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COMPUTING OSCILLATING VIBRATIONS EMPLOYING EXPONENTIALLY FITTED BLOCK MILNE'S DEVICE

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ABSTRACT

Background and Objectives: The idea of estimating oscillating vibration problems via multinomial basis function haven been seen by some authors as a convenient approach but not appropriate. This is as result of the behavior of the problem and as such depends largely on the step size and frequency. This research article is geared towards computing oscillating vibrations employing exponentially fitted block Milne's device (COVEFBMD). Materials and Methods: This is specifically designed using interpolation and collocation via exponentially fitted method as the approximate solution to generate COVEFBMD, thereby finding the tolerance level of the method. Results: Some numerical examples were selected and implemented on Mathematica kernel 9 to show speed, technicality and accuracy. Conclusion: The completed solutions show that COVEFBMD performs better than the existing methods because of its ability to design a worthy step size; decide the tolerance level resulting to maximized errors.

Keywords: COVEFBMD, exponentially fitted method, tolerance level, principal local truncation errors.

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1. INTRODUCTION

Computing oscillating vibrations employing exponentially fitted block Milne's device is built with the intent of approximating the oscillating vibrations. Particularly, when the final outcome shows oscillating vibrations. That is while exponentially fitted block Milne's device is more preferred to non-fitted methods as cited [2, 26]. In this study, effort is directed at finding suitable approximate solution of oscillating vibrations of the form [2, 11]

$$y'' = f(t, y), \quad y(\alpha) = y_0, \qquad y'(\alpha) = y'_0$$

for $t\epsilon[\alpha,\ T]$, (1)

where $f: R \times R^m \to R^m$, m is the magnitude of the forcible system.

Nevertheless, it is dared that $f \in R$ is sufficiently differentiable to a certain level on $t\epsilon[\alpha,\ T]$ and gratifies a world-wide Lipchitz consideration, i.e., there is an invariant quantity $L \geq 0$ such that

$$|f(t, y) - f(t, \bar{y}| \leq L|y - \bar{y}|, \ \forall y, \bar{y} \in R.$$

Below this given, par (1) checked the world-wide and singularity fixed on $t\epsilon[a,\ b]$ likewise viewed to satisfy the Weierstrass theorem, see[6, 8, 11, 28] for details.

Certainly, oscillating vibrations predominantly occurs in fields of scientific discipline and applied science such as Newton's laws of motion, celestial bodies and universe, quanta theory, control theory, electrical circuit and biologic science. Different techniques instituted on the use of multinomial expression have been discovered for evaluating oscillating vibrations. Trenchant techniques established on exponentially fitted method whose outcome is acknowledged beforehand have been proposed. Search[2, 16-20, 23-27] for more info. However, their implementation is been done using fixed step-size strategy aside adjusting to convergence. Look[2, 16-20, 23-27] for more items. The motivation of this study is based on a more amplified exponentially fitted method is require to address oscillatory vibrations problem unlike multinomial basis routine for approximation. Oscillatory vibration solvents guarantee step size and established frequence. Unique ODEs having oscillatory vibrations call for proficiency which is the characteristics of COVEFBMD. Thus, exponentially fitted block Milne's device becomes more advantageous. Examine[2, 16-20, 23-27]. Moreover, block Milne's device is designed for varying the step-size, deciding tolerance level and error control as cited[3-5, 11-12, 21-22, 29] for more items.

The primary objective of this study is to formulate a suited exponential fitted block Milne's device for oscillating vibrations which recognized the final outcome and frequency in anticipation. In addition, this novelty has been demonstrated on several literatures as discussed[3-5, 11-12, 21-22] for more items. Elements for developing this novelty includes; Adams type, block predictor-corrector formula of the same order and principal local truncation errors as remarked[3-5, 11-12, 21-22].

