types of shading could be used (Phong, Gouraud etc.) and the 3D data reversed to reverse depth. These methods should make it possible to discriminate between the effects of different aspects of negative images of the face. Hill at Stirling university is currently researching this.

### **9.2.3 Blurring Faces**

The recognition of blurred (ie. intensity averaged) images of faces was investigated by Harmon (1973) and Sergent (1986). They found that we can recognize blurred faces, providing the level of blurring is not too great. This suggests that the configural aspects of the face are used to facilitate recognition in these cases. Harmon (1973) also found that filtering out high spatial frequencies from coarsely quantized blurred images greatly enhances recognition, implying that configural information is carried by low frequency components. He also assessed the effect on recognition of adding noise of various frequencies to the image.

For upright, positive faces, no disruption to recognition performance has been found for differences in pose, expression, age, lighting or hairstyle (within reason) if the face is a familiar one. However, the recognition of once-viewed faces is disrupted by such factors (Bruce, personal communication). Dynamic information has not been found to have any significant advantage over statically presented images for either familiar or unfamiliar faces (Bruce and Valentine, 1988).

### **9.3 3D shape in human face recognition**

In 1988, Bruce hypothesized a possible role for 3D shape information in the recognition of faces (Bruce, 1988). She stated that up until that time research into the mechanisms of face processing had been based on treating the face as a "flat pattern" when in actual fact the face is a "lumpy surface". Speculating on the role played by the facial surface, she pointed out that photographs of faces are significantly more recognizable than line

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drawings (Davies et al, 1978) and that negation of a facial photograph which has been believed to disrupt depth cues from shading and shadow information, has a adverse effect on its recognition (Phillips, 1972).

In this section, the limitations of line drawings for recognising faces are described. This is followed by evidence which suggests that the *surface* of the face may play a role in face recognition tasks.

### **2.3.1 Line drawings**

Line drawings convey information about sharp turns in the surface away from or towards the observer and about surface discontinuities. Although they have conventionally been the domain of artists, recently a method has been developed for producing line drawings automatically from digitized images. In this method edges and valleys are extracted at sharp changes in the image intensity (Pearson, 1986; Hanna et al, 1985).

Although line drawings are sufficient to recognize an object as a face (Biederman and Ju, 1988), they have *not* been found to be adequate for *identifying* the represented face (Bruce et al, in press<sup>a</sup>). For instance, Davies and Colleagues (Davies et al, 1978) found that famous faces were very much more recognizable from photographs than line-drawings.

An evaluation of facial recognition from line-drawings suggested a reason why they are badly recognized (Laughery and Fowler, 1980). Laughery and Fowler found that faces reconstructed by artists provided better likenesses than reconstructions which were produced using the Identi-kit forensic package (which constructs faces from line segments). They postulated that the reason for this maybe due to the greater flexibility of the artist and to his addition of shading and age-lines. The addition of shading to line drawings has indeed been found to enhance recognition (Davies et al, 1978; Bruce et al, in press<sup>a</sup>) whilst the addition of wrinkles and blemishes to Photo-fit reconstructions has been shown to provide more believable representations (Kitson et al, 1978). These findings suggest that using changes in edge intensity information alone to extract facial features for automatic recognition systems is not likely to be produce a terribly accurate identification system.

### **2.3.2 Facial surfaces**

In order to investigate whether the shape of the face is important for recognition, information about the shape of the face must first be isolated from other psychological cues such as texture and colour. In order to achieve this a number of volunteers had their faces optically surface scanned at UCL with the hair concealed under a stocking. Images of the facial surface were obtained from the 3D data sets. These images were

devoid of texture and hair information and thus only contained information about the shape of the face. Psychological experiments were conducted using hard copies of full-face, profile and  $3/4$  views.

These "facial surface images" were found to be rather badly recognised for both familiar faces, where internal facial features, such as the eyes and mouth, are believed to be important for recognition (Ellis et al, 1979), and for unfamiliar faces. In neither case was recognition of these images 100% (ceiling) (Bruce et al, 1991c). The recognizability of the female heads was impaired the most, even when a list of same sex names were given to aid identification. Comparison of the facial surface images with photographs of the same individuals, again with their hair concealed, showed that the facial surface images were much harder to recognise (Bruce, 1991c). Thus these findings imply that some important recognition cue (maybe texture or eyebrows?) is lacking from facial surface images and that something is more salient for women's faces than men's.

The accuracy of judging the *sex* of these facial surfaces compared with photographs, where the hair is also concealed, has recently been reported (Bruce et al, in press<sup>b</sup>). In general, subjects were found to be less accurate at judging the facial surface images, performing best when the heads were displayed in  $3/4$  view. Bruce and colleagues suggested that this was because the  $3/4$  view is the viewpoint from which the 3D shape of the face is easiest to see and therefore these results imply that the 3D structure of the face is one source of information contributing to the perception of its sex. Other evidence that suggests 3D information is used in sex judgement was obtained from displaying the faces in photographic negative (see section 9.3.2) and altering the shape of the nose and chin (section 9.2.1). Some other concurrent work of the Nottingham group has also supported this theory. In this work, a discriminant function was sought to categorize faces according to sex using 2D and/or 3D measures (Burton et al, in press). It was found that both 2D and 3D measures were needed in order to obtain a function that performed as well at sex classification as humans (see chapter 4).

The majority of work on the recognition of faces and the design of automatic systems to recognise faces has used full-face views. However, for unfamiliar faces, a definite advantage for recognition has been found for  $3/4$  views compared with full-face views, - although for familiar faces  $3/4$  and full-face views are equally good. In both cases,  $3/4$  and full-face are better than profile views (Bruce et al, 1987). This is thought to be because familiar face representations rely more heavily on the "internal" features of the face, and  $3/4$  views provide no more information about these features than full-face views. The recognition of unfamiliar faces believed to be more dependant on "external" features and the  $3/4$  view may provide more information about those, eg. hairstyle. More

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information is provided about the shape of the nose and cheeks in  $3/4$  view. This provides a possible reason for its recognition advantage.

A recent study by Harries et al (1991) investigated the viewpoints from which subjects inspected 3D head models mounted on a turntable that could be rotated through 360 degrees. They found that most attention was paid to the full face and profile views. There are several possible interpretations of this observation. It may be that a longer time was needed to examine these views because the information contained in the view took longer to process or that these views were more important for recognition or simply that they were easiest to encode. It is not clear why subjects should pay particular attention to these viewpoints if the  $3/4$  view is the most useful for recognition. Harries and colleagues postulated that the reason why the  $3/4$  view is the most efficiently recognized maybe because it activates the structural descriptions of both the full face and profile views.

The *precise* role that 3D information may play in recognition is as yet unclear. It is possible that the visual system may represent the surface structure of a face explicitly. The work described in this thesis encoding the face into eight surface type primitives, will enable the importance of an *explicit encoding* of this sort to be investigated. Other possibilities are that an *implicit knowledge* of the 3D structure of faces is used or that a general knowledge of surfaces is used to invoke the operation of 2D algorithms (such as ones which extract line-drawing of objects eg. Hanna et al, 1985) (see Bruce et al, 1991b).

### **2.4 Preliminary investigations using surface type analysis**

Using the surface type methodology, we have begun to investigate the role (if any) that the facial surface shape may play in various facial recognition tasks. In this section, our preliminary investigations into the questions of sex judgement and distinctiveness are reported.

A set of 13 female faces and 14 male faces were optically scanned and facial surface images were produced. These images were rated on a scale of one to ten by unfamiliar observers for, *masculinity/femininity* (where 1 is female, 10 is male), *distinctiveness* (where 1 is typical and 10 is distinctive) and *attractiveness* (where 1 is unattractive and 10 is attractive).

#### **2.4.1 Investigation of sex judgement**

The surface type analysis was used to investigate the misclassification of the sex of a face. Three faces were selected from the 27 faces; one which was considered to be very feminine (rating 1.92), one which was considered to be very masculine (rating 9.17) and one with considerable confusion regarding sex (rating 5.83). The facial surface images



for these faces, together with their surface type encodings at a medium threshold level are shown in figure 9.1. Comparison of the surface type descriptions for these faces with each other were made for the nose, eyes and lower face regions (figure 9.2).

The surface type images shown in figure 9.1 show that the masculine face has more peaks (red) on the brow ridge than the feminine face. Other differences include, smaller peaks on the cheeks, and more saddle valleys (brown) at the sides of the nose, a larger nose tip peak and a larger saddle valley surface across the soft tissue B point, the upper and lower lip peak are more extended horizontally and a larger chin peak. The feminine face has saddle valley surfaces across the lower eye orbitals which are not present in the masculine face. These differences imply that the male face has more prominent brows, lips and chin than the female face but less prominent cheeks and a bulbous tip to the nose.

