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**TIME VARIATION AND ASYMMETRY IN THE WORLD PRICE OF  
COVARIANCE RISK: THE IMPLICATIONS FOR INTERNATIONAL  
DIVERSIFICATION**

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# Time Variation and Asymmetry in the World Price of Covariance Risk: The Implications for International Diversification\*

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## Abstract

The International Capital Asset Pricing Model measures country risk in terms of the conditional covariance of national returns with the world return. Using impulse responses from a multivariate non-linear model we provide evidence of time variation and asymmetry in the measure of country risk. and the implied benefit to international diversification. The evidence implies that the price of risk and the benefits from diversification may differ in a statistically and economically meaningful fashion across bull and bear markets.

*Keywords:* Generalised Impulse Responses; Asymmetry; International Capital Asset Pricing Model.

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# 1 Introduction

A diversified portfolio comprising assets with returns that move independently from each other provides a means of managing individuals' exposure to financial risk. Given that industry structures are often very different across countries, an internationally diversified portfolio, one where assets are sourced from a range of countries, is potentially an attractive risk management strategy. A large literature now exists that advocates the use of international diversification as a method of reducing the risk on a portfolio for a given level of expected return, see Grubel (1968), De Santis and Gerard (1997), Chang, Eun and Kolodny (1997) and Griffin and Karolyi (1998) *inter alia*. However, more recently, financial liberalisation, the globalisation of equity markets, the listing of foreign securities on domestic exchanges, and contagious crises may have reduced both the benefits and motivation to diversify internationally, see Longin and Solnick 1995, Van Royen 2002 *inter alia*. The benefits to international diversification may also be adversely affected by periods in which the correlation across markets is extremely high; the crash of October 1987 and the Asian Crisis of 1997 are obvious examples. Longin and Solnick (1995) and Karolyi and Stulz (1996), *inter alia*, suggest that correlations among asset returns vary systematically with market conditions, similarly Campbell, Koedijk, and Kofman (2002) argue that correlations increase in bear markets<sup>1</sup>. Bekaert and Harvey (1997), Brooks and Henry (2000) and Bekaert, Harvey and Ng (2002) illustrate the importance of this asymmetry in modelling the transmission of volatility across markets.

Country risk can be defined as the conditional sensitivity or covariance of the return to investing in a particular country with the world stock return. This concept of risk is a key feature of the International Capital Asset Pricing Model (ICAPM). The benefits to holding an internationally diversified portfolio will move inversely with the degree of this covariance. Should the covariance display time variation, as in Harvey's (1991) application of the ICAPM, the advantages conferred by international diversification will

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<sup>1</sup>There is widespread evidence that the volatility of equity returns is higher in a bull market than in a bear market. One potential explanation for such asymmetry in variance is the so-called 'leverage effect' of Black (1976) and Christie (1982). As equity values fall, the weight attached to debt in a firm's capital structure rises, *ceteris paribus*. This induces equity holders, who bear the residual risk of the firm, to perceive the stream of future income accruing to their portfolios as being relatively more risky. An alternative view is provided by the 'volatility-feedback' hypothesis of Campbell and Hentschel (1992). Assuming constant dividends, if expected returns increase when stock price volatility increases, then stock prices should fall when volatility rises.

also vary through time.<sup>2</sup> Therefore, understanding temporal changes in the conditional covariance may be an important consideration when measuring country risk and assigning appropriate country weights in portfolio allocation. An additional complication is the possibility that the covariance also displays an asymmetric response to newly acquired information. This would be the case if bad news, for example, has a larger effect on the covariance than does good news. Failure to account for any such asymmetry will also distort portfolio decisions, as well as leading to inaccurate and erroneous measures of the benefits to international diversification.<sup>3</sup>

This paper exploits the ICAPM methodology to investigate time variation and asymmetry in the ICAPM measure of country risk. As an application we estimate a very general multivariate model of world and country returns using data comprising returns from asset portfolios for the world, Hong Kong and Singapore. We adapt the Variance Impulse Response Function methodology of Shields, Olekalns, Henry and Brooks (2003) to illustrate the response of the variance covariance matrix of world and country returns to shocks, and we provide strong evidence of asymmetry in these impulse responses. Furthermore the paper presents a method for calculating impulse responses for the ICAPM measure of country risk and for the one-period benefit to diversification. This approach is used to investigate whether these variables respond asymmetrically in a statistically significant and economically important fashion to shocks.

One implication of our results is that we find that, for these countries, in periods of high volatility, there is increased correlation across markets. This is not surprising from a purely statistical perspective, see Forbes and Rigobon (2001) *inter alia*.<sup>4</sup> However, in addition to increased correlation, we find strong evidence that measures of risk increase in magnitude in response to return shocks. Hence, as country risk increases, the benefit to diversification may diminish. We also present evidence that the benefit to international diversification displays an asymmetric response to returns shocks. In partic-

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<sup>2</sup>The single factor ICAPM explains differences in country's returns performance as being due to differences in conditional covariances. Multifactor models such as those suggested by King, Sentana and Wadhvani (1994) examine how much of the observed variation in covariances is explained by measurable economic variables. Bekaert and Harvey (1997) examine similar questions for emerging equity market volatility, while Bekaert, Harvey and Ng (2002) explore the notion of contagion, the tendency of markets to move more closely together during periods of crisis.

<sup>3</sup>Commonly used measures of sovereign risk typically assume symmetry and often assume a constant variance covariance matrix of asset returns and therefore may provide misleading measures of exposure to risk.

<sup>4</sup>Forbes and Rigobon (2001) argue that after correcting for conditioning biases there is no evidence of contagion surrounding recent events such as the Asian Crisis.

ular a negative shock is shown to have greater long run impact on the benefit to diversification than a positive shock of equal magnitude.

This paper has six further sections. Section I outlines the ICAPM. The methods used to capture time variation and asymmetry in measures of sovereign risk are discussed in section II. The sources and time series characteristics of our data are presented in section III. Section IV presents the empirical model and the results. The impulse response analysis is discussed in the penultimate section. The final section presents a summary and some concluding comments.

## 2 The International Capital Asset Pricing Model

The basis for the ICAPM is the observation that asset returns reflect the risks arising from changes in the investment opportunity set over time; this is consistent with Merton's (1973) model, for example, which implies that the covariances between the return on a given asset and the return on a range of hedging portfolios determines the expected return to the asset and predicts a positive relationship between the market risk premium and the variance of the market portfolio. The degree of risk need not be constant over time. Bollerslev, Engle and Wooldridge (1988), Braun, Nelson and Sunier (1995), Engle and Cho (1999), and Brooks and Henry (2002) *inter alia*, report evidence of time varying variance-covariance structures in financial markets based upon the GARCH class of models. This implies that a conditional ICAPM, which allows for some predictability of the second moment of asset returns, might be an appropriate framework in which to capture the behaviour of agents who make their investment decisions on the basis of information available up to the immediate past time period and who maximize their utility on a period by period basis, see Attanasio 1991, and González-Rivera 1996 *inter alia*.

The conditional ICAPM assumes that the entire world operates under the same risk structure (Harvey 1991). If the markets are integrated, the conditional ICAPM implies that the expected return in country  $i$  conditional on the information set used by investors to determine prices,  $\Omega_{t-1}$ , is given by:

$$E [R_{i,t}|\Omega_{t-1}] = r_{f,t} + \frac{COV [R_{i,t}, R_{M,t}|\Omega_{t-1}]}{VAR [R_{M,t}|\Omega_{t-1}]} E [R_{M,t} - r_{f,t}|\Omega_{t-1}], \quad (1)$$

where  $E [R_{M,t} - r_{f,t}|\Omega_{t-1}]$  is the expected excess return to the World portfolio and  $r_{f,t}$  is the rate of return on the risk free asset.<sup>5</sup> The measure of

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<sup>5</sup>The World portfolio aggregates across countries such that all assets held in terms of

undiversifiable risk for county  $i$  is

$$\beta_{i,t} = \frac{COV [R_{i,t}, R_{M,t} | \Omega_{t-1}]}{VAR [R_{M,t} | \Omega_{t-1}]}.$$

Defining the time varying price per unit risk,  $\lambda_t$  as

$$\lambda_t = \frac{E [R_{M,t} - r_{f,t} | \Omega_{t-1}]}{VAR [R_{M,t} | \Omega_{t-1}]}, \quad (2)$$

we can write (1) as,

$$E [R_{i,t} | \Omega_{t-1}] = r_{f,t} + \lambda_t COV [R_{i,t}, R_{M,t} | \Omega_{t-1}]. \quad (3)$$

As equation (3) makes clear, the ICAPM predicts that risk depends solely on the conditional sensitivity or covariance of the return to investing in a particular country with the world stock return. In this context the expected return to the world portfolio would be:

$$E [R_{M,t} | \Omega_{t-1}] = r_{f,t} + \lambda_t COV [R_{M,t}, R_{M,t} | \Omega_{t-1}] = r_{f,t} + \lambda_t VAR [R_{M,t} | \Omega_{t-1}] \quad (4)$$

The ICAPM implies that the risk measure at the highest level of aggregation is the own variance of the world portfolio. Similar models are used by Giovannini and Jorion (1989), Harvey (1991), Chan, Karolyi and Stulz (1992), and De Santis and Gerard (1997).

