Reliability Assessment of BS 8110 (1997) Ultimate Limit State Design Requirements for Reinforced Concrete Columns

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ABSTRACT

This paper describes the reliability assessment of reinforced concrete columns designed according to the BS 81I0 (1997) ultimate limit state requirements. A typical cross-section (400 mm×400 mm) for three different commonly used columns was adopted and probabilistically assessed when all variables relating to the loading geometry and material properties are randomly distributed. First-Order Reliability Method (FORM) was employed to estimate the implied probability of failure for simulated loading and reinforcement quantities. The results showed that the cross-section (400 mm×400 mm) assessed could not sustain more than 40% of the expected ultimate design load before the violation of the limit state. In addition, the performance of reinforced concrete columns depends more on the applied load than on the amount of reinforcement used. The general inference from these results is that most of these types of columns designed according to BS 8110 (1997) have not failed, because they were carrying far less than their ultimate design loads.

KEYWORDS: BS 8110 (1997), Structural reliability analysis, Reinforced concrete columns, Ultimate limit state design requirements.

INTRODUCTION

Reinforced concrete structures designed before the introduction of modern seismic code in the early 1970s are vulnerable to damage and collapse during a natural disaster, seismic wave,… etc. Thus, it is vital that reinforced concrete structures designed not in accordance with modern BS code specifications or as a result of building column with unsuitable material that does not help in attaining its required characteristic strength be retrofitted to sustain seismic loading (Flores, 2007). The limit state design concept resulted from the probabilistic considerations and it is assumed to be more logical in its presentation of safety margin. The concept aims at achieving a consistent and acceptable probability that structures being designed will perform satisfactorily during their intended life (BS 8110, 1997).

Structural reliability is being defined as the probability that a structural system will survive the given load level. There is a counterpart to reliability called probability of failure (P_f). It is defined as the probability that a structural system will fail under the given loading conditions. The lack of reliability represents certain probability that failure can occur. Hence, reliability and probability of failure form two

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extremes related to the safety of structural systems. Probability theory states that the sum of reliability and probability of failure is always equal to unity. This rule makes it easy to evaluate one quantity if the other is known.

When the probability distribution of all the parameters involved in the design of a structure is known, the probability of failure can be determined as follows:

$$P_r = \int f_x\,(x).\mathrm{dx} \qquad (1)$$

The reliability index (β) is related to the probability of failure p_f through:

$$p_f = \emptyset(-\beta); \qquad (2)$$

where $\emptyset(-\beta)$ represents the cumulative distribution function of the standard normal distribution. Equations 1 and 2 have been developed to predict the level of probability of failure of several civil engineering structures (Afolayan et al., 2005; Kamiski and Trapko, 2006).To determine the probability of failure of different types of columns in this investigation, there are several types of methods which have been described in many papers, such as the first-order reliability method (FORM) (Au et al., 2007; Katafygiotis and Zuev, 2008; Kmet et al., 2011) and Monte Carlo simulation (MCS) (Schueller, 2009; Paik et al., 2009; Basaga et al., 2012). The Firsr-Order Reliability Method (FORM) can simplify the task and will be employed so as to estimate P_f (probability failure) and β (reliability index) for each situation.

There have been several research works involving different applications of reliability methods on various structures of different materials and structural elements, like: beams, slabs and columns. Afolayan and Opeyemi (2008) studied the reliability analysis of pile capacity of concrete in cohesive and cohesionless soils and suggested that steel piling should be encouraged in cohesive and cohesionless soil. Olanitori et al. (2014) studied the collapse mode of square single panel reinforced concrete space framed structures, with beam and column joint hinged, defined the effects of errors in detail and decided that there must be effective supervision of reinforcement details during construction. Abubakar (2006) studied the reliability analysis design parameters of strip footings, where the results showed that BS 8110 requirements for reinforcement under varying reinforcement ratios, effective depths and ratios of dead load to live load are fairly satisfactory. Abubakar (2014) used a FORTRAN-based reliability design program which was developed for the design of raft footings based on the ultimate and serviceability design requirements of BS 8110. Other applications of reliability assessments in civil engineering have been reported by Afolayan (2004a), Afolayan (2004b), Ayininuola and Olalusi (2004), Afolayan (2005), Afolayan et al. (2005), Akindahunsi and Afolayan (2009), Afolayan and Opeyemi (2010), Kaura and Afolayan (2011), Ugurhan et al. (2013), Fang et al. (2013) and Mohammed (2015).

METHODOLOGY

With the aid of a modern high speed computer, it is possible to probabilistically investigate failure of an existing building and/or proposed building.This research work is carried out by modelling reasonable different situations of loading reinforced concrete columns according to BS 8110 (1997). Three different reinforced concrete columns are probabilistically examined when all the relevant design variables are randomly distributed.

