Nuclear Symmetry Energy Probed by Neutron Skin Thickness of Nuclei

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We describe a relation between the symmetry energy coefficients $c_{\rm sym}(\rho)$ of nuclear matter and $a_{\rm sym}(A)$ of finite nuclei that accommodates other correlations of nuclear properties with the low-density behavior of $c_{\rm sym}(\rho)$. Here, we take advantage of this relation to explore the prospects for constraining $c_{\rm sym}(\rho)$ of systematic measurements of neutron skin sizes across the mass table, using as example present data from antiprotonic atoms. The found constraints from neutron skins are in harmony with the recent determinations from reactions and giant resonances.

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A wealth of measured data on densities, masses, and collective excitations of nuclei has allowed to resolve basic features of the equation of state (EOS) of nuclear matter, like the density $\rho_0 \approx 0.16$ fm^{-3}, energy per particle $a_v \approx -16$ MeV, and incompressibility $K_v \approx 230$ MeV [1] at saturation. However, the symmetry properties of the EOS due to differing neutron and proton numbers remain more elusive to date. The quintessential paradigm is the density dependence of the symmetry energy [1–10]. The accurate characterization of this property entails profound consequences in studying the neutron distribution in stable and exotic nuclei and neutron-rich matter [2–4]. It impacts on heavy ion reactions [5–9], nuclear astrophysics [3,4,10], and on diverse areas such as tests of the Standard Model via atomic parity violation [11].

The general expression $e(\rho,\delta) = e(\rho,0) + c_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4)$ for the energy per particle of nuclear matter of density $\rho = \rho_n + \rho_p$ and asymmetry $\delta = (\rho_n - \rho_p)/\rho$ defines the symmetry energy coefficient $c_{\rm sym}(\rho)$ of a nuclear EOS. It is customary and insightful to characterize the behavior of an EOS around the saturation density ρ_0 in terms of a few bulk parameters, like $e(\rho,0) \simeq a_v + \frac{1}{2}K_v\epsilon^2$ and $c_{\rm sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{\rm sym}\epsilon^2$ where $\epsilon = (\rho_0 - \rho)/(3\rho_0)$ [5–7,12]. The value of $J = c_{\rm sym}(\rho_0)$ is acknowledged to be about 32 MeV. The values of $L = 3\rho\partial c_{\rm sym}(\rho)/\partial\rho|_{\rho_0}$ and $K_{\rm sym} = 9\rho^2\partial^2 c_{\rm sym}(\rho)/\partial\rho^2|_{\rho_0}$ govern the density dependence of $c_{\rm sym}$ around ρ_0. They are less certain, and the predictions vary largely among nuclear theories, see, e.g., Ref. [7] for a review.

In experiment, recent research in intermediate-energy heavy ion collisions (HIC) is consistent with a dependence $c_{\rm sym}(\rho) = c_{\rm sym}(\rho_0)(\rho/\rho_0)^\gamma$ at $\rho < \rho_0$ [6–9]. Isospin diffusion predicts $\gamma = 0.7$–1.05 ($L = 88 \pm 25$ MeV) [6,7], isoscaling favors $\gamma = 0.69$ ($L \sim 65$ MeV) [8], and a value closer to 0.55 ($L \sim 55$ MeV) is inferred from nucleon emission ratios [9]. Nuclear resonances are another hopeful tool to calibrate $c_{\rm sym}(\rho)$ below ρ_0 [13–16]. Indeed, the giant dipole resonance (GDR) of ^{208}Pb analyzed with Skyrme forces suggests a constraint $c_{\rm sym}(0.1\ {\rm fm}^{-3}) = 23.3$–$24.9$ MeV [14], implying $\gamma \sim 0.5$–0.65. Note that the Thomas-Fermi model fitted very precisely to binding energies of 1654 nuclei [17] predicts an EOS that yields $\gamma = 0.51$. With the caveat that the connection of experiments to the EOS often is not at all trivial [6–9,13], it is important to seek further clues from the above and other isospin-sensitive signals, such as the neutron skin thickness $S = R_n - R_p$ of nuclei (difference of neutron and proton rms radii). Because S of heavy nuclei correlates linearly with the slope L of $c_{\rm sym}$ in mean field theories of nuclear structure [2–7,18,19], these studies have far-reaching implications for nuclear theory.

