Three Essays on Sovereign Debt and Stagnation Economics

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Preface

Until not long ago, the topics that motivated this thesis seemed to be of little relevance for policymakers in the United States and Europe. Theories of economic stagnation were considered barely more than remnants of a distant past with memories of the Great Depression fading away and economies growing at a healthy pace, apart from occasional fluctuations in the business cycle. Macroeconomic priorities were clearly directed towards supply-side policies, while problems of insufficient aggregate demand, if they existed at all, were thought to be well manageable.[1] Moreover, sovereign debt of advanced economies appeared to be as safe as houses, spreads among countries in the Euro Area virtually disappeared and sovereign debt crises were deemed to be not more than an occasional occurrence in lesser developed peripheral countries.

The aftermath of the global financial crisis of the late 2000s revealed that these views were fundamentally flawed. Recovery from the Great Recession has been disappointingly slow as inflation rates persistently fell below their targets, estimates of potential output were repeatedly lowered and unemployment reached levels not seen for decades in many advanced economies. All of this occurred despite unconventional monetary policy actions of unprecedented scales that pushed nominal interest rates to zero, reduced spreads in financial markets and contributed to a drastic increase in the values of stocks, land and other assets. Yet consumption and investment demand remained weak and worries that economies might persistently operate below their potential became widespread and expressed in the "secular stagnation" hypothesis.

In addition, sovereign debt lost its seemingly sacrosanct status as investors started to question the repayment ability of governments in the wake of soaring fiscal deficits.[2] A sovereign debt crisis emerged in the Euro Area, which further impeded recovery by limiting the scope for stabilizing fiscal policies. This culminated in the restructuring of Greece with its private creditors in 2012, which was the first major debt restructuring in Europe since the defaults preceding the Second World War (see Zettelmeyer, Trebesch, and Gulati, 2013). As a response to the sudden stop of private lending, official loans were made available on a large scale via the European Financial Stability Facility (EFSF) and its successor, the European Stability Mechanism (ESM).

[1] This culminated in the claim of Lucas (2003) during his presidential address to the American Economic Association that the "problem of depression prevention has been solved, for all practical purposes".

[2] More precisely, it was the commitment of the monetary authorities to back the fiscal authority in case of repayment problems that was questioned by investors. For example, the downgrade of the United States by several rating agencies in August 2011 did not seem to have a lasting effect on US bond yields. Moreover, changes in expectations about the ECB's commitment to provide liquidity to governments better account for the widening and subsequent contracting of bond spreads in the Euro Area than changes in economic fundamentals (see De Grauwe and Ji, 2013).

The unfolding of these events raises important new questions and challenges the conventional wisdom of the economics profession. The prolonged period of dismal economic growth, which cannot be explained by standard models, necessitates a new theory of aggregate demand shortage, while the increased importance of official lending requires a better understanding of sovereign debt markets, particularly with respect to the relative status of different claims in the presence of default risk. I will explain how this thesis contributes to both topics.

The economic slump following the global financial crisis is not without precedents in contemporary history, each of which has inspired theories of stagnation economics. In fact, the discipline of macroeconomics itself emerged as a direct response to the Great Depression of the 1930s, with Keynes (1936) in the General Theory as the first to point out how markets can fail to clear at full employment due to insufficient aggregate demand. The "secular stagnation" hypothesis, a term coined by Hansen (1939), considers such a failure a permanent state of affairs without any natural recovery. Long viewed as a mere theoretical possibility, the case of Japan shows that such a scenario can in fact occur in a modern economy. The post-war economic miracle of the then second-largest economy came to an abrupt halt in the early 1990s. What followed were decades of economic stagnation and deflation despite aggressive monetary policy actions by the Bank of Japan (see Ugai, 2007).[3] The Japanese experience has inspired a new theory of economic stagnation, starting with Ono (1994, 2001), which effectively reinterprets and formalizes the aforementioned ideas of Keynes and Hansen in a dynamic macroeconomic model.[4] Similar approaches have been developed to explain the weak recovery from the Great Recession in the United States and Europe after Larry Summers (2013) called for a resurrection of the secular stagnation hypothesis, mentioning the similarities with Japan.[5]

At the very core of stagnation economics is a discrepancy between the natural interest rate and the actual return on savings. The natural interest rate is a purely theoretical concept. It refers to the short-term real interest rate that balances savings and investment at full employment. This rate can be negative in case of low investment demand or a high desire for savings. In contrast, the actual saving incentives for households are determined by the real return on stocks, bonds and other assets, including holding money as cash. If this return is persistently higher than the natural rate of interest, the incentives to save are too strong and create an oversupply of savings, which depresses aggregate demand. Secular stagnation occurs.

[3] I describe this transition and the similarities to the situation in the United States and Europe during the Great Recession in greater detail in Chapter 2.

[4] Krugman (1998) also states Japan's stagnation as the motivation for his liquidity trap theory. In fact, stagnation occurs for similar reasons in liquidity trap models and models of secular stagnation. But while it is thought to be a temporary phenomenon by the former, the latter considers stagnation a permanent affair.

[5] Larry Summers (2013): "I wonder if a set of older and much more radical ideas [...] that went under the phrase secular stagnation, are not profoundly important in understanding Japan's experience in the 1990s, and may not be without relevance to America's experience today."

The typical explanation for this discrepancy is twofold. On the one hand, the natural interest rate in major advanced economies has substantially declined over the last decades, potentially into negative territory, due to demographic factors, a slowdown in productivity growth and global factors, like the opening of China to international capital markets.[6] On the other hand, the nominal interest rate cannot fall substantially below zero in a monetary economy. Hence household losses on savings are limited to the rate of inflation in real terms. Therefore the incentives for households to save might be too high in a low inflation environment and stagnation can occur as a monetary phenomenon.

In Chapter 1 of this thesis, I present a theory of secular stagnation that builds on these insights but adds the possibility of investment in land. Though the prime example of a productive real asset with highly inelastic supply and immense empirical relevance, land is usually ignored in models of secular stagnation. The availability of land as a store of value implies that the real return on savings cannot become negative as the rental yield from land ownership constitutes a strictly positive real return. Therefore the existence of land establishes a lower bound on the real interest rate similarly to the existence of money. Yet this is a real rather than a nominal constraint and hence independent of the inflation rate. In addition, households have wealth preferences, which create a strong desire for savings, on top of the impatience motive, and can result in a low or negative natural interest rate. I analyze this model under different assumptions on the utility from wealth and derive conditions under which full employment is not feasible and secular stagnation occurs.

The case for secular stagnation is substantially weakened in the economy with land and standard wealth preferences. The reason is that increases in the value of land stimulate household consumption and absorb excess savings. Full employment is always feasible at a positive real interest rate unless the central bank follows a deflationary policy or when there is a high risk premium on land. These conclusions change profoundly when I follow Ono (1994) and assume that households always value a further increase in wealth irrespective of how wealthy they are. Then higher asset prices cease to stimulate consumption sufficiently at some point. In fact, it is the very existence of land that prevents full employment from being feasible and secular stagnation occurs because land investment is too attractive to be compatible with full employment. Then an increase in the inflation target, which is the usual remedy in other models of stagnation, ceases to be effective. Somewhat surprisingly, full employment might not be feasible if the risk premium on land is very low.

[6] Holston, Laubach, and Williams (2017) estimate the natural interest rate for the United States, Canada, the Euro Area and the United Kingdom and observe a drastic decline during the Great Recession as well as substantial comovement over time, which indicates that global factors are the single most important determinant. Pescatori and Turunen (2016) come to similar conclusions.

This model provides a better understanding of why aggregate demand remains weak despite substantial increases in asset prices and reductions in both the nominal interest rate and risk premia in financial markets. It is, to the best of my knowledge, the first contribution that explains secular stagnation as a real phenomenon, rather than a monetary one, with a causal role of land in combination with strong wealth preferences.

An important aspect that the model of Chapter 1 does not capture is the role of household debt and its interaction with asset prices. All major episodes of economic stagnation were preceded by asset price and credit booms and busts, including Japan's Bubble Economy of the 1980s and the housing market boom fueled by sub-prime lending in the United States and several European countries. Credit to the private sector soared, then soured. And while the credit boom triggered unsustainable spending, which masked the underlying aggregate demand weakness, the subsequent debt overhang substantially depressed consumption and investment demand. In fact, Summers (2014a) argues that conditions of insufficient aggregate demand were already present in the United States for a long time before the global financial crisis, but had been masked by an expansion of household indebtedness. Therefore, "it has been close to 20 years since the American economy grew at a healthy pace supported by sustainable finance".[7]

In Chapter 2, I present a model of secular stagnation based on strong liquidity preferences of households that formalizes these claims. In the model, savers lend funds to borrowers, which in return have to pledge collateral in the form of housing. What follows are feedback loops between the real house price, the value of collateral and aggregate demand. In the model, a credit boom, which is triggered by financial liberalization, can temporarily stimulate aggregate demand in a deflationary economy by stimulating consumption spending of borrowers. However, the boom plants the seeds for its future destruction as borrowers end up with a higher debt burden, which depresses their spending. Savers in turn are not willing to increase their consumption accordingly but prefer to hoard money due to their strong liquidity preferences. The economy is trapped in a deflationary state of insufficient aggregate demand in which borrowers continuously delever, without however reducing their real debt burden due to the effects of debt deflation.

This model is among the first to shed light on the interconnections of secular stagnation and private sector debt, both from a static and a dynamic perspective.[8] It explains both the stimulative effect of credit booms and the dampening effect of debt overhang on aggregate demand and is consistent with the developments in Japan and other advanced economies.

[7] Summers (2017, 2018) also attributes the increasing speed of the recovery in the United States after 2016 towards full employment to a "sugar high" resulting from the stock market boom and other transitory factors.

[8] Eggertsson and Krugman (2012) and Eggertsson, Mehrotra, and Robbins (2017) also analyze the role of deleveraging and household debt in a model of stagnation. However, stagnation is temporary in Eggertsson and Krugman (2012), which is a model of the liquidity trap. Moreover, Eggertsson, Mehrotra, and Robbins (2017) assume an exogenous borrowing limit, whereas the borrowing limit is endogenous in the model of Chapter 2.

While the focus of Chapter 2 is on the role of private debt, much of the policy discussion following the Great Recession has been on public debt, particularly in the Euro Area. Governments typically borrow via different debt contracts from a variety of creditors, both official and private ones. And while the bulk of lending to advanced countries used to come from private bondholders, the sudden stop of lending to Southern European countries necessitated a greater reliance on official creditors. The International Monetary Fund and other European governments stepped in to substitute for the freeze in private lending.

Intended to provide relief for crisis-stricken countries, the increased presence of official lending has raised new concerns about repayment risks for taxpayers of creditor countries, particularly about the status of these claims relative to claims of private creditors. Do official creditors (implicitly) enjoy seniority in case of default as suggested by their historical insistence upon preferential treatment? Or are these claims in fact junior to bonds, thereby effectively reducing default risk for the remaining private creditors? These questions became particularly apparent when Greece missed several scheduled payments to the IMF in June and July 2015, effectively becoming the first advanced economy to default on the Fund. At the same time, however, the country decided to repay a “Samurai bond”, a bond denominated in Japanese Yen, to its private creditors (see Bloomberg, 2015). In the absence of an enforceable international insolvency scheme, sovereign states naturally have substantial discretionary power to discriminate against their creditors. However, surprisingly little is known about the outcome of these actions.

In Chapter 3, I provide the first systematic study of the implicit seniority structure in sovereign debt markets. Based on two new datasets on missed payments (arrears) and creditor losses during restructurings (haircuts) in over 100 countries since the late 1970s, I employ modern panel data techniques and counterfactual decomposition analysis to compare the performance of debt titles for up to six different groups of external creditors. The data reveals a clear pecking order of sovereign debt repayment and default, despite the lack of legal enforcement. While multilateral creditors, like the World Bank, and the International Monetary Fund are senior throughout the analysis, the status of bilateral loans is more dubious. Bilateral creditors seem junior to bondholders when it comes to missed payments and face substantially higher haircuts than private creditors in case of restructurings. Banks and trade creditors are consistently junior. This implicit seniority structure is confirmed even when accounting for economic fundamentals, like income per capita or the level and composition of a country’s public debt.

This contribution offers a new viewpoint on the heterogeneity of creditors in sovereign debt markets. It is the first systematic assessment of creditor seniority and can help to rethink both the theoretical literature on sovereign debt and policy interventions in these markets.

The United States and Europe have lived through turbulent times following the global financial crisis with economic recoveries being far from adequate and a sovereign debt crisis threatening the very existence of the Euro Area. Conventional economic wisdom has failed to provide sufficient answers to these problems. Persistent economic stagnation was considered barely more than an obscure theoretical possibility despite the experience of Japan and little attention was paid to the heterogeneity of creditors in sovereign debt markets as official debt played only a minor role in advanced economies before the crisis.

