Sustainable Development and Green National Accounts

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October 1995

Thesis submitted in part-fulfillment of the degree of Doctor of Philosophy in Economics
University College London

Abstract

Defining sustainable development as non-declining utility, the relationship between sustainable development and optimal growth is examined critically in Part 1. The operation of the Hartwick rule for an exhaustible resource is explored under different values of the elasticity of substitution between capital and resources. The Hartwick rule is then extended to the case of fossil fuels, where carbon dioxide emissions arise as an externality. Optimal growth paths with exhaustible resources are shown to be non-sustainable for positive pure rates of time preference or if produced capital depreciates. For linked environment-economy models where pollution stocks dissipate, the optimal steady state is characterized and feasibility conditions for the steady state derived. When resources are renewable and production leads to emissions that damage the resource, the restrictions on the feasible resource stock size in the steady state are determined. Part 2 considers the problem of measuring sustainable development, deriving 'green NNP' as a transformation of the Hamiltonian function for an optimal control problem. Two problems in accounting for exhaustible resources are developed: resource discoveries and heterogeneous resource deposits. The key issue of the treatment of pollution and pollution abatement in green national accounts is explored through a series of six models: flow pollutants, stock pollutants, impairment of pollution dissipation, fossil fuels and carbon dioxide, living resources and acid rain, and household defensive expenditures. The models of flow accounts are extended to green wealth accounting, where it is shown that stocks of pollution can be treated as liabilities in the national balance sheet. Empirical measures of sustainable development are presented in Part 3, with a discussion of the policy implications of green national accounting. Estimates of the value of pollution and 'genuine' savings rates are presented for the UK and selected European countries. The genuine savings analysis is extended to resource depletion and carbon emission damages for over 50 developing countries, revealing significant dissaving in Subsaharan Africa.

Acknowledgments. Thanks for guidance and insightful comments go first to my advisors, David Pearce and David Ulph. Malcolm Pemberton provided the initial encouragement for this research. This work has benefited greatly from discussions with Giles Atkinson, John Pezzey, Mohan Munasinghe and John O'Connor. As will be obvious in what follows, the work of John Hartwick has been a major inspiration. Any errors, of course, are solely those of the author.

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Chapters 4 and 10 of this thesis are based on co-authored papers. This is to certify that in each case more than 50% of the chapter was written by the author of this thesis.

Professor D.W. Pearce

1. Introduction

Economics has been concerned with sustainable development, in the sense of sustainable income, at least since Hicks' (1946) famous definition of income. Hicks' notion of sustainability was limited, however, since it considered only produced assets in the determination of the maximum amount that could be consumed while leaving future prospects for income unchanged. The contributions of environmental economics to thinking about sustainable development have been twofold: (i) the asset base has been expanded beyond produced capital to include natural resources and the environment; and (ii) the environment has been recognized as an explicit source of welfare. Both of these ideas will be exploited in this study.

Sustainable development as defined by Pearce *et al.* (1989) has two forms, strong and weak. *Strong* sustainability assumes that there are some environmental assets for which there are effectively no substitutes, and so these assets, or at least some portions of them, must be maintained intact if welfare is to be maintained over the indefinite future. *Weak* sustainability assumes that environmental assets are substitutable for produced assets and therefore that maintaining a non-decreasing total value of assets, both produced and natural, will ensure non-declining welfare over time. Weak sustainability should be viewed as a precondition for sustainable development: where it is possible, substitution must occur in order for sustainability to be achieved. It is therefore weak sustainability that will be the underlying paradigm in what follows.

Adopting sustainability as a goal is, of course, an ethical position rather than 'positive economics' - sustainability is not the logical outcome of the precepts of neo-classical economics. At the level of ethics it is possible to argue about whether non-declining utility could entail lower levels of per capita utility spread over more individuals or whether it should entail non-declining per capita utility; most governments, faced with the choice, would opt for the latter. The effect, within economics, of aiming for sustainable development is either to turn the classical problem of maximizing the present value of utility into a constrained maximization problem, or to lead to formal rules that characterize paths with constant utility.

Expanding the theoretical underpinnings of sustainable development is an important objective of this study. However, if government commitments to the goal of sustainable development are to be more than mere words, significant advances need to be made as well on the *measurement* of progress towards sustainable development. The second objective in what follows is therefore to extend both the theory and the practice of measuring sustainable development, with 'green' national accounting as the pre-eminent means of measuring progress. Of particular concern in this regard is the treatment of pollution emissions and abatement expenditures in the national accounts.

These motivations provide the plan of the study. Part 1 is concerned with the theory of sustainable development. Part 2 develops the theory of green national accounting. Part 3 presents empirical applications of green national accounting in order to measure whether, and to what extent, countries are behaving sustainably.

Outline and Linkage to the Literature

The exposition of the theory of sustainable development begins with a proof of the classic result in Dasgupta and Heal (1979, ch. 10), that in an economy with an exhaustible resource that is essential for production, positive rates of pure time preference lead to declining consumption along the optimal programme that maximizes the present value of utility. This result is extended in one important way by considering what happens if produced assets depreciate. By deriving the Hotelling rule for this model, it is shown that depreciation of produced capital generally leads either to non-sustainability or to the infeasibility of the Hotelling rule. Depreciation is therefore fatal both to sustainability along the optimal path, even if the pure rate of time preference is 0, and to the operation of the Hartwick (1977) rule, since the latter depends on the Hotelling rule. In a world where capital depreciates, therefore, technological change is necessary for the achievement of sustainability.

Setting aside the question of depreciation, the next stage in the analysis is the exploration of the Hartwick rule under different assumptions about the elasticity of substitution between capital and resources. While the degree of substitutability is obviously a key issue, the literature does not contain a full exposition of the role that it plays. It is proved that a 'generalized Hartwick rule,' analogous to that of Dixit, Hammond and Hoel (1980), that investment must equal resource rents times resource extraction plus a constant, is necessary and sufficient for constant consumption if the Hotelling rule holds. It is then shown that non-zero values of the constant lead to infeasibility of the Hartwick-Hotelling programme for constant elasticity of substitution (CES) production functions.

Hartwick (1978) begins but does not complete the analysis of the Hartwick-Hotelling programme for different values of the elasticity of substitution. This gap is filled when it is shown that constant consumption is infeasible if the elasticity is less than 1, and that the programme does not produce maximal consumption for values of the elasticity that are greater than and near to 1. The existence of a maximal constant consumption path requires that the elasticity of substitution be precisely 1. Linking this result back to the sustainability debate, it is clear that constant consumption, a type of 'minimal sustainability,' is impossible if one holds to a 'stronger' sustainability position, that the elasticity of substitution between resources and capital is less than 1.

The operation of the Hartwick rule is then extended to the case of fossil fuels, where the exhaustible resource in question is also the source of a global externality. The extended Hartwick rule that results requires that investment equal the diminution in the resource stock plus the change in the CO_2 stock, each valued at their shadow prices: unit resource rents net of an optimal carbon tax in the case of the fossil fuel, and the marginal social cost of carbon dioxide. While such a result is implicit in Dixit, Hammond and Hoel (1980), several new features are highlighted in this model: (i) the effect of the carbon externality on net resource rents; (ii) the operation of the Hartwick rule when carbon emissions are assumed to lead to catastrophic outcomes at some stock level; and (iii) the condition for the feasibility of the Hartwick programme, namely that marginal dissipation rates of the carbon stock must approach 0 as the total stock approaches its 'pre-industrial' level.

The question of sustainability is next extended to additional models with pollution externalities. A model of a pollutant with cumulative effects is shown not to have an optimal path that is sustainable. For a model of a stock pollutant that dissipates, the feasibility conditions for a long-run steady state are, jointly, that the willingness of consumers to pay for environmental services not be too high and that the pure rate of time preference not be too low. For the acid rain problem, the pollution externality leads to a smaller range of feasible stock sizes for a living resource than in the absence of acid emissions. Sufficient conditions for the optimal paths of these models to be sustainable are established.

Turning to the *measurement* of sustainable development, the 'traditional' literature on green national accounting, which eschews formal models, is shown to be inconsistent and unpersuasive - this covers the work in volumes edited by Ahmad *et al.* (1989) and Lutz (1993), as well as work by Repetto *et al.* (1989), Bartelmus *et al.* (1989), Hueting and Bosch (1990), Juster (1973) and Herfindahl and Kneese (1973).

The green national accounting work of Hartwick (1990, 1993) and Mäler (1991), building on the seminal work of Weitzman (1976), is extended in many directions. These studies all interpret some transformation of the current value Hamiltonian function for the optimal growth programme of a simple economy to be a measure of NNP. The result of extending these models is interpreted, initially, as a guide to optimal growth policy - a model with exhaustible resources is developed, to show that green NNP is a good indicator of the direction of movement of the Hamiltonian along the optimal path. Interpretations with regard to sustainable development are taken up later in the study.

With regard to exhaustible natural resources, a model is presented, developed independently by Hartwick (1993), of the treatment of resource discoveries in a deterministic framework. By making resource discoveries a function both of cumulative discoveries and of exploration effort - an approach similar to that of Pindyck (1978) - it follows that resource discoveries are properly added to green NNP valued at their marginal discovery cost, which is necessarily less than the unit rental rate. This contradicts the non-intuitive conclusion in Hartwick (1990) and Repetto *et al.* (1989) that resource discoveries, valued at the full rental rate, should be treated as income in the year of discovery. Treating resource exploration expenditures as investment, as is done in the standard national accounts (United Nations 1993a), is therefore roughly correct.

Another issue in accounting for exhaustible resources is the phenomenon of heterogeneous resource deposits. The models of Hartwick (1990) and Hartwick and Lindsey (1989) assume that the resource (oil) is, in effect, in one big pool, which suggests that frontier oil production costs represent the marginal cost of extraction. A model is developed in which simultaneous production takes place from heterogeneous resource deposits, each having its own extraction cost function (production is efficient as long as each specific unit rent follows the Hotelling rule). Green NNP for such a model deducts the rents from each individual deposit, giving a result differing from the 'one big pool' assumption.

The area with the greatest need for development in green national accounting is the treatment of pollution emissions and abatement expenditures. Hartwick (1990,1993) and Mäler (1991) both explicitly extend Weitzman's approach to look at maximizing the present value of utility under different presumptions about the depletion of natural resources and damage to the environment from pollution. Mäler constructs one large model that contains, in addition to consumption and investment goods, a flow resource that is damaged by pollution emissions, a living resource that is harvested and whose growth is affected by inputs of goods and labour, and a household production function through which, by inputting goods and labour, households can increase their benefits from the environment (i.e., the flow resource). The key result in Mäler (1991) is that deductions for defensive expenditures should not be made in the measure of national welfare derived from the model.

Hartwick (1990) presents two pollution-related models, one in which there is a stock pollutant that accumulates emissions and is subject to a natural dissipation process - this pollutant appears (negatively) in the production function - and a second in which the rate of change of the stock pollutant appears in the utility function as well. In these models pollution is mitigated by expenditures that affect the rate of the natural dissipation process, an unlikely form of mitigation. Hartwick (1993) offers a more intuitive model in which utility is related to the accumulated stock of pollutant and abatement expenditures limit the quantity of pollution emissions.

This study builds on and extends the Hartwick and Mäler models in several directions: (i) an explicit approach is taken to pollution abatement expenditures, and these are related to marginal social costs and optimal emission taxes; (ii) a series of models are constructed to examine individually the effects of flow pollutants, stock pollutants and pollutants with cumulative effects, degradation of natural dissipation processes, stock pollutants linked directly to exhaustible resources (the CO_2 problem), and flow pollutants that damage living resources (the acid rain problem); and (iii) the treatment of household defensive expenditures is re-examined.

A key conclusion of the pollution models is that pollution emissions in NNP should be valued at their marginal social costs (which equal marginal abatement costs and the level of a Pigovian tax on emissions at the optimum), which is in clear contrast to the suggestion of using 'restoration costs' in the UN System of Integrated Environmental and Economic Accounts (United Nations 1993b). Other general conclusions are that abatement expenditures should be treated as intermediate consumption, that adjustments need to be made for both pollution emissions and natural pollution dissipation processes, and that the level of environmental services must be valued in measuring welfare. Not only should household defensive expenditures not be subtracted from welfare, under plausible assumptions the adjustment to welfare includes a value greater than the level of household defensive expenditure. Because the value of environmental services appears naturally in these models, the resulting measure is best interpreted as a 'measure of economic welfare.'

These modeling approaches are extended to green wealth accounting, where it is shown that the value of pollution stocks should be treated as a liability in the national balance sheet, but that constant wealth at current prices is not synonymous with constant welfare. The wealth analysis helps to sharpen the interpretation of the measure of economic welfare in the previous models: taking the natural definition of green NNP as the sum of unconsumed output and the change in the real value of assets and liabilities, this measures the maximum amount that can be consumed while leaving utility *instantaneously* constant (this has similarities to the model of Pemberton *et al.* 1995). The measure of economic welfare (MEW) adds the value that consumers place on the flow of environmental services, a sort of expanded notion of consumption, to this green NNP measure. Green NNP is shown to be an imperfect indicator to guide policies for optimal growth, while the MEW is better, but under some restrictive assumptions. While Pezzey (1994) and Asheim (1994) show that positive net savings rates at a point in time are not sufficient to prove that the economy is on a sustainable path, it is proven that persistent negative net savings rates must lead to declining welfare in the long run. This rate of 'genuine' saving is therefore a one-sided indicator of sustainability, and this indicator is arguably the green national accounting aggregate with the greatest welfare significance.

The empirical portions of the study present new results on the measurement of genuine savings based on the preceding theoretical work. It is shown that the marginal social costs of pollution provide an upper-bound estimate of pollution damages that can be used in green accounting exercises. Genuine savings rates, accounting for resource depletion and the social costs of pollution emissions, are calculated for the UK over the decade of the 1980's, showing it to be marginally non-sustainable over this period; similar but cruder calculations are carried out for the other countries of OECD Europe. A new data set giving consistent estimates of the value of resource depletion over the 1980's, including oil, minerals and tropical forests, is used to estimate genuine saving rates for over 50 countries; Subsaharan African nations stand out as persistent dissavers.

The study concludes with a discussion of the policy implications of green national accounting, with particular emphasis on measures of genuine saving.

Part 1

Theory of Sustainable Development

2. Sustainability, Exhaustible Resources and Depreciation

One of the less satisfying aspects of the notion of *weak sustainability* (Pearce *et al.* 1989), that produced assets can substitute for natural resources, is the fact that produced assets depreciate and wear out. Intuitively, it would seem that the combination of a finite non-renewable resource base and depreciable capital should lead to unsustainability: consumption will decline in the long run. This intuition turns out to be correct under fairly general conditions.

Dasgupta and Heal (1979, ch. 7 endnote) state that exponential decay of capital is 'a positive embarrassment' to their model of maximal constant consumption in the face of exhaustible resources. They then simply claim that exponential decay is not very realistic and carry on. The discussion of the depreciation of capital is not continued in their presentation of the optimal growth model with exhaustible resources. The purpose of this note is to develop this model, and to draw conclusions for the sustainability of economic development.

To keep the argument to its essentials we assume no external trade and fixed technology. The optimal growth model involves utility function $U(C)$, consumption C, Cobb-Douglas production function $F(K,R,L)$, capital stock K, resource use R, labour L, resource stocks S, and capital depreciation $d(K)$. Resource extraction is costless. Given constant pure rate of time preference r (all other variables are implicitly functions of time), we wish to,

$$\max \int_0^\infty U(C)e^{-rt}dt \quad \text{such that:}$$

$$\dot{K} = F - C - d$$

$$\dot{S} = -R.$$

Here there are two state variables, K and S, and two control variables, C, and R. The current value Hamiltonian for this problem is,

$$H = U + \gamma_1(F - C - d) - \gamma_2 R$$

and the first order conditions for maximization are,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \quad \Rightarrow \quad \gamma_1 = U_C, \text{ and} \tag{2.1}$$

$$\frac{\partial H}{\partial R} = 0 = \gamma_1 F_R - \gamma_2 \quad \Rightarrow \quad \gamma_2 = U_C F_R. \tag{2.2}$$

It is the dynamic first order conditions that are of primary interest:

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} \quad \Rightarrow \quad \frac{\dot{U}_C}{U_C} = r - F_K + d_K, \text{ and} \tag{2.3}$$

$$\dot{\gamma}_2 = r\gamma_2 - \frac{\partial H}{\partial S} \quad \Rightarrow \quad \frac{\dot{\gamma}_2}{\gamma_2} = r. \tag{2.4}$$

Substituting (2.2) and (2.3) into (2.4) we get our analogue to the Hotelling rule,

$$\frac{\dot{F}_R}{F_R} = F_K - d_K. \tag{2.5}$$

The assumption of finite total resources implies that $R \to 0$ as $t \to \infty$. For Cobb-Douglas production technology, and assuming stable population and therefore employed labour, $\overline{L}$, we have

$$F = K^{\alpha} R^{\beta} \overline{L}^{\delta}, \quad \alpha + \beta + \delta = 1, \quad F_K = \alpha \overline{L}^{\delta} \frac{R^{\beta}}{K^{1-\alpha}}.$$

Now, $R \to 0$ implies that $\dot{F}_R > 0$ because $F_{RR} < 0$. As $R \to 0$ then either: (i) $K \to 0$ as well, in which case both production F and consumption C approach 0; or (ii) $F_K \to 0$, in which case, from expression (2.5), $\dot{F}_R < 0$, a contradiction. The latter contradiction arises if d_K is constant ($d(K) = \delta K$, exponential decay of capital) or decreases at a lower rate than F_K (there is no inherent reason for this to hold in the model). Development is not *sustainable* in this model, by Pezzey's (1989) definition, under most reasonable assumptions about the marginal rate of depreciation.

The elasticity of marginal utility of consumption is,

$$\eta(C) = -\frac{U_{CC}}{U_C} C,$$

so that expression (2.3) reduces to,

$$r + \eta(C)\frac{\dot{C}}{C} = F_K - d_K. \tag{2.6}$$

The preceding expression is the Ramsey (1928) rule: the social rate of return on investment must equal the return on capital. If there is no depreciation and a positive pure rate of time preference then, by the arguments presented for expression (2.5), either development is unsustainable because $K \to 0$ or it is unsustainable because eventually $F_K < r$ as $F_K \to 0$. This puts a 'sustainability' interpretation on this result from Dasgupta and Heal (1979, ch. 10).

If both the pure rate of time preference and depreciation of capital are zero then expression (2.6) implies that the optimal path is sustainable, under some non-intuitive restrictions on the integrability of the utility function that are derived in Dasgupta and Heal. Under these conditions resource rents will increase as the resource depletes, as governed by expression (2.5).

Both positive rates of pure time preference and depreciation of capital lead to optimal paths that are not sustainable. While this result is derived in the context of Cobb-Douglas production technology, it is clear that it should generalize to any technology where resources are essential and there are decreasing returns to their use.

If we reject the assumption of exponential decay of capital, although it is not obviously a bad assumption, is the alternative any more palatable? The alternative is declining marginal depreciation of capital, such that $d_K < F_K$ as $F_K \to 0$. First, this seems an arbitrary constraint on what is, at least partly, a physical process: the wearing out of buildings and machines. Secondly, it seems difficult to construct a plausible model of a process by which the marginal amount of depreciation *decreases* when the capital stock *increases*. One possibility would be if each additional unit of capital were more durable than its predecessor, which implies a specific form of technological change: asset lives would approach infinity in the limit rather than at the outset as in the standard model with zero depreciation.

Not only does depreciation of produced capital imply that the optimal path is not sustainable, it also implies, since the Hartwick rule relies on the Hotelling rule (expression (2.5)), that the Hartwick programme is infeasible as well.

If the pure rate of time preference is constant, produced assets depreciate and there are exhaustible natural resources that are essential for production, it seems clear that some form of technological change will be required if sustainability is to be possible. While this is the last time that we will discuss technological change or depreciation of capital in this study, this is a point that should be borne in mind in what follows.

3. Sustainability and the Elasticity of Substitution[1]

Concern about damage to the environment and depletion of resources has made sustainable economic development a concept with both wide currency and wide interpretation, as Pezzey's (1989) exposition demonstrates. Although various criticisms have been levelled at the notion of sustainable development (see, for instance, Nordhaus (1992a)), it is the goal of this chapter to explore a particularly simple definition, that per capita utility be non-declining, owing to Pezzey. Given that the sustainability criterion is, in effect, an ethical constraint on the classic economic problem of intertemporal optimization, the key question to be answered is whether, or under what conditions, sustainable development so defined is consistent with optimal growth and finite resources.

The problem to be examined in this chapter is that of finding a development path with maximal consumption that is minimally sustainable in the face of finite resources. Stated this way, it is clear that this is equivalent to a *maximin* programme, which has been widely studied in the literature. The starting ethical position in the preceding work was different, essentially a Rawlsian framework in which welfare across time is equal to that of the least well-off generation, but the end goal was the same: maximal constant consumption.

Solow (1974) proved the existence of a path with maximal constant consumption and finite resources, and this was elaborated in Dasgupta and Heal (1979, ch. 7). Both of these results required a Cobb-Douglas production function with the elasticity of output with respect to produced capital being greater than that of natural resources. The famous result in this literature is Hartwick (1977), who showed that the 'Hartwick rule', to invest resource rents, is a sufficient condition for a maximin programme for general production functions. Hartwick (1978) explored the Hartwick rule for several resources and raised unanswered questions about the existence of a maximin path for different values of the elasticity of substitution between capital and resources in a constant elasticity of substitution (CES) production function. Finally Dixit, Hammond and Hoel (hereafter DHH) (1980) showed in a very general framework that an extended Hartwick rule, in which capital accumulation equals unit resource rent times the quantity of resources used plus an arbitrary constant, was necessary and sufficient for the existence of a maximin path[2].

The substitution possibilities between capital and resources are clearly important in determining whether a maximin path exists. A substantial portion of this chapter therefore deals with characterizing the behaviour of both the standard and generalized Hartwick rules under varying values of the elasticity of substitution in a CES production function; this has not been resolved in the literature to date. The first section begins with a simple proof, for general production functions, that the generalized Hartwick rule combined with the Hotelling rule is necessary and sufficient for maximal constant consumption. It ends by tying together the various approaches to the maximin problem in the literature, including the derivation of a weaker condition for maximal consumption under the generalized Hartwick rule than that in DHH (1980) for CES production functions.

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[1] This chapter was originally published in Hamilton (1995).

[2] Dasgupta and Mitra (1983) showed that in a discrete time formulation constant maximal consumption requires that investment be less than resource rents. However, the standard Hartwick result is approached asymptotically as the time step nears 0.

The behaviour of a Hartwick-Hotelling programme under varying values of the elasticity of substitution, σ, is related to the debate concerning 'strong' versus 'weak' sustainability (Pearce *et al.* 1989). The proponents of weak sustainability argue that capital and resources are substitutable, and so identify sustainable development with maintaining total assets (produced capital and natural resources) constant or increasing. The strong sustainability position is that there is a critical quantity of at least some natural resources that must be maintained intact if utility is not to decline in the future - in the limit this would imply a zero elasticity of substitution for these resources. Without doing too much violence to the basic elements of this debate, it is possible to equate 'weaker' sustainability with substitution possibilities that are elastic ($\sigma > 1$), and 'stronger' sustainability with inelastic substitution possibilities ($\sigma < 1$). The result derived below, that constant consumption is not attainable for $\sigma < 1$, therefore implies that the Hartwick rule will not yield constant consumption if you subscribe to the 'stronger' sustainability position.

Maximal Constant Consumption Paths

While this section is primarily concerned with the behaviour of the Hartwick rule under different assumptions about the elasticity of substitution of capital and resources, the starting point is a simple proof that a generalized Hartwick rule, to use the terminology of DHH (1980), combined with the Hotelling rule, is necessary and sufficient for constant consumption.

We assume that there is constant population (so that labour can be treated implicitly in the production function), and no disembodied technological growth. The initial endowment is a stock S_0 of resources and K_0 of capital. Output F is produced from capital K and resources R according to the production function,

$F = F(K,R)$ such that $F_K,\ F_R > 0,\ \ F_{KK},\ F_{RR} < 0.$

Consumption C is defined by the following set of differential equations:

$$\begin{aligned} C &= F - \dot{K} \\ \dot{S} &= -R. \end{aligned} \tag{3.1}$$

Any efficient programme of production, investment and consumption must satisfy two criteria:

$$\frac{\dot{F}_R}{F_R} = F_K, \tag{3.2}$$

and

$$\lim S_t = 0 \text{ as } t \rightarrow \infty. \tag{3.3}$$

The first of these is the familiar Hotelling rule; in the economy postulated, holders of natural resource stocks must be indifferent between holding resources or the alternative asset, capital, which yields F_K. Any programme that left unexploited natural resources would clearly be inefficient, hence expression (3.3) which says that the programme must exhaust the initial resource stock.

The generalized Hartwick rule is given by:

$$\dot{K} = F_R(R + v), \quad v \text{ constant,} \tag{3.4}$$

where the return to resources, F_R, is the resource rental rate.

Sufficiency of the generalized Hartwick-Hotelling programme for constant consumption is proved as follows. Applying expressions (3.2) and (3.4) we have:

$$\begin{aligned}
\dot{C} &= \frac{d}{dt}(F - \dot{K}) \\
&= \frac{d}{dt}(F - F_R(R + v)) \\
&= \dot{F} - \dot{F}_R(R + v) - F_R\dot{R} \\
&= \dot{F} - F_K F_R(R + v) - F_R\dot{R} \\
&= \dot{F} - F_K\dot{K} - F_R\dot{R} \\
&= 0.
\end{aligned}$$

Necessity of the programme for constant consumption is shown by assuming $\dot{C} = 0$. We have,

$$\begin{aligned}
\ddot{K} &= \dot{F} - \dot{C} \\
&= F_R\dot{R} + F_K\dot{K} \\
&= F_R\dot{R} + \frac{\dot{F}_R}{F_R}\dot{K} \\
&= F_R\dot{R} + \dot{F}_R R + \frac{\dot{F}_R}{F_R}(\dot{K} - F_R R).
\end{aligned}$$

Now define $Z = \dot{K} - F_R R$, so that $\dot{Z} = \ddot{K} - F_R\dot{R} - \dot{F}_R R$, and therefore the preceding expression for $\ddot{K}$ can be written as $\dot{Z} = \frac{\dot{F}_R}{F_R}Z$. This equation has solution $Z = vF_R$ for constant v, and therefore $\dot{K} = F_R(R + v)$.

Having established that the Hotelling rule and the generalized Hartwick rule are together necessary and sufficient for constant consumption, the next question to be examined is the behaviour of the system under different assumptions about v. For $v \neq 0$ we will explore the generalized Hartwick rule, while the case $v = 0$ will be explored in a sub-section on the standard Hartwick rule.

The Generalized Hartwick Rule

We wish to derive the path for output and consumption under the generalized Hartwick rule. It is clear from the foregoing derivation that the parameter v is simply a constant of integration - it has no obvious economic interpretation, and it would be disturbing if constant consumption were feasible for any programme that added an arbitrary amount to the quantity of resource extracted.

The first case to be considered is $v < 0$. The efficiency condition that $S \rightarrow 0$ implies that $R \rightarrow 0$. If $v < 0$, eventually $\dot{K} < 0$, and therefore, assuming capital can be consumed and given the fixed initial endowment K_0, both R and K will tend to 0; assuming that no output is produced purely by labour, constant consumption is impossible.

To take the argument further requires more structure for the production function. Since it is clearly the degree of substitutability between capital and resources that is of key importance in models with exhaustible resources, the important functional form to consider is the class of constant elasticity of substitution (CES) production functions. Defining, as before, σ to be the elasticity of substitution, we have, assuming constant labour force and normalizing per unit of labour,

$$F = (\alpha K^{\frac{\sigma-1}{\sigma}} + \beta R^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta)^{\frac{\sigma}{\sigma-1}}, \quad \alpha, \beta > 0, \ \alpha + \beta < 1, \ \sigma > 0, \ \sigma \neq 1.$$

Note that F, K, and R are all functions of time, while α, β and σ are fixed parameters. For the case $\sigma = 1$ this reduces to the familiar Cobb-Douglas form.

It will be convenient in what follows to define the following expressions:

$$X = (\alpha K^{\frac{\sigma-1}{\sigma}} + \beta R^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta), \tag{3.5}$$

and

$$\gamma = \frac{\beta R^{\frac{\sigma-1}{\sigma}}}{X} < 1. \tag{3.6}$$

With these definitions we can derive,

$$F_R = \beta X^{\frac{1}{\sigma-1}} R^{-\frac{1}{\sigma}} = \beta \frac{F}{X} R^{-\frac{1}{\sigma}} = \beta \left(\frac{F}{R}\right)^{\frac{1}{\sigma}}.$$

The examination of the behaviour of the system for $v > 0$ will be divided into three parts, according to the assumptions about the elasticity of substitution. For the Cobb-Douglas function ($\sigma = 1$) we have,

$$\begin{aligned} C &= F - F_R R - F_R v \\ &= F\left(1 - \beta - \frac{\beta v}{R}\right). \end{aligned}$$

Therefore C becomes negative as $R \to 0$, contradicting $\dot{C} = 0$.

If $\sigma < 1$ then,

$$\begin{aligned} C &= F - F_R R - F_R v \\ &= F - \gamma F - v\beta \frac{F}{X} R^{-\frac{1}{\sigma}} \\ &= F\left(1 - \gamma - \frac{v\beta}{\alpha K^{\frac{\sigma-1}{\sigma}} R^{\frac{1}{\sigma}} + \beta R + (1 - \alpha - \beta) R^{\frac{1}{\sigma}}}\right). \end{aligned}$$

Again, C becomes negative as $R \to 0$ (note that $\gamma \to 1^-$), contradicting $\dot{C} = 0$.

If $\sigma > 1$ then resources are not essential for production. For large σ, i.e. as $\sigma \to \infty$, $F_R \to \beta$ and $F_K \to \alpha$, so that the Hotelling rule is violated. Efficient production is therefore impossible when capital and resources are perfect substitutes. As derived in the Appendix to this chapter, the rate of change of output for general σ can be shown to be given by:

$$\dot{F} = \dot{K}\frac{\dot{R}}{R + v}\left(\frac{R(\sigma - 1) - v}{R(\sigma - \gamma) - \gamma v}\right). \tag{3.7}$$

This expression reduces to the following for the case $v = 0$:

$$\dot{F} = \dot{K}\frac{\dot{R}}{R}\frac{(\sigma - 1)}{(\sigma - \gamma)}. \tag{3.8}$$

As $\sigma \to 1^+$, $\gamma \to \beta$ and, from expression (3.7),

$$\dot{F} \to \dot{K}\frac{\dot{R}}{R + v}\left(\frac{-v}{R(1 - \beta) - \beta v}\right).$$

We distinguish two cases according to the initial conditions in the preceding expression. If in the initial period $R_0(1 - \beta) - \beta v > 0$, then for some time beyond this period,

$$R = \frac{\beta}{1 - \beta} v,$$

at which point output is infinite (since the growth rate is positive and infinite - recall that $\dot{R} < 0$ because of efficiency condition (3.3)). The programme is not feasible.

Alternatively, if $R_0(1-\beta)-\beta v < 0$ in the initial period, then $\dot{F} < 0$. Note that

$$\frac{F}{R} = \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta R^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta\right)^{\frac{\sigma}{\sigma-1}} R^{-1}$$

$$= \left(\alpha\left(\frac{K}{R}\right)^{\frac{\sigma-1}{\sigma}} + \beta + \frac{1-\alpha-\beta}{R^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{\sigma}{\sigma-1}}$$

is unbounded as $R \to 0$. Consumption is given by,

$$C = F - F_R(R+v)$$

$$= (1-\gamma)F - \beta v\left(\frac{F}{R}\right)^{\frac{1}{\sigma}}.$$

Therefore if $R_0(1-\beta)-\beta v < 0$, consumption becomes negative as $R \to 0$.

These results for the generalized Hartwick rule are summarized in Table 3.1. Any non-zero choice of v, roughly speaking, leads to declining consumption, infinite output or a violation of the Hotelling rule, depending on the value of the elasticity of substitution σ.

Table 3.1 Results for the Generalized Hartwick Rule

$$\dot{K} = F_R(R+v)$$

	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$ $\sigma \approx 1$	$\sigma \to \infty$
$v < 0$	$F \to 0$ $C \to 0$ as $t \to \infty$	$F \to 0$ $C \to 0$ as $t \to \infty$	$F \to 0$ $C \to 0$ as $t \to \infty$	Hotelling Rule violated
$v > 0$	$C_T < 0$ for $T < \infty$	$C_T < 0$ for $T < \infty$	Either $\dot{F}(K_T, R_T) = \infty$ for $T < \infty$ or $C_T < 0$ for $T < \infty$	Hotelling Rule violated

The Standard Hartwick Rule

Having shown the generalized Hartwick rule to be infeasible, at least for CES production functions, in what follows we employ the widely known form of the Hartwick rule,

$$\dot{K} = F_R R \tag{3.9}$$

i.e., that investment equal resource rents. Now the question to be explored is the behaviour of output, consumption and investment, under the standard Hartwick-Hotelling programme, for different values of the elasticity of substitution. We proceed by considering three cases, according to whether this elasticity is less than, equal to, or greater than 1.

We first consider the case $\sigma < 1$. In this instance the marginal product of resources is bounded since,

$$\begin{aligned} F_R &= \beta\left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta R^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta\right)^{\frac{1}{\sigma-1}} R^{-\frac{1}{\sigma}} \\ &= \beta\left(\alpha\left(\frac{K}{R}\right)^{\frac{\sigma-1}{\sigma}} + \beta + \frac{1-\alpha-\beta}{R^{\frac{\sigma-1}{\sigma}}}\right)^{\frac{1}{\sigma-1}}. \end{aligned} \tag{3.10}$$

Thus, since K is non-decreasing under the Hartwick rule,

$$F_R \rightarrow \beta^{\frac{\sigma}{\sigma-1}} \text{ as } R \rightarrow 0.$$

This in turn implies that K is bounded because

$$K_T = K_0 + \int_0^T F_R R\, dt \text{ and } \int_0^\infty R\, dt = S_0 .$$

Consumption is given by,

$$\begin{aligned} C &= F - F_R R \\ &= X^{\frac{\sigma}{\sigma-1}} - \beta X^{\frac{1}{\sigma-1}} R^{-\frac{1}{\sigma}} R \\ &= X^{\frac{1}{\sigma-1}}(X - \beta R^{\frac{\sigma-1}{\sigma}}) \\ &= \left(\alpha K^{\frac{\sigma-1}{\sigma}} + \beta R^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta\right)^{\frac{1}{\sigma-1}} (\alpha K^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta). \end{aligned} \tag{3.11}$$

Therefore, since K is bounded, consumption tends to 0 as R tends to 0 when $\sigma < 1$.

This derivation can be compared with Dasgupta and Heal (1979, ch. 7), who show that if the elasticity of substitution is less than 1, then F / R is bounded, implying total output is bounded, and therefore that constant consumption is impossible.

The preceding derivation and the result from Dasgupta and Heal (1979) contradict Theorem 3 from Hartwick (1978). One component of this theorem implies that $\dot{F} > 0$ iff $\sigma < 1$. Recalling expression (3.8), note that $\gamma \to 1^-$ as $R \to 0$, so that, while F may initially be increasing (i e. for $\gamma < \sigma < 1$), eventually it must decrease. In fact, since K is bounded, eventually $\dot{K} \to 0$, so that $\dot{F} \to 0^-$, contradicting the first part of the above theorem. Because total output is bounded, we know that $F \to 0$ in the long run.

Next we consider the case $\sigma > 1$. Since resources are not essential in this instance, both Solow (1974) and Dasgupta and Heal (1979) dismiss this case. The behaviour of the Hartwick-Hotelling programme under these conditions needs to be clarified.

Since $\sigma > 1$ and $\gamma < 1$, expression (3.8) implies that $\dot{F} < 0$. Because consumption is constant, this in turn implies that $F_R R \to 0$ as $R \to 0$, since

$$F \to \left(\alpha K^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta\right)^{\frac{\sigma}{\sigma-1}} \quad \text{as } R \to 0.$$

A further conclusion from the preceding expression is that K must be bounded since $\dot{F} < 0$. This is in spite of F_R being unbounded as $R \to 0$, as is obvious from expression (3.10).

How does consumption vary with σ? Because $F \to \alpha K + \beta R + 1 - \alpha - \beta$ as $\sigma \to \infty$, the Hotelling rule is violated as capital and resources become perfect substitutes. For finite values of the elasticity of substitution we have, following from expression (3.11):

$$\begin{aligned}\frac{\partial C}{\partial \sigma} &= X^{\frac{1}{\sigma-1}} \ln(X) \frac{(-1)}{(\sigma-1)^2}\left(\alpha K^{\frac{\sigma-1}{\sigma}} + 1 - \alpha - \beta\right) + X^{\frac{\sigma-1}{\sigma}} \alpha K^{\frac{\sigma-1}{\sigma}} \ln(K) \frac{1}{\sigma^2} \\ &= X^{\frac{1}{\sigma-1}}\left(\alpha K^{\frac{\sigma-1}{\sigma}}\left(\frac{\ln(K)}{\sigma^2} - \frac{\ln(X)}{(\sigma-1)^2}\right) - (1-\alpha-\beta)\frac{\ln(X)}{(\sigma-1)^2}\right).\end{aligned}$$

The critical issue is therefore the behaviour of $\dfrac{\ln(X)}{(\sigma-1)^2}$.

Using l'Hôpital's rules, we take the derivatives of numerator and denominator with respect to σ,

$$\frac{\frac{\partial \ln(X)}{\partial \sigma}}{2(\sigma-1)} = \frac{\frac{\partial X}{\partial \sigma}}{2X(\sigma-1)}. \text{ Now } \frac{\partial X}{\partial \sigma} = \frac{\alpha K^{\frac{\sigma-1}{\sigma}} \ln(K)}{\sigma^2} + \frac{\beta R^{\frac{\sigma-1}{\sigma}} \ln(R)}{\sigma^2} \to \ln\left(K^{\alpha} R^{\beta}\right) \text{ as } \sigma \to 1^+.$$

Consumption is therefore a declining function of $\sigma > 1$. Constant consumption under the Hartwick rule consequently increases as $\sigma \rightarrow 1^{+}$. As Solow (1974) and Dasgupta and Heal (1979) show, the Cobb-Douglas production function yields maximal consumption.

