Evaluation of Tensile Strength of Brazilian Test under Solid and Ring Disks using Finite Element Analysis
(Penilaian Kekuatan Regangan bagi Uji Kaji Brazil di bawah Cakera Utuh dan Cincin menggunakan Analisis Unsur Terhingga)

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ABSTRACT

The tensile strength of intact rock materials has been determined by indirect method more frequent than the direct method. The most commonly used indirect method is Brazilian test. Stress and deformability undergo during the test reflected by geometry shape of the samples with respect to the different diameter ratio. This study focuses on influence of geometry shape in solid and ring disk with different diameter ratio on the stress distribution and deformations within sandstone subjected to indirect tensile loading by Brazilian test. Then, the finite element method in RS^2 software was utilised to simulate and gain in depth understanding the behaviour of Brazilian test. The analysis shown that the maximum tensile strength in a ring disk with diameter ratio of 0.1 is three times higher than in solid disk. Meanwhile, as the diameter ratio of ring disk increases, it produces lower tensile strength. The numerical simulation also has successfully illustrated the shear failure which observed near the loading platen of solid disk during Brazilian test. The finite element analysis utilised in this research has successfully enables the stress distribution and deformation behaviour of the rock under tension to be studied closely.

Keywords: Brazilian test; finite element method; ring test; RS^2; tensile strength

ABSTRAK

Kekuatan regangan batuan utuh ditentukan dengan kaedah tidak langsung lebih kerap berbanding daripada kaedah langsung. Kaedah tidak langsung yang paling biasa digunakan adalah uji kaji Brazil. Tekanan dan perubahan bentuk yang dialami semasa uji kaji merujuk kepada bentuk geometri sampel dengan perkaitan nisbah diameter yang berbeza. Kajian ini memberi tumpuan kepada pengaruh bentuk geometri dalam cakera utuh dan cincin dengan nisbah diameter yang berbeza pada agihan tegasan dan perubahan bentuk di dalam batu pasir yang tertakluk kepada bebanan regangan tidak langsung dari uji kaji Brazil. Kaedah unsur terhingga di dalam perisian RS^2 telah digunakan untuk mensimulasi dan memahami dengan lebih mendalam sifat uji kaji Brazil. Analisis menunjukkan bahawa kekuatan regangan maksima di dalam cakera cincin dengan nisbah diameter 0.1 adalah tiga kali lebih besar daripada cakera utuh. Sementara itu, semakin besar nisbah diameter cakera cincin, semakin kecil kekuatan regangan yang dihasilkan. Simulasi berangka telah berjaya menggambarkan tegasan ricih yang wujud berhampiran beban plat di dalam uji kaji Brazil pada cakera utuh. Analisis unsur terhingga yang digunakan dalam kajian ini telah berjaya membolehkan agihan tegasan dan perubahan sifat bentuk batuan di bawah tekanan untuk dikaji dengan teliti.

Kata kunci: Kaedah unsur terhingga; kekuatan regangan; RS^2; uji kaji Brazil; uji kaji cincin

INTRODUCTION

Rock material is strong in compression, yet weak in tension. According to the rule of thumb, the tensile strength is 10% of its unconfined compression strength (UCS). Thus, tensile strength plays more important role in the failure because rocks are weaker in tension than in compression (Goh et al. 2012). Regardless of its decisive, the tensile strength is usually measured using indirect method rather than directly because of difficulties in implementing the test. Hoek (1964) described, a valid direct tensile test should result in failure at the midpoint of the disk. With stress concentrations at the ends of the disk failure can initiate near the grips resulting in an invalid test.

For that, the most commonly used indirect test is the diametrical compression of thin disks, frequently referred as the Brazilian test. As mention in paper by Erarslan et al. (2012), the Brazilian test was officially proposed by the International Society for Rock Mechanics (ISRM) as a suggested method for determining the tensile strength of rock materials.

A study by Tsutsumi et al. (2016) shows that, the Brazilian test is performed by placing a disk-shaped sample with thickness twice its diameter in between two rigid platens. Compressive load is applied via steel platen which is in contact with the sample. Since tensile strength of rock is smaller than its compressive strength, therefore, the rock material fails under tensile manner, usually in the

form of vertical diametrical fracture. The test induces a biaxial state of stress in which the stress at the centre of the circular plane is compressive in x-direction (σ_x) and tensile in y-direction (σ_y). For the rocks tested, the solid disks failed along the loaded diameter at a higher load than the disks with a central hole. The shear stresses close to the loading platens at failure were therefore greater for the solid disks of rock (Chou & Chen 2008).

