Waveform Clipping in FSK Modulated Signal to Combat Impulse Noise

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***Abstract*—This article presents results on the pre-processing (clipping/nulling) of impulse noise corrupted signal that is digitally modulated. The novelty of the article is in performing the clipping technique on the waveform of the digitally modulated signal as opposed to working with the constellation of the modulated signal. We present bit error rate performance results of Frequency Shift Keying (FSK) modulation in the presence of impulse noise and AWGN, when clipping is performed. We furthermore, develop closed-form expressions for the bit error rates of FSK modulation in the presence of both AWGN and impulse noise, when clipping of the received signal has been performed.**

***Index Terms*—Frequency Shift Keying, Impulse noise, Clipping, Bit error rate expressions.**

I. INTRODUCTION

Power line communication (PLC) research has devoted a large body of work to the study of impulsive noise and methods to mitigate the effects of impulsive noise [1]–[7]. One of the widely used methods of combatting impulsive noise is clipping. Clipping has been observed to be very effective in OFDM systems where, the impulse noise energy is spread in the frequency domain, after the FFT. Clipping can also be performed in single carrier modulations like Frequency Shift Keying (FSK), Amplitude Shift Keying (ASK) and Phase Shift Keying (PSK). Two important points worth noting: 1.The introduction of OFDM in PLC has resulted in very few research on clipping for single carrier modulation. 2. When clipping is performed in the literature, it is conventionally done on the signal constellation of the digitally modulated signal (see [3]–[6]). In this paper, we address these two points by (a) performing clipping on single carrier modulated signal (specifically FSK), and secondly (b) performing clipping on the time-domain waveform signal, which is more realistic. We then go on to give analytical expressions for clipping of a FSK modulated signal that is affected by both impulse noise (IN) and additive white Gaussian noise (AWGN).

II. IMPULSE NOISE CLIPPING

Clipping of the received signal is performed before the demodulation process and it is called a pre-processing operation. For impulse noise clipping, a threshold, which is usually higher than the average amplitude of the transmitted signal and AWGN, is employed to detect impulse noise. We use the definition of the clipping technique described in [9], which is

$$\tilde{r}_k = \begin{cases} r_k, & \text{for } |r_k| \le T_h \\ T_h e^{j\arg(r_k)}, & \text{for } |r_k| > T_h \end{cases}, \tag{1}$$

where r_k is received sample, $\tilde{r}_k$ is clipped received sample and T_h is the so called clipping threshold. The above definition of impulse noise (1) has been widely used by many researchers who worked on impulse noise mitigation techniques [3], [4], [5], [6] and [8]. In this paper, we use the same process of amplitude clipping as given by (1). However, instead of working with signal constellation, we apply the clipping technique on a real sinusoidal FSK signal that is corrupted by AWGN and/or IN.

III. SYSTEM MODEL

The communication system model used in this paper is shown in Figure 1. For the IN effect, we adopt the Bernoulli-

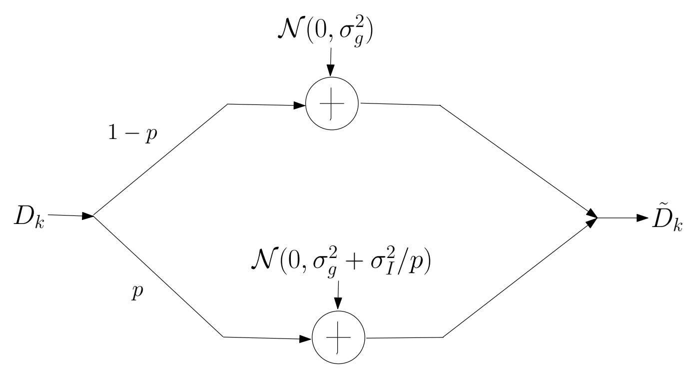

Fig. 1. Bernoulli-Gaussian impulse noise model. The parameter p is the probability of entering a state wih impulse noise and AWGN with variances σ_I^2 and σ_g^2, respectively.

Gaussian noise model in [10], [11] and [12], where p is the probability of IN in a symbol period as shown in Figure 1. The variance of IN is given by $\sigma_I^2 = K\sigma_g^2$, where $K > 1$. The IN noise samples follow a zero-mean Gaussian distribution ($\mathcal{N}(0,\sigma_I^2)$). D_k is the k^{th} sample of an FSK modulated symbol, S_l with symbol period T_b, and $\tilde{D}_k$ is the noisy version of D_k. This means that an FSK symbol S_l is a sequence of samples such that $S_l = \{D_1, D_2, \ldots D_L\}$, where L is the number of samples of the FSK symbol. Therefore, each sample D_k goes through the channel depicted by the model in Figure 1, and each sample has a probability p of encountering impulse noise.