In this thesis, I provide new perspectives on both issues. In Chapters 1 and 2, I offer a theory of secular stagnation that explains the experience of Japan during its lost decades and highlights the role of private sector debt in particular. In Chapter 3, I present a systematic empirical analysis of seniority in sovereign debt markets based on the historical experience of a broad set of developed and developing countries.

It is my hope that this thesis gives new insights to academics and policymakers alike and contributes to better understanding, and ultimately overcoming, the new challenges faced by modern economies.

Chapter 1

Secular Stagnation in An Economy with Land†

Can secular stagnation occur in an economy with land? Conventional wisdom negates this as the availability of land precludes a negative real interest rate in steady state. In fact, the case for secular stagnation is substantially weakened in a model with standard wealth preferences of households once investment in land is possible. Full employment is always feasible unless the central bank follows a deflationary policy. In contrast, secular stagnation can emerge as the unique equilibrium if wealth preferences are insatiable. Then it is the very existence of land itself that prevents full employment from being feasible. Increases in the inflation target are no longer effective as stagnation is a real phenomenon rather than a monetary one.

This chapter is organized as follows. The first section introduces and motivates my research question and gives an overview of the related literature. In section 1.2, I present a neoclassical model with wealth preferences and land, which I analyze in the following section for different assumptions on the utility from wealth. The model is extended in section 1.4 by introducing money and nominal wage rigidities. The monetary economy is then analyzed in section 1.5, which also discusses the case of a risk premium on land. The final section concludes.

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1.1 Introduction

The term "secular stagnation" describes the persistent failure of an economy to produce at full capacity as a result of an oversupply of savings. Originally postulated by Hansen (1939) in the wake of the Great Depression, the hypothesis seemed to have lost its relevance in the post-war economic system as downturns were limited to short-lived recessions that could be sufficiently well addressed by monetary policy. However, the unexpected fall of Japan, the then second-largest economy, into decades of inadequate growth and particularly the sluggish recovery in most economies during the Great Recession have led to a resurrection of this idea, most famously by Summers (2013).[1]

The secular stagnation hypothesis relies on two claims. First, the natural rate of interest, which is the short-term interest rate consistent with full employment, has permanently declined into negative territory as a result of high desired savings and low investment demand. Economies hence overaccumulate capital. Second, the real return on savings cannot fall sufficiently to accommodate the decline of the natural rate as a consequence of the zero lower bound on the nominal interest rate and the commitment of central banks to maintain price stability. This mismatch results in an oversupply of savings, which depresses aggregate demand and prevents markets from clearing at full employment. Therefore, stagnation is a monetary phenomenon.

There is, however, a major objection to this argument. The existence of a non-producible productive asset prevents overaccumulation and restricts the real interest rate to be positive as shown by Homburg (1991). The argument is simple: An asset with positive rent payments and finite price has a strictly positive yield, which constitutes a real return. Excess savings can be invested into this asset, the natural rate of interest remains positive in steady state and secular stagnation does not occur. Land is the prime example of such an asset with high empirical relevance. According to Homburg (2014), land values exceed GDP and public debt in many major economies and land is almost as important as capital from a macroeconomic perspective. However most models of secular stagnation exclude the possibility of investment in land.[2]

I propose a theory of secular stagnation in an economy with land based on strong wealth preferences of households. Specifically, I extend the standard neoclassical growth model with infinitely-lived agents along the following dimensions.

[1]In fact, Summers (2014a) argues that weak aggregate demand has been present in the United States for several decades but was masked by the stock market boom of the 1990s and the credit boom of the early 2000s, both of which initiated unsustainable consumption spending of households. Illing, Ono, and Schlegl (2017), in Chapter 2, formally show that credit booms can temporarily cure aggregate demand deficiency but come at the cost of more severe stagnation in the long-run due to the resulting debt overhang.

[2]The existence of equity also prevents overaccumulation of capital if the dividend on stocks is strictly positive. This is the case if labor is not an input factor in the production process as in Zhou (2013) or in case of imperfect financial markets that prevent negative dividend payments and stock dilution.

First, I introduce wealth preferences of households. The idea of wealth accumulation for the sake of itself as an aspiration of human behavior has a long tradition in economic thought and is embraced among others in the works of Adam Smith, Karl Marx, Max Weber and John Maynard Keynes as described in great detail by Zou (1994), who refers to the concept as "spirit of capitalism". In addition, it can be related to behavioral factors like status concerns. Wealth preferences have first been formally introduced into the neoclassical growth model by Kurz (1968) and Bose (1971) and their implications are analyzed in several contributions.[3] Wealth preferences create a strong desire for savings which lowers the natural interest rate below the time preference rate and potentially into negative territory. Preferences for wealth therefore present an alternative to the overlapping generations model that can also generate a negative real rate, as is well-known since Samuelson (1958). Recent contributions by Michaillat and Saez (2014) and Michau (2017) study similar models of secular stagnation based on wealth preferences. Ono (1994, 2015) offers a theory of stagnation based on the additional assumption that wealth preferences are insatiable. However, these papers do not account for the availability of a non-producible productive asset.

I extend these models by introducing land as a means of saving for households in addition to bonds, physical capital and money. Land is a non-producible asset in fixed supply and pays a positive rent.[4] The land value affects the natural rate of interest via the wealth preference and its rental yield represent a strictly positive real return on savings, which rules out a negative real interest rate in steady state.

I also introduce money into the model via a money-in-the-utility framework, in which the central bank directly controls the nominal money supply. In addition, I impose a limit on nominal wage declines in periods of aggregate demand shortage to prevent a deflationary spiral. This friction is borrowed from Schmitt-Grohé and Uribe (2016, 2017) and results in a non-linear inflation process.[5]

What determines the value of land? Under what conditions does secular stagnation occur in steady state? And what is the role of monetary policy? I analyze these questions in both a neoclassical model with land and its extended monetary version. The analysis focuses primarily on the steady state properties of these models. The conclusions differ markedly depending on the nature of the preference for wealth.

[3] Bakshi and Chen (1996) and Smith (2001) study the implications of wealth preferences for asset pricing. Kumhof, Rancière, and Winant (2015) explain the rise in inequality in recent decades in a model with wealth preferences of rich households. Michau, Ono, and Schlegl (2017) analyze their effects on the natural interest rate and the occurrence of rational bubbles in a model with infinitely-lived assets. Similarly, Kamihigashi (2008) and Zhou (2013, 2015) study rational equity bubbles in a model with wealth preferences.

[4] The constant supply property of land is one of the main reasons why it is typically omitted in standard growth models with technological progress. An exception is Nichols (1970), who incorporates land into a balanced growth model with land-augmenting technological progress.

[5] Similar nominal rigidities are used in other models of secular stagnation like Michau (2017), Eggertsson, Mehrotra, and Robbins (2017), Caballero and Farhi (Forthcoming) or Ono and Ishida (2014).

With standard wealth preferences, the case for secular stagnation is substantially weakened by the introduction of land as investments in land absorb the oversupply of savings. Full employment prevails in the neoclassical model as increases in the land value stimulate household consumption and increase the natural interest rate into positive territory. In the monetary version of this model, secular stagnation could in principle occur in steady state. However, this requires quite restrictive assumptions. Specifically, the economy has to be deflationary at full capacity to rule out full employment. In addition, it requires that the real money stock does not affect the desire for savings via the wealth preference. Then, the real interest rate is determined solely in the real economy. In contrast, stagnation cannot occur if the real money stock is considered net wealth by households as increases in the money supply stimulate consumption until full employment is restored.[6] In addition, the land price always equals its fundamental value and rational land price bubbles cannot occur.

These conclusions change profoundly if the desire for wealth accumulation is insatiable. Specifically, I follow the literature initiated by Ono (1994) and assume a strictly positive lower bound on the marginal utility from wealth. Then, full employment cannot be maintained in steady state if the desire for savings is sufficiently strong and secular stagnation occurs. This steady state is not well-defined in the neoclassical economy, but uniquely determined in the monetary economy by the binding nominal wage rigidity. Stagnation persists even though households can invest in land at a positive real rate. In fact, it is the very existence of land that creates stagnation as it prevents the actual return on savings from falling sufficiently to clear markets at full employment. As households become sufficiently wealthy, their consumption ceases to be affected by further increases in wealth, which is why the natural interest rate can permanently remain below the return on savings. In addition, land price bubbles can occur.

Consequently, secular stagnation is a real phenomenon and increases in the inflation target are no longer effective in restoring full employment. The steady state with aggregate demand shortage is unique and multiple equilibria are ruled out. This is in contrast to monetary models of secular stagnation like Michau (2017) or Caballero and Farhi (Forthcoming), in which both stagnation and full employment steady states coexist for a sufficiently high money growth rate.

These findings continue to hold when households require a risk premium on land investments. Increases in the inflation target are only effective if the return on money, rather than the return on land, constrains the return on savings. With standard wealth preferences, an increase in the risk premium can prevent the economy from operating at full employment. In contrast, a higher risk premium can help to restore full employment when wealth preferences are insatiable.

[6] This mechanism would also restore full employment in Michau (2017). However, money is not part of net household wealth in the steady state of his model and hence does not affect the natural rate.

Related Literature: This paper contributes to the growing literature on secular stagnation and offers a theory of stagnation based on the existence of a real asset, namely land. In contrast, stagnation is considered a monetary phenomenon throughout the literature starting with Keynes (1936), who argues in Chapter 17 of the General Theory that permanent demand shortage can exist as a steady state phenomenon in a monetary economy.[7]

In the liquidity trap literature, stagnation occurs temporarily following severe demand shocks that result in a negative natural rate of interest (see among others Eggertsson and Krugman, 2012; Eggertsson and Woodford, 2003; Krugman, 1998). As the zero lower bound and well-anchored inflation expectations prevent the real return on money from falling sufficiently, an oversupply of savings depresses demand. However, full employment is eventually restored even in the absence of policy measures.

In secular stagnation models, no natural recovery occurs because the natural interest rate is permanently negative as a result of structural factors.[8] In standard infinitely-lived agent models the natural interest rate in steady state is determined by the positive time preference rate of the least impatient agent as shown by Becker (1980). Models of secular stagnation therefore rely on an overlapping generations framework or wealth preferences, both of which allow for a negative natural rate in steady state.

In Eggertsson, Mehrotra, and Robbins (2017), the natural rate is negative as a consequence of deleveraging shocks, demographic developments, inequality or a fall in the price of capital goods. Kocherlakota (2013) highlights the role of asset price declines that can worsen stagnation if labor markets are incomplete, while Caballero and Farhi (Forthcoming) show that a shortage of safe assets depresses the natural interest rate in a perpetual youth model.[9] Alternatively, wealth preferences of households can result in a negative natural rate. Then, secular stagnation can occur in the steady state of a monetary economy as shown by Michaillat and Saez (2014) and Michau (2017). A related strand of stagnation models is based on the stronger assumption of insatiable wealth or liquidity preferences. Initiated by Ono (1994) and substantially extended by Ono and Ishida (2014), these contributions assume that there exists a strictly positive lower bound on the marginal utility from wealth. A limit on the feasible inflation rate, for example via a restrictive inflation target of the central bank, rules out full employment.

[7] A modern formalization of Chapter 17 is provided by Ono (2001) in a continuous dynamic optimization model in which stagnation occurs as a consequence of strong liquidity preferences of households.

[8] In Ono (2001) and Illing, Ono, and Schlegl (2017), it is not the zero lower bound but strong liquidity preferences that create a lower bound on the return on money holdings. Then stagnation can occur even at a positive natural rate of interest, but it remains a monetary phenomenon.

[9] The open economy versions of these secular stagnation models are analyzed in Eggertsson, Mehrotra, and Summers (2016) and Caballero, Farhi, and Gourinchas (2016). International considerations include the notion of a "global savings glut", postulated by Bernanke (2005). For an overview of these and other factors, see the VoxEU.org eBook edited by Baldwin and Teulings (2014).

Yet, what most of these contributions omit is the importance of non-producible and productive real assets, like land. There are two notable exceptions though.

In Kocherlakota (2013), the existence of land also implies a strictly positive real rate in steady state, which can result in stagnation if the wage inflation rate does not adjust sufficiently in case of imperfect labor markets. Then, exogenous land price declines further reduce the labor input. However, once capital is introduced into the model, stagnation requires the stronger assumption that the real rate cannot fall below the time preferences rate of the household. In contrast, I do not require any exogenous assumptions on the real rate. All restrictions are derived from the No-Arbitrage condition in asset markets.

The perpetual youth model of Caballero and Farhi (Forthcoming) features a non-producible Lucas tree. However, the return on this asset includes a risk-premium in the stagnation equilibrium. It is then again the existence of money that accounts for stagnation as the inflation rate restricts the real return on the safe asset. In contrast, I show that secular stagnation can result from the existence of both money and land when there is an exogenous risk premium on land.[10]

As stagnation is typically a monetary phenomenon, a sufficiently large and credible increase in the inflation target or the money growth rate is a common remedy (see for example Caballero and Farhi, Forthcoming; Eggertsson, Mehrotra, and Robbins, 2017; Michaillat and Saez, 2014; Michau, 2017). This lowers the return on money and can restore full employment. Then, both full employment and stagnation steady states can occur. In an economy with land, increases in the inflation target cease to affect the economy under stagnation and cannot restore full employment as stagnation is a real phenomenon.

