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Analytical Solutions of a 1D Time-fractional Coupled Burger Equation via Fractional Complex Transform

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Abstract: - In this paper, we obtain analytical solutions of a system of time-fractional coupled Burger equation of one-dimensional form via the application of Fractional Complex Transform (FCT) coupled with a modified differential transform method (MDTM) in comparison with Adomian Decomposition Method (ADM). The associated fractional derivatives are defined in terms of Jumarie's sense. Illustrative cases are considered in clarifying the effectiveness of the proposed technique. The method requires minimal knowledge of fractional calculus. Neither linearization nor discretization is involved. The results are also presented graphically for proper illustration and efficiency is ascertained. Hence, the recommendation of the method for linear and nonlinear space-fractional models.

Key-Words: Fractional calculus, Adomian decomposition method, fractional complex transform, MDTM, coupled Burger equation.

1 Introduction

Burger's equation appears to be a basic partial differential equation with copious applications in applied mathematics viz: modelling, gas dynamics, traffic flow, nonlinear acoustics and so on [1-3]. As regards stochastic dynamics, the applications of stochastic Burgers equation surface in mathematical finance, quantum physics, and financial physics [4-7]. The one-dimensional integer form of the coupled nonlinear Burger equation follows:

$$\left.\begin{aligned} \frac{\partial u}{\partial t}+\xi_1\frac{\partial^2 u}{\partial x^2}+\xi_2 u\frac{\partial u}{\partial x}+\gamma\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0\\ \frac{\partial v}{\partial t}+\mu_1\frac{\partial^2 v}{\partial x^2}+\mu_2 v\frac{\partial v}{\partial x}+\eta\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0 \end{aligned}\right\} \qquad (1)$$

subject to the following conditions (2) and (3) (that is, initial and Dirichlet boundary conditions respectively):

$$\left.\begin{aligned} u(x,0)=f_1(x)\\ v(x,0)=f_2(x) \end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} u(x,t)=e_1(x,t)\\ v(x,t)=e_2(x,t) \end{aligned}\right\} \qquad (3)$$

for $x\in\Omega$, $t>0$ where $\Omega=\{x:x\in[c,d]\}$ signifies a domain of computational interval, the constants ξ_1, ξ_2, μ_1, and μ_2 are real, while γ and η are arbitrary constants subject to the system's constraints.

A lot of analytical, semi-analytical, and numerical methods of solution appear in literature for solving PDEs such as the one-dimensional Burger, coupled Burger equations (1) and the likes [8-27].

Sequel to fractional calculus, this work considers a non-integer ordered form of (1) as an extension which is regarded as time-fractional order coupled nonlinear Burger equation of the form:

$$\left.\begin{array}{l} \dfrac{\partial^\alpha u}{\partial t^\alpha}+\xi_1\dfrac{\partial^2 u}{\partial x^2}+\xi_2 u\dfrac{\partial u}{\partial x}+\gamma\left(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial u}{\partial x}\right)=0,\\ \dfrac{\partial^\alpha v}{\partial t^\alpha}+\mu_1\dfrac{\partial^2 v}{\partial x^2}+\mu_2 v\dfrac{\partial v}{\partial x}+\eta\left(u\dfrac{\partial v}{\partial x}+v\dfrac{\partial u}{\partial x}\right)=0,\\ \alpha\in(0,1]. \end{array}\right\} \tag{4}$$

Recent work on fractional Burgers' equation include that of Momani [28] via the application of a semi-analytical approach: Adomian Decomposition Method (ADM). Fractional Burger model equation may be expressed in terms of time, space, or time-space order. For instance, the space-fractional Burger model is used for proper description of the physical processes of unidirectional propagation of weakly nonlinear acoustic waves via a gas-filled pipe. The view is the same for other systems like the shallow-water waves and so on [29].