The analytical expression of the FSK modulated signal is given in [13] by

$$S_l = \sqrt{\frac{2E_b}{T_b}} \sin(2\pi f_l t + \phi), \quad (2)$$

for $l = 1, 2, \ldots, M$ and $0 \leq t \leq T_b$, where f_l is any one of M frequencies. The amplitude of the FSK modulated carrier wave, A can be deduced from (2) as

$$A = \sqrt{\frac{2E_b}{T_b}},$$

E_b is the bit energy of the modulating signal and T_b is the bit duration. If the bit duration is fixed, then the amplitude depends on the bit energy. This definition of A will be useful later when we track the changes in signal-to-noise-ratio (SNR). In digital modulation, SNR is given as E_b/N_0, where N_0 is the noise power. Thus, for a fixed N_0, the SNR can be increased by increasing the value of E_b. Having defined the amplitude, we discuss clipping of the modulated carrier waveform at the receiver, which will be related to the amplitude of the carrier waveform.

For clipping at the receiver to mitigate noise, we set a clipping threshold, T_h that will be equal to the amplitude of the carrier waveform, $T_h = |A|$. This means that if there is no noise in the system, the modulated carrier wave will be preserved because the clipping threshold is equal to its amplitude.

IV. Clipping under AWGN

A. Square waveform plus AWGN

To understand the results of clipping for AWGN only, it is more informative to use a square wave signal for the carrier signal. Let us assume that the sinusoidal carrier waveform of amplitude A is approximated by a square waveform of the same amplitude A. Then we observe that when a zero mean AWGN with variance σ_g^2 is added to the square waveform of amplitude A, the AWGN takes on a new mean, A or $-A$ as shown in Figure 2 where $A = 1$. The total power, P_T will be the sum of the signal power and the noise power, $P_T = A^2 + \sigma_g^2$. This total power can easily be proved to be the sum of the signal power and noise power by considering the symmetry of the Gaussian distribution about its mean. This symmetry means that approximately half of the AWGN samples are above the mean A and the other half is below the mean A, therefore the total power of the signal and AWGN is given as

$$P_T = \frac{1}{2}(A + \sigma_g)^2 + \frac{1}{2}(A - \sigma_g)^2 = A^2 + \sigma_g^2. \quad (3)$$

If clipping is performed with threshold, $T_h = |A|$, Equation (3) can be adjusted to remove the contribution of the noise

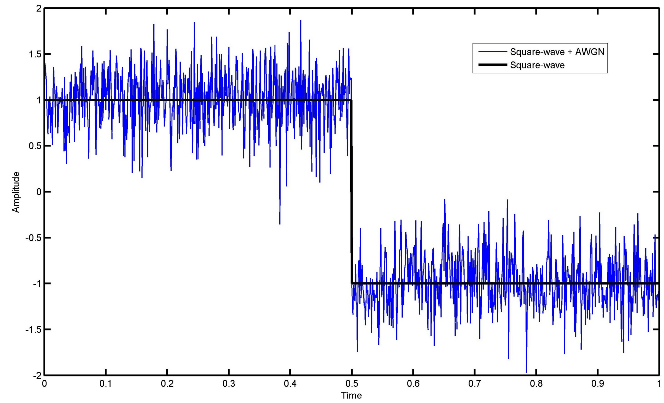

Fig. 2. Square waveform show the effect of AWGN.

above $|A|$, hence the new total power after clipping, P_c is given as

$$P_c = \frac{1}{2}(A)^2 + \frac{1}{2}(A - \bar{\sigma}_g)^2 = A^2 - A\bar{\sigma}_g + \bar{\sigma}_g^2/2, \quad (4)$$

where $\bar{\sigma}_g = \sigma_g\sqrt{2/\pi}$ because the noise follows the half normal distribution after clipping half of it.

Therefore, when clipping is performed with $T_h = |A|$ for the square wave signal transmission under AWGN, we can use (4) to derive the SNR which is given by

$$\begin{aligned} SNR_c &= \frac{E_b - A(\bar{\sigma}_g/\sqrt{T_b})}{\bar{\sigma}_g^2/2} \\ &= \frac{E_b - \sqrt{2E_b/T_b}(\bar{\sigma}_g/\sqrt{T_b})}{\bar{\sigma}_g^2/2} \\ &= \frac{E_b - \frac{1}{T_b}\sqrt{2E_b\bar{\sigma}_g^2}}{\bar{\sigma}_g^2/2}. \end{aligned} \quad (5)$$

Clipping when the transmitted signal is approximated by square waveform under AWGN presents a case of an upper bound on SNR_c due to the fact that a square wave has a flat amplitude which enables about half of the AWGN amplitudes to be clipped off.