In this paper, the returns in (1) and (4) are expressed in units of a common currency, the U.S. dollar. This is a trivial assumption in the case of Hong Kong which has a fixed exchange rate with the U.S. dollar. In markets with floating exchange rates, this approach assumes that investors do not hedge against currency fluctuations. Hence this approach can be viewed as a restricted version of an IAPM where the price of exchange rate exposure is zero.<sup>6</sup>

## 2.1 Benefits to Diversification

In the case where national financial markets are affected by country specific factors, correlations across markets are likely to be lower than correlations within markets. In this situation international diversification can be a practical strategy to improve portfolio performance. There are some caveats associated with taking a position that is exposed to international risks. Firstly,

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their value weights.

<sup>6</sup>Sercu (1980), Stulz (1981, 1985) and Adler and Dumas (1983), *inter alia*, consider more general models.

markets have become more integrated in recent years, increasing correlations across countries. Secondly, recent studies suggest that bear markets are contagious at an international level, see Lin, Engle and Ito (1994), De Santis and Gerard (1997), Brooks and Henry (2000) *inter alia*. In the case of contagion, benefits to diversification disappear just as they become most valuable to the investor.

The ICAPM framework can be used to assess the potential benefits from international diversification. Consider an internationally diversified portfolio,  $D$ , paying a return  $R_{D,t}$  with the same level of conditional volatility as a domestic portfolio paying return  $R_{i,t}$ . The expected benefit to diversification,  $E(BD_{i,t}|\Omega_{t-1})$ , can be defined as

$$E(BD_{i,t}|\Omega_{t-1}) = E[R_{D,t} - R_{i,t}|\Omega_{t-1}] \quad (5)$$

where  $R_{D,t} = \Psi_t R_{M,t} + (1 - \Psi_t) r_{f,t}$ . Here  $\Psi_t > 0$  is the optimal weight that satisfies  $\Psi_t^2 = VAR[R_{i,t}|\Omega_{t-1}] / VAR[R_{M,t}|\Omega_{t-1}]$ .<sup>7</sup> The ICAPM predicts the expected return to country  $i$  should satisfy (1) and that the expected return to  $D$  should satisfy

$$E[R_{D,t}|\Omega_{t-1}] = r_{f,t} + \lambda_t COV[\Psi_t R_{M,t}, R_{M,t}|\Omega_{t-1}] = r_{f,t} + \lambda_t \Psi_t VAR[R_{M,t}|\Omega_{t-1}] \quad (6)$$

Combining (1) and (6), the benefit to diversification implied by the ICAPM is

$$E[BD_{i,t}|\Omega_{t-1}] = \lambda_t [\Psi_t VAR[R_{M,t}|\Omega_{t-1}] - COV[R_{i,t}, R_{M,t}|\Omega_{t-1}]]. \quad (7)$$

Setting  $\Psi_t = 1$ , we can rewrite (7) as

$$E[BD_{i,t}|\Omega_{t-1}] = \lambda_t [VAR[R_{i,t}|\Omega_{t-1}] - COV[R_{i,t}, R_{M,t}|\Omega_{t-1}]]. \quad (8)$$

The term inside the brackets in (8) can be interpreted as a measure of the time varying non-systematic risk of country  $i$ , for which investors are not compensated. It is clear from (8) that the benefits to diversification are increasing in the exposure to country risk.

The conditional correlation between market  $i$  and the world portfolio  $M$ ,  $\rho_{i,M,t}$ , can be defined as

$$\rho_{i,M,t} = \frac{COV(R_{i,t}, R_{M,t}|\Omega_{t-1})}{\sqrt{VAR(R_{i,t}|\Omega_{t-1}) VAR(R_{M,t}|\Omega_{t-1})}}. \quad (9)$$

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<sup>7</sup>The weight  $\Psi_t$  is given by  $VAR(R_{i,t}|\Omega_{t-1}) = VAR(R_{D,t}|\Omega_{t-1})$  and  $VAR(R_{D,t}|\Omega_{t-1}) = \Psi_t^2 VAR(R_{M,t}|\Omega_{t-1})$ . Rearranging yields  $\Psi_t^2 = VAR[R_{i,t}|\Omega_{t-1}] / VAR[R_{M,t}|\Omega_{t-1}]$ .

Using (9) and again setting  $\Psi_t = 1$ , we can rewrite (7) as

$$E[BD_{i,t}|\Omega_{t-1}] = \lambda_t (1 - \rho_{iM,t}) VAR(R_{i,t}|\Omega_{t-1}). \quad (10)$$

Equation (10) shows that diversification benefits are decreasing in the level of correlation with the world as, *ceteris paribus*,  $\rho_{iM,t} \rightarrow 1$ , implies that  $E[BD_{i,t}|\Omega_{t-1}] \rightarrow 0$ ; there is no benefit to diversification if country  $i$  is perfectly correlated with the world. Furthermore, the benefit to hedging is increasing in  $\lambda_t$ , the price per unit risk, and in  $VAR(R_{i,t}|\Omega_{t-1})$ , the simple risk of the country.

### 3 Empirical Framework

Let  $\tilde{R}_{k,t} = R_{k,t} - r_{f,t}$  represent the excess return to the  $k^{th}$  asset or market. Consider the  $k$  dimensional vector of excess returns  $\tilde{R}_t$

$$\tilde{R}_t = \left( \tilde{R}_{M,t}, \tilde{R}_{1,t}, \dots, \tilde{R}_{k-1,t} \right)'. \quad (11)$$

We can write the conditional mean of our model as

$$\tilde{R}_t = \mu_t(\phi) + \varepsilon_t, \quad (12)$$

where  $\mu_t(\phi)$  is the conditional mean vector and  $\varepsilon_t = (\varepsilon_{M,t}, \varepsilon_{1,t}, \dots, \varepsilon_{k-1,t})'$ , is the innovation vector. Here  $\varepsilon_t = H_t^{1/2}(\phi) z_t$ , and  $H_t^{1/2}(\phi)$  is a  $k \times k$  positive definite matrix where  $H_t$  is the conditional variance matrix of  $\tilde{R}_t$  and  $z_t$  is the  $k \times 1$  vector of standardised innovations  $z_t = (z_{M,t}, z_{1,t}, \dots, z_{k-1,t})'$ . Note that  $H_t$  is

$$H_t = \begin{bmatrix} h_{M,t} & h_{M,1,t} & \dots & h_{M,k-1,t} \\ h_{M,1,t} & h_{1,t} & \dots & h_{1,k-1,t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,k-1,t} & h_{1,k-1,t} & \dots & h_{k-1,t} \end{bmatrix} \quad (13)$$

Consider  $\tilde{R}_{k,t}$  the  $k^{th}$  element of  $\tilde{R}_t$ . Holding  $|\varepsilon_t| = \varepsilon^*$  a variable is said to display own variance asymmetry if

$$VAR \left[ \tilde{R}_{k,t+1} | \Omega_t \right]_{|\varepsilon_{k,t} < 0} - \sigma_{k,t}^2 > VAR \left[ \tilde{R}_{k,t+1} | \Omega_t \right]_{|\varepsilon_{k,t} > 0} - \sigma_{k,t}^2. \quad (14)$$

for all values of  $\varepsilon^*$ . Here a negative excess return innovation for market  $k$  leads to an upward revision of the expected conditional variance of  $\tilde{R}_{k,t+1}$ . This increase in the expected conditional variance exceeds that for a shock of equal magnitude but opposite sign. Similarly, if a negative excess return innovation



for market  $j$  leads to an upward revision of the expected conditional variance of  $\tilde{R}_{k,t+1}$  then  $\tilde{R}_{k,t+1}$  is said to display cross variance asymmetry,

$$VAR \left[ \tilde{R}_{k,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} < 0} - \sigma_{k,t}^2 > VAR \left[ \tilde{R}_{k,t+1} | \Omega_t \right] |_{\varepsilon_{k,t} > 0} - \sigma_{k,t}^2. \quad (15)$$

Covariance asymmetry occurs if

$$COV \left[ \tilde{R}_{k,t+1}, \tilde{R}_{j,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} < 0} - \sigma_{jk,t}^2 \neq COV \left[ \tilde{R}_{k,t+1}, \tilde{R}_{j,t+1} | \Omega_t \right] |_{\varepsilon_{k,t} > 0} - \sigma_{jk,t}^2 \quad (16)$$

or

$$COV \left[ \tilde{R}_{k,t+1}, \tilde{R}_{j,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} < 0} - \sigma_{jk,t}^2 \neq COV \left[ \tilde{R}_{k,t+1}, \tilde{R}_{j,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} > 0} - \sigma_{jk,t}^2. \quad (17)$$

Given the definition of  $\beta_{i,t}$  from the ICAPM we may write

$$\beta_{i,t} = \frac{COV \left( \tilde{R}_{M,t}, \tilde{R}_{i,t} | \Omega_{t-1} \right)}{VAR \left( \tilde{R}_{M,t} | \Omega_{t-1} \right)} \quad (18)$$

for  $i = 1, \dots, k - 1$ .

If the data display own variance, cross variance or covariance asymmetry it follows that  $\beta_{i,t}$ , may respond asymmetrically to positive and negative return innovations. Holding  $|\varepsilon_t| = \varepsilon^*$  we define beta asymmetry as

$$E \left[ \beta_{i,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} < 0} - E \left[ \beta_{i,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} > 0} \neq 0, \quad (19)$$

for all values of  $\varepsilon^*$ . Here the impacts of positive and negative shocks of equal magnitude to the  $k^{th}$  market may lead to differing revisions to the conditional measure of risk.

