

# Control of Vehicle Driving Model By Non-linear Controller

M. Mubin\*, S. Ouchi\*\*, N. Kodani\*\*, N. Mokhtar\*, H. Arof\*

\*Department of Electrical Engineering, Faculty of Engineering,  
University of Malaya, 50603 Kuala Lumpur, Malaysia.  
E-mail: [marizan@um.edu.my](mailto:marizan@um.edu.my)

\*\*Department of Applied Computer Engineering, Tokai University,  
1117 Kitakaname, Hiratsuka-shi, Kanagawa 259-1292, Japan.  
E-mail: [ouchis@keyaki.cc.u-tokai.ac.jp](mailto:ouchis@keyaki.cc.u-tokai.ac.jp)

*Abstract*— A new drive control system which has an effect on controlling the vehicle slip during accelerating and braking is proposed in this paper. This drive control system uses a non-linear controller designed by following the Lyapunov theorem. The controller is designed in order that it can work at both conditions that is the slippery and non-slippery road. The effectiveness of this control system is proved by a basic experiments.

## I. INTRODUCTION

Traction control system is one of the safety developments that have reached the automobiles during this period. It works to ensure maximum contact between the tire and the road surface so that the wheels are prevented from losing grip especially when accelerating, decelerating or braking on the low friction road condition such as wet or icy road. As to date, many researches have been developed addressing this problem. [3],[4],[5],[6],[7],[8],[9] However, the traction control problems are still far from reaching the final solution and a lot of works must be done.

This paper proposes the control of vehicle driving model by a non-linear controller. This controller is designed by following the Lyapunov theorem, in order that it can work at both conditions: the slippery and non-slippery road. Then, the application of the designed controller on the drive-control of a vehicle is discussed, including the structure of the control system and it is followed by the implementation of this controller into a specially modified device. The discussion on the estimation of the car-body speed is also given in this paper. Finally, the feasibility of this controller has been proven by the experimental results.

## II. THE CONTROLLED OBJECT

A two-inertia system, which consists of two rotating wheels which represent the wheel and the car-body as shown in Fig. 1 is considered as a controlled object. The equation of motion of this system is expressed in Eq. (1).

$$\begin{cases} \dot{v} = a_1 v + b_{m1} \mu \\ \dot{\omega} = a_2 \omega + b_{m2} \mu + b_2 \tau \\ \dot{\tau} = a_3 \tau + b_u \mu \\ \lambda := \omega - v / \max(\omega, v) \\ \mu = f(\lambda) \end{cases} \quad (1)$$

where all the coefficients used are given as follows:

$$\begin{bmatrix} a_1 := -B_2/J_2 & b_{m1} := W(r_2/r_1)/J_2 \\ a_2 := -B_1/J_1 & b_{m2} := W(r_1/r_2)/J_1 & b_2 := 1/J_1 \\ a_3 := -1/T_f & b_u := 1/T_f \end{bmatrix}$$

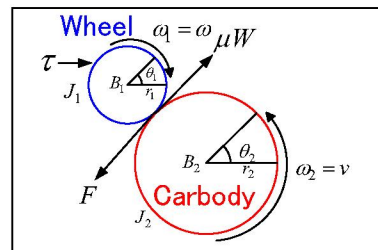


Fig. 1 Two-inertia system model

All the parameters of the controlled object are given in Table 1.

Table 1 Parameter of controlled object

$\tau$	Driving torque [Nm]	$J_1$	Moment of inertia of 1 <sup>st</sup> wheel $4.55 \times 10^{-2} [kgm^2]$
$\mu$	Road friction coefficient	$J_2$	Moment of inertia of 2 <sup>nd</sup> wheel $3.87179 [kgm^2]$
$W$	Weight of car-body 15.4872 [N]	$r_1$	Radius of 1 <sup>st</sup> wheel 0.137 [m]
$F$	Driving power [N]	$r_2$	Radius of 2 <sup>nd</sup> wheel 0.25 [m]
$B_1$	Friction of 1 <sup>st</sup> wheel $4.5 \times 10^{-2} [Nm/rad/s]$	$\omega$	Speed of 1 <sup>st</sup> wheel [rad/s]
$B_2$	Friction of 2 <sup>nd</sup> wheel $2.5 \times 10^{-2} [Nm/rad/s]$	$v$	Speed of 2 <sup>nd</sup> wheel [rad/s]
		$T_f$	Torque time constant 0.01 [s]

