REALISTIC MATHEMATICS EDUCATION AS A LENS TO EXPLORE TEACHERS' USE OF STUDENTS' OUT-OF-SCHOOL EXPERIENCES IN THE TEACHING OF TRANSFORMATION GEOMETRY IN ZIMBABWE'S RURAL SECONDARY SCHOOLS

BY

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Submitted in accordance with the requirements for

The degree of

DOCTOR OF EDUCATION

in the subject of

CURRICULUM STUDIES

at the

UNIVERSITY OF SOUTH AFRICA (UNISA)

SUPERVISOR: PROFESSOR MG NGOEPE

November 2017

DECLARATION

I, SAMUEL SIMBARASHE MASHINGAIDZE, declare that REALISTIC MATHEMATICS EDUCATION AS A LENS TO EXPLORE TEACHERS' USE OF STUDENTS' OUT-OF-SCHOOL EXPERIENCES IN THE TEACHING OF TRANSFORMATION GEOMETRY IN ZIMBABWE'S RURAL SECONDARY SCHOOLS is my own work and that all the sources I have used or quoted have been indicated and acknowledged by means of complete references.

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ABSTRACT

The study explores Mathematics educators' use of students' out-of-school experiences in the teaching of Transformation Geometry. This thesis focuses on an analysis of the extent to which students' out-of-school experiences are reflected in the actual teaching, textbook tasks and national examination items set and other resources used. Teachers' teaching practices are expected to support students' learning of concepts in mathematics. Freudenthal (1991) argues that students develop their mathematical understanding by working from contexts that make sense to them, contexts that are grounded in realistic settings.

ZIMSEC Examiners Reports (2010; 2011) reveal a low student performance in the topic of Transformation Geometry in Zimbabwe, yet, the topic has a close relationship with the environment in which students live (Purpura, Baroody & Lonigan, 2013). Thus, the main purpose of the study is to explore Mathematics teachers' use of students' out-of-school experiences in the teaching of Transformation Geometry at secondary school level.

The investigation encompassed; (a) teacher perceptions about transformation geometry concepts that have a close link with students' out-of-school experiences, (b) how teachers are teaching transformation geometry in Zimbabwe's rural secondary schools, (c) the extent to which students' out-of-school experiences are incorporated in Transformation Geometry tasks, and (d) the extent to which transformation geometry, as reflected in the official textbooks and suggested teaching models, is linked to students' out-of-school experiences.

Consistent with the interpretive qualitative research paradigm the transcendental phenomenology was used as the research design. Semi-structured interviews, Lesson observations, document analysis and a test were used as data gathering instruments. Data analysis, mainly for qualitative data, involved coding and categorising emerging themes from the different data sources. The key epistemological assumption was derived from the notion that knowing reality is through understanding the experiences of others found in a phenomenon of interest (Yuksel & Yildirim, 2015). In this study, the phenomenon of interest was the teaching of Transformation Geometry in rural secondary schools. In the same light, it meant observing teachers teaching the topic of Transformation Geometry, listening to their perceptions about the topic during interviews, and considering how they plan for their teaching as well as how students are assessed in transformation geometry.

The research site included 3 selected rural secondary schools; one Mission boarding high school, a Council run secondary school and a Government rural day secondary school. Purposive sampling technique was used carefully to come up with 3 different types of schools in a typical rural Zimbabwe. Purposive sampling technique was also used to choose the teacher participants, whereas learners who sat for the test were randomly selected from the ordinary level classes. The main criterion for including teacher participants was if they were currently teaching an Ordinary Level Mathematics class and had gained more experience in teaching Transformation Geometry. In total, six teachers and forty-five students were selected to participate in the study.

Results from the study reveal that some teachers have limited knowledge on transformation geometry concepts embedded in students' out-of-school experience. Using Freudenthal's (1968) RME Model to judge their effectiveness in teaching, the implication is teaching and learning would fail to utilise contexts familiar with the students and hence can hardly promote mastery of transformation geometry concepts. Data results also reveal some disconnect between teaching practices as espoused in curriculum documents and actual teaching practice. Although policy stipulates that concepts must be developed starting from concrete situations and moving to the abstract concepts, teachers seem to prefer starting with the formal Mathematics, giving students definitions and procedures for carrying out the different geometric transformations.

On the other hand, tasks in Transformation Geometry both at school level and the national examinations focus on testing learner's ability to define and use procedures for performing specific transformations at the expense of testing for real understanding of concepts. In view of these findings the study recommends the revision of the school Mathematics curriculum emphasising pre-service programmes for teacher professional knowledge to be built on features of contemporary learning theory, such as RME theory. Such as a revision can include the need to plan instruction so that students build models and representations rather than apply already developed ones.

KEYWORDS:

Image, object, Students' out-of-school experience, Realistic Mathematics Education, Informal mathematics, Formal mathematics, *Mathematising*, Transcendental phenomenology, Transformation Geometry, Secondary Education, Rural School, ZIMSEC,

DEDICATION

I would like to dedicate this thesis to my entire family for their love, support and patience during my studies.

ACKNOWLEDGEMENTS

I thank God, the Almighty for granting me health and strength to complete my thesis.

I would like to thank my supervisor, Professor M.G Ngoepe, for professional guidance and encouragement throughout this long academic journey.

My heartfelt gratitude goes to my colleague Erick Nyoni who continuously gave me valuable insight to the success of this project.

I am also very grateful to Mberengwa District Education Officers who were always helpful and provided me with their assistance throughout my fieldwork. My sincere appreciation goes to the school heads of the three secondary schools in Mberengwa district.

Finally, I want to thank all who contributed by encouraging and supporting me to accomplish my dreams.

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LIST OF ABBREVIATIONS

CDASSG	Cognitive Development and Achievement in Secondary School Geometry
GoZ	Government of Zimbabwe
HOD	Head of Department
IRME	Indonesian Realistic Mathematics Education
O' Level	Ordinary Level
PCK	Pedagogical Content Knowledge
SoSE	Students' out-of-school Experiences
SPSS	Statistical Package for the Social Sciences
STEM	Science Technology Engineering and Mathematics
RME	Realistic Mathematics Education
TG	Transformation Geometry
ZJC	Zimbabwe Juniour Certificate
ZIMSEC	Zimbabwe Schools Examination Council

CHAPTER ONE
THE RESEARCH PROBLEM AND ITS CONTEXT

1.0 INTRODUCTION

This study explores the extent to which teaching and learning in Transformation Geometry (TG) embraces students' out-of-school experiences in Zimbabwe's rural secondary schools. The creation of such synergies is believed to enhance students in acquiring a more inclusive and holistic knowledge of concepts in mathematics (Freudenthal, 1991; Gravemeijer & Terwel, 2000). While it is the scope of this thesis to explore the teaching and learning experiences, the study also highlights critical reasons for teaching and learning in transformation geometry. Further, it affords opportunities to identify and use students' out-of-school experiences (SoSE) for informal mathematical experiences.

In developing this chapter, a number of themes provide conceptual boundaries for the discussions. The following are the major themes explored in the chapter: The Nziramasanga (1999) Presidential Commission of inquiry into Mathematics Education in Zimbabwe, Motivation for the study, Background to the study, Explanatory framework, Significance of the study, Research design and methodology.

1.1 THE NZIRAMASANGA (1999) PRESIDENTIAL COMMISSION OF INQUIRY INTO MATHEMATICS EDUCATION IN ZIMBABWE

This section unpacks introspections that were conducted by the Government of Zimbabwe (GoZ) (Nziramasanga Commission, 1999) in an attempt to clean up the teaching and learning practices which were detrimental to students learning of concepts in mathematics. A rise of unemployment levels in Zimbabwe left many people querying the relevance of the curriculum in terms of meeting the country's expectations. Various sections of society (such as commerce and industry) questioned the relevance of the curriculum as evidenced by criticism in various forms of the media. The GoZ through the relevant ministry sanctioned a review of the curriculum. It instituted the Presidential Commission of Inquiry into Education and Training in 1999 that recommended the re-focusing of education on the sciences, mathematics, and technology and life skills. With regards to Mathematics teaching the following recommendations were made;

A complete reorientation was deemed necessary in the field of mathematics education. Both the curriculum and the teaching methodology required major changes. Hence, the focus in this study was to explore nature of teaching and learning and be convinced that it contributes to student mastery of concepts in Transformation Geometry.

a. The curriculum was too academic and did not teach problem-solving or mathematical reasoning. The revision of the secondary school level syllabus needed to be varied so that it could cater for different content for the three career pathways to be introduced (Nziramasanga Commission, 1999).

b. A new approach to teaching methodology was required at all levels. Mathematics should be taught experimentally, like science, with a specialist mathematics classroom resembling a laboratory. Hence, this study sought to gain understanding into how teachers' teaching is supportive for students' engagement in transformation geometry.

c. Mathematics should be an entry requirement for all teachers, and their training in the teaching of maths needs to be overhauled. In-service training will need to be given to all who teach Mathematics (Nziramasanga Commission, 1999). Thus, this study will unpack the situation on the ground and make possible recommendations based on the current state of teaching and learning in transformation geometry.

1.2 MOTIVATION FOR THE STUDY

Mathematics and science are the most crucial learning areas responsible for the growth and development of individuals as well as the driving forces of socio-economic development of nations (Baker, Goesling & LeTendre, 2002; Kozma, 2005). Mathematics also occupies a core status in the secondary school curriculum because it is the key to the opening of career opportunities for students. Achievement in Science, Technology and Mathematics (STM) is increasingly recognised as one of the most trusted indicators in measuring the socio-economic and geo-political development among nations (Atebe, 2009; Justina, 1991). In that view, mathematics teaching should be effectual, focused and relevant.

Zimbabwe as a nation continues to perform badly in mathematics achievement at Ordinary Level (henceforth O' Level or Form 4), particularly in topics like Transformation Geometry. The results for students completing Form 4 and writing Zimbabwe Schools Examinations Council (ZIMSEC) O' Level mathematics examinations indicate a failure to provide learners with a meaningful education (National Education Advisory Board, 2010). The topic, Transformation Geometry at O' Level in Zimbabwe is regarded as the most difficult for an average student, particularly in rural secondary schools (ZIMSEC Examination Report, 1991, 1998, 2006). The ZIMSEC O' Level Mathematics Examiners' Report (2010) outlines seven topics in the Mathematics syllabus which requires special attention from the teachers where performance is poor. In the same report, candidates were reportedly having problems with the question on Transformation Geometry with a good number of candidates not even attempting the whole question (ZIMSEC, 2010). Thus, this study stands out to be of substance as it reveals the real challenges facing the teaching and learning of the said topic.

ZIMSEC (2010 p.2) highlights that in Transformation Geometry, the following are problem areas:

- Candidates fail to identify the type of transformation given an object and its image in diagram form or from the given matrices
- Transformation Geometry questions are not so popular with candidates. Weak candidates simply copied the diagrams on the questions and so wasted time.
- Transformation descriptions given were incomplete in most cases

The above evidence shows the unprecedented challenges befalling students in the topic of Transformation Geometry. Some of the questions asked in this inquiry are, 'Why do students fail, particularly, in Transformation Geometry?' and 'What are the possible sources of their challenges?' Literature also shows that both Mathematics teachers and students experience problems in the topic Transformation Geometry, since it is a little more abstract than the other topics (Harper, 2002; Keleş, 2009).

In Zimbabwe, more than a decade and half after independence, students' performance in Mathematics in general has always been a cause for concern. According to *The Herald* (2013), the November 2012 O' Level results revealed low pass rates with Mathematics recording the lowest (13,91%) as compared to other subjects. Table 1.1 below shows the over-all Mathematics O-level pass rate for the period 2011-2015.

Table 1.1: Ordinary Level Mathematics pass rate 2011-2015

Year	2011	2012	2013	2014	2015
Registered Students	241 512	125 408	126 099	121 945	126 567
% Pass Rate	19.5	13.91	21.62	20.05	26

[Source: ZIMSEC, 2011; 2012; 2013; 2014; 2015]

Table 1.1 above shows the percentage pass rate of 13.9 % - 26 % and comparatively the rural student struggles in mathematics more than the urban student. Whilst these results combine the performance of students both from rural and urban secondary schools, a study by Chirume (2017) found out that on average more students at schools in rural areas made more errors in mathematics compared to their urban counterparts. The same study confirms that Children from rural schools are mostly disadvantaged because besides having to walk long distances to school, they do not have adequate facilities to enable them attain good results in their final examinations. Thus the current study explored possibilities of having the rural student find interest and increase one's performance in mathematics.

Generally the percentage pass rate above indicates a very low overall performance by students in Mathematics although the Table shows some positive improvement in pass rates over the five year period. According to ZIMSEC Results Analysis (2015) the slight improvements were attributed to Government's thrust in the teaching and learning of Science, Technology, Engineering and Mathematics (STEM) subjects. STEM is an acronym that stands for Science, Technology, Engineering and Mathematics programme. With this programme, the Government of Zimbabwe's thrust was to develop students' critical thinking and interest in solving Mathematics and Science problems (Gandawa, 2016).The STEM programme came as a revitalisation movement aimed at the proliferation of the uptake of science and mathematics subject disciplines by learners (Van der Wal, Bakker & Drivers, 2017).

However, the same report (ZIMSEC, 2010 p.1) confirms that the Mathematics pass rate is still very low compared to other subjects. Evidence of continual low Mathematics performance by students justifies why it is prudent to revisit the teaching and learning processes enacted by teachers in Zimbabwe.

The Minister of Primary and Secondary Education made a comment on the performance in Mathematics where he says,

> *There is a declining interest in Mathematics among our learners due to poor teaching, low numbers of learners continuing with mathematics at secondary level and beyond* (The Herald, 25 May 2017).

From the Minister's remark, two essential elements can be realised. Firstly, teachers are using teaching methods which can hardly promote student learning. Secondly, numbers of students advancing in Mathematics continues to dwindle. Essentially, the goal of teaching Mathematics should not only to make students become fluent in performing certain procedures and steps but also to create a web of the different but related procedures including Mathematical concepts as used in the real world (Hebert & Grows, 2007).

From the researcher's experience as well as validation from colleagues who are O' Level Mathematics teachers, Mathematics is a dreaded subject by students, particularly those in rural schools, and Transformation Geometry is one of the difficult topics where students' performance is always low. According to Ekawati and Lin (2014) if teaching and learning must go a milestone in terms of helping students in mastering concepts in Mathematics it must begin with contextual situations which are meaningful and familiar to students. It is against this background that this study makes an inquiry into whether the teaching and learning in Transformation Geometry in the rural school in Zimbabwe embodies aspects of students' life experiences, given that a handful of challenges, such as access to resources, seem to affect the rural students' performance compared to their urban counterpart. Baroody and Hume (1991) concur that instruction in Mathematics can foster mastery provided it focuses on understanding, promotes active and purposeful learning; fosters informal knowledge, and links formal instruction to informal knowledge.

Contextual situations which are meaningful and known to students must be elaborated with Mathematics learning (Ekawat & Lin, 2014; Gravemeijer, 2015). It is the opinion of the researcher that the low attainment in the topic of Transformation Geometry is as a result of many factors. These factors include the type of instruction learners receive and the teaching and learning environment (Akay, 2011) within which they experience the Mathematics (Barnes, 2004).

The hypothesis drawn out of this discussion was: teachers' instructional approaches in the teaching and learning of Transformation Geometry fails to uphold Baroody and Hume's (1991) features that support for the mastery of concepts shown above. Following Baroody and Hume's (1991) concurrence, this study explores Mathematics teachers' incorporation of students' out-of-school experiences (informal mathematics knowledge) in the teaching and learning of Transformation Geometry (formal knowledge).

1.3 BACKGROUND TO THE STUDY

The Zimbabwe school curriculum for O' Level Mathematics Syllabus consists of the learning outcomes for what students should be able to do and know. For example, teachers in Mathematics are expected to enable students to:

- appreciate, understand and converse mathematical information in everyday life
- acquire Mathematical skills for use in their everyday lives and in national development (ZIMSEC, 2012 p.3).

To achieve these, the curriculum aims of the syllabus stipulate several ways which include the development of concepts beginning from concrete situations (*the immediate environment*) and extending to abstract ones and that skills must be learnt only after prerequisite concepts and principles are mastered (ZIMSEC, 2012 p.4). In other words, the school curriculum refers to teaching practices which can manage the transition from students' informal everyday Mathematical knowledge to the formal school-taught Mathematical knowledge. This is an important aspect in the construction of Mathematical concepts (Greens, Ginsburg & Balfanz, 2004; Purpura, Baroody & Lonigan, 2013).

Despite all these clearly outlined guidelines, students continue to do badly in the topic, Transformation Geometry. The researcher became interested to examine the teaching and learning of the topic, Transformation Geometry, in the context of the curriculum expectations mentioned above. Thus, the aim of this study is twofold, which are to,

- Explore Mathematics teachers' perceptions on transformation geometry concepts which are contained in students' out-of-school experiences.
- Explore Mathematics teachers' use of students' out-of-school experiences (informal mathematics) in the teaching and learning of transformation geometry (from a conviction that this is one way of enhancing mastery of concepts in Mathematics (Ekawat & Lin, 2014; Gravemeijer, 2015).

Zimbabwe like any other country attempts to offer education that enforces socio-economic development in the country. Responding to low student uptake of Science and Mathematics subjects, GoZ introduced quite a number of initiatives. The most recent of such initiatives is the Science Technology Engineering and Mathematics (STEM) programme introduced in 2014. The programme was meant to foster student attitudes and interest towards science and mathematics. The STEM initiative means the functions of teaching and learning must shift from overemphasising knowledge delivery to putting emphasis on students' realities and their active participation to develop their competence in disciplines such as Mathematics (Gainsburg, 2008).

Teaching that is transitive and starts with definitions and formulae completely opposes what creative Mathematicians do (Gravemeijer, 2015). The researcher contends that the STEM initiative directly calls for a relook at teaching and learning processes enacted in the schools, particularly paying attention on how teachers embrace students' background knowledge systems. This means teaching and learning of Mathematics as implemented by teachers has to come under the microscope as a possible way to improve it. Since there is very poor performance in Transformation Geometry, the researcher maintains that there is need to explore if teaching and learning in the topic is as effective as can contribute to mastery of concepts.

The STEM initiative cemented the idea that subjects such as Mathematics have got to be compulsory to every school-going child in Zimbabwe from primary to secondary school level (Gandawa, 2016). Statistics about student performance in Mathematics referred to in Table 1.1 above is likely to affect initiatives such as STEM and others. These statistics on performance can be examined through an inquiry into the teaching and learning processes enacted by Mathematics educators (Zakaria & Syamann, 2017). The quality of what happens in a teaching and learning environment is a product of what teachers do in the classroom.

Transformation Geometry is a branch of Mathematics that has the closest link with the world around us and the space in which we live (Clements & Samara, 2010; Leitzel, 1991; NCTM, 1989), yet Mathematics teachers do not emphasise on application of its concepts in their teaching (Tate, 1994). For instance, in play and daily activities, children often explore Mathematical ideas when they seek, classify, compare and notice shapes and patterns (Naidoo, 2012).

Some researchers, for example Tate (1994) and Bansilal and Naidoo (2012) argue that the content of Mathematics taught in schools is so removed from students' everyday life experiences making it appear irrelevant. Taylor (2000) also highlights the mismatch between students' uses of Mathematics outside the school and the ways through which Mathematics is presented as a school-taught subject. Mathematics teaching in our schools emphasises repetition, drill, convergent, right answer thinking and inevitability (Hansen, 2015; Ladson-Billings, 1995). Students are asked to execute similar problem tasks over and over. They are seldom required to contest the rule or procedure of Mathematics. Rarely are their prior knowledge and experiences required to support or conflict with school practices (Purpura, Baroody & Lonigan, 2013). In other words, the teaching and learning practices that place emphasis on mechanical processes continue to dominate classroom practices.

Zimbabwean students, in particular those in rural schools, though they come from diverse home backgrounds, and go through many experiences, their experiences are neither reinforced nor represented in school Mathematics. According to the Nziramasanga commission (1999), students' experiences are seldom utilised in classroom situations in order to connect them to any mathematical foundations. In Geometry the most challenging area for teachers is in coming up with activities that can increase learning gains in students (Choi-Koh, 2000). Prominent in Mathematics teaching and learning reforms is the call not to make students memorise formulae and procedures but to be engaged in processes that Mathematicians went through (NCTM, 2000; 2006).

The rationale and inspiration behind this thesis stems from the researcher's interest in Geometric Transformations as a topic of the Mathematics curriculum. Further, back-dated to the time when the researcher was a student, the inspiration also stems from the researcher's personal as well as from other teachers' classroom experiences. In addition, there is very limited research on learners' understanding and learning of Transformation Geometry (Bansilal & Naidoo, 2012) warranting a need for research in this area. After graduating with a Teaching Diploma as a secondary school teacher the researcher was then deployed at a rural secondary school. During the researcher's tenure at the school, there were some Mathematics teachers who occasionally asked him to teach the topic, Geometric Transformations on their behalf. Further, some teachers would allocate time for teaching the topic towards the end of the O' Level course as a strategy to either prepare candidates for the national examinations or to avoid having to spend much time dealing with a topic proving otherwise challenging for them to teach. The other observations are that the strategy of delaying teaching the topic necessitated hurried coverage or omission of the topic altogether. This is evidence therefore, of existing challenges among teachers themselves in the teaching of Transformation Geometry.

Geometry has four conceptual aspects (Clements, Battista & Sarama, 2001; Usiskin, 1987). The first conceptual aspect, visualisation, depiction, and construction, focuses on visualisation, sequence of patterns, and physical drawings. However, a diagram given on the Cartesian plane, for instance, may elicit visualisation strategies provided the student is able to recognise what is given (Bansilal & Naidoo, 2012). Hence, by bringing real life situations in the classroom, it is hoped that this will provide students with something to recognise in as far as the aspects are concerned. In this study, the act of visualisation is mainly related to external constructions of objects, figures previously known to the student in the form of the student's real-life experiences (Bansilal & Naidoo, 2012).

Visualisation is a mental construction of external objects or processes. According to Zazkis et al. (1996), thinking begins as an act of visualisation. A student, who is able to connect a transformation problem to a real-life scenario, has greater chances of succeeding in a task. According to Duval (2006) conversion type activities which involve movement across different representation are essential for deepening of understanding. Thus, the study explores students' out-of-school experiences that can provide opportunities for students to engage in activities that emphasise conversion instead of concentrating on treatment – type problems (Bansilal & Naidoo, 2012).

Learning Transformation Geometry starts with the student's visualisation, mental manipulation as well as spatial orientation about figures and objects. Through the study of transformations, Clements, Battista and Sarama (2001) as well as Leitzel (1991), concur that students develop spatial visualisation and the ability to mentally transform two dimensional images. Two dimensional transformations are an important topic which all students must study. The recommendation is that all middle grades students study transformations (NCTM, 1989, 2000, 2006).

Geometric Transformations are one thread of geometry whose study should make abundant use of student experiences and their active involvement (Brown & Heywood, 2011). Constructing models, folding paper, using mirrors, geo-boards to mention a few, should all provide opportunities for students to learn by doing and communicate their observations and conclusions.

The researcher became deeply interested in the investigation of what situations/experiences of learners can teachers embrace to enhance Mathematics learning with Geometric Transformations? This takes into account students' informal solution strategies and interpretations through experientially real context problems (Duval, 2006). The heart of this approach lies in *mathematising* activities in problem contexts that are experientially real to students.

This research aims to establish the status quo in the teaching and learning of Transformation Geometry and come up with possible reasons that explain students' low performance in the topic.

Students' motivation increases considerably when they understand why they are learning the concepts and how those concepts become relevant outside the classroom (Cord, 1999; Purpura, Baroody, & Lonigan, 2013). All students could benefit when classroom mathematics reflected their everyday practices. When mathematical concepts such as rotation and shear are introduced in the classroom, the concepts are not completely novel to the students. Students already know several situations which have a relationship with the concepts of rotation and reflection although there are conceptualised by mathematicians as mathematical transformations, and eventually as elements of a group structure (Luneta, 2015). It is against this background that this study embarks on a research where the teaching of Transformation Geometry was put under a microscope to assess how it embraces students' real-life experiences.

1.4 STATEMENT OF THE PROBLEM

Education in Zimbabwe, particularly in Mathematics, continues to undergo a process of change where emphasis is geared towards making students independent and active learners (Zimbabwe New Curriculum Report, 2015). A Mathematics teacher's role is to simplify mathematics content and demystify the notion that it is difficult through showing students that the subject is quite meaningful to their experiences (Zakaria & Syamann, 2017). In spite of the changes being lobbied for in teacher pedagogy, many teachers today continue to use the traditional approach in their lesson delivery resulting in students memorising formulae in mathematics for them to pass (Zakaria & Syamann, 2017). The main challenge affecting teaching and learning in schools is the discord between teaching in theory (as enshrined in policy documents) and the actual teaching practice in schools (Zeichner, 2010). In other words, teachers concentrate more on learning outcomes at the expense of the learning process.

The country continues to experience low levels of achievement in topics such as Transformation Geometry in Mathematics. Little regard is given to how well the students understand the geometrical concepts. On the topic of transformations, students encounter difficulties in linking their experiences with what they have learned as they are not afforded opportunities to understand the concepts this way. Instead of capitalising on these experiences they always rush to traditional forms of teaching. Learning Transformational Geometry may not be easy, and a large number of the students fail to develop an adequate understanding of the concepts, geometry reasoning, and geometry problem solving skills (Battista, Clements, Arnoff, Battista & Borrow, 1998; Noraini, 1999).

The nature of examinations and teachers' teaching approaches put emphasis on how much the students can memorise and less on how well the students can think and be able to master (Sunzuma et al., 2013). Thus, learning becomes unnatural and rarely brings satisfaction to the students. According to Gravemeijer (1994, 2015) Realistic Mathematics Education (RME) is one of the many instructional theories which offer guidelines for instruction that support learners in the mastery of concepts in problem-based interactive instruction. However, students can only benefit from such instruction if their teachers are knowledgeable in the instructional theory.

A lot of research has concentrated on intervention strategies (Dobitsh, 2014; Ekowati & Nenohai, 2016; Zakaria & Syamann, 2017) where the effect of RME approaches on student learning is measured. However, very limited inquiry has gone back to the schools to find out whether the actual teaching and learning does in any way embrace RME principles such as the reality principle, which states that teaching and learning must aim to bridge students' informal mathematical knowledge with the school formal mathematics. Hence, this study focuses on teacher practices in the teaching of one of the most difficulty topics in mathematics, transformation geometry. The study explored the extent to which teachers utilise students' out-of-school experiences (one of the elements of the Realistic Mathematics Education theory) in teaching Transformation Geometry.

1.5 EXPLANATORY FRAMEWORK

This section presents the framework that guided the study. It discusses, in brief, how the theoretical framework underpinned this study.

1.5.1 Realistic Mathematics Education (RME) Theory

The study is underpinned on Freudenthal's (1991) Realistic Mathematics Education (RME). Freudenthal has made a huge impact with his RME model in Mathematics Education (de Lange, 1996; Gravemeijer, 2015; Ekowati & Nenohai, 2016; Zakaria & Syamann, 2017). RME is an instructional theory of teaching and learning mathematics which emphasises on increasing students' understanding and motivation in mathematics (de Lange, 1987; Freudenthal, 1991; Gravemeijer, 1994). The theory of RME has its own philosophical characteristics which mainly focus on what mathematics is and how it should be taught. The philosophy of RME is strongly influenced by

Freudenthal's notion of mathematics as a ‘human activity’ (Freudenthal, 1991). In other words, it calls for active participation of the student for his/her learning.

In its country of origin, the Netherlands, RME had a substantial impact on Mathematics learning programmes. The Netherlands scored very well in international benchmark tests, the International Mathematics and Science Study (TIMSS), and almost all Mathematics textbooks now embrace the RME theory (Van den Heuvel-Panhuizen, 2010). The traditional approaches to in-service teacher education have probably not achieved their intended goals due to a myriad of factors. For example, in the 1980s, the market share of primary education textbooks designed with a traditional, mechanistic approach was 95% and the textbooks with a reform-oriented approach (based on the notion of RME) had as low a market share as 5% (van den Heuvel-Panhuizen, 2010). However, in 2004, reform-oriented textbooks attained a 100% market share and the ones based on the mechanistic approach became very unpopular to fall to 0% market share.

Realistic Mathematics Education (RME) has its underlying principles as guided reinvention, didactical phenomenology, and emergent models. These themes are grounded on Freudenthal’s philosophical assumptions which emphasise the notion of reinvention through progressive *mathematisation* (Fredenthal, 1973, 1991). In RME, context specific problems are the foundation for progressive *mathematisation*, and through *mathematising*, where the students develop informal context-specific solution strategies from experientially realistic situations (Gravemeijer & Doorman, 1999). It is in the thrust of this study to explore how the teaching and learning of transformation geometry embraces context problems drawn from students’ out-of-school experiences.

Three guiding heuristics for RME instructional design must be incorporated (Gravemeijer, Cobb, Bowers, & Whitenack, 2000). The first of these heuristics is reinvention through progressive *mathematisation*. According to the reinvention principle, the students should be given a chance to practice a process similar to the process by which the Mathematics was invented. The re-invention principle means the coming up of teaching and learning activities that should present students with real situations where they (students) are likely to come up with informal solution strategies (Freudenthal, 1973). Thus, the teacher can look at the history of transformation geometry as a source of stimulation as well as focus at informal solution strategies of students who are solving experientially real problems for which they do not know the standard solution procedures yet

(Streefland, 1991; Gravemeijer, 1994). In this study, the process of progressive *mathematisation* in the teaching of Transformation Geometry was examined.

The second heuristic is didactical phenomenology. Freudenthal (1973) defines didactical phenomenology as the relationship between the phenomena in which the mathematical concept is represented and the concept itself. In this phenomenology, the focus was on how mathematical interpretations make phenomena accessible for reasoning and calculation. The didactical phenomenology focuses on possible instructional activities that might support both individual work and collaborative work in which the students engage in progressive *mathematisation* (Gravemeijer, 1994). Thus, its aim is to generate settings where students can cooperatively gain increasingly sophisticated solutions to realistic problems through individual activity and collaborative work (Gravemeijer, Cobb, Bowers & Whitenack, 2000).

RME's third heuristic for instructional design put emphasis on the role played by emergent models in bridging the gap between students' informal mathematical knowledge and the formal school mathematics. The term model is understood in a dynamic, holistic sense. As a result, the symbolizations that are embraced in the process of modelling and that make up the model can alter over time. Thus, students first develop a model of a situated activity, and this model later becomes a model for more advanced mathematical reasoning (Gravemeijer & Doorman, 1999).

Thus, RME's heuristics of reinvention, didactical phenomenology, and emergent models served to enlighten how effectual learning trajectories in Transformation Geometry classes were. In the same light, these heuristics served to show how teacher mathematics educators build connections between informal and formal mathematical knowledge in transformation geometry (Webb, Kooij and Geist, 2011).

1.6 RESEARCH QUESTIONS

Aligned with the research problem stated earlier, the main research question is: **To what extent do educators embrace students' out-of-school experiences in the teaching of geometric transformations at Secondary school ordinary level (O' Level)**?

The research question is further sub-divided giving the following:

1. What are teachers' perceptions about the mathematics involving transformational geometry concepts contained in the students' out-of-school activities?
2. How is the context of Transformation Geometry teaching implemented by practising teachers in Zimbabwe rural secondary schools?
3. To what extent are students' out-of-school experiences incorporated in Transformation Geometry tasks?
4. How is transformation geometry, as reflected in the official textbooks and suggested teaching models, linked to students' out-of-school experiences?

1.7 SIGNIFICANCE OF THE STUDY

The chief aim of this study is to explore the extent to which teachers of mathematics employ students' life experiences in the teaching of Geometric Transformations. The research has strategic importance since Transformation Geometry constitute one of the most important topics in mathematics that play a critical role in the social, political and economic development and transformation of society (Baykul, 2002; Gurbuz, 2008; NCTM, 2000).

This thesis complements the STEM initiative, where students are expected to demonstrate knowledge in Mathematics through using it in different facets of their life, by the Ministry of Primary and Secondary education by exploring a how Mathematics educators embrace SoSE into their teaching. Teachers' emphasis of SoSE in their teaching lies with this broader goal of STEM. The study of Transformation Geometry has improved geometry to a dynamic level through offering the student with a powerful problem-solving tool (NCTM, 1989). Spatial reasoning and spatial visualisation through transformations facilitate the construction and manipulation of mental representations of two dimensional objects (NCTM, 2000). Students need to investigate shapes, including their properties, attributes, and transformations. Hence, this study is one step amongst the many in increasing the student access to concepts in Transformation Geometry.

Geometric Transformations, for Secondary school students are composed of five basic concepts: translations (slides), reflections (flips or mirror images), rotations (turns), enlargement (size changes), shear and stretch, and the composite transformation of two or more (Wesslen & Fernandez, 2005). Transformation concepts provide background knowledge to develop new perspectives in visualisation skills to clarify the concepts of congruence and similarity in the development of spatial sense (NCTM, 1989). Spatial reasoning, including spatial orientation and spatial visualisation, is a characteristic feature that is related to one's mathematical ability (Brown & Wheatley, 1989; Clements & Sarama, 2010).

According to Hollebrands (2003), there are three significant reasons to justify the study of Geometric Transformations in school mathematics which are;

1. It offers opportunities to students to reason about important mathematical concepts (e.g., functions, symmetry, and similarity).
2. It offers a realistic context through which students can develop a perception of Mathematics as an unified discipline, and,
3. It offers opportunities to students for engagement in higher-level reasoning activities using numerous representations. Hence, carrying out a study such as this will increase students' chances to highly engage in mathematics concepts.

Fashioned by the instructional design theory of Realistic Mathematics Education, this research advocates for approaches in the learning and teaching of Geometric Transformations that are different from the traditional ones. Introducing a change to classroom discourse would mean a change in the nature of the classroom environment that is the way students learn Mathematics or interaction between teachers and students.

The findings of this study are intended to benefit Mathematics teaching and learning particularly in transformation geometry. To Mathematics teachers, the study serves as a call to introspect their effectiveness in teaching mathematics topics by reflecting on syllabus expectations. In other words, the teaching and learning in mathematics ought to be analysed in terms of how it matches with pedagogical demands enshrined in the syllabus document (*see Appendix N)*.

To the curriculum development unit of Zimbabwe, a study of this nature results in a need to seriously relook at possibilities of introducing RME-based curriculum in the teaching and learning of Mathematics and other disciplines in the country. In view of the outcomes of the study, educational policies and practices can be revisited particularly in the area of teacher capacitation as well as curriculum material development, such as textbooks. Thus, the study is of significance in that it may highlight the need for staff development in the form of Ministry of Education sponsored workshops at which teachers share ideas on the teaching and learning strategies of Geometric Transformations in particular and Mathematics in general. The study promotes further research on intervention strategies on the teaching of Geometrical Transformations that embrace the aspect of students' life experiences, thereby partly responding to a call for increased research on teaching and learning practices in transformation geometry (Kirby & Boulter, 1999, Hollebrands, 2003).

1.8 OBJECTIVES OF THE STUDY

The main aim is to explore Mathematics educators' use of students' real-life experiences in the teaching of Transformation Geometry in Zimbabwe's rural secondary schools. To achieve this goal, the main aim was divided into the following objectives:

1. To determine Mathematics teachers' perceptions about the mathematics involving Transformation Geometry concepts contained in the students' out-of-school experiences.
2. To analyse the context of Mathematics teaching in Transformation Geometry in Zimbabwean schools as implemented by mathematics teachers.
3. To determine the extent to which students' out-of-school experiences are incorporated in Transformation Geometry tasks.
4. To determine how the official textbooks and suggested teaching models used by teachers in the teaching of Geometric Transformations relate to students' out-of-school experiences.

1.9 DELIMITATIONS OF THE STUDY

The study focused on teacher practices in Transformation Geometry at three different types of rural secondary schools in Mberengwa district. Six teachers were selected to explore the extent to which their teaching utilises students' out-of-school experiences. One mission boarding high school, one Council-run secondary school and one Government day secondary school were selected to gain data for the research. The three schools were selected using purposive sampling.

1.10 RESEARCH DESIGN AND METHODOLOGY

The Transcendental phenomenological qualitative research design was used. This design was found suitable as justified in the sub-sections shown below;

1.10.1 Empirical inquiry

To establish the connection between students' out-of-school experiences and the teaching of Geometric Transformations the transcendental phenomenological research approach was initiated. Consistent with the postmodern qualitative paradigm the transcendental phenomenological approach focuses on the ways that the life world, the world every individual takes for granted is experienced by its members (Holliday, 2007 p.16). Phenomenology offers a descriptive, reflective, interpretive and engaging mode of inquiry from which the fundamental nature of teaching and learning of geometric transformations was elicited (Mutemeri, 2013).

The major aim is to understand and describe the teaching and learning of Transformation Geometry in the context of rural secondary schools in Zimbabwe within its natural context. The intention was to see through the eyes of the participants (Nieuwenhuis, 2007:51) so that the process of teaching and learning could be described in terms of the meanings by them.

The main epistemological assumption was that the way of capturing reality was through exploring the experiences of others regarding a specific phenomenon, in this case teaching and learning of transformational geometry. Richards and Morse (2007) are of the view that to a phenomenologist reality is dependent on human beings. In other words, there is no reality out there. Hence, the study capitalized on the meanings as experienced by teachers of mathematics. The aim was to determine what the experience means for the teachers who have had the experience (Moustakas, 1994).

In phenomenological terms, knowledge is socially constructed within the socio-cultural and historical context (Goulding, 2004; Yuksel & Yildirim, 2015). Individuals are the sources of knowledge, knowledge that they have built, that is, through lived experiences. In this case, the researcher obtained descriptions of experiences through first-person accounts in interviews with teachers of Mathematics (Moustakas, 1994). In the same light, the experiences and voices of the respondents were the medium through which the researcher explored and understood reality embedded in the teaching and learning of Geometric Transformations.

This study is underpinned on the naturalist or interpretive view of knowledge that says knowledge is gained by studying reading the meanings explain phenomena studied; a researcher interacts with the participants to obtain data (Krauss, 2005). The 'problem' for many researchers with phenomenological research is that it generates a large quantity of interview notes and tape recordings which have to be analysed. Analysis is also essentially untidy, as data does not easily fall into precise categories and there can be many ways of linking between different parts of discussions or observations. However, phenomenological approaches are superior at surfacing profound issues and making voices heard (Yuksel & Yildirim, 2015).

1.10.2 Sampling and sample composition

Two main participants for the study were teachers and students. The participants were chosen using purposive sampling techniques for teachers and simple random sampling for students. According to Silverman (2013), purposive sampling makes us select a case because it demonstrates some feature where our interest is. Purposive sampling technique was used to select a teacher who had some experience in teaching the topic of Transformation Geometry. A total of six teachers of Mathematics were selected to provide their experiences in teaching the topic through interviews and lesson observations. Of this number, three teachers already teaching an ordinary level class then were purposively selected to allow for lesson observations in transformation geometry. Collecting data from teachers of Mathematics was important so as to learn how they have been ensuring effective teaching and learning of Geometric Transformations.

On the other hand, students were selected using a simple random sampling technique. The sample of students comprised of three O' Level (Form 4) classes, one from a boarding high school another from a council run secondary school and the other from a rural day secondary school. The major reason is that form four students had gone through four years learning mathematics and were on the verge of writing their examinations in the topic. At the time of the study the students were attending lessons in Transformation Geometry.

1.10.3 Data collection instruments

For purposes of this research, data was collected using interviews with the six teachers and from observing the teachings of three teachers, one in a boarding high school, and another in a council run school and the third in a government day public school. Only two lessons per the three teachers were observed on different topics under Transformation Geometry. The data collected included classroom observation notes, audio-recordings of lessons which were then transcribed. Photographs were taken also to provide information on work presented on chalkboard and some demonstrations.

1.10.3.1 Interviews

The major means of data collection was the interview. Interviews were preferred as a tool for data collection because they allowed the researcher to tap into the experiences of mathematics teachers and their students in the topic of Transformation Geometry. Interviews provided rich data to build a solid basis for significant analysis of respondents' views and actions (Charmaz, 2006).

1.10.3.2 Observations

Heedful of the idea that there are variations of observer involvement, the researcher was a non-participant observer of the proceedings in the three O' Level (Form 4) classes led by their mathematics teacher in Transformational Geometry. The researcher, guided by an observation schedule (*see Appendix B)*, concentrated more on picking intervention strategies applied by teachers in teaching the concepts. During lesson observations the class sessions were audio-taped to obtain first-hand data on class discourse, from which interpretations based on the research problem were made. The naturalistic observation preserved rules of the game, that no manipulation of the data observed was required (Yuksel & Yildirim, 2015).

1.10.3.3 Document analysis

As Patton (1990) notes, a particularly rich source of information about many programmes derives from records and documents. In order to address the research problem, the following documents were identified to give information: schemes of work, students' daily exercise books, textbook illustrations and past exam question papers. The data was subjected to some content analysis based on the document analyses schedule (see *Appendix C*). This form of analysis was guided by the extent to which the information contained in the different sources embraced students' out-of-school experiences.

1.10.4 Data analysis

Data analysis is the process of making sense of the data and discovering what it had to say about how the teaching and learning of Geometric Transformation can utilise students' real-life experiences. It required an understanding of the phases in the Van Hiele model, that is, understands students' level of geometric thought based on the model. Further, analysis elicited the teacher's transition from forms of direct instruction towards the students' independence from the teacher, that is, how the teacher moves through the teaching phases and how he/she relates the content to students' out-of-school experiences.

In analyzing data an effort was made to establish how teachers made meaning of the relationship between content of Transformation Geometry and students' real life experiences by analyzing their perceptions, attitudes, understanding, knowledge, values, feelings and experiences in an attempt to approximate their reality (Giorgi, 2008). This was best achieved through inductive analysis of qualitative data where the frequent, dominant or significant themes that were inherent in the raw data were allowed to emerge (Hall, Chai & Albrecht, 2016). The researcher submitted himself to emerging patterns of data and he was free to engage strategically with realities that go beyond his initial themes (Holliday, 2007p.92). This was meant to provide parameters that ensure that the question that guided research was comprehensively explored. Holliday (2007 p.93) argues that, 'taking a purely thematic approach, in which data is taken holistically and rearranged under themes which emerge as running through its totality, is the classic way to maintain the principle of emergence.'

1.10.5 Ethical considerations

The researcher is mindful of ethical issues in phenomenological research such as not violating participants' rights. However, the ethical considerations for this study are discussed in detail in chapter three. Below, key ethical considerations are presented.

A wide range of data sources used in this study, that includes, teacher interviews, lesson observations and document analyses ensured the reliability of information got in the study as one source was verified across the various other sources of data (Shenton, 2004). Shenton goes on to advise that if a tape recorder has been used, the articulations themselves should at least have been accurately captured by thick descriptions of the phenomenon under scrutiny. Creswell (2013) also advises that for reliability, validity and trustworthiness of data to be achieved the researcher should be a good listener; information should be recorded accurately with early writing being initiated. To ensure that all this was realised, the researcher had both a tape recorder (for recording) and a notebook to jot down notes where possible, as the events unfolded. Consistent with the qualitative research, issues of credibility and dependability were considered as essential criteria in the attainment of trustworthiness, that is, the extent to which the conclusions will be trustworthy and could be depended upon as discussed in detail in chapter three.

1.11 DEFINITION OF KEY WORDS

1.11.1 Students' out-of-school Experience (SoSE)

This aspect defines students' informal Mathematical knowledge which for the purpose of this study is abbreviated SoSE. In this study, students' out-of-school experience is viewed the same way as students' life experiences. Informal Mathematical knowledge also refers to competencies generally learnt out-of-school settings, frequently in unprompted but significant everyday situations including play, and is characterised by employing nonconventional and even self-invented symbols, or procedures rather than conventional ones (Ginsburg, 1977; Purpura, Baroody, & Lonigan, 2013). In other words, the purpose of this study was to study how teachers provide opportunities to students for their transition from such experiences to the formal mathematical knowledge.

1.11.2 Realistic Mathematics Education

Realistic Mathematics Education (RME) is an instructional theory in Mathematics education that states that students develop their Mathematical understanding by working from contexts that make sense to them (Dickson et al., 2011). It emphasises the idea that Mathematics is a 'human activity' that must be connected to reality or lived experiences of participants.

1.11.3 Transformation Geometry

Transformation Geometry is a topic in Mathematics which has two main components, Isometric and Non-isometric transformations. It is also one topic in Mathematics that relates directly to students' spatial reasoning as an aptitude of an individual's mathematical ability (Brown & Wheatley, 1989; Clements & Sarama, 2010). Transformations also have a critical function in many of the artwork involved, for example, they appear in pottery patterns and tailings.

1.11.4 Secondary Education

Secondary education in Zimbabwe means a 4-year Ordinary Level programme of learning. There is unhindered advancement to the O' Level programme of learning but some schools set selection criteria on the basis of Grade 7 examinations (Zimbabwe National Commission for UNESCO, 2001).

1.11.5 Rural secondary school

In this study, a rural secondary school is that school in the countryside where its geographical area is located outside towns and cities. The term ‘rural’ encompasses all population, housing, and territory not included within an urban settlement.

1.11.6 ZIMSEC

The Zimbabwe School Examinations Council (ZIMSEC) is a Government of Zimbabwe institution responsible for decisions on assessment objectives and content of public national examinations, assessment and the awarding of end-of cycle grades (such as Grade seven, Ordinary and Advanced Levels). The Council can offer syllabus review suggestions.

Students sit for the General Certificate in Education Ordinary Level (O’ Level) at the end of four years of secondary education. This examination is comparable to the Cambridge University General Certificate of Education from which it originated. Zimbabwe has now localised its curriculum development and the setting (including marking) of examinations at this level. ZIMSEC certificates awarded at the end of a cycle determine admission into Advanced Level (A’ Level, Tertiary education and the labour market (Zimbabwe National Commission for UNESCO, 2001).

1.11.7 Informal mathematical knowledge

Informal mathematics knowledge is the students’ preconceived knowledge about mathematical tools which can help organise and solve a problem in a real-life situation. It is then the teacher’s role to facilitate build upon the student’s informal mathematics knowledge into the formal school mathematics knowledge.

1.11.8 Formal mathematics knowledge

This refers to Mathematics done within the axiomatic systems, for instance, mathematical rules, theorems etc (Schoenfeld, 2014). In this case, it refers to the procedures and rules used in performing specific transformations.

1.11.9 Horizontal Mathematisation

Horizontal mathematisation is the process of building mathematical tools to solve problems in realistic contexts (Webb, Koij & Geist, 2011). In other words, it is about solving problems given in a real-life context, moving from the real-life context to the world of symbols. In this case, the realistic contexts serve as supportive starting points that enhance student engagement and thinking.

1.11.10 Vertical Mathematisation

Vertical mathematisation is advancing within mathematical domains (Webb, Koij & Geist, 2011). In other words, it means reorganisation within the mathematical system, finding rules and procedures that connect between concepts.

1.12 OUTLINE OF THE STUDY

The study is made up of 5 chapters. Chapter 2 provides a detailed review of related literature as well as a description of the lens of the study, the Theoretical Framework which is the RME theory. Van Hiele's Model and Constructivism are major highlights of chapter 2 because they focus on how students meaningfully acquire mathematical concepts. This was then followed by chapter 3 on Research Methodology.

Chapter 3 outlines the Research Methodology. It provides grounding on the methods that underpinned the study. Chapter 4 presents; analyses and discusses the data as obtained. The outline of chapter 4 is such that emerging themes drawn from each research question are presented. The last chapter, chapter 5, is the Summary of the study. It also narrates on the study's Conclusions and Recommendations, including a section of the study's contribution to knowledge.

1.13 SUMMARY

Chapter 1 provided an advance organiser to the background of the study by highlighting issues about the research problem and its context as well as the explanatory framework. Some of the antecedents that prevail in the teaching of transformation geometry are briefly discussed. This chapter highlighted the study's motivation and elaborates on the background of the research problem. A brief description of some antecedents on the current situation surrounding teaching and learning of transformation geometry in Zimbabwe is provided. The significance of the study is also provided as well as making reference to methodological highlights. Finally, the chapter illuminates key terms and ends by giving an overview of the outline of the thesis. The next chapter discusses the Theoretical Framework as well as reviews literature informing the study.

CHAPTER TWO

THEORETICAL FRAMEWORK AND REVIEW OF RELATED LITERATURE

2.0 INTRODUCTION

In this chapter discusses theoretical perspectives which guide the study and reviews relevant literature. The main ideas pivotal to the study are discussed under the headings, Theoretical Framework and Conceptual Framework. The purpose of this chapter is to present related study findings and investigations to form the foundation on which this study was developed. The main focus of this study is to explore mathematics educators' use of students' real-life experiences in the teaching of transformation geometry at the secondary school level.

The review is divided into two major sections. The first section presents a discussion on the Theoretical Framework from which this study is developed. In this study the Realistic Mathematics Education Model (RME) is used as a framework to explain how learning in mathematics can exploit students' out-of-school experiences. The second section looks at the Conceptual Framework. It focuses on the tools or constructs on which the study was based. It mainly discusses tools that are used to measure students' knowledge levels and teacher practices. The historical perspectives of geometry in general and geometric transformations in particular are also analysed.

2.1 THEORETICAL FRAMEWORK

2.1.1 Realistic Mathematics Education (RME)

This section discusses the theoretical framework that was used as a lens to explore the main research question of the study. The study is underpinned on Hans Freudenthal's (1991) theoretical framework on Realistic Mathematics education model (RME). Hans Freudenthal has made a huge impact with his RME theory in Mathematics education (Freudenthal, 1968; 1971; 1991; Gravemeijer, 2015). He coined the principles of RME in order to explain how students can effectively acquire concepts in Mathematics.

Realistic Mathematics Education (RME) Model is enshrined in Freudenthal's (1971, 1991) ideas that Mathematics is a human activity and consequently must 'be connected to reality and should be relevant to society' (Ekawati & Lin, 2014 p.131). There are several models which can be used in the teaching and learning of mathematics, however this study is grounded in the RME model.

Realistic Mathematics Education Model is a theory in Mathematics education that was initially developed in the Netherlands. According to Dickson et al. (2011), Arsaythamby & Zubainur (2014) and Dickinson et al. (2012), central to the philosophy of Realistic Mathematics Education is that students develop their mathematical understanding by working from contexts that make sense to them. Such an approach is inclined to students' mastery of concepts in Mathematics since student learning is grounded in realistic or context-based settings (Searle & Barmby, 2012; Zakaria & Syamann, 2017). It emphasises the idea that mathematics is a human activity that must be connected to reality. By using what is real to the learner, the real-world context as a source of concept development and as an area application through process of *mathematisation* (both horizontal and vertical *mathematisation*) abstract mathematics become simpler. It was in the scope of this study to explore the extent to which teaching and learning in transformation geometry embraces what is real to the student.

According to Freudenthal (1991) Mathematics teaching and learning should be driven by setting 'mathematising' as a goal for Mathematics education, through both horizontal and vertical mathematisation (Gravemeijer, 1994; 2008). In Freudenthal view, the idea for making mathematising the key process in Mathematics education is based upon two reasons. Firstly, when students are able to use Mathematics in their everyday lives they get to understand and appreciate its value to their lives and society, and come to perceive mathematics as part of their own histories and lives (Gravemeijer, 1994; 2015). The main advantage of using 'real world' problems in teaching Mathematics is the natural way in which teaching takes place (Hoffmann, 2012).

Secondly, mathematising has links with reinvention. Freudenthal (1991) supports the idea of Mathematics education structured as a process of guided reinvention where students go through a similar process as the process by which mathematics was invented (Freudenthal, 1991; Gravemeijer, 1994). The study of Geometric Transformations enhances how we interpret and describe the physical environment we live in as well as provide us with the much needed tool of problem solving (NCTM, 2000). Thus, a lot from our physical environment can provide a platform for *mathematisation* in Transformation Geometry.

Realistic Mathematics Education theory is comprised of six essential principles. The next sub-sections present the six principles showing how this study benefits from them: use of contextual problems, use of models or bridging by vertical instruments, use of student's contribution, interactivity, intertwining of learning strands and role of contexts.

