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ON THE THEORY OF ELECTRICALLY CONTROLLED BIREFRINGENCE

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**CENTRAL
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BUDAPEST

ON THE THEORY OF ELECTRICALLY CONTROLLED
BIREFRINGENCE

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ABSTRACT

In the present paper we discuss the detailed theory of electrically controlled birefringence. It is taken into account that the Frank elastic constants are unequal and that the field induces electric current and polarisation in the sample. Experimental illustrations of the effect are also presented.

KIVONAT

A cikkben az elektromosan vezérelt kettőstörés elméletét tárgyaljuk. Figyelembe vesszük a Frank-féle rugalmas állandók különbözőségét, továbbá az alkalmazott tér által indukált áram és polarizáció hatását. Az effektust kisérletekkel is illusztráljuk.

РЕЗЮМЕ

В данной статье описывается теория управляемого электрическим полем двойного лучепреломления. Принимаются во внимание различия между упругими константами Франка, и также влияние тока и поляризации, индуцированных применяемым полем. Эффект иллюстрируется и экспериментом.

1. INTRODUCTION

The different electrooptic effects which arise when an electric
field is applied to a thin layer of a liquid crystal have been extensively
studied in the last few years. These effects can be utilized for example for
preparing displays. A well known example of these effects is the "dynamic
scattering" /see e.g. [1] /.

In this paper we discuss another electrooptic effect connected with
nematic liquid crystals which is also suitable for modulating light inten-
sity by electric fields. This effect is the so called electrically controlled
birefringence [2], [3]. The main point of this effect is as follows:

Let us investigate a cell with plane parallel walls, in which the
molecules are oriented parallel to the walls throughout the whole cell. The
dielectric anisotropy is assumed to be positive. Further, let us consider a
light beam that falls on the cell perpendicularly to the walls and which is
linearly polarised in a direction 45° from the direction of the molecular
alignment. The light beam will be divided into an ordinary component /i.e.
the electric field vector of the light is perpendicular to the director/ and
into an extraordinary component /electric vector parallel to the director/.
The two components propagate in the sample with different phase velocities.
Placing an analyzer behind the sample, the intensity of the light beam pass-
ing through it will depend on the phase difference between the two components.

As the molecules possesses positive dielectric anisotropy, therefore
by applying a gradually increasing electric field to the sample the molecules
can be gradually turned to a direction perpendicular to the walls. While the
molecules are turning, the phase difference between the two lightcomponents
continuously decreases, consequently the intensity behind the analyzer
continuously varies.

The experimental setup applied for the investigation of the effect
is shown on Fig.1. Fig.2. displays an experimental result. The material
used in this experiment was a mixture of methoxybenzilidene butilanilin /MBBA/,
and n 4-ethoxybenzilidene 4-amino-benzonitride /PEBAB/. The cell thickness
was approximately 12 μ. As it can be seen from the figure a threshold voltage
exists below which the original molecular arrangement remains stable,
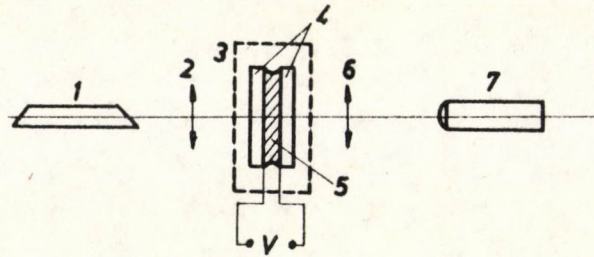similarly to the case of electrohydrodynamical instabilities [1]. Minima in

Fig. 1. Experimental setup applied for investigation
of electrically controlled birefringence

