International Journal of Engineering & Technology, 7 (2.14) (2018) 43-47

SPC

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research Paper

Analyzing and Optimizing Pedestrian Flow through a Single Route in a Topological Network

Ruzelan Khalid [1,*], Mohd. Kamal Mohd. Nawawi[2], Md Azizul Baten[3], Nurhanis Ishak[4]

[1,2]Institute of Strategic Industrial Decision Modelling (ISIDM), School of Quantitative Sciences, Universiti Utara Malaysia ,Sintok, Kedah, Malaysia
[3]Department of Statistics, School of Physical Sciences, Shahjalal University of Science and Technology, Sylhet, Bangladesh
[4]Faculty of Management and Information Technology, Universiti Islam Sultan Azlan Shah, Kuala Kangsar, Perak, Malaysia
**Email: ruzelan@uum.edu.my*

Abstract

In emergency cases, people are typically recommended to use the shortest route to minimize their travelling time. This recommendation may however not yield the optimal performance in the long run since the route may be over utilized after a certain point of time and this situation eventually causes heavy blockages. This paper thus measures the pedestrian flow performance through all available single routes in a topological network based on relevant arrival rates. The performance was measured using an *M/G/C/C* state dependent queuing approach which dynamically models pedestrians' walking speed in relation to their current density in a route. The analysis was based on an imaginary network consisting of various routes and topologies. For each route, its performance in terms of the throughput, blocking probability, expected number of pedestrians and expected travel time was first evaluated. The performance was then compared to each other and also compared to the flow performance if all available routes were utilized. The results indicated that the shortest route did not necessarily generate the optimal throughput and that the utilization of all available routes to flow pedestrians generated better performance. The optimal performance could be obtained if the arrival rate was controlled at a certain level.

Keywords: *M/G/C/C state dependent; pedestrian flow; performance evaluation; queuing system; topological network*

1. Introduction

Two or more locations are linked to each other using a network. The network can then be used to model a variety of problems. These include transportation, the minimal spanning tree, the shortest route problem and the maximal flow problem. The transportation deals with how to transport items with the minimum cost. The minimal spanning tree involves connecting all locations with the minimum distance. The shortest route problem finds the shortest route through a network. The maximal flow problem meanwhile finds the maximal flow of entities through a network with conditions that the flow is restricted to the capacity of a route and that flow must be conserved; i.e., the flow into each route must be the same with its flow in. This concept can be utilized to analyze the performance of evacuating people through a network.
The evacuation time also depends on the evacuation procedures (i.e., how an evacuation is executed) and the availability of safe egress during the evacuation process. The evacuation time can thus be reduced if pedestrian behavior can be adapted to a building environment and controlled through education and training. In quantitative studies, the pedestrian behavior is modelled in terms of walking speed. The walking speed of pedestrians through a network depends on its space size and current number of pedestrians [1, 2]. This dynamic pattern of speed and density has been replicated in various speed-density models. Examples include the *M/G/C/C* model [e.g., 3, 4, 5], the Greenberg model [6] and the Underwood model [2].

The speed-density models have been utilized to support the performance analysis of pedestrian flow through a complex network. For this, the flow from network to network is to be conserved [7, 8]. In a splitting topology, the throughput of a network is split to the number of its upstream networks to be their arrival rates. The arrival rate for a merging topology is the summation of the throughputs of its downstream networks. In a series topology, the throughput of a network is the arrival to its upstream network. In all topologies, the capacity of each network may vary and the states of downstream networks determine the states of their upstream networks. Thus, the recommendation of using the shortest route to minimize travelling time may not yield the optimal performance in the long run.

The objective of this paper is to analyze the flow performance of pedestrians through all available single routes in a topological network based on a relevant arrival rate using an *M/G/C/C* approach. The performance was then compared to each other to find the route with the best throughput. For each route, its optimal throughput was then obtained by deriving its optimal arrival rate. The throughput was next compared with the performance evaluated based on flowing the pedestrians using all available routes.

This paper is organized as follows. Section 2 briefly collects some relevant literature reviews of the *M/G/C/C* approach. Its mathematical background and how it can be used to analyze pedestrian flow are discussed in Section 3. Section 4 architects and describes our imaginary topological network as a platform for our analysis. Section 5 analyses the performance of the network based on two strategies; i.e., flowing pedestrians using single routes and using all of its available routes. Both of the performance are then compared and documented. Finally, some conclusions are given in Section 6.