Because resources are not essential for elasticities of substitution greater than 1, one strategy for achieving maximal consumption might be to consume all of the resource in the initial period. The derivation in the Appendix to this chapter shows, however, that such a strategy will not yield constant consumption under the Hartwick rule.

The operation of the standard Hartwick-Hotelling programme, where investment is precisely equal to current resource rents, is summarized for CES production functions in Table 3.2. Only the Cobb-Douglas production function, for which the elasticity of substitution is equal to 1, yields minimal sustainability at the maximum rate of consumption.

Table 3.2 Results for the Standard Hartwick Rule

$$\dot{K} = F_R R$$

$\sigma < 1$	$\sigma = 1$	$\sigma > 1$ $\sigma \approx 1$	$\sigma \rightarrow \infty$
$F \rightarrow 0$ $C \rightarrow 0$ $as\ t \rightarrow \infty$	$\dot{F} = 0$ $\dot{C} = 0$ *C is maximal*	$\dot{F} < 0$ $\dot{C} = 0$ $\partial C / \partial \sigma < 0$	Hotelling Rule violated

Links to the Literature

As noted in the introduction to this chapter, the literature on the Hartwick rule has several strands to it. This section identifies the common threads in the literature and provides a generalization of one result in DHH (1980).

The solution of the system for a Hartwick-Hotelling programme is remarkably simple in the Cobb-Douglas case, for constant consumption C_0:

$$F = K^{\alpha} R^{\beta}, \quad \alpha, \beta > 0, \quad \alpha + \beta < 1,$$
$$\dot{K} = F_R R = \beta F,$$

implying,

$$C_0 = (1-\beta)F \text{ constant } \Rightarrow F \text{ constant } \Rightarrow \frac{\dot{R}}{R} = -F_K \text{ from Appendix (A3.1),}$$

so that $K = K_0 + \frac{\beta C_0}{1-\beta}t$, and therefore, $R = \left(\frac{C_0}{1-\beta}\right)^{\frac{1}{\beta}}\left(K_0 + \frac{\beta C_0}{1-\beta}t\right)^{-\frac{\alpha}{\beta}}$.

The condition for the *existence* of a solution to the system is therefore $\alpha > \beta$, i.e. the elasticity of output with respect to capital must be greater than that with respect to resources, since R must have a finite integral equalling S_0. Performing the integration yields the value for maximal consumption,

$$C_0 = (1-\beta)(\alpha-\beta)^{\frac{\beta}{1-\beta}} S_0^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}}. \tag{3.12}$$

To tie the literature together, it is worth describing the solutions of Solow (1974) and Dasgupta and Heal (1979) to the maximin problem, which did not use the Hartwick rule explicitly. Both choose the Cobb-Douglas production function after rejecting CES functions where total output is bounded ($\sigma < 1$) and where resources are not essential ($\sigma > 1$). For this production function maintaining constant consumption C_0 implies that,

$$\begin{aligned} K &= K_0 + mt \\ R &= (C_0 + m)^{\frac{1}{\beta}}(K_0 + mt)^{-\frac{\alpha}{\beta}} \end{aligned} \tag{3.13}$$

where $m = \dot{K}$ is a constant. Both point out that efficiency requires that the integral of the above expression for R exist and be equal to S_0, so that the condition $\alpha > \beta$ is required. Performing this integration yields:

$$C_0 = m^{\beta}\left(\frac{\alpha-\beta}{\beta}S_0\right)^{\beta} K_0^{\alpha-\beta} - m.$$

Dasgupta and Heal then maximize this expression with respect to m to yield the optimal constant consumption,

$$C_0 = \left(\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right)\left(\frac{\alpha-\beta}{\beta}\right)^{\frac{\beta}{1-\beta}} S_0^{\frac{\beta}{1-\beta}} K_0^{\frac{\alpha-\beta}{1-\beta}},$$

which simplifies to expression (3.12).

Solow (1974) takes a different approach to deriving the optimum. By constructing phase diagrams for the problem, the following expression for the parameter m in the system of equations (3.13) is arrived at directly,

$$m = \dot{K} = \frac{\beta C_0}{1-\beta}, \tag{3.14}$$

for some fixed C_0. The equation for R in expression (3.13) is then integrated, and set equal to S_0 to arrive at the maximal level of consumption, as given by expression (3.12).

Hartwick's (1977) key insight, that expression (3.14) embodies the rule "invest resource rents", is not derived explicitly in the paper. Instead the sufficiency of this rule for a programme of constant consumption is proved for a general production function.

It is also worth linking what has been presented so far to the much more general framework employed by DHH (1980)[1]. If we assume the existence of a competitive output price path p and a positive constant exhaustion rent μ (i.e. assuming efficient resource extraction), then expression (3.4) may be re-written as,

$$\begin{aligned} p\dot{K} &= pF_R(R+v) \\ &= \mu(R+v) \\ &= -\mu\dot{S}+\mu v, \end{aligned}$$

where μv is constant. Rearranging terms then gives the analog of the 'generalized Hartwick rule' of DHH:

$$p\dot{K}+\mu\dot{S} = \mu v.$$

The authors prove that this rule is necessary and sufficient for constant utility in their general framework. They go on to show that any path with $v<0$ is infeasible.

A point of particular interest in the Dixit, Hammond and Hoel paper is their proof that, given any efficient path such that $v=0$, any other path with $v>0$ and a larger capital stock for all $t>0$ will yield a lower level of utility under the generalized Hartwick rule. Expressing their proposition in terms of the CES production function analysis presented so far, we can say that

$$F_{R_0'}(R_0'+v) > F_{R_0}R_0 \quad \text{for } v>0,$$

is a necessary condition for $C_0' < C_0$, where the 'unprimed' variables represent their values for $v=0$ (this is a necessary condition because the capital stock can only be greater than its previous value for all $t>0$ if its rate of change in the initial period is greater).

Changes in v will clearly affect R_0, because the capital stock level will be altered under the generalized Hartwick rule, and the efficient path must both exhaust the resource and satisfy the Hotelling rule. For infinitesimal changes in v it is shown in the Appendix to this chapter that the DHH condition for maximal utility is equivalent to

$$\frac{\partial R_0}{\partial v} < \frac{\sigma}{1-\gamma-\sigma}, \tag{3.15}$$

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[1] I am grateful to David Ulph for pointing out this connection.

while a more direct derivation of this condition, using the machinery of CES production functions presented so far, yields

$$\frac{\partial R_0}{\partial v} < \frac{\sigma}{1-\gamma}. \tag{3.16}$$

Recalling that the case $\sigma > 1$ was the most problematic in terms of analyzing its behaviour with respect to changes in v, and noting that $\gamma < 1$, we can conclude that the DHH condition is stronger than necessary for this case, since expression (3.15) is negative and expression (3.16) is positive. At least for infinitesimal changes in v the weaker condition (3.16) yields maximal consumption under the Hartwick-Hotelling programme when the elasticity of substitution is greater than 1.

Conclusions

If for reasons of intergenerational equity the desired goal is minimal sustainability with maximum consumption, then the Hartwick rule, to invest resource rents, is the keystone. Solow (1986) refers to it as a "rule of thumb" for growth policy.

This study has drawn together several strands from a diverse literature on the Hartwick rule. The analysis has shown that, given virtually unrestricted production functions, the generalized Hartwick rule in combination with the Hotelling rule is both necessary and sufficient for consumption to be constant. The Cobb-Douglas production function (out of the class of CES production functions), in which the elasticity of substitution between capital and resources is exactly 1, yields consumption that is constant, positive and maximal when a standard Hartwick programme is followed in combination with two efficiency conditions, the Hotelling rule and complete resource exhaustion.

The generalized Hartwick rule, $\dot{K} = F_R(R + v)$ for non-zero v, yields either declining consumption or infinite output for finite values of the elasticity of substitution. The standard Hartwick rule, where $v = 0$, yields either declining consumption or consumption less than that obtained under a Cobb-Douglas production function for finite elasticities of substitution that are not equal to 1. The derivations in this chapter therefore emphasize the "knife edge" role of the Cobb-Douglas production function. Although the generalized Hartwick rule promises constant consumption for general production functions, the requirement that a maximal constant consumption path *exist* places severe limits on the substitution possibilities inherent in the production function.

Appendix to Chapter 3

We begin with a basic result for the rate of change of output under the generalized Hartwick rule:

$$\begin{aligned}\dot{F} &= \ddot{K}+\dot{C} = \dot{F}_R(R+v)+F_R\dot{R} \\ &= \dot{K}\left(F_K+\frac{\dot{R}}{R+v}\right) \\ &= \dot{K}\left(\frac{\dot{F}_R}{F_R}+\frac{\dot{R}}{R+v}\right).\end{aligned} \tag{A3.1}$$

Recalling the definitions of X (expression (5)) and γ (expression (6)), a few results follow directly from the definition of the CES production function:

$$F_R = \beta X^{\frac{1}{\sigma-1}} R^{-\frac{1}{\sigma}} = \beta\frac{F}{X}R^{-\frac{1}{\sigma}} = \beta\left(\frac{F}{R}\right)^{\frac{1}{\sigma}} \tag{A3.2}$$

$$F_{RK} = \frac{\alpha\beta}{\sigma}X^{\left(\frac{1}{\sigma-1}-1\right)}R^{-\frac{1}{\sigma}}K^{-\frac{1}{\sigma}} = \frac{1}{\sigma}\frac{\beta R^{-\frac{1}{\sigma}}}{X}F_K = \frac{1}{\sigma}\frac{\gamma}{R}F_K \tag{A3.3}$$

$$F_{RR} = \frac{1}{\sigma}\frac{F_R}{R}(\gamma-1) < 0. \tag{A3.4}$$

Note as well that $\gamma < 1$ and that $\gamma \rightarrow \beta$ as $\sigma \rightarrow 1$.

The first item to be derived is the expression for $\dot{F}$. We begin with $\dot{F}_R = F_{RR}\dot{R}+F_{RK}\dot{K}$. Therefore,

$$\begin{aligned}\frac{\dot{F}_R}{F_R} &= \frac{1}{\sigma}\frac{\dot{R}}{R}(\gamma-1)+F_{RK}(R+v) \\ &= \frac{1}{\sigma}\frac{\dot{R}}{R}(\gamma-1)+\frac{1}{\sigma}\frac{\gamma}{R}(R+v)F_K \\ &= \frac{1}{\sigma}\frac{\dot{R}}{R}(\gamma-1)+\frac{1}{\sigma}\frac{\gamma}{R}(R+v)\frac{\dot{F}_R}{F_R},\end{aligned}$$

so that,

$$\frac{\dot{F}_R}{F_R} = \left(\frac{\dot{R}}{R+v}\right)\frac{(\gamma-1)(R+v)}{R\sigma-\gamma(R+v)}.$$

From expression (A3.1) we therefore derive,

$$\begin{aligned}\dot{F} &= \dot{K}\frac{\dot{R}}{R+v}\left(\frac{(\gamma-1)(R+v)}{R\sigma-\gamma(R+v)}+1\right)\\ &= \dot{K}\frac{\dot{R}}{R+v}\left(\frac{R(\sigma-1)-v}{R(\sigma-\gamma)-\gamma v}\right)\end{aligned}$$

and thus,

$$\dot{F} \rightarrow \dot{K}\frac{\dot{R}}{R+v}\left(\frac{-v}{R(1-\beta)-\beta v}\right) \quad \text{as} \quad \sigma \rightarrow 1^{+}.$$

The next issue to be considered is the behaviour of the Hartwick-Hotelling system when $\sigma > 1$ and, since resources are not essential for this value of the elasticity of substitution, when all of the resource is extracted in the base period. $\dot{C} = 0$ if and only if, given $F = F(K,R)$,

$$F(K_0+F_R(K_0,S_0),0) = F(K_0,S_0)-F_R(K_0,S_0)S_0. \tag{A3.5}$$

It will simplify the algebra considerably, without unduly affecting the generality of the argument, if we assume for this derivation that $\alpha+\beta=1$, so that

$$X = \alpha K^{\frac{\sigma-1}{\sigma}}+\beta R^{\frac{\sigma-1}{\sigma}}.$$

Expression (A3.5) may now be re-written as,

$$\left(\alpha\left(K_0+\beta X^{\frac{1}{\sigma-1}}S_0^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = X^{\frac{1}{\sigma-1}}\left(X-\beta S_0^{\frac{\sigma-1}{\sigma}}\right),$$

which implies that,

$$\alpha^{\frac{\sigma}{\sigma-1}}\left(K_0+\beta X^{\frac{1}{\sigma-1}}S_0^{\frac{\sigma-1}{\sigma}}\right) = X^{\frac{1}{\sigma-1}}\left(\alpha K_0^{\frac{\sigma-1}{\sigma}}\right),$$

and therefore that,

$$\alpha^{\frac{\sigma}{\sigma-1}} K_0 = X^{\frac{1}{\sigma-1}}\left(\alpha K_0^{\frac{\sigma-1}{\sigma}} - \alpha^{\frac{\sigma}{\sigma-1}}\beta S_0^{\frac{\sigma-1}{\sigma}}\right).$$

For consumption to be constant this latter expression should be an identity. However, there are clearly choices of K_0 and S_0 for which the right hand side is less than or equal to 0, so the identity does not hold.

Finally, we consider how the level of consumption varies with v in the generalized Hartwick rule. The analysis will be based on infinitesimal positive changes dv, to examine the transition from the standard to the generalized Hartwick rule - negative values have already been ruled out because they lead to declining consumption. In order for the capital stock to be greater for all time (after the initial period) in the transition to the generalized rule, as DHH (1980) hypothesize, a necessary condition is that $\dot{K}$ be greater under the generalized rule. This may be written as,

$$C_0' < C_0 \text{ if } dv > 0 \text{ and } F_{R_0'}(R_0' + dv) > F_{R_0}R_0.$$

However, for infinitesimal dv we may write,

$$F_{R_0'} = F_{R_0} + \frac{\partial F_{R_0}}{\partial v}dv, \text{ and } R_0' = R_0 + \frac{\partial R_0}{\partial v}dv.$$

Therefore,

$$\begin{aligned} F_{R_0'}(R_0' + dv) &= \left(F_{R_0} + \frac{\partial F_{R_0}}{\partial v}dv\right)\left(R_0 + \left(\frac{\partial R_0}{\partial v} + 1\right)dv\right) \\ &= F_{R_0}R_0 + \frac{\partial F_{R_0}}{\partial v}R_0 dv + F_{R_0}\left(\frac{\partial R_0}{\partial v} + 1\right)dv, \end{aligned}$$

where terms in dv^2 have been dropped because they will go to zero in the limit. The question is therefore reduced to whether the sum of the second two terms in the preceding expression is greater than 0. Note that, since we are dealing with the initial period, K_0 is independent of v, and therefore the relationship we wish to test for these terms may be written as,

$$F_{RR_0}\frac{\partial R_0}{\partial v}R_0 dv + F_{R_0}\left(\frac{\partial R_0}{\partial v} + 1\right)dv > 0.$$

Recalling expression (A3.4), this may be written for CES production functions as,

$$\frac{\gamma - 1}{\sigma}\frac{\partial R_0}{\partial v}dv + \left(\frac{\partial R_0}{\partial v} + 1\right)dv > 0,$$

so that, after dividing by dv, the Dixit, Hammond and Hoel condition reduces to

$$\frac{\partial R_0}{\partial v} < \frac{\sigma}{1-\gamma-\sigma}. \qquad \text{(A3.6)}$$

Because K_0 is given and independent of v, both C_0 and R_0 are functions of v under the generalized Hartwick rule. A more direct attack on this problem is therefore to evaluate

$$\begin{aligned}\frac{\partial C_0}{\partial v} &= \frac{\partial}{\partial v}\left(F_0 - F_{R_0}\left(R_0 + dv\right)\right)\\ &= -F_{RR_0}\frac{\partial R_0}{\partial v}\left(R_0 + dv\right) - F_{R_0}.\end{aligned}$$

In order for C_0 to vary negatively with v we require the latter expression to be less than 0, or, again employing expression (A3.4) and dividing by F_{R_0},

$$-\frac{\gamma-1}{\sigma}\frac{1}{R_0}\frac{\partial R_0}{\partial v}(R_0 + dv) < 1.$$

Taking the limit as dv tends to 0, this reduces to the condition

$$\frac{\partial R_0}{\partial v} < \frac{\sigma}{1-\gamma}. \qquad \text{(A3.7)}$$

For the case $\sigma > 1$, expression (A3.7) is less restrictive than (A3.6) because its right-hand side is positive, while that of (A3.6) is negative.

4. The Hartwick Rule in a Greenhouse World[1]

Different times, different problems. The origins of the Hartwick rule (Hartwick (1977)) lie in the 'energy crisis' of the early 1970's, when the depletion of resources seemed to present limits to economic development. This chapter looks at a problem for the 1990's: how economic well-being can be maintained when an exhaustible resource, fossil fuel, is also the direct source of a pollutant that reduces welfare. This, in essence, is the CO_2 problem.

There is a consensus in the scientific community that continued emissions of carbon dioxide will lead to greenhouse warming of the earth's atmosphere (IPCC (1992)). Economic approaches to the problem, attempting to measure the costs versus the benefits of limiting CO_2 emissions, include Nordhaus (1991, 1992b) and Cline (1992), with carbon taxes being the favoured means of achieving abatement of emissions. Dean and Hoeller (1992) summarize several studies of the level of carbon tax required to cap emission levels. Ulph and Ulph (1994) are interested in the time path of a carbon tax, rather than simply the level, while taking account of the exhaustibility of fossil fuels. Fankhauser (1994) estimates the level of global social costs associated with carbon emissions using a stochastic greenhouse damage model.

Given the current concern about the sustainability of economic development, the Hartwick rule occupies a central position: if resource rents are invested in produced capital, then constant consumption is possible even if resources are both exhaustible and essential for production. We wish to derive the analogue of the Hartwick rule when resource use leads to greenhouse warming. The standard rule has been widely studied, including Hartwick (1977, 1978), Dixit, Hammond and Hoel (1980), Dasgupta and Mitra (1983), as well as in Chapter 3.

Recognizing the greenhouse effect associated with fossil fuel consumption extends the standard analysis in three ways: (i) the usual efficiency condition, the Hotelling (1931) rule, has to be augmented by another describing the optimal time path of a carbon tax; (ii) keeping consumption constant will no longer keep utility constant, so sustainability has to be defined as constant utility (in keeping with Pezzey (1989)); and (iii) the notion of keeping total capital intact, which is Solow's (1986) interpretation of the standard Hartwick rule, has to be extended to include not only the depletion of the exhaustible resource but the changes in the stock of CO_2 as well.

We begin with the derivation of the efficiency conditions for the exploitation of fossil fuels when account is taken of the accumulation of carbon dioxide in the atmosphere.

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[1] This chapter is based on a paper of the same name co-written with David Ulph of University College London - see Hamilton and Ulph (1994).

Efficiency Conditions

We wish to derive the dynamics of optimal growth with exhaustible fossil fuel resources and carbon dioxide emissions linked to fuel use. To limit the problem to its essentials, several simplifying assumptions are made: (i) population is constant, so that labour can be factored out of the calculations; (ii) technology is assumed to be unchanging; (iii) the production function F is Cobb-Douglas and uses inputs of capital K and resources R; (iv) the discount rate r is constant; and (v) resources are costly to extract, as expressed by function $f(R)$. U refers to the utility function, and C consumption; utility is assumed to be an increasing function of both consumption and the flow of environmental services B, measured in appropriate (but not necessarily monetary) units. The choice of a Cobb-Douglas production function is not merely for convenience: Chapter 3 showed that it is precisely the unitary elasticity of substitution between capital and resources characterizing this function that leads to the existence of a maximal constant consumption path under the Hartwick rule.

Uncontrolled CO_2 emissions are assumed to be stoichiometrically related to the quantity of resource consumed, an accurate assumption for carbon-based fossil fuels. Actual emissions are assumed to be controlled (i.e. reduced) as a result of abatement expenditures a, and to be described by an emission function $e(R,a)$ such that $e_R > 0$ and $e_a < 0$.

The model of CO_2 accumulation is a simplified version of that of Nordhaus (1991). In that paper M represents the ratio of carbon dioxide concentration in the atmosphere to the assumed concentration in pre-industrial times, E emissions, β the marginal atmospheric retention ratio, and δ_M the transfer rate to the deep ocean, to yield the following relationship:

$$\dot{M} = \beta E - \delta_M M.$$

In the Nordhaus model emissions increase the atmospheric concentration ratio while mixing with the deep oceans reduces it. δ_M is equal to 1 over the mean atmospheric retention time, 120 years; since this parameter is constant, reducing emissions to zero would reduce the atmospheric concentration ratio to zero in the long run, which lacks plausibility since the natural carbon cycle will eventually lead to an equilibrium concentration.

The model we wish to explore for carbon dioxide accumulation involves a simple stock-flow relationship. We assume M to be the stock of CO_2 in the atmosphere, rather than a concentration ratio as in Nordhaus (1991). Emissions e are assumed to be stoichiometrically related to the quantity of resource consumed, as above. The stock of CO_2 dissipates, through natural processes such as deep ocean mixing, as defined by function $d(M)$ - this formulation allows for the possibility of the stock stabilizing, with d approaching 0 as M approaches M_0, the pre-industrial stock. The stock-flow model is therefore,

$$\dot{M} = e(R,a) - d(M). \tag{4.1}$$

We assume that the natural environment is a source of utility in the form of environmental services B, which are considered to be inversely related to the stock of CO_2 in the atmosphere. Greenhouse damages therefore appear as reduced environmental service flows according to,

$$B = B_0 - \beta(M - M_0),$$

where B_0 is the flow of services associated with the pre-industrial stock of carbon dioxide. β is a constant, the reduction in services associated with a unit of CO_2 in excess of pre-industrial levels. This formulation implies that $\dot{B} = -\beta\dot{M}$.

The natural resources we are concerned with are fossil fuels, whose stock S is depleted as quantities R are extracted. Given the usual assumption that we wish to maximize the present value of utility, the greenhouse model is therefore:

$$\begin{aligned} &\max \int_0^\infty U(C,B)e^{-rt}\,dt \quad \text{subject to:} \\ &\dot{K} = F - C - a - f \\ &\dot{M} = e - d \\ &\dot{B} = -\beta\dot{M} \\ &\dot{S} = -R \\ &S \rightarrow 0 \ \text{ as } \ t \rightarrow \infty. \end{aligned} \tag{4.2}$$

The final part of expression (4.2) just says that resources must be exhausted over the program; this is an efficiency condition. In this model K, M, B and S are the state variables and C, a and R are the control variables. This optimal control problem has the current value Hamiltonian function,

$$H = U + \gamma_1(F - C - a - f) + \gamma_2(e - d) - \gamma_3\beta(e - d) - \gamma_4 R.$$

where γ_1, γ_2, γ_3 and γ_4 are the co-state variables corresponding to capital, carbon dioxide stocks, environmental services and resource stocks respectively. If we define $b = -1/e_a$ (i.e., *the marginal cost of abatement*), then derivation of the first-order conditions for this problem yields,

$$\gamma_1 = U_C, \quad \gamma_2 - \gamma_3\beta = -U_C b, \quad \gamma_4 = U_C(F_R - f_R - b e_R).$$

The Hamiltonian is measured in utils and is maximized at each point in time under the optimal program - it is therefore a current measure of welfare. We may define a measure of economic welfare in consumption units as $MEW = H / U_C$. Substitution of the above expressions for the co-state variables into the Hamiltonian therefore gives,

$$MEW = \frac{U}{U_C} + \dot{K} - b(e - d) - (F_R - f_R - b e_R)R. \tag{4.3}$$

This expression says that emissions decrease welfare while regeneration of the environment, through the dissipation of CO_2, increases it (i.e., the environment is productive); in both cases the appropriate unit of valuation is the marginal cost of abatement, b. Assuming profit-maximizing producers, F_R is the market price of the resource and f_R its marginal cost of extraction. The final term in expression (4.3) therefore relates to the value of resource depletion[2], being of the form "price minus marginal cost". However, the unit resource rent $F_R - f_R$ is reduced by a Pigovian tax, at rate $b e_R$. This is a *carbon tax*, a specific tax required to achieve both the maximization of the present value of utility and, as will be seen below, the efficient extraction of the resource when its use leads to CO_2 emissions. The net rental value of fossil fuels decreases when account is taken of their environmental externalities.

For a general utility function the dynamic efficiency conditions for this problem yield further insights. The first of these conditions takes the form,

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} = \gamma_1(r - F_K) \Rightarrow \frac{\dot{U}_C}{U_C} = r - F_K . \tag{4.4}$$

In general we can say that $\dot{U}_C = U_{CC}\dot{C} + U_{CB}\dot{B}$. We will assume that as resource extraction declines exponentially (this results from the Cobb-Douglas production function, where resources are essential for production, and the efficiency condition that resources must be exhausted over the infinite time horizon), the stock of CO_2 will return to its pre-industrial level, implying that $\dot{B} \rightarrow 0$. Thus, if there is declining marginal utility of consumption (a standard assumption), we can define the elasticity of the marginal utility of consumption in the long run to be,

$$\eta(C) \equiv -C\frac{U_{CC}}{U_C},$$

and therefore expression (4.4) becomes,

$$\eta(C)\frac{\dot{C}}{C} + r = F_K . \tag{4.5}$$

This expression is an optimality condition, the Ramsey rule (after Ramsey (1928)). The left-hand side is the social rate of return on investment; at the optimum it must equal the rate of return on capital. For Cobb-Douglas production technology it is shown in Dasgupta and Heal (1979, ch. 10) and in Chapter 2 that any positive choice of discount rate r leads to declining consumption, essentially because F_K must approach 0 as resources deplete. Consumption is not ***sustainable*** in this model. Note that r is the pure rate of time preference - if this is 0 then consumption will increase continually, although some non-intuitive restrictions are needed on the model to ensure the integrability of the utility function.

The next dynamic efficiency condition is derived by combining those for the second and third co-state variables, so that,

$$\begin{aligned}\dot{\gamma}_2 - \beta\dot{\gamma}_3 &= r(\gamma_2 - \beta\gamma_3) - \left(\frac{\partial H}{\partial M} - \beta\frac{\partial H}{\partial B}\right) \\ &= r(\gamma_2 - \beta\gamma_3) + (\gamma_2 - \beta\gamma_3)d_M + \beta U_B .\end{aligned}$$

Recalling that $\gamma_2 - \beta\gamma_3 = -U_C b$ and expression (4.4), we therefore have,

---

[2] This is comparable to the value of depletion, based on resource rentals, that appears in Hartwick (1990) and in Chapter 7.

$$\dot{b} = (F_K + d_M)b - \beta\frac{U_B}{U_C}. \tag{4.6}$$

This defines the dynamic relationship between the marginal cost of abatement (and the emission tax rate, since the two are equal) and the price consumers would be willing to pay to avoid another unit of carbon dioxide in the atmosphere ($\beta U_B / U_C$). This is the relationship derived in Ulph and Ulph (1994).

If we define $n \equiv F_R - f_R - b e_R$ (the net rent per unit of resource), then the next dynamic efficiency condition is,

$$\frac{\dot{n}}{n} = F_K. \tag{4.7}$$

This expression says that the percentage rate of change in the difference between resource rents and carbon royalties must equal the rate of return on capital. This is a variant of the Hotelling rule (Hotelling (1931)), that the percentage rate of change in resource rents in an efficient extraction program will equal the rate of interest. Expression (4.7) implies that unit resource rents as traditionally measured may increase at a percentage rate less or greater than the rate of return on capital, but that *net* unit rents increase at a percentage rate precisely equal to this rate.

It may be argued that there are few viable options for abating carbon dioxide emissions, and therefore the model posits an unrealistic situation. We would hold that it is more general to assume some abatement effort in any model of pollution emissions. In practice the difference this makes to the model is small. It is straightforward to show that in the absence of abatement expenditures the model leads to precisely the same welfare measures and efficiency conditions, with a Pigovian tax on emissions (based on the shadow price of the stock of carbon dioxide in the atmosphere) required to achieve an optimum. In the model just presented the marginal cost of abatement is necessarily equal to the rate of tax on emissions.

This formulation of the optimal control programme is quite general. In particular, if it is assumed that some level of carbon accumulation in the atmosphere will lead to catastrophic results then the following analysis applies. Denote this critical stock M^c and note that there is a corresponding critical level of environmental services B^c. We can then assume a particular form of the utility function U as shown in Figure 4.1 (for constant consumption level C).

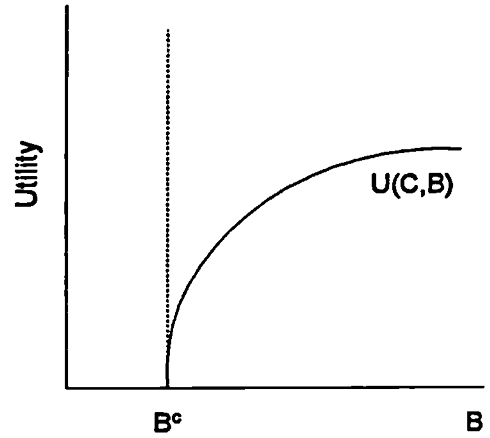

Figure 4.1. Utility function with a critical level.

The manifestation of catastrophic effects associated with critical levels of carbon is that the marginal utility of environmental services approaches infinity as the critical level of service is approached. The effect on marginal abatement costs (and optimal tax levels) can be seen by re-arranging expression (4.6) as follows:

$$\frac{\dot{b}+\beta\frac{U_B}{U_C}}{b}=F_K+d_M .$$

As $U_B \to \infty$, therefore, $b \to \infty$, so that the emission tax rate becomes arbitrarily large. This has the effect of maintaining the stock of carbon below its critical level - eventually the amount of fossil fuel used will decline to the point where the total carbon stock begins to diminish as a result of natural dissipation.

Dixit, Hammond and Hoel (1980) show that the general Hartwick rule requires maintaining the total value of the various stocks (produced assets, fossil fuel and carbon dioxide in our case) constant, with stocks valued at their shadow prices - we can posit, therefore, the result desired. But it is of some analytical interest to explore other investment rules as well.

Variations on the Hartwick Rule

In a useful exploration of the various economic interpretations of sustainable development, Pezzey (1989) provides a simple definition that we will adopt here: $\dot{U} \geq 0$. The standard Hartwick model is one in which it is assumed implicitly that consumption is the sole source of utility, and so the rule 'invest resource rents' yields both constant consumption and utility - the Introduction termed this *minimal* sustainability. Since the models we wish to develop include environmental services as a source of utility, we are seeking a variant of the Hartwick rule that will yield constant utility rather than constant consumption. It will be important to examine the path for consumption, however, if it is assumed that it is infeasible to derive welfare from environmental services alone.

Throughout what follows we assume that it is efficient to exhaust the finite stock of fuel resources, so that,

$$S \to 0 \text{ as } t \to \infty. \qquad (4.8)$$

Assuming resources are essential for production, this implies that R must decline to 0, along with its rate of change, as the resource is exhausted. We assume that there is a stoichiometric relationship between *uncontrolled* emissions and resource use, and in addition that,

$$e_R = \frac{e}{R}. \qquad (4.9)$$

As R declines to 0 then so must uncontrolled emissions, and, since controlled emissions must be less than uncontrolled, therefore e must decline to 0 as well.

We also introduce the Cobb-Douglas functional form at this point, although many of the results below apply to general production functions,

$$F(K,R) = K^{\kappa} R^{\rho}, \ \kappa + \rho < 1, \ \kappa, \rho > 0. \qquad (4.10)$$

Finally, it will be convenient to make some assumptions about the path of carbon dioxide stocks in the atmosphere. Given that resource use and emissions decline to 0, it is certainly possible that, if dissipation of the stock is greater than emissions from the outset, then the carbon stock will decline monotonically to its pre-industrial level. A more realistic assumption, for initially abundant fossil fuels and low rates of CO_2 dissipation, is that carbon stocks will rise at the beginning of the programme and then decline monotonically to pre-industrial levels as dissipation eventually exceeds emissions. If there are catastrophic effects associated with critical levels of carbon in the atmosphere, then the utility function of Figure 4.1 can be assumed in what follows.

The variations on the Hartwick rule we wish to examine are (i) investing resource rents - the standard Hartwick rule; (ii) investing rents net of carbon taxes; and (iii) investing net rents plus the rate of change in the carbon dioxide stock, valued at the marginal cost of abatement (or the emission tax rate) - an extended Hartwick rule.

Investing Resource Rents

The essential elements of this model are, for $n = F_R - f_R - be_R$,

$$\dot{K} = (F_R - f_R)R, \quad \frac{\dot{n}}{n} = F_K, \quad \dot{b} = (F_K + d_M)b - \beta\frac{U_B}{U_C}. \qquad (4.11)$$

We determine the rate of change of consumption as follows:

$$\begin{aligned} \dot{C} &= F_K\dot{K} + F_R\dot{R} - F_R\dot{R} + f_R\dot{R} - \dot{F}_R R + \dot{f}_R R - \dot{f} - \dot{a} \\ &= (F_K(F_R - f_R) - (\dot{F}_R - \dot{f}_R))R - \dot{a}. \end{aligned}$$

But $F_R - f_R = n + be_R \Rightarrow \dot{F}_R - \dot{f}_R = \dot{n} + \dot{b}e_R + b\dot{e}_R$, so that, using expression (4.6),

$$
\begin{aligned}
\dot{C} &= (F_K b e_R - \dot{b} e_R - b \dot{e}_R)R - \dot{a} \\
&= (\dot{b} e_R - d_M b e_R + \beta \frac{U_B}{U_C} e_R - \dot{b} e_R - b \dot{e}_R)R - \dot{a} \\
&= (\beta \frac{U_B}{U_C} - b(d_M + \frac{\dot{e}_R}{e_R}))e - \dot{a}.
\end{aligned}
$$

This can be simplified if we assume no abatement. In this case both $\dot{e}_R$ and $\dot{a}$ are equal to 0, so that,

$$\dot{C} = (\beta \frac{U_B}{U_C} - b d_M)e, \tag{4.12}$$

(this result also holds if the emission function is of the form $e = \phi R g(a)$, where ϕ is the uncontrolled rate of emissions and $g(a)$ is the abatement function). In general, therefore, it is difficult to draw conclusions about the rate of change of consumption. If the marginal rate of dissipation of the carbon stock, d_M, is less than the ratio of the price consumers are willing to pay for a unit of diminution of the carbon stock to the marginal cost of abating one unit, then the rate of change of consumption will be positive. This clearly depends on the nature of the dissipation process as well as the time path of the price and marginal abatement costs.

Applying expression (4.6) in expression (4.12) yields the following:

$$\dot{C} = \left(F_K + d_M - \frac{\dot{b}}{b} - d_M \right) be = \left(F_K - \frac{\dot{b}}{b} \right) be.$$

The direction of change of consumption is therefore determined by the difference between the marginal return on capital and the percentage rate of change of marginal abatement expenditures (i.e., the emissions tax). For resource extraction to be feasible we require $\dot{K} \geq 0$ which, for Cobb-Douglas production technology, implies that $F_K \to 0$ as $R \to 0$.

The rate of change of utility, from expression (4.12), is given by,

$$
\begin{aligned}
\dot{U} &= U_C \dot{C} - \beta U_B \dot{M} \\
&= U_C [(\beta \frac{U_B}{U_C} - b d_M)e - \beta \frac{U_B}{U_C}(e - d)] \\
&= U_C (\beta \frac{U_B}{U_C} d - b d_M e).
\end{aligned} \tag{4.13}
$$

If consumption is increasing then utility will be increasing as long as $d > e$, i.e. as long as the carbon stock in the atmosphere is decreasing.

Investing Net Rents

What happens if only rents net of carbon taxes are invested? The basic elements of this model are, for $n = F_R - f_R - b e_R$,

$$\dot{K} = (F_R - f_R)R, \quad \frac{\dot{n}}{n} = F_K, \quad \dot{b} = (F_K + d_M)b - \beta\frac{U_B}{U_C}.$$

The combined extended Hotelling and "invest net rent" rules imply that $F_K\dot{K} = \dot{n}R$. Given that $\dot{f} = f_R\dot{R}$ and $C = F - nR - f - a$, we therefore have,

$$\begin{aligned}
\dot{C} &= F_K\dot{K} + F_R\dot{R} - \dot{n}R - n\dot{R} - \dot{a} - \dot{f} \\
&= \dot{n}R + F_R\dot{R} - \dot{n}R - n\dot{R} - \dot{a} - \dot{f} \\
&= F_R\dot{R} - F_R\dot{R}b_{e_R}\dot{R} + f_R\dot{R} - \dot{a} - \dot{f} \\
&= b_{e_R}\dot{R} - \dot{a}.
\end{aligned}$$

Since $\dot{e} = e_R\dot{R} + e_a\dot{a}$, we see that $\dot{C} = b\dot{e}$. Consumption falls because emissions must decline along with resource extraction.

The path for utility is uncertain in this model. Clearly, if carbon dioxide stocks are increasing at the beginning of the programme, then utility is falling. If utility is rising later in the programme, consumption is still decreasing. Unless consumption approaches some positive asymptote, the long-run situation when utility is rising would be that environmental services B become the sole source of utility as the carbon stock stabilizes in the atmosphere. This is infeasible.