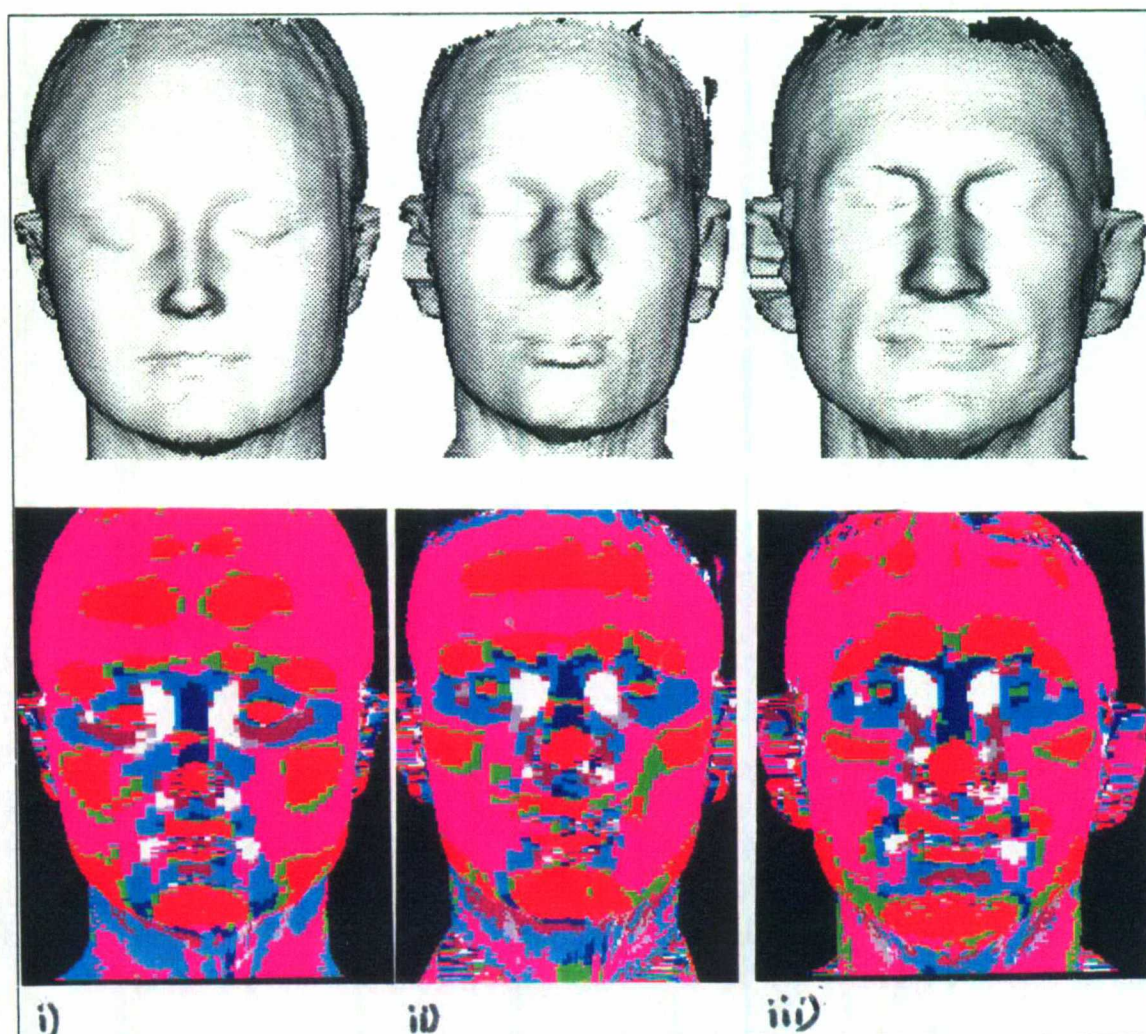


Figure 9.1: Faces that were rated as i) very feminine ii) sex unknown (actually female) iii) very masculine.

The face which suffers from confusion over sex (in the centre of figure 9.1) is actually female, but it has a large chin peak, a large amount of peak surface on the brows, a small amount of saddle valley surface at the sides of the nose and a large nose tip peak. In comparison with the masculine and feminine face, the presence of these surface types

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suggest that the face may be male. However, the face also has small peaks on the cheeks and a little saddle valley surface on the lower eye orbitals, these surface types suggest that face maybe female.

In the nose region and for all curvatures, the masculine face had more peak, ridge, pit, valley and saddle valley surfaces than the feminine face and less flat and minimal surfaces. At low curvatures, the masculine face had more saddle ridge surfaces but at high curvatures, the feminine face had more saddle ridge surface. This indicates that the nose of this man was more curved than this woman.

For the woman, the sex of whose face was found to be hard to judge, there was considerably more peak, ridge, pit and saddle valley surfaces than the feminine woman but somewhat less of these surface types than for the masculine man. There was also less minimal surfaces than the feminine woman but more than the masculine man. She also had less saddle ridges than either the masculine man or the feminine woman. The valley surfaces showed no clear trend. These results indicate that this woman's nose is more curved than the feminine woman, although not as curved as the masculine man, provided a possible explanation of the sex confusion.

In the lower face region, masculine face has more peak and ridge surfaces and less saddle ridges, flat, pit and saddle valleys than the feminine face. The indistinct face has more flat surface and less minimal, pit, valley and saddle valley surfaces than either the masculine or feminine face.

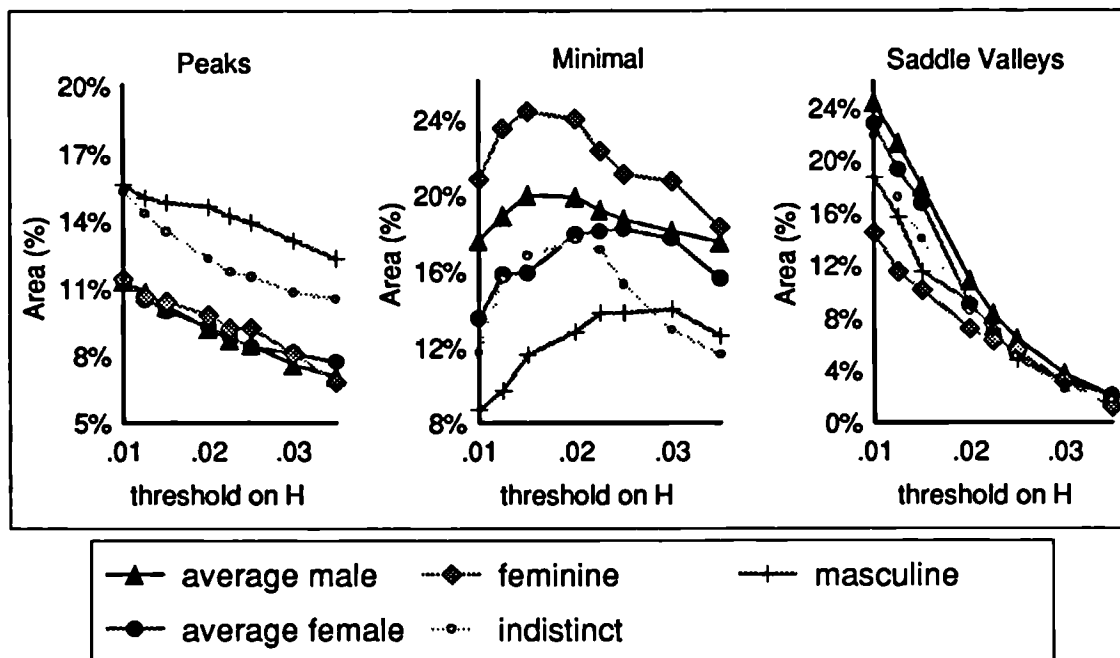


Figure 9.2: Comparison of the amount of peak (left), minimal (centre) and saddle valley (brown) surfaces in the nose region for faces that were rated as very feminine, very masculine and of indistinct sex (actually female).

In the eye region, the masculine face has more peak, highly curved ridges, saddle ridges and pits but less valleys and saddle valleys than the feminine face. The indistinct face is closer to the feminine face in terms of peak, ridge, saddle ridge, shallowly curved minimal and valley surfaces and closer to the masculine face in terms of saddle valley and highly curved minimal surfaces. She also has considerably less shallowly curved pits than either of the other faces. In this region, the indistinct face appears closer to the female face.

Comparison of surface types for these three faces has shown the ways in which the facial surfaces of the indistinct face resembles both the masculine and feminine faces. It provides a possible explanation for the confusion regarding its sex. It is interesting to note that, as one would imagine, the average male and average female appear to be closer to each other, in terms of their surface types, than the extremely feminine woman and extremely masculine man are to each other. However, these analysis must be treated with caution, due to the small sample size used for the averages and the small number of exemplars considered, and are not very conclusive. This is an area which would benefit from further research.

### **9.4.2 Investigation of facial distinctiveness.**

Extensive psychological research has established that faces rated as "distinctive" are recognised and remembered better than those rated as more typical in appearance. In related research Bruce and colleagues showed reasonably high correlations between the extent to which faces deviate from an "average" face (measured by summing deviations across a number of different measures) and their distinctiveness and memorability (Bruce et al, in press<sup>c</sup>). Research using computer-generated caricatures has shown that an individual face can be made more recognisable if it is made to deviate more from an "average" face (Rhodes et al, 1987; Benson and Perrett, 1991a). Considering these findings, we reasoned that distinctive faces should deviate more from average faces in terms of surface measures than faces rated more typical in appearance, to the extent that the surface types capture psychologically meaningful variations of a face.

For our preliminary investigation into what role may be played by the shape of the facial surface in facial distinctiveness four faces were selected from the set of optically scanned faces. These were one male and one female face that were rated as distinctive in appearance and one male and one female that were rated typical in appearance. On a scale where 10 is distinctive and 1 is typical, the ratings for these faces were as follows: distinctive male 7.2, distinctive female 7.0, typical male 4.3 and typical female 4.3. Surface type images for these faces were produced and these are shown, together with facial surface images of these faces, in figure 9.3. Graphical representations were produced to describe how the amount of each surface type varies with threshold level for the nose, eyes and lower face regions of the face. In these graphs, the distinctive and

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typical male faces were compared to the average male and the distinctive and typical female faces were compared to the average female. Figure 9.4 shows some of these comparisons. The comparisons made and described below are reported in Bruce et al (submitted).

It was hypothesized that if the proportion of a surface type, in a region, gives a quantitative measure of interfacial variation which is of psychological relevance, then the absolute deviation from the average face should be greater for distinctive faces than typical exemplars for that surface type. Comparing these exemplars with the average face eliminates any effect of averaging *per se* (such as smoothing), since a constant amount would be added to the deviations of both typical and distinctive exemplars.

We found that several of the comparisons *do* show greater deviations from the average for distinctive faces, eg. in the lower face region, distinctive faces have a smaller amount of flat surface and a greater amount of pit surfaces than typical ones. Also in the nose region, distinctive faces have a smaller amount of minimal surface and a greater amount of saddle valley surface than typical exemplars. However, not all the comparisons showed this consistency (eg. the peaks and saddle ridges in the eye region). Observations of the surface type images shows that some of these comparisons can be directly related to specific surface type patches, eg. the pits in the lower face region correspond to the corners of the mouth and the saddle valleys in the nose region correspond to the side of the nose.

An investigation was undertaken in order to discover if there was any statistically significant tendency for the distinctive faces to deviate more from the average than the typical faces. For all three regions and all eight surface types, the absolute deviation from the average was calculated at the (mean curvature) threshold levels 0.01, 0.02 and 0.03 for both the distinctive and typical faces. At the 0.01 threshold, 60% of the comparisons showed a greater deviation for the distinctive faces. This result is just statistically significant. At the 0.02 and 0.03 thresholds 64.5% of the comparisons showed a greater deviation for the distinctive faces. These results are statistically significant. At the lowest threshold, the surface type description is more susceptible to noise in the data and the analysis is least conclusive. At higher thresholds larger curvatures are detected. Analysis of the male and female faces separately also showed similar trends.

For the small number of faces examined, the hypothesis that distinctive faces deviate more from the average than typical faces was supported. These analyses are based on the relative surface deviations within a local region of the face and do not address any contribution to distinctiveness that may arise from the relationships between these parts of the face. Whilst it would certainly be valuable to examine the overall configuration of the face, a combinatorial explosion problem is encountered in attempting to quantify relationships between different measures in different regions.