2. Literature review

The pedestrian flow through a single network measured in terms of average speed is dynamically adjusted by its capacity and current density. The speed in turn determines the performance of the network which basically describes how smooth its pedestrians move. One of the approaches to measure the performance of such the state dependent network is by using an *M/G/C/C* approach. This approach measures the impact of various arrival rates to the blocking probability, throughput, expected number pedestrians in the system and expected travel (service) time. In the *M/G/C/C* approach, the network space acts as the server for its pedestrians. The capacity of the server is the maximum number of pedestrians that can accommodate the space. The server's service time representing the current walking speed depends on the current density of pedestrians. Full density of pedestrians blocks other incoming pedestrians who are then forced to queue until any of the pedestrians being serviced releases the space.

The logic of the *M/G/C/C* approach can be utilized to analyze the performance a queuing topological network. A queuing topological network is a series of networks consisting of one or several interconnected queues which later forms various types of topologies. These topologies can be seen in commercial service, social service, telecommunication, transportation and manufacturing systems. The *M/G/C/C* approach has been applied in evacuating occupants from a facility [9, 10, 11, 12, 13], flowing vehicles through a road [14, 15] and handling materials in an accumulating conveyer system [16].

Basically, a serviced pedestrian will be the arrival to the upstream queue or network. If the arrival rate is higher than the service time of the upstream network, the incoming pedestrians will be blocked. This will eventually cause congestion. This scenario happens when the capacity of the upstream network is smaller than the downstream network. Alternatively, if the arrival rate is smaller than the service time of the upstream network, the incoming pedestrians will move smoothly and cause no congestion. This will however generate a small throughput in the long run. This scenario happens when the capacity of the upstream network is bigger than the downstream network. It is thus crucial to derive the optimal arrival rate to the source network maximizing the overall throughput along all the travel route and control the arrival rate to satisfy this level. Simply flowing pedestrians based on the shortest route strategy does not necessarily generate the optimal throughput at the end.

3. An *M/G/C/C* analytical model

The *M/G/C/C* mathematical model has been presented in many previous literature [e.g., 4, 5, 13, 14, 17, 18, 19, 20, 21, 22]. It was first formulated by Yuhaski and Smith [22] relating the density of pedestrians to current walking speeds as follows:

$$V_n = A\exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right]$$

where

$$\gamma = \frac{\ln\left[\dfrac{\ln(V_a/V_1)}{\ln(V_b/V_1)}\right]}{\ln\left(\dfrac{a-1}{b-1}\right)},$$

$$\beta = \frac{a-1}{\left[\ln\left(\dfrac{V_1}{V_a}\right)\right]^{1/\gamma}} = \frac{b-1}{\left[\ln\left(\dfrac{V_1}{V_b}\right)\right]^{1/\gamma}},$$

γ, β = shape and scale parameters for the exponential model,

V_n = average walking speed for n pedestrians in a corridor,

V_a = average walking speed when crowd density is 2 peds/m^2 = 0.64 m/s,

V_b = average walking speed when crowd density is 4 peds/m^2 = 0.25 m/s,

V_1 = average walking speed for a single pedestrian = 1.5 m/s,

n = number of pedestrians in a corridor,

$a = 2 \times l \times w$,

$b = 4 \times l \times w$,

$c = 5 \times l \times w$,

l = route length in meters, and

w = route width in meters.

Based on this model, the limiting probabilities for the number of pedestrians are formulated as:

$$P_n = \frac{[\lambda E(S)]^n}{n!\, f(n) f(n-1) \ldots f(2) f(1)} P_0 \qquad n = 1, 2, 3, \ldots, c$$

where

$$P_0^{-1} = 1 + \sum_{n=1}^{C}\left[\frac{[\lambda E(S)]^i}{i!\, f(i) f(i-1) \ldots f(2) f(1)}\right]$$

λ = the arrival rate to a corridor

$E(S)$ = the expected service time of a single pedestrian in the corridor; i.e., $E(S) = l/1.5$,

P_n = the probability when there are n pedestrians in the corridor,

P_0 = the probability when there is no pedestrian in the corridor, and

$f(n)$ = the service rate and is given by $f(n) = \dfrac{V_n}{V_1}$.