The Extended Hartwick Rule

If investing resource rents leads to an indeterminate path for consumption, and investing rents net of carbon taxes yields declining consumption, the obvious question is what level of investment will yield constant utility. It turns out that for $n \equiv F_R - f_R - b_{e_R}$, we require the following three conditions:

$$\dot{K} = nR + b\dot{M}, \quad \frac{\dot{n}}{n} = F_K, \quad \dot{b} = (F_K + d_M)b - \beta\frac{U_B}{U_C}. \tag{4.14}$$

The first of these is the extended Hartwick rule, which says that investment equals net resource rents plus the rate of change of the carbon dioxide stock in the atmosphere, valued at the marginal cost of abatement

We first derive the rate of change of consumption as follows:

$$\begin{aligned}
\dot{C} &= F_R\dot{R} + F_K\dot{K} - \ddot{K} - \dot{f} - \dot{a} \\
&= F_R\dot{R} + F_K b\dot{M} - n\dot{R} - \dot{b}\dot{M} - b\ddot{M} - \dot{f} - \dot{a} \\
&= b_{e_R}\dot{R} + F_K b\dot{M} - \dot{b}\dot{M} - b\dot{e} + b\dot{d} - \dot{a}.
\end{aligned}$$

Now, $b\dot{e} = b_{e_R}\dot{R} - \dot{a}$ and $b\dot{d} = b d_M\dot{M}$, so that, using the third equation in expression (4.14),

$$\dot{C} = (F_K + d_M)b\dot{M} - \dot{b}\dot{M} = \beta\frac{U_B}{U_C}\dot{M}. \tag{4.15}$$

Therefore $\dot{U} = U_C\dot{C} + U_B\dot{B} = U_C(\dot{C} - \beta\frac{U_B}{U_C}\dot{M}) = 0.$

Since the assumed pattern would be for carbon stocks in the atmosphere to rise and then fall, expression (4.15) implies that consumption must also do the same, even though utility is constant. The question of feasibility, i.e., positive consumption in the long run, therefore arises.

The next thing to note is that investment is less than the resource rent in this model. To see this, recall that $e_R = e/R$. Therefore,

$$\dot{K} = (F_R - f_R - b e_R)R + b(e - d) = (F_R - f_R)R - bd.$$

Investment is equal to resource rent less CO_2 dissipation valued at the marginal cost of abatement.

There is nothing in principle to prevent investment becoming negative in this model, but this is key to the question of feasibility. If $\dot{K} \leq 0$, then output falls to 0 because $R \rightarrow 0$ (this is a consequence of resources being essential for production in the Cobb-Douglas production function). If $F \rightarrow 0$ then so must consumption. This would imply that utility is derived only from environmental services, which is infeasible. $\dot{K} > 0$ is therefore a necessary (but not a sufficient) condition for feasibility.

Because consumption falls at a rate proportional to the rate of decline of carbon stocks in the atmosphere, feasible programmes are those for which consumption approaches a positive asymptote as carbon stocks stabilize. This is only possible if $\dot{M} \rightarrow 0$ as $R \rightarrow 0$, which is a necessary condition for the stabilization of carbon stocks at pre-industrial levels. Our choice of this behaviour for the dissipation of carbon stocks as a 'realistic' feature of the model turns out to be essential for feasibility. Clearly, of the set of feasible paths for the model, the best path is the one yielding maximal constant utility.

Conclusions

The original work leading to the derivation of the Hartwick rule was concerned with a particular problem, how to achieve intergenerational equity in the form of constant consumption when resources are exhaustible - see, for instance, Solow (1974). In this chapter we eschew constant consumption as a goal, but seek a constant utility analogue to the Hartwick rule when resource use leads to greenhouse warming.

The model of optimal growth leads to the efficiency conditions necessary for the Hartwick rule with greenhouse gas emissions. A number of conclusions follow from this model. The first is that the externality from fuel consumption must be corrected by a Pigovian tax if an optimum is to be achieved, and the tax on emissions equals the marginal cost of abatement - this is as one would expect. The extended Hotelling rule that results from this model equates the percentage rate of change of resource rents net of carbon taxes to the marginal product of capital. The path of the emission tax is related to the marginal product of capital, the marginal rate of dissipation of the carbon stock, and the price consumers would be willing to pay for one less unit of carbon in the atmosphere. In the measure of economic welfare related to the optimal growth programme, the deleterious effects of carbon emissions are precisely balanced by the carbon tax, and the value dissipation of the carbon dioxide stock is added to welfare along with the value of the level of environmental services. And finally, consumption declines to 0 if the pure rate of time preference is positive, as in the standard model of exhaustible resources.

Three variants of investment rules were explored in this chapter. The standard Hartwick rule, to invest resource rents, yields the result that consumption will increase if marginal abatement costs (and abatement taxes, therefore) are decreasing or if their percentage rate of increase is less than the marginal product of capital - if consumption is increasing in the long run then so is utility. If, instead, resource rents net of carbon taxes are invested, consumption declines as resource extraction declines - irrespective of whether utility is increasing in the long run, this model is feasible only if consumption approaches a positive asymptote. Only the extended Hartwick rule unequivocally yields constant utility.

There are several points to note about the extended Hartwick rule. First, consumption typically rises then falls, in step with the change in the carbon stock in the atmosphere. Second, investment is always less than resource rents (in clear contrast with the standard Hartwick rule). And finally, a necessary condition for feasibility (consumption that is positive in the long run) is that dissipation of the carbon stock drop to 0 as the stock declines to its equilibrium at pre-industrial levels. These results hold in general and for specific assumptions about the catastrophic effects associated with given levels of carbon stocks.

Solow (1986) called the Hartwick rule a 'rule of thumb' for growth policy in a world of exhaustible resources. In a greenhouse world with finite fossil fuel resources the 'green' rule of thumb is to invest resource rents net of carbon taxes plus the value of the rate of change of the carbon stock.

5. Feasible Steady States for Environment-Economy Systems

While sustainable development emphatically does not imply a steady state economy, there are models where welfare may increase over time as the steady state is approached asymptotically. This chapter is concerned with the feasibility of such steady states.

There is one well-known example of an optimal programme, one that maximizes the present value of welfare, that is not sustainable: for non-renewable resources, Dasgupta and Heal (1979, ch. 10) show that a positive pure rate of time preference leads to declining utility along the optimal path. Chapter 2 showed that if produced assets depreciate then, even for a zero pure rate of time preference, an economy where non-renewable resources are essential for production will not be sustainable.

A second example of an unsustainable programme is provided by the model of Hamilton and Atkinson (1995), in which the goal is to maximize the present value of utility when there is a cumulative pollutant and the size of the stock of pollution emitted determines the level of welfare that may be drawn from the natural environment. The proof that this economy is not sustainable is shown in the Appendix. Both of these examples are characterized by the lack of a steady state solution, on the one hand because an essential input is depleted, on the other because a necessary byproduct of production, the pollutant, has cumulative effects that do not dissipate.

The 'answer' to the sustainability problem for these models is to eschew maximizing the present value of utility in favour of a programme in which welfare is held constant over time. Such a programme is logically equivalent to the Hartwick (1977) rule in the case of non-renewable resources and its extension in Hamilton and Atkinson (1995) to the case of cumulative pollutants. Under a Hartwick programme the essential idea is to ensure that the change in the volume of assets and liabilities, valued at current prices that are dynamically efficient, is zero.

This chapter constructs two models where the environment and the economy are linked and where a steady state is possible. One model involves pollutants that accumulate and dissipate, the other a living resource that is damaged by current emissions of a flow pollutant (i.e., the acid rain problem). Because a steady state is possible, both models may produce sustainable outcomes; the key results in what follows relate to the feasibility of the steady state.

The Models

We wish to portray the steady state of linked environmental and economic systems by deriving the steady state solutions of optimal growth models of these systems. Two such models will be explored: a general model with a stock pollutant that dissipates, in which both current and capital expenditures may reduce pollution emissions; and a model in which a living resource is damaged by the flow of pollution emissions (the acid rain problem).

The general model

We assume a simple economy, closed to trade, in which a single good is produced with fixed technology. That good can be consumed, invested in productive capital, invested in pollution abatement capital, or used as a current input to pollution abatement. A single stock pollutant is emitted as a result of production in the economy. Environmental services (e.g., the services provided by clean air) are negatively related to the level of stock pollutant; these services provide utility to consumers and are also productive. For productive capital K, labour L, and environmental services B, the production relationship is therefore,

$$F(K,L,B) = C + \dot{K} + \dot{K}_a + a,$$

where C is consumption, K_a is abatement capital, and a is current abatement expenditures. We will assume that investment in abatement capital is simply equal to the quantity m, so that

$$\dot{K}_a = m.$$

Pollution emissions are given by,

$$e = e(F, K_a, a), \text{ with } e_a < 0 \text{ and } e_{K_a} < 0, \qquad (5.1)$$

while the pollution stock X is assumed to dissipate as a result of natural processes, as represented by the dissipation function $d(X)$. The stock-flow relationship for the pollutant is therefore,

$$\dot{X} = e - d,$$

while environmental services are given by a linear relationship,

$$B = B_0 - \beta(X - X_0),$$

for some assumed level of services B_0 derived from the environment in its pristine state X_0. Note that this implies that $\dot{B} = -\beta\dot{X}$.

The problem we wish to characterize is that of maximizing the present value of utility U, which is assumed to be a function of consumption and the level of environmental services. This is expressed formally as the following optimal control problem:

$$\max \int_0^\infty U(C,B)e^{-rt}dt \quad \text{subject to:}$$
$$\dot{K} = F - C - m - a$$
$$\dot{K}_a = m$$
$$\dot{X} = e - d$$

Here r is the fixed discount rate (all other variables are functions of time), and C, m, and a, are the control variables.

This constitutes our linked environment-economy system. Current production leads to the accumulation of pollution which affects both production levels and the well-being of consumers. Natural processes dissipate the pollutant, while households can give up some consumption in order for pollution emissions to be abated. Determining the optimal path for the system requires, in addition to the control variables, a set of shadow prices as expressed in the following current value Hamiltonian function:

$$\begin{aligned} H &= U + \gamma_1 \dot{K} + \gamma_2 \dot{K}_a + \gamma_3 \dot{X} \\ &= U + \gamma_1 (F - C - m - a) + \gamma_2 m + \gamma_3 (e - d). \end{aligned} \tag{5.2}$$

Here the γ_i are the shadow prices of productive capital, abatement capital, the pollution stock and environmental services respectively. The solution to the optimal control problem requires a set of first order conditions on the Hamiltonian with respect to the control variables, as well as a set of dynamic conditions on the shadow prices.

As derived in the Appendix, the first order conditions yield the following relationships,

$$\gamma_1 = \gamma_2 = U_C, \text{ and } \gamma_3 = -U_C b, \tag{5.3}$$

where $b \equiv -1/e_a$ is the marginal cost of abatement; as shown in the Appendix, increasing marginal abatement costs are a necessary condition for a maximum. The measure of economic welfare (*MEW*) with a consumption numéraire is therefore,

$$\begin{aligned} MEW &= C + \frac{U_B}{U_C} B + \dot{K} + \dot{K}_a + \frac{b}{\beta} \dot{B} \\ &= C + \frac{U_B}{U_C} B + \dot{K} + \dot{K}_a - b(e - d). \end{aligned} \tag{5.4}$$

This is derived by valuing each flow underlying the current value Hamiltonian by its shadow price, yielding welfare measured in utils, and dividing the resulting expression by U_C to convert to consumption units. If the term in environmental services B is dropped (i.e., a more restricted notion of consumption is employed) then the resulting expression is ‘green’ NNP. Noting that U_B / U_C is the price a utility-maximizing consumer would be willing to pay for another unit of environmental service, expression (5.4) has a natural interpretation: economic welfare in this model consists of the proximate sources of welfare, consumption and the value of environmental services, plus a series of terms required to maximize welfare over time. The latter terms are of course investment in productive capital, investment in abatement, and the change in environmental services valued at the marginal cost of abatement (if emissions exceed natural dissipation of the pollution stock then this term will be negative). Expression (5.4) also yields the first result for the steady state: if a steady state is reached for this system, then economic welfare will consist solely of the proximate sources of utility, consumption and environmental services.

Determining the optimal path for the system also requires dynamic conditions on the shadow prices; these are derived in the Appendix. The first of these conditions is for the shadow price of productive capital, which yields the following expression:

$$\frac{\dot{U}_C}{U_C} = r - (1 - b e_F) F_K \tag{5.5}$$

If we assume that the utility function is additively separable, so that $U(C,B)=U_1(C)+U_2(B)$, then we can derive a variant on the Ramsey (1928) rule by defining,

$$\eta(C)=-\frac{U_{CC}}{U_C}C.$$

This is the elasticity of the marginal utility of consumption. As shown in the Appendix, we must have $U_{CC}<0$ (i.e., declining marginal utility of consumption) in order to maximize the Hamiltonian. Expression (5.5) then reduces to,

$$r+\eta(C)\frac{\dot{C}}{C}=(1-be_F)F_K. \tag{5.6}$$

The left-hand side of this expression is the 'consumption rate of interest', the social rate of return on investment. In the classic Ramsey rule the right-hand side is just the marginal product of capital, F_K. Here the social rate of return on investment is equated to the marginal product of capital net of pollution taxes - it is straightforward to show (see, for instance, Chapter 8) that the marginal cost of pollution abatement b is exactly equal to the level of an optimal emissions tax required to maximize utility, so that be_F is the effective tax rate on output.

The second dynamic optimality condition derived in the Appendix is as follows,

$$\frac{\dot{U}_C}{U_C}=r+be_{K_a}. \tag{5.7}$$

This yields another variant on the Ramsey rule,

$$r+\eta(C)\frac{\dot{C}}{C}=-be_{K_a}=\frac{e_{K_a}}{e_a}.$$

This expression links the environment and the economy with a vengeance: in order to maximize the present value of utility, the social rate of return on investment must be equal to the ratio of the reduction in pollution emissions associated with a marginal unit of abatement capital to the reduction in pollution emissions associated with a marginal unit of current abatement expenditure.

The third and final dynamic condition derived in the Appendix concerns the rate of change of marginal abatement costs,

$$\frac{\dot{U}_C}{U_C}+\frac{\dot{b}}{b}=r+d_X+\beta e_F F_B-\frac{\beta}{b}\left(F_B+\frac{U_B}{U_C}\right). \tag{5.8}$$

Here d_X is the marginal rate of dissipation of the pollution stock; it must be less than 1. F_B is the marginal product of environmental services, and so $\beta e_F F_B$ is a feedback term, the amount by which pollution emissions are increased as a result of a marginal increase in the level of environmental services (roughly speaking); this must be less than 1 for production to be feasible. The discount rate for utility is r and, as before, U_B / U_C is the price utility-maximizing consumers would be willing to pay for another unit of environmental service.

These conditions (along with certain transversality conditions concerning the limits approached as time t tends to infinity) completely characterize the optimal path for the linked environment-economy system. The optimal steady state is characterized setting all time rates of change to 0 in expressions (5.5), (5.7) and (5.8). Of the several possible ways to summarize what results from setting the rates to 0, perhaps the most intuitive is to examine the determinants of the level of marginal abatement costs in the steady state. As shown in the Appendix, a necessary condition for the Hamiltonian function to be maximized is that there be increasing marginal abatement costs; higher levels of abatement expenditure therefore entail higher marginal abatement costs.

From expression (5.5) the steady state level of marginal abatement costs is,

$$b = \frac{F_K - r}{e_F F_K}. \tag{5.9}$$

From expression (5.7) we obtain,

$$b = \frac{r}{-e_{K_a}}. \tag{5.10}$$

From expression (5.8) we derive,

$$b = \frac{\beta\left(F_B + \frac{U_B}{U_C}\right)}{r + d_X + \beta e_F F_B}. \tag{5.11}$$

Finally, expressions (5.5) and (5.6) together imply that,

$$b = \frac{F_K}{e_F F_K - e_{K_a}}. \tag{5.12}$$

The direction of change of marginal abatement costs with the various model parameters is obvious in most cases from expressions (5.9)-(5.12). Expressions (5.9) and (5.12) can each be differentiated with respect to F_K to show that the relationship is positive, and expression (5.11) can be differentiated with respect to F_B to show that this relationship is also positive (it is necessary to use expression (5.14), shown below, in this derivation). The overall picture is therefore as appears in the following table.

Table 5.1. Dependence of Marginal Abatement Costs on Model Parameters

	F_K	r	e_F	$\left\vert e_{K_a} \right\vert$	F_B	U_B / U_C	d_X
Direction of change	+	?	?	-	+	+	-

In this table a '+' indicates that marginal abatement costs increase when the parameter in question increases. The question mark for the pure rate of time preference is a result of the combination of expressions (5.9) and (5.10), which give different indications. In a model in which there is no abatement capital the variation of marginal abatement costs with r is unequivocally negative. Higher marginal emission rates e_F will in general lead to lower steady state levels of productive capital K, which in turn lead to a higher marginal product of capital F_K (given the usual assumption of declining marginal product of capital) - the direction of change with regard to marginal emission rates is therefore uncertain in Table 5.1.

The results in this table are intuitive to varying degrees. Clearly increasing the marginal product of environmental services, F_B, and the price consumers are willing to pay for these environmental services, U_B / U_C, will increase the steady state level of marginal pollution abatement costs. The positive effect of the marginal product of capital is less obvious: for a given capital stock a higher marginal product of capital implies more production, more pollution emitted and therefore higher marginal abatement costs. It is clear that the higher the marginal rate of dissipation of the pollution stock, d_X, the lower the marginal abatement cost will be, and that the more effective in reducing emissions the marginal unit of abatement capital is, again, the lower will be marginal abatement costs in terms of current expenditures.

This is a rich array of results and requires some discussion. The first point to note is that expression (5.12) holds whether there is a steady state or not. Secondly, expressions (5.10) through (5.12) all imply $b > 0$. This implies, from expression (5.9), that $F_K > r$, which is in clear contrast to standard models (without environmental linkages) in which the marginal product of capital must be precisely equal to the discount rate for the steady state to be attained. For very large endowments of capital which, other things being equal, will produce a small marginal product of capital, the steady state may be infeasible.

Expression (5.10) can also be expressed as,

$$r = \frac{e_{K_a}}{e_a}. \tag{5.13}$$

This implies that the steady state requires a particular balance of current and capital environmental protection expenditures: the emission reduction associated with a marginal unit of abatement capital as a proportion of the emission reduction associated with a marginal unit of current abatement expenditures must equal the discount rate for utility.

Expressions (5.9) and (5.11) can be combined as follows:

$$
\begin{aligned}
r &= (1 - be_F)F_K \\
&= \left(1 - \frac{\beta(F_B + U_B / U_C)e_F}{r + d_X + \beta e_F F_B}\right)F_K
\end{aligned}
$$

Since $be_F < 1$ is a necessary condition for production to be feasible (i.e., the optimal pollution tax rate on production must be less than 100%), this expression implies that,

$$\beta\frac{U_B}{U_C}e_F - r < d_X. \tag{5.14}$$

Note that βe_F is the change in environmental services resulting from a marginal unit of production, so that $\beta(U_B / U_C)e_F$ is the value of pollution disbenefits resulting from a marginal unit of production. This term should therefore be less or equal to 1 at the optimum.

We can take e_F, a technological parameter, and d_X, a physical parameter, as given. Expression (5.14) then has a striking interpretation: if the price consumers are willing to pay for environmental services (U_B / U_C) is too high, or their impatience too low (as measured by r, the rate of discount of utility, or pure rate of time preference), then the steady state is not feasible - this is explored in Figure 5.1 below. Both the price consumers are willing to pay for environmental services and the pure rate of time preference are behavioural variables.

Figure 5.1. Feasible range for steady state

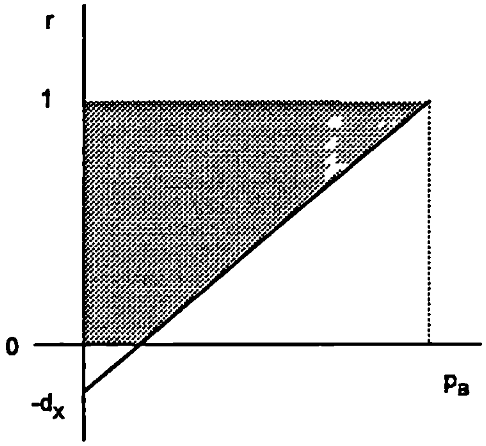

Figure 5.1 clarifies the question of feasibility. The line plotted is $r = \beta e_F p_B - d_X$, where $p_B \equiv U_B / U_C$. If we assume $0 \le r \le 1$, then the feasible set for willingness to pay and the pure rate of time preference, assuming fixed e_F and d_X, is given by the shaded polygon.

The usual constraints on achieving a steady state, essentially that investment in productive capital must have an appropriate initial value relative to the initial capital stock, hold for this model. What the foregoing analysis has shown is that there are a variety of additional constraints to be satisfied if the steady state is to be feasible.

If the steady state is *feasible*, there remains the question of whether the asymptotic approach to the steady state is *sustainable* - i.e., is utility non-decreasing along the optimal path? Given that there is an assumed initial stock of pollutant X_0, this may be less or greater than the steady state level of the pollution stock (we can therefore speak, loosely, of the initial state being 'under-polluted' or 'over-polluted'). The rate of change of utility is given by,

$$\dot{U} = U_C\dot{C} + U_B\dot{B} = U_C\left(\dot{C} - \beta\frac{U_B}{U_C}\dot{X}\right).$$

This can be combined with expression (5.6) to yield:

$$\dot{U} = U_C\left[\frac{C}{\eta(C)}\left[(1 - be_F)F_K - r\right] - \beta\frac{U_B}{U_C}(e - d)\right]. \tag{5.15}$$

From expression (5.15) we can conclude that the economy is unsustainable if there is a large initial capital stock (and therefore small F_K) or a high pure rate of time preference *r*, and the initial state is under-polluted. The economy is sustainable in the long run if there is a small initial capital stock or low rate of time preference and the initial state is over-polluted (the proviso 'in the long run' is required because it is possible that the initial level of emissions, while on a downward trend, may be higher than the initial value of dissipation, in which case the stock of pollution rises then falls asymptotically to its steady state value). Other combinations of initial conditions produce uncertain signs for the rate of change of utility.

The acid rain problem

In this model we assume a flow pollutant that does not accumulate is emitted as a byproduct of production[1]. This pollutant causes dieback of a living resource that is a productive input. The pollutant is assumed to have no direct effect on the utility of consumers. The relationships for production and utility are therefore,

$$F = F(K, L, R) \text{ and } U = U(C),$$

for resource harvest *R*; harvesting is assumed to be costless. The total resource stock *S* grows by an amount *g(S)* and dies back by an amount *w(S,e)* as a result of pollution emissions, such that,

$$w_S > 0 \text{ and } w_e > 0.$$

As in the previous model, emissions are given by *e(F,a)*, for abatement expenditures *a*. The optimal programme is therefore the solution to,

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[1] The green national accounting aspects of this model are presented in Hamilton (1994) and in Chapter 8.

$$\max \int_0^\infty U(C)e^{-rt}dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a$$
$$\dot{S} = -R - w + g.$$

Here the control variables are *C*, *a*, and *R*, and the current value Hamiltonian function to be maximized is,

$$H = U + \gamma_1(F - C - a) + \gamma_2(-R - w + g).$$

As derived in the Appendix, the first order conditions for a maximum are,

$$\gamma_1 = U_C \text{ and } \gamma_2 = \frac{U_C b}{w_a} = U_C F_R.$$

The dynamic conditions that the shadow prices on capital and resources must satisfy are shown in the Appendix to lead to the following conditions:

$$\frac{\dot{U}_C}{U_C} = r - F_K \tag{5.16}$$

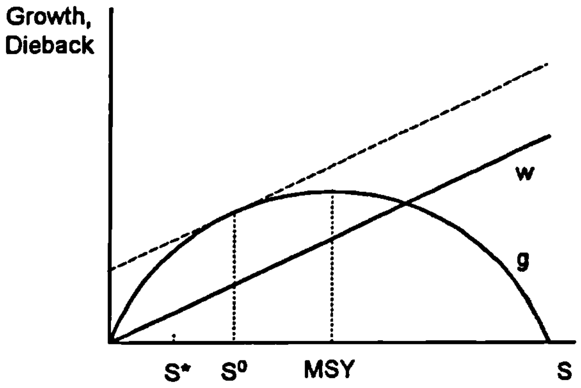
$$\frac{\dot{F}_R}{F_R} = F_K + w_S - g_S. \tag{5.17}$$

The first of these expressions implies the standard form of the Ramsey rule, and in the steady state we have the usual condition that the pure rate of time preference must precisely equal the marginal product of capital. The second expression is a modified Hotelling rule for the efficient harvest of the resource. The steady state for conditions (5.16) and (5.17) combined therefore implies that,

$$r = F_K = g_S - w_S. \tag{5.18}$$

We see the steady state is feasible only if $g_S > w_S$. The optimal steady state is best characterized using Figure 5.2.

Figure 5.2. Optimal stock size with acid rain.



Here the growth curve *g(S)* has the usual shape as a function of stock size, and MSY is the stock size at which the maximum sustainable yield is attained. The dieback function *w(S,e)* is plotted for a fixed value of emissions *e*; it is assumed that dieback is linearly related to the stock size, which is a reasonable presumption for fixed emissions of a well-mixed pollutant. Stock size S^0 is the point at which $g_S = w_S$. S* is the optimal steady state stock for an assumed value of *r*.

The model yields the expected result from standard models of living resources, that increasing values of *r* decrease the optimal stock size. Expression (5.18) implies three new properties for the acid rain model: (i) for a given value of *r* the optimum stock size is less than in the standard model; (ii) stock sizes greater than S^0 are infeasible (in the standard model stock sizes up to MSY are attainable as the pure rate of time preference approaches 0); and (iii) the feasible range of stock sizes decreases as emissions increase, since it is reasonable to assume that higher pollution emissions at the optimum will increase the slope of *w* (i.e., $w_{S_e} > 0$); in the limit, very high emissions of pollution will render the steady state infeasible.

Further to the latter point, we know from the first order conditions for a maximum that $b = w_e F_R$. Sufficient second order conditions for a maximum of the Hamiltonian are $U_{CC} < 0$ and $e_{aa} > 0$, so that there are increasing marginal costs of abatement. For a fixed stock size and constant w_e, increases in F_R imply increases in *b*, increases in abatement expenditures and therefore decreases in emissions. The optimum level of acid emissions will thus be inversely related to the marginal product of the living resource. For constant w_e and $w_{S_e} > 0$, therefore, the larger the marginal product of the living resource, the larger the optimal steady state stock will be, and the greater the range of feasible stock sizes.

Since $\dot{U} = U_C \dot{C}$ and $\eta(C)\frac{\dot{C}}{C} = F_K - r$ (the Ramsey rule), we can conclude that the optimal path that approaches the steady state asymptotically will be sustainable if the pure rate of time preference is low or if the initial endowment of capital is low. The obverse of these conditions would imply that the optimal path is unsustainable, so the usual intuition that high discount rates are inimical to sustainable development is borne out in this model.

Conclusions

To repeat a basic point from the beginning of this chapter, sustainable development does not imply a steady state economy. For the models examined here, where a stock pollutant dissipates or a living resource is damaged by acid emissions, the path to the optimal steady state may or may not be sustainable and the steady state itself may not be feasible.

The conditions governing the feasibility of the steady state are therefore particularly important. Three of the results of the models have a bearing on feasibility:

- If environmental services are productive and/or a source of utility, the steady state is feasible only if the marginal product of capital is greater than the pure rate of time preference. Other things being equal, this implies that the steady state stock of productive capital will be lower for linked environment-economy systems than for systems that optimize over time without considering the effects of environmental services on production or utility.

- For a given technological parameter, the marginal pollution emissions associated with a unit of production, and a given physical parameter, the marginal rate of dissipation of the pollution stock, the steady state will not be feasible if, in combination, the price consumers are willing to pay for environmental services is too high or their rate of impatience (i.e., pure rate of time preference) is too low.

- For the acid rain problem, the range of stock sizes of the living resource that is infeasible is larger than in the absence of acid rain. Sufficiently large acid emissions will render the steady state infeasible. The optimal steady state stock is smaller than in the absence of acid rain, but it increases with the marginal product of the living resource.

There are two determinants of the steady state level of marginal pollution abatement costs that are of interest. If there are only current expenditures to abate pollution (i.e., no abatement capital) then the higher the pure rate of time preference the lower is the marginal cost of abatement (MCA). And the higher the marginal product of capital the higher is the optimal steady state MCA.

Given the feasibility of the steady state, there is no guarantee the optimal path that approaches the steady state asymptotically will be sustainable. For the stock pollutant the optimal path will be sustainable in the long run if an appropriate combination of low initial capital stock and low pure rate of time preference is combined with an initial state that is 'over-polluted.' The acid rain economy is sustainable if there is the appropriate combination of low initial capital stock and low pure rate of time preference.

Appendix to Chapter 5

The model of Hamilton and Atkinson (1995).

For a pollutant with cumulative effects, or equivalently a stock pollutant that does not dissipate, we wish to establish that the optimal path is not sustainable. This corresponds to dropping both investment in pollution abatement capital (so that current expenditures only are used to abate emissions) and the dissipation function from the first model presented above. In addition, we assume that production is a function of capital and labour only, and that labour inputs are fixed.

Under these conditions the key relationships are the Ramsey rule,

$$\eta(C)\frac{\dot{C}}{C} = (1 - be_F)F_K - r, \tag{A5.1}$$

and the rate of change of utility along the optimal path,

$$\dot{U} = U_C\left(\dot{C} - \beta\frac{U_B}{U_C}e\right). \tag{A5.2}$$

Under the usual assumption of decreasing returns to factors, $\dot{C} > 0$ in the long run implies that $K \rightarrow \overline{K}$ (i.e., a fixed limit) such that $(1 - be_F)F_K > r$, from expression (A5.1). This implies that $F \rightarrow \overline{F}$, since labour is fixed, and therefore that $e \rightarrow \overline{e}$.

Because there is declining marginal utility of consumption, $\dot{C} > 0$ implies that U_C is declining. Since $e > 0$, it follows that U_B is increasing, under the assumption of declining marginal utility of environmental services. It is reasonable to assume that $U_B \rightarrow \infty$ as $B \rightarrow 0$ (this would hold, for instance, if a sufficient accumulation of pollution were life-threatening).

Therefore $\beta\frac{U_B}{U_C}e$ increases without bound and, by expression (A5.2), we conclude that $\dot{U} > 0$ iff $\ddot{C} > 0$, which is inconsistent with the fact that $F \rightarrow \overline{F}$. We know that $\dot{U} = 0$ holds only under the Hartwick rule (investment equals emissions valued at marginal social costs in this instance), which is inconsistent with the optimal programme under the Ramsey rule. The conclusion is that utility must decline in the long run under the optimal programme. Optimal growth with a pollutant with cumulative effects is not sustainable.

The general model.

For the general model the current value Hamiltonian function is given by,

$$H = U + \gamma_1 (F - C - m - a) + \gamma_2 m + \gamma_3 (e - d).$$

The static first order conditions for a maximum of this function are therefore,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \quad \Rightarrow \quad \gamma_1 = U_C \tag{A5.3}$$

$$\frac{\partial H}{\partial m} = 0 = -\gamma_1 + \gamma_2 \quad \Rightarrow \quad \gamma_2 = U_C$$

$$\frac{\partial H}{\partial a} = 0 = -\gamma_1 + \gamma_3 e_a \quad \Rightarrow \quad \gamma_3 = \frac{U_C}{e_a} \tag{A5.4}$$

Expression (A5.3) implies that $U_{CC} < 0$ is a sufficient condition for the Hamiltonian to be maximized, i.e., that there must be decreasing marginal utility of consumption. Expression (A5.4) implies that $e_{aa} > 0$ is a sufficient condition for the Hamiltonian to be maximized, i.e., that there must be increasing marginal costs of abatement.

The dynamic first order conditions for the shadow prices are as follows:

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} = r\gamma_1 - \gamma_1 F_K - \gamma_3 e_F F_K ,$$

so that,

$$\frac{\dot{U}_C}{U_C} = r - (1 - b e_F) F_K \ ; \tag{A5.5}$$

next,

$$\dot{\gamma}_2 = r\gamma_2 - \frac{\partial H}{\partial K_a} = r\gamma_2 - \gamma_3 e_{K_a} \ ,$$

so that, for $b \equiv -1/e_a$,

$$\frac{\dot{U}_C}{U_C} = r - b e_{K_a} \ ; \tag{A5.6}$$

and

$$\dot{\gamma}_3 = r\gamma_3 - \frac{\partial H}{\partial X} = r\gamma_3 - \left(-\beta U_B - \beta U_C F_B + U_C b \beta e_F F_B + U_C b d_X\right) ,$$

so that,

$$\frac{\dot{U}_C}{U_C}+\frac{\dot{b}}{b}=r+d_X+\beta e_F F_B-\frac{\beta}{b}\left(F_B+\frac{U_B}{U_C}\right). \tag{A5.7}$$

The acid rain model.

The current value Hamiltonian for this model is,

$$H=U+\gamma_1(F-C-a)+\gamma_2(-R-w+g) \ .$$

Therefore the first order conditions for a maximum are:

$$\frac{\partial H}{\partial C}=0=U_C-\gamma_1 \Rightarrow \gamma_1=U_C \ ; \tag{A5.8}$$

$$\frac{\partial H}{\partial a}=0=-\gamma_1-\gamma_2 w_e e_a \Rightarrow \gamma_2=\frac{U_C b}{w_e} \ ; \tag{A5.9}$$

$$\frac{\partial H}{\partial R}=0=\gamma_1 F_R-\gamma_2 \Rightarrow \gamma_2=U_C F_R \ . \tag{A5.10}$$

Expression (A5.8) implies that a sufficient condition for the Hamiltonian to be maximized is $U_{CC} < 0$, i.e., declining marginal utility of consumption. Similarly, expression (A5.9) implies that increasing marginal abatement costs, $e_{aa} > 0$, is a sufficient condition for a maximum if $w_{ee} \geq 0$, i.e., if damage to the living resource is a constant or increasing function of emissions. The dynamic first order conditions for the shadow prices are derived as,

$$\dot{\gamma}_1=r\gamma_1-\frac{\partial H}{\partial K}=r\gamma_1-\gamma_1 F_K \ ,$$

so that,

$$\frac{\dot{U}_C}{U_C}=r-F_K \ ; \tag{A5.11}$$

and,

$$\dot{\gamma}_2=r\gamma_2-\frac{\partial H}{\partial S}=r\gamma_2+\gamma_2(w_S-g_S) \ ,$$

so that

$$\frac{\dot{F}_R}{F_R} = F_K + w_S - g_S \ . \tag{A5.12}$$

Part 2

Theory of Green National Accounting

6. Synopsis of Proposals for Green National Accounts[1]

We now turn from the theory of sustainable development to 'green' national accounting. The traditional national accounts present the most comprehensive and integrated information concerning the state and operation of national economies. Many of the standard national accounting aggregates are explicitly concerned with measuring income, product, savings, investment and wealth net of the depreciation of produced assets. If produced capital were the only asset influencing human welfare, then it could be argued that the traditional national accounts are well suited to measuring progress towards, or away from, sustainable development. This is particularly so because market prices permit the aggregation of many types of assets in a consistent manner: one dollar's worth of housing stock yields, in principle at least, the equivalent stream of benefits as a dollar's worth of factory or production machinery.

It is on this basis that greener national accounts are, at least potentially, the most powerful indicators of sustainable development. The critical issue, to be considered in subsequent chapters, is the expansion of the asset base to include natural resources and other environmental assets.

There are several standard criticisms that environmental analysts make of the standard System of National Accounts (SNA): (i) the accounts measure the goods but not the 'bads' (in the form of pollution) associated with production activities; (ii) some environmental protection expenditures are measured as final output (the defensive expenditures issue); (iii) the depreciation of environmental assets and depletion of natural resources is not reflected in national income; (iv) environmental assets and natural resources are not measured in national wealth (although commercial resources, at least, do appear in the wealth accounts in the revised SNA - see United Nations 1993a); and (v) environmental liabilities, in the form of accumulations of pollutants, also do not appear anywhere in the accounts. However, the overriding concern with the standard SNA is that there is no means to determine if an economy is on a sustainable path.

Before describing the different approaches to green accounting in the literature, it is valuable to examine the ways in which natural resources and the environment are reflected in the existing national accounts. This breaks into two parts: commercial natural resources, where there is a market price, and environmental resources such as clean air or wildlife that lie outside the market system.

While commercial natural resources are measured directly in the accounts, in the sense that the value added associated with their exploitation is measured in national income, the economic value of these resources as assets appears only implicitly. The value of a subsoil resource deposit or standing forest as an asset is related to the flow of economic rent that results from its exploitation; for a given resource deposit this rent is measured as the difference between the market price of the resource and the full marginal cost of its extraction/harvest, including normal returns to capital. So resource rents show up as a portion of operating surplus for the resource sectors, but are not explicitly measured. Consequently the value of economic depreciation of a resource deposit as a result of exploitation is not measured either, which means that resource depletion does not enter into the calculation of net product, NNP or

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[1] Some of this material is published in Hamilton (1994)

NDP. While the guidelines for the balance sheet accounts in the SNA[2] call for the valuation of subsoil or standing natural resources, the change in value of these assets from year to year is recorded as a reconciliation item, and so again does not alter net product estimates.

Environmental resources are measured more indirectly in the accounts. To the extent that there is a commercial activity associated with an environmental asset, such as tourism or hunting, then the value added in this activity appears as part of national product. But the underlying asset, the pristine lake or wilderness, is not valued explicitly. When environmental quality deteriorates the effects show up indirectly in a variety of forms: loss of tourism industry income (as the lake is polluted, for instance); lost productivity of agriculture and living natural resources; increased repair and maintenance costs for buildings and other assets damaged by pollution; increased costs of inputs when water, for instance, must be cleaned prior to use in productive activities; increased health expenditures and lost productivity as a result of increasing morbidity and mortality; and diversion of resources from other valuable employment when accidents, such as oil spills, need to be cleaned up. All these effects are there in the accounts, but not directly and identifiably.

One aspect of government resource policy does show up directly in the accounts: since commercial resources are frequently government-owned, with the right to exploitation being leased, governments attempt to capture resource rents through royalty schemes, and these royalties are measured explicitly in the accounts. But broader environmental policies appear only indirectly. Whether through regulation or market-based instruments, policies aimed at abating pollution and preserving ecosystems affect the level of intermediate expenditures and the mix of investment between environmental protection facilities and conventional productive assets. The values associated with market-based instruments will show up in the accounts, as indirect taxes in the case of pollution taxes, or as investments (and corresponding assets in the balance sheet accounts) in the case of emissions permits[3]. Growth rates in national product are affected implicitly by environmental policies.