B. Sinusoidal waveform plus AWGN

If the transmitted signal is a sinusoidal waveform, as is usually the case for bandpass modulation, most of the AWGN amplitudes remain in the received clipped signal because the clipping is performed at the peak of the sinusoidal waveform. This is true especially when the sinusoidal waveform amplitude A is much larger than the noise power σ_g^2, that is at high SNR.

For sinusoidal waveform transmission under AWGN, and at high SNR, we can assume that the noise power is hardly affected/changed after clipping at the receiver. This means that the signal energy is more affected by clipping compared to the noise. With this assumption, the SNR in (5) can be modified to give an approximation of the SNR when a sinusoidal is used and clipping is performed at $T_h = |A|$:

$$SNR_c = \frac{E_b - \frac{1}{T_b}\sqrt{2E_b\sigma_g^2}}{N_0} = \frac{E_b - \frac{1}{T_b}\sqrt{2E_b\sigma_g^2}}{2\sigma_g^2}. \quad (6)$$

V. Clipping under AWGN and Impulse noise

Having discussed clipping of the received signal under AWGN, we now discuss clipping when impulse noise (IN) and AWGN are present in the channel. Impulse noise is usually of oder of magnitudes larger than AWGN and lasts for a short duration of time. This statement leads to the following definition of the probability of impulse noise occurrence: denote the impulse noise duration by τ, then $p = \tau/T_b$, where $\tau \leq T_b$ and T_b is the signal duration as defined in (2). Now, concerning the amplitude of impulse noise, recall that the variance of IN is given by σ_I^2 and the amplitude of the transmitted signal is given by A. For our analysis, we consider a case of IN of very large amplitudes compared to the transmitted signal, $\sigma_I \gg A$. In this case we can observe that when $T_h = |A|$, the resulting clipped signal affected by this IN will produce noise of amplitude A. The average energy of the clipped large IN amplitude will be zero, and its variance will be approximately $\sigma_{CI}^2 = p^2 E_b$, where p is the probability (fraction of the time) of impulse noise occurrence within the transmitted signal duration T_b. After clipping at the receiver, the SNR when both impulse noise and AWGN were added to the transmitted sinusoidal waveform is

$$\begin{aligned} \overline{SNR_c} &= \frac{(1-p)^2(E_b - \frac{1}{T_b}\sqrt{2E_b\sigma_g^2})}{(1-p)(\sigma_g^2) + (\sigma_{CI}^2)} \\ &= \frac{(1-p)^2(E_b - \frac{1}{T_b}\sqrt{2E_b\sigma_g^2})}{(1-p)(\sigma_g^2) + (p^2 E_b)} \end{aligned} \quad (7)$$

VI. Discussion of Results

The main aim of the article is to investigate the effect of clipping on a FSK modulated waveform that is corrupted by impulse noise. However, the effect of clipping under AWGN only is also very important, as evidenced by the analysis that resulted in (5) and (6), as well as the results in Figure 3. It can be seen in Figure 3 that the simulated and analytical results for $T_h = |A|$ are very close. Figure 3 shows the effect of various clipping thresholds on a FSK modulated waveform that is affected by AWGN only. The results show that clipping degrades the performance of FSK when only AWGN is present, and that the larger the clipping threshold the better the performance. The best performance is when no clipping is performed on the waveform. The benefit of clipping shows when IN is present together with the AWGN as shown in Figure 4.

Figure 4, 5 and 6 present results of the effect of clipping on a FSK modulated waveform that is affected by both IN and AWGN. In the simulation, we set $p = 0.01, 0.1$ and 0.2, and $\sigma_I^2 = 10^5$. We chose a large value of σ_I^2 in the simulation because in our analytical expression in (7) we assumed very large IN power.