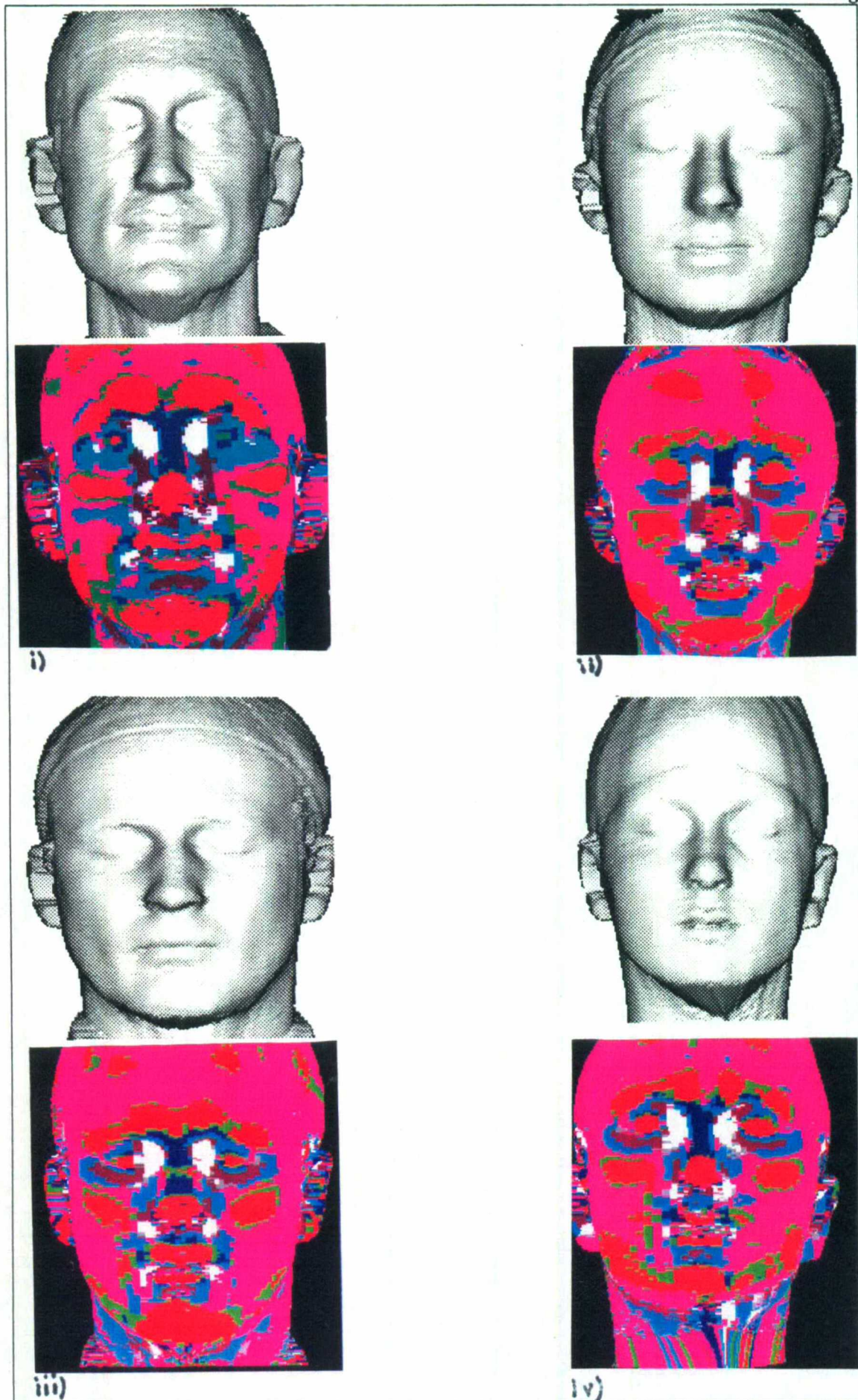


Figure 9.3: Facial surfaces images and corresponding surface type description for faces rated as i) distinctive male ii) distinctive female iii) typical male iv) typical female.

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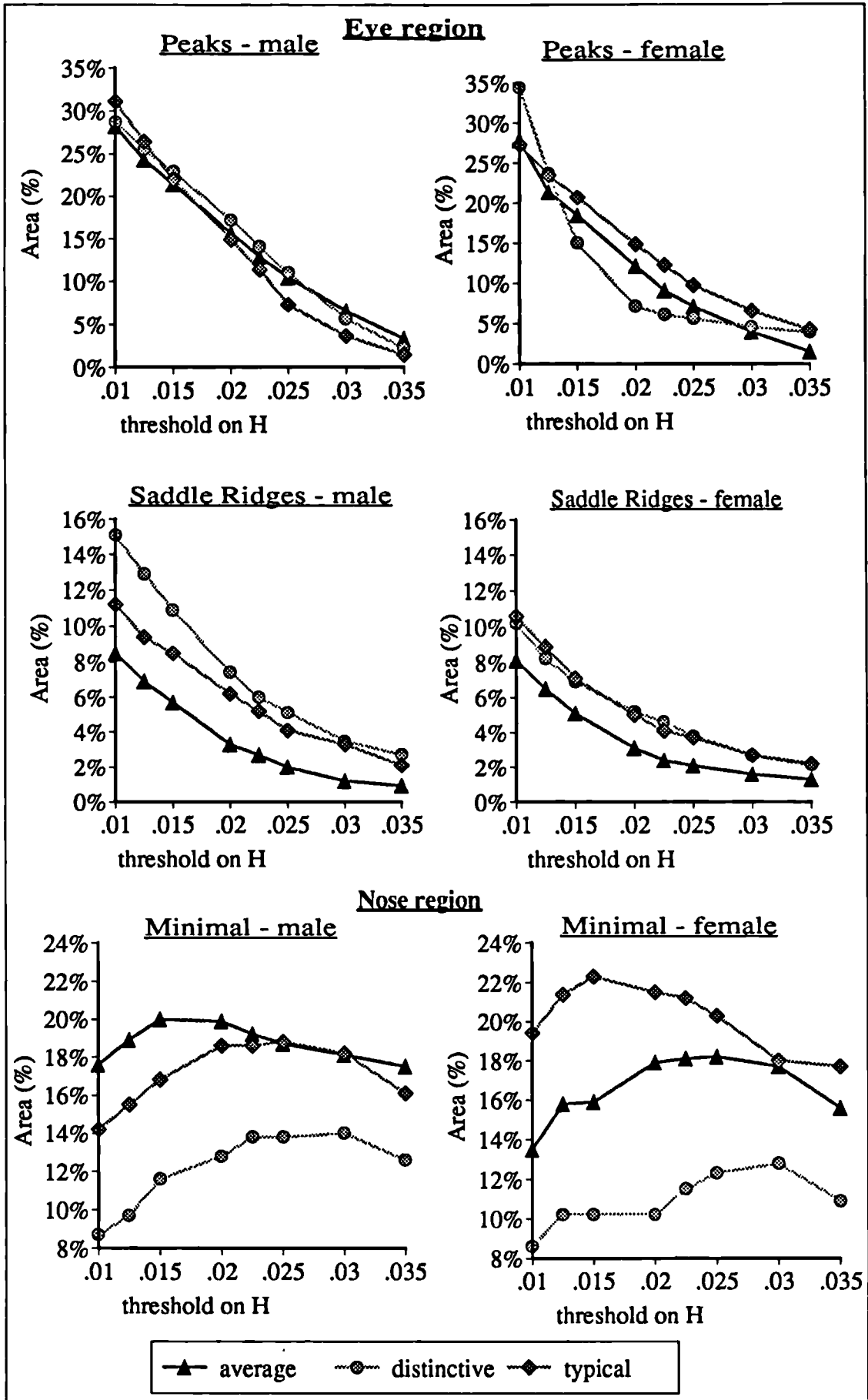


Figure 9.4: Comparison of the amount of surface types in three regions of the face for male (left) and female (right) faces rated as distinctive, typical and average.

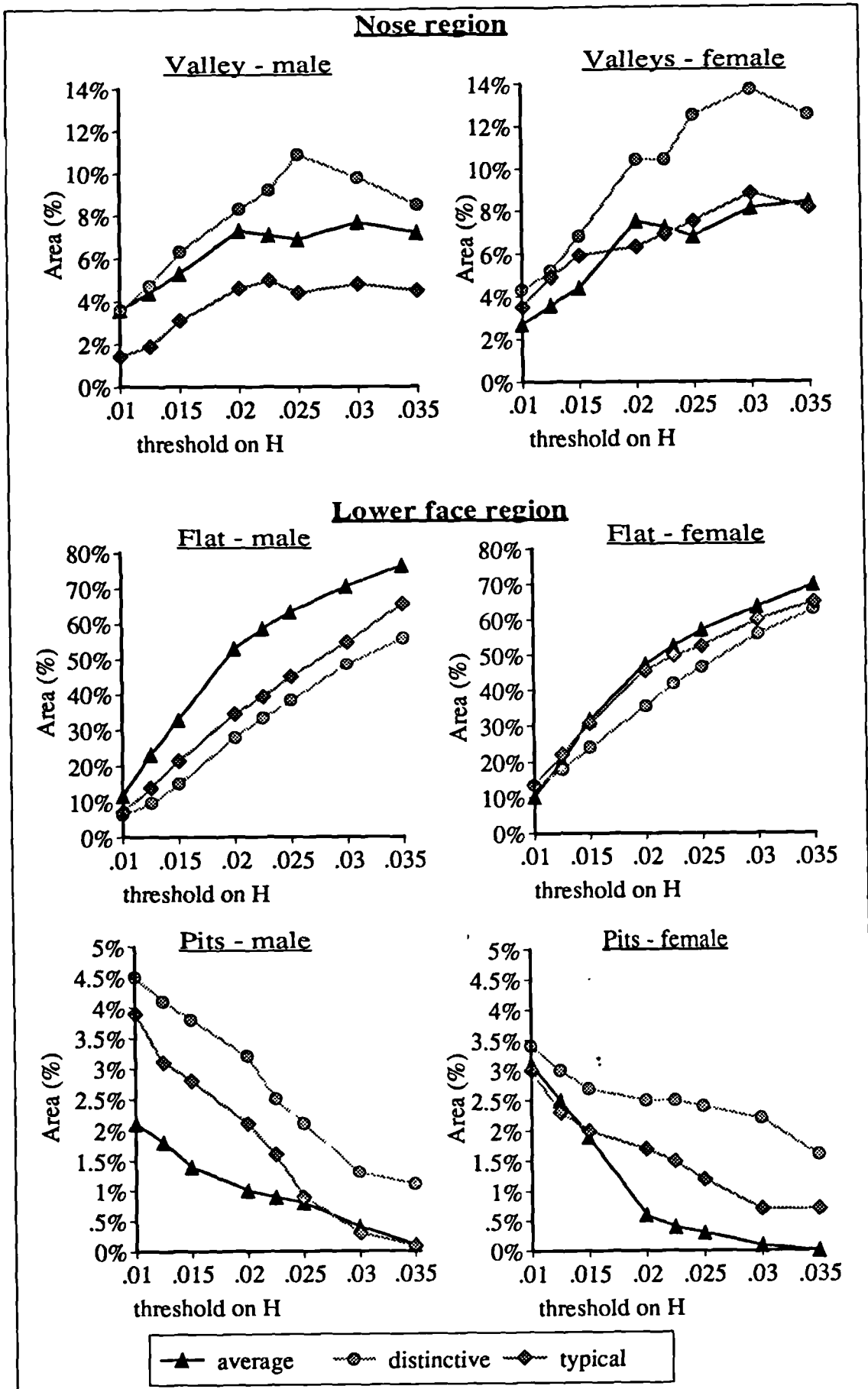


Figure 9.4 (continued): Comparison of the amount of surface types in three regions of the face for male (left) and female (right) faces rated as distinctive, typical and average.

