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DEFICIT, HUMAN CAPITAL AND ECONOMIC GROWTH DYNAMICS [1]

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Abstract: Long-run economic growth arouses a great interest since it can shed light on the income-path of an economy and try to explain the large differences in income we observe across countries and over time. The neoclassical model has been followed by several endogenous growth models which, contrarily to the former, seem to predict that economies with similar preferences and technological level, do not necessarily tend to converge to similar per capita income levels. This paper attempts to show a possible mechanism through which macroeconomic disequilibria and inefficiencies, represented by budget deficits, may hinder human capital accumulation and therefore economic growth. Using a mixed education system, deficit is characterized as a bug agent which may end up sharply reducing the resources devoted to education and training. The paper goes a step further from the literature on deficit by introducing a rich dynamic analysis of the effects of a deficit reduction on different economic aspects.

Following a simple growth model and allowing for slight changes in the law of human capital accumulation, we reach a point where deficit might sharply reduce human capital accumulation. On the other hand, a deficit reduction carried on for a long time, taking that reduction as a more efficient management of the economy, may prove useful in inducing endogenous growth. Empirical evidence for a sample of countries seems to support the theoretical assumptions in the model: (1) evidence on an inverse relationship between deficit and human capital accumulation, (2) presence of a strongly negative association between the quantity of deficit in the economy and the rate of growth. They may prove a certain role for budget deficit in economic growth.

Keywords: deficit, human capital accumulation, economic growth.
JEL classification: E62, H62, I22, O41

Resum: El creixement econòmic a llarg termini és un aspecte que ha desvetllat un gran interès atès que pot facilitar el coneixement del camí seguit per la renda en una economia, alhora que pot ajudar a explicar les grans diferències a nivell de renda que han existit i que encara avui romanen entre els diversos països. El model neoclàssic fou seguit per diferents models de creixement endogen que a diferència del primer semblen predir que les economies amb preferències i un nivell tecnològic semblants no han de convergir obligatòriament vers nivells de renda per càpita semblants. En aquest paper hem tractat de mostrar un possible mecanisme a través del qual els desequilibris macroeconòmics i les ineficiències, representats ambdós per la presència de dèficit, poden entorpir l'acumulació de capital humà i per tant acabar entorpint també el creixement econòmic. Així, mitjançant un sistema educatiu mixte, el dèficit és caracteritzat com un agent molest que pot acabar exercint una important reducció dels recursos destinats a educació. L'article dóna un pas més enllà de la literatura existent pel que fa a les qüestions sobre dèficit tot i introduint una rica anàlisi dinàmica dels efectes d'una reducció deficitària en nombrosos aspectes econòmics.

Partint d'un model de creixement simple i tot i afegint-hi certs canvis, especialment concentrats en la llei d'acumulació de capital humà, s'aconsegueix arribar a un punt on el dèficit redueix de manera important l'acumulació de capital humà. Per altra banda, una reducció del dèficit continuada, entenent aquesta reducció com una direcció més eficient de l'economia per part dels seus responsables, podria entendre's com una manera de facilitar el creixement. L'evidència empírica per una àmplia mostra de països sembla donar suport als supostos teòrics del model: (1) evidència d'una relació inversa entre el dèficit i l'acumulació de capital; (2) presència d'una forta associació negativa entre la quantitat de dèficit en una economia i la taxa de creixement. Ambdós resultats semblen concedir un paper destacat al dèficit en el procés de creixement econòmic.

1. Introduction

Deficits are an economic problem or may become so. Some questions we may wonder about their origin can be posed as follows. Are they generated in response to demand pressures from citizens? Alternatively, are they generated by interest groups that drive up the size of government as pointed out by Buchanan, Rowley and Tollison (1987)? Certainly, one of the main debates always has been the one about the size of the government and the necessity or viability of a strong state. But, deficits lead to major questions referring to the role of the government and how good/bad could be the creation of oversized governments, questions which became one of the important concerns of the major classical economists such as Adam Smith, David Ricardo, Thomas R. Malthus, or John S. Mill, whose ideas formed the basis of the political economy (so called dismal science by Carlyle). The classical model of macroeconomics is exclusively supply driven and aggregate demand adjusts essentially via interest rate. Hence, an increase in government purchases (or a decrease in taxes) would imply an increase in the interest rate, which would decrease physical capital formation and/or consumption. They claim that financing government expenditures issuing more bonds (creating deficit) would certainly divert agents' investment between capital and government claims, thus withdrawing resources from industry and productive investments. Underlying this mechanism there is the idea that private investment is more productive than public expenditures, sustained mainly by Smith and Ricardo. This last one linked taxes and debt finance under what came to be called, following Buchanan (1976), the Ricardian Equivalence Theorem. The opposite traditional view mainly assumes people as being shortsighted or myopic. Hence, they consider that current consumption does not move as much as Ricardians believe. Even though Ricardo (1817) himself seems to doubt that people were rational and farsighted enough, which turns out to be rather ironic.

Diamond's (1965) paper was one of the first efforts to formally study the effects of budget deficits in the context of neoclassical models. He argued that a permanent increase in the ratio of domestically held debt to national income depresses the steady state capital-labor ratio. At the beginning of the nineties, only Drazen (1978) had considered the consequences of deficit on human capital, given that the vast majority of studies dealing with deficit had mainly centered on physical capital. However, human capital may be an important aspect to take into account given that, as it follows from Trostel (1995), recent research suggests that human capital is the most important component of national wealth[3], in line with Romer (1989), Benhabib and Spiegel (1994) or Temple (1999a).

Moreover, the likely negative relationship between deficit and long-run growth may be interpreted, according to Easterly and Rebelo (1993), considering a tax smoothing, which would imply that large deficits would be associated with low growth periods. Most of the literature on this topic turns to the possibility that large deficits may simply be an indicator of a huge public debt, which, in turn, could imply the presence of larger taxes and less public capital in the future.

In this paper, we will intend to analyze the effects that macroeconomic disequilibria, represented by deficits, may have on the economy. More specifically, we will characterize them as a bug agent that slows both human and physical capital accumulation and thus economic growth. Our last goal will be to try to explain how the presence of macroeconomic disequilibria may influence growth and be able to explain the existence of different types of equilibria (ones with low levels of physical and human capital and high levels of deficit; others, with higher capital levels and lower values for deficit). We will do this within a framework where human capital accumulation depends positively on existing human capital mainly following the formulation by Lucas (1988) and also taking into account Azariadis and Drazen (1990).

Section 2 reviews the concept of deficit in the context of macroeconomics. First, we analyze different empirical aspects, revising some of the main empirical studies and results on the relation mainly between deficit and growth. Then, we introduce the concept of human capital glimpsing a likely relationship between deficit and human capital accumulation. Section 3 presents a model formalizing the analysis of the effects that

[3] Davies and Whalley (1989) suggest that the stock of human capital is about three times as large as the stock of physical capital.

macroeconomic disequilibria, represented by deficits, may have on the economy. It covers the equilibrium, the dynamics and the numerical analysis and discussion of the transitional path. Section 4 undertakes an empirical study and gives statistical evidence supporting the main proposition of the paper, that deficit may harm both human and physical capital accumulation, thus slowing down economic growth. Finally, section 5 concludes and discusses some possible extensions of the analysis presented before.

2. Deficits and macroeconomic analysis

2.1. Empirical aspects

There is an extensive literature dealing with deficit and its likely influence on an economy. For instance, Barro (1974) showed that government debt is neutral when private intergenerational transfers are positive and when the rate of growth is lower than the interest rate. Later, Carmichael (1982) extended debt neutrality when the rate of growth is greater than the interest rate suggesting as sources of non-neutrality of public debt heterogeneous tastes and uncertainty. Besides, Drazen (1978) argued that when intergenerational transfers take the form of investments in human capital, government bonds might affect the equilibrium employment and increase welfare. Barro (1989) gives some empirical evidence on the economic effects of budget deficits. He argues that deficits mainly support the Ricardian viewpoint. Eisner and Pieper (1988), Boskin (1988), Hansson and Stuart (1987), among others, emphasize the role of deficit in real economic activity and its effects on wealth. On the other hand, Ihori (1988), Tanzi and Blejer (1988), Eisner (1989), van der Ploeg and Alogoskoufis (1994), show evidence of the significance of the impact that deficit financing has exerted on certain economies.

In addition, Easterly and Rebelo (1993) provide a wide review of the statistical relations between different fiscal policy measures, the level of development of the economy and the rate of growth. Their results confirm the fact that the presence of a high correlation between most of the fiscal variables under study (different taxes and budget deficit) and initial income makes it difficult to isolate the effects of fiscal policy in the context of Barro regressions. The presence of this correlation leads them to study fiscal policy as being endogenous in the sense of being related to certain characteristics of an economy such as its level of development. Results show that deficit is one of the fiscal variables whose relation with growth is more robust. They also confirmed one of the stylized facts of the literature, which is that deficit was consistently correlated with economic growth and private investment. Fischer (1993b) results are in the same line, showing a robust correlation between deficit and growth. Hence, evidence seems to suggest that deficits exert some influence on factor accumulation, which becomes stronger when the variable under analysis is productivity growth.

During the nineties, the debate focused on how the persistence of big deficits during the last decades in most industrialized countries may partly account for the high interest rates observed during this period. The classics emphasized the negative effects of deficits on the economy mainly through the interest rate adjustment that operated reducing investment and capital accumulation. The link between interest rate and deficit may be relevant for different aspects. It may fall upon the interdependence of fiscal and monetary policy; it may introduce a feedback component that may influence the degree of sustainability of public debt; it may become a key part in the mechanism of fiscal shocks transmission between countries; or it may also affect the consumption decisions taken by different economic agents. The controversy on budget deficits exerting a significant or, on the contrary, a neutral effect on nominal and real interest rates has been subject to wide discussion in the literature.

Plenty of empirical studies have given proven evidence of the impact deficits exerted on interest rates (Tanzi and Lutz, 1985; Evans, 1985, 1987; Cebula, 1988, 1991; Cebula and Hung, 1992; Ballabriga and Sebastián, 1993; Esteve and Tamarit, 1996, Doménech et al., 2001). However, some others do not find a statistically significant relationship (Kormendi, 1983; Darrat, 1989).

The IS-LM model assumes that pressures exerted over interest rates reducing the effect of what some authors have called the Keynesian multiplier are a consequence of public spending increases that have not been financed by taxes. Except for the extreme case of a perfectly elastic LM curve, the effect is caused by the competition between public and private sector in capturing resources. Later extensions of the model, introducing variable prices, include the possibility of spending levels above the full employment, which implies inflationary pressures, as well as real money stock decreases. The upward pressures on interest rates induced by deficit, according to Goisis (1989), are more likely to happen in this context. He also asserts that both changes in interest rates as well as crowding out effects are larger the higher the rigidity of the monetary policy, the closer the economy is from full employment, and the more precise is the perception of inflation. The relationship between deficit and interest rates has also been studied applying equilibrium dynamic models with perfect markets (overlapping generations, Diamond, 1965 and Bowles et al., 1989; or random decease date, Blanchard, 1985). Most of them show that changes in the intertemporal structure of taxes influence real variables.

