

LOCAL NONSIMILARITY SOLUTION FOR THE IMPACT
OF THE BUOYANCY FORCE ON HEAT AND MASS TRANSFER
IN A FLOW OVER A POROUS WEDGE WITH A HEAT SOURCE
IN THE PRESENCE OF SUCTION/INJECTION

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Abstract: Combined heat and mass transfer in free, forced and mixed convection flows along a porous wedge with internal heat generation in the presence of uniform suction or injection is investigated. The boundary-layer analysis is formulated in terms of the combined thermal and solute buoyancy effect. The flow field characteristics are analyzed using the Runge–Kutta–Gill method, the shooting method, and the local nonsimilarity method. Due to the effect of the buoyancy force, power law of temperature and concentration, and suction/injection on the wall of the wedge, the flow field is locally nonsimilar. Numerical calculations up to third-order level of truncation are carried out for different values of dimensionless parameters as a special case. The effects of the buoyancy force, suction, heat generation, and variable wall temperature and concentration on the dimensionless velocity, temperature, and concentration profiles are studied. The results obtained are found to be in good agreement with previously published works.

Keywords: local nonsimilarity, suction, injection, buoyancy force, heat source.

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INTRODUCTION

The study of convection heat and mass transfer and fluid flow in porous media has received great attention in recent years. Most of the earlier studies were based on Darcy's law, which states that the volume-averaged velocity is proportional to the pressure gradient. Excellent reviews of natural convection flows in porous media have been presented by many authors (see, e.g., [1–7]). Many practical applications of convective heat transfer exist, for example, in chemical factories, in heaters and coolers of electrical and mechanical devices, in lubrication of machine parts, etc. The earliest activities in this field include the works of Gebhart and Pera [8] and Pera and Gebhart [9], where similarity solutions were obtained for a natural convection flow from a vertical surface and a horizontal surface, respectively. Chen and Yuh [10] and Kandasamy and Devi [11] studied this problem for the case of an inclined surface and a wedge surface. Many contemporary problems of heat and mass transfer do not admit similarity solutions [12–14]. The nonsimilarity of boundary layers may result from a variety of causes, such as surface mass transfer, nonuniform wall temperature and concentration, and variable pressure gradient.

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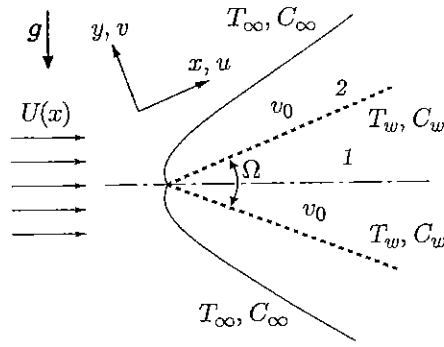


Fig. 1. Flow analysis along the wedge wall: (1) wedge; (2) porous wall.

Several numerical approaches have been developed for obtaining nonsimilar solutions in boundary layers. Among them, the local nonsimilarity method is one of the most well known methods. This method was developed by Sparrow and et al. [15, 16] and has been applied by many investigators to solve various nonsimilar boundary-layer problems [17–19]. The numerical scheme was also applied to several representative problems of boundary-layer analysis [20–22], and the results obtained were found to be in excellent agreement. However, all of the published works in this field did not reveal any applications of the local nonsimilarity method for solving the problem of nonsimilar convective heat and mass transfer in the boundary-layer flow over a wedge. Apparently, the effect of injection and suction on mixed convection along a permeable wedge with a variable surface heat flux embedded into a Darcian porous medium seems not to have been investigated.

The present study addresses the buoyancy force effects on the boundary-layer flow over a porous wedge with heat generation in the presence of uniform mass suction or injection with a nonuniform pressure gradient. The governing equations are obtained in terms of local nonsimilarity equations. Numerical solutions are obtained by employing the method of local nonsimilarity and the Runge-Kutta-Gill integration scheme in conjunction with the shooting method to satisfy the conditions at the boundary-layer edge. It is, thus, expected that the local nonsimilarity approach should yield more accurate results for the velocity, temperature, and concentration fields than those of the local similarity model. Results obtained from this study will be helpful in predicting flow, heat and mass transfer, and solute or concentration dispersion about intrusive bodies, such as salt domes, magnetic intrusions, piping, etc.

1. MATHEMATICAL ANALYSIS

A two-dimensional laminar boundary-layer flow of a viscous incompressible fluid past a porous wedge is considered (Fig. 1). The fluid is assumed to be Newtonian, and variations of its properties due to temperature depend on density and viscosity. The density variation and buoyancy effects are taken into account in the momentum equation (Boussinesq approximation). Let the x axis be taken along the wedge generatrix and the y axis be normal to it. The viscous dissipation effect and the Joule heat are neglected owing to finite conductivity of the fluid. A constant suction or injection is imposed on the wedge surface. Under these assumptions, the governing equations of the problem have the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + U_\infty \frac{dU_\infty}{dx} + [g\beta(T - T_\infty) + g\beta^*(C - C_\infty)] \sin \frac{\Omega}{2} - \frac{\nu}{K}(u - U); \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho c_p}(T - T_\infty), \quad u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}.$$

