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On Estimation of Autoregressive Conditional Duration (ACD) Models Based on Different Error Distributions

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ABSTRACT

Autoregressive Conditional Duration (ACD) models play a central role in modelling high frequency financial data. The Maximum Likelihood (ML) and Quasi Maximum Likelihood (QML) methods are widely used in parameter estimation. This paper considers a semi parametric approach based on the theory of Estimating Function (EF) in estimation of ACD models. We use a number of popular distributions with positive supports for errors and estimate the parameter(s) using the both EF and ML approaches. A simulation study is conducted to compare the performance of the EF and the corresponding ML estimates for ACD(1,1), ACD(1,2) and ACD(2,1) models. It is shown that the EF approach provides comparable estimates with the ML estimates using a shorter computation time. Finally, both methods are applied to model a real financial data set and provide empirical evidence to support the use EF approach in practice.

Keywords: Conditional duration, Estimating function, High Frequency data, Maximum likelihood

INTRODUCTION

In many financial modelling problems, we face with the problem of analyzing high frequency data. A class of high frequency data models are originally appeared as "fixed-interval" models, where all transactions are recorded as the fixed time intervals. However, one main drawback of these models is that they do not take into account the irregular spacing of the data. As a result, we may lose some useful information if the transactions cluster differently within a fixed interval. To avoid such loss of information, Engle and Russell (1998) had proposed the class of models called the autoregressive conditional duration (ACD) models. This class of models adapts the AR and GARCH theory to study the dynamic structure of the adjusted durations and can be used to analyze transaction data with irregular time intervals.

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Let x_i be the adjusted duration such that $x_i = t_i - t_{i-1}$, where t_i is the time of the ith transaction and let

$$\psi_i = E[x_i \mid x_{i-1}, \cdots, x_1] = E[x_i \mid F_{i-1}], \quad (1)$$

where F_{i-1} is the information set available at the $(i-1)$ th trade.

The basic ACD model is defined as

$$x_i = \psi_i \varepsilon_i, \quad (2)$$

where ε_i is a sequence of independently and identically distributed (iid) non-negative random variable's with density $f(.)$ and $E(\varepsilon_i) = 1$ and ε_i is independent of F_{i-1}.

From Equation (2) it is clear that a vast set of ACD model specifications can be defined by allowing different distributions for ε_i and specifications of ψ_i.

A general class of ACD models generated from (2) is called ACD (m,q),($m \geq 1, q \geq 0$) and is given by

$$\psi_i = \omega + \sum_{j=1}^{m} a_j x_{i-j} + \sum_{j=1}^{q} b_j \psi_{i-j}, \quad (3)$$

where $\omega > 0$, $a_j, b_j > 0$ and $\sum_{j=1}^{r} (a_j + b_j) < 1$, and $r = \max(m,q)$.

This paper focuses on the parameter estimation of ACD models based on a number of different distribution with positive support for ε_i. Engle and Russel (1998) used the ML method to estimate the parameters of the ACD model. Tsay (2002) and Maria Pacurar (2008) also discussed the usage of the ML method. Applications of ACD models are discussed by Allen *et al.*, (2008, 2009) using the Quasi Maximum

Likelihood (QML) methods. If the parameters in the model are not well-estimated, then the model may not be adequate for describing the behavior of the data. The accuracy of forecasts may also affect by a "poor" model.

In their paper, Peiris *et al.*, (2007) suggested the use of the estimating function (EF) approach to estimate the parameters in ACD models. Peiris (2008) shows a simulation result of the estimation of ACD models using estimating functions.

Since the error distribution of the model is not known in practice, this paper extends the work of Peiris et.al.(2007) and focuses on the estimating parameters of ACD models based on various possible error distributions. We assess the performance of EF and ML methods and compare the bias and the standard errors in each case. We consider four popular different non-negative distribution for ε_i including the Exponential, Rayleigh, Lognormal and Gamma to model ACD structures. Results show that both methods are comparable in estimating the parameters of the ACD models but the EF method proves to be faster and efficient than the ML for estimating the parameters when the distribution is unknown.