If there is a common thread running through the literature on green accounting, it is that use of the environment and natural resources represents asset consumption, and that one of the key problems with standard national accounts is that this is not reflected in the measures of income and product. Moreover, this literature is concerned with making explicit what is currently only implicit in the accounts with respect to natural resources and the environment.

With this as background, some notation is required in order to describe the various approaches to constructing green national accounts aggregates:

ES - environmental services
ED - environmental damages
DE - defensive expenditures
IR - invested resource rents
RD - resource discoveries
DEP - depletion of resources

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[2] The key elements of the System of National Accounts in what follows are the income and expenditure accounts, measuring current flows of economic activity (e.g., GDP), and the national balance sheet accounts measuring opening and closing stocks of assets (both financial and tangible) over the accounting period.

[3] Because they are assets with market values, tradable emission permits would be measured as intangible assets in the balance sheet accounts.

NFA - net financial assets
TA - tangible assets

What follows is a brief description and assessment of the main lines of thought on environmental national accounting. Three basic identities are presented below, followed by notes explaining and evaluating the key points. It is useful to summarize the approaches according to whether they are intended to alter GDP (gross domestic product as conventionally defined), NDP (conventional net domestic product), or national wealth (denoted NW) as measured in the National Balance Sheet Accounts, including a measure of natural wealth. gGDP, gNDP and gNW are the new green aggregates:

$$gGDP = GDP + ES \pm ED_1 - DE - IR$$

$$gNDP = NDP + RD - DEP - ED_2$$

$$gNW = NFA + TA_H + TA_N$$

Here ED_1 and ED_2 represent different approaches to valuing environmental damage. We consider the component parts of these expressions in sequence:

ES Peskin (1989) advocates augmenting GDP by a measure of environmental services, viewed chiefly as waste disposal services, which are provided free of cost by the environment. However, to the extent that producers use these services without paying for them, then it is arguable that their value already shows up in profits and therefore in GDP.

ED_1 Environmental damages can be either added or subtracted. Peskin views the negative externalities associated with producers availing themselves of the services of the environment as a deduction from GDP[4]. Harrison (1989) takes the opposite tack: since gross product includes the consumption of assets by definition, conventional GDP is understated because it does not measure the consumption of environmental assets. Note that this would require the estimation of a dollar value for total environmental deterioration, including that which was prevented as a result of current abatement expenditures.

DE Defensive expenditures are expenditures on environmental protection undertaken by households (Juster 1973) and governments (Herfindahl and Kneese 1973). It is argued that environmental expenditures by households do not increase welfare but merely preserve the status quo (e.g. not getting sick from environmental deterioration) and that government environmental protection expenditures (e.g. on waste management) are essentially intermediate in character.

IR El Serafy (1989) calls for the deduction of hypothetically invested resource rents from GDP, arguing that true income from a non-renewable natural resource is that constant stream of income that can be obtained from investing a portion of the rents from

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[4] Environmental services are, in effect, positive externalities provided by nature. Peskin is not explicit concerning whether environmental damage would appear subsequently as a decline in environmental services.

exploitation in a fund (a suitable program will ensure that rents in excess of the portion invested will be identically equal to interest on the fund at the point of exhaustion). Hartwick and Hageman (1993) show that this is equivalent to valuing the change in the present value of the resource stock as a result of its exploitation - i.e. it is a true user cost.

RD Turning to the measurement of net domestic product, Repetto *et al.* (1989) reason that in order to maintain consistency between product and wealth accounts, augmented to include natural resources, the full value of natural resource discoveries should be added to net product in the period in which they are made. Hartwick (1990) developed a model in which discoveries are similarly added to net product. Weitzman (1976) showed formally that an unanticipated resource discovery does indeed increase the amount of sustainable product and income, but by less than the full value of the discovery in the period it was made.

DEP Depletion of natural resources is the major adjustment to net product suggested by Repetto *et al.* (1989). Depletion is valued as the total of resource rents taken in the accounting period (the "net price" approach) or, in the case of soil erosion, as the present value of foregone production. Repetto notes that the Hotelling rule, that resource rents in an efficient market will increase at a percentage rate of change equal to the interest rate, will yield this valuation of resource depletion. The United Nations (1993b) suggests valuing depletion using either the user cost or net price approaches.

ED_2 As an alternative to deducting environmental damage from gross product, Bartelmus *et al.* (1989) suggest deducting it from net product as asset consumption. This asset consumption is valued as the cost of returning the environmental asset to its state at the beginning of the accounting period. Hueting and Bosch (1990) offer an alternative methodology in which environmental deterioration is valued as the costs that would be incurred to achieve sustainable use of the environment (rather than merely preserving its state, as in Bartelmus *et al.*). UN guidelines (United Nations 1993b) suggest contingent valuation as an alternative basis for valuing environmental degradation, but without discussing how, or whether, this can be applied to the environment as a whole.

NFA Turning finally to measures of national wealth, net financial assets are an important component of total wealth. For an open economy the difference between financial assets and liabilities is equal to either net claims on foreign assets or net foreign indebtedness. The scale of investment of resource rents by OPEC producers in Europe and North America indicates the significance of this type of wealth where domestic investment opportunities are limited.

TA_H Human-made tangible assets are the familiar elements of reproducible capital: machinery, equipment, buildings and infrastructure. The Hartwick rule (Hartwick 1977) states that, under suitable conditions of substitutability, investing resource rents in reproducible capital will permit a non-declining stream of consumption into the indefinite future. Building up human-made assets to match the drawing down of natural resources, thereby preserving wealth, fits the criterion for weak sustainability described by Pearce *et al.* (1989).

TA_N Natural tangible assets are measured by the dollar value of commercial resources (minerals, energy, forests and fish) and environmental resources (natural environments providing non-market services including waste disposal and amenity value). Scott (1956) first suggested expanding the national balance sheet account to include commercial resources. The problems in doing this include defining the appropriate measure of extent (proven reserves, i.e. those that can be produced profitably at current prices and costs, would be the correct measure) and, in the absence of markets for publicly held resource deposits, deriving values for these deposits. Hamilton (1991) argued that total national wealth per capita is a useful measure of sustainability. Pearce *et al.* (1989) point out that there is limited substitutability between certain critical natural assets and human-made assets, which argues for maintaining the value of at least some natural assets constant or increasing as a condition for sustainability.

This broad range of approaches to green national accounting is reflected in the activities of national statistical offices as well. Hamilton *et al.* (1993) review the efforts of Brazil, Canada, France, Germany, the Netherlands and Norway to develop new green accounts. The US Bureau of Economic Analysis has opted to concentrate on expanding the national balance sheet through an 'Integrated Economic and Environmental Satellite Account' - in addition, this account breaks out expenditures on environmental protection (Bureau of Economic Analysis 1994).

There is little unanimity, therefore, in the literature on green accounting. This is the prime motivation for taking a more formal approach to the problem. This in turn will permit an assessment of the main ideas in the foregoing literature.

Formal Models of National Accounts

By asking a simple question, why we measure both consumption and investment in national product when the economic goal is to maximize consumption, Weitzman (1976) provided the theoretical framework for a fruitful line of inquiry into the relationship between resources, the environment and national product, the prime examples being Solow (1986), Hartwick (1990, 1992, 1993), Mäler (1991) and Hamilton (1994).

Weitzman's answer to the question was that, if we assume we are on the optimal path of a dynamic competitive economy, then national product measured as the sum of consumption and investment in the current period is, if held constant and the present value taken, just equal to the present value of consumption along the optimal path - he calls it the *stationary equivalent* of future consumption. In an equally appealing interpretation of this framework, Solow (1986) showed that increases in national product from some assumed initial value are equal to the discount rate times the accumulation of capital from the initial period to the present - national product can thus be conceived as the interest on total accumulated wealth.

There is perhaps a simpler welfare interpretation of national product: as Weitzman (1976) noted, the current value Hamiltonian of an optimal control representation of the economy,

$$\max \int_0^\infty C e^{-rt}\, dt \quad \text{subject to}$$

$$F = C + p\dot{K},$$

is just $H = C + p\dot{K}$, i.e., it is equal to national product (in this formulation, *F* is production, *C* consumption, *K* capital and *p* the relative price of capital and consumption goods; *r* is the constant discount rate). From Pontryagin's Maximum Principle we know that the Hamiltonian is maximized at every point in time along the optimal path. Therefore, national product is simply that quantity that a social planner would choose to maximize in each period in order to maximize the present value of consumption.

The present value of future consumption is a wealth measure, and Usher (1994) shows that the Hamiltonian is the return to this wealth under assumptions of fixed technology and endogenous consumption and capital formation. Usher demonstrates that the Hamiltonian is not equal to the return on wealth so defined if: (i) consumption can increase autonomously; (ii) there is autonomous technological change; or (iii) there are tax distortions in the economy.

For a model with exhaustible resources we wish to compare the rate of change of NNP to that of the Hamiltonian function for an optimal growth problem with exhaustible resources. If the rate of change of NNP has the same sign as the rate of change of the Hamiltonian along the optimal path, then NNP can serve as a useful indicator to guide policies for optimal growth, in the sense that policies that increase the Hamiltonian along the optimal path also increase NNP.

For a simple economy where $F(K,R) = C + \dot{K}$ and resources are costless to extract, the current value Hamiltonian function,

$$H = U(C) + U_C\left(\dot{K} - F_R R\right)$$

is maximized at each point in time along the path that maximizes the present value of utility, using a constant utility discount factor *r*. The optimal path is determined by the initial conditions and the following dynamic efficiency conditions,

$\frac{\dot{F}_R}{F_R} = F_K$, the Hotelling rule, and $\frac{\dot{U}_C}{U_C} = r - F_K$, the Ramsey rule.

NNP for this economy is given by,

$$NNP = C + \dot{K} - F_R R = F - F_R R.$$

This corresponds to valuing each flow at the price that supports the optimum, and then converting to a consumption numeraire. The definition of NNP is derived more formally in Chapter 9.

Note that $\dot{K} - F_R R$ is the change in real wealth in this model. We first derive the expression for the rate of change of the Hamiltonian:

$$\begin{aligned}\dot{H} &= U_C\dot{C} + U_C\tfrac{d}{dt}\left(\dot{K} - F_R R\right) + \dot{U}_C\left(\dot{K} - F_R R\right)\\ &= U_C\left[\dot{NNP} + \left(r - F_K\right)\left(\dot{K} - F_R R\right)\right]\end{aligned}$$

This results from applying the Ramsey rule. Applying the Hotelling rule gives the following expression for the rate of change of NNP:

$$\begin{aligned}\dot{NNP} &= \dot{F} - \dot{F}_R R - F_R \dot{R} \\ &= F_K \dot{K} + F_R \dot{R} - \dot{F}_R R - F_R \dot{R} \\ &= F_K \left(\dot{K} - \dot{F}_R R \right)\end{aligned}$$

The rate of change of the Hamiltonian therefore simplifies to:

$$\dot{H} = rU_C \left(\dot{K} - \dot{F}_R R \right).$$

We conclude that both NNP and the Hamiltonian peak at the same time (when the change in real wealth is 0), and that their rates of change have the same sign at each point in time. NNP is a useful welfare indicator. Moreover,

$$\dot{NNP} = \frac{1}{r} \frac{F_K}{U_C} \dot{H}.$$

The rate of change of NNP is a simple scaling of that of the Hamiltonian, with the scale factor declining once consumption begins its decline along the optimal path.

For the next two chapters this will be the primary welfare interpretation that will be attributed to 'NNP-like' aggregates derived from the current value Hamiltonian. The relationship between these aggregates and sustainable development is explicitly considered in Chapter 9, which deals with wealth accounting.

7. Exhaustible Resources and Net National Product

As argued in Chapter 6, the recent enthusiasm for 'green' national accounting is aimed at better reflecting resource depletion and environmental damage in measures of economic development. A key issue in the treatment of exhaustible resources in the National Accounts is the source of much confusion: the method of valuing resource discoveries, with El Serafy (1989), Repetto *et al.* (1989) and Hartwick (1990) offering the chief variants. El Serafy, in common with United Nations (1993b), opts to add discoveries to the stock of resources, while Repetto *et al.* and Hartwick add discoveries to current Net National Product (NNP).

The lack of consensus on these varying methodologies has practical consequences. In studies employing both the El Serafy and Repetto valuation methods, for Australia (Young (1993)), and Brazil (Serôa da Motta and Young (1991)), widely divergent valuations of adjusted NNP based on the treatment of resource discoveries have led to discouragement concerning the usefulness and applicability of alternative national accounting aggregates. This chapter presents a further development of the model of Hartwick (1990) to clarify the approach to resource discoveries in extending the national accounts. The issue is clearly one of extending the accounts, since the recent revision to the System of National Accounts (United Nations (1993a)) recommends that resource discoveries appear in the 'other volume changes' component of the balance sheet accounts, so that there is no consequent effect on the measure of standard NNP.

Another matter of practical consequence for the development of extended national accounts concerns the valuation of resource depletion. Theoretical models such as Hartwick (1990) and Mäler (1991) suggest valuing depletion at the current rental rate, price minus marginal cost. However these models assume, in effect, a single pool of resource that is exhausted, with a well-defined cost function for this pool. In practice even homogeneous resources are found in deposits of varying quality and therefore cost of extraction. The effects of heterogeneous resource deposits on depletion-adjusted NNP are derived in the second part of this chapter.

Resource Discoveries

Weitzman (1976) was the first to show that the current value Hamiltonian of an optimal growth representation of the economy is, expressed in suitable units, the NNP of the economy. To be precise, Weitzman showed that to maximize the present value of consumption over an infinite time horizon, the current value Hamiltonian is given by $H = C + p\dot{K}$, where p is the relative price of capital and consumer goods and C and K are the amount of consumption and capital on the optimal path. This is clearly the measure of net national product, consumption plus net investment.

NNP so defined can be given several different welfare interpretations. Weitzman called it the 'stationary equivalent' of future consumption, since the value of NNP at any given point in time, if held constant and present value taken over an infinite horizon, can be shown to be equal to the present value of (optimal) consumption over the same horizon. Solow (1986) offered a more satisfying rendering, by showing that NNP is equal to the interest on net capital accumulation along an optimal path from some assumed initial year. As noted in the previous chapter, the Maximum Principle offers a more natural interpretation still: NNP is that quantity that a planner would choose to maximize at each point in time in order to maximize the present value of consumption.

Hartwick (1990) and Mäler (1991) extend the Weitzman approach to examine economies that include the depletion of natural resources and the environment. The Hamiltonian functions they derive embody conventional NNP and adjustments that reflect the changes in resource stocks - by the reasoning just given, the results should be viewed as the proper value of NNP since it is this quantity whose maximization leads to the maximization of the present value of consumption. It is to Hartwick's treatment of resource discoveries that we now turn.

For utility function $U(C)$, consumption C, production function $F(K,R,L)$, capital stock K, resource use R, labour L, resource discoveries D, and resource stocks S, Hartwick (1990) assumes a cost function for resource extraction $f(R,S)$, and a discovery cost function $g(D,S)$, so that both extraction and discoveries exhibit a stock effect. Given constant pure rate of time preference r (all other variables are implicitly functions of time), the model is,

$$\max \int_0^\infty U(C)e^{-rt}dt \quad \text{such that:}$$
$$\dot{K} = F - C - f - g$$
$$\dot{S} = -R + D.$$

Here there are two state variables, K and S, and three control variables, C, R, and D. While the relationship between exploration effort and discoveries is in general stochastic, Hartwick has chosen a deterministic form: the present and future benefits of discoveries D are traded off against the current cost of discovery g. The fact that the model is closed to trade does not affect the principal results of the model, at least for small open economies - Gomez-Lobo, in Gomez-Lobo *et al.* (1993), shows that accounting for external trade simply adds a term of the form iA to the resulting measure of NNP, where i is an international interest rate and A is the net accumulation of foreign assets.

The current value Hamiltonian for this problem is,

$$H = U + \gamma_1(F - C - f - g) + \gamma_2(-R + D)$$

and the first order conditions for maximization are,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \quad \Rightarrow \quad \gamma_1 = U_C \tag{7.1}$$

$$\frac{\partial H}{\partial R} = 0 = \gamma_1(F_R - f_R) - \gamma_2 \quad \Rightarrow \quad \gamma_2 = U_C(F_R - f_R) \tag{7.2}$$

$$\frac{\partial H}{\partial D} = 0 = -\gamma_1 g_D + \gamma_2 \quad \Rightarrow \quad \gamma_2 = U_C g_D. \tag{7.3}$$

A consequence of expressions (7.2) and (7.3), one that Hartwick does not draw, is that $g_D = (F_R - f_R)$. This is the result of having two control variables, R and D, associated with one state variable, S. The current value Hamiltonian resulting from expressions (7.1)-(7.3) is therefore,

$$H = U + U_C\dot{K} - U_C(F_R - f_R)(R - D).$$

The dynamic efficiency condition for U_C is given by,

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} \Rightarrow \frac{\dot{U}_C}{U_C} = r - F_K. \tag{7.4}$$

It is natural, given that the Hamiltonian is measured in utils, to define NNP as H / U_C, as Hartwick does. Mäler (1991) uses the linear support of the Hamiltonian function, which approximates the function at any point by its tangent hyperplane. The expedient used in Chapter 6 is simply to value each flow in the Hamiltonian at its shadow price and then to normalize by the marginal utility of consumption. This normalization preserves relative prices at each point in time and, as shown in Chapter 6, the NNP so derived is a valid indicator for the guidance of policies for optimal growth. In the present model NNP can therefore be defined as,

$$\text{NNP} = C + \dot{K} - (F_R - f_R)(R - D).$$

This is the formula for NNP that Hartwick (1990) derives. It says that adjusted net national product is equal to traditional NNP less resource depletion plus resource discoveries, valued at the unit rental rate. As a guide to practical national accounting it is the latter term that is disturbing - it suggests that even very large discoveries should be valued at the full rental rate as an addition to national product even though no production has taken place. As noted in the introduction, this is also the approach taken by Repetto *et al.* (1989), which has the effect of dramatically reducing the growth rate of adjusted NNP that they derive for Indonesia.

Because expressions (7.2) and (7.3) imply that marginal discovery costs must equal unit resource rents in this model, Hartwick could as easily have written NNP as,

$$\text{NNP} = C + \dot{K} - g_D(R - D).$$

This would imply that resource depletion should be valued at the marginal discovery cost. Using marginal discovery costs as an approximation to resource rents was suggested by Devarajan and Fisher (1982).

The final point to note about the Hartwick model, before developing an alternative, is that although f and g are both functions of the remaining stock S, this has no effect on the derivation of NNP. In fact, making discovery costs a function of the remaining stock seems implausible. Where the stock effects do show up is in the second dynamic efficiency condition,

$$\dot{\gamma}_2 = r\gamma_2 - \frac{\partial H}{\partial S} \quad \Rightarrow \quad \frac{\dot{\gamma}_2}{\gamma_2} = r + \frac{f_S + g_S}{g_D}. \tag{7.5}$$

Defining $n \equiv F_R - f_R$, and using expression (7.4), we therefore derive,

$$\frac{\dot{n}}{n} = F_K + \frac{f_S + g_S}{g_D}.$$

This is a Hotelling-type rule for resource rents. Stock effects therefore influence the time path of resource rents in this model.

This completes our dissection of Hartwick (1990). There are several points to note. First, the choice of a linear utility function has more expository value than inherent justification. Second, the fact that both R and D are control variables for the state variable S results in marginal discovery costs that must equal resource rental rates. Thus, although Hartwick uses resource rents as the basis of valuing both resource depletion and discoveries, an alternative would have been to value both at the marginal discovery cost in the formula for NNP. Finally, making resource discovery costs a function of the remaining stock of resource lacks intuitive appeal, although the effects of this assumption show up only indirectly in the time path of resource rents.

An alternative model[1]

A more appealing treatment of resource discoveries would be to consider the cost of discoveries to be an increasing function of both the amount discovered in the current period and cumulative discoveries to date[2] - i.e., the more resource that is discovered, the more expensive it is to discover the next marginal unit. This is related to the formulation of exploration given in Pindyck (1978). Representing cumulative discoveries as M, the definition of the cost of resource discoveries is therefore $g(D,M)$, where $g_D > 0$ and $g_M > 0$. For simplicity we drop stock effects in resource extraction, so that $f = f(R)$. The optimal growth model, where K, S, and M are the state variables, and C, R, and D are the control variables, therefore becomes,

$$\max \int_0^\infty U(C)e^{-rt}dt \text{ subject to:}$$
$$\dot{K} = F - C - f - g$$
$$\dot{S} = -R + D$$
$$\dot{M} = D.$$

We assume that there is a finite amount of resource known to exist in the initial period, S_0, and a finite amount remaining to be discovered, V_0. Assuming resources are essential for production, efficiency requires that $M \rightarrow V_0$, and $S \rightarrow 0$ as $t \rightarrow \infty$.

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[1] This model was developed independently by Hartwick and published in Hartwick (1993).

[2] I am indebted to John Livernois for suggesting this.

For this problem the current value Hamiltonian is,

$$H = U + \gamma_1(F - C - f - g) + \gamma_2(-R + D) + \gamma_3 D.$$

The first-order conditions for utility to be maximized are as follows:

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \;\Rightarrow\; \gamma_1 = U_C \tag{7.6}$$

$$\frac{\partial H}{\partial R} = 0 = \gamma_1(F_R - f_R) - \gamma_2 \;\Rightarrow\; \gamma_2 = U_C(F_R - f_R) \tag{7.7}$$

$$\frac{\partial H}{\partial D} = 0 = -\gamma_1 g_D + \gamma_2 + \gamma_3 \;\Rightarrow\; \gamma_3 = U_C\big(g_D - (F_R - f_R)\big)\,. \tag{7.8}$$

Collecting terms from expressions (7.6)-(7.8), the current value Hamiltonian becomes,

$$H = U + U_C(F - C - f - g) + U_C(F_R - f_R)(-R + D) + U_C(g_D - (F_R - f_R))D.$$

We therefore derive the following expressions for NNP,

$$\text{NNP} = C + \dot{K} + g_D D - (F_R - f_R)R.$$

In both Hartwick (1990) and the present derivation, resource depletion appears as a deduction from NNP, calculated as the value of current resource rents. Whereas Hartwick values all resource discoveries as unit rent times the amount discovered, the new expression includes only $g_D D$, the marginal cost of discovering resources times the amount discovered, as an addition to NNP. This term may be conceived to be a measure of investment in resource discoveries.

One effect of introducing cumulative discoveries into the model is to decouple marginal discovery costs and unit resource rents. In fact we can conclude more. Note that γ_3 is the shadow price of cumulative discoveries. Since we have assumed that $g_M > 0$, this implies that $\gamma_3 < 0$. From expression (7.8) we therefore conclude that $g_D < F_R - f_R$, i.e., marginal discovery costs must be less than resource rents.

This result may be compared with Hartwick (1991), where for a resource *firm* a component of resource rent is shown to be the marginal change in surplus associated with discovery (this is related in an obvious way to the marginal cost of discovery) times the amount discovered. It should also be compared with Lasserre (1985), who employed a stochastic model of exploration to show that marginal discovery costs must be less than current resource rents by the amount of the rent on resource prospects. In Lasserre's model resource prospects are exhausted by new discoveries (i.e., the amount of resource that can be found is finite) - this has at least the same flavour as the present model, where it is assumed that the greater the quantity of cumulative resource discoveries, the greater the cost of discovery.

The dynamic efficiency conditions for this model are also of interest. The first (corresponding to γ_1) is the same as that for the Hartwick (1990) model, expression (7.4). For a general utility function with declining marginal utility of consumption we can define the elasticity of the marginal utility of consumption to be,

$$\eta(C) = -\frac{U_{CC}}{U_C}C.$$

Expression (7.4) then reduces to,

$$r + \eta(C)\frac{\dot{C}}{C} = F_K. \tag{7.9}$$

As shown in Chapter 2, development is not *sustainable* in this model, by Pezzey's (1989) definition, as long as the pure rate of time preference is positive.

Because we have assumed no stock effects on resource extraction and consumption, the second efficiency condition is derived from expression (7.5) to yield, for unit resource rents *n*,

$$\frac{\dot{n}}{n} = F_K.$$

This is a standard form of the Hotelling (1931) rule, that unit resource rents must increase at a percentage rate of change equal to the return on capital.

The third dynamic efficiency condition is,

$$\dot{\gamma}_3 = r\gamma_3 - \frac{\partial H}{\partial M} = r\gamma_3 + \gamma_1 g_M.$$

Since expressions (7.6) - (7.8) imply that $\gamma_3 = \gamma_1 g_D - \gamma_2$, the preceding expression reduces to,

$$\dot{g}_D = F_K g_D + g_M.$$

But $\dot{g}_D = g_{DD}\dot{D} + g_{DM}\dot{M} = g_{DD}\dot{D} + g_{DM}D$, so that,

$$\dot{D} = \frac{1}{g_{DD}}\left(F_K g_D + g_M - g_{DM}D\right).$$

Resource discoveries eventually decline if either (i) $F_K g_D \to 0$, and suitable convexity conditions hold, $g_{DD} > 0$, $g_{DM}D > g_M$, or (ii) suitable concavity conditions hold, $g_{DD} < 0$, $g_{DM}D < g_M$. Of course we know that the finiteness of total resources also implies that discoveries must eventually decline, so these assumptions are necessary components of this model.

In terms of practical national accounting, the key results of this model are that resource depletion valued at the unit rental rate should be deducted from NNP, while resource discoveries, valued at the marginal discovery cost, should be added. Assuming resource discovery costs increase with cumulative discoveries, then marginal discovery costs will be less than unit resource rents.

Resource Extraction with Heterogeneous Deposits

If exhaustible resources are to be treated in extended or 'green' national accounts, there is another vexing question concerning precisely which unit rent to use in valuing depletion when there are several heterogeneous resource deposits. As noted earlier, the Hartwick (1990) model and the one presented in the previous section assume, in effect, one homogeneous pool of resource and so this question does not arise in these models. In a combined theoretical and empirical study Hartwick and Lindsey (1989) argue that the scarcity rent on US oil reserves is represented by price minus the marginal cost of frontier reserves, and so derive very small adjustments to NNP when account is taken of oil depletion. This would seem to be contrary to normal national accounting practice, where it is customary to value assets at their market price - extending this notion to heterogeneous resource deposits, where extraction costs and therefore unit rents vary from deposit to deposit, one would expect the market price of resource deposits of similar physical extent to vary according to the unit rents available. Given that the US has widely disparate oil resources - comparing production costs between continental and North Slope deposits for instance - standard national accounting would suggest that the value of depletion should reflect these varying unit rents.

We will formalize these ideas in a model of heterogeneous deposits of a homogeneous resource. Ignoring, for the sake of simplicity, discoveries and stock effects in the extraction cost function, assume that extraction R_i takes place in m deposits. Then, since the resource is itself homogeneous, we can write,

$$F = F(K,R,L), \quad \text{where } R = \sum_{i=1}^{m} R_i .$$

Heterogeneous resource deposits are reflected in the extraction cost functions,

$$f_i = f_i(R_i).$$

The dynamic constraints for the optimal control problem become,

$$\dot{K} = F - C - \sum_{i=1}^{m} f_i ,$$

and, for the m resource deposits,

$$\dot{S}_i = -R_i,\ i=1..m.$$

Again, the objective is to maximize the present value of utility over an infinite horizon, given a fixed pure rate of time preference r. The current value Hamiltonian for the problem is,

$$H = U + \gamma_1\left(F - C - \sum_{i=1}^{m} f_i\right) - \sum_{i=1}^{m} \lambda_i R_i,$$

where C and R_i are the control variables and γ_1 and λ_i are the corresponding co-state variables. The first order conditions for a maximum are,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \quad \Rightarrow \quad \gamma_1 = U_C, \text{ and}$$

$$\frac{\partial H}{\partial R_i} = 0 = \gamma_1 \frac{\partial F}{\partial R_i} - \gamma_1 \frac{df_i}{dR_i} - \lambda_i, \quad i = 1..m.$$

Because $\frac{\partial F}{\partial R_i} = F_R \frac{\partial R}{\partial R_i} = F_R$, the latter conditions reduce to,

$$\lambda_i = U_C\left(F_R - \frac{df_i}{dR_i}\right).$$

We therefore arrive at the following expression for net national product,

$$\text{NNP} = C + \dot{K} - \sum_{i=1}^{m}\left(F_R - \frac{df_i}{dR_i}\right)R_i.$$

Net national product is therefore the standard measure, consumption plus investment, less the sum of the resource rents on the individual resource deposits. This result therefore brings the treatment of heterogeneous resources into line with other assets in the national accounts.

One question that arises is why production should take place from deposits that yield less rent than the deposit with the maximum unit rent - why doesn't production proceed through merit order? The answer to this can be seen by examining the dynamic efficiency conditions. The condition for γ_1 is precisely that of expression (7.4). To examine those for the λ_i we first define the unit rent on the i-th deposit to be,

$$n_i \equiv \left(F_R - \frac{df_i}{dR_i}\right), \text{ and so } \lambda_i = U_C n_i.$$

The dynamic efficiency conditions therefore become, for each $i = 1..m$,

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial H}{\partial S_i} = r\lambda_i,$$

so that we have, using expression (7.4),

$$\frac{\dot{n}_i}{n_i} = F_K, \quad i = 1..m.$$

This says that the Hotelling rule must apply to each individual resource deposit. Resource rents may have different levels for different deposits, but each must increase with a percentage rate of change equal to the rate of return on capital. Under these conditions extraction will be efficient for each resource deposit, and the owners of the deposits will be indifferent between holding them and holding other assets yielding the market rate of return.

Rather than valuing depletion using the unit rent on the most marginal deposit, as in Hartwick and Lindsey (1989), this model says that depletion must be calculated for each distinct deposit and summed to arrive at the depletion adjustment to NNP. While this may mean more work for the national accountants, it does bring the treatment of resource deposits into congruence with the practice for other assets in the accounts.

Conclusions

Hartwick's (1990) specification of discovery costs depending on the amount of remaining stock S, it has been argued, lacks plausibility. It turns out, however, that this assumption affects only the dynamics of resource rents and not the derived NNP. Because both discoveries and extraction are control variables for S, the resulting shadow prices for these quantities are equal, so that Hartwick's result for NNP could also be expressed with discoveries and extraction valued at the marginal discovery cost rather than the unit rent.

Introducing a new state variable, cumulative discoveries, into the model provides a touch of realism (the notion that the more resource you find the more costly it is to find the next unit) and effectively decouples marginal discovery costs from unit rents - in fact marginal discovery costs must be less than unit rents. The result is a more satisfactory treatment of discoveries and depletion in adjusted NNP, with the former valued at the marginal discovery cost and the latter at the unit rental rate.

The dynamics of the model developed above have two points of interest. First, because total resources (discovered and undiscovered) are assumed to be fixed, the model shares with the standard model of exhaustible resources the characteristic that consumption must decline for positive rates of pure time preference. Positive time preference is inimical to sustainability when resources are finite. Secondly, the assumption of fixed total resources implies that discoveries must also decline over time, with the consequence that the discovery cost function must obey specified convexity conditions.

The model of heterogeneous resource deposits is also offered in the spirit of introducing some realism into the discussion of adjusted NNP. This is a matter of some practical consequence as well, as the valuation of US oil depletion at the rental rate for North Slope oil in Hartwick and Lindsey (1989) indicates. The key result is that resource rents calculated deposit by deposit must be deducted from NNP to measure true net product. In terms of the dynamics of the model, it was shown that the unit resource rents on each individual deposit must obey the Hotelling rule if extraction is to be efficient.

How much of this will 'green' national accountants take up? With regard to resource discoveries, the standard national accounts treat most exploration costs as investment. Where the model suggests using marginal discovery costs times the amount discovered, the standard accounts are, in effect, using average discovery costs times the amount discovered. Over a typical accounting period, one year, the divergence between the two measures may not be that great. This suggests that 'green' accounts need not be adjusted for resource discoveries. With regard to heterogeneous resource deposits, the practitioners are already working towards measuring different resource rental rates for different grades of deposits, as the work of Born (1992) indicates in the case of Canadian oil and gas. The practice in measuring depletion is to measure average rental rates (price minus average cost) for individual deposits, multiplied by the amount extracted. Again, for individual deposits and accounting periods of one year, the divergence between this measure and that suggested by theory, based on price minus marginal cost, may be small.

8. Pollution and Pollution Abatement in the National Accounts[1]

As discussed in Chapter 6, there is a pervasive sense that the conventional national accounts overstate the measurement of 'true' income and product because they do not account for the damage to the environment from pollution emissions. The basic notions are that the value of environmental damage should be deducted from domestic product and that at least some final expenditures on environmental protection, 'defensive expenditures', should not be considered to be final demand. This chapter develops a series of models to examine these claims and to suggest extensions to the standard accounts to account for environmental change.

As noted in the Introduction, Hartwick (1990,1993) and Mäler (1991) both explicitly extend Weitzman's (1976) approach to national accounting by maximizing the present value of utility under different presumptions about the depletion of natural resources and damage to the environment from pollution. Mäler constructs one large model that contains, in addition to consumption and investment goods, a flow resource that is damaged by pollution emissions, a living resource that is harvested and whose growth is affected by inputs of goods and labour, and a household production function through which, by inputting goods and labour, households can increase their benefits from the environment (i.e., the flow resource). The key result in Mäler (1991) is that deductions for defensive expenditures should not be made in the measure of national welfare derived from the model.

Hartwick (1990) presents two pollution-related models, one in which there is a stock pollutant that accumulates emissions and is subject to a natural dissipation process - this pollutant appears (negatively) in the production function - and a second in which the rate of change of the stock pollutant appears in the utility function as well. In these models pollution is mitigated by expenditures that affect the rate of the natural dissipation process, an unlikely form of mitigation. Hartwick (1993) offers a more intuitive model in which utility is related to the accumulated stock of pollutant and abatement expenditures limit the quantity of pollution emissions.

This chapter builds on and extends the Hartwick and Mäler models in several directions: (i) an explicit approach is taken to pollution abatement expenditures, and these are related to optimal emission taxes; (ii) a series of models are constructed to examine individually the effects of flow pollutants, stock pollutants, degradation of natural dissipation processes, stock pollutants linked directly to exhaustible resources (the CO_2 problem), and flow pollutants that damage living resources (the acid rain problem); and (iii) the treatment of household defensive expenditures is re-examined to yield a variation on Mäler's interpretation.

Chapter 6 evaluated how the traditional 'green national accounting' literature has approached pollution issues. As a brief summary of this literature, the general contention is that some measure of the cost of environmental protection should be deducted from GNP (or net product) to reflect damage to the environment, and that defensive expenditures should be deducted as well. In this view conventional national product is an overstatement of 'true' product. This chapter aims to provide a more rigorous basis for adjusting the accounts to

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[1] This material is published in Hamilton (forthcoming).

reflect a wide variety of pollution issues and to test the extent to which theory supports the conclusions of this literature.

Models of Green National Accounts

Each of the models presented below is designed to examine a particular facet of the treatment of pollution in the national accounts. A number of simplifying assumptions are made: (i) technology is assumed to be unchanging; (ii) the production function F exhibits declining returns to factors; (iii) there is a single product that may be consumed, invested or used in abating pollution; (iv) labour markets are assumed be in equilibrium, so that the welfare effects of labour do not figure in what follows (as was derived in Mäler (1991)); and (v) the discount rate r is constant. U is the utility function, and C consumption; in most of the models utility is assumed to be an increasing function of both consumption and the flow of environmental services B, measured in appropriate (but not necessarily monetary) units. B is assumed to measure pure *non-market* environmental services, so that there is no duplication with the indirect effects of environmental quality on production or asset values. With the exception of r, all variables are functions of time. Additional assumptions will be added as required.

The general ideas developed in the following models are that the natural environment provides a flow of non-market services that can be diminished by pollution emissions, that this flow of services yields utility, and that produced goods can be employed to abate pollution emissions.

Model 1: flow pollutant related to production.

A flow pollutant is a pollutant whose current level of *emissions* can be assumed to affect the level of services derived from the environment. Any pollutant with noxious effects that are not cumulative, such as a toxin, could serve as an example. The simple economy for the model of green national accounts is therefore one where emissions are assumed to be related to the level of production, $e = e(F)$, production is a function of produced capital and labour, $F = F(K,L)$, and output of the composite good can either be consumed or invested, so that,

$$F = C + \dot{K}.$$

The objective of the social planner for this economy is to maximize the present value of utility over an infinite time horizon, where utility U is a function of both consumption C and the level of environmental services B. Utility is assumed to be discounted at a fixed rate r. Environmental services are negatively related to pollution emissions as,

$$B(e) = B_0 - \alpha e.$$

Here B_0 is the level of environmental services that flow from a pristine environment, while α is the amount by which services decline when a unit of pollution is emitted. While it is not essential to specify a linear relationship between emissions and environmental services, it simplifies the exposition. The problem therefore is,

$$\max \int_0^\infty U(C,B)e^{-rt}dt \quad \text{subject to:}$$
$$\dot{K} = F - C$$

For γ_1 as the shadow price of capital, the current value Hamiltonian function for this programme is,

$$H = U + \gamma_1 \dot{K} = U + \gamma_1 (F - C).$$

The only control variable is consumption *C*, and therefore the first order condition for a maximum is,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \Rightarrow \gamma_1 = U_C.$$

The second order condition for the Hamiltonian to be maximized is $U_{CC} < 0$ (i.e., declining marginal utility of consumption).

The Hamiltonian function is measured in utils, and so must be transformed into consumption units in order to yield an expression that conforms more closely to conventional national accounting aggregates. As in Chapter 6, this is done in two steps: (i) each flow in the Hamiltonian - consumption, environmental services and investment - is valued at its shadow price in utils; and (ii) the resulting expression is divided by the marginal utility of consumption U_C to give a measure of economic welfare (*MEW*) in consumption units - again, this will be derived formally in Chapter 9. Scaling by the marginal utility of consumption yields the correct relative prices between flows at each point in time. The resulting expression is,

$$MEW = C + \dot{K} + \frac{U_B}{U_C} B.$$

Here economic welfare is measured as the sum of GNP ($C + \dot{K}$) and the value of the flow of environmental services. Note that U_B/U_C is the price that utility-maximizing consumers would be willing to pay for a marginal unit of environmental service. Pollution flows can be brought explicitly into the picture by substituting the expression for *B*,

$$MEW = C + \dot{K} - \alpha \frac{U_B}{U_C} e + \frac{U_B}{U_C} B_0. \tag{8.1}$$

Here $\alpha U_B/U_C$ is the marginal social cost of a unit of emissions, yielding the correct valuation of pollution in the aggregate welfare measure. This is also clearly the level of a Pigovian emissions tax sufficient to maximize welfare in each period. The last term in this expression is the (constant) environmental service flow from a pristine environment valued at (varying) current prices.