The boundary conditions are

$$u = 0, \quad v = -v_0, \quad T = T_w(x) = T_\infty + b_1 x^n, \quad C = C_w(x) = C_\infty + b_2 x^n \quad \text{at } y = 0,$$

$$u = U(x), \quad T = T_\infty, \quad C = C_\infty \quad \text{at } y = \infty.$$

In these equations, u and v are the corresponding velocity components in the x and y directions, respectively, U_∞ is the flow velocity on the outer edge of the boundary layer, ν is the kinematics viscosity, g is the acceleration due to gravity, β is the linear coefficient of thermal expansion, β^* is the volumetric coefficient of thermal expansion, T , T_w , and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature, and the free-stream fluid temperature, respectively, C , C_w , and C_∞ are the corresponding concentrations, Ω is the total angle of inclination of the wedge, K is the permeability of the wedge wall, α is the thermal conductivity of the fluid, D is the effective diffusion coefficient, v_0 is the suction (injection) velocity, the term $Q_0(T_\infty - T)$ is assumed to be the amount of heat generated (absorbed) per unit volume, and Q_0 is a constant, which may take either a positive or a negative value. If the wall temperature T_w exceeds the free-stream temperature T_∞ , the source term represents a heat source at $Q_0 < 0$ and a heat sink at $Q_0 > 0$. The third and the fourth terms on the right side of Eq. (2) are the buoyancy force and porosity of the wedge wall acting on the fluid elements.

Following [23], we perform the replacement of variables:

$$\psi(x, \eta) = \sqrt{\frac{2U\nu x}{1+m}} f(x, \eta), \quad \eta = y \sqrt{\frac{(1+m)U}{2\nu x}}, \quad \theta(x, \eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

($U = ax^m$, $m = \beta_1/(2 - \beta_1) \geq 0$, $\beta_1 = \Omega/\pi$ is the Hartree pressure gradient parameter, and Ω is the total angle of the wedge).

The continuity equation (1) is satisfied by defining a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.$$

The velocity components can be expressed as

$$u = U \frac{\partial f}{\partial \eta}, \quad v = -\left(\frac{2}{1+m} \frac{\nu U}{x}\right)^{1/2} \left(\frac{1}{2} f + \frac{1}{2} \frac{x}{U} \frac{dU}{dx} f + \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + x \frac{\partial f}{\partial x}\right).$$

Introducing a buoyancy parameter $\gamma_1 = Gr_x/Re_x^2$ and a wedge parameter $\xi = kx^{(1-m)/2} = | -v_0 | [(m+1)x/(2\nu U)]^{1/2}$, we can write the governing partial differential equations of the problem as

$$f''' + ff'' + \frac{2m}{1+m} (1 - f'^2) + \frac{2}{1+m} \gamma_1 \xi^{2(n+1-2m)/(1-m)} (\theta + N\varphi) \sin \frac{\Omega}{2} - \frac{2}{1+m} \xi^2 \lambda (f' - 1)$$

$$= \frac{1-m}{1+m} \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \xi \partial \eta} - \frac{\partial f}{\partial \xi} \frac{\partial^2 f}{\partial \eta^2} \right),$$

$$\theta'' - \frac{2n}{1+m} Pr f' \theta + Pr f \theta' - \frac{2}{1+m} Pr \delta \xi^2 \theta = \frac{1-m}{1+m} Pr \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \theta}{\partial \eta} \frac{\partial f}{\partial \xi} \right), \quad (3)$$

$$\varphi'' - \frac{2n}{1+m} Sc f' \varphi + Sc f \varphi' = \frac{1-m}{1+m} Sc \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial \varphi}{\partial \xi} - \frac{\partial \varphi}{\partial \eta} \frac{\partial f}{\partial \xi} \right).$$

The boundary conditions are

$$\eta = 0: \quad \frac{\partial f}{\partial \eta} = 0, \quad f = \frac{2}{1+m} \xi, \quad \theta(0) = 1, \quad \varphi(0) = 1,$$

$$\eta \rightarrow \infty: \quad \frac{\partial f}{\partial \eta} = 1, \quad \theta(\infty) = 0, \quad \varphi(\infty) = 0.$$

Here the primes denote partial derivatives with respect to η , Sc is the Schmidt number, Pr is the Prandtl number, Re_x is the Reynolds number, Gr_x is the Grashof number, Gr_C and Gr_T are the modified Grashof numbers for concentration and temperature, N is the buoyancy ratio, γ_1 is the buoyancy parameter, δ is the heat-source parameter, and λ is the porosity parameter:

$$\text{Gr}_C = \frac{\nu g \beta^* (C - C_\infty)}{U^3}, \quad \text{Gr}_T = \frac{\nu g \beta (T - T_\infty)}{U^3}, \quad \text{Gr}_x = \frac{g \beta (T_w - T_\infty) x^3}{\nu^2},$$

$$\text{Re}_x = \frac{U x}{\nu}, \quad N = \frac{\text{Gr}_C}{\text{Gr}_T}, \quad \gamma_1 = \frac{g \beta b_1}{a k^2 (n+1-2m)/(1-m)}, \quad \delta = \frac{Q_0}{a \rho c_p}, \quad \lambda = \frac{\nu}{K a}.$$

It may be observed that Eqs. (3) remain partial differential equations after the transformation, with terms containing $\partial/\partial\xi$ in the right side. In this system of equations, it is obvious that the nonsimilarity aspects of the problem are embodied in terms containing partial derivatives with respect to ξ . This problem does not admit similarity solutions. Thus, to obtain a solution of the system containing terms with derivatives with respect to ξ , it is necessary to employ a numerical scheme suitable for partial differential equations.