2 Fractional Derivative in the Sense of Jumarie

It is noted here that Jumarie's Fractional Derivative (JFD) is a modified form of the Riemann-Liouville derivatives [30]. Hence, the definition of JFD and its basic properties as follows:
Suppose $\sigma(z)$ is a continuous real valued function of z, and $D_z^\alpha\sigma=\dfrac{\partial^\alpha\sigma}{\partial z^\alpha}$ denoting JFD of σ, of order α w.r.t. z. Then,

$$D_z^\alpha\sigma=\begin{cases}\left(\begin{array}{l}\dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dz}\displaystyle\int_0^z(z-\zeta)^{-\alpha}\left(\sigma(\zeta)-\sigma(0)\right)d\zeta,\\ \text{for } \alpha\in(0,1)\end{array}\right),\\ \left(\begin{array}{l}\dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dz}\displaystyle\int_0^z(z-\zeta)^{-\alpha}\left(\sigma(\zeta)-\sigma(0)\right)d\zeta,\\ \text{for } \alpha\in(0,1)\end{array}\right),\\ \left(\begin{array}{l}\left(\sigma^{(\alpha-\phi)}(z)\right)^{(\phi)},\ \alpha\in[\phi,\phi+1),\\ \phi\geq 1\end{array}\right),\end{cases} \tag{5}$$

where $\Gamma(\cdot)$ represents a gamma function. The main features of JFD are as follows [31]:

(i) $D_z^\alpha c=0,\ \alpha>0$, for a constant c

(ii) $D_z^\alpha\left(c\sigma(z)\right)=cD_z^\alpha\sigma(z),\ \alpha>0$,

(iii) $D_z^\alpha z^\beta=\dfrac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)}z^{\beta-\alpha},\ \beta\geq\alpha>0,$

(iv) $D_z^\alpha\left(\sigma_1(z)\sigma_2(z)\right)=\left(\begin{array}{l}D_z^\alpha\sigma_1(z)\left(\sigma_2(z)\right)\\ +\sigma_1(z)D_z^\alpha\sigma_2(z)\end{array}\right),$

(v) $D_z^\alpha\left(\sigma(z(g))\right)=D_z^1\sigma\cdot D_g^\alpha z$,

The features (i)-(v) are fractional derivatives of: constant function, constant multiple function, power function, product function, and function of function respectively. Though, (v) can be associated to Jumarie's chain rule in terms of fractional derivative.

3 The Reduced Differential Transform [32-35]

Suppose $m(x,t)$ is an analytic and continuously differentiable function, defined on D (a given domain), then the differential transformation form of $m(x,t)$ is defined and expressed as:

$$M_k(x)=\frac{1}{k!}\left[\frac{\partial^k m(x,t)}{\partial t^k}\right]_{t=0} \tag{6}$$

where $M_k(x)$ and $m(x,t)$ are referred to as the transformed and the original functions respectively. Thus, the differential inverse transform (DIT) of $M_k(x)$ is defined and denoted as:

$$m(x,t)=\sum_{k=0}^{\infty}M_k(x)t^k. \tag{7}$$

3.1 The fundamentals properties of the DTM

D1: If $m(x,t)=\alpha p(x,t)\pm\beta q(x,t)$, then $M_k(x)=\alpha P_k(x)\pm\beta Q_k(x)$.

D2: If $m(x,t)=\dfrac{\alpha\partial^\eta h(x,t)}{\partial t^\eta},\ \eta\in\mathbb{N}$, then $M_k(x)=\dfrac{\alpha(k+\eta)!}{k!}H_{k+\eta}(x)$.

D3: If $m(x,t)=\dfrac{g(x)\partial^\eta h(x,t)}{\partial x^\eta},\ \eta\in\mathbb{N}$, then $M_k(x)=\dfrac{g(x)\partial^\eta H_k(x)}{\partial x^\eta},\ \eta\in\mathbb{N}$.

D4: If $m(x,t)=p(x,t)q(x,t)$, then $M_k(x)=\sum_{\eta=0}^{k}P_\eta(x)Q_{k-\eta}(x)$.

