On the Indoor VLC Link Evaluation Based on the Racian K-Factor

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***Abstract*—In visible light communications (VLC) technology, the channel is generally modeled as a multi-path propagation environment. This is due to the presence, on the photo-detector (PD), of specular and non-specular signal components. The Racian K-factor is exploited in such environments to measure the dominance of the line-of-sight (LoS) signal power over that of the non-LoS (NLoS) link. In this paper, K is exploited in the evaluation of the VLC link. Examples of the use of K is underlined in the computation of few link evaluation metrics including the signal-to-noise ratio (SNR), γ, the channel capacity, C, and the outage probability, p_{out}.**

***Index Terms*—VLC, K-factor, VLC link analysis, signal-to-noise ratio, channel capacity, outage probability.**

I. Introduction

VISIBLE light communications (VLC) technology is continuously gaining interests from researchers and industry. This can be illustrated by the number of research outputs found in the literature. In most cases, it is assumed that the line-of-sight (LoS) signal is dominant when compared to the non-LoS (NLoS) signal. This is valid in umpteen cases, but in a few situations such as in some indoor environments, the NLoS received power should not be completely neglected. This paper attempts to analyze the indoor VLC link using the Racian K-factor, which gives an idea of the power in the NLoS link when the power in the LoS link is known.

The indoor VLC channel is typically made of two main components: *(i)* The LoS sub-channel which represents the direct link between transmitter and receiver without an obstacle and *(ii)* the NLoS sub-channel representing the remaining paths used by the transmitted message to reach the receiver. The lighting comfort of the communication environment has naturally benefited from this structure as it may contribute to the harmony of the light over the environment, knowing that in VLC, both lighting and communication have to be optimized.

The Racian K-factor gives the ratio of the squared signal power of the LoS link, $(P_r^{(o)})^2$, over that of the signal from the NLoS link, $(P_r^{(k)})^2$. This paper defines a factor, $\Delta = 1 + 1/\sqrt{K}$, giving the ratio of the received total power by the power in the LoS link based on K. Both K and Δ are exploited to measure the non-specular signal power. K is given between 0 and $+\infty$ while Δ is defined between 1 and $+\infty$. For $\Delta = +\infty$, ($K = 0$), the fading channel is said to be Rayleigh distributed [1]. If $\Delta = 2$, ($K = 1$), equal powers are received from both LoS and NLoS links and $\Delta = 1$, ($K = +\infty$), invokes an overall Gaussian distributed link [1]. These variations of the value of K/Δ show that, the probability of facing a link with a high number of NLoS rays is high. Hence, the need of analyzing the indoor VLC channel based on the K-factor, because a great number of reflected rays may induce a high non-specular received power. To the best of the authors' knowledge, such analysis has not been done. With this in mind, in this paper, the main indoor VLC link evaluation metrics, which are the signal-to-noise ratio (SNR), γ, the channel capacity, C, and the outage probability, p_{out}, are analyzed based on K/Δ. This helps to underline the values of K/Δ that provide the best link structure. Results show the impacts of the NLoS channel gain on γ, C and p_{out}, and, finally on the performance of the system.

The remainder of the paper is organized as follows: Section II discusses the generalized indoor VLC system and looks at the ratio between LoS and NLoS received powers, $P_r^{(o)}$ and $P_r^{(k)}$. The link evaluation metrics are discussed and analyzed in Section III. These include the probability density function (PDF) and the moment generating function (MGF) of the received signal, the SNR, the channel capacity, and the outage probability. In Section IV, numerical results are given to highlight the behavior of γ, C and P_{out}. Finally, concluding remarks are given in Section V.

II. VLC channel and Racian K-factor

This section introduces the indoor VLC system and presents the impact of the Racian K-factor on the overall channel gain.

