International Journal of Civil Engineering and Technology (IJCIET)
Volume 9, Issue 8, August 2018, pp. 332–340, Article ID: IJCIET_09_08_033
Available online at http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=9&IType=8
ISSN Print: 0976-6308 and ISSN Online: 0976-6316

 Scopus Indexed

THE REVERSED ESTIMATION OF VARIABLE STEP SIZE IMPLEMENTATION FOR SOLVING NONSTIFF ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

This study is design to examine the reversed estimation of variable step-size implementation for solving nonstiff ordinary differential equations. This is exclusively dependent on the principal local truncation error of both predictor and corrector formulae of the same order. Collocation and interpolation methods with the aid of power series as the approximate function is utilized in the construction of a class of predictor and corrector formulae of the same order with distinct. The computed results existed in literatures demonstrated the performance of the method over existing methods. The reversed estimation of predictor and corrector formulae is solely the predictor formulae and also, draws a lot of computational benefits which insures convergence, tolerance level, monitoring the step size and maximum errors.

Key words: Predictor and corrector formulae, Tolerance level, Maximum errors, Distinct k-step, Principal local truncation error.

Cite this Article: J.G. Oghonyon, O.F. Imaga, P.O. Ogunniyi, The Reversed Estimation of Variable Step Size Implementation for Solving Nonstiff Ordinary Differential Equations. *International Journal of Civil Engineering and Technology*, 9(8), 2018, pp. 332-340.
http://www.iaeme.com/IJCIET/issues.asp?JType=IJCIET&VType=9&IType=8

1. INTRODUCTION

Concurring with[5], several techniques have been devised to produce global error estimation. A distinctive approach, frequently employed if local error control is expended, is called tolerance reduction. This banks on the presumption of tolerance balance being correct. In solving a differential equation over the necessitated interval, a new result is achieved employing a decreased or increased tolerance. The deviations in the result, obtained at like points, can be used to approximate the global error.

Computational methods for providing the solution of ODEs are ordinarily divided into single-step or multistep processes. From each one has its pros and cons, and many numerical analysts favour one or the other technique. Moreover, such a choice may originate from the needs of the problem being worked out. Authors viewed generally that several types of numeric methods had better be equated to the user aims. See[5].

Considering the initial value problem of a first-order differential equation of the form

$$y'(x) = f(x, y),\ y(a) = \alpha,\ x \in [a,b] \text{ and } f : R \times R^m \to R^m \qquad (1)$$

The solution to (1) is broadly written as

$$\sum_{i=0}^{j} \alpha_i y_{n+i} = h \sum_{i=0}^{j} \beta_i f_{n+i} \qquad (2)$$

where the step size is h, $\alpha_j = 1$, α_i, $i = 0,1,..., j$, β_j are unknown constants which are uniquely defined such that the formulae is of order j as discussed[2].

It is presumed that $f \in R$ is sufficiently differentiable on $x \in [a,b]$ and satisfies a global Lipchitz condition, i.e., there is a constant $L \geq 0$ such that

$$\left| f(x,y) - f(x,\bar{y}) \right| \leq L \left| y - \bar{y} \right|, \quad \forall y, \bar{y} \in R.$$

Under this presumption, equation (1) assured the existence and uniqueness set on $x \in [a,b]$ as well as viewed to fulfill the Weierstrass theorem, see for instance "[12], [15], [21], [23]" for details.

Where a and b are finite and $y^{(i)}[y_1^{(i)}, y_2^{(i)}, \ldots, y_n^{(i)}]^T$ for $i = 0(1)3$ and $f = [f_1, f_1, \ldots, f_n]^T$, originate in real life applications for problems in science and engineering such as fluid dynamics and motion of rocket as presented[19].

However, authors "[1], [6], [8], [11], [13], [14], [20], [21]",proposed block multistep methods which were employed in predictor-corrector mode. Block multistep methods have the vantage of assessing at the same time at all points with the integration interval, thereby reducing the computational encumbrance when an evaluation is demanded at more than one point within the grid. Again, one step methods are employ starting values in order to estimate the corrector.

Bookmen have proposed block predictor and corrector formulae for the numeric solution of nonstiff ODEs in the simple form of Adams type united with $P(EC)^m$ and $P(EC)^m E$ mode implemented using variable step size appear for example[3, 4, 17-18, 22]. Still, their implementation was geared towards Backward Differentiation Formula (BDF). This paper presents Milne's implementation on block predictor-corrector method for solving nonstiff ODEs of (1) founded on variable step size technique implemented in $P(EC)^m$ or $P(EC)^m E$ mode. The Reversed Estimation of Variable Step Size Implementation for Solving Nonstiff ODEs comes with many numeric vantages as cited[21].

