

Nuclear surface properties in relativistic effective field theory

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Abstract

We perform Hartree calculations of symmetric and asymmetric semi-infinite nuclear matter in the framework of relativistic models based on effective hadronic field theories as recently proposed in the literature. In addition to the conventional cubic and quartic scalar self-interactions, the extended models incorporate a quartic vector self-interaction, scalar-vector non-linearities and tensor couplings of the vector mesons. We investigate the implications of these terms on nuclear surface properties such as the surface energy coefficient, surface thickness, surface stiffness coefficient, neutron skin thickness and the spin-orbit force.

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1 Introduction

Quantum hadrodynamics (QHD) and the relativistic treatment of nuclear systems has been a subject of growing interest during recent years [1,2,3,4,5]. The σ - ω model of Walecka [1] and its non-linear extensions with cubic and quartic self-interactions of the scalar-meson field [6] have been widely used to this end. This model contains Dirac nucleons together with neutral scalar and vector mesons as well as isovector-vector ρ mesons. At the mean field (Hartree) level, it already includes the spin-orbit force, the finite range and the density dependence which are essential ingredients of the nuclear interaction. This simple model has become very popular in relativistic calculations and describes successfully many properties of the atomic nucleus.

From a theoretical point of view, the non-linear σ - ω model with cubic and quartic scalar self-interactions was classed within renormalizable field theories which can be characterized by a finite number of coupling constants. However, very recently, generalizations of this model that include other non-linear interactions among the meson fields and tensor couplings have been presented on the basis of effective field theories by Serot et al. [5,7,8,9]. The effective theory contains many couplings of non-renormalizable form that are consistent with the underlying symmetries of QCD. Consequently, one must find some suitable expansion parameters and develop a systematic truncation scheme. For this purpose the concept of naturalness has been employed: it means that the unknown couplings of the theory should all be of the order of unity when written in appropriate dimensionless form using naive dimensional analysis [5,7,8,9]. Then, one can estimate the contributions coming from different terms by counting powers in the expansion parameters and truncating the Lagrangian at a given level of accuracy.

One important fact is the observation that at normal nuclear densities the scalar and vector meson fields, denoted by Φ and W , are small as compared with the nucleon mass M and that they change slowly in finite nuclei. This implies that the ratios Φ/M , W/M , $|\nabla\Phi|/M^2$ and $|\nabla W|/M^2$ are useful expansion parameters when the effective field theory is applied to the nuclear many-body problem. From this viewpoint, if all the terms involving scalar and meson self-interactions are retained in the Lagrangian up to fourth order, one

recovers the well-known non-linear $\sigma-\omega$ model plus some additional terms [5,7,9]. For the truncation to be consistent, the corresponding coupling constants should exhibit naturalness and cannot be arbitrarily dropped out without an additional symmetry argument. The effective Lagrangian truncated at fourth order contains thirteen free parameters that have been fitted to reproduce twenty-nine finite nuclei observables [5,9]. Remarkably, the fitted parameters turn out to be natural and the results are not dominated by the last terms retained. This evidence confirms the utility of the principles of naive dimensional analysis and naturalness and shows that truncating the effective Lagrangian at the first lower orders is justified.

The term with a vector-meson quartic self-interaction has been considered previously in relativistic mean field (RMF) calculations from a phenomenological point of view. Bodmer [10] considered this coupling to avoid the negative coefficient of the quartic scalar self-interaction that appears in many non-linear $\sigma-\omega$ parametrizations that correctly describe the atomic nucleus [11]. In some special situations this negative term can lead to a pathological behaviour of the scalar potential. On the other hand, the equation of state is softened at moderate high densities when the vector non-linearity is taken into account. The quartic vector self-interaction has also been phenomenologically used by Gmuca [12,13] in a non-linear $\sigma-\omega$ model for parametrizing Dirac–Brueckner–Hartree–Fock calculations of nuclear matter. The same idea was developed by Toki et al. and applied to study finite nuclei [14] and neutron stars [15]. Recently, the properties of high-density nuclear and neutron matter have been analyzed in the RMF approach taking into account scalar and vector non-linearities [8].

The tensor couplings of the vector ω and ρ mesons to the nucleon were investigated by Reinhard et al. [3,16] as an extension of the RMF model, and more recently by Furnstahl et al. [9,17] from the point of view of relativistic effective field theory. In these works it was shown that the tensor coupling of the ω meson has an important bearing on the nuclear spin–orbit splitting.

The surface properties of nuclei play a crucial role in certain situations. This is the case, for instance, of saddle-point configurations in nuclear fission or fragment distributions

in heavy-ion collisions. Within a context related to the liquid droplet model (LDM) and the leptodermous expansion [18], the surface properties can be extracted from semi-infinite nuclear matter calculations either quantally or semiclassically (though the total curvature energy coefficient can only be computed semiclassically [19]). In the non-relativistic case most of the calculations of the surface properties have been carried out using Skyrme forces, quantally [20] or semiclassically with the help of the extended Thomas–Fermi (ETF) method [20,21]. In the relativistic case the nuclear surface has been analyzed within the σ – ω model since a long time ago. The calculations have been performed semiclassically using the relativistic Thomas–Fermi (TF) method or its extensions (RETF), for symmetric [1,6,22,23,24] and asymmetric [25,26] matter, and also in the quantal Hartree approach [27,28,29].

In the framework of the relativistic model and effective field theory, the main purpose of the present work is to carefully analyze the influence on surface properties of the quartic vector non-linearity, of the newly proposed scalar-vector self-interactions and of the tensor coupling. We shall investigate quantities such as the surface energy coefficient, surface thickness, spin–orbit strength, surface stiffness coefficient and neutron skin thickness obtained from Hartree calculations of symmetric and asymmetric semi-infinite nuclear matter.

The paper is organized as follows. Section 2 is devoted to the basic theory. The results on the surface properties of symmetric matter are discussed in Section 3. Section 4 addresses the case of asymmetric systems. The summary and conclusions are given in the last section.

2 Mean field equations for symmetric semi-infinite nuclear matter

Following Ref. [7], to derive the mean field equations one starts from an energy functional containing Dirac baryons and classical scalar and vector mesons. The energy functional can be obtained from the effective Lagrangian in the Hartree approach using many-body techniques [5,9]. However, this energy functional can also be considered as an expansion in Φ/M , W/M , $|\nabla\Phi|/M^2$ and $|\nabla W|/M^2$ of a general energy density functional that contains all the correlation effects. The theoretical basis of this functional lies on the extension of the Hohenberg–Kohn theorem [30] to QHD [31]. Using the Kohn–Sham scheme [32] with the mean fields playing the role of Kohn–Sham potentials, one finds similar mean field equations to those obtained from the Lagrangian [31], but including effects beyond the Hartree approach through the non-linear couplings [4,7,8,9].

A semi-infinite system of uncharged nucleons corresponds to a one-dimensional geometry where half the space is filled with nuclear matter at saturation and the other half is empty, so that a surface develops around the interface. The fields and densities change only along the direction perpendicular to the medium. Specifying the energy density functional considered in Refs. [5] and [9] to symmetric semi-infinite nuclear matter with the surface normal pointing into the z direction one has

$$\begin{aligned}
\mathcal{E}(z) = & \sum_{\alpha} \varphi_{\alpha}^{\dagger}(z) \left\{ -i\boldsymbol{\alpha} \cdot \nabla + \beta[M - \Phi(z)] + W(z) - \frac{if_v}{2M} \beta\boldsymbol{\alpha} \cdot \nabla W(z) \right\} \varphi_{\alpha}(z) \\
& + \frac{1}{2g_s^2} \left(1 + \alpha_1 \frac{\Phi(z)}{M} \right) (\nabla\Phi(z))^2 + \left(\frac{1}{2} + \frac{\kappa_3}{3!} \frac{\Phi(z)}{M} + \frac{\kappa_4}{4!} \frac{\Phi^2(z)}{M^2} \right) \frac{m_s^2}{g_s^2} \Phi^2(z) \\
& - \frac{1}{2g_v^2} \left(1 + \alpha_2 \frac{\Phi(z)}{M} \right) (\nabla W(z))^2 - \frac{\zeta_0}{4!} \frac{1}{g_v^2} W^4(z) \\
& - \frac{1}{2} \left(1 + \eta_1 \frac{\Phi(z)}{M} + \frac{\eta_2}{2} \frac{\Phi^2(z)}{M^2} \right) \frac{m_v^2}{g_v^2} W^2(z), \tag{2.1}
\end{aligned}$$

where the index α runs over all occupied states of the positive energy spectrum, $\Phi \equiv g_s\phi_0$ and $W \equiv g_v V_0$ (notation as in Ref. [1]). Except for the terms with α_1 and α_2 , the functional (2.1) is of fourth order in the expansion. We retain the fifth-order terms α_1 and α_2 because

in Refs. [5] and [9] they have been estimated to be numerically of the same magnitude as the quartic scalar term in the nuclear surface energy.