A. The VLC system

A generalized VLC system is shown in Fig. 1. The incoming bits are coded, then mapped to a VLC signal constellation. Each signal point is converted from digital to analog (D/A), and the resulting signal is applied to a light source (LS), which may be a light emitting diode (LED) or a laser diode (LD). The LS is in charge of coupling the message signal to the channel while simultaneously serving as an illumination device. On the receiver side, the broadcast light is detected by a photo-detector (PD) and converted into an electric pulse. The latest is digitalized (A/D) and then de-mapped to produce bits which are decoded and sent to the data recipient. The channel in Fig. 1 is made up of three main elements. The LoS rays going from the transmitter to the receiver without deviation, the NLoS rays, also generated by the same LS as the LoS rays, but reflected by objects such as walls, ceilings, etc., and the unwanted rays provided by additional sources of light such as the sun or incandescent and luminescent lights. Based on the above description, the entire link is Racian distributed. The unwanted signal is considered as noise or interference sources and the transmission is governed by

$$\mathbf{y} = \lambda \mathbf{h} \mathbf{x} + \mathbf{n}, \qquad (1)$$

where $\mathbf{y}$ and $\mathbf{x}$ are the received and the transmitted vectors, respectively. λ is the responsivity of the PD, $\mathbf{h}$ the overall

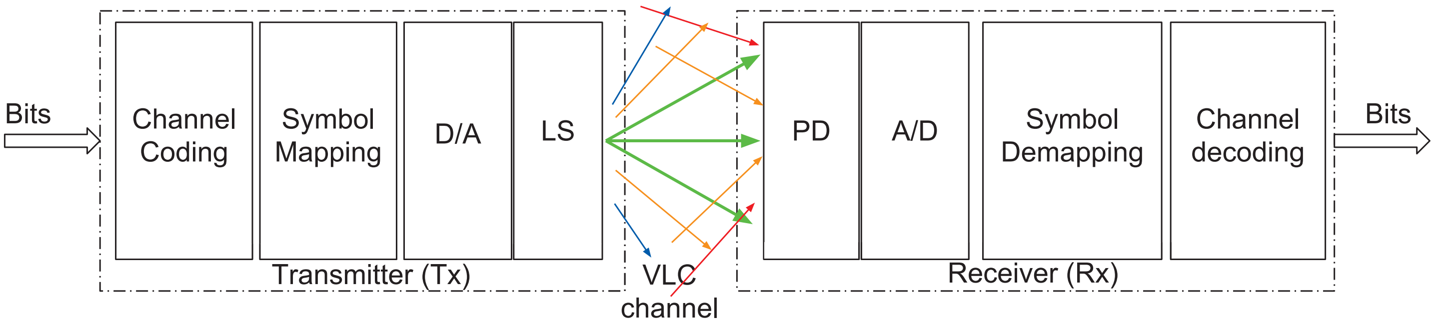

Fig. 1: A VLC system model.

channel gain and **n**, given by $n \sim \mathcal{N}(0, \sigma^2)$, $\sigma_2 = N_0/2$, is the additive noise vector. Note that N_0 is the single-sided power spectral density of the total noise at the PD. Taking into account both LoS and NLoS paths, the indoor VLC channel impulse response, $h(t)$, is defined as [2]

$$h(t) = h^{(o)}(t) + h^{(k)}(t), \tag{2}$$

where $h^{(o)}(t)$ is the impulse response of the LoS link while $h^{(k)}(t)$ is the sum of impulse responses of the remaining paths exploited by the transmitted signal to reach the receiver.

B. Link powers and the K-factor

The transmitted power is split into three part: The first part is directly detected by the PD with no reflection (LoS), the second part reaches the PD after one or multiple reflections (NLoS) and the third part does not reach the photo-detector. This paper considers the parts of the transmitted signal that reach the receiver. Let P_t be the power at the transmitter, $P_r^{(o)}$ the power at the received from the LoS link and $P_r^{(k)}$, the power at the receiver due to the NLoS link. The Racian K-factor is defined by

$$K = \left(\frac{P_r^{(o)}}{P_r^{(k)}}\right)^2 = \left(\frac{\lambda P_t H^{(o)}}{\lambda P_t H^{(k)}}\right)^2 = \left(\frac{H^{(o)}}{H^{(k)}}\right)^2, \tag{3}$$

where $H^{(o)}$ and $H^{(k)}$ are LoS and NLoS links' DC gains. From Eq. (3), H^o is readily calculated as $H^{(o)} = H^{(k)}\sqrt{K}$. Beside, in [3], $H^{(o)}$ is also given as a temporal function of the minimum delay, t_0, as

$$H^o = \int_{t_0}^{\frac{t_0}{\cos(\theta)}} \frac{2t_0}{t^3 \sin^2(\theta)} dt = \frac{1}{2t_0^2}. \tag{4}$$