In a more empirical level, if potential disequilibrium between supply of funds and required investment is large it should be easy to foresee a strong reaction of long-run interest rates given that agents would anticipate the lack of funds. The main path through which this mechanism would act would be the temporary structure of interest rates. Following the model by Blanchard and Fischer (1989), the effect of deficits on short-run interest rates is small at the beginning. However, and due to the fact that agents anticipate the increase in the level of debt (additional deficit), the effect is larger in the anticipated future short-run interest rates. Turnovsky (1989) reaches a similar result using a complete macroeconomic model, assuming that agents maintain rational expectations. In this model, the behavior of the temporary structure of interest rates depends on whether fiscal policies are permanent of temporary as well as anticipated or not. Whenever fiscal policy is unanticipated, the most significant result is the fact that a permanent fiscal expansion exerts a larger effect on expected future long-run interest rates then on present short-run interest rates. This would imply, by means of the temporary structure of interest rate, a larger increase in the present long-run interest rate. On the other hand, an anticipated fiscal expansion is likely to increase both short-run and long-run interest rates by the same quantity.

2.2. Human capital and deficit

According to macroeconomic theory, when there is a change in government spending it affects the demand for the economy's production of goods and services, altering the national saving. Following Mankiw (1992), if one considers that output is initially fixed by the factors of production, an increase in government spending must be offset by a decrease in any other of the demand components. Then, assuming that the disposable income is unchanged, consumption is also unchanged; hence, an increase in government purchases that is not accompanied with a tax increase should be offset by a decrease in private investment. Considering that private savings are unchanged, government borrowing decreases national saving, thus leading to an increase in the equilibrium interest rate of the economy given that government needs to capture investors' resources in order for these to absorb new debt issuance (crowding-out). Under this situation, capital stock would grow slower than in a balanced one, hence reducing the capacity of an economy to produce goods and services and so depressing the national income growth. These conditions, as we have previously said, would probably translate into an upward pressure on interest rates in such a way that long-run interest rates could pass on the effects of deficits to the real side of the economy. This is so since private sector expenditure components are especially sensitive to interest rates (i.e. house or plant building) and even more to long-run interest rate changes.

When reducing investment by increasing interest rates, deficits may not only depress physical capital accumulation but also human capital accumulation. If we assume an education system where people have to finance at least part of their own education, then, there might be an important role for interest rates in human capital accumulation too, in the sense that it would be more costly for people to ask for loans in order to be able to pay for their education (i.e. Sánchez-Losada, 1998). Hence, the agents' decision of how many resources devoted to education would depend negatively on the cost of funding it. We could think of this one as the value of the interest rate they would have to pay on the funds borrowed to finance education, that is, the interest rate in the economy. Certainly, a higher rate of interest would make investment in both human and physical capital more costly; apart from this, it would reduce the present value of the returns on human capital investment, that is, future wages. If the present cost to invest in human capital were too high, given that (future) human capital cannot be used as a collateral, then agents would probably reduce their optimal resource allocation to education, even if human capital is most times seen as an embodiment of skills, being both a source of new knowledge as well as a factor of production.

Over the 1970's and the 1980's, growth of public spending has generated large fiscal deficits in both industrial and developing countries. In several economies, further borrowing has no longer been a viable possibility, forcing the country to either decrease non-interest public spending or to increase taxes. Nevertheless, spending reduction, in most cases, has not followed efficiency considerations but political ones, resulting in a structure of public expenditures less conducive to growth, further depressing the economy. On the other hand, in low developed countries, increasing taxes is very difficult. Empirical evidence shows that attempts to increase taxes have not proved very successful. What is more, when fiscal authorities have been able to increase taxes they have induced large distortions as well as a reduction in the growth potential of the country. An increase in taxes reduces households' income both directly through tax payments and indirectly through deadweight losses due to distortions arisen by taxes. According to Trostel (1995), if taxes are based on income, an increase in future tax rates decreases future net wage, which can be seen as the return on current human capital investment, decreasing the benefits of investing in human capital. Thus, investment in human capital would be discouraged during a deficit.

3. Model with deficit

3.1. Putting down the model

We start assuming that government expenditures are financed either by taxes or deficit. Hence, a change in deficit, taking taxes as constant, would be linked directly to a change in government spending. Under this situation, in order to illustrate the difference that the presence of different values of deficit can make in an economy we will simulate a change from a high to a low value of deficit in a certain economy *ceteris paribus.*

In order to formalize this argument, we will use a simple model where households choose their private consumption time path according to their preferences represented by the following intertemporal isoelastic utility function:

$$\Pi = \int_0^{\infty} \left(\frac{1}{V} \right) \left(C G^{q} \right)^{V} e^{-rt} dt \qquad q > 0; \qquad -\infty < V \leq 1; \qquad V(1+q) < 1 \tag{1}$$

where C denotes aggregate consumption, G denotes the government investment in public goods other than education, r is the rate of time preference and the parameter q measures the impact of this public consumption on the agent's welfare[4].

For simplicity, we will assume that the population growth rate is null. On the other hand, firms operate combining physical and human capital. We will assume a Cobb-Douglas function of the form:

$$Y = A_F K^{a_K} \left[(1-l) H \right]^{a_H} N^{a_N} G^{a_G} \qquad 0 < a_i < 1; \quad i = K, H, G, N \tag{2}$$

The agents' objective is to maximize their utility (1) subject to both the laws for physical and human capital accumulation. The former one will be formalized as follows:

$$\dot{K} = Y - C - G - E - d_K K \tag{3}$$

where E[5] is an education transfer from the government.

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[4] Following Turnovsky (2000a,b), the parameter V is related to the intertemporal elasticity of substitution, s say by $s = 1/(1-V)$.

[5] We have not introduced E in the utility function to avoid a duplication since the education transfer should be used to acquire education, which would likely translate into a higher level of future consumption.

The law for human capital accumulation is formalized as:

$$J - d_H H \equiv \dot{H} = a_J\, l^{h_l} X^{h_X} H^{h_H} E^{h_E} - d_H H \qquad 0 < h_i < 1; \quad \text{with} \quad i = l, X, H, E \tag{4}$$

where a_J is the productivity of schooling (an efficiency parameter); *H* is the stock of human capital already existing in the economy (human capital stock of parents); *X* are the private expenditure agents make in education (it could be seen as a loan asked to the financial sector), which would partly determine the quality of individual specific education; individuals devote *l* part of their endowment of time to education; and d_H is the depreciation rate for human capital. All factors exhibit decreasing returns, but the learning sector is subject to increasing returns to human capital, education transfer and education expenditure. With the introduction of *H,* we are assuming that already existing human capital positively determines the accumulation of future human capital (with decreasing returns, though). We can interpret *E* as a proxy for the quality of public schools; all individuals face the same quality coming from *E*, which is outside the control of one agent. This is an argument in the learning technology that is consistent with Card and Krueger (1992) and Glomm and Ravikumar (1992). Furthermore, with the incorporation of both *X* and *E*, we are introducing a mixed system of education.

The analytical framework for the deficit follows that in Ihori (1988), which contains basic principals from Diamond (1965) and Gale (1973) as well as an extension of the Samuelson consumption loans model.

$$total\ deficit = D' = G + E + rB - t_y Y = \dot{B} \tag{5}$$

$$primary\ deficit = D = G + E - t_y Y = \dot{B} \tag{6}$$

where *B,* is the quantity of government bonds existing in the economy, t_y is the income tax rate and *D* is a measure of current fiscal imbalance (budget deficit).

Some authors have examined a variety of alternative measures of deficit, most of them concluding that for the purpose of conducting macroeconomic analysis the deficit that makes sense is the one that Sargent and Wallace (1994) propose, the *primary deficit*, namely the *total deficit* excluding debt payments. According to this, and given its higher simplicity, in our model we will use the primary deficit.

Following Turnovsky (2000b)[6], we will assume that the government sets its current gross expenditures on education, E, and other public investments, G, as fixed fractions of output, namely:

$$E = eY \tag{7}$$

$$G = gY \tag{8}$$

where g and e are fixed policy parameters. Using (7) and (8) and considering government deficit, d, as a fraction of output, we may set the government budget constraint as follows:

$$[g + e - t]\ Y = dY \Rightarrow [g + e - t] = d \tag{9}$$

Plugging (8) into (2) and rearranging, we get:

$$Y = A_F K^{a_K} [(1-l)H]^{a_H} (gY)^{a_G} N^{a_N}$$

$$Y = A_F^{\frac{1}{1-a_G}} g^{\frac{a_G}{1-a_G}} K^{\frac{a_K}{1-a_G}} [(1-l)H]^{\frac{a_H}{1-a_G}} N^{\frac{a_N}{1-a_G}}$$

$$Y = A\, K^{s_K} [(1-l)H]^{s_H} N^{s_N} \tag{10}$$

where we have redefined:

$$A \equiv A_F^{\frac{1}{1-a_G}} g^{\frac{a_G}{1-a_G}};\ s_K \equiv a_K/(1-a_G);\ s_H \equiv a_H/(1-a_G);\ s_N \equiv a_N/(1-a_G)$$

[6] This specification is equivalent to setting G/Y constant. It is adopted by Barro (1990), or Devereux and Love (1995), or Turnovsky (1997).

Individuals will take their decision of how much to invest in education depending on two aspects, the additional wage they are going to get thanks to their stock of human capital, as well as the cost to finance their investment in education (i.e. interest rate of the economy or interest rate on bonds, *R*). Hence, agents will face a trade off between: (i) investing in physical capital and get, $s_K Y/K$; (ii) investing in bonds and get R; and (iii) asking for a loan in order to invest in human capital during the present period and get a higher salary than unskilled or uneducated people in the future period once interest payments, RX, are discounted, $w_H - RX > w_N$. In any case, we assume that agents take their decision in a context of perfect capital markets. The value $w_H - w_N - RX > 0$ would represent the skill premium, where w_H and w_N are the rewards to skilled and unskilled workers respectively. Essentially, the total reward to skilled workers is the wage received by an unskilled worker (rewarding the raw labour) plus the marginal product derived from skills of a worker (rewarding the skills each educated worker has) in the final goods sector. We will define the marginal product of skills (or human capital, *H*), as $s_H Y/H$, and the marginal product of raw labor as $s_N Y/N$. In addition, defining the wage each skilled individual gets in per hour terms, we are left with the following expression for skilled wage:

$$w_H = w_N + s_H \frac{Y}{H} H \frac{1}{N(1-l)} = w_N + s_H \frac{Y}{N(1-l)} \quad \text{with} \quad w_N = s_N \frac{Y}{N} \tag{11}$$

where the second term in the right hand side of the equality can be considered as the skill premium and it comes from dividing the reward to the whole bunch of skills, $s_H Y$, by the number of skilled hours worked in the economy, $N(1-l)$.

We could represent the trade off in terms of some arbitrage conditions, which will take the following form:

$$s_H Y \frac{1}{N(1-l)} - RX = RK \tag{12a}$$

$$R = s_K \frac{Y}{K} \tag{12b}$$

It says that the money privately invested in education, X, will end up depending positively on the additional salary due to human capital as well as negatively on the interest rate value of the economy.

When the returns to physical capital, represented by the nominal interest rate in the economy, are greater than the returns to human capital, agents will decide not to invest at all in human capital, thus devoting no resources to studying. We will refer to any equilibrium that satisfies this condition as an *underdevelopment trap*. Thus, in an economy with no human capital accumulation, even if it accumulates physical capital, output is not likely to grow. On the contrary, when returns to physical capital are equal to returns to human capital, we reach an interior equilibrium as detailed below.