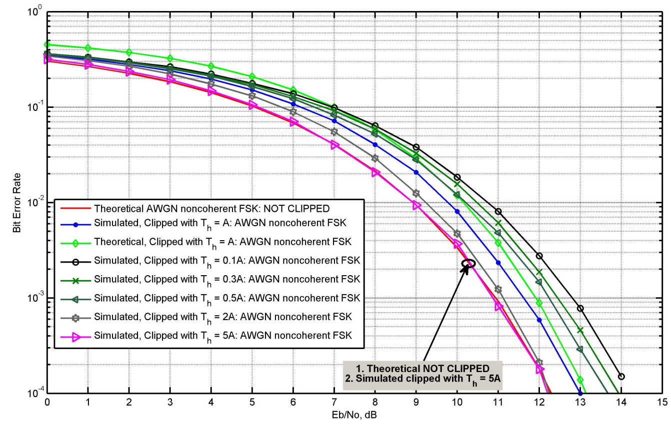

Fig. 3. The effect of clipping a FSK modulated waveform that is affected by AWGN only. Various clipping thresholds are tested.

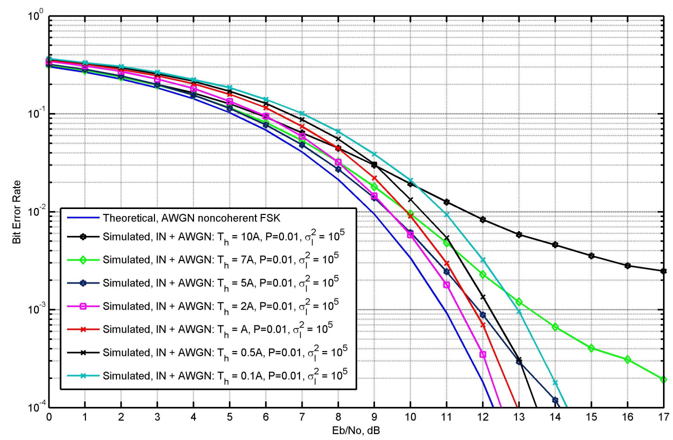

Fig. 4. The effect of clipping a FSK modulated waveform that is affected by AWGN and IN. Various clipping thresholds are tested, for $p = 0.01$ and $\sigma_I^2 = 10^5$.

In Figure 4, where $p = 0.01$, it can be observed that $T_h = |A|$ is not the best threshold, while in Figures 5 and 6 the best threshold is $T_h = |A|$. It is also observed from both Figures 3 and 4 that a threshold of $T_h = 2|A|$ is good for both cases. The reason for this is that in Figure 4 the probability of IN , $p = 0.01$ is low enough for the performance to be comparable to the AWGN only case in Figure 3. When the probability of IN starts getting higher, it is observed that the best threshold is $T_h = |A|$ as seen in Figures 5 and 6, where $p = 0.1$ and $p = 0.2$, respectively.

Figures 7 and 8 show the expected result that increasing the variance of IN or the probability of IN deteriorates the performance. Figure 7 presents performance results of increasing σ_I^2 for a fixed probability of IN $p = 0.01$. In Figure 8, the results show that increasing p, for a fixed threshold ($T_h = |A|$) and fixed large variance ($\sigma_I^2 = 10^5$), deteriorates the performance.

The results show the following: (a) While a clipping threshold equal to the amplitude of the FSK modulated carrier ($T_h = |A|$), dramatically reduces the effect of impulse noise

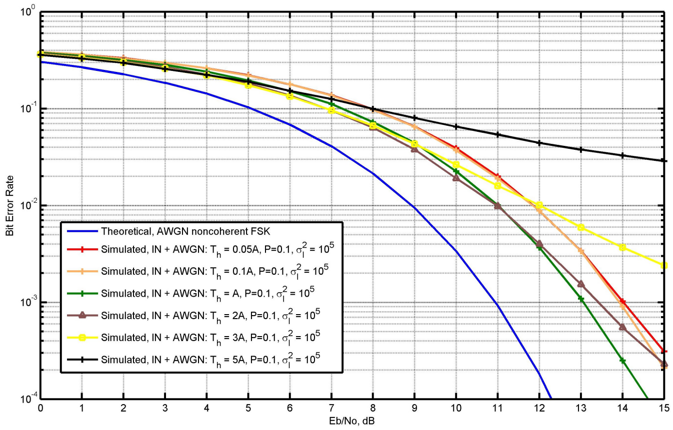

Fig. 5. The effect of clipping a FSK modulated waveform that is affected by AWGN and IN. Various clipping thresholds are tested, for $p = 0.1$ and $\sigma_I^2 = 10^5$.