2.1.2 Contextual problems

In Realistic Mathematics Education, the starting point of instructional experience should be real and familiar to the students (Gravemeijer, 2008; Webb, Koij & Geist, 2011). This allows them to immediately become engaged in the teaching and learning process. According to Freudenthal (1991), Mathematics must be connected to reality and also regarded as a human activity. So, students should learn Transformation Geometry concepts by developing and applying mathematical concepts and tools in daily life problem situations that make sense to them (Van Den Heuvel-Panhuizen, 2010).

According to the model, the statement '*mathematics must be connected to reality*' means that Mathematics must be close to learners and must be relevant to everyday life situations (Barnes, 2004; Arsaythamy & Zubainur, 2014). Contexts used should be meaningful to students, and may as equally base on fantasy as a 'real word' scenario. Thus, teaching Mathematics should be as practical and feasible as possible particularly in areas such as transformation geometry. For example, if we translate a shape, we move it up or down or from side to side, then questions like: Does this change its appearance? When we translate a shape? Does each of the vertices (corners) move in exactly the same way?, could be experientially explored.

The above questions can be based on the Realistic Mathematical Model if they involve processes that are familiar to learners. Students will start thinking the solution to the given problem and the teacher in this scenario will be more of a facilitator. Considering the above example, learners work with reality and manage to solve problem unconsciously but are working on a Transformation Geometry task. With realistic mathematical education, Math-phobia will be counter attacked. The RME model in this case stresses on learning as a process rather than as outcomes, learning algorithms (Posnanski, 2010). Learners are able to give reasons for their answers which are far ahead of memorisation of facts and formulae (Gravemeijer, 2008). Using realistic mathematics problems encourages learners to use their own methods at hand. Learners can explain their method to peers and defend themselves. The starting contexts are rich (Posnanski, 2010; Freudenthal, 1991) and sometimes low-ability students do not realise they are doing maths (Barnes, 2004) and this is good for students with little confidence in the subject.

With such an approach to the subject of Mathematics, teaching and learning are organised as a process of guided reinvention, where students experience similar processes by which mathematics is invented (Gravemeijer, 2008; Dickinson & Hough, 2012). Invention in this case refers to the steps in the learning process while guided explains the instructional environment of the learning process. Encouragement of guided reinvention implies building on the range of informal strategies provided by students to promote materialisation of more sophisticated ways of symbolisation and understanding. Realistic Mathematics Education requires highly constructivists approach to teaching, in which children are no longer seen as receivers of knowledge but makers of it (Nickson, 2000). The current study, concurs in these elements of teaching and learning practices and these elements were used to explore how effective teachers of Mathematics are in teaching Transformation Geometry.

2.1.3 The use of models or bridging by vertical instruments

Realistic Mathematics Education involves the use of mathematical models which bridges the gap between abstract and real contexts that help students realise the mathematics (Hansen, 2015; Purpura, Baroody & Lonigan, 2013; Freudenthal, 1991). Here broad attention is paid to the development of models and schemers rather than being offered the rule of formal mathematics. According to Zulkardi (1999) the term ‘model’ refers to situations and mathematical models that students develop themselves. These models, which ought to be familiar to the learner, are used to solve mathematical problems. Later, through the process of generalisation and formalisation, the model eventually becomes an ethnicity on its own (Zulkardi, 1999). On the same vein, Van Den Huvel-Panhuizen (2010) echoed that progressive formalisation is mirrored in the use of models, which starts at the situational level, where the specifics of the content is then modelled, at the referential of a ‘model’ of the situation created.

At the general level, the model is increasingly abstracted, becoming a model for the type of a problem. Thus, Realistic Mathematics model makes students solve problems in transformation geometry unconsciously. In the first-place, learners will have to learn transformation geometry in school by reflecting on activities they do at home and then develop until they begin to work on complex algorithm problems (Purpura, Baroody & Lonigan, 2013). The advantage here is that learners are not only likely to solve problems correctly but they also show considerable understanding (Zulkardi, 1999; Purpura, Baroody & Lonigan, 2013). The Realistic Mathematics Education in this case improves learners' problem-solving abilities and helps them understand and approach any questions. In line with this principle, the current study explores opportunities similar to the ones discussed above as created in a Transformation Geometry class.

2.1.4 The use of learners' contributions

Realistic Mathematics Education is also characterised with the use of students' own productions and constructions. With Realistic Mathematics Education, learners are asked to produce more concrete objects. In the case of Transformation Geometry learners could come up with concrete objects that demonstrate, for example, the notion of enlargement. Such should come from learners' own constructions. Lange (1998) postulate that, by making free productions, students are made to reflect on the path they have gone through in their learning process and at the same time, to anticipate continuation.

In Mathematics at secondary level, learners are engaged in setting Mathematics tasks for other learners and also giving homework (Purpura, Baroody & Lonigan, 2013). In case of learners being able to set tests for their colleagues, they produce model shapes connected through some transformations. Using free hand in their drawings implies that Realistic Mathematics Education is practically in the teaching of Transformation Geometry. Realistic Mathematics Education assessment is also given in form of tests during teaching and learning process in addition to end-of-unit or course assessment. Here, assessment materials should be developed in the form of open-ended questions which lead the students to free productions. These assessments should be given to learners either during or after the instructional process as homework. In the same line, the current study explored the nature of assessment conducted in Transformation Geometry classes.

2.1.5 Interactivity

Furthermore, Realistic Mathematics Education is characterised by interactivity between students and between students and teachers. This is a critical component of learning in Mathematics, which shows the benefits of working together to achieve a common goal. They encompass explicit negation, intervention, discussion, co-operation and evaluation among students and teachers which are also essential elements in constructive learning process in which students' strategies are used as a lever to attain formal ones (Purpura, Baroody & Lonigan, 2013; Vygotsky, 1978). Thus, the principle of Realistic Mathematics Education gives more emphasis on student-to-student interaction as well as teacher-student interaction. In the teaching and learning of Mathematics at secondary level, it is very crucial for learners to interact by sharing knowledge they have through activities of collaborative work. Zulkardi (1999) posits that in interactivity, students are engaged in explaining, justifying agreeing and disagreeing, questioning and reflecting. During interactive activities learners get clues in class, as they compare their solution strategies.

In secondary school Mathematics, we can talk of realistic Transformation Geometry which calls for work to be done in groups where investigations, experimentations, discussions and reflections are the core of the teaching and the learning process. The realistic Transformation Geometry deals with a kind of instruction which differs largely from well-known deductive Transformation Geometry (Gravemeijer, 1997). The role of the teacher in this principle is a facilitator, organiser, guide and evaluator (Purpura, Baroody & Lonigan, 2013).

Today, with new teaching and learning models, it is possible to transform traditional learning styles into a more relevant and powerful classroom practices and deliver a rich experience to students regardless of location (Van den Heuvel-Panhuizen, 2014; Gravemeijer, 2008). Such approaches to teaching the topic do reflect dynamic changes in students learning styles and employer expectations. It is clear that higher education cannot meet student needs through traditional learning styles. Students should be more self-reliant, they can turn to the teacher for validation of their answers or for direction for a standard solution procedure. This resonated well with the thrust of this current study to find out if teaching and learning in transformation geometry is mechanistic or promotes self-reliance in students' interactions.

2.1.6 The intertwining of various learning strands

This entails the holistic approach implying that learning strands cannot be dealt with as separate entities, instead an intertwining of learning strands is exploited in problem-solving. Gravemeijer (2013) echoed that the integration of mathematic concepts is essential. The above statement brings us to the teaching principle that learning strands in mathematics must be intertwined with each other. For example, Geometry encompasses topics like transformation, locus, mensuration of solid and plane shapes and scale. All these topics need not to be treated with isolation since one topic may work as a basis of the other.

2.1.7 The role of contexts in Mathematics teaching

There is a correlation between Mathematics taught at school and Mathematics in application (Boaler, 2002; Posnanski, 2010). If this assertion holds, that is, if the context of a mathematics task can match the application of the task in real life then this can be examined with the aim of increasing awareness for teachers to embrace students' out-of-school experiences linked to Transformational Geometry.

It is widely thought that reducing school Mathematics to the level of real life contexts and the links between the requirements of Mathematics in school and in real life results in the chances of situation specificity weakened (Boaler, 2002; Scribner, 1984). However, a study conducted by Lave (1988) found out that use of students' life experiences does not enhance learning compared to an emphasis of the underlying principles and processes which form mathematics. One disagrees with the above assertions but agree with Gravemeijer (2008) who suggests that contexts are critical in facilitating learning transfer although as they are generally used, there are not useful.

The use of contexts in teaching Transformation Geometry has the motivational effect on students' learning (Boaler, 2002; Gravemeijer, 2008). In the early 1970s, increasing awareness of learners as well as general reports of adults' ability to transfer Mathematics learned in schools prompted a vocational shift towards the 'everyday' use of Mathematics. Advocates of everyday Mathematics argue that the impact of use of contexts lies not only in enhancing content learnt but also in providing students the bridge between the abstract role of Mathematics and their role as community members (Purpura, Baroody & Lonigan, 2013). In other words, critics of the teaching of Mathematics as a formal discipline have support from those with beliefs of increasing learning gains in Mathematics.

It is believed that the use of contexts such as real world and local community examples do simplify the abstractness in Mathematics (Boaler, 2002). Such beliefs which include a consciousness of the value of Mathematics and its role in students' lives is known to positively boast students' interest (Walkerdine, 2003). Contexts provide the motivation needed to fully engage the student and also help students relate the real-world events to the abstract Mathematics. In this study using real world, local community and even individualised examples which students may understand was investigated through lesson observations, interviews and document analyses. The next section unpacks the Van den Heuvel-Panhuizen's (2010) level principle.

2.1.8 Van den Heuvel – Panhuizen's Level Principle

Van den Heuvel-Panhuizen (2010) identifies six principles underpinning RME-based pedagogy. These are, the Activity Principle, Reality Principle, Level Principle, Intertwinement Principle, Interactivity Principle, and the Guidance Principle. Although all six are critical aspects of RME-based practices, the purpose of this section is to expand on the Level Principle.

The principle is grounded on the notion of Mathematics learning where students move from one level of understanding to the next (Van den Heuvel-Panhuizen, 2010). According to this principle, the first level stresses on teaching which makes use of examples drawn from students' out-of-school experiences. In other words, teaching and learning should prioritise students' informal knowledge as scaffolds for mastery of concepts. In the second level, the teacher then introduces Mathematical models representing mathematical objects. The third level is the level which facilitates transition into formal mathematical knowledge. The fourth or formal level encompasses cognitive thinking of formal mathematical reasoning and reflection (Cheng & Hung, 2005).

The significance of the Level Principle in this study is to evaluate the teaching of Transformation Geometry and see if it accounts for growth from the concrete, up to the symbolic mathematical level (Bruner, 1960). For example, a contextual scenario should not end at the informal level but extend to the more formal school Mathematics.

2.2 CONCEPTUAL FRAMEWORK

In this segment, the conceptual framework informing the study is presented. First, is the van Hiele Model which is used to discern students' level of Geometry thought so as to use appropriate instruction at their level. A Cognitive Development and Achievement in Secondary School Geometry CDASSG test, which is a product of the van Hiele's Model, was used to determine the level of Geometry thought of the learner participants.

2.2.1 The van Hiele model

2.2.1.1 Historical development

Two Dutch educators, husband and a wife, Dina van Hiele-Geldof and Pierre van Hiele teamed-up to develop a theory involving the levels of mental development in Geometry (van Hiele, 1999). The primacy of the theory, the van Hiele framework, attests to the very special status of Geometry in Mathematics as an essential component of school mathematics curricula all over the world.

Van Hiele (1990) proposed five hierarchical levels that describe growth in student thinking in Geometry. Although van Hiele (1986) claimed that the roots of the theory are found in the theories of Piaget, progression from one level to the next is not the result of maturation or natural development. All depends on the quality of the experience that one is exposed to (Dindyal, 2007).

The reform of the 1960s in Mathematics education brought major changes in the school geometry content (Crowley, 1987). New approaches to Geometry such as co-ordinate, Transformational, and vector approaches were emphasised in the school curriculum. Duval (2002) has claimed that there is no direct access to Mathematical objects other than through their representations, and thus we can only work on and from semiotic representations, because they provide a means of processing. In Geometry, this implies working in different registers (natural language, symbolic, and figurative) and moving in between registers (Crowley, 1987). Transformations offer Geometry a powerful form of figurative representation. It became the thrust of this study to investigate the nature of Mathematical objects used in transformation geometry classes.

The ZIMSEC national syllabus highlights the following aspects of school Transformation Geometry for O' Level:

- ✓ analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- ✓ specify locations and describe spatial relationships using coordinate Geometry and other representational systems;
- ✓ apply transformations and use symmetry to analyze mathematical situations; and
- ✓ use visualisation, spatial reasoning, and Geometric modelling to solve problem

(ZGCE, 2012).

Constituent elements of the national syllabus focus in Transformation Geometry had their prerequisite skills assessed through a CDASSG test to establish students' level of readiness to receive instruction in transformation geometry.

2.2.1.2 Van Hiele levels of mental development in Geometry

There are five sequential phases in studying geometry as informed by the van Hiele's Model. The Van Hiele's Model has had applications in a number of studies done in the area of Geometry (Clement & Battista, 2001, Battista, 2002; Noraini, 2005; Halat, 2008). The van Hiele Model has been in use from as early as the 1980s to explain difficulties students experience in Geometry. The theory claims that learners taught at a van Hiele level higher than they have achieved face difficulties in any high school Geometry concepts (Clement & Battista, 2001; Bansilal & Naidoo, 2012). In the same vein, it was important to administer a test of this model in this study to detect a student's level of geometric thought. Similarly, this exercise tells us about the prerequisite skills students have for learning new topics such as transformation geometry.

The van Hiele's Model's levels of geometric thought are denoted *a* to *e*, and which are,

a. *the recognition or visualisation level,*
b. *the analysis level,*
c. *the order (or informal deduction) level,*
d. *the deduction level, and*
e. *the rigor level*

(Crowley, 1987; Fuys, 1985; Usiskin, 1982).

Each one of the levels has characteristic features that describe a level and help in deducing the achievement of the level. Advancement from one level to another is determined by a learner's experience and not on his/her age.

At the first level is the visualisation or recognition level, *Level 1* (Crowley, 1987; Usiskin, 1982). It describes the ability to name figures and to recognise different shapes of figures by their appearance (but not by their properties) (Guven, 2012). The learner recognises a Geometric shape based on the entity of the object and not on its components. The student learns the geometric language but not the full understanding of the definition (Atebe, 2009). Learners operating at *Level 1* identify a shape given in any specified orientation. For example, a learner can recognise a figure as a rectangle by the four sides with two opposite sides of the same length and the four “corners” even if the figure has been altered such that the sides appear to be angled. Students at this level can recognise any figure be it a square, rectangle or parallelogram. However, the same student cannot describe a square as a special case of a rectangle, a square or rectangle as a special case of a parallelogram (Battista, 2001; 2002).

At the *Level 2* (analysis), the learner can identify a figure, say a square (Guven, 2012), as having sides of equal-length and parallel sides with each of the four corners as 90 degrees. A learner in *Level 2* is able to identify the characteristics features of figures in order to form classes of figures but cannot describe the relationship between different properties of shapes (Guven, 2012). For example, are able to create classes of figures that have different characteristics in common such as all triangles have three sides and all quadrilaterals have four sides (Battista, 2001; 2002).

At *Level 3, (informal deduction),* the learner can identify the relation between shapes and then the student creates that relation (Fuys, 1985). *Level 3* is defined as the learner’s ability to start noticing an interrelationship between properties, either within a class of figures, or among a class of figures (Guven, 2012). Student can follow formal proofs but cannot reproduce the proof when starting from an unfamiliar premise. For example, learners can now recognise a square as a special case of rectangles since it has all the properties of a rectangle, but a rectangle is not a square because it lacks the property that all four sides must be congruent (Hollebrands, 2003).

At *Level 4,* (*deduction*), the learner appreciates the meaning and importance of deduction (Guven, 2012). At *Level 4,* it is possible for the learner to develop proofs from more than one premise and understands the difference between necessary and sufficient information (Halat & Peker, 2008). For example, it is sufficient for a figure to be four-sided if it is to be recognised as a quadrilateral but it is necessary for the sides to be of the same length if it is to be a square and it is necessary that all the four angles be right angles for it to be a square.

Level 5, (rigor), the learner understands working in axiomatic system (Guven, 2012). They can form a more abstract deduction. *Level 5* is defined as the ability to transfer understanding and compare different axiomatic systems. Since this final level does not concern the student like the ones in this study, the discussion of the level was not provided. Usually, lower secondary students can only reach up to *Level 3*, of Van Hiele’s Model, which is informal deduction (van Hiele, 1986;

Halat & Peker, 2008). Translated to Geometric Transformations, all the four levels address the following.

Van Hiele (1986) identified five characteristics of the levels which are:

1. *Sequential.* The levels are orderly, meaning students should receive adequate and effective learning experiences at lower levels in order to learn how to operate at higher levels.
2. *Intrinsic and extrinsic.* Geometric concepts that are implicitly understood at one level become explicitly understood at the next level.
3. *Linguistics.* Each level has its own vocabulary, set of symbols and network of relations.
4. *Mismatch.* If students are at a level lower than the teacher, instructional materials, and content then students will not be able to learn effectively and not much progress would be anticipated, because they will not be able to understand the thought processes being used.
5. *Advancement.* Transition from one level to another is not automatic because it is determined by the teaching and learning programmes (van Hiele, 1986 p.50).

(Adapted from Meng & Idris, 2012, p.21)

Thus, in this study the Van Hiele Model was used to determine learner's level of Geometric thought necessary for comprehension of transformation geometry concepts. A CDASSG test was used to identify the developmental level or geometric reasoning of participating students. This allowed the researcher to assess students' readiness to acquire concepts taught. For instance, students at the visualisation/recognition level know nothing beyond the properties of the figures and can only begin their exploration of transformations using tracing paper (Meng, 2009). Learners can draw a figure and then use the paper to do the transformation.

According to Crowley (1987) progress from one level to the next is not through biological development but rather depends on instruction. A number of assumptions are basic to the van Hiele Model. These are: students' levels are not affected by their age, students must master each developmental level to progress in their geometric understanding and level is determined by concepts that have been taught to the students (Crowley, 1987).

In order for teaching and learning to be very successful, the developmental level or geometric reasoning of students must be determined as this will subsequently inform instruction. This allows teachers to differentiate instruction based on student readiness (Meng, 2009). Having got the developmental levels of the students in the class, students then receive instruction at their level. For instance, learners at the visualisation/recognition level focus on properties of the whole figure and can begin their exploration of transformations using tracing paper. Students can draw a figure and then use the paper to do the transformation. If a student is at one developmental level and the teacher instructs concepts at a different developmental level, it is very likely that the student will not grasp and retain the information (Crowley, 1987). By understanding where students are in their geometric comprehension, teachers can best meet their students' needs, and will be more successful in teaching Transformational Geometry.

2.2.1.3 Implications of the van Hiele levels for Transformation Geometry

Table 2.1: Levels of understanding in Transformation Geometry

LEVEL	CHARACTERISTICS: THE STUDENT
Level 1	• Recognises a transformation by the changes in the figure; (a) in simple drawings of figures and images; and (b) in pictures of everyday applications. • Recognises a transformation by observing actual movement; names and discriminates the transformation. • Names transformations using basic labels
Level 2	• Uses properties of changes to draw the object or image of a geometric transformation. • Identifies properties of changes to figures as a result of a geometric transformation. • Uses transformation vocabulary appropriately • Can locate reflection lines, centre of rotation, translation vector and centre of enlargement. • Relates well with transformations involving coordinates of points. • Can solve tasks that use known properties of transformations.
Level 3	• Performs and describes composition of simple transformations. • Can use coordinates and matrices to represent transformations. • Inter-relates the properties of changes of a figure resulting from transformations. • Can name a single transformation given the object and its image. • Can define a transformation as a composition of simple transformations given the objects and their images.
Level 4	• Can give a geometric proof based on transformational logic. • Can present a proof using the coordinates and matrices. • Argues through multi-step problems and gives reasons for problems.
Level 5	• Can argue using laws of associative, commutative, inverse, identity with respect to a composite transformation operation. • Can relate to groups of transformations. • Proves or disproves subsets of transformation from group structures.

[Adapted from Guven, 2012:375]

The five van Hiele levels described above have specific properties common to them. These properties are identified as: (a) fixed sequence, (b) adjacency, (c) distinction, (d) separation and (e) attainment (Guven, 2012). The fixed sequence property describes the student's inability to advance to a level *n+1* before having attained level *n*. This explains Vygotsky's (1978) process of moving from spontaneous concepts to scientific concepts (Byrnes, 2001). The 'adjacency' property describes the learner's ability to describe the properties of an object, which are intrinsic at one level, and extrinsic at the next level. For example, at Level 1 a learner recognises that a parallelogram is a parallelogram because of its shape and appearance. However, at Level 2 a parallelogram is defined by its two pairs of parallel sides that are the same length and at Level 3 by its opposite parallel sides which form a pair of allied angles that add up to 180 degrees (Guven, 2012).

The 'distinction' property defines the student's ability to use the vocabulary associated with the level (Guven, 2012). For example, a learner at *Level 1* will not be able to equate a square to a rectangle because they are yet to start analyzing the properties of figures. However, a student at *Level 2* can begin to realise that a square is a special type of a rectangle because a square also has all the properties that make a rectangle a rectangle.

The 'separation' property defines the inability of two students at different levels to argue the same. Many researchers (Mayberry, 1983; Senk, 1989; Usiskin, 1982) believe that it is this property that explains why most secondary school geometry students fail to succeed in geometry and transformation geometry. Since most of the material for secondary school geometry is at a Level 3, students who have not attained that level of understanding in geometry will not progress to the next level. For example, a learner who explained to his instructor, "I can follow a proof when you do it in class, but I can't do it at home" (Usiskin, 1982 p.5), shows that the student operates at a level below Level 3.

In the last and fifth property, 'attainment,' the learning process that leads to complete understanding at the next higher level is outlined (Guven, 2012). The key elements of this property are that understanding depends on the content and methods of instructions received more than on age (Crowley, 1987). Progression is more dependent on choice of instructional methods. For example, teaching learners to memorise a procedures or formulae without the student being able to reason why the procedure or formula works is detrimental to understanding.

The next sections, 2.2.1.4 through 2.2.1.5, analyze three essential components of the van Hiele Model. The components are phases of the attainment property; Mathematical level Raising and Expectations of students.

2.2.1.4 The Phases of the Attainment Property

The attainment property, described previously, is a process that consists of five phases that lead to the attainment of understanding at the next level. These phases are :(i) *inquiry*, (ii) *directed orientation*, (iii) *explanation*, (iv) *free orientation* and (v) *integration* (Usiskin, 1982; Crowley, 1987). In the inquiry phase, the teacher introduces the new vocabulary and activities that instigate observation and questioning. This allows the teacher to gauge learner's level of comprehension.

In directed orientation, the teacher gives materials that are structured strategically to allow the students to become steadily aware of the situation under investigation. The third phase calls for students' explanations of their previous experiences. Except for helping students use correct and precise vocabulary, the teacher is a by-stander in the dialogue and all observations and explanations are considered valid.

The fourth phase involves tasks that are more complex and are more open-ended (Crowley, 1987). Once done, the final phase, integration, is implemented. The learners review, collectively, the work done and the observations made in the first four phases and create a summary that provides an overview of the new concepts (Crowley, 1987). The teacher assists by proctoring the discussions and ensures no new information is introduced at this phase. At the conclusion of this phase students will have attained this level of understanding and the phases can begin anew to raise the level of understanding to the next level. In line with this study, elements of the five phases of the attainment property were analysed to understand the discourses employed in transformation geometry classes.

2.2.1.5 Mathematical level Raising

Raising students' mathematical level is the aim of all Mathematics teaching and learning programmes. The term 'level' in this context is used to refer to the model of Van Hiele (1986), who identified three distinct levels of mathematical understanding and ability. The first level is a pre-scientific perceptual (visual) level underpinned by concrete operations. The second level is a conceptual (descriptive) level underpinned by the use of mathematical concepts and the mutual relations between these concepts. The third level is underpinned by formal operations on mathematical concepts and mathematical principles (Cowley, 1987).

Introducing mathematical concepts such as translation and stretch in the classroom does not mean that the concepts are completely new to the students. Students are normally familiar with related phenomena in everyday life, which may have been investigated, for instance, in the tangible context of patterns. What is unknown to them is the rotation and reflection which mathematicians have called geometric transformations. For the present study, the transition between the first and second level is the most relevant. Level Raising within this range is achieved by growing aptitude in discerning *aspects* of transformation Geometry (as concrete operations) and application of descriptive knowledge, for instance in solving construction problems (Crowley, 1987; Hershkowitz et al., 2001).

2.2.1.6 Expectations of Learners

The NCTM Teaching Principle (2000: p.16) states that, “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well”. A teacher is expected to possess and demonstrate content knowledge at or above what is expected of the student. NCTM (2000) provides a description for the Geometry Standard that states, for example, that: “Instructional programmes for all students to emphasise on the students being able to apply transformations and use symmetry to analyse mathematical situations; and use visualisation, spatial reasoning, and geometric modelling to solve problems” (p. 41). In such standards and principles, it shows that students are expected to master transformation geometry in order to enhance their survival skills. Thus, it is critical for teaching and learning to draw from life experiences.

Previous studies in Geometry (for example, Senk, 1989; Usiskin,1982) have shown that learners who are not yet at a van Hiele’s *Level 2* of understanding of Geometry before enrolling for a secondary school Geometry programme have a level of understanding too low to ensure success. As a result, the successful completion of informal Geometry at their secondary school level can only be achieved if the students attain the simple deduction level (*Level 3*) of understanding Geometry upon completion of elementary and middle school. According to this model, progression in between Van Hiele's levels depends on teaching method more than on chronological age (Crowley, 1987). Thus, using traditional teaching methods, according to research (for example Bansilal & Naidoo, 2012), leaves many lower secondary students performing at Levels 1 or 2 where nearly forty percent of learners completing secondary school are below Level 2. The reason for this, according to the van

Hiele Model, is that teaching continues to emphasise a curriculum that is at a higher level than the student (Bansilal & Naidoo, 2012) and hence not contribute to learner's mastery of concepts.

2.3 THE CONCEPT OF TRANSFORMATION GEOMETRY

Transformation Geometry was first introduced by Christian Felix Klein in 1872 during a seminar named Erlangen programmes. Klein (182) defined Geometry as the shapes whose properties remain stable under a transformation (Burton, 2011).

Geometry is comprised of four conceptual aspects. The first aspect is called visualisation, depiction, and construction; then the second focuses on the study of the concrete situations presented in the real-life contexts where students are linked up with geometric concepts; and the third aspect defines non-physical or non-visual representations (Clements, 2003; Usiskin, 1987). The fourth aspect embraces the mathematical system with its logical organisation and proofs.

The first three conceptual aspects focus on the use of spatial sense, which can be enhanced through studying geometric transformations (Clements, 2003). Studying Transformation Geometry supports our understanding of the physical environment and equips us with a valuable tool in problem solving (NCTM, 2000). In other words, there is a strong link between transformation geometry concepts and the real-world experiences. Thus, teaching these concepts requires a solid reference to students' real-world experiences. This realisation resonates well with the thrust of this study, to explore mathematics educators' use of students' out-of-school experiences in Transformation Geometry classes.

Spatial reasoning and visualisation enable one to form mental representations of two-dimensional figures (NCTM, 2000) through examining the different shapes, their properties and transformations. The Geometric Transformation curriculum is mainly made up of five basic concepts: translations (slides), reflections (flips or mirror images), rotations (turns), dilations (size changes), and the composite transformation of two or more of the first three (Wesslen & Fernandez, 2005). These Transformation Geometry concepts provide a foundation to the congruence and similarity (NCTM, 1989). Thus, this becomes important since school mathematics covers topics like congruence and similarity of shapes. In other words, understanding these concepts increases understanding of successive concepts in mathematics.

According to Chagwiza et al. (2013 p.229) the aims of teaching geometry can be summarised as "to develop skills of applying geometry through modelling and problem solving in real world contexts." Chagwiza et al. (2013) also point out that Geometrical Transformation help students to develop the skills of visualisation, critical thinking, intuition, perception, problem solving, deductive reasoning, logical argument and proof. Since visual images can be manipulated on graphs, geo-boards or computer screens, students are invited to observe and draw generalisations. Such generalisations can easily be understood and applied in their learning. De Villiers (1997) indicates that proving conjectures require students to understand how the observed images are related to one another and are linked to fundamental building blocks and that much of our experience is through visual stimulus. This means that the ability to interpret visual information is fundamental to human existence. De Villiers notes that,

> *Much of our cultural life is visual, aesthetic appreciation of art, architecture music and many cultural artifacts involve geometric principles, symmetric perspective scale and orientation. Understanding many scientific principles and technological phenomena also require geometry awareness as do navigation and map reading*. (De Villiers, 1997).

Transformations permit students to develop broad concepts of congruency and similarity and apply them to all figures. Chagwiza et al. (2013) cited that similar figures are always related either by a reflection, rotation, and slide or glide reflections. This implies that recognition of the familiar and the unfamiliar; similar and the not similar, require an ability to characterise and note key features. Thompson (1993) points out that studying transformations can enable students to realise that photographs are geometric objects and that all parabolas are related because they can be mapped out on each other. The graphs of y=$\cos x$ and y=$\sin x$ are congruent and have powerful geometric applications (de Villiers, 1996). Transformations also play a major role in artwork of many cultures, for example, they appear in pottery and patterns, tiling and friezes.

Chagwiza et al. (2013) also points out that Geometry offers a rich way of developing visualisation skills. Transformation Geometry "dominates almost every field of one's activities" (Mahanta and Islam, 2013:713). According to Mahanta and Islam (2013), Geometry disciplines the mind, systematizes one's thought and reasoning. In this era of Science and Technology, "mathematics is considered to be the father of all sciences" (Mahanta and Islam, 2013:713). Napolean, (cited in Mahanta & Islam, 2013:713) remarked that "The progress and improvements of mathematics (and transformation geometry) is linked to the prosperity of the state."

Mandebvu (1996) acknowledges that most employers in Zimbabwe expect job-seeking school leavers to have passed Mathematics, English and Science among others subjects at Ordinary Level. Tecla (2007) emphasises that nations match towards scientific and technological advancement and people should have nothing short of a good performance in mathematics at all levels of schooling. In Zimbabwe, mathematics competence is a critical determinant of the post-secondary educational and career opportunities available to young people (Woods & Barrow, 2006). Since Geometry is a major component of Mathematics, it is therefore equally influential in all the faculties underpinned by Mathematics.

2.4 A HISTORICAL PERSPECTIVE OF THE STRUCTURE OF EUCLIDEAN GEOMETRY

Geometry is one of the oldest subjects that got inspired by practical needs (Morrow, 1970). The word 'geometry' means earth measurement and comes from two ancient Greek words, one meaning *earth* and the other meaning to *measure*. These Greek words, as well as the word 'Geometry', may themselves be derived from the Sanskrit word '*Jyamiti*' (in Sanskrit, 'Jy a' means an arc or curve and '*Miti'* means correct perception or measurement) (Jones, 2002). The origin of Geometry is as ancient as several ancient cultures such as Indian and Babylonian. It is in Geometry where relationships between lengths, areas, and volumes of physical objects became meaningful. In these olden days, Geometry was used to measure land and also in constructing religious and cultural artefacts, such as the Hindu Vedas, the ancient Egyptian pyramids, Celtic knots (Jones, 2002).

The Celts became popular in Europe during the 4th and 5th centuries CE (Jones, 2002). An example given below, in Fig. 2.1, is of the Celtic knot pattern.

Figure.2.1. Simple Celtic knot patterns.

[Adapted from, Jones, 2002, p.34]

In the Western world, the ancient Greeks made a huge contribution to the discipline of geometry. For instance, they developed geometry in the form of menstruation of shapes and (Stahl, 1993). About 300 BCE Geometry was developed into thirteen books which comprised axioms, postulates and theorems. The Geometry of Euclid grew to become highly rated among mathematical theories of space (Jones, 2002).

Geometry is a critical discipline which is the foundation of a more robust understanding of mathematics such as transformation geometry. According to Freudenthal (1973) Geometry is about exploring space, the space in which the child lives and breathes. It is in this space that the child ought to be a master explorer and conqueror, so that he lives a better life. Geometry is one of the topics that offer students opportunities to learn how to *mathematise* reality; hence no successful teaching of it can avoid reality. Hence, this study explores the extent to which teaching and learning of Transformation Geometry embrace learner's reality as well as their out-of-school experiences.

In everyday life and employment careers, there are geometric concepts and skills that can be transferred from the Geometry classroom for use in the outside world. The building of a round hut requires the marking of a centre of the hut before drawing a circle, the foundation on which the hut's walls will be built. The circle is drawn in the same manner as the geometry construction technique of drawing a locus of points equidistant from a fixed point. In the forestry industries the notion of similar shapes is useful in identifying heights of trees. In these few examples, the value of Geometry can be discerned and hence suggesting strongly that the secondary school Mathematics curriculum must incorporate Geometry because it's teaching lands a lot from real life experiences of students. The teaching of geometry, however, ought to bring out key elements of geometry, and these are: Invariance, Symmetry, and transformation.

2.5 CREATING SIGNIFICANT LEARNING EXPERIENCES

This section presents how the teaching and learning of Transformation Geometry can create significant learning experiences for students. Effective teachers support students to make connections by providing them with opportunities to engage in complex tasks and by setting expectations that they explain their thinking and solution strategies, and that they listen to the thinking of others (Anghileri, 2006). Teachers can assist students to make connections by using carefully sequenced examples, including examples of students' own solution strategies, to illustrate key mathematical ideas (Watson & Mason, 2007).

Hill, Schilling and Bell (2004) extended Shulman (1986)'s original ideas about pedagogical content knowledge and developed a model for mathematics teachers' knowledge referred to as mathematics knowledge for teaching. In their model the three knowledge domains most central to mathematics teaching are common knowledge of mathematics, specialised knowledge of content, and knowledge of students and their ways of thinking about the content.

Common knowledge is the knowledge that any adult not necessarily educated needs to possess to provide correct mathematical solutions. Specialised knowledge of content is being able to provide students with multiple representations addressing diverse and learning styles (Hill, Schilling & Bell, 2004). Thus, teachers need to have both common and specialised knowledge to enhance their teaching of transformation geometry along with their pedagogical knowledge. It was in the thrust of this study to examine if Mathematics teacher-participants had common knowledge or specialised knowledge or both in teaching transformation geometry. The next discussion unpacks the interactions between teacher knowledge of subject and knowledge of teaching in a different but related model.

To engage in this discussion a model adapted from Danielson's (2007) framework for teaching was used. The framework has four domains however a more emphatic version of the framework with three domains was adapted for the purpose of this study as shown in Table 2.2 below.

Table 2.2: Framework for the teaching of Mathematics

Domain	**Description**
1. Planning and Preparation	• Knowledge of content • Knowledge of related students' informal mathematical knowledge • Knowledge of resources
2. The Classroom Environment	• Coherent instruction • Environment of respect and rapport • Managing classroom procedure
3. Instruction	• Clear and accurate communication • Use of question and discussion techniques • Nature of tasks and student feedback

[Adapted from Danielson, 2007]

The framework alludes to three key domains in the teaching and learning process. In the first domain, a teacher has got to engage in *planning and preparation* for future lessons through considering the content to be taught, students' informal mathematical knowledge related to the content, as well as the available resources (Danielson, 2013). Transformation Geometry provides a culturally and historically rich context within which to do mathematics. Teachers have a critical role to play in ensuring that resources used effectively support students to organise their mathematical reasoning and support their sense-making (Blanton & Kaput, 2005).

There are many interesting results in transformational geometry that can enhance students' learning when relevant resources and approaches are selected for use (Mahanta & Islam, 2013). The teacher must also have a command of the content they teach. In line with the first domain, the current study used document analyses to examine the form of teacher planning and preparation.

The second domain refers to the *classroom environment*. It is an essential skill for teachers to manage a positive classroom environment. Teachers create and maintain an environment that is conducive for creating significant learning experiences for students. Presenting transformation geometry in a way that increases students' curiosity and enhances exploration can boost student's learning in the topic (Chigwiza et al., 2013). Also, patterns of interactions are critical for the overall tone of the class. In this study, lesson observations were used to discern the tone in Transformation Geometry classes.

The third domain is about instruction. Teachers who are competent use clear and imaginative analogies and metaphors to increase the bond between students' informal mathematical knowledge and the formal-taught mathematics (Danielson, 2007; 2013; Purpura, Baroody & Lonigan, 2013). Demonstrating the links among mathematical topics is important for enhancing conceptual understanding. Teachers and learning must encourage students to make connections with their world of experience. Ready-made tools, effective teachers should acknowledge the importance of students generating and employing their own representations, such as in notation, or graphical form (Chick, Pfannkuch, & Watson, 2005).

When learners discover that they can manipulate mathematics as a tool for solving problems in real life, they start to perceive the subject as of value. The focus of this study is to explore teaching of Transformation Geometry is dependent on life experiences of students. Thus, building on students' existing understanding of concepts can help teachers emphasise the links between different ideas in mathematics (Arsathamby & Zubainur, 2014; Gravemeijer, 2013; Posnanski, 2010). Student experiences are an essential element of a rich instructional environment, if not used students continue to guess and not learn.

Mathematical tasks also offer opportunities for students to engage in thought provoking and reasoning activities (Henningsen & Stein, 1997; Stein & Smith, 1998). According to (Stein & Lane, 1996; Stein & Smith, 1998) the greatest learning gains on mathematics assessments occur when students are engaged in high level cognitive tasks. Thus, to improve learners' performance in Transformation Geometry students must engage in cognitively demanding activities (Boston & Smith, 2009) that foster the development of concepts (Jupri, 2017). It was the thrust of the current study to examine the nature of tasks used in transformation geometry classes and evaluate their contribution to significant learning experiences.

2.6 RESEARCH FINDINGS ON RME-BASED TEACHING AND LEARNING

Teachers' knowledge of facts and procedures has little effect on students' achievement than teachers' knowledge of connections and concepts (Fauskanger, 2015). An implication drawn out of this statement is that traditional teaching methods have become less and less effective than approaches which value learner participation and contribution. Realistic Mathematics Education is one of the tried and tested instructional theories that have resulted in a great positive impact on students' mastery of concepts (Gravemeijer, 2015). RME has been implemented in the Netherlands for almost three decades and has positively improved performance in mathematics (de Lange, 1996). This section highlights research findings on RME-based approaches as well as showing gaps which this study was poised to address.

Results by many studies have revealed great strides that can be realised through RME-based teaching practices as shown below. A study by Zakaria and Syamann (2017) focusing on the impact of RME approach on student achievement, concluded that RME approach compared to the traditional approach is more effective in enhancing students' mathematics achievement. The approach, according to Zakaria and Syamann (2017), encourages student participation and interest in a subject discipline like Mathematics. The study used a quasi-experimental design with two groups of students. Thus, the current study explored the implementation of RME's key element of use of students' out-of-school experiences in teaching.

Another study by Ekowati & Nenohai (2016) focused on the implementation of RME in the teaching of LCM and greatest common divisor. The study based on an intervention mode involved forty-six students in a fifth Grade class. The study results proved that RME is effective in boosting comprehension of the concepts. Ali, Bhagawati and Sarmah (2014) conducted a study on the challenges faced by learners in performing transformation geometry. They found out the following as aggravating towards low performance in transformation geometry: imbalance student ratio, students coming for concepts in transformation geometry without basic knowledge in geometry from lower forms. In the current study the van Hiele test (CDASSG), was used to detect if students learn Transformation Geometry concepts after acquiring the prerequisite skills in Geometry.

Another study by Arsaythamby and Zubainur (2014) focused on how a Realistic Mathematics Education approach affects students' activities in primary schools. The learning and teaching of Mathematics in Indonesia has always been teacher-centred, and mechanistic. This study argues that the Indonesian Realistic Mathematics Education (IRME) approach promotes students' learning in Mathematics classrooms. The study involved lesson observations of students' mathematics activities with an IRME (Indonesia Realistic Mathematics Education) approach in the classroom. It emerged from Arsaythamby and Zubainur (2014) study that mathematics activities for those taught using IRME were greater than for those using the conventional approach. The study recommends to the Indonesian Aceh Education department an increase in the implementation of IRME in all their primary schools to make learning of Mathematics more effectual.

All these studies, however, concentrated on intervention strategies where the impact of RME instructional designs was measured. Limited studies focused on detecting the extent to which teaching and learning embrace elements of RME – which is the focus of the current study. The next section discusses how Constructivism contributes towards the teaching and learning of Transformational Geometry.

2.7 CONSTRUCTIVIST THEORY IN THE TEACHING OF TRANSFORMATIONAL GEOMETRY

Constructivism (Freudental, 1991) is a critical departure in thinking about nature of knowing, hence of teaching and learning. Constructivism has links with RME in a number of ways. For instance, RME is a neo-constructivist approach, which emphasises on the teaching of mathematics that should stress the connection with reality in order for the content to be of human value (Freudenthal, 1977). Constructivism is opposed to the teachers' role of transmitting knowledge to passive students. Central to Constructivism is the notion that learners should be active players in their learning (Duffy & Jonassen, 2013). Students make concepts their own through manipulating concrete objects thereby creating a contextual fabric to their learning.

The Constructivist view defines learning as change resulting from meaning constructions (Newby et al., 1996). Constructivism as a theory of knowledge is opposed to the view where knowledge is received passively by students from authoritative sources (Maclellan and Soden as cited in Yilmaz, 2008). According to Ernest (1991) a Mathematics teacher who believes in the transmission of knowledge to students subscribes to the instrumentalist view. Such a teacher is regarded as an industrial trainer. Conversely, the Constructivist learning theory attributed to the works of Lev Vygotsky (1978), Jean Piaget and John Dewey, argues that knowledge is not static but is mediated and formed in ways that are dynamic and critical as the knowledge itself (Hirtle, 1996).

Constructivists say that knowledge and truth are dependent on an individual' view and do not exist outside the human mind (Duffy and Jonassen, 2013). This view, however, is opposed to the objectivists' view that, 'knowledge and truth exist outside the human mind of the individual and are therefore objective' (Runes, 2001). The purpose of education according to the objectivists' view is to assist students to acquire knowledge about the real world. Learning is thus perceived as the attainment and accumulation of a fixed set of skills and facts.

According to Von Glaserfeld (1984:104)

> *...learners construct understanding. They not only mirror and reflect what they are told but look for meaning and will try to find uniformity and order in the events of the world even in the absence of full information.*

Constructivism is all about knowledge constructions while objectivism is concerned with the objective of knowing. It is the essential distinction with reference to knowledge and learning that separates the two concerning both the fundamental principles and implications in designing instruction. The most important principle underpinning constructivism is notion of active learning. Whilst information may be transmitted understanding cannot be achieved since it must originate from within. Thus, by embracing students' experiences during the course of teaching geometric transformations allows students to swiftly adapt to any new content. Powel and Kalian (2009) say Constructivist learning is a view of learning where students actively create their own knowledge, with the mind of the student mediating what comes from the outside world to decide on what the student will learn. Learning is thus viewed as an active mental work, where the role of teaching is not about transmission of knowledge.

However, learners may converse with others their understandings about a subject and end up developing shared understandings (Cognition and Technology Group, 1991). The most important principle central to constructivism, as characterised by the Piagetian approach to constructivism, concerns the collaboration among learners, working together for a common goal (Duffy and Jonassen, 2013). Rather, it promotes the creation of a social context where collaboration builds a sense of community, and that teachers and students are active participants in the whole process of learning.

Accordingly, the constructivist perspective, argues that learning involves the complex interaction among learners' prior knowledge, the social context, and the nature of problems to be solved. A number of authors have described the characteristics of Constructivist teaching (Brooks & Brooks, 2001; Cognition and Technology Group, 1993; Collins, Brown, & Holum 1991; Honebein, Duffy, & Jonassen, 2013). Two features seem to be key to these Constructivist descriptions of the learning process:

(a) 'Good' problems

Instruction grounded in Constructivist approaches calls for students to employ their own knowledge and skills to solve problems within a meaningful and realistic context. The nature of the tasks invites students to use their knowledge and become masters for their learning. High-quality problems are necessary to arouse the students' energy in the exploration and reflection required for meaning building. Brooks and Brooks (2001), describe good problems as the ones that,

- call for students to construct and check a prediction
- can be solved with economical apparatus
- are practically multifaceted
- flourish from efforts of different groups
- are viewed as significant and motivating by students.

(b) Collaboration

The Constructivist viewpoint believes that students learn by sharing with others. Learners work together as peers, making use of their collective knowledge in solving problems. The discourse that results from the collective effort presents learners with the chance to verify and improve their understanding developmentally. However, this does not leave out the teacher's responsibility. There is one more facet of collaboration in a constructivist learning environment where the teacher has a role to play.

Vygotsky's (1978) theory of Social Constructivism, which is different from Piaget's individualistic approach to Constructivism, emphasises the interface of learners with their peer for cognitive development. His theoretical notion of the zone of proximal development, his conviction that learning is directly linked to social development (Rice & Wilson, 1999) defines his rational dimension. Vygotsky (1978 p.187) says, 'The discrepancy between a child's actual mental age and the level he reaches in solving problems with assistance indicates the zone of his proximal development.' Vygotsky felt that effective instruction may possibly be enacted by establishing first where a child is in his or her mental growth and building on the child's experiences.

Copley (1992), says that constructivism calls for a teacher to act as a facilitator with the main role to help out students in becoming active participants in their learning and build significant connections between their prior understandings in a field, new knowledge, and the processes implicated in learning. As Jazima and Rahmawatia (2017) noted, a Constructivist learning environment is branded by collective knowledge among teachers and students; collective authority and responsibility among teachers and students and the teacher's revised role of guiding and promoting learning. Constructivism entails a situation where the teachers act as models and guides, demonstrating to students how they reflect on their evolving insights and giving direction where necessary (for example, Collins et al., 1991; Rogoff, 1990). Learning is a shared vision and responsibility for the instruction is also shared. The amount of supervision provided by the teacher is dependent on the students' knowledge level as well as experience (Fosnot, 2013).

Brooks and Brooks (2001) sum up a large section of the literature on descriptions of a 'constructivist teacher'. They envisage a Constructivist teacher as an individual whose role is to:

- foster and acknowledge student autonomy and initiative

- employ a wide variety of materials, that includes raw data, primary sources, and interactive materials and promote students in using them;
- find out about students' understandings of concepts before contributing their own understanding of those concepts;
- promote students' engagement in discussions with their teacher as well as with others;
- promote student inquiry by asking thought provoking and open-ended questions
- promote student to student by encouraging them to ask one another questions and seeking clarity from peers' contributions;
- engage students in dialogues that promote augmentation in order to refine their initial understandings and thus promote meaningful learning;
- create opportune time for students to build relationships and generate metaphors; and
- evaluate students' comprehension through assessing their performance in open-structured tasks.

Hence, from a Constructivist viewpoint, the main role of the teacher is to form and preserve a collective and interactive problem-solving environment, where students are sanctioned to build their own knowledge, with the teacher acting as a facilitator and guide. The Constructivist propositions outlined above propose a set of instructional principles that can nurture, guide effective teaching practice and design conducive learning environments. It is imperative that design practices ought to offer more than merely the constructivist perspectives; they should also nurture the creation of effective learning environments that utilise the main underlying epistemological principles.

Lebow (1993), says traditional educational technology statutes of replicability, reliability, communication, and control (Heinich, 1984) are opposed to the seven main constructivist values of collaboration, personal autonomy, generativity, reflectivity, active engagement, personal relevance, and pluralism. Such a mismatch between the traditional instructional design practice and the constructivist principles in the design of instruction arises from the epistemological distinctions between the two contrasting theories of instruction.

To teach Transformational Geometry effectively to learners of any age or ability, it is critical to ensure that students master the concepts they are taught and know why they have to follow certain steps involved in particular processes. More effective teaching approaches can foster students to recognise connections between different ways of representing geometric transformations and between the Geometry and other areas of Mathematics, such as similarity and congruence. The foregoing discussion suggests that this is inclined to helping students retain learned knowledge and skills and hence enable students to approach new transformation geometry problems with some confidence.

The Constructivist instructional design principles, implemented within the Realistic Mathematics Education framework, can lead to rich learning environments in Transformation Geometry. Typical cases of these constructivist instructional designs include promoting student cognition in contexts that are real and meaningful to the student, reflexive learning, collaborative learning, etc. In order to transform the principles of Constructivism into a real classroom practice, quite a number of instructional designers are in the course of developing more constructivist environments and instructional prescriptions. A key element of these prescriptions is the condition that instruction be situated in relevant contexts. Situated cognition (Brown, Collins, & Duguid, 1989) stipulates that knowledge and the conditions guiding its use are inseparably linked. Learning takes place most effectively in a context, and it is the notion of relevantly chosen contexts that is a critical part of the knowledge base associated with learning, an important foundation for which this study is premised (Duffy and Jonassen, 2013).

An approach in teaching called ‘Anchored Instruction’ places more or less the same emphasis in teaching as constructivism (Cognition and Technology Group at Vanderbilt, 1992). It puts emphasis on teaching and learning which embrace skills and knowledge in practical and realistic contexts. Anchored contexts employ complex and contextual problems which make learners create new knowledge whilst they determine how and when the knowledge become useful (Chen, 2013; Hannafin et al., 1997). Work related learning models are correspondingly allied as they encourage scaffolding and training in skills, heuristics, and approaches, as the student carries out genuine tasks (Chen, 2013; Collins, Brown and Newman, 1989). More linked approaches include the problem-solving model by the likes of Poyla (Polya, 2014; Misnasanti, Utami & Suwanto, 2017) and case-based learning environments (Choi & lee, 2009) where learners engage in solving authentic and contextual problems. These are problem tasks which are real and meaningful to students, and thus the focus this study to explore the extent to which task design either in teacher made exercises or examination questions embraces students’ life experiences.

Presenting several perspectives to learners is also another critical strategy that enhances students’ mastery of concepts. According to the Constructivist perceptive, learners should learn how to build a variety of perspectives on a subject of study (Fosnot, 2013). Students should make an attempt to view a concept from different angles so that they make the best out of the different perspectives. In other words, creating learning environments such as those involving constructivists approaches create a collaborative learning environment where learners to develop, compare, and understand multiple views on a subject. It is the thorough process of building and analysing the arguments that makes the goal in collaborative learning (Barkley, Cross & Major, 2014; Bednar et al., 1992). In this study, the component of collaborative learning was explored in the teaching and learning of Transformation Geometry.

Thus, Constructivist learning environment mirrors significant fundamental principles in the of transformation geometry. The task of Mathematics educators is to assess and review teaching and learning theories, tools and resources at their disposal, and to consider (if appropriate) how constructivist learning with transformation geometry may be facilitated, and how instructional designing responds to Constructivism.

2.8 RME PRINCIPLES FOR TASK DESIGN IN TRANSFORMATION GEOMETRY

RME (Realistic Mathematics Education) is an instructional theory that states that students should be active participants for their learning where they develop mathematical tools and insights for themselves (Freudenthal, 1991). RME theory provides principles for designing tasks in mathematics (Lin & Tsai, 2013). In this section, I investigate how these principles could be applied to design Transformation Geometry tasks. Three core principles of RME; Guided Reinvention, Didactical Phenomenology and Emergent Modelling, are discussed to show how they inform the designing of Transformation Geometry tasks.

2.8.1 Guided reinvention

The Guided Reinvention Principle sates the importance for students to experience a process similar to how the mathematical topic or concept was invented (Freudenthal, 1991 & Drijvers et al., 2016). Although this perspective of RME is essentially a teaching principle, it can be used to develop mathematical tasks and exercises for students. The task designer should come up with question items that provide students with the opportunity to reinvent solution strategies (Drijvers et al., 2016).

Students who explore the geometrical properties of objects and be able to arrive at targeted rules have engaged in the principle of guided reinvention. For instance, when learners explore properties of shapes (between an object and its image under a transformation) they pay attention on the dimensions of figures which help them identify the relevant geometric transformation (for example Isometric or Non-Isometric Transformations).