From Eq. (1), friction coefficient  $\mu$  is a nonlinear function and it is different for the various road conditions, depends on the slip ratio  $\lambda$ . The  $\mu-\lambda$  characteristics, generated by Pacejka model for the normal and icy road is shown in Fig. 2. Meanwhile, the relationship between the driving torque and the reference torque is given as

$$\tau = \tau_r + u \quad (2)$$

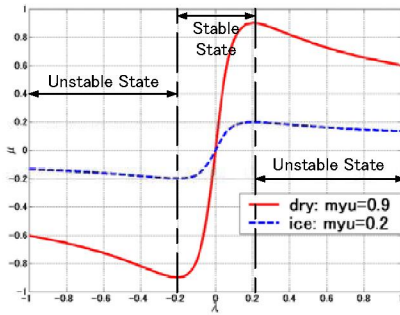


Fig. 2  $\mu-\lambda$  characteristic

### III. ESTIMATION OF CAR-BODY SPEED USING DISTURBANCE OBSERVER

Initially, the car-body speed is estimated by using the disturbance observer. The minimal order observer is applied to the system; instead of full order one, because the system also has measurable states. The result of the car-body speed that estimated using the observer is shown in Fig. 3.

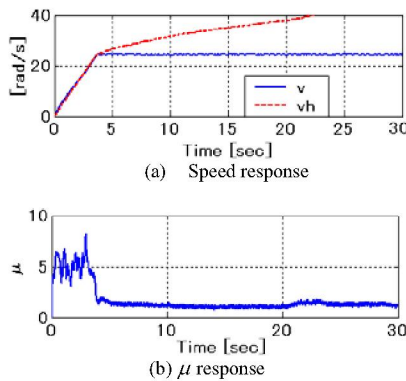


Fig. 3 Results of estimation of car-body speed

However, from Fig. 3, the accuracy of the results obtained is not promising. It is because of many factors such as the temperature characteristic change, the running resistance and the modeling error margin of accurate system identification.

### IV. CONTROL SYSTEM DESIGN

Since the results obtained from the observer is not promising, the encoder is used to measure the car-body speed. To achieve the objective that is to control the slip ratio, the theory of the control system is restructured.

#### A. $\tau-\lambda$ Control

In order to stabilize the controlled object expressed in Eq. (1), the following performance index is considered:

$$S = v - a\omega \quad (3)$$

where

$$\begin{cases} a := 1 - \lambda_0 & (\omega \geq v) \\ a := 1/1 - \lambda_0 & (\omega < v) \end{cases}$$

In order to obtain  $S \rightarrow 0$  (at  $t \rightarrow \infty$ ), the following Lyapunov function candidate is considered:

$$V = S^2 \quad (S \neq 0) \quad (4)$$

Here, the time derivation for  $V$  can be obtained as

$$\begin{aligned} \dot{V} &= 2S\{b_m\mu + a_1v - aa_2\omega - ab_2(u + \tau_r)\} \\ b_m &:= b_{m1} - ab_{m2} \end{aligned} \quad (5)$$

where the control law is considered as

$$u := (a_1\hat{v} - aa_2\omega + k_1S/|S| + k_2S)/ab_2 \quad (6)$$

To satisfy the Lyapunov stability theorem,  $\dot{V} < 0$  must be obtained. Therefore,  $\dot{V}$  must satisfy the following condition:

$$\dot{V} < 2\{(|b_m\mu - ab_2\tau_r| - k_1)|S| - k_2|S|^2\} \quad (7)$$

Here, in order to obtain  $S \rightarrow 0$  (at  $t \rightarrow \infty$ ), a reference value for torque  $\tau_r$  is defined as given in Eq. (8), where  $\mu_r$  is a reference value for friction coefficient and  $b_{mr}$  ( $b_{mr} \geq b_m$ ) is a standard value for the change of car-body's weight.

$$\tau_r \approx (b_{mr}/ab_2)\mu_r \quad (8)$$

Therefore, Eq. (7) can be written as

$$\dot{V} < 2\{(b_{mr}|\mu - \mu_r| - k_1)|S| - k_2|S|^2\} \quad (9)$$

When the following conditions are satisfied

$$|\mu - \mu_r| < \gamma, \quad |b_{mr}|\gamma \leq k_1, \quad 0 < k_2,$$

the control law is obtained as expressed in Eq. (11).

$$u = F_1\omega + F_2v + K_1S/|S| + K_2S \quad (10)$$

where

$$\begin{bmatrix} F_1 := -a_2/b_2, & F_2 := a_1/ab_2 \\ K_1 := k_1/ab_2, & K_2 := k_2/ab_2 \end{bmatrix}$$

The block diagram of  $\tau-\lambda$  control system is shown in Fig. 4.