2. LOCAL NONSIMILARITY SOLUTION

This Section deals with the local nonsimilarity method initiated by Sparrow and Yu [16] and applied by many investigators to solve various nonsimilarity boundary-value problems. Let us formulate the system of equations for the local nonsimilarity model with reference to the present problem.

At the first level of truncation of system (3), the terms containing $\xi \partial/\partial\xi$ are small. This is particularly true for $\xi \ll 1$. Thus, the terms with $\xi \partial/\partial\xi$ in the right sides of Eqs. (3) can be deleted. As a result, we obtain the system of equations

$$f''' + f f'' + \frac{2m}{1+m} (1 - f'^2) + \frac{2}{1+m} \gamma_1 \xi^{2(n+1-2m)/(1-m)} (\theta + N\varphi) \sin \frac{\Omega}{2} - \frac{2}{1+m} \xi^2 \lambda (f' - 1) = 0,$$

$$\theta'' - \frac{2n}{1+m} \text{Pr} f' \theta + \text{Pr} f \theta' - \frac{2}{1+m} \text{Pr} \delta \xi^2 \theta = 0, \quad (4)$$

$$\varphi'' - \frac{2n}{1+m} \text{Sc} f' \varphi + \text{Sc} f \varphi' = 0.$$

Equations (4) can be regarded as a system of ordinary differential equations for the functions f , θ , and φ . The variable ξ is considered as a parameter. For the next level of truncation, we introduce the functions

$$f_1 = \frac{\partial f}{\partial \xi}, \quad \theta_1 = \frac{\partial \theta}{\partial \xi}, \quad \varphi_1 = \frac{\partial \varphi}{\partial \xi},$$

$$f_2 = \frac{\partial f_1}{\partial \xi}, \quad \theta_2 = \frac{\partial \theta_1}{\partial \xi}, \quad \varphi_2 = \frac{\partial \varphi_1}{\partial \xi}.$$

As a result, we obtain equations up to the third level of truncation:

$$f''' + f f'' + \frac{2m}{1+m} (1 - f'^2) + \frac{2}{1+m} \gamma_1 \xi^{2(n+1-2m)/(1-m)} (\theta + N\varphi) \sin \frac{\Omega}{2}$$

$$- \frac{2}{1+m} \xi^2 \lambda (f' - 1) = \frac{1-m}{1+m} \xi (f' f'_1 - f_1 f''),$$

$$\theta'' - \frac{2n}{1+m} \text{Pr} f' \theta + \text{Pr} f \theta' - \frac{2}{1+m} \text{Pr} \delta \xi^2 \theta = \frac{1-m}{1+m} \text{Pr} \xi (f' \theta_1 - \theta' f_1), \quad (5)$$

$$\varphi'' - \frac{2n}{1+m} \text{Sc} f' \varphi + \text{Sc} f \varphi' = \frac{1-m}{1+m} \text{Sc} \xi (f' \varphi_1 - \varphi' f_1);$$

$$f_1''' + f_1 f_1'' + f f_1'' - \frac{4m}{1+m} f' f_1' + \frac{2}{1+m} \gamma_1 \xi^{2(n+1-2m)/(1-m)} \left(\frac{2(n+1-2m)}{1-m} \xi^{-1} (\theta + N\varphi) + (\theta_1 + N\varphi_1) \right) \sin \frac{\Omega}{2}$$

$$- \frac{2}{1+m} \lambda (2\xi (f' - 1) + \xi^2 f_1') = \frac{1-m}{1+m} (f' f_1' - f'' f_1 + \xi (f_1' f_1' - f_1'' f_1)),$$

$$\theta_1'' - \frac{2n}{m+1} \text{Pr} (f_1' \theta + f' \theta_1) + \text{Pr} (f_1 \theta' + f \theta_1') - \frac{2}{1+m} \text{Pr} \delta(\xi^2 \theta_1 + 2\xi \theta) \quad (6)$$

$$= \frac{1-m}{1+m} \text{Pr} (f' \theta_1 - \theta' f_1 + \xi(f_1' \theta_1 - \theta_1' f_1)),$$

$$\varphi_1'' - \frac{2n}{m+1} \text{Sc} (f_1' \varphi + f' \varphi_1) + \text{Sc} (f_1 \varphi' + f \varphi_1') = \frac{1-m}{1+m} \text{Sc} (f' \varphi_1 - \varphi' f_1 + \xi(f_1' \varphi_1 - \varphi_1' f_1));$$

$$f_2''' + f_2 f_2'' + 2f_1 f_1'' + f f_2'' - \frac{4m}{1+m} (f' f_2' + f_1' f_1')$$

$$+ \frac{2}{1+m} \gamma_1 \frac{2(n+1-2m)}{1-m} \xi^{2(n+1-2m)/(1-m)} \xi^{-1}$$