The mean field equations are obtained by minimizing with respect to φ_α^\dagger , Φ and W :

$$\left\{ -i\boldsymbol{\alpha}\cdot\nabla + \beta[M - \Phi(z)] + W(z) - \frac{if_v}{2M}\beta\boldsymbol{\alpha}\cdot\nabla W(z) \right\} \varphi_\alpha(z) = \varepsilon_\alpha \varphi_\alpha(z), \quad (2.2)$$

$$\begin{aligned} -\Delta\Phi(z) + m_s^2\Phi(z) &= g_s^2\rho_s(z) - \frac{m_s^2}{M}\Phi^2(z) \left(\frac{\kappa_3}{2} + \frac{\kappa_4}{3!} \frac{\Phi(z)}{M} \right) \\ &+ \frac{g_s^2}{2M} \left(\eta_1 + \eta_2 \frac{\Phi(z)}{M} \right) \frac{m_v^2}{g_v^2} W^2(z) \\ &+ \frac{\alpha_1}{2M} [(\nabla\Phi(z))^2 + 2\Phi(z)\Delta\Phi(z)] + \frac{\alpha_2}{2M} \frac{g_s^2}{g_v^2} (\nabla W(z))^2, \end{aligned} \quad (2.3)$$

$$\begin{aligned} -\Delta W(z) + m_v^2 W(z) &= g_v^2 \left(\rho(z) + \frac{f_v}{2} \rho_T(z) \right) - \left(\eta_1 + \frac{\eta_2}{2} \frac{\Phi(z)}{M} \right) \frac{\Phi(z)}{M} m_v^2 W(z) \\ &- \frac{1}{3!} \zeta_0 W^3(z) + \frac{\alpha_2}{M} [\nabla\Phi(z) \cdot \nabla W(z) + \Phi(z)\Delta W(z)]. \end{aligned} \quad (2.4)$$

The baryon, scalar and tensor densities are respectively

$$\rho(z) = \sum_\alpha \varphi_\alpha^\dagger(z) \varphi_\alpha(z), \quad (2.5)$$

$$\rho_s(z) = \sum_\alpha \varphi_\alpha^\dagger(z) \beta \varphi_\alpha(z), \quad (2.6)$$

$$\rho_T(z) = \sum_\alpha \frac{i}{M} \nabla \cdot [\varphi_\alpha^\dagger(z) \beta \boldsymbol{\alpha} \varphi_\alpha(z)]. \quad (2.7)$$

The expression of the four-component spinors $\varphi_\alpha(z)$ in the semi-infinite medium was given by Hofer and Stocker in Ref. [27].

In a semi-infinite nuclear matter calculation the sum over the single-particle states is replaced by an integration over momenta:

$$\sum_\alpha \longrightarrow 2 \frac{\Omega}{(2\pi)^3} \sum_\lambda \int d\mathbf{k}, \quad (2.8)$$

where Ω stands for the volume of the box, the factor 2 takes into account the isospin degree of freedom and λ describes the spin orientation of the nucleons. Introducing the Fermi

momentum k_F , the integration domain is restricted to $k_x^2 + k_y^2 + k_z^2 = k_\perp^2 + k_z^2 \leq k_F^2$, with $k_z \geq 0$ if the bulk nuclear matter is located at $z = -\infty$. Following the method outlined in Ref. [27] one finds two sets ($\lambda = \pm 1$) of first-order differential equations for the orbital part of the upper and lower components of the Dirac spinors:

$$\frac{dG_a(z)}{dz} - \left[\lambda k_\perp + \frac{f_v}{2M} \frac{dW(z)}{dz} \right] G_a(z) = [\varepsilon_a - W(z) + M^*(z)] F_a(z), \quad (2.9)$$

$$-\frac{dF_a(z)}{dz} - \left[\lambda k_\perp + \frac{f_v}{2M} \frac{dW(z)}{dz} \right] F_a(z) = [\varepsilon_a - W(z) - M^*(z)] G_a(z), \quad (2.10)$$

where $a = (k_z, k_\perp, \lambda)$ and $M^*(z) = M - \Phi(z)$ is the Dirac effective mass of the nucleons. From the asymptotic behaviour at $z = -\infty$ (bulk nuclear matter), the condition on the energy eigenvalues is $\varepsilon_a = \sqrt{k_\perp^2 + k_z^2 + M_\infty^{*2}} + W_\infty$, with M_∞^* and W_∞ being the nuclear matter values of M^* and W . The densities for each spin orientation $\lambda = \pm 1$ read

$$\rho^\lambda(z) = \frac{2}{\pi^2} \int_0^{k_F} dk_z \int_0^{\sqrt{k_F^2 - k_z^2}} dk_\perp k_\perp (|G_a(z)|^2 + |F_a(z)|^2), \quad (2.11)$$

$$\rho_s^\lambda(z) = \frac{2}{\pi^2} \int_0^{k_F} dk_z \int_0^{\sqrt{k_F^2 - k_z^2}} dk_\perp k_\perp (|G_a(z)|^2 - |F_a(z)|^2), \quad (2.12)$$

$$\rho_T^\lambda(z) = \frac{2}{\pi^2} \int_0^{k_F} dk_z \int_0^{\sqrt{k_F^2 - k_z^2}} dk_\perp k_\perp \frac{d}{dz} \left(\frac{2}{M} F_a(z) G_a(z) \right), \quad (2.13)$$

and the total densities are given by

$$\rho(z) = \sum_\lambda \rho^\lambda(z), \quad \rho_s(z) = \sum_\lambda \rho_s^\lambda(z), \quad \rho_T(z) = \sum_\lambda \rho_T^\lambda(z). \quad (2.14)$$

Using the equations of motion the energy density of the semi-infinite nuclear matter system can be written as follows:

$$\begin{aligned} \mathcal{E}(z) &= \frac{2}{\pi^2} \sum_\lambda \int_0^{k_F} dk_z \int_0^{\sqrt{k_F^2 - k_z^2}} dk_\perp k_\perp \left(\sqrt{k_\perp^2 + k_z^2 + M_\infty^{*2}} + W_\infty \right) (|G_a(z)|^2 + |F_a(z)|^2) \\ &+ \frac{1}{2} \Phi(z) \rho_s(z) - \frac{1}{2} W(z) \left(\rho(z) + \frac{f_v}{2} \rho_T(z) \right) - \frac{\Phi(z)}{4M} \left(\frac{\kappa_3}{3} + \frac{\kappa_4}{6} \frac{\Phi(z)}{M} \right) \frac{m_s^2}{g_s^2} \Phi^2(z) \\ &+ \frac{\Phi(z)}{4M} \left(\eta_1 + \eta_2 \frac{\Phi(z)}{M} \right) \frac{m_v^2}{g_v^2} W^2(z) + \frac{\zeta_0}{4!} \frac{1}{g_v^2} W^4(z) \\ &- \frac{\alpha_1}{4g_s^2} \frac{\Phi(z)}{M} (\nabla \Phi(z))^2 + \frac{\alpha_2}{4g_v^2} \frac{\Phi(z)}{M} (\nabla W(z))^2. \end{aligned} \quad (2.15)$$

Finally, the surface energy coefficient E_s is obtained from the expression [18]

$$E_s = 4\pi r_0^2 \int_{-\infty}^{\infty} dz [\mathcal{E}(z) - (a_v + M)\rho(z)], \quad (2.16)$$

where a_v is the energy per particle in bulk nuclear matter and r_0 is the nuclear radius constant: $r_0 = (3/4\pi\rho_0)^{1/3}$, with ρ_0 the nuclear matter density.