Taking this into account, we can rewrite the law for physical capital, (3) as:

$$\dot{K} + RX = Y - C - G - E - d_K K \tag{3'}$$

Performing the optimization by using a *discounted Hamiltonian* with *costate* variables m for physical capital and q for human capital we obtain:

$$\begin{aligned}\Im = \int_0^{\infty} (1/V)\left(CG^{q}\right)^{V} e^{-rt} + me^{-rt}\left[(1-t_y-d)AK^{s_K}(1-l)^{s_H} H^{s_H} N^{s_N} - C - d_K K - RX - \dot{K}\right] + \\ qe^{-rt}\left[a_H X^{h_X} l^{h_l} H^{h_H} E^{h_E} - d_H H - \dot{H}\right]\end{aligned} \tag{13}$$

and taking into account (9), it yields the following first order conditions:

$$\frac{\partial \Im}{\partial C} = 0 \Rightarrow C^{V-1} G^{qV} = m \tag{14a}$$

$$\frac{\partial \Im}{\partial l} = -m s_H \left(1 - t_y - d\right) A K^{s_K} H^{s_H} N^{s_N} \left(1 - l\right)^{s_N - 1} + q\left(a_H X^{h_X} l^{h_l} H^{h_H} E^{h_E} \left[\frac{h_l}{l} - \frac{h_E s_H}{(1-l)} \right] \right) = 0 \qquad (14b)$$

$$\Rightarrow \frac{m}{q} = \frac{J\left[\frac{h_l}{l} - \frac{h_E s_H}{(1-l)} \right]}{s_H \left(1 - t_y - d\right) Y / (1 - l)} \qquad (14b')$$

with J as defined in (4)

$$\frac{\partial \Im}{\partial X} = 0 \Rightarrow \frac{m}{q} = \frac{h_X \ J/X}{R} \qquad (14c)$$

The two laws of motion for the *costate* variables according to the *Maximum Principle,* will be the following:

$$\dot{m} = -\frac{\partial \Im}{\partial K} + mr \Rightarrow \frac{\dot{m}}{m} = d_K + r - \left(1 - t_y - d\right) s_K \frac{Y}{K} - \frac{q}{m} h_E s_K \frac{J}{K} \qquad (15a)$$

$$\xrightarrow{with\ (14b')} \frac{\dot{m}}{m} = d_K + r - \left(1 - t_y - d\right) s_K \frac{Y}{K} \left(1 + \frac{h_E s_H}{\frac{(1-l)}{l} h_l - h_E s_H} \right)$$

$$\dot{q} = -\frac{\partial \Im}{\partial H} + qr \Rightarrow \frac{\dot{q}}{q} = d_H + r - \left(1 - t_y - d\right) \frac{Y}{H} s_H \frac{m}{q} - \frac{J}{H} \left(h_H + h_E s_H \right) \qquad (15b)$$

$$\xrightarrow{with\ (14b')} \frac{\dot{q}}{q} = d_H + r - \frac{J}{H} \left((1-l) \frac{h_l}{l} + h_H \right)$$

In order to ensure that the individuals' intertemporal budget constraint is met, we will impose the following transversality conditions:

$$\lim_{t \to \infty} \ mKe^{-rt} = 0 \qquad (16a)$$

$$\lim_{t \to \infty} \ qHe^{-rt} = 0 \qquad (16b)$$

3.2. Balanced Growth Equilibrium

Before describing the dynamics, we will characterize the balanced growth equilibrium. This one is defined to be a growth path along which all variables grow at constant (not necessarily equal) rates. Along the balanced growth path, (Lucas, 1988), aggregate output, private physical capital stock, private schooling expenditure and consumption are assumed to grow at the same constant rate, g , and the fraction of time devoted to education remains constant. In accordance with the stylized empirical facts (Romer 1986), we assume that the output/capital ratio, *Y/K*, is constant.

For simplicity, we will normalize the population value to unity. Totally differentiating (14a), taking into account (8) and defining the output (physical capital) growth rate at equilibrium as $\hat{Y} \equiv g_K$, we get:

$$\hat{m} = [V(1+q)-1]g_K \tag{17}$$

Equalizing (14b') with (14c) and taking into account (12a-12b), we get:

$$\frac{s_H(1-t_y-d)}{\left(\frac{h_l}{l}-\frac{h_E s_H}{1-l}\right)(1-l)}\frac{h_X}{s_K}=\frac{s_H}{s_K}\frac{1}{1-l}-1 \tag{18a}$$

Finally, solving for l, we obtain:

$$l^2\left(s_K h_E s_H + s_K h_l - (1-t_y-d)s_H h_X\right) + l\left[s_H h_X(1-t_y-d) - s_K h_l + (s_H - s_K)(h_l + h_E s_H)\right] - (s_H - s_K)h_l = 0$$

$$aaa \equiv \left(s_K h_E s_H + s_K h_l - (1-t_y-d)s_H h_X\right)$$

$$bbb \equiv \left[s_H h_X(1-t_y-d) - s_K h_l + (s_H - s_K)(h_l + h_E s_H)\right]$$

$$l=\frac{-bbb \mp \sqrt{bbb^2-4aaa(s_K-s_H)h_l}}{2aaa} \tag{18b}$$

In order to find a solution for time devoted to education, $\tilde{l}$, we have to impose the condition that $bbb^2 > 4(s_K - s_H)\, h_l\, aaa$.

Using the optimality conditions in (14) and (15), it can be shown that the growth rate of the shadow values of physical capital and knowledge, *m, q*, grow in accordance with:

$$\hat{m} - \hat{q} = g_H - g_K \tag{18c}$$

Using the Pontryagin equality, (Pontryagin, 1962), which guarantees the same shadow value for the rates of physical and human capital, the equilibrium relationship would be:

$$\hat{m} = \hat{q} \Rightarrow g_H = g_K \equiv g \tag{18c'}$$

Hence, the growth rates of physical and human capital, in this case, are equalized in the long-run.

Equalizing (15b) and (17), (Pontryagin equality), using the equilibrium value for the schooling time as found using (18b) and taking into account (18c'), we get:

$$\begin{aligned} &[V(1+q)-1]\, g_K = d_H + r - (g_H + d_H)\left[\frac{(1-l)}{l}h_l + h_H\right] \\ &\Rightarrow g_K j + g_H x = d_H(1-x) + r \end{aligned} \tag{18d}$$

where $j \equiv V(1+q) - 1 < 0$ and $x \equiv \frac{1-\tilde{l}}{\tilde{l}}h_l + h_H$

To ensure that we have a rate of growth which does not decline with schooling, we need to further assume that the elasticity of intertemporal substitution, $\frac{1}{1-V}$, is high enough, which requires that $1-V < \frac{r}{d_H}\frac{1}{1+q}+1$.

3.3. Dynamics of a Two-Sector Model

To derive the equilibrium dynamics around the balanced growth path we define the following stationary variables:

$$y \equiv Y \big/ \exp^{g_K t};\ \ k \equiv K \big/ \exp^{g_K t};\ \ c \equiv C \big/ \exp^{g_K t};\ \ h \equiv H \big/ \exp^{g_H t};\ \ ee \equiv E \big/ \exp^{g_K t};\ \ x \equiv X \big/ \exp^{g_K t};\ \ j \equiv J \big/ \exp^{g_H t};$$

For convenience, we shall refer to y, k, c, x, j and h as *scale-adjusted* quantities. This allows us to rewrite scale-adjusted output and human capital as:

$$y = A(1-l)^{s_H} h^{s_H} k^{s_K} \tag{19a}$$

$$j = a_H l^{h_l} x^{h_X} h^{h_H} e^{h_E} y^{h_E} \tag{19b}$$

Considering (18e'), we can rewrite (18d) as follows:

$$\hat{K} = \hat{Y} = \hat{C} = \hat{H} = \frac{(1-x)d_H + r}{j + x} = g \tag{18d'}$$

The equilibrium percentage growth rate of human capital (knowledge) and physical capital, g, is determined by production parameters as well as the elasticity of intertemporal substitution, the discount factor, the depreciation rate of knowledge and the government expenditures (that is to say taxes as well as the value of deficit in the economy). Equation (18c') implies that countries converge to identical output per capita and human capital per capita growth rates if their production technologies are identical as well as their fundamental parameters referring to human capital depreciation, discount rate, intertemporal elasticity of substitution, and taxes and deficit (or government expenditures); otherwise, they will differ in their long-run growth. Intuitively, a decrease in deficit should lead to lower interest rate values, making investment in physical capital more appealing, which would attract labor to the output sector. Once physical capital has increased sufficiently, it will be high time for human capital to start accumulating in order to take advantage of the new physical capital previously accumulated.

Using the optimality conditions, the dynamics of the system can be expressed in terms of the redefined stationary variables as:

$$\dot{k} = k\left[\left(1 - t_y - d\right)A\left(1 - l\right)^{s_H} h^{s_H} k^{s_K - 1} - d_K - \frac{c}{k} - \frac{x}{k} s_K \frac{y}{k} - g_K\right] \tag{20a}$$

$$\dot{h} = h\left[a_H l^{h_l} x^{h_X} h^{h_H - 1} e^{h_E} y^{h_E} - d_H - g_H\right] \tag{20b}$$

$$\dot{l} = \frac{\frac{j}{h}\left(\frac{(1-l)}{l}h_l + h_H\right) - \left(1 - t_y - d\right)s_K \frac{y}{k} BB + \frac{\dot{h}}{h}\left(\left(1 - h_E\right)s_H - h_H\right) + \frac{\dot{k}}{k}\left(\left(1 - h_E\right)s_K - h_X\right) - \left(d_H - d_K\right) - \left(g_H - g_K\right)}{\frac{h_l}{l} + \left(1 - h_E\right)\frac{s_H}{1 - l} - \left(1 - h_X\right)FF} \tag{20c}$$

$$FF \equiv \frac{\frac{h_l}{l^2}}{\frac{h_l}{l} - h_l - h_E s_H}; \qquad BB \equiv \frac{\frac{1-l}{l}h_l}{\frac{1-l}{l}h_l - h_E s_H}$$

To the extent that we are interested in the per capita growth rates of physical and human capital, they are given by $\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + g$; $\frac{\dot{H}}{H} = \frac{\dot{h}}{h} + g$.

The steady state to this system, denoted by "~" superscripts, can be summarized by:

$$\left(1 - t_y - d\right)\frac{\tilde{y}}{\tilde{k}} - \frac{\tilde{c}}{\tilde{k}} - \frac{\tilde{x}}{\tilde{k}} s_K \frac{\tilde{y}}{\tilde{k}} = d_K + g \tag{21a}$$

$$\frac{\tilde{j}}{\tilde{h}} = d_H + g \tag{21b}$$

$$\left(1 - t_y - d\right) s_K \frac{\tilde{y}}{\tilde{k}} \left(\frac{\frac{\left(1 - \tilde{l}\right)}{\tilde{l}} h_l}{\frac{\left(1 - \tilde{l}\right)}{\tilde{l}} h_l - h_E s_H} \right) = \frac{\tilde{j}}{\tilde{h}} \left(\left(\left(1 - \tilde{l}\right) \frac{h_l}{\tilde{l}} + h_H \right) - \left(d_H - d_K\right) \right) \tag{21c}$$

These equations determine the steady-state equilibrium[7] in the following sequential manner. First, equation (18b) yields the equilibrium value for schooling time, in terms of the elasticities in both production functions and the income tax and deficit value in the economy. Next, given $\tilde{l}$, equations (18c') and (18d') respectively give us the *gross* equilibrium growth rates of knowledge, $\tilde{j}/\tilde{h} = \tilde{J}/\tilde{H} = g_H$, and physical capital/ output, $\frac{\dot{Y}}{Y} = g_K$, in terms of the elasticities, the rate of human capital depreciation, the discount rate, and the weight given to government goods as well as the intertemporal elasticity of substitution. Next, given $\tilde{j}/\tilde{h}$, and $\tilde{l}$, equation (21c) determines the output-capital ratio, such that the net rates of return to investing in physical capital and human capital are equalized. Given $\tilde{l}$ and $\tilde{j}/\tilde{h}$, the scale-adjusted production functions determine the stocks of physical capital, $\tilde{k}$, and human capital, $\tilde{h}$. Having derived $\tilde{l}, \tilde{h}, \tilde{k}$, using (14b') and (14c), we will obtain the value for private expenditures on schooling, $\tilde{x}$. Finally, having

[7] See appendix 3 for more details on the linearization process.

obtained the output-capital ratio, as well as the private expenditure in schooling, (21a) determines the consumption-capital ratio consistent with the growth rate of capital necessary to replace depreciation.