$$\times \left(\frac{2(n+1-2m)}{1-m} \xi^{-1} (\theta + N\varphi) + (\theta_1 + N\varphi_1) \right) \sin \frac{\Omega}{2} + \frac{2}{1+m} \gamma_1 \xi^{2(n+1-2m)/(1-m)}$$

$$\times \left(- \frac{2(n+1-2m)}{1-m} \xi^{-2} (\theta + N\varphi) + (\theta_1 + N\varphi_1) \right)$$

$$+ \frac{2(n+1-2m)}{1-m} \xi^{-1} (\theta_1 + N\varphi_1) + (\theta_2 + N\varphi_2) \Big) \sin \frac{\Omega}{2}$$

$$- \frac{2}{1+m} \lambda (2(f' - 1) + 4\xi f_1' + \xi^2 f_2')$$

$$= \frac{1-m}{1+m} (f' f_2' + f_1' f_1' - f_1'' f_1 - f'' f_2 + \xi(2f_1' f_2' - f_1'' f_2 - f_2'' f_1)),$$

$$\theta_2'' - \frac{2n}{m+1} \text{Pr} (2f_1' \theta_1 + f_2' \theta + f' \theta_2) + \text{Pr} (2f_1 \theta_1' + f_2 \theta' + f \theta_2') \quad (7)$$

$$- \frac{2}{1+m} \text{Pr} \delta(\xi^2 \theta_2 + 4\xi \theta_1 + 2\theta)$$

$$= \frac{1-m}{1+m} \text{Pr} (f' \theta_2 + f_1' \theta_1 - \theta' f_2 - \theta_1' f_1 + \xi(f_1' \theta_2 + f_2' \theta_1 - \theta_1' f_2 - \theta_2' f_1)),$$

$$\varphi_2'' - \frac{2n}{m+1} \text{Sc} (2f_1' \varphi_1 + f_2' \varphi + f' \varphi_2) + \text{Sc} (2f_1 \varphi_1' + f_2 \varphi' + f \varphi_2')$$

$$= \frac{1-m}{1+m} \text{Sc} (f' \varphi_2 + f_1' \varphi_1 - \varphi' f_2 - \varphi_1' f_1 + \xi(f_1' \varphi_2 + f_2' \varphi_1 - \varphi_1' f_2 - \varphi_2' f_1)).$$

System (6) is obtained by differentiating system (5) with respect to ξ . In this set, all the terms in the right side are retained. Differentiating system (6) with respect to ξ again, we obtain system (7), in which the derivatives of the functions f_2 , θ_2 , and φ_2 with respect to ξ are neglected. The systems obtained satisfy the following boundary conditions:

$$f(\xi, 0) = \frac{2}{1+m} s, \quad f'(\xi, 0) = 0, \quad \theta(\xi, 0) = \varphi(\xi, 0) = 1,$$

$$f_1(\xi, 0) = f_1'(\xi, 0) = \theta_1(\xi, 0) = \varphi_1(\xi, 0) = 0,$$

$$f_2(\xi, 0) = f_2'(\xi, 0) = \theta_2(\xi, 0) = \varphi_2(\xi, 0) = 0, \quad (8)$$

$$f'(\xi, \infty) = 1, \quad \theta(\xi, \infty) = \varphi(\xi, \infty) = f_1'(\xi, \infty) = \theta_1(\xi, \infty) = \varphi_1(\xi, \infty) = 0,$$

$$f_2'(\xi, \infty) = \theta_2(\xi, \infty) = \varphi_2(\xi, \infty) = 0.$$

As at the lower levels of truncation, system (5)–(7) together with the boundary conditions (8) contains nine functions f , f_1 , f_2 , θ , θ_1 , θ_2 , φ , φ_1 , and φ_2 , which are mutually coupled. The total order of this system is now 21. For given values of the pertinent parameters, these equations can be treated as ordinary differential equations that contain the parameter ξ . From the solutions of the above-derived equations, we are interested in obtaining solutions only for the functions f , θ , and φ and their derivatives. System (5)–(7) is solved by the Runge–Kutta–Gill integration method in conjunction with the shooting technique. The details of the computational technique have already been discussed in [24, 25]. It should be noted here that five to seven iterations were sufficient to ensure convergence of the solutions within 10^{-6} .

Table 1. Comparison of the values of $f''(\xi, 0)$ and $-\theta'(\xi, 0)$ for different values of Gr_x/Re_x^2

Gr_x/Re_x^2	Data of [26]		Present work	
	$f''(\xi, 0)$	$-\theta'(\xi, 0)$	$f''(\xi, 0)$	$-\theta'(\xi, 0)$
0	0.33206	0.29268	0.33206	0.29268
0.2	0.55713	0.33213	0.55707	0.33225
0.4	0.75041	0.35879	0.75007	0.35910
0.6	0.92525	0.37937	0.92449	0.37986
0.8	1.08792	0.39640	1.08700	0.39685
1.0	1.24170	0.41106	1.24062	0.41149
2.0	1.92815	0.46524	1.92689	0.46551
10.0	5.93727	0.64956	5.93665	0.64959