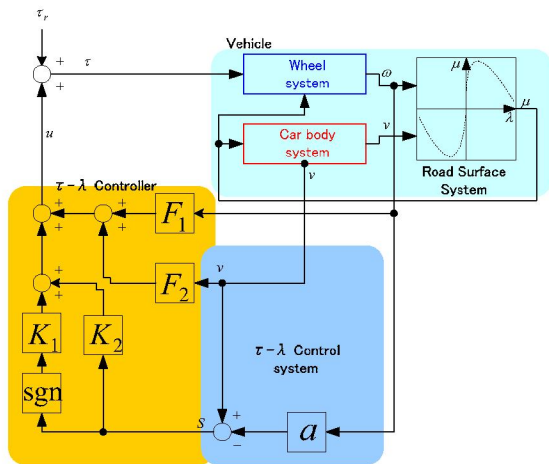


Fig. 4  $\tau - \lambda$  control system

### B. Structure of Control System

Next, the structure of the control system is discussed. From Eq. (11), the equation can be divided into two terms that is the linear feedback term  $u_l$  and also the non-linear feedback term  $u_{nl}$ , as shown in the following equation:

$$\begin{aligned} u_l &:= F_1\omega + F_2v + K_2S \\ u_{nl} &:= K_1S/|S| \end{aligned} \quad (11)$$

For the non-linear feedback term  $u_{nl}$ ,

$$K_1 \frac{S}{|S|} \begin{cases} +K_1 (S > 0) \\ -K_1 (S < 0) \end{cases} \quad (12)$$

At  $S \neq 0$ , the driving torque  $\tau$  can be expressed as

$$\tau = F_1\omega + F_2v \pm K_1 + K_2S + \tau_r \quad (13)$$

#### B.1 During Non-slipping

At  $S \neq 0$  that is when slip does not occur, the time variation for the estimated value of car-body speed  $\hat{v}$  and the vehicle speed  $\omega$  are small. Therefore, it can be considered that  $\dot{\hat{v}}, \dot{\omega} \rightarrow 0$ .

$$\mu = (|b_{mr}|/b_m)(\mu_r \pm \gamma), \quad |\mu - \mu_r| < \gamma \quad (14)$$

From Eq. (14), at  $b_{mr} = b_m$ , the friction coefficient  $\mu$  is equivalent to the reference value of friction coefficient  $\mu_r$  with the addition of limit value  $\gamma$ .

The block diagram of the control system for  $S \neq 0$  case is shown in Fig. 5. This system is known as  $\tau$  control system.

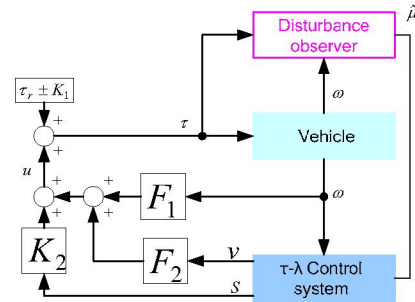


Fig.5  $\tau$  control system

#### B.2 During Slipping

On the other hand, for the case of  $S \approx 0$ ,  $\lambda = \lambda_0$  is obtained at  $\omega \geq v$ , where  $\lambda_0$  is the reference value of slip ratio. Meanwhile, at  $\omega < v$ ,  $\lambda = -\lambda_0$  is obtained where full control can be done. This control system is called  $\lambda$  control system.

The block diagram for the  $\lambda$  control system is shown in Fig. 6.

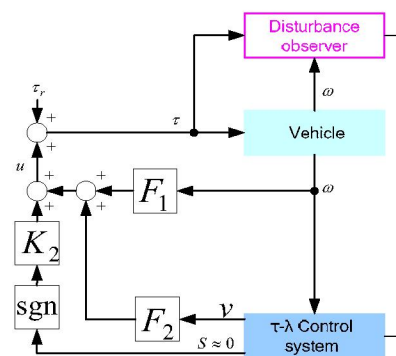


Fig. 6  $\lambda$  control system

## V. EXPERIMENTAL RESULTS

In order to check the feasibility of the designed controller, it is implemented into a specially modified experimental device as shown in Fig. 7. The device consists of two rotating wheels, which represent the vehicle wheel and the car-body as labeled.