A block-by-block method is a method for computing vectors $Y_0, Y_1, \ldots$ in sequence. Let the r-vector (r is the number of points within the block) Y_μ, F_μ, and G_μ, for n=mr, m=0, 1,. . . be given as $Y_w = (y_{n+1}, \ldots, y_{n+r})^T, F_w = (f_{n+1}, \ldots, f_{n+r})^T$, then the l-block r-point methods for (1) are given by

$$Y_w = \sum_{i=0}^{j} A^{(i)} Y_{w-i} + h \sum_{i=0}^{j} B^{(i)} F_{w-i}$$

where $A^{(i)}$, $B^{(i)}$, $i = 0, \ldots, j$ are r by r matrices as introduced[2, 8, 10].

Thusly, from the above account, a block method has the numeric vantage that in each practical application program, the solution is estimated at more than one point concurrently. The number of points depends on the construction of the block method. Hence, employing these methods can give quicker and faster solutions to the problem which can be managed to generate the desired accuracy. See[17, 19]. Therefore, the main objective of this paper is to propose the reversed estimation of variable step size implementation for solving nonstiff ODEs.

The reversed estimation of variable step size implementation for solving nonstiff ODES is alone estimated in predictor alone and following variable step size technique. This technique possesses the vantages as stated[21].

Hence, the residual of this paper is harsh out as follows: in Section 2 Materials and Methods. Section 3 Results. Section 4 Conclusion as seen [2].

2. MATERIALS AND METHODS

Development of the Methods: Employing[2] in this section, the main aim is to derive the chief explicit block method of the form (2). To advance by seeking an approximation of the exact solution $y(x)$ by presuming a continuous solution $Y(x)$ of the form

$$Y(x) = \sum_{i=0}^{q-k} d_i \vartheta_i(x) \tag{3}$$

such that $x \in [a, b]$, d_i are unknown coefficients and $\vartheta_i(x)$ are polynomial basis functions of degree $q-k$, where q is the number of interpolation point and the collocation points k are respectively chosen to satisfy $q = j \geq 3$ and $k \geq 1$. The integer $j \geq 1$ denotes the step number of the method. Thus, we construct a $j+1$-step explicit block multistep method with $\vartheta_i(x) = \left(\frac{x - x_i}{h} \right)^i$ by imposing the following restrictions

$$\sum_{i=0}^{q} d_i \left(\frac{x - x_i}{h} \right) = y_{n-i}, \quad i = 0, \ldots, q-1 \tag{4}$$

$$\sum_{i=0}^{q} d_i i \left(\frac{x - x_i}{h} \right)^{i-1} = f_{n-i}, \quad i \in Z, \tag{5}$$

where y_{n+i} is he approximation for the exact solution $y(x_{n+i})$, $f_{n+i} = f(x_{n+i}, y_{n+i})$, n is the grid index and $x_{n+i} = x_n + ih$. It should be observed that equations (4) and (5) leads to a system of $q+1$ equations of the AX=B where

$$A = \begin{vmatrix} x_n^0 & x_n^1 & x_n^2 & x_n^3 & x_n^4 & \cdot \;\cdot\; \cdot & x_n^q \\ & & \cdots & \cdots & & & \\ x_{n\text{-k}}^0 & x_{n\text{-k}}^1 & x_{n\text{-k}}^2 & x_{n\text{-k}}^3 & x_{n\text{-k}}^4 & & x_{n\text{-k}}^q \\ 0 & 0 & 0 & k(k-1)(k-2)\,x_{n\text{-k}}^3 & 0 & 0\;0\;0 & 0 \\ & & & \cdots & \cdots & & \\ 0 & 0 & 0 & k(k-1)(k-2)\,x_{n\text{-k}}^3 & k(k-1)(k-2)\,x_{n\text{-k}}^4 & \cdot \;\cdot\; \cdot & k(k-1)(k-2)\,x_{n\text{-k}}^q \\ 0 & 0 & 0 & k(k-1)(k-2)\,x_{n-k-1}^3 & k(k-1)(k-2)\,x_{n-k-1}^4 & \cdot \;\cdot\; \cdot & k(k-1)(k-2)\,x_{n-k-1}^q \\ & & & \cdots & \cdots & \cdot \;\cdot\; \cdot & \\ 0 & 0 & 0 & k(k-1)(k-2)\,x_{n-\text{k-r}}^3 & k(k-1)(k-2)\,x_{n-\text{k-r}}^4 & \cdot \;\cdot\; \cdot & k(k-1)(k-2)\,x_{n-k-r}^q \end{vmatrix}$$

$$X = \left[x_0, x_1, x_2, x_3 \cdots, x_k\right]^T$$
$$U = \left[f_n, f_{n\text{-}1,\ldots,}\, f_{n\text{-k-}1}, y_n, y_{n\text{-}1}, \ldots, y_{n\text{-}k\text{-}1}\right]^T \tag{6}$$

Evaluating equation (6) using Mathematica, we get the coefficients of d_i and replacing the values of d_i into (4) and after some algebraic computation, the explicit block multistep method is obtained as

$$\sum_{i=0}^{q} \alpha_i y_{n-i} = h\left[\sum_{i=0}^{q} \beta_i f_{n\text{-i}} + \sum_{i=0}^{q} \beta_i f_{n-i}\right] \tag{7}$$

where α_i and β_i are continuous coefficients.