In order to adequately estimate the failure probabilities of the columns, the relevant design equations for reinforced concrete columns according to BS 8110 (1997) are reviewed and all variables governing their performance established. Consequently, data on the probability density functions of the variables are sourced from literature. The design equations based on BS 8110 (1997) are transferred to the CalREL (calculation of reliability) platform, a coded algorithm for the computation of multi-dimensional integral of failure surface. The aspect of CalREL adopted in the study can be summarized as follows.

A performance function and limit state is:

$$G\ (X) = G\ \left(\sigma_{x_1}.X'_1 + \mu_{x_1}, \sigma_{x_2}.X'_2 + \mu_{x_2}, \ldots., \sigma_{x_n}.X'_n + \mu_{x_n}\right) = 0. \quad (3)$$

The reliability index β associated with Equation 3 can be calculated either using the invariant solution by Hasofer and Lind (1974) or the Mean-Value First-Order Second-Moment (MFOSM), also known as the Mean-First-Order Reliability Method (MFORM). The value of β based on the FORM model is given by:

$$\beta = \min_{x \varepsilon F} \sqrt{(X'_1)^2 + (X'_2)^2 + \cdots + (X'_n)^2} \quad (4)$$

where X'_1, X'_2,…, X'_n are the random variables in the limit state function given by G(x) =0.

The minimization of Equation 3 is performed through an optimization procedure over the failure domain F corresponding to the region G(x) ≤ 0. This can be accomplished using FORM5 (Gollwitzer et al., 1988). FORM5 has the following advantages (Juang et al., 1999):

i. Solution to the problem can be obtained by working with original rather than previously transformed or reduced random variable space.
ii. The partial derivatives of G(x) need not be provided.
iii. Correlated and non-normal variables are handled easily through transformations.

Generally, FORM provides an approximation to:

$$P_f = P(X \in F) = P(G(X) \leq 0) = \int_{G(X) \leq 0}^{0} dF_x\ (X), \quad (5)$$

by (i) transforming non-Gaussian (non-normal) variables into independent standard normal variables, (ii) locating the β-point (most likely failure point) through an optimization procedure, linearizing the limit state function in that point and then estimating the failure probability using the standard normal integral (Gollwitzer et al., 1988). A first-order approximation to P_f =P (G(x) ≤ 0) is given by Thoft-Christensen and Baker (1982):

$$P_f = \emptyset(-\beta); \quad (6)$$

where Ø is the standard normal integral and β is the (geometric) safety index or reliability index (Gollwitzer et al., 1988). It then follows that (Thoft-Christensen and Baker, 1982):

$$\beta = -\emptyset^{-1}(P_f). \quad (7)$$

The value of β estimated in Equation 7 is used to adjudge the adequacy or inadequacy of the ultimate limit state design requirements in BS 8110 (1997) for reinforced concrete columns in Nigerian environment.

RESULTS AND DISCUSSION

Results of Different Column Loading Conditions when f_{cu} = 20N/mm²

Short Braced Axially Loaded Column (SBC)

The ultimate loading capacity as given by BS 8110 (1997) is:

$$N_{uz} = \left(0.67 \frac{f_{cu}}{\gamma_{mc}}\right) A_c + \left(\frac{f_y}{\gamma_{ms}}\right) A_{sc}. \quad (8)$$

With $A_{sc} = \frac{\rho bh}{100}$, A_C =bh, factor of safety for concrete (γ_{mc}) =1.5 and for steel (γ_{ms}) = 1.05,

Equation (8) becomes:

$$N_{uz} = 0.45 f_{cu} bh + 0.0095 \rho bh f_y. \quad (9)$$

Then, for a rectangular section:

$$N_{uz} = 0.45 bh\left(f_{cu} + 2.11\text{x}10^{-2} f_y \rho\right). \quad (10)$$

For a particular column design, the following data was generated:

b = 400mm, h = 400mm, A_{sc} =6261mm^2, f_{cu} = 20 N/mm^2, f_y = 410 N/mm^2.

When the values are substituted in Equation 9, we obtain N_{uz} =3875.14 kN.

We assume this value N_{uz} =3875.14 kN as the demand (D) on a particular column. That is D = 3875.14 kN. Let C= Capacity = Ultimate load a column can carry using BS 8110 (1997) design equation. That is:

$$C = 0.45bh\left(f_{cu} + 2.11\text{x}10^{-2}f_y\rho\right). \quad (11)$$

Conditions for checking the performance of a column are stated as:

When G = performance function

G = C – D = 0 LIMITING STATE

G = C – D > 0 SAFE STATE

G = C – D < 0 FAILURE STATE

In order to estimate the implied probability of failure, we need to compute:

$$P_f = P\,(G = C - D < 0), \quad (12)$$

which can be expressed as:

$$P_f = P\left[0.45bh\left(f_{cu} + 2.11x10^{-2}f_y\rho\right) - \alpha N_A < 0\right]; \quad (13)$$

where:

α = percentage of ultimate load and

N_A = applied ultimate load.