Definition 1: $b - block, m - point\ method$. If k refers to the block size and h is the pace size, then block size in time is mh. Let $r = 0,1,2, \ldots$ form the block number and let $n = rm$, then the $b - block, m - point$ method can be composed in the next general class:

$$Y_\beta = \sum_{u=1}^{b} A_u Y_{\beta-u} + h \sum_{u=0}^{b} B_u F_{\beta-u}, \tag{2}$$

where

$$Y_\beta = [y_{n+1}, \ldots, y_{n+i}, \ldots, y_{n+m}]^T$$

$$F_\beta = [f_{n+1}, \ldots, f_{n+i}, \ldots, f_{n+m}]^T$$

A_u and B_u are $m \times m$ constants matrices. See[7, 10, 22].

Therefore, commencing from the above report, a block method possesses the numerical vantages that each one taking off from the supra account, a block method has the computational benefits that for each practical application, the end result is valuated to a greater extent at the same time. The amount of values relies on the structure of the block method. Hence, applying these techniques can generate more immediate and quicker results which can be dealt to provide the needed accuracy. See[13-15, 21-22] for more information.

2. MATERIALS AND METHODS

The goal of this section is to develop exponentially fitted block Milne's device. This block Milne's device is a collection of $v - step$ explicit (block predictor) method and $v - 1 - step$ implicit (block corrector) method of ilk order. This collection can be represented as

$$y(t) = \sum_{j=0}^{k} \alpha_j y_{n-j} + h^2 \sum_{j=0}^{k} \beta_j(v) f_{n-j}, \tag{3}$$

$$y(t) = \sum_{j=0}^{k} \alpha_j y_{n-j} + h^2 \sum_{j=1}^{k} \boldsymbol{\beta}_j^*(v) f_{n+j}. \tag{4}$$

Par (3) and (4) defines the block predictor-corrector method of exponentially block Milne's device with $v = wh,\ \beta_j(v),\ j = 0, 1, 2$ containing features that depends on changing the step size and frequence. Noting that y_{n+j} is the numerical approximate to the exact solutions $y(t_{n+j})$ i.e. $y(t_{n+j}) \approx y_{n+j}$, and $f_{n+j} \approx f(t_{n+j}, y_{n+j})$ possessing $j = 0, 1, 2$. To achieve par (3) and (4), the exponentially fitted method is spelt below to approximate the exact solution $y(t)$ on clear-cut time intervals of $[t_n, t_{n-j}]$ via interpolation of the form

$$y(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 \exp(wt). \tag{5}$$

Rewriting (5) gives rise to the exponentially fitted method as

$$y(t) = a_0 + a_1\left(\frac{t-t_n}{h}\right) + a_2\left(\frac{t-t_n}{h}\right)^2 + a_3\left(\frac{t-t_n}{h}\right)^3 + a_4\left(1 + w\left(\frac{t-t_n}{h}\right) + \frac{w^2}{2}\left(\frac{t-t_n}{h}\right)^2 + a_3\frac{w^2}{6}\left(\frac{t-t_n}{h}\right)^3 + \frac{w^4}{24}\left(\frac{t-t_n}{h}\right)^4\right), \tag{6}$$

where a_0, a_1, a_2, a_3 and a_4 are parameters required to be settle in a special manner. Presuming that par (6) corresponds with the exact solution at some selected time interval t_n, t_{n-j} to yield the approximation as

$$y(t_n) \approx y_n, \quad y(t_{n-j}) \approx y_{n-j}. \tag{7}$$

Taking that the approximating function (6) gratifies par (1) some chosen points $t_{n+j}, j = 0, 1, 2$ to obtain the following approximates as

$$y'(t_{n+j}) \approx f_{n+j}, \qquad y''(t_{n+j}) \approx f_{n+j}, \qquad j = 0, 1, 2. \tag{8}$$