By analogy to Eq. (2), the overall channel DC gain is given by

$$H = H^{(o)} + H^{(k)} = \left(1 + \frac{1}{\sqrt{K}}\right) H^{(o)}. \tag{5}$$

The factor $\Delta = 1 + 1/\sqrt{K}$ is seen as a modified Racian K-factor, defining the ratio of the overall gain by that of the LoS link. Δ is defined between 1 and $+\infty$ and totally describes the fading aspect of the VLC channel. It provides a feeling that the link is always LoS scaled by Δ since the LoS component is always the strongest. It is therefore kept close to 1, knowing that $\Delta > 2$ implies $P_r^{(k)} > P_r^{(o)}$.

III. VLC Link Evaluation

In this section, the link evaluation metrics are underlined and the impact of the Racian K-factor and Δ on the link performance is analyzed.

Let $a(t)$ be the received temporal signal. Since the channel is Racian fading, it is fully described by the Racian K-factor and the total received signal power $P_r^2 = |a(t)|^2$. This received power, also known as the total mean power, is the sum of powers in LoS and NLoS links.

$$P_r^2 = (P_r^{(o)})^2 + (P_r^{(k)})^2, \tag{6}$$

where $(P_r^{(o)})^2 = \frac{1}{1+(\Delta-1)^2} P_r^2$ and $(P_r^{(k)})^2 = \frac{(\Delta-1)^2}{(\Delta-1)^2+1} P_r^2$.

A. PDF and MGF

The probability density function of the received signal over the considered fading channel, $f(a)$, is given by [4]

$$f(a) = \frac{2a + (\Delta-1)^2)}{(\Delta-1)^2 \lambda^2 H^2 P_t^2} e^{-\frac{1}{(\Delta-1)^2} - \frac{a^2+(\Delta-1)^2)}{(\Delta-1)^2\lambda^2 H^2 P_t^2}} \times I_0\left(2a\sqrt{\frac{1+(\Delta-1)^2}{(\Delta-1)^4 \lambda^2 H^2 P_t^2}}\right), \tag{7}$$

where $I_0(\cdot)$ is the 0^{th} order modified Bessel function of the first kind. The SNR based moment generating function of this fading transmission can be expressed as [5]

$$M(a) = \frac{1+(\Delta-1)^2}{1+(\Delta-1)^2(1-a\bar{\gamma})} \exp\left\{\frac{a\bar{\gamma}}{1+(\Delta-1)^2(1-a\bar{\gamma})}\right\}, \tag{8}$$

where the average SNR, $\bar{\gamma}$, is defined as

$$\bar{\gamma} = \frac{2\lambda^2 H^2 P_t (1 + (\Delta-1)^2)}{B(\Delta-1)^2} \frac{P_t}{N_0}, \tag{9}$$

B being the bandwidth in Hz.

B. Channel capacity

Two main types of fading channel can be defined: The ergodic fast fading and the bloc fading channels. This implies two main types of channel capacities. In this paper, it is assumed that the codeword is long enough so that the transmitted symbols experience all state of the channel. Hence, the considered channel is ergodic fast fading. In this case, the capacity is generally defined as

$$C = \max_{(px)} I(X;Y), \tag{10}$$

where (px) is the input distribution and $I(X;Y)$ is the mutual information and, X and Y are two discrete variables. This capacity can be evaluated in terms of the PDF, $f(\cdot)$, of the SNR and is expressed as [6]