With that view in mind the following section reviews the parameter estimation problem based on the ML and EF approaches.

ESTIMATION

This section considers the ML and EF approaches to estimate the parameters of ACD models as well as the parameters of the corresponding error distributions.

The ML approach

Let $i_0 = \max(m,q)$ and $\mathbf{x}_n = (x_1,\cdots,x_n)'$ for an ACD model, where n is the sample size. The likelihood function for the durations is

$$L(\mathbf{x}_n \mid \boldsymbol{\theta}, \mathbf{x}_{i_0}) = \prod_{i=1}^{n} f(x_i \mid F_{i-1}, \boldsymbol{\theta})$$

$$= \left[\prod_{i=i_0+1}^{n} f(x_i \mid F_{i-1}, \boldsymbol{\theta})\right] f(\mathbf{x}_{i_0} \mid \boldsymbol{\theta}),$$

where $\boldsymbol{\theta}$ denotes the vector of model parameters, $\mathbf{x}_{i_0} = (x_1, \cdots, x_{i_0})$ and $f(\mathbf{x}_{i_0} \mid \boldsymbol{\theta}) = \prod_{i=1}^{i_0} f(x_i)$.

As the sample size n increases, the impact of the marginal probability density function (pdf), $f(\mathbf{x}_{i_0} \mid \boldsymbol{\theta})$ on the likelihood function diminishes. Thus, the marginal density can be ignored resulting in the conditional likelihood function as below:

$$L(\mathbf{x}_n \mid \boldsymbol{\theta}, \mathbf{x}_{i_0}) = \prod_{i=i_0+1}^{n} f(x_i \mid F_{i-1}, \boldsymbol{\theta}). \tag{4}$$

For some selected distributions of ε_i, the likelihood functions are as shown below:

1. Standardized Exponential distribution : $L(\mathbf{x} \mid \mathbf{x}_{i_0}) = \prod_{i=i_0+1}^{n} \left\{ \frac{1}{\psi_i} e^{-\left(\frac{x_i}{\psi_i}\right)} \right\}$

2. Standardized Rayleigh distribution : $L(\mathbf{x} \mid \mathbf{x}_{i_0}) = \prod_{i=i_0+1}^{n} \left\{ \frac{\pi}{2}\left(\frac{x_i}{\psi_i^2}\right) e^{-\left(\frac{\pi}{4}\right)\left(\frac{x_i^2}{\psi_i^2}\right)} \right\}$

3. Standardized Lognormal distribution :

$$L(\mathbf{x} \mid \sigma^2, \mathbf{x}_{i_0}) = \prod_{i=i_0+1}^{n} \left\{ \frac{1}{x_i}\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left\{-\left[\log\left(\frac{x_i}{\psi_i}\right)+\frac{\sigma^2}{2}\right]^2 / 2\sigma^2\right\}} \right\}$$

4. Standardized Gamma distribution : $L(\mathbf{x} \mid \kappa, \mathbf{x}_{i_0}) = \prod_{i=i_0+1}^{n} \left\{ \left(\frac{\kappa^{\kappa}}{\Gamma(\kappa)}\left(\frac{x_i}{\psi_i}\right)^{\kappa-1} e^{-\frac{x_i \kappa}{\psi_i}}\right) \frac{1}{\psi_i} \right\}$

Now we consider the theory of estimating functions (EF) as an alternative semi-parameter approach for parameter estimation.

The Estimating Function Approach

Let $\{y_1, y_2, \cdots\}$ be a discrete stochastic process. We are interested of fitting a suitable model for a sample of size n from this process. Let Θ be a class of probability distributions F on R^n and $\theta = \theta(F)$, $F \in \Theta$ be a vector of real parameters.