Asymmetry in one or all of the elements of  $H_t$  has potentially important implications for measures of exposure. If the return to the world portfolio displays own or cross variance asymmetry, and/or if covariance asymmetry exists between the returns to country  $i$  and the world portfolio, then  $\beta_{i,t}$  will display asymmetry.

Similarly, own or cross variance asymmetry to the returns to market  $i$  and the world portfolio and/or covariance asymmetry will give rise to asymmetry in the measure of benefits to diversification. Diversification asymmetry may be defined as:

$$E \left[ BD_{i,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} < 0} - E \left[ BD_{i,t+1} | \Omega_t \right] |_{\varepsilon_{j,t} > 0} \neq 0 \quad (20)$$

A method of modelling the responses of the joint distribution of world and country returns and detecting asymmetric responses to positive and negative shocks is central to this study and has potentially important implications for risk estimation and portfolio allocation. This paper presents a unified framework for this task.

### 3.1 Generalised Impulse Responses

Define the vector  $\Lambda_t = \text{vech}(H_t)$ , where  $\text{vech}$  is the column stacking operator of a lower triangular matrix;  $\Lambda_t$  is of dimension  $k(k+1)/2$ . Stacking  $\beta_{i,t}$ ,  $\rho_{i,t}$  and  $BD_{i,t}$  into the vector  $\Xi_t$ . We can now define the  $4k + k(k+1)/2 - 2$  dimensional vector  $Q_t$  as

$$Q_t = \left( \tilde{R}_t, \Lambda_t, \Xi_t \right). \quad (21)$$

The generalised impulse function,  $GIRF$ , for a specific shock  $v_t$  and history  $\omega_{t-1}$  can then be given as

$$GIRF_Q(n, v_t, \omega_{t-1}) = E[Q_{t+n}|v_t, \omega_{t-1}] - E[Q_{t+n}|\omega_{t-1}] \quad (22)$$

for  $n = 0, 1, 2, \dots$ . Hence the  $GIRF$  is conditional on  $v_t$  and  $\omega_{t-1}$  and constructs the response by averaging out future shocks given the past and present. A natural reference point for the impulse response function is the conditional expectation of  $Q_{t+n}$  given only the history  $\omega_{t-1}$ . In this benchmark case the current shock is also averaged out. Assuming that  $v_t$  and  $\omega_{t-1}$  are realisations of the random variables  $V_t$  and  $\Omega_{t-1}$  that generate realisations of  $\{Q_t\}$  then, (following the ideas proposed in Koop *et al* (1996)) the  $GIRF$  defined in (22) can be considered to be the realisation of a random variable given by

$$GIRF_Q(n, V_t, \Omega_{t-1}) = E[Q_{t+n}|V_t, \Omega_{t-1}] - E[Q_{t+n}|\Omega_{t-1}]. \quad (23)$$

Note that the first  $k$  elements of  $GIRF_Q(n, V_t, \Omega_{t-1})$  contain the impulse responses for the excess returns, the next remaining  $k(k+1)/2$  elements contain the variance impulse responses,  $VIRF_\Lambda(n, V_t, \Omega_{t-1})$ <sup>8</sup>, while the remaining  $3k - 2$  elements are the impulse responses for the elements of  $\Xi_t$ ,  $IRF_\beta(n, v_t, \omega_{t-1})$ ,  $IRF_\rho(n, v_t, \omega_{t-1})$ , and  $IRF_{BD}(n, v_t, \omega_{t-1})$ , respectively.

A number of alternative conditional versions of  $GIRF_Q(n, V_t, \Omega_{t-1})$  can be defined.<sup>9</sup> In this study we are particularly interested in the evaluation of the significance of the asymmetric effects of positive and negative world and country shocks on the elements of  $Q_t$ . For instance, the response functions can be used to measure the extent to which negative shocks may (or may not) be more persistent than positive shocks. It is also possible to assess the

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<sup>8</sup>Hafner and Herwartz (2001) also consider such an extension and derive analytical expressions for the VIRF's for the case of symmetric multivariate GARCH models.

<sup>9</sup>For instance, it is possible to condition on a particular shock and treat the variables generating the history as random, or, condition on a particular history and allow the shocks to be the random variables. Alternatively, particular subsets of shocks/histories could be conditioned on, see Koop, Peseran and Potter (1996) for further details.

potential diversity in the dynamic effects of positive and negative shocks on the conditional volatilities and on the conditional covariances.

van Dijk *et al* (2000) present a measure of asymmetry in the response of the conditional mean to positive and negative innovations. Let  $GIRF_Q(n, V_t^+, \Omega_{t-1})$  denote the impulse response function from conditioning on the set of all possible positive shocks, where  $V_t^+ = \{v_t | v_t > 0\}$  and  $GIRF_Q(n, -V_t^+, \Omega_{t-1})$  denote the response from conditioning on the set of all possible negative shocks. The distribution of the random asymmetry measure,

$$ASY_{\tilde{R}}(n, V_t, \Omega_{t-1}) = GIRF_{\tilde{R}}(n, V_t^+, \Omega_{t-1}) + GIRF_{\tilde{R}}(n, -V_t^+, \Omega_{t-1}), \quad (24)$$

will be zero if positive and negative shocks have exactly the same effect on the conditional mean vector,  $\tilde{R}_t$ . Grier, Henry Olekalns and Shields (2004) describe the application of (23) and (24) for multivariate asymmetric GARCH in mean models.

Shields, Henry, Olekalns and Brooks (2003) present a measure of asymmetry in the response of the conditional variance-covariance matrix to shocks. Let  $VIRF_{\Lambda}(n, V_t^+, \Omega_{t-1})$  denote the variance impulse response function from conditioning on the set of all possible positive shocks, where  $V_t^+ = \{v_t | v_t > 0\}$  and  $VIRF_{\Lambda}(n, -V_t^+, \Omega_{t-1})$  denote the response from conditioning on the set of all possible negative shocks. The distribution of the random asymmetry measure,

$$ASY_{\Lambda}(n, V_t, \Omega_{t-1}) = VIRF_{\Lambda}(n, V_t^+, \Omega_{t-1}) - VIRF_{\Lambda}(n, -V_t^+, \Omega_{t-1}), \quad (25)$$

will be zero if positive and negative shocks have exactly the same effect on the conditional variance. The distribution of (25) can provide an indication of the asymmetric effects of positive and negative shocks. The asymmetry measure  $ASY_{\Lambda}$  is analogous to the measure proposed in van Dijk *et al* (2000) for the case of  $GIRFs$ . However, a notable distinction is that the measure in (25) is comprised of the difference between the variance response functions,  $VIRF_{\Lambda}(n, V_t^+, \Omega_{t-1})$  and  $VIRF_{\Lambda}(n, -V_t^+, \Omega_{t-1})$ , in contrast to the summation of the corresponding generalised impulse response versions in (24). This distinction arises because  $VIRFs$  are made up of the squares of the innovations (and therefore will be of the same sign), in contrast to the case of  $GIRFs$ , where positive and negative shocks cause the response functions to take opposite signs.

The distribution of the random asymmetry measure,

$$ASY_{\beta}(n, V_t, \Omega_{t-1}) = IRF_{\beta}(n, V_t^+, \Omega_{t-1}) - IRF_{\beta}(n, -V_t^+, \Omega_{t-1}), \quad (26)$$

will be zero if positive and negative shocks have exactly the same effect. The distribution of (26) can provide an indication of the asymmetric effects of

positive and negative shocks to  $\beta_{i,t}$ . Similarly, the asymmetry measure

$$ASY_{\rho_{jk}}(n, V_t, \Omega_{t-1}) = IRF_{\rho_{jk}}(n, V_t^+, \Omega_{t-1}) - IRF_{\rho_{jk}}(n, -V_t^+, \Omega_{t-1}), \quad (27)$$

can be used to evaluate the asymmetric effects of positive and negative return realisations to markets  $j$  and  $k$  on  $\rho_{jk}$ . Note that (26) and (27) are composed of the elements of  $\Lambda_t$  and therefore the asymmetry measures, analogous to the *VIRFs*, will be made up of the *difference* between the respective impulse responses for positive and negative shocks. Finally, the asymmetry measure

$$ASY_{BD_t}(n, V_t, \Omega_{t-1}) = IRF_{BD_t}(n, V_t^+, \Omega_{t-1}) + IRF_{BD_t}(n, -V_t^+, \Omega_{t-1}), \quad (28)$$

can be used to evaluate the asymmetry effects of positive and negative return realisations to markets  $i$  and  $M$  on the benefit to diversification. In other words we may evaluate whether the one period benefit to hedging displays asymmetry.

## 4 Data Description

Weekly price index data,  $P_t$ , denominated in \$US for Hong Kong, (*HK*), Singapore, (*SP*) and the World (*M*) were downloaded from Datastream. The sample runs from January 1<sup>st</sup> 1973 to July 28<sup>th</sup> 2003, a total of 1597 observations.<sup>10</sup> The continuously compounded returns to each index were calculated using

$$R_{k,t} = 100 \times \log \left( \frac{P_{k,t}}{P_{k,t-1}} \right) \quad (29)$$

for  $k = M, HK, SP$ .

