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Exact Solution of a Linear-Quadratic Inverse Eigenvalue Problem on a Certain Hamiltonian Symmetric Matrices

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Abstract**-- This paper investigate the exact solution of an inverse eigenvalue problem (IEP) on a certain Hamiltonian symmetric matrices namely singular symmetric matrices of rank 1 and non-singular symmetric matrices in the neighborhood of the first type of matrices via the Newton's iterative method.**

Keywords**-- Inverse, Hermiltian, eigenvalue, exact, symmetric, iterative**

I. Introduction

Various theoretical results on the solvability of the inverse eigenvalue problem for Hamilton matrices together with numerical examples are systematically reviewed and discussed in respect of the inverse eigenvalue problems for certain singular and non-singular Hamilton matrices in Oduro et al (2012),Oduro (2012a, b), Baah Gyamfi (2012) as well as Oladejo et.al (2014) and (2015).This paper investigate and established the exact solution of an inverse eigenvalue problem (IEP) on a certain Hamilton matrices consist of both singular and non-singular symmetric matrices of rank 1 via Newton's iterative method.

Linear-quadratic optimal control system

Throughout this paper we define our notation as follows: We let

$A \in \Re^{n\times n}, B \in \Re^{n\times m}, Q \in \Re^{n\times n} : Q = Q^T \geq 0$ and $R \in \Re^{m\times m} : R = R^T > 0$

Where Q is a symmetric positive semi definite matrix and R is a symmetric positive definite matrix

We find the linear- quadratic optimal control for the functional:

$$Ix_i(u) = \int_{t_0}^{t_1} \frac{1}{2}\left[x(t)^T Qx(t) + u(t)^T Ru(t)\right]dt \qquad (1)$$

Subject to differential equation

$$\dot{X}(t) = Ax(t) + Bu(t), t \in [t_0, t_1], x(t_i) = x_i \qquad (2)$$

Constructing the Hamiltonian equation from equation (1) and (2) yields:

$$H(p,x,u,t) = \frac{1}{2}\left[x^T Qx + u^T Ru\right] + p^T[Ax + Bu] \quad (3)$$

Given any optimal input $u_\bullet$ and the corresponding state $x_\bullet$

$$\frac{\partial H}{\partial u}(p_\bullet(t), x_\bullet(t), u_\bullet(t), t) = 0$$

$$\Rightarrow u_\bullet(t)^T R + p_\bullet(t)^T B = 0 \qquad (4)$$

Thus: $u_\bullet(t) = -R^{-1}B^T p_\bullet(t)$ (5)

and the adjoint equation yields:

$$\left[\frac{\partial H}{\partial x}(p_\bullet(t), k_\bullet(t), u_\bullet(t), t)\right]^T$$

$$\Rightarrow \left(x_\bullet(t)^T Q + p_\bullet(t)^T A\right)^T = -\dot{p}_\bullet(t), t \in [t_0, t_1],$$
$$p_\bullet(t_1) = 0 \qquad (6)$$

Thus;

$$\dot{p}_\bullet(t) = A^T p_\bullet(t) - Qx_\bullet(t), t \in [t_0, t_1], p_\bullet(t_1) = 0 \ (7)$$

Consequently;

$$\frac{d}{dt}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix},$$
$$t \in [t_0, t_1], x_\bullet(t_0) = x_i, p_\bullet(t_1) = 0 \quad (8)$$

Equation (8) is then a linear, time variant differential equation in $(x_\bullet, p_\bullet)$

Inverse Eigenvalue-Problem for singular $2n \times 2n$ *symmetric matrix of rank 1*

Consider the following singular symmetric matrix of order 4×4 when $n = 2$

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$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