1. He-Ne laser 2. polarisator 3. thermostat
4. transparent electrodes 5. liquid crystal
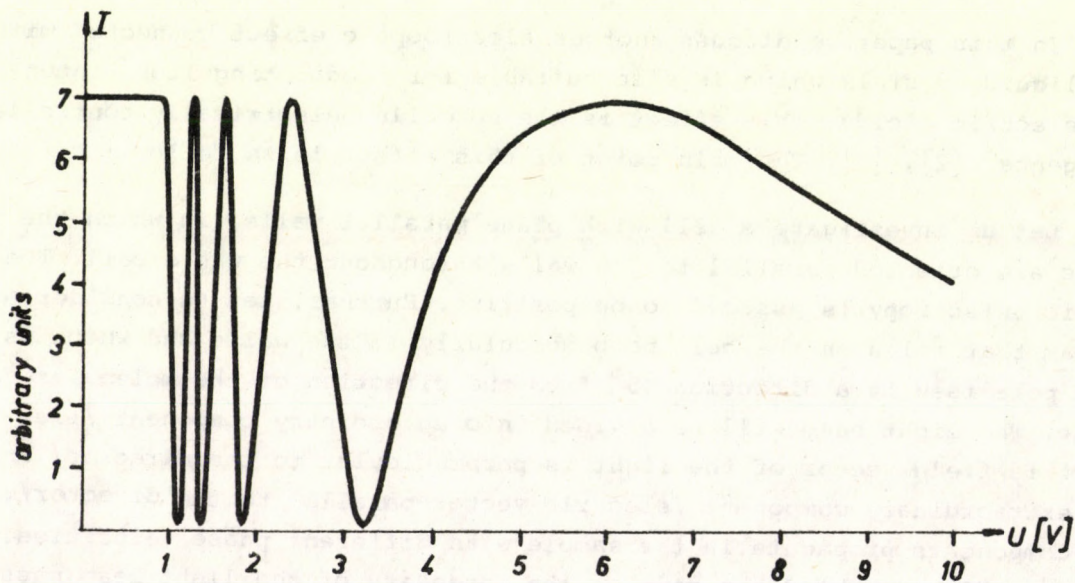6. analyzer 7. detector

Fig. 2. Intensity of the light at the detector as a function of the applied
voltage

the curve occurs when the phase difference between the two components is $2k\Pi$
/k integer/, maxima corresponds to phase difference of (2k+1)π. On Fig. 3.
the phase difference is plotted as a function of the applied voltage.

In the following we shall discuss the detailed theory of electrically
controlled birefringence. Our approach to the problem is a modification of
the calculation previously described in [2] and [3]. We do not assume that
the Frank elastic constants are equal, further we take into account that the
field strength inside the sample is not constant because the applied field in-
duces electric current and polarisation. In sec. 2 the molecular alignment is
calculated as a function of the applied voltage, in Sec. 3. the phase differ-
ence between the ordinary and extraordinary component is presented for special
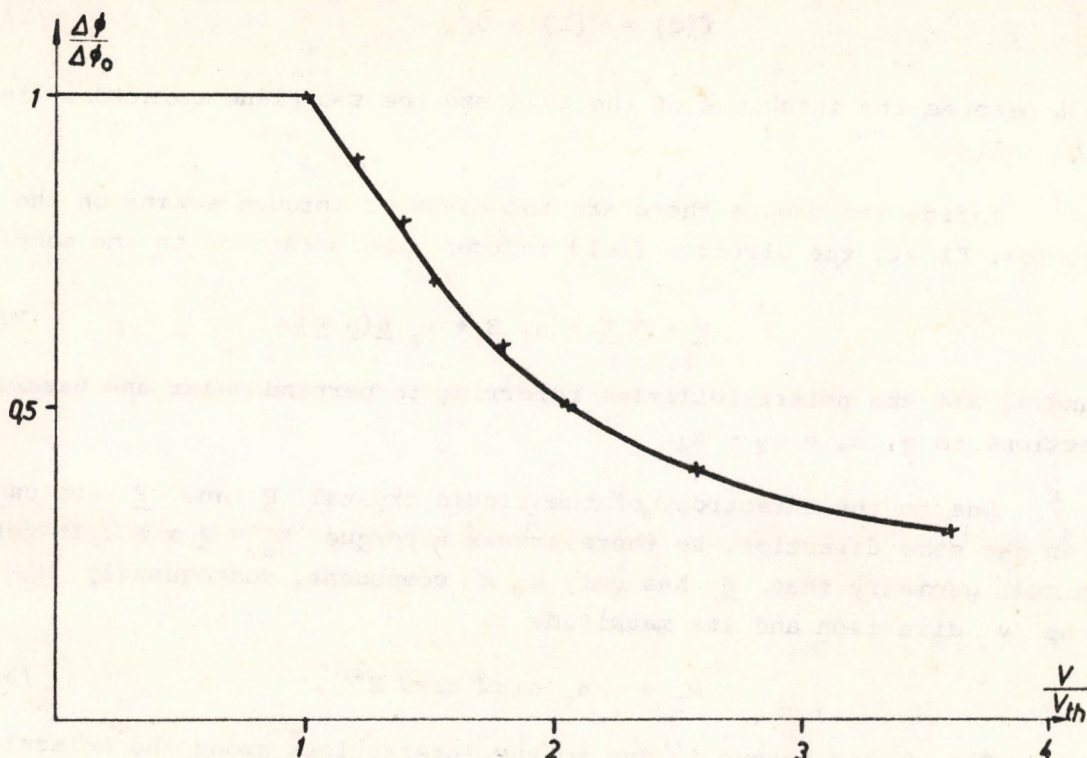cases.