Another important quantity in the study of the nuclear surface structure is the spin-orbit interaction. By elimination of the lower spinor in terms of the upper spinor, one obtains a Schrödinger-type equation with a term $V_{so}(z)$ that has the structure of the single-particle spin-orbit potential for the non-relativistic case [27,29]. In our present model the orbital part of the spin-orbit potential reads as

$$V_{so}(z) = \frac{1}{2M} \left[\frac{1}{\varepsilon_a - W(z) + M^*(z)} \left(\frac{dW(z)}{dz} + \frac{d\Phi(z)}{dz} \right) + \frac{f_v}{M} \frac{dW(z)}{dz} \right]. \quad (2.17)$$

In the non-relativistic limit, by means of a Foldy-Wouthuysen reduction, Eq. (2.17) becomes

$$V_{so}^{FW}(z) = \frac{1}{4M^2} \left[(1 + 2f_v) \frac{dW(z)}{dz} + \frac{d\Phi(z)}{dz} \right] \quad (2.18)$$

and the nucleons are then moving in a central potential of the form

$$V_c(z) = W(z) - \Phi(z). \quad (2.19)$$

3 Surface properties in the symmetric case

Although previous works [23,24,25,26,27,28,29] have thoroughly investigated the properties of the nuclear surface in the standard non-linear $\sigma-\omega$ model, for which κ_3 and κ_4 are the only non-linearities in (2.1), here we wish to enlarge this study by including the additional non-linear and tensor couplings considered in Refs. [5,7,9]. In concrete, we want to study the role of the quartic vector self-interaction ζ_0 that has been used after the work of Bodmer [10], the role of the terms with η_1 and η_2 that couple the scalar and vector fields, of the terms with α_1 and α_2 that imply the gradients of the fields, and of the tensor coupling f_v of the vector ω meson to the nucleon. While ζ_0 , η_1 and η_2 can be classified as volume contributions, the couplings α_1 , α_2 and f_v are genuine surface terms. On the basis of the concept of naturalness there is no reason to omit any of these terms in the energy density functional (2.1), unless there exists a symmetry principle to forbid it. However, to clarify the impact on the surface properties of the aforementioned couplings we will analyze each one separately, as it has been similarly done in Ref. [7]. In this section we shall study symmetric systems, while in Section 4 we shall address the case of asymmetric matter.

3.1 Effect of the quartic vector self-interaction

In the conventional non-linear $\sigma-\omega$ model the value of the coefficients g_s^2/m_s^2 , g_v^2/m_v^2 , κ_3 and κ_4 can be univocally obtained by imposing that for nuclear matter at saturation the density ρ_0 , energy per particle a_v , effective mass M_∞^*/M and incompressibility modulus K take given values. When the vector-meson quartic self-interaction is switched on, the Dirac equation for the baryons and the Klein–Gordon equation for the vector field in infinite nuclear matter become (throughout this subsection we set $\eta_1 = \eta_2 = \alpha_1 = \alpha_2 = f_v = 0$):

$$a_v = \sqrt{k_F^2 + M_\infty^{*2}} + W_\infty - M, \quad (3.1)$$

$$m_v^2 W_\infty = g_v^2 \rho_0 - \frac{1}{6} \zeta_0 W_\infty^3. \quad (3.2)$$

The saturation density ρ_0 and the Fermi momentum k_F are related as usual by $\rho_0 = 2k_F^3/3\pi^2$. Specifying ρ_0 , a_v and M_∞^*/M , from the above equations one extracts the coupling constant

g_v as a function of ζ_0 (the nucleon and ω masses take their empirical values: $M = 939$ MeV and $m_v = 783$ MeV). The steps to calculate g_s^2/m_s^2 , κ_3 and κ_4 are then the same as when $\zeta_0 = 0$, see e.g. Refs. [10] and [11], but now these coefficients become functions of ζ_0 . The reader will find a detailed study of the implications of the vector non-linearity ζ_0 in nuclear matter in Refs. [7] and [10].

The assumption of naturalness requires that the couplings $g_s/4\pi$, $g_v/4\pi$, κ_3 , κ_4 and ζ_0 should all be roughly of the order of unity. Figure 1 illustrates the variation of these couplings as a function of the non-dimensional parameter

$$\eta_0 = \frac{m_v^2}{g_v^2} \sqrt{\frac{6m_v^2}{\zeta_0 \rho_0^2}} \quad (3.3)$$

used by Bodmer¹ in Ref. [10]. With $m_s = 490$ MeV, the figure presents the results for four sets of nuclear matter properties: $\rho_0 = 0.152$ fm⁻³ ($k_F = 1.31$ fm⁻¹), $a_v = -16.42$ MeV, $K = 200$ and 350 MeV, and $M_\infty^*/M = 0.6$ and 0.7 . When $K = 200$ MeV and $M_\infty^*/M = 0.6$ the equilibrium properties of the interaction and the scalar mass m_s are very close to those of the non-linear parametrization NL1 [33].

We realize that g_s and g_v are only weakly affected by η_0 (the change is not appreciable in the scale of Figure 1). However, η_0 has a direct effect on the scalar-meson quartic self-interaction κ_4 , moving it from a negative value when the quartic vector term is absent ($\eta_0 = \infty$) to a desirable positive value when $\eta_0 \sim 2$. The change of sign of κ_4 takes place at larger values of η_0 for larger K and M_∞^*/M ; in fact, κ_4 is already positive at $\eta_0 = \infty$ if $K = 350$ MeV and $M_\infty^*/M = 0.7$. Both non-linearities of the scalar field κ_3 and κ_4 remain in the natural zone for $2 \leq \eta_0 \leq \infty$ approximately, excepting the case of $K = 350$ MeV and $M_\infty^*/M = 0.7$, but they start to depart appreciably from their natural values when $\eta_0 \leq 2$. These trends fairly agree with the assumption of naturalness: $\eta_0 \geq 2$ corresponds to the region where ζ_0 can be considered as natural (see the figure), and for this range of η_0 values also the rest of coupling constants are natural. The behaviour gleaned from Figure 1 is rather independent of the saturation density ρ_0 and energy a_v , and e.g. we have found similar trends with the specific nuclear matter properties used by Bodmer in Ref. [10].

¹Notice that in Ref. [10] the parameter η_0 was called z .

Next we turn our attention to the surface properties. In Figures 2 and 3 we have plotted, respectively, the surface energy coefficient E_s and the surface thickness t of the semi-infinite density profile (the 90%–10% fall-off distance) against the η_0 parameter. The selected values of η_0 are those employed in Table 1 of Ref. [10]. With $\rho_0 = 0.152 \text{ fm}^{-3}$ and $a_v = -16.42 \text{ MeV}$, we have performed the calculations for a few incompressibilities ($K = 125, 200$ and 350 MeV) and effective masses ($M_\infty^*/M = 0.6$ and 0.7). Furthermore, we have considered two values of the mass of the scalar meson ($m_s = 490$ and 525 MeV). This quantity governs the range of the attractive interaction and determines the surface fall-off: a larger m_s results in a steeper surface and a reduction of E_s and t .