The negative real rate in secular stagnation models implies that economies accumulate too much capital, a state known as dynamic inefficiency.[11] Under this condition, a rise in government debt can be a Pareto improvement, as shown by Diamond (1965). In fact, Michau (2017) argues that public debt cannot be unsustainable under secular stagnation. Moreover, rational asset price bubbles can occur as analyzed by Tirole (1985). In this paper, dynamic efficiency is restored once land is available. However, land price bubbles can still occur and are not ruled out by the transversality condition when the marginal utility of wealth is bounded from zero either in absolute terms or relative to consumption as also shown by Ono (1994), Kamihigashi (2008) and Zhou (2013, 2015)

[10] In this paper, agents have perfect foresight, which excludes stagnation equilibria based on self-fulfilling expectations as in Benhabib, Schmitt-Grohé, and Uribe (2001). Secular stagnation results from fundamental factors.

[11] Empirical tests of dynamic inefficiency are imperfect as temporary shortfalls of the real rate below the growth rate are possible in a dynamically efficient economy as argued by Homburg (2014). In a seminal paper, Abel et al. (1989) compare capital income with investment expenses as a criterion for efficiency and conclude that all major economies are dynamically efficient. This result is challenged by Geerolf (2017) after adjusting the efficiency criterion for rent income from land.

1.2 A Neoclassical Economy with Land

Throughout this paper, I assume perfect foresight and abstract from uncertainty and technological progress. Time is continuous and denoted by t. The model consists of homogeneous households and competitive firms, which produce the single output good using labor and capital. There is no government. I make two extensions to the standard neoclassical model. First, households have preferences for wealth. Second, there is the possibility of investment in land.

1.2.1 Asset Markets and the Real Return on Savings

There are three assets in the neoclassical model: Bonds b_t, physical capital K_t and land H_t. All of them serve as a store of value for households to transfer income over time.

Bonds are financial claims between households that yield a real return of r_t. Financial markets are perfect and there are no asymmetric information or contract enforcement problems. Bond market equilibrium requires $b_t = 0$ as bonds are in zero net supply.

Physical capital is an input in the production process. It is owned by households, who rent it at price R_t to firms. Capital depreciates at rate $\delta > 0$. One unit of output can be transformed into one unit of consumption or one unit of investment and there are no adjustment costs. Therefore, the relative price of capital is one.[12] The return on capital r_t^K is then given by

$$r_t^K = R_t - \delta \ . \tag{1.1}$$

Land is a non-producible productive asset. Unlike physical capital, land cannot be produced and does not depreciate. It is in fixed supply, normalized to $H_t = 1$, and trades at real price q_t. Each unit of land pays a real rent $z > 0$ to its owner, which is exogenous. Households can freely dispose of land, which implies that its price cannot become negative, i.e. $q_t \geq 0$. The return on land r_t^L consists of a rental yield and potential capital gains or losses with

$$r_t^L = \frac{\dot{q}_t}{q_t} + \frac{z}{q_t} \ . \tag{1.2}$$

Consider the decomposition of the land price q_t into a fundamental component q_t^F and a bubble component q_t^B, i.e.

$$q_t = q_t^F + q_t^B \ , \tag{1.3}$$

[12]If investment is irreversible, we have $\dot{K}_t + \delta K_t \geq 0$. Then, the price of capital equals one whenever investment is positive, but it can drop below one once this constraint is binding. Changes in the price of capital then affect the return on capital, which is analyzed in Michau (2017). I abstract from this occasionally binding constraint as I focus on steady state analysis, in which investment is strictly positive due to depreciation. The same holds during the adjustment dynamics if the initial capital stock is below the steady state level.

where the fundamental price is associated with rent payments and the bubble is a speculative price component, which is not backed by claims on income. The fundamental return is given by r_t^L in (1.2) whereas the return on the bubble equals the capital gain or loss given by

$$r_t^B = \frac{\dot{q}_t^B}{q_t^B} \ . \tag{1.4}$$

Equilibrium in the bond market, the market for physical capital and the market for land requires the equalization of returns across these assets.[13] If bubbles exist, the bubble component has to grow with this rate as well, which is a standard requirement in the literature (see Tirole, 1985). This is formalized in the No-Arbitrage condition

$$r_t = r_t^K = r_t^L \ (= r_t^B) \ , \tag{1.5}$$

where r_t^K, r_t^L and r_t^B are defined in (1.1), (1.2) and (1.4) respectively. Condition (1.5) implies that households are indifferent between bonds, physical capital and investment in land as a means of savings in equilibrium. All assets yield the same real return, which I will denote by r_t^S. This return determines the actual incentives of households to save.

1.2.2 Competitive Firms

Firms rent physical capital K_t from households at rental price R_t and employ labor L_t at real wage w_t. They produce output Y_t with the neoclassical production function

$$Y_t = F\left(K_t, L_t\right) \ , \tag{1.6}$$

which exhibits constant returns to scale and fulfills the standard assumptions of strictly positive and declining marginal products of both factors. Let $k_t = K_t/L_t$ and $y_t = Y_t/L_t$ denote the capital stock per worker and output per worker respectively. Using (1.6), I rewrite output per worker as $y_t = F(k_t, 1) = f(k_t)$ with $f'(k_t) > 0$ and $f''(k_t) < 0$.

Firms take w_t and R_t as given. In competitive factor markets, capital and labor are paid their marginal products, which implies that the rental price and the real wage are given by

$$R_t = F_K\left(K_t, L_t\right) = f'(k_t) \tag{1.7}$$

$$w_t = F_L\left(K_t, L_t\right) = f(k_t) - k_t f'(k_t) \tag{1.8}$$

[13] The models of Caballero, Farhi, and Gourinchas (2016) and Caballero and Farhi (Forthcoming) also assume a non-producible productive asset. However, the return on this asset can include a premium in the presence of aggregate risk depending on the marginal investor. I discuss the case of a risk premium in section 1.5.

Since the rental price always equals the marginal product of capital, the return on physical capital r_t^K is given by its net marginal product. Using (1.7), r_t^K in (1.1) is rewritten as

$$r_t^K = f'(k_t) - \delta \ . \tag{1.9}$$

By the No-Arbitrage condition (1.5), this return equals the real rate on bonds and the return on land r_t^L. Hence, (1.9) constitutes the actual real return on savings r_t^S for households.[14]

1.2.3 Households and the Natural Rate of Interest

There is a mass one of identical and infinitely-lived households and no population growth. Households own physical capital, which they rent to firms. Each household is endowed with one unit of labor L_t, which is supplied inelastically at wage w_t. Therefore, $L_t = 1$ represents full employment. Households can save via bonds b_t, investments in physical capital K_t or investment in land at price q_t. Hence, total real wealth a_t is decomposed as

$$a_t = b_t + K_t + q_t H_t \ . \tag{1.10}$$

Households receive wage income and interest or rental income on their investments. This income is used to finance consumption spending c_t. Real wealth evolves as

$$\dot{a}_t = r_t a_t + \left(r_t^K - r_t\right) K_t + \left(r_t^L - r_t\right) q_t H_t - c_t + w_t L_t \ . \tag{1.11}$$

By the No-Arbitrage condition (1.5), all assets have to yield the same return in equilibrium. Then, (1.11) simplifies to

$$\dot{a}_t = r_t a_t - c_t + w_t L_t \ . \tag{1.12}$$

The representative household has preferences over consumption and wealth. His lifetime utility is given by

$$U_0 = \int_0^\infty \left[u(c_t) + v(a_t)\right] e^{-\rho t} dt \ , \tag{1.13}$$

where $\rho > 0$ is the time preference rate. Utility $u(c)$ satisfies the standard assumptions $u'(c) > 0$, $u''(c) < 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. In contrast, the desire for wealth accumulation is insatiable. Specifically, I follow the literature initiated by Ono (1994) and assume

$$v'(a) > 0 \ , \quad v''(a) \leq 0 \ , \quad \lim_{a \to 0} v'(a) = v_0 \ , \quad \lim_{a \to \infty} v'(a) = \beta > 0 \ .$$

[14]Note that the same result is obtained when firms own the capital stock directly. In this case, households are shareholders and receive dividend income equal to firm profits. As firms have to replace the depreciated capital stock, the net marginal product still determines the return on household savings r_t^S.

There exists a lower bound β on the marginal utility from wealth, which stays positive even as wealth becomes infinitely high. This assumption is borrowed from behavioral economics and related to status preferences as discussed in Murota and Ono (2011).[15]

Maximizing (1.13) subject to (1.12) yields the Euler Equation and the transversality condition as

$$\eta_c \frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{v'(a_t)}{u'(c_t)} \ , \quad \text{where } \eta_c \equiv -\frac{u''(c_t)c_t}{u'(c_t)} \ , \tag{1.14}$$

$$\lim_{t \to \infty} a_t u'(c_t) e^{-\rho t} = 0 \ . \tag{1.15}$$

Compared to the standard Euler Equation, wealth preferences create an incentive for savings in addition to the impatience motive. I define the natural rate of interest r_t^N as the real interest rate that clears the goods market at full employment. It is given by (1.14) with $L_t = 1$. As saving becomes a virtue by itself, goods market clearing requires a real rate below the time preference rate. The natural rate can even become negative if the preference for wealth is sufficiently strong.

Finally, the transversality condition (1.15) implies that household wealth cannot grow at a rate greater or equal to ρ in a stationary steady state.

1.2.4 Market Clearing Conditions

The real wage perfectly adjusts to clear the labor market with $L_t = 1$ at full employment. The No-Arbitrage condition (1.5) implies bond market clearing with $b_t = 0$, land market clearing with $H_t = 1$ and equilibrium in the market for physical capital. Then real wealth is given by

$$a_t = K_t + q_t = k_t L_t + q_t \ . \tag{1.16}$$

Aggregate supply consists of output Y_t and the rent z from land. Aggregate demand equals consumption demand c_t and gross investment in capital $\dot{K}_t + \delta K_t$. Goods market clearing requires

$$Y_t + z = c_t + \dot{K}_t + \delta K_t \ , \tag{1.17}$$

which can be rearranged and expressed in per worker terms as

$$\dot{K}_t = \dot{k}_t L_t + k_t \dot{L}_t = \left[f(k_t) - \delta k_t \right] L_t + z - c_t \ . \tag{1.18}$$

Under full employment, it holds that $L_t = 1$ and $\dot{L}_t = 0$. Then $k_t = K_t$ and $\dot{K}_t = \dot{k}_t$. In the next section, I analyze the conditions, under which the full employment steady state exists.

[15] Ono, Ogawa, and Yoshida (2004) offer empirical support for the insatiability of preferences for monetary wealth based on quarterly data in Japan using parametric and non-parametric methods.

1.3 Analysis of the Neoclassical Economy

The neoclassical model economy is characterized by the Euler Equation (1.14), the wealth composition (1.16), the return on physical capital (1.9), the return on land investment (1.2), the goods market clearing condition (1.18) and the No-Arbitrage condition (1.5) together with the transversality condition (1.15) for any given initial value of capital $K_0 > 0$. In case of a bubbly equilibrium, equations (1.3) and (1.4) also apply for a given initial bubble value $q_0^B > 0$. Under full employment, it holds that $L_t = 1$ and consequently $K_t = k_t$. Each stationary steady state is characterized by $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

The feasibility of full employment in steady state is determined by the actual real return on savings r^S and the natural rate of interest r^N. The actual return on savings follows from the No-Arbitrage condition (1.5), which requires the equalization of the real returns of all available assets. Using the definitions of the returns on land in (1.2) and capital in (1.9), the return on savings is given by

$$r^S = f'(k) - \delta = \frac{z}{q_t} + \frac{\dot{q}_t}{q_t} \ , \tag{1.19}$$

where $\dot{q}_t \geq 0$ in steady state depends on the presence of land price bubbles, which in turn depends on the specification of the utility from wealth. The natural rate of interest r^N is derived from (1.14) with $\dot{c}_t = 0$ as

$$r^N = \rho - \frac{v'(a)}{u'(f(k) - \delta k + z)} \ , \tag{1.20}$$

where I use the goods market clearing condition (1.18) with $\dot{k}_t = 0$ and $L = 1$ to substitute for consumption under full employment as a function of the capital stock, i.e.

$$c = f(k) - \delta k + z \ . \tag{1.21}$$

Note that consumption is maximized when the capital stock is at the Golden Rule level $\tilde{k}$, which is defined by $f'(\tilde{k}) = \delta$.