Various performance measures of the corridor can then be computed as:

$$\theta = \lambda(1 - P_{balk}), \qquad E(N) = \sum_{n=1}^{C} nP_n \qquad and \qquad E(T) = \frac{E(N)}{\theta}$$

θ is the throughput of the route (in pedestrians per second; i.e., peds/s), $E(N)$ is the expected number of pedestrians in the route and $E(T)$ is the expected service time in seconds.

4. A considered topological network

Our considered topological network is as in Fig. 1. The network consists of 13 corridors connected to each other to form three types of topologies; i.e., series, merging and splitting. Each corridor has its own length and width as in Table 1. Based on the dimension, the capacity of a corridor (i.e., 5 x its space) can then be calculated. Subsequently, its optimal arrival rate can also be derived using formula discussed in in Khalid, Baten, Nawawi, & Ishak [19]. The topologies can then be converted to nodes and edges as illustrated in Fig. 2 to ease its performance analysis.

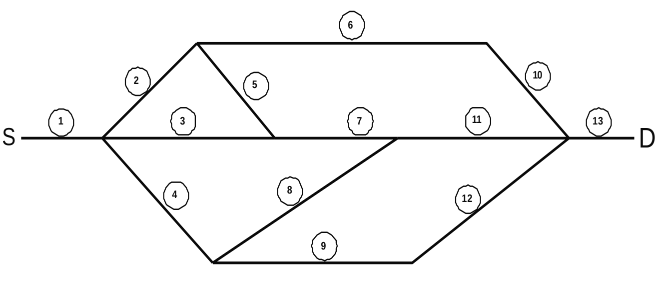

Fig. 1: Topological Network

Table 1: Dimensions, Capacities and the Optimal Arrival Rates of Corridors

Cor	Length	Width	Capacity	$\lambda_{optimal}$	P_C	$\theta_{optimal}$
1	8.0	4.0	160	4.3378	0.0085	4.3012
2	8.0	2.5	100	2.6983	0.0139	2.6608
3	12.0	2.0	120	2.1627	0.0114	2.1380
4	12.0	2.6	156	2.8189	0.0087	2.7944
5	10.0	2.5	125	2.7045	0.0109	2.6749
6	18.0	1.5	135	1.6240	0.0101	1.6076
7	10.0	2.0	100	2.1587	0.0139	2.1287
8	18.0	1.8	162	1.9523	0.0083	1.9360
9	16.0	2.1	168	2.4303	0.0080	2.4108
10	10.0	1.5	75	1.6147	0.0190	1.5839
11	10.0	2.0	100	2.1587	0.0139	2.1287
12	14.0	2.1	147	2.1238	0.0092	2.1042
13	8.0	4.0	160	4.3378	0.0085	4.3012

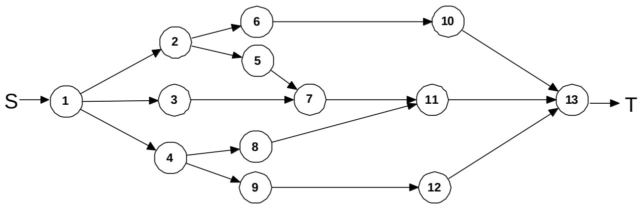

Fig. 2: Nodes and Edges of the Topological Network

5. The performance of the network

5.1 Flow based on a single route

In order to find the flow based on the shortest route, we first have to find the shortest route from the source corridor to the sink corridor in the network. Basically, the shortest-route problem is the problem of finding the shortest distance from an originating location to various destination locations. In a topological network, this involves determining the shortest route from node to node so that the total distance from any starting node to a final node can be minimized. The shortest route problem can be solved either by the shortest-route technique or by modelling the problem as a linear program [23, 24]. The linear program for the shortest route is given by:

$$\min Z = \sum_i \sum_j c_{i\to j} x_{i\to j}$$

subject to

$$\sum_j x_{i\to j} - \sum_k x_{k\to i} = \begin{cases} 1 & if\ i = S \\ -1 & if\ i = T \\ 0 & otherwise \end{cases}$$

$$x_{i\to j} \geq 0$$

where

$c_{i\to j}$ = the distance from node *i* to node *j*

$x_{i\to j}$ = the flow from node *i* to node *j*

Based on the linear program, the Lingo script for the shortest route program from corridor 1 to corridor 13 is given in Table 2.