2.8.2 Didactical Phenomenology

The Didactical Phenomenological Principle addresses the issue on how the thought object can be used to organise and structure phenomena in reality. The task designer should identify meaningful phenomena that can be organised and structured by relevant mathematical knowledge. A question may be good or poor on the didactical phenomenology perspective if in the question there is or there is no phenomenon at stake (Drijvers et al., 2016). The presence of a phenomenon, for instance, drawn from a learner's everyday experiences, can motivate a student's engagement in a task. For example, a task for Geometric reflection can be designed such that it factors in some components of everyday experiences of students (for instance finding a phenomenon in graphic design of fabrics or floor tiling).

2.8.3 Emergent modelling

According to Drivers et al. (2016 p.55) the Emergent Modelling Principle requires that, "the task designer should find relevant situations that asks for students to develop models and allow for a process of progressive abstraction." Students go through a process of developing and refining models that allow them to bridge the gap between their intuitive understanding of real situations and their understanding of the more formal mathematics systems. Problems designed with the three principles in mind may bring about the development of new solution strategies different from those available already.

2.9 THE CONTEXT OF THE STUDY AND TRANSFORMATION GEOMETRY CURRICULUM

The main focus of the section is to articulate the context of the study so as to gain more insight about mathematics education in Zimbabwean rural secondary schools. This will shed more light on the nature of challenges surrounding mathematics in general and Transformation Geometry in particular. The section also discusses the importance of the Transformation Geometry component in the school mathematics curriculum

2.9.1 The context of the study

The particular community that is the focus of this study centred around three rural secondary schools. In the rural settings there are different types of schools. The GoZ established the rural day secondary schools which largely benefits learners within walking distances. There are also Mission boarding schools and Council run schools located in township areas of the rural Zimbabwe. For the purpose of this study, the three rural schools are classified as Mission boarding secondary school (School A), Council run school (School B) and rural Government day secondary school (School C).

There has been expansion of education in Zimbabwe resulting in more secondary schools built in both urban and rural settings (Mugomba, 2016). This, however, resulted in problems of allocation of resources. According to Ersado (2005) there is an uneven distribution of resources where the government seems to spend more in the urban school compared to rural schools.

2.10 IMPORTANCE OF TRANSFORMATION GEOMETRY IN SCHOOL MATHEMATICS

In this section, I spearhead a discussion that justifies the inclusion of Transformation Geometry in the school Mathematics syllabus.

According to Hollebrands (2003), there are three important reasons to studying Geometric Transformations in school Mathematics. It provides opportunities for learners to think about important mathematical concepts (e.g. symmetry, congruence), it provides a context within which students can view Mathematics as an interconnected discipline, and it provides opportunities for students to engage in higher-level reasoning activities using a variety of representations. Peterson (1973) points out that Transformation Geometry encourages students to investigate geometric ideas through an informal and intuitive approach. We see symmetry everyday but often do not realise it. People use concepts of symmetry including rotations and translations as part of our careers, for example, artists, craftspeople and musicians (Dobitsh, 2014). Thus, it is important for learners to learn the concepts of Transformation Geometry as a means of exposing them to situations they meet with everyday that might have a strong foundation in Mathematics (Dobitsh, 2014).

Transformation can lead students to exploration of the abstract mathematical concepts of congruence, symmetry, similarity, and parallelism; enrich students' geometrical experience, thought and imagination; and thereby enhance their spatial abilities. Research suggests that learners should have sufficient knowledge of geometric transformations by the end of eighth grade in order to be successful in higher level Mathematics studies (Carraher & Schlieman, 2007; NCTM, 2000). For these reasons there is significant support for the incorporation of Geometric Transformations in a school Geometry courses (Hollebrands, 2003).

However, studies show that learners have difficulties in understanding the concepts and variations in performing and identifying transformations including translation, reflection, rotation and combinations of transformations of these types (Clements & Burns, 2000; Edwards, 1990; Olson, Zenigami& Okazaki, 2008; Rollick, 2009). For example, Edwards (1989) found that middle school students encounter difficulties in both executing and identifying transformations. Execution errors include drawing images of reflections in the wrong orientation and out of scale. In these studies, it was concluded that whilst most students have an operational understanding of transformations, most have not developed a conceptual understanding. In other words, they have not developed deeper structural relationships between concepts whose establishment result in students growing full mathematical power. Some researchers such as Edwards (1989) have seen dynamic representations as a powerful tool to improve students' understanding from operational to conceptual.

The second justification is the strong similarity between Transformation Geometry and the natural world and the recognition of the vast learning experiences that can be drawn from, and related to, the physical world these students live in (Trafton & LeBlanc, 1973). The National Council of Teachers of Mathematics Principles and Standards for School Mathematics (2000 p.52) states that, "By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom."

Hershkowitz, Bruckheimer, and Vinner (1987:222) state, "This basic knowledge, which comprises geometric concepts, their attributes, and simple relationships, should, in general, be acquired through geometrical experiences prior to secondary school." Furthermore, in reconceptualising the geometry curriculum it should be noted that, while the traditional approach to geometry as described above is undoubtedly important, there is much more to the study of geometry than this and this can realistically be explored at school level:

Malkevitch (1998) notes that nowadays most Geometers are not professionally interested in the axiomatic development of Geometry but rather, that for most Geometers, “geometry has become the study one is led to by mathematical training when one studies visual phenomena”. Geometry has led to a number of rich applications currently used in modern technology, for example, in Computer Technology, Medical Imaging, Communications Technology (codes in fax technology, etc.) and Image Processing. Malkevitch has suggests the following topics for study at school: Graph theory; Compression Codes and Error-Correcting Codes; Frieze Patterns, Wallpaper Patterns, Fabric Patterns; Knots; and Polyhedra and Tilings. While acknowledging the implications of the inclusion of such topics in the curriculum for teacher development it is imperative that learners be afforded the opportunity to study these topics in preparation for participation in a technological society

2.11 THE MATHEMATICS TEXTBOOK

Textbooks are indispensable for the learning environments as well as teachers. In this section the researcher discusses the key resource, the textbook. According to Senk and Thompson (2003) textbooks are structured so that topics are introduced by stating a rule, showing an example and then offering end-of-unit exercises for student practice. Right through the 20th century and the 21st century, the most common textbook presentation style followed the sequence of offering exposition, examples, and exercises (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Hence, the present type of textbook used in schools needs to be examined to determine its alignment with the national syllabus requirements.

Research suggest that the textbook is a key resource that has a striking influence on the teaching and learning in the classroom (Reys, Lapan, Holliday, & Wasman 2003; Schmidt et al., 2001; Schmidt, 2002; Tornroos, 2005). Learners typically do not learn what is not in the textbook (Jones, 2004; Reys, Reys, & Lapan, 2003; Schmidt, 2002) and teachers also teach what is in the textbook (Reys, Reys, & Lapan, 2003). Therefore, textbook content analyses are needed.

Teachers rely heavily on the textbook for curriculum design, scope, and sequence (Stein, Remillard & Smith, 2007) and sometimes on guidance on pedagogy. Thus, the textbook is the most common channel through which teachers are exposed to the communications from professional organisations in reference to mathematics standards and to recommendations from the research community (Collopy, 2003); both standards and recommendations translate into immediate determinants for teaching practices (Ginsbury, Klein, & Starkey, 1998). For purposes of this study, various textbooks, including national official texts, were subjected to some critical analysis (for example Porter, 2006) to examine the extent to which they support student comprehension of transformations concepts.

According to Grouws and Smith (2000) as well as Tarr et al. (2006), throughout Mathematics classrooms in the United States, the textbook holds a prominent position and represents the expression of the implicit curriculum requirements. These various educators suggest that the mathematics textbook is regarded as the authoritative voice that directs the specified mathematics curriculum content in the classroom (Haggarty & Pepin, 2002; Olson, 1989). The influence that the textbook maintains is related to most of the teaching and learning activities that take place in the mathematics classroom (Howson, 1995).

The development of the structure and content of the textbook is done by textbook authors and publishing staff. The problem with some texts is that the quantity of topics presented (Jones, 2004; Snider, 2004; Valverde et al., 2002) is such that they lack the depth of study for specific topics (Jones, 2004; Snider, 2004; Tarr, Reys, Barker, & Billstein, 2006; Valverde et al., 2002). The number of breaks between mathematics topics (Valverde et al., 2002) and the contextual features and problems related to performance are not properly addressed.

Although, reports indicate that Mathematics textbooks are frequently used in classrooms for teaching practices and student activities (Grouws & Smith, 2000; Tarr, Reys, Barker & Billstein, 2006), inconsistency and weak coverage of mathematical concepts were found in most of the textbooks examined (Valverde, 2002). Teachers use the textbook as the main source for lesson presentations and student exercises (Grouws & Smith, 2000).

Studies show that the textbook has become the formal curriculum and in that case, it dominates what goes on in the classroom and students opportunity to learn (Braswell et al., 2001; Grouws & Smith, 2000; Tarr, Reys, Barker & Billstein, 2006). Hence, because the textbook is used to determine classroom curriculum it is important to analyse the content of textbooks used in facilitating the teaching of transformations in Zimbabwe. For purposes of this study, an investigation was carried out to analyse the textbook structure and impact.

2.12 DIFFICULTIES EXPERIENCED BY LEARNERS WITH TRANSFORMATIONAL GEOMETRY CONCEPTS

2.12.1 Challenges impacting teaching and learning in Transformation Geometry.

The following are some of the challenges compromising effective teaching of transformation geometry:

(1) Poverty has become more acute and widespread, leading to many parents finding it difficult to afford their children's learning resources and school fees. This includes resources such as the mathematical set, the calculator, and the graph books which are useful in the teaching of transformation geometry.

(2) Some teachers never got a chance to learn transformation geometry while at school and as a result are not teaching their students the topic (Von, 2006). This has resulted in situations where some schools end up hiring relief teachers, who have not gone through training, indefinitely. Such a development has affected the teaching of long topics such as transformation geometry where either teachers skip the topics or simply teach the basics.

(3) Low morale within the teaching profession has led to staff exodus from the teaching profession (Financial Gazette, 2003). The unsatisfactory commitment of teachers has been exacerbated by their poor remuneration and conditions of service. Many teachers, especially for Mathematics and Science, have left the teaching profession to escape the worsening economic situation. Most found employment in neighbouring countries. Furthermore, supervision is lax due to lack of resources for Education Officers to visit schools (National Education Advisory Board, 2010). And with topics like transformations what can be expected from the calibre of such teachers. This brain drain seems to be reversing the gains attained over the past two decades of providing trained teachers to the system (Nziramasanga, 1999).

(4) Some teachers teach for examinations and as a result fail to develop in their students a mastery of concepts in transformation geometry. In this area it is necessary to come up with assessment techniques that strike a balance between the affective and cognitive domains (Zimbabwe National Commission for UNESCO, 2001). Examinations have

tended to require acquired knowledge other than a demonstration of an ability to apply knowledge.

2.12.2 Levels of Cognitive Demand for students involved in transformation tasks

Stein, Schwan-Smith, Henningsen and Silver (2000) came up with a framework (*see Appendix Q*) to identify a student's level of cognitive demand needed to complete mathematics exercises and tasks. In this framework the level of cognitive demand in mathematical tasks is documented by giving an assessment of a student's thinking and reasoning needed by the different questions posed (Kessler, Stein & Schunn, 2015).

This framework was adapted for use in evaluating the level of cognitive demand in student textbook exercises as well as in the national examination past papers. According to this framework, questions can be classified as those that call for memorisation or the application of algorithms or rules into a category of tasks that require *lower-level* demands. Questions that demanded students to use *higher-level* thinking were rather unstructured or semi structured, and often had more than a single solution, or was more sophisticated or non-algorithmic.

In this framework, four categories of levels of cognitive demand students are clarified. The outline suggested by Stein, Schwan-Smith, Henningsen, and Silver (2000); Stein, Grover, and Henningsen (1996); Stein, Lane, and Silver (1996); Stein and Smith (1998) provided suggestions for discerning the level of cognitive demand of mathematical tasks in transformation geometry. This understanding of levels of cognitive demand was used in this study to decide the level of cognitive demand required for student performance in transformation geometry tasks examined.

2.12.3 Research findings on Transformation Geometry Tasks and Common Student misconceptions

The purpose of this section is to review background information on the nature of challenges students experienced when performing two-dimensional transformation geometry tasks. The section exposes the specific challenges and misconceptions displayed by either teachers of Mathematics or students of Mathematics. According to Soon (1989), students aged between 15 and 16, successfully perform Transformation Geometry tasks in this order: reflections, rotations, translations, and dilations/ enlargement.

Contrary to Soon (1989), Kidder (1976), Moyer (1978), and Shah (1969) report that translation is the easiest transformation for students. Soon (1989) and Zorin (2011) both found that learners do not instinctively use particular or exact vocabulary when reporting about translations, instead they used the finger movements or terms such as “move” or “opposite” to demonstrate the direction of change. Thus, Zorin stresses the importance of emphasising vocabulary and the skills in drawing shapes and their images during teaching and learning in transformation geometry.

Students require concrete opportunities to augment the terms or vocabulary used in transformational geometry (Martinie & Stramel, 2004; Stein & Bovalino, 2001). The use of manipulative provides students with a concrete opportunity for mastering abstract concepts. Martinie and Stramel (2004 p.260).

> *Transformation geometry topics may be approached through the manipulation of concrete objects. But eventually, when the object becomes a distinct image, the child can then perform mental transformations (actions) concerning these images. Imagery evolves from initially a level of reproductive images based upon past perceptions to a level of true anticipatory images which are the results of an unforeseen transformation*.

Learners often exhibit numerous misconceptions when performing Transformation Geometry tasks. A number of studies found that students who focused on the whole figure being transformed, instead of focusing on each point being moved to its corresponding location, are bound to demonstrate misunderstandings (Hollebrands, 2003, 2004; Laborde, 1993; Soon, 1989), since they experience problems in visualising the features of the figures on their own (Kidder, 1976; Laborde, 1993).

According to Kidder (1976) learners in Grades 4, 6, through 8 experience difficulties with the property of preservation of length. They focus more on the visual features and the movement of the shape under the transformation and not on the properties of the transformation itself (Soon, 1989; Soon & Flake, 1989). Laborde went further to propose higher level reasoning powers as a requirement in the mastery for preservation of properties of figures. The following sections discuss the misconceptions and errors shown by students in performing the different transformations.

In the discussions, issues pertaining to how students experience the four principle types of transformations (translations, reflections, rotations and dilations) and composite transformations are considered. Literature reviewed articulated on the particular challenges displayed by students in the different forms of transformations within transformation tasks. The structure of the presentation takes the form of presenting the background and reasoning in the collection of specific performance tasks for each of the transformation types.

Translations

A Translation Transformation is the image found by a function describing a straight-line movement in the same direction of a vector or a geometric object (Akkaya, Tatar & Kagizmanli, 2011; Zembat, 2013). It is the movement of a geometric shape from one place of location to another in a specified direction which defines a geometric translation (Aksoy & Bayazit, 2009; Channon et al., 2004). In other words, in a Translation Transformation three essential properties are critical for the mastery of the related concepts. The first and most important property for a Translation Transformation concerns the internal dynamics of a shape, that is, the edge length, angles and orientation of a geometric shape. The second property stipulates that every point on Geometric shapes is identical to the matched points after the transformation. Thirdly, a translation transformation which uses a zero vector results in the image of the geometric object being the object itself and on the same location as the object (Zembat, 2013).

Shah (1978) says translations are usually the easiest geometric transformations for students. In their study with 3rd and 4th Graders, Schultz and Austin (1983) and Shultz (1978) discovered that the direction of the movements of objects in translation geometry transformation was the source of difficulty. They found out that translation movements to the right and to the left were easier compared to movements in an inclined direction. However, according to ZIMSEC (2010) candidates also experience difficulties in appreciating the effect in negative sign in a translation vector. For instance, $\binom{a}{b}$ and $\binom{-a}{-b}$ are vectors representing opposite directions.

Additionally, they found out that by increasing the distance between the object and its image in the translation, increases students' difficulty in performing the translation tasks. Flanagan (2001) found out that learners experience problems in recognising the movement of the object in a translation in terms of the magnitude of movement and how it is related to the magnitude of the vector given on the coordinate graph. According to Hollebrands (2003) students must realise that an object and its image can be seen as parallel figures and that the magnitude of the gaps between the object and its image points are identical and of the same magnitude as the translation vector. Flanagan (2001) as well as Wesslen and Fernandez (2005) concur that students generally failed to recognise that by translating an object every point on the object moves the same distance and in a parallel and matching orientation.

The research findings illustrated above show how important it is to emphasise the direction of the translation of the objects as certain directions in movements are easily recognisable by some students than others, particularly the movement of an object in a translation that is in an inclined direction. Thus, it was in the thrust of this study to identify concrete manipulative that are real in the mind of the students that are at teachers' disposal and have a relationship with the notion of translation. Therefore, by relating these concepts to learner experiences, it is hoped, it can bring about conceptual understanding of the concept translation and not operational/procedural understanding.

Reflections

The notion of Symmetry is one of the fundamental application fields in the real world of geometric reflection. It is a critical tool in the understanding of nature and hence the environment and is useful in numerous fields such as art and architecture (Aksoy & Bayazit, 2009). Symmetry can be viewed in two different ways. One of them is linked to the order of symmetry, aesthetics, beauty and perfection (Yavuzsoy, 2012). A geometric transformation is also the foundation for the comprehension of the topics in Analytic Geometry.

Studies reveal that there are numerous challenges posed by pre-service teachers with geometric reflections (Rollick, 2009). The only reflection task that these teachers found the simplest was of a shape moving from the left to the right side over the *y-axis* or a vertical line. The teacher participants revealed problems to do with performing a reflection from the right to left and they would classify the movement as from top to bottom instead. A number of the participants recognised a reflection as a translation especially when symmetric shapes were used. Moreover, sometimes they confused reflections for rotations where they would perform a rotation in place of a reflection and vice versa (ZIMSEC, 2010). According to Rollick (2009) developing the notion of an invariant relationship between the object and its image is critical in mitigating these misconceptions.

Yanik and Flores (2009) as well as Edwards and Zazkis (1993) in their studies both concur that pre-service elementary teachers of mathematics interpreted the line of the mirror of reflection as slicing the figure in two equal halves, or alternatively interpreted one of the edges of the shape as the predetermined mirror line of reflection. In other words, if pre-service elementary teachers of mathematics struggle with the concepts of reflection what more with students who rely on their teachers for expert advice?

Kuchemann (1980; 1981) discovered that students had the challenges particularly when a figure is reflected over an inclined line, the students were found to often disregard the angle or gradient of the line of reflection and perform a horizontal or vertical reflection they are used to instead. This realisation was also evident in the works of Burger and Shaugnessy (1986), Perham, Perham, and Perham, (1976), as well as Schultz (1978). The other difficulty experienced by learners was in reflecting a figure over a line of reflection that intersects the object, in this transformation the image overlaps the object (Edwards & Zazkis, 1993; Soon, 1989; Yanik & Flores, 2009). In such cases, using tracing paper (*Patty paper*) is useful as it assist students to visualise the transformation in a practical sense (Serra, 1994). The axes and the object are traced; then, the tracing paper is flipped over and aligned to bring out the position of the image.

Thus, these research findings on reflections reveal challenges that students have in performing a geometric reflection and that it is critical important to mention the direction of movement of the figure since reflection right to left, over an inclined line, and of a figure over itself are common challenges. The use of manipulative or concrete objects was recommended in clarifying these challenges in students with reflection problem tasks.

Rotation

Rotation transformation is one of the subjects of geometry useful in the interpretation of solids. Learners who can visualise a cone after rotating a right angled-triangle through 360° about one of its legs, visualise a cylinder after the rotation of 360° of a rectangle around one of its lines, and visualise a sphere after the rotation of a semicircle through an angle of 360° around its diameter, can easily learn solids with understanding (Aksoy & Bayazit, 2009).

Clements and Burns (2000) in their study involving fourth graders noted that students were made to understand the notion of rotation through experiencing the physical turning with their own bodies. Additionally, the concept of clockwise and anti-clockwise was developed through practising a turn to the right and a turn to the left, and then followed by noting the amount of turn. Out of all, Isometric Transformation, learners in their early learning years at school would experience challenges when working with rotations (Moyer, 1975; 1978).

Kidder (1976) administered a test to students who were in the age range of nine to thirteen years. It was discovered that learners could not envisage the presence of the angle and its rays necessary for a rotation. Learners could not keep some factors fixed as they were varying others in performing a rotation. Kidder also observed that learners had challenges in holding the distance from the point of rotation to the vertices of the shape fixed as they were performing a rotation. The learner could not realise that angle measures of the shapes remain unaffected under the rotation.

Olson, Zenigami, and Okazaki (2008) observed that learners were unable to see that when rays of different lengths are rotated through the same angle the same number of degrees resulted. Students revealed common misconceptions about the size of an angle determined by the lengths of the rays that form up the angle (Mmarella & Caviola, 2017; Clements, & Battista, 1990). Moreover, Clements, Battista and Sarama (1998) discovered that students had challenges in deciding on the size of an angle of rotation, but they were seen showing confidence using such measures like 90 and 180 degrees.

Yanik and Flores (2009), as well as Wesslen and Fernandez (2005) agree that students mental image of rotation is normally at the centre of the figure being rotated, and students did well in this type of rotation. Wesslen and Fernandez (2005) found out that students were least confident when rotating figures whose centre of rotation was defined as other than the centre of the shape or a vertex of the figure; however, students also had challenges in using the figures' vertices for centre of rotation and had problems when it comes to clockwise and anti-clockwise directionality. Soon (1989) and Soon and Flake (1989) discovered that students have serious challenges especially in the rotation of a figure whose centre of rotation is given as any point external to the figure. Students tend to disregard the prescribed centre of rotation and instead went on to rotate the figure about the centre of the figure or using any one vertex of the figure. They often ignored the prescribed direction of rotation in the transformation question (Soon & Flake, 1989). Soon (1989) as well as Wesslen and Fernandez (2005) concur that students do not demonstrate knowledge to do with angle of rotation or centre of rotation or both.

Clements and Burns (2000) as well as Clements and Battista (1992) also concur that, students generally depict many misconceptions and difficulties in mastering the concepts of angle of rotation as well as direction of rotation. These concepts are critical in the mastery of rotation. Clements and Burns believe that the static meaning used for an angle (An angle is the section of the plane in between two rays that meet at a vertex) may be the source of the misconception. Clements et al. (1996) observed that students do not realise the importance of noting the direction of a rotation, that is, whether it is clockwise or ant-clockwise when performing a rotation. According to ZIMSEC (2010) candidates also confuse a rotation with a reflection where they often regard one as the other. Studies shown above portray the numerous challenges that students go through when performing rotational tasks. Common errors made by students concern the meaning of a measure of angle of rotation, and centre of rotation. Additionally, Clements and Burns (2000) in their study involving fourth graders noted that students were made to understand the notion of rotation through experiencing the physical turning with their own bodies. Distinction between factors that are invariant and those that are not during a rotation seems to create further problems for students when performing tasks in rotation. Thus, this study is anchored on the position that embracing contexts that are real and meaningful to students in Transformation Geometry is bound to enhance mastery. Hence, the focus of this study was to explore mathematics teachers' use of students' out-of-school experiences in the teaching transformation geometry.

Dilations/Enlargement

According to Soon (1989) the Transformation Geometry in the form of dilation is one of the most difficult concepts for students as indicated by examiners' reports. Learners show a lot of confusion with the scale factor of enlargement. They think that a positive scale factor means an enlargement and a negative factor means a reduction in size of the figure. Many students fail to recognise the centre of enlargement given the object and the image of figures related by a geometric enlargement and some failing to appreciate the meaning of a minus scale factor (ZIMSEC, 2014).

Further, learners are hesitant to use the exact terminology for centre of enlargement or for enlargement scale factor. They instead use, for example, "equal angles but sides enlarged two times" (Soon, 1989, p. 173). In addition, students would anticipate a change to happen even if the scale factor is given as 1. They do not recognise the situation where a zero resembles an identity property (Soon, 1989). Hence, it would look like there is confusion surrounding the topics of scale factor, similarity, and identity, with evidence of terminology use also posing challenges for the students in mathematics.

Composite Transformations

According to Wesslen and Fernandez (2005) the study of composite transformations enhances understanding for concepts such as similarity and congruence of two-dimensional objects and gives meaning to the mathematical system of transformations (Wesslen & Fernandez, 2005). Composite transformation entail that two transformations are combined to form a compound transformation, and this results in an image that can be redefined as one of the original transformations (Wesslen & Fernandez, 2005). A section on composite transformations is included in the national syllabus for the ordinary level mathematics in Zimbabwe. For example, one of the content objectives stipulates that, "all students should be able to apply combinations of the different transformation", (ZIMSEC**,** 2012 p.13).

According to Wesslen & Fernandez (2005) the inclusion of composites to the topic of geometric transformations, makes Mathematics interesting because it has become a complete system with plenty of patterns to be discovered. For example, the two authors' sentiments that any two transformations combined seem always to be like one of the already existing ones speaks to interesting discoveries. The role of composite transformations in the school mathematics curriculum is overemphasised by several educators in Mathematics (for example Burke, Cowen, Fernandez & Wesslen, 2006; Schattschneider, 2009; Wesslen & Fernandez, 2005).

When students fail to visualise figures which are congruent to one another when the figures are placed in different orientations, they would need to have considered, for example, that a composite of isometric transformations (translations, reflections and reflections) would still yield the same resulting figure (Usiskin et al., 2003), and hence this yields various possible conjectures for students to make. moreover, problems that students experience include finding the distance a shape was moved for a transformation on a coordinate plane; the students seem to experience difficulty in finding the distance as well as direction which the figure moved (Usiskin et al., 2003).

2.13 CHAPTER SUMMARY

This chapter has reviewed literature linked to the present study. It focused on two major categories: the Theoretical Framework which defined the model which guided the development of this study as well as the Conceptual Framework which helped identify the constructs relevant in this study. This included an articulation on the historical perspectives of both Geometry and Transformation Geometry; theories that guide the teaching and learning of Transformation Geometry concepts, and related study findings on the misconceptions and errors that are committed by students in performing transformation geometry concepts.

The chapter also made a description of the curriculum content and the debate on textbook use. It also discussed how these resources contribute to student comprehension of the transformation geometry concepts, that is, the impact the textbook has on students' mastery in the topic, criticisms of the curriculum and the textbook, and the need for content analysis. The findings were presented on an in-depth delineation of the Transformation Geometry constructs related to this study and the nature of difficulties that students experience when learning transformation concepts.

This particular consideration of relevant literature has defined the need for analysis of the content on transformation geometry and has provided background for the structure of the conceptual framework for this study. Chapter Three, presents a framework for the study's methodological approach including the methods used in obtaining relevant data.

CHAPTER THREE
RESEARCH DESIGN AND METHODOLOGY

3.0 INTRODUCTION

The preceding chapter presented the theoretical underpinnings of this enquiry. In order to fully address the study objectives, this chapter starts with a synopsis of the research paradigm and research design, and then it provides a description of the research site and participants. In addition, instruments, data collection and analysis procedures, validity and reliability, and ethical issues will be discussed.

The major question that guided the study was, “To what extent does teaching and learning of transformation geometry utilise students’ lived experiences?” This research study has been undertaken primarily to identify and suggest how Transformation Geometry thinking can be enhanced in learners through incorporating their real-life experiences. The study is significant in that incorporating students’ lived experiences in transformation geometry classes is bound to help teachers deliver lessons in the subject topic in a way that will excite students, assist their connection and application of "real world" settings to the concepts and extend students' abilities to solve mathematics problems in other context (Dickson et al., 2011). For that purpose, it was important that the selected research design and methods be relevant and appropriate in answering the research questions. The following are the research questions that guided the study:

1. What are teachers’ perceptions about the mathematics involving transformation geometry concepts contained in the students’ out-of-school activities?
2. How is the context of transformation geometry teaching implemented by practising teachers in Zimbabwean rural secondary schools?
3. To what extent are students’ out-of-school experiences incorporated in transformation geometry tasks?
4. How is transformation geometry, as reflected in official textbooks and suggested teaching models, linked to students’ out-of-school experiences?

3.1 RESEARCH PARADIGM

Denzin and Lincoln (2000:157) define a research paradigm as, “a basic set of beliefs that guide action, dealing with first principles, ‘ultimate’s or the researcher’s worldviews”. In other words, a paradigm is an action of submitting to a view. In this study it was important to define the researcher's paradigm choice as it served to guide in exploring the extent to which teachers use learners' real-life experiences to enhance learners' transformation geometric thinking.

This study is oriented in the interpretive research paradigm. The interpretive paradigm holds the view that people have reasons why they act the way they do, and that to understand the reasons behind human action requires not detachment from, but rather direct interaction with the people concerned (Connole, 1998; Schwandt, 2000). Like other research paradigms, the interpretive paradigm is characterised by its own ontology, epistemology and methodology (Terre Blanche & Kelly, 1999).

The interpretive tradition assumes that people's subjective experiences are real and should not be overlooked (ontology), that these experiences can be understood by interacting with the people concerned and listening to what they have to say (epistemology), and that qualitative research techniques are best suited to gaining an understanding of the subjective experiences of others (methodology) (Blanche & Kelly, 1999). Ontology defines the nature of reality that is to be studied and what can be known about it; epistemology defines the nature of the relationship between the researcher (knower) and what can be known; and "methodology specifies how the researcher may go about practically studying what can be known" (Blanche & Durrheim, 1999:6).

The epistemological position regarding the current study was formulated as follows:

a) data are contained within the perspectives of people that are involved with the teaching (teachers of mathematics) and how students learn the geometric transformations in rural secondary schools in the district of Mberengwa, Zimbabwe, through observations and interrogation; and

b) to be engaged with the participants (teachers of mathematics and ordinary level students of mathematics) in collecting the data.

3.2 METHODOLOGICAL DESIGN

Based on the qualitative research paradigm I took the transcendental phenomenology approach by Moustakas (1994), adapted from Husserl (1931) to generate an essence of the lived experience of participants. The general purpose of the phenomenological study is to understand and describe a specific phenomenon in- depth and reach at the essence of participants' lived experience of the phenomenon (Yuksel & Yildirim, 2015). The intention of the study therefore was to explore Mathematics teachers' use of students' out-of-school experiences in the teaching and learning of Geometric Transformations and the associated benefits, through use of different data collection techniques such as lesson observations and post lesson interviews (Yin, 2003: Creswell, 2013).

In this study, the researcher was able to see and hear mathematics teacher practices with transformation geometry. The use of this design ensured the researcher to arrive at answers to questions "What, how and to what extent" teachers use students' real life experiences in teaching transformation geometry concepts (Creswell, 2013).

In this study, the *object of the phenomena* is use of real life experience in the teaching of Transformation Geometry in Mathematics classes. The subject is teachers of mathematics. Therefore, the study explored how teachers use learners' real-life experience in Transformation Geometry (T.G) Mathematics classes. In this study, the act of experience which is the meaning of the essence occurred after the *imagination variation* is using real life in Transformation geometry (T.G) teaching in the classroom (Yuksel & Yildirim, 2015).

3.2.1 Structure of study design

In order to understand the phenomenological idea, the following key concepts of the phenomenology philosophy are examined: lived experience, intentionality, *noema-noesis,* epoché and co-researchers.

Lived Experience

Phenomenological research investigates the lived experience of participants with a phenomenon. Phenomenological studies start and stop with lived experience which should be meaningful and significant experience of the phenomenon (Creswell 2007; Moustakas, 1994, Thani, 2012; van Manen, 1990).

In this study, the researcher was interested directly with related lived experiences of the phenomenon, that is, teachers' use of real life experiences of the learner. Therefore, participants demonstrated some meaningful and significant experience of how they use learner experiences in Transformation Geometry teaching.

Intentionality

Husserl (1970) argues that there is a positive relationship between perception and objects. The object of the experiences is actively created by human consciousness as we always use our consciousness in thinking. It needs perceiving or conceiving an object or an event (Holstein & Gubrium, 2000). Therefore, for Husserl (1931), intentionality is one of the fundamental characteristics of the phenomenology that is directly related to the consciousness.

Intentionality refers to doing something deliberate. For example, in this study, using learners' real-life experience for enhancing Teaching and learning in TG is an intentional experience of teachers' non-mental activities (Yuksel & Yildirim, 2015). Teachers' examples of TG concepts in their teaching are intentional acts dependent on teachers' consciousness. Therefore, the act of experience is related to the meaning of a phenomenon. Thus, the essence of the phenomenon is derived from the act of teachers experiencing perceived real-life examples of TG concepts in the classroom. This means that "the object exists in the mind in an intentional way" (Kolkelman, 1967; Moustakas, 1994:28). Therefore, intentionality reflects the relationship between the object and the appearance of the object in one's consciousness.

In the transcendental phenomenology design, the intentionality has two dimensions, *noema* and *noesis* (Yuksel & Yildirim, 2015). *Noema* is the object of experience or action, reflecting the perceptions and feelings, thoughts and memories, and judgments regarding the object. *Noesis* is the act of experience, such as perceiving, feeling, thinking, remembering, or judging (Yuksel & Yildirim, 2015). The act of experience is related to the meaning of a phenomenon. In this study, while real life learner experiences related to TG concepts is the *noema* of the experience, using the real-life experiences for purposes of teaching concepts is the noesis of the experiences. *Noema* and *noesis* are interrelated and cannot exist independently or be studied without the other (Cilesiz, 2010).

Epoché

Epoché is a Greek word used by Husserl (1931) meaning to stay away or abstain from presupposition or judgments about the phenomena under the investigation (Langdridge, 2007). Basically, *Epoché* allows the researcher to be bias-free to describe the reality from an objective perspective. For example, from previous experiences of the phenomena as a mathematics educator, the researcher bracketed his own experience and knowledge concerning the phenomena under study in order to understand the participants' experiences entirely by staying away from prejudgment results. In other words, the researcher bracketed his own views about real life examples on Teaching TG and relied on statements supplied by participants.

Phenomenological Reduction

In phenomenological reduction, the task is to describe individual experiences through textural language. In order to describe the general features of the phenomenon, elements that are not directly within conscious experience were left out (Yuksel & Yildirim, 2015). This was achieved by eliminating overlapping, repetitive, and vague expressions i.e. cleaning the raw data. In this study, there was need to clean the participants' interview on responses which were not directly linked to the focus of the study.

Imaginative Variation

Imaginative variation is a phenomenological analysis process that follows phenomenological reduction and depends purely on researchers' imagination rather than empirical data (Yuksel & Yildirim, 2015). The aim was to arrive at structural descriptions of an experience, the underlying and precipitating factors that account for what is being experienced; in other words the "how" that speaks to conditions that illuminate the "what" of experience" (Moustakas, 1994: 85).

Co-researchers

Moustakas (1994) defined all research participants as co-researchers because the essence of the phenomena is derived from participants' perceptions and experiences, regardless of the interpretation of the researcher. The participants' narratives of experiences provide the meaning of the phenomena. It is the role of the researchers to create the textural, structural, and textural-structural narratives without including their subjectivity (Yuksel & Yildirim, 2015). This means the transcendental analysis requires no interpretation by the researchers.

3.2.2 Structure of the research process

Though qualitative studies are not generalised in the traditional sense, some have redeeming qualities that set them above the requirement (Myers, 2000). According to Yin (2013), qualitative research findings can be transferred to similar contexts. Analytic data cannot be generalised to some defined population that has been sampled, but to a theory of the phenomenon being studied, a theory that may have much wider applicability than the particular phenomenon studied. In this study it resembles experiments in the physical sciences, which make no claim to statistical representativeness, but instead assumes that their results contribute to a general theory of the phenomenon (Yin, 2013). Since the study focused on teachers' use of students' real-life experiences in the teaching and learning of geometric transformations it assumes that failure by teachers in using learner experiences in teaching is detrimental to their understanding.

The following is a basic model which was used in the total research process.

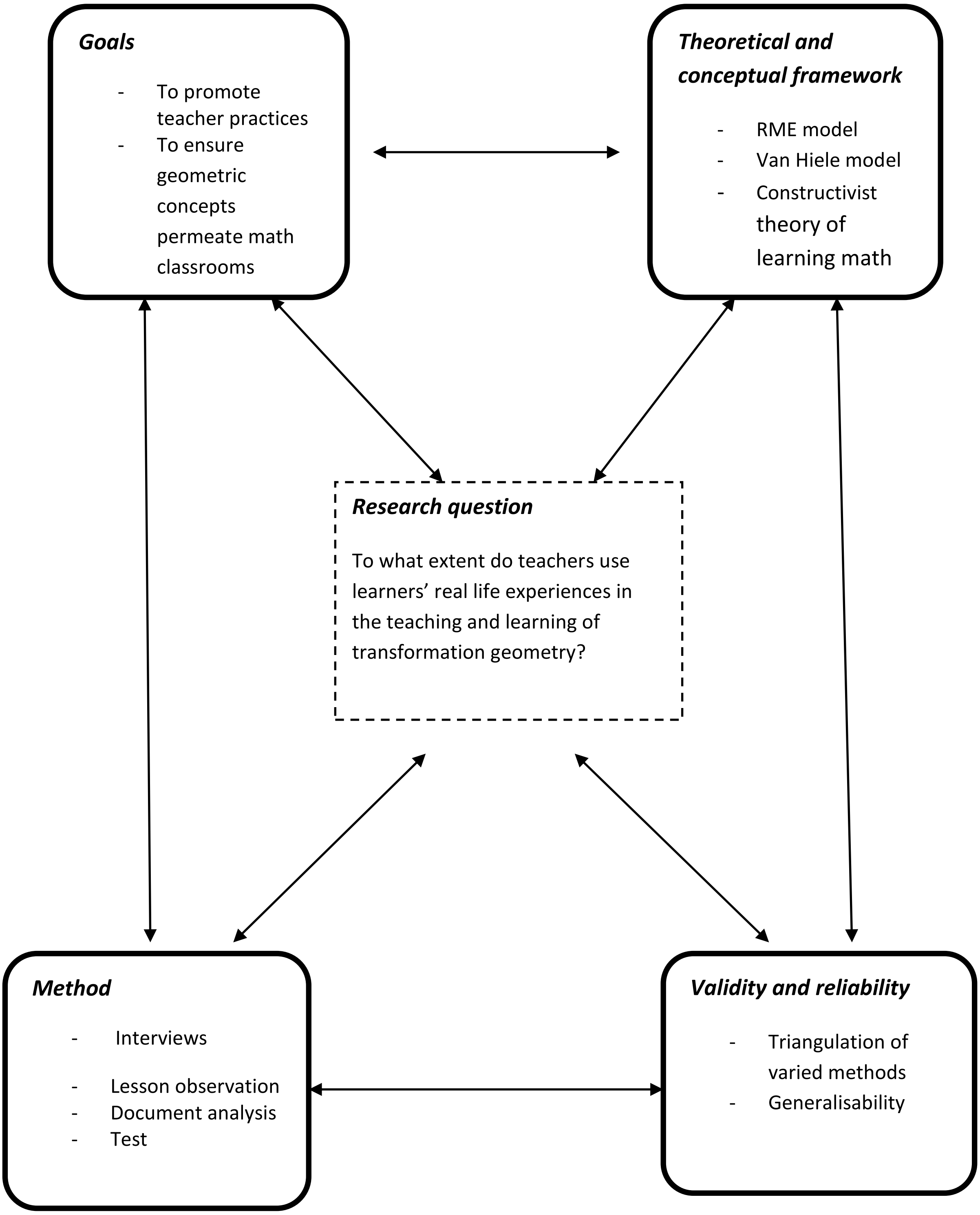


Fig. 3.1: Interactive model of the phenomenology strategy

(Adapted from Maxwell 2005:9)

The above model was used in an attempt to provide links between components of the research process. For example, theoretical and conceptual frameworks were understood and used in the context of the study's research questions as well as the goals of the study. Thus, every component was influenced by and influenced at least two other components.

3.3 POPULATION AND SAMPLING TECHNIQUES

Selection of the schools and participants was done with a number of considerations. In the following section an account of the research site and the selection of participants is given.

3.3.1 The research site

According to Creswell (2012*)* a study population refers to a complete group of entities that share a set of characteristics that are similar. The population of this study constituted secondary schools in Mberengwa district. Mberengwa district is one among 10 districts in the midlands province of Zimbabwe. There are 9 provinces in Zimbabwe, giving an estimate of *72* districts in the country. The study purposively selected Mberengwa district schools mainly because of the schools diversity. Basically, they are three types of secondary schools in the district; mission owned schools, government owned schools and council run schools. Three schools were selected purposively, however which the choice resembling the schools diversity. Thus, there are different factors that influence learning in these schools in a significant way, such as students' home and social life, resources available to the school, and the type of community in which it is situated. The schools were assigned arbitrary names for anonymity; *School A, School B and School C*.

School A

School A is a co-educational Mission boarding high school of the Evangelical Lutheran Church in Zimbabwe. It is located some *24* km away from a residential township area in the rural Zimbabwe. The school was established a very long time ago. It enrols both boys and girls Forms 1 up to 6. Forms 1 up to 4 have about 4 classes per Form level which are not streamed according to ability. Since it is a mission school the church is responsible for financing all its operations. The school has an average enrolment of about 900 students.

There are four Mathematics teachers only two of whom are professionally qualified. School has boarding facilities that house about *500* boys and girls, with the remainder as 'day scholars'. The school has a computer laboratory and three separate science laboratories. The computer laboratory has about 20 functional computers. The average number of students is (45) per mathematics class which is tolerable. *School A* is comparatively well resourced with white boards, fairly well-equipped science laboratories for biology, chemistry and physics practicals. Although School A is a fee-paying school, parents generally can afford the fees. In School A students have their own permanent classes and the teachers move to teach the students during each change-of-lesson time.

School B

School B, is located in a formal residential township. The school was established just after Zimbabwe gained independence in 1980.The school has an average enrolment of about *700* learners in forms 1 – 6. Forms 1 - 4 have each, 3 classes which are not streamed according to ability. Despite its large student population, the school has only 3 mathematics teachers all not qualified *(see Table 3.1 for the teachers' general demographic data).*

The Mathematics classes are fairly large, with about *55* learners, typically of many government run schools in Zimbabwe. As with all Government schools students are expected to pay tuition fees and levies. School B is relatively well resourced as it has a computer laboratory with 10 computers donated by the Honourable president of the country, and one science laboratory for the lower classes meant for practicals in integrated science only.

School C

School C, is a 'day' secondary school which however is located in a very remote area of the district. It is a school which is located in an area where most parents struggle to raise fees to send their children to school. The school is in a location often hard hit with drought. An average parent in the area is a peasant farmer where proceeds of their sale of agriculture produce would cater for all family expenses.

It enrols both boys and girls Forms 1 – 4 with an average enrolment of *300* students. Each form level has about 2 classes which are not streamed according to ability. The school is classified as a council school and it depends on students' fees on all its operations. The school is under-resourced with however only 2 qualified maths teachers *(see table 3.1 for the teachers' general demographic data).*

3.3.2 Study participants

The researcher chose mainly purposive sampling (Groenewald, 2004; McMillan & Schumacher, 2006; Teddlie & Yu, 2007) in selecting the research participants. According to Welman and Kruger (2008) purposive sampling is the most important kind of non-probability sampling, to identify the primary participants. Purposive sampling was used to select the mathematics teachers *(see Table 3.2 below)*. According to Richards and Morse (2007) qualitative researchers seek valid representation when they employ non-random sampling techniques such as purposive sampling where participants are chosen based on certain characteristics.

The basic criterion for selection was to look for a mathematics teacher who at that time was teaching an ordinary level class. A sample of participants was selected based on the nature of the research, looking for those who "have had experiences relating to the phenomenon ..." of teaching transformation geometry (Teddlie & Yu, 2007). However, in all three schools only one teacher was teaching the ordinary level mathematics classes, so that a second teacher was then selected based on experience in having taught the topic of TG before. Thus a total of six teachers participated in this study.

However, simple random sampling technique was used in selecting students in the ordinary level stream who took the test *(see table 3.2 below)*. Two lessons were observed each for three teachers of mathematics, one on a unit of isometric transformations *(translation, reflection or rotation)* and the other on non-isometric transformation *(shear, stretch or enlargement).* The reason being, the researcher wanted to have a feel of the teaching and learning experiences for both types of transformations. However selection of which lessons to observe was somehow a random process so that the flow of lessons at the different schools is not interrupted.

Participation in the study was on voluntary basis and the participants would end their participation in the study at any time without risk or harm. All six teachers participated in the study until it ended. Table 3.1 below shows a summary of the six Mathematics teachers' demographic data.

Table 3.1: The teachers' demographic data

Name of school	School A		School B		School C	
Characteristic	Teacher A1	Teacher A2	Teacher B1	Teacher B2	Teacher C1	Teacher C2
Sex of teacher	Male	Male	Male	Male	Male	Male
Professional qualification	B.Ed Math	B.Sc Math (no teaching qualification)	B.Sc Math (with education)	B.Sc Math (no teaching qualification)	Diploma in Educ. Math	Diploma in Educ. Math
Subject major	Math	Math	Math	Math and Statistics	Math	Math
Mathematics teaching exp.	25	10	8	3	21	16

NB: Teacher A1 means teacher 1 at school A and teacher A2 means teacher 2 at school A. Teacher A1is for identification only.

The distribution of teachers by gender is biased, showing that all six were male teachers. Of the six four have a teaching qualification. Whilst school A and B have degreed teachers for Mathematics, one of the two at each school has no teaching qualification. According to Shulman (1986) qualified teachers receive training in pedagogical content knowledge necessary to provide a bridge between the subject matter and the knowledge of teaching. This means teacher A2 and teacher B2 are likely not to provide such a bridge in their teaching of concepts in mathematics.

Teachers at School C, whilst they hold a diploma qualification in teaching their long teaching experience might be compromised by the absence of in-service teacher professional development.

Table 3.2: Summary selection criteria of the different participants

Participant	Instrument used	Selection criteria
Teacher A1	- Interview - Lesson observation	- Purposive sampling - Purposive sampling
Teacher A2	- Interview	- Purposive sampling
Teacher B1	- Interview - Lesson observation	- Purposive sampling - Purposive sampling
Teacher B2	- Interview	- Purposive sampling
Teacher C1	- Interview - Lesson observation	- Purposive sampling - Purposive sampling
Teacher C2	- Interview	- Purposive sampling
Students (35)	- Test	- Simple random

3.4 INSTRUMENTATION

3.4.1 Mathematics Teacher interview guide

A semi-structured interview for mathematics teachers was used in this study. Open – ended type of questions which were fairly specific in intent constituted the instrument (McMillan & Schumacher, 2006). The use of semi-structured interview schedules helped maintain the focus of each interview and at the same time allowing the teachers the flexibility to provide alternative and detailed responses to the questions (Opie, 2004). Interviews were seemingly vital as the teacher respondents openly voiced their opinions, beliefs and views (Nieuwenhuis, 2007) tied to the teaching and learning of transformation geometry. The interview was divided into two parts, A and B.
Part A focused on teacher experiences with transformation geometry teaching. It aimed to unpack the teachers' views about their own teaching, their understandings and ways of dealing with learners in the teaching of transformation geometry. In this section the first set of questions made the participant talk about self. For example, "For how long have you been teaching transformation geometry?", "Do you find it interesting to teach?" In these questions a relaxed atmosphere was created whereby respondents would express themselves freely.

A total of eleven questions constituted this part A of the interview. The following are examples of questions in the section;

- Do your students perform well in this topic compared to other topics?
  If no, what do you think contributes to their poor performance?

- What other aspects do you exploit with your students to enhance effective teaching of transformation geometry?

Part B focused on teacher utilisation of 'real' mathematics in the teaching of Transformation geometry. Following RME model the statement '*mathematics must be connected to reality*' (Freudenthal, 1991) informed the construction of items in the section of the guide. The statement means that mathematics must be close to learners and must be relevant to everyday life situations (Freudenthal, 1991). In this section it was important to understand how teachers use learners' world of experiences in the teaching of transformation geometry. Thirteen questions constituted this section. These are examples of questions in part B;

- Does the topic relate to students' real-life experiences, their culture etc?
  If yes, in which areas?
- How do you make your instruction in transformation geometry 'real' to the learners?
- What experiences of the learner do you consider relevant for incorporation in teaching transformation geometry concepts?
- To what extent can teaching for applications be included in transformation geometry instruction?

Central to the philosophy of Realistic Mathematics Education is that students develop their mathematical understanding by working from contexts that make sense to them (Dickson et al., 2011). Hence, these questions emphasised the idea that mathematics is a human activity that must be connected to reality. Tapping from this model questions were devised that called for an in-depth discussion on the issues around real mathematics education in transformation geometry.

The last phase of the interview solicited information to do with participants' recommendations. For example, "What do you think teachers need to adopt in order to teach transformation geometry effectively?" This helped bring the interview to an end. *(See Appendix D)*

A substantial amount of information was accessed through interviews. One question or answer led to another which is not the case with other instruments like questionnaire (Creswell, 2009). For example, questions like; *do your students perform well in this topic compared to other topics?* Lead to questions such as; *if no, what do you think contributes to their poor performance?* However, since these interviews were done during a normal school session where a teacher had an average of 3 lessons to teach per day it proved very taxing to organise meetings with interviewees in between their lesson slots. All interviews were audio-recorded and later transcribed for analysis

Pilot testing of the interview guide

The purpose of the pilot study was to ascertain the validity of the instrument before use. A pilot study is needed to detect flaws in measurement procedures and as a basis to identify unclear or ambiguous items in an instrument. Burns and Grove (2001) describe a pilot study as a smaller form of future study which is meant to redefine methodology. A pilot study was conducted with colleagues who are *PhD* holders who were not part of the participants for this study. The pilot test results revealed that the RME model, as the theoretical framework, should strictly inform all the instruments, for instance, the interview and observation guides. With this discovered deficiency, the researcher aligned all items in line with the Realistic Mathematics Education model. For the item *how do you make your instruction in transformation geometry interesting,* was later changed to read *how do you make your instruction in transformation geometry 'real' to the learners?*

3.4.2 Lesson observation guide

Observations were conducted with teachers during their regular transformation geometry instruction to gather direct observational data and better illustrate the overall experience of transformation geometry education. The researcher was a non-participant observer, involved in listening, observing and recording information without participating in mathematics lessons under observation (Creswell, 2013). The purpose of the lesson observations was to collect data about each of the three teachers' teaching practices in the topic of geometric transformations and to explicate the possible approaches to instruction that can enhance '*real*' teaching and learning in the topic.

Teachers were, for instance, observed on their choice and use of examples in transformation geometry teaching, incorporation of learners' real-life experiences, attitude and demeanour while teaching, strategies to promote transformation geometry mastery. Detailed field notes were recorded and transferred to an observational matrix following the observation (Hall et al., 2016). Lesson observation schedule was divided into three sections. Part A focused on the use of realistic contexts in developing transformation geometry concepts, where aspects such as the following were observed;

Table 3.3: Lesson Observation Sample 1

1.	Are new concepts presented in real-life (outside the classroom) situations and experiences that are familiar to the student?
2.	Do examples and student exercises include many real, believable problem-solving situations that students can recognise as being important to their current or possible future lives?
3.	Do lessons and activities encourage the student to apply concepts and information in useful contexts, projecting the student into imagined futures (e.g., possible careers) and unfamiliar locations (e.g., workplaces)?

Part B aspects were developed around the students' engagement on lesson activities. Mainly the objective was to find out if students learn transformation geometry concepts by developing and applying mathematical concepts and tools in daily life problem situations that make sense to them (Van Den Heuvel: Panhuizen, 2003). The following are examples of aspects that were observed during lessons.

Table 3.4: Lesson Observation Sample 2

1.	Teacher used most time explaining and solving mathematical problems
2.	Students freely discussed among themselves
3.	Students were challenged to solve real problems in transformation geometry

Part C concentrated on classroom assessment on transformation geometry as enacted for the teaching and learning purposes**.** Here assessment should be developed in the form of open-ended questions which lead the students to free productions (Lange, 1998). The following are examples of aspects observed during lessons.