In this Letter, we show that $c_{\rm sym}(\rho)$ of the EOS equals at $\rho \approx 0.1$ fm^{-3} the value of the symmetry energy coefficient $a_{\rm sym}(A)$ of heavy *finite* nuclei, universally in mean field theories. The observed correlations of S [2–7] and of the excitation energy of the GDR [14] with the density dependence of $c_{\rm sym}$ can be deduced naturally from this relation. We resort to the nuclear droplet model (DM) [12] to work out the analytical formulas. The result derived for S is applied to investigate limits to the slope and curvature of $c_{\rm sym}$ from neutron skins measured for 26 stable nuclei, from ^{40}Ca to ^{238}U, in antiprotonic atoms [20]. A main point is ascertaining how far *uniformly* measured neutron skins *over* the periodic table may help constrain the density dependence of $c_{\rm sym}$. We provide the first evidence that the constraints from skins are in consonance with the recent observations from reactions and giant resonances, though the probed densities and energies are not necessarily the same.

The symmetry energy coefficient $a_{\rm sym}(A)$ of finite nuclei is smaller than the bulk value J. Given a nuclear force, the DM allows one to extract $a_{\rm sym}(A)$ as [12,21]

$$a_{\rm sym}(A) = \frac{J}{1 + x_A}, \quad \text{with} \quad x_A = \frac{9J}{4Q}A^{-1/3}. \qquad (1)$$

The so-called surface stiffness Q measures the resistance of the nucleus against separation of neutrons from protons to form a neutron skin. One can obtain Q of nuclear forces by asymmetric semi-infinite nuclear matter (ASINM) calculations [12,21,22]. The contribution of $a_{\rm sym}(A)$ to the nucleus energy is $a_{\rm sym}(A)(I + x_A I_C)^2 A$, where $I = (N - Z)/A$ and $I_C = e^2 Z/(20JR)$ is due to Coulomb. One has $R = r_0 A^{1/3}$. A small correction to $a_{\rm sym}(A)$ from surface compression [12] is neglected here. Let us mention that (1) may be derived also from the notion of surface symmetry energy [4,19].

The neutron skin thickness of nuclei is obtained as

$$S = \sqrt{3/5}[t - e^2 Z/(70J)] + S_{\rm sw} \tag{2}$$

in the DM [12,23]. The quantity t gives the distance between the neutron and proton mean surface locations:

$$t = \frac{3r_0}{2}\frac{J/Q}{1 + x_A}(I - I_C) = \frac{2r_0}{3J}[J - a_{\rm sym}(A)]A^{1/3}(I - I_C), \tag{3}$$

where in the right side we have introduced the surface symmetry term $a_{\rm ss}(A) = [J - a_{\rm sym}(A)]A^{1/3}$ using Eq. (1). The second term in Eq. (2) is due to Coulomb repulsion, and $S_{\rm sw} = \sqrt{3/5}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by an eventual difference in the surface widths b_n and b_p of the neutron and proton density profiles.

We first illustrate the aforesaid correlation of S of heavy nuclei with L in Fig. 1(a). It depicts the *quantal* self-consistent value of S in ^{208}Pb against L for multiple Skyrme, Gogny, and covariant models of different nature [2–7,18,21,24]. In Fig. 1(b), we show that a similar correlation exists with the ratio L/J, which is proportional to γ if a scaling $(\rho/\rho_0)^\gamma$ holds for $c_{\rm sym}(\rho)$. And in Fig. 1(c), we show that the close dependence of S on $J - a_{\rm sym}(A)$ predicted by the DM is borne out in the quantal S value, using

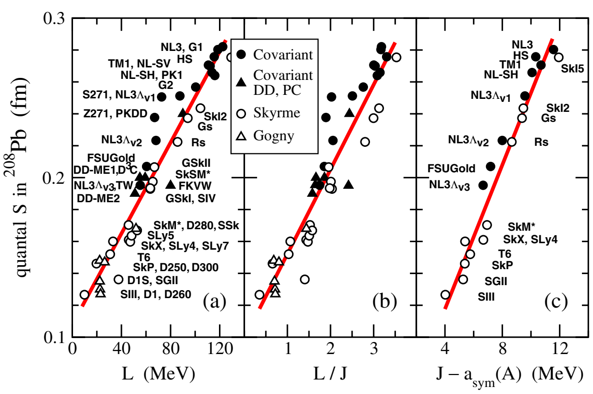

FIG. 1 (color online). Correlation of the quantal self-consistent S value in ^{208}Pb with the slope of the symmetry energy L (a), the ratio L/J (b), and with $J - a_{\rm sym}(A)$ (c), for various nuclear models (DD and PC stand for density dependent and point coupling models). From left to right, the correlation factors are $r = 0.961$, 0.945, and 0.970.