Let h_i be a real valued function of $y_1, y_2, \cdots, y_i$ and θ such that

$$E_{i-1,F}[h_i\{y_1, y_2, \cdots, y_i; \theta(F)\}] = 0\,,\ (i = 1,2,\cdots,n; F \in \Theta)$$

and

$$E(h_i h_j) = 0,\ (i \neq j),$$

where $E_{i-1,F}(.)$ denotes the expectation holding the first $i-1$ values $y_1, y_2, \cdots, y_{i-1}$ fixed and $E_{i-1,F}(.) \equiv E_{i-1}$, $E_{0,F}(.) \equiv E_F(.) \equiv E(.)$ (unconditional mean).

Estimating Functions

Any real valued function $g(.)$ of the random variates $y_1, y_2, \cdots, y_n$ and the parameter θ, that can be used to estimate θ is called an estimating function.

If $g(.)$ satisfies regularity conditions (i) the first and the second derivatives of $g(.)$, $g'(.)$ and $g''(.)$ exist, and (ii) $E[g^2(.)]$ is non-zero

and

$$E[g\{y_1, y_2, \cdots, y_n; \theta(F)\}] = 0$$

then $g(.)$ is called a regular unbiased estimating function.

Among all regular unbiased estimating functions g, g^* is said to be optimum if

$$\frac{E[g^2\{y_1, y_2, \cdots, y_n; \theta(F)\}]}{\left\{E\left(\left[\frac{\partial(y_1, y_2, \cdots, y_n; \theta)}{\partial\theta}\right]_{\theta=\theta(F)}\right)\right\}^2} \tag{5}$$

is minimized for all $F \in \Theta$ at $g = g^*$.

Then, we estimate θ by solving the optimum estimating equation

$g^*(y_1, y_2, \cdots, y_n; \theta) = 0$.

Main Results

We restrict initially to estimating functions g of the form

$$g = \sum_{i=1}^{n} h_i a_{i-1}$$

where the functions h_i are as defined before and a_{i-1} is a function of the random variates $y_1, y_2, \cdots, y_{i-1}$ and the parameter θ for all $i = 1, 2, \cdots, n$. We consider the class of linear estimating functions L generated by g. Note that g being linear in h_i, the class L corresponds to linear functions in Gauss-Markov set-up for linear models.

Clearly,

$E(g) = 0,\ g \in L$.

Now we state the following theorem due to Godambe(1985):

Theorem

In the class L of estimating functions g, the function g^* minimizing (5) is given by

$$g^* = \sum_{i=1}^{n} h_i a_{i-1}^*,$$

where

$$a_{i-1}^* = \frac{E_{i-1}\left[\dfrac{\partial h_i}{\partial \theta}\right]}{E_{i-1}[h_i^2]}.$$

Notes:

1. The function g^* is called the optimum estimating function.
2. Based on Godambe (1985), an optimal estimate of θ can be obtained by solving the equation(s) $g^* = 0$.

See Thavaneswaran and Abraham (1988) and Grahramani and Thavaneswaran (2009) for theory and various applications of estimating functions.

Estimation of ACD (m,q) Using the EF Approach

Consider the ACD (m,q) model given by Equations (2) and (3). It is clear that the conditional distribution

$$x_i \mid \Omega_{i-1} \sim (\psi_i, \psi_i^2 \sigma_\varepsilon^2),$$

where Ω_{i-1} is the information set available at time $i-1$ and σ_ε^2 is the variance of ε_i.

Let $h_i = \psi_i - x_i$. It is obvious that h_i is an unbiased estimating function. Now, a linear unbiased estimating function is constructed such that

$$g = \sum_{i=1}^{n} h_i a_i^* ,$$

where $a_i^* = \dfrac{\frac{\partial \psi_i}{\partial \theta}}{\psi_i^2 V}$ and θ is a parameter.