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### **2.5 Automatic face recognition systems**

So far in this chapter, the psychological literature has been reviewed which describes the importance of various features in the perception of faces and the configural arrangement of these features which places emphasis on the need to treat the face as a whole entity. In the last section, it was shown that the surface type analysis could be used to investigate the role played by facial shape in specific face recognition tasks. In this section, the methods which have been devised by computer vision scientists for automatically recognising, or distinguishing between, faces are briefly described. This task involves differentiating between faces which differ from each other in subtle ways.

All of the systems devised to date have worked with 2D images, taking little or no account of the surface structure, consequently individual variation is considered only in terms of measurements on the picture plane. The psychological evidence to date implies that automatic recognition systems will need to incorporate a greater degree of knowledge about the structure of faces (Burton et al, in press). This was also the conclusion reached by Samal and Iyengar (1992) in their recent review of the subject. It is important to take great care in interpreting research on these systems since *picture* recognition can become confounded with *face* recognition (Gordon, 1991a). Thus extreme care must be taken to standardize photography conditions and cues such as expression, clothing, jewellery and even hair style which would allow recognition to be made on a picture difference basis.

#### **2.5.1 Explicit measurement of facial features**

A number of computer-based systems have been devised for automatic face recognition. The goal of these systems is the unambiguous identification of a target face for security and forensic purposes. They involve a classification stage followed by a recognition stage. These systems have been based either on *measuring*, or otherwise describing, *facial features* to form "feature lists" which can be compared with a stored database of such measures, or, more recently, on *globally extracted information*. In this section, the first kind of approach is described. The second approach is reviewed in the following section.

Initially, explicit measurements of the face were made manually on photographs but later digitized images were used and to extract measurements automatically. A prerequisite for interfacial comparison is that the facial size should be standardized. This has mainly been achieved using the interocular distance. The measures used for comparison have been either *geometric* - measuring specific distances between landmark points, or *syntactic* - classifying a feature by some aspect of it (such as the shape of the nose). In face recognition systems, the measures used have usually been taken from anterior views of the face (Burton et al, in press) and have been difficult to



extend to multiple viewpoints. Comparison of salient measures to a database has been made either sequentially or by matching one extreme measure first, followed by the rest. This latter method enables the size of the search to be reduced and results in a faster convergence on a candidate match. Unfortunately, it is also more likely to produce false matches. The number of features needed to distinguish between individual faces has been found to be a logarithmic function of the size of the database (Goldstein et al, 1971).

### a) Descriptions from an anterior view

The first attempts at producing a facial recognition system were made by Bledsoe (1964; 1966) and Bisson (1965). Bledsoe manually marked the coordinates of various points on a photograph and tried to recognise an individual from a database of 2000 by computing a pseudo-distance between two points. His best results were obtained when the measurements were first assigned to groups representing the eyes, the nose etc. and the pseudo-distance between those groups was computed. He observed that the locating and classification of facial features can be affected by rotation, tilt, lighting conditions, aging and expression but he only corrected for tilt and rotation. Bledsoe also pointed out the need for invariance in viewpoint for the comparison of two faces.

Bisson's (1965) method searched a photograph horizontally and vertically for the maximum value of the "average intensity difference". This was then used to extract the edges of features effectively. Measurements were then made between these edges. One weakness in this method, and other intensity based methods, is that the presence of shadows affects the image intensity and leads to false measurements.

The idea of using a *syntactic* description to classify the shape of facial features was introduced by a team working at the Bell Labs (Goldstein et al, 1971; Harmon, 1973). A panel of assessors were used to assign ratings on a scale of 1-5 to 21 features and the average rating for each feature was stored for each face. An operator would then describe a face to the computer, which would search for the best match within the database. The model was later refined to allow for errors in the description of the face (Goldstein et al, 1972). In this refinement questions such as "is the nose long?", which has only a binary answer, thereby giving a poor separation of the population, were rephrased to provide answers with a rank-order (ie. the answer would contain categories such as very long, longish, middle, shortish and short rather than yes or no), thus providing a better discrimination for well distributed features. Goldstein and colleagues stated that a face could be recognised by the *shape* and *size* of its features and especially by any extreme or "distinguishing" features. It is interesting to recall in this context the difficulty of distinguishing between two identical twins, especially if they are encountered separately. Twins have many features in common which are almost exactly the same shape and size. The main disadvantage of this method is that the

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computational cost increases with increasing population size and hence gets very large for a reasonable sized database. However, the ability of the system to reduce the set of 255 faces to 10 including the target face with 99% success and identify the target face with 70% accuracy demonstrated that the explicit encoding of shape information may well be a useful method for producing an automatic system. If this method were supported by a parallel computing facility the computational cost could well be made sufficiently small.

A similar combination of geometric and syntactic measures were used by Israeli police researchers to construct an Identi-kit image of a crime suspect (Riccia and Iserles; 1977). A similarity coefficient was calculated between the Identi-kit reconstruction and mugshots of criminals to identified possible suspects.

#### **b) Descriptions based on the facial profile**

All the above methods have used anterior views of the face. A few researchers have attempted to recognise facial profiles by segmenting the profile and classifying the profile according to the curvature of the segments. These systems seem to work quite well.

Kaufman and Breeding (1976), working with silhouettes, obtained 12 profile components using a circular autocorrelation function. From these they were able to obtain 90% accuracy for 12 profiles of 10 subjects (120 total).

In 1910, Francis Galton discussed the possibility of classifying midline profiles using a numerical coding scheme for various shapes of noses, chins etc. These ideas were used by Harmon and his colleagues for a scheme for the recognition of midline profiles (Harmon et al, 1978; 1981). They identified curvature extrema along the profile and calculated various distances and angles between them. This method enabled the identification a target face within a database, implying that curvature extrema can be used to distinguish between faces. Later, Wu and Huang (1990) used a similar method to recognise profile, extracting curvature extrema by B spline fitting. They commented that it was difficult to produce algorithms which are stable to noise. Wu and Huang's database consisted of faces which were as dissimilar as possible (all ages and sizes) which rather defeats the object of their research, to investigate the use of profile recognition in security systems, since it is vital for such systems to be able to distinguish between similar faces! Neither of these two approaches tested whether the method could recognise a "foreign" face ie. one not included in the database.

One problem with the kinds of explicit descriptions described above is the difficulty of locating features, such as the tip of nose, which are easily identified by humans (Craw and Cameron, 1991). An advantage of working with 3D data is that these landmark

points can be located precisely, as points of maximum and minimum curvature, either by looking directly at the horizontal and vertical profiles through the surface (see section-7.1) or by using the Gaussian and mean curvatures, used in calculation of the surface type description (see section 8.3). Another problem for this method is that the inherent mobility of the face means that it is extremely difficult to use exact distance measures for systems designed for security purposes (Sherman, 1990). This difficulty led to interest in methods for automatically detecting facial features and in more statistical and global approaches to face recognition.

### **9.5.2 Automatic location of facial features**

The automatic extraction of facial features is not simple, and moreover the more specific the information used for the feature identification, the easier the task becomes. Unfortunately, this leads to increasingly rigid algorithms which are unable to cope with novel faces!

Facial features have been located and classified on digitized photographs or frame-grabbed images using a number of different techniques. Early methods used low-level vision techniques to identify individual features in a predetermined order (Sakai et al, 1969; 1972; Kanade, 1977; Craw et al, 1987). Others have included statistical and probabilistic methods, these include; peak and valley detectors derived from morphological filters (Serra, 1982), template matching (Kelly, 1970; Baron, 1981) and geometric models called deformable templates (Yuille et al, 1989; Shackleton and Welsh, 1991; Hallinan, 1991; Bennett and Craw, 1991). Techniques which explicitly use the image intensity have also been used. These include intensity graphs (Nixon and Jia, private communication; Buhr, 1986) isodensity maps (Nakamura, 1991) and the use of Gaussian maps (Pearson, 1991). The Hough transform has been used to measure eye spacing (Nixon, 1985) and edge detection has been used to identify features by their distance from the head outline (Wong et al, 1989).

The extraction of the head outline has been attempted using "Snakes" or adaptive contour models (Waite and Welsh, 1990). These were developed by Kass, Witkin and Terzopoulos (1987). The results of this application are not impressive. This maybe because snakes are not really suitable for this task since they do not take into account the *a priori* knowledge available and, additionally, are quite computationally expensive (Yuille et al, 1989).

Of all the methods proposed, deformable templates have provided the best method of extracting facial features. Deformable templates act over the entire template region, compared to snakes which are local in range and need some time for the propagation of a deformation from one end of the snake to the other. They work by defining an energy function which contains terms that incorporate *a priori* knowledge about the shape of

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peaks, valleys, edges and intensities of the feature. This energy function is minimized to give the best fit of the template to the image. These systems have performed well to date, locating eyes with a reliability of 90% (Hallinan, 1991; Bennett and Craw, 1991).

### **9.5.3 Systems utilizing automatic feature extraction**

A number of systems have been developed using the automatic feature extraction methods listed above. These systems are briefly described here, in chronological order.

Sakai and colleagues (1972) extracted line-drawing type pictures from grey-level photographic images by convolution with a Laplacian filter. From these line-drawings, features were sequentially searched for by making use of *a priori* knowledge about their position and shape. This method involved placing templates of certain sizes over portions of the image to define a search region (Sakai et al, 1969). Characteristic contours were found within these regions which located the required feature. The success rate obtained was 552 out of 607, with the failures being evenly distributed between the features used. However, this method fails completely for faces with glasses or beards.