The real rate r^S in (1.19) determines the actual return on assets and hence the saving incentives for households, while the natural rate r^N in (1.20) describes the real return on savings that is necessary to achieve full employment. Hence, full employment requires $r^S = r^N$. An persistent oversupply of savings occurs for $r^S > r^N$ in steady state as the incentives to save are too high. This depresses aggregate demand and prevents full employment from materializing.

In the following, I analyze the feasibility of full employment and the role of land for different assumptions on the preference for wealth. I start with the neoclassical model without land as a benchmark. Most of the derivations and proofs are relegated to Appendix A.

1.3.1 The Benchmark Model without Land

Consider first the neoclassical model augmented for wealth preferences but without land. In equilibrium, household wealth is given by the capital stock as bonds are in zero net supply and $a = k$ in (1.20).[16] The return on savings r^S in (1.19) equals the net marginal product of capital without any further restrictions. Both r^N and r^S are continuous functions of the capital stock and illustrated in Figure 1.1 for the case of standard wealth preference (left panel) and a constant marginal utility of wealth $v'(a) = \beta$ (right panel). Full employment requires $r^S = r^N$ or equivalently

$$f'(k) - \delta = \rho - \frac{v'(k)}{u'\left(f(k) - \delta k + z\right)} \ . \tag{1.22}$$

Let $\bar{k}$ and $\bar{r}$ denote the steady state values of the capital stock and the real interest rate. The following proposition shows that the full employment steady state is always realized in this economy irrespective of the nature of the preference for wealth.

Proposition 1.1 *The steady state under full employment, characterized by (1.22), always exists irrespective of the lower bound on the marginal utility of wealth. The steady state is unique under mild restrictions on preferences or model parameters. The dynamic system exhibits saddle-point stability around this steady state.*

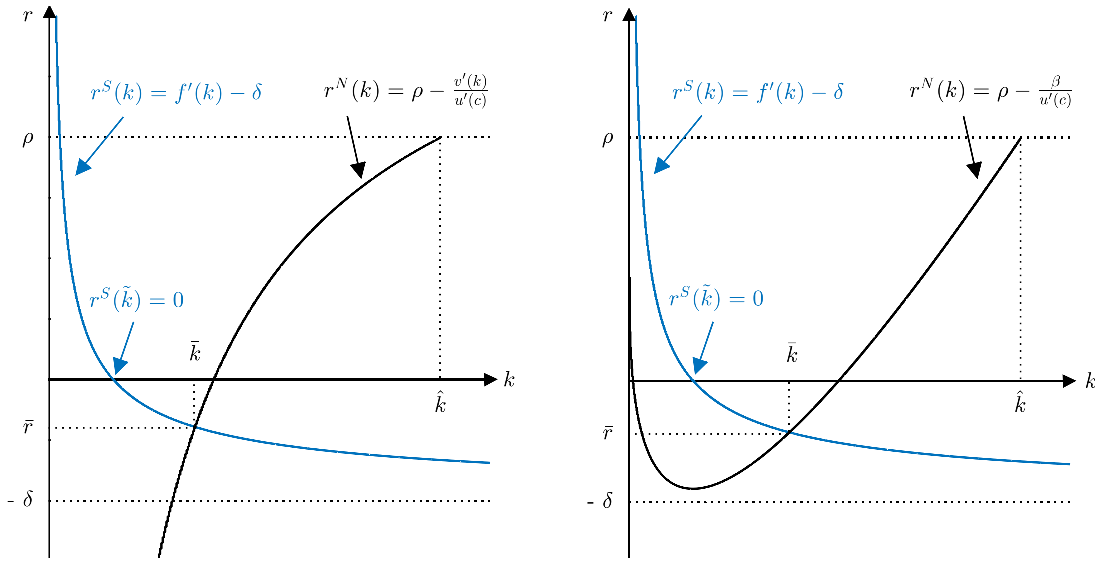
The proof is in Appendix A.1. I assume that the steady state is unique, which holds under mild restrictions as discussed in Kurz (1968) and Bose (1971). Graphically, uniqueness requires that the slope of r^N exceeds the slope of r^S at each intersection.

Full employment is sustainable even with insatiable wealth preferences because physical capital is both a store of value and a producible asset whose replacement affects aggregate demand. Suppose there was an excess supply of savings, i.e. $r^N < r^S$. As no other assets are available to absorb the excess savings, these have to be invested in bonds and physical capital. As a consequence, the capital stock increases and the net marginal product of capital declines as does the return on savings r^S. The depreciation rate constitutes a lower bound this return, i.e. $r^S > -\delta$.

In addition, the higher capital stock affects the natural rate via two channels. As households become wealthier, they prefer lower savings and higher consumption. The natural rate increases via this wealth channel as $v'(a)$ declines. Higher values of the capital stock also affect the supply of goods. Increase in k increase the net supply if k is below the Golden Rule level $\tilde{k}$, which lowers the natural rate. The opposite holds if k is above the Golden Rule, since more and more output is used for the replacement of the depreciating capital stock.

[16]Note that I assume that households receive the income z despite the exclusion of land. In this case, z should be interpreted as a lump-sum transfer. Hence, the expression for consumption in (1.21) continues to hold and there are no further changes to the natural rate in (1.20).

Figure 1.1: The Steady State of the Benchmark Model



Note: This figure shows the natural interest rate r^N and the real return on savings r^S as a function of the capital stock k. The left graph shows the case of a declining marginal utility of wealth, i.e. $v''(a) < 0$. The right graph shows the case of a constant marginal utility, i.e. $v'(a) = \beta$.

For $k < \tilde{k}$, the net effect depends on the relative strength of these channels and the natural rate might increase (left panel) or decrease (right panel) for higher k. For $k > \tilde{k}$, the natural rate increases because of both wealth and income effects. Eventually, output is fully used up for investment and the natural rate approaches the time preference rate ρ (at $\hat{k}$ in Figure 1.1). With a constant marginal utility of wealth, the wealth channel ceases to operate and increases in wealth no longer stimulate consumption. In contrast, the income channel continues to operate and the natural rate increases with the capital stock once it exceeds the Golden Rule level.

Taken together, any excess supply of savings is invested into physical capital and bonds. The associated increase in the capital stock reduces the return on savings and eventually increases the natural interest rate. Market clearing (1.22) is always feasible at full employment, even for a constant marginal utility of wealth. The steady state real rate satisfies $\bar{r} \in (-\delta, \rho)$, which can be negative for sufficiently strong wealth preferences. Then overaccumulation of capital above the Golden Rule level occurs.

Secular stagnation results from the existence of a non-producible store of value that affects the incentives for savings without, however, affecting full employment consumption.[17] Money is an example of such an asset in a monetary economy, while land is an example of such an asset in a real economy.

[17] This is also the reason why the possibility of storage or the existence of non-depreciable capital does not allow for deviations from full employment. In this case, the lack of demand would be substituted by investment demand and $\dot{K}_t > 0$ in steady state, which is sustainable if wealth preferences are insatiable.

1.3.2 The Neoclassical Model with Land

Consider first the immediate effect of introducing land, which pays a fixed rent z to its owner, into the neoclassical model. The existence of land prevents the real rate of interest from becoming negative in steady state as summarized in the following proposition.

Proposition 1.2 *The steady state real interest rate in the presence of land cannot become negative, i.e. $r > 0$, as a consequence of the No-Arbitrage condition (1.5) and the possibility of free disposal.*

The intuition is as follows (see Appendix A.2 for the proof). The land price cannot become negative due to free disposal, which also excludes the possibility of a declining price in steady state. As land yields a positive rent to its owner, the rental yield on land remains strictly positive and constitutes a real return which households can earn on their savings. The No-Arbitrage condition then implies that the returns on other assets cannot become negative as well. This result does not depend on the exogenous income component z, but holds as long as there is positive income or utility associated with ownership of the asset in steady state.[18]

It follows that the actual return on savings r^S in (1.19) and hence the net marginal product of capital have to be strictly positive in an economy with land. Thus, the steady state capital stock will always be strictly below the Golden Rule level $\tilde{k}$ and I obtain the same result as Homburg (1991): The existence of a non-producible productive asset rules out dynamic inefficiency as well as the possibility of Golden Rule growth. Any excess savings can be invested in land at a positive rate of return. This result does not depend on the specification of the wealth preference.

In a purely fundamental steady state with $\dot{q}_t = 0$, households earn the rental yield on land and the land value q adjusts to equalize this yield to the net marginal product of capital. In a bubbly steady state with $\dot{q}_t > 0$, the rental yield asymptotically approaches zero and the return on land is purely driven by the capital gain.

In addition, private wealth no longer coincides with the capital stock as households can invest in land. The land value then affects the natural interest rate r_t^N via the wealth preference as $v(a) = v(k + q)$. As households are more wealthy, their desire for savings is reduced compared to the benchmark model and the natural rate increases for a given value of the capital stock.

Both the feasibility of full employment and the possibility of rational bubbles in the land price depend on the nature of the wealth preference.

[18]Alternatively, land can be modeled as a factor input in the production function (1.6) similar to capital, as in Homburg (1991, 2014). Then, z is the rental price of land and equals the marginal product of land, which is strictly positive. Similarly, the real interest rate cannot become negative in the presence of housing that yields a positive user cost in terms of utility as in Illing, Ono, and Schlegl (2017) or in the presence of positive dividends from stock ownership as in Zhou (2013).

1.3.3 The Steady State with Standard Wealth Preferences

Consider first the case of standard wealth preferences. The land price is constant in steady state and bubbles do not occur as summarized in the following proposition.

Proposition 1.3 *With $\beta = 0$, there cannot be an ever-growing or ever-decreasing land price. Therefore, we must have $\dot{q}_t = 0$ in steady state. Moreover, the land price is purely fundamental and bubbles cannot exist, i.e. $q_t = q_t^F$ and $q_t^B = 0$.*

An ever-declining land price is not possible as the land price would eventually become negative as shown in the proof of Proposition 1.2 in Appendix A. In contrast, an ever-growing land price implies that the land value becomes infinitely high and so does household wealth. With $\beta = 0$, the marginal utility of wealth decreases to $v'(a) = 0$ and the natural rate converges to the time preference rate ρ as can be seen in (1.20). The return on land eventually consists purely of the capital gain in (1.2), which also approaches ρ. As a consequence, the growth rate of both the land price and household wealth converges to ρ as the capital stock is constant. However, this violates the transversality condition (1.15). Therefore, we must have $\dot{q}_t = 0$ in steady state. It directly follows that there cannot be a bubble in equilibrium. Any bubble has to grow with the real interest rate as is clear from (1.4). This rate is strictly positive in steady state by Proposition (1.2). This implies $\dot{q}_t^B > 0$, which is not possible.[19]

With $\dot{q}_t = 0$, the return on land is given by the rental yield in (1.2). The land price adjusts such that this return equals the net marginal product of capital. Full employment requires

$$f'(k) - \delta = \rho - \frac{v'\left(k + \frac{z}{f'(k)-\delta}\right)}{u'\left(f(k) - \delta k + z\right)} > 0 \ . \tag{1.23}$$

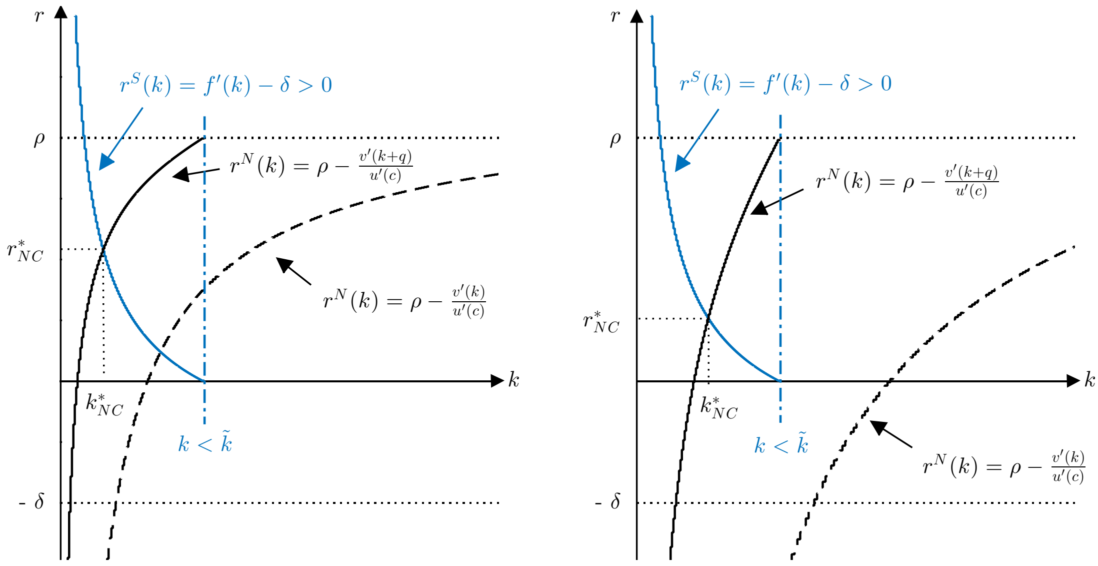
This is the equivalent to condition (1.22) in the benchmark model. The return on savings is restricted to be positive due to the availability of land as a store of value.