Table 3.5: Lesson Observation Sample 3

1.	Assessment is an integral and indispensable part of the teaching-learning Process
2.	Assessment activities focused on both procedural and conceptual proficiency
3.	Assessment is conscious of the objectives of learning that utilises students' real life experiences

Transformation geometry classrooms described in this study are not necessarily representative of transformation geometry instruction in Zimbabwe. They, however, offer some insight into the conduct of instruction in transformation geometry classrooms in the country. From these classroom rich descriptions and analyses of the instructional methods that were observed, revelations were gained into what prospects observed instructional methods hold for the learners to learn transformation geometry. *(See Appendix B for the complete observation schedule)*

3.4.3 Document analysis schedule

The study employed primary sources as part of document analyses. The records and documentation used provided sustenance of the arguments used in this study to either support or refute the philosophy behind the teaching and learning practices with transformation geometry employed in rural secondary schools in Zimbabwe (Mpofu, 2013). School official documents such as the national syllabi, the New General Mathematics (Book 3) and mathematics teachers' scheme – cum plans were analysed for their incorporation of the phenomenon under investigation. Document review was done to gather background information to determine if implementation of curriculum in transformation geometry reflects programme plans as alluded to in the school official documents. The review process concentrated on two sections. The first section looked at how far the document addresses teaching and learning that incorporates students' real-world experiences (Freudenthal, 1971). The following examples of aspects were explored;

Table 3.6: Document Analysis Sample1

1.	Teaching and learning objectives/methods/activities foster deep learning strategies that place 'emphasis on use of students' real-life experiences
2.	Examples used have a link with students' real-life experiences
3.	Comments/evaluation made commensurate with objectives

The second section focused on the nature of assessment/exercises on transformation geometry. The following are examples of aspects covered. *(See Appendix C for the full document analysis schedule)*

Table 3.7: Document Analysis Sample2

1.	Assessment questions are merely routine problems
2.	Students are challenged to solve real problems in transformation geometry
4.	The marking schemes were flexible and allowed for a variety of solution methods
5.	Comments in students' exercise books foster deep inner connections between concepts and real-life experiences

3.4.4 Van Hiele Geometric Test (VHGT)

In line with the van Hiele (1999) theory of the levels of thought in geometry, achievement tests that measure the attainment of the van Hiele levels among student participants were adapted (Hoffer, 1983). The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) is one such test which was adapted and used in this study. The CDASSG was used to classify learners in this study into distinct van Hiele levels of geometric thought.

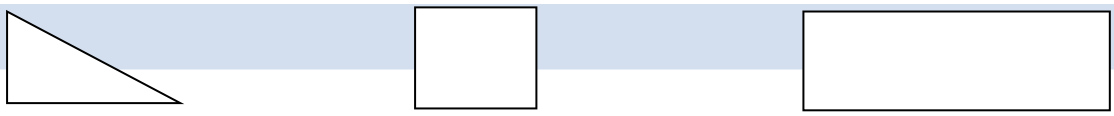
The van Hiele Test is a 25-question multiple choice test. The van Hiele Test is organised in blocks of five questions that were created using behaviours identified from the nine writings published by the van Hieles about their theory (Knight, 2006). The questions are arranged sequentially, in blocks of 5 questions each, such that questions 1-5 measure student understanding at Level 1, questions 6-10 measure student understanding at Level 2, questions 11-15 measure student understanding at Level 3, questions 16-20 measure student understanding at Level 4, and questions 21-25 measure student understanding at Level 5.

The test helped establish the level at which they are in terms of the van Hiele model. The CDASSG test items "were based on direct quotations from the van Hieles' writings and were piloted extensively" (Senk, 1989 p.312). From quotes of the van Hieles regarding what could reasonably be expected of student behaviours at the various levels, questions were written by the CDASSG project personnel for each level that would test students' attainment of specific levels (Usiskin, 1982).

The reason for adapting the CDASSG test was that learners do not think at the same van Hiele levels in all areas of geometry contents (Senk, 1989). Therefore, van Hiele (1986) and Senk (1989) suggest that studies that seek understanding of the thinking processes that characterise the van Hiele levels should be content specific. This CDASSG test was adapted to mirror geometry thinking as reflected in the Zimbabwean curriculum. For instance, Item 1 and 2 of the test instrument read;

Table 3.8: VHGT test items Sample1

1. Which of these are squares?



K **L** **M**

A. K only

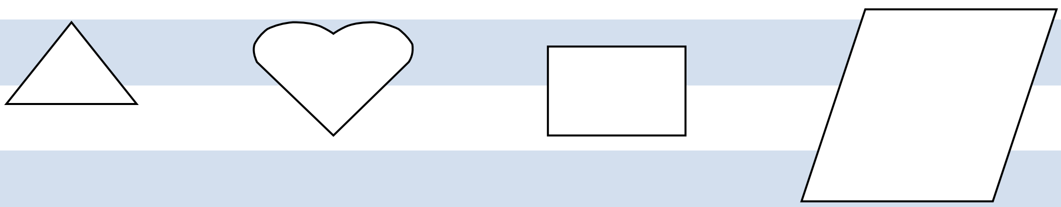
B. L only

C. M only

D. L and M only

E. All are squares

2. Which of these are triangles?



A. None of these are triangles.

B. V only

C. W only

D. W and X only

E. V and W only

The rationale for the VHGT is based on the notion that students' understanding of geometry can be described largely by their relative positions in the van Hiele scale of geometric thinking levels (Atebe, 2009). As with the CDASSG van Hiele test (see Usiskin, 1982), the VHGT was designed to determine the van Hiele levels of the participating learners. Thus, the instrument was to assign learners to various levels of geometric thinking in transformation geometry so as to determine how achievement in this topic is related to students' van Hiele levels.

The assumption made was that these learners would have acquired a significant proportion of the learning experiences intended for them in their mathematics curriculum. Therefore, students' performances in these tests were interpreted as a true reflection of the achieved aspects of the mathematics curricula to which this group of learners was exposed (Atebe, 2009). Thus, students' achievements in this test reflected their abilities in transformation geometry. (*See Appendix E for the CDASSG van Hiele test)*

3.5 ETHICS AND NEGOTIATING ACCESS

Research that involves humans may be personally invasive to the participants (McMillan & Schumacher, 2006). Any research undertaking must therefore observe ethics in its conduct. Ethics have to do with the respect for the rights of participants in research. In this study permission to carry out research was granted by the provincial education department in Zimbabwe as well as ethical clearance from the UNISA Ethics Committee. Additional ethical considerations taken into account are discussed below.

3.5.1 Informed consent

The respondents received an overview of the research undertaking. The informed consent document communicated to the prospective research participants the purpose, procedures including time commitment of the subject, and the confidentiality of their information. The participants had the right to participate in the research, and the freedom to turn down/withdraw at any time.

The informed consent 'agreement' form was designed mainly on the following items:

(a) That they are participating in the research
(b) The purpose of the research (without stating the central research question)

(c) The procedures of the research

(d) The risk and benefits of the research

(e) The voluntary nature of research participation

(f) The subject's (informant's) right to stop the research at any time

(f) The procedures used to protect confidentiality (Bless, Higson-Smith & Kagee, 2006)

The respondents signed an informed consent form before the interview, to give full assurance of the confidentiality of their responses. Sending consent forms to the parents (or guardians) of 'minors' did not happen as per initial plan. However, for School A the Mathematics H.O.D signed on behalf of the parents since the school being a boarding station most of the parents of participating learners were far and wide. The signed informed consent forms were retained and are kept in a locked cabinet. The collected information was stored in an Excel file maintained on a password protected flash memory data storage device. The hardcopies of the transcripts including the signed consent form and instrument paper which include the participant feedback was kept in a sealed envelope and stored in a locked cabinet, which only the researcher had the access to.

3.5.2 Confidentiality

The identity of the participants remained confidential and was not directly associated with any data. In ensuring that ethical standards were maintained during the course of this study, the participants were informed about the purpose of the study so that their informed consent can be obtained before pursuing the study.

Secondly, the privacy and confidentiality of the participants were ensured by not requiring them to divulge their names in order to ensure anonymity of their responses and protect them from any retributive action. Care was taken to minimize any harm caused to the respondents, by ascertaining at the onset whether they have any objections to participating in the study or whether they foresee any negative impact being caused to them by participating in it.

Also as part of ensuring the observation of ethics in the study, the researcher applied for ethics clearance from the University of South Africa (UNISA) Ethics Committee. The process of negotiating access involved applying to the Midlands Provision Department of Education for approval to conduct the research in the province. After identifying the research site, meetings were held with the school administrators and mathematics teachers to negotiate access into the school and informed consent to participate in the study. The herd teachers of the schools gave their verbal approval and helped set up the meeting with the mathematics teachers.

The selected students and their parents or guardians were asked to complete and sign informed consent forms. In the assent form, each learner was asked to give his or her assent to participate in the study; to be audio-recorded; and to be video-recorded during mathematics lessons. The parents or guardians were asked to give similar consent for their children. In both the teachers' and learners' consent forms the right of the participants to anonymity was assured and no real names were used in this study and any other papers written about the findings of the study.

3.5.3 Gaining access

This research study adopted an interpretive qualitative approach which meant establishing a direct personal contact with the participants. The process of negotiating access to the study sites was initiated by visiting the ministry of primary and secondary education with an ethics certificate generated from UNISA college of Education and a letter directed to the director of education seeking for permission to do research in secondary schools in Mberengwa district. Upon getting approval, the researcher then visited the three schools one after the other, with an approval note from the national director of education.

The researcher briefly explained the nature of the study to the three school heads and their mathematics (HODs), after which a nod to proceed was given. The school heads through heads of mathematics department (H.O.D) then introduced the researcher to both the mathematics teachers and groups of students and spelt out the intensions of the visit. This welcoming support was pivotal in gaining the respect and cooperation that the researcher needed for the rest of the time he was in the schools. Two teachers and not more than 15 students per school were selected to participate in the study.

In all three schools a time-table showing how the study should be conducted in a school in order for it not to interrupt the normal school running. The potential significance of the study was explained to the participants during the initial contacts. For example, the researcher explained to mathematics teachers that this study is aimed to bring a positive difference in transformation geometry teaching and learning.

3.6 DATA COLLECTION PROCEDURES

This phenomenological study involved three secondary schools where detailed descriptions of phenomena under study were collected through interviews, observations, document analysis and tests as data collection methods. The study aimed to explore how mathematics teachers utilised students' lived experiences in the teaching and learning of Transformation Geometry. The process of collecting data depended on meticulous time keeping and constant planning and re-planning (Mutemeri, 2013). The researcher managed to come up with a tentative time table of appointments with participating schools.

The first phase of the data collection involved observations of teaching and learning sessions. The purpose of this phase was to examine the extent and teachers' use of students' lived experiences in transformation geometry classes. The second phase of data collection involved the phenomenological interviews. During this phase data was collected through the one – one interviews with the six mathematics teachers. The main purpose was to explore teacher beliefs, exposure and attitudes towards the use of students' lived experiences in the teaching and learning of transformation geometry. As a way of motivating teachers to participate the researcher introduced some refreshments; biscuits and soft drinks during and after interviews. It was amazing how much more relaxed and informative the interviews turned out to be. Frazer and Lawley (2000 p.74) argue that consequently, the researcher needs to do all that is possible to encourage a better response. During the interviews a voice recorder was used. The gadget enabled the researcher to give full attention to the interviewees. Every bit of the interview was recorded.

The third phase of data collection involved an assessment of the official school documents that report and guide teaching and learning in transformation geometry. Attention needed was to focus on the extent to which they highlight the significance of students' lived experiences. Documents which include scheme-cum plans, mathematics textbooks and students' exercise books were availed for analysis. *(See Appendix C for the document analysis schedule)* The last phase of the data collection utilised Usikin's (1982) CDASSG test which was administered to a total of 45 ordinary level students. The test was written simultaneously by the three groups of students to minimise chances of dilution by the students.

3.7 PHENOMENOLOGICAL DATA ANALYSIS AND REPRESENTATION

Data analysis in this study used largely the qualitative techniques. This was because of a variety of research tools which were used, such as interviews, observations and tests. The process of qualitative data analysis involved a process of categorizing data and identifying relationships (McMillan & Schumacher, 2006), looking for patterns, themes, consistencies and exceptions in the data. A more detailed approach employed is elaborated below.

Although this study followed the interpretive paradigm test scores were analysed following the descriptive data analysis procedures where the participants were described in terms of their levels of geometric conceptualisation. According to Van Hiele (cited in de Lange, 1996) the process of learning proceeds through three levels:

Level (1): A student reaches the first level of thinking as soon as he can manipulate the known characteristics of a pattern that is familiar to him/her;

Level (2): As soon as he/she learns to manipulate the interrelatedness of the characteristics he/she will have reached the second level;

Level (3): He/she will reach the third level of thinking when he/she starts manipulating the intrinsic characteristics of relations.

Thus, descriptive statistics, based on the SPSS statistical package, (frequency distributions, bar charts and measures of averages) were used to analyse and compare performance of students in the 5-different van Hiele levels of geometry thinking.

As Moustakas (1994) indicated, the research procedure starts with identifying the phenomenon under the investigation. After collecting data through phenomenological interviews with co-researchers who had experienced the phenomenon, the data was analysed by following Moustakas' phenomenological data analysing procedure. This section describes the procedure of preparing and analysing the data. The general procedures include preparing data for the analyses, reducing the data phenomenologically, engaging in imaginative variation, and uncovering the essence of the experience (See Fig 3.2 for the steps of data analysis).

The phenomenological analysis starts with bracketing the researcher's subjectivity which refers to clarify preconception throughout the study. This process is described as Epoché, and it refers to setting aside the researcher's prejudgments and predispositions towards the phenomenon. This process begins with the writing a complete description of the phenomenon by the researchers. Before starting the data analysis, researchers should read their subjectivity statement, including the description of their own experience with the phenomena.

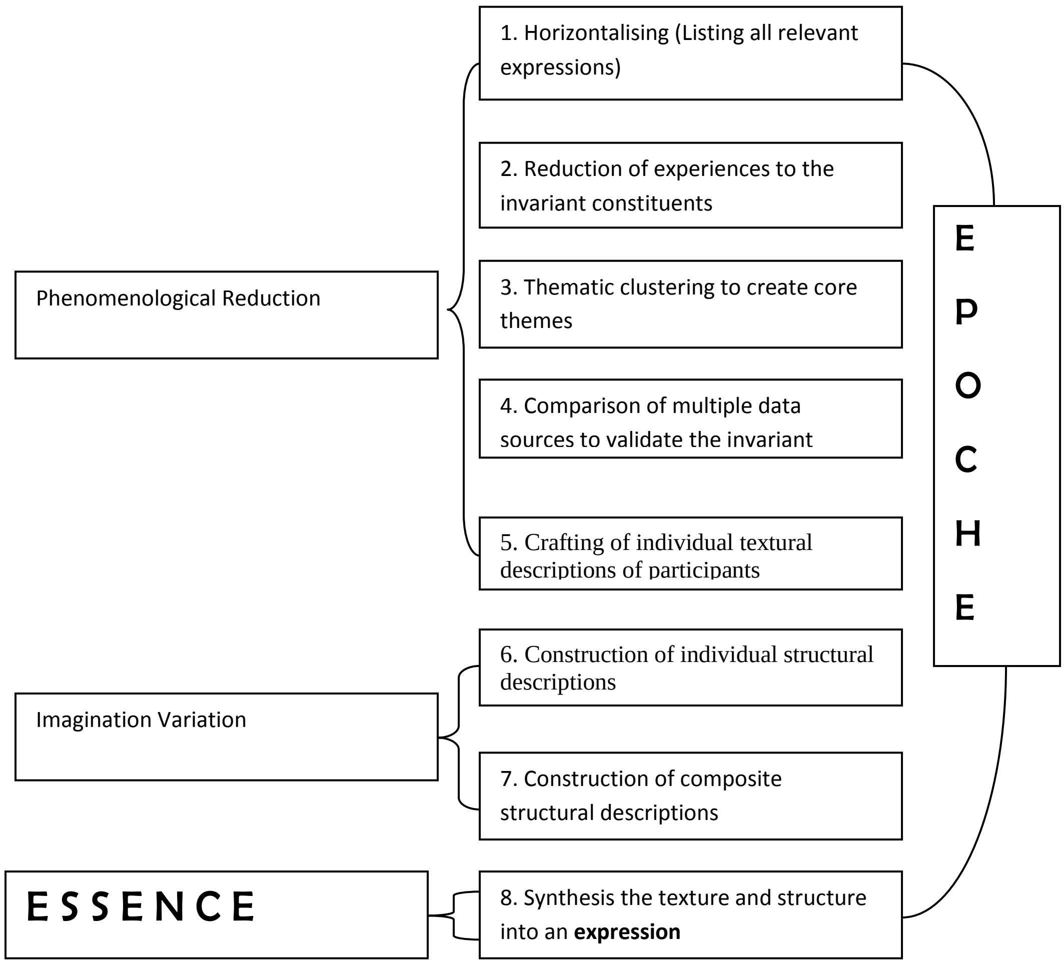


Figure 3.2: Steps of data analysis

(Adapted from Cilesiz, 2010)

3.9 THE STEPS OF DATA ANALYSIS

1. *Horizontalising, or listing all relevant expressions:*

In this part of the data analysis, all data was scrutinised as every statement had equal value. Statements which were irrelevant to the investigating phenomena and were repetitive or overlapping were ignored. In other words, a list was created from the verbatim transcripts of co-researchers and deleted all irrelevant expression. For example, if the co-researcher explained the phenomena that are outside of the scope of the investigation, parts of the verbatim were deleted. After cleaning the data, the remaining parts of the data are called as *horizons*. Horizons are the textural meanings or constituent parts of the phenomenon. Moustakas (1994) said that horizons are unlimited and horizonalisation is a never-ending process.

2. ***Reduction of experiences to the invariant constituents:***

In this step, the researcher clustered horizons into themes. Then read through the transcripts to try and get a holistic picture across all transcripts. This meant reading more than once in order to get closer to the data (Richards and Morse, 2007). Reading through the transcripts/horizons led to the emergence of themes. Participants' responses were critical in their relationship to the research questions. At this point I began to highlight segments of data and also started to reflect on the meanings and implications of the text divisions. Using highlighting made it possible to determine data that supported or contradicted each other in terms of the themes that emerged (Hramiak, 2005:88).

Then, data was sort into theme 1, theme 2, theme 3 or theme 4 relating correspondingly to research question 1, 2, 3 or 4. For instance, the research question: *What mathematics involving transformation geometry concepts are contained in the students' out-of-school activities?* resulted in a theme: students' out-of-school activities involving transformation geometry. This meant reading from interview transcripts the examples teachers would use in teaching transformation geometry. This step of the phenomenological reduction describes the phenomena in "*textural language*".

3. ***Thematic clustering to create core themes:***

In this step, the invariant constituents which are the horizons defined as the "*core themes of the experience*" of the phenomenon (Moustakas, 1994 p. 121) were clustered and thematised. For example, analyses of lessons included the analyses of the questions teachers asked students and these were divided into such categories as *probing*, *extending* and *orienting*. This level of analysis was designed in response to the awareness that the teachers' questions were an important indicator of the transformation geometry on which students and teachers worked (Boaler & Brodie, 2004).

4. ***Comparison of multiple data sources to validate the invariant constituents:***

The themes derived from participants' experiences collected by a particular data collection method, such as interview, are compared to other methods, such as researcher observation, field notes, focus group interviews, and literature to verify accuracy and clear representation across the data sources.

5. *Constructing of individual textural descriptions of participants:*

The textural description is a narrative that explains participants' perceptions of a phenomenon. In this step, the experiences of co-researchers were described using verbatim excerpts from their interview. Moreover, the meaning of units in a narrative format was explained to facilitate the understanding of participants' experiences.

6. *Construction of individual structural descriptions.*

This step is based on the textural descriptions and imaginative variation. By using imaginative variation, the researcher imagined how experience occurred and then, created the structures.

7. *Construction of composite structural descriptions:*

After writing the textural description for each co-researcher, the textural description was incorporated into a structure explaining how the experience occurred by adding the structures at the end of each paragraph in order to create structural description. This process helps to understand co-researchers' experiences with the phenomena under the investigation.

8. *Synthesising the texture and structure into an expression:*

Two narratives for each co-researcher were created, including textural describing "what" occurred and structural describing "how" it occurred. Then the meaning units for each co-researcher were listed. After that, meaning units common to all co-researchers were created and composite textural and structural descriptions based on these shared meaning units were created. In the composite textural and structural descriptions, individual meaning units were eliminated in order to create the essence of the phenomena. Then composite narratives from the third person perspective representing the group as a whole were noted down. This step is the synthesis of the all narratives for the group as a whole. The composite structural description is combined into the composite textural description to create a universal description of the phenomenon of the investigation.

The purpose of the step is to reach the essence of the experience of the phenomenon. This last step provided a link from data to literature. In presenting data, thick descriptions (as in line with RME *model dimensions shown in Table 3.9) were then achieved through expression of interconnections of different data extracts from these sources, namely test results, teacher interviews, document analysis, and lesson observations. This was meant to triangulate findings from the different sources of data as well as from the literature review in order to strengthen the research findings and conclusions (Spring, 2016), showing how these contributed to the argument. For instance, with focus on research question 1 the information gathered through the different instruments linked to the research question was considered. The actual qualitative data analysis used the RME model as the theoretical framework as illustrated in Table 3.9 below. For instance, with data gathered from interviews with classroom teachers under dimension 1 shown in the table introspection was made to explain whether or not lessons were relevantly introduced. Was it mechanical or whether it was real to the students? To what extent did it motivate the students to become more and more engaged in the learning process as shown by their participation and attentiveness?

Table 3.9: Summary of RME dimensions used in data analysis

Dimension	**Description**
1. Phenomeno-logical exploration	- The researcher was looking at the classroom instruction paying attention on the start of instruction, whether it was `real' to the students; allowing them to immediately become engaged in the situation. - Thus, noting the nature of the introduction to the lessons.
2. Use of models	- Focus was at the choice of models, whether the *model of* a situation is familiar to the students or not. - Thus, noting any models or objects brought into class as media, how related there are to learners' real-life experiences
3. Use of students' own productions	- Focus was on how students reflect on the path they themselves have taken in their learning process and, at the same time, to anticipate its continuation. - This requires a critical observation of the teaching and learning processes paying attention on learners' independent contributions
4. The interactive character of the teaching process	- Interactive instruction engages students in explaining, justifying, agreeing and disagreeing, questioning alternatives and reflecting. - Thus, noting active participation of the learners
5. The intertwining of various learning strands.	- The holistic approach, which incorporates applications of transformation geometry concepts, implies that learning strands cannot be dealt with as separate entities-but as connected web - Instead as an intertwining of learning strands exploited in problem-solving.

(Adapted from Freudenthal, 1991)

The dimensions in the Table above provided an essential guide to lessons observed and document analysis. For instance, the last dimension, the intertwining of various learning strands, was used to analyse if teaching and learning emphasised on the connection between Transformation Geometry and other related topics. In this section attention was given to how teaching connects the topic to other topics which students had covered.

The model proved suitable for measuring the teachers' teaching because of its coverage of essential dimensions of mathematics lessons. For instance, the dimension on 'the interactive character of the teaching process' provided a description of how interactive instruction was in terms of engaging students in explaining, justifying and questioning. Each lesson observed was coded based on the separate dimensions. Coding each dimension was done by judging the extent to which each teaching and learning activities measured up to the standards of RME. In which case the following coding system was adopted 'low' - (1); 'Ave' - (2); and 'High' - (3). For instance, where classroom discourse was more teacher – centred on dimension 4 the code (1) - 'low' was assigned to show that learners were rather inactive in the lesson.

Thus, it played a major role in the process of understanding most of the empirical data in this study. This was best achieved through inductive analysis (moving from the particular cases to a more general understanding of phenomena). In this study the different views by participants were noted allowing the frequent, dominant or significant themes that were inherent in the raw data emerge. The purpose of this was to try and understand the learner conceptions of Transformation Geometry in terms of the kind of instruction they received.

The degree of conformity with, or deviation from, the Realistic Mathematics model of the learning phases as exemplified by the checklist in Table 3.9 above, therefore, provided a measure of the learning opportunities offered to the learners in transformation geometry classrooms. In total, the process became of turning lesson observations into information produced data that indicates the degree to which observed teaching methods conform to the Realistic Mathematics Education model on instruction.

3.9 VALIDITY CONSIDERATIONS

The trustworthiness of the study was answered by the following 4 accountability standards: credibility, dependability, transferability, and confirmability. Trustworthiness refers to rigor/rigidity in a study that comes from the validity of the research process and the use of triangulation in data collection (Trochim & Donnelly, 2006). 'Validity' refers to the best estimate of the truth of any proposition or conclusion or inference described in the research (Trochim and Donnelly, 2006; Yuksel & Yildirim. 2015).

3.9.1 Credibility

This aspect of accountability was ensured through the technique of data triangulation (Creswel, 2013; Moustakas, 1994) where more than one method of data collection was used. Cilesiz (2006 p. 60) states that, "collecting data from two sources from the same participants enables the researcher to compare the information from both data sources and to eliminate any inconsistencies, which would indicate untruthful data."

In this study, some teachers who were interviewed had their lessons observed too. Thus, for the purposes of triangulation an alternative data collection method, observation, was used to verify the data from phenomenological interviews. Lesson observation, thus, proved fundamental in approving or disproving certain responses by teachers during interviews (Smith, 2015). The researcher drew common themes from the different data instruments as experiences emerging.

3.9.2 Dependability

To achieve the dependability of the study, the researcher used member checks as a measure of validity (Creswel, 2013; Merriam, 1995). This process is the *horizontalisation* step of the data analysis including the process of removing the irrelevant statement of the phenomenon (Yuksel & Yildirim. 2015). For example, where a response from an interviewee ended up including aspects other than those to do with transformation geometry data cleaning was done to retain responses only focused on the study.

3.9.3 Transferability

The researcher provided full details about the participant's background information as well as the research site. This helps map contexts where the study results can be generalised (Yuksel & Yildirim. 2015) and to enable readers to understand how the data was interpreted. Thus, External

validity was achieved, which addresses the generalisability of the research finding to other situations or people (Merriam, 1995).
Phenomenological research aims to gain an in-depth description of the experience of specific group. The findings can be extended for the obtaining reasons including providing detail information, selecting sample strategies, providing objectivity of researcher, and researchers avoiding presupposition (Cilesiz, 2009).

3.9.4 Confirmability

To achieve confirmability a colleague who had just completed his DED thesis and the supervisor helped in certifying the validity of the data analysis plan. As for the validation of instruments the Theoretical Framework, the RME model, was used to inform instrumentation. Realistic Mathematics Education is comprised of. This included the use of contextual problems, models or bridging by vertical instruments, use of student's contribution, interactivity and intertwining of learning as strands. Since these strands were the focus of this study the observation guide, interview guide and document analysis were developed in line with the RME model. For the test instrument Usikin's (1982) CDASSG test was adapted. Thus, construct validity was ensured where the instrument measured what it purported to measure (Creswell, 2012).

3.10 LIMITATION

All methods have limitations in their nature. The issues of bias and generalisability are quickly noticeable. Concerning bias, the argument is personal experiences and beliefs are very subjective. During interviews some interviewees responded by telling the ideal and not their personal experience and practice. This was noted particularly with teachers whose lessons were observed. What came out in the interviews was rather different from what was observed during lessons. Participants possibly feared exposing their weaknesses. However, use of both observation and interview tools helped alleviate some of the differences.

Since the programme of classroom observations was organised well in advance and in liason with teachers, there is a likelihood that teachers conducted lessons which mirrored more than their usual conduct. Differences in teachers' approaches in teaching Transformation Geometry could be explained by their beliefs and content knowledge base however the impetus in this study was teaching practices not necessarily teacher characteristics (Hiebert & Morris, 2017). In other words, there could be other variables that explain the differences of the teachers' approaches.

Within the qualitative research framework, subjectivity is strength because truth is relative, no story can have more credibility than any other; all stories are equally valid. Nieuwenhuis (2007 p.52) contends that qualitative researchers accept value laden narratives as true for those who have lived through the experiences. Focus was on the depth and quality of information provided by respondents pertaining to teaching and learning of transformation geometry, with major emphasis being on the uniqueness of each particular contribution.

The initial plan was to have every teacher who was interviewed, observed whilst teaching a class. However, only 3 teachers out of the total six were both interviewed and observed teaching. As far as generalisability is concerned the major observation was that, the researcher restricted participation to Mathematics teachers and their students in the three different orientations of rural secondary schools. However, Zientek (2007:962) echoes the sentiment that of course such samples are not without limitation but can yield some insights when sample characteristics reasonably well match those of a targeted population.

3.11 CHAPTER SUMMARY

In this chapter the research paradigm and research design of the study were discussed. The intention of this chapter was to describe the Research Methodology. It was explained that the study is oriented within the interpretive research paradigm. The chapter outlined the research methods used, the data collection and analysis, and how ethical issues were addressed in the study. Within the qualitative paradigm the transcendental phenomenology approach by Moustakas (1994) was used to generate an essence of the lived experience of participants.

The discussion showed how the data was collected using a phenomenological approach in order to answer the question that guided the study, that is, 'To what extent do teachers embrace students' out-of-school experiences in the teaching and learning of transformation geometry?' A total of 35 students and 6 Mathematics teachers participated in this study. The sample and sampling procedures were elaborated together with the research ethics. The research process was expounded with a focus on procedures for data collection, analysis and validity measures. Data was presented and analysed by following Moustakas' phenomenological data analysis procedure. The data gathering tools included phenomenological interviews, observation guide, document analysis guide, and a test. The next chapter, Chapter 4, presents; analyses and discusses data in order of the themes as derived from research questions.

CHAPTER FOUR
DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 INTRODUCTION

The purpose of this qualitative study is to explore the extent to which teaching and learning of Transformation Geometry embraces learners' related real life experiences at secondary school level. Anecdotal evidence shows that the majority of students in schools generally are unable to make connections between what they are learning and how that knowledge will be used and this has denied their mastery of the concepts in the topic. This is because the way they process information and their motivation for learning are undermined by the traditional methods of classroom teaching (Gravemeijer, 2016).

Moustakas (1994) suggests many angles and perspectives of examining an experience in order to understand the entire phenomena being investigated. In line with this recommendation, this study employed the transcendental phenomenological design. Semi-structured interviews, lesson observations, test and document analyses were used to gain an in-depth understanding and compile a well-rounded description of the study (Creswell, 2013).

Data collected through audio files from the interviews and field notes from lesson observations were transcribed. Interviews were conducted with six Mathematics teachers so as to gain information regarding how they understand the topic of Transformation Geometry and how this understanding shapes their practice. Three schools A, B and C were involved in this study. The six teachers interviewed are named Teacher A1, Teacher A2, Teacher B1, Teacher B2, Teacher C1 and Teacher C2 and three lessons were observed with Teacher A1, Teacher B1 and Teacher C1 (see summary Tables 3.1 and 3.2). Four research questions guided the study. All four questions were based on the theoretical framework, the Realistic Mathematics Education model (RME), developed by Hans Freudenthal (1991) and his team at the University of Utrecht.

Data collected was mainly qualitative, that is, non-numeric (Devos et al., 2002) and was presented and analysed to address the following research questions:

1. What are teachers' perceptions about the mathematics involving transformation geometry concepts contained in students' out-of-school activities?

2. How is the context of transformation geometry teaching implemented by practising teachers in Zimbabwean rural secondary schools?
3. To what extent are students' out-of-school experiences incorporated in transformation geometry tasks?
4. How is transformation geometry, as reflected in official textbooks and suggested teaching models, linked to students' out-of-school experiences?

This chapter is organised into sections. Section 4.1 through 4.4 presents results of the study in the order of the research questions above. The results from the teacher interviews, the lesson observations and document analyses on each research question are presented, analysed and discussed below. A brief summary is provided at the end of each research question. Finally, the chapter ends with an overall conclusion.

4.1 RESEARCH QUESTION 1:

What are teachers' perceptions about the mathematics involving transformation geometry concepts contained in students' out-of-school activities?

In this Section, data is presented, analysed and discussed under the following subthemes: Movements and Patterns (As Translations); Reflections and Symmetry; Turns and Rotations; Enlargement; Shear and Stretch. The Section unpacks the Mathematics involving Transformation Geometry that is contained in students' out-of-school experiences. Data to answer research Question 1 was gathered through a semi–structured interview with six Mathematics teachers.

There are a handful of concepts as revealed by the different teacher participants embedded in students' out-of-school activities that have a link with the topic of transformation geometry as shown below.

4.1.1 Movements and patterns

In this subtheme, teacher responses that speak to the concept of geometric translations are presented. The findings from the interviews revealed ideas about movements in objects and patterns as 'Translation concepts' found in students' out-of-school experiences. The following are results from the teacher interviews.

Results from Teacher Interviews

Interviewer: (Q.6). *What experiences of the learner do you consider relevant for incorporation in teaching **Translations**?*

Teacher A1: *I use the notion of movement of objects to illustrate a translation. ... That is when an object moves from one position to the other and that is an experience which students are familiar with... or I can talk about decorations or patterns they see on traditional objects that's a translation when one shape gets repeated many times*.

Teacher A2: *With translations I simply apply the translation vector. I don't have clear experiences on this one related to learner experiences out there. That's why we end up resorting to theory*

Teacher B1: *Usually in translation I just consider the movement of an object, like the movement northwards ...so I just refer to movements from one place to another.*

Teacher B2: *As for translation I refer to it as a displacement. I will be trying to show students that when we have an object on point A and it has been displaced to point B... and now that it is on point B students should see how the object has moved in terms of x-coordinate and y-coordinate.*

Teacher C1: *ok, translation i would talk about movement in a straight line in a particular direction...and I mention that you have been translated nothing has changed... I have seen students actually enjoying that.*

Teacher C2: *I use the example of his (student) movement from home to school. That is a translation.*

Table 4.1 below summarises teachers' responses on the Mathematics involving translations that are found in students' out-of-school experiences. There were mixed responses from the participants particularly based on whether a participant did a teacher training course or not.

Table 4.1: Showing Teacher conception of out-of-school concepts related to a translation

Name of teacher	School station type	Out-of-school concept
Teacher A1	Mission boarding school	- *Movements of objects* - *Patterns on traditional objects*
Teacher A2	Mission boarding school	- *Nil*
Teacher B1	Council run school	- *Movements of objects*
Teacher B2	Council run school	- *Displacement of object*
Teacher C1	Rural day secondary school	- *Straight line movement*
Teacher C2	Rural day secondary school	- *Movement*

The summary table above shows the different teacher responses to the interview question, item 6. The Mission boarding school is well resourced compared to the Council and rural day secondary schools in this study (see Section 3.1.1 in Chapter 3). Teacher A1 and Teacher A2 were both from a Mission boarding school (School A). Teacher A1, reported on aspects of object movements and patterns that appear on traditional objects as resembling typical translation concepts. Teacher A2, however, could not find a link between students' out-of-school experiences with the translation concept. He referred to no experiences known to him that are related to translations.

In Table 3.1, which shows teachers' demographic data, Teacher A1 holds a Teaching degree in Mathematics whilst Teacher A2 has no teaching qualification. In other words, Teacher A1 had been exposed to the pedagogy of teaching in his training, and hence was aware of the significance of building the linkage between the formal and informal mathematics, whilst Teacher A2 had not got a similar exposure. Shulman and Grossman (1988) clarify pedagogy as the science and art of education whose role is to make teachers see and describe Mathematics in ways that can support learning. Accordingly, pedagogy is the understanding in a field that are essential for teachers but may not be important for non-teachers (Teachers without training), like Teacher A2.

The Council run school (School B) is located in a township area and is better resourced compared to the rural day school (School C*)* (see Section 3.1.1 in Chapter3). Teacher B1 and Teacher B2 both come from the Council run school (School B). Teacher B1 mentioned ‘the movements of objects’ as concepts of translation embedded in students’ out-of-school experiences, while Teacher B2 talked about a displacement of an object from one point to another. The two teachers literally referred to one and the same idea of the concept but teacher B1’s explanation was more grounded in the practical displacement than Teacher B2 who referred to the movement in terms of the cardinal points in a Cartesian plane.

Although both Teacher B1 and Teacher B2 hold degree qualifications (see Table 3.1) only Teacher B1 had a teaching qualification. This confirms again some difference in how a qualified and an unqualified teacher can view concepts in mathematics for teaching purposes. Shulman and Grossman (1988) argue about the importance of pedagogy within a teacher to be able to represent and model Mathematics ideas and concepts using objects and situations familiar to students.

Whilst all three schools studied in this research are rural bound, School C, a rural day secondary school is in the category of little or no resource support that could enhance students’ mastery of concepts (see Section 3.1.1). Both Teacher C1 and Teacher C2 were from the rural day secondary school. Teacher C1 and Teacher C2 gave more or less practical explanations of ‘movements’ as resembling translations (see Table 4.1). The two teachers both hold a minimum teacher’s qualification, a diploma teaching. In other words, Teacher C1 and Teacher C2 had examples of translations linked to student experiences they could use in the teaching and learning situation.

Of the six teachers, it emerges that only those who went through teacher training like Teacher A1 were more likely to see value in identifying and using examples for translation that are drawn from students’ out-of-school experiences. Thus, a teacher to be well equipped in terms of teaching and learning skills that value students’ out-of-school experiences in translation concepts they must have undergone some training in pedagogy.

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The study findings resonate well with results from the Centre of Development in Education (2010) study where it was found that an average mathematics teacher needs to be equipped with requisite skills and concepts to be effective in a mathematics classroom. According to Mtetwa (2017) students in the traditional classes complain of failing to see meaningful connections of mathematics concepts and procedures with their life worlds. This suggests teaching and learning that is more robust and is directed at helping students relate their experiences with the topics in mathematics.

4.1.2 Reflections and symmetry

In this Section, six teachers reported on out-of-school experiences of students they likened with the notion of a reflection transformation. There were noted similarities in the teachers' responses. This is what the teachers said in the interviews:

Interviewer: (Q.6). *What experiences of the learner do you consider relevant for incorporation in teaching* ***reflections****?*

Teacher A1*: When teaching reflections, I ask my students to bring mirrors from home so that they see their reflections in the mirror. They get the concept of a reflection i.e. same distance of object from the mirror as the distance of image from the same mirror... if the object is 2cm from the mirror then the image should also be 2cm from the same mirror.*

Teacher A2*: I teach reflections theoretically although I mention about mirrors and reflections on mirrors.*

Teacher B1*: I talk about a mirror when teaching reflections... I don't bring a mirror in class because it's an experience they know. The image comes out in the mirror exactly identical to the object.*

Teacher B2*: When teaching reflections, I use mirrors because I would be trying to illustrate reflection in more practical and familiar way*

Teacher C1*: Reflection! I usually talk about the mirror... usually I ask questions like: How many of them looked in the mirror before coming to school? What did you see? What happens if you move closer or away from the mirror? And so on... so I normally refer to the girls as the ones who spend more time on the mirror before they come to school. What you see is the image of your reflection in the mirror.*

Teacher C2: *I demonstrate using a mirror that when you look into the mirror you see exactly yourself but facing one another.*

Table 4.2 below summarises teacher responses on the Mathematics involving reflections that are found in students' out-of-school experiences. There were almost similar responses from the six teacher participants.

Table 4.2: Teacher conception of out-of-school concepts related to reflection

Name of teacher	School type	Out-of-school concept
Teacher A1	Mission boarding school	- *Mirror reflections*
Teacher A2	Mission boarding school	- *Mirror reflections*
Teacher B1	Council run school	- *Mirror reflections*
Teacher B2	Council run school	- *Mirror reflections*
Teacher C1	Rural day secondary school	- *Mirror reflections*
Teacher C2	Rural day secondary school	- *Mirror reflections*

From the table above, all six teachers responded the same by citing the mirror reflections as synonymous with geometric reflections. However, responses given by teacher A1 and teacher A2 differed in that Teacher A1 (a qualified teacher) gave a more detailed explanation of how the mirror produces an image in a practical sense by demonstrating the concept of same distance between object and image in a reflection. Teacher A2 (no teacher qualification) although acknowledging the importance of a mirror when teaching reflections said it was not important to bring the actual mirror into the class to develop the concepts. Teacher A2 rather prefers teaching reflections using procedures only (Teacher A2 said, "*I teach reflections theoretically").*

From the council run school, teacher responses were identical in that both teachers said they use a mirror in the teaching and learning of the topic of *reflections*. However, one teacher in this school has no teacher qualification, whilst the other is a qualified teacher. Similarly, the two teachers at the rural day-secondary school both suggested use of a mirror to expound on the topic of geometric reflections. The foregoing sentiments by teachers reveal some commonalities in what they consider to be a relevant example from students' real-life experiences linked to the concepts of reflection. Following these results and the fact that these two teachers (at the rural day secondary school) both hold a diploma in education, it means they are aware of the significance of drawing from learner experiences when developing concepts in geometry reflections.

Thus, all six teachers said they use the mirror, which is a common object known by students, in illustrating geometric reflections. A mirror is also recognised as critical in bringing up the meaning of a reflection as shown in the New General Mathematics Book 3. A reflection is, “the image you see when you look in a mirror” (Channon, Smith, Head, Macrae & Chasakara, 2004; p.30). According to Dobitsch (2014) a reflection is a transformation where a figure is flipped about a line, known as a line of reflection. When the figure is mapped to the opposite side of the line of Reflection, the perpendicular distances between any point on the figure and the mirror line and between corresponding image points on one side of the line and line of reflection are the same.

Geometric Reflections permit students to develop broad concepts of congruency and similarity. Chagwiza et al. (2013) cited that similar figures are always related either by a reflection or rotation, and there are many instances out there with which students have had experience, such as reflections on water levels. This implies that recognition of the familiar and the unfamiliar; similar and the not similar, require an ability to characterise and note key features between objects, a critical component of the level two of the van Hiele’s Model (Guven, 2012). This characterisation which can be enhanced through exploring with a mirror noticing and describing reflections should be the starting point for teaching and learning.

However, out of the six teachers, only Teacher A2 and Teacher C1 mention about the mirror in passing and thus concentrate on teaching for the procedural fluency. Contrary to RME philosophy students should be afforded more opportunities to explore problems in depth with their own objects rather than when they simply follow as a teacher leads (Jung, 2002:20). Thus, Teacher A2 and Teacher C1 do not emphasise so much on the experiences of students which is critical for mastery of the concept.

4.1.3 Turns and Rotations

In this Section, teachers were asked to talk about students’ out-of-school experiences linked to the topic of rotations. The findings from the interviews revealed differences in the way the six teachers relate to the notion of a rotation as shown below. This is what the teachers said in the interviews:

Interviewer: (Q.6) *What experiences of the learner do you consider relevant for incorporation in teaching **rotations**?*

Teacher A1: *When teaching rotations, I normally talk about the opening of a door or window as an example. This way they get to master the idea of same distance of object from centre of rotation as of image from the same centre of rotation.*

Teacher A2: *I don't have examples for rotations*

Teacher B1: *there are no examples I can draw from learner experiences*

Teacher B2: *I don't have any ...ah I just use the matrix method although it is difficult for learners to comprehend.*

Teacher C1: *Rotation... I usually turn or ask a student to stand up and turn whilst in the same position and face me again.*

Teacher C2: *Rotations... I just teach the procedure involved in the rotation of a figure... I don't have any experiences linked to learners.*

Table 4.3 below summarises teachers' responses on the mathematics involving geometric rotations that are common in students' out-of-school experiences. Some teachers could give examples whilst others could not.

Table 4.3: Teacher conception of out-of-school concepts related to a rotation

Name of teacher	School type	Out-of-school concept	
Teacher A1	Mission boarding school	-	*Opening a door or window*
Teacher A2	Mission boarding school	-	*No examples*
Teacher B1	Council run school	-	*No examples*
Teacher B2	Council run school	-	*No examples*
Teacher C1	Rural day secondary school	-	*When a student turns*
Teacher C2	Rural day secondary school	-	*No examples*

Of the six teacher participants only two (Teacher A1 and Teacher C1) had examples of the concept of a rotation in students' out-of-school experiences. In other words, it is hard for the teachers to come up with an example of this concept from students' out-of-school experiences. This means for the four teachers (Teacher A2, B1, B2 and C2) emphasis in their teaching of a rotation is on the mechanical processes only, that is, the procedures. In the three school types used in this study at least one teacher is not aware of any mathematics involving geometric rotations linked to students' out-of-school experiences. Although there were some noted differences across the three school types it was not in the interest of this study to test if the differences noted across the different schools could be ascribed to school type.

A rotation is a transformation of the plane where a point/ figure is turned at a certain angle about a point that remains fixed (Doditsch, 2014). Rotations are transformations that preserve distance and have exactly one fixed point. From the responses above only teacher A1 and Teacher C1 gave relevant practical examples of experiences that could elicit the concept of a rotation. The examples of the movement of a door and a window given by Teacher A1 are typical cases that can bring out the idea of a fixed point and a moving part that maintains same distance from the fixed point (centre of rotation).

4.1.4 Enlargement

The findings from the interviews revealed differences in how teachers perceive geometric enlargements. This is what the teachers said in the interviews:

Interviewer: (Q.6) *What experiences of the learner do you consider relevant for incorporation in teaching enlargement?*

Teacher A1*: I use photographs to explain the concept of enlargement. Sometimes I bring bottles of the same type but of different sizes... students will be able to see that it's the same bottle but in different sizes... You can see the smaller one, the bigger one, you can visualise some enlargement and so on... The ratio of enlargement can be calculated using ratio of proportional sides...*

Teacher A2*: I normally use photographs to talk about enlargement...because they depict a similarity between the real person and their photography...*

Teacher B1*: I normally mention photographs as examples of enlargements*

Teacher B2*: P**hotographs are good learning aids for the transformation of enlargement... they are relevant especially when introducing the topic*

Teacher C1*:* *As for enlargement I normally bring my photographs, one smaller and the other one bigger or the portrait... to clearly portray the notion of enlargement. Students would realise that, say, three photographs are the same but differ in size.*

Teacher C2*: I can use photographs that can be enlarged for illustration.*

Table 4.4 below summarises teachers' responses on the mathematics involving geometric enlargement that are found in students' out-of-school experiences. Photographs were recognised as very critical objects which can help illustrate the notion of an enlargement. The six teachers, all, mentioned the photograph.

Table 4.4: Teacher conception of out-of-school concepts related to enlargement

Name of teacher	School type	Out-of-school concept
Teacher A1	Mission boarding school	- *Photographs* - *Similar objects*
Teacher A2	Mission boarding school	- *Photographs*
Teacher B1	Council run school	- *Photographs*
Teacher B2	Council run school	- *Photographs*
Teacher C1	Rural day secondary school	- *Photographs*
Teacher C2	Rural day secondary school	- *Photographs*

All six teachers from the three different school types had the example of a photograph as an example of an enlargement. In the topic of enlargement there were no differences noted according to school type. According to Channon, Smith, Head, Macrae and Chasakara (2004:170), an

"Enlargement is a transformation in which a shape is magnified (made larger) or diminished (made smaller)". A photograph is a typical example of an enlargement as the resultant image is similar to the original object but different in size (NGM Book3, 1999). Although the participants cited a photograph as an example in which the students are familiar with, teachers like Teacher C1 said they only refer to photographs and not directly use them to derive the concepts. Teacher A1 was more explicit as he went on to give another useful illustration of similar bottles of different sizes.

The results demonstrate that the participants are aware of contexts found in students' out-of-school experiences that are relevant and related to the topic of enlargement. Thompson (1993) points out that studying a transformation of enlargement can enable students to realise that objects such as photographs are geometric objects. Photographs, therefore, are geometric objects which teachers can use when teaching the notion of enlargement. It is the nearest example that a teacher can imagine with a very close appeal to students' world of experience.

4.1.5 Shear

In this Section teachers were responding to a question about the mathematics involving a shear transformation that is found in students' out-of-school experiences. Some teachers had typical examples of shear whilst others did not have as shown below. The following is what the six teachers had to say.

Interviewer: (Q.6) *What experiences of the learner do you consider relevant for incorporation in teaching **shear**?*

Teacher A1*:* *For shear I sometimes use a pile of their exercise books. Put it neatly and then tilt it to change its initial order. It is in the tilting where students are made to realise a shear.*

Teacher A2*: This is a problematic topic of transformation geometry... I do not have experiences I can imagine from students' background linked to shear.*

Teacher B1*: For shear I use a sheet of paper, cut a triangular piece from one end and place it on the other end.... students notice the concept of shear practically.*

Teacher B2: *I don't normally teach for shear I find it difficult to teach to my normally weak students.*

Teacher C1: *With shear, usually, I demonstrate with a pile of books which they are familiar with. I usually demonstrate by first arranging the pile neatly when someone upsets the pile slightly it slants in some direction and that is what is called a shear.*

Teacher C2: *Examples for shear are usually a problem to me.*

Table 4.5 below summarises participants' responses on the mathematics involving a geometric shear found in students' out-of-school experiences. Teacher responses above demonstrate why it is one of the difficult topics as revealed by the teacher participants.

Table 4.5: Teacher conception of out-of-school concepts related to shear

Name of teacher	School type	Out-of-school concept	
Teacher A1	Mission boarding school	-	*Pile of exercise books*
Teacher A2	Mission boarding school	-	*No examples*
Teacher B1	Council run school	-	*Use of sheet of paper*
Teacher B2	Council run school	-	*No examples*
Teacher C1	Rural day secondary school	-	*Pile of books*
Teacher C2	Rural day secondary school	-	*No examples*

Of the six teacher participants half had no examples to use in teaching, whilst two teachers could only think of a pile of exercise books. Teacher A2, B2, and C2 expressed their uneasiness with the concepts related to a geometric shear. They indicated that they were not sure whether they understood the important aspects of shear transformation in everyday life. As expressed during interviews these three teachers had no examples from students' out-of-school experience that relate to the topic of shear (see comments above). Teacher A2 indicated that he normally skips the section during his teaching.

However, Teacher A1 and Teacher C1 said they use the example of a pile of students' exercise books to demonstrate a shear. Teacher B1's example of using a piece of paper that is cut on one end to fill up the other end was also another example given. The example, however, is not necessarily an example from students' out-of-school experiences. This can only prove how difficult the topic is for teachers of Mathematics.

4.1.6 Stretch

In this Section, teachers were asked to give examples, drawn from students' out-of-school experiences, which they use in teaching geometric stretch. The findings from the interviews revealed mixed reactions from the participants. Just like in a shear some teachers had examples whilst others did not have. This is what the teachers said in the interviews:

Interviewer: (Q.6) *What experiences of the learner do you consider relevant for incorporation in teaching stretch?*

Teacher A1*:* *For stretch I use elastic rubbers or a catapult... the concept of stretch is linked to the simple stretching of the rubbers...*

Teacher A2*: these are problematic aspects of the topic... I don't have experiences i can imagine from students' background linked to stretch*

Teacher B1*: For stretch I use an elastic band to show the movements.*

Teacher B2*: since I don't normally teach for stretch ... its difficulty to come up with learner experiences in these sections. I don't waste my time on topics like stretch personally i don't have confidence in the topic.*

Teacher C1*: As for stretch I normally refer to the under garments that when u buy a smaller size and when you put it on you it stretches in order to fit.*

Teacher C2*: ... as for stretch I can use a balloon to show the effect of a stretch.*

Table 4.6 below summarises teacher responses on the mathematics involving a geometric stretch found in students' out-of-school experiences. This is another difficult topic as revealed by the teacher participants below.

Table 4.6: Teacher conception of out-of-school concepts related to stretch

Name of teacher	School type	Out-of-school concept
Teacher A1	Mission boarding school	- *Stretching of elastic bands* - *Catapult*
Teacher A2	Mission boarding school	- *No examples*
Teacher B1	Council run school	- *No examples*
Teacher B2	Council run school	- *No examples*
Teacher C1	Rural day secondary school	- *Fitting clothes of a smaller size*
Teacher C2	Rural day secondary school	- *Balloon*

Out of the six teacher participants, Teacher A1, Teacher C1 and Teacher C2, gave some examples from students' out-of-school experiences. For example, teacher A1 referred to the stretches done with elastic bands and the catapult. However, Teacher A2, B1and B2 expressed their agitation with the concept of stretch. As expressed by some of the participants stretch is one of the dreaded topics by teachers where some teachers feel they cannot teach the topic (Teacher B2 said, "Since I don't normally teach for stretch ... its difficulty to come up with learner experiences in these sections").