Fig. 3. Phase difference between the ordinary and extraordinary component as a function of the applied voltage. $\Delta\Phi_0$ is the phase difference belonging to $V=0$. V_{th} is the threshold voltage.

2. THE MOLECULAR ALIGNMENT IN THE CELL

In the present paper we shall use the same notations as in [1]; the direction in which the molecules are oriented when no field is applied is denoted x; the direction perpendicular to the walls z; the direction perpendicular to both x and z is y.

As the applied field is parallel to z, it can be assumed that the molecules lie in the x,z plane. This means that the direction of the molecular alignment /i.e. the director/ can be unambiguously characterised by the angle between the long axis of the molecules and the x axis /ϑ/. In terms of ϑ the director, $\underline{n}$ can be expressed as $\underline{n} = /\cos\vartheta , o, \sin\vartheta /$.

It follows from the geometry of the cell, that ϑ can be only a function of z. Furthermore, we assume that the interaction between the molecules and the boundary of the cell, which is responsible for the alignment when there is no field, is strong enough to keep the molecular alignment on the boundaries unchanged even when voltage is applied. This assumption can be mathematically expressed as

$$\vartheta(0) = \vartheta(L) = 0 \qquad\qquad /1/$$

Here L denotes the thickness of the cell and the z=o plane coincides with one of the walls.

Inside the sample there are two kinds of torques acting on the molecules. First, the electric field induces a polarisation in the sample

$$\underline{P} = \hat{\alpha}\,\underline{E} = \alpha_\perp\,\underline{E} + \alpha_a\,\underline{n}(\underline{n}\,\underline{E}) \qquad\qquad /2/$$

$\alpha_\perp$ and $\alpha_\parallel$ are the polarisibilities referring to perpendicular and parallel directions to $\underline{n}$, $\alpha_a = \alpha_\parallel - \alpha_\perp$.