ISRM (2007) suggested formula for calculating the splitting tensile strength σ_t (MPa) based on the Brazilian test is formulated as:

$$\sigma_t = 2P/\pi Dt = 0.636\ P/Dt \qquad (1)$$

where P is the load at failure (N); D is the diameter of the test specimen; and t is the thickness of the test specimen measured at the centre.

Analytical study showed that the tensile stress generated was influenced by geometry of the samples (solid and ring) disk in Brazilian test. Hudson (1969) reported that the formula for the ring test tensile strength is express as in (2) below and the average tensile strength varies with the disk geometry and loading configuration. An approximate value for K mentioned by Hobbs (1964) as in (3).

$$\sigma_t = 2\ PK/\pi Dt \qquad (2)$$
$$K = 6 + 38\ \bar{r}^2 \text{ for } 0 > \bar{r} > 0.1 \qquad (3)$$

where K is a stress concentration factor; and ($\bar{r}$) is a function of the relative hole radius.

For initiation and propagation of cracks discussed by Chen and Hsu (2001), the results of the ring shows cracks does not start at the platen contacts since in many tests both ring and solid Brazilian, the diametrical crack in some solid disks tend to 'gape' at the centre, suggesting that failure probably started in the mid-section. According to Gramberg (1965), the crack surfaces of failed ring disks in Lucite, provided clear evidence in the form of 'ray patterns' that failure originated near the hole walls. A variety of secondary crack patterns were formed in ring disks when loading was inadvertently continued after primary failure (Yanagidani 1978). When analysed in accordance with existing theory, results of ring tests give tensile strengths which are, in general far higher than the uniaxial values (Hague 2010).

Research into behaviour of rock fracture is somewhat more complex and not well understood. This makes the field of Rock Mechanics in particular use of Finite Element Analysis (FEA) in studying the behaviour of the rock during fracture where many parameters are involved. The Finite Element used in the proposed modelling enables the facture process of the rock to be studied closely in relation to the force component causing the start of fracture and the behaviour of the propagating crack during this process (Cheng 2000). Yet, the experimental results are needed to justify and verify the results from FEA modelling (Aresh et al. 2016). Behaviour of rock fractures and the growth of cracks or micro-cracks in rocks are strongly influenced by heterogeneous microstructures (Li et al. 2013). The numerical methods used for studying the fracture behaviour

of rocks should take heterogeneity into consideration. Kennedy (1972) also mentioned in numerical simulations, rock heterogeneity was found as a stress concentrator, similar to the presence of pre-existing cracks under loading. In front of the crack propagation direction, if the strength of the elements is not too high, the crack propagates in a straight manner, which may be considered as the transgranular failures observed in experiments.

This study, make use of the RS^2, a powerful two-dimensional finite element program for soil and rock applications (Rocscience 2016). RS^2 is used for a wide range in engineering projects such as analysis and design of excavation and slope stability (Abdullah et al. 2015; Al-Bared et al. 2015). According to Khalfalla (2016), 2D finite element analyses were carried out in the research in order to establish how the stresses and strains in the homogenous and isotropic Acrylic disk change with different ratios of contact area by using RS^2 software.

Materials and Methods

Sandstone samples were taken from Kg. Bakong, Pahang. The petrography analysis is important to identify types of mineral in rock (Azlan et al. 2017). The result showed that, percentage of mineral composition obtained from rough counting on thin section denotes 92% of Quartz, 5% of Alkali Feldspar, 3% of Lithic Fragment and 5% of Muscovite. The thin section was analysed under cross and plane polarized light with 10 times magnification to identify each mineral under the microscope (Figure 1). Thus, from the rough counting, this sample was named as Sandstone.

Other than Brazilian test, Uniaxial Compressive Test (UCT) and Triaxial Compression Test (TCT) were performed to determine the engineering properties of sandstone for
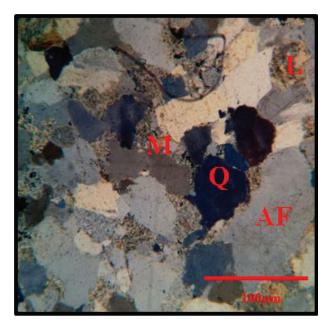

FIGURE 1. Lithic fragment formed due to sedimentation process under cross polarized light
Q = Quartz, L = Lithic Fragment, M = Muscovite, AF = Alkali Feldspar

the input data in numerical modelling. The tests were carried out in Rock Mechanics Laboratory, Faculty of Civil Engineering, Universiti Teknologi Malaysia, Johor Bahru.