3.4. Numerical analysis of transitional path

Table 1 shows that the values we employ for our fundamental parameters are in line with those suggested by previous calibration exercises (Lucas, 1988; Jones, 1995; Ortigueira and Santos, 1997). The final goods sector exhibits increasing returns to scale in labor and physical capital (and knowledge). The learning sector is subject to increasing returns to scale in human capital and government education transfers (and private expenditures).

Table 1. Benchmark parameters.

Production parameters	$A=1,\ A_F=1.46,\ s_K=0.45,\ s_N=0.6,\ s_H=0.335,$ $a_J=1,\ h_l=0.425,\ h_H=0.65,\ h_E=0.45,\ h_X=0.1246$ $a_G=0.2,\ a_K=0.36,\ a_H=0.268,\ a_N=0.48,$
Preference parameters	$r=0.05,\ q=0.25,\ g=0.7,\ V=0.30$
Depreciation and population parameters	$d_K=0.05,\ d_H=0.05,\ n=0$
Fiscal policy parameters	$g_1=0.175,\ g_1=0.075,\ e=0.10,\ t=0.15$

Table 2. Equilibrium values before the shock.

$Y\hat{/}N$	$H\hat{/}N$	$\tilde{l}$	$Y\tilde{/}K$	$C\tilde{/}Y$	$\tilde{R}$	$X\tilde{/}K$	Relative wage $= w_H/w_N$
2.248%	2.248%	0.315	0.292	0.515	13.16%	0.225	1.815

Table 3. Equilibrium values after the shock.

$Y\hat{/}N$	$H\hat{/}N$	$\tilde{l}$	$Y\tilde{/}K$	$C\tilde{/}Y$	$\tilde{R}$	$X\tilde{/}K$	Relative wage $= w_H/w_N$
2.627%	2.627%	0.326	0.259	0.569	11.68%	0.244	1.829

We group the resulting endogenous variables into three categories. The balanced per capita growth of capital, output and knowledge; and key equilibrium ratios, including the output capital ratio, the share of consumption in output, time devoted to education, private expenditure in education as a percentage of physical capital; as well as the interest rate value and the relative wage. The values for all these variables both before and after the shock are presented in *table 2* and *table 3* respectively. According to them, per capita output and knowledge growth at equilibrium, defined as $Y \hat{} N$ and $H \hat{} N$ respectively in the tables, is slightly above 2.6%, 0.4 percentage points more than before the deficit cut. The equilibrium capital output ratio, $Y \tilde{} K$ in the tables, is slightly below four, that is half a point higher than the value before the shock. Moreover, 57% of output is devoted to consumption, compared to the previous 51%. Around 33% of the endowment of time is devoted to schooling, 5% more than the value they had before the shock, with 67% devoted to production.

To provide clear intuition into the adjustment process, we conduct a numerical simulation[8] where deficit is reduced from 12.5% to 2.5% by means of a reduction of 10 points in the government expenditure as a percentage of GDP, g. The phase diagram (see *figure 1E* in page 26) shows that the decrease in deficit and subsequent decrease in the interest rates attracts resources first to the output production sector and away from the knowledge production sector, with physical capital accumulating at the expense of human capital. The figure shows how the scale-adjusted physical capital accumulates accompanied by a reduction in the scale adjusted knowledge due to the fact that per capita rate of accumulation of knowledge is insufficient to cover the depreciation rate. *Figures 1A* and *1C* show that adjusted per capita capital growth rises to above 7% during the early stages. Within 20 periods, the growth rate of capital is reduced to half after which it falls rapidly down towards the long-run steady state of just above 2.5%. Scale-adjusted knowledge growth has a completely different pattern, as seen in *figures 1.A* and *1.D*. After its initial decline, mainly because of the rapid increase in physical capital together with the reduction in knowledge during the early stages, it overshoots its long-run growth rate and eventually raises the return on investing in knowledge sufficiently, relative to the return on physical capital. With respect to the evolution of scale adjusted output growth (see *figures 1A* and *1B* in page 26), it is worth noting that it is a combination of the two scale adjusted variables (physical capital and knowledge). Therefore, it follows a path that lies between both of them, adjusting faster than knowledge but slower than capital for the first 40 periods and

[8] The simulations are based on the *Mathematica* algorithm developed by Eicher and Turnovsky (2001).

reversing this trend from then onwards. Scale adjusted output growth initially rises to above 3.75% but it falls sharply to around 2.75% in 20 periods, just when the decrease in physical capital is higher. After that, it starts a mild increase during the next 30 periods, followed by a decrease until reaching its long-run value.

The initial accumulation of physical capital attracts more time devoted to working in the output sector, reducing the time people devote to schooling. Over time, as the growth rate of physical capital declines and knowledge increases, the allocation of time to the knowledge sector increases. In addition, under the assumption $s_H < h_H$, this rise in knowledge further serves to drive the allocation of time from the final output sector to schooling (knowledge producing sector). In the long-run, we end up with a higher allocation of time to the knowledge (schooling) sector (see *figure 2* in page 27). On the other hand, the growth rate of schooling expenditures follows, in essence, the same evolution as the allocation of time to schooling. The value of private expenditure on education jumps down dramatically immediately after the shock and starts increasing up to its long-run value, overshooting its previous value, hence people end up devoting more resources to education after the shock, which changes from representing a value of 22.5% of total physical capital up to 24.4%. The corresponding evolution is illustrated in *figure 3* (page 28).

Setting life expectancy equal to its average over the sample of 90 countries that is used in the empirical analysis of the following section[9], and subtracting 6 years to the original number so as to take into account that for the first six years of life children do not go to school, we get a value of 55.25. Using this value, our model would predict an increase of approximately one year of schooling in the long-run, which represents a 5% increase with respect to the original value.

Regarding the evolution of the interest rate, it starts decreasing immediately after the shock from 13.16% down to 8.75% after 50 periods. Then, it starts increasing up to its new equilibrium value, which is as high as 11.7%, hence around 1.5 points lower than the initial

[9] See section 4.

one, as showed in *figure 4A*. It is also worth noting that although both the new raw labor wages and skilled wages are higher after the decrease in the value of deficit. Skilled wages have increased more relatively to unskilled wages, which translates into larger inequality (1.5% higher measured in terms of relative wages) once the economy performs better. This may be attributable to the fact that skilled workers have become relatively more productive, as their stock of skills has increased, and we are considering workers being paid their productivity. This fact is in line with the increasing inequality in salaries observed in several economics, especially during the nineties[10] and in countries whose labor market is characterized by a higher flexibility (in countries with highly regulated labor markets, labor market pressures have more likely translated into a higher rate of unemployment for the less skilled).

Figure 5 shows the evolution of long-run growth as deficit changes. It is interesting noting that once deficit has reached a very large value, further accumulation of deficit does not imply a further decrease in growth rates, which stabilizes around -5%. In addition, we can appreciate how a surplus situation leads to higher growth but only up to some point. Again, this time it is worth pointing out that once surplus has reached a certain large value, it leads to negative growth and further increases in surplus do imply further decreases in growth.

Hence these simulations give us a flavor of how damaging deficits may become, and also how a policy that pursues larger surpluses may not be an optimal solution either, since both deficit and surplus can be seen as two forms of disequilibrium, i.e. excessive spending and idle resources respectively, these latter ones translated into working at a lower level than the potential one.

3.5. Discussion

One could view long-run deficits as being determined by the economic, social or political characteristics of a country in such a way that better fiscal and monetary policies (or with

[10] Manacorda and Petrongolo, 1999; Mortensen and Pissarides, 1999.

better results), better management of the economy, less corruption, greater productivity, etc., would bring an economy to a situation with lower values for deficit and, consequently, towards a greater level of growth at equilibrium. Our work provides an enriched view of the consequences of deficit introducing the dynamics of the economy following a change in deficit. We believe that large values for deficit would determine a value for interest rate, R, large enough for the agents to believe that human and physical capital accumulation was not worth enough. In an analogous way, a value for deficit that was sufficiently low or a certain surplus, associated with low values for R, could determine large positive values for schooling time, l, and private schooling expenditures, x, followed by positive and optimal growth.

Deficits are defined as the part of government expenses that are not covered with taxes. If we think of them as a percentage of GDP it is easy to see that we are not saying that running a certain level of deficit is largely deleterious for an economy or that an economy should always try to avoid an increase of deficit in absolute terms. As long as the growth rate of the economy is higher than the absolute deficit growth and if the economy has low deficit values these latter ones should not be, in principle, a major problem, since they would be decreasing in relative terms as a percentage of GDP. Problems arise when the ratio of deficit with respect to GDP, d, is increasing over time and especially in the long-run. Then, deficits may start strongly harming the economy. We do not mean that governments should reduce their spending by all means at any time, we are only saying that when public spending were increasing more than GDP for some periods of time, thus generating more deficit, governments should start taking more care about public spending because it may be that it is undertaken without following efficient requirements or that certain expenses are under productive, hence generating macroeconomic disequilibria. In that case governments should reduce the unproductive spending and enhance the productive part of it, to pursue economic growth.

The model provides a long-run value for deficit, that is a steady state value reached after one (like this case) or more shocks. The idea underlying the model is to capture a stable value for macroeconomic disequilibria (represented by deficits) which is consistent with reasonable economic parameters and especially with a reasonable rate of growth. If we look at *figure 5*, we can see how deficits and surpluses are related to growth. The way we have elaborated *figure 5* is the following one. Taking (18d') as the long-run relationship between deficit and growth, keeping all parameters constant but deficit, we have plotted different values for deficit with their corresponding growth values. We get the standard results for the positive values of deficit saying that the higher the deficit, the lower the economic growth; and that low absolute values for deficit (or even the presence of surplus) are related to positive values for growth. In any case, the value at which deficit is related to positive values of growth as well as the value at which surplus starts exerting a negative influence on growth varies according to the parameters in the economy.

Given this, it is interesting to know if even small quantities of deficit resulting from efficient government spending (i.e. infrastructures) might have a negative effect on growth. When analyzing this, we should bear in mind that deficits are taken as the mean of several years, thus including up and down parts of the cycle (deficitarian and surplus years). Hence, it would be likely that we ended up with a very low value for deficit in the long-run in order for it to be efficient. Under this situation, even current deficits initially acting as buffers in the economic cycle may have negative effects if they become permanent throughout the whole cycle. Consequently, a fiscal behavior like this one, which may be beneficial at certain points of the cycle, cannot certainly be sustainable in the long-run.

At this point, we would like to cast some doubts on the goodness of surplus. The case with the presence of surplus could be viewed as a positive situation where governments might decide to give back the surplus into the form of transfers to agents, which could stimulate larger values for l as well as x, (up to a certain point). Nevertheless, our present model would not predict that any larger surplus increase translated into a better economic situation. Indeed, a high surplus situation could also represent a sub optimal situation given that there could be some idle resources. According to our model a budget surplus should be related to low interest rate values and this, in turn, would make the acquisition of education more attractive (less costly). However, increasing values of time devoted to education have their counterpart in decreasing values of time devoted to working. As long as productivity of working time is able to compensate for the reduced number of working hours, the economy will go on growing. There will be a point, though, where the economy will require more labor not to have idle resources (let us say capital). It will be at this point, where surpluses may start exerting a negative influence on growth. Considering that we are assuming the existence of complemetarity of the different productive factors, which is quite plausible if we consider the functioning of an industry, the lack of one of these factors may slow down the production of final output impinging a lower rate of growth to the economy.