From Figures 2 and 3 we realize that the quartic vector self-interaction scarcely alters the values of the surface energy coefficient and of the surface thickness if ζ_0 remains in the natural domain, i.e., if $2 \leq \eta_0 \leq \infty$. At $K = 350 \text{ MeV}$ and $M^*/M \geq 0.6$, E_s is raised by decreasing η_0 . This tendency may be inverted at smaller values of the incompressibility, depending also on the value of the nucleon effective mass, but the global trends are practically independent of the mass of the scalar meson. The thickness t exhibits a more monotonous behaviour: in all cases it stays almost equal to its value at $\eta_0 = \infty$ and goes down slightly for $\eta_0 \leq 2$. Noticeable departures of the surface energy coefficient and thickness from the $\eta_0 = \infty$ values can be found only if η_0 is decreased beyond the natural limit, a situation where the interaction is mainly ruled by the vector-meson quartic self-interaction.

Even though the impact of the vector-meson quartic self-interaction is small, it can help to find parameter sets for which both the surface energy coefficient and the surface thickness lie in the empirical region. This fact is illustrated in Figure 4, where E_s and t are drawn versus M_∞^*/M for $\eta_0 = \infty, 2$ and 0.5 and for $m_s = 450, 500$ and 550 MeV , using the nuclear matter conditions of Ref. [10]: $\rho_0 = 0.1484 \text{ fm}^{-3}$, $a_v = -15.75 \text{ MeV}$ and $K = 200 \text{ MeV}$. (We also performed the calculations for $\eta_0 = 5$ and $\eta_0 = 1$ to confirm the trends we discuss below.) The horizontal dashed lines in Figure 4 serve to indicate the empirical region for the surface energy and thickness.

If at $\eta_0 = \infty$ we concentrate, for instance, on the parametrizations with $m_s = 450 \text{ MeV}$ we see that the one with $M_\infty^*/M = 0.7$ yields, simultaneously, E_s and t within the empirical

region. The agreement between the calculated values with $m_s = 450$ MeV and the empirical region improves for $\eta_0 = 2$, where both E_s and t are acceptable for practically all the values of the effective mass considered.

For $\eta_0 \geq 2$ the dependence of E_s and t upon the nucleon effective mass is similar to that found in the usual σ - ω model without a quartic vector self-interaction [23,28]. The general tendencies start to change when η_0 is lowered and leaves the natural region. As η_0 becomes smaller Figure 4 shows that the slope of the curves of E_s and t as a function of M_∞^*/M changes, and that the curves for the different m_s come closer together.

To get more insight about the influence of the vector-meson quartic self-interaction, we display in Figure 5 the profiles of the baryon density $\rho(z)$ and of the surface tension density (Swiatecki integrand) $\sigma(z) = \mathcal{E}(z) - (a_v + M)\rho(z)$, Eq. (2.16). In turn, we have represented in Figure 6 the orbital part of the spin-orbit potential $V_{so}(z)$, Eq. (2.17), at the Fermi surface (i.e., evaluated at $k = k_F$) and the central mean field $V_c(z)$ defined in Eq. (2.19). The properties of the interactions used in these figures are $\rho_0 = 0.152$ fm⁻³, $a_v = -16.42$ MeV, $K = 200$ MeV, $M_\infty^*/M = 0.6$ and 0.7 , and $m_s = 490$ MeV. Results are shown for $\eta_0 = \infty, 2$ and 0.5 . The corresponding values of the surface energy coefficient and surface thickness can be read from Figures 2 and 3.

It can be seen that the local quantities depicted in Figures 5 and 6 oscillate as functions of z (Friedel oscillations). Both the surface tension density and the single-particle spin-orbit potential are confined to the surface and average to zero inwards as the bulk matter is approached. The local profiles for $\eta_0 = 2$, which somehow marks the limit of naturalness as we have commented, are almost equal to those obtained in the absence of the quartic vector self-interaction. In agreement with Figures 2 and 3, only when η_0 is decreased to non-natural values one can notice some changes in the profiles, which are more visible for $M_\infty^*/M = 0.6$ than for $M_\infty^*/M = 0.7$. Decreasing η_0 makes the surface steeper and the thickness t smaller, produces an enhancement in the surface region of the density $\rho(z)$ and of the mean field $V_c(z)$, and builds up Friedel oscillations in $\sigma(z)$ and in the spin-orbit potential $V_{so}(z)$.

It is well known that the experimental spin-orbit splittings require within narrow bounds a Dirac effective mass M_∞^*/M around 0.6 in the conventional relativistic model [11,7]. Figure

6 shows that introducing a quartic vector self-interaction makes the spin-orbit well deeper. However, it is only a minor effect: with $M_\infty^*/M = 0.7$ it is not possible to reproduce the spin-orbit interaction of the case $M_\infty^*/M = 0.6$ at $\eta_0 = \infty$, not even if one sets $\eta_0 = 0.5$ (which in addition brings about an unreasonably small t , see Figure 3). A similar conclusion was drawn in Ref. [10] from an analysis in nuclear matter. We also have computed the non-relativistic limit $V_{\text{so}}^{\text{FW}}(z)$ of the spin-orbit potential given by the expression (2.18). In agreement with Ref. [29] we have found that while $V_{\text{so}}^{\text{FW}}(z)$ qualitatively reproduces the behaviour of $V_{\text{so}}(z)$, it strongly underestimates the quantitative depth of the fully relativistic spin-orbit strength (by $\sim 20\%$ for $M_\infty^*/M = 0.6$ and by $\sim 15\%$ for $M_\infty^*/M = 0.7$).

3.2 Influence of the volume cubic and quartic scalar-vector interactions

The next bulk terms in the energy density (2.1) that contain non-linear meson interactions are

$$-\frac{1}{2} \frac{\Phi}{M} \left(\eta_1 + \frac{\eta_2}{2} \frac{\Phi}{M} \right) \frac{m_v^2}{g_v^2} W^2. \quad (3.4)$$

In analogy to Figure 1 for η_0 , in Figure 7 we study the change of the couplings $g_s/4\pi$, $g_v/4\pi$, κ_3 and κ_4 with the parameters η_1 and η_2 (introducing each one separately) for some specific equilibrium properties. With $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42$ and $m_s = 490 \text{ MeV}$, in part (a) of Figure 7 it is $K = 200 \text{ MeV}$ and $M_\infty^*/M = 0.6$, while in part (b) it is $K = 350 \text{ MeV}$ and $M_\infty^*/M = 0.7$.

For $K = 200 \text{ MeV}$ and $M_\infty^*/M = 0.6$ both η_1 and η_2 have a considerable effect on the scalar non-linearities κ_3 and κ_4 . The coupling κ_4 changes sign for $\eta_2 \approx 1$, but it remains negative in the interval of η_1 values used. To keep all the coupling constants within natural values we see that the range for η_1 and η_2 (when introduced separately) is restricted to run roughly from -0.5 to 2.5 . Consider now different values of the incompressibility and effective mass, as in part (b) of Figure 7. The dependence of the couplings on η_2 is not significantly altered. However, increasing either K or M_∞^*/M results in a smoother slope of κ_3 with η_1 , while it makes κ_4 grow steadily with η_1 and become positive at some value of this parameter. We have checked for $K = 200 \text{ MeV}$ and $M_\infty^*/M = 0.55$, and for $K = 125$

MeV and $M_\infty^*/M = 0.6$, that κ_4 remains negative at all values of η_1 and that κ_3 grows with η_1 faster than in part (a) of Figure 7.

The influence of the non-linear interactions η_1 and η_2 on the surface properties is analyzed in Figure 8. With $\zeta_0 = \alpha_1 = \alpha_2 = f_v = 0$, in this figure we have computed E_s and t taking into account the terms (3.4). The saturation conditions of the interaction and the scalar mass are the same as in part (a) of Figure 7. The results are displayed in the plane η_1 - η_2 in the form of contour plots of constant E_s (solid lines) and of constant t (dashed lines). The range of variation of η_1 and η_2 lies in the region imposed by naturalness, and yields values of E_s and t within reasonable limits.