Both, r^N and r^S are continuous functions of k and illustrated in Figure 1.2. Let k_{NC}^* and r_{NC}^* denote the steady state values of the capital stock and the real rate. The following proposition shows that full employment is always feasible in this economy.

Proposition 1.4 *The steady state under full employment, characterized by (1.23), always exists if $\beta = 0$. This steady state is unique under mild restrictions on preferences or model parameters. It is characterized by a lower capital stock and a higher real rate than in the benchmark model without land. In addition, the model exhibits saddle-point stability around this steady state.*

[19] This result depends on the assumption of no depreciation in the value of land. I do not consider the possibility of depreciation in the rent z as the focus is on the nature of the wealth preferences. For the general case with depreciation, see Michau, Ono, and Schlegl (2017).

Figure 1.2: The Steady State of the Neoclassical Model with Land for $\beta = 0$



Note: This figure shows the natural rate r^N for the model with land (solid line) and the benchmark model (dotted line) as well as the return on savings r^S as a function of the capital stock k. The left graph shows the case of a positive real interest rate $\bar{r} > 0$ in the benchmark model, whereas the right graph shows the case of a negative real interest rate $\bar{r} < 0$ in the benchmark model. The return on savings is strictly positive as explained in Proposition 1.2, which is illustrated by the vertical line at $\tilde{k}$.

The proof is in Appendix A.3. As before, I assume uniqueness of the steady state.

For a given value of k, the return on savings r^S is unaffected by the introduction of land, but restricted to be positive, which is represented by the vertical line at $\tilde{k}$. In contrast, household wealth is higher in the presence of land, which reduces the desire for savings and increases the natural rate r^N. Hence, the r^N curve shifts upwards in Figure 1.2, which results in a lower capital stock and a higher real rate. Hence, $k^*_{NC} < \bar{k}$ and $r^*_{NC} > \bar{r}$. This holds for both a positive (left panel) and a negative (right panel) real rate $\bar{r}$ in the benchmark model.

The existence of land establishes a zero lower bound on the real return on savings as households can always invest in land at a positive return. Excess savings that could only be invested in physical capital in the benchmark model are channeled into land investment. Therefore, overaccumulation of capital cannot occur despite the preference for wealth.

In addition, the value of land also affects the natural rate. As the capital stock increases, the fundamental value of land increases and so does household wealth. As households become wealthier, their desire for savings declines and the natural rate increases. In fact, the land price becomes infinitely high when k approaches $\tilde{k}$ and the natural rate r^N approaches ρ, while the return on savings converges to zero. It follows that there always exists a capital stock $k^*_{NC} < \tilde{k}$ for which the return on savings matches the natural rate at full employment. This real rate is strictly positive at $r^*_{NC} \in (0, \rho)$.

1.3.4 The Steady State with Insatiable Wealth Preferences

With $\beta > 0$, a negative real interest rate is still ruled out by Proposition 1.2. However, the insatiability of wealth preferences gives rise to a continuum of land price paths that are compatible with equilibrium, including paths with exploding land price bubbles.

Proposition 1.5 *With insatiable wealth preferences, there is a continuum of equilibrium paths for the land price. These include a constant steady state land price* $\dot{q}_t = 0$ *as well as steady states with exploding land prices* $\dot{q}_t > 0$.

Similar to before, we cannot have a steady state with a declining land price as $q_t \geq 0$. In case of $\dot{q}_t = 0$, the land price is simply given by its fundamental value, namely the capitalized value of rents z. This is the only solution if $\beta = 0$. An ever-increasing land price in steady state implies that household wealth expands indefinitely. However, the marginal utility of wealth now converges to the lower bound $\beta > 0$. As a consequence, the real interest rate remains strictly below the time preference rate as is clear from (1.20) with $v'(a) = \beta$. Then the return on land is purely given by the capital gains component in (1.19) as the rental yield becomes infinitely small. Therefore, the growth rates of the land price and household wealth are below the time preference rate ρ. The transversality condition (1.15) is not violated despite the expansion of the land value. Intuitively, this is because the strong preference for wealth prevents households from expanding their desired consumption spending in response to an increase in wealth. Hence, the resource constraint of the economy is not violated as wealth is accumulated without expanding consumption. Households prefer to keep their land investments rather than selling it to realize parts of the increase in value.[20]

This implies that rational bubbles are sustainable, which is in stark contrast to the case of $\beta = 0$. For any initial bubble value $q_0^B > 0$, the bubble component of the land price will grow with the real return on savings as can be seen from (1.4). As this rate is strictly positive by Proposition 1.2, the bubble component will grow in steady state, thereby expanding household wealth. Eventually, the lower bound on the wealth preference is obtained and we have $v'(a) = \beta$.

In the steady state of the model with land and insatiable wealth preferences, full employment cannot always be realized. In that case, there exist a range of values $L < 1$ that are compatible with market clearing but the resulting secular stagnation steady state is not well-defined. This result is summarized in the following proposition.

[20] Similar conclusions are obtained by Ono (1994), Kamihigashi (2008) and Zhou (2013, 2015) about the dynamics of equity prices based on non-standard behavior of wealth preferences. The first two papers also assume a lower bound on the marginal utility of wealth, while the last one restricts the behavior of the marginal rate of substitution between wealth and consumption. In all cases, the real rate does not converge to the time preference rate as wealth expands and the transversality condition is not violated.

Proposition 1.6 *In the model with land and $\beta > 0$, the economy operates at full employment in steady state if*

$$\beta < \rho u'\left(c(\tilde{k})\right) \;, \tag{1.24}$$

where $\tilde{k}$ is the Golden Rule capital stock. The steady state then has the same stability properties as in the case of standard wealth preferences. In contrast, the economy fails to operate at full employment and there exist a range of values $L < 1$ compatible with market clearing if condition (1.24) is violated and if the rent from land ownership z is sufficiently low, specifically if

$$\rho u'(z) > \beta \geq \rho u'\left(c(\tilde{k})\right) \;. \tag{1.25}$$

This result holds for both a constant land price and in the presence of land price bubbles. However, the secular stagnation steady state with $L < 1$ is not well-defined.

I give an intuitive outline here (see Appendix A.4 for the proof). Consider first the case of land price bubbles. In a bubbly steady state, we have $\dot{q}_t^B > 0$ in (1.4) as $r^S > 0$ by Proposition 1.2. Both the land price and real wealth expand indefinitely and $v'(a) = \beta$ in the asymptotic steady state. Hence, the natural rate in (1.20) no longer depends on household wealth and attains a minimum at the Golden Rule capital stock $\tilde{k}$. In contrast, the return on savings is unaffected compared to the previous case. Both returns are illustrated in Figure 1.3.[21] Full employment requires

$$f'(k) - \delta = \rho - \frac{\beta}{u'\left(f(k) - \delta k + z\right)} > 0 \;. \tag{1.26}$$

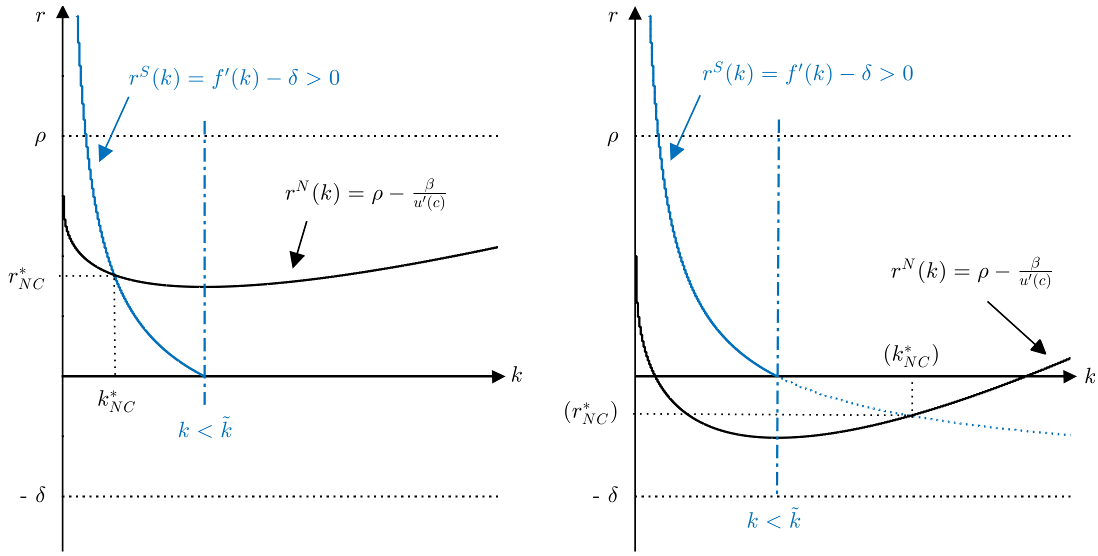
This is the equivalent to conditions (1.22) and (1.23) before. Full employment is not feasible if the preference for wealth is sufficiently strong. With $\beta = 0$, increases in the land price reduce the desire of households for savings and increase the natural rate into positive territory. In contrast, with insatiable wealth preferences, increases in the land price at some point cease to stimulate consumption. The desire for saving remains sufficiently strong. Investment demand does not expand as excess savings are invested in land instead of capital. As a consequence, an oversupply of saving occurs and the goods market cannot clear at full employment.

The same argument holds for the fundamental steady state with a constant land price. In this case, households wealth is finite and we have $v''(a) < 0$. Full employment requires

$$f'(k) - \delta = \rho - \frac{v'\left(k + \frac{z}{f'(k) - \delta}\right)}{u'(f(k) - \delta k + z)} > 0 \;, \tag{1.27}$$

[21] Note that the bubbly steady state with insatiable wealth preferences is identical to the benchmark model with a constant marginal utility of wealth that is depicted in the right panel of Figure 1.1, except for the additional restriction of $r^S > 0$. In the model without land, we have instead $\bar{r} \in (-\delta, \rho)$.

Figure 1.3: The Steady State of the Neoclassical Model with Land for $\beta > 0$



Note: This figure shows the natural interest rate r^N for the model with land and $v'(a) = \beta$ in the bubbly steady state as well as the return on savings r^S as a function of the capital stock k. In the left graph, condition (1.24) holds and $r^S > 0$ is not violated under full employment since β is sufficiently low. In the right graph, condition (1.24) is violated. Market clearing at full employment would require a negative interest rate, which is precluded by the possibility of land investment.

which is identical to (1.23) and the dynamics and stability properties of the model economy are the same as in the case of $\beta = 0$. Yet, the realization of full employment depends on the lower bound β. Specifically, the marginal utility of wealth approaches its lower bound once the steady state capital stock approaches $\tilde{k}$ since the value of land becomes infinitely high. Full employment at a positive real rate can only occur if the natural rate at $\tilde{k}$ exceeds the actual return on savings, which is zero at the Golden Rule level. Existence of the full employment steady state then depends on the same condition as in the bubbly steady state since the value of the natural rate at $\tilde{k}$ is identical in both cases.

Hence, condition (1.24) determines the feasibility of full employment in the steady state with $\beta > 0$. In Figure 1.3, this condition holds in the left panel, while it is violated in the right panel.

Consider a concrete example to illustrate the factors that determine the feasibility of full employment. Assume the standard production function $f(k) = k^\alpha$ and a constant intertemporal elasticity of substitution $\sigma > 0$ with $u'(c) = c^{-\sigma}$. The Golden Rule level of the capital stock is given by $f'(\tilde{k}) = \delta$ and the Golden Rule level of consumption satisfies $\tilde{c} = f(\tilde{k}) - \delta\tilde{k} + z$. Full employment is no longer feasible if condition (1.24) is violated or alternatively if condition (1.25) holds. In this example, full employment is not feasible if

$$\beta \geq \rho \left[(1-\alpha) \left(\frac{\alpha}{\delta} \right)^{\frac{\alpha}{1-\alpha}} + z \right]^{-\sigma} .$$

Intuitively, factors that lower the desire for savings and increase the natural rate make full employment easier to sustain, while factors that reduce the natural rate make it more difficult.

Household preferences affect the feasibility of full employment. Specifically, a lower time preference rate ρ, a more concave utility function $u(c)$ and a stronger preference for wealth β increase the desire of households for savings and reduce the natural rate.

In addition, a higher depreciation rate δ implies that more output is used for investment to replace the depreciated capital stock, which lowers consumption under full employment for a given k. This increases the natural interest rate. In contrast, a higher capital intensity of production α increases the supply of goods available for consumption, which lowers the natural rate and makes full employment harder to sustain.[22]

Finally, higher rental income z from land ownership increases consumption under full employment as well as desired savings. The natural rate decreases.