forces where we have computed Q in ASINM. It reassures one that the DM expression incorporates the proper elements for the study. Many of the given nuclear interactions are accurately fitted to experimental binding energies, single-particle data, and charge radii of a variety of nuclei. However, their isospin structure is not sufficiently firmed up as seen, e.g., in the differing predictions for $S(^{208}\text{Pb})$. There is thus a need to deepen our knowledge of isospin-sensitive observables like S and of their constraints on $c_{\rm sym}(\rho)$.

We bring into notice a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{\rm sym}(\rho)$ equals $a_{\rm sym}(A)$ of a heavy nucleus like ^{208}Pb at a density $\rho \approx 0.1$ fm^{-3}. Indeed, the relation holds similarly down to medium mass numbers, at lower ρ values and a little more spread. Table I exemplifies this fact with several nuclear models, where we show the density fulfilling $c_{\rm sym}(\rho) = a_{\rm sym}(A)$ for $A = 208$, 116, and 40. We find that this density can be parametrized as

$$\rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}) \tag{4}$$

with c fixed by $\rho_{208} = 0.1$ fm^{-3} (which gives $\rho_{116} \approx 0.093$ fm^{-3} and $\rho_{40} \approx 0.08$ fm^{-3} for the models of Table I).

The relation "$c_{\rm sym}(\rho) = a_{\rm sym}(A)$" can be very helpful to elucidate other correlations of isospin observables with $c_{\rm sym}(\rho)$ and to gain deeper insights into them. For example, it allows one to replace $a_{\rm sym}(A)$ in Eq. (3) for a heavy nucleus by $c_{\rm sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{\rm sym}\epsilon^2$ with ϵ computed at $\rho \approx 0.1$ fm^{-3} [25]:

$$t = \frac{2r_0}{3J}L\left(1 - \epsilon\frac{K_{\rm sym}}{2L}\right)\epsilon A^{1/3}(I - I_C). \tag{5}$$

The imprint of the density content of the symmetry energy on the neutron skin appears now explicitly. The leading proportionality of (5) with L explains the observed linear-

TABLE I. Value of J, $a_{\rm sym}(A)$, and density ρ that fulfils $c_{\rm sym}(\rho) = a_{\rm sym}(A)$ for $A = 208$, 116, and 40, in various nuclear models. J and $a_{\rm sym}$ are in MeV and ρ is in fm^{-3}. Here, $c_{\rm sym}(\rho)$ was computed exactly as $\frac{1}{2}\partial^2 e(\rho,\delta)/\partial\delta^2|_{\delta=0}$ from the EOS of the models.

		$A = 208$		$A = 116$		$A = 40$	
Model	J	$a_{\rm sym}$	ρ	$a_{\rm sym}$	ρ	$a_{\rm sym}$	ρ
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	26.0	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.099	24.2	0.092	21.9	0.078
TF [17]	32.6	24.2	0.094	22.9	0.086	20.3	0.071
SLy4	32.0	25.3	0.100	24.2	0.093	22.0	0.079
SkX	31.1	25.7	0.103	24.8	0.096	22.8	0.084
SkM*	30.0	23.2	0.101	22.0	0.094	19.9	0.079
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.078
SGII	26.8	21.6	0.104	20.7	0.098	18.9	0.084

ity of S of a heavy nucleus with L in nuclear models [2,4,7]. The correction with K_{sym} does not alter the situation as $\epsilon \sim 1/9$ is small. One can use Eq. (5) in other mass regions by calculating ϵ from ρ_A of Eq. (4). We have checked numerically in multiple forces that the results closely agree with Eq. (3) for the $40 \le A \le 238$ stable nuclei given in Fig. 2.