The first system to be fully automated was Kanade's (1977). He extracted a binary image from digitized photographs using a Laplacian filter. A top-down control strategy was used to locating features using *a priori* knowledge about their location and shape. Failures were able to be recovered from at the later stages of analysis by instructing the earlier stages re-analyse the image. After the eye, nose and mouth region were identified, a higher frequency filter was applied to the original image in these areas to obtain a more detailed description. A nearest neighbour algorithm was used to derive several different distance metrics, based on 16 parameters. The best of these recognised 75% of his set of 20 faces. Pattern classification was used to match the target face to a known set using statistics.

Baron's model (1981) combined a global description with local parts in a hierarchical template matching scheme. This first sought to match anterior views and then went on to compare the features. He found that lighting had a significant effect on the ability to extract the right target face and that rotations of more than 20 degrees led to failure. His model was a serious attempt to relate face recognition to neural processes.

Another method developed by Craw and colleagues started out by using a coarsely quantized image to locate the more global features and proceed through a series of successively higher resolution images to find more local features (Craw et al, 1987).

Deformable template methods have recently been incorporated into a more flexible approaches (Tock et al, 1990). Here a model of the features to be found is described.

These features are then located on the photograph and the found feature is then matched to the modelled feature using a control module to execute, and refine, a matching strategy and to determine the accuracy of the match. Potential applications of such techniques in facial recognition include encoding facial measurements onto identity cards (Tock et al, 1990).

A recognition method using isodensity maps extracted from photographs at a hierarchy of intensity levels has recently been reported (Nakamura et al, 1991). Nakamura and colleagues report that this method is able to correctly recognise, and discriminate between, known individuals with 100% accuracy. It will, of course, be strongly affected by changes in lighting conditions.

Nixon and Jia (private communication) have used the image intensity to extract head, eyes and mouth regions. Various features have been located by a variety of methods including curve fitting to extract the chin and moments to find the eyes. Buhr (1986) analysed grey-level images applying a spatial filtering mask within a search area. This allowed him to detect zero-crossings in intensity which he used to identify features. Jia and Nixon's work builds on Buhr's (1986) work and on Wong's outline detection (Wong et al, 1989) research.

### **9.5.4 Connectionist models**

The first statistically based approaches to automated face recognition examined the entire facial pattern on a pixel by pixel basis using neural net based systems. Again, these treated the face as a 2D pattern but sought to capture the configural aspect of the face (the "gestalt").

The first of these was an associative network, termed a "matrix memory model" developed by Kohonen and colleagues (Kohonen, 1977; 1984; Kohonen et al, 1981). This system was able to recognise face images and recover a face from incomplete or noisy images. Their ideas were later extended by Cottrell and Fleming (1990).

A system with a similar performance was WISARD developed by Wilke, Stonham and Aleksander (Aleksander, 1983; Stonham, 1986). This could also discriminate between different faces, given fairly consistent viewpoint and lighting conditions and a flexible rejection criteria. WISARD could also be trained to recognise faces in different facial expressions, but it required a larger number of training instances than Kohonen system. The cost to the neural net in terms of increasing complexity for every new face learnt limits the usefulness of this method (see Bruce; 1988 p.103-107, for a further critique). However, there is continuing interest in this method and one company (SD-Scicon UK Ltd) are currently attempting to use it to produce "smart cards" to allow automatic verification of identity.

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The argument behind these systems is that it is *not necessary* to specify the *nature of the representations* abstracted and constructed from face as somehow the human visual systems extracts statistical regularities from the patterns with which it is presented and that is really all that is necessary to understand about the basis of recognition (Bruce et al, 1991d). In response to this, Bruce and colleagues point out that the "hidden units" in the neural nets systems end up looking very much like "feature detectors" and also that humans describe faces in terms of features even if they are not explicitly encoding them (Bruce et al, 1991d)!

#### **2.5.5 Principal component analysis (PCA)**

Another approach to building a face recognition system was proposed by Kaya and Kobayashi (1972). They described the application of principal component analysis, essentially an information theory method, to face recognition (see appendix 1 of their paper for the mathematical detail of this method). Kaya and Kobayashi calculated the number of faces that can be distinguished between using nine geometric parameters by assessing the amount of information carried by them. They found that, in theory, the correct face could be found from a set of 5 000 faces with a probability of 92% using an average of six parameters.

The potential of this technique for recognition systems has recently led to its implementation on digitized images (Kaya and Kobayashi had made measurements on photographs manually). PCA involves the derivation of a set of principal components, which turn out to be eigenvectors, each of which describe an independent source of variation amongst the face images. Each image location contributes an amount to each eigenvector, which can be displayed as an "eigenface". The subspace spanned by the eigenvectors is referred to as "face space" (Turk and Pentland, 1991). A face is characterised as a weighted sum across the eigenvectors and can be approximated by the "best" eigenface. An individual face can be identified by comparison of the weights of each component to the corresponding weight values for each of the faces in the database. Although this method does not naturally capture salient psychological information, it has nevertheless been demonstrated to be able to recognise a small number of individuals (Turk and Pentland, 1991; Kirby and Sirovich, 1990). However, Kirby and Sirovich (1990) found that 50 "eigenfaces" were needed to account for 95% of the total variance in a fairly homogeneous set of 100 faces. This method is amenable to implementation on a simple connectionist network (O'Toole et al, 1991).

In order for a PCA to be made, it is essential that the face images are normalised for differences in size, location and orientation within the image. This is because the method has the effect of averaging between the faces. This normalization requires a certain amount of *a priori* knowledge about the images being used. One method of ensuring coincidence of the facial features is to select a number of control points and

map them, and the face, onto a standard position (Benson and Perrett, 1991b; Tock et al, 1990). This normalization has enabled unknown faces to be reconstructed using the PCA methodology (Craw and Cameron, 1991). Another way of ensuring the coincidence of features has been to locate the head in the image either as a blob template (Turk and Pentland, 1991) or by finding its outline using snakes (Waite and Welsh, 1990) or deformable templates (Bennett and Craw, 1991). If facial features can be successfully isolated, then PCA can be used to compare them also.

### **2.5.6 Depth-based comparisons**

Despite the abundance of different techniques that have been used to generate automatic face recognition systems, all of which have been based on intensity images, none of these has been very successful at producing a general purpose solution for automated face recognition (Gordon, 1991a). Techniques such as principal component analysis which appear to work well, are in fact too inflexible to use with large databases and require a substantial amount of standardization of the images used for recognition. Techniques that have been based on feature extraction have met with varied success, but often neglect to report the consistency with which the features can be extracted.

Identification schemes based on these methods rely on the extraction of edges which correspond to the boundaries of facial features such as the eyes from two dimensional images. These "boundaries" are actually three dimensional in nature. Thus the accuracy of these systems will be substantially affected by variation in lighting conditions and viewing angle. Extracting features based on properties of the surface avoids this difficulty and should lead to a more robust system since these are independent of viewpoint and lighting conditions. Intensity based methods have also been limited by their ability to identify features, usually only locating the eyes, mouth, nose etc. Surface-based descriptions of the face allow less obvious features such as the cheeks and forehead to be located and described consistently adding new features which can be used for identification purposes.

Recently, interest has been shown in using a depth based approach to face recognition. Researchers at the Nippon Telephone and Telegraph's Human Interface Lab have used a depth comparison over the entire head, matching between two normalised face data sets with a minimised distance criteria (Masui et al, 1990). They report a success rate for recognition of between 94% and 99% for a set of 20 faces. At present the method's usefulness is restricted by its non-exclusion of differences in hair or clothing and the fact that the depth comparison is made on a cartesian grid, implying that differences in facial widths will contribute more strongly than differences in feature shape.

Gordon (1991a) described two methods for recognising faces based on range data. In the first, various features such as the nose base, inner canthii and nose ridge, are used to

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register two facial surfaces. Comparison can then be made between faces based on a depth map of entire facial surface, with similarity being measured by the approximate volume of space between the facial surfaces. She tested this method using 3 views for each of 24 faces and achieved a 97% correct recognition rate. Errors that occurred were due to either error in extraction of the features used for registration or from the inclusion of the neck area in the comparison.

Her second comparison method was based solely on the facial features. Each face was described by a vector of scalar features containing both absolute distances, of eyes and nose widths and heights, and curvature measures; the maximum Gaussian curvature of nose ridge (ie. tip of nose), the average minimum curvature on ridge above tip of nose (the naision) and the Gaussian curvature of the nose bridge and the nose base. Comparison was made based on the Euclidean distance between points and cluster analysis was used to evaluate the results of the comparison (Duda and Hart, 1979). Gordon investigated different methods of dealing with missing features and the use of specific sub-sets of features. The sets of features she used were very primitive and did not fully exploit the curvature information available to her. Her comparison concentrated only on the curvature of the nose and nose and eye geometric measures. Nevertheless, she achieved a recognition of better than 79% even using only a small set of features.

Gordon concluded that descriptors based on depth and curvature data have the following advantages over intensity based descriptors: they are potentially more accurate in characterizing surface events, they make available a larger feature set through their ability to describe even low contrast areas on the face and they are naturally invariant to viewpoint and lighting changes.

Gordon's work could be built on by using a more extensive sets of curvature and geometric measures. Appropriate curvature measures could be extracted from the surface type representation of a face (see section 8.3).

### **2.6 Implications for forensic identification**

The ability to reconstruct a face, seen only briefly, is of tremendous importance to forensic scientists, the police and the legal profession. To aid the eye-witness in his/her recollection of a face, a number of face retrieval systems have been developed to construct the faces of criminal suspects. These systems must be based on the explicit encoding of facial features and dimensions. Facial features are construed as two-dimensional parts embedded in a two-dimensional configuration (Bruce et al, 1991d). Those systems that have been used in the U.K. are Identi-kit (based on line drawings of facial features), Photo-fit (based on photographs) and E-fit (a computerised version of Photo-fit). However, the face representations produced by these systems have been



found to be inadequate for consistent identification (Ellis et al, 1975a; 1978; Laughery and Fowler, 1980).