Due to the anisotropy of the liquid crystal $\underline{P}$ and $\underline{E}$ are usually not in the same direction, so there arises a torque $\underline{M}_E = \underline{P} \times \underline{E}$. It follows from cell geometry that $\underline{E}$ has only a z component, consequently $\underline{M}_E$ is in the y direction and its magnitude is

$$M_E = -\alpha_a\,\sin\vartheta\,\cos\vartheta\,E^2 \qquad\qquad /3/$$

The second torque is due to the interactions among the molecules which interactions tend to align the molecules parallel to each other in the liquid crystal phase. On the basis of Franks elastic theory [4] this torque is equal to [5]

$$M_D = -\left\{ (K_{11}\cos^2\vartheta + K_{33}\sin^2\vartheta)\frac{d^2\vartheta}{dz^2} + (K_{33} - K_{11})\sin\vartheta\,\cos\vartheta\left(\frac{d\vartheta}{dz}\right)^2 \right\} \qquad /4/$$

M_D is also in the y direction, K_{11} and K_{33} denote the elastic constants related to "splay" and "bend" deformations [4].

The equilibrium alignment of the molecules is determined by the differential equation

$$M_E + M_D = 0 \qquad\qquad /5/$$

After a short calculation, from eq. /3/, /4/ and /5/ we obtain

$$\frac{1}{2}\left(\frac{d\vartheta}{dz}\right)^2 = \frac{C - F(\vartheta)}{K_{11}\cos^2\vartheta + K_{33}\sin^2\vartheta} \qquad\qquad /6/$$

with

$$F(\vartheta) = \int_0^\vartheta \alpha_a\,\sin\vartheta\,\cos\vartheta\,E^2\,d\vartheta \qquad\qquad /7/$$

C is a constant, its value will be determined later on.

When calculating the function $F(\vartheta)$, it must be taken into account that the field strength inside the cell is not only determined by the applied voltage, but by the dielectric and conductivity properties of the liquid crystal also. When a static field is applied the current will be /assuming that electron emission from the electrodes is negligible/

$$\underline{j} = \hat{\sigma} \underline{E} = \sigma_\perp \underline{E} + \sigma_a \underline{n} (\underline{n} \underline{E}) \qquad /8/$$

where $\hat{\sigma}$ denotes the conductivity tensor.

As in steady state j_z must be constant all over the cell, E is determined as

$$E = \frac{j_z}{\sigma_\perp + \sigma_a \sin^2\vartheta} = E_o \frac{1}{1 + \beta \sin^2\vartheta}; \quad \beta = \frac{\sigma_a}{\sigma_\perp} \qquad /9/$$

E_o is the field strength acting at the walls, however inside the sample E usually deviates from E_o.

If the sample contains no charges at all, or if we apply an alternating field with a frequency high enough to avoid the development of space charges, then E is determined by the continuity of the z component of the displacement vector. In this case we obtain again eq. /9/, with $\beta = \frac{\varepsilon_a}{\varepsilon_\perp}$.

Insertion of eq/9/ to eq. /7/ gives

$$F(\vartheta) = -\frac{1}{2}\alpha_a \frac{E_o^2}{\beta} \frac{1}{1 + \beta \sin^2\vartheta} \qquad /10/$$

C in eq. /6/ is a constant, its value is determined by the boundary conditions. As electron emission is negligable $\vartheta(z)$ is symmetric to the z = $\frac{L}{2}$ plane, hence ϑ has a maximum at $z = \frac{L}{2}$:

$$\frac{d\vartheta}{dz}\bigg|_{z = \frac{L}{2}} = 0$$

From eq. /6/ we obtain

$$C = F(\vartheta_m); \quad \vartheta_m = \vartheta(\frac{L}{2}) \qquad /11/$$

/6/, /10/ and /11/ gives

$$\frac{dz}{d\vartheta} = \frac{1}{E_0} \sqrt{\frac{K_{11}}{\alpha_a}} \sqrt{\frac{(1 + \beta\sin^2\vartheta_m)(1 + \beta\sin^2\vartheta)(1-\{1 - K_{33}/K_{11}\}\sin^2\vartheta)}{\sin^2\vartheta_m - \sin^2\vartheta}} \qquad /12/$$

The alignment of the molecules is calculated by

$$z(\vartheta) = \int_0^\vartheta \frac{dz}{d\vartheta}\, d\vartheta \qquad /13/$$

$\frac{dz}{d\vartheta}$ is to be inserted from eq. /12/.