As it can be inferred from the nearly vertical lines in the η_1 - η_2 plane, the surface energy coefficient and thickness depend mostly on η_1 and are rather independent of η_2 . The consequence of increasing η_1 is a reduction of the values of E_s and t . The lines of constant t turn out to be, roughly speaking, parallel to the lines of constant E_s . This means that from the interplay of the parameters η_1 and η_2 it is not possible to change the value of t relative to that of E_s (for example, we see in Figure 8 that $t \sim 2.2$ fm if $E_s = 18$ MeV). We have calculated the spin-orbit potential $V_{so}(z)$ at the Fermi surface for several values of η_1 and η_2 . We have found that these couplings have a marginal effect on the spin-orbit strength, as it happened to be the case with the other bulk non-linearity η_0 .

To get some information about the incidence on the surface energy and thickness of all the volume non-linear meson interactions together, we have repeated the calculations in the η_1 - η_2 plane setting $\eta_0 = 2$ for the quartic vector self-interaction. One finds similar features to those of Figure 8. The effect of $\eta_0 = 2$ is just shifting E_s and t towards smaller values as compared with the case $\eta_0 = \infty$ ($\zeta_0 = 0$), which is in accordance with what was found in Figures 2 and 3 at $K = 200$ MeV and $M_\infty^*/M = 0.6$.

3.3 Influence of the non-linear terms with gradients

Now we discuss the non-linear interactions

$$\frac{1}{2} \frac{\Phi}{M} \left[\frac{\alpha_1}{g_s^2} (\nabla\Phi)^2 - \frac{\alpha_2}{g_v^2} (\nabla W)^2 \right] \quad (3.5)$$

that vanish in infinite nuclear matter. We recall that these terms are actually of order 5 in the expansion of the effective Lagrangian but, following Refs. [5] and [9], we include them because they can be relevant in the surface due to their gradient structure.

Using the same saturation properties and scalar mass of Figure 8, in Figure 9 we have calculated E_s and t for several values of α_1 and α_2 with $\zeta_0 = \eta_1 = \eta_2 = f_v = 0$. One observes that the curves of constant E_s are projected onto the plane α_1 - α_2 as almost parallel straight lines (at least in the analyzed region, corresponding to natural values of α_1 and α_2). The same happens to the curves of constant t . But in contrast with the situation found in the plane η_1 - η_2 (Figure 8), the slope of the lines of constant t is different from that of the lines of constant E_s . This means that by varying α_1 and α_2 one can achieve some modification on the surface thickness while keeping the same surface energy. For example, if we consider the contour line of $E_s = 18$ MeV we find that for $\alpha_2 = 2.0$ it is $t \sim 2.05$ fm, whereas for $\alpha_2 = -1.5$ it is $t \sim 2.25$ fm. From Figure 9 we also see that increasing α_1 at constant α_2 brings about larger values of E_s and t , and that the opposite happens if one increases α_2 at constant α_1 .

We have repeated the calculations of Figure 9 ($K = 200$ MeV, $M_\infty^*/M = 0.6$) for $K = 350$ MeV and for $M_\infty^*/M = 0.7$, to verify to which extent the behaviour in the α_1 - α_2 plane is affected by the incompressibility and effective mass of the interaction. Certainly, the contour lines of E_s and t are shifted with respect to Figure 9, but the trends with α_1 and α_2 turn out to be qualitatively the same. The range of variation of the surface energy and thickness in the α_1 - α_2 region we are considering is shorter when $M_\infty^*/M = 0.7$, while it is more or less the same when $K = 350$ MeV.

To assess the importance of the bulk non-linear meson interactions on our study on α_1 and α_2 , we have performed calculations as in Figure 9 but setting $\eta_0 = 2$ with $\eta_1 = \eta_2 = 0$, and setting $\eta_1 = 1$ with $\eta_0 = \infty$ and $\eta_2 = 0$ (as indicated, the effect of η_2 is much smaller than that of η_1). The results show a completely similar behaviour to Figure 9. Even the slope of the contour lines of E_s and t in the α_1 - α_2 plane changes only slightly. Comparing with Figure 9, when $\eta_0 = 2$ one finds that E_s is shifted by approximately -1 MeV, and that when $\eta_1 = 1$ then E_s is shifted by around -3 MeV. The shifts of the surface thickness t are

less regular and their magnitude depends on the value of α_1 and α_2 .

In order to investigate the impact of the gradient interactions α_1 and α_2 on the spin-orbit potential, in Figure 10 we have plotted $V_{\text{so}}(z)$ at the Fermi surface for a few selected values of α_1 and α_2 . The nuclear matter properties and the scalar mass are the same as in Figure 6, where we studied the dependence of $V_{\text{so}}(z)$ on η_0 . One can see that the meson interaction with $\alpha_1 = 1$ and $\alpha_2 = 0$ reduces the strength of $V_{\text{so}}(z)$ and shifts the position of the minimum slightly to the exterior. On the contrary, the interaction with $\alpha_1 = 0$ and $\alpha_2 = 1$ makes the potential well deeper. The combined effect is probed in the case $\alpha_1 = \alpha_2 = 1$. Since in the relativistic model the spin-orbit force is strongly correlated with the Dirac effective mass, we compare in Figure 10 the situation at $M_\infty^*/M = 0.6$ and at $M_\infty^*/M = 0.7$. We realize that the incidence of α_1 on $V_{\text{so}}(z)$ is weaker for $M_\infty^*/M = 0.7$. The small perturbations arising from the gradient interactions when $M_\infty^*/M = 0.7$ are not sufficient to produce a spin-orbit strength equivalent to that of the case $M_\infty^*/M = 0.6$.

3.4 Role of the tensor coupling of the omega meson

To conclude this section we investigate the influence of the ω tensor coupling

$$\sum_{\alpha} \varphi_{\alpha}^{\dagger}(z) \left[-\frac{if_v}{2M} \beta \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} W(z) \right] \varphi_{\alpha}(z) \quad (3.6)$$

which adds some momentum and spin dependence to the interaction. The natural combination for this coupling is $f_v/4$.

Well known from one-boson-exchange potentials (where f_v above is commonly written as f_v/g_v), the tensor coupling was included in the fits to nuclear properties of Refs. [3,16] (conventional QHD) and [5,9] (effective field theory), and in the study of the nuclear spin-orbit force in chiral effective field theories carried out in Ref. [17]. These works noticed the existence of a trade-off between the size of the ω tensor coupling and the size of the scalar field. In other words, the tensor coupling breaks the tight connection existing in relativistic models between the value of the nucleon effective mass at saturation and the empirical spin-orbit splitting in finite nuclei (which constrains M_∞^*/M to lie between 0.58 and 0.64 [7]). Including a tensor coupling the authors of Refs. [5,9,17] were able to obtain natural

parameter sets that provide excellent fits to nuclear properties and spin-orbit splittings with an equilibrium effective mass remarkably higher ($M_\infty^*/M \sim 0.7$) than in models that ignore such coupling. We want to analyze the nature of this effect in the simpler but more transparent framework of semi-infinite nuclear matter.

In Figure 11 we have drawn the surface energy coefficient and the surface thickness as functions of f_v in the range $[-0.6, 0.9]$ for two values of the effective mass and of the incompressibility, having set $m_s = 490$ MeV, $\rho_0 = 0.152$ fm $^{-3}$ and $a_v = -16.42$ MeV. To exemplify the incidence of f_v on the spin-orbit potential, Figure 12 displays $V_{so}(z)$ at the Fermi surface for a few of the cases of Figure 11. We also performed the calculations for $m_s = 525$ MeV: E_s and t are shifted downwards with respect to Figure 11 and $V_{so}(z)$ is deeper than in Figure 12, but the global trends with f_v are the same.

Figure 11 shows the strong reduction of E_s and t as f_v increases (the slope of the curves is milder for $M_\infty^*/M = 0.7$ than for $M_\infty^*/M = 0.6$). Figure 12 reveals that this fact is associated with a deeper and wider spin-orbit potential. This agrees with the results of Hofer and Stocker [27] who showed in the standard RMF model that the spin-orbit coupling reduces the surface energy and thickness. At variance with the individual values of E_s and t , the ratio E_s/t stays to a certain extent constant with f_v .