Merging par (7) and (8) will lead to quintuplicate systems of equation which produces At=b. Solving the systems of equation applying Mathematica 9 kernel will give $a_j, j = 0, 1, 2, 3, 4$ and replacing the values of $a_j's$ into (6) will generate the continuous block Milne's device. Valuating the continuous block Milne's device at some favoured points of $t_{n+j},\ j = 1, 2, 3$ will develop the exponentially fitted block Milne's device as

$$y(x) = y_n + y_{i-1} + h^2(\mu_1(w,x) f_i + \mu_2(w,x) f_{i-1} + \mu_3(w,x) f_{i-2}), \tag{9}$$

$$y(x) = y_n + y_{i-1} + h^2(\beta_1(w,x) f_{i+1} + \beta_2(w,x) f_{i+2} + \beta_3(w,x) f_{i+3}),$$

where w is the frequency, $\beta_1(w,x)$, $\beta_2(w,x)$, $\beta_3(w,x)$, $\mu_1(w,x)$, $\mu_2(w,x)$ and $\mu_3(w,x)$ are parameters. See[4-5, 11-12, 16-20] for more details.

2.1. Developing Tolerance level for Exponentially Fitted block Milne's Device:

To set up the mathematical operation of exponential fitted block Milne's device, the $v-step$ explicit (block predictor) method and $v-1-step$ implicit (block implicit) method are utilized as the block predictor-corrector method possessing ilk order as situated[3-5, 11-12, 21-22]. Merging[3-5, 11-12, 21-22], it is practicable to approximate the principal local truncation error of the block predictor-corrector method in absence of finding the first and second derivatives of $y(x)$. Again, considering the fact that $\tilde{p}=\bar{\bar{p}}$, where $\bar{\bar{p}}$ and $\tilde{p}$ shows the order of the block predictor and block corrector methods. Right away, for a method of order $\tilde{p}$, the analysis of the block predictor $k-step$ gives birth to the principal local truncation errors as

$$\tilde{C}^{[1]}_{\tilde{p}+5}h^{\tilde{p}+5}y^{(\tilde{p}+5)}(\tilde{t}_n)=y(t_{n+1})-y^{[q_1]}_{n+1}+O\left(h^{\tilde{p}+6}\right),$$

$$\tilde{C}^{[2]}_{\tilde{p}+5}h^{\tilde{p}+5}y^{(p+5)}(\tilde{t}_n)=y(t_{n+2})-y^{[q_2]}_{n+2}+O\left(h^{\tilde{p}+6}\right) \tag{10}$$

$$\tilde{C}^{[3]}_{\tilde{p}+5}h^{\tilde{p}+5}y^{(p+5)}(\tilde{t}_n)=y(t_{n+3})-y^{[q_3]}_{n+3}+O\left(h^{\tilde{p}+6}\right).$$

However, the same investigation of the block corrector method $v-1-step$ produces the principal local truncation errors as

$$\bar{C}^{[1]}_{\bar{\bar{p}}+5}h^{p+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)=y(t_{n+1})-y^{[l_1]}_{n+1}+O\left(h^{\bar{\bar{p}}+6}\right),$$

$$\bar{C}^{[2]}_{\bar{\bar{p}}+5}h^{p+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)=y(t_{n+2})-y^{[l_2]}_{n+2}+O\left(h^{\bar{\bar{p}}+6}\right) \tag{11}$$

$$\bar{C}^{[3]}_{\bar{\bar{p}}+5}h^{p+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)=y(t_{n+3})-y^{[l_3]}_{n+3}+O\left(h^{\bar{\bar{p}}+6}\right),$$

where $\tilde{C}^{[1]}_{p+5}, \tilde{C}^{[2]}_{p+5}, \tilde{C}^{[1]}_{p+5}$, $\bar{C}^{[1]}_{\bar{\bar{p}}+5}, \bar{C}^{[2]}_{\bar{\bar{p}}+5}$ and $\bar{C}^{[3]}_{\bar{\bar{p}}+5}$ exists as separate entity of the step-size h and $y(x)$ behave as the exact solution to the higher derivatives gratifying the initial precondition $y(t_n)\approx y_n$. Search into[3-5, 11-12, 21-22] for more particulars.