We assume that the singularity is due to the row dependence relations specified below:

$$R_{i+1}=k_i R_1$$

$$\Rightarrow a_{21}=k_1a_{11}; a_{22}=k_1a_{12}, a_{23}=k_1a_{13}, a_{24}=k_1a_{14}$$
$$a_{31}=k_2a_{11}; a_{32}=k_2a_{12}, a_{33}=k_2a_{13}, a_{34}=k_2a_{14}$$
$$a_{41}=k_3a_{11}; a_{42}=k_3a_{12}, a_{43}=k_3a_{13}, a_{44}=k_3a_{14}$$

$$\Rightarrow a_{22}=k_1(a_{12})=k_1(a_{21})=k_1{}^2a_{11}$$
$$a_{23}=k_1(a_{13})=k_1(a_{31})=k_1k_2a_{11}$$
$$a_{24}=k_1(a_{14})=k_1(a_{41})=k_1k_3a_{11}$$
$$a_{33}=k_2(a_{13})=k_2(a_{31})=k_2{}^2a_{11}$$
$$a_{34}=k_2(a_{14})=k_2(a_{41})=k_2k_3a_{11}$$
$$a_{44}=k_3(a_{14})=k_3(a_{41})=k_3{}^2a_{11}$$

Thus:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}=\begin{bmatrix} 1 & k_1 & k_2 & k_3 \\ k_1 & k_1{}^2 & k_1k_2 & k_1k_3 \\ k_2 & k_1k_2 & k_2{}^2 & k_2k_3 \\ k_3 & k_1k_3 & k_2k_3 & k_3{}^2 \end{bmatrix}$$

To solve the inverse eigenvalue problem (IEP) we use the given nonzero eigenvalue as follows

$$tr(A)=\lambda=a_{11}(1+k_1{}^2+k_2{}^2+k_3{}^2)$$

$$a_{11}=\frac{\lambda}{1+k_1{}^2+k_2{}^2+k_3{}^2}$$

$$\Rightarrow A=\frac{1}{tr(A)}\begin{bmatrix} 1 & k_1 & k_2 & k_3 \\ k_1 & k_1{}^2 & k_1k_2 & k_1k_3 \\ k_2 & k_1k_2 & k_2{}^2 & k_2k_3 \\ k_3 & k_1k_3 & k_2k_3 & k_3{}^2 \end{bmatrix}$$

Illustration

Given that $\lambda=30, k_1=2, k_2=3, k_3=4$ b $\Rightarrow$

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

Hence, matrix A is a 4×4 singular symmetric matrix which has been reconstructed from the given nonzero eigenvalue and the prescribed dependence relation parameters.

Inverse Eigenvalue Problem for a non-singular 4×4 symmetric matrix- Newton's method

We construct a characteristic (Polynomial) function of the diagonal elements of the matrix

$A=$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}=\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33}-\lambda & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44}-\lambda \end{bmatrix}$$

In other words, consider the function with independent variables defined on 4 selected elements of matrix A, precisely, the diagonal elements:

$$f(a_{11},a_{22},a_{33},a_{44})=\lambda^2-(trA)\lambda+\det A$$

Thus, given four distinct eigenvalues $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ we have the following four (4) functions with 4 independent variables being the diagonal element of A

$$f_1(a_{11},a_{22},a_{33},a_{44})=$$
$$\begin{pmatrix} \lambda^4{}_1-(a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_1+(a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44} \\ +a_{22}a_{33}+a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_1-(a_{11}a_{22}a_{33}+ \\ a_{11}a_{21}a_{44}+a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}- \\ a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44}-a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_1+a_{11}a_{22}a_{33}a_{44} \\ -a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44}-a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}- \\ a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{pmatrix}$$

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$f_2(a_{11}, a_{22}, a_{33}, a_{44}) =$

$$\begin{pmatrix} \lambda^4{}_2 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_2 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44} \\ +a_{22}a_{33}+a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_2 - (a_{11}a_{22}a_{33} \\ +a_{11}a_{21}a_{44}+a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43} - \\ a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44}-a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_2 + a_{11}a_{22}a_{33}a_{44} \\ -a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44}-a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44} - \\ a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{pmatrix}$$