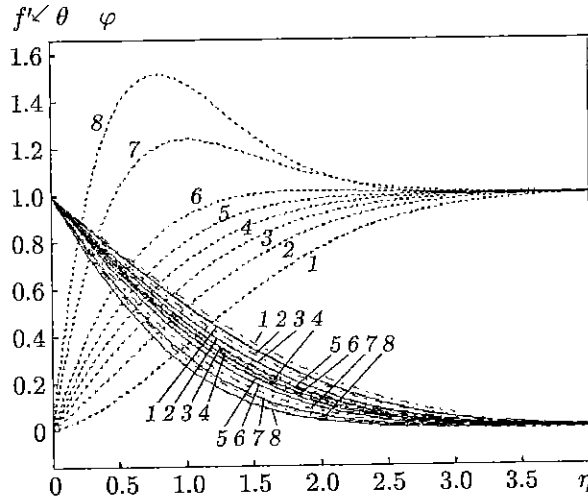


Fig. 2. Effect of the buoyancy parameter on the velocity (dotted curves), temperature (solid curves), and concentration (dot-and-dashed curves) profiles at $m = 0.6667$, $S = \xi = 0.1$, $\delta = 0.1$ and $\gamma_1 = -2$ (1), -1.5 (2), -1 (3), -0.5 (4), 0.1 (5), 1 (6), 5 (7), and 10 (8).

3. RESULTS AND DISCUSSION

In the present investigation, the results are obtained by two different methodologies, namely, the Runge-Kutta-Gill method in conjunction with the shooting technique and the local nonsimilarity method with the third level of truncation. The two-point boundary-value problem of the nonsimilar system of ordinary differential equations is obtained by the Falkner-Skan transformation. This system of ordinary differential equations with initial conditions that take into account the skin friction and the rate of heat and mass transfer is integrated by the Runge-Kutta-Gill method. The results are presented graphically for the dimensionless velocity, temperature, and concentration distributions as functions of η for various prescribed parameters. The numerical computation is carried out for $Pr = 0.72$, $N = 1$, $Sc = 0.62$, $\lambda = 0.1$, and $n = 0.4$ in the power law of temperature and concentration. In order to validate our method, we compared the skin friction $f''(0)$ and the rate of heat transfer $-\theta'(0)$ obtained for different values of Gr_x/Re_x^2 (Table 1) with those of Minkowycz [26] and found them to be in excellent agreement.

The effect of the buoyancy parameter on the dimensionless velocity and temperature distributions inside the boundary layer in the presence of a heat source and suction aligned at an angle of 72° to the horizontal axis is illustrated in Fig. 2. The positive values of the buoyancy parameter correspond to an assisting flow past the wedge, while the negative values of the buoyancy parameter correspond to an opposing flow past the wedge. In the assisting flow, the convection mode is dominated by forced convection at $\gamma_1 < 1$, by free convection at $\gamma_1 > 1$, and by mixed convection at $\gamma_1 = 1$. It is seen in Fig. 2 that an increase in the buoyancy parameter γ_1 increases the fluid velocity inside the boundary layer, but decreases the fluid temperature and concentration, whereas a decrease in the buoyancy parameter decreases the fluid velocity inside boundary layer, but increases the fluid temperature and concentration. It is also noted that the mode dominated by free convection exerts a substantially greater effect on the velocity profile than the modes dominated by mixed and forced convection.

Table 2. Analysis of skin friction and rate of heat and mass transfer at $Pr = 0.72$, $N = 1$, $Sc = 0.62$, $m = 2/3$, $n = 0.4$, $S = 0.1$, $\xi = 0.1$, $\lambda = 0.1$, and $\delta = 0.1$

γ_1	$f''(\xi, 0)$	$\theta'(\xi, 0)$	$\varphi'(\xi, 0)$	$f''(0, \eta)$	$\theta'(0, \eta)$	$\varphi'(0, \eta)$
-2.0	0.123672	-0.504939	-0.476211	-6.836902	1.475289	1.357969
-1.5	0.480123	-0.569138	-0.534981	-3.822628	0.602907	0.563315
-1.0	0.776502	-0.611620	-0.573934	-2.225439	0.291086	0.277659
-0.5	1.043462	-0.644848	-0.604438	-1.018297	0.111728	0.112787
0.1	1.339282	-0.677709	-0.634625	0.188519	-0.030944	-0.018700
1.0	1.749301	-0.718189	-0.671859	1.732609	-0.174579	-0.151439
5.0	3.313301	-0.839868	-0.783977	7.045659	-0.485509	-0.440071
10.0	4.968368	-0.938448	-0.876607	12.331609	-0.663532	-0.602591