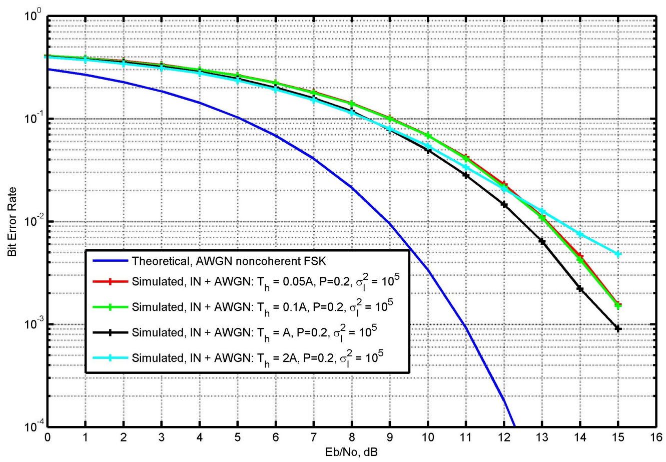

Fig. 6. The effect of clipping a FSK modulated waveform that is affected by AWGN and IN. Various clipping thresholds are tested, for $p = 0.2$ and $\sigma_I^2 = 10^5$.

and result in an analytical expression for the SNR, it is not the optimal clipping threshold. This conclusion can be drawn from Figures 3 and 4. (b) Figure 3 shows that for the case when only AWGN is present, clipping makes the performance worse than when no clipping is performed. This is due to the fact that the FSK signal's energy is reduced significantly, due to clipped, compared to the AWGN's energy.

VII. Conclusion

The paper has presented results on impulse noise clipping in FSK modulated signal, showing the effect of clipping the time-domain FSK signal waveform as opposed its constellation. The performance results showed that clipping is beneficial when there is impulse noise in the communication system, giving a performance that is close to the AWGN case with a difference of about 0.9 dB for $p = 0.001$, 1 dB for $p = 0.01$, 2.5 dB for $p = 0.1$, and 4 dB for $p = 0.2$. The results also show that while we used $T_h = |A|$ to obtain the analytical expression,

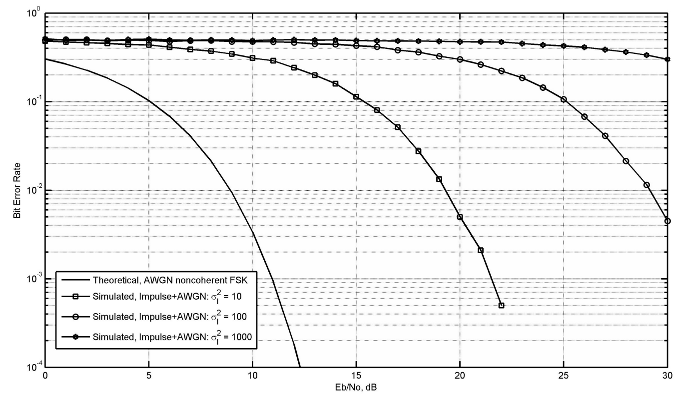

Fig. 7. The effect of IN and AWGN on a FSK modulated waveform. The results are for $p = 0.01$ and various values of the IN variance ($\sigma_I^2 = 10$, 100 or 100).

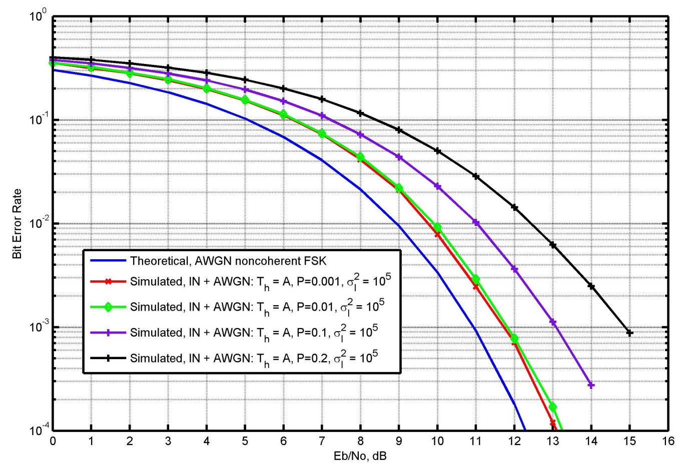

Fig. 8. The effect of IN and AWGN on a FSK modulated waveform. The results are for $T_h = |A|$, $\sigma_I^2 = 10^5$ and various values of the probability of IN occurrence ($p = 0.001$, 0.01, 0.1 or 0.2).

it is not always the best clipping threshold. $T_h = |A|$ is the best threshold for higher p, and for very low values of p the threshold has to be larger than A, $T_h > |A|$.

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