In spite of the challenges mentioned, Teacher A1 and Teacher C1 said they do find the topic embracing relevant and interesting experiences of the students. The teachers talked about gadgets such as elastic bands and a catapult, common in students' out-of-school experiences, which they use in teaching the related concepts. Of the three school types at School B, the Council school, not one teacher new of experiences of students related to stretch. Teacher A1 from School A, the mission boarding school, had more examples compared to all the other teachers. This teacher is the most experienced of the six teachers and his qualifications are aligned to the mathematics teacher profession (see Table 3.1).

4.1.7 Discussion of Research question 1

The purpose of research question 1 was to explore Mathematics Teachers' conception of student experiences that have grounding in transformation geometry concepts. This was meant to ascertain the extent to which teaching and learning in Transformation Geometry values students' out-of-school experiences. The results on research question 1 revealed some concepts in Transformation Geometry linked to students' out-of-school experiences. In this presentation, such experiences were deduced from objects like mirrors, elastic bands, and catapults. Research has it that most learners' interest and achievement in Mathematics improve dramatically when they are helped to make connections between new information and experiences they have had (Cord, 1999 & Gravemeijer, 2008). In other words, bringing objects like these that bear the life experiences of students into the teaching and learning context is commensurate with ideal teaching and learning dynamics.

Teachers' responses highlighted some students' out-of-school experiences that relate to translation concepts, that is, the general movements of objects and pattern building on some traditional objects. These responses were however coming from teachers who had undergone a teacher training course. In other words, teachers with a bit of pedagogical training were aware of the value of such knowledge for teaching and learning purposes. The unqualified teacher only knew the more mechanical features of teaching concepts, for example, Teacher A2 who says "I don't have clear experiences on this one related to learner experiences out there. That's why we end up resorting to theory". Teacher A2 and B2 described a Translation based on the procedural fluency of the topic. For instance, Teacher B2 illustrated it as the notion of a displacement of an object from point A to point B in terms of changes on the x-coordinate and y-coordinate. They described a 'Translation' as a movement in a straight line in a particular direction. While their explanations were correct, they were very mechanical and had difficulties in visualising a translation in the mind of the student or in real life contexts of the learner.

Nevertheless, the qualified teachers' explanations exhibited a superficial illustration of a translation as resembling a movement of an object without specifying whether the movement is in a straight line or not. According to Jung (2002) a 'Translation' is a construction where an original figure is translated or moved or displaced and its original size, shape and orientation is preserved. In other words, a Translation has got to be a movement of a figure in a straight line without altering its compass reading. The four teachers (Teacher B1, B2, C1 and C2) seem to envisage a translation as simply a movement of an object without putting emphasis on preserving orientation. This might mean that some teachers are not fully aware of the meaning of a translation.

In contrast to the above, Teacher A1 mentioned decorations made on traditional objects, where patterns symbolise shapes repeated through translations. The example used by Teacher A1 gives a more precise model of a Translation, which demonstrates a movement where size, shape and orientation are preserved. Teacher A1's illustration of a translation here resonates well with RME's principle on guided reinvention through progressive mathematisation, which requires the choosing of relevant contexts that offer students opportunities to see value in their informal knowledge (Doorman, 2001).

Teaching and learning need to afford students an opportunity to bridge the gap between their informal, what they are used at home, and the formal knowledge, the school mathematics (Barnes, 2004). Learners' active participation in class and schoolwork is more dependent on teachers' utilisation of students' informal knowledge. These, in turn, become intrinsic motivators for further learning and resiliency. Teachers must be able to draw a lot of transformation geometry from the physical environment, pattern repetitions, object movement and photographs (Einsten, 2014).

Rivet and Krajcik (2008) highlights that if students are taught abstract ideas without meaning, they may not develop their understanding. Although participants demonstrated an appreciation of the fact that practically relevant examples are critical in making learners realise the link between Transformation Geometry and students' world of experience, and hence increase comprehension of the concept, teachers referred did not refer to many examples. In this study teachers referred to the following among other examples; the patterns (Translation); mirrors (Reflection); door movement (Rotation); photographs (Enlargement); pile of books (shear) and catapult (stretch). This is an indication that some teachers, in theory, are aware of the importance of manipulative in their teaching that expose students to real life situations. By using what is real to the learner, the real-world context as a source of concept development and as an area application through process of *mathematisation* both horizontal and vertical, abstract mathematics become simpler (Freudenthal, 1977).

It also emerged in this study, that the more experienced and more qualified a teacher is (Teacher A1 and Teacher C1 compared to Teacher B1 and Teacher B2) the more likely is the teacher able to value students' informal mathematics knowledge. There are however many aspects of real life that contain mathematics involving Transformation Geometry, such as in music. A classroom practitioner should be able to build a vast knowledge base on real world elements containing transformation geometry, which according to this study can build over a period of time. For example, a class can discuss objects that rotate. Rotations are also compositions (in the mathematical sense) of reflections (Usiskin et al. 2003, p. 315). Only Teacher A1 and C1 gave an example of Rotation.

The music connection also holds great potential for the high school geometry classroom. A geometric translation is like sliding an object from one place to another without changing orientation (Usikin et al., 2003). In music there is a horizontal translation where a melody shifts to later time. Anytime a melody line repeats in music a translation in time has occurred (Cooper and Barger, 2009). That's the translation will be noted in the tunes. During interviews with teachers, Teacher A1 gave an example of decorations on clay pots as a representation of geometric translations. using examples that directly have to do with student experiences, be it in their play or in certain menial tasks they do outside of school (Naidoo, 2012) makes students more likely to experience academic success (Ladson-Billings, 1995).

Teacher inability to imagine and use students' world of experience attribute to low student performance in the topic. Freudenthal (1991) says if learners are made to process new information in a way that does not make sense to them mastery of concepts will be a challenge. According to literature, use of learners' experiences is pivotal for student success in Transformation Geometry (Walkerdine, 2003). Teachers need to choose and design learning environments that incorporate as many different forms of learner experience as possible – social, cultural, physical and psychological. In line with the study's Theoretical Framework and Realistic Mathematics Education Model, it is pivotal for teaching and learning to encourage students' comprehension of concepts through recognising connections. This is what Loewenberg et al. (2008) refers to as SCK (specialised Content Knowledge) for teachers.

4.1.8 Summary to research Question 1

In this section, an attempt was made to analyse teachers' conception of Transformation Geometry contained in students' world of experience. Learners acquire concepts by going through different levels of mastery (Van – den Heuvel – Panhuizen, 2010). This means at the very first stage learning should be built on related students' informal knowledge. Thus, it was necessary to inquire on what teachers know as learners' out-of-school experiences related to Transformation Geometry concepts. In particular, this section described teachers' knowledge of the Transformation Geometry related to students' out-of-school experiences. This helped in bringing up evidence of teaching and learning which utilises students' world of experience.

Results from the study revealed that teachers had limited knowledge of the Transformation Geometry (Translation, Rotation, Reflection, Enlargement, Shear and Stretch) that relate to students' out-of-school experiences. As a result, they have a limited fall-back on students' informal knowledge when teaching transformations forcing teachers to teach using procedural fluency. This was compounded by the fact that some teachers had not received training in the pedagogy of teaching. With such a limitation it meant that students missed significant aspects of mathematical experiences. They (students) approach tasks with a very narrow frame of mind that keeps them from developing personal methods and build confidence in dealing with transformation geometry concepts (Boaler and Brodie, 2004).

4.2 RESEARCH QUESTION 2:

How is the context of transformation geometry teaching implemented by practicing teachers in Zimbabwean rural secondary schools?

In this Section, the researcher presents results from lesson observations, document analyses and teacher interviews. Three teachers were observed teaching at three different school types. The lessons observed highlight different approaches used by teachers in teaching Transformation Geometry as shown in the sections below. Contexts of Mathematics teaching used by teachers when dealing with transformations geometry are demonstrated in sections 4.2.1 through 4.2.4. Sections 4.2.5 and 4.2.6 discuss the justification for the inclusion of transformation geometry in the school mathematics curriculum and challenges faced by teachers when teaching transformation geometry, respectively.

4.2.1 Using the procedural approaches in concept development.

In this Section, data is presented and analysed as coming from the three sources; lesson observations, document analyses and teacher interviews. For the purpose of this section lesson observations for only Teacher B1and C1 were considered. This also includes document analyses of Teacher B1's schemes of work as well as teacher interview responses.

Results from Lesson Observations

This Section presents and analyses classroom contexts of Mathematics teaching under teacher B1 and Teacher C1. The two teachers used approaches which were largely procedural in nature as shown below. Contrary to curriculum policy on teaching and learning that says preference be given to conceptual than procedural knowledge forms (Zimbabwe School Examination Council (*ZIMSEC*, 2012 – 2017) some teachers prefer teaching for procedural understanding in Transformation Geometry.

Below, is an episode of a lesson the researcher observed showing interactions between a teacher and his class on the topic of rotation:

Teacher C1's lesson

Name of school: School C (Rural day secondary school)

Subject: Mathematics

Class: Form 4Y

Observed lesson topic 2: Transformation Geometry (Rotation)

Period: 8

Duration: 40 minutes

Date: 18 May 2016

Teacher C1: *Take out your maths exercise books and mathematical sets. Please if you don't have a mathematical set like yesterday leave my class, I don't want spectators here I want participants.*

(Class jostling as students reach out for their bags)

Teacher C1*: Can someone summarise what we covered yesterday with reflection? How do we reflect a shape?*

(There was silence in class)

Teacher C1*: Ok you tell me you have forgotten already? Ha-a-a-a please let's be serious.*

(There was silence again)

Teacher C1*: Ok without wasting time i will move to the next topic ...i will not summarise for you guys. I expect you to read and master these things. You just need to practice these transformations otherwise you will not make it...*

(Then a student's hand was up)

Teacher C1*: Yes! (Student 1)*

Student 1*: We looked at how to reflect a shape given a mirror line ...where we use a ruler and campus to measure equal distances between image point and mirror line and object point and the* *same mirror line. The straight line between image and object points must be perpendicular to the mirror line... and that can be shown using a set square.*

Teacher C1*: Good! Clap hands for him... so please practice tasks that are given in our NGM textbook...There are many of them.*

The teacher then introduced the topic **rotation** by first asking students to explain what they understand by a rotation. This time a handful of students had their hands up. Students responded differently to the question as shown below:

Student2*: rotation is turning of a wheel*

Student3*: rotation is movement right round a fixed point...e.g. the minute hand of a clock moves from the 12 point right round and back.*

Teacher C1*: yes, rotation is turning or movement about a fixed point... but the turning doesn't have to be right round always... sometimes even a little movement or turn which is not right round is still a rotation. A rotation is called an isometric transformation because it does not change the lengths and angles of a figure. When a shape rotates its dimensions are not affected.*

Teacher C1*: Can you give me examples from real life where a rotation can still occur apart from the clock and wheel?*

(A number of students had hands up)

Teacher C1*: Yes*
Student 4*: The opening of a door*
Teacher C1*: Good! Clap hands for her.*

After student 4's response the teacher then highlighted the key concepts of a rotation which are centre of rotation, direction of rotation and angle of rotation.

Teacher C1*: So, to rotate a shape you need the centre of rotation, angle of rotation and direction of rotation (clockwise or anti-clockwise). Do you know these directions? Yes student1...*

Student 1: *Clockwise is the movement of a second hand of a clock in a normal way whilst anticlockwise is when movement is backwards.*

Teacher C1: *Yes, that's correct. Now look at the triangle drawn on the blackboard.*
(Teacher moving closer to the blackboard carrying board instruments: ruler, a set square and a pair of campus)... *First of all* (Teacher demonstrating) *You draw a line from point A to the centre of rotation... with line OA measure an angle of 90 degrees clockwise using a protractor. Then draw an adjacent side making an angle of 90 degrees with OA to produce* OA^1. *Make sure* $OA=OA^1$ (Class paying attention)

Teacher C1: *Can you copy in pairs what I have demonstrated here with point A and do the same with points B and C.*

The students started working on the activity as given in their pairs. There were students who could follow the teacher's steps correctly although there were some who did not produce the correct positions for the image points B^1 and C^1 (see Fig. 4.2 below). For example, there was one group where positions for B^1 and C^1 were very queerly determined as shown.

Nearly half the class did not have the mathematical instruments for drawing the constructions and so they had to wait for one pair to finish before they could draw theirs. The main objective of the lesson was for students to practice how to rotate a shape through 90, 180, 270 degrees both clockwise and anticlockwise, given the centre of rotation. An important goal of the lesson was to see how students demonstrated mastery in rotating the different shapes.

Fig.4.1 below shows the steps followed by Teacher 5 in his demonstrations. Triangle ABC is the original shape. The teacher illustrated a clockwise rotation of a shape through 90 degrees, centre the origin. The teacher drew a triangle on an $x - y$ plane as shown in **Fig**. 4.1 below. Using a ruler to join OA and OA^1, OB and OB^1, OC and $OC^1$and protractor to measure out 90 degrees the image points were deduced (see **Fig**. 4.1 below).

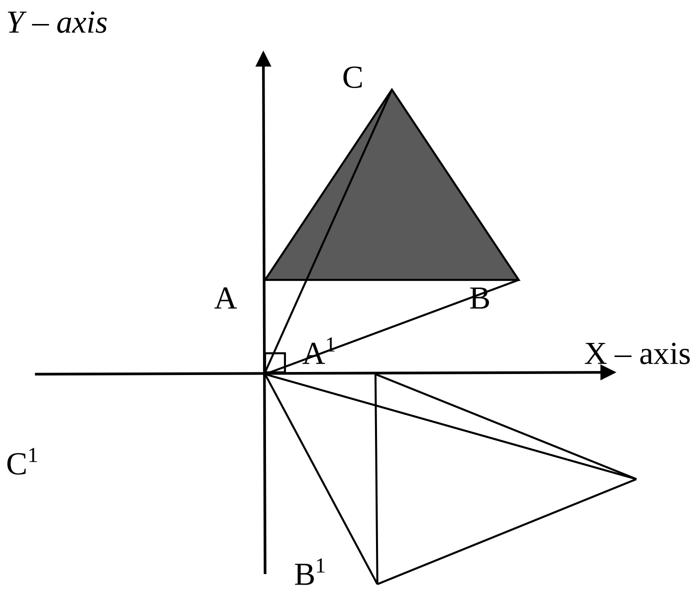

Figure 4.1: Steps in rotating triangle ABC

Although in this lesson the teacher had asked for students to provide real life examples of a rotation Teacher C1 then dominated proceedings when showing students how to use ruler, setsquare and campus to locate the image of an object under a rotation. Such use of direct instruction does not give students chance to employ critical thinking skills (Loewenberg et al., 2008).

Fig. 4.2 shows how some group got the image for triangle ABC under this rotation. The group, because they were asked to draw the image triangle got the three image points in a straight line.

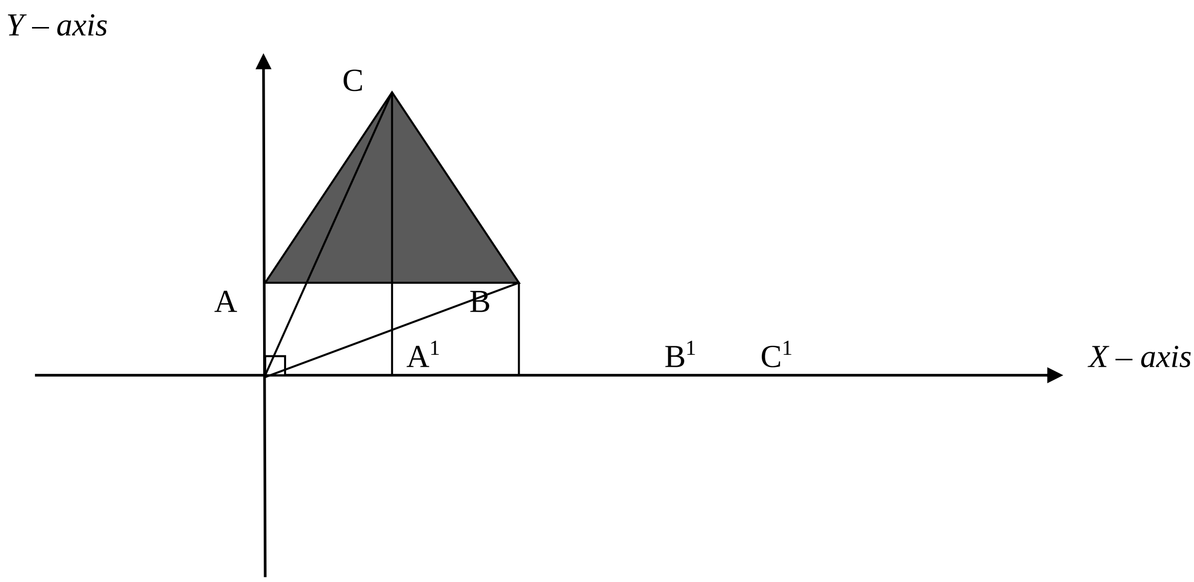

Figure 4.2: Group 1's solution to a rotation task

On **Fig.** 4.2 above positions for B^1 and C^1 are incorrect. Students seem to have incorrectly followed the steps as required when the teacher demonstrated with point A. Although these students were given all the steps they demonstrated limited understanding of what is required in rotation.

The chalkboard as a resource looked very old fashioned and blurred such that it was hardly possible to read the located points on the chalkboard graph. There was no further reference to students' world of experience made during the lesson but instead the teacher emphasised mastery of the procedures.

Generally, this lesson went on without any teacher's reference to students' real-life experiences. It was mainly grounded on transfer of ready-made mathematics, which includes the steps and procedures for performing a rotation. Such an approach to the teaching of mathematics is perceived as an 'anti-didactic inversion' (Freudenthal, 1971) that is detrimental to real learning. The teacher's experience is one that describes the struggle that most teachers come across when teaching transformational geometry and also points to the source of student's struggles in understanding and reasoning with the concepts, that is, when students fail to make connections between what they know and school mathematics. The teacher however did emphasise on the preservation of lengths and angles under rotation. This is revealed in the following extract: **Teacher C1***: ... a rotation ... does not change the lengths and angles of a figure.*

In a different lesson observed at school B teacher B1 was teaching enlargement to a Form 4 class. Below is an extract of the lesson.

Teacher B1's lesson

Name of school: School B

Subject: Mathematics

Class: Form 4 WEST

Observed lesson topic 3: Transformation geometry (Enlargement)

Period: 1

Duration: 35 minutes

Date: 24 May 2016

The topic for the lesson was "Enlargement" where students learnt how to enlarge a shape given a scale factor. The teacher started with a recap on previously covered concepts of translation, reflection and rotation. He then explained what the day's lesson was going to cover. Emphasis was made on the fact that the latter group of concepts focused on isometric transformations and they were now moving onto non-isometric transformations.

Teacher B1: *Today we look at non-isometric transformations where a transformation results in a change of size and/ shape of an object.*

The teacher then gave students steps involved in enlarging shapes, as shown below:

Step *1: Draw a triangle and label it ABC*
Step *2: Mark a point on the graph P (2; 4)*
Step *3: Using a scale factor of 2* $A^1P/AP = 2$, $B^1P/BP = 2$ *and* $C^1P/P = 2$
Step *4: Mark points* $A\ {}^1B^1C^1$ *along lines PA, PB and PC extended respectively such that* $A^1P=2AP$, $B^1P=2BP$ *and* $C^1P=2CP$
Step *5: Join* $A^1B^1C^1$ *to form new triangle, the image of triangle ABC under an enlargement centre P and scale factor 2.*

After giving out these steps, an example was presented for demonstration. Then students were jotting down the teacher's example. After this the teacher worked out for the class a task that was extracted from the textbook (NGM Book3). Students later got involved in tasks given to them in groups. They spent time practising the steps involved which the teacher demonstrated on chalkboard. At School B the teacher focused mainly at the more routine type of questions (*see Appendix J*). A critical feature of this Lesson was the manner in which students attempted to understand the teacher's demonstrations and explanations of an enlargement. In this lesson no attempt was made by the teacher to allow students to make connections with their prior knowledge of, say, *similarity* which is related to the notion of enlargement. Students could be seen attempting to follow step-by-step instructions given for mapping a figure through given conditions of enlargement.

Student 1: *How do we decide on the position of* A^1, B^1 *and* C^1*?*

Such questions meant that although the steps given were correct, students had no further clue as to the meaning of some steps. The increasing number of steps that learners need to commit to memory in mathematics often results in learners becoming confused (Passolumghi & Mammarella, 2012).

From observations, it was evident that Teacher B1 made the activities of a procedural (routine) type without any conceptual emphasis. The teacher demonstrated an enlargement by giving steps involved as shown above. In other words, although students were given steps to use in this transformation, they did not understand why the steps work and this limited the transfer of the procedures (Barnes, 2004).

In the exercise students were involved in trying to imitate the steps given by their teacher however some were failing to *operationalise* the steps. For example, there was one group which the researcher visited where instead of joining C to the centre of rotation, the origin, they took A^1 as the centre to come up with the image C^1. In short for the three points A, B and C they ended up with three different centres for rotating each of them (see **Fig.** 4.2). Students were not given enough time to think about the operations with a rotation since the teacher was more active than them during the teaching and learning process. In this lesson students exhibited a passive role. Generally, the teacher's approach fell into the procedural category (Boaler & Brodie, 2004).

The teacher consumed too much time talking to the whole class, through giving out demonstrations with minimal contributions from the students. The teacher, however, remained somewhat sceptical, behaving in class as if students could easily follow what the teacher was presenting.

Teaching approaches followed by these two teachers (Teacher B1 and Teacher C1) show that traditional instruction still dominates secondary school education in Zimbabwe. Students in teacher C1's class didn't appear motivated to learn, especially girls. They showed little interest in the concepts being taught. Because of this, they were not paying much attention to the lesson. Unfortunately, the teacher was not sensitive to the reactions and actions by the students. Some students never bothered to capture notes even though the teacher stressed the importance of jotting down the steps. Some students could be seen doing the work in a sloppy and incomplete manner. In mathematics, studies have shown that instruction, especially at the secondary school level, remains overwhelmingly teacher-centred, with greater emphasis placed on lecturing than on helping students to think critically and apply their knowledge to real-world situations (Cobb, Wood, Yackel, & McNeal, 1992)

Teachers B1 and C1 used approaches characterised by routine tasks that are completed by mechanical reproduction of procedures without deep thinking. The critical features of these Lessons were first, the manner in which learners perceived the role of the teacher, as the sole authority in the classroom. Instead of attempting the tasks, students waited for the teacher to start talking. Students seemed to equate *teaching* with *telling*. Teaching and learning was structured around the discussion conducted by the teacher. The main conclusions were written on the chalkboard and then copied down by the students to their notebooks. At *school B* it appeared that there was one practice that was valued quite often – that of executing procedures correctly and accurately. Teacher B1 at *school B* taught in a more rigid manner, structured with absolutely no chance of contextualised learning.

Thus, the interactions above show lessons whose proceedings were dominated by the teacher. The majority of the learners seemed struggling to cope with the approach, particularly noting how they performed during some pair work. The teacher moving about had to assist nearly every pair of students in class. The narrowness by which success in Transformation Geometry is judged means that few capable students rise to the top of class, whilst the majority sink to the bottom (Boaler & Stapples, 2008 p.629).

Results from Teacher Interviews

In this Section, teachers were asked to explain and justify the approaches they used in teaching transformation geometry. Contrary to curriculum policy on teaching and learning that says preference be given to conceptual than procedural knowledge forms (Zimbabwe school Examination Council (ZIMSEC, 2012 – 2017) interviewed teachers said they prefer teaching for procedural understanding in transformation geometry. This is how teacher participants responded.

Teacher A2: *I just start by defining concepts, say translation, and then move straight into the procedure involved in translating objects i.e. Object + Translation = Image. I find this approach easier to follow.*

Teacher B1: *Even if you try to explain to students using the practical way they won't understand. I have realised that the best is to teach them the procedure so that they memorise for understanding.*

Teacher B2: *normally I teach transformations using matrices, because they will have mastered the topic of Matrices (Addition, subtraction and multiplication of matrices). So I use the identity matrix* $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ *to derive the matrix of say reflection about the x – axis, as an example.*

From teacher comments above there is evidence of teaching for procedures in transformation geometry, as indicated by comments such as: *I start by defining concepts, and then move straight into the procedure involved; the best is to teach them the procedure*.

An analysis of teachers' scheme plan work revealed the same emphasis on procedures. Below is an exposition on the contents of a teacher's scheme:

Results from Document Analysis

School documents were selected to analyse the extent to which teachers incorporate students' out-of-school experiences in the teaching of transformation geometry. In this Section, teacher planning for teaching and learning in transformation geometry is scrutinised because what the teacher plans has such an influential factor on student learning. It is thus important to document the opportunities presented in the teachers' schemes of work for learners to gain competency in Transformation Geometry. It is also important to identify what content is presented in the *Scheme* and how the processes are utilised to assist students to attain highest achievement. This study's focus is to explore the extent to which teaching and learning of transformation geometry utilise students' out-of-school experiences in increasing student achievement in the area.

Teachers' plan for teaching and learning is compiled in a document called schemes of work (ZGCE, 2012). The *Scheme of work* has sections for objectives, teaching and learning activities and lesson evaluation (see Table. 4.7 below).

Table 4.7: Sample of a Scheme work structure

Week Ending	***Topics and objectives***	***Methods/Approaches***	***Aids***	***Assignments***	***General evaluation***	***Individual evaluation***

(Source: Mathematics teachers' Scheme of work)

Teachers communicate their plan for teaching through filling in sections of the *Scheme of work* shown above. This plan acts as a lesson in theory before the actual lesson delivery. It provides among others a guide as to what content is covered, what objectives to be achieved and the nature of activities lined up in pursuit of the stated objectives.

After every lesson taught teachers are expected to complete the evaluation section. Ideally, it is supposed to be a report of how well the students learned and how effective the teaching was. Teachers can then use this information to refocus their teaching to help students make their learning more efficient and meaningful. Table 4.8 and Table 4.10 below show extracts of the *Scheme of work* used by Teacher A1 and Teacher B1 at *School A and School B* respectively.

Table 4.8: Teacher A1's week scheme for Transformation Geometry at school A

Week End	Topic and Objectives	Methods or Approaches	Aids	Assignments
9/9/16	Topic: **Geometric Transformations** By the end of the week students should be able to: 1. Enlarge simple plane figures using a rational scale factor 2. Shear simple plane figures using a rational scale factor 3. Stretch simple plane figures using a rational scale factor.	- Teacher guides students to enlarge/shear/stretch simple plane figures using a rational scale factor. - Group work by students as they enlarge/shear/stretch simple plane figures using a rational scale factor - Individual work by students	NGM BK. 3 Textbook Graph books Chalk board Ruler	NGM BK. 3 EX.

The above *Schemes of work* include the topics of Enlargement, Shear and Stretch. A closer look at the *Methods* or *Approaches* column reveals a lot in terms of how activities are spread between the teacher and their students. Students seem to have a larger share of activities as compared to the teacher. *The Scheme of work* speaks to the following as evidence: *group work by students as they enlarge/shear/stretch simple plane figures, Individual work by students as they shear simple plane figures using a rational scale factor and teacher guides learners how to enlarge a simple figure using a rational scale factor*.

Below is a summary of the findings of document analyses with special reference to teacher's *Schemes of work.*

Table 4.9: Summary results of the Schemes of work analysis for School A

Item	YES	PARTLY	NO
Objectives look for use of real life contexts			X
Objectives look for students' own solution methods		X	
Objectives look for active interaction among students (to communicate, argue against and justify their solutions).	X		
Teaching and learning methods foster deep learning strategies that place 'emphasis on use of students' real-life experiences		X	
Teaching and learning activities designed allow for high student – student interaction	X		
Teaching and learning resources involved have a direct link with students' real-life experiences		X	
Activities provided for students' own solution methods		X	

The statement of objectives only speaks to the concepts to be achieved with no reference made to use of real life contexts. However, teaching and learning activities are designed with a high student involvement and partly provide for students' own solution methods. Thus, planning for teaching and learning in this case, although it values student involvement, does not speak to students' world of experience in Transformation Geometry.

The key source in teachers' lesson planning is the national syllabus. A deliberate attempt was made to match teachers' planning against curriculum policy expectations. The following is what the national syllabus says teachers of mathematics need to incorporate in their plan for teaching and learning processes.

> *Section 5.8*
>
> *"a deliberate attempt be made to teach problem-solving as a skill, with students being exposed to non-routine problem-solving situations";*
>
> *Section 5.9*
>
> *"students to be taught to identify problems in their environment, put them in a mathematical form and solve them e.g. through project work".*
>
> (Source: Zimbabwe General Certificate of Education, 2012:5)

Teacher A1's *Scheme of work* does not clearly reveal the above policy expectations. This goes to show that lesson planning is not taken as equally important as the actual teaching. Teachers simply plan to fill the void of teacher documentation as per requirement without putting a serious thought into the whole exercise.

Table 4.10 below shows a scheme of work designed by Teacher B1. In this *Scheme of work*, the teacher planned for the topics of translation, reflection and rotation for completion in one week.

Table 4.10: Teacher B1's week scheme for Transformation Geometry at School B

Week End	**Topic and Objectives**	**Methods or Approaches**	**Aids**	**Assignments**
16/9/16	Topic: **Geometric Transformations** By the end of the week students should be able to: 1. Translate objects in x-y plane 2. Describe the translation given the object and the image 3. Reflect simple figures in the x-y plane 4. Describe the reflection fully given the object and image 5. Rotate simple figures about the origin through different angles.	- Teacher demonstrates translation, reflection and rotation on squared board - Teacher helps students describe a translation, reflection and rotation fully - Students carry out translation, reflection and rotation of plane shapes in graph books	NGM Bk. 3 Mathematical set Graph book	NGM BK. 3 EX. Past exam paper

In the *Methods/Approaches* section (the third column from the right), the planned classroom activities are largely centred on the teacher, for instance, *teacher demonstrates translation, rotation, reflection on squared C/B and teacher helps students.* In spite of the curriculum policy guidelines *(*see *Appendix N* on expected Methodologies*)*, the teacher shows preference to traditional approaches as shown in his planning.

Below is a summary of the findings of the analyses done with special reference to the schemes of work for teacher B1 at school B.

Table 4.11: Showing Summary results of the Schemes of work analysis for Teacher B1

Item	YES	PARTLY	NO
Objectives look for use of real life contexts			X
Objectives look for students' own solution methods			X
Objectives look for active interaction among students (to communicate, argue against and justify their solutions).			X
Teaching and learning methods foster deep learning strategies that place 'emphasis on use of students' real-life experiences			X
Teaching and learning activities designed allow for high student – student interaction		X	
Teaching and learning resources involved have a direct link with students' real-life experiences			X
Activities provided for students' own solution methods			X

Table 4.11 above shows that Teacher B1's planning is rather unlikely to foster deep learning in students as it does not put emphasis on students' out-of-school experiences in developing concepts for transformation geometry. For instance, there is no deliberate attempt to involve objectives whose focus is on developing students' informal mathematical knowledge in transformation nor is there an attempt to utilise resources which have grounding on learners' real-life experiences.

From the teaching and learning activities section, phrases such as; *teacher demonstrates, teacher guides, teacher illustrates and teacher helps students* (see Table 4.10) are commonly used. Such teaching and learning activities result in classroom practice where students are passive throughout the lesson, chalk and talk is the preferred teaching style and more emphasis is placed on factual knowledge (Ottevanger, 2001). Such an approach to teaching and learning is procedural in nature and thus traditional. For example, as the teacher demonstrated how to perform a rotation. Students were passively observing as the procedure was performed. Such practices mean there is no active engagement of the student in the learning process since the teacher is the one doing virtually everything for the student. These problems are more restrictive in the sense that only the teacher's way is correct, and students are forced to follow explicitly (Dekker & Elshout-Mohr, 2004). For most teachers teaching mathematics follow the routine method in which the same topics are taught or re-taught the same way year after year (Fauzan, 2002). Traditional methods of teaching mathematics not only are they ineffective but severely stunt the growth of students' mathematical reasoning and problem-solving skills (Fauzan, 2000:27).

In addition, traditional methods differ with the recommendations by modern theory involved in mathematics education like Realistic Mathematics Education (RME). Teachers do not pay attention to how students learn concepts, for instance, the fact that a Teacher C1 demonstrated an enlargement on squared chalkboard and then asked students to answer questions speak to this development. Teacher B1 and Teacher C1 focused on their teaching more than on how students learn. They aimed to teach topics in the allocated time. This is directly opposed to RME philosophy.

Planning for teaching and learning should recognise the fact that students have prior knowledge as a result of their contact with the environment (Gravemeijer, 2008), which teaching and learning must value (see RME theory in chapter two). In other words, teaching and learning mustn't treat students as tabular Rasa that is as if students come to learn with empty minds (Freudenthal, 1991). Teachers have got to consider the prior knowledge their students bring, such as knowledge of their environment, as a strong base on which to build new understanding (Fauzan, 2002).

The specific methods or approaches used in Table 4.8 show dominance on students in the application of given procedures. Transformation Geometry is a branch of mathematics that should provide a way to understand and reason about our environment (Denton, 2017; Moeharty, 1993). If learning objectives are designed in the way outlined above, that is, where instructional objectives put more value on students remembering and applying procedures, then the usefulness of the topic cannot be realised through the form of teaching and learning.

4.2.2 Mathematical problems with contexts meaningful to learners

One of the goals of Mathematics teaching and learning as stipulated in the Mathematics syllabus is that concepts must be developed starting from concrete situations (in the immediate environment) and moving to abstract ones (Zimbabwe School Examination Council (ZIMSEC), 2012 – 2017). The thrust of this section is to report on teaching and learning in transformation geometry that included problems/ contexts reflecting students' interest and cultural backgrounds.

Data presented in this section is taken from lesson observations. There was evidence of teaching that drew from situations and contexts meaningful to students as demonstrated below. A detailed report on Teacher A1's Lesson is given below. An analysis of the teacher's use of realistic contexts in concept development in transformation geometry, the students' engagement on lesson activities and whether students were challenged to solve real problems is also given.

4.2.3 Results from Lesson Observation

Teacher A1's lesson

Name of School: School A

Subject: Mathematics

Class: Form 4A

Observed lesson topic 1: Transformation geometry (Stretch)

Period: 5

Duration: 40 minutes

Date: 10 May 2016

The researcher observed the teacher teaching the notion of stretch. He introduced the lesson with a recap of the concept of enlargement covered in previous lessons.

Teacher A1: *Good morning class! Eeeeh today we are going to discuss a new topic. Yesterday we ended on Enlargement. Today we wish to move on and focus our attention on a topic called Stretch. I have brought this gadget today. Have you ever stretched anything in your life?*

Teacher A1: *Susan...*

Student 1: *Yes Sir. I have done that normally with elastic bands.*

Teacher A1: *Good! What name do you give to the gadget I am holding?* (Teacher showing the gadget to the class)
(The majority in class had their hands up wanting to respond)

Student 2: *It's an object that looks like a catapult.*

Teacher A1: *Good! Now today I want us to watch very closely what will happen with this catapult and I want us to discuss the different changes you will observe as we use this object*

A catapult is an object well known by students particularly in rural areas, where it is used by young people to aim and shoot a target such as a bird. The object is elastic in nature. It can be stretched in any direction as illustrated in Fig. 4.4 below. How far it can be stretched depends on a number of factors as illustrated in the lesson dialogue below.

Teacher A1: *What do we use it for and have we done that before?*

Student 3: *Kupfura shiri kazhinji ...ehee tinozviita kumba kana takarisa mombe zvedu* (Quite often we use it to shoot a bird... we do this when at home looking after a head of cattle)

Teacher A1: *Good! Now when you stretch out this object* (Teacher demonstrating) *what determines the extent of my stretch?*

Student 4: *It is determined by how far away the target is*

Student 5: *Also by the size of the target.*

These questions prepared students for the related concepts to come. Fig 4.3 below summarises the kind of questions asked to explore the catapult problem.

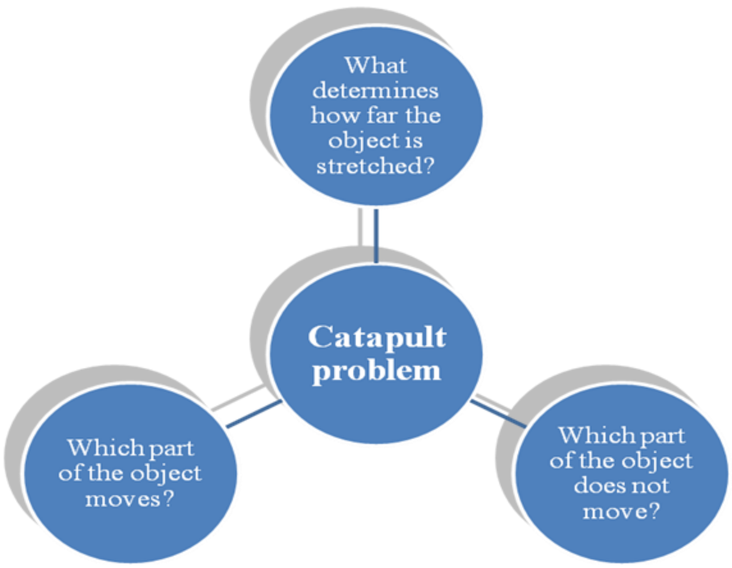

Figure 4.3: Modelling the catapult problem

Below is how students responded to questions in Fig. 4.3 above.

Teacher A1: *Which part of the object does not move?*

Student 6*: The wood part does not move*

Teacher A1*: Which part is moving?*

Student 7*: The part carrying the stone moves due to the elastic ends.*

Appendix I shows Teacher A1 illustrating a stretch using a catapult. The lesson objectives included students learning how to perform a geometric stretch. In this lesson, the teacher used an exciting example of a stretch with a familiar object. In other words, the concept of stretch was not entirely treated as new to students when the teacher used a catapult, which students particularly boys have great exposure to. The teacher used an example drawn from students' play and this aimed to create a more integrated, holistic knowledge of the concepts (Gravemeijer & Terwel, 2000).

This signalled the beginning of informal *mathematising*; horizontal *mathematisation*. "In horizontal mathematisation, the students come up with mathematical tools which can help to organise and solve a problem located in a real-life situation", (Makonye, 2014:656). Horizontal *mathematising* in this case involved students moving from the world of life into the world of Mathematics. Table 4.12 below provides an analysis of the concepts of stretch inbuilt in the catapult problem.

Table 4.12: Conjecturing the stretch concept inbuilt in the visual

Parts on the catapult	inbuilt concepts
1. The wood section	- The invariant line
2. The elastic band	- Move in a direction perpendicular to the wood section (The invariant line) - L1/L2 = stretch factor (where L1 is New dist. after stretch and L2 original dist. before stretch)
3. Shape of object (before and after stretch)	- Area and shape of image are different from area and shape of object - A1/A2 = L1xW1 / L2xW1 = L1/L2 (Stretch factor) (Where A1 is Area of image shape and A2 is Area of object shape)

Table 4.12 above demonstrates a transition into vertical *mathematisation*. The lesson upheld the power of Realistic Mathematics Education which is to bridge the informal and the formal

mathematics. The teacher tried to come up with a problem task that lead students to a series of processes that together resulted in the reinvention of the intended mathematics (Doorman, 2001) – the notion of stretch. Exploring transformation geometry concepts using learners' out-of-school experiences is very useful as a foundation of learning and can result in mastery of abstract concepts. In other words, the lesson aimed at building from students' world of experience. Fig 4.4 below shows some of the different postures that came out of stretching the gadget.

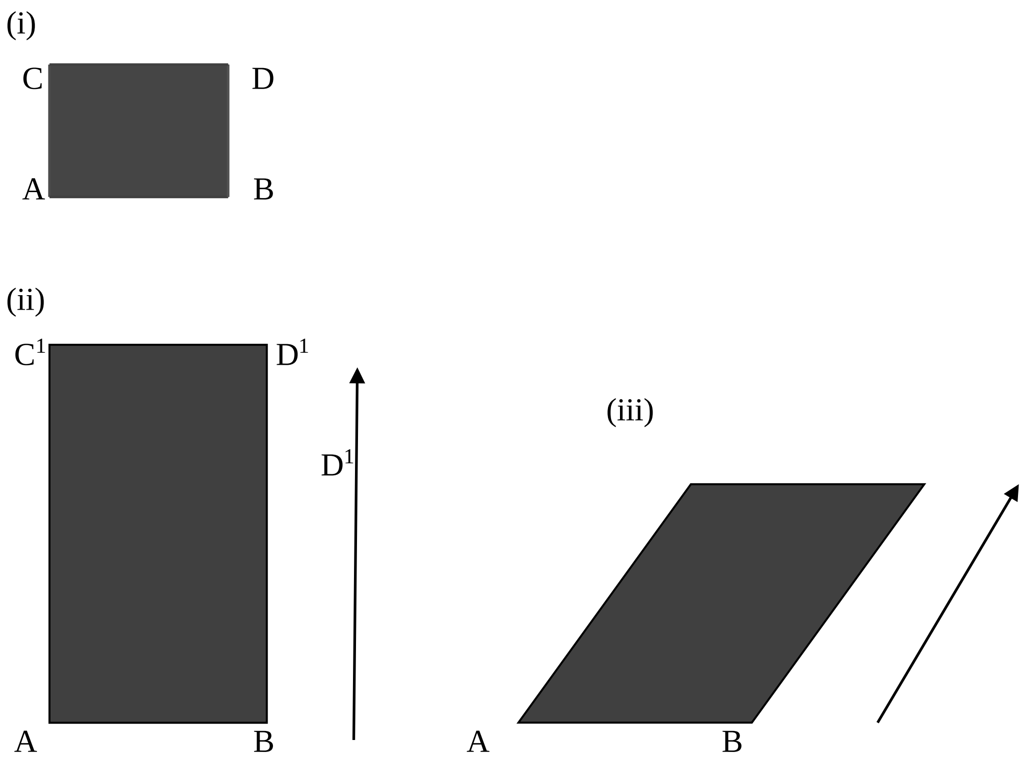

Figure 4.4 Different postures assumed by stretching (i) with AB fixed

Fig. 4.4 illustrates some of the different postures assumed by stretching the original catapult. (i) is the original posture of the catapult, (ii) is a new posture of the catapult after pulling it in the vertical direction with AB fixed and then (iii) is another new posture of the catapult when it was pulled in an inclined direction as shown above. Following Polya's (2014) problem solving framework drawing replica shapes provides useful heuristics for understanding the problem.

Students were put into groups of about four or five, and were asked to note the changes from the original catapult in terms of direction of movement, shape and size on the new shape. Students were noting and recording the changes in groups, such as the shape has changed from the original and there is a part of the model that is not changing (which the teacher later introduced as the Invariant line). The teacher made a decision to give students ample time to work on their tasks. An interesting process of constructive debate and self-correcting took place.

Later after some interesting class discussions, the teacher introduced the concept of invariant line and the notion of stretch, stretch factor, direction of stretch. Students were then given a task to show more or less similar movements with a triangle of their choice, labelling the invariant line and showing the direction of the stretch. They then later compared their drawings with that of their peers. One volunteer was asked to present their solution to the class. He took a simple approach entirely based on the student’s informal reasoning. This then degenerated into interesting discussions which helped to cement the notion of stretch.

Teacher A1 gave the class a group activity where they were asked to match an object with its image under stretch. Students were given two sheets of paper. On Sheet A there were three shapes; a right-angled triangle, an isosceles triangle and a square. While on Sheet B were different sizes of three-sided and four-sided shapes. The task was for the students to identify shapes on Sheet B which were a possible result of stretching shapes on Sheet A. The teacher would move about in the classroom attending to questions from groups and also monitoring students’ interactions. Learners showed a lot of interest in the lesson. They were very much inquisitive in what was unfolding during the lesson noting by their level of participation in the class.

Tasks given to groups were later presented by students’ representatives before the class. Students were matching a shape on Sheet A with a shape on Sheet B justifying their choices. Group presentations were followed by general class discussions. At School A, students could draw effectively on the collective resources of the group (Horn, 2005). Thus, for students to learn effectively quality teaching and interaction are fundamental to developing the new generation of learners. This provided evidence to the effect that students had assumed some pattern of thinking and this was supporting them in the mathematical reasoning.

In this class, Transformation Geometry was seen as practical and hands on. Active construction of mathematical concepts was observed because the mathematical concepts involved contexts that originate from human activities (Makonye, 2014). Learners were more captivated by the different displays reading from the level of motivation in the class. According to Realistic Mathematics Education model (RME) the statement ‘*mathematics must be connected to reality*’ means that Mathematics must be close to learners and must be relevant to everyday life situations. The teacher used an approach that valued student experiences and environment they come from and this really helped the teacher to cultivate students' interest and attention to the mathematics under discussion (Bosco, 2015; Tapper, 2010).

The context used was meaningful to students, e.g. in this case use of a catapult. Students were learning Mathematics by *mathematising* subject matter from real contexts rather than from the traditional view of presenting mathematics to them as a ready – made system with general applicability (Gravemeijer, 1994). In other words, teaching and learning should consider contextual problems or mathematically genuine contexts which students have experience in.

Teacher A1 above showed an open enthusiasm for mathematics teaching. This was manifest in the way he placed emphasis in illustrating the transformation of a stretch as shown in Fig 4.4 using familiar gadgets. Contextualised teaching which resonates well with Realistic Mathematics Education is known to have a positive influence towards students' ability to understand concepts in Mathematics (Bonotto, 2011). RME stresses that teaching and learning aids should be related to students' daily lives and experience. This is important to arouse students' interest and motivate them on the importance of transformation geometry (Arsaythamby & Zubainur, 2014).

4.2.3.1 RME elements in the conducted lessons

The lesson by Teacher A1 showed some elements of the RME model. Three key principles of RME: guided reinvention and progressive *mathematising*, didactical phenomenology and emergent models (Gravemeijer: 1994, 1999) are going to be explored.

a. Guided reinvention through progressive *mathematisation*

According to Gravenmeijer (1994) Mathematics education should be a process of reinvention where students act as a mathematician to acquire mathematical concepts as illustrated in figure 4.8 below. The role of guided reinvention principle was only visible in Teacher 1's lesson on Stretch. The principles were reflected in the activities involving exploring different illustrations of a stretch with a catapult (Freudenthal, 1991). In this case, the class were given the opportunity to experience processes of discovering different forms of stretch. For example, before the students construe the concept of stretch, they experienced how to represent different forms of stretch with a catapult. Firstly, they were experiencing stretch in their informal knowledge. At this stage the students dealt with the concept of stretch intuitively. Learners were stimulated by the different shapes assumed by the same catapult as a result of stretching in a specific direction (to *mathematise* the situation). Finally, the class named the different shapes they were forming in the process and compared areas of the original and transformed shape that helped clarify the concept of a stretch.

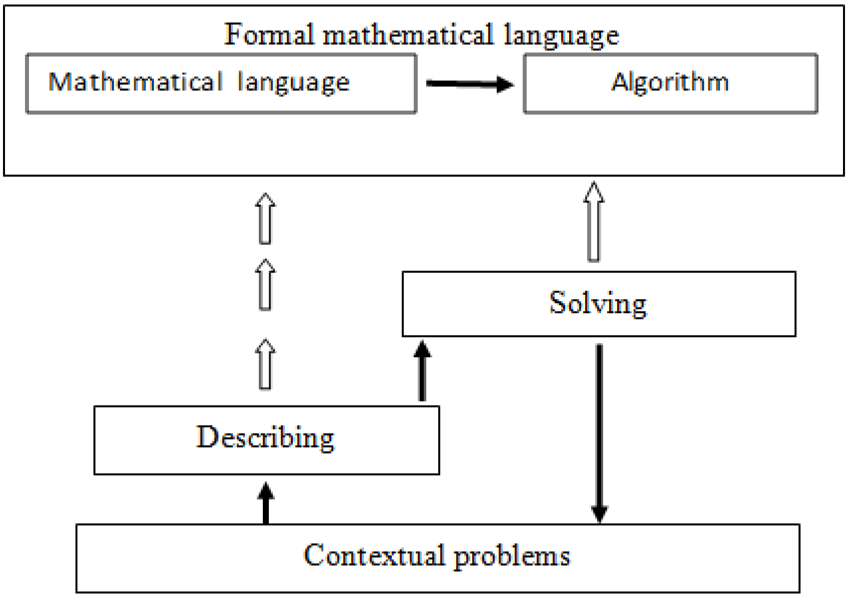

Figure 4.5: Guided reinvention model (Gravenmeijer, 1994)

b. Didactical phenomenology

Of the three lessons the principle of didactical phenomenology was more visible in the lesson conducted by Teacher A1 (on Stretch) compared to the two other lessons for Teacher B1 and Teacher C1 as attested below. The principle of didactical phenomenology relates to contextualised teaching (Gravemeijer, 1999). In this contextual based activity, the lesson (by Teacher A1) was designed based on the phenomenon involving a catapult that is meaningful for the students. Moreover, the contexts that emerged when using the catapult not only were meaningful but also gave the students the opportunity to *mathematise* them. These conditions are in line with the intention of the didactical phenomenology mentioned by Gravemeijer (1994, 1999). He mentions that the goal of a phenomenological investigation is to find contextual problems for which a situation-specific approach can be generalised, and to find contexts that lead to similar solution procedures that can be taken as the basis for vertical *mathematisation*. Makonye (2014:3) notes that "The approach may facilitate learners' readiness to accept mathematical symbols on stretch when they are eventually introduced because learners may have seen the necessity for the symbols."

c. Emerging models

Teacher A1 created opportunities for the development of models. By noticing the different forms of stretch and comparing the areas of the original and the transformed shapes comes to the fore a model of iterating the notion of stretch (Gravemeijer, 1994, 1999). Later, noting that with the so-called stretch when one side of the catapult is fixed as the opposite side moves *(*see **Fig.4.4**) a model for reasoning about the notion of a stretch emerges and can be applied to various shapes such as square, rectangle, triangle and parallelogram (refer to Teacher A1 Lesson). In this case, the stretches of these shapes will be understood on the basis of the imagery of the relationship between the area of the original shape and that of the transformed shape. Unlike in other lessons, where, for instance, demonstration of rotating a figure was done, a model would hardly emerge.

Thus, when one compares these lessons, elements of the RME model were more evident in the lesson produced by Teacher A1 on stretch. The traditional and authoritarian approach to teaching mathematics that has dominated in classrooms for years has not afforded learners opportunities to make use of horizontal *mathematisation* (Barnes, 2004). Lessons are taught by way of introducing the relevant concepts to the learners and then show with a few examples before giving an exercise or worksheet, which was more evident in the lessons of the other two teachers. According to RME theory this type of approach puts learners immediately in a more formal vertical *mathematisation* process where they would have omitted horizontal *mathematisation* (Barnes, 2004).

4.2.4 Providing opportunities for students to work interdependently

Social interaction remains an integral part of learning. Interactions with peers cause learning to occur through creating opportunities for learners to share knowledge. According to the fourth learning principle of RME (social context and interactivity), learning is not a solo activity but it is achieved through making students work in groups (Fauzan, 2002).

The purpose of this Section is to report on how teachers teaching provide learners with the opportunities to work collaboratively with their peers. The following is what teachers said about students working interdependently with peers, that is, use of interactive teaching in Transformation Geometry.

Results from Teacher Interviews

Interviewer: Do you use interactive instruction in teaching transformation geometry concepts?

Teacher A1*:* *I ask the students to experiment among themselves. They get into groups; argue about the aspects under discussion.*

Teacher A2*:* *I make use of groups whereby students will be able to interact and try to solve problems on Transformation geometry. However due to class sizes it's sometimes difficult to employ it.*

Teacher B1*:* *Usually we try to do that but the major limitation is calibre of students and class sizes. You cannot have a class discussion with students who are not on the same page as you are.*

Teacher B2*:* *If there is one student who understands the topic better he/she will explain to others. And it will make it easier for others' understanding. However, with big classes it is impossible to employ interactive teaching.*

Teacher C1*:* *interactive teaching is rather time consuming and if used always can derail completion of the syllabus.*

Teacher C2*:* *Interactive teaching!!!!We rarely use it.*

Group work, sharing and discussion strategies are important characteristics of RME. This gives students the opportunity for the exchange of ideas so that they learn from one another. However, according to participants, it is a challenge to employ interactive teaching with transformation geometry.

Results from Lesson Observation

An attempt was made to employ interactive instruction by both Teacher A1 and Teacher B1. The evidence is given below.