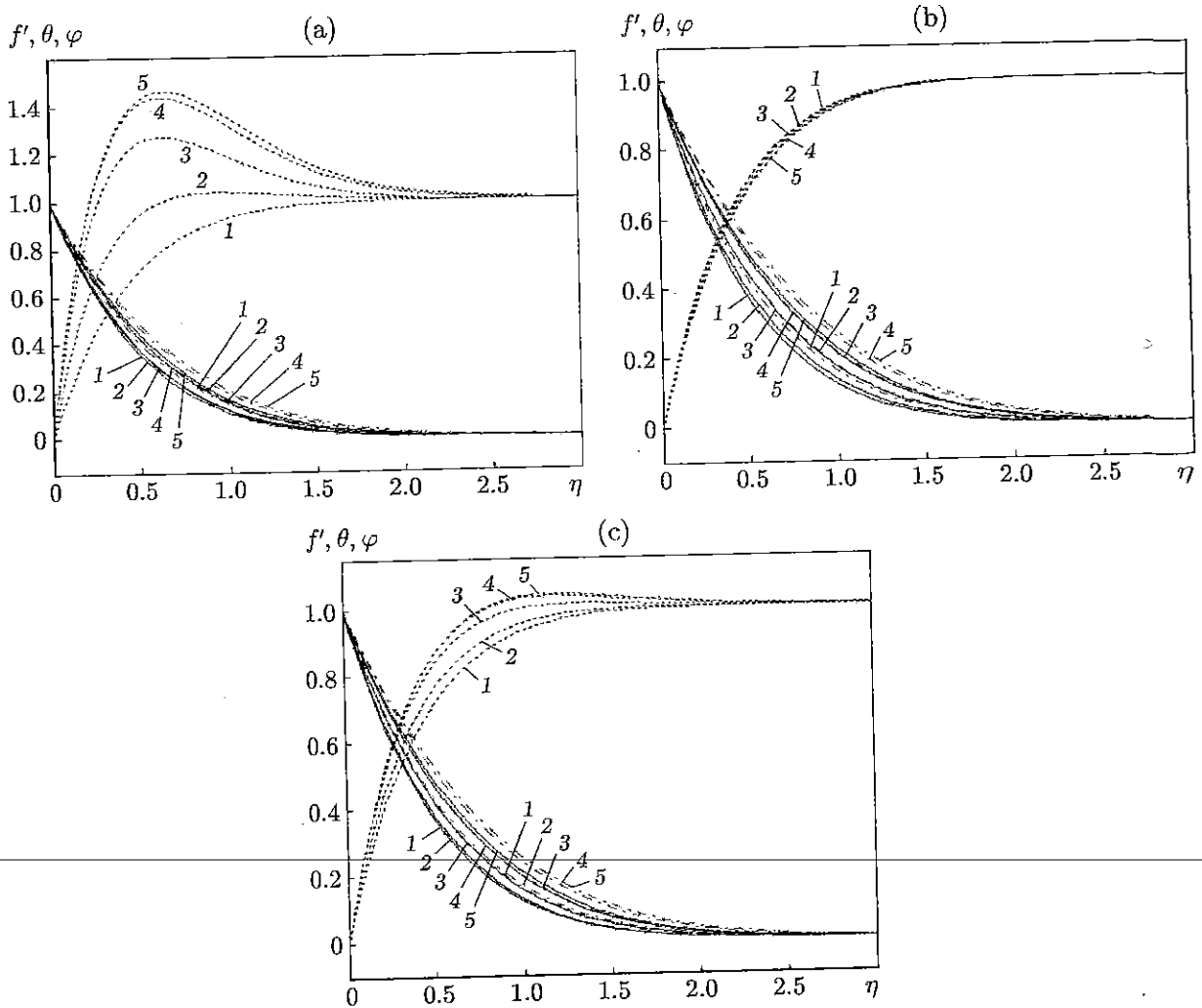


Fig. 3. Effect of the index in the power law of the free-stream velocity on the velocity (dotted curves), temperature (solid curves), and concentration (dot-and-dashed curves) profiles at $\lambda = 0.1$ and $\delta = 0.1$: (a) free convection ($\gamma_1 = 5$); (b) forced convection ($\gamma_1 = 0.1$); (c) mixed convection ($\gamma_1 = 1$); $m = 0$ (1), 0.0909 (2), 0.3333 (3), 0.6667 (4), and 0.8000 (5).