$$C = B \int_0^{\infty} \log_2(1+\gamma) f(\gamma) d\gamma. \tag{11}$$

For a Racian ergodic fading channel, the PDF in Eq. (11) can be expressed as

$$\begin{aligned} f(\gamma) = \frac{1+(\Delta-1)^2}{\bar{\gamma}(\Delta-1)^2} e^{-\frac{1}{(\Delta-1)^2}} e^{-(\frac{1}{(\Delta-1)^2}+1)\frac{\gamma}{\bar{\gamma}}} \\ \times I_0\left[\sqrt{\frac{4\gamma}{\bar{\gamma}} \frac{1}{(\Delta-1)^2}\Big(\frac{1}{(\Delta-1)^2}+1\Big)}\right]. \end{aligned} \tag{12}$$

Substituting Eq. (12) in Eq. (11) leads to a closed-form of the VLC Racian ergodic fading channel capacity [7]. Adapting Δ, the capacity, C, becomes

$$\begin{aligned} C = \frac{e^{\frac{(\Delta-1)^2+1}{\bar{\gamma}(\Delta-1)^2}}}{\ln 2} E_1\Big(\frac{(\Delta-1)^2+1}{\bar{\gamma}(\Delta-1)^2}\Big) + \log_2\Big(1+\frac{\bar{\gamma}}{(\Delta-1)^2+1}\Big) \\ + \frac{1}{\ln 2} \frac{\bar{\gamma}((\Delta-1)^2+1)}{((\Delta-1)^2+1+\bar{\gamma})^2}, \end{aligned} \tag{13}$$

where $E_1(\cdot)$ is an exponential integral and $\bar{\gamma}$ defined in Eq. (9). For values of Δ close to 1 (high values of K), this capacity reduces to

$$C = \frac{1}{2}\log_2(1+\bar{\gamma}). \tag{14}$$

C. Outage probability

This metric measures the probability that the SNR, γ, falls under a threshold SNR, γ_{th}, set for a specific quality of service (QoS). It is given by

$$p_{out} = p_r[\gamma < \gamma_{th}] = \int_0^{\gamma_{th}} f(\gamma) d\gamma, \tag{15}$$

where $f(\cdot)$ is the PDF of the SNR given in Eq. (12). Based on Eq. (12), Eq. (15) can also be rearranged to obtain a closed-form expression. It is shown in [5] that for a Racian fading channel, the cumulative distribution function (CDF), which is another expression of Eq. (15) (outage probability), can be represented using the Marcum Q-function. This is exploited to derive the outage probability based on Eq. (9), which is expressed as

$$F(\gamma) = 1 - \chi\Bigg(\frac{\sqrt{2}}{|\Delta-1|}, \frac{4\lambda^2 H^2 P_t(1+(\Delta-1)^2)}{BP_r^{(k)}(\Delta-1)^2}\frac{P_t}{N_0}\Bigg), \tag{16}$$

where $\chi(\cdot,\cdot)$ is the Marcum Q-function given by

$$\chi(a,b) = \exp\Big\{-\frac{a^2+b^2}{2}\Big\} \sum_{q=1-M}^{\infty} \Big(\frac{a}{b}\Big)^q I_q(ab), \tag{17}$$

$I_q(\cdot)$ being the q^{th} order Bessel function of the first kind.

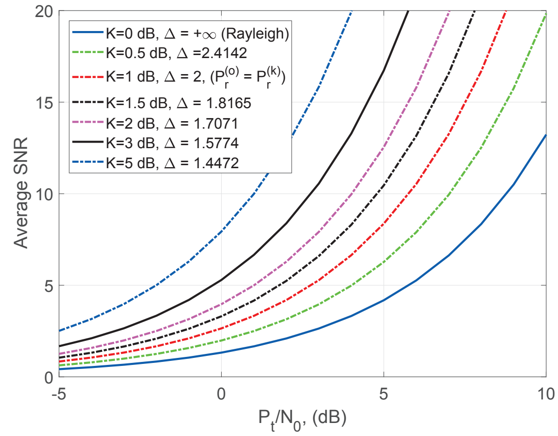

Fig. 2: Average SNR, $\bar{\gamma}$, of an indoor VLC link in terms of P_t/N_0 for multiple values of Δ; $\Delta \in \{1.4472, 1.5774, 1.7071, 1.8165, 2, 2.4142, +\infty\}$.