When $z(\vartheta)$ is actually calculated it is convenient to regard ϑ_m as a parameter. The applied voltage, as a function of ϑ_m, is to be calculated from the condition

$$V = \int_0^L E\, dz = 2\int_0^{\vartheta_m} E\, \frac{dz}{d\vartheta}\, d\vartheta \qquad /14/$$

where E is given by eq. /9/, $dz/d\vartheta$ by eq. /12/. E_0 is determined by the condition

$$L = \int_0^L dz = 2\int_0^{\vartheta_m} \frac{dz}{d\vartheta}\, d\vartheta \qquad /15/$$

For $\vartheta_m = 0$ eq. /12/-/15/ are meaningless. However for $\vartheta_m \to 0$ we get

$$V \to V_{th} = \pi\sqrt{\frac{K_{11}}{\alpha_a}}; \quad E_0 \to \frac{V_{th}}{L}$$

V_{th} can be interpreted as a threshold voltage; applying voltages smaller than V_{th} the original molecular alignment remains stable. As we have presented in Sec. 1. the existence of the threshold is experimentally verified.

On fig. 4.a. we present V as a function of ϑ_m for different K_{11}/K_{33} ratios, taking $\beta=0$. On fig. 4.b. the same function is presented, inserting $K_{11}/K_{33} = 1$, and taking β as a parameter.

3. CALCULATION OF THE PHASE DIFFERENCE BETWEEN THE ORDINARY AND EXTRAORDINARY COMPONENT

In the experiments monochromatic, lineary polarised light beams are used, falling perpendicularly onto the walls of the cell. The angle between the electric field vector and the x axis is usually 45°. In the sample the beam is divided into an ordinary and extraordinary component, which leave the cell with a phase difference of /see. e.g. /3/ /
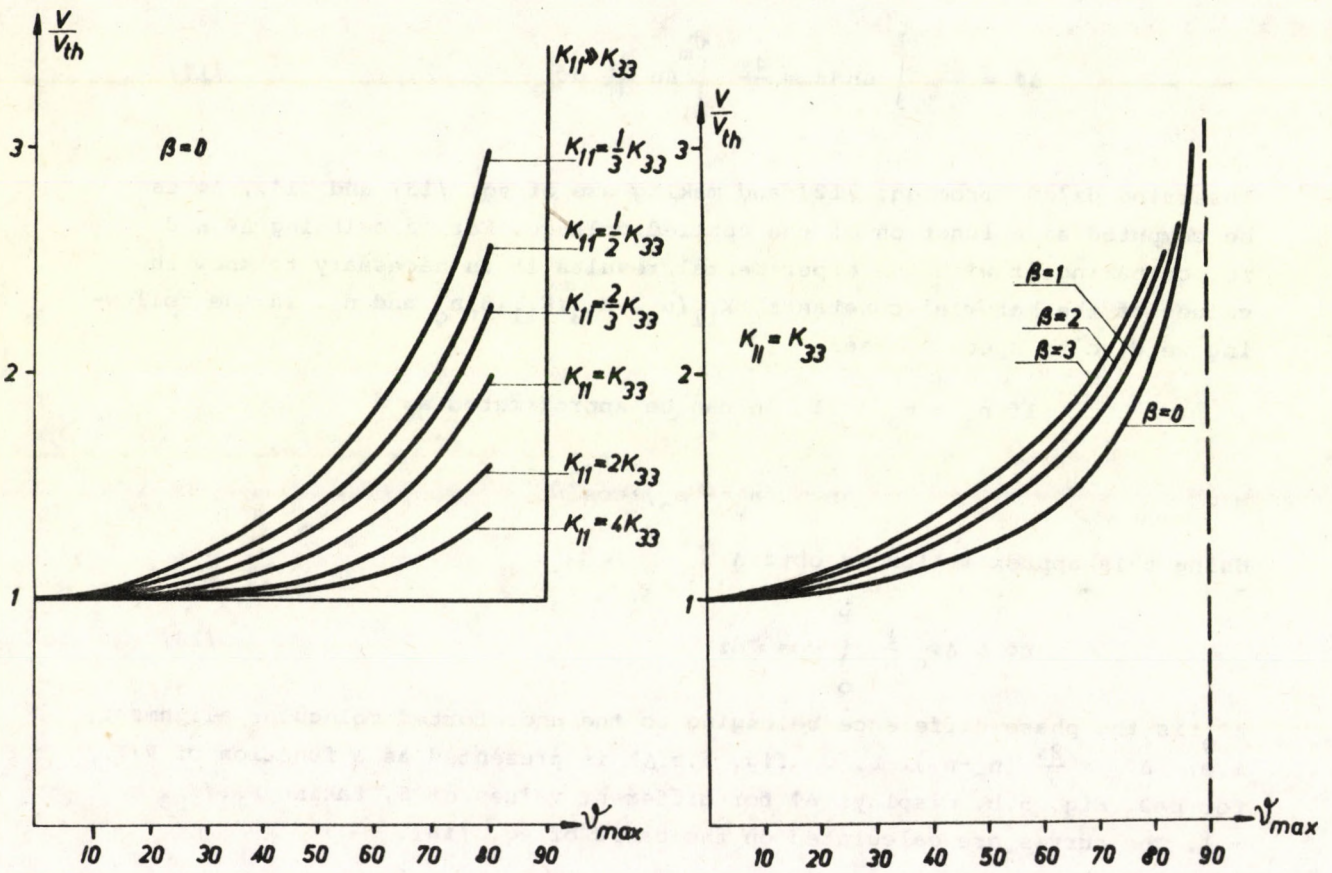
Fig. 4. V/V_{th} as a function of ϑ_{max}