The statistics of the variables in Equation 13 are presented in Table 1.

Table 1. Statistics of design variables for a short braced reinforced concrete column

Variables	Probability density function	Mean	Standard deviation
Breadth, B(mm)	Normal	400.00	40.00
Height, H(mm)	Normal	400.00	40.00
Strength of concrete,f_{cu}(N/mm^2)	Lognormal	20.00	6.00
Strength of steel,f_y(N/mm^2)	Lognormal	410.00	123.00
Reinforcement ratio, ρ	Lognormal	3.90	1.17
Ultimate load, N_l (kN)	Lognormal	3875	1162.5

With the statistical models in Table 1, the implied probability of failure was computed and a typical output was printed after a complete processing of a few cycles of the CalREL program. Figure 1 shows the plot of reliability index (β) against the percentage of the ultimate load. It can be observed that a short braced reinforced concrete column under pure axial load will completely lose its carrying capacity if it carries about 40% of its design load. This implies that most of the short braced reinforced columns in service have not failed because they were carrying far less than their designed loads. It may be concluded that such columns are not economically designed. It is also obvious from the plot that the amount of reinforcement is not a serious factor in the performance of short braced reinforced columns. In Figure 2, the interaction of β, α and ρ is plotted. It is also confined that the safety of short braced reinforced columns is grossly dependent on the magnitude of the applied load.

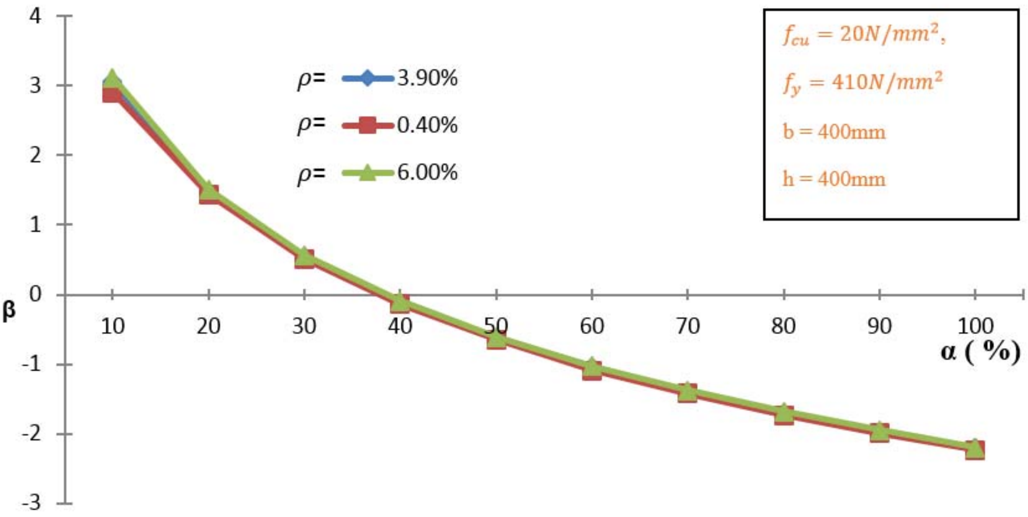

Figure (1): Reliability index (β) against percentage of expected ultimate load (α) for short braced reinforced concrete columns