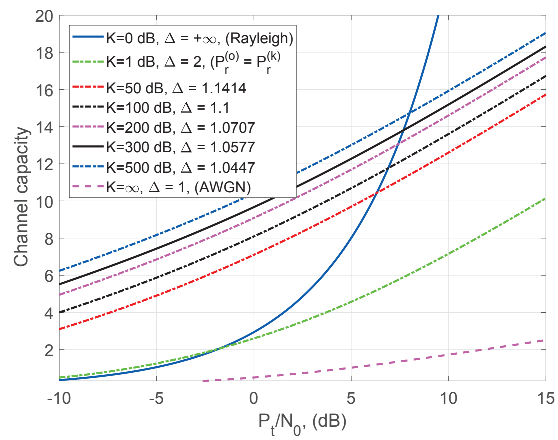

Fig. 3: Ergodic channel capacity of an indoor VLC link in terms of P_t/N_0 for multiple values of Δ; $\Delta \in \{1, 1.0447, 1.0577, 1.0707, 1.1, 1.1414, 2, +\infty\}$.

IV. Numerical Results

This section provides some results related to the analysis proposed in the previous section on the link evaluation metrics. Note that the values of Δ are selected between 1 and 2 to keep $P_r^{(k)} \leq P_r^{(o)}$ valid, with an exception for $\Delta = +\infty$ to emphasize the case of a Rayleigh distributed channel. The first metric analyzed here is the SNR. The results are shown in Fig. 2. The average SNR varies with the ratio P_t/N_0 for $\Delta \in \{1.4472, 1.5774, 1.7071, 1.8165, 2, 2.4142, +\infty\}$. Notice the Rayleigh behavior of the SNR for $\Delta = +\infty$ ($K = 0$). It is also important to highlight the case where $\Delta = 2$ (K = 1), which corresponds to the situation where both LoS and NLoS links are transmitting the same amount of power to the receiver. All curves of Fig. 2 have the same shape. The difference between them is related to the value of Δ. They show that P_t/N_0 is exponentially proportional to the SNR.

Based on Eqs. (13) and (14), the indoor VLC ergodic chan-

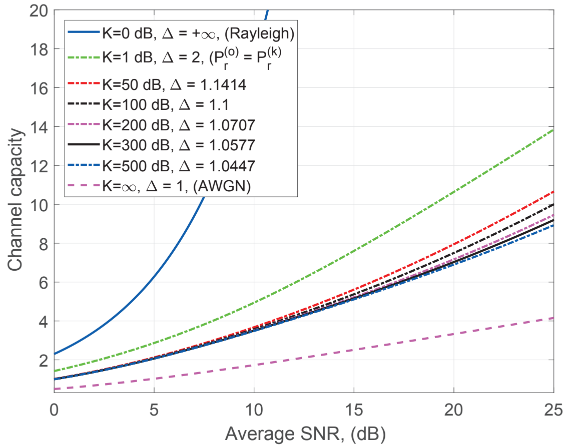

Fig. 4: Ergodic channel capacity of an indoor VLC link in terms of average SNR for multiple values of Δ; $\Delta \in \{1, 1.0447, 1.0577, 1.0707, 1.1, 1.1414, 2, +\infty\}$.

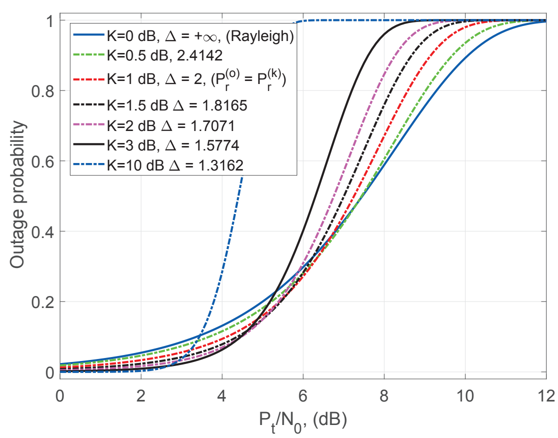

Fig. 5: Outage probability of the VLC indoor link in terms of P_t/N_0 for multiple values of Δ; $\Delta \in \{1.3162, 1.5574, 1.7071, 1.8165, 2, 2.4142, +\infty\}$.