Solving the following system of Equation (6) for θ and the optimal set of estimates can be obtained:

$$\sum_{i=1}^{n} \frac{1}{\sigma_\varepsilon^2 \psi_i^2} \frac{\partial \psi_i}{\partial \theta}(\psi_i - x_i) = 0 \qquad (6)$$

The following derivatives under the conditions of i-th order stationarity can be used:

- $\dfrac{\partial \psi_i}{\partial \omega} = 1 + \sum_{j=1}^{q} b_j \dfrac{\partial \psi_{i-j}}{\partial \omega}$
- $\dfrac{\partial \psi_i}{\partial a_k} = x_{i-k} + \sum_{j=1}^{q} b_j \dfrac{\partial \psi_{i-j}}{\partial a_k}$
- $\dfrac{\partial \psi_i}{\partial b_l} = \psi_{i-l} + \sum_{j=1,\, j \neq l}^{q} b_j \dfrac{\partial \psi_{i-j}}{\partial b_j}$

where k and l are respectively the subscripts of the parameter of interest for a and b.

It is easy to see that for ACD(1,1) model, we have

$$Var(x_i) = \frac{(s-1)\mu_x^2[1 - b_1^2 - 2a_1 b_1]}{1 - s a_1^2 - b_1^2 - 2a_1 b_1} \qquad (7)$$

where s is the variance of ε_i.

For example,

a) $s = 2$ for standardized Exponential distribution

b) $s = \frac{4}{\pi}$ for standardized Rayleigh distribution

c) $s = e^{\sigma^2}$ for standardized Lognormal distribution

d) $s = \frac{\kappa + 1}{\kappa}$ for standardized Gamma distribution

SIMULATION

This section considers a large scale simulation study in order to compare the performances of the MLE and EF approaches for an ACD(1,1) model based on a number of standardised distributions as mentioned above in (a) to (e). Let $\hat{\theta}$ be an estimator of parameter θ. Suppose that we simulate a series of length n and estimate respective parameters. Repeat these simulation and estimation steps N times and calculate the following:

i) Mean, $\overline{\hat{\theta}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}$

ii) Estimated Bias $= \overline{\hat{\theta}} - \theta$

iii) Absolute Relative Estimated Bias (%) $= \left(\frac{|\text{Estimated Bias}|}{\theta}\right) \times 100\%$

iv) Estimated Standard Deviation (S.D) $= \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\hat{\theta} - \overline{\hat{\theta}})^2}$

v) Mean Squared Error, $MSE(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right]$

$$= Var(\hat{\theta}) + (bias)^2$$

vi) Relative Efficiency $= \frac{\text{Estimated SD of EF}}{\text{Estimated SD of ML}}$

For the Exponential ACD (EACD) case EACD(1, 1), EACD(2, 1), EACD(1, 2) and EACD(2, 2) have been studied and the corresponding estimating results have been reported in Tables 1 to 4. It seems that the results obtained using the ML and EF methods are comparable. However, the EF method provides smaller estimated standard error in estimating the parameters $\hat{b}_1$ and $\hat{b}_2$ for both EACD(1, 2) (in Table

3) and EACD(2, 2) (in Table 4) models. Although the absolute relative bias, the means of $\hat{b}_1$ and $\hat{b}_2$ seem to be slightly better in ML estimates than the EF method, the computation time is relatively smaller in the EF approach. When the Mean Squared Error (MSE) is scrutinized for the case involving the EACD(1, 2) model, the results for $\hat{b}_1$ and $\hat{b}_2$ appear to be comparable. The MSE for $\hat{\omega}$ is slightly larger when EF is applied. The MSE computed for $\hat{\varpi}$, $\hat{b}_1$ and $\hat{b}_2$ are smaller when the EF approach is utilized in estimating the parameters of the EACD(2, 2) model.

A similar conclusion can be drawn for Lognormal ACD (1, 1) model, Rayleigh ACD (1, 1) model and Gamma ACD model(GACD(1, 1)) (Refer to Tables 5 to 7) the estimates of the parameters are somewhat comparable for both EF and ML methods. In general, the EF method's computation time is 5 times faster compared to the ML method.

Table 1: Estimated Results for Simulated Exponential ACD (1, 1) Series with 500 observations ($\omega = 0.20$, $a_1 = 0.30$, $b_1 = 0.60$ and $\psi_1 = 0.50$).