Table 2: Lingo Script for Deriving the Shortest Route

```
1    MODEL:
2    MIN = 8 * XS_Corr1 + 8 * X_Corr1_Corr2 + 12 * X_Corr1_Corr3 + 12 * X_Corr1_Corr4 + 10 * X_Corr2_Corr5 +
3    18 * X_Corr2_Corr6 + 10 * X_Corr3_Corr7 + 18 * X_Corr4_Corr8 + 16 * X_Corr4_Corr9 + 10 * X_Corr5_Corr7 +
4    10 * X_Corr6_Corr10 + 10 * X_Corr7_Corr11 + 10 * X_Corr8_Corr11 + 14 * X_Corr9_Corr12 + 8 * X_Corr10_Corr13+
5    8 * X_Corr11_Corr13 + 8 * X_Corr12_Corr13+ 0 * X_Corr13_T;
6
7        XS_Corr1 = 1;
8        XS_Corr1 = X_Corr1_Corr2 + X_Corr1_Corr3 + X_Corr1_Corr4;
9        X_Corr1_Corr2 = X_Corr2_Corr5 + X_Corr2_Corr6;
10       X_Corr1_Corr3 = X_Corr3_Corr7;
11       X_Corr1_Corr4 = X_Corr4_Corr8 + X_Corr4_Corr9;
12       X_Corr2_Corr5 = X_Corr5_Corr7;
13       X_Corr2_Corr6 = X_Corr6_Corr10;
14       X_Corr3_Corr7 + X_Corr5_Corr7 = X_Corr7_Corr11;
15       X_Corr4_Corr8 = X_Corr8_Corr11;
16       X_Corr4_Corr9 = X_Corr9_Corr12;
18       X_Corr6_Corr10 = X_Corr10_Corr13;
19       X_Corr7_Corr11 + X_Corr8_Corr11 = X_Corr11_Corr13;
20       X_Corr9_Corr12 = X_Corr12_Corr13;
21       X_Corr10_Corr13 + X_Corr11_Corr13 + X_Corr12_Corr13 = X_Corr13_T;
22       X_Corr13_T=1;
23   END
24
25   SET GLOBAL 1
26   GO
27   QUIT
```

The script was then run by Lingo. Lingo reported that the shortest route from corridor 1 to corridor 13 was 48 meters. The route used by pedestrians to travel was through corridor 1 (8 meters), corridor 3 (12 meters), corridor 7 (10 meters), corridor 11 (10 meters) and corridor 13 (8 meters). To find the second shortest route from corridor 1 to the corridor 13, corridor 3 was removed from the script. The script in line 7 should thus be changed to XS_Corr1 = X_Corr1_Corr2 + X_Corr1_Corr4 to prohibit pedestrians to travel using corridor 3. The script was then resolved by Lingo. Lingo reported that the second shortest route was 52 meters. The route was corridors 1−2−6−10−13. The third shortest route could be extracted by removing corridor 6. This was done by changing the script in line 8 to X_Corr1_Corr2 = X_Corr2_Corr5. The script was then run again. The process was repeated until all routes from corridors 1 to 13 were evaluated. All available routes and their distances are as in Table 3.

Table 3: Available Routes and Their Distances

Rank	Route	Distance	∑ Distance
1	1−3−7−11−13	8 + 12 + 10 + 10 + 8	48
2	1−2−6−10−13	8 + 8 + 18 + 10 + 8	52
3	1−2−5−7−11−13	8 + 8 + 10 + 10 + 10 + 8	54
4	1−4−8−11−13	8 + 12 + 18 + 10 + 8	56
5	1−4−9−12−13	8 + 12 + 16 + 14 + 8	58

For each route, its performance was then analysed. The performance of the route based on the arrival rates of 3.000 peds/s for the shortest route of 1−3−7−11−13 is shown in Table 4.