D5: If $m(x,t)=x^{n_1}t^{n_2}$, then

$$M_c = x^{n_1}\delta(c-n_2), \delta(c)=\begin{cases}0, & c\neq 0,\\ 1, & c=0\end{cases}.$$

3.2 The Fractional complex transform [29, 30]

Suppose we consider a general fractional differential equation of the form:

$$h\left(\upsilon, D_t^{\alpha}\upsilon, D_x^{\beta}\upsilon, D_y^{\lambda}\upsilon, D_z^{\gamma}\upsilon\right)=0,\ \upsilon=\upsilon(t,x,y,z), \tag{8}$$

and define the Fractional Complex Transform (FCT) as follows:

$$T=\frac{at^{\alpha}}{\Gamma(1+\alpha)},\ \alpha\in(0,1], \tag{9}$$

where a is an unknown constant, then from (iii), we have:

$$D_v^{\alpha}z^{\beta}=\frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)}z^{\beta-\alpha},\ \beta\geq\alpha>0,$$

$$\therefore\ D_t^{\alpha}T == \frac{a}{\Gamma(1+\alpha)}\left[\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha-\alpha)}\right]t^{\alpha-\alpha}=a. \tag{10}$$

Hence,

$$D_t^{\alpha}\upsilon = D_t^{\alpha}\upsilon\left(T(t)\right)=D_T^{1}\upsilon\cdot D_t^{\alpha}T = a\frac{\partial\upsilon}{\partial T}. \tag{11}$$

3.3 Adomian decomposition method

The Adomian decomposition method is one of the numerical methods which can be applied to both linear and non-linear differential problems (i.e. both ordinary differential equations (ODE) and partial differential equation (PDE)) [36, 37]. It is in the form of algorithms. It permits researchers to solve initial value problems (IVP) and boundary value problems (BVP) without assuming the initial term, or some basis function.

The ADM represents the nonlinear equation in this form:

$$Lu + Ru + Nu = g \tag{12}$$

where L represents a linear operator, R represents a remainder and N represents a non-linear operator.

Generally, we choose $L=\frac{d^n}{dx^n}(\cdot)$, to be the nth-order differential operator and thus its inverse L^{-1} follows as the nth-order integration operator.

Therefore, applying the inverse linear operator L^{-1} to both sides of (12), we have:

$$L^{-1}Lu + L^{-1}Ru + L^{-1}Nu = L^{-1}g \tag{13}$$

where;

$$L^{-1}Lu = u-\phi \tag{14}$$

and ϕ signifies the initial value.

Therefore, (13) becomes;

$$u-\phi + L^{-1}Ru + L^{-1}Nu = L^{-1}g \tag{15}$$

or

$$u = L^{-1}g + \phi - [L^{-1}Ru + L^{-1}Nu] \tag{16}$$

or

$$u = \varphi(u) - [L^{-1}Ru + L^{-1}Nu] \tag{17}$$

where

$$\varphi(u) = L^{-1}g + \phi\ , \tag{18}$$

signifies the initial values of the nonlinear equation.

The ADM expresses the solution $y(t)$ in series form;

$$u=\sum_{n=0}^{\infty}u_n \tag{19}$$

Also, the non-linear operator can be expressed into a series of Adomain polynomials;

$$Nu=\sum_{n=0}^{\infty}A_n \tag{20}$$

The Adomain polynomials A_n are dependent on the values of $u_0,u_1,u_2,...,u_n$ and are obtained for the nonlinearity $Nu = f(u)$ by the formula:

$$A_n=\begin{cases}\left.\dfrac{1}{n!}\dfrac{\partial^n}{\partial\lambda^n}\left(N\left(\sum_{k=0}^{\infty}\lambda^k y_k\right)\right)\right|_{\lambda=0}\\ n=0,1,...\end{cases} \tag{21}$$

4 Examples/Applications

Here, the concerned methods of solution are used for a nonlinear time-fractional coupled Burger equation as follows.

Suppose we take:

$\xi_1=-1,\ \xi_2=-2,\ \mu_1=-1,\ \mu_2=-2$, & $\gamma=\eta=1$, then we consider (4) of the form:

$$\left.\begin{aligned}\frac{\partial^\alpha u}{\partial t^\alpha}-\frac{\partial^2 u}{\partial x^2}-2u\frac{\partial u}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0\\ \frac{\partial^\alpha v}{\partial t^\alpha}-\frac{\partial^2 v}{\partial x^2}-2v\frac{\partial v}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0\end{aligned}\right\} \quad (22)$$

subject to:

$$u(x,0)=v(x,0)=\sin(x). \quad (23)$$

Case I Method I (FCT-RDTM)

Solution Steps:

By FCT, $T=\frac{at^\alpha}{\Gamma(1+\alpha)}$, this according to section 3 gives $\frac{\partial^\alpha u}{\partial t^\alpha}=\frac{\partial u}{\partial T}$ and $\frac{\partial^\alpha v}{\partial t^\alpha}=\frac{\partial v}{\partial T}$ for $a=1$. Hence, (22) becomes:

$$\left.\begin{aligned}\frac{\partial u}{\partial T}-\frac{\partial^2 u}{\partial x^2}-2u\frac{\partial u}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0\\ \frac{\partial v}{\partial T}-\frac{\partial^2 v}{\partial x^2}-2v\frac{\partial v}{\partial x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial u}{\partial x}\right)=0\end{aligned}\right\} \quad (24)$$

subject to:

$$u(x,0)=\sin x=v(x,0).$$

By the RDTM in section 3, we have the recurrence relation from (24) as:

$$\begin{cases}U_{\upsilon+1}=\frac{1}{(1+\upsilon)}\left(U''_{x,\upsilon}+2\sum_{r=0}^{\upsilon}U_{x,p}U'_{x,\upsilon-p}-\frac{\partial}{\partial x}\sum_{p=0}^{\upsilon}U_pV_{\upsilon-p}\right),\\ V_{\upsilon+1}=\frac{1}{(1+\upsilon)}\left(V''_{x,\upsilon}+2\sum_{r=0}^{\upsilon}V_{x,p}V'_{x,\upsilon-p}-\frac{\partial}{\partial x}\sum_{p=0}^{\upsilon}U_pV_{\upsilon-p}\right),\end{cases}$$

$$\upsilon\geq 0. \quad (25)$$

Hence, using the initial condition: $u(x,0)=\sin x=v(x,0)$ we obtain:

$$\begin{cases}U_0=V_0=\sin x,\ U_2=\frac{\sin x}{2!}=V_2,\\ U_4=\frac{\sin x}{4!}=V_4,\cdots\\ U_1=-\sin x=V_1,\ U_3=\frac{-\sin x}{3!}=V_3,\\ U_5=\frac{-\sin x}{5!}=V_5,\cdots\end{cases} \quad (26)$$

In general, we have:

$$U_3=V_3=\frac{(-1)^p\sin x}{p!},\ p\in\mathbb{N}\cup(0). \quad (27)$$

$\therefore$

$$u(x,T)=\sum_{h=0}^{\infty}U_hT^{\alpha h}=\begin{pmatrix}\sin x-(\sin x)T+\frac{\sin x}{2!}T^2\\ -\frac{\sin x}{3!}T^3+\frac{\sin x}{4!}T^4+\cdots\end{pmatrix}$$
$$=\sin x\sum_{n=0}^{\infty}\frac{(-1)^nT^n}{n!}=\sin x\sum_{n=0}^{\infty}\frac{(-T)^n}{n!} \quad (28)$$
$$=\sin(x)\exp(-T).$$

Similarly,

$$v(x,T)=\sum_{\hbar=0}^{\infty}V_\hbar T^\hbar=\sin(x)\exp(-T). \quad (29)$$

Hence, the exact solution of (4.1) is:

$$\begin{cases}u(x,t)=\sin(x)\exp\left(-\frac{t^\alpha}{\Gamma(1+\alpha)}\right),\\ v(x,t)=\sin(x)\exp\left(-\frac{t^\alpha}{\Gamma(1+\alpha)}\right).\end{cases} \quad (30)$$

Note**:** when $\alpha=1$ in (30), we have $u(x,t)=\sin(x)\exp(-t)=v(x,t)$ yielding the exact solution of the classical coupled nonlinear Burgers equation in line with the result in [1], [8], and [25].

Case 1 Method II (FCT-ADM)

Here, we consider the integer order form in (24) via the decomposition method as follows. Suppose $\tilde{u}$ and $\tilde{v}$ satisfy (24), then in an operator form, (24) is expressed as:

$$\left.\begin{aligned}L_T\tilde{u}=L_{xx}\tilde{u}+2\tilde{u}L_x\tilde{u}-L_x(\tilde{u}\tilde{v})\\ L_T\tilde{v}=L_{xx}\tilde{v}+2\tilde{v}L_x\tilde{v}-L_x(\tilde{u}\tilde{v})\\ \tilde{u}(x,0)=\tilde{v}(x,0)=\sin(x)\end{aligned}\right\} \quad (31)$$

where

$$L_x(\tilde{u}\tilde{v})=\left(\tilde{u}\frac{\partial\tilde{v}}{\partial x}+\tilde{v}\frac{\partial\tilde{u}}{\partial x}\right) \quad (32)$$

and

$$\begin{cases}L_T(\cdot)=\frac{\partial}{\partial x}(\cdot),\\ L_T^{-1}(\cdot)=\int_0^T(\cdot)\,ds.\end{cases} \quad (33)$$