These face retrieval systems involved a witness laboriously searching through a database of many hundreds or thousands of photographs for appropriate features and faces. Large searches such as these have been shown to severely fatigue the witness, strongly impairing his/her ability to retrieve the correct face (Davies et al, 1979c). A system originally rather unfortunately named FRAME (for Face Retrieval And Matching Equipment) is currently being developed to *automatically* retrieve suitable photographs from a database (Shepherd, 1986; Ellis et al, 1989). In this system, certain parameters of a face (eg, hair style, face length or nose shape) are described on a scale by a witness (similar to Goldstein's system). These are fed into the computer which searches the database for close matches. For typical target faces, FRAME has been found to perform better than album searches but for distinctive faces, which are generally better remembered (Valentine and Bruce, 1986), no difference in performance was noticed. This system was later re-named FACES and underwent a 2 year Home Office trial using a database of 8000 faces.

Another concern that has been expressed about these systems is the assumption that witnesses can remember separate parts of the face. The Israeli police devised a system for retrieving faces based on rating the overall similarity of each photograph in their database to every other photograph. A witness then selects the most similar faces from a small subgroup of the database and similar photographs are retrieved. Averages of these photographs are then computed in order to improve the match (Levi et al, 1990). Neural nets systems have also been tested in the context of face retrieval (Starkey and Aleksander, 1990).

It has been postulated that the facial characteristics used to positively identify an individual are similar in only a small number of attributes (Harmon, 1973; Davies et al, 1978). Davies and his colleagues pointed out that faces often share similarity in certain facial *dimensions* but subjects are infrequently confused by these similarities (Davies et al, 1979b). It seems that there are certain salient attributes which distinguish between faces. These having been suggested to be face shape, age, hairstyle and eyes (Davies et al, 1979b). This has grave implications for the identification of suspects by witnesses to a crime.

Some faces are better remembered than others. A number of factors may affect the accuracy of eyewitness descriptions and identification. These have been discussed by Deffenbacher and colleagues (Deffenbacher et al, 1978), Davies (1978) and Clifford (1978) and others. Lord Devlin's report on Identification Evidence in Criminal Cases drew attention to unreliability of eyewitness evidence and especially to the problem for

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the witness of comparing similar faces in the absence of distracting faces (Devlin, 1976).

Recently, methods for the comparison of faces of suspects and robbers, caught on security video systems have been devised. Usually this takes the form of comparing facial proportions and making explicit measurement of a few distances and angles salient for the particular case. One method that Linney and I have used in preparing expert evidence in these cases is to optically scan the accused. The head can then be orientated to match the viewpoint from which the robber's head is depicted, using interactive software, and thus compared.

Currently, progress is being made towards providing photographic based representations of a suspects face from a different viewpoints. This may be done by texture-mapping a photograph onto a 3D head model (Duffy and Yau, 1988; Yau and Duffy, 1988; Duffy, 1990; Aitchison and Craw, 1991). A suitable head model can be provided by optically scanning the suspect. This system is likely to be of substantial aid to witnesses asked to recognize criminals, since the number of photographs available of an individual suspect are usually small and not taken from the angle from which witnesses has observed the criminal. Difference in viewpoint, as well as changes in expression, has been shown to considerably reduce recognition accuracy for once-viewed faces (Bruce, 1982).

One of the weaknesses of all the face reconstruction and retrieval systems produced to date, may be due to no account being taken of the 3D surface of the faces. They make no allowance for the case where a witness thinks that the eyebrows, for example, were more protuberant than a reconstruction shows. This is because no methodology exists for either automatically searching for more protuberant brows within the database or for automatically altering the existing brows interactively. A description method which took greater account of the 3D shape of features, together with the kind of software described in this thesis for altering the facial surface in a manner which corresponds to facial features and utilization of texture mapping techniques would enable this sort of requirement to be meet.

Finally, two other applications that have made use of 3D facial data. Firstly, a 3D model of the face has been manipulated to create different facial expressions and simulate speech, altering the 3D shape of the face in accordance with muscle movements (Waters, 1987; 1989; Waters and Terzopoulos, 1990; 1991). Secondly, assessment of the identifiability of unknown faces from a 3D model is important for determining the identity of some deceased individuals. Attempts at producing likenesses of murder victims have been made by combination of 3D data obtained for a found skull and a known face of similar age, sex and build to the victim (Vanezis et al, 1989). However, it

is not known as yet how well such a model conveys identity or whether the addition of texture or hair, that may be incorrect, would aid or hinder recognition.

## **9.7 Summary**

In this chapter, 3D data and shape analysis of the facial surface have been demonstrated to be useful for addressing a number of psychological questions about face recognition which have previously been limited by 2D material. The application of these techniques has been initiated by myself and my collaborators.

Specifically, the surface type description method has enabled investigations to be carried out into the role played by the shape of the face in sex judgement and in making a face distinctive. In the first investigation, possible reasons for the misclassification of a female face as male were suggested by the surface type analysis. In the second investigation, analysis of the surface type composition of the face, in the eyes, nose and lower face regions, showed the distinctive faces tend to deviate from the average by a greater amount than typical faces.

In addition to these investigations, the technique described in chapter 7 for altering the 3D data represented by a surface type patch may allow the relative importance of each facial feature for recognition tasks to be assessed.

To date no recognition systems have been built based on 3D data and little account of the surface differences in faces has been taken. The advantages of a surface based approach are that a larger feature set is available and the description produced is invariant to viewpoint and illuminations.

A better understanding of the projective geometry of the face, afforded by the ability to view it from any chosen direction and the ability to perform systematic alterations to its surface, as described at the end of chapter 7, should enable the limitations of today's face retrieval systems to be better understood. The 3D surface type description produced in this thesis may be viewed from any angle, enabling the limits of validity of 2D comparisons to be determined.

3D data has been found to be useful in two forensic areas: the reconstruction of found skulls and the generation of viewpoint determined silhouettes.

## DISCUSSION AND CONCLUSION

*"For now we see but a poor reflection, but then we shall see face to face".*

1 Corinthians 13 v 25

In this thesis, a methodology has been developed for the mathematical description of the shape of the facial surface and the quantification of changes in facial shape arising from reconstructive surgery or differences due to interfacial variation.

From examination of the literature concerning shape, I have found common ground between the work reported by computer vision scientists dealing with the mathematical description of shape and the concepts that have been formed by psychologists regarding how the brain *perceives* shape. This has allowed me to look at shape description in a novel way. The tenets of one field of study have been shown to reinforce those of the other. For instance, the idea of abstracting shape information from edges and parts has been related to mathematical descriptions of shape that are global or local in nature.

The discovery of a close relationship between the perception of an object's shape and the recognition of that object led me, in the latter stages of this work, to address questions regarding the recognition of faces.

Mathematical descriptions of shape in three dimensions have not been as well developed as those in two dimensions. Nevertheless some characteristics of a good 3D shape descriptor have been realised and these have served as pointers for my own work.

The method that I have developed for describing the shape of the face answers some of the criticisms raised against other methods. The principal criticisms concerned lack of objectivity and degree of abstraction. Landmarks, for example, have to be identified by an observer and represent the surface by only a very small sample of points. Some highly complex methods have been used to analyse the landmarks and to establish connections between them, but these do not go any way to answering the principal objections of landmark methods. A further criticism of current methods is the lack of statistical theory associated with them. Some statisticians have taken an interest in developing this theory, but their progress has so far been limited.

Methods which show potential for describing biological surfaces, but have not yet been developed in 3D, include Leyton's process history of shape changes and techniques based on bending energy.

One of the restrictions affecting the development of other ideas for characterizing 3D shape and change has been the absence of a method for recording sufficient data on surfaces. Fortunately for this work, methods for acquiring 3D measurements of the

human body, and in particular the face have now become available. The method chosen for collecting 3D facial data sets was optical surface scanning. This method is non-invasive, quick and provides structured digital data. I had easy access to a system which had already been evaluated and its stability and reproducibility of data sets has been demonstrated. The data it produced were sufficiently accurate for unambiguous results to be obtained from the surface type description method. The advantage of deriving 3D shape from range data compared to intensity data is that it is not affected by illumination conditions.

The use of differential geometry for describing surfaces, has been explored by other researchers. They have proposed segmenting the surface using lines and patches of curvature. Their investigations have influenced my own work in pioneering the application of differential geometry to descriptions of the face.

Selection of an approach for describing facial shape which is based on differential geometry *inherently* met some of the requirements that I identified as important for a successful facial shape descriptor. These requirements included describing the geometry of the *surface* and producing a description of the face which is independent of the viewpoint from which it is seen. Other important criteria were that the description produced should be robust against noise in the acquired data, have a hierarchical nature, be easy to visualize and be sensitive to small scale changes.

In seeking to produce a description of the face that met these requirements, three algorithms for calculating the Gaussian and mean curvatures of a surface were considered. These were Besl and Jain's convolution filters, Yokoya and Levine's bi-quadratic surface fit and a least squares algorithm for irregularly sampled surfaces. The latter is a novel algorithm. The first two are based on the extraction of a depth map from the surface range data whereas the latter computes the surface curvatures directly from the acquired data. Although, the latter algorithm takes a far longer time to compute, it is *truly* independent of the direction from which the surface is viewed and free from "edge" effects. The problem of the long computation time is presently a hindrance to the general clinical application of this method. However, this problem is surmountable by the parallelization of the algorithm, implemented on an array of Transputers.

The eight *fundamental surface types*, that were defined from the sign of the Gaussian and mean curvatures, allowed a surface type image (STI) representation to be produced. This representation provides a visual representation of a large amount of facial data. It was encouraging to find that the patches of the different surface types, formed on the representation, have perceptually meaningful interpretations and, likewise, that changes in the patches between two STI's are understandable in terms of movements of the face.

### ***Discussion***

Because hierarchical methods have been so successful, I was interested to try to obtain a hierarchical description of the facial surface. This objective was achieved by alteration of the thresholds on the surface curvatures, which delineate between zero and other curvatures. The hierarchical structure enabled small scale changes in shape to be described, as well as changes to the major facial features such as the nose and eyes.

The STI and KH-map representations were used to evaluate the performance of the three curvature algorithms for describing both geometrical regular objects and the facial surface. This assessment had not previously been undertaken and clearly demonstrated the advantage of the least squares algorithm in terms of accuracy.

Confirmation of our belief that the optical scan data was good enough to enable facial shape to be described came from the assessment which was made of the amount of noise in the optical surface scan data and the subsequent demonstration that the STI representation was robust against noise in the acquired data. The STI representation was also shown to not be drastically affected by smoothing the data. Although the STI representation is affected by facial expression, it was shown to be reproducible for four different scans of an individual face with a neutral expression. The demonstration of the reproducibility of the STI and its stability against noise indicate that a relatively high degree of confidence can be placed in the surface type description to faithfully represent the surface shape.

Because of the great value for many applications, of being able to compare two or more surfaces, methods for the comparison of surface type descriptions were considered. The development of these methods help answer the question "What are the most significant differences (similarities) between faces?" in an objective rather than subjective manner.