	$\hat{\omega}$		$\hat{a}_1$		$\hat{b}_1$	
	ML	**EF**	**ML**	**EF**	**ML**	**EF**
Mean	0.2198	0.2205	0.2977	0.2949	0.5893	0.5905
Estimated Bias	0.0198	0.0205	-0.0023	-0.0051	-0.0107	-0.0095
Abs. Rel. Est. Bias	9.90%	10.25%	0.77%	1.70%	1.78%	1.58%
Estimated S.D.	0.0715	0.0695	0.0527	0.0521	0.0686	0.0689
MSE	0.0055	0.0053	0.0028	0.0027	0.0048	0.0048
Relative Efficiency	1.0000	0.9720	1.0000	0.9886	1.0000	1.0044

Table 2: Estimated Results for Simulated Exponential ACD (2, 1) Series with 500 observations ($\omega = 0.10, a_1 = 0.20, a_2 = 0.30, b_1 = 0.40$, $\psi_1 = 0.40$ and $\psi_2 = 0.60$).

	$\hat{\omega}$		$\hat{a}_1$		$\hat{a}_2$		$\hat{b}_1$	
	ML	**EF**	**ML**	**EF**	**ML**	**EF**	**ML**	**EF**
Mean	0.1082	0.1082	0.1961	0.1959	0.2953	0.2957	0.3928	0.3924
Estimated Bias	0.0082	0.0082	-0.0039	-0.0041	-0.0047	-0.0043	-0.0072	-0.0076
Abs. Rel. Est. Bias	8.20%	8.20%	1.95%	2.05%	1.57%	1.43%	1.80%	1.90%
Estimated S.D.	0.0322	0.0321	0.0564	0.0563	0.0815	0.0817	0.0917	0.0913
MSE	0.0011	0.0011	0.0032	0.0032	0.0067	0.0067	0.0085	0.0084
Relative Efficiency	1.0000	0.9969	1.0000	0.9982	1.0000	1.0025	1.0000	0.9956

Table 3: Estimated Results for Simulated Exponential ACD (1, 2) Series with 500 observations ($\omega = 0.10, a_1 = 0.20, b_1 = 0.30, b_2 = 0.40, \psi_1 = 0.40$ and $\psi_2 = 0.60$)

	$\hat{\omega}$		$\hat{a}_1$		$\hat{b}_1$		$\hat{b}_2$	
	ML	**EF**	**ML**	**EF**	**ML**	**EF**	**ML**	**EF**
Mean	0.1249	0.1236	0.1924	0.1827	0.3516	0.4238	0.3281	0.2652
Estimated Bias	0.0249	0.0236	-0.0076	-0.0173	0.0516	0.1238	-0.0719	-0.1348
Abs. Rel. Est. Bias	24.90%	23.60%	3.80%	8.65%	17.20%	41.27%	17.98%	33.70%
Estimated S.D.	0.0817	0.1107	0.0552	0.0551	0.3245	0.3101	0.2870	0.2586
MSE	0.0073	0.0128	0.0031	0.0033	0.1080	0.1115	0.0875	0.0850
Relative Efficiency	1.0000	1.3550	1.0000	0.9982	1.0000	0.9556	1.0000	0.9010

Table 4: Estimated Results for Simulated Exponential ACD (2, 2) Series with 500 observations ($\omega = 0.40, a_1 = 0.10, a_2 = 0.20, b_1 = 0.20, b_2 = 0.40, \psi_1 = 0.50$ and $\psi_2 = 0.50$)

	$\hat{\omega}$		$\hat{a}_1$		$\hat{a}_2$		$\hat{b}_1$		$\hat{b}_2$	
	ML	EF	ML	EF	ML	EF	ML	EF	ML	EF
Mean	0.4428	0.4192	0.0933	0.0967	0.1807	0.1641	0.3161	0.3870	0.2938	0.2425
Estimated Bias	0.0428	0.0192	-0.0067	-0.0033	-0.0193	-0.0359	0.1161	0.1870	-0.1062	-0.1575
Abs. Rel. Est. Bias	10.70%	4.80%	6.70%	3.30%	9.65%	17.95%	58.05%	93.50%	26.55%	39.38%
Estimated S.D.	0.2064	0.1849	0.0524	0.0515	0.0831	0.0824	0.4155	0.3593	0.3308	0.2817
MSE	0.0444	0.0346	0.0028	0.0027	0.0073	0.0081	0.1861	0.1641	0.1207	0.1042
Relative Efficiency	1.0000	0.8958	1.0000	0.9828	1.0000	0.9916	1.0000	0.8647	1.0000	0.8516