Table 4: The Performance of the Shortest Route

Cor	λ	Θ	P_c	*L*	*W*
Corr1	3.0000	3.0000	0.0000	20.9090	6.9697
Corr3	3.0000	1.5654	0.4782	118.8760	75.9420
Corr7	1.5654	1.5654	0.0000	14.0287	8.9620
Corr11	1.5654	1.5654	0.0000	14.0287	8.9620
Corr13	1.5654	1.5654	0.0000	9.3564	5.9772
Total throughput of the network: 1.5654 peds/s					

This setting generated the throughput of 1.5624 peds/s. The throughput of the upstream nodes would be the arrival rate for their downstream nodes. Generally, for any nodes, flow balance must be conserved. In a splitting topology, the throughput of a node is split to be the arrival rates for their downstream nodes. In a merging topology, the sum of the throughputs of upstream nodes will be the arrival rate for their downstream node. Observe that high blocking probability of 0.4782 occurred at corridor 3 since the capacity of corridor 3 (i.e., 120 pedestrians) was much smaller than corridor 1 (160 pedestrians). Substituting $P_{balk} = 0.4782$ and 0.4782 $\lambda = 3.0000$ to $\theta = \lambda(1 - P_{balk})$ generated the throughput of 1.5654 peds/s. The arrival rates of 1.5654 peds/s to corridors 7, 11 and 13 did not generate any blockings to these corridors. The final throughput was 1.5654 peds/s.

The performance of the longest route 1−4−9−12−13 based on the arrival rates of 3.000 peds/s is shown in Table 5. As observed, blocking probability occurring in corridor 4 reduced the throughput of the corridor which would then be the throughput for corridor 9. The final throughput for the route was 2.0745 peds/s.

This performance was much better than that of the shortest route. This shows that flowing pedestrians using the shortest route does not necessarily generate the optimal throughput. For comparison, Table 6 shows the performance of the available routes based on the same arrival rate of 3.0000 peds/s. Observe the second shortest route was the lowest throughput under the arrival rate. Table 6 also derives the optimal arrival rate to achieve the optimal throughput. As observed, the shortest route could be the best route if we could only control the arrival of pedestrians at a certain level. In our case, the arrival of pedestrians should be controlled to 2.1587 peds/s.

Table 5: The Performance of the Longest Route

Cor	λ	Θ	P_c	L	W
Corr1	3.0000	3.0000	0.0000	20.9090	6.9697
Corr4	3.0000	2.0760	0.3080	150.6983	72.5906
Corr9	2.0760	2.0760	0.0000	30.2251	14.5593
Corr12	2.0760	2.0745	0.0007	34.4654	16.6142
Corr13	2.0745	2.0745	0.0000	13.0037	6.2685
Total throughput of the network: 2.0745 peds/s					

Table 6: The Performance of All Available Routes

Shortest Route	Route	θ	$\lambda_{Optimal}$	$\theta_{Optimal}$
1	1−3−7−11−13	1.5654	2.1587	2.1143
2	1−2−6−10−13	1.1762	1.6147	1.5828
3	1−2−5−7−11−13	2.0068	2.1587	2.1180
4	1−4−8−11−13	1.4334	1.6147	1.5839
5	1−4−9−12−13	2.0745	2.1238	2.1042

5.2 Optimal flow based on a single route

The linear program for finding the optimal flow for any single route is given by:

$$\max Z = x_{T\to S}$$

subject to

$$x_{i\to j} - x_{k\to i} = 0$$

$$0 \le x_{i\to j} \le \lambda_{optimal,i\to j}$$

$$x_{i\to j} \ge 0$$

where

$x_{i\to j}$ = the flow from node *i* to node *j*

$\lambda_{optimal,\, i\to j}$ = the optimal arrival rate of node *i* to node *j*

The optimal arrival of each of available corridor is as in Table 1. Using the information, the linear program for each route can be modelled. For example, the Lingo script for deriving the optimal arrival for the shortest route of 1−3−7−11−13 is shown in Table 7.

Table 7: Lingo Script for Deriving the Optimal Arrival Rate of the Shortest Route

```
MODEL:
    [R_OBJ] MAX = XT_S;

    !Flow out of a super source node equals its flow in;
    XT_S = XS_Corr1;

    !Flow out of each node equals its flow in;
    XS_Corr1 = X_Corr1_Corr3;
    X_Corr1_Corr3 = X_Corr3_Corr7;
    X_Corr3_Corr7 = X_Corr7_Corr11;
    X_Corr7_Corr11 = X_Corr11_Corr13;
    X_Corr11_Corr13 = X_Corr13_T;

    !Flow out of a super sink node equals its flow in;
    X_Corr13_T = XT_S;
    XS_Corr1 <= 4.3378;
    X_Corr1_Corr3 <= 2.1627;
    X_Corr3_Corr7 <= 2.1587;
    X_Corr7_Corr11 <= 2.1587;
    X_Corr11_Corr13 <= 4.3378;
    DATA:
        @POINTER( 1) = @STATUS();
        @POINTER( 2) = R_OBJ;
        @POINTER( 3) = XS_Corr1;
    END DATA
END
SET GLOBAL 1
GO
QUIT
```

Running the script generated the results as in Table 8.