Two useful prerequisites for a comparison of facial surfaces are registering the two data sets to be compared together and obtaining an average face in order to provide a standard against which comparisons can be made. A qualitative description of the differences between two facial scans, as represented by the surface type images (STIs), allowed shape differences to be related to facial features. In addition to this, a method was described for the quantitative comparison of portions of the STIs using graphical representations of the amount of each surface type in a defined region. The patches formed by different surface types on the STI can also be compared and with the shape of the patch boundaries being quantified using some of the computer vision methods described in chapter 1, and the concept of bending energy. In order for this technique to be viable for a large number of patches, a method for *automatically* comparing corresponding patches must be found. This relies on the automatic *identification* of corresponding patches. An approach based on the artificial intelligence techniques of

frame representation and rule-based comparisons and on fuzzy logic was outlined which may ultimately provide a solution to this problem. This development, together with reducing the run-time of the surface curvatures algorithm, would enable the method described here to be implemented for clinical applications.

The development of these methods for comparing STIs enabled an evaluation to be made of its sensitivity for comparing *global* changes to the data (produced by 3D caricatures) and *local* changes to the data (produced by altering portions of the data with a B spline or by altering portions of the data set that correspond to a particular surface type patch). The perceptibility of these changes, together with the quantification of the change to the surface as measured in terms of surface type composition, will be of benefit in face recognition questions and assessing the aesthetic impact of surgery.

Many groups of people have been interested in measuring the face and for different reasons. Orthodontists and facial surgeons wish to know the way in which a face changes with growth and due to disease in order to improve the timing and precision of their operations. Anthropologists have been concerned with the interfacial variation between cultures, people-groups and races as well as within families. Psychologists have been interested in the way in which faces are recognized and the accurate identification of faces is of benefit to police, security and forensic services. Previous analyses of the face, have been limited by the lack of availability of 3D data. The recent availability of this data led maxillo-facial surgeons to articulate their need for a more quantitative description of how the face changes due to surgery. This plea provided an impetus for this work. The availability of large amounts of 3D data has allowed the face to be more easily measured and analysed and has enabled us to make progress with some of these questions.

The methodology developed has been applied to clinical cases. The potential of this method for meeting the challenge, laid down by facial surgeons, for analysing the complex changes in facial shape, has been demonstrated. Quantitative analyses of the changes to different types of faces with reconstructive surgery were made, using the region based graphical method described in chapter 7. The hierarchical nature of this method allows an assessment of the *importance* of the shape changes to be made. However, its use for charting the growth of the face has not yet been demonstrated.

The method has also been used to define landmark points consistently, free from observer error, from the surface type patches. This development may enable an *automatic* method for registering two 3D data sets to be produced.

## ***Discussion***

There are different perceptions of an aesthetically pleasing face and difficulties have been experienced in defining an aesthetic standard or standards for faces. The surface type analysis may eventually allow a mathematical basis for facial aesthetics to be defined.

Considering the application of this method to questions regarding the recognition of faces, we have seen that cognitive psychologists have hypothesized a role for the shape of the face in a variety of face recognition tasks. The description of facial shape in terms of surface type components, has enabled us to begin investigating whether facial surface shape does play a role in face recognition. In the first of two preliminary investigations we found that the perceived ambiguity regarding the sex of a facial image might be explicable in terms of facial shape. These shapes shared characteristics of masculine and feminine faces. Secondly, we found that in terms of their surface type composition, distinctive faces deviated more from an average face, of the same sex, than the typical faces. Further work along these lines of enquiry, with larger data sets and improved comparison techniques should allow firmer conclusions to be reached about the role that is played by 3D facial shape in face recognition.

## **Conclusions**

From this work, the following have been found:

- 1) Differential geometry techniques are useful for describing the facial surface.
- 2) 3D data obtained from optical surface scanning is a suitable database for the production of a description of the shape of the face.
- 3) Segmentation into surface types (shapes) provides a useful way of producing an objective description of the face which is stable and repeatable.
- 4) The concept of surface types appears to be closely related to the human perception of facial features. Thus the description of facial shape produced by segmenting the facial surface in surface types is perceptually meaningful, and its correspondence to facial features is apparent.
- 5) The method allows a *hierarchical* description of facial shape.
- 6) The surface type method is useful for analysing changes in facial shape and allows changes in facial shape to be appreciated both qualitatively and quantitatively.



- 7) The method is suitable for evaluating the effects of facial surgery on facial shape.
- 8) The method provides a way of investigating the role that 3D shape plays in human perception and facial recognition.
- 9) It provides a way of objectively locating landmark points on a surface. This could enable the automatic registration of the 3D data sets.
- 10) It has potential for developing an automatic facial recognition system.

### **Areas for future research**

A number of areas for further work have been identified. These are listed below in no particular order.

- 1) The objective definition of landmark points, as the maximum curvature of a surface type patch, could enable a method for *automatically registering* the optical surface scan data. This method would improve on Fright's current technique which relies on the *interactive* marking of landmark points.
- 2) Further work is needed on *automatically* locating the surface type patches on the surface type image. The development of a robust method for this, will enable the shape of the face before and after surgery, or between two individuals, to be automatically compared and will enable this technique to be more widely used. An outline of a possible approach to this problem was presented in chapter 7.
- 3) The development of a method for automatically recognizing faces, based on the position, extent etc. of different surface type patches. This may eventually enable an automatic facial recognition system to be produced.
- 4) The development of a statistical theory for the method. Statisticians at Leeds university have become interested in the surface type representations and their attempts to attach statistical meaning to the distribution of the surface type patches between faces may enable statistical rules to be established with regard to the surface type representation.
- 5) The application of this technique to describing the shape of other parts of the body.

### ***Discussion***

- 6) The application of this method to produce a classification of facial features based on shape.
  
- 7) The production of a set of images of the same individual's face, where the 3D data has been systematically altered in a known manner to produce *slightly* different faces. Smooth, well-constrained alterations to the 3D data could be made by altering the portion of the data which corresponds to one or more surface type(s) using the method described in chapter 7. This will be of benefit for psychological research into the relative importance of the shape of facial features for recognition.
  
- 8) The systematic alteration of the facial data and the visualization of the changed face could be of benefit in improving both clinicians and patients awareness of facial aesthetics and in the planning of operations.

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## APPENDIX A

Equations of fundamental solids

Sphere:

$$r^2 = (x - h)^2 + (y - k)^2 + (z - l)^2$$

for a sphere is centred on (h,k,l) where r is its radius.

Ellipsoid:

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

Elliptic Paraboloid: (This was used to generate a peak or pit surface)

$$cz = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Hyperbolic Paraboloid: (This was used to generate a saddle surface)

$$-cz = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Plane:

$$ax + by + cz + d = 0$$

Ridge or Valley surface:

$$z = x^2 : z = y^2 : x = z^2 : x = y^2 : y = x^2 : y = z^2$$

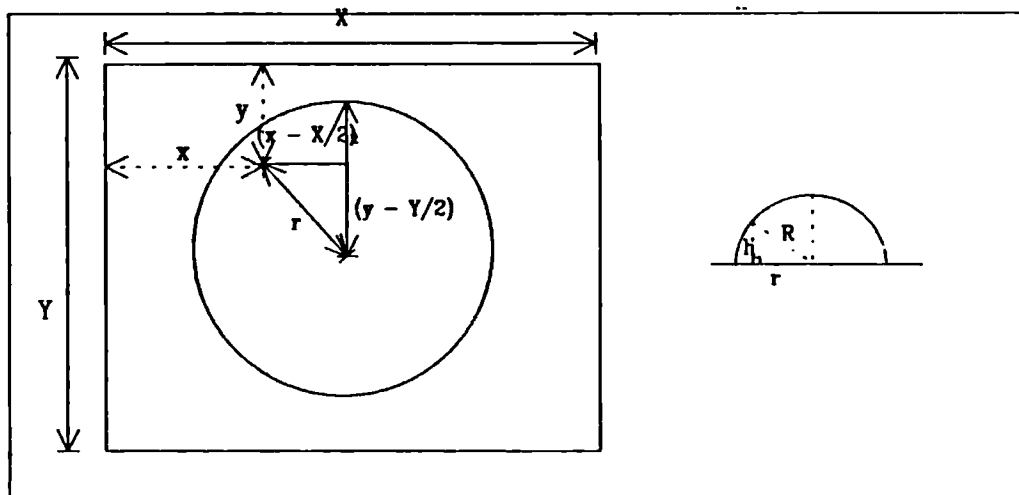


Figure A1: Calculation of a depth map of a sphere.

**OCCAM Code to generate mathematical models of geometrically regular objects****1. A plane**

```

--{{{
-- NB must be positive.
VAL scanlines IS 100:
REAL32 rad,dist,ang.deg: --dist of plane from centre
SEQ
  nprof := 64
  centre := 500.0(REAL32)
  rad := 50.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishft
      ishft1[A] := ((nprof-A)*256)/nprof
      -- between profs: ldeg := 1024/360
      ishft2[A] := ((nprof-A)*256)/nprof
      ang.deg := (PI*(REAL32 ROUND A))/(REAL32 ROUND (nprof*2))
      dist := (rad/(SQRT(2.0(REAL32)))) /
              (COS((PI/4.0(REAL32)) - ang.deg)) --x + r
      --{{{ check dist very small, like zero
      IF
        dist < 0.01 (REAL32)
          dist := 0.01(REAL32)
          TRUE
          SKIP
      --}})
      SEQ B=0 FOR scanlines
        SEQ
          hsiv[A][B] := (centre - dist)
      done := TRUE
    --}})
--}}}

```

**2. A paraboloid**

```

--{{{
-- NB must be positive
VAL scanlines IS 100:
VAL min.point IS 50.0(REAL32): --a
VAL offset IS 50.0(REAL32):
SEQ
  nprof := 128
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishft
      ishft1[A] := (nprof-A)*8 -- between profs: ldeg := 1024/360
      ishft2[A] := (nprof-A)*8
      REAL32 x, ysquared,top:
      SEQ B=0 FOR scanlines/2
        SEQ
          ysquared := ((REAL32 ROUND((scanlines/2)-B))*
                      (REAL32 ROUND((scanlines/2)-B)))
          x := (ysquared/(4.0(REAL32)*min.point))
          --{{{ check x very small, like zero

```

## Appendix A

```
      IF
        x < 0.1 (REAL32)
          x := 0.1(REAL32)
        TRUE
        SKIP
      --}}
      hsiv[A][B] := (centre - (x+offset))
      hsiv[A][(scanlines-1)-B] := hsiv[A][B]
    done := TRUE
  --}}}
```