$f_3(a_{11}, a_{22}, a_{33}, a_{44}) =$

$$\begin{pmatrix} \lambda^4{}_3 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_3 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_3 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44} \\ +a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33} - \\ a_{12}a_{21}a_{44}-a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_3 + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43} \\ -a_{12}a_{21}a_{33}a_{44}-a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42} - \\ a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{pmatrix}$$

$f_4(a_{11}, a_{22}, a_{33}, a_{44}) =$

$$\begin{pmatrix} \lambda^4{}_4 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_4 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_4 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44} \\ +a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33} - \\ a_{12}a_{21}a_{44}-a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_4 + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43} \\ -a_{12}a_{21}a_{33}a_{44}-a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42} - \\ a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{pmatrix}$$

Derivation of an Explicit Formula for the Jacobian in the 4 × 4 case

$$\frac{\partial f_1}{\partial a_{11}} = -\lambda^3{}_1 + \lambda^2{}_1(a_{22}+a_{33}+a_{44}) - \lambda_1(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44})$$

$$\frac{\partial f_1}{\partial a_{22}} = -\lambda^3{}_1 + \lambda^2{}_1(a_{11}+a_{33}+a_{44}) - \lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_1}{\partial a_{33}} = -\lambda^3{}_1 + \lambda^2{}_1(a_{11}+a_{22}+a_{44}) - \lambda_1(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_1}{\partial a_{44}} = -\lambda^3{}_1 + \lambda^2{}_1(a_{11}+a_{22}+a_{33}) - \lambda_1(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_2}{\partial a_{11}} = -\lambda^3{}_2 + \lambda^2{}_2(a_{22}+a_{33}+a_{44}) - \lambda_2(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44})$$

$$\frac{\partial f_2}{\partial a_{22}} = -\lambda^3{}_2 + \lambda^2{}_2(a_{11}+a_{33}+a_{44}) - \lambda_2(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_2}{\partial a_{33}} = -\lambda^3{}_2 + \lambda^2{}_2(a_{11}+a_{22}+a_{44}) - \lambda_2(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44})$$

:

$$\frac{\partial f_2}{\partial a_{44}} = -\lambda^3{}_2 + \lambda^2{}_2(a_{11}+a_{22}+a_{33}) - \lambda_2(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_3}{\partial a_{11}} = -\lambda^3{}_3 + \lambda^2{}_3(a_{22}+a_{33}+a_{44}) - \lambda_3(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44})$$

:

$$\frac{\partial f_3}{\partial a_{22}} = -\lambda^3{}_3 + \lambda^2{}_3(a_{11}+a_{33}+a_{44}) - \lambda_3(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_3}{\partial a_{33}} = -\lambda^3{}_3 + \lambda^2{}_3(a_{11}+a_{22}+a_{44}) - \lambda_3(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_3}{\partial a_{44}} = -\lambda^3{}_3 + \lambda^2{}_3(a_{11}+a_{22}+a_{33}) - \lambda_3(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_4}{\partial a_{11}} = -\lambda^3{}_4 + \lambda^2{}_4(a_{22}+a_{33}+a_{44}) - \lambda_4(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44})$$

$$\frac{\partial f_4}{\partial a_{22}} = -\lambda^3{}_4 + \lambda^2{}_4(a_{11}+a_{33}+a_{44}) - \lambda_4(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_4}{\partial a_{33}} = -\lambda^3{}_4 + \lambda^2{}_4(a_{11}+a_{22}+a_{44}) - \lambda_4(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44})$$