This model can be made more realistic and more general if we assume that the composite good can both be invested in pollution abatement capital K_a and spent on current abatement expenditures a in order to reduce pollution emissions to welfare-maximizing levels. The emission function therefore becomes,

$$e = e(F, K_a, a), \text{ with } e_a < 0 \text{ and } e_{K_a} < 0.$$

Introducing a new control variable m for investment in pollution abatement capital, the maximization problem becomes,

$$\max \int_0^\infty U(C,B)e^{-rt}dt \quad \text{subject to:}$$

$$\dot{K} = F - C - a - m$$

$$\dot{K}_a = m$$

The current value Hamiltonian for this programme is,

$$\begin{aligned} H &= U + \gamma_1 \dot{K} + \gamma_2 \dot{K}_a \\ &= U + \gamma_1 (F - C - a - m) + \gamma_2 m \end{aligned}$$

and the first order conditions for a maximum are:

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \Rightarrow \gamma_1 = U_C$$

$$\frac{\partial H}{\partial m} = 0 = -\gamma_1 + \gamma_2 \Rightarrow \gamma_2 = U_C$$

$$\frac{\partial H}{\partial a} = 0 = -\alpha U_B e_a - \gamma_1 \Rightarrow -\alpha U_B e_a = U_C$$

The additional second order condition for a maximum is therefore $e_{aa} > 0$. Defining $b \equiv -1/e_a$ as the *marginal cost of pollution abatement*, this condition amounts to increasing marginal abatement costs. This marginal cost, from the first order condition on a, is equal to the marginal social cost of emissions,

$$b = \alpha \frac{U_B}{U_C}.$$

Transforming the Hamiltonian as in the model without abatement expenditures yields,

$$MEW = C + \dot{K} + \dot{K}_a - be + \frac{U_B}{U_C} B_0. \tag{8.2}$$

There are several points to note about this expression. First, all investment, whether in productive capital or in abatement capital, is counted in the aggregate welfare measure. Second, current abatement expenditure a is not measured in welfare - these expenditures are essentially intermediate in character. Third, current pollution emissions are represented as a deduction from welfare, valued either at marginal abatement costs or marginal social costs, both of which in turn are equal to the level of a Pigovian emissions tax. The equivalence of these marginal costs is, of course, a consequence of MEW being measured on the optimum path.

Model 2: a cumulative pollutant and a stock pollutant.

We next wish to model a pollutant whose effects are cumulative[2]. The level of the flow of environmental services is therefore related negatively to the cumulative amount of pollution emitted, X, so that

$$\dot{B} = B_0 - \beta X.$$

We first assume no abatement expenditures, so that $e = e(F)$. The model is therefore,

$$\max \int_0^\infty U(C,B)e^{-rt}dt \text{ subject to:}$$
$$\dot{K} = F - C$$
$$\dot{X} = e.$$

Here C is the only control variable. The current value Hamiltonian for this problem is,

$$H = U + \gamma_1(F - C) + \gamma_2 e,$$

for co-state variables γ_1 and γ_2, and the first order condition for a maximum (ignoring the dynamic conditions for the moment) is,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_1 \Rightarrow \gamma_1 = U_C.$$

For the first order condition to yield a maximum, a necessary condition is that $U_{CC} < 0$, i.e., that there be declining marginal utility of consumption.

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[2] As a simplification in this and all subsequent models, investment in pollution abatement will be ignored, since its effects on the welfare measure have been explained in Model 1.

Note that $\gamma_2 < 0$, since increases in the accumulation of the pollutant decrease welfare. The measure of economic welfare is obtained by transforming the Hamiltonian as in Model 1, to yield,

$$MEW = C + \dot{K} + \frac{\gamma_2}{U_C} e + \frac{U_B}{U_C} B.$$

There are several things to note about this expression, beginning with why it should be interpreted as a welfare measure rather than net national product. The terms in emissions e and environmental services B provide the answer: as purely external phenomena they reflect adjustments to utility rather than to market production. This expression should be interpreted as what a planner should maximize at each point in time in order to maximize the present value of utility, in keeping with our earlier interpretation of the Weitzman model. The expression U_B / U_C is the price that utility-maximizing consumers would be willing to pay for a unit of environmental service, and so a key component of welfare in this model, as with the flow pollutant, is the monetized value of the *level* of environmental services.

Since γ_2 is the shadow price of the accumulation of the pollutant measured in utils, it is natural to define $\sigma \equiv -\gamma_2 / U_C$ as the marginal social cost of a unit of the pollutant, and, as in Model 1, this will equal the Pigovian tax required to maximize utility. If $p_B \equiv U_B / U_C$, then the expression for economic welfare becomes,

$$MEW = C + \dot{K} - \sigma e + p_B B. \tag{8.3}$$

Economic welfare is therefore measured as consumption plus investment less the value of an optimal emissions tax plus the value of the level of environmental services.

Abatement expenditures, a, are introduced into this model as the use of current production to reduce the level of emissions, so that the emission function is re-defined as follows:

$$e = e(F,a), \quad e_F > 0, \quad e_a < 0.$$

The maximization problem is now specified as:

$\max \int_0^\infty U(C,B) e^{-rt} dt$ subject to:

$\dot{K} = F - C - a$

$\dot{X} = e.$

The control variables are C and a and the current value Hamiltonian is as specified above. The first order condition for γ_1 is again that it should equal the marginal utility of consumption. For γ_2 we now have,

$$\frac{\partial H}{\partial a} = 0 = -\gamma_1 + \gamma_2 e_a \Rightarrow \gamma_2 = \frac{U_C}{e_a}. \tag{8.4}$$

It will be useful in what follows to define $b \equiv -1/e_a$; this is just the *marginal cost of pollution abatement.* Transforming the Hamiltonian into consumption units, we therefore derive,

$$MEW = C + \dot{K} - be + p_B B. \tag{8.5}$$

Expression (8.4) implies that $b = -\gamma_2 / U_C$. The marginal cost of abatement is identically equal to the marginal social costs of emissions and to the value of the optimal unit emissions tax. Given that $\gamma_1 > 0$ and $\gamma_2 > 0$, a sufficient condition for the Hamiltonian to produce a maximum of utility is $e_{aa} > 0$, i.e., increasing marginal abatement costs.

Economic welfare, therefore, is measured as consumption plus investment, less the value of pollution, plus the value of environmental services. Note the valuation of pollution in expression (8.5). While this may appear similar to valuing environmental damage as the current cost of abatement, a moment's reflection shows that this is not so: valuation is based on the *marginal* cost of abatement, and emissions are implicitly held to their optimal value, because welfare is being maximized.

If we consider the accumulation of pollutant X to be a liability in the national balance sheet, then $\dot{K} - be$ is equal to what Hamilton (1994) calls *genuine saving* - the change in the real value of assets when all assets, including environmental ones, are taken into account. Net national product is therefore derived as,

$$NNP = C + \dot{K} - be,$$

and it is the addition of value of environmental services, $p_B B$, that produces a welfare measure.

Expression (8.5) can also be written as,

$$MEW = C + p_B B + \dot{K} + \frac{b}{\beta}\dot{B}, \tag{8.6}$$

so that economic welfare consists of the proximate sources of utility, C and $p_B B$, plus the adjustments required to ensure utility maximization over time, $\dot{K}$ and $(b/\beta)\dot{B}$. Note that $\dot{B} < 0$ for any non-zero production level because pollution accumulates.

Expression (8.5) yields another interpretation. First, $GNP = F = C + \dot{K} + a$. This implies that

$$MEW = GNP - a - be + p_B B.$$

So we conclude that, in order to arrive at a welfare measure, abatement expenditures should be subtracted from GNP - they become, in effect, intermediate consumption[3]. This is consistent with the notions of Juster (1973) and Leipert (1989). What goes beyond the conclusions in these studies is the subtraction of emissions valued at the marginal cost of abatement and the addition of the value of environmental services. This is not an argument for changing NNP, but rather a prescription for measuring welfare.

One unsatisfactory aspect of the previous model is that it treats the environment as purely exhaustible: the flow of environmental services can only decline for any non-zero level of output. In the following model we assume a simple representation of a pollutant that both accumulates and dissipates as follows:

$$\dot{X} = e - d(X)$$
$$B = B_0 - \beta(X - X_0)$$

Here X_0 is the initial stock of the pollutant and $d(X)$ is the dissipation function for this stock, representing physical processes that reduce and render harmless some amount of the accumulation of the pollutant. Environmental services B are assumed to be negatively related to the stock of pollutant, with β being the fixed rate at which services decrease with accumulation of the stock. As a consequence, $\dot{B} = -\beta\dot{X}$. As in the preceding model, $e = e(F, a)$. The overall model therefore becomes[4], for control variables C and a,

$$\max \int_0^\infty U(C,B) e^{-rt} dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a$$
$$\dot{X} = e - d.$$

The current value Hamiltonian for this problem is given by,

$$H = U + \gamma_1(F - C - a) + \gamma_2(e - d).$$

As in the previous model, the first order conditions for a maximum give $\gamma_1 = U_C$. For marginal abatement cost $b \equiv -1/e_a$, the first order conditions then imply that $\gamma_2 = -U_C b$. Transforming the Hamiltonian into consumption units, the measure of economic welfare therefore reduces to:

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[3] This is true for all the following models. The interpretation of household defensive expenditures will be derived in Model 6.

[4] This model is formally similar to one in Hartwick (1993). It is here given a more careful interpretation and is used to set the stage for models that follow.

$$MEW = C + \dot{K} - be + bd + p_B B. \tag{8.7}$$

In this expression the term *bd* represents the dissipation of the pollutant stock valued at the marginal cost of abatement. Emissions are a deduction from welfare in this model, while dissipation of the stock of pollutant represents an increase in welfare

The value of the flow of environmental services is again included in economic welfare, owing to the fact that environmental services are a direct source of utility. It is simple to show that if environmental services enter the production function as well, so that $F = F(K,L,B)$, then the expression for welfare that results is formally the same as expression (8.7) - however, NNP will reflect the level of production associated with the optimal level of environmental services *B* that is attained. The marginal product of environmental services, F_B, and the marginal dissipation rate, d_X, affect the time path of the marginal cost of abatement, as will be briefly explored in a later section on model dynamics.

Model 3: emissions impair dissipation.

An interesting variant on the previous model is to examine what happens when current pollution emissions impair the ability of the environment to dissipate the pollution stock. We assume therefore that $d = d(X,e)$, $d_X > 0$, $d_e < 0$. This implies that d_X and d_e are dimensionless (i.e., they are scalars), and both must be less than 1 in absolute value.

The optimization problem can be expressed precisely as in the previous model, and the current value Hamiltonian is again given by,

$$H = U + \gamma_1(F - C - a) + \gamma_2(e - d).$$

The first order condition on consumption yields, as before, $\gamma_1 = U_C$. For abatement expenditures the condition is,

$$\frac{\partial H}{\partial a} = 0 = -\gamma_1 + \gamma_2(e_a - d_e e_a) = -U_C + \gamma_2\left(-\frac{1}{b} + \frac{d_e}{b}\right)$$
$$\Rightarrow \gamma_2 = \frac{b}{d_e - 1}U_C.$$

The measure of economic welfare is therefore given by,

$$MEW = C + \dot{K} - \frac{b}{1 - d_e}(e - d) + p_B B. \tag{8.8}$$

This bears an obvious resemblance to expression (8.7) from the previous model, with the exception that the value of the net change in the pollutant stock (*e* - *d*) is attenuated by the factor $1/(1 - d_e)$. Recalling that $d_e < 0$, this leads to the mildly paradoxical conclusion that the greater is the marginal impairment of the dissipation of pollution associated with current emissions, the smaller is the effect of pollution emissions on welfare (other things being equal).

Model 4: pollution is linked to exhaustible resource use.

This can be dubbed 'the CO_2 problem' because we view the level of pollution emissions as being linked directly to the quantity of resource use, much as carbon dioxide emissions are related stoichiometrically to the carbon content of fossil fuels[5]. We will assume in what follows that we are dealing with an exhaustible fossil fuel resource. As in Model 3, utility is a function of both consumption and environmental services, and the environment regenerates as a result of dissipation processes. The key differences are that $e = e(R, a)$, $e_R > 0$, R measures the quantity of resource extracted and used, S is the resource stock, resources are inputs into production, so that $F = F(K, L, R)$, $F_R > 0$, and resources are costly to produce, so that $f = f(R)$, $f_R > 0$ specifies the cost of resource extraction. This treatment of exhaustible resources follows Hartwick (1990) and Chapter 7.

The following stock-flow relationships for pollution stock and environmental services hold,

$$\dot{X} = e(R, a) - d(X), \text{ and } B = B_0 - \beta(X - X_0).$$

The model is specified as:

$$\max \int_0^\infty U(C, B) e^{-rt} dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a - f$$
$$\dot{X} = e - d$$
$$\dot{S} = -R$$
$$S \to 0 \text{ as } t \to \infty.$$

The final part of this expression just says that resources must be exhausted over the program; this is an efficiency condition. In this model K, X, and S are the state variables and C, a and R are the control variables. This optimal control problem has the current value Hamiltonian function,

$$H = U + \gamma_1(F - C - a - f) + \gamma_2(e - d) - \gamma_3 R.$$

where γ_1, γ_2 and γ_3 are the co-state variables corresponding to capital, carbon dioxide stocks, resource stocks respectively. If we define $b = -1/e_a$ to be the marginal cost of abatement as before, then derivation of the first-order conditions for this problem yields,

$$\gamma_1 = U_C, \quad \gamma_2 = -U_C b, \quad \gamma_3 = U_C(F_R - f_R - b e_R).$$

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[5] This model is similar to that developed in Chapter 4 and in Hamilton and Ulph (1994).

The Hamiltonian is measured in utils and is maximized at each point in time under the optimal program - it is therefore a current measure of welfare. The conditions for a maximum include $F_{RR} < 0$, so resources have declining marginal product, and $f_{RR} \geq 0$, i.e., constant or increasing marginal extraction costs. As in previous models, we define the measure of economic welfare by transforming the Hamiltonian into consumption units. Substitution of the above expressions for the co-state variables into the Hamiltonian therefore gives,

$$MEW = C + \dot{K} - b(e-d) - (F_R - f_R - b e_R)R + p_B B. \tag{8.9}$$

This expression says that emissions decrease welfare while regeneration of the environment, through the dissipation of CO_2, increases it (i.e., the environment is productive); in both cases the appropriate unit of valuation is the marginal cost of abatement, *b*. Assuming profit-maximizing producers, F_R is the market price of the resource and f_R its marginal cost of extraction. The next-to-last term in this expression therefore relates to the value of resource depletion[6], being of the form 'price minus marginal cost'. However, the unit resource rent $F_R - f_R$ is reduced by a Pigovian tax, at rate $b e_R$. This is a *carbon tax on resource use*, a specific tax required to achieve both the maximization of the present value of utility and, as will be seen in the discussion of dynamics, the efficient extraction of the resource when its use leads to CO_2 emissions. The net rental value of fossil fuels decreases when account is taken of their environmental externalities.

It might be argued that carbon emissions cannot be abated in any practical manner, calling the dependence of expression (8.9) on marginal abatement costs into question. One response to this is to argue that in any model of pollution emissions it is more general to assume some level of abatement effort. The second response is that, as in Models 1 and 2, the same results can be obtained by assuming no abatement effort - what results is an expression containing the level of the marginal social costs of emissions or an optimal carbon emissions tax, σ, in place of the marginal abatement cost *b* in expression (8.9) (and the preceding expressions as well).

As a final note, this model assumes a single country dealing with the welfare effects of its own carbon emissions. The situation in reality, of course, is much more complex, with multiple emitting countries facing different marginal abatement cost schedules, so that finding a global optimum would require, for example, some form of emission trading.

Model 5: living resources are damaged by pollution.

This model examines the 'acid rain' problem: it is assumed that living resources with economic value are damaged by emissions resulting from production. To keep the analysis to its essentials we will make a couple of simplifying assumptions. First, harvest of the living resource is assumed to be costless. And secondly, utility is derived only from consumption and not from the resource or the quality of the environment in general.

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[6] This is comparable to the value of depletion, based on resource rentals, that appears in Hartwick (1990) and Chapter 7.

Production is characterized by the production function $F = F(K,L,R)$, where R is the quantity of resource harvested, and emissions (as in Models 1-3) are related to the level of production and abatement expenditures a, so that $e = e(F,a)$. The resource stock S is augmented by natural growth $g(S)$ and diminished by harvesting and the amount of damage resulting from pollution emissions w, such that,

$$w = w(S,e),\ w_S > 0 \text{ and } w_e > 0.$$

This formulation implies that acid emissions cause direct damage only, and have no cumulative effect. This is obviously another simplification.

The model therefore becomes,

$$\max \int_0^{\infty} U(C)e^{-rt}dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a$$
$$\dot{S} = -R - w + g.$$

Here C, a, and R are the control variables. The current value Hamiltonian for this problem is,

$$H = U + \gamma_1(F - C - a) + \gamma_2(-R - w + g).$$

As in previous models, the efficiency condition on consumption implies that $\gamma_1 = U_C$. Optimality also requires, assuming marginal abatement costs $b \equiv -1/e_a$,

$$\frac{\partial H}{\partial a} = 0 = -\gamma_1 - \gamma_2 w_e e_a \Rightarrow \gamma_2 = \frac{U_C b}{w_e}, \text{ and}$$

$$\frac{\partial H}{\partial R} = 0 = \gamma_1 F_R - \gamma_2 \Rightarrow \gamma_2 = U_C F_R.$$

These expressions imply the interdependence of abatement costs, marginal emission damages and resource rents, so that $F_R = b / w_e$. This interdependence arises from having three control variables but only two state variables, K and S, in the model. Assuming a linear homogeneous utility function $U = U_C C$, the measure of economic welfare is,

$$\begin{aligned} MEW &= C + \dot{K} - F_R(R + w - g) \\ &= C + \dot{K} - \frac{b}{w_e}(R + w - g). \end{aligned} \tag{8.10}$$

Note that these expressions can be considered to be a measure of net national product, since there are no terms representing household welfare. The first is similar to that of Hartwick (1990) - when living resources are exploited, net national product is adjusted by deducting resource harvest and dieback and adding resource growth, all valued at the resource rental rate. The second expression shares with Model 3 a mildly counter-intuitive result: other things being equal, the greater the marginal damage associated with emissions, the smaller is the adjustment (associated in this case with resource harvest, dieback and growth) to national product.

Model 6: household defensive expenditures.

This model explores the situation where households make expenditures that directly affect the benefits obtained from the environment. We assume, therefore, that rather than deriving utility from environmental services B directly, utility is obtained via some *benefit* function Φ, that is in turn a function of environmental services and household expenditures that enhance benefits from the environment (or, equivalently, that can compensate for decreases in flows of environmental services - e.g., as suggested earlier, purchasing a water filter to purify drinking water that is declining in quality). The model is otherwise very similar to Model 2 for a pollutant with cumulative effects.

We therefore have $U = U(C, \Phi)$, $U_\Phi > 0$, $\Phi = \Phi(B, h)$, $\Phi_B > 0$, and $\Phi_h > 0$ for household defensive expenditures h. As in the first models, emissions are given by $e = e(F, a)$. For state variables K and X (the stock of pollutant) and control variables C, a, and h, therefore, the optimal control program is:

$$\max \int_0^\infty U(C,\Phi) e^{-rt} dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a - h$$
$$\dot{X} = e.$$

The current value Hamiltonian for the optimal control program is:

$$H = U + \gamma_1 (F - C - a - h) + \gamma_2 e.$$

The first order conditions for the optimum yield $\gamma_1 = U_C = U_\Phi \Phi_h$. At the optimum we therefore have the following constraint,

$$\frac{1}{\Phi_h} = \frac{U_\Phi}{U_C},$$

where $1 / \Phi_h$ is the *marginal defensive cost*. Note that this means that the price of environmental benefits just equals the marginal defensive cost, which is to be expected for a utility-maximizing consumer[7]. Sufficient conditions for a maximum are $U_{\Phi\Phi} < 0$, so there is declining marginal utility of environmental benefits, and $\Phi_{hh} < 0$, which implies increasing

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[7] This is obviously related to the notion of using avertive expenditures to value environmental benefits as described in Smith (1991).

marginal defensive costs. As in Model 2, $\gamma_2 = -U_C b$. As a result, the measure of economic welfare is:

$$MEW = C + \dot{K} - be + \frac{\Phi}{\Phi_h}. \tag{8.11}$$

Where households can compensate for changing environmental service flows, therefore, economic welfare is measured as consumption plus investment, less the value of pollution emissions (where pollution is priced at the marginal abatement cost), plus the value of environmental benefits to households (where benefits are priced at the marginal defensive cost). The term Φ / Φ_h is conceptually similar to $p_B B$, the value of environmental services, that appears in the other models.

This result requires careful interpretation. The measure of economic welfare can also be written as,

$$MEW = GNP - a - h - be + \frac{\Phi}{\Phi_h}. \tag{8.12}$$

Because $\Phi_{hh} < 0$, i.e., increasing marginal defensive costs, is part of the sufficient condition for a maximum, we can conclude that the welfare measure includes a value larger than household defensive expenditures *h*.

This should be compared with Mäler's (1991) interpretation of household defensive expenditures, which is basically that they should not be deducted in arriving at a 'green' welfare measure. Rather than deducting defensive expenditures, this model suggests that the welfare measure should include some value greater than defensive expenditures. In addition, the model says that abatement expenditures and emissions valued at their marginal cost of abatement (or, equivalently, the value of an optimal emissions tax) should be deducted from GNP in arriving at welfare.

Dynamics

The optimal control problems presented above also require a set of dynamic first order conditions in order to achieve an optimum. Further insight into the welfare measures derived can be gained by examining these conditions.

Since productive capital is the only stock in the Model 1 without abatement, the dynamic optimality condition is simple to derive as,

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} = r\gamma_1 - \gamma_1 F_K \Rightarrow \frac{\dot{U}_C}{U_C} = r - F_K.$$

If we assume that the utility function is additively separable, so that $U(C,B)=U_1(C)+U_2(B)$, then we can derive a variant on the Ramsey (1928) rule by defining,

$$\eta(C)=-\frac{U_{CC}}{U_C}C.$$

This is the elasticity of the marginal utility of consumption. The dynamic optimality condition then reduces to the Ramsey rule,

$$r+\eta(C)\frac{\dot{C}}{C}=F_K.$$

The second version of Model 1 has two stocks, productive capital and abatement capital. The optimality condition for productive capital is as just derived. However, since $U_C=\alpha U_B/b$, this can also be written as,

$$\frac{\dot{U}_B}{U_B}-\frac{\dot{b}}{b}=r-F_K,$$

so that the dynamics of the marginal utility of environmental services and marginal abatement costs are linked.

For abatement capital the dynamic first order condition is,

$$\dot{\gamma}_2=r\gamma_2-\frac{\partial H}{\partial K_a}=r\gamma_2+\alpha U_B e_{K_a},$$

so that $\frac{\dot{U}_C}{U_C}=r+be_{K_a}$ and therefore,

$$r+\eta(C)\frac{\dot{C}}{C}=-be_{K_a}=\frac{e_{K_a}}{e_a}.$$

These two variants on the Ramsey rule therefore imply that the ratio of the marginal emission reduction associated with a unit of abatement capital to the marginal emission reduction associated with a unit of current abatement expenditures must equal the marginal product of capital,

$$\frac{e_{K_a}}{e_a}=F_K.$$

Next we consider a generalization of Model 2, the stock pollutant. Here we assume that environmental services B are a productive input, so that $F=F(K,L,B)$. The Hamiltonian remains the same as in Model 2, as do the values of the co-state variables: $\gamma_1 = U_C$ and $\gamma_2 = -U_C b$. The first dynamic condition is,

$$\dot{\gamma}_1 = r\gamma_1 - \frac{\partial H}{\partial K} = r\gamma_1 - \gamma_1 F_K - \gamma_2 e_F F_K .$$

This implies that,

$$\frac{\dot{U}_C}{U_C} = r - (1 - be_F)F_K . \tag{8.13}$$

Here be_F is the effective rate of pollution tax on output - for the model to be feasible this must be less than 1. If we assume that the utility function is additively separable, so that $U(C,B) = U_1(C) + U_2(B)$, then expression (8.13) produces,

$$r + \eta(C)\frac{\dot{C}}{C} = (1 - be_F)F_K . \tag{8.14}$$

Other things being equal, therefore, the pollution externality reduces the consumption rate of interest when compared with the standard Ramsey rule (as derived in Model 1 above) - growth in consumption is slower and capital is, in effect, less productive. This fits with intuition. The result follows in the absence of pollution abatement as well, with the marginal social cost σ replacing the marginal abatement cost b.

The second and third dynamic efficiency conditions for this model are derived as follows:

$$\dot{\gamma}_2 = r\gamma_2 - \frac{\partial H}{\partial X} = r\gamma_2 - (-\beta U_B - \gamma_1 \beta F_B + \gamma_1 b \beta e_F F_B + \gamma_1 b d_X),$$

so that, for $p_B \equiv U_B / U_C$,

$$\dot{b} = ((1 - be_F)F_K + d_X + \beta e_F F_B)b - \beta(F_B + p_B). \tag{8.15}$$

This expression defines the time path for the optimal marginal abatement costs. Since the marginal abatement cost must be positive for the model to be feasible, its rate of change of varies positively with the net marginal product of capital, the marginal dissipation rate of the pollution stock, and a term relating to the feedback of environmental services on emissions (this term, $\beta e_F F_B$, must less than 1 in order for production to be feasible); the rate of change varies negatively with the sum of the marginal valuation of environmental services by producers and consumers[8].

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[8] This suggests that environmental services have the character of public goods in this model.

When environmental services are not productive the time path of marginal abatement costs is given by setting the terms in F_B in expression (8.15) to 0; if it is assumed that the pollution stock does not dissipate (the cumulative pollutant version of Model 2), then setting the term in d_X to 0 in addition yields the optimal time path.

If current pollution emissions impair the dissipation of the pollutant stock (Model 3), it is straightforward to show that under the assumptions employed above the Ramsey rule becomes,

$$r + \eta(C)\frac{\dot{C}}{C} = \left(1 - \frac{b}{1-d_e} e_F\right) F_K .$$

This yields the mildly surprising result that, other things being equal, the larger is $|d_e|$ (the marginal impairment of dissipation from emissions *e*), the higher is the growth rate of consumption. This can be disentangled to some extent by examining the second dynamic efficiency condition. Derivations similar to those above yield,

$$\frac{\dot{\gamma}_2}{\gamma_2} = r + d_X - \frac{1-d_e}{b}\beta\frac{U_B}{U_C},$$

so that,

$$\frac{\dot{b}}{b} + \frac{\dot{d}_e}{1-d_e} = \left(1 - \frac{b}{1-d_e} e_F\right) F_K + d_X - \frac{1-d_e}{b}\beta p_B .$$

This says that the time paths of marginal abatement costs and the marginal impairment of dissipation are linked.

In Model 4, the CO_2 problem, because emissions are linked to resource use rather than the level of production, the regular Ramsey rule applies. The time path that the unit carbon tax follows is governed by,

$$\dot{b} = (F_K + d_X)b - \beta p_B .$$

The new element of this model concerns the behaviour of resource rents net of carbon taxes. If we define $n \equiv F_R - f_R - b\, e_R$, then the earlier first order conditions for this problem imply that the co-state variable of the resource stock is given by $\gamma_3 = U_C n$. The dynamic efficiency condition for this variable is,

$$\frac{\frac{d}{dt}(U_C n)}{U_C n} = \frac{\dot{U}_C}{U_C} + \frac{\dot{n}}{n} = r,$$

which in turn implies that,

$$\frac{\dot{n}}{n} = F_K .$$

This is an extended Hotelling rule: the percentage rate of change of resource rental rates less the pollution tax rate must equal the marginal product of capital.

In Model 5, the acid rain problem, the standard Ramsey rule again applies, in this case because there is no state variable corresponding to environmental services. However, another variant on the Hotelling rule results,

$$\frac{\dot{F}_R}{F_R} = F_K + w_S - g_S .$$

Here the rate of change of resource rents is increased by the marginal product of capital and stock effects on resource dieback (w_S), and decreased by stock effects on resource growth.

Finally, the model of household defensive expenditures (Model 6) yields the modified Ramsey rule of expression (8.14), because emissions affect the flow of environmental services and are again a function of the level of production. The time path of marginal abatement costs is given by,

$$\dot{b} = \left(1 - b e_F\right) F_k b - \alpha \frac{\Phi_B}{\Phi_h} .$$

Here the term Φ_B / Φ_h plays the same role, that of a unit value for environmental services as valued by households, that the term p_B plays in the previous models.

This analysis leads to several general conclusions. First, in models where the rate of change of environmental services is linked to pollution emissions, which are in turn linked to the level of production, the rate of growth of consumption is lowered as a result of the pollution externality, and the capital stock is, in effect, less productive. Second, in models with marginal abatement costs or unit emission taxes, the rate of change of these abatement costs is generally increased by the marginal product of capital and the marginal rate of dissipation of the pollution stock, and decreased by the price consumers would be willing to pay for environmental services. Third, if environmental services are productive then these effects only show up in the dynamic equation for the rate of change of marginal abatement costs. Finally, for models involving natural resources, pollution externalities affect the time path of resource rents, yielding a modified Hotelling rule.

Conclusions

The analysis in this chapter has considered models where explicit current and capital expenditures are made to abate pollution. In the real world, of course, many capital investments jointly increase productivity and reduce the uncontrolled level of pollution emissions. Under these circumstances the notion of 'marginal cost of abatement' is not well defined, but it is still possible to measure the marginal social costs of pollution emissions.

Introducing external trade in the composite good into the models has no major effect on the results. Gomez-Lobo in Gomez-Lobo *et al.* (1993) shows that when there are exports and imports of the produced good, a term of the form iA must be added to the derived welfare measure, where i is a fixed international interest rate and A the net accumulation of foreign assets resulting from external trade. Of course this assumes a small open economy, so that the interest rate can be taken as given, and it does not deal with imports and exports of pollutants.

It should be obvious that these separate models could be combined to represent the more realistic assumption of multiple pollutants and multiple control effort. Each pollutant would have its own emissions function (including abatement, where appropriate), separate accumulation of stock pollutant, with its own dissipation function, and distinct environmental service flow that is associated with the level of emission (in the case of flow pollutants) or the level of the pollutant stock. Alternatively, a single environmental service flow could be the result of taking a weighted combination of the separate emissions and dissipation of the various pollutants. Living resources, fossil fuels, and other exhaustible resources could be added as well, including the effects of acid rain and CO_2, as long as the cross-effects are accounted for in the analysis (e.g., the reduction in the level of natural resource rents resulting from pollution emissions associated with production).

The following is a brief summary of the model results, concentrating on the versions of the models with abatement expenditures. In each model the starting point in measuring welfare is GNP less abatement expenditures.

Model 1. For flow pollutants, deduct emissions valued at the marginal cost of abatement, and add the level of environmental services that would flow from a pristine environment, valued at consumers' marginal willingness to pay.

Model 2. For the cumulative pollutant, rather than adding the value of the service flow from a pristine environment, add instead the value of the current level of environmental services. For a stock pollutant that dissipates, in addition to the preceding adjustments you must add the dissipation of the pollutant stock, valued at the marginal cost of abatement.

Model 3. When current emissions impair the dissipation of the pollution stock, the net accumulation of pollutant is deducted valued at the marginal cost of abatement divided by one plus the marginal rate of impairment of dissipation. This apparent attenuation of the deduction for net pollutant accumulation is clarified by the analysis of the model dynamics, in which it is shown that the time paths of marginal abatement costs and marginal impairment rates are linked.

Model 4. For the CO_2 problem, in addition to the adjustments in Model 2, you must subtract the value of net fossil fuel rents - price minus marginal cost of extraction less the value of an optimal carbon tax.

Model 5. The acid rain problem yields the standard adjustment for living resources: from GNP less abatement expenditures subtract net resource depletion (harvest minus growth) valued at the resource rental rate. However, the resource rental rate must be equal to the marginal cost of abatement divided by the marginal dieback rate for the living resource.

Model 6. For household defensive expenditures, the result is formally the same as for the cumulative pollutant of Model 2, except that the flow of environmental services valued at marginal willingness to pay is replaced by the flow of environmental benefits (i.e., service levels as affected by defensive expenditures) valued at the marginal defensive cost. Assuming increasing marginal defensive costs, this term is in fact greater than the level of household defensive expenditures.

A key aspect of these results is that in each case some valuation of the level of environmental services features in the measure of economic welfare. This is in contrast to most of the traditional literature reviewed Chapter 6, with the exception of Peskin (1989). It therefore needs to be asked whether, in adjusting national accounts for environmental effects, it makes sense to deduct the value of current damage to the environment if you have not first added the value of the flow of environmental services. For calculations of residuals, such as a 'green' net savings rate (referred to as 'genuine' savings in Hamilton (1994)) this would not be a problem. The result for household defensive expenditures is also at variance with the traditional literature.

This chapter has presented a series of theoretical models. Since national accounting is a practical exercise, built upon measurement and estimation as well as a core of theory, it is fair to ask what practical consequences these results might have for 'green' national accounting.

One set of practical questions revolves around measurability, in particular how we should measure the flow of environmental services. The appropriate measure of the services provided by clean air, for instance, is not obvious. But one proxy might be to look at air quality indices and to measure the willingness of consumers to pay for marginal changes in these indices. Where such indices have been reported for a long period of time and consumers have developed a sense of what subjective environmental quality they associate with given index levels, this might be a practical approach. Alternatively, there is by now a substantial literature on how to measure willingness to pay for environmental amenities (see, for instance, Cropper and Oates (1992) for a review), including travel cost methods, hedonic pricing and contingent valuation, that could be used in environmental national accounting. Much of this literature is concerned with valuing individual sites or environmental assets, however - how to add up across the myriad environmental assets within a country in a consistent, non-duplicating manner is an unanswered question.

The models point to the need to value pollution emissions and the regeneration of the environment (through dissipation of pollution stocks) at the marginal abatement cost or the marginal social cost. Measuring pollution emissions is in principle a straightforward matter. Measuring regeneration is more problematic - it might be necessary to handle this using models of the physical processes involved. It would also be difficult to measure *marginal*, as opposed to average, abatement costs in the current period, but measuring marginal social costs is increasingly viable - see, for instance, CEC/US (1993).

Measurement problems abound, therefore. There is the overriding fact that the models all restrict the economy to be on the optimal path. But the models presented suggest the way to think clearly and consistently about how conventional national accounts can be extended to account for the welfare effects of environmental change.

9. National Wealth and Sustainable Income

As noted in previous chapters, economic notions of sustainability have been with us for some time, most obviously in the Hicksian definition of income. An important strand of environmental and resource economics seeks to broaden the notion of sustainable development to include changes in the natural environment. The traditional admonition to 'maintain capital intact' has been extended to maintaining the value of total capital, both produced and natural, constant - see Pearce *et al.* (1989). Interest in the policy prescriptions required to achieve sustainable development has led to a natural concern with the measurements our statistical systems provide, and whether these systems, notably the System of National Accounts (SNA), can indicate whether we are on a sustainable path.

Much of the recent work in this domain has been concerned with adjustments to GNP or NNP, as suggested in the papers in Ahmad *et al.* (1989) and Lutz (1993). This work and some of the outstanding issues are reviewed briefly in Chapter 6. There has been a parallel development of theoretical approaches to the measurement of national product, work that started with Weitzman (1976) and has been extended into the environmental domain by Hartwick (1990), Mäler (1991) and Hamilton (1994). Working within this theoretical domain, Asheim (1994) and Pezzey (1994) have raised questions about whether current flow measures can indicate whether an economy is on a sustainable path.

Since concerns about sustainability are primarily concerns about future flows of well-being, this suggests that a fruitful line of inquiry would be to look at the measurement problem from the point of view of total wealth rather than current income. This was the motivation in Hamilton (1994) for the proposal that total wealth per capita would be a superior measure of sustainability. Suggestions to expand the measure of national wealth to include natural resources go back at least to Scott (1956). The latest revision to the SNA (United Nations 1993a) standardizes the inclusion of commercial natural resources in the balance sheet accounts but does not prescribe adjustments to national income to reflect resource depletion.

This chapter aims to develop a theoretical approach to the measurement of national wealth, where the notion of wealth is suitably expanded to include both natural resources and environmental pollution. This provides the basis for the definition of an extended notion of net national product, and proof of the critical role that measuring 'genuine savings' plays in policies to achieve sustainable development. It is also shown under what conditions 'greener' measures of net national product can serve as indicators to guide policies for optimal growth.

Modeling National Wealth

We assume a simple closed economy with a single resource producing a composite good that may be consumed, invested or used to abate pollution, so that $F(K,R) = C + \dot{K} + a$, where R is resource use and a is pollution abatement expenditures. This defines GNP in our economy. Labour is fixed and is therefore factored out of the production function.

Pollution emissions are a function of production and abatement, $e = e(F,a)$, and pollutants accumulate in a stock M such that $\dot{M} = e - d(M)$, where d is the quantity of natural dissipation of the pollution stock. The flow of environmental services B is negatively related to the size of the pollution stock, so that $B = q(M)$, $q_M < 0$. Resource stocks S grow by an amount g and are depleted by extraction R, so that $\dot{S} = -R + g(S)$, and resources are assumed to be costless to produce. The utility of consumers is assumed to be a function of consumption and environmental services, $U = U(C,B)$.