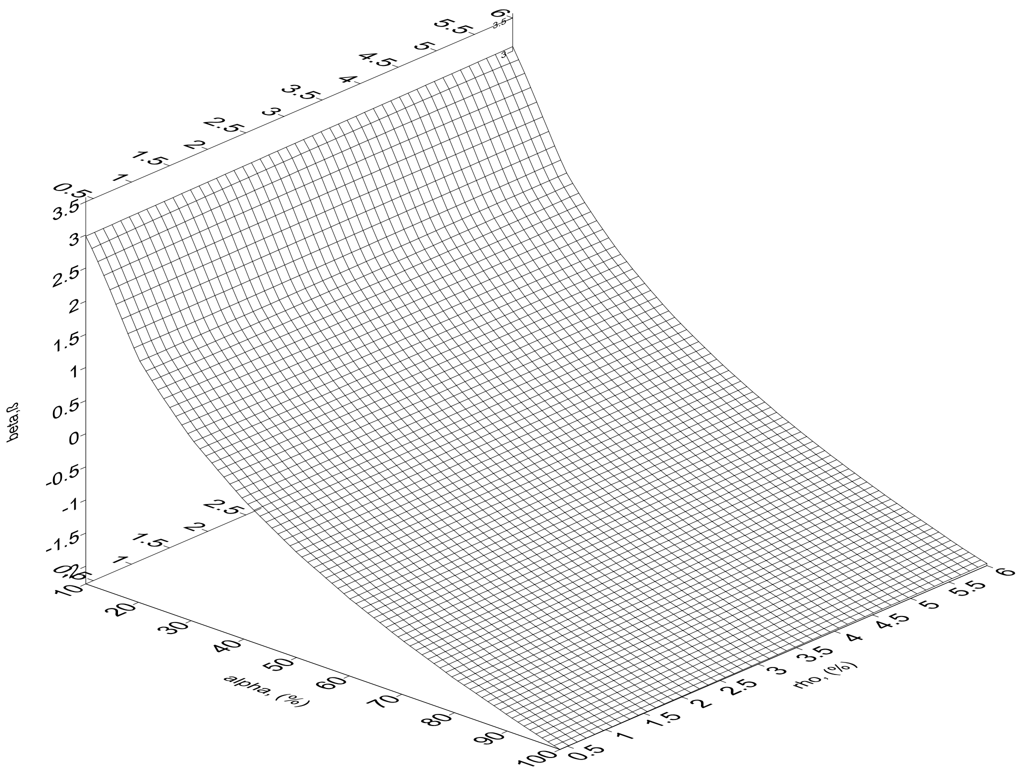

Figure (2): Variation of β with α and ρ for a short braced reinforced concrete column

Nominal Eccentricity of a Short Column Resisting Moments and Axial Forces (NE)

Recall that according to BS 8110 (1997), the ultimate capacity of a short column with a nominal eccentricity can be taken as:

$N_{uz}= 0.4f_{cu}A_c + 0.8A_{sc}f_y$. (14)

When $A_{sc} = \frac{\rho bh}{100}$, A_C =bh, Equation 14 becomes:

$N_{uz} = 0.4bh\left(f_{cu} + 2.00\text{x}10^{-2}f_y\rho\right)$. (15)

For a typical design, the following data was generated:

b = 400mm, h = 400mm, A_{sc} =6261mm^2, f_{cu} = 20 N/mm^2, f_y = 410 N/mm^2.

When these values are substituted in Equation 15, we have N_{uz} =3333.61kN.

We assume this value N_{uz} = 3333.61kN to be the demand (D) on the column, so that D = 3333.61kN.

Let C = Capacity = Ultimate load the column can carry using BS 8110 (1997) design equation. Then:

C = $0.4bh\left(f_{cu} + 2.00\text{x}10^{-2}f_y\rho\right)$. (16)

In order to estimate the implied probability of failure, we need to compute:

When G = performance function

P_f = P (G = C – D < 0), in which

G = C - D

G = $0.4bh\left(f_{cu} + 2.00\text{x}10^{-2}f_y\rho\right) - \alpha N_A$. (17)

The statistics of the variables in Equation 17 are presented in Table 2.

Table 2. Statistics of variables in the design of short columns resisting moments and axial forces

Variables	Probability density function	Mean	Standard deviation
Breadth, B (mm)	Normal	400.00	40.00
Height, H (mm)	Normal	400.00	40.00
Strength of concrete,f_{cu}(N/mm^2)	Lognormal	20.00	6.00
Strength of steel, f_y (N/mm^2)	Lognormal	410.00	123.00
Reinforcement ratio, ρ	Lognormal	3.90	1.17
Ultimate load, N_l (kN)	Lognormal	3333	999.9

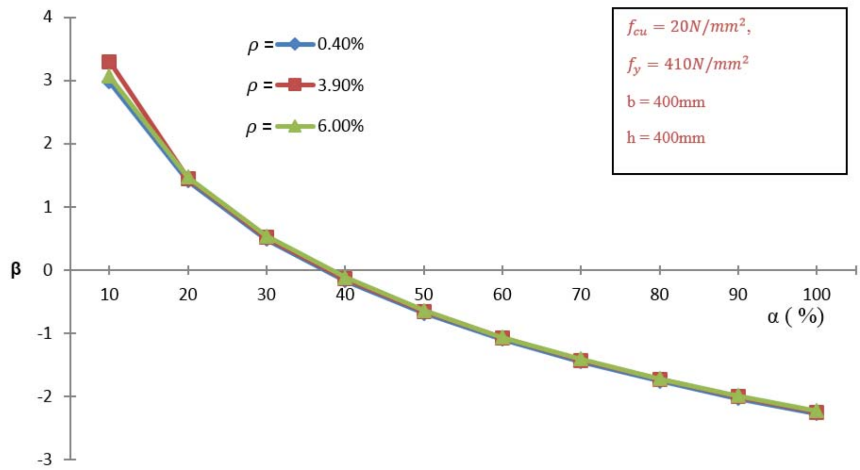

Figure (3): Reliability index (β) against percentage of expected ultimate load (α) for nominal eccentricity of a short column resisting moments and axial forces