The continuously compounded risk free return,  $r_{f,t}$  was calculated from secondary market yields on 3-month US Treasury Bills obtained from the FRED II database at the Federal Reserve Bank of Saint Louis<sup>11</sup>. Our analysis is performed on the returns in excess of the riskless rate for each index,  $\tilde{R}_{k,t} = R_{k,t} - r_{f,t}$ . The price and excess return data are plotted in Figure 1. In particular the excess returns data appears to display the volatility clustering usually associated with returns data. Large (small) shocks of either sign tend to follow large (small) shocks.

- Figure 1 here -

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<sup>10</sup>The datastream codes are TOTMHK\$, TOTMSG\$ and TOTMKWD for the Hong Kong, Singapore and World Indices, respectively.

<sup>11</sup><http://research.stlouisfed.org/fred2/>. The secondary market yields are contained in the file WTB3MS.

Table 1 presents summary statistics for the excess returns. The data are non-normal with clear evidence of negative skewness and excess kurtosis. The Bera-Jarque (1982) test rejects the null of normality at all usual levels of significance. There is strong evidence of conditional heteroscedasticity in the data with Engle's (1982) LM test for up to fifth order ARCH rejecting the null of no ARCH at all usual levels of significance.

**-Table I about here-**

There is also evidence of asymmetry in volatility for each of the series. Engle and Ng (1993) present tests of the null hypothesis of own variance asymmetry, however this test cannot detect cross variance or covariance asymmetry. For Hong Kong and Singapore the negative sign bias test of Engle and Ng (1993) suggests that negative innovations will lead to higher levels of conditional volatility than positive innovations of equal magnitude. This implies that a symmetric model would tend to systematically under forecast volatility when prices are trending downwards and over forecast volatility in an environment where prices are appreciating. Furthermore time variation and asymmetry in  $VAR [R_{M,t}|\Omega_{t-1}]$  implies that  $\lambda_t$ , the price per unit risk,  $\beta_{i,t}$ , the measure of risk for county  $i$ , and  $BD_{i,t}$ , the benefit to diversifying out of country  $i$ , are likely to display time variation and asymmetry unless  $E [R_{M,t} - r_{f,t}|\Omega_{t-1}]$ ,  $E [R_{i,t} - r_{f,t}|\Omega_{t-1}]$  and  $COV [R_{i,t}, R_{M,t}|\Omega_{t-1}]$  display sufficient offsetting asymmetry and time variation. Our empirical model, described below is a trivariate model, and allows for all three types of asymmetry.

## 5 The statistical model

We illustrate our methodology using a multivariate asymmetric GARCH model. However, the approach is sufficiently general to apply to a far wider class of multivariate non linear models.

Consider the  $3 \times 1$  excess return vector  $\tilde{R}_t = (\tilde{R}_{M,t}, \tilde{R}_{HK,t}, \tilde{R}_{SP,t})'$ , the conditional mean of our model is written as

$$\tilde{R}_t = \mu_t(\phi) + \varepsilon_t, \quad (30)$$

where  $\mu_t(\phi)$  is the conditional mean vector and  $\varepsilon_t = (\varepsilon_{M,t}, \varepsilon_{HK,t}, \varepsilon_{SP,t})'$ , is the innovation vector where

$$\varepsilon_t = H_t^{1/2}(\phi) z_t, \quad (31)$$

and  $H_t^{1/2}(\phi)$  is a  $3 \times 3$  positive definite matrix with  $H_t$  being the conditional variance matrix of  $\tilde{R}_t$  and  $z_t$  is the  $3 \times 1$  vector of standardised residuals  $z_t = (z_{M,t}, z_{HK,t}, z_{SP,t})'$ . Here  $H_t$  is

$$H_t = \begin{bmatrix} h_{M,t} & h_{M,HK,t} & h_{M,SP,t} \\ h_{M,HK,t} & h_{HK,t} & h_{SP,HK,t} \\ h_{M,SP,t} & h_{SP,HK,t} & h_{SP,t} \end{bmatrix}. \quad (32)$$

We assume that the excess return to the market portfolio follows a GARCH-M process written as

$$\tilde{R}_{M,t} = \phi_0 + \phi_1 h_{M,t} + \varepsilon_{M,t}. \quad (33)$$

The country returns, given by the ICAPM, are written as

$$\tilde{R}_{i,t} = \beta_{i,t} \tilde{R}_{M,t} + \varepsilon_{i,t}, \quad (34)$$

for  $i = HK, SP$ . We condition on the sigma field generated by all the information available until week  $t-1$ , contained in the information set  $\Omega_{t-1}$ .

It is possible to assume that  $\{z_t\}$  is *i.i.d.* with  $E(z_t) = 0$  and  $Var(z_t) = I_3$  where  $I_3$  is a  $3 \times 3$  identity matrix. Maximum likelihood estimation is then possible under the assumption of conditional normality of  $z_t$ . However, such an assumption must be considered tenuous given the extreme levels of non-normality present in the data as reported in Table 1. Our approach is to assume that the data follows a conditional Student-t density with unknown degrees of freedom  $\eta$ . As  $\eta$  tends to infinity the Student density converges on the normal distribution. We further assume that  $\eta > 2$  to ensure the existence of the first and second order moments and to retain the interpretation of  $H_t$  as a conditional variance covariance matrix. The Student density for our case is

$$g(z_t | \phi, \Omega_{t-1}, \eta) = \frac{\Gamma(\frac{\eta+3}{2})}{\Gamma(\frac{\eta}{2}) [\pi(\eta-2)]^{3/2}} \left[ 1 + \frac{z_t' z_t}{\eta-2} \right]^{-\frac{3+\eta}{2}}. \quad (35)$$

The conditional variance matrix  $H_t$  is parameterised as

$$H_t = C_0^{*'} C_0^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}' D_{11}^*, \quad (36)$$

where  $C_0^*$  is a  $3 \times 3$  upper triangular parameter matrix to ensure that the model is identified, and  $A_{11}^*, B_{11}^*$  and  $D_{11}^*$  are  $3 \times 3$  parameter matrices with elements  $a_{jk}, b_{jk}$ , and  $d_{jk}$ , respectively for all combinations of  $j, k = 1, 2, 3$ . Defining  $\xi_{i,t} = \min\{0, \varepsilon_{i,t}\}$ , and  $\xi_t = (\xi_{M,t}, \xi_{HK,t}, \xi_{SP,t})'$ , our model captures the negative size bias evident in Table 1 through the main diagonal elements

of the  $D_{11}^*$  matrix. Significance of the off-diagonal elements of  $D_{11}^*$  indicates the presence of cross-variance asymmetry and/or covariance asymmetry.

To close the model we require a definition of  $\beta_{i,t}$  the time varying measure of undiversifiable risk for each country. Given the definition of  $\beta_{i,t}$  from the ICAPM we write

$$\beta_{i,t} = \frac{COV\left(\tilde{R}_{M,t}, \tilde{R}_{i,t} | \Omega_{t-1}\right)}{VAR\left(\tilde{R}_{M,t} | \Omega_{t-1}\right)} = \frac{h_{M,i,t}}{h_{M,t}} \quad (37)$$

for  $i = HK, SP$ .

The conditional variance-covariance structure allows for asymmetry to enter through the elements of the outer product matrix  $\xi_{t-1}\xi'_{t-1}$  in (36). Hence, if the matrix of coefficients,  $D_{11}^*$ , defined in (36) is statistically insignificantly different from zero, then the *VIRF* will not distinguish between a positive or negative shock. If, on the other hand,  $D_{11}^*$  is significant, then the possibility of asymmetric responses to positive and negative shocks arises.

Table 2 presents parameter estimates of the full model. Consistent with the results displayed in Table 1 there is strong evidence of GARCH in the data. The estimates of main diagonal elements of the  $\hat{A}_{11}^*$  coefficient matrix are all strongly significant at all usual levels of confidence. Conversely the off-diagonal elements are insignificant. This suggests that persistence in variance is largely due to own market effects. All the elements of the first row of the  $\hat{B}_{11}^*$  matrix are significant indicating the presence of possible spillover effects between the World index and the Hong Kong and Singapore indices. Additionally  $\hat{d}_{11}$  and  $\hat{d}_{33}$  are significant indicating own variance and cross variance asymmetry in the World and Singapore returns. The significance of the off-diagonal elements of  $\hat{D}_{11}^*$  is consistent with the presence of cross variance and covariance asymmetry.

**-Table II about here-**

The model appears well specified. Table 3 presents specification test results for the model based on orthogonality conditions suggested by Nelson (1991). The standardised residuals from the model,  $z_{it}$ , display dramatically reduced levels of skewness and kurtosis and are largely free from serial correlation and conditional heteroscedasticity.

**-Table III about here-**

In addition the moment conditions  $E(\hat{\varepsilon}_{j,t}\hat{\varepsilon}_{k,t}) = \hat{h}_{jk,t}$  were satisfied for all combinations of  $j$  and  $k$ . To conserve space these results are not reported but are available upon request.