Table 5: Estimated Results for Simulated Rayleigh ACD (1, 1) Series with 500 observations ($\omega = 0.05, a = 0.30, b_1 = 0.60$ and $\psi_1 = 0.50$)

	$\hat{\omega}$		$\hat{a}_1$		$\hat{b}_1$	
	ML	EF	ML	EF	ML	EF
Mean	0.0559	0.0555	0.2986	0.2966	0.5877	0.5912
Estimated Bias	0.0059	0.0055	-0.0014	-0.0034	-0.0123	-0.0088
Abs. Rel. Est. Bias	11.80%	11.00%	0.47%	1.13%	2.05%	1.47%
Estimated S.D.	0.0169	0.0171	0.0386	0.0403	0.0577	0.0589
MSE	0.0003	0.0003	0.0015	0.0016	0.0035	0.0035
Relative Efficiency	1.0000	1.0118	1.0000	1.0440	1.0000	1.0208

Table 6: Estimated Results for Simulated Lognormal ACD (1, 1) Series with 500 observations ($\omega = 0.10, a = 0.20, b_1 = 0.70, \sigma = 1.50$ and $\psi_1 = 0.50$)

	$\hat{\omega}$		$\hat{a}_1$		$\hat{b}_1$		$\hat{\sigma}$	
	ML	**EF**	**ML**	**EF**	**ML**	**EF**	**ML**	**EF**
Mean	0.1151	0.1160	0.1985	0.1974	0.6855	0.6857	0.4983	0.4913
Estimated Bias	0.0151	0.0160	-0.0015	-0.0026	-0.0145	-0.0143	-0.0017	-0.0087
Abs. Rel. Est. Bias	15.10%	16.00%	0.75%	1.30%	2.07%	2.04%	0.34%	1.74%
Estimated S.D.	0.0401	0.0442	0.0372	0.0400	0.0637	0.0690	0.0157	0.0267
MSE	0.0018	0.0022	0.0014	0.0016	0.0043	0.0050	0.0002	0.0008
Relative Efficiency	1.0000	1.1022	1.0000	1.0753	1.0000	1.0832	1.0000	1.7006

Table 7: Estimated Results for Simulated Gamma ACD (1, 1) Series with 500 observations ($\omega = 0.05, a = 0.20, b_1 = 0.70, \kappa = 1.50$ and $\psi_1 = 0.50$)

	$\hat{\omega}$		$\hat{a}_1$		$\hat{b}_1$		$\hat{\kappa}$	
	ML	**EF**	**ML**	**EF**	**ML**	**EF**	**ML**	**EF**
Mean	0.0584	0.0590	0.2004	0.2010	0.6820	0.6797	1.5169	1.6055
Estimated Bias	0.0084	0.0090	0.0004	0.0010	-0.0180	-0.0203	0.0169	0.1055
Abs. Rel. Est. Bias	16.80%	18.00%	0.20%	0.50%	2.57%	2.90%	1.13%	7.03%
Estimated S.D.	0.0230	0.0238	0.0420	0.0419	0.0721	0.0736	0.0881	0.2518
MSE	0.0006	0.0006	0.0018	0.0018	0.0055	0.0058	0.0080	0.0745
Relative Efficiency	1.0000	1.0348	1.0000	0.9976	1.0000	1.0208	1.0000	2.8581

APPLICATION OF ACD MODELS IN FINANCIAL DATA

As an analogy of duration models in financial data set, the transaction durations of IBM stock on five consecutive trading days from November 1 to November 7, 1990 was considered. This data was obtained from *(Tsay,2002)*. We have 3534 observations where the positive transaction durations were focused on. In a nutshell, we employ 3534 positive adjusted durations. Figures 1 to 3 are respectively the series, the histogram of the series and the autocorrelation (ACF) of the series. Based on Figure 3, there exist some serial correlations in the adjusted durations.