Teacher B1*: Can you copy and complete this exercise... What I have demonstrated with point A I want you to do the same with points B and C in your pairs.*

Learners worked on the activity in their pairs. Nearly half the class did not have the mathematical instruments for drawing the constructions but however managed to share as they were working collaboratively.

Teacher A1*: I want you to get into group***s** *of 4 or 5 students.*

The teacher gave the class a group activity where they were asked to match an object with its image under stretch. Students were given two sheets of paper. On Sheet A there were three shapes; a right-angled triangle, an isosceles triangle and a square. While on Sheet B were different sizes of three-sided and four-sided shapes. The task was for the students to identify shapes on sheet B which were a possible result of stretching shapes on Sheet A. Students showed a lot of interest in the lesson noting by their level of participation in the different groups.

Tasks given to groups were later presented by students' representatives before the class. Group presentations were followed by general class discussions. At School A, students could draw effectively on the collective resources of the group (Horn, 2005). Thus, for learners to learn effectively quality teaching and interaction are fundamental to developing the new generation of learners.

However, at School B, the researcher observed that since the class was too big, the teacher could not effectively move from one group to another. It made it difficult for the teacher to be effectual in this approach.

Teacher A1 says: *"I ask the students to experiment among themselves.... They get into groups; argue about the aspects under discussion in transformation geometry".*

Based on this point, Junkins (2017) as well as Adler and Sfard (2016) agree that it is important for teachers to set up learning opportunities that encourage students' interaction to use mathematical language themselves, so as to better grasp the underlying mathematical meaning of Transformation Geometry concepts (as cited in Kotsopoulos, 2007). To achieve these benefits outlined by Adler and Sfard (2016), teachers must create environments free of hierarchies and encourage collaborations amongst students. In the same token, they must remain mindful of their use of vocabulary because they directly contribute to students' understanding or misunderstanding of concepts (Gay, 2010).

Teacher B2 says, *"if there is one student who understands the topic in transformation geometry better he/she will explain to others. And it will make it easier for others' understanding"*.

In other words, concepts such as in transformation geometry call for concerted efforts even to involve students helping out one another. The more able student may help his/her less able peer in reinforcing transformation geometry concepts, and in providing reinforcement of the concepts already learned (Erbah & Yenmez, 2011). The fundamental effect of enhancing mastery of concepts through stimulation of the learning path is when students become aware of the drawbacks or disadvantages of their own productions during group tasks (Treffers, 1987). In other words, learners need to be involved in their own learning and have opportunities to discuss their difficulties. Thus, learning takes place when individual work is combined with consulting peers during group discussions (Manouchehri & St. John, 2006).

According to Wachira (2016) learning must be viewed as an active activity where students are encouraged to discuss and communicate their ideas and results, as part of a community of learners, often within small, cooperative groups. In this view, Mathematics teaching and learning is highly interactive because teachers are building upon the ideas of the students (Fauzan, 2002). Effective teachers gather information about students by watching students as they engage in group work and by talking with them. They monitor their students' understanding, notice the strategies that they prefer, and listen to the language that they use (Erbas & Yenmez, 2011).

According to RME the interactivity principle symbolises the learning Mathematics as a social activity (Freudenthal, 1991). Thus, it recognises whole-class discussions and group work which offer students chances to share their contributions with others. In this *modus operandi,* learners get ideas from peers to improve on their strategies, thereby enhancing students to reach higher levels of comprehension.

4.2.5 The inclusion of Transformation Geometry in school Mathematics curriculum

This Section provides a discussion on the reasons for inclusion of the topic in the ordinary level mathematics curriculum. Analysis of policy that guides teaching and learning of Transformation

Geometry was done. Teachers were asked to provide their opinions as to why they think transformation geometry is a fundamental topic for the ordinary level student.

Like in any other discipline the teaching and learning of the topic transformation geometry is guided by legislation and policy. This comes in a package known as the national syllabus, in this case, the ordinary level ZIMSEC Mathematics syllabus, Forms 1 – 4; 2012 – 2017 (see *Appendix. N*). The Mathematics syllabus is guided by seven curriculum aims. Of the seven aims, three stress the importance of any topic in mathematics to be in sync with students' lived experiences (see *Appendix. N*). The following is what teachers had to say about the inclusion of transformation geometry in the syllabus:

Interviewer: *Explain why this topic should be included in the mathematics curriculum?*

Teacher A1: *Topic should be included in the mathematics curriculum since it has aspects that are real to the students. The movement of objects is an experience in life that we always encounter. Objects move from one position to another... the topic gives visual impression of objects.*

Teacher A2: *it must be included because some of its activities apply to real life situations such as mirror reflections*

Teacher B1: *I prefer it to be included in the curriculum because it invokes critical thinking. It enables learners to tackle real life situations and it dignifies the subject due to its challenging nature.*

Teacher B2: *It is an important topic in that it provides a link between topics such as matrices and vectors. Also, students will gain the practical aspect of matrices and vectors.*

Teacher C1: *It is quite important in that it involves change which students experience in life. Life involves a lot of changes and they will be actually seeing that.*

Teacher C2: *Haaa!! The Topic is too long and rather very difficult for the average student. It must be trimmed to the level of students.*

One of the six teachers, Teacher C2, thought that the topic is rather too long and cumbersome for an average student. He proposed that the topic be spruced up to cover aspects at the level of the learner. In other words, the teacher although recommending for inclusion of the topic he felt that it might be covering too many aspects at the level of an average student.

Although participants had mixed opinions on how best the topic should be mirrored in the syllabus, where they expressed their feelings about the length and level of difficulty of the topic, they still felt the topic is justified for inclusion in the syllabus. Below is what emerged from teachers as points in support for inclusion of the topic.

Invokes critical thinking

Teacher C1 believes the topic is important because it invokes critical thinking in the students. According to Hollebrands (2003) one of the important reasons why students should study geometric transformations in school Mathematics is because it provides them with opportunities to engage in higher level reasoning activities using a variety of representations. Thus, the key to improving the performance of students is to engage students in more cognitively demanding activities (Boston & Smith, 2009) and hence provide the foundation for mathematical learning.

Interlinks with other topics

Teacher B2 was of the view that transformation geometry is pivotal because it provides a strong practical link between topics of mathematics such as matrices and vectors. The compartmentalisation of the subject into different branches has outlived its utility. In schools the idea of teaching topics separately has to be given up. The topic provides students with opportunities to think about important mathematical concepts (e.g., symmetry) (Hollerbrands, 2003).

Transformation concepts provide background knowledge to develop new perspectives in visualisation skills to illuminate the concepts of congruence and similarity in the development of spatial sense (NCTM, 1989). In other words, it provides students with a context within which they can view mathematics as an interconnected discipline.

Empowers learners to tackle real problems

Teacher A1, Teacher A2, Teacher C1 and Teacher C2 all seem to agree that the topic has strong links with learners' world of experiences. According to Teacher B1 such is important as it empowers learners to tackle real life situations. Mathematical empowerment concerns the role of mathematics in the life of the individual learner and its impact on their school and wider social life, both in the present and in the future (Ernest, 1999). The study of transformations supports the interpretation and description of our physical environment as well as provides us with a valuable tool in problem solving in many areas of mathematics and in real world situations (NCTM, 2000).

Elements of these teachers' views are in agreement with curriculum expectations, shown in the three curriculum aims below:

- develop an understanding of mathematical concepts and processes in a way that encourages confidence, enjoyment and interest
- further acquire appropriate mathematical skills and knowledge
- apply mathematics in other learning areas and in life
- develop an appreciation of the role of mathematics in personal, community and National development (Source: ZIMSEC, 2012:5)

The curriculum aims above demonstrate the importance for teaching and learning that provides students the opportunity to make connections with their real-life experiences (ZGCE, 2012). In line with this requirement, most of the teachers felt that the inclusion of this topic transformation geometry is critical because the topic allows students to connect with the real world through their own experiences and actions *(*see Teacher A1, A2, B1, B2, C1 comments above*)*. By making learners visualise concepts of Mathematics in their world of experiences, learning is supported by making it explicit since transfer of learning does not always take place automatically. Thus, by studying Transformation Geometry learners appreciate the relevance and value of Mathematics in real life (Gainsburg, 2008)

Dignifies the subject due to its challenging nature

Teacher B1 said transformation geometry is important because it dignifies the subject due to its challenging nature. The conception of mathematics that teachers hold may have a great deal to do with the way in which mathematics is characterised in classroom teaching (Cooney, 2002). The subtle messages communicated to students about mathematics and its nature affect the way they grow to view mathematics and its role in their world. Teacher B1 feels that being regarded as difficult by many makes the topic valuable. Too easy work leads to little learning and minimal pleasure and although work that is too hard leads to continual failure and subsequent lack of commitment (Freudenthal, 1991).

To engage fully in learning, the student needs to be convinced that doing the tasks is pleasurable. The unique contribution of mathematics to curriculum is what it offers for intellectual satisfaction, which can only result from successful problem solving. Giving problems challenging enough to permit a reasonable chance of success, thus resulting in increased satisfaction and significant learning is beneficial. According to Teacher B1, the topic is important because it provides these qualities for the subject.

Fig. 4.6 below is a summary of the teachers' points on why Transformation Geometry should be a part of the syllabus for the ordinary level Mathematics.

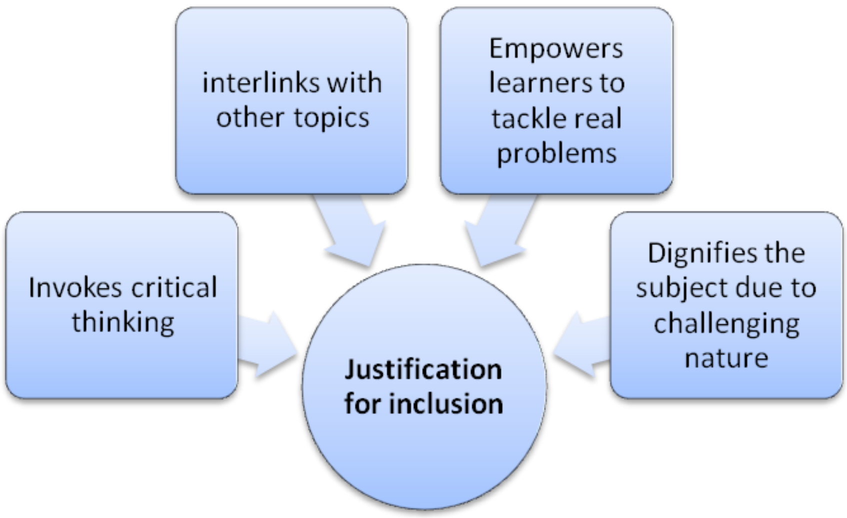

Figure 4.6 Justification for inclusion – Mathematics Teachers' perspectives

Collectively, participants referenced the four points as the benefits accrued by learners from studying the topic of transformation geometry as shown in fig 4.6. The topic invokes critical thinking, interlinks with other topics, empowers learners to tackle real life challenges and it dignifies the subject of mathematics due to its challenging nature. Thus, according to Mathematics teachers it is justified for its inclusion in the school Mathematics curriculum.

The purpose of this section was to assess the extent to which teaching and learning encourages students to connect transformation geometry with real life experience in line with current school mathematics curriculum. Curriculum in Zimbabwe is reviewed periodically. Currently, a new curriculum on Mathematics was enacted and is scheduled to run for the period 2012 – 2017. The goal of the systematic reviews is to keep up to changing times and eventually shift from a curriculum of its colonisers, Britain, which is Eurocentric to a more problem – based curriculum which should benefit the local indigenous people and eventually the nation. The envisioned curriculum promotes problem solving and critical thinking the country needed.

4.2.6 Challenges affecting the teaching of Transformation Geometry

This Section looks at the challenges that restrain teaching and learning from using the context – based approach. There were no visible aspects of RME seen in lessons observed at both School B and School C; such as teacher's use of real – life contexts, or use of a more learner –centred approach. Curriculum aims (see *Appendix N)* place greater emphasis on use of contexts familiar to student experiences. The Section, thus, attempts to bring out challenges that deny contextualised teaching and learning approaches.

The Section discusses teachers' challenges when teaching Transformation Geometry in relation to use of students' real world experiences. Teachers spoke about difficulties they witnessed for the past years they have taught Transformation Geometry. The question asked was:

Interviewer*: What in your view makes learners fail to grasp concepts in Transformation Geometry?*
The following are points raised by the different teacher participants:

Teacher A1 *Learners come to a lesson without graph books and end up being spectators rather than participants... and this limits their practice*

Teacher A2: *Teacher-student ratio is too high to make it impossible to employ learner -centred Approaches ... resources such as the mathematical set and graph books are expensive for parents in the rural areas... The dissolution of ZJC meant reduced practice of concepts, i.e. they lack background knowledge from lower classes... Teachers lack real life exposure.*

Teacher B1: *Teaching ends up not employing models due to limitation of time and shortage of resources, for instance not all students will have graph books...Generally, teaching is more theoretical than practical. At my school we use only one type of textbook and with very few copies... The other problem is we teach these concepts in the same way we were taught by our teachers, that is, traditionally. However, our teaching rarely makes students solve real life problems.*

Teacher B2: *Teaching a big class makes teachers teach only the fast learners at the expense of the slow learners...Teachers are not knowledgeable enough to handle transformation geometry concepts due to their limited exposure on the topic.*

Teacher C1: *The problem is as the teacher we fail to involve students a lot... Teachers face limited resources (graph books) in their pursuit for effective teaching.* *There is a tendency to resort to teaching following the textbook approach which does not have examples which are real life.*

Teacher C1: *We do not use ICT resources in the teaching and learning since we only have four computers at the school. Classes we teach are too big, around 50,*

The excerpts above show a number of challenges facing teaching and learning in transformation geometry. A myriad of factors militate against implementation of teaching and learning that embrace students' world of experience as revealed by teacher responses above. The following challenges emerged from the discussions with the teachers and were divided into subthemes: teacher-student ratio, lack of relevant materials, Traditional teaching approaches, teachers' lack of depth in transformation geometry concepts, and teachers' lack of out-of-school application of concepts. The ensuing discussion provides detail on the various points highlighted above.

Teacher-student ratio

Teacher A2, Teacher B2 and Teacher C2 complained about the sizes of classes they teach and they said it compromises using ideal teaching approaches that are practical in nature. The participants said they would appreciate a situation where the number of students in a class was reduced. In other words, teachers complained about classes which are too big for effective teaching of transformation geometry. This is important because the teacher can then successfully employ learner-based instruction. One of the teachers (Teacher A2) echoed, "Class sizes make it impossible to employ effective methods".

Class size is surmountable to the way teachers teach and managed their classes. Because of high number of students in classes teachers complained about limited leeway in terms of choosing learner – centred approaches, leaving the teacher to employ chalk and talk. Teaching and learning in Transformation Geometry is difficult to implement in crowded classrooms (Ball et al., 2008). Thus, teaching normal class sizes constitute a major pillar and modality of effective teaching in transformation geometry.

Lack of relevant material

One of the most important findings of the present study is that there are not enough hands-on and technological materials in schools to support the teaching of topics like Transformation Geometry. When asked about the resources they use that support their teaching, teacher responses indicated that resources weren't adequate. The resources are related to both material and immaterial things ranging from stationery and technological ones. Teacher emphasis was on lack of resources that can drive effective teaching and learning in transformation geometry. Opportunities to practise mathematics skills and concepts were hampered by lack of relevant learning material that enables students to consolidate their learning. Such includes the pair of campus (mostly used in Reflections and rotations), the graph books etc.

Implementation of the envisioned curriculum is a problem because it requires resources (Stols, Ono & Rogan, 2015). Learners who are taught skills and concepts theoretically may have difficulties with transfer of what is learnt to other settings including real life. In such cases use of real-life objects is quite helpful. When teaching challenging areas such as shear and stretch students should be exposed to concrete material (for instance the catapult used by Teacher A1) until the concepts are well grounded.

Traditional teaching approaches

A challenge noted by Teacher A1, A2 and B1 was that teachers tend to teach in a more procedural way focusing on steps and rules with little emphasis put on problem solving. Teachers echoed that due to the challenges they highlighted they end up teaching the topic as if Mathematics was a rigid and fixed body of knowledge where their responsibility is to transmit the knowledge to students (Stodolsky & Grossman; 1995 cited in Staples, 2007). In other words, teachers were able to identify their own weaknesses in teaching.

As a result of the traditional teaching methods in schools, including shortage of resources, research suggests that about 40 % of students are below van Hiele Level 2 (Staples, 2007). Since no one student passed beyond van Hiele Level 2 for school B and C it means students in respective schools can hardly operate at the level where students perceive geometric objects as determined by their properties and the relationships between properties and figures evolve. It means that learners cannot recognise relationships between geometric figures and their properties (Hoffer, 1981; van Hiele, 1986; Mason, 1998), an important feature in the study of Transformation Geometry. Students at Level 2 of the van Hiele theory are yet to master properties necessary and sufficient to describe geometric figures (Mason, 1998).

Teachers' teaching influence the ways in which students think about the concepts taught. Realistic Mathematics Education requires highly constructivist approaches to teaching, in which children are no longer seen as receivers of knowledge but makers of it (Nickson, 2000).

Teachers' lack of depth in transformation geometry concepts

Teachers are the most important resource for developing students' mathematical identities (Cobb & Hodge, 2002). Indeed, trained teachers are a necessity in some parts of Africa that are more rural and have no access to amenities of life. It emerged in this study that teaching in schools is rather far from being effective as teachers lack application of transformation geometry concepts.

Teachers are also not knowledgeable enough to handle Transformation geometry concepts because they lack exposure on the topic. Effectively applying context-based practices requires a teacher to possess a deep understanding of Mathematics, a teacher who knows the mathematics concepts in the context of students' world of experience. Literature shows that both learners and instructors have difficulties in understanding the Transformation Geometry since this is a little more abstract than the other topics (Harper, 2002). In light of this, Freudenthal (1991) suggests that mathematics education has to be organised as a process of guided reinvention where students can experience a similar process to the process in which mathematics was invented by mathematicians (Fauzan, 2002).

Teachers' lack of out-of-school application of concepts

Another problem that became evident as affecting implementation of context-based approach in Transformational Geometry was in its connection with learners' real world of experience. The majority of the participating teachers (5 out of 6) confessed to their failure to connect the subject with students' real world of experience (see interview excerpts above). One of the teachers (Teacher B1) said, "...generally our teaching is more theoretical than practical."

Thus, teachers give little attention to approaches that embrace students' out-of-school experiences; they teach and explain, and give exercises to be done as class work/homework. This justifies why Teacher B1 and Teacher C1's approaches in teaching Transformation Geometry were detrimental to students' mastery of concepts. Teachers put considerable emphases on procedures. Freudenthal (1991) argues that starting with formula or already laid down procedures is an anti-didactical inversion because the process by which mathematicians come to their conclusions is the reverse (Fauzan, 2002). Teachers need to create more supportive learning environments rather than just giving procedures and notes to students. The first learning principle of RME, constructing and concretising, states that learning mathematics is a constructive activity, and such contradicts the idea of learning as absorbing knowledge which is presented or transmitted (Treffers, 1991).

Fig 4.7 below summarises participants' views on the kind of challenges that deny them in taking advantage of students' out-of-school experiences in the teaching of Transformation Geometry.

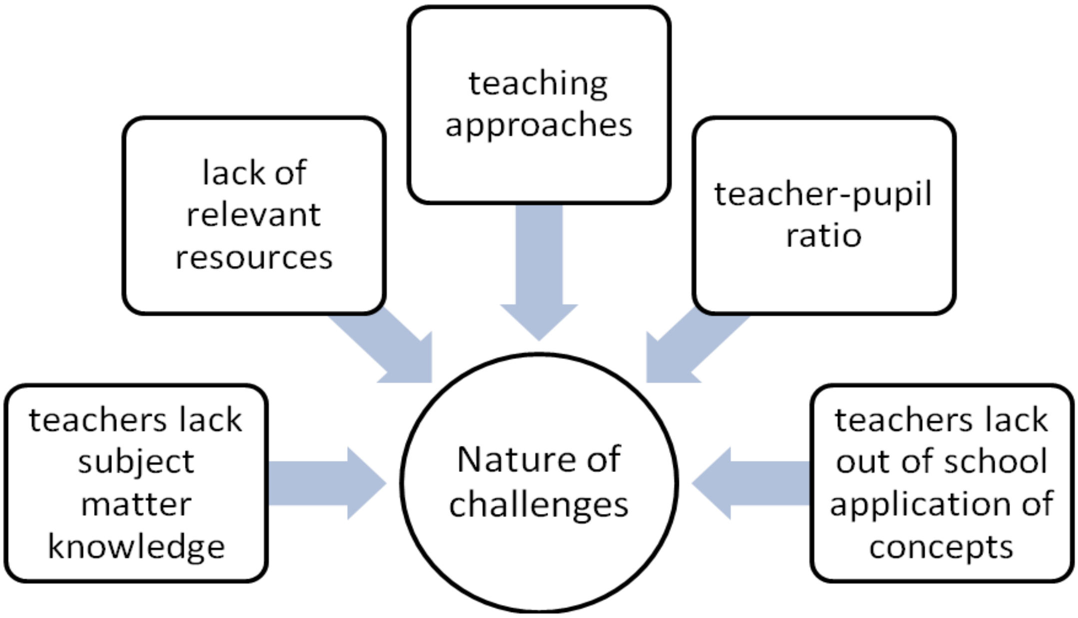

Figure 4.7 Factors restraining teachers from using of students' out-of-school experiences in teaching

4.2.7 Discussion of Research Question 2

The indicator used as an analysis factor was the extent to which the teachers were able to embrace students' out-of-school experiences in the teaching of Transformation Geometry. In this part lessons conducted by teacher A1, B1 & C1 were evaluated as shown in the ensuing discussion.

The researcher first of all looked at whether the teachers used meaningful contexts in introducing their lessons. Teacher A1 used the context of catapult to talk about stretch and teacher B1 introduced by recapping on previously covered topics whilst Teacher C1 introduced by asking students to give a bit of account about how the mirror operates. Teacher C1, when teaching rotation, had instances where a connection was sought between mathematics concepts and students experiences out-of-school, for instance where the teacher asked for examples that depict a rotation.

Teacher C1*: Can you tell me examples in real life where a rotation can still occur apart from the clock and wheel?*

Learners responded by citing examples they have had experiences with. This, according to Freudenthal (1991), is critical in the teaching and learning of mathematics concepts. Of the three teachers Teacher A1 used a meaningful context that exposes the notion of a stretch. In teacher B1's however there was no meaningful context used. Teacher C1 used a meaningful context where he was talking about what happens when one looks into a mirror.

Secondly, the researcher looked at integration of the topic with other topics and in all three topics taught there was some evident of the reference to other relevant units for example teacher C2 referred to symmetry in elaborating about reflections.

Thirdly, the researcher looked at whether the nature of problems given to students invited learners to discuss their solutions critically. At School A and School C the two teachers used group work and that provided room for students to be highly interactive during the tasks. There was evidence of participation in some demanding exercise.

Finally, were the problems guiding students to use their informal methods or strategies instead of directly using the formal ones. Elements of this principle were only prevalent in teacher A1's lesson, where students were asked questions like: what determines the extent of the stretch. Lessons for Teacher B1 and Teacher C1 did not prove any prevalence of the principle. At School B the Teacher B1 used a clear exposition which limited students' chances to debate, argue and discuss.

Thus, of the three teachers Teacher A1 was more inclined to a lesson that incorporates RME elements. In other words, the observed lessons at School B and C demonstrated a situation where the teaching of concepts was very deductive. The philosophy behind RME theory is that students must be given the opportunity to reinvent Mathematics. In other words, students would need a chance to follow the footsteps of the inventor.

The notion of *mathematising* is important as it familiarises students with a mathematical approach to everyday life situations. That is, it offers possibilities and limitations of knowing when a mathematical approach is appropriate and when it is not (Fauzan, 2002). Such approaches develop in learners, strategies based on their own experiences and informal knowledge and invite them to solve the problems (motivational factor) (Freudenthal, 1991).

However, in general Teacher B1 and Teacher C1's approaches in teaching Transformation Geometry were rather far from contributing to students' mastery of concepts as they valued mastery of procedures. This is a setback to success in transformation geometry. The idea of approaches that are more learner-centred is a directive from policy documents (the National syllabus) rather than from teachers' own beliefs (Stols, Ono & Rogan, 2015). According to Ersoy and Duatepe (2003) the Transformation topic in Geometry is rather enjoyable for children and bears some features that can promote their creative thinking. For example, a rug pattern which is repetitive, shifted, or rotated, will help them to become aware of the geometry around them.

According to Freudenthal (1991), Mathematics must be connected to reality and also regarded as a human activity. Only at School A were students accorded a chance to view Transformation Geometry in a real world of experience. From the findings it can be concluded that Teacher A1 used an approach in teaching Transformation Geometry based on some key elements of RME as shown above. According to the second learning principle of RME, Gravemeijer (1994) advocates for a broad attention to be given to visual models, model situations (in this case the catapult) that arise from problem solving activities because it will help students move through various levels of abstraction. In the RME Model, the statement '*mathematics must be connected to reality*' means that Mathematics must be close to learners and must be relevant to everyday life situations. At School A Transformation Geometry concepts were not taught directly but the intention was to derive them from the reality by means of adequate contexts and in an informal manner (Purpura, Baroody & Lonigan, 2013). This encouraged stimulation of learners' understanding of Transformation Geometry.

Learners increase understanding when taught how the concepts acquired can be used outside classroom. Contexts used should be meaningful to students. Without the ability of teachers to support learning by simplifying concepts via effective use of students' world experiences, Africa's education efforts will stagnate and eventually retrogress (Wachira, 2014)

Challenges cited above have a negative effect on learner comprehension of transformation geometry as experienced by mathematics teachers. Usually, teaching transformation geometry is limited to informing students what is meant by a particular transformation, how it is used to transform a shape (Jones, 2002). This kind of approach does not encourage students to make logical connections and explain their reasoning.

4.2.8 Summary to research Question 2

In light of the above report on findings under research question 2, the following summary is made. Teachers whose lessons were observed at School B and School C were more teacher dominant compared to their students who were very passive. Teaching observed was centred on explaining procedures and demonstrating to students contrary to national syllabus aims guiding teaching and learning practices. One can describe mathematics as a discipline comprised of procedures.

4.3 RESEARCH QUESTION 3:

To what extent are students' out-of-school experiences incorporated in transformation geometry tasks?

In this section data is presented, analysed and discussed under the following subthemes: Nature of Textbook Tasks, Nature of Examination Items and Students' General Aptitude in Geometry. Data to answer research question 3 was gathered mainly through document analysis.

In this segment an attempt was made to report on the type of questions developed by either teachers or teaching and learning resources of mathematics. Two past examination questions were purposively picked for analysis including the New General Mathematics Textbook: Book 3. The main aim was to explain the extent to which tasks incorporates students' out-of-school experiences in transformation geometry.

Results from Document Analysis

4.3.1 Nature of textbook tasks

In this Section, the textbook tasks were selected for analyses. The official textbooks for mathematics teaching and learning is the New General Mathematics (NGM) series books, used for all secondary school mathematics. The textbook normally follows a layout where concepts are introduced first, then developed and finally students are tested in the concepts (NGM, 2009).
Generally textbooks are regarded as important resources for students to learn and practise mathematics concepts (Lepik et al, 2015). Thus the textbook played a central role in teachers' preparation for teaching as well as in the selection of practice exercises for the students.

Four question tasks were selected randomly to analyse the nature of tasks designed for students. The four questions are represented as *Appendix O (a)* to *(d)* and *Appendix* R (a) is a question on concepts of a reflection, (b) is about stretch, (c) is about enlargement and (d) is about stretch. All 4 questions invite students to practice procedures in performing the different transformations (see *Appendix O*). Through working out such questions learners gain the mechanical processes of performing the transformations, they hardly can realise any real-world applications in the concepts. Although the nature of tasks used reveal some level of difficulty necessary for the students to move from one level of mastery to another they are not presented in a real-life context. Thus, questions such as these, where the real-life application is not valued, will seldom motivate students to want to learn Mathematics (Wachira, 2014).

In many Mathematics classes, teachers believe that students need to be told how to solve a problem. Students' watching a teacher work through several examples is still the principal method of instruction in mathematics classrooms (Webb, Kooij & Geist, 2011). As a result, questions set often focus on student training in formal Mathematics without including contexts, which is counterproductive for many students who desire to make sense of the mathematics they encounter (Webb, Van Der Kooij & Geist, 2011).

For such students, the lack of relevance and mathematical sense making often results in frustration, disengagement and/or failure in Transformation Geometry (Wachira, 2014). Problem contexts are critical for successful implementation of curriculum and instruction that values students' informal mathematics knowledge, like in RME.

4.3.2 Nature of the Ordinary Level Examination items

An O' Level examination in mathematics is tested in two papers; Paper 1 and Paper 2 (see full details in Chapter 3). An item on Transformation Geometry rarely misses a slot in both examination papers. Appendices *K, L and M* show typical questions taken from past O' Level Mathematics examination papers for the years from 2013 and 2014.

All questions focus on the mechanical aspects of Transformation Geometry. They request students to draw and label shapes, to map a shape under a given transformation, to describe a transformation by studying the posture of the object and its image. With such questions students are not tested on the application of concepts in real world situations. Thus, although curriculum expectations stress the importance of an inclination towards problem solving (which is a critical strand for RME philosophy), examination items are little more than the traditional type of problems that most can be solved by applying formulas and procedures.

This means solving most of these questions appear as a routine process in which students go over a fixed order of procedures. The problem with the current assessment and examination methods which emphasise on mathematics as a formal discipline (*see Appendix L*), is that they value students' ability to recall and apply formal Mathematics (formulae and procedures). The principle of guided reinvention (one of the key aspects of RME) stipulates that carefully selected contextual problems must be made accessible to learners because they offer them opportunities to develop highly context - specific solution strategies (Doorman, 2001).

There are ten assessment objectives stipulated in the Mathematics O' Level syllabus document (*see Appendix N*). However, of the ten assessment objectives only two stress emphasis on students' real-life experience. In other words, the principle in testing is not in line with RME theory. The majority of the objectives concentrate on mathematical procedures (conventions). For example;

> *Students are to be assessed on their ability to carry out calculations and algebraic and geometric manipulations accurately*. (Zimbabwe General Certificate of Education, 2012:3).

An analysis of the questions set at the national level (see *Appendices K, L & M*) shows none of the question items being grounded on real life applications of transformation geometry. It also emerges that whilst the assessment aims of the syllabus put credence on applications of concepts, for example one reads: "Students to be assessed on their ability to apply and interpret mathematics in daily life situation" (Zimbabwe General Certificate of Education, 2012:3), the assessment items are not grounded on the real-world applications. This means that there is no policy correlation between assessment objectives and examination questions. There must be some proportion of items that test students on their ability to identify and tackle with real life challenges than there are on procedural fluency.

As a result, teachers whose teaching approaches are more traditional could be drawing their practice from the structure of the examination papers and not from the national syllabus knowing that what counts in the end is how students perform in final examinations. In view of this, one wonders how the items would match the requirements of the syllabus as enshrined in the curriculum aims, that is, 'to acquire mathematical skills for use in their everyday lives and in national development' (Zimbabwe General Certificate of Education, 2012:3)

In other words, items set do not accurately measure up to the stipulated expectations of the syllabus. Students are made to memorise a lot of procedures and they should be able to regurgitate them for the examinations (Fauzan, 2002), which does not generally resonate with RME philosophy. Learning mathematics becomes more effective if students are tested on their ability to process and transform information actively. A curriculum has got to depart from overemphasis on knowledge delivery to putting emphasis on students' active participation. Such an attempt to make more connections between mathematics and real-life situations is expected to help students appreciate the relevance and value of Mathematics in real life (Gainsburg, 2008)

4.3.3 Students' General Aptitude in Geometry

This Section, presents students' performance in a test to measure out their general aptitude in geometry necessary for understanding transformation geometry. The van Hiele (1999) test used is based on geometry. It was chosen primarily because Transformation Geometry involves transformations of geometric shapes, such as triangles, quadrilaterals etc. The van Hiele test was relevant in this study because transformations of shapes call for one to recognise the properties of shapes (Jones, 2002). This helped to comprehend students' level of understanding in geometry thought which is relevant for mastery of transformation geometry concepts.

A CDASSG test *(*see *Appendix E)* was administered to student participants drawn from the three different schools. It was a 25-multiple choice item test which measured their self-efficacy in geometry. The researcher chose these selections randomly (Tobin & Kincheloe, 2006) so as to get an impression of how the students from the three schools perform in geometry according to the van Hiele Model. The model enables revelations into why students encounter difficulties in their transformation geometry concepts. The model also offers an approach of teaching that teachers could apply in order to promote their learners' levels of understanding in transformation geometry (van Hiele, 1986; Fuys et al., 1988; Pegg, 1995). In this study, the aspect of the van Hiele Model, that is, the level of geometric thinking, was utilised to explore teaching and learning in transformation geometry in Zimbabwe.

Central to the philosophy of Realistic Mathematics Education (RME) is that students can easily develop their mathematical understanding provided the mathematics involved has concepts embedded in students' out-of-school activities (Dickson et al., 2011). That is, students grasp concepts by working from contexts that make sense to them. Based on this connotation a low performance by students in the test suggests limited teacher emphasis of students' world of experience in the teaching and learning of transformation geometry.

The van Hiele Test (CDASSG test) was used to determine student understanding at five levels. The test used is divided into sections with five questions each designed following the van Hiele Model (see Section 3.4.4 in Chapter 3*)*. Learners were expected to demonstrate knowledge of a number of concepts, such as being able to recognise properties of shapes (Level 2) and the relationships between properties of different shapes (Level 3). The results from the test are represented below in Table 4.13 and Table 4.14

Table 4.13: Overall performances of students in the CDASSG 25-item test

(N=35)

Name of School	**Score out of 25**				
	1 – 5	**6 – 10**	**11 – 15**	**16 – 20**	**21 – 25**
A	**2**	**4**	**9**	**0**	**0**
B	**1**	**9**	**0**	**0**	**0**
C	**1**	**9**	**0**	**0**	**0**

Students' general performance in the test was described in terms of the overall participants' mean score obtained in this test (Creswell, 2013). Table 4.13 summarises participants' performance in the

test whose possible total score was 25. School A had two students with scores in the range 1 – 5, four in the range 6 – 10, nine in the range 11 – 15, and none for the ranges 16 – 20 and 21 – 25 respectively. School B had one student who scored marks in the range 1 – 5, nine in the range 6 – 10, and none for the ranges11 – 15, 16 – 20 and 21 – 25. School C had one student with a score in the range 1 – 5, nine in the range6 – 10, and none for the ranges 11 – 15, 16 – 20 and 21 – 25, similar to school B results. Only students from School A scored highest marks in the range 11 – 15.
From School B and C, the highest scores were in the range 6 – 10 as shown in Table 4.13. In other words, performance was lower in the two schools, B and C. However, in all three schools no student scored a mark beyond 15 out of 25. These results show that students from school B and C can operate up to Level 2 of the van Hiele Model. Only in School A are there learners who can go up to Level 3. The results show that students have problems with higher order questions, such as being able to give geometric proofs using transformational approaches (Level 4). Since these students are in the ordinary secondary school level of education they still lack in terms of geometry concepts necessary for Transformation Geometry.

Table 4.14 Mean and standard deviation on students' performance (N=35)

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
School A	15	4.00	15.00	10.3333	3.43650
School B	10	5.00	10.00	7.6000	1.50555
School C	10	3.00	9.00	7.4000	1.95505
Valid N (*listwise*)	10				

Table 4.14 summarises the mean and standard deviation of the performances per school. As shown in Table 4.14, the mean score obtained by learners from School A was 10.3, School B obtained 7.6 and that for learners from School C was 7.4. This confirms that performance was below average for all the three schools, indicating that this cohort of high school learners had a low level of knowledge in geometric thought. That is, learners in this study had a weak understanding of basic geometry necessary for mastery of concepts in Transformation Geometry (Naidoo, 2012). According to the sixth property of the van Hiele theory, *ascendancy*, progress from one level to the next is more dependent on instructional experience than on age or biological maturation (Clements, 2004). Results in this study, point at limited progression. Thus, according to Clements (2004) there must be a mismatch between these teachers' instruction and their students' capacity to master concepts in geometry.

In his theory of cognitive thinking levels van Hiele (1986) states that when students learn to understand a structure by direct contact with reality, they increase their chances of mastery of concepts. This is a phase in the learning process that van Hiele calls *explicitation* (see Section 2.4.1 in Chapter 2). In other words, teaching and learning must be in sync with the student's reality. Therefore, results of this study can either confirm that the concepts of transformation geometry have limited link with students' reality or teachers teaching does not provide the link.
Van Hiele explains that at Level 3, learners should be able to define a figure using minimum sets of properties, gives informal arguments and discovers new properties by deduction or sees the interrelationships between networks of theorems (Van Hiele, 1999). Since no student passed beyond Level 2 for School B and C, it means students in the respective schools can hardly operate at a level where learners perceive geometric objects as determined by their properties and the relationships between properties and figures evolve. Students at Level 2 of the van Hiele theory are yet to master properties necessary and sufficient to describe geometric figures (Mason, 1998; van Hiele, 1986). Such is an important feature in the study of Transformation Geometry, where for instance, one can tell that 'figure B' is as a result of a one-way stretch on 'figure A' by introspection of the properties of the two.

Consequently, the expectation of the successful completion of a course informal geometry at the secondary school level can only be realised if learners have attained the simple deduction level (Level 3) of understanding geometry upon completion of elementary and middle school. This being the case, it is reasonable to assume that for students to be prepared for success in secondary school geometry they must achieve the level of understanding identified as simple deduction (Usiskin, 1982), abstraction (Burger & Shaughnessy, 1986), or informal deduction (Crowley, 1987), so that they can mature to the level of understanding identified as deduction (Usiskin, 1982; Burger & Shaughnessy, 1986; Crowley, 1987) upon completion of a secondary school Geometry course. According to this model, progress from one of Van Hiele's levels to the next is more dependent upon teaching method than on age (Crowley, 1987). Given traditional teaching methods, research suggests that lower secondary students perform at levels one or two with almost 40% of students completing secondary school below level two (Bansilal & Naidoo, 2012). The explanation for this, according to the van Hiele model, is that teachers are asked to teach a curriculum that is at a higher level than the student (Bansilal & Naidoo, 2012).

4.3.4 Summary to Research Question 3

In respect to research question 3, a number of conclusions can be made. Examination tasks and tasks from the official textbook do not value the learner experiences in their composition. Question items set are of the routine type. They call for learner's procedural fluency in Transformation Geometry. It also emerged that the performance of students in these classes was generally low in geometry concepts. Selected students from all the three schools performed very low in the CDASSSG test. The finding gleaned from the data shows that many students lacked understanding in basic geometry which is a prerequisite for further concepts such as Transformation Geometry.

4.4 RESEARCH QUESTION 4

How is transformation geometry, as reflected in official textbooks and suggested teaching models, linked to students' out-of-school experiences?

In this Section, data is presented, analysed and discussed under the following subthemes: The Mathematics Textbook and Models that Represent Transformation Geometry Concepts. Data to answer research question 4 was obtained through document analyses involving the new general mathematics textbook (NGM Book 3) and semi–structured interviews with six Mathematics teachers.

4.4.1 The Mathematics textbook

The main purpose of this section was to explore how teachers describe the usefulness of the textbook used in schools in terms of how it utilises students' out-of-school experiences in developing concepts. The inspection considered the following aspects: the way topics are introduced and the process in the development of concepts, including the nature of examples used. In the first section, however, we hear what teachers said about the textbook they use, its suitability in enhancing their teaching in Transformation Geometry.

Results from Teacher Interviews

Interviewer: *Do you find teaching resource (the textbook) relevant in enhancing comprehension of transformation geometry concepts?*

Teacher B1: *the problem at my school is that we use only one type of textbook, the new general mathematics book series, and we have very few copies such that students would share, sometimes about 5 per copy and because we use it to give students homework some students may fail to complete homework saying so and so went with the textbook home and we didn't have access to it.*

Teacher C1: *There is a tendency to resort to teaching following the textbook approach which to me is still very old-fashioned way of teaching ... the examples used in the textbook do not always appeal to learners' (students) real life experiences.*

The interview results with teachers revealed that the textbook is the major source of their (teachers) teaching in schools. One teacher claimed to use it religiously by following on its approach (Teacher C1). In this study, it was observed that in all three schools the New General Mathematics Textbooks Series (NGM) was the main text used for the teaching and learning of Mathematics. Teachers used the NGM textbook to extract examples and tasks for students' practices (Teacher B1 referred students to an exercise in the textbook as homework).

Results from Document Analysis

In this Section, the textbook is analysed to justify its relevance in developing concepts drawing from student real life experiences.

Translation

The topic, 'Ttranslation' is introduced by bringing in the notion of patterns. A pattern is defined as, "made by taking a basic shape and repeating to build a pattern", (NGM, 1999:28). After several examples on pattern building an exercise is given (Exercise 3a.) where students are asked to copy and extend patterns.

In another question students are asked to name the basic shape which makes the patterns (see Exercise 3b). The concept of a translation is then introduced by linking it to the formation of patterns. A translation occurs when an object moves in a straight line or when a basic shape repeats itself to form a pattern (NGM, 1996:30). In a given exercise (Exercise 3b), (a) the learner is asked to, "use translation to draw additional shapes on each pattern", (p30), (b) learners are asked plot the image of a translation given some graph space.

Reflection

The topic is introduced by relating it closely to the functions of a mirror (students' out-of-school experiences) wherein the example given a letter 'P' and its mirror reflection in a line of symmetry are shown, but in a two-dimensional frame (NGM, 1999:30). Then the next illustration shows how two or more mirror lines reflect the same letter several times. In the exercise the questions are more inclined to making students demonstarte their informal reasoning with functions of a mirror before the formal notion of a geometric reflection is introduced.

Rotation

The illustration of a geometric rotation is more or less simliar in terms of detail as in a reflection. In other words, the same approach of starting from students' informal reasoning with rotations is used. However the extent of detail is as limited as in reflection. Questions designed ask students, "to use the given shapes and mirror lines to make reflection patterns" (NGM, 1999:31).

Enlargement

The topic is introduced by way of defining an enalrgement, to mean," a transformation in which a shape is magnified or diminished" (NGM, 1999:170). Then a figure is given to show the different forms of enlargements. The next Section describes in stages the process of enlarging quad ABCD into 3 different images using different scale factors of enlargement. In this case there is no dileberate attempt to make this relevant to students' out-of-school experiences. It's all done perfunctorily.

Shear and stretch

The topics of shear and stretch are presented in the same way. for shear an illustration given shows how the shape of a book is changed by pushing its top surface (see *Appendix G*). The resulting shape of the book is said to have undergone the transfornaion of a shear (NGM, 1999 p.172). Then what follows is the introduction of the many concepts surrounding the transfromation of a shear (e.g. the invariant line, the shear factor etc).

Similiarly, with stretch a rubber sheet showing the picture of a dog is used to demonstrate the different appearances assumed by the same dog when the rubber sheet is stretched in different ways (see *Appendix F*). Then what follows again are definitions of the different concepts linked to a geometric stretch (e.g. stretch factor, one-way stretch and two-way stretch).

An analysis of the official textbook used in the three schools revealed an approach where the textbook first introduces facts, be it definitions of concepts, then followed by formula and procedures for practising different forms of transformations. The topic presented is on translation, where its definition is given, and then an illustration of how to translate a point A to point B is given. This is then followed by many exercises involving largely applying the given procedures. Such an approach, however, is restrictive and not in tandem with the RME philosophy of drawing examples from student experiences. Generally, abstract concepts are introduced with limited attention paid to real life application of concepts, reasoning and understanding (Soedjadi, 2000).

In contrast, *Appendix G,* shows an approach where illustrations that are related to students' world of experience are displayed first. The topic being presented is on shear. The illustration is of the top of a book which is pushed and the resulting effect is a shear. Such ways of illustrating concepts promote use of students' background experiences in the teaching and learning of concepts.

Of the six teachers interviewed, two shared their experiences with the 'New General Mathematics textbook' used in the three schools. Below is what teachers were saying in relation to the relevance of the resource for effective teaching and learning.

Some teachers were found using the textbook to introduce and develop new concepts (for instance in the lesson taught by Teacher C1 at School C, hence analysis of the textbooks used by teachers. However, adherence to the textbook approach has a great number of challenges bound to impact effective teaching of transformation geometry. For example, the way the concepts are illustrated is far removed from students' world of experience, as demonstrated earlier on. Even the way questions are structured is in a very mechanistic way (Treffers, 1987).

Illustrations of concepts which are more inclined to the traditional values of teaching and learning of the subject are linked to an instrumental understanding of mathematics. A textbook needs to provide illustrations and explanations with detail which must be in line with students' background experiences. Such has the advantage of attracting students' attention, by bringing the mathematics to life and motivate them to want to learn more.

4.4.2 Models used for Transformation Geometry concepts

This Section looked at the nature of resources teachers used in a Transformation Geometry class. The following is a summary table showing the models used in the three schools during lessons.

Results from Lesson Observations

Table 4.15: Models used in the Transformation Geometry classes

Name of teacher	Name of School	Topic	Model Linked with SoSE
Teacher A1	A	Stretch	Catapult
Teacher B1	B	Rotation	None
Teacher C1	C	Enlargement	None

In all the three lessons shown in Table 4.15, only Teacher A1 used a model. At School A, Teacher A1 used a catapult (as a model to illustrate a stretch). Students at School A, all had each a textbook, New General Mathematics Book 4. At School B, Teacher B1 didn't have a model in his lesson, although he had mathematical board instruments, the campus, ruler and protractor, and one textbook. Learners also had textbooks but were sharing since there were not enough copies available for every student.

At School C, Teacher C1 also didn't have a model in his lesson. He had a campus and a ruler for the board and one textbook. Students didn't have textbooks or anywhere to refer to for transformation geometry. The teacher would spend time copying a question on the board, a very time-consuming exercise.

Teachers' presentation of mathematics concepts is strongly dependent on the availability of models which can help simplify abstract concepts. Since Mathematics is abstract, teachers have to be creative enough in coming up with relevant and effective models that relate at least to the learners' experience. Teacher A1 used a model of a catapult to introduce and develop the concepts of a stretch. The example of a catapult used by Teacher A1 had characteristic features known to students. The other two teachers, Teacher B1 and C1 did not have any models for their lessons.

However, research has shown that teachers suffer from a lack of knowledge in creating and using media (Mukni, 2002). Below are teachers' comments in relation to models they have used in the teaching and learning of transformation geometry.

Results from Teacher Interviews

Interviewer: *In a Transformation Geometry class, what models can a teacher use?*

Teacher A2: *It's important to identify relevant models for use in teaching such as mirrors... As teachers we lack practical application of the concepts and hence we cannot think of suitable models*

Teacher B1: *Teaching ends up not employing models due to limitation of time and shortage of resources for instance not all students will have graph books. And this is a limiting factor to a large extent because mathematics is a subject that relies more on practice.*

Teacher B2: *Teachers are not knowledgeable enough with the topic of Transformation Geometry as a result they have difficulties in coming up with relevant models.*

Teacher C1: *Teachers face limited resources in their pursuit for effective teaching.*

Interview data above indicate that teachers face problems in the teaching of transformation geometry as a result of limited resources. All participating teachers mentioned lack of concrete materials as the major hindrance to teaching and learning that embrace students' out-of-school experiences. Teacher B1 said they lack practical application of the concepts and hence cannot think of suitable models. In other words, teachers lack skills in visualising Transformation Geometry concepts in relevant situations.

4.4.3 Discussion of Research Question 4

The foregoing data presentation and analysis under research question 4 shows how the textbook approach presents topics in Transformation Geometry. The first topic presented is about geometric translations.

The topic is presented in a somewhat developmental manner as shown above. The approach used by the textbook, in this case, started from students' informal/ real-life experiences with patterns. The problem context used provided starting points to elicit students informal reasoning in how to generate/extend the patterns (Webb, Van Der Kooij & Geist, 2011). Thus, the concept of patterns is used as a source for introducing the notion of a translation. Also, the chosen context provides a smooth transition from students' informal everyday Mathematical knowledge to the formal school-taught mathematics, and such is critical in the development of concepts (Freudental, 1991). Hence, the textbook approach, with geometric translations, is giving evidence of both horizontal and vertical *mathematisation* which are key conceptions of the RME theory.

Then on the next topics of reflection and rotation, although the illustrations augur very well with how concepts are introduced in an RME-based context, they are not as detailed as in a translation. However, there is evidence of starting from students' informal understanding (e.g. the mirror in a reflection) before the formal school-taught mathematics knowledge is introduced.

With enlargement the starting point are steps and procedures to processes involved. However, this is clearly opposed to Freudental's RME-based philosophy , and he calls it an, "anti-didactical inversion". In other words, in Enlargement, the formal representations are taught first and then used to solve problems in a given exercise. The theoretical emphasis used in this case makes students struggle in learning mathematics concepts (Von, 2006). Thus, the view of context-based learning should be considered seriously in textbook development.

In shear and stretch, the starting point of learning is in students' out-of-school experiences (that is in illustrating the movement of the top surface of a book or in the way the dog's picture was changing) then followed by introducing the formal Mathematics (e.g. the invariant line, shear factor etc). Students' out-of-school experiences are recognised in terms of the role they play in the development of concepts. Such an approach, according to the RME model, offers a way of supporting students transtion from the concrete (everyday experiences) to the abstract (the formal mathematics) (Webb, Van Der Kooij & Geist, 2011; Freudenthal, 1991).

Teaching Mathematics should be as practical and feasible as possible particularly in areas such as Transformation Geometry. RME offers more than a way to support student transition from the concrete to the abstract. RME instructional sequences are conceived as "learning lines" in which problem contexts are used as starting points to elicit students' informal reasoning (Webb, Van Der Kooij & Geist, 2011). That is, the context is a source for new mathematics.

More and more teachers today are discovering that most students' interest and achievement in mathematics improve dramatically when they are helped to make connections between new information and experience they have had (Wachira, 2014). Students should learn Transformation Geometry concepts by developing and applying mathematical concepts and tools in daily life problem situations that make sense to them (Van Den Heuvel: Panhuizen, 2003).

4.4.4 Summary to Research Question 4

Based on the findings presented in Section 4.4 the following conclusions can be drawn. Lesson observations and teacher interviews revealed affect teaching in transformation geometry. These factors include teaching and learning resources and class sizes. The three schools mainly used the New General Mathematics textbook series. The use of a 'one size fits all' approach where there is a single official textbook in the schools is detrimental for students' success since they are of varying degrees of abilities.

Although there is evidence of some topics of Transformation Geometry that are introduced with contexts related to students' experiences, in other cases the opposite is true, where formal mathematics is introduced then used to solve problems. Freudenthal's (1991) RME philosophy is opposed to this approach. However, on the whole, the textbook presents its topics in a manner that values the students’ out-of-school experiences. There was evidence of great strides in this direction on topics such as Translation, Shear and stretch. In the three schools the NGM Book 3 was in short supply especially at School C. Further, some students came for lessons some without the mathematical instruments.

4.5 CONCLUSION OF THE CHAPTER

In order to determine the extent of teachers’ use of students’ real-life experiences in teaching transformation geometry lesson observations, teacher interviews and document analysis were instituted. In analysing the data, the RME model by Freudenthal (1991) was used. Noticeable differences were seen in the way Teacher A1, B1 and C1 taught concepts in Transformation Geometry. Teacher B1 and C1 were seen to emphasise more on correct use of procedures of a given transformation without paying attention to conceptual understanding. For example, in Teacher B1’s lesson it proved that the steps given were not meant to help students understand the process of rotation but for the students to memorise the procedures.

Teacher A1 emphasised on some problem-solving task with his class. The different teaching styles appeared to contribute to different learning opportunities, levels of participation and opportunities for mastery. Teacher A1 provided a highly complex endeavour of mathematics teaching with very interesting illustrations. The teacher’s approach resonated well with the RME philosophy. The idea is not that students are expected to reinvent concepts on their own, but that Freudenthal's (1991) concept of "guided reinvention" should apply (Barnes, 2004). This strategy should in turn allow learners to regard the knowledge they acquire as knowledge for which they have been responsible and which belongs to them.