With the help of Eq. (5) for t (using ρ_A to compute ϵ), we next analyze constraints on the density dependence of the symmetry energy by optimization of (2) to experimental S data. We employ $c_{\mathrm{sym}}(\rho) = 31.6(\rho/\rho_0)^\gamma$ MeV [6–9] and take as experimental baseline the neutron skins measured in 26 antiprotonic atoms [20] (see Fig. 2). These data constitute the largest set of uniformly measured neutron skins over the mass table till date. With allowance for the error bars, they are fitted linearly by $S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)$ fm [20]. This systematics renders comparisons of skin data with DM formulas, which by construction average the microscopic shell effect, more meaningful [26]. We first set $b_n = b_p$ (i.e., $S_{\mathrm{sw}} = 0$) as done in the DM [12,23,26] and in the analysis of data in Ref. [19]. Following the above, we find $L = 75 \pm 25$ MeV ($\gamma = 0.79 \pm 0.25$). The range $\Delta L = 25$ MeV stems from the window of the linear averages of experiment. The L value and its uncertainty obtained from neutron skins with $S_{\mathrm{sw}} = 0$ is thus quite compatible with the quoted constraints from isospin diffusion and isoscaling observables in HIC [6–8]. On the other hand, the symmetry term of the incompressibility of the nuclear EOS around equilibrium ($K = K_v + K_\tau \delta^2$) can be estimated using information of the symmetry energy as $K_\tau \approx K_{\mathrm{sym}} - 6L$ [5–7]. The constraint $K_\tau = -500 \pm 50$ MeV is found from isospin diffusion [6,7], whereas our study of neutron skins leads to $K_\tau = -500^{+125}_{-100}$ MeV. A value $K_\tau = -550 \pm 100$ MeV seems to be favored by the giant monopole resonance (GMR) measured in Sn isotopes as is described in [13]. Even if the present analyses may not be called definitive, significant consistency arises among the values extracted for L and K_τ from seemingly unrelated sets of data from reactions, ground-states of nuclei, and collective excitations.

To assess the influence of the correction S_{sw} in (2), we compute the surface widths b_n and b_p in ASINM [22]. This yields the $b_{n(p)}$ values of a finite nucleus if we relate the asymmetry δ_0 in the bulk of ASINM to I by $\delta_0(1 + x_A) = I + x_A I_C$ [21–23]. In doing so, we find that Eq. (2) reproduces trustingly S (and its change with I) of self-consistent Thomas-Fermi calculations of finite nuclei made with the same nuclear force. Also, S_{sw} is very well fitted by $S_{\mathrm{sw}} = \sigma_{\mathrm{sw}} I$. All slopes σ_{sw} of the forces of Fig. 1(c) lie between $\sigma_{\mathrm{sw}}^{\min} = 0.15$ fm (SGII) and $\sigma_{\mathrm{sw}}^{\max} = 0.31$ fm (NL3). We then reanalyze the experimental neutron skins including $S_{\mathrm{sw}}^{\min}$ and $S_{\mathrm{sw}}^{\max}$ in Eq. (2) to simulate the two conceivable extremes of S_{sw} according to mean field models. The results are shown in Fig. 3. Our above estimates of L and K_τ could be shifted by up to -25 and $+125$ MeV, respectively, by nonzero S_{sw}. This is on the soft side of the HIC [6–8] and GMR [13] analyses of the symmetry energy, but closer to the alluded predictions from nucleon emission ratios [9], the GDR [14], and nuclear binding systematics [17]. One should mention that the properties of $c_{\mathrm{sym}}(\rho)$ derived from terrestrial nuclei have intimate connections to astrophysics [3,4,10]. As an example, we can estimate the transition density ρ_t between the crust and the core of a neutron star [3,10] as $\rho_t/\rho_0 \sim 2/3 + (2/3)^\gamma K_{\mathrm{sym}}/2K_v$, following the model of Sect. 5.1 of Ref. [10]. The constraints from neutron skins hereby yield $\rho_t \sim 0.095 \pm 0.01\ \mathrm{fm}^{-3}$. This value would not support the direct URCA process of cooling of a neutron star that requires a higher ρ_t [3,10]. The result is in accord with $\rho_t \sim 0.096\ \mathrm{fm}^{-3}$ of the microscopic EOS of Friedman and Pandharipande [27], as well as with $\rho_t \sim 0.09\ \mathrm{fm}^{-3}$ pre-