To summarize the analysis so far: Full employment is always feasible in a purely neoclassical economy with land and standard wealth preferences. In contrast, full employment cannot always be sustained despite the existence of land if wealth preferences are insatiable. In fact, it is the very existence of land that rules out the full employment steady state by preventing the real return on savings from becoming negative.

In a monetary economy, the inflation rate also creates a lower bound on the real return on savings due to the zero lower bound on the nominal rate. Yet, secular stagnation can occur in the monetary economy with standard wealth preferences as shown by Michaillat and Saez (2014) and Michau (2017), while it is not possible in the economy with land unless wealth preferences are insatiable. The reason is that money does not affect the utility from wealth in these models as it is not considered net wealth. The real money stock then does not affect the natural rate.

In the neoclassical economy with land, it is not the preference for wealth itself that prevents full employment in steady state but its insatiable nature.[23] It results in a desire for savings that is too strong to clear markets at full employment. The oversupply of savings then causes a shortfall of the labor supply. In fact, there are multiple levels of the labor supply that are consistent with market clearing. Specifically, any $L < 1$ that fulfills condition (1.25) in Proposition 1.6 defines a secular stagnation steady state. Secular stagnation is then not a monetary, but a real phenomenon. The steady state is however indeterminate in the absence of real wage rigidities. Therefore, I introduce money and downward nominal wage rigidity to determine the steady state.

[22]Note that higher depreciation or a lower capital intensity also lower the net marginal product of capital and shift the r^S curve. However, it is the return on savings at the Golden Rule level that determines the feasibility of full employment. This return is always zero.

[23]Note that the insatiability of wealth preferences in the benchmark model is consistent with full employment as there is no non-producible productive store of value.

1.4 A Monetary Economy with Land

I introduce money via a money-in-the-utility framework following Sidrauski (1967), where the central bank controls the nominal money supply. In addition, nominal rigidities limit the downward adjustment of wages and prevent deflationary wage-price spirals.

1.4.1 Monetary Policy, Wage Setting Frictions and Inflation

Money serves as a means of savings for households, in addition to bonds, capital and land, and it is a unit of account for the price level. Money does not pay interest, but provides households with a convenience yield due to its liquidity and transaction value.

The central bank directly controls the nominal money supply M_t^S, while the nominal interest rate i_t is endogenously determined in the money market. Let P_t denote the price level of the output good and $m_t^s \equiv M_t^S/P_t$ the real money supply. Similarly let m_t^d denote the real money demand of households. In equilibrium, the money market clears such that

$$\frac{M_t^S}{P_t} \equiv m_t^s = m_t^d = m_t \tag{1.28}$$

Let μ denote the growth rate of the nominal money supply. The real money supply then evolves as

$$\dot{m}_t = (\mu - \pi_t)m_t \ , \tag{1.29}$$

where π_t denotes the rate of inflation. In the monetary economy, bonds pay a nominal interest rate i_t, which is related to the real rate of return via the Fisher Equation as

$$r_t = i_t - \pi_t \ . \tag{1.30}$$

As land and capital are real assets, their returns are unaffected by the introduction of money and the No-Arbitrage condition (1.5) continues to hold, with the return on bonds given by (1.30).

In addition, the firm problem is unaffected and the rental price of capital and the real wage are still determined by the marginal products of capital and labor in (1.7) and (1.8). The latter implies that the real wage is constant in steady state. Therefore, the inflation rate always equals the change in the nominal wage W_t in steady state.

I assume downward nominal wage stickiness. Specifically, I follow Schmitt-Grohé and Uribe (2016, 2017) and postulate that nominal wages cannot fall by more than an exogenous rate γ, i.e.

$$\frac{\dot{W}_t}{W_t} \geq -\gamma \ . \tag{1.31}$$

A value of $\gamma = 0$ implies that nominal wages cannot fall at all, whereas $\gamma = 1$ allows for perfectly flexible wages.[24] Equation (1.31) is only binding in case of stagnation, hence the complementary slackness constraint

$$(1 - L_t)\left(\frac{\dot{W}_t}{W_t} + \gamma\right) = 0 \ . \tag{1.32}$$

Under full employment we have $L_t = 1$ and (1.31) is not binding. The inflation rate is determined by the money supply growth rate μ as can be seen from (1.29) with $\dot{m}_t = 0$. The central bank can perfectly control the inflation rate and money is fully neutral. The price level adjusts to clear the money market in (1.28). It is then the dynamics of the price level that determine nominal wage dynamics.

In contrast, we have $L_t < 1$ under stagnation and (1.31) is binding. Nominal wages decline at a constant rate γ. The dynamics of the nominal wage determine the inflation rate π as the real wage is given by (1.8). Taken together, the rate of inflation is determined as

$$\pi_t \ = \ \frac{\dot{P}_t}{P_t} \ = \begin{cases} \mu \ \text{ if } \ L_t = 1 \ , \\ -\gamma - \dfrac{\dot{w}_t}{w_t} \ \text{ if } \ L_t < 1 \ . \end{cases} \tag{1.33}$$

I assume $\mu \geq 0 > -\gamma$. Hence, the economy is deflationary under stagnation, but not under full employment. Under stagnation, inflation is related to changes in the real wage, which are determined by changes in the capital-labor ratio as $\dot{w}_t = -f''(k_t)k_t\dot{k}_t$ from (1.8). In steady state, the inflation rate is constant as $\dot{k}_t = 0$ and hence $\dot{w}_t = 0$.[25]

1.4.2 Households and the Natural Rate of Interest (Revisited)

Money is a store of value for households. Total real wealth a_t is now given by the household's real bond holdings $b_t \equiv B_t/P_t$, with $b_t = 0$ in equilibrium, the value of real capital, the value of land and the real money holdings as

$$a_t = b_t + k_t L_t + q_t H_t + m_t \ , \tag{1.34}$$

[24] The same assumption is adopted by Hanson and Phan (2017) and Biswas, Hanson, and Phan (2017). Similar relationship are used among others in Ono and Ishida (2014), Eggertsson, Mehrotra, and Robbins (2017), Caballero and Farhi (Forthcoming) or Michau (2017). They are derived from nominal frictions in the wage setting process like fairness concerns of workers or the prevalence of a wage norm or a reference wage. For a review of the empirical evidence on wage stickiness, see section 8 in Schmitt-Grohé and Uribe (2016).

[25] The asymmetry in the inflation process based on nominal wage stickiness is a common feature of models of secular stagnation and can be found among others in Ono and Ishida (2014), Michau (2017) or Illing, Ono, and Schlegl (2017). In this setting, the inflation rate in the secular stagnation steady state is determined by structural factors, which affect the degree of wage stickiness γ, and hence unaffected by the labor supply gap. In contrast, the inflation rate depends on the steady state output gap in Ono and Ishida (2014), Illing, Ono, and Schlegl (2017) and Schmitt-Grohé and Uribe (2017).

and, using the No-Arbitrage condition (1.5) for the returns on capital and land, real wealth evolves as

$$\dot{a}_t = r_t a_t - i_t m_t - c_t + w_t L_t + \tilde{z}_t \ , \tag{1.35}$$

where I assume that all seignorage income $\tilde{z}_t \equiv \mu m_t$ is redistributed to households in a lump-sum fashion. Holding money creates opportunity losses equal to the nominal interest rate i_t.

I follow the money-in-the-utility literature and I assume that real money balances generate utility $\omega(m_t)$ to households as they are a liquid means of payment. This function fulfills the standard assumptions of $\omega'(m) > 0$, $\omega''(m) < 0$, $\lim_{m \to 0} \omega'(m) = \infty$ and $\lim_{m \to \infty} \omega'(m) = 0$. The lifetime utility of households is given by

$$U_0 = \int_0^\infty \left[u(c_t) + \omega(m_t) + v(\tilde{a}_t)\right] e^{-\rho t} dt \ , \tag{1.36}$$

which is the standard Sidrauski (1967) model except for the wealth preference $v(\tilde{a}_t)$. Households maximize (1.36) subject to (1.35), which gives the optimality condition for the intertemporal allocation of consumption and optimal money demand as

$$\eta_c \frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{v'(\tilde{a}_t)}{u'(c_t)} \ , \quad \text{where } \eta_c \equiv -\frac{u''(c_t) c_t}{u'(c_t)} \ , \tag{1.37}$$

$$\frac{\omega'(m_t)}{u'(c_t)} = i_t \ . \tag{1.38}$$

In addition, the transversality condition (1.15) continues to apply to real household wealth.

The Euler Equation in (1.37) is unaffected by the introduction of money, except for the difference in the wealth composition in (1.34). It is up for debate if the real money stock constitutes net wealth of households and generates utility via the wealth preference. Both views can be found in the literature. Ono (1994, 2015) makes the case for including money in the utility from wealth. Then $v(\tilde{a}_t) = v(a_t)$ as before and the natural rate increases for a given k compared to the neoclassical model. As households feel wealthier, their desire to save declines. In contrast, Michaillat and Saez (2014) and Michau (2017) argue that only wealth generated by real assets affects the utility from wealth.[26] Then $v(\tilde{a}_t) = v(a_t - m_t^S)$ and the real money stock does not affect the wealth preference. The natural rate is the same as in the purely neoclassical economy. From the perspective of the individual household, the assumption $v(\tilde{a}_t) = v(a_t)$ seems more natural. However, I will analyze the implications of both views.

[26] Michaillat and Saez (2014) assume that the central bank issues money to buy household bonds. The wealth created by money is therefore equal to the bond liability and net wealth equals the stock of physical capital. Similarly, Michau (2017) argues that both government debt and money do not constitute net wealth of households as both need to be redeemed eventually.

The nominal rate i_t determines optimal money demand in (1.38). Households equate the liquidity premium, which is the marginal rate of substitution between money and consumption, to the nominal interest rate on bonds. In fact, (1.38) represents a No-Arbitrage relation between money and bonds and establishes a zero lower bound on the nominal interest rate. The wealth preference itself does not affect money demand directly.

The real return on money r_t^M affects the return on savings r_t^S as money is a store of value for households. The nominal return on money consists of the liquidity premium in (1.38). Using the Fisher Equation (1.30) and inflation in (1.32), the real return on money satisfies

$$r_t^M = \frac{\omega'(m_t)}{u'(c_t)} - \pi_t = \begin{cases} \dfrac{\omega'(m_t)}{u'(c_t)} - \mu \quad \text{if} \quad L_t = 1 \ , \\ \dfrac{\omega'(m_t)}{u'(c_t)} + \gamma + \dfrac{\dot{w}_t}{w_t} \quad \text{if} \quad L_t < 1 \ . \end{cases} \tag{1.39}$$

Asset market clearing requires the equalization of the real returns on money, bonds, land and capital. The No-Arbitrage condition (1.5) is extended to include money as

$$r_t = r_t^M = r_t^K = r_t^L \ (= r_t^B) \ , \tag{1.40}$$

where r_t^K, r_t^L, r_t^B and r_t^M are given by (1.9), (1.2), (1.4) and (1.39). This constitutes the real return on savings r_t^S, which households receive irrespective of the asset in which they invest.

1.5 Analysis of the Monetary Economy

The equilibrium of the economy is characterized by the Euler Equation (1.38), the return on land (1.2), the return on capital (1.9), the return on money (1.39), the goods market clearing condition (1.18), the money supply process in (1.29) and the No-Arbitrage condition (1.40), together with the transversality condition (1.15), where household wealth is given by (1.34), for a given initial value of capital $K_0 > 0$. Under full employment $L_t = 1$ and $K_t = k_t$. Under stagnation $L_t < 1$ and $K_0 > 0$ is given while $k_t > K_t$. In steady state, it holds that $\dot{c}_t = 0$ and $\dot{k}_t = 0$, which implies $\dot{K}_t = 0$ and $\dot{L}_t = 0$. The properties of $\dot{q}_t$ and $\dot{m}_t$ depend on $v(\tilde{a})$.

The actual return on savings r^S in steady state is determined by the No-Arbitrage condition (1.40) that requires the equalization of real returns across assets as

$$r^S = \left\{ \begin{array}{l} \dfrac{\omega'(M^S/P)}{u'(f(k) - \delta k + z)} - \mu \quad \text{if} \quad L_t = 1 \\ \\ \gamma \quad \text{if} \quad L_t < 1 \end{array} \right\} = f'(k) - \delta = \frac{z}{q_t} + \frac{\dot{q}_t}{q_t} > 0 \ , \tag{1.41}$$

where I used money market equilibrium (1.28) and goods market equilibrium (1.18) to substitute for optimal money demand and consumption. Note that Proposition 1.2 continues to hold, which is why r^S is required to be strictly positive.