For the ring disk sample, three diameter ratios were prepared which are 0.1, 0.2 and 0.3, with diameter ratio is calculated as inner diameter divided by the outer diameter. Noted that, the sandstone samples were sprayed in black and white dotted used for the Particle Image Velocimetry (PIV) analysis, which will not be discussed in this paper. Strain gauges were attached at the centre of solid disk sample and at the periphery for the ring samples as to measure the displacement.

Table 1 and Figure 2 show the results and mode of failure in Brazilian test, respectively. It can be seen the maximum tensile strength in a ring disk with 0.1 diameter ratio is three times as great as the maximum tensile strength in a solid disk subjected to the same diametrical load. Meanwhile in ring Brazilian test, the tabulated results showed that values of tensile strength in ring disk decreases as the diameter of the hole increases. Since the laboratory result can only determine the engineering values such as tensile strength, further investigation on the failure mechanism and deformation behaviour in tension was carried out in the numerical modelling.

FINITE ELEMENT MODEL

The finite element model was generated using RS^2 program by Rocscience (Rocscience 2016). The objective of this modelling was to simulate and gain in depth understanding of the Brazilian test performed on solid and ring samples.

In this study, uniform 6-noded triangles mesh type with 25000 number of mesh elements was used for efficiency purpose. Figure 3(a)-3(b) shows the constructed geometry model and the loading condition, respectively. The dimension of the solid and ring model is 49 mm in diameter. 'Add Excavation Boundaries' option was used to construct the excavation at the centre of the disk model.

Meanwhile, for the boundary condition, the 'Loads and Restraints' option was utilised, where the top surfaces were kept free from any restraints to allow displacement in this surface. The bottom surfaces were fully restrained in all directions so that the model remains stable under loading, and the sides were restrained in *x*-direction (Figure 3(b)). The loading was applied under displacement loading type and 30 stages of loading were assigned to observe the stresses within the model. The input parameters of sandstone and steel plates are shown in Table 2, respectively. The models were then monitored in terms of horizontal stress and yield element.

RESULT AND DISCUSSION

STRESS DISTRIBUTION

From the numerical analysis, the distribution of tensile stress occurs along the horizontal diameter determine that ring disk has maximum tensile stress three times higher than maximum tensile stress of solid (Table 3).

TABLE 1. Solid and ring Brazilian test results

Sample	A	B	C	D
External diameter, Ø (mm)	49	49	49	49
Internal diameter, Ø (mm)	0	3	10	15
Diameter ratio	0	0.1	0.2	0.3
$K = (6+38\Gamma^2)$	0	6.142	7.583	9.561
Thickness, t (mm)	25	25	25	25
Failure load (N)	11420	9950	4790	2880
Tensile strength, σ_t (MPa)	5.707	31.76	18.88	14.31
Horizontal strain at failure (%)	−0.0611	−0.077	−0.009	−0.057
Vertical strain at failure (%)	0.0994	0.1714	0.050	0.012
Modulus of elasticity in tension, E_t (GPa)	5.5	16.29	39.63	123.3

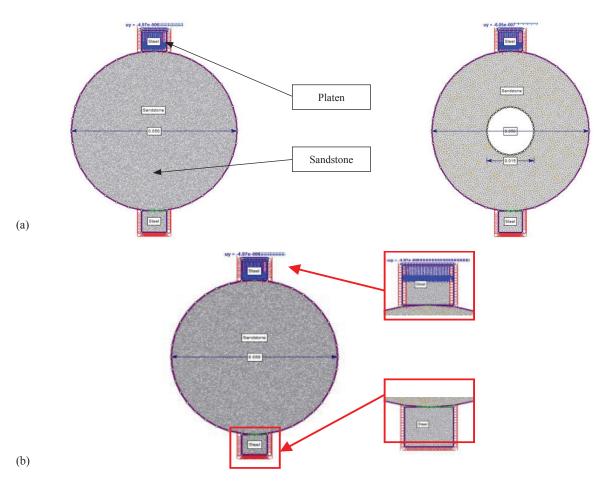FIGURE 2. Mode of failure in a) solid, b) 0.1, c) 0.2 and d) 0.3 diameter ratio in ring disk under Brazilian test