a; K_{11}/K_{33} parameter, $\beta = 0$.

b; $K_{11}/K_{33} = 1$, β parameter.

$$\Delta\Phi = \frac{2\pi}{\lambda} \int_0^L \left(\frac{n_e}{\sqrt{1 + \frac{n_e^2 - n_o^2}{n_o^2} \cdot \sin^2\vartheta}} - n_o \right) dz \qquad /16/$$

n_o and n_e are the ordinary and extraordinary index of refraction. Denoting the expression in the bracket Δn we can write

$$\Delta\Phi = \frac{2\pi}{\lambda} \int_0^L \Delta n \, dz = \frac{4\pi}{\lambda} \int_0^{\vartheta_m} \Delta n \, \frac{dz}{d\vartheta} \, d\vartheta \qquad /17/$$

Inserting $dz/d\vartheta$ from eq. /12/ and making use of eq. /13/ and /14/, $\Delta\Phi$ can be computed as a function of the applied voltage. For calculating $\Delta\Phi$ and for comparing it with the experimental results it is necessary to know the values of the material constants, K_{11}/α_a, K_{33}/K_{11}, β, n_o and n_e. In the following we discuss special cases.

If $n_e - n_o \ll 1$, Δn can be approximated as

$$\Delta n \sim (n_e - n_o) \cdot \cos^2\vartheta$$

Using this approximation we obtain

$$\Delta\Phi \approx \Delta\Phi_o \frac{1}{L} \int_0^L \cos^2\vartheta \, dz \qquad /18/$$

$\Delta\Phi_o$ is the phase difference belonging to the undistorted molecular alignment, i.e. $\Delta\Phi_o = \frac{2\pi}{\lambda} (n_e - n_o) \cdot L$. On fig. 5.a. $\Delta\Phi$ is presented as a function of V/V_{th} for $\beta = 0$. Fig. 5.b. displays $\Delta\Phi$ for different values of β, taking $K_{11}/K_{33} = 1$. The curves are calculated on the basis of eq. /18/.