One further implication of our model is the existence of a poverty trap: for some conditions, the economy evolves to a low (or null) growth situation. We could think that these economies burdening too big deficits and finishing with a low value for human capital and growth, could adopt a different technology, which might allow them to initially increase their production without requiring so much human capital (e.g. working in the primary sector with a production function with physical capital as its sole inputs). Later, they could adopt a higher technology including human capital, which would allow them to grow faster once the economy had already taken off.

Notice that, according to our model, it is not impossible for a poor country to join the richer ones since it only needs a favorable mix of deficit, schooling and human capital. On the other hand, the equilibrium with growth may not be fully optimal because of the different value given to deficit [by different agents, i.e. consumers and authorities]. So, individuals may not be aware of the positive influence a small quantity of deficit may exert on the economy. The performance of a Social Planner (or a fiscal authority) imposing a certain level of deficit (optimal), larger than the one in equilibrium, depending on the characteristics of the economy, may lead to a *Balanced Growth Path* where the variables H, Y, C and K grow at constant rates different from zero and larger than the ones obtained in this paper; and where the shadow values of both human and physical capital (m and q) slow down at constant rates lower than the ones obtained here. At this point we could mention some of the policies recently undertaken by the IMF, especially in low developed countries, seeking the control of deficits, which, in most times, are tied to investments in human capital in order to foster a higher level of economic development and growth.

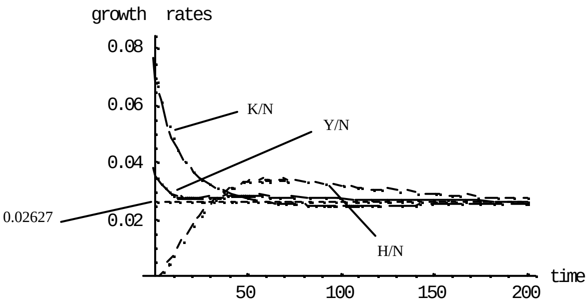

Figure 1A. Growth rates of output, physical capital and human capital

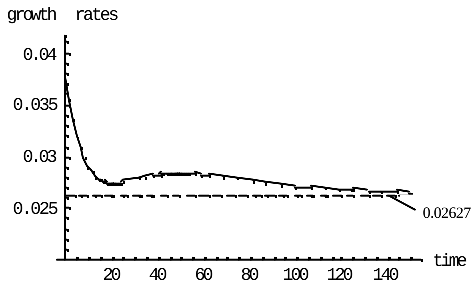

Figure 1B. Growth rate of output

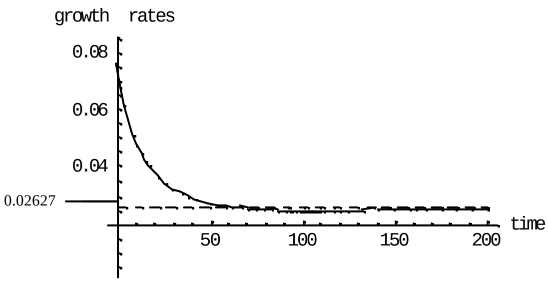

Figure 1C. Growth rate of physical capital

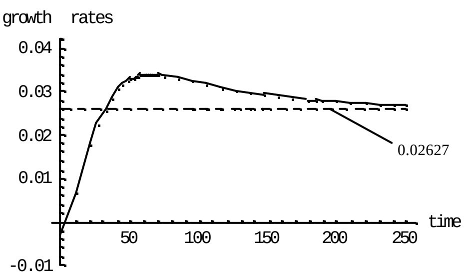

Figure 1D. Growth rate of human capital

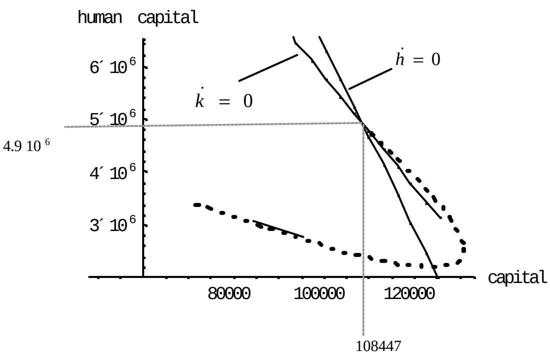

Figure 1E. Adjustment dynamics

*Figure 1.*Growth rates of scale adjusted output, capital, knowledge and adjustment dynamics

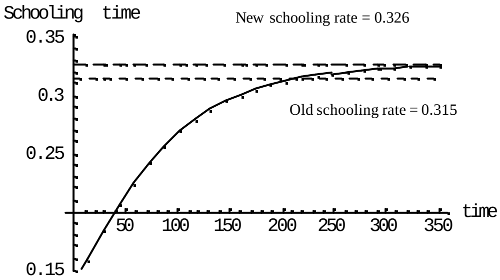

Figure 2A. Schooling time adjustment and compared

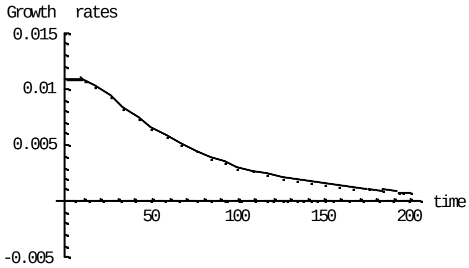

Figure 2B. Growth rate of schooling time

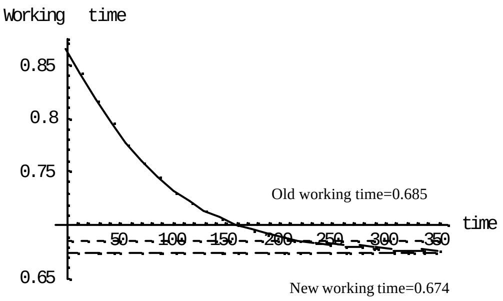

Figure 2C. Working time adjustment

Figure 2. Schooling and working dynamics.

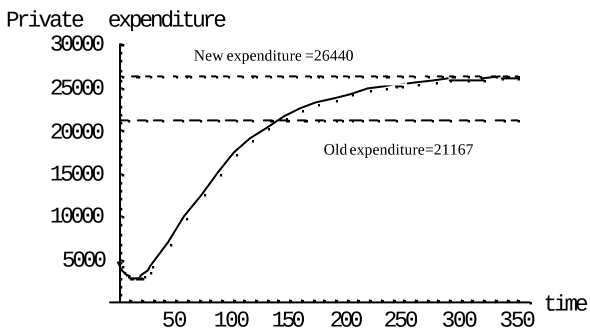

Figure 3A. Private expenditure adjustment and compared

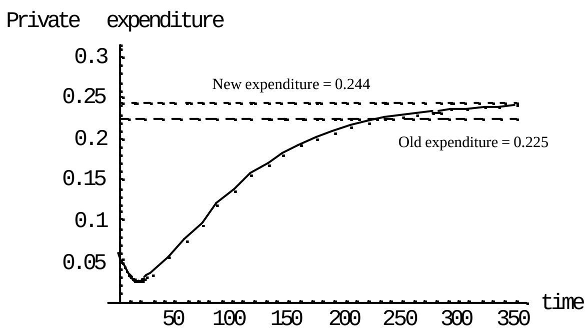

Figure 3B. Private schooling expenditure as a percentage of capital

Figure 3. Private expenditure on schooling and dynamics.

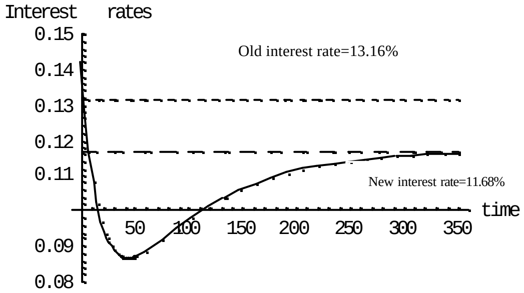

Figure 4A. Interest rate adjustment

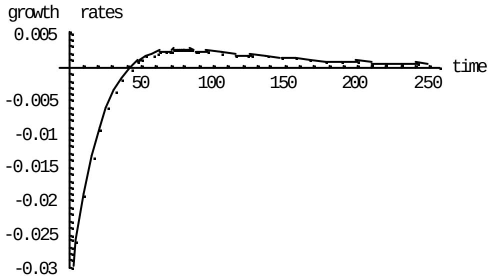

Figure 4B. Interest rate growth

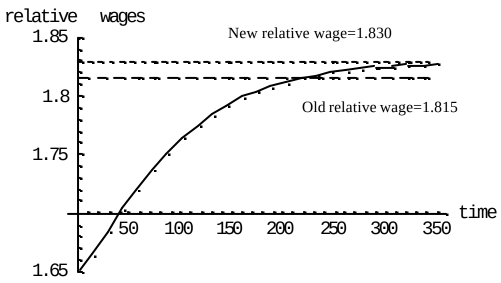

Figure 4C. Relative wages adjustment

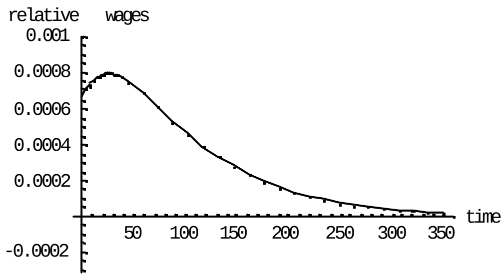

Figure 4D. Relative wages growth

Figure 4. Relative wages and interest rates

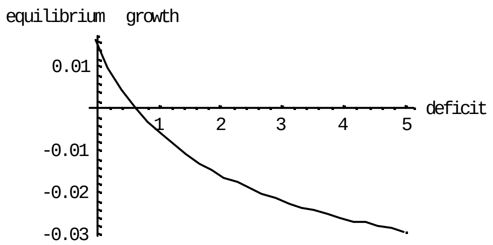

Figure 5A. Long-run growth values and deficit

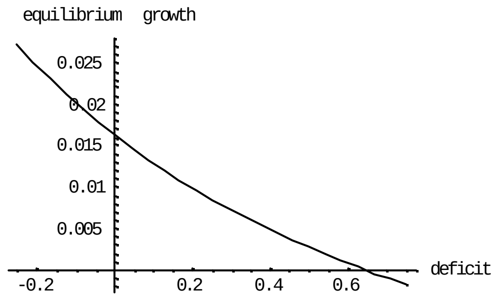

Figure 5B. Long-run growth values, deficit and surplus

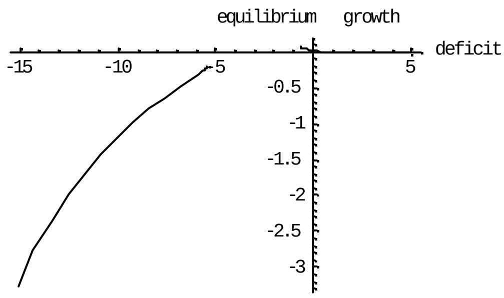

Figure 5C. Long-run growth values and surplus

Figure 5. Deficit and growth

Summing up, depending on long-run structural disequilibria, an economy may finish with different equilibrium values for growth. Following that, a possible way to foster long-run growth could come from keeping low values for deficit. These low values could be partly determined both by a restrictive fiscal policy as well as a positive interaction between other determinants of macroeconomic stability such as a restrictive monetary policy or a major spending control, among other possibilities.