The stocks for a candidate measure of national wealth are clearly K, M, and S. The issue is the choice of prices for each stock. Because wealth is inherently a measure of the potential for income in the future, the obvious prices are the shadow prices of these stocks in a programme to maximize the present value of welfare (utility) - the classic utilitarian maximand. To calculate these prices we assume that there is a fixed pure rate of time preference r.

The maximization problem is therefore,

$$\max \int_0^\infty U(C,B)e^{-rt}dt \quad \text{subject to:}$$
$$\dot{K} = F - C - a$$
$$\dot{M} = e - d$$
$$\dot{S} = -R + g$$

and the Hamiltonian function, which is maximized at each point in time, is given by,

$$H = U + \gamma_K \dot{K} + \gamma_B \dot{B} + \gamma_S \dot{S}, \tag{9.1}$$

where γ_K is the shadow price of capital, in utils, γ_B the shadow price of environmental services, and γ_S is the shadow price of the resource. The first order conditions for maximizing the Hamiltonian, setting the partial derivatives with respect to the control variables C, a, and R to 0, yield the following:

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_K \quad \Rightarrow \quad \gamma_K = U_C. \tag{9.2}$$

$$\frac{\partial H}{\partial a} = 0 = -\gamma_K + \gamma_M e_a \quad \Rightarrow \quad \gamma_M = -U_C b \quad \text{for} \quad b \equiv -\frac{1}{e_a}. \tag{9.3}$$

$$\frac{\partial H}{\partial R} = 0 = \gamma_K F_R + \gamma_M e_F F_R - \gamma_S \quad \Rightarrow \quad \gamma_S = U_C(1 - b e_F)F_R. \tag{9.4}$$

Because the optimization problem is expressed in terms of maximizing *utility*, these prices are measured in utils. These prices can be transformed into consumption units by dividing through by the marginal utility of consumption U_C in each of these expressions - this will always give the correct relative prices for the stocks in each time period, but the scaling factor U_C will vary over time.

Note that b is the marginal cost of pollution abatement. It was shown in Chapter 8 that this is precisely equal to the marginal social costs of pollution emissions, and that this in turn is equal to the level of a tax - the Pigovian tax required to maximize welfare - on emissions. These equalities hold because the economy is at the optimum.

The term be_F is the effective tax rate on production. Therefore, although we have started with an optimal control problem, the prices that result are those that would prevail in a competitive economy with a Pigovian tax on pollution. In such a competitive economy the measure of national wealth (*NW*) is,

$$NW = K - bM + (1 - be_F)F_R S. \tag{9.5}$$

There are at least two striking aspects to this expression. First, the correct treatment of the pollutant is as a liability in the national balance sheet - the stock of pollution is valued at the marginal social cost of a unit of pollutant. Secondly, although resources are fundamentally valued at their rental rate (as other papers, e.g. Hartwick 1990, suggest), the unit rent is attenuated to reflect the disbenefits associated pollution emissions - resources are therefore not as valuable as they might seem at first glance, because of pollution byproducts.

The link between this wealth measure and sustainable development is provided by defining the *change in real wealth* (*dW*) to be:

$$dW \equiv \dot{K} - b\dot{M} + (1 - be_F)F_R\dot{S} = \dot{K} - b(e - d) - (1 - be_F)F_R(R - g). \tag{9.6}$$

This is not equal to the rate of change of national wealth, which would include capital gains terms involving the rates of change of the various prices.

As will be shown below, if the prices satisfy the efficiency conditions for the optimal programme then *dW* has two key properties:

(P1) if $dW = 0$ at a point in time, then $\dot{U} = 0$ at that time if and only if $\dot{dW} = 0$;

(P2) if $dW < 0$ for all time, then eventually $\dot{U} < 0$.

The first property holds for a given point in time. Asheim (1994) and Pezzey (1994) show that $dW > 0$ at a point in time is not a sufficient condition for sustainable development.[1] If the change in real wealth is equal to 0 over all future time, then utility will be constant; this is an extension of the Hartwick rule, implicit in the results of Dixit, Hammond and Hoel (1980). The second property says that persistent declines in real wealth are not sustainable.

For the problem of maximizing the present value of utility, U_C is the shadow price of capital (as well as consumption). The optimality condition is,

$$\dot{\gamma}_K = r\gamma_K - \frac{\partial H}{\partial K} \quad \Rightarrow \quad r - \frac{\dot{U}_C}{U_C} = (1 - be_F)F_K. \tag{9.7}$$

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[1] Asheim and Pezzey both examine the important sub-case of an exhaustible resource and no pollution emissions.

This is the Ramsey rule: the social rate of return on investment (on the left-hand side) must be equal to the interest rate, which is the net rate of return on capital allowing for the effects of pollution, $(1-be_F)F_K$. More germane to the problem at hand are the efficiency conditions on the other prices, derived as follows:

$$\dot{\gamma}_S = r\gamma_S - \frac{\partial H}{\partial S} \quad \Rightarrow \quad \frac{\frac{d}{dt}\left[(1-be_F)F_R\right]}{(1-be_F)F_R} = (1-be_F)F_K - g_S, \tag{9.8}$$

$$\dot{\gamma}_M = r\gamma_M - \frac{\partial H}{\partial M} \quad \Rightarrow \quad \dot{b} = \left((1-be_F)F_K + d_M\right)b + q_M\frac{U_B}{U_C}. \tag{9.9}$$

Expression (9.8) is an extended Hotelling rule: efficient resource pricing requires that the percentage rate of change in net resource rents be equal to the interest rate less the marginal effects of stock size on growth. Expression (9.9) is essentially the same as that derived in Ulph and Ulph (1994).

The proof of property (P1) is as follows. First, denote environmental depletion, the rate of change of in the real value of the total stock of natural assets, as $\dot{N}$, so that,

$$\dot{N} \equiv -(1-be_F)F_R\dot{S} + b\dot{M}. \tag{9.10}$$

Given that $\ddot{S} = -\dot{R} + g_S\dot{S}$, $\ddot{M} = \dot{e} - d_M\dot{M}$ and $b\dot{e} = be_F\dot{F} - \dot{a}$, substituting from expressions (9.8) and (9.9) yields:

$$\begin{aligned}
\ddot{N} &= \tfrac{d}{dt}\left(-(1-be_F)F_R\dot{S} + b\dot{M}\right) \\
&= -\tfrac{d}{dt}\left[(1-be_F)F_R\right]\dot{S} - (1-be_F)F_R\ddot{S} + \dot{b}\dot{M} + b\ddot{M} \\
&= -(1-be_F)F_R\left[(1-be_F)F_K\dot{S} - \dot{R}\right] + \left(\dot{b} - bd_M\right)\dot{M} + be_F\dot{F} - \dot{a} \\
&= -(1-be_F)F_R\left[(1-be_F)F_K\dot{S} - \dot{R}\right] + b\left[(1-be_F)F_K + \frac{q_M}{b}\frac{U_B}{U_C}\right]\dot{M} + be_F\dot{F} - \dot{a}.
\end{aligned}$$

Since $\dot{F} = F_K\dot{K} + F_R\dot{R}$ and $dW=0$, this yields,

$$\begin{aligned}
\ddot{N} &= F_R\dot{R} - (1-be_F)F_R(1-be_F)F_K\dot{S} + b\left[(1-be_F)F_K + \frac{q_M}{b}\frac{U_B}{U_C}\right]\dot{M} + be_FF_K\dot{K} - \dot{a} \\
&= F_R\dot{R} + (1-be_F)F_K\dot{K} + q_M\frac{U_B}{U_C}\dot{M} + be_FF_K\dot{K} - \dot{a} \\
&= \dot{F} + q_M\frac{U_B}{U_C}\dot{M} - \dot{a} \\
&= \dot{C} + \ddot{K} + q_M\frac{U_B}{U_C}\dot{M}.
\end{aligned}$$

Therefore,

$$\dot{C} = \ddot{N} - \ddot{K} - q_M \frac{U_B}{U_C}\dot{M}. \tag{9.11}$$

The expression $-q_M U_B / U_C$ is the price that a utility-maximizing consumer would be willing to pay for a unit reduction in the pollution stock. The rate of change of consumption is therefore this price times the rate of change of the pollution stock (which may be rising or falling depending on whether the pollution stock was above or below the efficient level at the outset of the programme) plus the difference between the rates of change of depletion and investment.

The rate of change of utility is derived as follows:

$$\dot{U} = U_C\dot{C} + U_B\dot{B} = U_C\left(\dot{C} + q_M \frac{U_B}{U_C}\dot{M}\right). \tag{9.12}$$

Since $\dot{dW} = \ddot{K} - \ddot{N}$, property (P1) is proved. Again, it should be emphasized that this property holds at a given point in time. Only policies that ensure that changes in real wealth are zero at each point in time will ensure sustainable development (i.e., constant utility).

Property (P2), that persistent decreases in real wealth is sufficient for a negative rate of change of utility, was proven by Pezzey (1994) for the case of an exhaustible resource with no pollution. For the current problem, assume that

$$\dot{K} = -(1 - be_F)F_R\dot{S} + b\dot{M} - \varepsilon, \text{ for } \varepsilon_t > 0,$$

so that the change in real wealth is negative at each point in time. Then derivations along the lines of the proof of property (P1) yield the result that,

$$\dot{U} = U_C\varepsilon\left(\frac{\dot{\varepsilon}}{\varepsilon} - (1 - be_F)F_K\right). \tag{9.13}$$

Therefore, if $\frac{\dot{\varepsilon}}{\varepsilon} < (1 - be_F)F_K$, so that the percentage rate of change of the decline in real wealth is less than the interest rate (the marginal product of capital net of pollution emissions taxes), then utility will be declining.

If $\frac{\dot{\varepsilon}}{\varepsilon} = (1 - be_F)F_K$ at each point in time then expression (9.13) appears to imply that utility will be constant. However, the proof of property (P1) implies that utility can only be constant if the change in real wealth is zero at each point in time (this is just the Hartwick rule). This equality is therefore infeasible. The reason for this infeasibility is that lower levels of investment entail both lower emissions, and therefore lower emission taxes, and higher marginal product of capital. This implies that it is impossible for the percentage rate of change of the decline in real wealth to converge to the interest rate from below, which implies that a growth rate that is everywhere greater than the interest rate is also infeasible.

Property (P2) is therefore proved. While substituting consumption for investment in the short run can lead to increasing utility, in the long run persistent negative changes in real wealth must lead to decreasing utility.

Expression (9.13) also yields a useful interpretation of property (P1): assume that $d\dot{W} = 0$ at a point in time but that $dW = -\varepsilon$ for *constant* ε (here ε can be either positive or negative). Then expression (9.13) implies that utility is not constant at that instant. Zero change in real wealth is therefore a necessary condition for utility to be instantaneously constant.

The next question of interest is whether the rate of change of national wealth is zero along a constant utility path - are constant wealth and constant utility the same thing? Since $dW = 0$ at each point in time is both necessary and sufficient for constant utility (this is generalized in Dixit, Hammond and Hoel 1980 and Hamilton 1995), we can take the derivative of expression (9.5) for national wealth and substitute the expression for constant real wealth to yield,

$$\dot{NW} = \ddot{K} - \dot{K} - \dot{b}M + \tfrac{d}{dt}\left(\left(1 - be_F\right)F_R\right)S .$$

Suppose for the moment that national wealth is constant. This would imply that,

$$\ddot{K} = \dot{K} + \dot{b}M - \tfrac{d}{dt}\left(\left(1 - be_F\right)F_R\right)S . \qquad (9.14)$$

If we turn to the rate of change of consumption, substituting expressions (9.14) and that for zero change in real wealth successively yields,

$$\begin{aligned}\dot{C} &= \dot{F} - \ddot{K} - \dot{a} \\ &= \dot{F} + \left(1 - be_F\right)F_R\dot{S} - b\dot{M} + \dot{b}M - \tfrac{d}{dt}\left(\left(1 - be_F\right)F_R\right)S - \dot{a}\end{aligned}$$

We know that utility is constant if and only if $\dot{C} = -q_M \frac{U_B}{U_C}\dot{M}$. But this expression will only hold in general if there is a term consisting of $\dot{b}\dot{M}$ in the expression for the rate of change of consumption. Assuming expression (9.14) has therefore led to a contradiction, and we conclude that national wealth is generally not constant along a sustainable (constant utility) path.

Net National Product

Having elucidated the relationship between total national wealth, the change in real wealth and sustainable development, the next step is to draw conclusions about the measurement of net national product (NNP). The natural definition of NNP is that it is the sum of (i) any production that is not invested or spent on abatement and (ii) the change in real national wealth. The logic of this definition follows from the previous section: if the change in real wealth is positive, then the maximum amount that can be consumed and still have utility be *instantaneously* constant is the sum of current consumption and the increment in real wealth; if the change in real wealth is negative then current consumption would have to be reduced and the residual invested in order to yield constant utility. Algebraically we have,

$$
\begin{aligned}
NNP &= \left(F - \dot{K} - a\right) + dW \\
&= GNP - a - b\dot{M} + \left(1 - be_F\right)F_R\dot{S} \\
&= C + \dot{K} - b\left(e - d\right) - \left(1 - be_F\right)F_R\left(R - g\right).
\end{aligned}
$$

The second of these expressions echoes one in Chapter 8: abatement expenditures are essentially intermediate and should be deducted from GNP in arriving at an NNP figure. The third expression yields further insights: dissipation of pollution stocks and growth of natural resources are both productive and their value should be included in the measure of NNP; the value of pollution emissions and resource depletion should both be deducted from NNP. This measure of net national product corresponds to that developed, using a different approach, by Pemberton *et al.* (1995).

There is also insight to be gained by extending this measure to include the level of environmental service flows as valued by consumers (an extended notion of consumption) to yield what is called the 'measure of economic welfare' (*MEW*) in Chapter 8:

$$
MEW = C + \dot{K} - b\left(e - d\right) - \left(1 - be_F\right)F_R\left(R - g\right) + \frac{U_B}{U_C}B. \tag{9.15}
$$

This expression for welfare provides a link back to the literature deriving national income measures from the Hamiltonian of an optimal control problem (see Weitzman 1976 and the other articles referred to in the Introduction). The Hamiltonian function for the current problem is:

$$
H = U + U_C\dot{K} - U_C b\left(e - d\right) - U_C\left(1 - be_F\right)F_R\left(R - g\right).
$$

Expression (9.15), the measure of economic welfare, can be derived from the Hamiltonian by: (i) multiplying each flow by its shadow price in utils (so that, in particular, the shadow price of environmental services is U_B); and (ii) converting to consumption units by dividing each term in the expression by the marginal utility of consumption U_C.

Thus there is a well-defined series of steps linking the Hamiltonian to the welfare measure in consumption units and, by dropping the term measuring consumers' valuation of environmental services, net national product. This is important because, by the Maximum Principle, we know that the Hamiltonian is what a social planner would choose to maximize at each point in time in order to maximize the present value of utility.

Taking this perspective, an interesting question is the extent to which changes in NNP or *MEW* are good indicators of the direction of change in the Hamiltonian. Chapter 6 answered this question in the affirmative for the case of an economy with an exhaustible resource and no environmental externalities. Algebra very similar to that used in the proof of property (P1) for the change in real wealth yields the following results:

$$\dot{H} = rU_C dW;$$

$$\dot{NNP} = (1 - be_F)F_K dW - q_M \frac{U_B}{U_C}\dot{M};$$

$$\dot{MEW} = (1 - be_F)F_K dW + \tfrac{d}{dt}\left(\frac{U_B}{U_C}\right)B.$$

NNP, while having the relevance noted earlier in the context of sustainability, is less exact as an indicator of the effectiveness of policies designed to keep the economy on the optimal path, since, generally speaking, decreases in the pollution stock ($\dot{M} < 0$) will lead to increases in real wealth ($dW > 0$). For the measure of economic welfare, if the price consumers are willing to pay for a unit of environmental service is constant (or at least constant along the optimal path), then the direction of change of *MEW* will be the same as the Hamiltonian.

The economy modeled here can be simplified in many ways. If there is no living resource then any terms in g above drop out. If pollution does not dissipate naturally then terms in d and its derivative disappear. If pollution externalities are ignored then, in addition to the disappearance of the obvious terms, the effect of the Pigovian tax as expressed in the term $(1 - be_F)$ also disappears.

It is worth noting that the change in real wealth, *dW*, is equal to what Hamilton (1994) and World Bank (1995) refer to as 'genuine saving.' The model employed here assumes no depreciation of produced assets or foreign trade, so that *dW* is equal to net savings less the value of depletion of natural resources and the value of net increments to the pollution stock.

Finally, it is important to note that although NNP measures the maximum amount that could be consumed at a point in time and still have utility be instantaneously constant, it does not measure the maximum level of consumption attainable if the economy were actually on a sustainable path, i.e., if the change in real wealth were zero at each point in time. The intuition behind this is that the constant utility path is uniquely determined by the initial stocks of capital, resources and pollution (and, in the case of a living resource, by the nature of the growth curve). Although prices will follow the same trajectories as on the optimal (present value of utility maximizing) path, their initial levels and the initial quantities of resource extraction and pollution emission will in general be different than on the optimal path. A policy choice to achieve sustainability by setting the change in real wealth to zero in all future years will generally involve a jump, either upwards or downward, in the measured level of NNP[2]. It is for this reason that we have consistently referred to NNP rather than 'sustainable national product' or 'sustainable national income.'

Conclusions

A strong conclusion from this model is that sustainable development, at least in the limited form of constant utility, is not synonymous with constant national wealth, even when the notion of wealth is expanded to include natural resources and stocks of pollution. This should not be surprising, considering that there are both price effects associated with the efficient

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[2] This point is brought out neatly by Pezzey and Withagen (1994) for the case of exhaustible resources and no pollution emissions.

price paths for resources and pollution and quantity effects corresponding to the growth and diminution of the various stocks - produced assets, natural resources, and pollution.

The model does offer concrete guidance for compilers of national balance sheet accounts by suggesting, first, that when there is a pollution externality the value of natural resources in the balance sheet should be reduced by the effective emission tax rate on output and, secondly, that stocks of pollutants should appear as liabilities in the accounts, valued at their marginal social costs. Hamilton and Atkinson (1995) discuss how to measure these shadow prices when the economy is not at the optimum.

The key measure resulting from this analysis is clearly *dW*, the change in real wealth or 'genuine saving', measured as the changes in the three stocks valued at their current shadow prices. This is linked to the question of sustainable development in the following sense: *dW* being instantaneously zero is a necessary condition for utility to be instantaneously constant. It needs to be asked, however, how useful a guide to sustainability an NNP measured on this basis would be, given that it does not measure what the maximum level of consumption would be on a constant utility path. It can be questioned how useful a guide to an optimal growth policy such a measure would be as well, given the results for the rate of change of NNP along the optimal path. The measure of economic welfare (*MEW*), which adds the value of environmental services to NNP, is a useful guide to optimal growth policies under moderately restrictive assumptions.

What is a useful indicator for sustainability policy is the measure of genuine saving. Persistent negative changes in real wealth were shown above to lead to eventual declines in utility. The welfare interpretation of this result is clear: genuine savings provide a one-sided measure of sustainability. This is the crucial link between green national accounting and sustainable development.

Part 3

Measuring Sustainable Development

10. Valuing Air Pollution in the National Accounts[1]

Attempts to develop 'green' national accounting aggregates fall into two distinct classes: those valuing depletion of natural resources and those valuing degradation of the environment. The strides that have been made in adjusting the accounts to reflect resource depletion[2] are beginning to have an effect on the policy debate concerning the sustainability of development (see, for instance, World Bank (1995)). Given the dependence of developing economies on natural resources, the greatest impact from valuing resource depletion will be felt there.

Valuing pollution emissions in the accounts will be important both for developed countries and the most rapidly urbanizing and industrializing developing countries. This study presents a theoretical approach to the treatment of pollution and pollution abatement in the national accounts, and a first attempt to apply this approach for the countries of OECD Europe. The empirical results are presented in the context of savings rules (Pearce and Atkinson 1993) designed to measure whether economies are on a path of sustainable economic development.

Placing a dollar value on emissions has proven to be the most difficult and contentious aspect of greening the national accounts. A number of issues can be identified: (i) should the value of environmental services be added to national income (Peskin 1989)? (ii) should the value of damages to the environment be subtracted from national income (Peskin 1989) or added (Harrison 1989)? and (iii) should all defensive expenditures, in the form of abatement and protection costs, be subtracted from national income (Juster 1973; Leipert 1989)? Theoretical papers by Mäler (1991) and Chapter 8 of this study answer the final question in the negative where *household* defensive expenditures are concerned.

Beyond the structural aspects of what should be added to or subtracted from national income to reflect pollution emissions, there are difficult questions concerning the valuation of pollution emissions. The UN guidelines for the System of Integrated Environmental and Economic Accounts (SEEA; United Nations 1993b) favour maintenance costs as the basis for valuing environmental degradation - this implies a valuation of the cost of returning the environment to its state at the beginning of the accounting period. Hueting and Bosch (1990) set a more ambitious goal: to value the cost of achieving environmental goals that are consistent with the sustainable use of the environment and natural resources.

Environmental degradation can take many forms, with complex causes. In this paper we wish to concentrate on the narrower question of treating pollution emissions within the national accounts. To do this we develop a theoretical model extending those of Hartwick (1993) and Chapter 8. The results of this model are then applied to valuing air pollution in a number of European OECD countries.

Modeling Multiple Pollutants with Cumulative Effects

In the spirit of the models developed in Chapter 8, we construct an optimal control model of an economy where the object is to maximize the present value of *utility*. Utility is assumed to

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[1] Much of this material is published in Hamilton and Atkinson (1995).

[2] Papers by El Serafy (1989), Hartwick (1990, 1993), Hartwick and Hageman (1993), Hamilton (1994), and Hill and Harrison (1994) have largely settled both the theoretical approach and the practicalities of adjusting national accounts for the discovery, depletion and growth of commercial natural resources.

be a function of both consumption and the services provided by the environment. The solution to the optimal control problem is characterized by the Hamiltonian function, involving both utility and the rate of change of the state variables of the system that are affected by the path of evolution of the economy.

Weitzman (1976) showed that the Hamiltonian function of a simple economy without environmental externalities is equal to GNP, i.e., consumption plus investment. Because the Hamiltonian is maximized at each point in time along the optimal path, this means that GNP is what a planner would choose to maximize if the goal was to maximize the present value of utility. As we will see below, a transformation of the Hamiltonian when environmental externalities are present consists of a GNP adjusted to reflect the effects of these externalities. Thus a seemingly abstruse problem in optimal control yields both a satisfying welfare interpretation of GNP and a consistent way to think about the welfare effects of environmental change. While the model is expressed formally as an optimal control problem, the results are equivalent to those attained in a competitive equilibrium with an environmental externality - factors are priced at their marginal product and Pigovian taxes are required to achieve an optimum.

We assume an economy that is closed to trade, with fixed technology and where the discount rate for utility r (i.e., the pure rate of time preference) is fixed. Gross national product is given by a production function $F(K,L)$, where K denotes the capital stock and L the input of labour. Labour supply is assumed to be unconstrained in the model. GNP may either be consumed or invested, so that,

$$F = C + \dot{K},$$

where C denotes consumption.

The extension of the model of a cumulative pollutant presented in Chapter 8 to multiple pollutants is straightforward if two key simplifications are made: that there are no cross-effects on welfare associated with the interaction of several different pollutants, and that there are no cross-effects on emission reductions associated with different abatement expenditures (the model can encompass these things without difficulty, but at the cost of expositional clarity). We therefore assume that there are N distinct pollutants, generated by N emission/abatement functions $e_i(F, a_i)$. Each pollutant diminishes an individual flow of environmental services according to the equations,

$$\dot{M}_i = e_i \quad \text{and} \quad B_i = B_{0i} - \alpha_i M_i \; .$$

This is not as unrealistic as it might seem: particulates degrade lung function, photochemical smog reduces visibility, and so on. So there are at least some instances in which distinct environmental services are affected by individual pollutants.

The optimal control model is now,

$$\max \int_0^\infty U(C,B_1,\ldots,B_N)e^{-rt}dt \quad \text{subject to:}$$

$$\dot{K} = F - C - \sum_{i=1}^{N} a_i$$

$$\dot{M}_i = e_i .$$

The current value Hamiltonian is given by,

$$H = U + \gamma_0 \dot{K} + \sum_{i=1}^{N} \gamma_i \dot{M}_i ,$$

where again the γ_i are the shadow prices. The first-order conditions for a maximum are,

$$\frac{\partial H}{\partial C} = 0 = U_C - \gamma_0 \Rightarrow \gamma_0 = U_C$$

$$\frac{\partial H}{\partial a_i} = 0 = -\gamma_0 + \frac{\partial e_i}{\partial a_i}\gamma_i \Rightarrow \gamma_i = U_C\left(\frac{\partial e_i}{\partial a_i}\right)^{-1} .$$

If we define $b_i \equiv -\left(\frac{\partial e_i}{\partial a_i}\right)^{-1}$ as the marginal abatement cost for pollutant *i* and $p_{B_i} \equiv \frac{U_{B_i}}{U_C}$, then the measure of economic welfare at shadow prices with a consumption numeraire is,

$$MEW = C + \dot{K} - \sum_{i=1}^{N} b_i e_i + \sum_{i=1}^{N} p_{B_i} B_i . \qquad (10.1)$$

This fits nicely with intuition: economic welfare in the multiple pollutant case consists of consumption plus investment, minus the sum of pollution emissions valued at their marginal abatement costs (or marginal social costs), plus the sum of environmental services valued at consumers' willingness to pay.

As in the model of a single cumulative pollutant developed in Chapter 8, the marginal cost of abatement of each pollutant equals its marginal social cost.

Measurement

Having derived a theoretically correct approach to measuring economic welfare, it is important to consider how these expressions for economic welfare might actually be measured. This breaks down into two sets of questions: (i) how to measure the value of the level of environmental services; and (ii) whether to use marginal social costs or marginal abatement costs as the basis of valuing pollution emissions. Note that the latter question only makes sense away from the optimum, since it was shown above that the two valuations must be equal at the optimum.

To the extent that environmental services can be discretely compartmentalized as in the preceding model, then the standard methods of measuring willingness to pay in environmental economics apply (e.g., hedonic pricing, household production functions, contingent valuation). However, it should be noted that willingness to pay will likely vary from location to location within a country, thereby complicating the measurement problem considerably. The biggest problem is one of aggregation: how to aggregate across the myriad services provided by the environment, for the whole population of a country, without under- or over-counting.

Turning to the valuation of the costs of pollution, the question of measurement away from the optimum is an interesting one, given that most real economies would not be expected to be operating at the environmental optimum. Figure 10.1 provides a way to think about this issue.

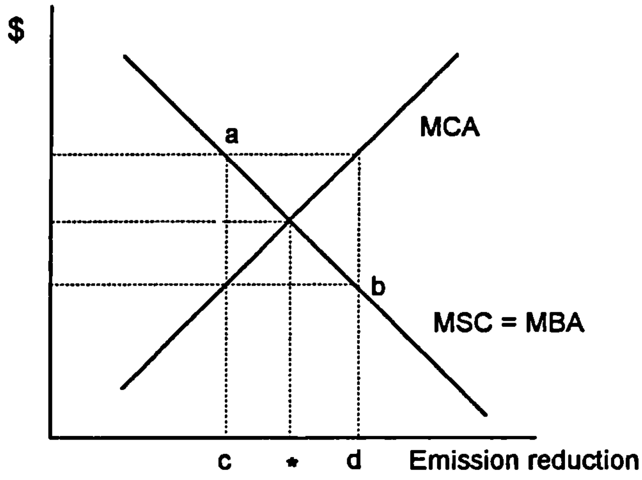

Figure 10.1 Marginal Costs

This is the canonical diagram used to derive the notion of optimal pollution. In this figure the horizontal axis refers to reductions in pollution emissions. 'MCA' is the curve for marginal cost of abatement. 'MSC' is the marginal social cost curve, which is equal to the marginal benefit of abatement (MBA). The optimal emission reduction occurs at level '*', while level 'c' represents over-polluting and 'd' under-polluting.

If we assume that the current state of the economy is one of over-polluting, then marginal social costs at level 'a' will provide an upper bound on the value of optimal pollution emissions; if the current state is one of under-polluting then marginal social costs at level 'b' will provide a lower bound on the optimal emission value. As long as one is reasonably certain that the economy is over-polluting, therefore, using marginal social costs to value emissions should be viewed as an upper limit estimate and interpreted accordingly in the results of the green accounting exercise. This implies in addition that the deduction for pollution emissions in the welfare measure will decrease as the optimum is approached, which is a desirable property.

Figure 10.1 also makes it clear that using marginal abatement costs to value emissions will not lead to an unequivocal direction of bias in the estimates of the value of pollution. If the economy is over-polluting then marginal abatement costs will be below the optimum, while emissions are above their optimal level; the opposite applies to an economy that is under-polluting.

As shown in the empirical part of this study, data on the marginal social costs of *air* pollution emissions, based on willingness to pay concepts, are increasingly available for developed economies, as are data on the levels of air pollution emissions. One of the challenges for practitioners will be to adapt these marginal social cost figures for use in developing countries, or better, to begin to measure willingness to pay for pollution reductions in these countries[3]. Data on marginal abatement costs are generally not collected, but models embodying the appropriate technological and economic information to estimate marginal costs are becoming available.

Savings Rules and Sustainable Development

Hamilton (1994) argues that 'green GNP' *per se* is not particularly useful for policy applications, even though it is important to know the true level of income in an economy. What does have much greater policy salience is the rate of *genuine saving*, i.e., net savings adjusted for environmental degradation. While Asheim (1994) and Pezzey (1994) have pointed out that a positive genuine savings rate at any given point in time does not necessarily indicate that an economy is on a sustainable path, it can be shown that persistent negative saving behaviour is not sustainable.

In the context of our model of a cumulative pollutant, the development path that yields constant utility over all time can be shown to be characterized by an extended Hartwick rule (after Hartwick 1977; see Chapter 4 as well). This can be stated as follows:

If (i) $\dot{K} = be$, so that net investment equals the value of pollution emissions,

and (ii) $\dot{b} = (1 - be_F)F_K b - \alpha U_B / U_C$ defines the time path for abatement expenditures,

then $\dot{U} = 0$, i.e., utility is constant.

This result follows from the proof in Chapter 9. Along the constant utility path we can say that,

$$\dot{U} = U_C\dot{C} + U_B\dot{B} = U_C\left(\dot{C} + \frac{U_B}{U_C}\dot{B}\right) = U_C\left(\dot{C} - \alpha\frac{U_B}{U_C}e\right), \tag{10.2}$$

so that consumption is increasing . The intuition behind this is clear: because environmental services are continually declining as a result of pollution emissions in this model, consumption must compensate for this decline.

The key point in all of this is to note that if $\dot{K} < be$ everywhere along the development path, so that genuine saving is negative, then utility must decline along this path, as was shown in

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[3] There is a growing literature, reviewed in CSERGE/UNC Chapel Hill (1994), on measuring willingness to pay for forest functions, ecotourism, sanitation, water supply and land degradation in developing countries, but the studies valuing the social costs of pollution *per se* are still quite rare.

Chapter 9. An economy with negative genuine saving is not sustainable. In the empirical section that follows, therefore, we will be looking for evidence of genuine savings rates that are persistently negative or near zero as an indicator of marginal or non-sustainability.

Transboundary Pollution

The models presented have, for the sake of simplicity, ignored geography, but the question of transboundary pollution is clearly important in the case of air pollution. Without developing a formal model, the following line of argument is offered.

First, regarding adjustments to income, an extension of the *polluter pays principle* to the domain of national accounting seems appropriate. In other words, pollution damage caused in country B by emissions from country A should appear as a deduction from income in country A. In practice, this means that the estimates of the unit marginal social costs of pollution in a given country should include all costs, including those in other nations. These unit marginal costs should then by multiplied by the total level of emissions in the given country, as given by expression (10.1).

The argument for this treatment of transboundary pollution in the case of savings rules is, if anything, even stronger. Some portion of a given country's total savings should, at least notionally, be set aside in order to compensate the recipients of the pollution emitted and transferred across international boundaries.

Empirical Measures of the Value of Air Pollution Emissions

Using the preceding model of multiple pollutants, we wish to estimate the value of air pollution in Europe caused by carbon dioxide (CO_2), sulphur dioxide (SO_2), nitrogen oxides (NO_X) and particulate matter (PM_{10}). Data on the physical quantity of emissions of key pollutants are relatively easy to obtain in developed countries (see OECD 1994). However, these data alone can tell us little about what is economic, in the sense of optimally balancing the costs and benefits of pollution abatement. As the model implies, to achieve this we need to value units of emissions using marginal social costs. Some estimates of the marginal social costs per tonne emitted are shown in Table 10.1. These are drawn from an evaluation of the social cost of fuel cycles within Europe (CEC/US 1993). The CO_2 estimate is from Fankhauser (1994).

Table 10.1 Marginal Social Costs Per Tonne of Air Pollutant Emitted ($)

	CO_2	SO_2	NO_X	PM_{10}
Health		1530	470	10350
Forestry		1760	1220	
Materials/ buildings		480	320	320
Climate Change	7			
Total	7	3770	2010	10670

Notes: values refer to damage caused by a unit of UK emissions across Europe.
Source: CEC/US (1993), converted to dollars per tonne.

The rows of Table 10.1 are the receptors (i.e., receiving agents). These represent the ultimate effects of polluting activities on human health (respiratory problems), forestry (forest death), material and buildings damage (general soiling and corrosion). The corresponding prices indicate the marginal social cost of a tonne of pollutant vis-à-vis its impact on each receptor[4]. Note that the figures refer to damages both inside and outside the polluting country.

Before combining these data with information on emissions, the methodology used in the CEC/US (1993) study will be described. The first issue is how emissions of pollutants are traced through to their impacts on receptors, and second, how these impacts are valued. The estimates in Table 1 are, insofar as is possible, approximations of marginal social costs and not simply the derivation of average damage costs caused by current emissions. The first point of discussion relates to the scientific part of CEC/US's work, analysing the *incremental* emissions from two hypothetical representative power stations in specific locations in Germany and in the United Kingdom. Incremental emissions are linked to impacts via dose-response functions. Although problems with this approach persist (see Pearce and ApSimon 1995), with the aid of dispersion modelling techniques the transboundary effects of emissions in particular regions can be accounted for, when combined with data on the relevant population densities and

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[4] The absence of an entry indicates either no effect or no quantifiable effect.

characteristics in the areas where pollutants are deposited. The result is that the dose-response function expresses damage to receptors as a function of emissions: e.g., an x% increase in emissions of PM_{10} causes y deaths.

The role of economic valuation techniques is to derive an appropriate value of these impacts, which can in turn be used to derive per unit social costs. This aspect of the CEC/US study will be described by receptor. Since the value of human health effects is derived solely with reference to studies using non-market valuation techniques we discuss this in more detail.

Human health. The basis of estimation for health effects is the value of a statistical life (VSL). What is estimated is the willingness of individuals to pay to obtain a reduction in the probability of mortality. For example, if a person is willing to pay $1000 to secure a reduction in the risk of dying by 1/1000 we would conclude that the VSL in this instance is $1 million, i.e., the willingness to pay (WTP) multiplied by the change in risk. It should be noted that such estimates do not attempt to take account of the likely presence of altruistic motives. VSL can be estimated by any of three main methodologies: (i) contingent valuation; (ii) wage-risk studies; and (iii) avertive behaviour (see Jones-Lee, 1976 and 1989). The first method asks individuals to state their WTP using a range of survey techniques, while the latter two methods infer WTP from actual market behaviour. From an appraisal of available estimates CEC/US (1993) choose a central estimate of the value of a statistical life of approximately US$ 3 .2 million (in 1991 prices). For a critique of this approach see Broome (1978). Morbidity impacts were not valued; it should be borne in mind, however, that these may be significant (Pearce and Crowards 1995).

Forestry. The most obvious economic damage is forgone timber harvest. In addition, carbon sequestration services derived from the growing stock of timber are also lost. Both of these effects are accounted for here. The basis of the assessment of timber losses is the international timber price. The estimation behind the latter forgone benefit is explained below in the description of damage caused by emissions of carbon dioxide (CO_2).

Materials and buildings damage. Estimation of these effects compares the costs of repair or replacement of building materials with and without the corrosion or soiling caused by air pollution emissions. Effects on welfare caused by damage to buildings that form the cultural heritage of a region are not considered.

Climate Change. Climate change will have an impact on receptors such as human health and biodiversity. Using an estimate from Fankhauser (1994) we have accounted for global warming damage in the form of the social costs of carbon dioxide (CO_2) emissions. The absence of adequate time series data on other greenhouse gases precluded their consideration. Although the mix of greenhouse gases varies from country to country, as a general rule carbon dioxide is the most important in terms of its total contribution to global warming.

Strictly speaking, these estimated damages are specific to emissions from the countries doing the polluting (in the case of Table 10.1, the UK). However, willingness to pay is closely linked to ability to pay. In order to value social costs in additional European countries, the unit marginal values in Table 10.1 are scaled across countries using the simple assumption that marginal valuations depend linearly on differences in income per capita. In reality the valuations in different countries will also depend on factors such as baseline pollution levels, the typical pollution content of a country's emissions and the population density in affected areas. There is ample scope, therefore, to improve these estimates as new data become available. In the case of the climate change problem, Fankhauser's estimate refers to the globalised social costs of CO_2 emissions and hence we do not adjust this estimate for different countries.

In addition, while it would have been possible to adopt other receptor damage estimates presented in CEC/US (1993), these would not have been relevant for present purposes. We are interested in those costs which affect future welfare, i.e., the effects of air pollution emissions on assets. This is one rationale for excluding damage to crops from the analysis, since this is a loss of current output and does not directly affect future welfare[5]. Within the receptors chosen, damages can legitimately refer to *either* marketed or non-marketed damages. This is worth stressing because, in the case of damage to marketed assets, it is extremely unlikely that any account has already been taken for these losses in conventional estimates of capital consumption allowances.

Using these prices for pollution emissions, we estimate both the value of the loss of environmental services as a proportion of total output and the measure of genuine saving.