FIGURE 3. Models generation in RS^2 (a) solid and ring and (b) loading condition

TABLE 2. Input properties in finite element model

Sandstone properties	Input parameter	Steel platen properties	Input parameter
Unit weight (MN/m^3)	0.027	Tensile strength (MPa)	100
Tensile strength (MPa)	5.7	Poisson's ratio	0.3
Poisson's ratio	0.32	Young Modulus (MPa)	200000
Young Modulus (MPa)	5500	Friction angle (°)	35
Friction angle (°)	46.32	Cohesion (MPa)	10.5
Cohesion (MPa)	9.83	Material type	Elastic
Material type	Plastic	Initial element loading	Field stress only
Initial element loading	Field stress and body force		

TABLE 3. Comparison tensile strength result in experimental and numerical test

Sandstone disk	Diameter ratio	Failure load (N)	Experimental tensile strength at failure (MPa)	Numerical tensile strength at the centre of disk (MPa)
Solid	0	11420	5.7	1.2
Ring A	0.1	9950	31.76	6.9
Ring B	0.2	4790	18.88	3.1
Ring C	0.3	2880	14.31	1.5

It was found that, ring model produced lower value of compressive stress, where it requires higher stress in tension to fail the sample. This explained the result obtained in ring disk is always higher in tensile strength than those from solid. It also can be rationalised by an equation, in which, the tensile strength is proportioned to the maximum tensile stress at the centre, and the lower compressive stress at the centre will increase the tensile stress, thus, it increases the tensile strength.

Figure 4 illustrates graph of tensile strength against diameter ratio for laboratory and numerical studies, respectively. It can be seen that both results show a
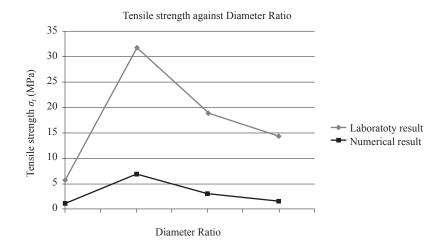
Tensile strength against Diameter Ratio

FIGURE 4. Graph tensile strength versus diameter ratio, zeroed diameter ratio
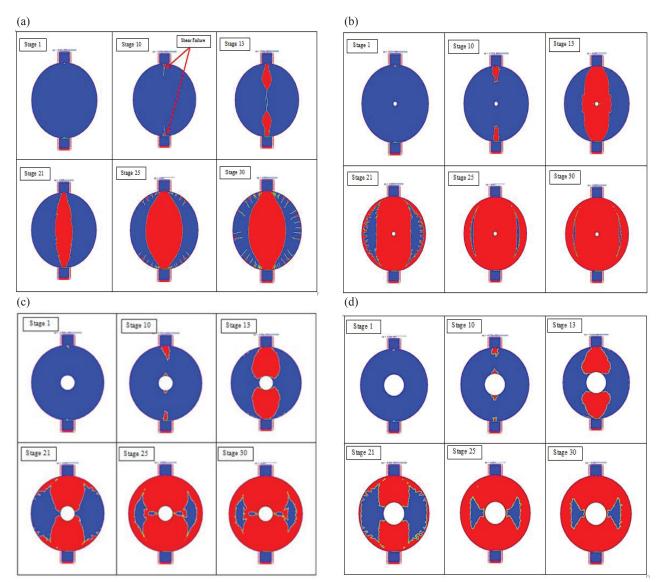reflects to the solid sample

FIGURE 5. Yielded element evolution process in (a) solid disk, (b) 0.1 diameter ratio, (c) 0.2 diameter ratio and (d) 0.3 diameter ratio
according to time cycle

similar pattern, in which the values of tensile strength in ring disk decreases as the diameter ratio increases and the solid disk shows the lowest tensile strength amongst all conditions. It also indicates the maximum tensile stress in a disk with 0.1 diameter ratio is three times as great as the maximum tensile stress in a solid disk subjected to the same diametrical load. The differences in values found may be due to the selection of failure criterion or meshing condition, which requires more extensive study in the numerical model.