4. Empirical evidence

Obviously, one of the first things one comes out with when thinking about analyzing the relationship of a macroeconomic variable or group of variables with growth is to check, in a very preliminary way, the empirical validity of that relationship for a large number of economies to see what the reality tells about the idea you just came out with. That is exactly what happened when we started sketching the analysis undertaken in this paper. The first results we got were quite satisfactory, showing a high correlation between deficit and growth, which allowed us to go on building the theoretical model that tried to find a satisfactory and somehow well-founded explanation for the relationships we were interested in analyzing. Furthermore, an extensive empirical analysis should help us to validate our theoretical results. Actually, statistical evidence seems to support our basic proposition that deficit may harm human capital accumulation and thus slow down growth.

The data we used have been obtained from two different sources. First, educational attainment comes from Barro-Lee data set (1993). We have used the variable defined as the average schooling years in the total population over age 25. Secondly, the rest of the variables included come from the World Bank macroeconomic data sets (25/05/1999), (*http://www.worldbank.org/html/prdmg/grthweb/growth_t.htm*). Although the World Bank provides data for 212 countries, our sample of countries was firstly reduced to approximately one half since data on education cover roughly one hundred of them. We also eliminated those countries that lacked all data necessary to obtain all the variables we required. In addition, some authors have studied the problems arising in gauging human capital data, such as Temple (1999b) and de la Fuente and Doménech (2000, 2001). As we proceeded with the first empirical analysis, we came out with some extreme values for a few countries, which motivated an outlier residual analysis that allowed us to capture some of the outliers. By means of the Cook distance criteria, we captured the extreme values for some countries in the sample. More especifically, for Togo, Nepal and Niger, which we also removed out from the sample. After this, we ended up with 90 countries (see *table A.3* in appendix 2).

Human capital growth is obtained as the difference between the logarithm of 1990 human capital and the logarithm of 1970 human capital. Output growth rate has been obtained in a similar way, taking the difference of the logs for 1990 and 1970 real per capita output. The rest of the variables included in the analysis have been obtained as the mean for the period 1970-1990. Deficit, exports, imports and government spending have been taken as a percentage of the GDP in each country. We have used the series *budget surplus including grants* from the World Bank for the deficit.

Goetz and Hu (1996) estimate human capital growth as a function of what they came to call *environmental variables*, which they chose based on previous studies. Most of those studies were based on individual polls. In addition, Cameron and Heckman (1993) assert that the decision to undertake university studies depend on some aspects such as the father's occupation or the house property; Cohn and Hughes (1994) add the unemployment tax, the mean salary and the size of the city, the latter one in order to capture the likely agglomeration effects. Goetz and Hu say that taking countries as an aggregate, there are variables such as employment, which are difficult to obtain, therefore the regressors they end up using are urbanization, in order to capture the effects coming from the size of the city, unemployment, the percentage of owned houses, the size of the family, the level of local taxes per capita, the education expenditures, as well as the percentage of professionals in the labour market.

In order to test for the influence as well as the robustness of deficit as an important determinant for human capital growth and economic growth, following the literature (Goetz and Hu, 1996; Easterly and Rebelo, 1993; Fisher, 1993), we have taken into account different groups of regressors related to different aspects of the economy that may influence this relationship. More specifically, we have introduced production (*initial output and output growth*) and trade (*exports plus imports*), Rivera-Batiz and Romer, 1991. The inclusion of trade relies on the assumption that a larger openness degree may have stimulative effects on the acquisition of human capital. We decided to introduce political and social instability variables (*coups, assassinations, revolts*) as a factor which may disincentive training, given that the economy may allocate resources away from education. Other variables introduce were location variables (*urbanization* percentage and *latitude*, as well as a dummy variable, *R4*, which tries to capture the likely regional effects coming from eastern Asia); government spending (*education, health, defense*), not only direct government spending on *education* but also on *defense*, as a possible deterrent of other types of expenditure, which may report higher social benefits, and *health*, given its direct relationship with human capital accumulation, especially in low developed countries. Besides, there are some further aspects relating health with growth, such as the demographic structure or various sanitary issues; monetary aspects (*M2*, real *interest rates, exchange rates, inflation and black market premium (BMP)),* all of them trying to capture the necessary stability required by the agents to form favorable future expectations that bring them to choose a higher value for schooling at present; *demographic aspects* (*life expectancy*) in order to capture the fact that a higher availability of time might increase the time agents devote to education; and government *tax collection,* following Nerlove *et al.* (1993). Given that we have introduced several variables, we should note that this might be quite a rigorous exercise; since one has to bear in mind that deficit may be influencing some of these variables we have used as regressors.

Table A.1 summarizes the results on human capital accumulation. *Equation 1* shows the positive relationship between human capital growth and surplus (negative relation with deficit, which is taken with a negative sign), being significant at 10% level. However, this equation may be subject to a wrong specification given that we have omitted the value for initial human capital. Hence, we would not be taking into account the possibility of human capital convergence across countries. *Equation 2*, with initial human capital, shows its importance as an explanatory variable in the acquisition of education. Most of the regressions onwards show the likely presence of catching-up in terms of education as its coefficient takes a highly significant negative value. This value shows the existence of convergence in human capital terms among countries, which could be understood as the presence of decreasing returns to scale of human capital stock in the knowledge equation, thus validating the assumption made in the theoretical part of the paper. Besides, *equation 2* allows us to test for the influence of deficit in human capital growth, being significant at 1.1% level. Hence, the first two equations might be confirming the discouraging effect that deficit seems to generate when accumulating human capital. The results we got for the various regressions seem to confirm the robustness of deficit to the inclusion of different economic aspects with respect to its significance level, except for the variables related to output (initial output and output growth), location (urbanization), and life expectancy, despite the fact that, as seen in the various specifications, the size of the coefficient for the deficit varies depending on the specification of the regression. On the other hand, the sign of the deficit is the expected one in all the cases analyzed with the exception of output growth and initial output. With respect to the hypothesis of trade, data does not seem to confirm it, since the coefficient on trade is not significant at all. Besides, it is quite surprising the lack of significance of the coefficient of government spending on education as well as health, given the likely *a priori* influence of both on schooling. Finally, we have introduced the last regression including all the regressors as a matter of completeness. We should highlight the solely significance of the following variables: initial human capital (1%), deficit (1%), trade (5%), assassinations (1%), riots (5%), latitude (5%), monetary aggregate (1%), life expectancy (1%) and taxes (1%). However, it is worth mentioning that the introduction of the whole range of variables may be hindering some of the interinfluences, and thus a likely high degree of multicollinearity with the subsequent problems related to it, which somehow reduces the confidence level of this last regression.

To sum up, we could say that deficits have a strong negative relationship with human capital growth in such a way that a 1 percentage point increase in deficit would translate into a reduction of 0.3-0.6 percentage points in human capital growth. Besides, it is worth noting that we have undertaken an empirical linear analysis. Actually we pretend it to be the starting point of a likely more extended analysis, which might be able to capture, in the near future, some of the non-linearities previously presented in the theoretical part.

In order to validate our theoretical model as well as to test for the influence of deficits on economic growth and compare our results to the ones previously obtained (Fischer, 1993a,b; Easterly and Rebelo, 1993), we have undertaken a cross-section analysis for the GDP growth. The dependent variable used has been output growth, whereas the rest of the variables are the ones used in the previous analyses. In the extensions of the neoclassical model (Mankiw, Romer and Weil, 1992) as well as in endogenous growth models (Lucas, 1988), economic growth rate is a function of two state variables, named physical capital and human capital. In later models, Becker, Murphy and Tamura (1990) and Azariadis and Drazen (1990), the initial value for human capital is taken as an important determinant for future growth. Easterly and Rebelo (1993) include monetary aspects (*M2*) as well as trade and try to keep constant the effects of some policies which other authors such as Levine and Renelt (1992) or King and Levine (1993) have shown to be robustly correlated with economic growth.

Given that the choice of regressors used in the literature in order to analyze growth varies quite a lot with respect to the framework of study as well as to the limits coming from the availability of the data, we have undertaken the inclusion of the variables according to the various empirical studies in the literature. Thus, deficit has been included in order to test for the economic growth distortions that deficit can induce basically when taken as an indicator for macroeconomic disequilibria, (Levine and Renelt, 1992; Levine and Zervos, 1994; Easterly and Rebelo, 1993; Fischer, 1993a,b; Esterly and Levine, 1997). Based on previous analyses, we have introduced *human capital growth* as well as *initial human capital,* since the first one has been identified as one of the aspects that exerts a larger influence on economic growth (e. g. Denison, 1984), whereas the value for initial human capital allows us to test for the influence of starting points. This last one would be related to the so called level effects, which can be seen as a requirement to adopt ongoing innovations; *trade* (Cornwall, 1976; Harrison, 1996; Frankel and Romer, 1996; Miller and Russek, 1997); *social instability*, (Alesina et al., 1996; Caselli et al., 1996; Sachs and Warner, 1995); *latitude* (Sala-i-Martín, 1997); *government expenditures*, (Levine and Renelt, 1992; Knowles and Owen, 1995; Barro, 1997); *monetary aspects*, (Kormendi and Meguire, 1985; Dollar, 1992; Fischer, 1993b; Englander and Gurney, 1994; Harrison, 1996; de Gregorio, 1996; Andrés and Hernando, 1997); *life expectancy* and *population growth*, (Kormendi and

Meguire, 1985; Dowrick and Nguyen, 1989; Mankiw, Romer and Weil, 1992; Barro and Lee, 1994; Lee, 1995); and *taxes* (Miller and Russek, 1997; Kneller *et al.*, 1998).

Results on annual average growth rates of per capita real GDP are summarized in *table A.2*. We can observe, as shown by *equations 14* and *15*, the presence of a large influence of deficit on growth. This result would confirm the theoretical long-run relationship, where long-run growth was established to depend negatively on the deficit value. From *equation 15* onwards we have introduced a convergence mechanism including initial output in order to capture the possible convergence process. We should note that *equation 15* tells us that the introduction of initial output solely, without initial human capital, does not work as a convergence mechanism, as shown by Baumol (1986), among other authors. Certainly, from there onwards, with the initial values for both variables, output and human capital, we do obtain convergence.

In order to test for the robustness of the deficit, we have added some other variables referred to different aspects of the economy. The results we obtained confirm the robustness of deficit with respect to its significance level. Our results allow us to capture the fact that the size of the influence of the deficit on growth does not change in the different specifications, with the exception of monetary variables that seem to relax a bit the significance of deficit and specially inflation. On the other hand, the sign of the coefficient is always the right one. The values we get for the coefficient on deficit in the various regressions undertaken are between 0.10 and 0.17, in line with the ones obtained in the previous empirical studies, (Easterly and Levine, 1997; Fischer, 1993b), which are between 0.1 and 0.25. This would mean that a one-percentage point increase in the value of deficit translates into around 0.15 percentage points decrease in growth. Finally, we have introduced all the regressors. In this case, the coefficient of deficit is significant only at 17%, and it even has a much lower magnitude. However, the exclusion of monetary variables, as showed in *equation 26*, leads to a highly significant coefficient for the deficit. That could be somehow showing the likely presence of correlation between fiscal and monetary policies, which could be distorting the results. On the other hand, it should be mentioned that the only significant variables appear to be initial income (1%), deficit (5%), latitude (5%), defense expenditure (5%), life expectancy (1%) and population growth (10%).