These figures demonstrate that the character of the theoretically calculated curves agrees with the experimental results. For accurate comparison a knowledge of the material parameters is necessary. However some general conclusions can be drawn from the theory. For example theory predicts that $\Delta\Phi/\Delta\Phi_o$ as a function of the voltage should be idependent from the cell thickness. Further, the threshold voltage obtained from the calculations does not depend on the value of β, consequently the threshold belonging to static and to alternating field should be equal. The experimental results, presented on fig. 6. show that the first conclusion is verified by the experimental results. However, for static field we obtained a much higher threshold as for alternating field. This fact may be due to the influence of electron emission and chemical polarisation. This point will be investigated further.
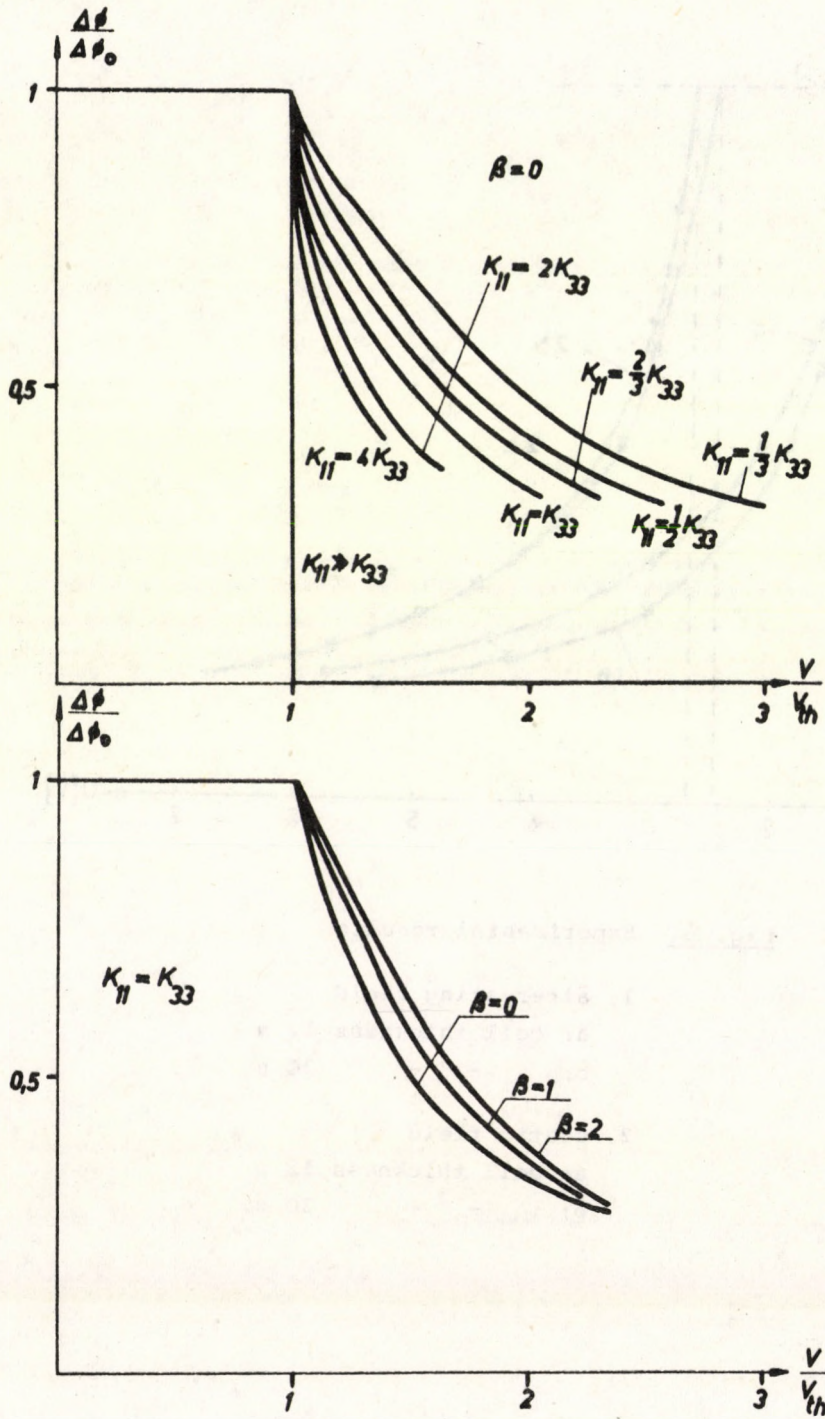
Fig. 5. $\Delta\Phi/\Delta\Phi_0$ as a function of V/V_{th}