To advance further, the consideration for small amounts of h is attained as

$$y^{(5)}(\tilde{t}_n)\approx y^{(5)}(\bar{t}_n),$$

and the authority of the exponentially fitted block Milne's device relies at once on this consideration.

Simplifying further the principal local truncation errors of (10) and (11) supra as well as ignoring terms of degree $O\left(h^{\bar{\bar{p}}+6}\right)$, it poses no difficulty to arrive at the mathematical expression of the principal local truncation errors of exponentially fitted block Milne's device as

$$\bar{C}^{[1]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)\approx\frac{\bar{C}^{[1]}_{\bar{\bar{p}}+5}}{\tilde{C}^{[1]}_{\tilde{p}+5}-\bar{C}^{[1]}_{\bar{\bar{p}}+5}}\left[y^{[q_1]}_{n+j}-y^{[l_1]}_{n+j}\right]<\varepsilon_1,$$

$$\bar{C}^{[2]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)\approx\frac{\bar{C}^{[2]}_{\bar{\bar{p}}+5}}{\tilde{C}^{[2]}_{\tilde{p}+5}-\bar{C}^{[2]}_{\bar{\bar{p}}+5}}\left[y^{[q_2]}_{n+j}-y^{[l_2]}_{n+j}\right]<\varepsilon_2, \tag{12}$$

$$\bar{C}^{[3]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)\approx\frac{\bar{C}^{[3]}_{\bar{\bar{p}}+5}}{\tilde{C}^{[3]}_{\tilde{p}+5}-\bar{C}^{[3]}_{\bar{\bar{p}}+5}}\left[y^{[q_3]}_{n+j}-y^{[l_3]}_{n+j}\right]<\varepsilon_3\ .$$

Referring the avouchment that $y^{[q_1]}_{n+j}\neq y^{[l_1]}_{n+j}$, $y^{[q_2]}_{n+j}\neq y^{[l_2]}_{n+j}$ and $y^{[q_3]}_{n+j}\neq y^{[l_3]}_{n+j}$ are called the predicted and corrected approximations established by the exponentially fitted block Milne's device of order p, altho $\bar{C}^{[1]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)$, $\bar{C}^{[2]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)$ and $\bar{C}^{[3]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}y^{(\bar{\bar{p}}+5)}(\bar{t}_n)$

are distinctly named the principal local truncation errors. $\varepsilon_1, \varepsilon_2$ and ε_3 are the bounds of the tolerance level or tolerance level of the exponentially fitted block Milne's device.

In addition, the approximate of the principal local truncation error (12) is used to decide either to accept the iterated results or redo the iteration with a smaller changing step-size. The step is maintained based on a test established by par (12). See[3-5, 11-12, 21-22] for more details. The principal local truncation errors (12) is the tolerance level of the exponentially fitted block Milne's, device distinctly denoted as referred to as exponentially fitted block Milne's device (approximate) for correcting to convergence

Numerical examples: Two numerical examples were tested and worked out using COVEFBMD at distinctly tolerance level of 10^{-3}, 10^{-4} , 10^{-5}, 10^{-6} , 10^{-7}, 10^{-8} , 10^{-9}, 10^{-10}, 10^{-11} , 10^{-12} and 10^{-13}. Find9, 16-20 for more actions. A programming codes on exponentially fitted block Milne's is composed utilizing Mathematica 9 kernel 64. This programming codes is implemented in a block by block fashion combine with the exponentially fitted block Milne's device.

Numerical example 1: Consider the following mildly stiff IVP

$$y'' = -1001y' - 1000y,\ y(0) = 1, y'(0) - 1, \qquad 0 \le x \le 10.$$

Analytical Solution: $y(x) = e^{-x}$.

Test problem 2: Consider the inhomogeneous IVP:

$$y''(x) = -100y + 99\sin(x), y(0) = 1, y'(0) = 11, 0 \le x \le 1000.$$

Analytical Solution: $y(x) = \cos(10x) + \sin(10x) + \sin(x)$.