**-Figure 2 about here-**

Finally, Figure 2 displays the estimated benefits to diversification for Hong Kong and Singapore. Here,  $\Psi_t$ , the weight attached to portfolio  $M$  in the diversified portfolio paying return  $R_{D,t} = \Psi_t R_{M,t} + (1 - \Psi_t) R_{f,t}$ , is set optimally satisfying  $\Psi_t^2 = VAR[R_{i,t}|\Omega_{t-1}] / VAR[R_{M,t}|\Omega_{t-1}]$ . Clearly  $BD_{i,t}$  displays time variation and sometimes sharp reaction to shocks. It is interesting to note that  $BD_{i,t}$  is uniformly positive for both countries suggesting that a diversified strategy should be the norm for investors in these countries.

## 6 The Impulse Response Analysis

It is impossible to construct analytical expressions for the conditional expectations for the non-linear structure proposed in this paper. Therefore, Monte Carlo methods of stochastic simulation need to be used. Following the algorithm described in Koop *et al* (1996), impulse responses are computed for all 1597 histories in the sample for horizons  $n = 0, 1, \dots, N$ , with  $N = 50$ . At each history, 500 draws are made from the joint distribution of the innovations and  $R = 100$  replications are used to average out the effects of the shocks.

Following Herwartz and Hafner (2002), one can define news in terms of the *i.i.d.* innovation  $z_t$  and use a decomposition strategy to overcome the general problem that the error vector shows contemporaneous correlation. The Jordan decomposition of  $H_t$  can be used to obtain the symmetric matrix

$$H_t^{1/2} = \Upsilon_t \Psi_t^{1/2} \Upsilon_t'$$

with  $\Upsilon_t = (v_{t1}, \dots, v_{tN})$  and  $\Psi_t^{1/2} = \text{diag}(\psi_{t1}, \dots, \psi_{tN})$ , where  $v_{ti}$ ,  $i = 1, \dots, N$  denote the eigenvalues of  $H_t$  with corresponding eigenvectors  $\psi_{t1}$ . Using  $z_t = H_t^{-1/2} \varepsilon_t$  to identify the independent news requires no zero restrictions and is independent of the ordering of the variables in the state vector. In the case where  $\varepsilon_t$  is Gaussian,  $z_t$  is not unique. However if  $z_t$  is a vector of independent standardised variates, the only occasion where non-identifiability occurs is where  $\varepsilon_t$  is normally distributed. News can be considered to be identified if the innovation vector is not normally distributed.

Generalised impulse responses and associated asymmetry measures were calculated for the elements of  $Q_t$ . To conserve space we report a selection of the results.



## 6.1 Generalised impulse responses

Figures 3, 4 and 5 display the cumulative dynamic response of  $\beta_{i,t}$ ,  $\rho_{i,k,t}$  and  $BD_{i,t}$  for  $i = HK$  and  $SP$  and for  $k = M, HK$  and  $SP$  to orthogonal shocks to each market. The responses are scaled to have unit impact on each of the respective measures. The figures are drawn for  $N=8$  horizons which was sufficient for the long run response to the shock to be achieved in each case.

**-Figure 3 about here-**

Cumulative GIRFs for  $\beta_{HK,t}$  and  $\beta_{SP,t}$  are displayed in the upper and lower panels of Figure 3, respectively. The long run impact of an orthogonalised shock to the market portfolio that causes  $\beta_{HK,t}$  to rise by 1 unit on impact is essentially zero. Moreover, the news about the world dissipates within two periods. In contrast, a world shock that causes  $\beta_{SP,t}$  (in the lower panel) to rise by one percent on impact leads to a greater than one percent long-run effect. This result is driven by the higher persistence of the covariance of Singapore returns with the world,  $h_{M,SP,t}$ , compared with  $h_{M,HK,t}$ , in response to an orthogonal shock to the world portfolio.

News about Singapore or Hong Kong that causes  $\beta_{HK,t}$  or  $\beta_{SP,t}$  to rise by one unit on impact have approximately one unit impact in the long run. This implies that country-specific news leads to a persistent increase in the measure of risk for country  $i$ . Here, diversification of this country-specific risk is desirable.

**-Figure 4 about here-**

Figure 4 displays GIRFs for  $\rho_{iM,t}$ . In all but one case a shock to the system that causes the correlation to rise by 1 unit on impact results in greater than one unit long run increase in the level of correlation across the individual countries and with the world index. News about Singapore which causes the correlation between  $R_{SP,t}$  and  $R_{M,t}$  to rise by 1 unit on impact leads to a 0.95 unit long run increase in the correlation. Recall that if  $\Psi_t = 1$ , and we invest 100% of our wealth in the diversified portfolio into  $M$ , we can rewrite (7) as  $E[BD_{i,t}|\Omega_{t-1}] = \lambda_t (1 - \rho_{iM,t}) VAR(R_{i,t}|\Omega_{t-1})$ . Our results imply that, *ceteris paribus*, any news will increase correlation leading to a reduction in the benefit to diversification across markets.

The upper panel of Figure 5 implies that the long run impact of an orthogonal shock to  $R_{M,t}$  that causes  $BD_{HK,t}$  to rise by one unit on impact is zero. Furthermore the impulse dissipates after 3 periods. On the other hand the long run response to an orthogonal shock to  $R_{HK,t}$ , or  $R_{SP,t}$  that causes  $BD_{HK,t}$  to rise by one unit on impact is almost three units. The system

achieves this long run level after 3 periods. A similar pattern is evident in the lower panel of Figure 5. Shocks to  $R_{M,t}$  have zero long run impact on  $BD_{SP,t}$  while shocks to Hong Kong and in particular Singapore have lasting impact on the benefit to diversification. International diversification reduces exposure to country specific shocks for any one market and these benefits to diversification appear to be lasting

**-Figure 5 about here-**

Unlike for GIRFs, the property of linearity in the impulse no longer holds for VIRFs and correspondingly for the impulse responses for  $\beta_{i,t}$ ,  $\rho_{iM,t}$ , and  $BD_{i,t}$ . Therefore, an innovation of  $\varrho v_t$ , where  $\varrho$  is a scalar, will not have  $\varrho$  times the effect of  $v_t$ , if we consider conditional volatility responses. Given the quadratic nature of the VIRFs, the magnitude of the response will be in terms of the square of  $\varrho$ , implying that the larger is the shock, the greater will be the correlation between the variables and so the smaller will be the benefit to diversification. For large shocks to Hong Kong and Singapore there will be an increasingly larger benefit to holding a diversified portfolio.

## 6.2 Measuring asymmetry in the response to news

Tables 4-7 display asymmetry measures for  $\tilde{R}_{k,t}$ ,  $\beta_{i,t}$ ,  $\rho_{i,k,t}$  and  $BD_{i,t}$  for  $i = HK$  and  $SP$  and for  $k = M, HK$  and  $SP$ , respectively. These measures are designed to highlight differences in average responses to positive and negative orthogonal shocks to each market. The random asymmetry measures will be zero in expectation if positive and negative shocks have equal effect.

**-Table IV about here-**

There is no evidence that the return to the world portfolio responds asymmetrically to positive and negative orthogonal shocks to  $R_{M,t}$  of equal magnitude. Conversely both  $R_{HK,t}$  and  $R_{SP,t}$  respond asymmetrically to news about the world portfolio, with bad news about  $R_{M,t}$  having greater impact than good news. There is some statistical evidence that good news about  $R_{HK,t}$  has greater long run impact than bad news, but the magnitude of the effect, at approximately 2 basis points is unlikely to be significant economically.

**-Table V about here-**

In Table 5 there is evidence of asymmetric response in  $\beta_{i,t}$  to news. The total impact of a negative shock to  $R_{M,t}$  on  $\beta_{HK,t}$  and  $\beta_{SP,t}$  will be greater than

the total impact of a positive shock of similar magnitude. Positive shocks to  $R_{HK,t}$  and  $R_{SP,t}$  have greater long run impact on  $\beta_{HK,t}$  and  $\beta_{SP,t}$  although the asymmetric response of  $\beta_{SP,t}$  to news about Singapore is not statistically significant.

**-Table VI about here-**

The results in Table 6 suggest that positive shocks to Hong Kong lead to a greater long run response in  $\rho_{HK,M,t}$ ,  $\rho_{SP,M,t}$  and  $\rho_{HK,SP,t}$  than negative shocks of equal magnitude. However only  $\rho_{HK,M,t}$  responds in a statistically significant fashion to news about Singapore. Bad news about the world portfolio has greater long run impact on  $\rho_{HK,M,t}$  and  $\rho_{HK,SP,t}$  than good news of equal magnitude. The effect of a positive shock to  $R_{M,t}$  on  $\rho_{M,SP,t}$  in the long run exceeds the impact of a negative shock of equal magnitude.

Finally, Table 7 presents asymmetry measures for  $BD_{i,t}$ . With the exception of the response of  $BD_{SP,t}$  to news about Singapore, the evidence suggests that positive shocks have greater long run impact on the benefit to diversification than negative shocks of similar magnitude. This suggests that as markets trend downwards, the benefit to international diversification is eroded.

## 7 Summary and Conclusions

In the International Capital Asset Pricing Model, risk depends on the conditional covariance of the return to investing in a particular country with the return on the world portfolio. The benefits from holding an internationally diversified portfolio decrease if this covariance increases. International diversification allows the investor to hedge against unexpected changes in the opportunity set associated with the arrival of new information. Over time, changes in the information set cause the market price of risk and the benefit to diversification to also display time variation. Furthermore, markets may respond asymmetrically to news, that is, both the sign and size of the innovation in the information set are important. In such a situation the market price of risk and the benefit to diversification may display time variation and asymmetry in response to positive and negative innovations of equal magnitude.