Reliability analysis was performed. After the seventh iteration, the system converged, with the result that for every 20 columns built only two are likely to fail. The same procedure was repeated using incremental expected ultimate load from 10% to 100%. Figure 3 shows that a short column resisting moments as a result of a nominal eccentricity will completely lose its carrying capacity if loaded to about 40% of its ultimate capacity. This performance is regardless the amount of reinforcement used. This is also established from the iteration plot in Figure 4.

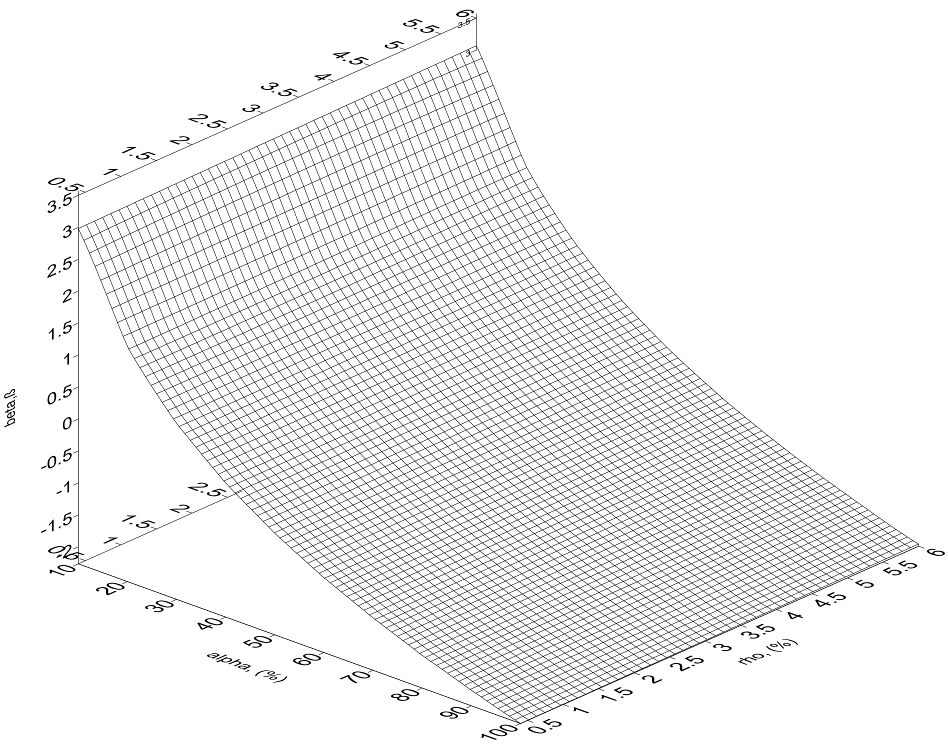

Figure (4): Variation of β with α and ρ for nominal eccentricity of a short column resisting moments and axial forces

Short Braced Column Separated on an Approximately Symmetrical Arrangement of Beams (SBCS)

The ultimate carrying capacity of a short braced column on an approximately symmetrical arrangement of beams is determined using BS 8110 (1997) as:

$$N_{uz} = 0.35 f_{cu} A_c + 0.7 A_{sc} f_y. \quad (18)$$

When $A_{sc} = \frac{\rho bh}{100}$ and A_C =bh, Equation 18 becomes:

$$N_{uz} = 0.35bh\left(f_{cu} + 2.00\text{x}10^{-2} f_y \rho\right). \quad (19)$$

For a typical design, the following data was generated:
b = 400mm, h = 400mm, A_{sc} =6261mm^2, f_{cu} = 20 N/mm^2, f_y = 410 N/mm^2.

When these values are substituted in Equation 15, we have N_{uz} =2916.91kN.

We assume this value N_{uz} = 2916.91kN to be the demand (D) on the column, so that D = 2916.91kN.

Let C = Capacity = Ultimate load the column can carry using BS 8110 (1997) design equation. Then:

$$C = 0.35bh\left(f_{cu} + 2.00\text{x}10^{-2} f_y\rho\right). \quad (20)$$

In order to estimate the implied probability of failure, we need to compute:
P_f = P (G = C – D < 0), in which G = C – D.