For further steps, we operate (33) on (31) yielding:

$$\left.\begin{aligned}\tilde{u}=\tilde{u}(x,0)+L_T^{-1}\{L_{xx}\tilde{u}+2\tilde{u}L_x\tilde{u}-L_x(\tilde{u}\tilde{v})\}\\ \tilde{v}=\tilde{v}(x,0)+L_T^{-1}\{L_{xx}\tilde{v}+2\tilde{v}L_x\tilde{v}-L_x(\tilde{u}\tilde{v})\}\end{aligned}\right\} \quad (34)$$

when $\tilde{u}$ and $\tilde{v}$ are decomposed as:

$$\left.\begin{array}{l}\tilde{u}=\sum_{i=0}^{\infty}\tilde{u}_i\\ \tilde{v}=\sum_{i=0}^{\infty}\tilde{v}_i\end{array}\right\} \quad (35)$$

then (34) becomes:

$$\left.\begin{array}{l}\sum_{i=0}^{\infty}\tilde{u}_i=\tilde{u}(x,0)+L_T^{-1}\left\{L_{xx}\sum_{i=0}^{\infty}\tilde{u}_i+2\sum_{i=0}^{\infty}P_i-L_x\left(\sum_{i=0}^{\infty}Q_i\right)\right\}\\ \sum_{i=0}^{\infty}\tilde{v}_i=\tilde{v}(x,0)+L_T^{-1}\left\{L_{xx}\sum_{i=0}^{\infty}\tilde{v}_i+2\sum_{i=0}^{\infty}H_i-L_x\left(\sum_{i=0}^{\infty}Q_i\right)\right\}\end{array}\right\} \quad (36)$$

where the nonlinear terms are expressed in infinite series (Adomian polynomials) as:

$$\left.\begin{array}{l}\sum_{i=0}^{\infty}P_i=\tilde{u}\tilde{u}_x\\ \sum_{i=0}^{\infty}H_i=\tilde{v}\tilde{v}_x\\ \sum_{i=0}^{\infty}Q_i=\tilde{u}\tilde{v}\end{array}\right\}. \quad (37)$$

For computational sake, we express few terms of (37) as:

$$\begin{cases}P_0=\tilde{u}_{0x}\tilde{u}_0\\ P_1=\tilde{u}_{0x}\tilde{u}_1+\tilde{u}_0\tilde{u}_{1x}\\ P_2=\tilde{u}_{0x}\tilde{u}_2+\tilde{u}_{1x}\tilde{u}_1+\tilde{u}_{2x}\tilde{u}_0\\ \vdots\end{cases}, \quad (38)$$

$$\begin{cases}H_0=\tilde{v}_{0x}\tilde{v}_0\\ H_1=\tilde{v}_{0x}\tilde{v}_1+\tilde{v}_0\tilde{v}_{1x}\\ H_2=\tilde{v}_{0x}\tilde{v}_2+\tilde{v}_{1x}\tilde{v}_1+\tilde{v}_{2x}\tilde{v}_0\\ \vdots\end{cases}, \quad (39)$$

$$\begin{cases}Q_0=\tilde{u}_0\tilde{v}_0\\ Q_1=\tilde{u}_0\tilde{v}_1+\tilde{u}_1\tilde{v}_0\\ Q_2=\tilde{u}_0\tilde{v}_2+\tilde{u}_1\tilde{v}_1+\tilde{u}_2\tilde{v}_0\\ \vdots\end{cases}. \quad (40)$$

Hence, the associated relation is given as:

$$\begin{cases}\tilde{u}_0=\tilde{u}(x,0)\\ \tilde{u}_{i+1}=L_T^{-1}\left\{\tilde{u}_{ixx}+2P_i-L_x(Q_i)\right\}\end{cases}, \quad (41)$$

$$\begin{cases}\tilde{v}_0=\tilde{v}(x,0)\\ \tilde{v}_{i+1}=L_T^{-1}\left\{\tilde{v}_{ixx}+2H_i-L_x(Q_i)\right\}\end{cases}. \quad (42)$$