### 3. A hyperboloid

```
--{{{
-- NB must be positive
VAL scanlines IS 100:
VAL min.point IS 50.0(REAL32): --a
VAL b IS 25.0(REAL32): --b
SEQ
  nprof := 128
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishaft
      ishaft1[A] := (nprof-A)*8 -- between profs: 1deg := 1024/360
      ishaft2[A] := (nprof-A)*8
      REAL32 x, ysquared,top:
      SEQ B=0 FOR scanlines/2
        SEQ
          ysquared := ((REAL32 ROUND((scanlines/2)-B))*
            (REAL32 ROUND((scanlines/2)-B)))
          x := ((min.point/b)*(SQRT((b*b) + ysquared)))
          --{{{ check x very small, like zero
          IF
            x < 0.01 (REAL32)
              x := 0.01(REAL32)
            TRUE
            SKIP
          --}}}
          hsiv[A][B] := (centre - x)
          hsiv[A][(scanlines-1)-B] := hsiv[A][B]
        done := TRUE
      --}}}
```

### 4. An ellipsoid

```
--{{{
-- NB a*a(1-e*e) >= ysquared!!!! or top is -ve
VAL scanlines IS 100:
VAL maj.axis.len IS 76.0(REAL32): --a
VAL eccent IS 0.75(REAL32): --e
SEQ
  nprof := 128
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishaft
```



```

ishaft1[A] := (nprof-A)*8 -- between profs: ldeg := 1024/360
ishaft2[A] := (nprof-A)*8 --(8 = 360 object if nprof= 128)
REAL32 x, ysqared,top:
SEQ B=0 FOR scanlines/2
  SEQ
    ysqared := ((REAL32 ROUND((scanlines/2)-B))*
      (REAL32 ROUND((scanlines/2)-B)))
    top := ((maj.axis.len*maj.axis.len)*
      (1.0(REAL32)-(eccent*eccent)))-ysquared)
    x := (top/(1.0(REAL32)-(eccent*eccent)))
    x := SQRT(x)
    --{{{ check x very small, like zero
    IF
      x < 0.01 (REAL32)
        x := 0.01(REAL32)
      TRUE
        SKIP
    --}}})
    hziv[A][B] := (centre - x)
    hziv[A][(scanlines-1)-B] := hziv[A][B]
done := TRUE
--}}})

```

## 5. A sphere

```

--{{{
-- contains modifications to simulate noise
SEQ
  INT RADIUS,scanlines:
  REAL32 radius:
  SEQ
    --{{{ set radius and scanlines required
    write.text.line(scr,"Enter radius of sphere required in mm
      [REAL] ")
    read.echo.real32(key,scr,radius,result)
    RADIUS := INT ROUND(radius)
    IF
      (RADIUS = 45)
        scanlines := 90
      (RADIUS = 90)
        scanlines := 180
      (RADIUS = 125)
        scanlines := 250
    TRUE
    SEQ
      write.text.line(scr,"Enter no of scanlines [INT] ")
      read.echo.int(key,scr,scanlines,result)
    --VAL scanlines IS 180:
    --VAL radius IS 90.0(REAL32):
    --}}})
  BOOL noise,q.noise:
  INT respond,n1,n2,quan:
  REAL32 sd:
  SEQ
    nprof := 128
    centre := 500.0(REAL32)
    --{{{ ask about noise
    noise := FALSE
    q.noise := FALSE
    write.text.line(scr, "Do you wish to add noise to object?
      [Y] ")

```

## Appendix A

```
key ? respond
IF
  respond = (INT'y')
  SEQ
    write.text.line(scr, "Gaussian noise? [Y]")
    key ? n1
    --{{{
    IF
      n1 = (INT'y')
      SEQ
        noise := TRUE
        write.text.line(scr, "Enter standard deviation of
                          noise in mm [REAL32]")
        read.echo.real32(key,scr,sd,result)
      TRUE
      SKIP
    --}}})
  write.text.line(scr, "Quantization noise? [Y]")
  key ? n2
  --{{{
  IF
    n2 = (INT'y')
    SEQ
      q.noise := TRUE
      write.text.line(scr, "Select option for standard
                          deviation")
      write.text.line(scr, " of noise in mm [REAL32]")
      write.text.line(scr, " 1 = 1 mm")
      write.text.line(scr, " 2 = 0.5 mm")
      write.text.line(scr, " 3 = 0.25 mm")
      key ? quan
    TRUE
    SKIP
  --}}})
  TRUE
  SKIP
--}}})
SEQ A=0 FOR nprof -- profiles
  SEQ
    pist[A] := 10
    plst[A] := scanlines+9
    -- angle in deg * 1024/360 := ishaft
    ishaft1[A] := (nprof-A)*8
    -- between profs: 1deg := 1024/360
    ishaft2[A] := (nprof-A)*8
  REAL32 x:
  SEQ B=0 FOR scanlines/2
    SEQ
      x := (radius *radius) -
            ((REAL32 ROUND((scanlines/2)-B))*
             (REAL32 ROUND((scanlines/2) - B)))
      x := SQRT(x)
      --{{{ add normal noise if needed
      IF
        noise
        --{{{ add noise
        SEQ
          INT32 seed:
          REAL32 random1,random2,sd,error:
          SEQ
            seed := 42(INT32)
```

```

random1,seed := RAN(seed)
random2,seed := RAN(seed)
error :=
    (SQRT((2.0(REAL32))*ALOG10(random1)))*
        COS((2.0(REAL32)*PI)*random2)
x := x + (sd*error)
--}})
TRUE
SKIP
--}})
--{{{ add quantization noise if needed
IF
q.noise
--{{{
INT x:
SEQ
X := INT TRUNC(x) --trunc rounds towards zero
                -- (1.6 ->1, -1.6 ->-1)
CASE quan
INT('1')
--{{{ to nearest mm

IF
((x-(REAL32 ROUND(X))) <= 0.5(REAL32))
x := REAL32 ROUND (X)
((x-(REAL32 ROUND(X))) > 0.5(REAL32))
x := REAL32 ROUND((X+1))
TRUE
write.text.line(scr,"error in
quantization noise")
--}})
INT('2')
--{{{ to nearest 0.5 mm
IF
((x-(REAL32 ROUND(X))) <=0.25(REAL32))
x := REAL32 ROUND (X)
(((x-(REAL32 ROUND(X))) > 0.25(REAL32))AND
((x-(REAL32 ROUND(X))) <
0.75(REAL32)))
x := (REAL32 ROUND (X)) + 0.5(REAL32)
((x-(REAL32 ROUND(X))) >= 0.75(REAL32))
x := REAL32 ROUND((X+1))
TRUE
write.text.line(scr,"error in
quantization noise")
--}})
INT('3')
--{{{ to nearest 0.25 mm
IF
((x-(REAL32 ROUND(X))) <=0.125(REAL32))
x := REAL32 ROUND (X)
(((x-(REAL32 ROUND(X))) >
0.125(REAL32))AND
((x-(REAL32 ROUND(X))) <=
0.375(REAL32)))
x := (REAL32 ROUND (X)) + 0.25(REAL32)
(((x-(REAL32 ROUND(X))) >
0.375(REAL32))AND
((x-(REAL32 ROUND(X))) <=
0.625(REAL32)))
x := (REAL32 ROUND (X)) + 0.5(REAL32)

```

## Appendix A

```
        ((x-(REAL32 ROUND(X))) >
0.625(REAL32)) AND
        ((x-(REAL32 ROUND(X))) <=
0.875(REAL32))
        x := (REAL32 ROUND (X)) + 0.75(REAL32)
        ((x-(REAL32 ROUND(X))) >= 0.875(REAL32))
        x := REAL32 ROUND((X+1))
        TRUE
        write.text.line(scr,"error in
        quantization noise")
        --}}
    ELSE
    SKIP
    --write.real32(scr,x,3,4)
    --}}
    TRUE
    SKIP
    --}}
    --{{{ check x very small, like zero
    IF
    x < 0.01 (REAL32)
    x := 0.01(REAL32)
    TRUE
    SKIP
    --}}
    hsiv[A][B] := (centre - x)
    hsiv[A][({scanlines-1)-B] := hsiv[A][B]
done := TRUE

--}}}
```

## 6. A Saddle

```
--{{{
VAL scanlines IS 100:
VAL radius IS 50.0(REAL32):
SEQ
  nprof := 120
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
  SEQ
    pist[A] := 10
    plst[A] := scanlines+9
    -- angle in deg * 1024/360 := ishaft
    ishaft1[A] := (nprof-A)*2 -- between profs: ldeg := 1024/360
    ishaft2[A] := (nprof-A)*2
    REAL32 temp:
    SEQ B=0 FOR scanlines/2
    SEQ
      temp := (REAL32 ROUND((scanlines/2)-B))
      temp := (temp*temp)/(REAL32 ROUND (scanlines/2))
      --{{{ check temp very small, like zero
      IF
      temp < 0.01 (REAL32)
      temp := 0.01(REAL32)
      TRUE
      SKIP
      --}}}
      hsiv[A][B] := centre - (temp+radius)
      hsiv[A][({scanlines-1)-B] := hsiv[A][B]
done := TRUE
```

```
--}}}
```

## 7. A Cylinder

```
--{{{
VAL scanlines IS 100:
VAL radius IS 35.0(REAL32):
SEQ
  nprof := 128
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishft
      ishft1[A] := (nprof-A)*8 -- between profs: ldeg := 1024/360
      ishft2[A] := (nprof-A)*8
      SEQ B=0 FOR scanlines
        hsiv[A][B] := centre - radius
      done := TRUE
  --}}}
```

## 8. A Cone

```
--{{{
VAL scanlines IS 100:
VAL radius IS 50.0(REAL32):
REAL32 x:
SEQ
  nprof := 128
  centre := 500.0(REAL32)
  SEQ A=0 FOR nprof -- profiles
    SEQ
      pist[A] := 10
      plst[A] := scanlines+9
      -- angle in deg * 1024/360 := ishft
      ishft1[A] := (nprof-A)*8 -- between profs: ldeg := 1024/360
      ishft2[A] := (nprof-A)*8
      SEQ B=0 FOR scanlines
        SEQ
          x:= (REAL32 ROUND(B))
          --{{{ check x very small, like zero
          IF
            x < 0.1 (REAL32)
              x := 0.1(REAL32)
            TRUE
              SKIP
          --}}}
          hsiv[A][B] := centre - x
        --{{{ COMMENT fill gap !!!
        --}}}
      done := TRUE
    --}}}
  --}}}
```