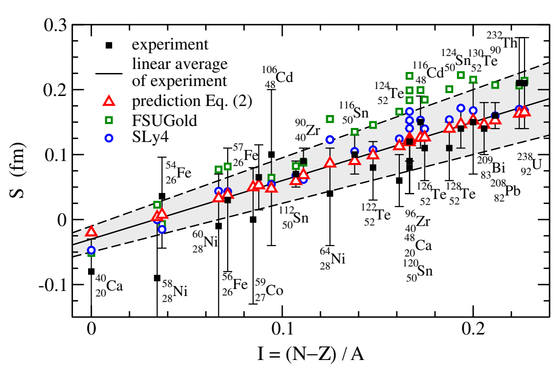

FIG. 2 (color online). Comparison of the fit described in the text of Eq. (2) with the experimental neutron skins from antiprotonic measurements and their linear average $S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)$ fm [20]. Results of the modern Skyrme SLy4 and relativistic FSUGold forces are also shown.

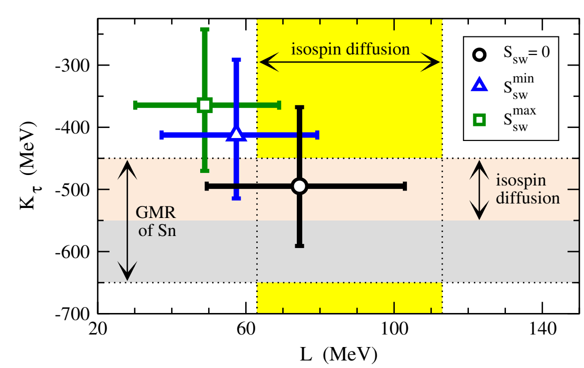

FIG. 3 (color online). Constraints on L and K_τ from neutron skins and their dependence on the S_{sw} correction of Eq. (2). The crosses express the L and K_τ ranges compatible with the uncertainties in the skin data. The shaded regions depict the constraints on L and K_τ from isospin diffusion [6,7] and on K_τ as determined in [13] from the GMR of Sn isotopes.

dicted by a recent analysis of pygmy dipole resonances in nuclei [15].

We would like to close with a brief comment regarding the GDR. As mentioned, Ref. [14] very interestingly constrains $c_{\rm sym}(0.1)$ from the GDR of ^{208}Pb. The analysis notes that the mean excitation energy of the GDR depends on $g(A) = J/\{1 + \frac{5}{3} a_{\rm ss}(A) A^{-1/3}/J\}$ [4,14] and shows numerically that the values of $g(208)$ and $c_{\rm sym}(0.1)$ are correlated in Skyrme forces. Inserting $a_{\rm ss}(A)$ given below Eq. (3), one has $g(A) = J/\{1 + \frac{5}{3}[J - a_{\rm sym}(A)]/J\}$. Immediately, the equivalence $a_{\rm sym}(208) \approx c_{\rm sym}(0.1)$ explains why $g(208)$ has a dependence on $c_{\rm sym}(0.1)$, gives it analytically, and validates it for any type of mean field model [28]. One could extend it to other A values through Eq. (4). In conclusion, the discussed relation of $c_{\rm sym}(\rho)$ with $a_{\rm sym}(A)$ can be much valuable to link different problems depending upon $a_{\rm sym}(A)$ of nuclei to the symmetry properties of the EOS.

Summarizing, we have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation. It plausibly encompasses other prime correlations of nuclear observables with the density content of the symmetry energy. We take advantage of this relation to explore constraints on $c_{\rm sym}(\rho)$ from neutron skins measured in antiprotonic atoms [20]. We discuss the L and K_τ values that skins favor vis-à-vis most recent observations from reactions and giant resonances. The difficult experimental extraction of neutron skins limits their potential to constrain $c_{\rm sym}(\rho)$. Interestingly, we learn that in spite of present error bars in the data of [20], the size of the final uncertainties in L or K_τ is comparable to the other analyses. This highlights the value of having skin data consistently measured across the mass table, and calls for pursuing extended measurements of neutron radii and skins with ''conventional'' hadronic probes. Combined with a precision extraction of R_n of ^{208}Pb through electroweak probes [29], they would contribute to cast uniquely tight constraints on $c_{\rm sym}(\rho)$.

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