Table 8: The Optimal Arrival Rate of the Shortest Route

Cor	λ	θ	P_C	L	W
Corr1	2.1587	2.1587	0.0000	13.6472	6.3220
Corr3	2.1587	2.1377	0.0097	32.8167	15.3516
Corr7	2.1377	2.1230	0.0068	26.4377	12.4528
Corr11	2.1230	2.1143	0.0041	25.2463	11.9407
Corr13	2.1143	2.1143	0.0000	13.3065	6.2936
Total throughput of the network: 2.1143					

Using the same approach, the optimal arrival of each of available routes is as in Table 6. As observed, in case we can control the arrival rate of pedestrians, the shortest route was still the best with the optimal throughput was 2.1143 peds/s. The lowest throughput was generated by the second shortest distance route with the throughput of 1.5828 peds/s.

5.3 Flow based on all available routes

We have seen the performance of each single route from a source corridor to a sink corridor. In this strategy, pedestrians are forced to only use a single route. Logically, if we can flow them to use all available corridors, the throughput of the network could be improved. The linear program of network flow programming is as follows:

$$\max Z = x_{T\to S}$$

subject to

$$\sum_j x_{i\to j} - \sum_i x_{j\to i} = 0 \qquad \text{[flow balance]}$$

$$0 \le x_{i\to j} \le \lambda_{opt,i\to j} \qquad \text{[flow capacities]}$$

where

$x_{i\to j}$ = the flow from origin node *i* to destination node *j*

$\lambda^{opt}_{i\to j}$ = the optimal arrival rate of origin node *i* to destination node *j*

Table 9 shows the performance of the network based on the linear program. As observed, the throughput was higher than the optimal throughput of the single route of 1−3−7−11−13; i.e., 2.1143 peds/s. Our analysis also showed that even with the arrival rate of 3.0000 peds/s, the route could generate the output of 2.1974 peds/s.

Table 9: The Performance of the Available Routes

Cor	λ	θ	P_C	L	W
Corr1	2.4221	2.4221	0.0000	15.7449	6.5006
Corr2	0.8074	0.8074	0.0000	4.7304	5.8592
Corr3	0.8074	0.8074	0.0000	7.2868	9.0256
Corr4	0.8074	0.8074	0.0000	7.0446	8.7256
Corr5	0.4037	0.4037	0.0000	2.8074	6.9546
Corr6	0.4037	0.4037	0.0000	5.2179	12.9260
Corr7	1.2110	1.2110	0.0000	9.9081	8.1816
Corr8	0.4037	0.4037	0.0000	5.1421	12.7382
Corr9	0.4037	0.4037	0.0000	4.2296	10.4776
Corr10	0.4037	0.4037	0.0000	2.9116	7.2128
Corr11	1.6147	1.5839	0.0191	23.4985	14.8355
Corr12	0.4037	0.4037	0.0000	4.2634	10.5614
Corr13	2.3913	2.3913	0.0000	15.4927	6.4788
Total throughput of the network: 2.3913					

6. Conclusion

This paper evaluates the impact of relevant arrival rates to the performance of all single routes in a topological network using a state dependent *M/G/C/C* network. Their performances in terms of the throughput and blocking are then analysed and compared. The analyses show that a shorter route does not necessarily generate better performance. This implicates that flowing pedestrians using a shorter route deemed as a better approach for moving people from location to location is not always true. To optimize the flow, the optimal arrival rate is then derived. By controlling the arrival rate to a certain level, the throughput of the route can be maximized.