3. RESULTS AND DISCUSSION

Underneath this section, the mathematical final output demonstrates the performance of the exponentially block Milne's device for resolving oscillatory vibrations. The terminal output provided were obtained with the technical support of Mathematica 9 Kernel 64 on Microsoft windows (64-bit) to show the viable efficiency and accuracy of the exponentially fitted block Milne's device. The terminologies utilized are listed under:

Table 1 and Table 2 proves the numerical final results of problems 1 and 2 using COVEFBMD equated with existing methods. The descriptors named on table 1 are located under.

Table 1

$\mathbf{M_{employed}}$	$\mathbf{Max_{errors}}$	$C_{criteria}$
BHT	$2.23e-04$	10^{-4}
COVEFBMD	$5.15962e-06$	10^{-4}
COVEFBMD	$1.06227e-05$	
COVEFBMD	$2.73213e-05$	
HLMMs	$4.28437e-05$	10^{-5}
COVEBMD	$4.94747e-07$	10^{-5}
COVEBMD	$1.12438e-06$	
COVEBMD	$1.34966e-06$	
BHT	$3.36e-06$	10^{-6}
COVEFBMD	$5.16334e-08$	10^{-6}
COVEFBMD	$1.06304e-07$	
COVEFBMD	$2.73359e-07$	

HLMMs	$2.33590e-06$	10^{-7}
COVEFBMD	$4.94975e-09$	10^{-7}
COVEFBMD	$1.12494e-08$	
COVEFBMD	$1.34997e-08$	
BHT	$2.44e-08$	10^{-8}
COVEFBMD	$5.16369e-10$	10^{-8}
COVEFBMD	$1.06311e-9$	
COVEFBMD	$2.73372e-9$	
BHT	$1.96e-10$	10^{-10}
COVEFBMD	$5.16132e-12$	10^{-10}
COVEFBMD	$1.06263e-11$	
COVEFBMD	$2.7321e-11$	
BHT	$2.13e-12$	10^{-12}
HLMMs	$1.33620e-12$	
COVEFBMD	$4.92939e-14$	10^{-12}
COVEFBMD	$1.01474e-13$	
COVEFBMD	$2.59015e-13$	

Table 2

$\mathbf{M}_{\mathbf{employed}}$	$\mathbf{Max}_{\mathbf{errors}}$	$\boldsymbol{C_{criteria}}$
TSDM	$1.7e-03$	10^{-3}
BHTFM	$1.2e-03$	
BHT	$1.9e-03$	
BHTRKKNM	$2.14e-03$	
COVEFBMD	$4.97089e-04$	10^{-3}
COVEFBMD	$1.13021e-04$	
COVEFBMD	$1.35216e-04$	
TSDM	$2.7e-05$	10^{-5}
BHTFM	$1.4e-05$	
BHMTB	$3.9e-05$	
BHTRKKNM	$5.98e-05$	
BHTRKKNM	$2.06e-05$	
COVEFBMD	$4.94813e-7$	10^{-5}
COVEFBMD	$1.12462e-6$	
COVEFBMD	$1.34914e-6$	
TSDM	$1.0e-07$	10^{-7}
BHTFM	$1.5e-07$	
BHMTB	$1.4e-07$	
COVEFBMD	$4.94585e-9$	10^{-7}
COVEFBMD	$1.12406e-8$	
COVEFBMD	$1.34883e-8$	
TSDM	$6.3e-09$	10^{-9}
BHTFM	$4.94556e-11$	10^{-9}
BHTFM	$1.124e-10$	

BHTRKKNM	$1.34876e-10$	
BHT	$9.7e-11$	10^{-11}
BHT	$6.7e-11$	
COVEFBMD	$4.95826e-13$	10^{-11}
COVEFBMD	$1.12643e-12$	
COVEFBMD	$1.35136e-12$	
BHT	$4.3e-13$	10^{-13}
COVEFBMD	$4.66294e-15$	10^{-13}
COVEFBMD	$1.06581e-14$	
COVEFBMD	$1.46549e-14$	

COVEFBMD: errors in COVEFBMD (computing oscillating vibrations employing exponentially fitted block Milne's device) for tested problems 1 and 2.