In the other teachers’ classes most students had a very dependent attitude. They lacked initiative. Classroom instruction could not give students a chance to build their own understanding and thus students became passive learners. The focus of the instruction has been in the vertical *mathematisation* component, which explains dominance of instrumental rather than relational understanding (Barnes, 2004).

Accordingly, where RME aspects were visible the researcher observed, through student participation, a better mastery of the concepts by students in geometry and where RME aspects were not visible students seemed to struggle with understanding concepts taught. Thus, teaching and learning of concepts in mathematics is largely using approaches that are theoretical and many abstract concepts and formulas are introduced without paying attention on aspects of logic, reasoning and understanding. These conditions make Mathematics, particularly Transformation Geometry, more difficult to learn and understand and students become afraid of Mathematics (Fausan, Slettenhaar & Plomp, 2002), yet the syllabus has clearly given teachers adequate information or suggestions on how to effectively engage students for their learning (see syllabus on methodology section).

Based on the findings of this study, it is clear that RME philosophy is more inclined towards relational understanding than to rote learning. Learners need to be afforded a chance to bridge the gap between their informal understanding and the formal knowledge. The next Chapter summarises the study, concludes and provides recommendations.

CHAPTER FIVE

SUMMARY OF THE STUDY, RECOMMENDATIONS AND CONCLUSION

5.0 INTRODUCTION

The thrust of this chapter is premised on winding up the study whose focus was to explore the extent to which teaching and learning of Transformation Geometry embraces students' out-of-school experiences. This chapter summarises the study as derived from the problem statement, the Literature Review, Methodology, Results and Discussion. It is hoped that the study findings benefit curriculum development in Mathematics education in Zimbabwe, and also teaching pedagogy in Mathematics at secondary school level, particularly in Transformation Geometry. It is also hoped that teacher practices will shift from the traditional teaching perspective to the more contemporary practices where the student must be seen as a key player.

It has emerged from other researches that teaching and learning that is underpinned on RME-based practices has a great positive effect on learners' performance in Mathematics (e.g. Zakaria & Syamann 2017). The summary of the findings is presented to show how the four research questions were answered. The recommendations and conclusion are presented later respectively.

5.1 Summary of the findings in this study

The purpose of this section is to summarise the findings of the study under the four research questions.

Like in other countries (see for example NCTM, 2000), the Mathematics curriculum for secondary schools in Zimbabwe values quite a number of very important aspects such as developing learners' reasoning, creativity and attitude, and providing students with mathematics skills so that they can handle real-world problems mathematically. These aspects are enshrined in the Mathematics curriculum aims for the Zimbabwe secondary school mathematics syllabus as follows: develop the ability to reason and present arguments logically; develop good habits such as thoroughness and neatness, and positive attitudes such as an enquiring spirit, open-mindedness, self-reliance, resourcefulness, critical and creative thinking, cooperation and persistence; acquire mathematical skills for use in their everyday lives and in national development (ZIMSEC, 2012).

Despite these highly rated targets the curriculum appears to have fallen short of its aims, giving rise to the following questions: Why is the quality of Mathematics teaching and learning in Transformation Geometry in secondary schools still low? Why students' achievements in mathematics are poor from year to year? These questions indicate too many problems in mathematics education, especially regarding the curriculum and the teaching and learning process.

One of the explanations for the problems mentioned above is that there seems to be a contrast between how teachers claim to teach and their actual teaching in a typical class session. For instance, Teacher B1 said, "when teaching enlargement, I use photographs." However, in teaching the same topic the teacher made no attempt to make students appreciate the notion of enlargements in photographing, which is related to the notion of enlargement. In other words, while from interviews teachers espouse a learner-centred class session, the opposite was true in the real class.

A weakness noted is the lack of connection between the topics in the curriculum. As a result, teachers perceive the curriculum as a set of unrelated topics that they have to teach, while students experience the topics as a number of separate units that they have to learn.

Secondly, the curriculum lacks examples in students' world of experience. In line with the curriculum aims mentioned earlier in this section, the content of the curriculum is supposed to be very rich in practical and meaningful applications of Transformation Geometry. In fact, the content is divorced from students' out-of-school experiences and follows mainly an approach that focuses on introducing and memorising abstract concepts, applying formulas and practising computational skills (see some examples in Chapter 4).

The learning and teaching process in Zimbabwean rural secondary schools largely follows the traditional way. Teachers are at the centre of almost all activities in the classrooms (through making illustrations, demonstrations to students) in which the students are treated as tabula Rasa. Generally, teaching and learning can be described in the following phrases:

- students are passive throughout the lesson;
- chalk and talk is the preferred teaching style;
- the emphasis is on factual knowledge;
- Whole-class activities of writing
- no practical work is carried out.

The problem with these classroom practices is the fact that most students won't be mastering concepts in mathematics. The results of the study show that generally teachers do not have confidence in dealing with Transformation Geometry. In other words, learners are not given the chance to learn significant Mathematics. Meanwhile, teachers do not want to distance themselves from their traditional methods. Based on the above we can then summarise some challenges linked to mathematics teaching and learning in Transformation Geometry in Zimbabwe:

1. The teaching and learning process values only the learning objectives and their outcomes at the expense of one who is learning these. As a result, learning can only be achieved through memorising facts and concepts, as well as computational procedures.
2. The approach to teaching transformation geometry is largely mechanistic and conventional.
3. Teachers demonstrated a deficiency in their content knowledge of Transformation Geometry and thus end up teaching parts of the topic not the whole of it.

5.1.1 Research question 1

What are teachers' perceptions about the mathematics involving transformation geometry concepts are contained in the students' out-of-school activities?

From the teacher interviews data revealed some examples which have a close relationship with transformation geometry concepts. Teachers were able to relate the concepts of:

- Translation to the movement of objects and decorations or patterns on cultural objects like pots;
- Reflection to mirror reflections,
- Rotation to the movement of doors or windows,
- Enlargements to the process in photographing;
- Shear to the tilting of a pile of books by changing its initial order, and
- Stretch to the pulling of a catapult or elastic bands

With the forgoing findings whilst teachers are aware of these students' out-of-school experiences it appears they lack the confidence in embracing them for teaching purposes. Teachers also need more of the experiences so that their teaching starts from the pre-scientific perceptual level (van Hiele, 1987) dominated by *concrete operations.*

According to van Den Heuval-Panhuizan (2002) teachers must demonstrate knowledge of situations familiar with students in transformation geometry to increase their performance. The Reality principle, grounded on RME theory, emphasises that Mathematics must start with a connection to reality so that it is close to students (Van den Heuvel-Panhuizen, 2010) rather than commencing with certain abstractions or definitions to be applied later. Teaching must start with rich contexts calling for both horizontal and vertical *mathematisation* (Freudenthal, 1991; Arsathamby & Zubainur, 2014).

From the findings of this study, it emerged that there were teachers who could not come up with student experiences, for example, Teacher B2 said, *"its difficulty to come up with learner experiences in these sections".* Thus, the cohort of teachers in this study had a limited knowledge base on the mathematics involving transformation geometry contained in students' out-of-school activities. Teachers' teaching is thus restrictive since teachers have no student experiences to fall-back on for stimulation of students' interest (Mahanta & Islam, 2013). This causes students to approach tasks with a very narrow frame of mind that keeps them from developing personal methods and build confidence in dealing with mathematical ideas (Boaler & Brodie, 2004).

5.1.2 Research Question 2

How is the context of transformation geometry teaching implemented by practising teachers in Zimbabwean rural secondary schools?

In analysing how teaching and learning is done in transformation geometry data revealed differences in teacher approaches. Differences were realised in the three different schools used in this study; the mission boarding secondary school, the council-run secondary school and the government rural day secondary school. Context-based teaching was more prevalent in the more resourced schools, e.g. the mission boarding school and mechanistic teaching was practised in the more remote and under-resourced schools.

Two kinds of classroom discourses characterised teaching and learning in transformation geometry in these three schools. On one hand teaching was characterised with hands-on approaches, interactivity and high student involvement while on the other it was the teacher mainly giving out instructions for example teacher giving out the steps in performing a rotation and then demonstrating every bit. Teaching and learning under this teacher was highly teacher-centred with very limited student participation and contribution. The teaching was grounded in the traditional perspective where students are viewed as receivers of knowledge than makers of it (Nickson, 2000).

Thus, with this type of a teacher whilst the ZIMSEC **(**2012) clearly stipulates that in teaching, 'concepts must be developed starting from concrete situations (in the immediate environment) and moving to abstract ones', the students do not benefit from the contexts which they know. In other words, teacher dominance remains exceptional in some classes. This was confirmed during an interview by Teacher B1, who said, "I have realised that the best is to teach them the procedures so that they memorise for understanding".

However, with the other type of teacher who valued students' background experiences, students' interests were realised. With this kind of a teacher active learning was key during the learning process. According to Dickson et al. (2012) as well as Searle and Barmby (2012) learners develop their mathematical understanding by working from contexts that make sense to them, that is, where learning is grounded in settings that are real to students. It emerges in this study that students' performance in transformation geometry is generally low in Zimbabwe, as revealed by examiner reports, and that the teaching and learning in the topic is more underpinned by the traditional teaching perspective (as revealed in the study by 2 out of 3 teachers who taught the procedural way). In other words, the kinds of contexts in mathematics teaching hardly support mastery of concepts in transformation geometry.

The problem with these classroom practices is the fact that most learners do not master concepts in Mathematics (Clements and Burns, 2000). The results of the study show that generally teachers do not have confidence in dealing with Transformation Geometry as a topic. In other words, they are not given the chance to learn significant Mathematics. Meanwhile, teachers do not want to distance themselves from their traditional methods. Based on the above we can then summarise some challenges linked to mathematics teaching and learning in transformation geometry in Zimbabwe:

1. The teaching and learning process values only the learning objectives and their outcomes at the expense of one who is learning these. As a result, learning can only be achieved through memorising facts and concepts, as well as computational procedures.

2. The approach to teaching transformation geometry is largely mechanistic and conventional.
3. Teachers demonstrated a deficiency in their content knowledge of Transformation Geometry and thus end up teaching parts of the topic not the whole of it.

The following model emerged from this study on Transformation Geometry teaching in Zimbabwe;

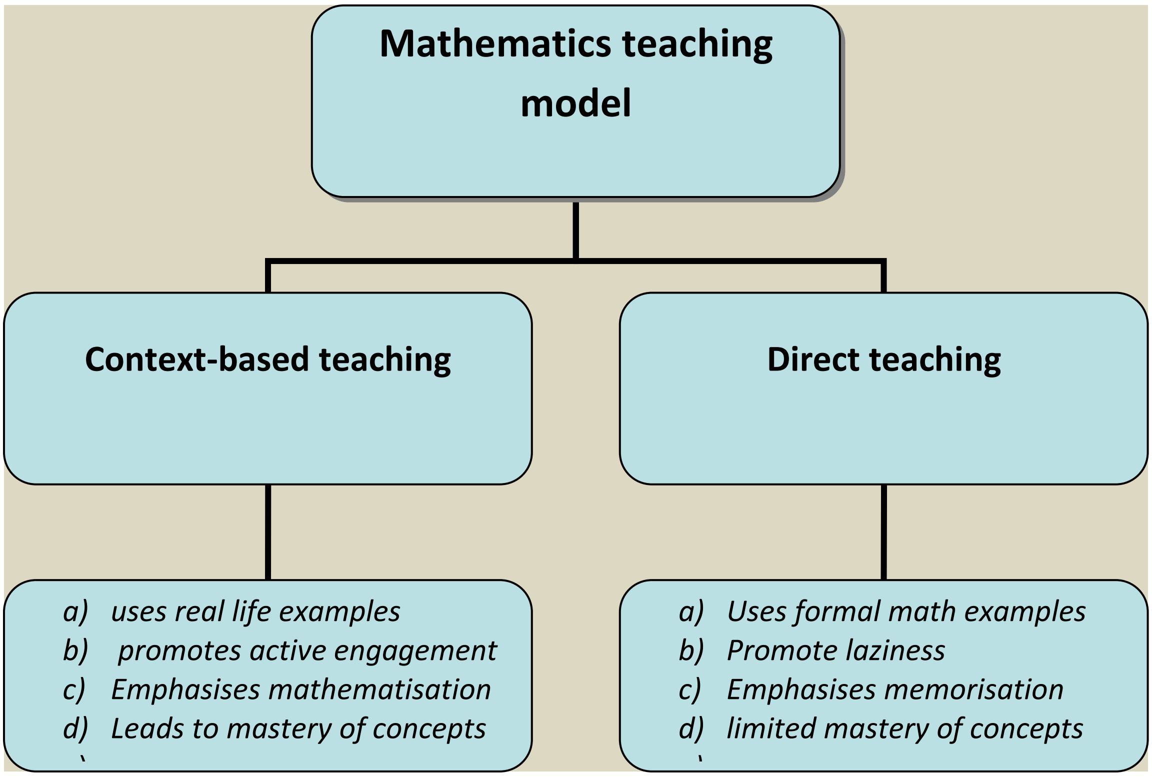

Figure. 5.1: Relational-Instrumental model in Mathematics teaching

The model, which is a two-way path system, resembles the nature of teaching practices implemented by teachers in Mathematics in this study. The model represents two types of teachers who emerged in this study. The teacher who employs context-based teaching promotes relational understanding of concepts. In this case students were given a contextual scenario where they had to explore some geometrical characteristics. It is in such situations where students develop different mathematical tools and insights on their own, and mathematise everyday contexts involving transformation geometry (Freudenthal, 1991).

While on the other hand there was a teacher who used direct teaching approaches which resulted in instrumental understanding of the concepts. Instrumental understanding arises where students are first taught the formal Mathematics (definitions, formulae, procedures) in an exposition lesson.

They are then taught how to apply the procedures in solving tasks indicated as exercises in the textbooks. This approach hardly promotes real student learning.

5.1.3 Research question 3

To what extent are students' out-of-school experiences incorporated in transformation geometry tasks?

Data suggests that tasks set both for use in the classroom and at national examination levels continue to put emphasis on the routine tasks. Tasks in the textbooks and national tests mainly assess students' ability to recall and use steps necessary to solve a particular transformation problem. Limited or no call is geared towards, 'application and interpretation of mathematics in daily life situations'.

Thus, although curriculum expectations stress the importance of an inclination towards problem solving (which is a critical strand for RME philosophy), tasks in the classroom and the national examination are little more than the traditional type of problems that most can be solved by applying formulas and procedures. Thus, solving most of these questions appear as a routine process in which students go over a fixed order of procedures. Questions such as these, where the real-life application is not valued, will seldom motivate students to want to learn mathematics.

With the forgoing realisation this means testing at both national and classroom levels is still grounded in the traditional perspective where emphasis is only on whether students can remember, that is, "recall, recognise and use mathematical formulae, rules and definitions" (ZIMSEC, 2012). Tasks should however be designed in such a way that they provide opportunities for students to be active by provoking thought and reasoning in meaningful ways (Stein & Smith, 1998) so that students start to view the subject as a tool for solving significant problems in their everyday life (Fruedenthal, 1991).

5.1.4 Research Question 4

How is transformation geometry, as reflected in official textbooks and suggested teaching models, linked to students' out-of-school experiences?

From teacher interviews participants argued that lack of relevant concrete materials and technological materials was the major hindrance for teaching and learning that embrace students' out-of-school experiences. Schools located in the rural areas of the country are the most affected in terms of teaching and learning resources. However, the most important challenge with media in the teaching of transformational geometry originated mostly from teacher beliefs. Teachers find teaching the topic of transformation geometry quite challenging as alluded to by teacher participants. Teacher B2 said, *"I don't normally teach for shear. I find it difficult to teach to my normally weak students."*

Teachers lack examples in students' world of experience. In line with the curriculum aims mentioned earlier in Chapter 4, the content of the curriculum is supposed to be very rich in practical and meaningful applications of Transformation Geometry. In fact, teachers do not have examples from students' out-of-school experiences and follow mainly an approach that focuses on introducing and memorising abstract concepts, applying formulas and practising computational skills (see some examples in Chapter 4).

In general, the new general mathematics textbook values students' life experiences in the development of concepts. This approach is emphasised under the syllabus aims and teaching methodologies. The approaches encouraged are in line with RME theory where students' life experiences form the base of mathematics teaching and learning (Gravemeijer, 2008; 2016).

5.2 CONTRIBUTION

This section highlights the study's contribution to knowledge. The study is well aligned with contemporary debates in teaching practices that harness quality and rigorous learning practices. There is a continued call to make teaching and learning more relevant, productive and driven by national goals and challenges in order to contribute to the national and economic development of the country.

Although the research does not address every aspect in the teaching and learning of transformation geometry it however will be the first step towards developing an empirical base that aims to introduce RME teaching approaches across the different curriculum subjects. Hence, findings contribute towards addressing the issues on low performance in mathematics, and in transformation

geometry in particular. In this view, the researcher believes findings of the research add to the development of curriculum in Zimbabwe that embraces RME theory.
Thus, the study's contribution is in the following areas:

- Teaching and learning of transformation geometry
- Teacher professional development
- Curriculum development

5.2.1 The teaching and learning of Transformation Geometry

Some teachers did not have an opportunity to be taught in the topic of transformation geometry during their time at school. Often, the teachers did not know of or expect the areas in which learners would have difficulties. When the situations did not arise, the teachers did not realise and anticipate the difficulties that learners can have in the topic. To support planning and success in the teaching of transformation geometry, teacher-learning opportunities need to capitalise on pedagogical content knowledge inside the transformation geometry investigation process. Some basic ideas and concepts teachers need to learn are identified in this study. They include:

- What aspects of students' experiences are relevantly related to transformation geometry concepts?
- How can teaching and learning in Transformation Geometry utilise students' experiences to increase mastery of the concepts?

In other words, a contribution made by this study in this regard is that teachers of mathematics need to see value in embracing students' out-of-school experiences in transformation geometry classes and that this can address low student performance in the topic in Zimbabwe.

5.2.2 Teacher Professional development

The teaching knowledge is insufficient. Teachers were aware that learners generally had difficulties with the topic of transformation geometry. However, they realised that they needed to improve their strategies in order to help learners overcome some of their difficulties. Thus the study contributes to the contemporary debate about the teacher competences in the subject of Mathematics. It provides ways that can be used to facilitate teacher professional growth in the area of Transformation Geometry.

First and foremost, effective teaching and learning of transformation geometry requires a teacher who is cognisant of the aspects of transformation geometry that are common in students' out-of-school experiences (SoSE). A mathematics teacher should have an informed knowledge base of what experiences of students have a close relationship with concepts in transformation geometry. In literature chapter of this study a theoretical framework on Realistic Mathematics Education provided a solid framework on which such kind of teaching can be successful. Teachers in mathematics need a consciousness of the connection between students' out-of-school experiences and concepts in mathematics. The RME model elaborated in Chapter two explicates how SoSE enhances real and meaningful learning of concepts in mathematics and how such an approach to teaching concepts in mathematics can shape the teaching and learning environment to ensure permanence of content learnt in mathematics.

The RME model is a theory that explains the teaching and learning of mathematics grounded on real and practical experiences that have a mathematical base. The model is built on a theoretical framework whose emphasis is on developing concepts from an informal standpoint to the formal mathematics

A study such as this is an important tool in the area of teacher professional development. It is used to represent and help teachers understand the complexity of teaching and that students master concepts when teaching and learning utilise their out-of-school experiences in concept development. The study offers a blend of different teaching and learning contexts so that teachers can see how particular pedagogical decisions can positively and negatively affect teaching and learning in mathematics in general. Further, this study's findings provide guidance for what aspects of teacher knowledge in Transformation Geometry should be the focus of teacher development programmes.

5.2.3 Curriculum development

An interpretation of the national syllabus revealed some parts of it that put emphasis on use of SoSE particularly in teaching, for instance, in the objective "understand, interpret and communicate mathematical information in everyday life" (ZGCE, 2012:3) whilst other parts value application of already developed models, for instance, in the objective "choose and use appropriate formulae, algorithms and strategies to solve a wide variety of problems" (ZGCE, 2012:4). The implication of this study, therefore, is that the syllabus could be wholly grounded on RME principles that would emphasise on the development of mathematical models during problem solving (Boaler, 2002). This includes the formulation of instructional objectives, the instructional media as well as the assessment objectives in line with RME theory. Teachers who approach the teaching of mathematics particularly transformation geometry by working with already developed models or procedures will continue to make concepts in mathematics a nightmare for students. Such teachers are constrained in enhancing mastery of concepts.

Thus, according to this study, it is a requirement whenever revisions are done to the curriculum in mathematics to come up with curriculum requirements that compel teachers to see value of students' out-of-school experiences in the teaching and learning process. This study's findings form a foundation for the development of a curriculum grounded in teaching and learning that embraces students' out-of-school experiences.

5.3 RECOMMENDATIONS OF THE STUDY

5.3.1 Recommendations and implications of the development of pedagogical content knowledge

The study has revealed that not all teachers know much or do care about the value of students' out-of-school experiences in motivating students towards acquiring Transformation Geometry concepts. It is critical for teacher education programmes to be streamlined in courses which embrace RME theory.

This study contributes to the debate about the measures that can be used to determine professional development needs of teachers in the area of Transformation Geometry. Further, it stimulates national and international dialogue among policy makers and educators regarding programmes and curricular to improve preparation and practice in secondary school mathematics teaching.

The study provides a wake-up call to teachers to expand their perception of the topic and assist the teachers in their personal development as professionals. One way of supporting and developing educators is a clear understanding of their problems with the topic and addressing these issues (Moodley, Njisane & Presmeg, 1992).

In view of the findings reported in Chapter 4, it was suggested that special attention should be given to developing necessary skills and knowledge for running workshops with teachers on the topic of transformation geometry. Based on the findings of the study, in this Section, the researcher presents Recommendations for policy and practice, further research and further development work.

5.3.2 Recommendations for policy and practice

RME theory is a teaching and learning approach for mathematics which was originally developed in the Netherlands, with a potential to address fundamental problems in mathematics education (Armanto, 2002; Hadi, 2002; Zulkardi, 2002). However, successful implementation of RME requires efforts to revisit the following areas; curriculum development, assessment practices, teacher training and material development. It is necessary that stakeholders appreciate the fact that not only is it necessary to develop a new curriculum and new pedagogy, but the notion of what effective mathematics education is has to change (see Fullan, 2001).

The findings from lesson observations revealed teaching and learning that is far from embracing SoSE and such classroom practice has no motivational effect on students' learning (Gravemeijer, 2008; Boaler, 2002). The findings on this aspect have serious implications on curriculum formulation and policy. Thus findings of the study have implications for policy revision. The ministry of primary and secondary education in Zimbabwe has got to come up with policy which emphasises and monitors classroom teaching in mathematics to be grounded on use of SoSE in order to optimise learning in mathematics. A policy framework should be put in place to ensure teachers function within the restriction of such policies to guarantee meaningful learning in transformation geometry and mathematics in general.

Curriculum revision would entail in-service training of teachers. It emerged through the use of the many data collection tools in this study that some teachers lacked the appreciation and hence an understanding of why teaching has to embrace SoSE. The study implication in line with this realisation is about teachers having to be re-schooled on the value of SoSE in ensuring mastery of concepts.

It is a process to initiate and implement change in the mathematics curriculum and is only possible with the support of the responsible ministry of primary and secondary education. The GoZ through the respective Ministry of Primary and Secondary Education has to develop a policy on Mathematics education that makes it a requirement for teaching and learning to take heed of RME theory.

Teacher training institutes should enhance the teachers' capacity to teach Mathematics using, among other useful theories, the RME approach. Some teachers' existing knowledge in the teaching of Transformation Geometry can be attributed to the way they were taught whilst at school. According to literature, some teachers do not possess adequate skills for teaching, which could mean that they continue to rely upon the way they were taught by their teachers (Fennema & Franke, 1992; Ball et al., 2008). This means the role and responsibility of teacher professional development programmes is still crucial.

Future research should investigate pedagogical content knowledge frameworks that can be used in varied contextual backgrounds. This should help with the revisiting of teacher-education training programmes. Thus, teacher training institutions must teach and disseminate RME to teacher trainees.

5.3.3 Recommendation for task design

Findings from national and textbook task analysis have implications on the nature of tasks set at both the school and the national levels. The analysis of the national and textbook tasks led to the conclusion that it's not only the teaching which has to be grounded on the RME model, but also the nature of questions set should be derived from and bring out relevant experiences of students that have a grounding in the mathematical concepts. Teachers must be trained in designing Mathematics tasks that embrace students' out-of-school experiences for the tasks to be interesting.

5.3.4 Recommendation for further research

The study provided experiences of male teachers only since participating teachers were all male. Future studies could engage both sexes and come up with a comparative analysis, which could explain differential preferences of the two sexes. Because this study focused on rural secondary schools, future studies could focus on a comparative analysis between urban and rural secondary schools.

The teaching and learning of topics other than transformation geometry can be explored in the context of RME approach. As an example: To what extent does teaching and learning utilise students' world of experience? What impact does RME theory have on teaching and learning of mathematics? What is the impact of RME curriculum on the students' understanding of concepts in Mathematics?
In this study focus was placed on investigating the extent to which teaching and learning takes advantage of students' world of experience in the teaching and learning of Transformation Geometry. It however, did not concentrate on how teachers can use such an experience to boost their pedagogical skills, in which case it could have employed intervention strategies. Therefore, further research could tackle on intervention strategies that demonstrate the effectiveness of RME approaches in Mathematics teaching and learning.

There is need for research in the area of developing pedagogical strategies that take advantage of what the student brings as experiences related to transformation geometry. Due to the study's time limit the study could not test and assess the impact of a developed pedagogy. In addition to time limit, each of the three teachers had a lesson observed in one area of Transformation Geometry. It is therefore important for research in future to observe teaching and learning involving all the 6 units under Transformation Geometry, that is, Translation, Reflection, Rotation, Shear, Enlargement, Shear and stretch.

Results of this study are meant to report whether teaching and learning in the topic uses RME theory or not. Then subsequent studies can work towards strategies that can aim to bring RME in the classroom. It is recommended, therefore, that the effect of the RME curriculum on teaching and learning outcomes should be investigated more thoroughly.

5.4 LIMITATIONS OF THE STUDY

A number of limiting factors could have an effect on the study findings. Data collected was subjected to the interpretations of one researcher with possibilities of biased explanations. However, with the researcher's experience as a teacher and the role he played in this study's methodological design meant that the potential for imperfection in interpretation be been minimised. Noteworthy, is the fact that over and above possible limitations as a result of the sample size are the advantages obtained from examining data over a short space of time. This had implications in the generalisation of the experiences to all other parts of Zimbabwe.

Questions like “Didn’t the camera influence some of the actions taken by participants? What was the general effect of the researcher’s presence on classroom discourse and with teachers’ interviews? A possibility could be that the researcher’s presence brought a “know-it-all” kind of attitude from the teachers making it difficult for participants to share actual practice. In order to minimise and counter this limitation, the researcher compared interviews responses of teachers whose lessons were observed (Teacher A1, Teacher B3 and Teacher C5). Further, the study was limited to studying one aspect of RME, students’ out-of-school experiences, although RME embodies a number of its critical elements.

5.5 CONCLUSION

The study is grounded on a transcendental phenomenological research design and it unpacked the extent to which teaching in Transformation Geometry embraces students’ out-of-school experiences. The study lays its focus on classroom teaching, nature of classroom activities, teachers’ planning for teaching, nature of textbooks and national examination tasks and resources. The interview and lesson observation were the main tools of data collection targeting the teaching of transformation geometry.

Based on this study’s findings, a number of conclusions can be realised. Use of learners’ out-of-school experiences in the teaching of Transformation Geometry has the ability to empower the secondary school students in transformation geometry. However, teachers are not ready to employ it. The study results are useful for in-service teachers, learners preparing to be teachers, people involved in curricular development in mathematics education. The use of learner experiences in a Transformation Geometry class proved to be a success in as far as students’ mastery of the concepts is concerned.

This thesis concludes that when use of students’ real-life experiences is missing in a Mathematics class, then learners’ interest and learning opportunities are hindered. If this aspect of knowledge is not embraced for use by teachers then learner success in Transformation Geometry teaching is hindered. Through this thesis knowledge gained from the study can be applied to professional development of pre-and in-service teachers in Mathematics. Professional development programmes that emphasise how use of students' world of experience can enhance mastery of concepts result in very dynamic teachers with an understanding of both the subject content knowledge and the pedagogic knowledge. The development is however demanding in that teachers need to develop visualisation skills in the topic of Transformation Geometry.

The learning and teaching process in Zimbabwean rural secondary schools largely follows the traditional way, as noted in two of the three teachers' lessons observed. Teachers are at the centre of almost all activities in the classrooms (through making illustrations, demonstrations to students) in which the students are treated as *tabula rasa*. Generally, teaching and learning can be described in the following phrases:

- students are passive throughout the lesson;
- chalk and talk' is the preferred teaching style;
- the emphasis is on factual knowledge;
- A whole-class activity of writing/there is no practical work carried out.

5.6 FINAL WORD

The main objective of this study was to find out the extent to which teachers of Mathematics use students' out-of-school experience in enhancing mastery of Transformation Geometry concepts.

In line with the study focus, the first objective of the study was to find out the Mathematics involving transformation geometry contained in students' out-of-school experiences. This was achieved mainly through teacher interviews and lesson observation among others.

Secondly**,** the study analysed and explored the correlation between teaching practices and curriculum policy prescribed. In this objective, the study examined challenges hindering teachers' use of learners' world of experience in their teaching practices. Document analysis, Lesson observation and interviews were the major tools used to gain this understanding.

The third objective of the study was to explore the extent to which tasks both teacher-made and national examination questions, incorporate students' world of experience. This was meant to establish the extent to which testing in the area of transformation geometry speaks to the same goal.

The fourth and final objective involved analysis of the nature of media, classroom examples and resources in as far as being linked to students' world of experience. This was achieved through analysing classroom discourse during lesson observations and as well as listening to teacher comments during interviews.

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APPENDICES

APPENDIX A: UNISA ETHICS APPROVAL

COLLEGE OF EDUCATION RESEARCH ETHICS REVIEW COMMITTEE

18 November 2015

Ref# **2015/11/18/50791389/05/MC**
Student#: Mr 55 Mashingaidze
Student Number #: 50791389

Dear Mr Mashingaidze

Decision: Ethics Approval

Researcher: Mr SS Mashingaidze
Tel: 00263773464334
Email: mashingaidzess@msu.ac.zw

Supervisor: Prof MG Ngoepe
College of Education
Department of Mathematics Education
Tel: +2712 429 8375
Email: ngoepmg@unisa.ac.za

Proposal: Exploring educators' use of students' real life experiences in the teaching of geometric transformations: A case study at the secondary school *of* Mberengwa district, Zimbabwe.

Qualification: D Ed in Mathematics Education

Thank you for the application for research ethics clearance by the College of Education Research Ethics Review Committee for the above mentioned research. Final approval is granted for the duration of the research.

The application was reviewed in compliance with the Unisa Policy on Research Ethics by the College of Education Research Ethics Review Committee on 18 November 2015.

The proposed research may now commence with the proviso that:

1) *The researcherjs will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.*
2) *Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study, as well as changes in the methodology, should be communicated in writing to the College of Education Ethics Review Committee.*

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An amended application could be requested if there are substantial changes from the existing proposal, especially if those changes affect any of the study-related risks for the research participants.

3) *The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study.*

Note:

The reference number **2015/11/18/50791389/05/MC** *should be clearly indicated on all forms of communication [e.g. Webmail, E-mail messages, letters] with the intended research participants, as well as with the College of Education RERC.*

Kind regards,

Dr M Claassens

CHAIRPERSON: CEDU RERC

APPENDIX B: LESSON OBSERVATION GUIDE

LESSON OBSERVATION GUIDE

Introspect

(To be completed by the researcher)

School: --

Class: ------------------------ Date: ---------------------

Topic / Content: --

A. The use of realistic contexts in developing **transformation geometry** concepts

	YES	PARTLY	NO
1. Teacher introduced the lesson by means of a contextual problem			
2. The context used was relevant i.e. it matched the concept/topic for the lesson			
3. Are new concepts presented in real-life (outside theclassroom) situations and experiences that are familiar to the student?			
4. Are concepts in examples and student exercises presented in the context of their use?			
5. Are new concepts presented in the context of what the student already knows?			
6. Do examples and student exercises include many real, believable problem-solving situations that students can recognise as being important to their current or possible future lives?			
7. Do examples and student exercises cultivate an attitude that says, "I need to learn this"?			
8. Do students gather and analyze their own data as they are guided in discovery of the important concepts?			
9. Do lessons and activities encourage the student to apply concepts and information in useful contexts, projecting the student into imagined futures (e.g., possible careers)and unfamiliar locations (e.g., workplaces)?			
10. Are students expected to participate regularly in interactive groups where sharing, communicating, and responding to the important concepts and decision			

making occur?			

Any other observations:

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B. The students' engagement on lesson activities

	YES	PARTLY	NO
Teacher dominated the lesson			
Teacher used most time explaining and solving mathematical problems			
Students were restricted to particular solution methods			
Students freely discussed among themselves			
Students were challenged to solve real problems in transformation geometry			
Students were encouraged to justify their solutions			

Any other observations:

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C. Classroom assessment on transformation geometry.

	YES	PARTLY	NO
Assessment is an integral and indispensable part of the teaching-learning Process			
Assessment activities focused merely on algorithms and procedures			
Assessment activities focused on both procedural and conceptual proficiency			
Assessment is conscious of the objectives of learning that utilises			

students' real life experiences			
Students were challenged to solve real problems in transformation geometry			

Any other observations:

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APPENDIX C: DOCUMENT ANALYSIS

DOCUMENT ANALYSIS SCHEDULE

(To be completed by the researcher)

School: --

Class: ------------------------ Date: ---------------------

A. Textbooks used in teaching and learning of transformation geometry

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B. Scheme/plan books

	YES	PARTLY	NO
Objectives look for use of real life contexts			
Objectives look for students' own solution methods			
Objectives look for active interaction among students (to communicate, argue against and justify their solutions).			
Teaching and learning methods foster deep learning strategies that place 'emphasis on use of students' real life experiences			
Teaching and learning activities designed allow for high student – student interaction			
Teaching and learning resources involved have a direct link with students' real life experiences			
Comments/evaluation made commensurate with objectives			
Activities provided for students' own solution methods			

Any other observations:

………
………

B. Nature of exercises on transformation geometry

	YES	PARTLY	NO
1. Assessment questions were merely routine problems			
2. Students were challenged to solve real problems in transformation geometry			
3. The marking schemes focused merely on algorithms and procedures			
4. The marking schemes were flexible and allowed for a variety of solution methods			
5. Comments in students' exercise books foster deep inner connections between concepts and real life experiences			

Any other observations:

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....…………………………
………

APPENDIX D: TEACHER INTERVIEW GUIDE

Individual Teacher Interview Guides

Part A: Teachers' experiences with transformation geometry teaching.

Purpose: To get the teachers' views about their own teaching; their understandings and ways of dealing with learners in the teaching of transformation geometry.

Questions:

1. (i) For how long have you been teaching transformation geometry?
   (ii) Do you find it interesting to teach?
   (iii) If yes, how do you make it interesting?
   (iv) If no, explain why?

2. (i) In your own words define transformation geometry.
   (ii) What do you consider as key aspects of transformation geometry?

3. Do students enjoy learning this topic?

4 Do your students perform well in this topic compared to other topics?
If no, what do you think contributes to their poor performance?

5. Explain why the topic should be included in the mathematics curriculum?

6. How can this topic be taught effectively?

7. (i) Students sometimes have their photographs taken. Are these photographs geometric objects?
If yes explain.........
   (ii) What other aspects do you exploit with your students to enhance effective teaching of transformation geometry?

8. (i) Do you find teaching resources relevant in enhancing comprehension of transformation geometry concepts?

(ii) If yes, what resources can a teacher bring in the classroom?

9. Briefly explain how you would approach the teaching of any one of the following by way of embracing students' life experiences:
Translation, rotation, reflection, shear, stretch and enlargement

10. Explain how you would teach for the following:
Definitions, axioms, laws in transformation geometry, construction of proofs

11. How would you engage ICT resources in teaching transformation geometry?

Part B: Teacher utilisation of 'real' mathematics in the teaching of Transformation geometry

Purpose: to measure the quality of mathematics teachers' instruction in transformation geometry. In this study it is important to come up with a measure of teachers' instruction in line with exploitation of learners' real life experiences.

Questions:

1. Does the topic relate to students' real life experiences, their culture etc
If yes, in which areas?

2. In the classroom are there any features that relate to concepts in transformation geometry?

3. How do you make your instruction 'real' to the learners?

4. For transformation geometry teaching what models/ media do you employ?

5. What models /media do you think is more relevant for effective mastery of the concepts?

6. What experiences of the learner do you consider relevant for incorporation in teaching and learning of transformation geometry concepts?

7. (i) Do you use interactive instruction in teaching transformation geometry concepts?

(ii) What affects implementation of interactive teaching at your school?

8. To what extent can teaching for applications be included in transformation geometry instruction?

9. What in your view makes learners fail to understand concepts in Transformation geometry?

10. In general what do you think teachers need to do in order to teach the concepts effectively?

11. Is it possible to make students recognise connections between transformation geometry and their real life experiences?
If yes explain....

13. Do you make your students solve real life problems in transformation geometry?
If yes, give examples.....

APPENDIX E: VHG TEST

The van Hiele Test is composed of 25 multiple choice questions. The test is used to determine student understanding at each level. The van Hiele Test is divided into sections of five questions each designed following the theory. The questions are arranged in this order: questions 1- 5 measure student understanding at Level 1, questions 6 -10 measure student understanding at Level 2, questions 11 -15 measure student understanding at Level 3, questions 16 -20 measure student understanding at Level 4, and questions 21-25 measure student understanding at Level 5. According to Usiskin (1982) the first three van Hiele Levels are sufficient in detail so that test questions can be developed easily.

[Adapted from CDASSG Van Hiele Geometry Test]

VAN HIELE GEOMETRY TEST FOR STUDENTS

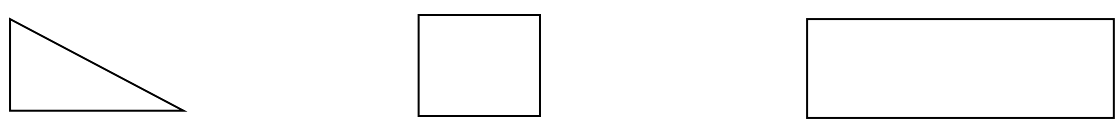
Directions

This test contains 25 questions. When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
4. If you want to change an answer, completely erase the first answer.
6. You will have 20 minutes for this test.

This test is based on the work of P.M. van Hiele.

1. Which of these are squares?



K L M

A. K only

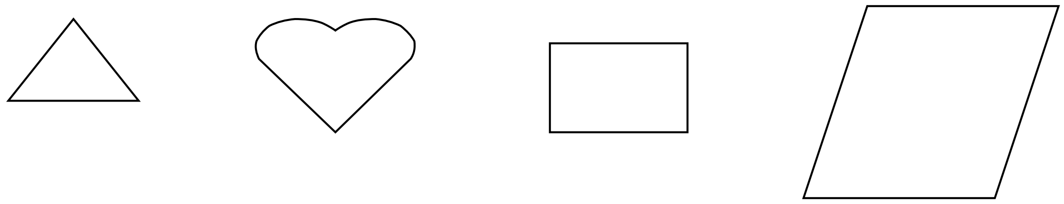
B. L only

C. M only

D. L and M only

E. All are squares

2. Which of these are triangles?



A. None of these are triangles.

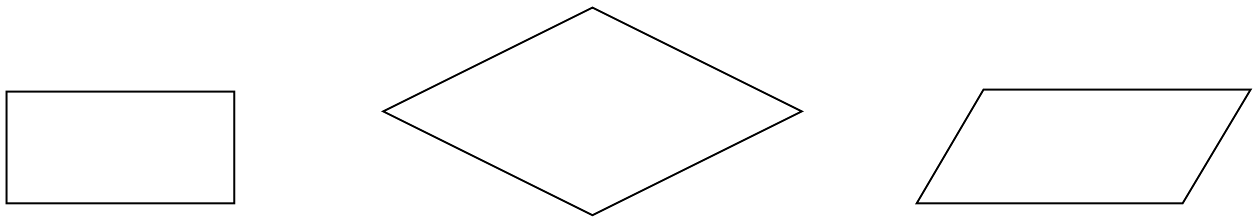
B. V only

C. W only

D. W and X only

E. V and W only

3. Which of these are rectangles?



A. S only

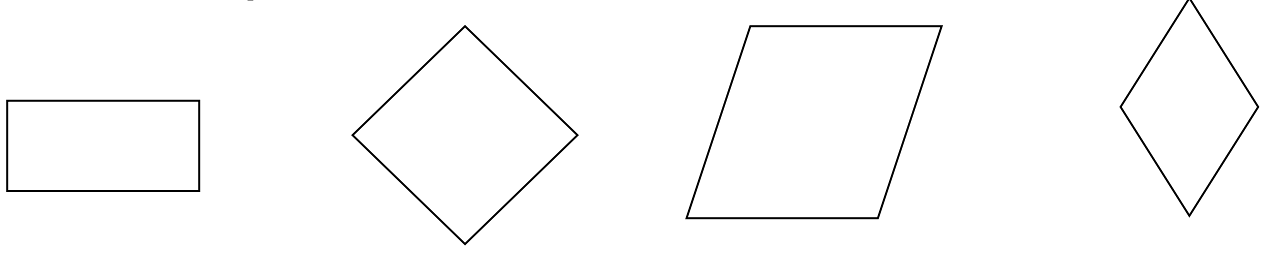
B. T only

C. S and T only

D. S and U only

E. All are rectangles.

4. Which of these are squares?



A. None of these are squares.

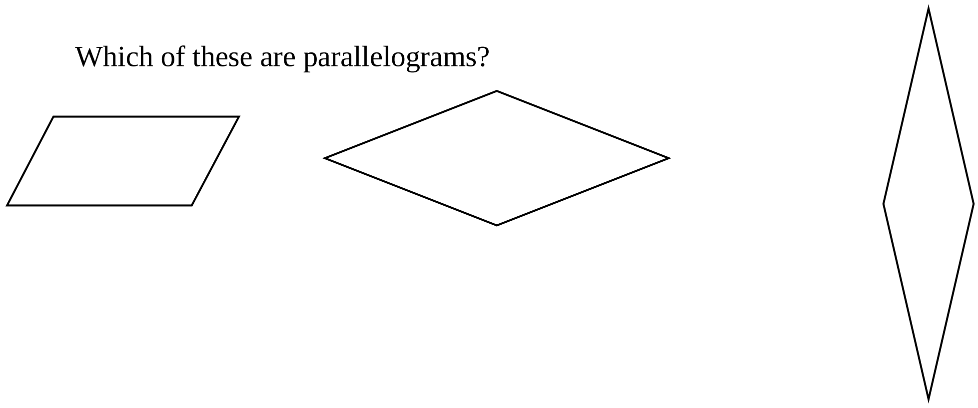
B. G only

C. F and G only

D. G and I only

E. All are squares.

5. Which of these are parallelograms?



A. J only

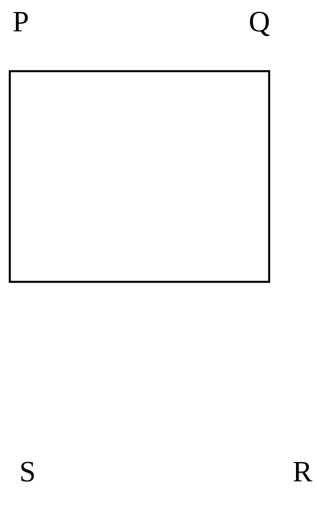
B. L only

C. J and M only

D. None of these are parallelograms.

E. All are parallelograms.

6. PQRS is a square.



Which relationship is true in all squares?

A. PR and RS have the same length.

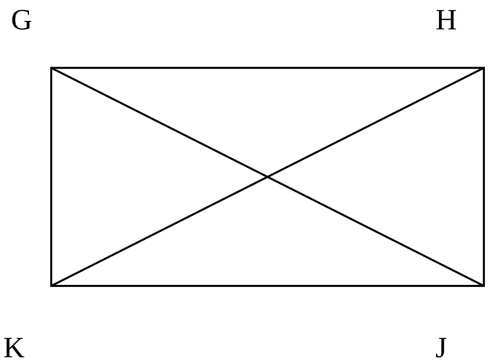
B. QS and PR are perpendicular.

C. PS and QR are perpendicular.

D. PS and QS have the same length.

E. Angle Q is larger than angle R.

7. In the rectangle GHJK, GJ and HK are the diagonals.



Which of (A) - (D) is not true in every rectangle?

A. There are four right angles.

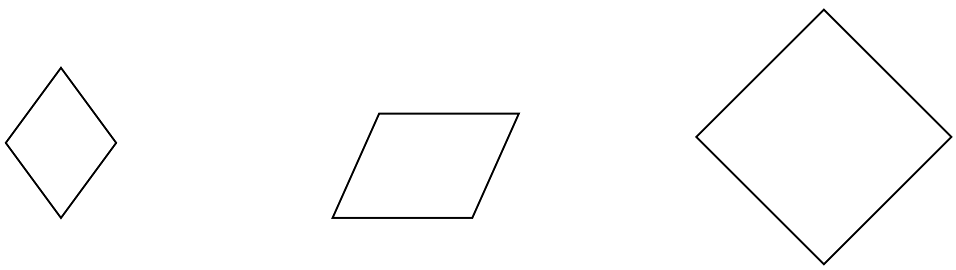
B. There are four sides.

C. The diagonals have the same length.

D. The opposite sides have the same length.

E. All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.
Here are three examples.



Which of (A)-(D) is not true in every rhombus?

A. The two diagonals have the same length.

B. Each diagonal bisects two angles of the rhombus.

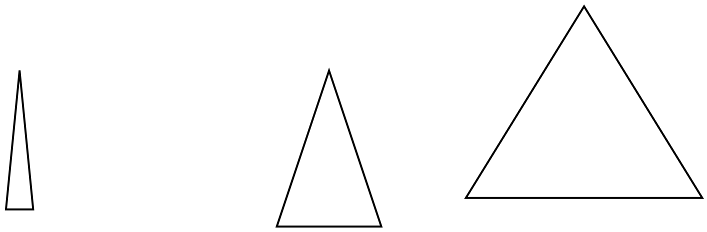
C. The two diagonals are perpendicular.

D. The opposite angles have the same measure.

E. All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length.

Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

A. The three sides must have the same length.

B. One side must have twice the length of another side.

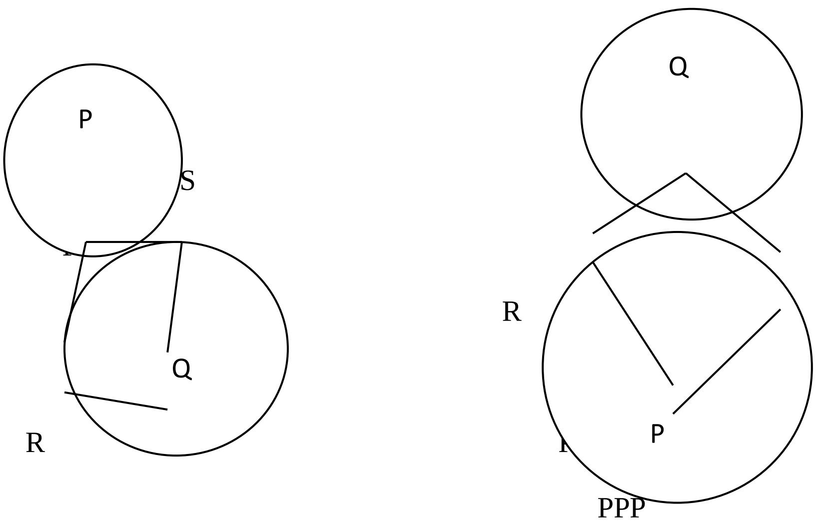
C. There must be at least two angles with the same measure.

D. The three angles must have the same measure.

E. None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centres P and Q intersect at R and S to form a 4-sided figure PRQS.

Here are two examples.



Which of (A)-(D) is not always true?

A. PRQS will have two pairs of sides of equal length.

B. PRQS will have at least two angles of equal measure.

C. The lines PQ and RS will be perpendicular.

D. Angles P and Q will have the same measure.

E. All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

A. If 1 is true, then 2 is true.

B. If 1 is false, then 2 is true.

C. 1 and 2 cannot both be true.

D. 1 and 2 cannot both be false.

E. None of (A)-(D) is correct.

12. Here are two statements.

Statement S: $\triangle$ABC has three sides of the same length

Statement T: In $\triangle$ABC, ^B and ^C have the same measure.

Which is correct?

A. Statement S and T cannot both be true.

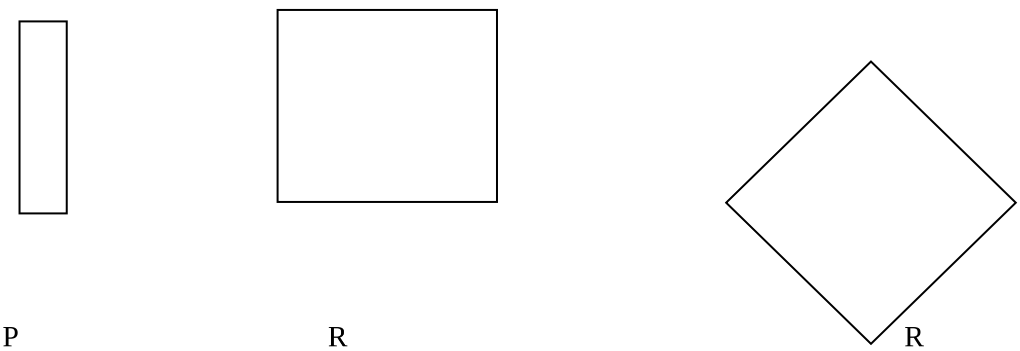
B. If S is true, then T is true.

C. If T is true, then S is true.

D. If S is false, then T is false.

E. None of (A)-(D) is correct.

13. Which of these can be called rectangles?



P R R

A. All can.

B. Q only

C. R only

D. P and Q only

E. Q and R only

14. Which is true?

A. All properties of rectangles are properties of all squares.

B. All properties of squares are properties of rectangles.

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms.

E. None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

A. Opposite sides equal

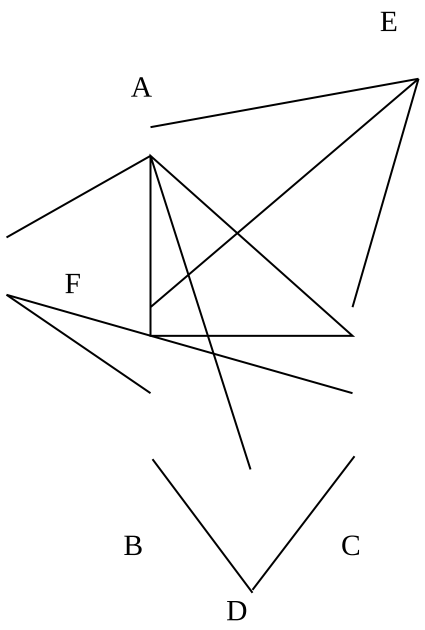
B. Diagonals equal

C. Opposite sides parallel

D. Opposite angles equal

E. None of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

A. Only in this triangle drawn can we be sure that AD, BE and CF have a point in common.

B. In some but not all right triangles, AD, BE and CF have a point in common.

C. In any right triangle, AD, BE and CF have a point in common.

D. In any triangle, AD, BE and CF have a point in common.

E. In any equilateral triangle, AD, BE and CF have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

A. D implies S which implies R.
B. D implies R which implies S.
C. S implies R which implies D.
D. R implies D which implies S.
E. R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.
II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.
B. To prove II is true, it is enough to prove that I is true.
C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.
D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
E. None of (A)-(D) is correct.

19. In geometry:

A. Every term can be defined and every true statement can be proved true.

B. Every term can be defined but it is necessary to assume that certain statements are true.

C. Some terms must be left undefined but every true statement can be proved true.

D. Some terms must be left undefined and it is necessary to have some statements which are assumed true.

E. None of (A)-(D) is correct.

20. Examine these three sentences.

1. Two lines perpendicular to the same line are parallel.
2. A line that is perpendicular to one of two parallel lines is perpendicular to the other
3. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular.