Using the ICAPM as a framework, we develop impulse response functions for the first and second moments of the joint distribution of country and world returns. We illustrate how stochastic simulation techniques may be used to obtain impulse responses for important risk management measures such as the conditional beta, the conditional correlation and the conditional benefit to diversification. This allows an illustration of the dynamic response of these

measure to such shocks. Using these dynamic responses we develop a metric for measuring the degree of asymmetry in the reaction of these measures to positive and negative shocks.

Using weekly returns data for three markets, the World, Hong Kong and Singapore we illustrate our methodology using a multivariate asymmetric GARCH model. We provide strong evidence that these markets respond asymmetrically to shocks, and importantly that news raises the conditional sensitivity of each country's return with the world return, raising the price of risk and reducing the conditional benefit to diversification. We further provide strong evidence of asymmetry in the response; the market distinguishes between good and bad news. The implication is that when these markets are trending downwards sharply the degree of correlation between the country and world return increases. This implies that the price of risk and the benefits from diversification may differ in a statistically and economically meaningful fashion across bull and bear markets.

Our approach is sufficiently general to apply to a wide class assets and to a range of multivariate non linear models including GARCH and non-parametric models.

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Table I: Data Description

Continuously compounded returns for each market were calculated as  $R_{k,t} = 100 \times \log(P_{k,t}/P_{k,t-1})$ , using data collected from Datastream. The risk free return  $r_{f,t}$  was calculated from secondary market yields on 3-month US Treasury Bills obtained from the FRED II database at the Federal Reserve Bank of Saint Louis. Summary statistics are reported for returns in excess of the riskless rate for each market,  $\tilde{R}_{k,t} = R_{k,t} - r_{f,t}$ . Marginal significance levels are reported as [.]. SK and EK measure the skewness and excess kurtosis of each series, respectively. JB is the Jarque-Bera test for normality. ARCH(5) is a Lagrange Multiplier test for Autoregressive Conditional Heteroscedastity suggested by Engle (1982). The test is performed using the squared residual from a fifth order autoregression. The squared residual is regressed on a constant and five lags of the squared residual. The Sign Bias, Negative Size Bias, Positive Size Bias and Joint tests are those of Engle and Ng (1993). We report the t-ratio and marginal significance level from the regression of the squared return to each market on (respectively) (i) an indicator variable that is unity if the return is negative and zero otherwise, (ii) the product of this indicator variable and the squared return (iii) the product of the squared return and an indicator variable which is unity if the return is positive and zero otherwise.

|                    | World                | Hong Kong            | Singapore             |
|--------------------|----------------------|----------------------|-----------------------|
| Mean               | 0.1299<br>[0.0110]   | 0.1470<br>[0.2322]   | 0.0822<br>[0.4058]    |
| Variance           | 4.1572               | 24.1506              | 15.5973               |
| SK                 | -0.3635<br>[0.0000]  | -1.0331<br>[0.0000]  | -0.6126<br>[0.0000]   |
| EK                 | 2.7221<br>[0.0000]   | 8.3991<br>[0.0000]   | 7.5541<br>[0.0000]    |
| JB                 | 527.5898<br>[0.0000] | 497.9738<br>[0.0000] | 3892.1574<br>[0.0000] |
| ARCH(5)            | 384.5812<br>[0.0000] | 239.6484<br>[0.0000] | 314.0226<br>[0.0000]  |
| Sign Bias          | 0.7288<br>[0.4662]   | -2.3748<br>[0.0177]  | -2.4684<br>[0.0137]   |
| Negative Size Bias | -1.2710<br>[0.2039]  | 0.3309<br>[0.7407]   | 0.3595<br>[0.7193]    |
| Positive Size Bias | 0.0058<br>[0.9953]   | 1.5663<br>[0.1175]   | 1.8013<br>[0.0718]    |
| Joint Test         | 8.2601<br>[0.0409]   | 8.1399<br>[0.0432]   | 8.1316<br>[0.0434]    |

Table II: Maximum Likelihood Estimates of the Model

Continuously compounded returns for each market were calculated as  $R_{k,t} = 100 \times \log(P_{k,t}/P_{k,t-1})$ , using data collected from Datastream. The risk free return  $r_{f,t}$  was calculated from secondary market yields on 3-month US Treasury Bills obtained from the FRED II database at the Federal Reserve Bank of Saint Louis. Summary statistics are reported for returns in excess of the riskless rate for each market,  $\tilde{R}_{k,t} = R_{k,t} - r_{f,t}$ . Marginal significance levels are reported as [.]. Maximum likelihood estimates and standard errors are obtained under the assumption that the data follows a conditional Student-t density with unknown degrees of freedom  $\eta$ .

$$\tilde{R}_{M,t} = \begin{matrix} 0.1185 \\ \{0.1603\} \end{matrix} + \begin{matrix} 0.0209 \\ \{0.0874\} \end{matrix} h_{M,t} + \varepsilon_{M,t}$$

$$\tilde{R}_{i,t} = \beta_{i,t} \tilde{R}_{M,t} + \varepsilon_{i,t}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{HK,t} \\ \varepsilon_{SP,t} \end{bmatrix}; \quad z_t \sim ST(0, H_t, \eta) \quad \hat{\eta} = \begin{matrix} 8.4884 \\ \{0.6934\} \end{matrix}$$

$$H_t = C_0^{*'} C_0^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}' D_{11}^*$$

$$\xi_t = \begin{bmatrix} \xi_{M,t} \\ \xi_{HK,t} \\ \xi_{SP,t} \end{bmatrix}; \quad \xi_{k,t} = \min\{0, \varepsilon_t\}$$


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|                    |   |                    |   |
|--------------------|---|--------------------|---|
| $\hat{C}_0^* =$    | $\begin{bmatrix} 0.3150 & 0.1116 & 0.0863 \\ \{0.0596\} & \{0.0538\} & \{0.0533\} \\ 0 & 0.6461 & 0.1708 \\ & \{0.0517\} & \{0.0540\} \\ 0 & 0 & 0.4149 \\ & & \{0.0323\} \end{bmatrix}$  | $\hat{A}_{11}^* =$ | $\begin{bmatrix} 0.9618 & 0.0088 & -0.0112 \\ \{0.0092\} & \{0.0095\} & \{0.0088\} \\ 0.0004 & 0.9235 & 0.0023 \\ \{0.0015\} & \{0.0045\} & \{0.0033\} \\ -0.0026 & 0.0041 & 0.9474 \\ \{0.0026\} & \{0.0057\} & \{0.0020\} \end{bmatrix}$  |
| $\hat{B}_{11}^* =$ | $\begin{bmatrix} 0.0510 & -0.1251 & -0.0611 \\ \{0.0199\} & \{0.0371\} & \{0.0322\} \\ 0.0014 & 0.3444 & 0.0012 \\ \{0.0034\} & \{0.0132\} & \{0.0076\} \\ 0.0140 & -0.0072 & 0.2740 \\ \{0.0069\} & \{0.0254\} & \{0.0134\} \end{bmatrix}$ | $\hat{D}_{11}^* =$ | $\begin{bmatrix} 0.2990 & 0.1346 & 0.2264 \\ \{0.0296\} & \{0.0348\} & \{0.0334\} \\ 0.0384 & 0.0028 & -0.0179 \\ \{0.0117\} & \{0.0268\} & \{0.0178\} \\ -0.0707 & 0.0840 & -0.0859 \\ \{0.0204\} & \{0.0395\} & \{0.0439\} \end{bmatrix}$ |

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Table III: Specification Test Results

$\hat{z}_{k,t}$  represents the standardised residual from the mean equation for each market  $k$ . Asymptotic  $t$ -ratios and marginal significance levels are reported for each moment condition.

| Orthogonality Condition                               | World               | Hong Kong           | Singapore           |
|---|---------------------|---------------------|---------------------|
| $E[\hat{z}_{k,t}] = 0$                                | -0.0145<br>[0.5614] | 0.0464<br>[0.0698]  | 0.0179<br>[0.4798]  |
| $E[\hat{z}_{k,t}^2 - 1] = 0$                          | -0.0114<br>[0.8241] | 0.0440<br>[0.4439]  | 0.0196<br>[0.7272]  |
| $E[\hat{z}_{k,t}^3] = 0$                              | -0.5299<br>[0.0315] | -0.5442<br>[0.0383] | -0.3480<br>[0.1776] |
| $E[\hat{z}_{k,t}^4 - 3] = 0$                          | 2.1501<br>[0.01373] | 3.3389<br>[0.0132]  | 3.0893<br>[0.0277]  |
| $E[(\hat{z}_{k,t}^2 - 1)(\hat{z}_{k,t-1}^2 - 1)] = 0$ | 0.3204<br>[0.3146]  | 0.4395<br>[0.2030]  | 0.3901<br>[0.0769]  |
| $E[(\hat{z}_{k,t}^2 - 1)(\hat{z}_{k,t-2}^2 - 1)] = 0$ | 0.1133<br>[0.1612]  | 0.7237<br>[0.2148]  | 0.0064<br>[0.9518]  |
| $E[(\hat{z}_{k,t}^2 - 1)(\hat{z}_{k,t-3}^2 - 1)] = 0$ | 0.0907<br>[0.3401]  | -0.0834<br>[0.2251] | 0.1091<br>[0.2517]  |
| $E[(\hat{z}_{k,t}^2 - 1)(\hat{z}_{k,t-4}^2 - 1)] = 0$ | -0.0803<br>[0.2171] | 0.0466<br>[0.5883]  | 0.3314<br>[0.3412]  |
| $E[(\hat{z}_{k,t}^2 - 1)(\hat{z}_{k,t-5}^2 - 1)] = 0$ | 0.0366<br>[0.6431]  | -0.1332<br>[0.0586] | 0.0212<br>[0.8099]  |
| $E[\hat{z}_{k,t}\hat{z}_{k,t-1}] = 0$                 | 0.0001<br>[0.9986]  | 0.0739<br>[0.0995]  | 0.0457<br>[0.1271]  |
| $E[\hat{z}_{k,t}\hat{z}_{k,t-2}] = 0$                 | 0.0455<br>[0.0823]  | 0.0360<br>[0.1520]  | 0.0741<br>[0.0038]  |
| $E[\hat{z}_{k,t}\hat{z}_{k,t-3}] = 0$                 | 0.0343<br>[0.1862]  | 0.0017<br>[0.9485]  | 0.0252<br>[0.3498]  |
| $E[\hat{z}_{k,t}\hat{z}_{k,t-4}] = 0$                 | 0.0004<br>[0.9850]  | -0.0248<br>[0.3083] | 0.0437<br>[0.1367]  |
| $E[\hat{z}_{k,t}\hat{z}_{k,t-5}] = 0$                 | 0.0077<br>[0.7606]  | 0.6727<br>[0.4121]  | 0.0054<br>[0.8232]  |