Hence, using $\tilde{u}(x,0)=\tilde{v}(x,0)=\sin(x)$, we have:

$$\begin{cases}\tilde{u}_1=L_T^{-1}\left\{\tilde{u}_{0xx}+2P_0-L_x(Q_0)\right\}\\ \quad=-T\sin x\\ \tilde{v}_1=L_T^{-1}\left\{\tilde{v}_{0xx}+2H_0-L_x(Q_0)\right\}\\ \quad=-T\sin x\end{cases}$$

$$\begin{cases}\tilde{u}_2=L_T^{-1}\left\{\tilde{u}_{1xx}+2P_1-L_x(Q_1)\right\}\\ \quad=\frac{T^2}{2!}\sin x\\ \tilde{v}_2=L_T^{-1}\left\{\tilde{v}_{1xx}+2H_1-L_x(Q_1)\right\}\\ \quad=\frac{T^2}{2!}\sin x\end{cases}$$

$$\begin{cases}\tilde{u}_3=L_T^{-1}\left\{\tilde{u}_{2xx}+2P_2-L_x(Q_2)\right\}\\ \quad=\frac{-T^3}{3!}\sin x\\ \tilde{v}_3=L_T^{-1}\left\{\tilde{v}_{2xx}+2H_2-L_x(Q_2)\right\}\\ \quad=\frac{-T^3}{3!}\sin x\end{cases}$$

$$\begin{cases}\tilde{u}_4=L_T^{-1}\left\{\tilde{u}_{3xx}+2P_3-L_x(Q_3)\right\}\\ \quad=\frac{T^4}{4!}\sin x\\ \tilde{v}_4=L_T^{-1}\left\{\tilde{v}_{3xx}+2H_3-L_x(Q_3)\right\}\\ \quad=\frac{T^4}{4!}\sin x\end{cases}$$

$$\begin{cases}\tilde{u}_5=L_T^{-1}\left\{\tilde{u}_{4xx}+2P_4-L_x(Q_4)\right\}\\ \quad=\frac{-T^5}{5!}\sin x\\ \tilde{v}_5=L_T^{-1}\left\{\tilde{v}_{4xx}+2H_4-L_x(Q_4)\right\}\\ \quad=\frac{-T^5}{5!}\sin x\end{cases}$$

$$\begin{cases} \tilde{u}_6 = L_T^{-1}\left\{\tilde{u}_{5xx} + 2P_5 - L_x\left(Q_5\right)\right\} \\ \quad = \dfrac{T^6}{6!}\sin x \\ \tilde{v}_6 = L_T^{-1}\left\{\tilde{v}_{5xx} + 2H_5 - L_x\left(Q_5\right)\right\} \\ \quad = \dfrac{T^6}{6!}\sin x \end{cases}$$

$$\vdots$$

$$\begin{cases} \tilde{u}_m = L_T^{-1}\left\{\tilde{u}_{(m-1)xx} + 2P_{(m-1)} - L_x\left(Q_{(m-1)}\right)\right\}, m \geq 1 \\ \quad = \dfrac{(-T)^m}{m!}\sin x \\ \tilde{v}_m = L_T^{-1}\left\{\tilde{v}_{(m-1)xx} + 2H_{(m-1)} - L_x\left(Q_{(m-1)}\right)\right\} \\ \quad = \dfrac{(-T)^m}{m!}\sin x \end{cases}$$

Hence,

$$\begin{cases} \tilde{u}(x,T) = \sum_{i=0}^{\infty} \tilde{u}_i(x,T) \\ \tilde{v}(x,T) = \sum_{i=0}^{\infty} \tilde{v}_i(x,T) \end{cases}$$

implies that:

$$\begin{aligned} \tilde{u}(x,T) &= \left(\begin{array}{l} \sin x - (\sin x)T + \dfrac{\sin x}{2!}T^2 \\ -\dfrac{\sin x}{3!}T^3 + \dfrac{\sin x}{4!}T^4 + \cdots \end{array} \right) \\ &= \left(1 - T + \frac{T^2}{2!} - T^3 + \frac{T^4}{4!} + \cdots\right)\sin x \qquad (43) \\ &= \sin x \exp(-T). \end{aligned}$$