Overview of Block Predictor and Corrector Formulae: Concurring with[15-16], the Predictor-Corrector Formulae is

$$\left.\begin{aligned} \sum_{j=0}^{k} \alpha_j^* y_{n+j} &= \sum_{j=0}^{k} \beta_j^* f_{n+j} \\ \sum_{j=0}^{k} \alpha_j y_{n+j} &= \sum_{j=0}^{k} \beta_j^* f_{n+j} \end{aligned}\right\} \tag{8}$$

There are different ways or modes, in which the pair (8) can be applied. First, utilize the predictor to provide the initial guess $y_{n+k}^{[0]}$, then permit the looping (2) to continue till convergence is attained (in practical applications, some criterion comparable $\left\| y_{n+k}^{[v+1]} + y_{n+k}^{[v]} \right\| < \varepsilon$, where ε is of the order of round-off error, is met). This is called the mode of correcting to convergence. In this mode, the predictor represents a very auxiliary role, and the local truncation error and stability characteristics of the predictor-corrector pair are those of the corrector exclusively. Nevertheless, this mode is unattempting in practical applications because one cannot assure ahead the looping numbers of the corrector and thus the numbers of function evaluations $-$will be needed at each step.

A practically more satisfactory process is to express ahead the numbers of looping of the corrector are to be allowed at each step. Ordinarily this number is small, commonly 1 or 2. The local truncation error and stability characteristics of the predictor and corrector method in such a bounded mode depend on both the predictor and the corrector formulae. A reliable mnemonic for depicting modes of this form can be built by applying P and C to indicate single application program of the predictor or corrector respectively, and P to represent a single evaluation of the function f, given x and y. Presuppose the predictor is employ to appraise $y^{[0]}_{n+k}$, appraise $f^{[0]}_{n+k} = f(x_{n+k}, y_{n+k})$ and then use (2) at one time to get $y^{[1]}_{n+k}$. The mode is then named as PEC. When the looping is done a second time to incur $y^{[2]}_{n+k}$, which apparently implies the advance evaluation $f^{[1]}_{n+k} = f\left(x_{n+k}, y^{[1]}_{n+k}\right)$, then the mode is depicted as $PECEC$ or $P(EC)^2$. There is one father conclusion to make. At the final of the $P(EC)^2$ step obtain a value $y^{[2]}_{n+k}$ for y_{n+k} and a value $f^{[1]}_{n+k}$ for $f(x_{n+k}, y_{n+k})$. There is a choice to modify the value of f by making a farther evaluation $f^{[2]}_{n+k} = f\left(x_{n+k}, y^{[2]}_{n+k}\right)$, the mode will then be reported as $P(EC)^2$. The two categories of modes $P(EC)^m$ and $P(EC)^m E$ can be spelt as a single mode $P(EC)^m E^{1-t}$, where m is positive integer and $t = 0$ or 1, and specified by

$P(EC)^m E^{1-t}$.

$$P: \qquad y^{[0]}_{n+k} + \sum_{j=0}^{k-1} \alpha^*_j y^{[m]}_{n+j} = h \sum_{j=0}^{k-1} \beta^*_j f^{[m-t]}_{n+j}$$

$$(EC)^m. \qquad \left.\left.\begin{array}{l} f^{[v]}_{n+k} = f\left(x_{n+k}, y^{[v]}_{n+k}\right) \\ y^{[v+1]}_{n+k} + \sum_{j=0}^{k-1} \alpha_j y^{[m]}_{n+j} = h\beta_k f^{[v]}_{n+k} + h\sum_{j=0}^{k-1} \beta_j f^{[m-t]}_{n+j} \end{array}\right\} \quad v = 0,1,\dots,m-1 \right\} \qquad (9)$$

$$E^{(1-t)}. \qquad f^{[m]}_{n+k} = f\left(x_{n+k}, y^{[m]}_{n+k}\right), \qquad \text{if } t = 0.$$