Under full employment, the real money stock increases in the capital stock of the economy. An increase in k lowers the net marginal product of capital and increases consumption as $k < \tilde{k}$. This makes holding money more attractive and increases money demand of households. For a given money supply M^S, the price level P declines to clear the money market. As a consequence, the real money stock increases and the nominal interest rate is lowered until $r^M = r^K$. As the nominal interest rate is strictly positive, the money growth rate μ - or equivalently the inflation target of the central bank - represents a lower bound on the return on savings when the economy operates at full employment. This lower bound is what prevents the return on savings from adjusting to the natural rate in other models of secular stagnation. In the model with land, this lower bound is only binding in an economy that is deflationary under full employment, i.e. if $\mu < 0$. For positive inflation rates, it is always the non-negativity restriction imposed by the existence of land that represents the minimum return on household savings.

In a secular stagnation steady state, the economy is deflationary and nominal wages and prices decline with rate γ. As a consequence, the real money supply expands indefinitely since $\mu + \gamma > 0$. In this asymptotic steady state, money demand of households becomes satiated and the nominal interest rate converges to zero. The return on savings is then determined by the rate of deflation and the capital stock adjusts accordingly.

The existence of a steady state requires the equalization of the return on savings r^S defined in (1.41) with the natural rate of interest r^N in (1.37) with $\dot{c}_t = 0$, taking into account the wealth composition. The full employment steady state exists if

$$\frac{\omega'(M^S/P)}{u'(f(k) - \delta k + z)} - \mu = f'(k) - \delta = \rho - \frac{v'(\tilde{a})}{u'(f(k) - \delta k + z)} > 0 \ , \tag{1.42}$$

where I use the goods market clearing condition under full employment (1.18) to substitute for consumption in (1.37) and where wealth is defined by the wealth composition (1.34) with $b_t = 0$, $L_t = 1$ and $H_t = 1$. For $v(\tilde{a}) = v(a)$, the natural rate increases for a given k compared to the previous section as money is part of net wealth in $v(a)$. In contrast, for $v(\tilde{a}) = v(a - m^S)$, the real rate is determined in the real economy and unaffected by the introduction of money. Once the real rate is determined, and strictly positive for full employment to be feasible, households adjust their desired money holdings such that $r^M = r^S$ from (1.40) and the price level adjusts to clear the money market in (1.28). Money is fully neutral as an exogenous increase in the money supply will simply result in a proportional increase in the price level.

In addition, for a secular stagnation steady state to exist, it has to hold that

$$\gamma = f'(k) - \delta = \rho - \frac{v'(\tilde{a})}{u'([f(k) - \delta k]L + z)} > 0 \ , \tag{1.43}$$

where I use goods market clearing condition (1.18) with $L < 1$ to substitute for consumption in (1.37). Under stagnation, the inflation rate is determined by the binding wage rigidity and the return on money equals the rate of deflation as the nominal rate becomes asymptotically zero. The capital-labor ratio is determined by the No-Arbitrage condition between the net marginal product of capital and the return on money. The expansion of the money supply reduces the marginal utility of wealth to its lower bound in the asymptotic steady state for $v(\tilde{a}) = v(a)$.

The feasibility of full employment and stagnation depend on the nature of the utility from wealth. As before, I distinguish again between standard and insatiable wealth preferences.

1.5.1 The Case of Standard Wealth Preferences

The steady state with standard wealth preferences is characterized by a constant land price as Proposition 1.3 continues to hold. Hence, the land price is given by the capitalized value of rents and bubbles do not exist. The same argument applies to the real money stock. Under full employment, we have $\pi_t = \mu$ from (1.33) and the real money supply is constant from (1.29). In a secular stagnation steady state, the real money supply expands indefinitely since $\mu + \gamma > 0$. The expansion of the money supply increases the natural rate to ρ, which eventually violates the transversality condition for $v(\tilde{a}) = v(a)$.[27]

Full employment is always feasible in a neoclassical economy with land and $\beta = 0$ as shown in Proposition 1.4. The respective condition for the monetary economy is as follows.

Proposition 1.7 *In the monetary model with $\beta = 0$, the full employment steady state in (1.42) is the unique equilibrium if the real money stock affects the utility from wealth, i.e. if $v(\tilde{a}) = v(a)$. The secular stagnation steady state in (1.43) cannot exist for $v(\tilde{a}) = v(a)$.*

For the case of $v(\tilde{a}) = v(a - m^S)$, the full employment steady state is not feasible if

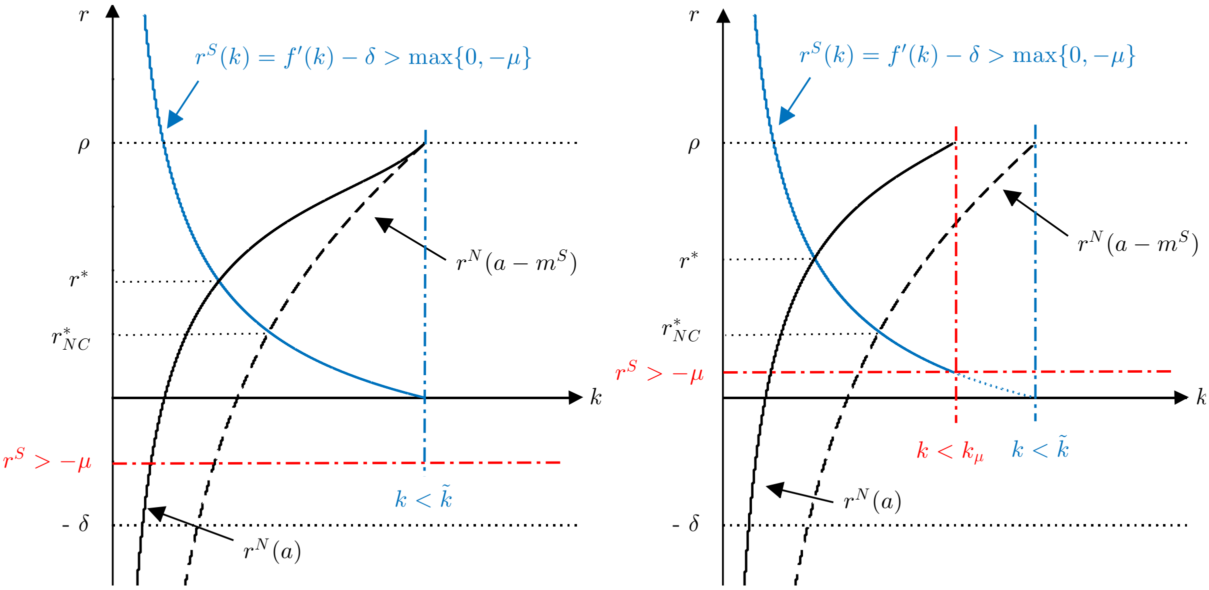
$$\mu < -r^*_{NC} < 0 \tag{1.44}$$

where $r^*_{NC} > 0$ denotes the steady state real rate in the neoclassical model with land.

The full employment steady state is unique under mild restrictions and the dynamic system exhibits saddle-point stability around this steady state.

[27] The expansion of the real money supply under stagnation does not affect the real interest rate for $v(\tilde{a}) = v(a - m^S)$. Then, money can expand without violating the transversality condition if $\mu + \gamma < \rho$.

Figure 1.4: The Steady State of the Monetary Model with Land for $\beta = 0$



Note: This figure shows the natural rate r^N for the monetary economy (solid line) and the neoclassical model with land (dotted line) as well as the return on savings r^S as a function of the capital stock k. The dotted line also represents the natural rate for $v(\tilde{a}) = v(a - m^S)$. The left graph assumes $\mu \geq 0$, whereas the right graph shows $0 > \mu > -\gamma$. $\tilde{k}$ is the Golden Rule capital stock, k_μ is defined by $f'(k_\mu) = \delta - \mu$.

The proof is relegated to Appendix A.5. The intuition is illustrated in Figure 1.4, which shows the natural rate for the neoclassical economy with land (dotted line) and the monetary economy with $v(\tilde{a}) = v(a)$ (solid line). The dotted line also represents the natural rate for the case of $v(\tilde{a}) = v(a - m^S)$. The natural rate in the monetary economy exceeds the natural rate in the neoclassical economy due to the higher wealth level for a given k if $v(\tilde{a}) = v(a)$. The return on savings is unaffected, but potentially restricted by the introduction of money. The restrictions $r^S > 0$ from land ($k < \tilde{k}$) and $r^S > -\mu$ from money ($k < k_\mu$) are also illustrated.

Consider first the case of an inflationary economy with $\mu \geq 0$, which is shown in the left panel of Figure 1.4. It is the existence of land that provides a lower bound on the real interest rate as the nominal interest rate exceeds the real rate. Full employment is always feasible as land price increases stimulate the natural rate towards ρ once the capital stock approaches the Golden Rule level. This is the same mechanism as in the neoclassical model with $\beta = 0$.

In a deflationary economy with $\mu < 0$ (right panel), the existence of money creates a lower bound on the return on savings. Yet, increases in the real money stock have a similar effect as land for $v(\tilde{a}) = v(a)$. A lower real rate increases money demand to lower the nominal rate for a given inflation rate. This requires an increase in the real money stock to clear the money market. The corresponding increase in monetary wealth stimulates the natural rate towards ρ (for $k = k_\mu$) and full employment is always feasible.

However, this channel only works when real money balances affect the natural rate in steady state, which is illustrated by the curve $r^N(a)$. If households do not derive utility from monetary wealth, the natural rate is independent of the real money stock, which is curve $r^N(a - m^S)$. Then, the steady state real interest rate is unaffected by the introduction of money and given by r^*_{NC}, which is the same rate as in the neoclassical model with land of section 1.3. This rate satisfies $r^*_{NC} \in (0, \rho)$. If the economy is sufficiently deflationary, the real return on savings can be persistently above the natural rate. This requires $\mu < -r^*_{NC}$. Then full employment is not feasible despite the existence of land as the incentives for savings by holding cash are too high.[28]

As long as the economy under full employment is characterized by price stability or positive trend inflation, the full employment equilibrium is always feasible with standard wealth preferences once land is available as a means of investment. In fact, it is the only feasible equilibrium of the model economy when money affects the utility from wealth. Then, secular stagnation cannot occur as either the increase in land prices for $\mu \geq 0$ or the expansion of the real money supply for $\rho > -\mu > 0$ absorbs any excess savings.

If wealth preferences are instead given by $v(\tilde{a}) = v(a - m^s)$, the possibility of stagnation is not ruled out by the presence of land. Then the full employment and a secular stagnation steady state can coexist and it is unclear which one is realized. In a monetary economy without land, this secular stagnation steady state occurs as the unique outcome if the central bank follows a low inflation target as in Michaillat and Saez (2014) and Michau (2017). Secular stagnation then becomes a policy choice and full employment can be restored by choosing a sufficiently high inflation target. In contrast, full employment is always feasible in an economy with land and standard wealth preferences unless in the implausible scenario of the central bank following a strong deflation target. The secular stagnation steady state can, however, exist as the unique equilibrium of the economy when wealth preferences are insatiable.

1.5.2 The Case of Insatiable Wealth Preferences

With a lower bound on the marginal utility of wealth, rational bubbles with $\dot{q}_t > 0$ are possible in a stationary steady state with $\dot{c}_t = \dot{k}_t = 0$ as shown in Proposition 1.5. The same argument implies that an infinite expansion of the money supply, i.e. $\dot{m}_t > 0$, is consistent with the transversality condition in a secular stagnation steady state if the rate of expansion is below the time preference rate ρ. For simplicity, I will assume in the following analysis that there are no bubbles, i.e. $q^B_t = 0$. The steady state with bubbles is identical to the case of $v'(\tilde{a}) = \beta$ due to an expanding real money stock for $v(\tilde{a}) = v(a)$.

[28]Note that the existence of a full employment equilibrium always requires $\mu > -\rho$ since the natural rate cannot exceed the time preference rate ρ.

The following proposition is the equivalent to Proposition 1.6 in the neoclassical economy.