When G = performance function

$$G = 0.35bh\left(f_{cu} + 2.00\text{x}10^{-2} f_y\rho\right) - \alpha N_A. \quad (21)$$

The statistics of the variables in Equation 21 are presented in Table 3.

Table 3. Statistics of variables in the design of nominal eccentricity of a short column resisting moments and axial forces

Variables	Probability density function	Mean	Standard deviation
Breadth, B(mm)	Normal	400.00	40.00
Height, H(mm)	Normal	400.00	40.00
Strength of concrete f_{cu}(N/mm^2)	Lognormal	20.00	6.00
Strength of steel f_y(N/mm^2)	Lognormal	410.00	123.00
Reinforcement ratio, ρ	Lognormal	3.90	1.17
Ultimate load, N_l (kN)	Lognormal	2916	874.8

The statistical models in Table 3 are used to estimate the safety of the column. Figure 5 shows a plot of reliability index (β) against the percentage increase in ultimate load (α) for a short braced column separated on an approximately symmetrical arrangement of beams. Such columns are not able to withstand up to 40% of the ultimate design load before they completely violate the ultimate limit state requirement. Similar to the other types of columns, the amount of reinforcement is not as critical to their safety as the applied load. The observation is obvious from Figure 5 and Figure 6.

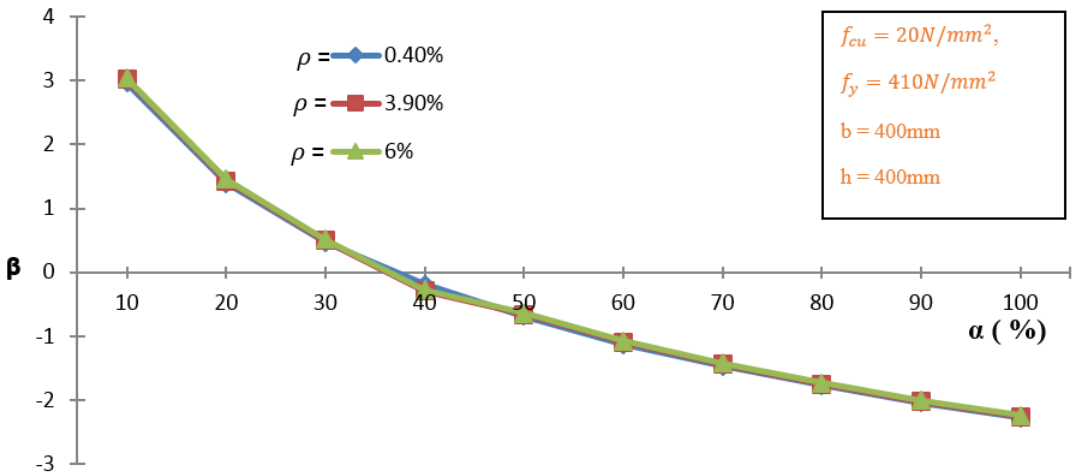

Figure (5): Reliability index (β) against percentage of expected ultimate load (α) for a short braced column separated on an approximately symmetrical arrangement of beams

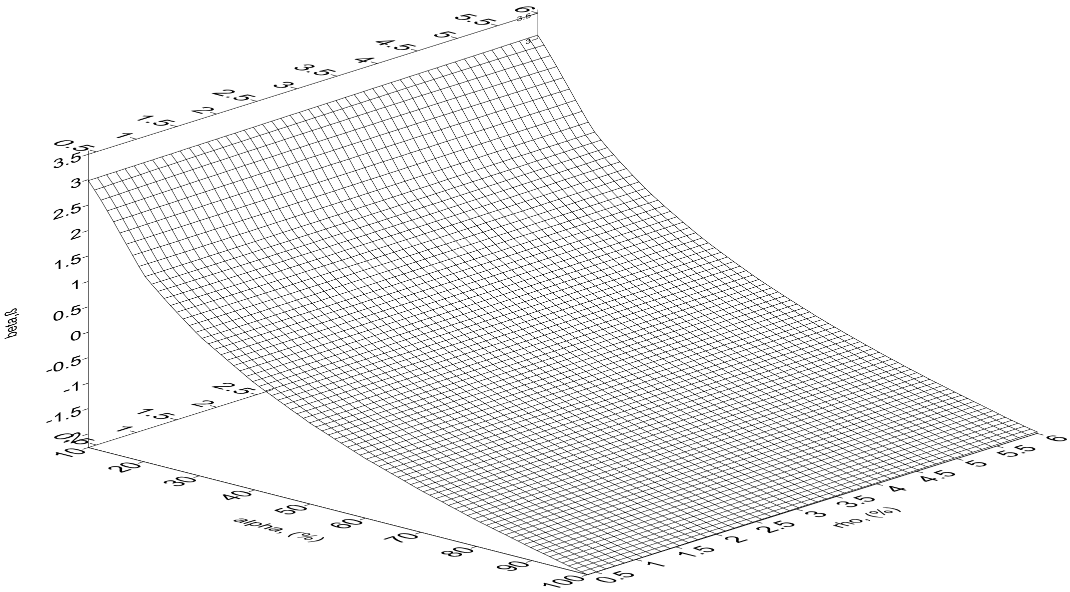

Figure (6): Variation of β with α and ρ for a short braced column separated on an approximately symmetrical arrangement of beams