Similarly,

$$\begin{aligned} \tilde{v}(x,T) &= \left(\begin{array}{l} \sin x - (\sin x)T + \dfrac{\sin x}{2!}T^2 \\ -\dfrac{\sin x}{3!}T^3 + \dfrac{\sin x}{4!}T^4 + \cdots \end{array} \right) \\ &= \left(1 - T + \frac{T^2}{2!} - T^3 + \frac{T^4}{4!} + \cdots\right)\sin x \qquad (44) \\ &= \sin x \exp(-T). \end{aligned}$$

As a result,

$$\begin{cases} u(x,t) = \sin(x)\exp\left(-\dfrac{t^\alpha}{\Gamma(1+\alpha)}\right), \\ v(x,t) = \sin(x)\exp\left(-\dfrac{t^\alpha}{\Gamma(1+\alpha)}\right). \end{cases} \qquad (45)$$

The solutions at different values of t and α are presented in Fig. 1 and Fig. 2.

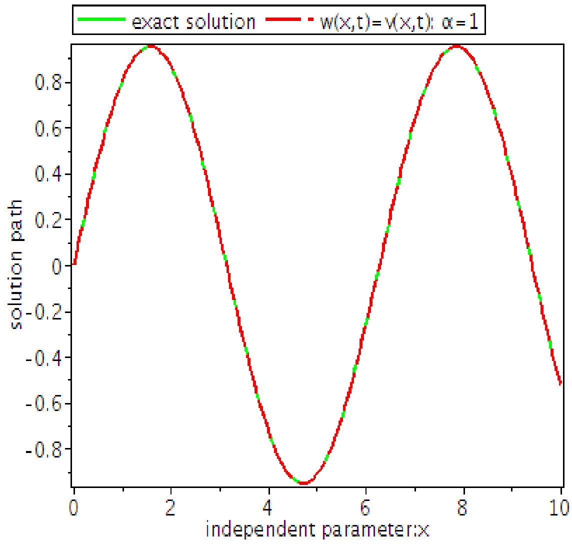

Fig. 1: Graphical solution for at $\alpha = 1$, $(t = 1)$

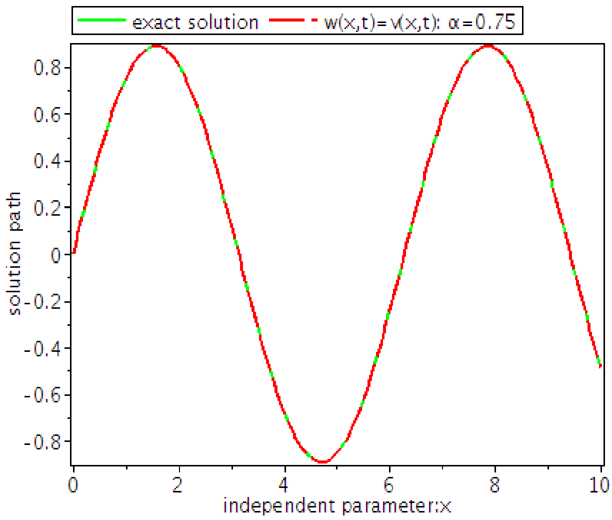

Fig. 2: Graphical solution for $\alpha = 0.75$, $(t = 1)$

5 Conclusion

We obtained exact solutions of a system type of one-dimensional time-fractional nonlinear coupled Burger equations via the application of FCT coupled with reduced differential transform method (FCT-

RDTM) and FCT coupled with ADM (FCT-ADM). Both methods: FCT-RDTM and FCT-ADM yielded same results in series forms; the former involved less iterations. The FCT is indeed simple but effective and accurate for the solutions of fractional differential equations. The associated derivatives were defined in terms of Jumarie's sense. It is noted that basic knowledge of advanced calculus is more required than that of fractional calculus while obtaining exact solutions of fractional equations with high level of accuracy not being compromised. This can therefore be extended to space-fractional derivatives of higher orders both in linear and nonlinear forms.

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Conflict of Interests

The authors declare that they have no conflict of interest regarding the publication of this paper.

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