Figure 12 evinces the sensitivity of $V_{so}(z)$ to f_v . The lower the nucleon effective mass is, the larger the effect. For $M_\infty^*/M = 0.7$ we realize that with positive values of f_v (~ 0.3 in the present case) one can get a spin-orbit strength comparable, or even stronger, to that of the case $M_\infty^*/M = 0.6$ and $f_v = 0$, something that could not be achieved with natural values of the couplings studied in the previous sections. Since our parametrization with $M_\infty^*/M = 0.7$ and $K = 200$ MeV at $f_v = 0$ already has reasonable surface energy and thickness ($E_s = 16.6$ MeV and $t = 1.97$ fm), increasing f_v results in smaller values of E_s and t . This should be compensated with the other couplings (especially α_1 and α_2) that modify the spin-orbit strength to a lesser degree than f_v , or the starting point should have other values of the incompressibility K and the scalar mass m_s .

The spin-orbit effect has to do with the explicit dependence of the nucleon orbital wave functions on the spin orientation λ . As described in Ref. [27] nucleons with $\lambda = +1$ feel

an attractive spin-orbit potential and are pushed to the exterior of the surface, whereas the spin-orbit force is repulsive for nucleons with $\lambda = -1$ which are pushed to the interior. As a consequence of this a depletion of particles with $\lambda = -1$ occurs at the surface. This behaviour is contrasted in Figure 13 for $f_v = 0$ and $f_v = 0.6$ in the case $M_\infty^*/M = 0.7$. The figure depicts the profiles of the total baryon and tensor densities as well as those of their spin components $\rho^\lambda(z)$ and $\rho_T^\lambda(z)$ for $\lambda = \pm 1$, Eqs. (2.11)–(2.14). When the spin-orbit strength is large, attraction dominates over repulsion and more particles accumulate at the surface than particles are removed from it. Then the total baryon density is enhanced at the surface region and it falls down more steeply.

4 Surface properties in the asymmetric case

We briefly recall some basic definitions concerning nuclear surface symmetry properties (further details on the relativistic treatment of asymmetric infinite and semi-infinite nuclear matter can be found in Refs. [25,26,29]). For a bulk neutron excess $\delta_0 = (\rho_{n0} - \rho_{p0}) / (\rho_{n0} + \rho_{p0})$ (i.e., the asymptotic asymmetry far from the surface), a surface energy coefficient can be computed as

$$E_s(\delta_0) = 4\pi r_0^2 \int_{-\infty}^{\infty} dz [\mathcal{E}(z) - (a_v(\delta_0) + M) \rho(z)], \quad (4.1)$$

where $\mathcal{E}(z)$ is the total energy density of the system of neutrons and protons, $a_v(\delta_0)$ denotes the energy per particle in nuclear matter of asymmetry δ_0 , and $\rho(z) = \rho_n(z) + \rho_p(z)$ with ρ_n and ρ_p referring to the neutron and proton densities, respectively. According to the liquid droplet model (LDM) [18], for small values of the neutron excess $E_s(\delta_0)$ can be expanded as follows:

$$E_s(\delta_0) = E_s + \left(\frac{9J^2}{4Q} + \frac{2E_s L}{K} \right) \delta_0^2 + \dots \quad (4.2)$$

In this equation J stands for the bulk symmetry energy coefficient, L reads for the LDM coefficient that expresses the density dependence of the symmetry energy, and Q is the so-called surface stiffness coefficient that measures the resistance of the system against pulling the neutron and proton surfaces apart. All of these macroscopic coefficients are familiar from semi-empirical LDM mass formulae.

Another quantity of interest is the neutron skin thickness Θ , namely the separation between the neutron and proton surface locations:

$$\Theta = \int_{-\infty}^{\infty} dz [\rho_n(z) / \rho_{n0} - \rho_p(z) / \rho_{p0}]. \quad (4.3)$$

In finite nuclei Θ would correspond to the difference between the equivalent sharp radii of the neutron and proton distributions. In the small asymmetry limit the LDM predicts a linear behaviour of Θ with δ_0 :

$$\Theta = \frac{3r_0}{2} \frac{J}{Q} \delta_0. \quad (4.4)$$

For calculations of finite nuclei of small overall asymmetry $I = (N - Z)/A$, the LDM

expansion of the energy can be written as

$$E = (a_v + JI^2) A + \left[E_s - \left(\frac{9J^2}{4Q} - \frac{2E_s L}{K} \right) I^2 \right] A^{2/3} + a_C Z^2 A^{-1/3} + \dots, \quad (4.5)$$

where a_C is the Coulomb energy coefficient. Notice that $I \neq \delta_0$ in finite nuclei.

To describe asymmetric matter in the relativistic approach we need to generalize the energy density (2.1) by including the isovector ρ meson. In terms of the mean field $R = g_\rho b_0$, with b_0 the time-like neutral component of the ρ -meson field, the additional contributions to Eq. (2.1) read

$$\begin{aligned} & \sum_\alpha \varphi_\alpha^\dagger(z) \left[-\frac{if_\rho}{4M} \tau_3 \beta \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} R(z) \right] \varphi_\alpha(z) + \frac{1}{2} R(z) [\rho_p(z) - \rho_n(z)] \\ & - \frac{1}{2g_\rho^2} (\boldsymbol{\nabla} R(z))^2 - \frac{1}{2} \left(1 + \eta_\rho \frac{\Phi(z)}{M} \right) \frac{m_\rho^2}{g_\rho^2} R^2(z). \end{aligned} \quad (4.6)$$

The symmetry energy coefficient turns out to be

$$J = \frac{k_F^2}{6(k_F^2 + M_\infty^{*2})^{1/2}} + \frac{g_\rho^2 k_F^3}{12\pi^2 m_\rho^2} \frac{1}{1 + \eta_\rho(1 - M_\infty^*/M)}. \quad (4.7)$$

In the conventional model one has $f_\rho = \eta_\rho = 0$. The isovector tensor coupling f_ρ was included in the calculations of Refs. [3,5,9,16]. The new non-linear coupling η_ρ between the ρ - and σ -meson fields is of order 3 in the expansion and it has been introduced in Refs. [5,9]. We will not consider higher-order non-linear couplings involving the ρ meson since the expectation value of the ρ field is typically an order of magnitude smaller than that of the ω field [5,9]. For example, in calculations of the high-density nuclear equation of state, Müller and Serot [8] found the effects of a quartic ρ meson coupling (R^4) to be only appreciable in stars made of pure neutron matter. On the other hand, in analogy to the couplings α_1 and α_2 for the σ and ω fields, we also tested a surface contribution $-\alpha_3 \Phi (\boldsymbol{\nabla} R)^2 / (2g_\rho^2 M)$ and found that the impact it has on the properties we will study in this section is absolutely negligible.

As we have seen, the quantity that governs the surface properties in the regime of low asymmetries is the surface stiffness Q . Table 1 analyzes the effect on Q and L of the couplings discussed in the preceding sections and of the f_ρ and η_ρ parameters. On the basis of Eq. (4.4), we have extracted Q from a linear regression in δ_0 to fit our results for Θ up

to $\delta_0 = 0.1$. We have set the equilibrium properties to $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$, $K = 200 \text{ MeV}$, $M_\infty^*/M = 0.6$ and $J = 30 \text{ MeV}$, and have used a scalar mass $m_s = 490 \text{ MeV}$ and a ρ -meson mass $m_\rho = 763 \text{ MeV}$. Though here we are interested in tendencies rather than in absolute values, for comparison we mention that NL1 has (units in MeV) $J = 43.5$, $L = 140$ and $Q = 27$ [29], the sophisticated droplet-model mass formula FRDM [34] implies $J = 33$, $L = 0$ and $Q = 29$, and the ETFSI-1 mass formula [35] based on microscopic forces predicts $J = 27$, $L = -9$ and $Q = 112$.