Which of the above sentences could be the reason that line m is parallel to line n?

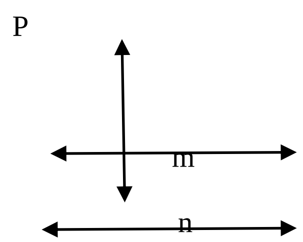
A. (1) only

B. (2) only

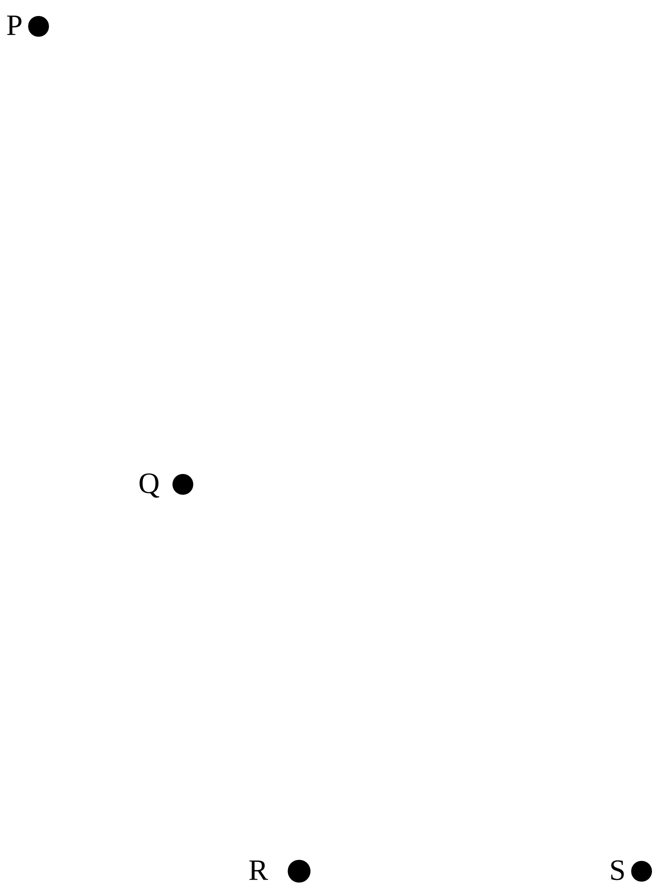
C. (3) only

D. Either (1) or (2)

E. Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.



Here are how the words “intersect” and “parallel” are used in F-geometry.

The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

A. {P,R} and {Q,S} intersect.

B. {P,R} and {Q,S} are parallel.

C. {Q,R} and {R,S} are parallel.

D. {P,S} and {Q,R} intersect.

E. None of (A)-(D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

B. In general, it is impossible to trisect angles using only a compass and a marked ruler.

C. In general, it is impossible to trisect angles using any drawing instruments.

D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180□.

Which is correct?

A. J made a mistake in measuring the angles of the triangle.

B. J made a mistake in logical reasoning.

C. J has a wrong idea of what is meant by “true”.

D. J started with different assumptions than those in the usual geometry.

E. None of (A)-(D) is correct.

24. The geometry books define the word rectangle in different ways.

Which is true?

A. One of the books has an error.

B. One of the definitions is wrong. There cannot be two different definitions for rectangle.

C. The rectangles in one of the books must have different properties from those in the other book.

D. The rectangles in one of the books must have the same properties as those in the other book.

E. The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I: If p, then q.

II: If s, then not q.

Which statement follows from statements I and II?

A. If p, then s.

B. If not p, then not q.

C. If p or q, then s.

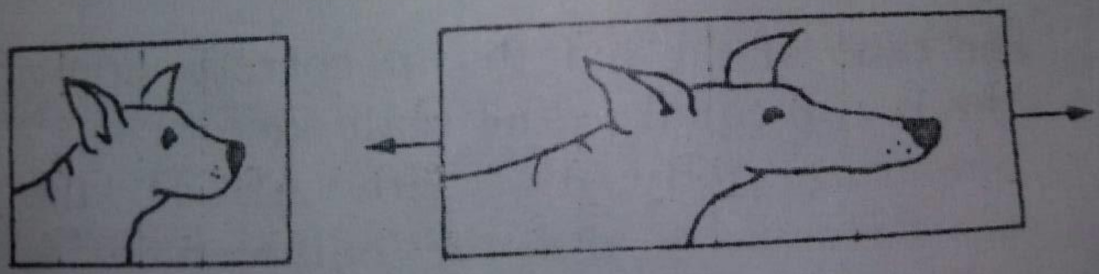
D. If s, then not p.

E. If not s, then p

APPENDIX F: TEXTBOOK Illustration of stretch

Stretch

In Fig. 19.14(a) an animal has been drawn on a rubber sheet, such as a piece of a car inner tube.



(a) (b)

APPENDIX G: TEXTBOOK ILLUSTRATION OF SHEAR

Shear

Fig. 19.11 shows a way of transforming the shape of a book by pushing its top surface.

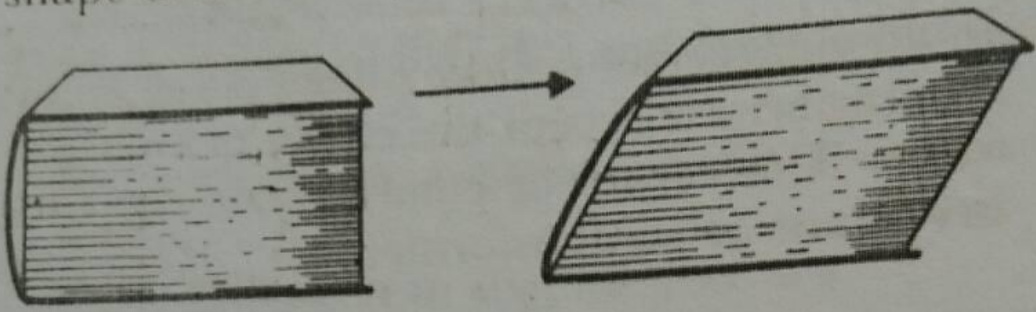
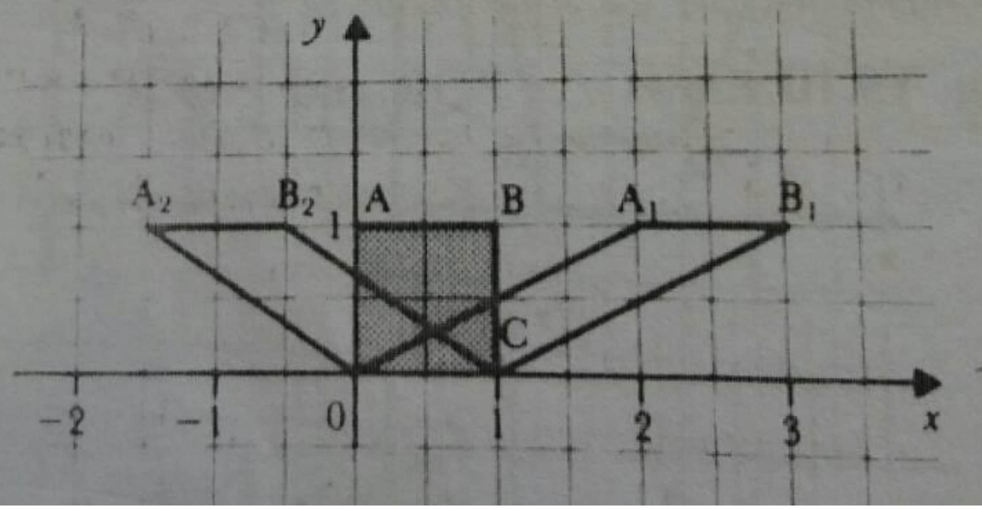
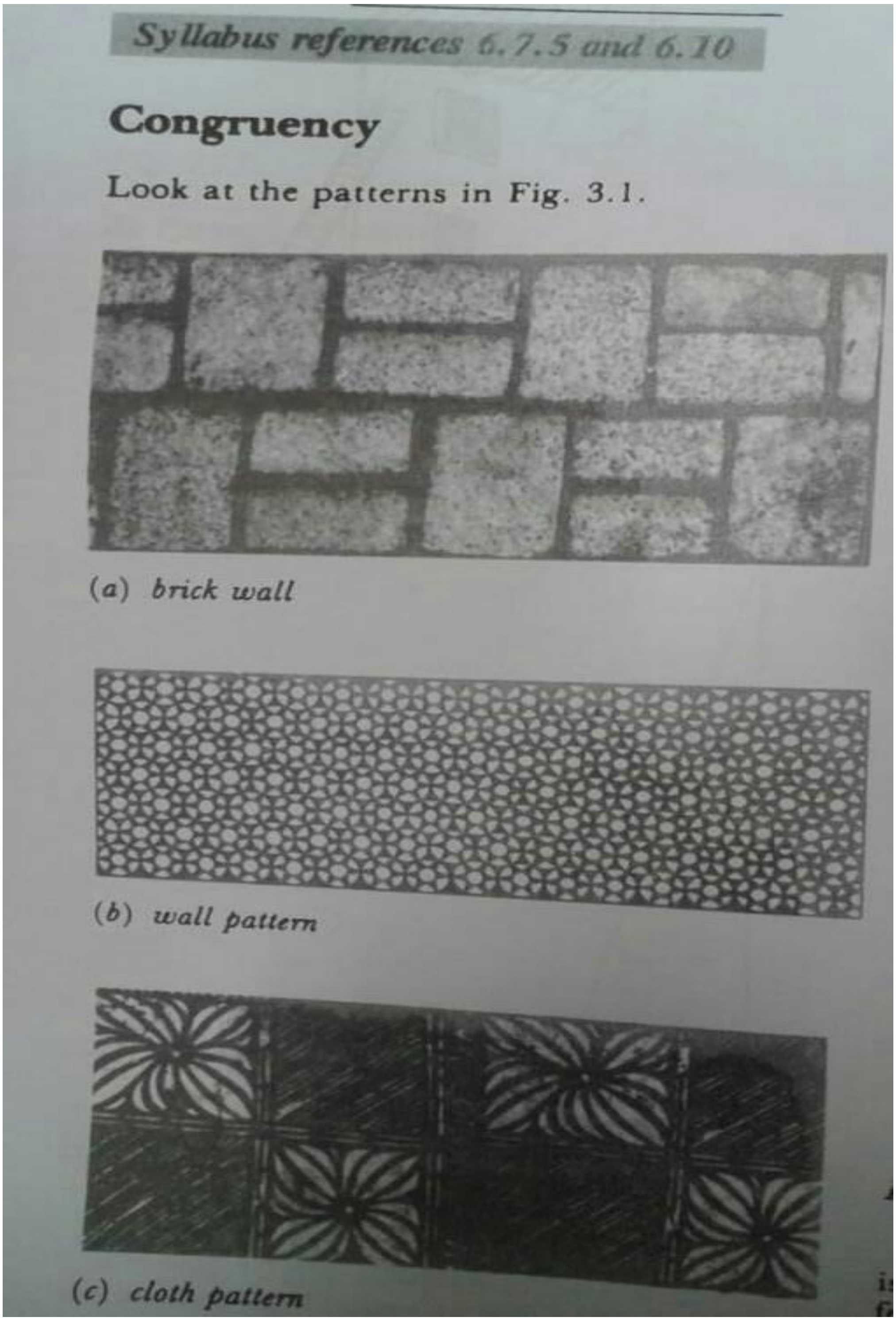


Fig. 19.11

This kind of transformation is called a **shear**. Fig. 19.12 shows two shears of a unit square OABC.



APPENDIX H: TEXTBOOK ILLUSTRATION OF PATTERNS



Syllabus references 6.7.5 and 6.10

Congruency

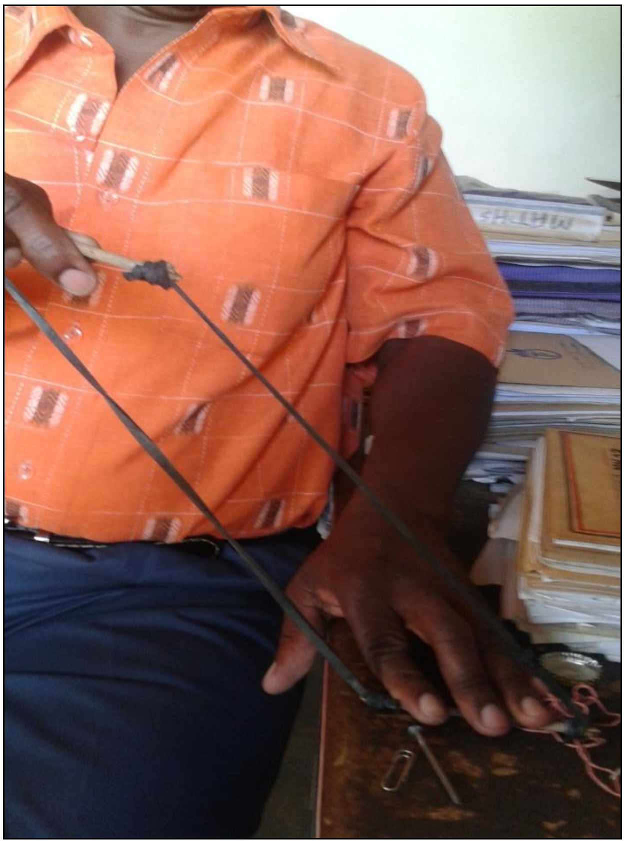
Look at the patterns in Fig. 3.1.

(*a*) *brick wall*

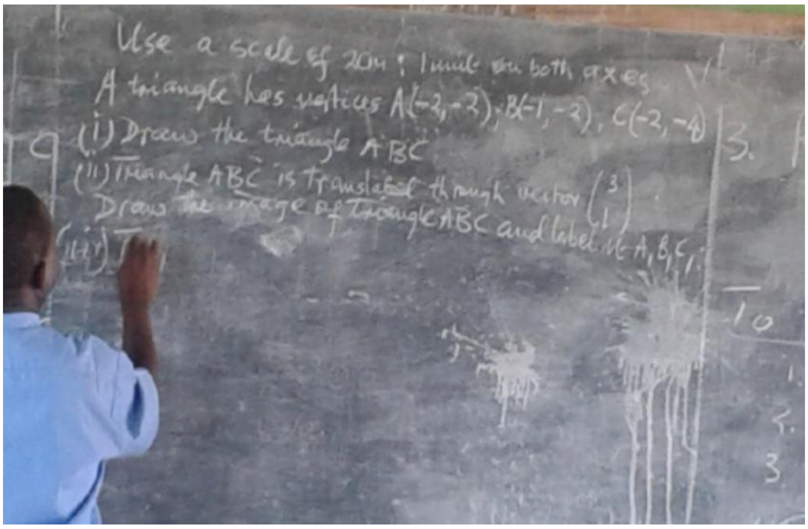
(*b*) *wall pattern*

(*c*) *cloth pattern*

APPENDIX I: TEACHER ILLUSTRATION OF A CATAPULT STRETCH



APPENDIX J: CONDITION OF CLASSROOM BOARDS



APPENDIX K: NATIONAL EXAMINATION QUESTION 1

[3]

9 **Answer the whole of this question on a sheet of graph paper.**

Quadrilateral ABCD has vertices at A(1; 0), B(2; 0) C(2; 2) and D(1; 2)

Using a scale of 2 cm to represent 1 unit on each axes, draw the x and y axes for $-4 \le x \le 6$ and $-5 \le y \le 5$.

(a) **(i)** Draw and label ABCD.

(ii) State the special name given to quadrilateral ABCD.

[2]

(b) Quadrilateral ABC_1D_1 has coordinates at A(1;0), B(2;0), $C_1(6;2)$ and $D_1(5;2)$.

(i) Draw and label quadrilateral ABC_1D_1.

(ii) Describe **fully the single** transformation that maps ABCD onto ABC_1D_1.

[4]

4028/2 J2015

[Turn over

APPENDIX L: NATIONAL EXAMINATION QUESTION 2

10 **Answer the whole of this question on page 24.**

The triangle LMN has vertices at L (3; 1), M (2; 2) and N (0; 1).

(a) Draw and label triangle LMN. [1]

(b) Triangle LMN is mapped onto triangle $L_1M_1N_1$ by a reflection in the line $y = 1$.

Draw and label triangle $L_1M_1N_1$. [3]

(c) Triangle LMN is mapped onto triangle $L_2M_2N_2$ by a rotation through $180°$ about the point $(-1; 0)$.

Draw and label triangle $L_2M_2N_2$. [3]

(d) Triangle $L_3M_3N_3$ is the image of triangle LMN under a transformation P, represented by the matrix $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Draw and label triangle $L_3M_3N_3$.

(iii) Describe fully the single transformation P, which maps triangle LMN onto triangle $L_3M_3N_3$. [5]

APPENDIX M: NATIONAL EXAMINATION QUESTION 3

9 **Answer the whole of this question on a sheet of graph paper.**

(i) Using a scale of 2 cm to represent 2 units on each axis, draw axes for values of x and y in the ranges $-12 \leqslant x \leqslant 6$ and $-10 \leqslant y \leqslant 8$. Draw triangle ABC whose vertices are at the points $A(1, 1)$, $B(4, 1)$ and $C(2, 3)$. Label the triangle T. [1]

(ii) (*a*) **M** is the matrix $\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$.

Calculate the coordinates of the points onto which the points $A(1, 1)$, $B(4, 1)$ and $C(2, 3)$ are mapped by the matrix **M**. [3]

(*b*) Draw the image of triangle ABC under the transformation represented by **M** and label the triangle T_1. [1]

(iii) Triangle T_1 is mapped onto triangle T_2 by an anticlockwise rotation through 90° about the point $(-2, 0)$. Draw this triangle and label it T_2. [2]

(iv) (*a*) T_3 is a triangle whose vertices are at the points $(-3, -3)$, $(-12, -3)$ and $(-6, -9)$.

Draw the triangle and label it T_3. [1]

(*b*) Describe fully the single transformation which maps T onto T_3. [3]

(*c*) Write down the matrix which represents this transformation. [1]

APPENDIX N: ORDINARY LEVEL MATHEMATICS SYLLABUS (2012 – 2017)

ZIMSEC

For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL (ZIMSEC)

ZIMBABWE GENERAL CERTIFICATE OF EDUCATION (ZGCE)

For Examination in November 2012 – 2017

O Level Syllabus

MATHEMATICS (4008/4028)

Subjects 4008/4028. MATHEMATICS

4008.This version is for candidates not using calculators

4028.This version is for candidates using calculators in Paper 2

Subjects 4008/4028 MATHEMATICS

1.0 **PREAMBLE**

This syllabus caters for those who intend to study mathematics and/or related subjects up to and beyond `O' level and for the mathematical requirements of a wide range of professions. The syllabus assumes the mastery of the Z.J.C. mathematics syllabus.

The syllabus is in two versions 4008 and 4028. Syllabus 4008 is the non-calculator version and syllabus 4028 is the calculator version.

2.0 **THE SYLLABUS AIMS**

To enable students to:

2.1 understand, interpret and communicate mathematical information in everyday life;

2.2 acquire mathematical skills for use in their everyday lives and in national development;

2.3 appreciate the crucial role of mathematics in national development and in the country's socialist ideology;

2.4 acquire a firm mathematical foundation for further studies and/or vocational training;

2.5 develop the ability to apply mathematics in other subjects;

2.6 develop the ability to reason and present arguments logically;

2.7 develop the ability to apply mathematical knowledge and techniques in a wide variety of situations, both familiar and unfamiliar;

2.8 find joy and self-fulfilment in mathematics and related activities, and appreciate the beauty of mathematics;

2.9 develop good habits such as thoroughness and neatness, and positive attitudes such as an enquiring spirit, open-mindedness, self-reliance, resourcefulness, critical and creative thinking, cooperation and persistence;

2.10 appreciate the process of discovery and the historical development of mathematics as an integral part of human culture.

3.0 **ASSESSMENT OBJECTIVES**

Students will be assessed on their ability to:

3.1 recall, recognise and use mathematical symbols, terms and definitions;

3.2 carry out calculations and algebraic and geometric manipulations accurately; check the correctness of solutions;

3.3 estimate, approximate and use appropriate degrees of accuracy;

3.4 read, interpret and use tables, charts and graphs accurately;

3.5 draw graphs, diagrams and constructions to given appropriate specifications and measure to a suitable degree of accuracy;

3.6 translate mathematical information from one form into another (e.g. from a verbal form to a symbolic or diagrammatic form);

3.7 predict, draw inferences, make generalisations and establish mathematical relationships from provided data;

3.8 give steps and/or information necessary to solve a problem;

3.9 choose and use appropriate formulae, algorithms and strategies to solve a wide variety of problems (e.g. agriculture, technology, science and purely mathematical contexts);

3.10 apply and interpret mathematics in daily life situations.

4.0 NOTES

4.1 MATHEMATICAL TABLES AND ELECTRONIC CALCULATORS

Mathematical tables and electronic calculators are prohibited in 4008/1 and 4028/1. However, the efficient use of mathematical tables is expected in 4008/2 and the efficient use of electronic calculators is expected in 4028/2. In 4028/2 mathematical tables may be used to supplement the use of the calculator.

Mathematical tables will be provided in the examination. A scientific calculator with trigonometric functions is strongly recommended.

4.2 MATHEMATICAL INSTRUMENTS

Candidates are expected to bring their own mathematical instruments to the examination. Flexi curves are not allowed.

UNITS

4.3.1. SI units will be used in questions involving mass and measures; the use of the centimetre will continue.

4.3.2. The time of day may be quoted by using either the 12-hour or the 24-hour clock,

e.g. quarter past three in the morning may be stated as either 3.15 a.m. or 03 15;
quarter past three in the afternoon may be stated as either 3.15 p.m. or 15 15.

4.3.3. Candidates will be expected to be familiar with the solidus notation for the expression of compound units e.g. 5 cm/s for 5 centimetres per second, 13/gcm^3 for 13 grams per cubic centimetre.

5.0 METHODOLOGY

In this syllabus, teaching approaches in which mathematics is seen as a process and which build an interest and confidence in tackling problems both in familiar and unfamiliar contexts are recommended.

It is suggested that:

5.1 concepts be developed starting from concrete situations (in the immediate environment) and moving to abstract ones;

5.2 principles be based on sound understanding of related concepts; and whenever possible, be learnt through activity based and/or guided discovery;

5.3 skills be learnt only after relevant concepts and principles have been mastered;

5.4 the human element in the process of mathematical discoveries be emphasised;

5.5 an effort be made to reinforce relevant skills taught in other subjects;

5.6 students be taught to check and criticise their own and one another's work;

5.7 group work be organised regularly;

5.8 a deliberate attempt be made to teach problem-solving as a skill, with students being exposed to non- routine problem solving situations;

5.9 students be taught to identify problems in their environment, put them in a mathematical form and solve them e.g. through project work.

6.0 CONTENT/TEACHING OBJECTIVES

TOPIC	OBJECTIVES All students should be able to:
6.1 NUMBER	
6.1.1.1 Number concepts and operations.	
number types (including: directed numbers, fractions and percentages)	- demonstrate familiarity with the notion of odd, even, prime, natural, integer, rational and irrational numbers, including surds,
	- use of the number line;
	- recognise equivalence between common/decimal fractions and percentages, convert from one to the other and use these three forms in appropriate contexts;
	- use directed numbers in practical situations (e.g. temperature, financial loss/gain);
factors, multiples, HCF, LCM	- find and use common factors/multiples, HCFs and LCMs of given natural numbers;
the four operations (+, −, ×,) and rules of precedence	- apply the four operations and rules of precedence on natural numbers, common/ decimal fractions, percentages, integers, surds and directed numbers (including use of brackets);
6.1.2. Approximations and estimates	- use the approximation sign (≏, ≃ or) appropriately
	- make estimates of numbers and quantities, and of results in calculations;
	- give approximations to a specified number of significant figures and decimal places;
	- round off to a given accuracy;
	- round off to a reasonable accuracy in the context of a given problem;

6.1.3. Limits of accuracy	-	obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy(e.g. calculation of area of a rectangle).
6.1.4. Standard form	-	express in, and use the standard form $A \times 10^n$ where is an integer (including zero) and 1 A 10;
6.1.5. Number bases, - bases 2,3,4,5,6,7,8, 9 and 10	-	do the following: - state and use place value; - add and subtract; - convert from one base to another;
6.1.6. Ratio, proportion and rates	-	use ratio, direct and inverse proportion (including use of unitary method) and rates (e.g. speed, cost per unit area);
6.1.7. Scales and simple map problems	-	find scales from given information;
	-	use given scales to calculate distances and areas;

6.2 **SETS**

6.2.1. Language and notation

definition of a set	-	define sets by listing and describing e.g.,V={a,e,i,o,u} or V={vowels};
	-	define sets using the set builder notation e.g.A={x:x is a natural number}, B={(x,y): y=mx+c} , C={x:a < x < b}
notation	-	correctly use symbols as follows: - is an element , - is not an element of, , - number of elements in set A,n(A), - complement of set A, A´, - the universal set, , - the null set,{} or , - A is a proper subset of B, A B, - A is contained in B, A B,

- B contains A, $B \supset A$

- A is a subset of A, A A,

- Φ is a subset of A, Φ A,

- union of A and B, A B

- intersection of A and B, A B;

- use the idea of complement of a union or an intersection;

- use the following symbols , , ⊅, and ⊄,

- use sets and Venn diagrams to solve problems involving no more than three sets and the universal set;

6.3 CONSUMER ARITHMETIC

6.3.1.

- interpret data (including data on real life documents like water/electricity bills, bank statements, mortgages and information in the media);

- solve problems on budgets (e.g. household, cooperative and state budgets), rates (including foreign exchange and household rates), insurance premiums, wages, simple interest, discount, commission, depreciation, sales/income tax, hire purchase and bank accounts (savings and current accounts);

- read, interpret and use data presented in charts, tables, maps and graphs (e.g. ready reckoners, road maps, charts and graphs in newspapers);

6.4 MEASURES AND MENSURATION

6.4.1. Measures

time

- read time on both the 12 and 24 hour clock(e.g. 7.35 p.m or 19 35).

SI units

- use Sl units of mass, temperature in degrees celsius length/ distance, area, volume/capacity and density in practical situations,

- express quantities in terms of larger or smaller units;

6.4.2. Mensuration

perimeter
density
area
volume/capacity

- carry out calculations involving:
  - the perimeter and area of a rectangle, triangle, parallelogram and trapezium;
  - density
  - the circumference of a circle and the length of a circular arc;
  - the area of a (circle including sector and segment); rectangle, triangle, parallelogram and trapezium.
  - the surface area and volume of a cylinder, cuboid, prism of uniform cross-section, pyramid, cone and sphere;

(formulae for surface areas and volumes of pyramid, cone and sphere will be provided);

(units of area to include the hectare);

6.5 GRAPHS AND VARIATION

6.5.1. Coordinates	-	use Cartesian coordinates in two dimensions to interpret and infer from graphs and to draw graphs from given data;
6.5.2. Kinematics		
travel graphs speed/velocity distance/displacement acceleration	-	draw and interpret displacement-time and velocity-time graphs and solve problems involving acceleration, velocity and distance.
6.5.3. Variation		
direct inverse joint	-	express direct, inverse, joint and partial variation in algebraic terms and hence solve problems in variation;
partial	-	draw and interpret graphs showing direct, inverse and partial variation;
6.5.4. Functional graphs	-	construct tables of values, draw and interpret given functions which include graphs of the form $ax + by + c = 0$, $y = mx + c$, $y = ax^2 + bx + c$ and $y = ax^n$ where $n = -2, -1, 0, 1, 2$, and 3 and simple sums of these;
	-	use the f(x) notation;
solution of equations	-	solve linear simultaneous equations graphically;
	-	solve equations using points of intersection of graphs (e.g. drawing $y = 1/x$ and $y = 2x + 3$ to solve $2x^2 + 3x - 1 = 0$);
gradients and rates of change	-	estimate gradients of curves by drawing tangents and hence estimate rates of change (e.g. speed, acceleration);
	-	find turning points (maxima and minima) of graphs (calculus methods not required);
	-	calculate the gradient of a straight line from the coordinates of points on it, interpret and obtain the equation of a straight line in the form $y = mx + c$;
	-	identify parallel straight lines using gradients;
area under a curve	-	estimate area under a curve by counting squares and by dividing into trapezia (trapezium rule not to be used);

6.6 ALGEBRAIC CONCEPTS AND TECHNIQUES

6.6.1. Symbolic expression	-	express basic arithmetic processes in letter symbols;
formulae	-	substitute numbers for words and letters in algebraic expressions (including formulae);

change of subject	-	change the subject of a formula and substitute in formulae including those from other subjects (e.g. science);

6.6.2. Algebraic manipulation

operations	-	use the four operations and rules of precedence to manipulate: - directed numbers, - monomials (including use of like and unlike terms), - simple algebraic fractions;
factors, multiples, HCF, LCM	-	find and use common factors, common multiples, HCF and LCM;
expansion	-	expand expressions of the forms $a(x + y)$, $(ax + by)(cx + d)$, $(ax + by)(cx + dy)$; etc where a, b, c and d are rational numbers;
factors		factorise expressions of the form $ax + bx$, $ax + bx + ay + by$, $ka^2 - kb^2$, $ax^2 + bx + c$; where a, b, c and k are intergers

6.6.3. Indices

laws of indices	-	use the following laws of indices (where m and n are rational other than zero): - $a^m \times a^n = a^{m+n}$ - $a^m \div a^n = a^{m-n}$; - $a^o = 1$; - $(a^m)^n = a^{mn}$; - $a^{1/n} = \sqrt[n]{a}$; - $a^{-n} = 1/a^n$; - $a^{m/n} = \sqrt[n]{a}^{\,m} = (\sqrt[n]{a})^m$;
squares/square roots cubes/cube roots	-	calculate squares and use factors to find roots and cube roots;

6.6.4. Equations	-	solve the following:
linear equations		- simple linear equations (including those involving algebraic fractions);
simultaneous equations		- simple linear simultaneous equations (by graphs, by substitution and by elimination);
quadratic equations		- quadratic equations of the form $ax^2 + bx + c = 0$ (by factorisation by graphs and by formula);
6.6.5. Logarithms	-	use the following basic ideas of the theory of logarithms: $\log_b MN = \mathrm{Log}_b M + \log N$, $\log_b \dfrac{M}{N} = \log_b M - \log_b N$ and $\log_b M^p = p\log_b M$ where b and p are rational numbers and M and N are greater than zero.

	-	use common logarithms in calculations (including finding powers and roots);
6.6.6. Inequalities		
signs	-	use the following in appropriate situations: $=, >, <, \geq, \leq$ $, \ngtr, \nless$;
linear inequalities	-	solve linear inequalities e.g. of the form $ax + b \quad c$, $ax + b < cx < dx$, $ax + b < cx + d < ex + f$, $c < ax + b < d$ etc where a, b, c, d, e and f are rational;
linear programming	-	represent inequalities and their solutions on a number line;
	-	use simple linear programming methods to solve problems (unwanted regions to be shaded, with inequality boundaries shown by broken lines);

6.7 GEOMETRIC CONCEPTS AND TECHNIQUES

6.7.1. Points, lines and angles	-	identify interpret and apply the following concepts: point, line, parallel, perpendicular;
types of angles	-	right angle, acute, obtuse, reflex, complementary, supplementary, vertically opposite angles, angles at a point, angles on a straight line;
parallel lines	-	transversal, allied or co-interior angles, corresponding angles, interior opposite or alternative angles;
angles of elevation and depression	-	angles of elevation and depression;
6.7.2. Bearings	-	interpret and use three-figure bearings measured clockwise from north, (i.e. from 000° to 360°) and compass bearings (e.g. N 47° E or 47°E of N);
6.7.3. Polygons		
triangles quadrilaterals	-	use properties of: triangles (including isosceles and equilateral), quadrilaterals (including kites, parallelograms, rectangles, rhombi, squares, trapezia);
n-sided polygons	-	regular and irregular n-sided polygons,
	-	state the special names of n-sided polygons (up to n=10),
parallel lines and area	-	use the area property of triangles and parallelograms between the same parallels;
6.7.4. Circles	-	use the properties: - radius - diameter - chord - tangent - cyclic quadrilateral

	-	use the following circle theorems: - angle subtended at the centre and on the circumference - angle in a semi-circle - angles in the same segment - angle in the alternate segment;
6.7.5. Similarity and Congruency	-	identify similar and congruent figures and solve problems on similar and congruent triangles;
		solve problems on:- - areas of similar plane figures, - volumes and masses of similar solids;
6.7.6. Constructions	-	construct the following using ruler and compasses only:
		- angle bisector, perpendicular bisector, angles of 30°, 45°, 60°; and 90°; and single combination of these;
	-	construct a perpendicular: - from a given point to a given line - through a given point on a given line;
triangles parallelograms regular polygons	-	construct triangles, parallelograms and simple n-sided polygons (protractors may be used where necessary);
scale drawings	-	produce scale drawings using an appropriate/given scale;
6.7.7. Loci	-	construct and use the locus (in two-dimensions) of a point - equidistant from - two given points, - two intersecting lines,
		- at a given distance from, - a fixed point, - a given straight line;
6.7.8. Symmetry	-	identify line symmetry in two dimensions;
line symmetry	-	balance properties of isosceles triangles, equilateral triangles, regular polygons, parallelograms and circles directly related to their symmetries;
identify symmetry	-	identify rotational symmetry (including order of rotational symmetry) in two dimensions;

6.8 TRIGONOMETRY

6.8.1. Pythagoras theorem and trigonometrical ratios	-	apply Pythagoras theorem, sine, cosine and tangent for acute angles to solve simple problems involving right-angled triangles in two dimensions;
	-	use and interpret sine, cosine and tangent of obtuse angles, use the sine and cosine rules for the solution of triangles (angles in either degrees/minutes or degrees to 1 decimal place);

three dimensional problems	-	solve three-dimensional problems involving the angle between a line and a plane;
6.8.2. Area of a triangle	-	use the formula Area = ½absin C for the area of a triangle;

6.9 VECTORS AND MATRICES

6.9.1. Vectors in two dimensions

translation and notation	-	represent a translation by a column vector and by a directed line segment and use the notation $\overrightarrow{AB}$ or AB or $\underline{a}$ or a or $\vec{a}$ or **a**;
operations	-	add and subtract vectors and multiply by a scalar;
	-	calculate the magnitude of a vector and use the notation $\left\|\overrightarrow{AB}\right\|$ or $\|a\|$; etc
position vectors equal vectors parallel vectors	-	identify and use the concepts of - position vectors, - equal vectors, - parallel vectors,

6.9.2. Matrices

dimension/order	-	use and interpret a matrix as a store of information and show familiarity with the idea of dimension/order of a matrix;
operations	-	add and subtract matrices (where appropriate) and multiply by a scalar;
	-	multiply matrices (of order 2 x 2 or less) where appropriate;
identity matrix	-	use the property of identity and zero matrix for 2 x 2 matrices;
determinant	-	find the determinant of a 2 x 2 matrix and distinguish between singular and non-singular matrices and use the notation determinant A or Det A or $\|A\|$
inverse matrix	-	find and use the inverse of a 2 x 2 non-singular matrix; (e.g solving simultaneous linear equations) and use the rotation A^{-1} ;

6.10 TRANSFORMATIONS

6.10.1.	-	carry out the following transformations in x-y plane:
6.10.2.		
translation	-	translate (T) simple plane figures;
reflection	-	reflect(M) simple plane figures in the axes and in any line;
rotation	-	rotate (R) about any point clockwise or anti-clockwise through 90° and 180°,
enlargement	-	enlarge(s) about any point using a rational scale factor;

stretch	-	stretch (S); both one way and two way stretch using the axes as the invariant stretch lines and rational stretch factor,
shear	-	shear (H), using the axes as the invariant lines and rational shear factor.
	-	apply combinations of the above (e.g. if M(a)=b and R(b)=c then RM(a)=c);
	-	describe transformations fully;
6.10.2 Matrices as operators	-	identify interpret and/or use matrices which represent the above transformations,
	-	describe transformations using coordinates and matrices (singular matrices are excluded);

6.11 STATISTICS AND PROBABILITY

6.11.1. Statistics

collection and classification	-	collect, classify and tabulate statistical data;
data representation	-	read, interpret, draw and make simple inferences from bar charts, pie charts, histograms and frequency tables/charts and frequency polygons (see also 6.3.1.);
measures of central tendency	-	calculate the mean, mode, median from given data and distinguish between the purposes for which they are used;
	-	use an assumed mean where appropriate;
	-	read and interpret data presented in classes and determine the modal class;
cumulative frequency	-	draw and use a cumulative frequency curve/orgive;

6.11.2. Probability

terms	-	use the terms: random, certain, impossible event, trial, sample space, equally likely, mutually exclusive, independent events;
experimental probability	-	distinguish between experimental and
theoretical probability	-	theoretical probability;
probability of - single events	-	solve simple problems involving the probability of a single event;
- combined events	-	calculate the probability of and solve simple problems involving combined events e.g. mutually exclusive and independent events (use of tree diagrams and outcome tables is recommended).

7.0 SCHEME OF ASSESSMENT

	PAPER 1	PAPER 2
WEIGHTING	50%	50%
TYPE OF PAPER	Approximately 30 short answer questions	Structured Questions Section A (6 compulsory questions) Section B (3 questions out of 6)
TIME ALLOWED	$2\frac{1}{2}$ hours	$2\frac{1}{2}$ hours

7.0 SCHEME OF ASSESSMENT

	PAPER 1	PAPER 2
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TIME ALLOWED	$2\frac{1}{2}$ hours	$2\frac{1}{2}$ hours

APPENDIX O (a): Textbook task 1

7 For each part of this question, trace the letter K and points P_1 and P_2 onto a sheet of plain paper.

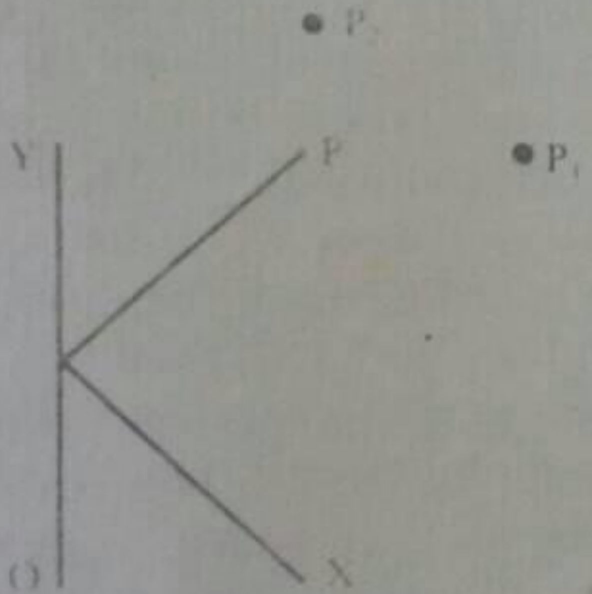


Fig. 19.22

Draw the image of K after

(a) a shear which maps P onto P_1 with OX invariant,

(b) a one-way stretch which maps P onto P_1 with OY invariant,

(c) a shear which maps P onto P_2 with OY invariant,

(d) a one-way stretch which maps P onto P_2 with OX invariant.

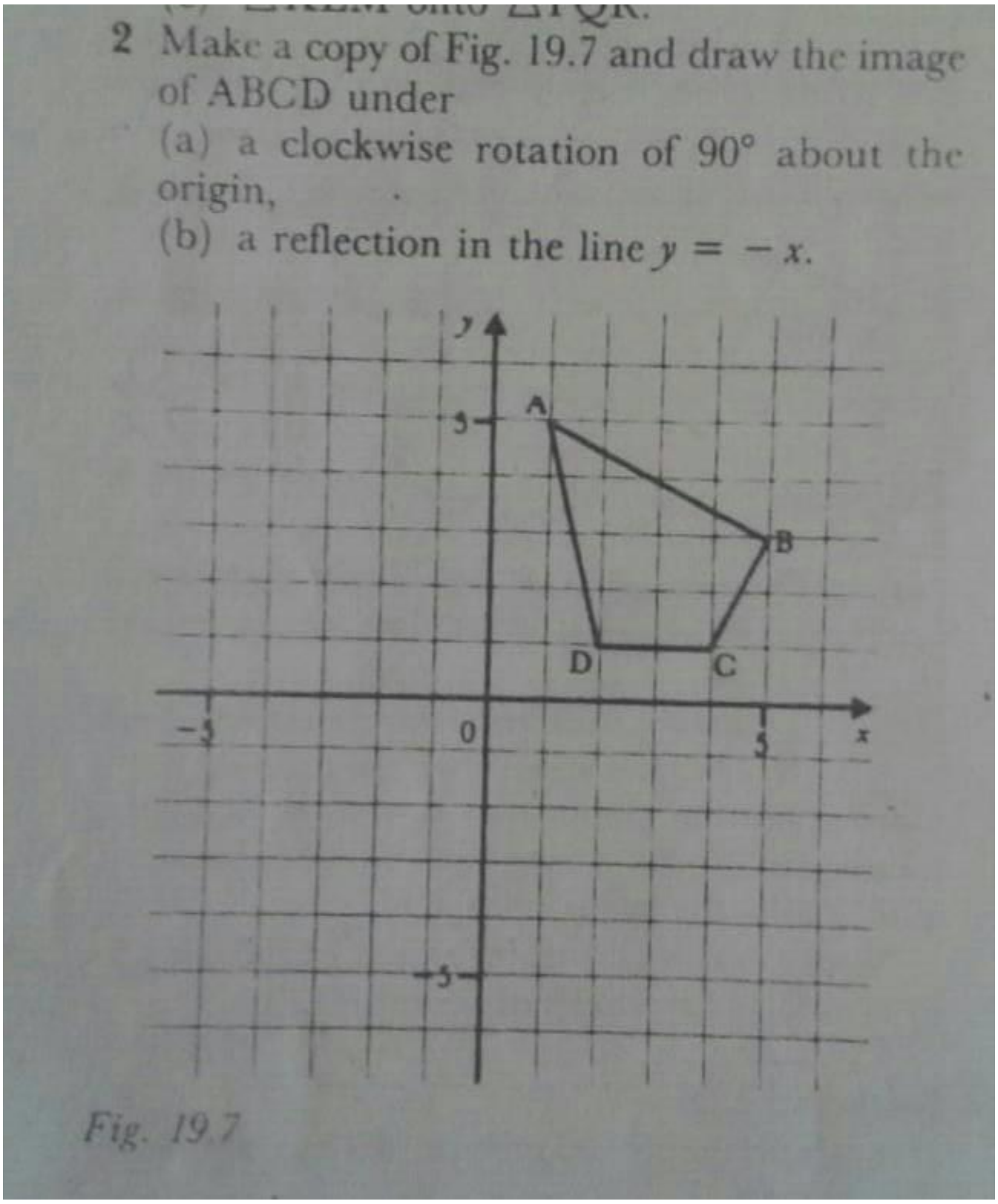
APPENDIX O (b): Textbook task 2

5 M is a reflection in the line $x = 2$ and R is a clockwise rotation of $90°$ about the origin. A is the point $(1; 2)$, B is $(4; 6)$ and C is $(7; 1)$.

(a) Find the coordinates of (i) M(A), (ii) R(B).

(b) Find the coordinates of the point D if M(D) = C.

APPENDIX O (c): Textbook task 3



2 Make a copy of Fig. 19.7 and draw the image of ABCD under

(a) a clockwise rotation of 90° about the origin,

(b) a reflection in the line $y = -x$.

Fig. 19.7

APPENDIX O (d): Textbook task 4

3. Each part of Fig. 19.20 contains a unit square and its image after a one-way stretch. Each image has been shaded.

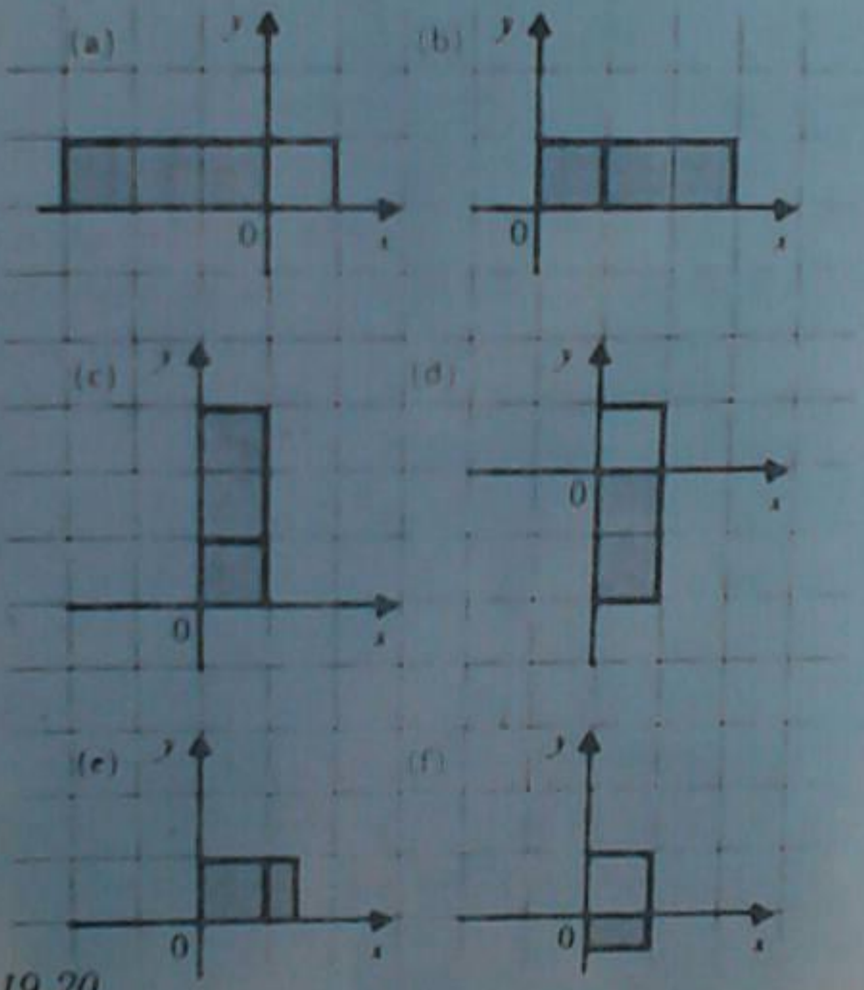


Fig. 19.20

Describe each stretch fully, giving the stretch factor, its direction and the equation of the invariant line.

APPENDIX P: Marked exercise in transformation geometry

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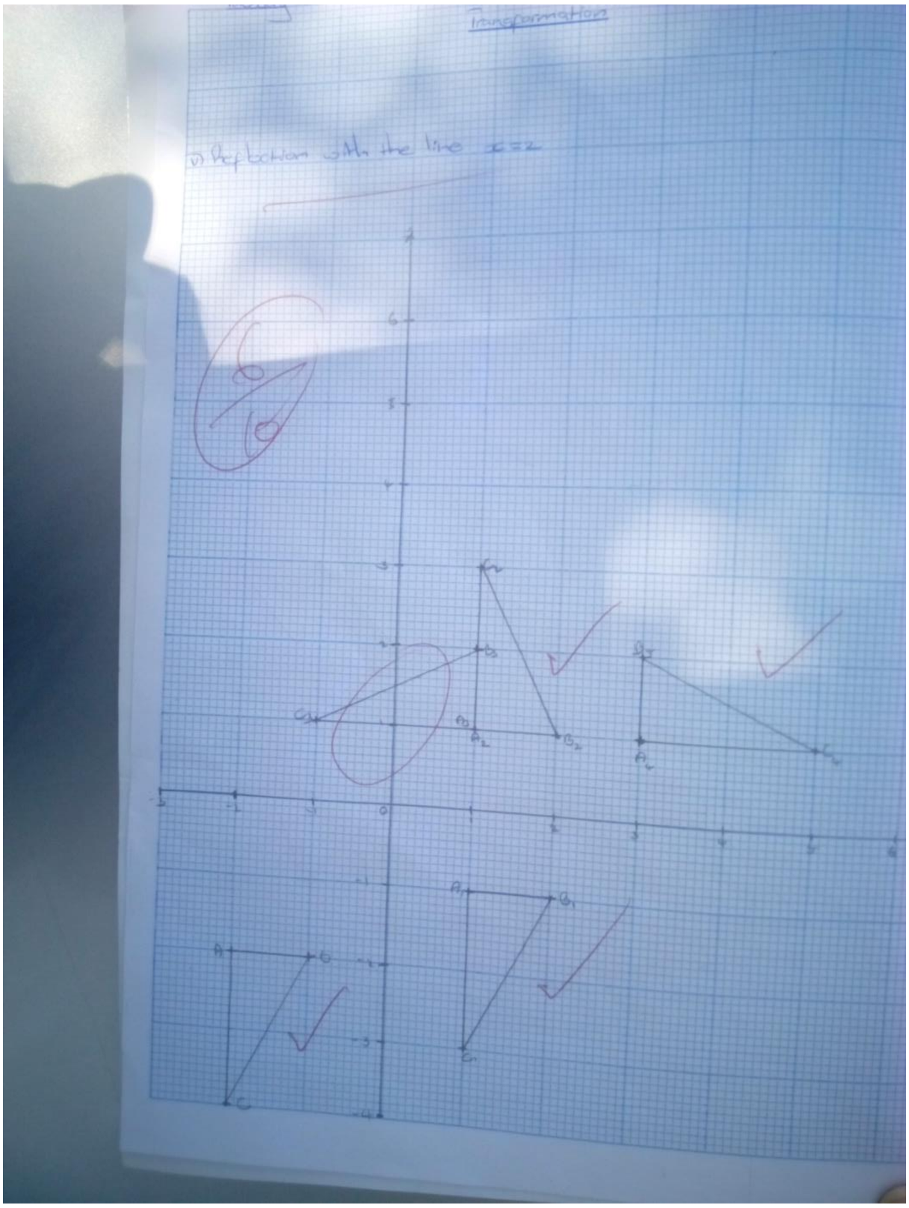
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APPENDIX Q: Ministry Permission letter of Research

All communications should be addressed to "The Provincial Education Director"
Telephone:054- 222460

Fax: 054- 226482

Ministry of Primary and Secondary Education
P.O Box 737
GWERU

12 September 2016

Mr/~~Mrs~~/~~Miss~~: SAMUEL S MASHINGAIDZE
1643 MKOBA 12
GWERU
ZIMBABWE

Dear Sir/Madam

APPLICATION FOR PERMISSION TO CARRY OUT AN EDUCATIONAL RESEARCH IN SELECTED SCHOOLS IN MIDLANDS PROVINCE

Permission to carry out a Research on:-
Realistic mathematics education as a lens to explore teachers use of students' out of school experiences in teaching transformation geometry in Zimbabwe's Rural Secondary schools.

In the Midlands Province has been granted on these conditions.

1. That in carrying out this you do not disturb the learning and teaching programmes in schools.
2. That you avail the Ministry of Primary and Secondary Education with a copy of your research findings.
3. That this permission can be withdrawn at anytime by the Provincial Education Director or by any higher officer.

The Education Director wishes you success in your research work and in your University College studies.

SHIRICHENA .C
For: PROVINCIAL EDUCATION DIRECTOR: MIDLANDS PROVINCE

MIN OF PRY. & SEC. EDUCATION
HUMAN RESOURCES (DISCIPLINE)
MIDLANDS
12 SEP 2016
P.O. BOX 737, GWERU
ZIMBABWE

APPENDIX R: Letter from the Editor

Midlands State University
Senga Road
P. Bag 9055
Gweru
Zimbabwe

5th October 2017

TO WHOM IT MAY CONCERN

EDITING OF MASHINGAIDZE SAMUEL SIMBARASHE'S THESIS TITLED: **REALISTIC MATHEMATICS EDUCATION AS A LENS TO EXPLORE TEACHERS' USE OF STUDENTS' OUT-OF-SCHOOL EXPERIENCES IN THE TEACHING OF TRANSFORMATION GEOMETRY IN ZIMBABWE'S RURAL SECONDARY SCHOOLS**

This is to certify that I have edited the above thesis and recommendations have been communicated to the respective author for further attention.

Should you require further information on the above, do not hesitate to contact the undersigned.

Yours Sincerely

Dr. U. Saidi (Ph. D Linguistics)
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saidiu@staff.msu.ac.zw, saeedu2010@gmail.com