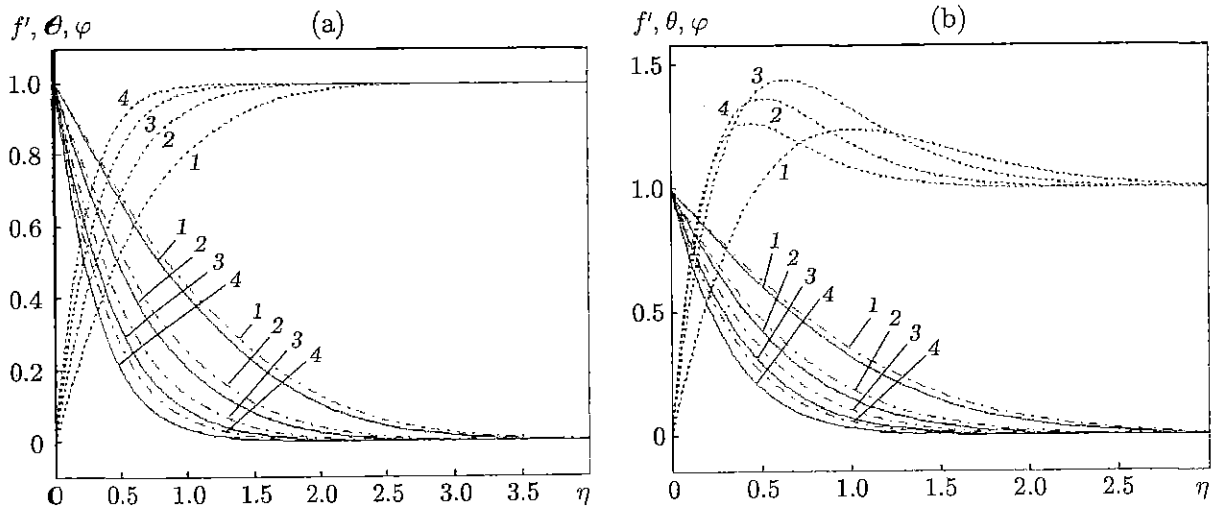


Fig. 4. Effect of suction on the velocity (dotted curves), temperature (solid curves), and concentration (dot-and-dashed curves) profiles at $n = 0.4$, $\lambda = 0.1$, and $\delta = 0.1$: (a) forced convection; (b) free convection; $S = 0.1$ (1), 1 (2), 2 (3), and 3 (4).

The values of the skin friction and the rate of heat and mass transfer are listed in Table 2. It follows from Table 2 that the skin friction increases with increasing buoyancy parameter, whereas the rate of heat and mass transfer decreases.

Figure 3 illustrates the effect of the index in the power law of the free-stream velocity near the leading edge ($\xi = 1$) on heat and mass transfer in the convective flow past a porous wedge in the presence of the buoyancy force and suction. The index m in the power law of the free-stream velocity has different effects on velocity inside the boundary layer in the case of the free, forced, and mixed convection modes. It is seen that an increase in m increases the fluid velocity inside the boundary layer in the case of the free and mixed convection modes (see Figs. 3a and 3b). On the contrary, in the forced convection mode, an increase in m decreases slightly the fluid velocity inside the boundary layer (see Fig. 3b). In addition, an increase in the power index m decreases both the temperature and concentration inside the boundary layer for all convection modes (free, forced, and mixed convection).

Figure 4 shows the variations of the velocity, temperature, and concentration profiles in the forced and free convection modes in the presence of the heat source for different values of suction $S > 0$. The dimensionless wedge parameter ξ is related to the suction parameter as $\xi = |S|$, where $S = -v_0((m+1)/(2\nu a))^{1/2} x^{(1-m)/2}$ and $v_0 > 0$. It is seen in Fig. 4 that an increase in the suction parameter S increases the fluid velocity inside the boundary layer, but decreases the fluid temperature and concentration.

Figure 5 presents the heat-source effect on the velocity, temperature, and concentration profiles near the leading edge ($\xi = 1$) on heat and mass transfer in the cases of forced, free, and mixed convection past the porous wedge in the presence of suction ($S = 1$). It is observed that an increase in the heat-source parameter decreases the fluid velocity inside the boundary layer in the free and mixed convection modes (see Figs. 5b and 5c), but the velocity profile in the forced convection mode remains constant (see Fig. 5a). In addition, an increase in the heat-source parameter decreases significantly the fluid temperature inside the boundary layer for all modes of convection, but the fluid concentration in the boundary layer remains constant (see Fig. 5).

The effect of the Schmidt number on the velocity, temperature, and concentration profiles is illustrated in Fig. 6. It is seen from this figure that the fluid concentration decreases with an increase in the Schmidt number, whereas there are only minor changes in the velocity and concentration profiles. This causes the concentration buoyancy to decrease, yielding a small reduction in the fluid velocity. The decrease in the concentration field due to the increase in the Schmidt number can be illustrated, for instance, with hydrogen ($Sc = 0.32$) being replaced by water vapour ($Sc = 0.62$) and then by ammonia ($Sc = 1.00$) in the said sequence. The decrease in concentration due to the increase in Sc can be explained by the combined effect of the magnetic field and the buoyancy force on the wedge wall.