Figure 7 shows the safety indices for the three types of columns compared. A short braced reinforced concrete column under pure axial load will lose its carrying capacity at about 40% of the required ultimate capacity. A braced short column with nominal eccentricity will completely lose its carrying capacity if loaded to about 39% of the ultimate capacity. A short braced column separated on an approximately symmetrical arrangement of beams will lose its carrying capacity if loaded to about 37% of its ultimate capacity. The general inference from these results is that most of these types of columns designed according to BS 8110 (1997) have not failed, because they were carrying far less than their ultimate design loads.

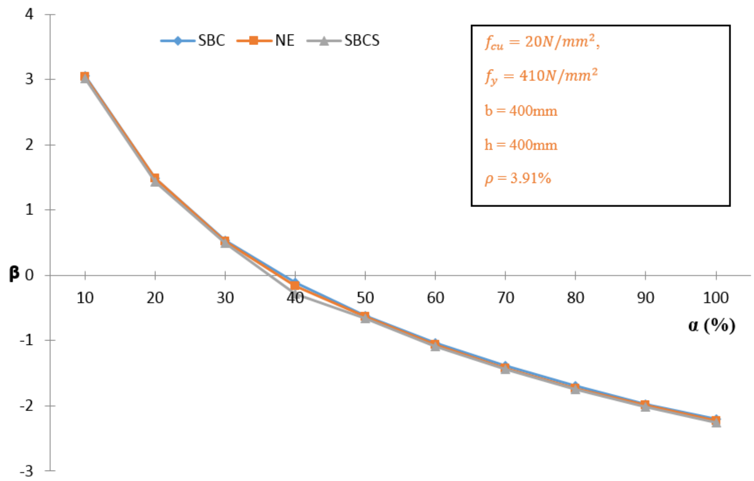

Figure (7): Comparison of safety indices of SBC, NE and SBCS

Influence of Material Properties on Safety

According to Alabi (2012), the characteristic strength of concrete, when pit sand is used (as commonly done), is about 10.44N/mm^2. On the basis of this information, it is necessary to recompute the implied safety levels for the different columns under study. Figure 8 shows the variation in reliability index (β) with the percentage of the ultimate load (α). It can be observed that a short braced reinforced concrete column under a pure axial load will completely lose its carrying capacity if loaded at about 40% of the required ultimate carrying capacity when f_{cu} = 20 N/mm^2 and at 22% of the required ultimate carrying capacity when f_{cu} = 10.44 N/mm^2. This implies that the reinforced concrete columns produced from pit sand as fine aggregate are grossly endangered.

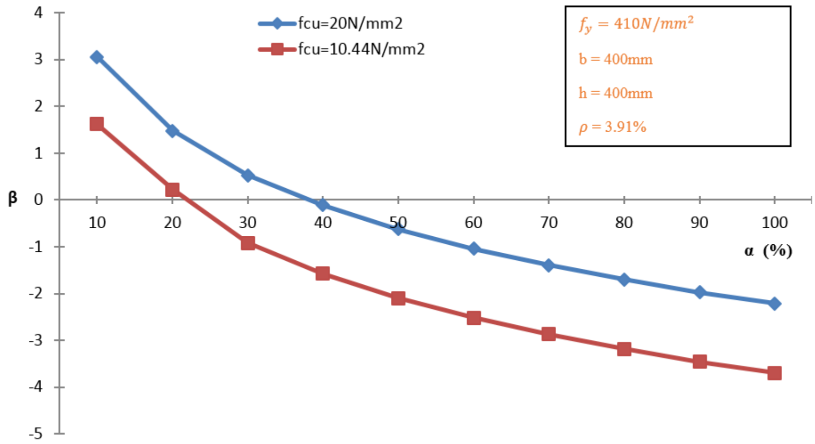

Figure (8): Variation of β with percentage of expected ultimate load (α) for short braced axially loaded columns

Effect of Variability in Loading on Safety of Columns

The effect of variability in the ultimate design load on the safety of short braced axially loaded columns is simulated. A range of (5-40%) was ensured. Figure 9 shows that the variability in loading has a very little influence on the performance of other types of columns that exhibit the same information (see Figures 10 and 11).