In sum, our results confirm the hypothesis presented in the theoretical model when we considered a situation where economic disequilibria, as represented by deficits, could exert a negative influence on economic growth in the sense that smaller deficits are associated with faster growth by means of higher human and physical capital accumulation.

5. Conclusion

When looking at the evolution of the economy, one observes how several countries have experienced large periods of sustained growth (Kaldor, 1961), although one can also see the presence of a large dispersion in income across countries or geographical locations (Galor and Zeira, 1993). Several studies have tried to shed light on this fact by using models dealing with capital varieties, improvements in the quality of products or diffusion of technology, among others. Differences in monetary and fiscal policies that have been applied by different economies may also partly explain the different economic performance of several countries during the last decades. Nevertheless, as posed by Romer (1989), in order to explain increases in income as well as in growth rates, it may be enough if we concentrate on variables such as schooling and human capital accumulation. In this paper, we have been mainly concerned about the behavior of the government and its fiscal policy, more specifically, about the presence of macroeconomic disequilibria in a country that we have defined in terms of budget deficits or fiscal imbalances that are capturing government mismanagement of an economy, and sub-optimal allocation of resources. Given the difficulty in defining and gauging [in]stability in a macroeconomic framework, some authors like Fischer (1993b) have specified some stability indicators, mainly centered on deficitarian aspects of an economy. In fact, budget deficits seem to be a macroeconomic indicator that appears in most of the analysis performed to capture the influence of macroeconomic stability on economic growth and it turns out to be one of the most influencing economic variables in this respect.

If we graph the evolution of long-run deficits for different countries (taking deficit as a percentage of GDP) we could appreciate how most of them decrease over time and tend to stabilize at different positive values within different groups of countries. That is to say that there seems to be different convergence values for deficit, depending on the parameters of the economy. According to this, and taking into account that different values of deficit may be related to different interest rate values, in this paper, we have introduced the possibility that the choice of schooling private investment depended on the presence of macroeconomic disequilibria. A simple model of growth that links deficit and human capital with economic growth has been posed. It considers deficit as a harmful aspect for an economy mainly when characterized by persistency and largeness. We have shown that long-run growth differences across countries may depend on their different disequilibria, thus determining the existence of poverty traps for economies with high budget imbalances, and positive rates of growth for those with low (or null) imbalances. This paper goes further away introducing the dynamics of the economy following a change in deficit. It shows the evolution of human and physical capital as well as the change in schooling time, interest rates or relative wages after a fiscal shock. In this sense, we could conclude that a value for deficit that were sufficiently low would be associated with low interest rate values, which would determine high values for private schooling expenditures (and schooling time).

Large deficits that have been reached after periods of excessive spending by the government, which may not disappear and become permanent, due to the nature of spending, are the most likely to exert deleterious effects on growth. Investing in wrong conceived projects (e.g. defense programs, temporary consumption) may not induce economic growth either. Therefore, under these circumstances, large permanent deficits could be weakening the economy. According to our model and taking into account the set of parameter values we used, a decrease of 10 basis points in deficit from 12.5 to 2.5 would finish increasing by around 5% the initial time people devoted to schooling. Besides, underlying our results, in high deficit economies there could be a reduction not only in resources and time itself but also in the efficiency of time devoted to education either because of a lack of resources or because it is simply approaching the mismanagement of the economy. Furthermore, low values for deficit that could be acknowledging a superior management of an economy, or a better distribution of resources, may translate into a better working of the education system, meaning greater efficiency of human capital accumulation and larger per capita income growth.

On the other hand, the broad range of evidence reviewed and presented here seems to support such effects. Regarding the empirical evidence, using data for a large sample of countries during the period 1970-1990, the results we obtained seem to support the idea that deficits may be negatively related to human and physical capital accumulation, mainly due to the burden impinged on both physical and human capital. Our results would be in line with the ones previously obtained in the literature, relating deficitarian situations negatively with growth. More specifically, we found that a 1 percentage point increase in deficit would translate into a human capital growth reduction of about 0.5 percentage points. On the other hand, regressing deficit on output growth gives that a one percentage point increase in deficit is related to approximately 0.15 percentage points decrease in growth, in line with the results obtained in the literature (Easterly and Levine, 1997; Fischer, 1993b).

Taking all this into account, one of the possible solutions to reduce income and growth inequality could be to provide external economic finance for the countries facing high deficits and low growth values under the form of subsidies to interest rates in order to finance a percentage of them when they happen to be far above a certain level. In any case, aid renewal should be tied down to periodic revisions, seeking the fulfillment of certain goals, mainly to devote the subsidies to finance schooling as well as to a better macroeconomic performance of the economy.

Furthermore, despite the generally attributed goodness of surplus, our present model would not predict that larger surpluses translated into a better economic situation. Indeed, a high surplus situation could also represent a sub optimal situation given that there could be some idle resources in the economy. This could mean that the economy was not working at its full potential level, thus wasting resources.

Passing now to other matters, one could believe that it is not only time devoted to education what helps improving human capital of agents who get education, but also the quality of the education acquired. We suspect that quality or effectiveness of training might be closely related to macroeconomic stability. Also, the distribution of government budget and the good or bad management of the economy could be an interesting aspect to go deep into in future research so as to find possible answers as well as solutions for the bad performance of some economies, once macroeconomic disequilibria are defined in more detail.

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Appendix 1. Empirical results.

Table A.1. Human capital growth equation. Dependent variable, GH: ln(H90)-ln(H70)*, **.

	Eq. 1	Eq.2	Eq.3	Eq.4	Eq.5	Eq.6	Eq.7	Eq.8	Eq.9	Eq.10	Eq.11	Eq.12	Eq.13	
Constant	0.5^{a} (0)	0.8^{a} (0)	-0.1 (0.65)	0.79^{a} (0)	0.87^{a} (0)	0.73^{a} (0)	0.81^{a} (0)	0.8^{a} (0)	1.12^{a} (0.0022)	0.77^{a} (0)	-3.04^{a} (0)	0.81^{a} (0)	-3.37^{a} (0.0093)	**Constant**
H70		-0.32^{a} (0)	-0.44^{a} (0)	-0.32^{a} (0)	-0.34^{a} (0)	-0.38^{a} (0)	-0.34^{a} (0)	-0.35^{a} (0)	-0.32^{a} (0)	-0.33^{a} (0)	-0.49^{a} (0)	-0.35^{a} (0)	-0.54^{a} (0)	**H70**
Surplus (Deficit)	0.7^{c} (0.08)	0.58^{b} (0.011)	-0.11 (0.79)	0.56^{b} (0.011)	0.6^{a} (0.0069)	0.32 (0.17)	0.53^{b} (0.014)	0.78^{a} (0.0009)	0.51^{b} (0.045)	0.64^{a} (0.0028)	0.29 (0.2)	0.51^{b} (0.026)	0.94^{a} (0.003)	**Surplus (Deficit)**
Y70			0.12^{a} (0.0001)											**GY**
GY			2.5^{b} (0.03)											**Y70**
Trade				0.0005 (0.2)									0.002^{b} (0.01)	**Trade**
Coups					-0.56 (0.17)								0.08 (0.86)	**Coups**
Assas					-0.0098 (0.68)								-0.073^{a} (0.009)	**Assas**
Revols					-0.033 (0.67)								0.14 (0.58)	**Revols**
Riots					-0.006 (0.65)								0.038^{b} (0.02)	**Riots**
Urban						0.0023^{b} (0.02)							0.0023 (0.11)	**Urban**
Latitude						0.0016^{a} (0.0039)	0.002^{a} (0.0015)						0.0017 (0.01)^{b}	**Latitude**
R4						-0.031 (0.74)	-0.08 (0.33)						-0.14 (0.31)	**R4**
Education								0.0029 (0.83)					-0.02 (0.12)	**Education**
Defense								0.01^{c} (0.08)					0.009 (0.18)	**Defense**
Health								0.0084 (0.47)					0.014 (0.31)	**Health**
M2									0.0005 (0.56)	0.001^{c} (0.08)			-0.003^{a} (0.005)	**M2**
Interest									0.0014 (0.5)	-0.0003 (0.67)			$2.25\cdot10^{-6}$ (0.99)	**Interest**
XR									-0.067 (0.33)				-0.15 (0.05)	**XR**
BMP									-0.00005 (0.78)				$6.27\cdot10^{-5}$ (0.81)	**BMP**
Inflation									0.0002^{c} (0.10)				-0.0002 (0.16)	**Inflation**
Life											0.97^{a} (0)		1.26^{a} (0.0002)	**Life**
Taxes												-0.0011 (0.63)	-0.008^{a} (0.0005)	**Taxes**
N	90	90	86	88	90	89	89	70	72	82	90	74	55	**N**
R2	0.01	0.75	0.79	0.74	0.76	0.78	0.77	0.77	0.72	0.75	0.85	0.76	0.87	**R2**
F	0.98	131	80	83	45	60	71	44	24	58	126	75	12	**F**

Note: OLS results are robust to White (1980) heteroscedasticity correction method.
Note: ^{a} 1%confidence level; ^{b} 5% confidence level; ^{c} 10% confidence level.
* Deficit values have been taken with a negative sign, whereas surplus values have been taken with a positive sign.
** P -values of the regression coefficients are in parenthesis.

Table A.2 .Output growth equation. Dependent variable, GY : ln(Y90)-ln(Y70), **.*