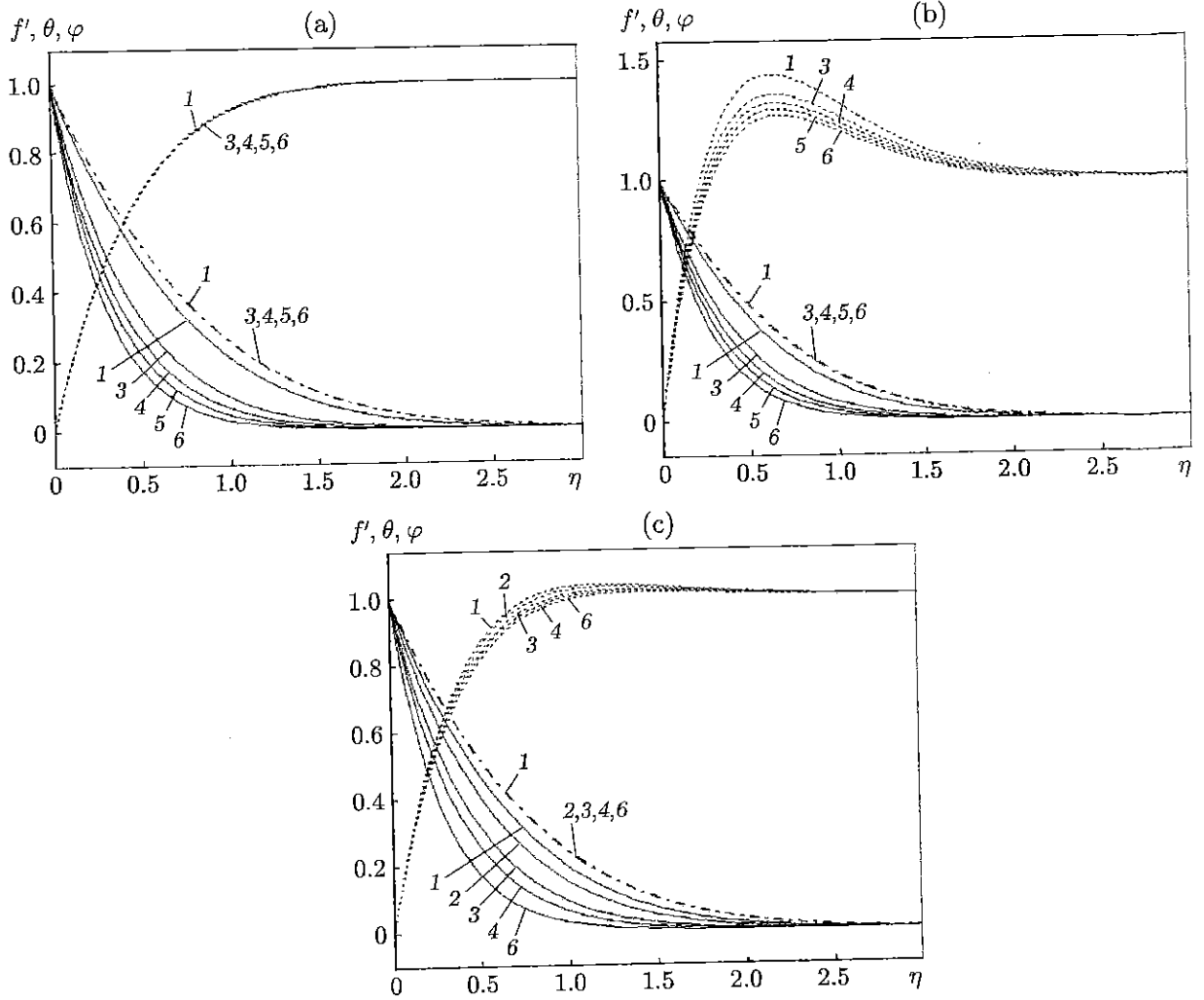


Fig. 5. Effect of the heat source on the velocity (dotted curves), temperature (solid curves), and concentration (dot-and-dashed curves) profiles ($m = 0.6667$, $n = 0.4$, and $\lambda = 0.1$) at $\gamma_1 = 0.1$ (a), 5 (b), and 1 (c): $\delta = 0.1$ (1), 1 (2), 3 (3), 5 (4), 7 (5), and 10 (6).

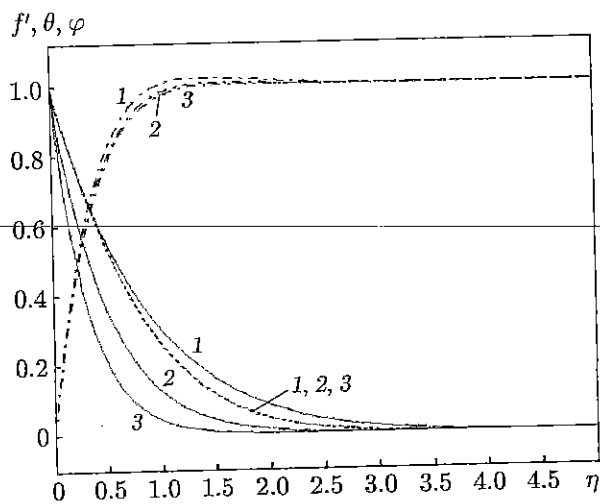


Fig. 6. Effect of the Schmidt number on the velocity (dot-and-dashed curves), temperature (dotted curves), and concentration (solid curves) profiles at $N = 1$, $\lambda = 0.1$, $\gamma_1 = 1.0$, $n = 0.4$, and $\xi = 0.1$: $Sc = 0.32$ (1), 0.62 (2), and 1.00 (3).

CONCLUSIONS

Extensive numerical integrations are carried out with the use of the Runge–Kutta–Gill method in conjunction with the shooting method and the local nonsimilarity method with the third-order level of truncation. Results of numerical calculations in wide ranges of the problem parameters are presented. In particular, it is found that the positive sign of the buoyancy parameter leads to acceleration of the fluid flow, whereas the negative sign of the buoyancy parameter means deceleration of the fluid flow. The opposing flow causes a decrease in the pressure gradient and the boundary-layer separation. Furthermore, the temperature and concentration inside the boundary layer increase with increasing power index in the law of the free-stream velocity for all convection modes. With increasing heat-source parameter, the fluid velocity inside the boundary layer substantially decreases in the free convection mode, slightly decreases in the mixed convection mode, and remains constant in the forced convection mode. The buoyancy force with the heat source and suction on the wall has a substantial effect on the flow field and, thus, on the rate of heat and mass transfer from the sheet to the fluid. A method of solving the nonlinear Falkner–Skan boundary-layer problem is proposed in the paper. Such a numerical solution with the third level of truncation for the flow past a porous wedge is obtained for the first time.

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