Genuine Saving and Air Pollution Emissions

We wish to extend the concept of 'genuine' saving appearing in Hamilton (1994) to include the value of pollution emissions, as suggested in the earlier section on 'Savings Rules and Sustainable Development.' Conventionally defined net savings deducts the value of depreciation of reproducible capital (dK) from gross saving GS. From this traditional net saving two further deductions are made: the depletion of commercial natural resources, valued as the unit resource rent n (net of the level of the effective emission tax rate on output - see Chapter 9) times the amount of resource extraction R and the degradation of the environment valued as the unit marginal social cost σ times the amount of pollution emissions e.

Genuine savings are therefore defined as,

$$S_g = GS - dK - nR - \sigma e.$$

In the following we express GS as a proportion of gross output (GDP).

The period of analysis is the decade of the 1980's. The values in Table 10.1 are in 1990 prices, so in order translate these to previous years a deflator is required. Since CEC/US (1993) present prices in ECUs, the deflator chosen is an index of European Union (EU) GDP deflators weighted by the ratio of each member's GDP to the EU total. Not surprisingly, this index is dominated by price changes occurring in France, Germany, Italy and the UK. This adjustment does not account for a possibility suggested by Figure 10.1 - since countries typically over-polluted even more in the past, the marginal social costs were likely higher than the simple deflation of a 1990 price would suggest. As noted previously, these deflated unit values are scaled by the individual countries' per capita GDP in order to account for differing ability (and willingness) to pay.

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[5] However, the position may be more complicated if the damage is a result of soil acidification that affects output in future periods.

Genuine Saving in the United Kingdom

Savings ratios for the UK are plotted in Figure 10.2 for the period 1980 to 1990. Shown as a proportion of GDP, these display the successive deductions from gross saving proposed above, with 'Res. Net Saving' indicating traditional net saving less the value of resource depletion[6].

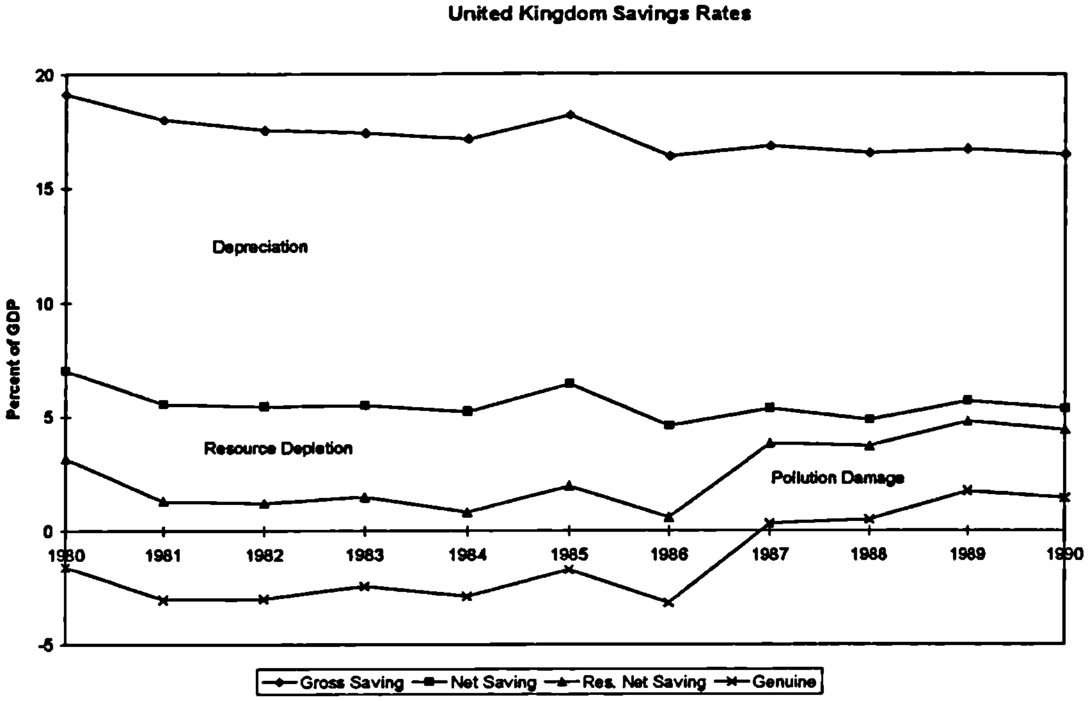

Figure 10.2

On this measure the UK appears to have persistently under-saved during much of the 1980s. In the period 1980 to 1986 genuine savings rates were between -1.6% and -3.1% of GDP. This is a striking result and shows that, by simply beginning the process of redefining a nation's savings rates to be net of the depletion of non-renewable resources and the value of air pollution, inadequate provision to offset asset loss was made during the 1980s in the UK. Of course, the caveat should be added that these numbers could be refined and that marginal social costs give an upwardly-biased estimate of the value of pollution emissions.

However, there are recent indications that the marginal social costs of PM_{10} emissions could be in excess of the levels used here (see Pearce and Crowards 1995). These estimates also exclude the value of water pollution - the physical indications are that UK water quality declined somewhat during the latter part of the 1980s. Figure 2 also indicates that the gross savings ratio in the UK is below 20% of GDP for the entire period. This is low relative to the ratios prevailing in other European countries.

The measure of genuine saving indicates that the UK stopped dissaving towards the end of the period. Some of this increase is attributable to a reduction in the value of resource depletion

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[6] Rough estimates of the value of resource depletion, using current unit rents, are based on the data reported in Chapter 11.

which in turn is in large part due to the reduction in world oil prices after 1986. Paradoxically, this conveys the impression that, other things being equal, a decrease in the price of oil raises genuine saving. Offsetting this is the fact that the remaining oil reserves are now a less valuable form of wealth, which could conceivably result in a lower level of future welfare. It can be shown that the capitalized value of these increased savings exactly equals the decrease in resource wealth associated with the fall in resource price.

Figure 3 displays in more detail the degree to which the value of air pollution damage has changed over the period (note that the curves are not cumulative). While damage caused by NO_X and CO_2 damages are a slightly declining proportion of GDP, PM_{10} and SO_2 damages fall more dramatically. In the latter case, these costs decrease from 2.6% of GDP in 1980 to 1.5% in 1990. This trend is largely attributable to a 23% fall in the quantity of emissions over that time. In the next section we investigate the extent to which these trends were experienced in other European countries.

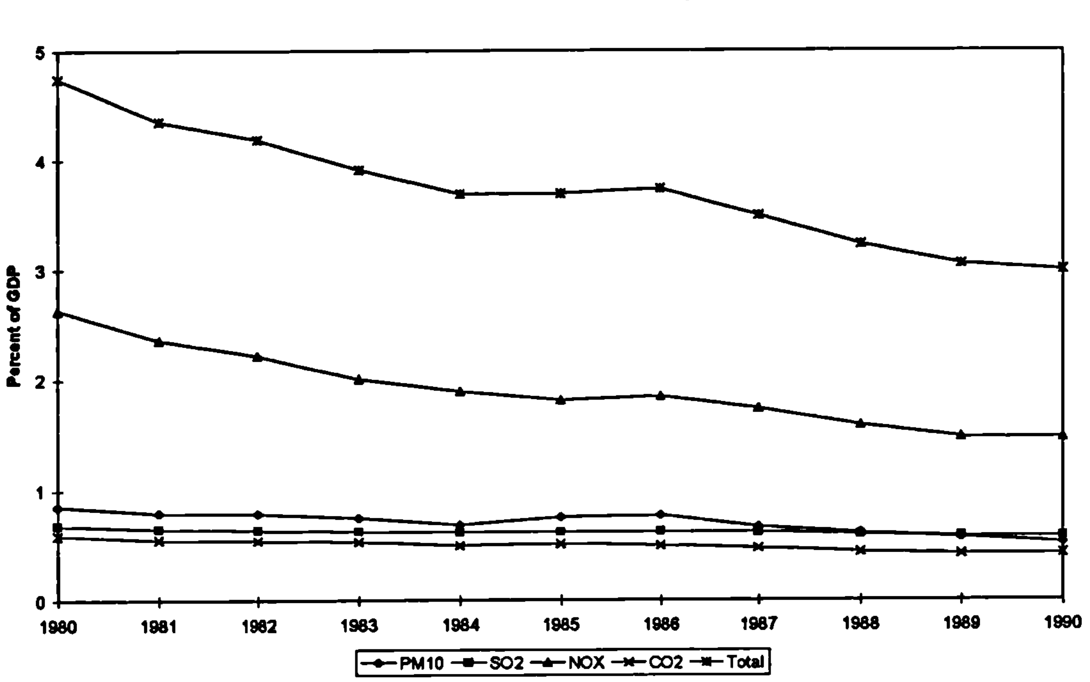

Figure 10.3

Saving Experience in the rest of OECD Europe

The calculations in this section are based on the crude scaling, from the UK base of Table 10.1, of marginal social costs by per capita income in the countries of OECD Europe. The earlier caveats about these figures should therefore be recalled.

Table 10.2 shows genuine savings rates for sixteen European countries[7]. These rates have been increasing in most of these countries, although the experience of Norway is one of a fall between the years 1980 and 1985 and a subsequent increase to 1990. Exceptions to this broad

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[7] Gross savings rates are taken from World Bank (1993), while depreciation is from United Nations (1992).

trend are (in addition to Norway) France and Greece, while Finland remains largely unchanged. Some of the reasons for increasing genuine saving rates are obvious. For those countries with relatively abundant oil (and gas) resources - the Netherlands, Norway and the UK - the fall in the oil price in the latter half of the 1980s is a significant factor. The key factor in most countries, however, was the decline in the value of emissions, as seen in Table 10.3.

Table 10.2 Genuine Saving as percent of GDP in Europe

Country	1980	1985	1990
Austria	11.4	8.8	13.1
Belgium	7.2	5.7	11.9
Luxembourg	8.4	10.3	15.6
Denmark	4.7	7.7	11.5
Finland	8.4	7.3	8.2
France	11.4	5.2	7.9
Germany	8.1	7.5	13.8
Greece	4.1	-3.4	-2.8
Ireland	-3.3	6.3	14.2
Italy	7.5	6.6	7.4
Netherlands	6.0	5.4	13.3
Norway	13.4	5.0	8.5
Portugal	-4.2	3.0	4.4
Spain	4.8	3.4	9.2
Switzerland	2.5	7.3	8.3
UK	-1.6	-1.7	1 4

Table 10.3 expresses the value of air pollution emissions as a proportion of GDP, thus giving an indication of the size of these losses across OECD Europe. As in the case of the UK, the picture is one of a declining value of pollution relative to output across the region, owing to decreasing emissions of air pollutants during the period. In terms of marginal impacts per unit emitted, Table 10.1 indicated that PM_{10} is the most damaging pollutant. However, the magnitude of the total values of pollution emissions will obviously also depend on the quantity of each pollutant emitted. In this respect, although not shown in Table 10.3, SO_2 is the most significant pollutant over the period. Emissions of this pollutant fall quite steeply in all countries between 1980 and 1990 (as is obvious for the UK from Figure 10.3), probably owing to country commitments with respect to the First Sulphur Protocol.

Returning to the estimates of net saving in Table 10.2, the large negative net savings ratio indicated for Portugal in 1980 largely reflects the high value of PM_{10} emissions relative to GDP in that year - the return to positive net savings in subsequent periods is owing to the absence of data for emissions of PM_{10} in the years 1985 and 1990. (These data are also lacking for Denmark [1980] and Spain [1980 and 1990].) Table 10.3 shows Portuguese air pollution damages at some 8.7% of GDP. This seems high, with the probable cause lying in our adoption of the estimates of marginal social costs based on UK emissions (weighted for differences in per capita income). We would also expect differences in population densities to play a significant role in explaining differences in these costs; the population density in the UK is about double that in Portugal. Furthermore, the dispersion of pollutants is also a key factor. The consideration of similar additional factors would go some way to correct this apparent anomaly. Similar comments apply to Ireland and Greece.

Table 10.3 Air Pollution Damage as percent of GDP in Europe

Country	1980	1985	1990
Austria	2.7	1.9	1.2
Belgium	3.1	2.0	1.8
Luxembourg	4.1	3.6	2.0
Denmark	3.7	2.7	1.8
Finland	4.6	2.7	1.8
France	3.3	1.6	1.3
Germany	2.5	2.4	1.6
Greece	7.0	3.7	2.5
Ireland	5.9	4.3	4.0
Italy	4.7	2.6	2.3
Netherlands	2.1	1.6	1.5
Norway	2.5	1.7	1.5
Portugal	8.7	1.9	1.3
Spain	5.1	4.8	2.0
Switzerland	4.9	1.6	1.0
UK	4.7	3.7	3.0

Greece exhibits negative genuine savings in 1985 and 1990. In contrast, Ireland shows an apparent increase in genuine savings of 17.5% from 1980 to 1990. This latter finding is largely due to a transition from a relatively low gross savings ratio of 14% in 1980 to 28% in 1990. The former West Germany increases its genuine savings rate owing to a large fall in emissions of SO_2 and PM_{10}. While not part of this analysis, it is noteworthy that the former East Germany emitted PM_{10} at about three to four times West German levels.

Conclusions

In this chapter we have developed a simple model of green national accounting and then applied the results to the valuation of pollution emissions in European OECD countries. A number of strong conclusions follow.

First, the model suggests that attempts to account for pollution result in a welfare measure rather than green GNP *per se*. An important element of this welfare measure, the value that consumers place on the current level of all non-market services provided by the environment, presents severe measurement difficulties. Marginal social costs or marginal abatement costs, rather than maintenance costs, represent the correct price to place on pollution emissions. Estimates of marginal social costs (based for the most part on the measure required by theory, the willingness to pay) are increasingly available for developed countries, and the estimates are becoming more refined.

Second, this treatment of pollution emissions in expanded national accounts fits well with 'savings rules' as policy prescriptions for sustainable development. If net investment in produced assets is persistently less than the value of pollution emissions then welfare will decline. The opposite is not necessarily true: positive genuine savings rates at a given point in time do not provide unequivocal evidence for sustainability.

The empirical application of the model suggests that the value of air pollution emissions is a significant percentage of GDP for the countries of OECD Europe. While the use of marginal social costs to value pollution leads to upward bias in the estimates for recent years (assuming countries are over-polluting), there is reason to believe that this is offset at least partially in the estimates for the early 1980's by the higher marginal social costs associated with higher pollution emissions.

Finally, when these pollution value estimates are combined with the value of resource depletion, the resulting genuine savings rates are negative for several European OECD countries and years. This suggests that concerns about the sustainability of development should be taken seriously for developed, as well as developing, countries. The good news is that pollution emissions are falling. An important question, therefore, is how close these countries are to the optimum level of pollution.

In terms of next steps there are obvious refinements that can be envisioned, including expanding the country coverage of data on willingness to pay for pollution reductions. This is particularly important for developing countries. In many of these countries rapid urbanization, industrialization and motor vehicle usage is leading to severe pollution problems. Placing a value on these emissions will help to provide the information and the motivation for governments to act.

11. Cross-Country Estimates of Genuine Saving

Given the centrality of savings and investment in economic theory, it is perhaps surprising that the effects of depleting natural resources and degrading the environment have not, until recently, been considered in the measurement of national savings. This omission may be explained both by the tools economists use, which emphasize gross measures, and the fact that the System of National Accounts (SNA) ignores depletion and degradation of the natural environment. This is not intended to be excessively critical of the SNA - it measures market activity very well, which is its intent. It is nonetheless true that the tools economists use tend to colour or restrict the view of the problems to be faced.

Valuing depletion and degradation within a national accounting framework is an increasingly viable proposition, both as a result of the significant progress made in the techniques of valuation of environmental resources (see, for a recent example, Freeman 1994) and as a result of the expanding foundation that theoretical developments are placing under the methods of 'green' national accounting (Hartwick 1990; Mäler 1991; Hamilton 1994). The first application of these accounting methods to the measurement of net savings appeared in Pearce and Atkinson (1993) - this study combined published estimates of depletion and degradation for 20 countries with standard national accounting data to examine true savings behaviour. By this measure many countries appear to be unsustainable, since their savings rates were less than the sum of conventional capital depreciation and natural resource depletion.

Enlarging the concept of net saving to include the depletion of natural resources is in many ways the most natural alteration of traditional savings concepts. This is because the depletion of a natural resource is, in effect, the liquidation of an asset and so should not appear as a positive contribution to net national product or, by extension, net savings. While minor technical issues remain, the methods of valuing the depletion, discovery and growth of commercial natural resources in the context of the SNA are by now well developed (Hamilton 1994; Hill and Harrison 1994).

More problematic is the valuation of environmental degradation. While UN guidelines for environmental accounting (United Nations 1993b) favour valuing this degradation in terms of maintenance costs (the cost of restoring the environment to its state at the beginning of the accounting period), the latest theoretical approaches, from Chapters 8 and 10, suggest that the marginal social costs of pollution are the correct basis for valuing waste emissions to the environment.

The Savings Rule

To give the flavour of what results from the formal approach to green national accounting, the following equation adapts the expression for economic welfare from Chapter 9:

$$MEW = C + I - n(R - g) - \sigma(e - d) + p_B B.$$

Here C is consumption, I net investment, n the unit resource rental rate less the value of the implicit pollution tax on production, R resource extraction, g resource growth, σ the marginal social costs of pollution emissions e, the natural dissipation of pollution d, and p_B the willingness of consumers to pay for environmental services B. For non-living natural resources the term in g is zero, while d is zero for pollutants with cumulative effects.

The measure of net national product simply drops the last welfare term from this expression. The intuition behind this is clear: $I - n(R-g) - \sigma(e-d)$ is the value of net investment when changes in natural resource stocks and stocks of pollutants, appropriately shadow priced, are included in addition to increments to the stock of produced assets.

However, while it is important to know what constitutes the sustainable level of national income, this measure is not particularly relevant for policy purposes. A shift in the level of national income does not carry a policy signal with regard to sustainable development, while the relative growth rates of sustainable income and GNP, for instance, are liable to give equivocal signals. Given that concerns about sustainable development are fundamentally concerns about the future, this suggests that adjusted measures of savings and wealth are more fertile territory for policy purposes.

The expression for genuine saving follows directly from the preceding:

$$S_g = GNP - C - dK - n(R-g) - \sigma(e-d).$$

Here *GNP - C* is gross saving as traditionally defined, with *C* being the sum of public and private consumption; gross saving includes the level of foreign saving as well. *dK* is the value of depreciation of produced assets, so that *GNP - C - dK* is conventional net saving. The last two terms represent the value of net depletion of natural resources and net accumulation of pollutants.

The importance of this measure of genuine saving is that it is a one-sided indicator of sustainability, in that persistent negative genuine savings are not sustainable, as shown in Chapter 9. Note that it is possible to have apparently robust gross saving and negative genuine saving. So while it is easy to calculate gross savings rates from published national accounts data, this may give little indication of whether the economy is on a sustainable path. This reinforces the point made earlier that the tools economists use may bias their conclusions with regard to economic performance.

In what follows we present a set of estimates of the value of resource depletion and environmental degradation for a range of developing countries, expressed in terms of genuine savings rates. What is presented is necessarily limited by the available data. The natural resource estimates span crude oil, the major metallic minerals, phosphate rock and tropical forests. The only environmental pollutant considered is carbon dioxide, the principal contributor to the greenhouse effect. While it would be highly desirable to expand the coverage of other pollutants, data on both the level of pollution emissions and their marginal social costs are lacking in many developing countries.

Measurement Issues

The problems in the measurement of depletion and degradation of the environment break down into several distinct pieces: (i) the valuation of resource rents for non-renewable resources; (ii) valuing depletion of tropical forests; and (iii) valuing the marginal social costs of CO_2 emissions. These will be described in turn.

The basic approach to calculating resource rents for non-renewable resources is to subtract country- or region-specific average costs of extraction from the world price for the resource in question, all expressed in current US dollars. World prices were derived from World Bank (1993) - where multiple markets, e.g., London and New York, are reported, a simple average of these market prices serves as the world price. For minerals the levels of resource rents are thus calculated as:

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Rent = World price - mining cost - milling and beneficiation costs
                   - smelting costs - transport to port
                   - 'normal' return to capital.
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Rents on crude petroleum are calculated as world price minus lifting costs. There are several things to note about this methodology:

- From a theoretical viewpoint, resource rents should be measured as price minus *marginal* cost of extraction (including a normal return to capital). In practice, marginal production costs are almost never available and practitioners (as evidenced by the 'green national accounting' literature) fall back on using average extraction costs. This will tend to overstate calculated resource rents.

- Countries may or may not be selling their natural resources (for internal consumption or export) at the world market price, although one would expect that they have every incentive to do so. This methodology therefore can be viewed as shadow pricing of natural resources.

- Extraction costs are measured at a fixed point in time, 1985 for minerals (Bureau of Mines 1987) and 1990 for crude oil (Dept. of Energy 1994), and held constant over the period 1980-1990. World prices vary over time, leading to corresponding variations in calculated rental rates.

- Where the extraction cost data were region- rather than country-specific, the regional cost structure was applied to all of the producing countries in the region.

- Rents are generally viewed as accruing to the resource owner for the production of the crude form of the material in question, typically an ore. In practice, most mineral operations are vertically integrated to a considerable extent and so the only price and cost data are for refined forms of the materials. Measuring resource rents as described above for these vertically integrated mineral operations therefore implicitly ascribes any excess returns to capital for the milling and refining stages to the resource rent.

- In some cases, such as for lateritic nickel deposits, the rental rates are negative for at least part of the time. This may represent, of course, a situation in which producers actually managed to decrease average extraction costs in line with price movements, so that rents 'in reality' are not negative, a phenomenon that is masked by the above methodology; it may also simply be the case that firms continued to operate, in spite of reduced or negative rates of return on capital, while they were meeting their variable costs and in the expectation of improved market conditions. Negative unit rents are set to zero in the calculations below.

Table 11.1 presents the calculated average rental rates for minerals and crude oil. The coefficients of variation are high for zinc and nickel. The table also shows which cost components, subject to data availability, went into the calculation of rental rates. In most (but not all) cases an explicit rate of return on capital appears as a cost component. Missing cost components lead, of course, to over-estimates of resource rents.

Table 11.1 Rental rates for minerals and crude oil.
Unweighted pooled data, 1980-1990, excluding negative values.

	Mean rate	Standard deviation	Cost components
Zinc	0.49	0.32	Mining, milling, smelting, transport, 15% ROC
Iron ore	0.56	0.20	Mining, beneficiation, transport
Phosphate rock	0.33	0.14	Mining, milling, transport, 15% ROC
Bauxite	0.59	0.18	Mining, milling
Copper	0.43	0.16	Mining, milling, smelting, 15% ROC
Tin	0.30	0.13	Mining, milling, 15% ROC
Lead	0.56	0.27	Mining, milling, smelting, transport, 15% ROC
Nickel	0.34	0.23	Mining, refining, smelting
Crude oil	0.74	0.14	Lifting costs

ROC: return on capital.

In line with the theoretical models, the country-specific unit resource rents in each year are multiplied by the quantities of resource extraction for each of the minerals in Table 11.1, to arrive at the total value of resource depletion.

For tropical forest resources the situation is much more complicated with regard to the valuation of depletion. The issue is essentially one of land use, with standing forests being one use among many for a particular land area. This suggests that the correct way to value deforestation is to measure the change in land value (which should represent the present value of the net returns under the chosen land use) - this is essentially the result in Hartwick (1992). The formal models of the preceding chapter suggest that, where deforestation is not occurring but harvest exceeds growth, it is the net depletion of the resource that should be valued.

Because there are few data on rates of harvest and natural growth, the estimates presented below are confined to the valuation of deforestation, based on the latest decadal FAO forestry assessment (FAO 1993). Since there are virtually no data on the value of forested land before and after clearance, the deforestation is simply valued as the stumpage value of the volume of commercial timber on each hectare cleared. Stocking rates (the volume of commercial timber per hectare) by country are as given in FAO (1993). The stumpage rate is assumed to be 50% of the market price, a crude assumption but consistent with studies such as Sadoff (1992) and Repetto *et al.* (1989); market prices are from World Bank (1993). Deforestation rates are linearly interpolated from the decadal estimates of forest cover given in FAO (1993).

The foregoing description of the valuation of forest depletion suggests that the estimates are quite rough. It should also be obvious that the only values being calculated are commercial, so that the values of biodiversity, carbon sequestration and other losses are not captured.

Turning to the value of carbon emissions, the basic emissions data employed are from the Carbon Dioxide Information and Analysis Center (Marland *et al.* 1989), covering fossil fuel combustion and cement manufacture. The global marginal social cost of a metric ton of CO_2 is assumed to be $20 US in 1990, taken from Fankhauser (1993). Global costs are assigned to emitting countries as an extension of the 'polluter pays principle' into the domain of environmental accounting - this seems consistent with the notion of genuine saving, in that the value of the external costs of emissions imposed on other countries should be, at least notionally, set aside in a fund to compensate those countries negatively affected.

The formal model of carbon emissions from fossil fuels in Chapter 8 suggests that the value of an optimal carbon tax should be deducted from fossil fuel rents, to account for the pollution externality, and that the value of natural dissipation of the carbon stock in the atmosphere should be deducted from emissions. Given that the carbon tax, based on marginal social costs, would be less than $5.50 US per metric ton of carbon in fuel, and that the mean residency time of carbon in the atmosphere is roughly 120 years, these effects have been ignored in the calculations.

A key missing element in the estimates is any valuation of soil erosion, owing to the lack of comprehensive data sets on either physical erosion or its value. This is an important gap considering the importance of agriculture in most developing countries - erosion is considered to be a major problem in Subsaharan African countries in particular.

Measures of Depletion and Genuine Saving

The basic national accounts data used to arrive at genuine savings rates are as given in the World Bank's *World Tables* (World Bank 1994). However, these data do not include the value of depreciation of produced assets. Accordingly, unofficial World Bank estimates of depreciation, as calculated from perpetual inventory models, are taken from Nehru and Dhareshwar (1993). Each of the data sets employed in this chapter, the *World Tables* data, the depreciation estimates, and the resource depletion and degradation estimates have various gaps in their coverage[1]. The result is that there are 56 developing countries for which complete times series of the basic data exist over the period 1980 to 1990.

The first question to be answered is whether the calculation of depletion and degradation adds substantially to the picture of whether countries are on a sustainable path. This reduces to the question of whether there are countries whose net savings rates (as conventionally defined, gross saving minus the value of depreciation of produced assets) are positive but whose genuine savings rates are negative. This is examined in Figure 11.1.

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[1] Note that resource extraction data in physical quantity are taken from the World Bank's Economic and Social Data Base (BESD).

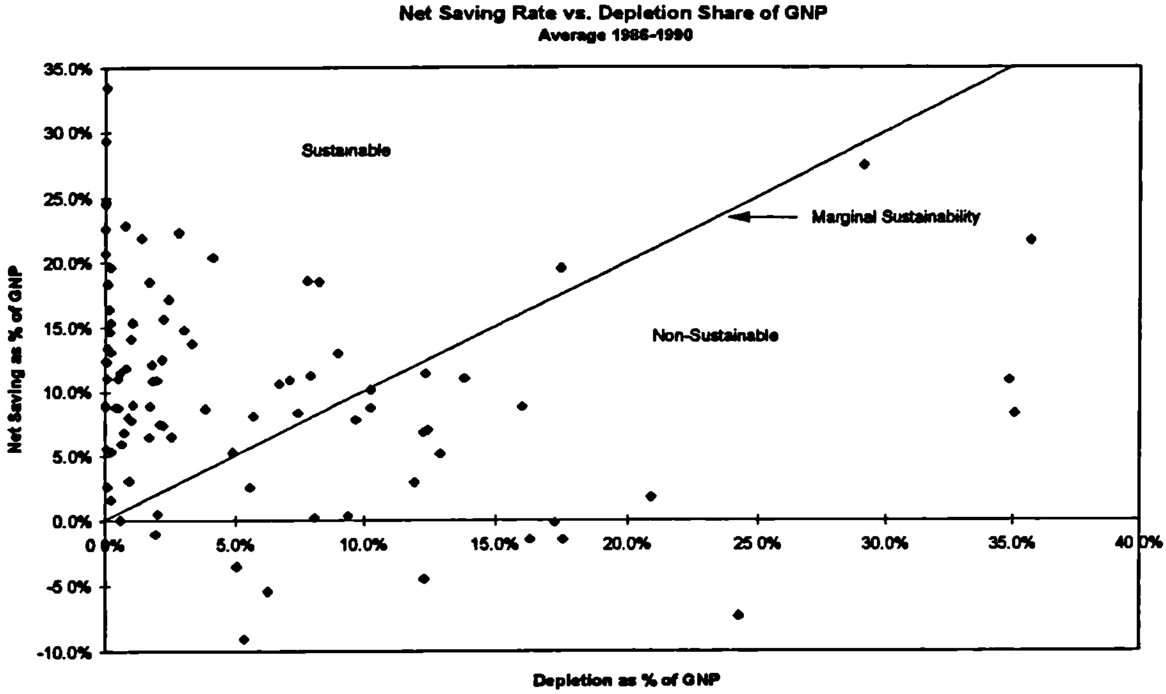

Figure 11.1

In this figure[2] the net saving rate (for developed and developing countries) is scatter-plotted against the value of depletion (and CO_2 emissions) as a percentage of GNP, using average figures for the period 1986-1990. The line labelled 'Marginal Sustainability' is the 45° line - countries falling above this line have genuine savings rates that are positive (since genuine saving is just net saving less the value of depletion and degradation), while those falling below have negative genuine savings rates. While there are several countries that have negative net savings rates, and so are unsustainable even by conventional national accounts measures, there are clearly a considerable number of countries with positive net savings but negative genuine savings. Measuring genuine saving provides useful new information therefore.

Figures 11.2 and 11.3 summarize the genuine savings rates for countries aggregated into regions. Note that the calculated regional savings rates are the net savings for the countries in the sample - they are therefore weighted towards the largest countries (which tend to have the largest absolute amounts of saving or dissaving) and do not estimate or 'gap-fill' for countries in the region but not in the sample. The countries in the sample are presented below in Table 11.2.

The first thing to note from these figures is that OECD countries and South Asian countries had broadly similar rates of genuine saving through the 1980's - in the neighbourhood of 10% of GNP. To match this, growth in per capita GNP was slightly higher in South Asian countries over this decade, 3.1% compared with 2.3% in OECD countries (World Bank 1994). So the savings figures and the growth figures tell a basically consistent story for these groups of countries.

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[2] For presentational purposes, a few countries that had net savings rates less than -10% and/or depletion greater than 40% of GNP have been excluded from this figure.

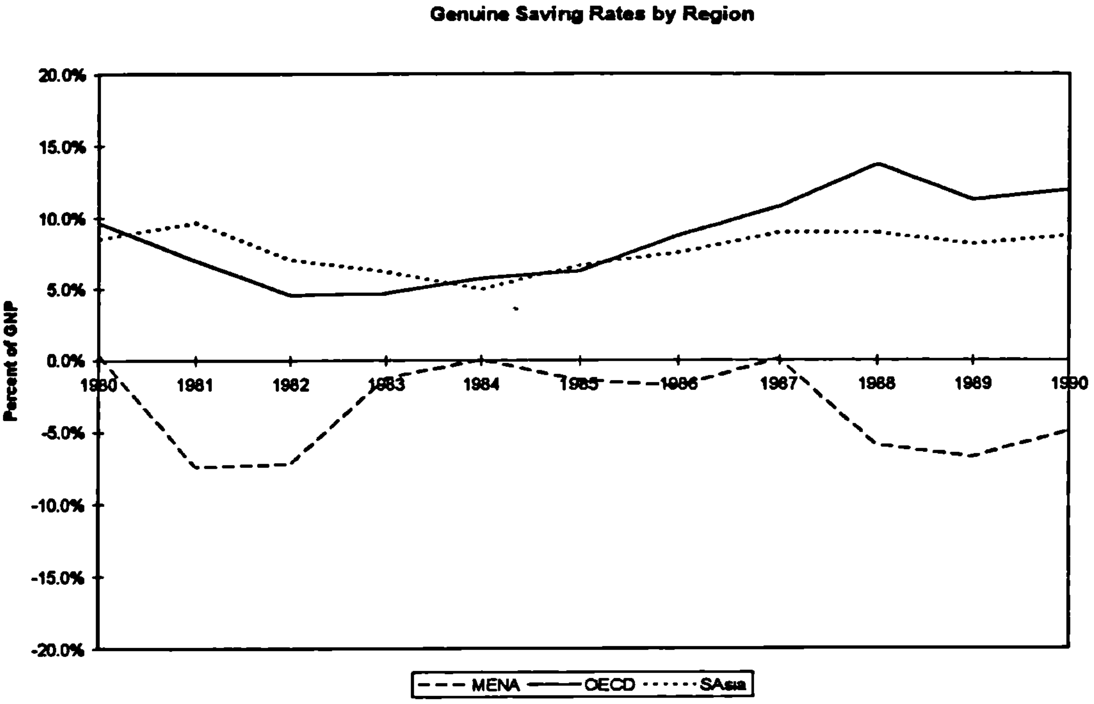

Figure 11.2 Abbreviations: Middle East and North Africa (MENA), South Asia (SAsia)

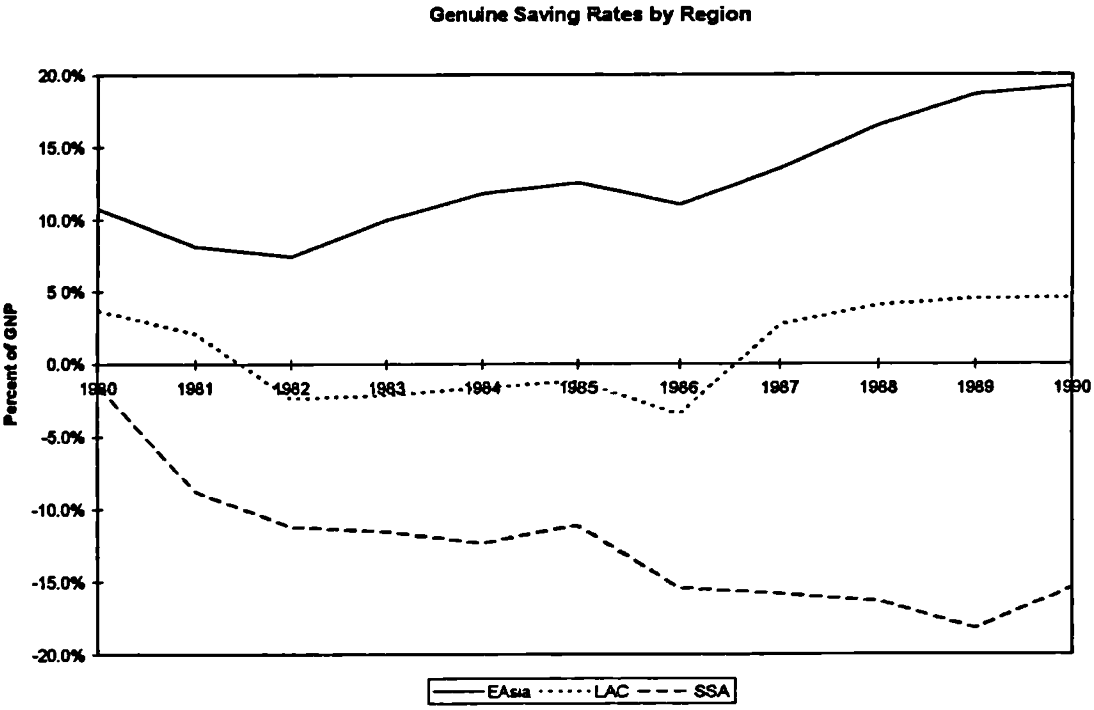

Figure 11.3 Abbreviations: East Asia (Easia), Latin America and the Caribbean (LAC), Subsaharan Africa (SSA)

The next point to note is that the countries of the Middle East and North Africa, basically the oil exporters, were marginal and sometimes substantial dissavers. However, this is a result both of oil constituting a substantial portion of economic activity in these countries and the fact that resource rents are not discounted in the calculation of depletion. While the theoretical models suggest that this is the correct valuation of depletion, the results of the models are the product of optimization, so that unit resource rents are assumed to increase at the rate of interest (i.e., the Hotelling rule). The effects of relaxing the Hotelling assumption in calculating the value of crude oil depletion are presented below.

The sampled countries of Latin America and the Caribbean were marginal dissavers during the years of the debt crisis (1982-1986). This largely reflects a dip in the gross saving rate over these years, so there is no strong evidence that these countries stripped resource assets in order to pay off debts (although the linear interpolation of the deforestation estimates would mask this effect in the case of forest depletion). From 1987 onwards there is a marked improvement in savings behaviour.

The most striking contrast in genuine saving rates is between East Asia and Subsaharan Africa, a contrast that is also vividly reflected in growth rates in per capita GNP. East Asia was consistently the strongest region in terms of any measure of saving: gross, net or genuine. Subsaharan Africa shows a near-steady decline in genuine saving over the course of the decade. The steep drop in genuine saving from 1980 to 1981 reflects Nigeria's transition from a sizable positive to sizable negative current account balance. The composition of savings and investment in these two regions is shown in Figures 11.4 and 11.5.