	Eq.14	Eq.15	Eq.16	Eq.17	Eq.18	Eq.19	Eq.20	Eq.21	Eq.22	Eq.23	Eq24	Eq.25	Eq.26	
Constant	0.018^{a}	0.0006	0.072^{a}	0.064^{a}	0.09^{a}	0.126^{a}	0.095^{a}	0.12^{b}	0.11^{a}	-0.306^{a}	0.08^{a}	-0.070	-0.23^{c}	**Constant**
	(0)	(0.96)	(0.0012)	(0.0043)	(0.0003)	(0)	(0)	(0.012)	(0)	(0.0006)	(0.0036)	(0.63)	(0.01)	
Y70		0.0023	-0.012^{a}	-0.008^{a}	-0.010^{a}	-0.0171^{a}	-0.012^{a}	-0.015^{a}	-0.015^{a}	-0.019^{a}	-0.01^{c}	-0.025^{a}	-0.024^{a}	**Y70**
		(0.16)	(0)	(0.0052)	(0.0014)	(0.0001)	(0.0006)	(0)	(0)	(0)	(0.063)	(0.0001)	(0)	
Surplus (Deficit)	0.1^{b}	0.097^{b}	0.12^{b}	0.13^{b}	0.15^{b}	0.14^{b}	0.16^{c}	0.07	0.11^{b}	0.13^{b}	0.14^{b}	0.019	0.17^{b}	**Surplus (Deficit)**
	(0.02)	(0.036)	(0.046)	(0.0388)	(0.02)	(0.0119)	(0.055)	(0.18)	(0.025)	(0.019)	(0.045)	(0.78)	(0.04)	
H70			0.029^{a}	0.013^{a}	0.015^{a}	0.016^{a}	0.016^{a}	0.015^{a}	0.015^{a}	0.0006	0.014^{a}	0.016	0.0033	**H70**
			(0.0001)	(0)	(0)	(0)	(0.0001)	(0)	(0)	(0.84)	(0.0007)	(0.17)	(0.75)	
GH			0.032^{a}									0.013	-0.0043	**GH**
			(0.01)									(0.43)	(0.8)	
Trade				0.0001^{a}								$-7.6\cdot10^{-5}$	$-7\cdot10^{-5c}$	**Trade**
				(0.0001)								(0.60)	(0.15)	
Coups					-0.003							-0.091	-0.21	**Coups**
					(0.25)							(0.15)	(0.65)	
Assas					-0.002							-0.002	-0.001	**Assas**
					(0.42)							(0.33)	(0.63)	
Revols					-0.015^{c}							0.027	-0.015	**Revols**
					(0.08)							(0.32)	(0.37)	
Riots					0.0011							0.0002	0.002^{c}	**Riots**
					(0.41)							(0.87)	(0.07)	
Urban						0.0001						$-7.5\cdot10^{-5}$	$-6.48\cdot10^{-}$	**Urban**
						(0.33)						(0.68)	(0.71)	
Latitude						0.0002^{a}						0.0002^{c}	0.0002^{c}	**Latitude**
						(0.0003)						(0.06)	(0.04)	
R4						-0.0007						-0.02^{c}	-0.014^{c}	**R4**
						(0.87)						(0.07)	(0.07)	
Education							0.0004					0.0003	$-1.3\ 10^{-5}$	**Education**
							(0.68)					(0.86)	(0.99)	
Defense							0.001^{c}					-0.0003	0.0001^{c}	**Defense**
							(0.07)					(0.81)	(0.013)	
Health							0.001					-0.001	-0.0007	**Health**
							(0.28)					(0.49)	(0.59)	
M2								0.0003^{a}	0.0003^{a}			0.0002		**M2**
								(0.0002)	(0.0002)			(0.19)		
Interest								0.0004^{a}	0.0005^{a}			0.0004		**Interest**
								(0.01)	(0)			(0.28)		
XR								-0.002				0.0005		**XR**
								(0.68)				(0.96)		
BMP								$-3.7\cdot10^{-5}$	$-4.7\cdot10^{-5}$			$-4.3\cdot10^{-5}$		**BMP**
								(0.029)	(0)			(0.15)		
Inflation								$-9\cdot10^{-6}$				$1.17\cdot10^{-5}$		**Inflation**
								(0.58)				(0.67)		
Life										0.118^{a}		0.066^{b}	0.11^{a}	**Life**
										(0)		(0.04)	(0.006)	
GPOP										-0.004^{c}		-0.002	-0.006^{c}	**GPOP**
										(0.07)		(0.63)	(0.08)	
Taxes											0.0003	0.0001	-0.0005	**Taxes**
											(0.23)	(0.77)	(0.16)	
N	90	90	86	85	86	85	66	71	75	86	70	54	65	**N**
R2	0.04	0.04	0.32	0.34	0.33	0.37	0.26	0.54	0.54	0.48	0.22	0.72	0.64	**R2**
F	5.55	3.5	9.4	10.3	5.4	7.6	3.4	9	13.6	15	4.7	3.4	4.7	**F**

Note: OLS results are robust to White (1980) heteroscedasticity correction method.
Note: ^{a} 1%confidence level;^{b} 5% confidence level; ^{c} 10% confidence level.
* Deficit values have been taken with a negative sign, whereas surplus values have been taken with a positive sign.
** P -values of the regression coefficients are in parenthesis.

Appendix 2. Sample countries

Table A.3. Countries included in the sample

Argentina	Haiti	Panama
Australia	Honduras	Papua New Guinea
Austria	Hungary	Paraguay
Bahrain	Iceland	Peru
Bangladesh	India	Philippines
Barbados	Indonesia	Poland
Belgium	Iran, Islamic Rep.	Portugal
Benin	Ireland	Rwanda
Bolivia	Israel	Senegal
Botswana	Italy	Sierra Leone
Brazil	Jamaica	Singapore
Cameroon	Japan	South Africa
Canada	Jordan	Spain
Central African Republic	Kenya	Sri Lanka
Chile	Korea, Rep.	Sudan
Colombia	Kuwait	Swaziland
Congo, Dem. Rep.	Lesotho	Sweden
Costa Rica	Liberia	Switzerland
Cyprus	Malawi	Syrian Arab Republic
Denmark	Malaysia	Thailand
Dominican Republic	Mali	Trinidad and Tobago
Ecuador	Malta	Tunisia
El Salvador	Mauritius	Turkey
Fiji	Mexico	Uganda
Finland	Myanmar	United Kingdom
France	Netherlands	United States
Ghana	New Zealand	Uruguay
Greece	Nicaragua	Venezuela
Guatemala	Norway	Zambia
Guyana	Pakistan	Zimbabwe

Appendix 3. Linearization

Linearizing around the steady state denoted by $\tilde{k}, \tilde{h}, \tilde{l}$, the dynamics of the system might be approximated by:

$$\begin{pmatrix} \dot{k} \\ \dot{h} \\ \dot{l} \end{pmatrix} = \begin{pmatrix} s_K\left(1-t_y-d-\frac{\tilde{x}}{\tilde{k}}s_K\right)\frac{\tilde{y}}{\tilde{k}}-d_K-g_K & s_H\left(1-t_y-d-\frac{\tilde{x}}{\tilde{k}}s_K\right)\frac{\tilde{y}}{\tilde{h}} & \left\{-s_H\frac{\tilde{y}}{1-\tilde{l}}\left(1-t_y-d-\frac{\tilde{x}}{\tilde{k}}s_K\right)-s_H\frac{\tilde{y}}{\left(1-\tilde{l}\right)^2}\right\} \\ h_X\frac{\tilde{j}}{\tilde{x}}\frac{\tilde{x}}{\tilde{k}}+h_E s_K\frac{\tilde{j}}{\tilde{k}} & \left(h_E s_H+h_H\right)\frac{\tilde{j}}{\tilde{h}}-d_H-g_H & \left\{h_l\frac{\tilde{j}}{\tilde{l}}-\frac{\tilde{j}}{1-\tilde{l}}h_E s_H+h_X\frac{1}{\tilde{x}/\tilde{j}}\frac{s_H}{s_K}\frac{1}{\left(1-\tilde{l}\right)^2}\right\} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad (A.3.1)$$

where:

$$a_{31} \equiv \left\{ \begin{aligned} &(s_K-1)\frac{\tilde{y}}{\tilde{k}^2}\left(-\left(1-t_y-d\right)s_K BB+\left(\left(1-t_y-d\right)-s_K\frac{\tilde{x}}{\tilde{k}}\right)\left[\left(1-h_E\right)s_K-h_X\right]\right)+\left[\left(1-h_E\right)s_K-h_X\right]\frac{\tilde{c}}{\tilde{k}^2}+ \\ &\frac{\tilde{j}}{\tilde{h}\tilde{k}}\left(\left(s_K h_E+h_X\left(\frac{s_H}{s_K}\frac{1}{1-\tilde{l}}-1\right)\right)\left[\frac{1-\tilde{l}}{\tilde{l}}h_l+\left(1-h_E\right)s_H\right]\right) \end{aligned} \right\} \frac{1}{DEN}$$

$$a_{32} \equiv \left\{(s_H)\frac{\tilde{y}}{\tilde{k}\tilde{h}}\left(-\left(1-t_y-d\right)s_K BB+\left(\left(1-t_y-d\right)-s_K\frac{\tilde{x}}{\tilde{k}}\right)\left[\left(1-h_E\right)s_K-h_X\right]\right)+\left[\left(1-h_E\right)s_H+\left(\frac{\left(1-\tilde{l}\right)}{\tilde{l}}h_l\right)\right]\left(\left(h_H-1+h_E s_H\right)\frac{\tilde{j}}{\tilde{h}^2}\right)\right\}\frac{1}{DEN}$$

$$a_{33} \equiv \left\{ \begin{aligned} &\left[\left(s_H\left(1-h_E\right)\right)+\left(\frac{1-\tilde{l}}{\tilde{l}}h_l\right)\right]\left(h_l\frac{\tilde{j}}{\tilde{h}\tilde{l}}+h_X\frac{\tilde{j}}{\tilde{h}}\tilde{k}\frac{s_H}{s_K}\frac{1}{\left(1-\tilde{l}\right)^2}-h_E s_H\frac{\tilde{j}}{\tilde{h}}\frac{1}{\left(1-\tilde{l}\right)}\right)+s_H s_K\left(1-t_y-d\right)\frac{\tilde{y}}{\tilde{k}}\frac{1}{\left(1-\tilde{l}\right)}BB- \\ &-s_K\left(1-t_y-d\right)\frac{\tilde{y}}{\tilde{k}}\frac{h_E h_l s_H/\tilde{l}^2}{\left(\frac{h_l}{l}-h_l-h_E s_H\right)^2}+\left(\left[\left(1-h_E\right)s_K-h_X\right]\left\{\left(-\left(1-t_y-d\right)-s_K\frac{\tilde{x}}{\tilde{k}}\right)(s_H)\frac{\tilde{y}}{\tilde{k}}\frac{1}{\left(1-\tilde{l}\right)}-s_K\frac{\tilde{y}}{\tilde{k}}\frac{s_H}{s_K}\frac{1}{\left(1-\tilde{l}\right)^2}\right\}\right)-\frac{\tilde{j}}{\tilde{h}}\frac{h_l}{\tilde{l}^2} \end{aligned} \right\}\frac{1}{DEN}$$

with:

$$DEN \equiv \left(\left(1-h_E\right)\frac{s_H}{1-\tilde{l}}+\frac{h_l}{\tilde{l}}-\left(1-h_X\right)FF\right)$$

The determinant of the matrix is proportional to the value *DEN*. According to this, one of the conditions we can find for the determinant to be positive is to impose the following:

$(1-h_X)\frac{h/l^2}{(h_l/\tilde{l}-h_l-h_E s_H)} < \frac{h_l}{\tilde{l}} + (1-h_E)\frac{s_H}{1-\tilde{l}}$. Having a positive determinant means that it has either 1 or 3 positive roots. Due to the complexity of the system, we cannot find as general a condition to rule out the case of three unstable roots. However, the condition $s_K \leq \frac{1}{2-V}$ seems a plausible restriction that may suffice to eliminate the first case of explosive roots. This condition is met in our simulations. Since l can jump instantaneously, while the variables (k, h) are constrained to adjust continuously, the number of unstable roots in our equation equals the number of jump variables. Henceforth, we assume that the stability properties are ensured so that we can proceed with the case with two negative roots (one positive root), which we will denote by $m_2 < m_1 < 0$, and it is defined as follows. The key variables of interest are physical capital and human capital. The generic form for the stable solution for our variables is given by:

$$k(t) - \tilde{k} = B_1 e^{m_1 t} + B_2 e^{m_2 t} \tag{A.3.2}$$

$$h(t) - \tilde{h} = B_1 v_{21} e^{m_1 t} + B_2 v_{22} e^{m_2 t} \tag{A.3.3}$$

$$l(t) - \tilde{l} = B_1 v_{31} e^{m_1 t} + B_2 v_{32} e^{m_2 t} \tag{A.3.4}$$

where B_1, B_2 are constants that depend on the initial conditions in the economy as well as upon the specific shocks, (i.e. an increase in the policy parameter *g*, which would in turn increase *d*). The normalized eigenvector associated with the stable eigenvalue m_i, is the vector *i=1,2* is $(1 \quad v_{2i} \quad v_{3i})$. With equations (A.3.2) and (A.3.3), setting *t=0*, we obtain:

$$B_1 = \frac{(k(0)-\tilde{k})v_{22} - (h(0)-\tilde{h})}{v_{22}-v_{21}} \tag{A.3.5}$$

$$B_2 = \frac{(h(0)-\tilde{h}) - v_{21}(k(0)-\tilde{k})}{v_{22}-v_{21}} \tag{A.3.6}$$

The value for $l(0)$ would be determined in response to the shock in the following way: $l(0) = \tilde{l} + B_1 v_{31} + B_2 v_{32}$.