Table 1 shows that the influence on the surface stiffness of the volume self-interactions η_0 , η_1 and η_2 is not very large for natural values of these couplings. In the present case Q is slightly increased by decreasing η_0 (i.e., by increasing the quartic vector coupling ζ_0). For $\eta_1 = 1$ we find a non-negligible increase of Q , which signals a larger rigidity of the nuclear system against the separation of the neutron and proton surfaces. The effect of η_2 is again moderate as compared to that of η_1 . Q is augmented by a positive η_ρ coupling, while a negative η_ρ induces a lower value of Q . Some visible changes in Q take place when the α_1 and α_2 gradient interactions are taken into account. Due to the opposite behaviour of Q with α_1 and α_2 , the tendencies compensate in a case like $\alpha_1 = \alpha_2 = 1$, but the net effect is reinforced e.g. if $\alpha_1 = -\alpha_2 = 1$.

As one could expect the isoscalar tensor coupling f_v has a notable effect on Q , even for the relatively small value $f_v = 0.3$ that we have used in Table 1. On the contrary, Q is virtually insensitive to the isovector tensor coupling f_ρ . The reason is that the derivative of the $R(z)$ field is much smaller than that of the $W(z)$ field. In the least-square fits to ground-state properties of Refs. [3,16] nothing was gained by the ρ tensor coupling. In any case, the best fits of Refs. [5,9] have $f_\rho \approx 4$.

From Table 1 we recognize that the main changes in the coefficient L arise from the η_ρ coupling. As a rule of thumb, increasing values of Q are associated with decreasing values of L for the bulk couplings η_0 , η_1 , η_2 and η_ρ . Since L is a bulk quantity, it is not modified by the surface interactions.

Figures 14 and 15 illustrate the dependence on asymmetry of the neutron skin thickness, surface energy and surface thickness for some of the cases considered in Table 1. The figures

extend up to $\delta_0 = 0.3$, which widely covers the range relevant for laboratory nuclei ($\delta_0 \leq 0.2$). As the system becomes neutron rich we can appreciate how a neutron skin develops and Θ grows from its vanishing value at $\delta_0 = 0$. For small asymmetries the growth is linear in δ_0 , as predicted by the LDM. The surface energy coefficient grows quadratically with increasing neutron excess and the LDM equation (4.2) is clearly a good approximation. In general, the interactions having thicker neutron skins (smaller values of Q) also have larger surface energies.

The parameter η_ρ can be used for the fine tuning of the symmetry properties of the interaction without spoiling the predictions for symmetric systems. If in the conventional ansatz g_ρ is fixed by the value of the symmetry energy J , in the extended model J depends on a combination of g_ρ and η_ρ , Eq. (4.7). Therefore, η_ρ provides in practice a mechanism that can help to simultaneously adjust Q (to get the required neutron skin Θ) and J (to keep the fit to the masses) preserving the symmetric surface properties.

5 Summary

Within relativistic mean field theory, we have investigated the influence on nuclear surface properties of the non-linear meson interactions and tensor couplings recently considered in the literature. These interactions, beyond standard QHD, are based on effective field theories. The effective field theory approach allows one to expand the non-renormalizable couplings, which are consistent with the underlying QCD symmetries, using naive dimensional analysis and the naturalness assumption [5,7,8,9].

The quartic vector self-interaction ζ_0 makes it possible to obtain a desirable positive value of the coupling constant κ_4 of the quartic scalar self-interaction, for realistic nuclear matter properties and within the bounds of naturalness. This ζ_0 coupling has only a slight impact on the surface properties. Nevertheless, it helps to find parametrizations where both the surface energy coefficient E_s and the surface thickness t lie in the empirical region. The ζ_0 vector non-linearity makes the spin-orbit potential well deeper, although the effect is almost negligible. Concerning the volume non-linear couplings η_1 and η_2 , they also allow one to obtain positive values of κ_4 in the region of naturalness, depending somewhat on the saturation properties (incompressibility and effective mass). The surface properties are not much affected by these bulk terms either, and it turns out that η_2 has a marginal effect as compared to that of η_1 .

The equilibrium properties do not depend on the couplings α_1 and α_2 that involve the gradients of the fields. Thus, these couplings serve to improve the quality of the surface properties without changing the bulk matter. In the conventional $\sigma-\omega$ model the only parameter not fixed by the saturation conditions is the mass of the scalar meson. In the $\alpha_1-\alpha_2$ plane the lines of constant E_s have a different slope than those of constant t . It is then possible to keep a fixed value of E_s and to modify the value of t by choosing α_1 and α_2 appropriately. The range of variation of E_s and t with α_1 and α_2 is wider than with the volume couplings. This justifies including these gradient terms in the energy functional in spite of being of order 5 in the expansion. The α_1 and α_2 surface meson interactions also influence the spin-orbit potential, but the effect is not extremely significant.

The effective model is augmented with a tensor coupling of the ω meson to the nucleon.

An outstanding feature is the drastic consequences it has for the spin–orbit force. We have emphasized how inclusion of f_v permits to obtain a spin–orbit strength similar to that of $M_\infty^*/M \sim 0.6$ with larger values of the equilibrium nucleon effective mass, contrary to the phenomenology known from models without such a coupling.

We have discussed the implications of the extra couplings of the extended model on various surface symmetry properties. We have restricted ourselves to the regime of low asymmetries, where the liquid droplet model can be applied and the surface stiffness coefficient Q is the key quantity. In particular we have pointed out the role that the non-linearity η_ρ of the isovector ρ -meson field may play in the details of the symmetry properties.

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References

- [1] B.D. Serot and J.D. Walecka, *Adv. Nucl. Phys.* 16 (1986) 1.
- [2] L.S. Celenza and C.M. Shakin, *Relativistic nuclear physics: theories of structure and scattering* (World Scientific, Singapore, 1986).
- [3] P.-G. Reinhard, *Rep. Prog. Phys.* 52 (1989) 439.
- [4] B.D. Serot, *Rep. Prog. Phys.* 55 (1992) 1855.
- [5] B.D. Serot and J.D. Walecka, *Int. J. of Mod. Phys. E* 6 (1997) 515.
- [6] J. Boguta and A.R. Bodmer, *Nucl. Phys. A* 292 (1977) 413.
- [7] R.J. Furnstahl, B.D. Serot and H.B. Tang, *Nucl. Phys. A* 598 (1996) 539.
- [8] H. Müller and B.D. Serot, *Nucl. Phys. A* 606 (1996) 508.
- [9] R.J. Furnstahl, B.D. Serot and H.B. Tang, *Nucl. Phys. A* 615 (1997) 441.
- [10] A.R. Bodmer, *Nucl. Phys. A* 526 (1991) 703.
- [11] A.R. Bodmer and C.E. Price, *Nucl. Phys. A* 505 (1989) 123, and references therein.
- [12] S. Gmuca, *Z. Phys. A* 342 (1992) 387.
- [13] S. Gmuca, *Nucl. Phys. A* 547 (1992) 447.
- [14] Y. Sugahara and H. Toki, *Nucl. Phys. A* 579 (1994) 557.
- [15] K. Sumiyoshi, H. Kuwabara and H. Toki, *Nucl. Phys. A* 581 (1995) 725.
- [16] M. Rufa, P.-G. Reinhard, J.A. Maruhn, W. Greiner and M.R. Strayer, *Phys. Rev. C* 38 (1988) 390.
- [17] R.J. Furnstahl, J.J. Rusnak and B.D. Serot, *Nucl. Phys. A* 632 (1998) 607.
- [18] W.D. Myers and W.J. Swiatecki, *Ann. Phys. (N.Y.)* 55 (1969) 395; *Ann. Phys. (N.Y.)* 84 (1974) 186; W.D. Myers, *Droplet model of atomic nuclei* (Plenum, New York, 1977).
- [19] M. Centelles, X. Viñas and P. Schuck, *Phys. Rev. C* 53 (1996) 1018.
- [20] M. Brack, C. Guet and H.-B. Håkansson, *Phys. Rep.* 123 (1985) 275; J. Treiner and H. Krivine, *Ann. Phys. (N.Y.)* 170 (1986) 406; K. Kolehmainen, M. Prakash, J.M. Lattimer and J. Treiner, *Nucl. Phys. A* 439 (1985) 537.
- [21] W. Stocker, J. Bartel, J.R. Nix and A.J. Sierk, *Nucl. Phys. A* 489 (1988) 252.