Instead, the predictor and corrector formulae may be composed as

$$\rho^*(E)\, y_n = h\sigma^*(E)\, f_n, \qquad \rho(E)\, y_n = h\sigma(E)\, f_n,$$

respectively, where ρ^*, ρ and σ possess a degree k and σ^* owns degree $k-1$ at most. With this notational system, the mode $P(EC)^m E^{1-t}$ may be redefined as

$$P: \quad E^k y_n^{[0]} + \left[\rho^*(E) - E^k\right] y_n^{[m]} = h\sigma^*(E) f_n^{[m-t]}$$

$$\left.\begin{array}{l} (EC)^m: \quad E^k f_n^{[v]} = f\left(x_{n+k}, E^k y_n^{[v]}\right) \\ E^k y_n^{[v+1]} + \left[\rho(E) - E^k\right] y_n^{[m]} = h\beta_k E^k f_n^{[v]} + h\left[\sigma(E) - \beta_k E^k\right] f_n^{[m-t]} \\ v = 0,1,\ldots,m-1 \end{array}\right\} \quad (10)$$

$$E^{(1-t)}: \quad E^k f_n^{[m]} = f\left(x_{n+k}, E^k y_n^{[m]}\right), \text{if } t = 0.$$

Theorem 1: If the multistep method (2) is convergent for pth order equations, then the order of (2) is at least p. See [9].

Theorem 2: The order of a predictor-corrector method for first order equations must be ⩾ 1 if it is convergent. See [9].

Theorems 1 and 2 draws the conclusion that the order and convergence of the method holds.

3. RESULTS AND DISCUSSION

Reverse Implementation of Predictor and Corrector Formulae: Following[5, 7], as with former methods for resolving differential equations, the approximation of error in a Predictor-Corrector method is all important for the decision of a suitable step-size. Luckily, in this instance where there are two approximates of the solution at each step, this is rather square. Distinct approaches to the error approximation process rely on the selection of Predictor-Corrector formulae pairs. Together with an addition parameter available (b_{-1}) the Corrector can be constructed to possess a higher order than the Predictor, the deviation between them constituting an error approximate alike to that established on an embedded RK pair. Such a pair would, effectively, be executed in the local extrapolation mode with the higher order output supplying the starting value for the next step. The choice systematic plan of action is to select Predictor and Corrector formulae to be of the same order as sited[20-21]. In this example, the principal error terms of each formulae can be approximated by viewing the deviation between the predicted and the corrected values. Afterward, the Corrector error is then available, it is common to perform local extrapolation.

Considering the error approximation process when both formulae possess the same order. Imagine that the local true solution fulfilling (x_n, y_n) is $u(x)$; then the local errors of the predictor and the corrector values are

$$P_{n+1} - C_{n+1} = (A_P - A_C) h^{q+1} y_n^{(q+1)} O\left(h^{q+1}\right) \quad (15)$$

$$h^{q+1} y_n^{(q+1)} = \frac{P_{n+1} - C_{n+1}}{(A_P - A_C)} + O(h^{q+1}) \tag{16}$$

Neglecting terms of degree $q+2$ and above, it is well-situated to constitute approximates of the local errors of the predicted and corrected values. These are

$$e_{n+1}^{(P)} \cong \frac{A_P}{(A_P - A_C)} (P_{n+1} - C_{n+1}) < \varepsilon \tag{17}$$

$$e_{n+1}^{(C)} \cong \frac{A_C}{(A_P - A_C)} (P_{n+1} - C_{n+1}) < \varepsilon \tag{18}$$

Since equation (17) is entirely the predictor then it is referred to as the reversed estimation of correcting to convergence which is bounded by a prescribed tolerance ε. While equation (18) is exclusively the corrector, this is called the estimation of correcting to convergence which is restricted by a prescribed tolerance ε otherwise known as stopping criteria (tolerance level).

Nevertheless, the procedure of summing on the error approximate or modifier is sometimes called Milne's device but, to be uniform with more former terminology, it is more practiced identified as local extrapolation.

4. CONCLUSIONS

The reversed estimation of variable step size implementation for solving nonstiff ordinary differential equations has been analyzed. Block predictor-corrector formulae is a compendium of Adams family of the predictor-corrector formula which can be executed in $P(EC)^m$ and $P(EC)^m E$ mode. The essential implications of this work is as follows:

- Block predictor-corrector formulae are both of the same order.
- The step-number of the predictor method is one step greater than the step-number of the corrector method.
- The principal local truncation of both the predictor-corrector formulae is employed in the execution and evaluation of the maximum errors.
- The convergence criteria or tolerance level is a requirement for convergence and as well, decide whether the result is admitted or not.
- The implementation of this method comes with many computational advantages as mentioned previously.

ACKNOWLEDGEMENT

The authors would like to thank the Covenant University for providing financial support through grants during the study period.

A CONFLICT OF INTEREST

The Author(s) proclaim that there are no contending interests regarding the publication of this research paper.

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