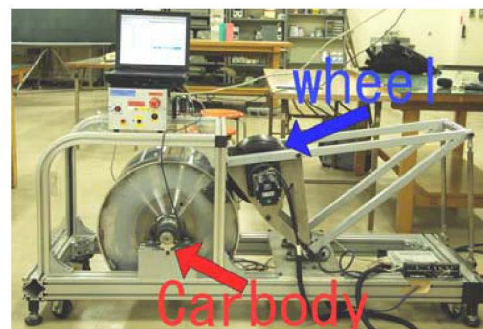


Fig.7 Experimental device of 2 inertia system

The controller gains that are used in this control system are given in Table 2. Different values of gains are used during acceleration, constant speed and also deceleration.

	Acceleration	Constant	Deceleration
$F_1$	$4.5 \times 10^{-2}$	$4.5 \times 10^{-2}$	$4.5 \times 10^{-2}$
$F_2$	$-3.6724 \times 10^{-4}$	$-2.9409 \times 10^{-4}$	$-2.3503 \times 10^{-4}$
$K_1$	$3.6 \times 10^{-2}$	$3.42 \times 10^{-4}$	$2.1 \times 10^{-4}$
$K_2$	3.3	2.85	2.1
$\lambda_0$	0.2	0.001	-0.2
$\gamma$	0.17	0.001	0.18

The experimental results for non-control and control cases are shown in Fig. (8) and Fig. (9), respectively. Experimental time is set to 15 sec, where the vehicle accelerates for the first 3 sec, travels at a constant speed for the next 9 sec, and decelerates for the last 3 sec. Here, it is assumed that the vehicle is moving on the low friction road condition.

The speed response for the non-control and control case are plotted in Fig. 8(a) and Fig. 9(a), respectively. The wheel speed is represented by 'w', while the car-body speed is represented by 'v'. For the non-control case, it can be observed that the wheel speed goes extremely high during acceleration and extremely low during deceleration. This indicates that slip occurs and the vehicle is running under unstable condition. However, for the control case, it can be seen that the tire slip is controllable.

From Fig. 9(c), it is shown that during acceleration, the slip ratio is controlled at  $\lambda = 0.2$ . And when the vehicle starts slipping again during deceleration, the slip ratio is controlled at  $\lambda = -0.2$ .

The switching response is shown in Fig. 9(e). It can be observed that the switching only occurs when the wheel is accelerating and decelerating. When it is running at the constant speed, the switching does not occur as shown in the figures. These results strengthen the previous given theory, which stated that the  $\lambda$  control system with the switching function is working when slip occurs to control the slip ratio and the  $\tau$  control system is working when slip does not occur.

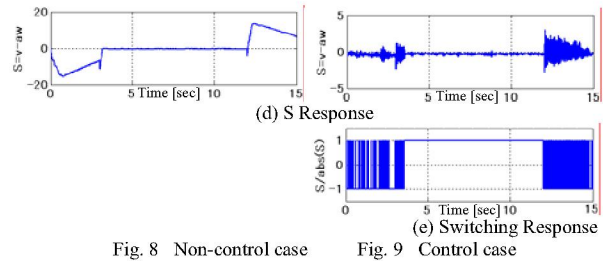
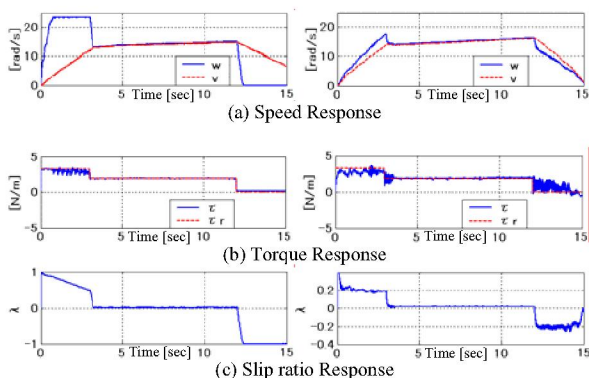


Fig. 8 Non-control case

Fig. 9 Control case

## VI. CONCLUSIONS

The proposed nonlinear controller for the vehicle driving model gives satisfactory experimental results. The works presented in this paper will be continued in the future where an amendment regarding the temperature characteristic will be made to increase the accuracy of the disturbance observer as an estimation system for the car-body speed.

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