Table IV: Measures of Asymmetry in Return

The random asymmetry measure,  $ASY_{\tilde{R}}(n, V_t, \Omega_{t-1}) = GIRF_{\tilde{R}}(n, V_t^+, \Omega_{t-1}) + GIRF_{\tilde{R}}(n, -V_t^+, \Omega_{t-1})$ , will be insignificantly different from zero if positive and negative shocks have exactly the same effect on the conditional mean vector,  $\tilde{R}_t$ . The asymmetry measure and its associated standard error are obtained using stochastic simulation.

|           | $ASY_{\tilde{R}_M}$  | $ASY_{\tilde{R}_{HK}}$ | $ASY_{\tilde{R}_{SP}}$ |
|-----------|----------------------|------------------------|------------------------|
| <i>M</i>  | -0.0011<br>(-0.0828) | -0.1167<br>(-22.7761)  | -0.1146<br>(-16.3076)  |
| <i>HK</i> | 0.0244<br>(3.3600)   | 0.0221<br>(2.8325)     | -0.0178<br>(-2.2751)   |
| <i>SP</i> | 0.0181<br>(2.0001)   | -0.1350<br>(-19.0217)  | -0.0903<br>(-12.9588)  |

Table V: Measures of Asymmetry in  $\beta_{i,t}$

The random asymmetry measure,  $ASY_{\beta}(n, V_t, \Omega_{t-1}) = IRF_{\beta}(n, V_t^+, \Omega_{t-1}) - IRF_{\beta}(n, -V_t^+, \Omega_{t-1})$ , will be insignificantly different from zero if positive and negative shocks have exactly the same effect on the conditional measure of undiversifiable risk,  $\beta_{it}$ . The asymmetry measure and its associated  $t$ -ratio are obtained using stochastic simulaion.

|      | $ASY_{\beta_{HK,t}}$  | $ASY_{\beta_{SP,t}}$ |
|------|-----------------------|----------------------|
| $M$  | -0.0292<br>(-13.8511) | -0.0264<br>(9.5003)  |
| $HK$ | 0.0595<br>(24.7067)   | 0.0119<br>(12.1141)  |
| $SP$ | 0.0068<br>(5.8772)    | 0.0012<br>(1.0318)   |

Table VI: Measures of Asymmetry in  $\rho_{i,k,t}$

The random asymmetry measure,  $ASY_{\rho_{jk}}(n, V_t, \Omega_{t-1}) = IRF_{\rho_{jk}}(n, V_t^+, \Omega_{t-1}) - IRF_{\rho_{jk}}(n, -V_t^+, \Omega_{t-1})$ , will be insignificantly different from zero if positive and negative shocks have exactly the same effect on the conditional correlation,  $\rho_{jkt}$ . The asymmetry measure and its associated  $t$ -ratio are obtained using stochastic simulation.

|           | $ASY_{\rho_{M,HK,t}}$ | $ASY_{\rho_{M,SP,t}}$ | $ASY_{\rho_{HK,SP,t}}$ |
|-----------|-----------------------|-----------------------|------------------------|
| <i>M</i>  | -0.0031<br>(-4.3452)  | 0.0063<br>(12.4159)   | -0.0025<br>(-10.0956)  |
| <i>HK</i> | 0.0133<br>(22.6024)   | 0.0085<br>(18.5639)   | 0.0111<br>(21.9407)    |
| <i>SP</i> | 0.0013<br>(4.0402)    | 0.0004<br>(1.2603)    | 0.0001<br>(0.5321)     |

Table VII: Measures of Asymmetry in  $BD_{i,t}$

The random asymmetry measure,  $ASY_{BD_t}(n, V_t, \Omega_{t-1}) = IRF_{BD_t}(n, V_t^+, \Omega_{t-1}) + IRF_{BD_t}(n, -V_t^+, \Omega_{t-1})$ , will be insignificantly different from zero if positive and negative shocks have exactly the same effect on the conditional benefit to diversification,  $BD_{it}$ . The asymmetry measure and its associated  $t$ -ratio are obtained using stochastic simulation.

|      | $ASY_{BD_{HK,t}}$   | $ASY_{BD_{SP,t}}$     |
|------|---------------------|-----------------------|
| $M$  | 0.0361<br>(8.0606)  | 0.0132<br>(2.9235)    |
| $HK$ | 0.0789<br>(27.3461) | 0.1149<br>(42.6970)   |
| $SP$ | 0.0333<br>(9.2520)  | -0.1031<br>(-27.6489) |