a; K_{11}/K_{33} parameter, $\beta = 0$.
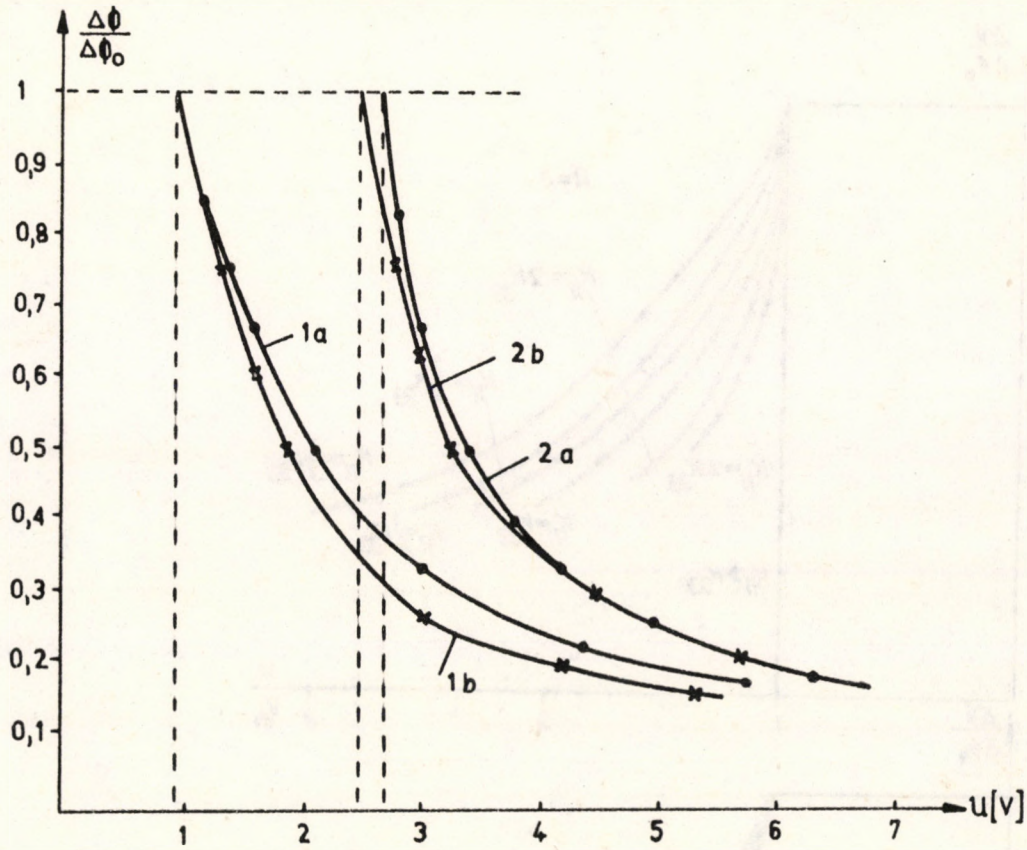
b; $K_{11}/K_{33} = 1$, β parameter.

Fig. 6. Experimental results

1. alternating field
   a; cell thickness 12 μ
   b;   - " -   30 μ

2. static field
   a; cell thickness 12 μ
   b;   - " -   30 μ

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