Acknowledgement

This study was supported by the University Research Grant Scheme [account number 13417], Universiti Utara Malaysia. We wish to thank Universiti Utara Malaysia for the financial support. The funder had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

References

[1] Smith, JM. (1991). State-dependent queueing models in emergency evacuation networks. *Transportation Research Part B: Methodological, 25*(6), 373-389.
[2] Underwood, RT. (1960). *Speed, Volume and Density Relationships*. Victoria: Bureau of Highway Traffic, Yale University.
[3] Smith, JM. (2001). Evacuation Networks. In CA Floudas & PM Pardalos (Eds.), *Encyclopedia of Optimization* (pp. 576-584). Dordrecht: Kluwer Academic Publishers.
[4] Smith, JM, & Cruz, FRB. (2014). M/G/c/c state dependent travel time models and properties. *Physica A: Statistical Mechanics and its Applications, 395*, 560-579.
[5] Weiss, A, Williams, L, & Smith, JM. (2012). Performance & optimization of M/G/c/c building evacuation networks. *Journal of Mathematical Modelling and Algorithms, 11*(4), 361-386.
[6] Greenberg, H. (1959). An Analysis of Traffic Flow. *Operations Research, 7*(1), 79-85.
[7] Kachroo, P. (2009). *Pedestrian Dynamics: Mathematical Theory and Evacuation Control*. Boca Raton: Taylor and Francis Group.
[8] Kachroo, P, Al-nasur, SJ, Wadoo, SA, & Shende, A. (2008). *Pedestrian Dynamics: Feedback Control of Crowd Evacuation*. Berlin: Springer-Verlag.
[9] Cruz, FRB, & Smith, JM. (2007). Approximate analysis of M/G/c/c state-dependent queueing networks. *Computers & Operations Research, 34*(8), 2332-2344.
[10] Cruz, FRB, Smith, JM, & Medeiros, RO. (2005). An M/G/C/C state dependent network simulation model. *Computers & Operations Research, 32*(4), 919-941.
[11] Mitchell, DH, & Smith, JM. (2001). Topological network design of pedestrian networks. *Transportation Research Part B: Methodological, 35*(2), 107-135.
[12] Kawsar, LA, Ghani, NA, Kamil, AA, & Mustafa, A. (2012). Restricted pedestrian flow performance measures during egress from a complex facility. *World Academy of Science, Engineering and Technology, 67*, 399-404.
[13] Kawsar, LA, Ghani, NA, Kamil, AA, & Mustafa, A. (2013). Pedestrian performance measures of an M/G/C/C state dependent queueing network in emergency. *Journal of Applied Sciences, 13*(3), 437-443.
[14] Jain, R, & Smith, JM. (1997). Modeling vehicular traffic flow using M/G/C/C state dependent queueing models. *Transportation Science, 31*(4), 324-336.
[15] Stepanov, A, & Smith, JM. (2009). Multi-objective evacuation routing in transportation networks. *European Journal of Operational Research, 198*(2), 435-446.
[16] Smith, JM. (2013). Optimal workload allocation in closed queueing networks with state dependent queues. *Annals of Operations Research*, 1-27. doi: 10.1007/s10479-013-1418-0
[17] Cheah, J, & Smith, JM. (1994). Generalized M/G/C/C state dependent queueing models and pedestrian traffic flows. *Queueing Systems, 15*(1), 365-386.
[18] Cruz, FRB, MacGregor Smith, J, & Queiroz, DC. (2005). Service and capacity allocation in M/G/c/c state-dependent queueing networks. *Computers & Operations Research, 32*(6), 1545-1563.
[19] Khalid, R, Baten, MA, Nawawi, MKM, & Ishak, N. (2016). Analyzing and optimizing pedestrian flow through a topological network based on M/G/C/C and network flow approaches. *Journal of Advanced Transportation, 50*(1), 96–119. doi: 10.1002/atr.1330
[20] Khalid, R, Nawawi, MK, Kawsar, L, Ghani, N, Kamil, A, & Mustafa, A. (2016). The evaluation of pedestrians behavior using M/G/C/C analytical, weighted distance and real distance simulation models. *Discrete Event Dynamic Systems, 26*(3), 439-476. doi: 10.1007/s10626-015-0215-0
[21] Khalid, R, Nawawi, MKM, Kawsar, LA, Ghani, NA, Kamil, AA, & Mustafa, A. (2013). A discrete event simulation model for evaluating the performances of an M/G/C/C state dependent queuing system. *PLos ONE, 8*(4), 1-9. doi: 10.1371/journal.pone.0058402
[22] Yuhaski, SJ, & Smith, JM. (1989). Modeling circulation systems in buildings using state dependent queueing models. *Queueing Systems, 4*(4), 319-338.
[23] Taylor, BW. (2016). *Introduction to Management Science*. London: Pearson.
[24] Render, B, Stair, RM, Hanna, ME, & Hale, TS. (2015). *Quantitative Analysis for Management* London: Pearson.