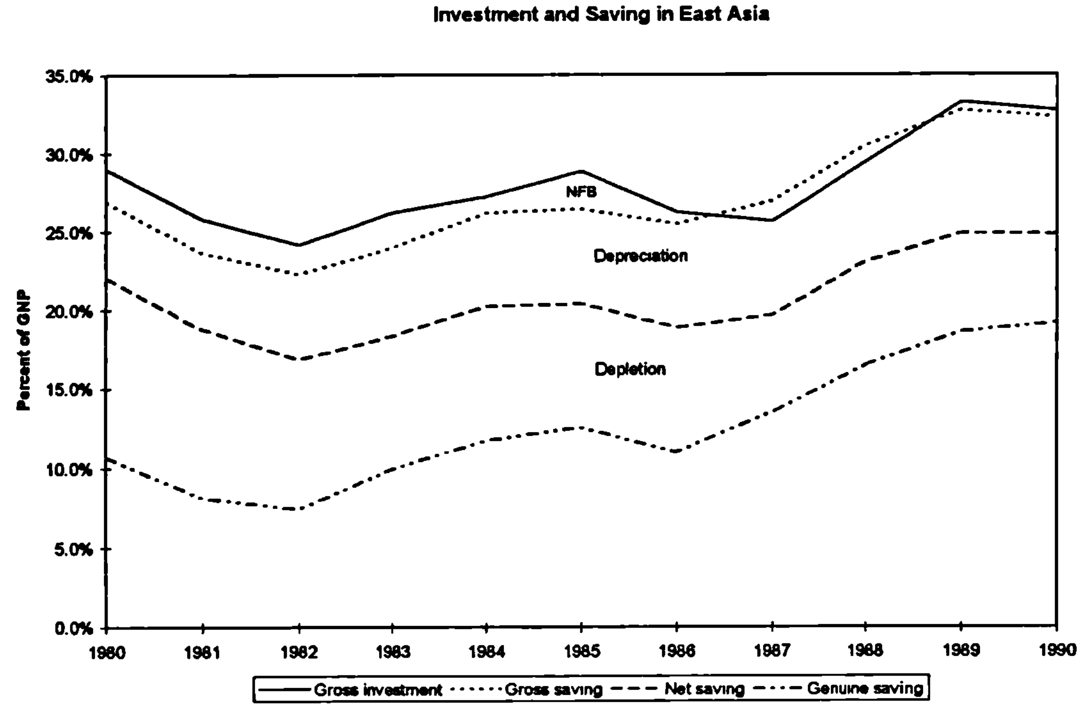

Figure 11.4

In these charts the difference between gross investment and gross saving rates is the ratio of net foreign borrowing (NFB) to GNP. If net borrowing is positive then, obviously, gross investment is greater than gross saving.

In East Asia the declining ratio of depletion (and degradation - recall that this includes the value CO_2 emissions) to GNP is mirrored by an increase in the share of depreciation of produced assets in GNP. The latter reflects the effects of industrialization. The falling ratio for depletion reflects a near-constant total value of depletion. This may, again, be an artefact of the linear interpolation of deforestation, an important component of depletion for this region. There is little doubt, however, that the growth in industrial (and service) output in this region is leading to a decline in the relative importance of natural resources.

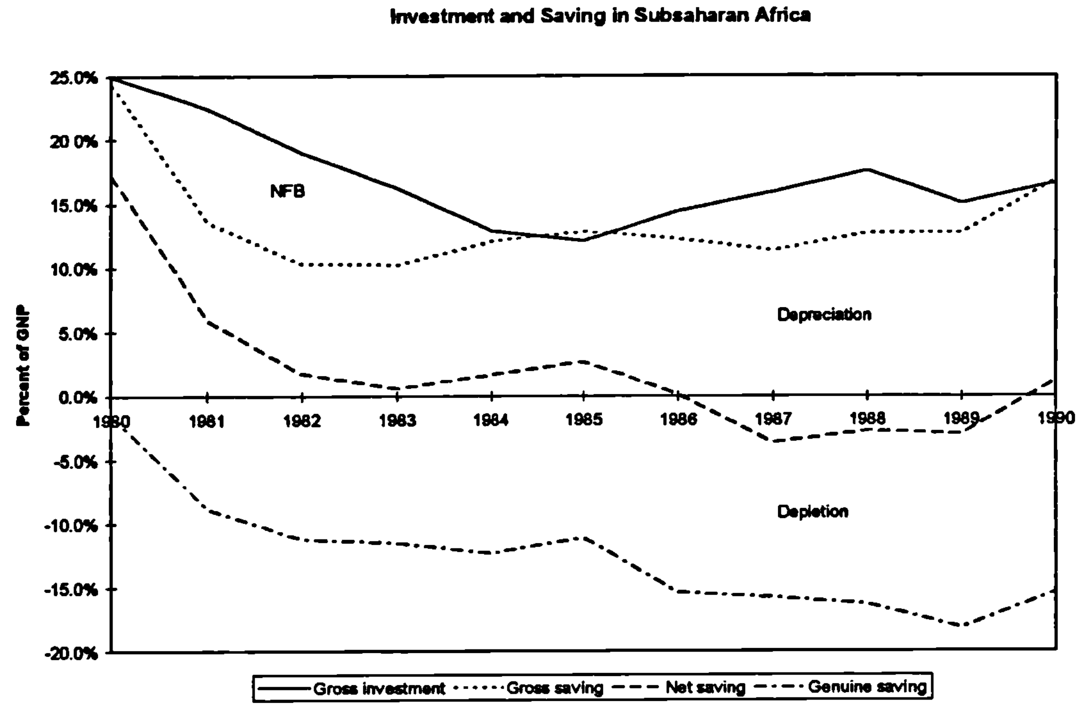

Figure 11.5

As noted above, there was a steep drop in all savings rates in Subsaharan Africa from 1980 to 1981, reflecting the dominance of Nigeria in the regional aggregate. Net foreign borrowing was significant in the early 1980's. From a net savings perspective, the sampled countries in this region were unsustainable from 1986 to 1989. The effect of including depletion in the genuine savings measure is to sharply accentuate this effect, and to show that the region was not on a sustainable path right through the decade[3].

The 'bottom line' in these two figures is clear: there was strong growth in genuine saving in East Asia over the 1980's, while there was strong growth in dissaving in Subsaharan Africa.

The Subsaharan African Experience

The results for Subsaharan Africa (SSA) are so striking that they merit a closer examination. Country-level estimates of genuine savings rates for the 1980's are given the Appendix to this chapter, in Table A11.1.

In a paper reviewing the literature on long term development and growth in SSA, Ndulu and Elbadawi (1994) cite a number of broad conclusions: (i) SSA has grown more slowly than other developing countries since the mid-1970's; (ii) lower savings rates and levels of human

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[3] Regional aggregates are a convenient way to summarize the behaviour of countries that are, to a degree, similar in their endowments. It is up to individual countries, however, to design and implement policies for the achievement of sustainable development.

capital have helped prevent it from catching up with other developing countries; (iii) the policy climate in SSA has not been conducive to sustained growth, characterized as it has been by disincentives to save, over-valued and variable exchange rates, high public consumption, and under-developed financial systems; and (iv) the economies of SSA have been subject to elevated levels of external shocks, both economic and physical (in the form of drought and other severe weather patterns), and political instability.

The pattern presented in Table A11.1 appears to be 'the curse of the mineral-rich' (cf. Gelb 1988). Countries such as Kenya, Rwanda, Burundi and Niger, with relatively few exports of oil and minerals, have the most promising saving performance. On the other hand, resource-rich Zimbabwe exhibited positive genuine savings. As noted earlier, the figures for Nigeria (and the Congo) are probably skewed by the substantial size of the deposits of crude oil. Zambia is the other anomalously large dissaver - while these effects may again be overstated for technical reasons having to do with the valuation of resource depletion, it is also true that the economic policy climate was particularly unfavourable in Zambia for many years.

The sorts of issues raised by Gelb (1988) about the nature and effects of oil windfalls in developing countries are particularly relevant in dealing with this question. Without sound policies, both macroeconomic and with regard to economic development, and prudent allocation of public resources, the effects of reliance upon large resource endowments can be negative for many countries.

It would of course be incorrect to conclude from Table A11.1 that mineral wealth is necessarily a curse. Growth theory tells us that a windfall increment in wealth leads to a permanent increase in sustainable income (see Weitzman 1976). However, growth theory assumes that resources are efficiently priced and that an optimal extraction programme will be combined with an optimal investment strategy. Good policy, therefore, turns resource wealth into sustained increases in income.

The striking result from accounting for resource depletion is the extent of dissaving in most Sub-Saharan African countries. The traditional macroeconomic data and indicators for SSA countries have been uniformly disappointing for two decades, with decline and stagnation being the general picture that results. What a bit of 'green' accounting tells us is that the situation with regard to future well-being is worse than we thought. Not only has SSA performed badly by conventional measures, it is clear that the wealth inherent in the resource stocks of these countries is simply being liquidated and dissipated. Not only have the trends in current indicators been downward, but total wealth, especially on a per-capita basis, has been declining as well.

User Cost vs. Net Price

As noted above, there is a potentially important divergence between theory and practice in the valuation of resource depletion. Because the models of the preceding chapter require an efficient time path for resource rents (the Hotelling rule), the change in the present value of a resource deposit as a result of current production - i.e., the user cost - is precisely equal to the value of rents on this production - i.e., resource depletion valued at its 'net price', the market price less the (marginal) cost of extraction (see Repetto *et al.* 1989). If efficient resource pricing is not assumed then some non-zero rate of discount should be used in valuing depletion; the formula for the user cost (UC) of resource extraction becomes:

$$UC_t = \frac{n_t Q_t}{(1+i)^N},$$

where n is the unit resource rent (the net price), Q is the quantity of resource extracted, i is the rate of discount and N is the reserve life[4].

This distinction between net price and user cost methods is not just of theoretical interest: for countries with very long-lived deposits, even small discount rates will yield user costs that are much smaller than current rents. Since it is the user cost that must be re-invested in order to maintain a constant value of capital, this has important consequences for countries' incentives to consume or invest resource rents. Examples of these effects are shown in Table 11.2 for some of the oil producers appearing in this chapter.

Table 11.2 Selected Oil Producers:
Depletion adjusted for reserve life, 3% discount rate, 1990

	Avg. production 1980-90 mmt	Reserves mmt	R/P ratio	Depletion: Net Price \$ m	Depletion: User Cost \$ m	Ratio UC/NP
Algeria	48.6	1800	37	6051	2024	33%
Congo	5.9	110	19	6051	3478	57%
Indonesia	70.2	726	10	6051	4458	74%
Iran	110.9	12700	115	17617	596	3%
Mexico	130.3	6079	47	13451	3388	25%
Nigeria	74.9	2400	32	9861	3825	39%
U. Arab Emirates	72.5	1300	18	11668	6868	59%
United Kingdom	104.0	535	5	5727	4919	86%
Venezuela	100.2	8604	86	11142	881	8%

Abbreviations: Reserves/Production (R/P) ratio, User Cost (UC), Net Price (NP)

For this 3% rate of discount the difference between current rents and the user cost is small for R/P ratios (N in the above formula) less than 10 years. For very large resource endowments, however, such as for Iran and Venezuela in the above table, the user cost of resource extraction is almost negligible.

The choice of discount rate is clearly key in this calculation. While a range of discount rates is possible, growth theory suggests that the social rate of return on investment (*SRRI*) (or consumption rate of interest, as it is sometimes called) is the fundamental discount rate. It is the maximum amount of extra consumption made possible by foregoing a unit of consumption in the current period, and is expressed by the following formula[5]:

$$SRRI = r + \eta \frac{\dot{C}}{C}.$$

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[4] Note that we have shifted to discrete time, rather than continuous time as in the previous chapter, for this formula. The formula is an adaptation of one in El Serafy (1989) and, as in that paper, implicitly assumes that the product of n and Q is constant over the life of the resource deposit.

[5] This is, of course, the Ramsey rule, derived in Chapter 3.

Here r is the pure rate of time preference (or rate of impatience, the rate at which future utility is discounted), η the elasticity of the marginal utility of consumption, and $\dot{C}/C$ the percentage rate of growth in per capita consumption. The social rate of return on investment is thus the sum of the rate of impatience and the rate of decline in the marginal utility of consumption associated with an extra unit of consumption. Estimates of the various components of this formula for the UK, reported in Pearce and Ulph (1995), show r to lie between 0 and 1.7%, and η to lie between 0.7 and 1.5. With a long-run growth rate in per capita consumption of 1.3%, Pearce and Ulph estimate a 'best' value of the *SRRI* of 2.4% for the UK, and a likely range of 2-4%. The 3% figure used in Table 11.2 is solidly within this range.

Another important consideration in the choice of discount rate for valuing the user cost of resource depletion is the fact that the optimal growth path of an economy with an exhaustble resource is not sustainable if the rate of impatience r is positive (as shown in Chapter 2) - that is, any positive rate of discounting of utility leads to an optimal path along which utility eventually declines. If sustainability is a goal, this argues for low rates of discounting.

The practical consequence of these calculations is that there is at least some argument for adjusting the value of depletion to reflect the size of the resource deposits. This would have obvious consequences for most of the oil-producing states and countries such as Guyana (with its bauxite) that have long-lived mineral deposits.

Such divergences between theory and practice represent difficult issues for the practitioner. Given the lack of empirical evidence for efficient resource rents (see, for instance, Adelman 1990), a low rate of discount in the calculation of resource rents may be preferable to the net price based calculations presented above. However, the choice of discount rate and the uncertainty of the size of resource deposits (to say nothing about varying resource grades - Chapter 7) present new difficulties in practical measurement.

The empirical estimates of this chapter are best viewed, therefore, as a rapid assessment of where the combination of resource depletion and deficient saving may be a significant policy concern.

Human Capital and Genuine Saving

The basic notion behind 'genuine' saving is to measure the change in value of the underlying assets (and liabilities, in the case of stocks of pollutants) upon which welfare depends. The basic contribution of the recent work on environmental accounting, upon which the foregoing estimates depend, has been to establish the proper measurement of natural assets. It is natural, therefore, to consider what other assets could be brought into this framework, and it is obvious that human capital is the missing element. This is the motivation in Hamilton (1994) for including current expenditures on education in the genuine savings measure.

It may be argued that this is superfluous, given that the returns to human capital are already measured implicitly in GNP. This misses the point, however. The goal of the genuine savings measure is to make explicit the true level of output that is not consumed and is therefore available to provide welfare in the future.

One problem with the standard national accounts from the human capital perspective is that, while capital expenditures on education (for buildings and equipment for instance) are treated as investment, current expenditures, both public and private, are not. From the viewpoint of the creation of human capital it seems clear that these current expenditures should also be considered to be investment.

The question of valuing human capital is complex. The literature on this topic (see, for instance, Jorgenson and Fraumeni 1992) has typically been concerned with valuing the returns to human effort beyond those provided by unskilled labour, and so questions of the measurement of the output of the education sector have predominated. This can be simplified in the context of measures of genuine saving by considering the goal just stipulated, to measure the true level of unconsumed output. From this standpoint it is sufficient to include current expenditures on education E as an addition to genuine saving, so that the expression for genuine saving becomes,

$$S_g = GNP - C - dK - n(R - g) - \sigma(e - d) + E.$$

International data on educational expenditures are fragmentary, presumably owing to the problem of consolidating expenditures by different levels of government. There is also some risk of double-counting because expenditures data typically do not distinguish between current and capital expenditures. Because of this poor coverage, a complete set of genuine savings measures adjusted for education expenditures will not be presented here. As an example of the difference this could make, Figure 11.6 presents the adjusted savings rates for Chile from 1980 to 1990[6].

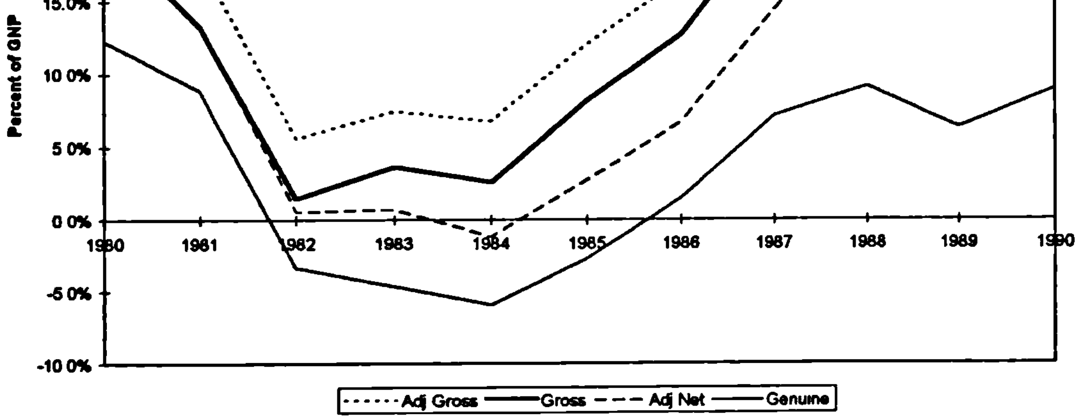

Figure 11.6

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[6] The data are from the World Bank's Economic and Social Database (BESD).

This figure requires careful interpretation. The curve for Gross saving measures the ratio of traditional gross saving to GNP. The 'Adj.Gross' curve adds the share of educational expenditures to the Gross measure. The 'Adj.Net' curve then subtracts the depreciation of produced assets share from the adjusted gross curve. Finally, the Genuine saving curve subtracts the share of depletion of resources and the social costs of carbon emissions from the adjusted net curve.

The effect of including educational expenditures in the genuine saving measure is therefore to improve the picture of saving in each country - in the case of Chile, this amounts to a little less than 5% of GNP, and in most countries it would fall between 2% and 8% of national product. As was noted when savings measures were introduced above, however, the effectiveness of these expenditures in fostering economic growth will vary widely. For many developing countries investments in primary education will pay higher dividends than those in higher-profile sectors such as universities.

Conclusions

We have argued that the most policy-relevant measure of progress towards sustainable development is the level of genuine savings. As the next chapter will show, the concomitant range of policy issues is wide-ranging, as befits a concept as all-embracing as sustainable development.

Given the complexity of the real world, as opposed to the models we tend to use as economists, it is often difficult to say what is the optimal savings rate for a given country. And all the caveats about the efficiency and effectiveness of investment bear repeating. That said, it remains true that savings rules provide useful one-sided tests of sustainability, in the sense that persistent negative genuine savings must lead to eventual declines in welfare.

The omissions in this empirical analysis are many: soils, gold, diamonds, natural gas, and pollution, to name a few. Notwithstanding these omissions, the empirical evidence is that genuine levels of saving are negative in a wide range of developing countries when the environment and natural resources are included in the savings measure. Negative genuine saving is more than a theoretical possibility, therefore, and the evidence is that many countries are being progressively impoverished as a result of poor government policies.

As countries develop we have seen an increasing tendency towards urbanization and the development of problem levels of pollution in these urban areas. The extension of this empirical work into the realm of air and water pollution will be increasingly important. There is some evidence from panel data[7] that there is an 'environmental Kuznets curve,' that historically there has been a rise and fall in pollution levels as per capita income has increased. However, there is absolutely no evidence that this was the optimal development path, in the sense that good policies could have increased overall welfare by 'flattening' the Kuznets curve. Valuing pollution emissions and including them in measures of genuine saving will have an important role to play in ensuring that past policy mistakes are not repeated.

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[7] See, for instance, Shafik (1994) and Seldon and Song (1994).

Appendix to Chapter 11

This table presents the country-level estimates of genuine saving rates over the decade of the 1980's.

Table A11.1 Genuine Saving Rates by Country

	Average 1980-85	1986	1987	1988	1989	1990
East Asia						
China	11.9%	12.1%	11.5%	12.1%	15.2%	17.0%
Indonesia	-2.3%	-6.0%	-0.7%	1.8%	8.1%	3.4%
South Korea	18.6%	25.2%	32.1%	38.8%	34.6%	32.7%
Malaysia	6.7%	-1.1%	13.0%	13.6%	11.1%	11.5%
P. New Guinea	-7.9%	-4.7%	-10.9%	-4.0%	-9.3%	-0.2%
Philippines	9.7%	5.6%	3.5%	5.6%	7.0%	6.2%
Thailand	11.6%	13.6%	15.8%	19.7%	21.8%	23.7%
Latin America and the Caribbean						
Argentina	2.5%	1.5%	4.1%	7.7%	-3.5%	7.3%
Belize	9.6%	13.3%	12.8%	12.5%	12.8%	20.4%
Bolivia	-46.2%	-49.3%	-43.5%	-35.9%	-35.4%	-31.8%
Brazil	4.4%	3.5%	8.8%	12.3%	15.7%	10.4%
Chile	-2.9%	-2.4%	3.4%	5.9%	3.5%	6.2%
Colombia	2.1%	1.4%	4.9%	5.4%	2.1%	2.2%
Costa Rica	4.9%	14.1%	10.3%	8.5%	7.7%	9.4%
Dominican Republic	8.6%	7.1%	9.7%	13.8%	18.0%	13.9%
Ecuador	-14.8%	-21.9%	-14.0%	-17.0%	-18.6%	-19.6%
Guatemala	0.7%	-1.8%	-4.8%	-4.1%	-2.8%	-2.4%
Guyana	-26.1%	-17.1%	-52.6%	-51.9%	-46.6%	-71.7%
Haiti	1.9%	1.7%	3.5%	2.4%	1.6%	2.6%
Honduras	-4.8%	-1.8%	0.2%	3.5%	-2.3%	1.0%
Jamaica	-22.4%	-17.6%	-9.9%	0.9%	-11.1%	-11.7%
Mexico	-1.1%	-9.1%	0.9%	-0.8%	1.3%	2.4%
Panama	13.2%	11.2%	10.7%	5.0%	-8.9%	6.2%
Paraguay	5.5%	-1.1%	-9.1%	-2.8%	6.6%	0.6%
Peru	-1.1%	-6.4%	-3.0%	-14.6%	-11.2%	-11.9%
Suriname	-2.7%	-15.2%	18.6%	7.5%	3.2%	-6.1%
Trinidad and Tobago	-9.7%	-25.1%	-17.4%	-25.4%	-22.6%	-13.7%
Venezuela	-13.9%	-23.4%	-14.7%	-15.7%	-27.8%	-19.1%
Middle East and North Africa						
Algeria	0.4%	-1.0%	4.4%	-5.6%	-5.5%	-1.9%
Egypt	-11.1%	-11.2%	-7.2%	-2.9%	-8.4%	-12.1%
Iran	-3.9%	3.6%	0.4%	-7.2%	-5.6%	-3.4%
Morocco	4.5%	11.1%	12.9%	11.0%	7.4%	13.8%
Oman	-24.7%	-51.3%	-22.0%	-43.9%	-36.7%	-27.3%
Syria	0.2%	-4.0%	-6.5%	-10.7%	-0.5%	1.6%
Tunisia	0.4%	-5.2%	3.7%	5.7%	4.1%	7.3%
U. Arab Emirates	3.6%	-19.1%	-2.4%	-8.0%	-22.0%	-23.1%
South Asia						
Bangladesh	1.1%	0.3%	2.8%	2.8%	1.5%	1.7%
India	7.4%	8.3%	9.5%	9.7%	8.8%	9.8%
Pakistan	7.7%	6.3%	8.1%	6.1%	6.3%	5.2%

Table A11.1 Genuine Saving Rates by Country

	Average 1980-85	1986	1987	1988	1989	1990
Sub-Saharan Africa						
Burundi	-2.9%	-1.5%	3.4%	-1.4%	4.8%	1.3%
Cameroon	6.1%	-3.2%	-0.5%	-4.3%	-9.4%	-10.2%
Congo	-28.5%	-68.7%	-36.2%	-51.0%	-41.6%	-44.9%
Cote d'Ivoire	-5.8%	-9.7%	-13.7%	-12.9%	-16.8%	-20.8%
Ethiopia	3.0%	7.8%	6.4%	7.4%	2.6%	1.4%
Ghana	-12.4%	-3.9%	-3.3%	-2.3%	-2.2%	-4.7%
Kenya	4.6%	6.9%	4.5%	6.1%	3.1%	3.3%
Madagascar	-13.3%	-14.3%	-19.0%	-19.0%	-17.3%	-20.2%
Mali	-8.9%	-4.6%	1.4%	1.3%	2.2%	3.8%
Mauritania	-13.3%	-17.7%	-9.5%	-2.9%	-8.3%	-7.7%
Niger	2.4%	4.6%	-5.3%	6.0%	-3.5%	-5.7%
Nigeria	-12.8%	-23.8%	-31.3%	-38.0%	-41.2%	-28.7%
Rwanda	7.7%	8.3%	5.3%	5.1%	4.2%	4.1%
Senegal	-12.4%	-7.1%	-2.8%	-0.5%	-1.6%	3.0%
Sierra Leone	-1.5%	-6.8%	-10.7%	-7.6%	-9.1%	-10.4%
Zambia	-36.6%	-70.4%	-84.5%	-96.3%	-65.8%	-65.1%
Zimbabwe	1.8%	3.6%	3.7%	9.7%	5.5%	4.4%

12. The Policy Implications of Green National Accounting

Governments are increasingly concerned with the extent to which the current path of economic development, including the exploitation of the natural environment, affects the potential for the future welfare of their country. Moreover, governments have been charged with considering and promoting the sustainability of development, in response both to the Brundtland Commission and to the United Nations Conference on Environment and Development's Agenda 21.

The variety of forms that resource and environmental accounting can take, as outlined in the United Nations' (1993b) SEEA, leads to a range of potential policy applications. These can range from sectoral concerns (e.g., the measurement of productivity in the extractive sectors), to broader questions of the relationship between pollution emissions, economic activity and the design of efficient policies to reduce emissions. As was argued in the introduction, however, there is a fundamental question that green accounting must be able to answer: are development and growth sustainable? The policy implications follow from the answer to this question.

Many would argue that obtaining measures of Hicksian or sustainable income is intrinsically important. However, this is not the same thing as saying that green measures of income have policy relevance with regard to sustainable development.

Part of the problem is that producing a new figure for the *level* of national income does not readily translate into a policy signal about the sustainability of development. The fact that a green national income series is, say, 10% lower than the traditional measure does not, in itself, tell you what policy prescriptions should follow, particularly since green income is necessarily lower than the standard income measure where exhaustible resources and pollution emissions are concerned. Most finance (or treasury) departments and development planners use rates of change to indicate where the economy is going and whether it is responding to policy stimuli. Whether the growth rate of green national income can provide a useful policy signal is open to question. For example, if we imagine a country with a fixed growth rate of standard gross domestic product[1] and no depreciation of produced assets, then for the growth rate of green national income it follows that: (i) if the value of resource and environmental depletion is constant each year, green national income will grow faster than GDP; and (ii) if the value of resource and environmental depletion is a constant proportion of GDP, the growth rate of green national income is necessarily the same as that of GDP. So a comparison of the growth rates in the two income measures does not automatically translate into a message about sustainability.

Green NNP measures *potentially* sustainable income. It does not in itself answer the question of whether the rate of saving is sufficient to maintain this income indefinitely. There is the additional difficulty, discussed in Chapter 9, that green NNP represents the amount that could be consumed while leaving the rate of change of utility *instantaneously* constant. As a

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[1] The national accounting identity implies that income equals expenditure equals product, so it is common to refer to GDP rather loosely as 'income'. We have followed this convention here, but it should be noted that the strict definition of National Income in the accounts is as a net concept, GDP plus net factor flows from abroad, less depreciation of produced assets, all valued at factor cost.

sustainability indicator, therefore, green NNP is unsatisfactory because it typically does not measure the amount that could be consumed if the economy were actually on a constant-utility path.

One of the key conlusions from Chapter 9 is that genuine savings is a one-sided indicator of sustainability, in the sense that persistent negative genuine savings must lead to declines in utility. If genuine savings is a superior sustainable development indicator, what policy implications follow from its measurement? To answer this question we will concentrate on the situation of developing countries because, on the empirical evidence, this is where issues of sustainability are most urgent.

The standard model of economic development[2] is a so-called 'two-gap' model: developing countries typically have a savings-investment gap, with investment exceeding savings, that is matched by an export-import gap, with imports exceeding exports. The role of development lending and grants as provided bilaterally and by agencies such as the World Bank has been to finance levels of investment that exceed the limited savings of developing countries.

At the heart of this model is a concern with gross levels of saving and investment, since it is the gap between the gross levels that must be financed. The effect of calculating genuine savings levels for developing countries is therefore not to deflect attention from this fundamental issue in development finance, but rather to give a new focus to the question of how much net wealth is being created and, critically, how domestic savings levels compare to the depreciation, depletion and degradation of a country's assets.

As Hamilton and O'Connor (1994) point out, Figures 11.4 and 11.5 can be interpreted in terms of how investment is financed. Starting with gross investment in East Asia in, say, 1981 as a benchmark, we see that the 29% of GNP that was invested was financed by a small amount of foreign borrowing, by a larger depreciation allowance, by a still larger depletion allowance (including, as argued previously, the amount that should be set aside to compensate other countries for East Asia's contribution to global warming damages), and by a substantial amount of genuine savings, nearly 11% of GNP. By analogy with a private firm, the depreciation and depletion allowances represent funds that a firm that wished to be sustainable would set against the erosion of its capital base. Thinking of the development problem in this manner clarifies exactly in what sense genuine savings are 'genuine.'

An important determinant of genuine savings rates for developing countries is the value of resource depletion. However, it would be wrong to conclude that the policy response regarding savings and natural resources is to boost genuine savings by restricting resource exploitation - this is clearly incorrect since it ignores the lessons from growth theory alluded to earlier, that the discovery of a natural resource, properly managed, leads to a permanent increase in the sustainable stream of income for a country. The question with regard to natural resources is therefore one of what constitutes 'proper management.' Part of this concerns the investment of resource rents and is therefore an element of the broader question of saving discussed below. But an important policy concern is the achievement of efficient levels of resource exploitation.

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[2] The standard example is the World Bank's RMSIM model.

The basic components of natural resources policy, royalty regimes and tenurial arrangements, are therefore relevant to the genuine savings issue. If government royalties on natural resources are set too high, then this will be a significant disincentive to resource exploitation, with less than optimal extraction/harvest rates; if royalties are set too low then natural resource firms will have an incentive to over-exploit resources in order to capture rents. With regard to tenure and property rights, the issues are well-known: open access to a resource such as a fishery where property rights are not established will lead to over-exploitation, generally leading to the requirement for second-best policies regarding restrictions on exploitation effort; similarly, resource leases that are too short will lead to over-exploitation because extracting firms will either have an incentive to exhaust more quickly than the efficient level, in the case of an exhaustible resource, or will lack an incentive to manage a resource for its efficient sustainable yield in the case of forest resources.

Consideration of environmental depletion within the genuine saving framework also casts a somewhat different light on resource exports. When a natural resource is sold at the border price in international markets, the full value of this sale shows up in the conventionally measured national income of the exporting country. However, a part of this income is in fact the liquidation of an asset, as measured by the value of depletion. This suggests that investment policies (in terms of investing resource rents) should also form a component of policies aimed at trade expansion and that the foregoing concerns about efficient exploitation rates for resources need to be considered as well. The bottom line is that the net benefit of exporting a natural resource commodity is not as great as conventional accounting implies.

The treatment of pollution in the genuine savings calculation raises issues that are similar to the exploitation of natural resources. First, because increments to pollution stocks have some analogues with depletion of natural resources, there is the need to see that investments in produced assets offset these increments. Secondly, there is the issue of achieving efficient levels of pollution emissions. Part of this involves the design of policies that attempt to equate pollution damages and abatement costs at the margin, and part of it is the cost-effectiveness of the policies themselves, with market-based approaches being the instruments of choice.

The generation of royalties from natural resources raises the question of public investment, since the 'rule of thumb' for sustainable development is to invest resource rents. Prudent government policies would aim to ensure that public investment, in education, infrastructure or other assets, at least matches the value of depletion of natural resources (assuming, of course, that the usual situation of resource ownership lying with the government, with the government then leasing resource lands for exploitation). If the arguments in Chapter 11 about the treatment of current educational expenditures are accepted, then these expenditures should be considered to be investment. These sorts of considerations reinforce the notion that it is important to distinguish between current and capital expenditures by governments when judging their fiscal stance and role in the economy. Questions of the appropriate scale of public versus private investment in the economy, and the effectiveness of public investments, are difficult to deal with, but necessarily part of the considerations in this domain.

Thinking about government expenditures raises the broader issue of consumption levels. Negative genuine savings rates imply, by definition, excessive consumption whether by governments or households. Extreme poverty plays a role in this picture, because at the margin the poor have little option but to consume all their income and, often, to run down their assets. So policies that promote growth and the alleviation of poverty will, in general, lead to a more favourable climate for generating private savings.

Reducing consumption expenditure by governments is one policy approach to boosting genuine savings. The government's fiscal stance is therefore a legitimate concern in this regard - deficit financing of public consumption, for instance, has the effect of both boosting overall consumption and crowding out private investment. But the lessons from the structural adjustment literature suggest that indiscriminate cutting of government expenditures, e.g. on primary health care and education, is likely to be harmful. The question, as always, is one of the appropriate role for the public sector and its appropriate scale. The situation with respect to the redistributive effects of government taxation is more complex. If redistribution is based on a progressive income tax system then it necessarily involves transferring income from households with high marginal propensities to save to ones with low propensities, reducing savings in the aggregate. However, redistribution may contribute to the *social* aspects of sustainable development, not discussed here, in that equity is increased; redistribution may also aid in the alleviation of poverty, with positive effects on saving in the longer run.

Promoting private savings is a complex affair, involving both the creation of a viable financial sector that can attract savings and mediate between savers and investors, and the establishment of a macro-policy climate that encourages savings. One essential feature of this macro climate must be positive real interest rates, which governments can set through their monetary policies.

Investment in new assets is not the only way to increase production in the short run - increasing X-efficiency, for example, can lead to significant gains - but it is the only way in the limit. More important is the effectiveness of investment. While each unit of savings should be put to its most productive use in principle, in practice many investments, especially in developing countries, have been wasteful. So while the analysis of genuine savings has an important role to play in focusing governments' attention on the net creation of wealth, it should also encourage increased concentration on the return to investment.

Thinking about sustainable development and its measurement inevitably leads to a conception of the process of development as one of portfolio management. Prudent governments will not only consider natural resources as assets, and pollution stocks as liabilities, in the national balance sheet, they will be concerned with the appropriate mix of produced assets and human capital as well.

Questions of the 'appropriate mix' of assets are inherently questions about returns on the marginal investment. This marginal investment may be in better resource management, boosting the value of natural resources in the national balance sheet; it may be in pollution control, decreasing the size of the pollution liability to its efficient level; it may be in infrastructure, as has traditionally been the case; and it may be in primary education, as an essential building block in increasing human capital.

13. Conclusions

Exhaustible resources are in many ways the cornerstone of the problem of sustainable development. If the pure rate of time preference is positive then the optimal development path is not sustainable. And while the 'solution' to this problem is the application of the Hartwick rule, to invest resource rents, the elasticity of substitution between capital and resources limits what can be achieved: constant consumption is not feasible if the elasticity is less than 1, and it is not maximal if the elasticity is greater than 1. If produced capital depreciates and the pure rate of time preference is constant then even the 'solution' of the Hartwick rule is not available - only technological change can ensure sustainability.

The optimal path for an economy with a pollutant with cumulative effects is also not sustainable, largely because consumers' valuation of environmental services increases without bound. For a stock pollutant that dissipates naturally, the marginal rates of dissipation (per unit of pollutant stock) and of pollution emission (per unit of production) limit the feasible range of pure rates of time preference and consumers' valuation of environmental services that will lead to a long run steady state. Acid rain limits the feasible range of stock sizes for a long run steady state with a living resource.

For fossil fuels and CO_2, the extended Hartwick rule is that investment must equal the sum of resource depletion, valued at the net rental rate after subtracting the value of a carbon tax, and the net change in the stock of carbon dioxide in the atmosphere, valued at the marginal social cost of carbon emissions. This result holds even if catastrophic effects are associated with a given level of carbon in the atmosphere.

Expanding the asset boundary of the system of national accounts to include natural resources and pollutants is the key to producing indicators of sustainable development. The value of genuine savings, defined as traditional net savings less the value of resource depletion and the value of pollution emissions, is a critical indicator of sustainability, in that persistent negative genuine savings must eventually lead to declines in welfare.

Formal approaches to green national accounts with such an expansion of the asset base lead to a range of conclusions about the measurement of welfare and net product. For exhaustible resources, green NNP should deduct resource depletion at its rental value. It should also include resource discoveries valued at their marginal discovery cost, so the treatment of discoveries in the standard SNA is roughly correct. Where there are heterogeneous resource deposits, the deduction for resource depletion should value each quantity extracted at the rental rate specific to its deposit of origin.

A number of conclusions for green national accounting follow from treating the environment as a source of welfare. First, pollution emissions should be subtracted from, and dissipation of pollution stocks should be added to, the measure of net product, with both valued either at marginal social costs or marginal abatement costs. Second, because fossil fuels are also the source of carbon emissions that decrease welfare, their depletion should be valued at the rental rate less the rate of carbon taxation required to maximize welfare. Third, where living resources are concerned, resource harvest minus growth should be deducted, as well as any damage to the resource as a result of pollution emissions, valued at the unit rental rate. Fourth, while capital investments in pollution abatement should be included in net product, current expenditures should not. Fifth, household defensive expenditures should not be deducted from green NNP and in fact typically understate their welfare effect. Finally, by including the value that households place on the flow of environmental services, a measure of welfare rather than simply net product results.

Green NNP *per se* has limited implications for sustainability because it measures only the level of consumption that is consistent with holding utility instantaneously constant. Actually moving the economy to a sustainable (i.e., constant utility) path will typically result in a different level of measured NNP, which may be higher or lower. Green NNP can be an indicator to guide optimal growth policies in that, at least for exhaustible resources, its rate of change has the same sign as the Hamiltonian function that the social planner is aiming to maximize.

While constant welfare is not synonymous with constant national wealth, the level of genuine savings is an important indicator of the sustainability of development. This idea, as well as the results just noted regarding the treatment of pollution and pollution abatement in the national accounts, was applied in several empirical tests of the sustainability of national economies. Among the conclusions of this work is the fact that the value of pollution emissions represents a significant proportion of GDP in most European countries, and that the UK economy is marginally unsustainable based on figures for the 1980's. Genuine savings estimates including the value of resource depletion for a sample of over 100 countries during the 1980's reveal wide regional variations, with East Asian countries exhibiting exceptionally strong saving performance while Subsaharan African countries were, for the most part, persistent and significant dissavers. This sheds new light on the economic circumstances of African nations.

Genuine savings, rather than green NNP, is the national accounting aggregate with the clearest policy implications for sustainable development. For governments concerned with sustainability, virtually all of natural resource policy, the targets for environmental quality, public investment programmes (particularly for the development of human capital), and large elements of monetary and fiscal policy, are all germane to the question of avoiding negative genuine savings. By expanding the asset base under consideration, green national accounting leads to a conception of economic development as a process of portfolio management. The efficient exploitation of resource assets, the shrinking of pollution liabilities to efficient levels, and changing the mix of produced and human capital in line with the highest rates of return on the marginal investment, all should become components of development policy.

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