YIELDED ELEMENT EVOLUTION PROCESS

One of the advantages in numerical analysis is that, it is capable of simulating the yield element. An overview of yielded element evolution process by time cycle shows how stresses inside the model throughout loading was applied from initial until the failure stages (Figure 5(a)-5(d)). Figure 5(a) depicts the cracks evolution process in the solid Brazilian disk simulated from RS^2 software. As the displacement loading applied to the model, the compressive stress initially develops at points close to both top and bottom of the loading platen. Once yielding of the element continues, this vertical stress in compression, propagate vertically from loading points towards the centre of the disk.

Interestingly, this resulting to the primary crack with the coalescence and shear failure can be observed at the top and bottom of the disk at Stage 10. The yield element distribution at the top and bottom of the solid disk sample propagated until both reach the centre of the disk (Stage 13). Here, the yield element which initially concentrated at centre of the disk start to propagate horizontally, thus produced yield in tension which can be observed during Stage 21 and it continues to propagate along the horizontal direction until the model entirely fail in tension and this condition is kwon as diametrical fracture (Stages 23 and 30). This condition is well-agreed of the bi-axial state of stress occurred in the Brazilian test.

In ring model, the circular excavation at the centre disturbed the stress distribution, thus stresses in the model and close to the excavation changed its direction and value. Figure 5(b) illustrates yield element evolution process in ring disk with 0.1 diameter ratio. Initially, as the displacement loading applied to the model, yield element evolution follow the similar pattern as the solid model, except the primary crack initiated at the bigger magnitude observed in Stage 10.

On the other hand, Figure 5(c) illustrates yield element evolution process in ring disk with 0.2 diameter ratio. Initially, as the loading applied to the ring model, yield element evolution follows similar pattern as the previous ring model (0.1 diameter ratio). However, it can be observed in Stage 10 that, as the yield elements propagated vertically from loading points towards the centre of disk, another yield element were initiated at the sidewall of the excavation simultaneously. Due to hollow part at the centre of the model, there is no medium for stress to concentrate at the centre, rather continues to propagate horizontally, thus produced the yield element distribution which concentrated on the sidewalls of the disk hole (Stage 13).

Then, the yield element distribution continues to propagate along the horizontal direction until the model entirely fail. It also can be observed that the yield element has progressively and fully developed around the sidewall of the excavation hole (Stages 21 to 30). This condition was not examined in the previous models, which justified that the compressive stress value in 0.2 diameter ratio is higher than 0.1 diameter ratio, thus it produces lower tensile strength.

Again, a similar condition was noted in the ring model of 0.3 diameter ratio (Figure 5(d)). As the diameter ratio increases, less force needed to fail the sample and require low tensile stress, thus produces lower tensile strength (Stages 25 and 30).

In conclusion, process of yield element evolution in solid and ring disk with different diameter ratio shows variation in stress distribution and also deformation behaviour under tension. The main difference was observed in solid model, where it shown the presence of shear failure but none in ring model of diameter ratio of 0.2 and 0.3. The models have successfully illustrated the shear failure witnessed in the laboratory work (Figure 6). As the diameter ratio increases in ring model, the yield element in tension become decreases and produce lower tensile strength. Moreover, results of the ring model also shown
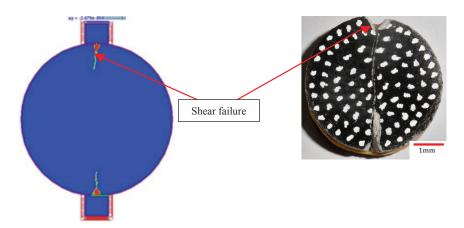

FIGURE 6. Shear failure observed in a) RS^2 analysis and b) laboratory work of solid Brazilian test

that the cracks do not initiated at the platen contacts since in both solid and ring Brazilian test, the diametric cracks in disks tend to gape in the mid-section. It can be concluded that, the objective of modelling has been achieved.

CONCLUSION

The tensile strength and crack evolution are considerably affected by geometry of the sample. Results of laboratory and numerical studies indicated the maximum tensile strength in a ring disk with 0.1 diameter ratio is three times higher than the maximum tensile strength in a solid disk subjected to the same diametrical load. Meanwhile, as diameter ratio of the ring disk increases, it produced smaller tensile strength.

The objective of numerical modelling has been successfully achieved. The stress distribution and yield element simulated have gain in depth understanding the behaviour of Brazilian test under solid and ring disk with different diameter ratio. Shear failure observed in the laboratory wok also being observed and explained by the numerical model.

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