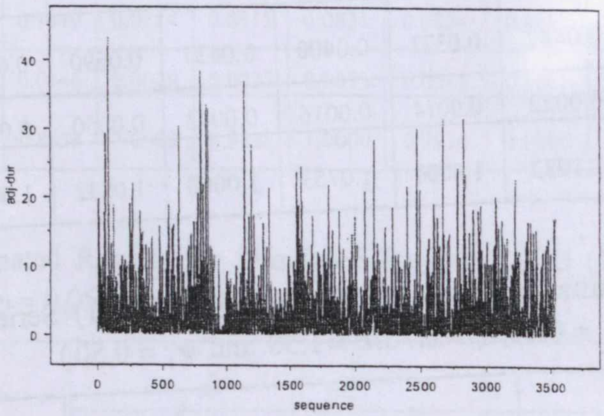

Figure 1: Time plots of durations for IBM stock traded in the first five trading days of November 1990: the adjusted series.

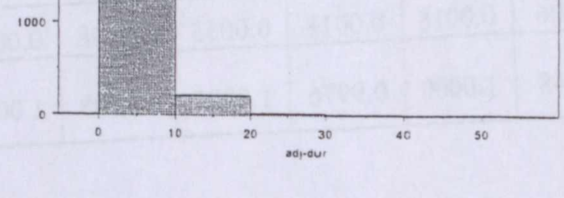

Figure 2: The histogram of the adjusted series.

Adjusted series

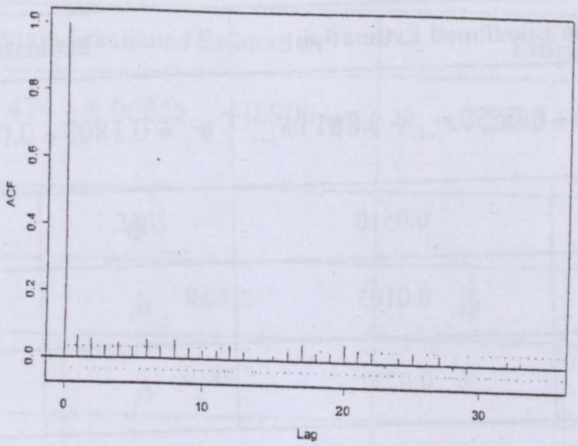

Figure 3: ACF of the adjusted series

Based on the above analysis, we have fitted the ACD(1,1) model from the distributions proposed earlier using both ML and EF methods for each of the distribution. In all cases we have use $\psi_1 = 1.0$ as the initial value.

The ACD(1,1) model is represented by:

$$x_i = \psi_i \varepsilon_i \qquad \text{and} \qquad \psi_i = \omega + a_1 x_{i-1} + b_1 \psi_{i-1}$$

where $\{\varepsilon_i\}$ is a sequence of independent and identical non-negative random variables with density $f(.)$ and $E(\varepsilon_i) = 1$.

The corresponding results are shown in Tables 8 to 11. As before, for this data set, the results for the parameter estimates are comparable except for the Rayleigh distribution.

Table 8: EACD(1,1) model fitted to the data

		Maximum Likelihood Estimation		Estimating Function
EACD (1,1)	$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$		$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$	
Standard Error	$\hat{\omega}$	0.0510	$\hat{\omega}$	0.0534
	$\hat{a}_1$	0.0103	$\hat{a}_1$	0.0111
	$\hat{b}_1$	0.0222	$\hat{b}_1$	0.0239