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$$J=\begin{bmatrix} \frac{\partial f_1}{\partial a_{11}} & \frac{\partial f_1}{\partial a_{22}} & \frac{\partial f_1}{\partial a_{33}} & \frac{\partial f_1}{\partial a_{44}} \\ \frac{\partial f_2}{\partial a_{11}} & \frac{\partial f_2}{\partial a_{22}} & \frac{\partial f_2}{\partial a_{33}} & \frac{\partial f_2}{\partial a_{44}} \\ \frac{\partial f_3}{\partial a_{11}} & \frac{\partial f_3}{\partial a_{22}} & \frac{\partial f_3}{\partial a_{33}} & \frac{\partial f_3}{\partial a_{44}} \\ \frac{\partial f_4}{\partial a_{11}} & \frac{\partial f_4}{\partial a_{22}} & \frac{\partial f_4}{\partial a_{33}} & \frac{\partial f_4}{\partial a_{44}} \end{bmatrix}$$

$$\begin{bmatrix} -\lambda^3{}_1+\lambda^2{}_1(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{33})- \\ \lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \\ -\lambda^3{}_3+\lambda^2{}_3(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{33})- \\ \lambda_2(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_2(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_2(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_2(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \\ -\lambda^3{}_3+\lambda^2{}_3(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{33})- \\ \lambda_3(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_3(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_3(a_{11}a_{22}+a_{11}a_{44}+a_{22}a_{44}) & \lambda_3(a_{22}a_{33}+a_{11}a_{22}+a_{11}a_{33}) \\ -\lambda^3{}_4+\lambda^2{}_4(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_2(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{22}+a_{33})- \\ \lambda_4(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_4(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_4(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_4(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \end{bmatrix}$$

II. NUMERICAL EXAMPLE

Consider the nonsingular symmetric
Coefficient matrix of the system:

$$\dot{x}_1 = a_{11}x_1 + 2x_2$$
$$\dot{x}_2 = 2x_1 + 4a_{22}x_2$$

Given the eigenvalues $\lambda_1 = -1, \lambda_2 = 3$

i.e., given a target solution of the form

$$x = c_1ue^{-t} + c_2ve^{3t}$$

Assuming the initializing matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $X^{(0)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Moreover, $A^{(0)}$ is singular with $k = 2, a_{11} = 1$

Also $\lambda = trA^{(0)} = 5$

We first compute the values of the functions at the initial point:

$$f_1(a_{11}, a_{22}) = \lambda_1^{\ 2} - (a_{11} + a_{22})\lambda_1 + (a_{11}a_{22} - a_{12}^{\ 2})$$
$$\Rightarrow 1 - 5(-1) + 0 = 6$$
$$f_2(a_{11}, a_{22}) = \lambda_2^{\ 2} - (a_{11} + a_{22})\lambda_2 + (a_{11}a_{22} - a_{12}^{\ 2})$$

$$\Rightarrow 9 - 5(3) + 0 = -6$$

$$\underline{f}(X^{(0)}) = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

Using the formula for the inverse of the Jacobian matrix, we have:

$$J^{-1} = \frac{1}{(\lambda_1 - \lambda_2)(a_{22} - a_{11})}\begin{bmatrix} a_{22} - \lambda_1 & a_{11} - \lambda_1 \\ a_{22} - \lambda_2 & a_{11} - \lambda_2 \end{bmatrix}$$

$$J^{-1} = \frac{1}{(\lambda_1 - \lambda_2)(a_{22} - a_{11})}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix}$$

Substitute the values into the Newton's iterative equation. i.e.

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(n)})\underline{f}(X^{(n)})$$

Then:

$$X^{(1)} = X^{(0)} - J^{-1}(X^{(0)})\underline{f}(X^{(0)})$$

$$X^{(0)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \frac{1}{12}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad X^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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$$\Rightarrow \qquad A(X^1) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Hence, we obtain the **(exact**) solution.

III. CONCLUSION

Theoretical results on the solvability of the inverse eigenvalue problem for Hamilton matrices was systematically reviewed and discussed in respect of the inverse eigenvalue problems (IEP) for certain singular and non-singular Hermitian matrices. Base on this we have successfully investigate and established the exact solution of an inverse eigenvalue problem (IEP) on a certain Hermiltian matrices consist of both singular and non-singular symmetric matrices of rank 1 via Newton's iterative method.

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