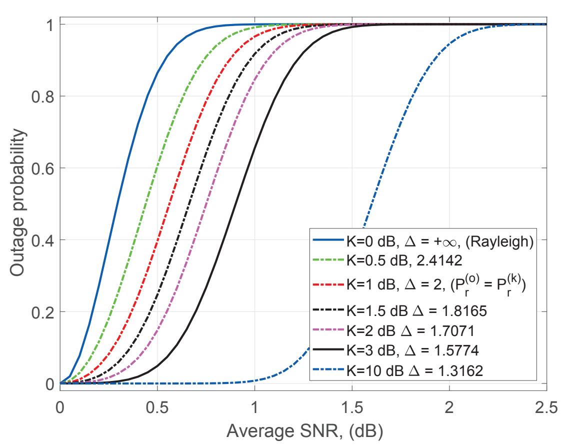

Fig. 6: Outage probability of the VLC indoor link in terms of average SNR, for multiple values of Δ; $\Delta \in \{1.3162, 1.5574, 1.7071, 1.8165, 2, 2.4142, +\infty\}$.

nel capacity is provided in Figs. 3 and 4, for multiple values of K/Δ. Fig. 3 gives the channel capacity in terms of P_t/N_0 while Fig. 4 gives the capacity in terms of average SNR. Notice in Fig. 3 the case where $\Delta = +\infty$ (Rayleigh channel) and the case where Δ = 2, corresponding to $P_r^{(o)} = P_r^{(k)}$. For $\Delta \in \{1, 1.0447, 1.0577, 1.0707, 1.1, 1.1414\}$, the channel capacity follows a similar pattern as Δ is seen as the main factor affecting the channel. This explains the fact that the curves look parallel. In Fig. 4, for each value of Δ corresponds a different curve with a different asymptote. Nevertheless, it also presents particularities in cases where $\Delta = +\infty$ (Rayleigh channel), Δ = 2 ($P_r^{(o)} = P_r^{(k)}$) and the case where $\Delta = 1$ (AWGN). In both Figs. 3 and 4, the cases where $\Delta = 1$ and $\Delta = +\infty$ are particular as the curves experience a different fading distribution (Rayleigh and Gaussian).

Figs. 5 and 6 are plots of the outage probability. They show plots of Eq. (16) for multiple values of Δ. They give an idea on the outage probability of the indoor VLC fading channel. Considering for example a 0.4 probability of failure of the SNR, at least about 4.2 dB of P_t/N_0 is required when Δ = 1.3162. If Δ increases (more NLoS power), it is required to increase P_t/N_0 as shown in Fig. 5. For the same outage probability expectation, the average SNR is less when the channel is Rayleigh distributed and grows progressively with the value of Δ as depicted in Fig. 6.

V. Conclusion

This paper aimed to discuss indoor VLC link evaluation metrics based on the Racian K-factor and Δ. A generalized VLC transmission system is presented and the impact of K and Δ on the DC channel gain is discussed. The paper looked at few communication link evaluation metrics including the SNR, the channel capacity, and the outage probability. They are all analytically given in terms of P_t/N_0 and the average SNR for multiple values of K/Δ. These metrics all give a general and precise idea on how K/Δ impact the indoor VLC channel, especially when the value of K/Δ change the fading distribution of the channel from Racian to Rayleigh (K = 0, $\Delta = +\infty$) or to Gaussian ($K = +\infty$, Δ = 1). Finally, the analysis proposed in this paper shows how the NLoS link can impact the performance of an indoor VLC system.

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