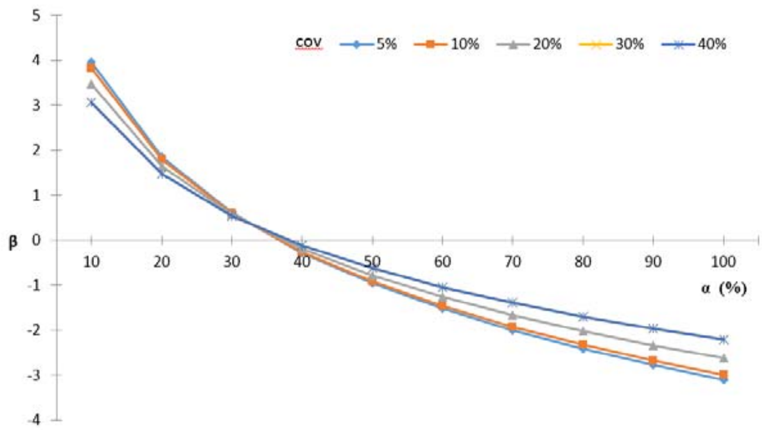

Figure (9): Effect of variability in applied load on reliability index (β) in short braced axially loaded columns

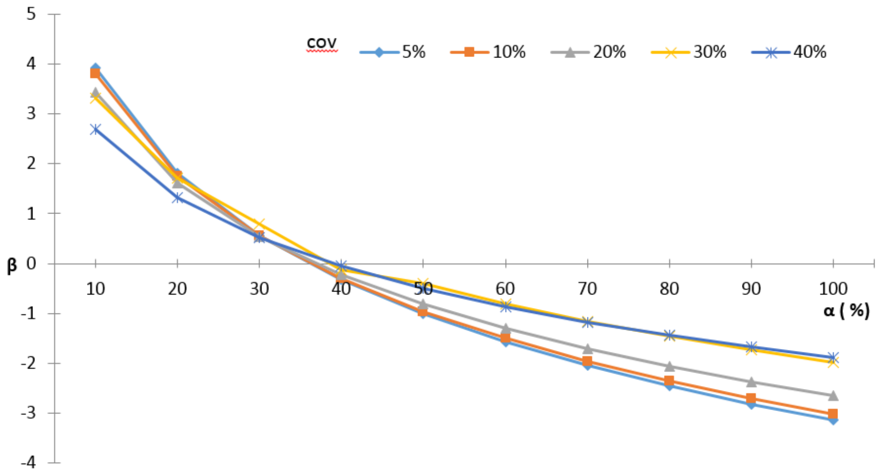

Figure (10): Effect of variability in applied load on reliability index (β) in nominal eccentricity of short column resisting moments and axial forces

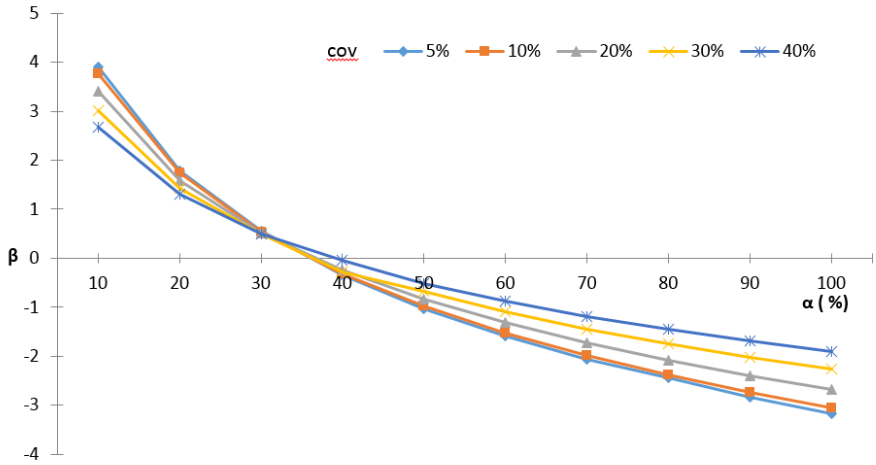

Figure (11): Effect of variability in applied load on reliability index (β) for short braced columns separated on an approximately symmetrical arrangement of beams

CONCLUSION

The ultimate limit state design requirements of BS 8110 (1997) for reinforced concrete columns have been probabilistically examined. Three different practical types of columns were assessed under varying percentage of the expected ultimate design load when all the relevant variables were assumed random. The First-Order Reliability Method (FORM) was employed in the determination of the measure of safety. For the three types of columns studied, the result of the simulation showed that none of the columns could withstand up to 40% of the expected ultimate design load under certain conditions of loading and material properties.

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