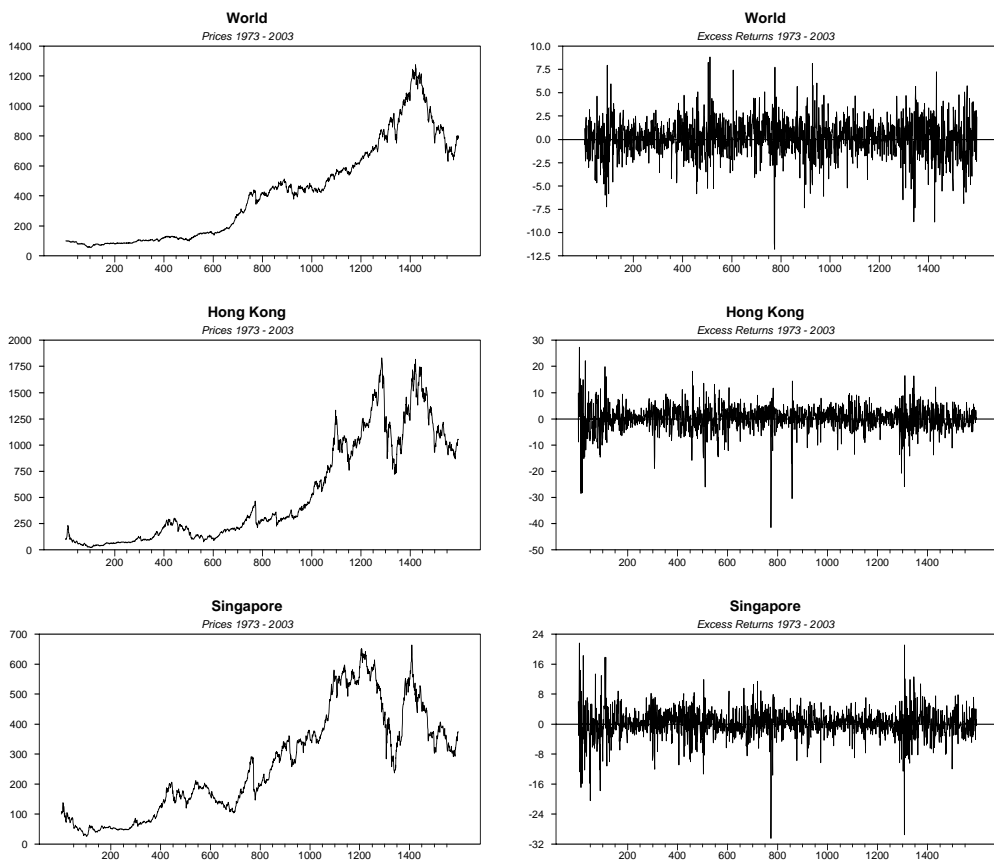


Figure 1: The Data



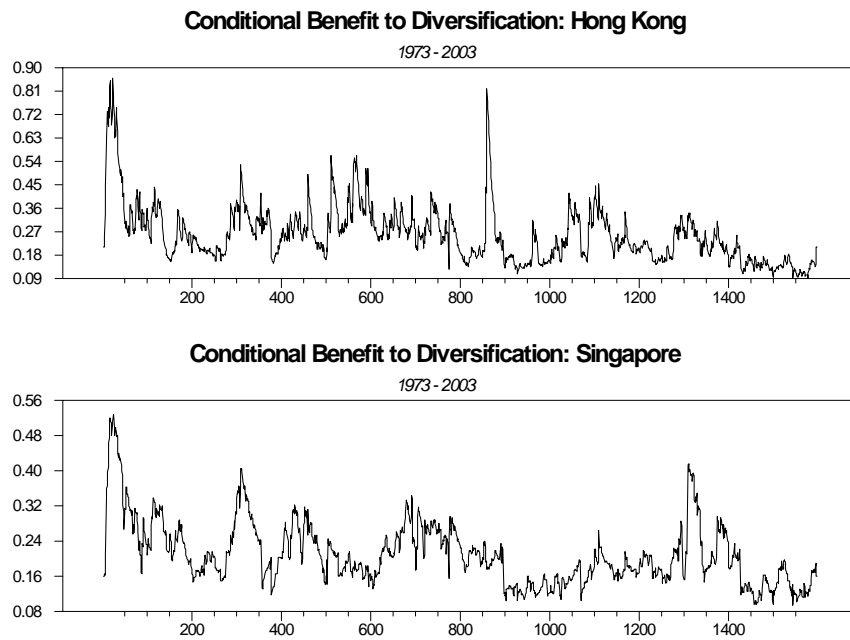


Figure 2: Time series plots for  $\widehat{BD}_{i,t}$

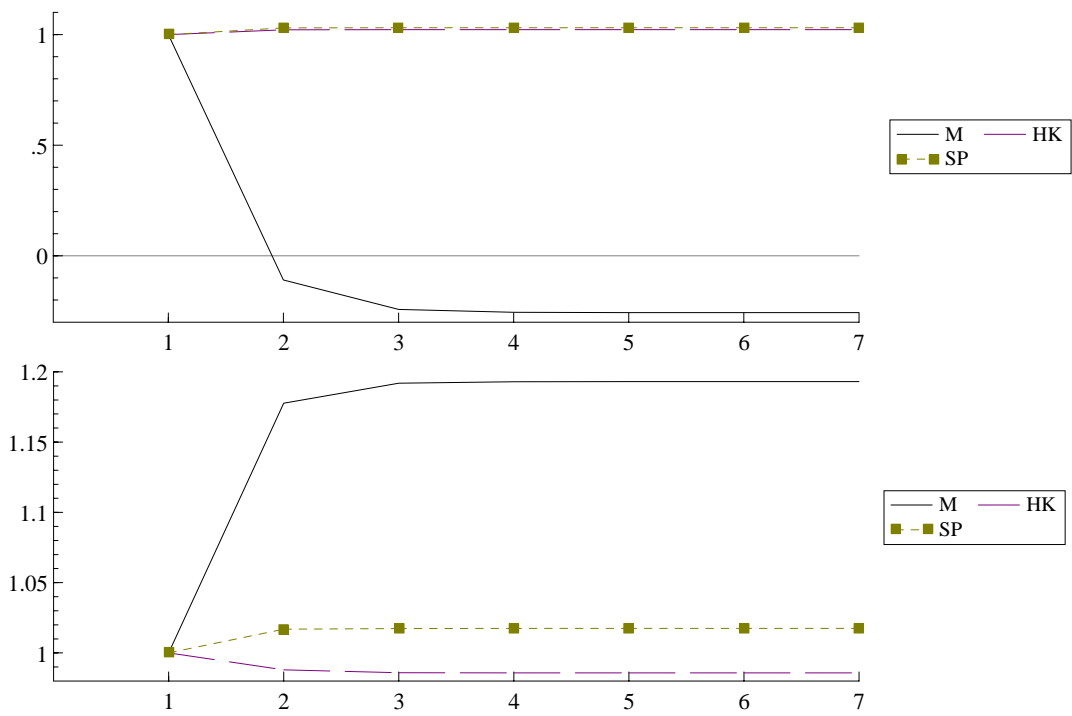


Figure 3: Cumulative Generalised Impulse Responses:  $\beta_{HK,t}$  Upper Panel;  $\beta_{SP,t}$  Lower Panel

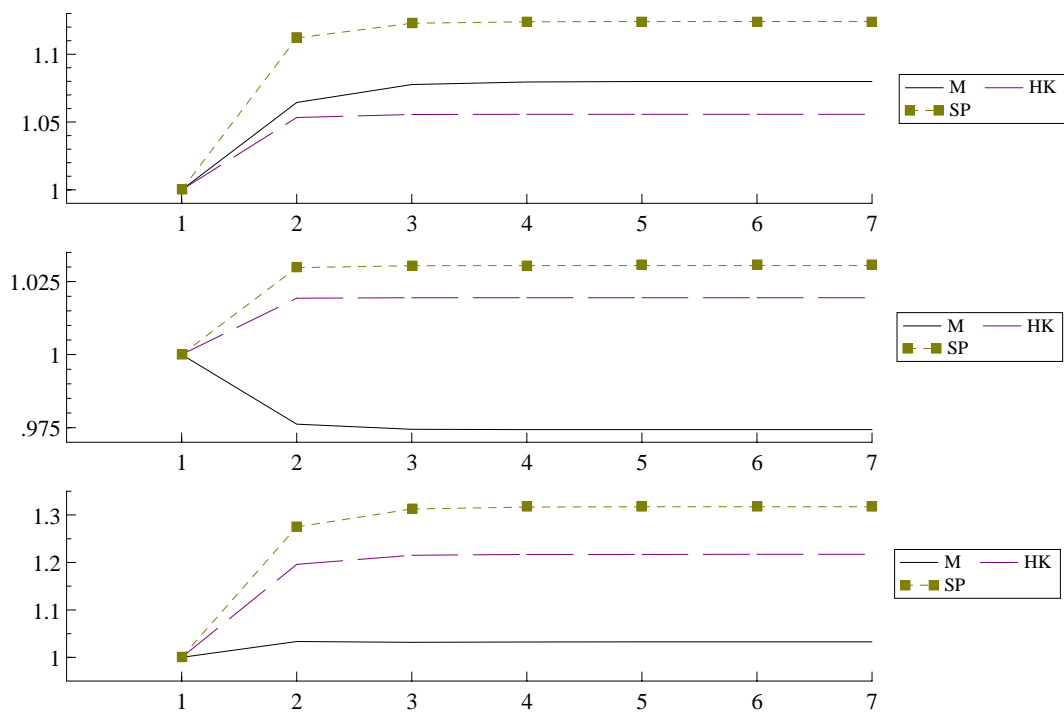


Figure 4: Cumulative Generalised Impulse Responses:  $\rho_{HK,M}$  Upper Panel;  $\rho_{SP,M}$  Middle Panel;  $\rho_{HK,SP}$  Lower Panel

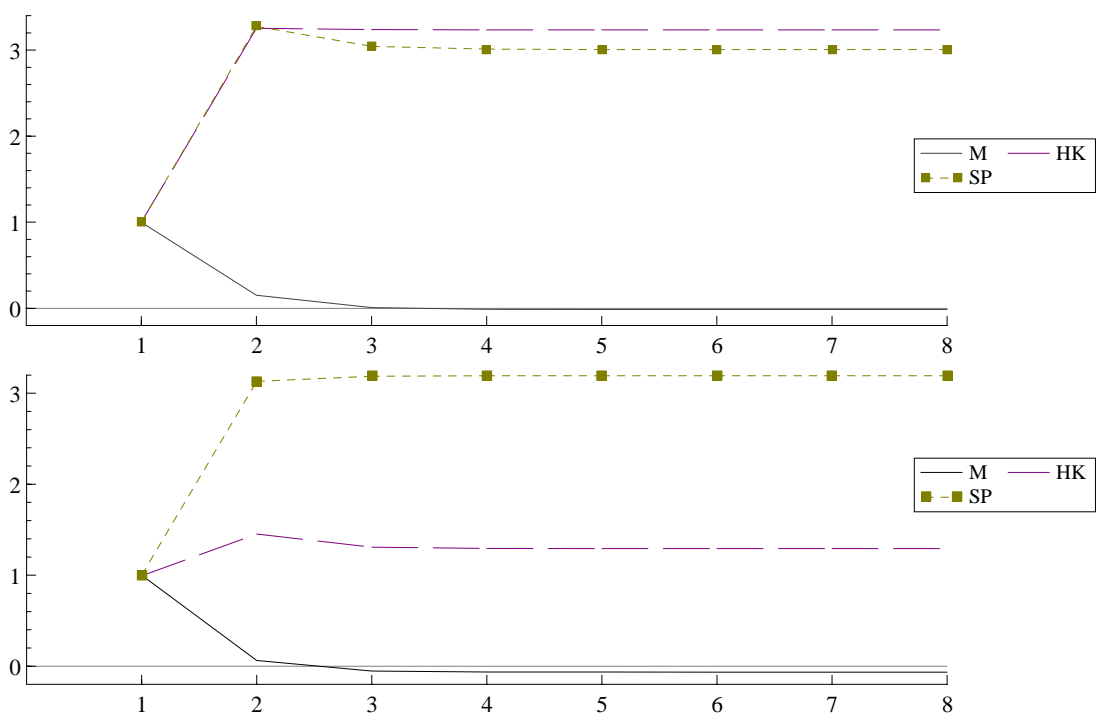


Figure 5: Cumulative Generalised Impulse Responses:  $BD_{HK,t}$  Upper Panel;  $BD_{SP,t}$  Lower Panel



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