Table 9: Rayleigh ACD(1,1) model fitted to the data

		Maximum Likelihood Estimation		Estimating Function
Rayleigh ACD (1,1)	$\psi_i = 0.7760 + 0.1338x_{i-1} + 0.73661\psi_{i-1}$		$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$	
Standard Error	$\hat{\omega}$	0.1206	$\hat{\omega}$	0.0478
	$\hat{a}_1$	0.0124	$\hat{a}_1$	0.0097
	$\hat{b}_1$	0.0277	$\hat{b}_1$	0.0209

Table 10: Lognormal ACD(1,1) model fitted to the data

	Maximum Likelihood Estimation		Estimating Function	
Lognormal ACD(1,1)	$\psi_i = 0.1474 + 0.0682x_{i-1} + 0.9034\psi_{i-1}$		$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$	
$\hat{\sigma}$	1.2963		0.9240	
Standard Error	$\hat{\omega}$	0.0317	$\hat{\omega}$	0.1424
	$\hat{a}_1$	0.0082	$\hat{a}_1$	0.0142
	$\hat{b}_1$	0.0119	$\hat{b}_1$	0.0519
	$\hat{\sigma}$	0.0157	$\hat{\sigma}$	0.0364

Table 11: GACD(1,1) model fitted to the data

	Maximum Likelihood Estimation		Estimating Function	
GACD (1,1)	$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$		$\psi_i = 0.1803 + 0.0650x_{i-1} + 0.8811\psi_{i-1}$	
$\hat{\kappa}$	0.8479		0.7415	
Standard Error	$\hat{\omega}$	0.0522	$\hat{\omega}$	0.1447
	$\hat{a}_1$	0.0102	$\hat{a}_1$	0.0189
	$\hat{b}_1$	0.0226	$\hat{b}_1$	0.0492
	$\hat{\kappa}$	0.0172	$\hat{\kappa}$	0.0453

CONCLUDING REMARKS

This paper reviews the theory of ACD models and two estimations methods based on the MLE and EF approaches. Based on a simulation study we have noticed that both methods are comparable but the EF method is computationally efficient.

Although the EF approach is easy to apply in practice, the ML estimates are better than the EF estimates when the true distribution is known. In practice, the EF approach gives reliable estimates as the true distribution is unknown. Using the recent result of Grahramani and Thavaneswaran (2009) it can be shown that the EF approach is superior and this will be further investigated in a future paper.

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REFERENCES

Allen, D., F. Chan, M. McAleer and M.S. Peiris (2008). Finite sample properties of QMLE for the Log-ACD model: Application to Australian stocks. *Journal of Econometrics*, 147: 163-185.

Allen, D., D. Zdravetz and M.S. Peiris (2009). Comparison of alternative ACD models via density and interval forecasts: Evidence from the Australian Stock Market. *Mathematics and Computers in Simulation,* 79(8): 2535-2555.

Engle R.F. and J.R. Russell (1998). Autoregressive conditional duration: A new model for irregularly spaced transaction data. *Econometrica,* 66(5): 1127-1162.

Godambe, V.P. (1985). The foundations of finite sample estimation in Stochastic processes. *Biometrika,* 72(2): 419-428.

Grahramani, M. and A. Thavaneswaran (2009). Combining estimating functions for volatility. *Journal of Statistical Planning and Inference*, 139(4): 1449-1461.

Pacurar, M. (2008). Autoregressive conditional duration models in finance: A survey of the theoretical and empirical literature. *Journal of Economic Surveys,* 22(4): 711-751.

Peiris, M.S., D. Allen and K.H. Ng (2008). Estimation of ACD models using estimating functions: A simulation study. Modelling and Managing Ultra High Frequency Data: An International Conference (MMUHFDIC 2008), 13-14th Feb 2008, Perth, Australia.

Peiris, M.S., K.H. Ng and I.B. Mohamed (2007). A review of recent developments of financial time series: ACD modelling using the estimating function approach. *Sri Lankan Journal of Applied Statistics*, 8: 1-17

Thavaneswaran and Abraham (1988). Estimation of nonlinear time series models using estimating equations. *Journal of Time Series Analysis*, 9: 99-108.

Tsay, R.S. (2002). *Analysis of Financial Time Series*. John Wiley & Inc., New York.