$M_{employed}$: method employed.

Max_{errors}: the magnitude of the maximum errors of ETMBVSST.

$C_{criteria}$: convergence criteria.

BHT: errors in BHT (block hybrid trigonometrically fitted of $\delta = 10^{-6}$) for numerical tested problems 1 and 2. See[19].

BHMTB: errors in BHMTB (block hybrid method with trigonometric basis) for numerical tested problem 2. See[16].

BHTFM: errors in BHTFM (block hybrid trigonometrically fitted method) for numerical tested problem 2. See[17].

BHTRKNM: errors in BHTRKNM (block hybrid trigonometrically fitted Runge-Kutta-Nystrom method of $\delta = 10^{-6}$) for numerical tested problem 2. See[20].

HLMMs: errors in HLMMs (hybrid linear multistep methods) for tested problem 1. See[9].

TSDM: errors in TSDM (trigonometrically-fitted second derivative method) for numerical tested problem 2. See[18].

Algorithm: A scripted algorithmic program that will implement the block Milne's device and valuate the maximum errors of the block Milne's device in the class of $P(EC)^m$ or $P(EC)^m E$ mode, if the mode is executed m times. Check out[20].

Step 1: Choose a step size for h.

Step 2: The block Milne's device of the block predictor-corrector method must possess the same.

Step 3: The stepnumber of the predictor method must be one step greater than the corrector method.

Step 4: Estimate the principal local truncation errors of the block Milne's device after the principal local truncation errors are achieved.

Step 5: Set the convergence criteria

Step 6: Write the code of the block Milne's device using Mathematica 9 kernel 64.

Step 7: Apply any one step method to initialize the process when needed, if not omit step 7 and move on to step 8.

Step 8: Execute the block Milne's device in the class of $P(EC)^m$ or $P(EC)^m E$ mode as m gains.

Step 9: If step 8 fails to reach convergence, reiterate the operation again and divide the step size (h) by 2 from step 0 or if not, move forward to step 10.

Step 10: Valuate the maximum errors after convergence is attained.

Step 11: Print maximum errors.

Step 12: Utilize this formula posited under to invent a new step size after convergence is arrived at

$$qh = \left|\frac{\delta}{2(\tilde{C}_{p+5} - \bar{C}_{p+5})}\right|^{\frac{1}{4}}.$$

4. CONCLUSION

The superiority of this numerical result has shown that COVEFBMD is arrived at with the help of the tolerance level. Again, the tolerance levels examine whether the iterations should be accepted or reiterated again with a lesser step size. The computational results instituted the efficiency of the COVEFBMD is observed to show better maximum errors at all tolerance levels. This is as a result of the suited/varying step size, finding out the tolerance level by that means maximize errors than existing methods like the TSDM, BHMTM, BHT, BHTRKNM, BHTFM and HLMMs in all analyzed tolerance levels of 10^{-3}, 10^{-4} , 10^{-5}, 10^{-6} , 10^{-7}, 10^{-8} , 10^{-9}, 10^{-10}, 10^{-11} , 10^{-12} and 10^{-13} as stated[9, 16-20]. More work can be done in the area of increasing the order of COVEFBMD to test and improve efficiency.

5. SIGNIFICANT STATEMENT

The significant of this research work is presented as follows:

- A more suitable exponentially fitted method is preferable to deal with oscillatory vibrations problem than considering any multinomial as basis function to approximate oscillatory vibration problems.
- Oscillatory vibration solutions trust on step size and frequency. The COVEFBMD posses the vantage of designing suited step size and inbuilt frequence established from the exponentially fitted method.
- Special ODEs in form of oscillatory vibration problems require technic which is the attributes of COVEFBMD.

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