- [22] W. Stocker and M.M. Sharma, *Z. Phys. A* 339 (1991) 147; M.M. Sharma, S.A. Moszkowski and P. Ring, *Phys. Rev. C* 44 (1991) 2493.
- [23] M. Centelles, X. Viñas, M. Barranco and P. Schuck, *Ann. Phys. (N.Y.)* 221 (1993) 165; M. Centelles and X. Viñas, *Nucl. Phys. A* 563 (1993) 173; M. Del Estal, M. Centelles and X. Viñas, *Phys. Rev. C* 56 (1997) 1774.
- [24] C. Speicher, R.M. Dreizler and E. Engel, *Nucl. Phys. A* 562 (1993) 569.
- [25] D. Von-Eiff, J.M. Pearson, W. Stocker and M.K. Weigel, *Phys. Lett. B* 324 (1994) 279.
- [26] M. Centelles, M. Del Estal and X. Viñas, *Nucl. Phys. A* 635 (1998) 193.
- [27] D. Hofer and W. Stocker, *Nucl. Phys. A* 492 (1989) 637.
- [28] D. Von-Eiff, W. Stocker and M.K. Weigel, *Phys. Rev. C* 50 (1994) 1436.
- [29] D. Von-Eiff, H. Freyer, W. Stocker and M.K. Weigel, *Phys. Lett. B* 344 (1995) 11.
- [30] P. Hohenberg and W. Kohn, *Phys. Rev. B* 136 (1964) 864.
- [31] C. Speicher, R.M. Dreizler and E. Engel, *Ann. Phys. (N.Y.)* 213 (1992) 312.
- [32] W. Kohn and L.J. Sham, *Phys. Rev. A* 140 (1965) 1133.
- [33] P.-G. Reinhard, M. Rufa, J. Maruhn, W. Greiner and J. Friedrich, *Z. Phys. A* 323 (1986) 13.
- [34] P. Möller, J.R. Nix, W.D. Myers and W.J. Swiatecki, *At. Data Nucl. Data Tables* 59 (1995) 185.
- [35] Y. Aboussir, J.M. Pearson, A.K. Dutta and F. Tondeur, *Nucl. Phys. A* 549 (1992) 155.

Table captions

Table 1. The surface stiffness coefficient Q and the coefficient L for several values of the couplings analyzed in the text. We have set $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$, $K = 200 \text{ MeV}$, $M_\infty^*/M = 0.6$, $J = 30 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Table 1

η_0	η_1	η_2	η_ρ	α_1	α_2	f_ν	f_ρ	Q (MeV)	L (MeV)
∞	0	0	0	0	0	0	0	21	96
5								21.5	95
2								22	90
∞	1	0	0	0	0	0	0	24.5	91
	0	1						22	93
	1	1						25.5	89
∞	0	0	1	0	0	0	0	23	87
			-1					18	119
∞	0	0	0	1	0	0	0	17	96
				0	1			25	96
				1	1			19	96
				1	-1			16	96
∞	0	0	0	0	0	0.3	0	24	96
						-0.3		19	96
∞	0	0	0	0	0	0	5	21	96

Figure captions

Figure 1. The couplings $g_s/4\pi$, $g_v/4\pi$, κ_3 , κ_4 and ζ_0 versus the parameter η_0 defined in Eq. (3.3). The naturalness assumption requires all these couplings to be of order unity. We have taken $\rho_0 = 0.152 \text{ fm}^{-3}$ ($k_F = 1.31 \text{ fm}^{-1}$), $a_v = -16.42 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Figure 2. Surface energy coefficient E_s for several values of the parameter η_0 , K , M_∞^*/M and m_s , with $\rho_0 = 0.152 \text{ fm}^{-3}$ and $a_v = -16.42 \text{ MeV}$.

Figure 3. Surface thickness t of the baryon density profile for several values of the parameter η_0 , K , M_∞^*/M and m_s , with $\rho_0 = 0.152 \text{ fm}^{-3}$ and $a_v = -16.42 \text{ MeV}$.

Figure 4. Surface energy coefficient E_s and surface thickness t for several values of the parameter η_0 , M_∞^*/M and m_s , with $\rho_0 = 0.1484 \text{ fm}^{-3}$, $a_v = -15.75 \text{ MeV}$ and $K = 200 \text{ MeV}$ (Ref. [10]).

Figure 5. Baryon density $\rho(z)$ and surface tension density $\sigma(z) = \mathcal{E}(z) - (a_v + M)\rho(z)$ of semi-infinite nuclear matter for some values of the parameter η_0 . It is $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$, $K = 200 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Figure 6. Orbital part of the spin-orbit potential $V_{so}(z)$ at the Fermi surface and central mean field $V_c(z)$, Eqs. (2.17) and (2.19) respectively, for some values of the parameter η_0 . It is $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$, $K = 200 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Figure 7. The couplings $g_s/4\pi$, $g_v/4\pi$, κ_3 and κ_4 against the parameters η_1 (left) and η_2 (right). With $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$ and $m_s = 490 \text{ MeV}$, results are shown for $K = 200 \text{ MeV}$ and $M_\infty^*/M = 0.6$ in part (a), and for $K = 350 \text{ MeV}$ and $M_\infty^*/M = 0.7$ in part (b).

Figure 8. Level curves in the plane η_1 - η_2 of the surface energy coefficient E_s (in MeV, solid lines) and of the surface thickness t (in fm, dashed lines), with $\zeta_0 = \alpha_1 = \alpha_2 = f_v = 0$. The point $\eta_1 = \eta_2 = 0$ is marked by a cross. It is $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$, $K = 200 \text{ MeV}$, $M_\infty^*/M = 0.6$ and $m_s = 490 \text{ MeV}$.

Figure 9. Same as Figure 8 in the plane α_1 - α_2 , with $\zeta_0 = \eta_1 = \eta_2 = f_v = 0$. The point $\alpha_1 = \alpha_2 = 0$ is marked by a cross.

Figure 10. Orbital part of the spin-orbit potential $V_{\text{so}}(z)$ at the Fermi surface for some values of the couplings α_1 and α_2 . The equilibrium properties of nuclear matter and the scalar mass are the same of Figure 6.

Figure 11. Surface energy coefficient E_s and surface thickness t as functions of the strength f_v of the ω -meson tensor coupling, with $\zeta_0 = \eta_1 = \eta_2 = \alpha_1 = \alpha_2 = 0$. We have set $\rho_0 = 0.152 \text{ fm}^{-3}$, $a_v = -16.42 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Figure 12. Orbital part of the spin-orbit potential $V_{\text{so}}(z)$ at the Fermi surface for some values of the tensor coupling f_v . The equilibrium properties of nuclear matter and the scalar mass are the same of Figures 6 and 10.

Figure 13. Total baryon density $\rho(z)$, total tensor density $\rho_{\text{T}}(z)$ and their components $\rho^\lambda(z)$ and $\rho_{\text{T}}^\lambda(z)$ for the two spin orientations $\lambda = \pm 1$. They have been calculated for $f_v = 0$ and $f_v = 0.6$, with $M_\infty^*/M = 0.7$, $K = 200 \text{ MeV}$ and $m_s = 490 \text{ MeV}$.

Figure 14. Neutron skin thickness Θ as a function of the bulk neutron excess δ_0 . The solid line is the result of the conventional model ($\zeta_0 = \eta_1 = \eta_2 = \eta_\rho = \alpha_1 = \alpha_2 = f_v = f_\rho = 0$). The other lines differ from the latter in the indicated parameter.

Figure 15. Same as Figure 14 for the surface energy coefficient E_s and the surface thickness t as functions of the bulk neutron excess squared δ_0^2 .