Proposition 1.8 *In the monetary economy with $\beta > 0$, full employment is not feasible if*

$$\begin{aligned} &\beta > \rho u'\left(c(\tilde{k})\right) && \text{for } \mu \geq 0 \ , \\ &\beta > (\rho+\mu)u'\left(c(k_\mu)\right) && \text{for } \mu < 0 \text{ and } v(\tilde{a}) = v(a) \ , \\ &\mu < -r^*_{NC} && \text{for } \mu < 0 \text{ and } v(\tilde{a}) = v(a - m^S) \ , \end{aligned} \tag{1.45}$$

where $\tilde{k}$ and k_μ are defined by $f'(\tilde{k}) = \delta$ and $f'(k_\mu) = \delta - \mu$. In addition, there exists a secular stagnation steady state with $0 < L < 1$ if

$$\begin{aligned} &\frac{\beta}{u'\left(f(k^*) - \delta k^* + z\right)} > \rho - \gamma > \frac{\beta}{u'\left(z\right)} && \text{for } v(\tilde{a}) = v(a) \ , \\ &\frac{v'\left(k^* + \frac{z}{\gamma}\right)}{u'\left(f(k^*) - \delta k^* + z\right)} > \rho - \gamma > \frac{v'\left(\frac{z}{\gamma}\right)}{u'\left(z\right)} && \text{for } \mu < 0 \text{ and } v(\tilde{a}) = v(a - m^S) \ , \end{aligned} \tag{1.46}$$

where k^* is given by $f'(k^*) = \delta + \gamma$, if the money growth rate is small and if nominal wages are sticky, specifically

$$\mu < \rho - \gamma \ , \tag{1.47}$$

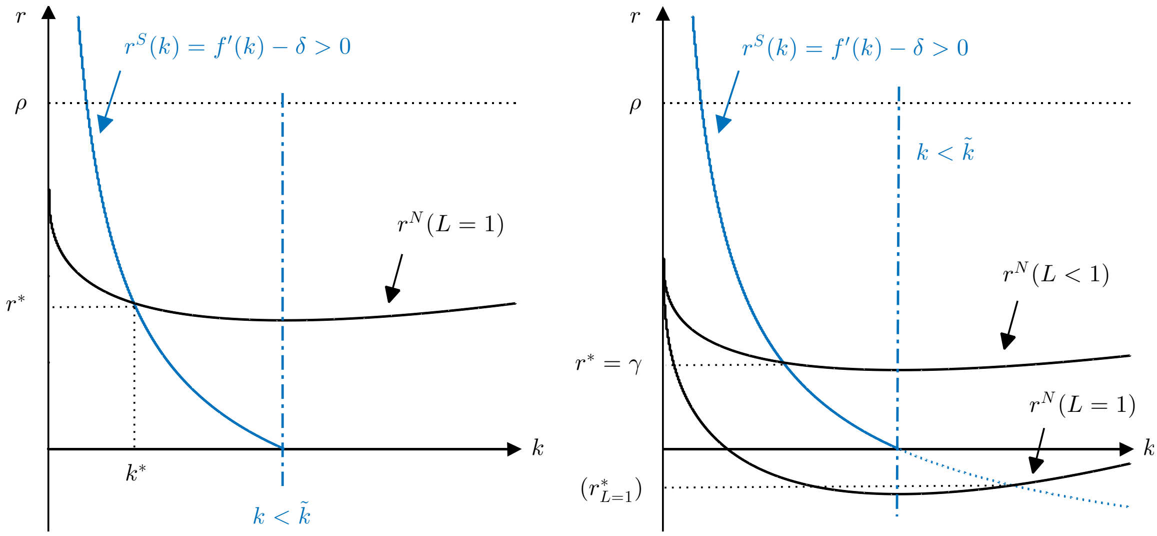
$$\gamma < \rho \ . \tag{1.48}$$

The full employment and secular stagnation steady states are unique under mild restrictions and the dynamic systems satisfy saddle-point stability around these steady states.

Consider first the case of $\mu \geq 0$, for which (1.45) is identical to (1.24) in Proposition 1.6. The lower bound on the return on savings r^S always results from the availability of land and money does not impose a constraint. The existence of land implies that the real return on savings cannot become negative and that the capital stock cannot exceed the Golden Rule level. Once it approaches $\tilde{k}$, the land price becomes infinitely high but fails to stimulate the natural interest rate as the lower bound β is binding. Hence, if the natural rate at the Golden Rule capital stock is positive, full employment is feasible. This is illustrated in the left panel of Figure 1.5.

In contrast, if the natural rate is negative at the Golden Rule level, an oversupply of savings occurs at full employment. Land prevents the real return on saving from falling below zero, while increases in the land price cease to stimulate the natural rate. The oversupply of savings depresses demand, the realized labor supply drops below the potential labor supply and secular stagnation occurs. This is illustrated in the right panel. The drop in the labor supply shifts the natural rate upwards and restores goods market equilibrium at a positive real rate.

Figure 1.5: The Steady State of the Monetary Model with Land for $\beta > 0$



Note: This figure shows the natural rate r^N for the monetary economy with $\beta > 0$ as well as the return on savings r^S as a function of the capital stock k for $\mu \geq 0$ and $v'(\tilde{a}) = \beta$. The vertical lines represent the lower bound on the real rate imposed by the existence of land. Full employment condition (1.45) holds in the left graph, while it is violated in the right graph.

In a deflationary economy with $\mu < 0$, the lower bound of the return on money exceeds the lower bound of the return on land. Therefore, money constrains the actual return on savings from falling below the rate of deflation. This implies that the capital stock under full employment cannot exceed $k_\mu < \tilde{k}$, with $f'(k_\mu) = \delta - \mu$, at which both physical capital and money yield the same real return. Full employment is only feasible if the natural rate exceeds the return on savings at k_μ. Otherwise, secular stagnation occurs. Note that condition (1.45) is independent of the lower bound β for $v(\tilde{a}) = v(a - m^S)$ and simply requires sufficiently weak wealth preferences.[29]

As the real money stock is part of household wealth, the transversality condition (1.15) is affected. In steady state, the growth rate of the real money supply $\mu - \pi$ cannot exceed the time preference rate ρ. Under full employment, the real money supply is constant and this always holds. In contrast, the growth rate under stagnation is given by $\mu - \gamma > 0$. Hence, condition (1.47) requires $\rho > \mu - \gamma$.

Finally, condition (1.48) simply implies that the stagnation steady state is well-defined. If the rate of deflation under stagnation exceeds the time preference rate ρ, the return on savings is persistently higher than the natural interest rate, whose maximum is ρ. Hence, the good market does not clear for any level of the labor supply L and a steady state therefore does not exist. Note that conditions (1.47) and (1.48) are identical for $\mu = 0$.

[29]This condition is identical to condition (1.44) in Proposition 1.7. Under these assumptions, the secular stagnation steady state also exists for $\beta = 0$. Yet, full employment is always possible in that case, while it is not feasible with $\beta > 0$ although $v''(a) < 0$ in steady state. Hence, it is the lower bound on the marginal utility of wealth that can result in secular stagnation as the unique feasible equilibrium.

It is easy to see from Proposition 1.8 that secular stagnation is a monetary phenomenon only to the extent that the economy under full employment is deflationary. Then increases in the money growth rate μ can restore full employment. However, secular stagnation becomes a real phenomenon once $\mu \geq 0$. It is the result of the existence of land in combination with insatiable wealth preferences. Further increases in the money growth rate cannot restore full employment. Consider briefly the features of the full employment and the secular stagnation steady states.

The Full Employment Steady State: The economy behaves similar to the standard money-in-the utility model. We have $L = 1$ and $\pi = \mu$ from (1.33), which implies a constant real money supply. The land price can include a bubble as shown in Proposition (1.5), in which case $v'(a) = \beta$. Without bubbles, the land price is finite. The capital stock is determined by $r^S = r^N$ in (1.42). From the capital stock, the real rate is derived as the net marginal product of capital. The land price is the rent z, capitalized with this rate, while consumption is determined from the market clearing condition. Given c and k, households adjust their money demand such that $r^M = r^S$ from (1.40) and the price level adjusts to clear the money market in (1.28). Money is neutral as an increase in the money supply results in a proportional increase in the price level.

How does the land value affect the steady state? An exogenous increase in z has two effects for a given k. On the one hand, the higher rent implies a higher fundamental land price, which increases household wealth and lowers the desire for savings. Hence, the natural rate increases. On the other hand, consumption under full employment increases, which reduces the natural rate. Hence, there is a wealth effect via higher land prices that causes the natural rate to increase and an income effect that lowers the natural rate.[30] The wealth effect is stronger the higher the elasticity of the utility from wealth $\eta_a \equiv -\frac{v''(a)a}{v'(a)}$. For high values of η_a, the increase in land prices causes a crowding out of investment and the capital stock declines. For sufficiently low values of η_a, the capital stock increases as the income effect dominates and there is crowding-in.[31]

The Secular Stagnation Steady State: The realized labor supply falls short of the potential labor supply, i.e. $L < 1$, and the downward nominal wage constraint is binding. Hence, inflation is given by $\pi = -\gamma < 0$ as can be seen in (1.33). The real money supply expands at rate $\mu + \gamma$, given that (1.47) holds. The liquidity demand of households is satiated and the nominal rate converges to $i_t = 0$ from (1.38). Then, the real return on money in (1.39) is simply given by the

[30] In addition, the increase in consumption increases money demand for a given nominal rate. The resulting increase in the real money stock increases household wealth unless we have $v(\tilde{a}) = v(a - m^S)$, which reinforces the wealth effect as it increases η_a.

[31] Note that the crowding out effect always dominates if the asset does not provided income directly. For instance, the value of land in Kocherlakota (2013) is derived from the utility it yields to households. Similarly, the real house price in Illing, Ono, and Schlegl (2017) depends on the convenience yield from housing.

rate of deflation as $r^M = \gamma > 0$ and, together with the No-Arbitrage condition (1.40), the steady state capital stock of the economy is uniquely determined as

$$f'(k^*) = \delta + \gamma > \delta \ , \tag{1.49}$$

which is above the Golden Rule level and hence consistent with the requirement of Proposition 1.2. In addition, the fundamental value of the land price equals the capitalized value of rents as

$$q^* = \frac{z}{\gamma} > 0 \ , \tag{1.50}$$

although by Proposition 1.5 several bubbly steady states with $\dot{q}_t > 0$ exist, where the bubble grows at rate γ without violating the transversality condition as $\rho > \gamma$ by (1.48).

Market clearing requires the real return on savings to be consistent with the natural rate r^N. Since this is not feasible at full employment, households adjust their labor supply according to (1.43) such that

$$\frac{\beta}{u'([f(k^*) - \delta k^*]L^* + z)} = \rho - \gamma \ , \tag{1.51}$$

where k^* is defined in (1.49). Note that we have $v'(a) = \beta$ as $\dot{m}_t > 0$ in the asymptotic steady state with $v(\tilde{a}) = v(a)$ and $\dot{q}_t^B > 0$ in the presence of bubbles. This condition does not depend on the money growth rate, but only on the degree of wage rigidity γ. The existence of the steady state requires wages to be sufficiently sticky, specifically $\rho > \gamma$. Condition (1.51) uniquely determines the labor supply gap under secular stagnation. In case of $v(\tilde{a}) = v(a - m^S)$ and in the absence of bubbles, the respective condition is given by

$$\frac{v'(k^* L^* + q^*)}{u'([f(k^*) - \delta k^*]L^* + z)} = \rho - \gamma \ , \tag{1.52}$$

where $v''(a) < 0$ and q^* is defined in (1.50). How do changes in the model parameters affect the secular stagnation steady state?

Higher wage flexibility worsens realized output under stagnation. An increase in γ causes higher deflation and a higher return on savings r^S. The capital stock per worker decreases to provide for a higher return on capital in (1.49). This causes the natural rate r^N to increase for a given labor input. The net effect is an excess supply of savings, which worsens the labor supply gap. The effect that increasing wage flexibility is detrimental under stagnation is common in models of stagnation and can be found among others in Eggertsson and Krugman (2012), Eggertsson, Mehrotra, and Robbins (2017) and Michau (2017).[32]

[32]Similarly, an increase in the depreciation rate increases the capital stock per worker in (1.49). Yet, it also lowers the supply of goods available for consumption. It is unclear which effect dominates.

In addition, a higher time preference rate ρ implies a higher realized labor supply. As households are less patient, their desire for saving decreases and the natural rate increases. Since the capital-labor ratio is unchanged, this causes excess demand. Moreover, stronger preferences for wealth reduce the natural rate without affecting the steady state capital-labor ratio. Consequently, a higher value of β causes the labor supply to decline.

A higher land rent z causes the fundamental value of land to increase. In case of $v''(a) < 0$, the net effect of the opposing income and wealth effects depends on the elasticities of $u(c)$ and $v(a)$. However, the wealth effect ceases to affect the natural rate if $v'(a) = \beta$. Therefore, the income effect dominates. As aggregate supply increases, the natural rate is lowered, which requires a lower labor input in steady state. Hence, land price increases worsen stagnation, though via the indirect effect on aggregate supply.[33]

To summarize, it is the existence of land together with the insatiable nature of the utility from wealth that results in the secular stagnation steady state as the unique equilibrium given the conditions in Proposition 1.8. Stagnation becomes a real phenomenon for $\mu \geq 0$, which in contrast to other stagnation models does not rely on restrictive inflation targets of the monetary authority. It is the possibility of investment in land that prevents the central bank from restoring full employment. As a consequence, an increase in the inflation target, which is a popular policy recommendation of other secular stagnation models, ceases to be effective.

1.5.3 Extension: A Risk Premium on Land

So far, I have assumed that households require the same real returns on all assets in equilibrium. However, land typically is less liquid and subject to more macroeconomic risk as modeled by Caballero and Farhi (Forthcoming) or political risk than bonds and physical capital as argued by von Weizsäcker (2014). Therefore, I assume in this section that households require a real risk premium $\Delta > 0$ for land investments above the risk-free return on bonds or physical capital. This risk premium is exogenous.

It follows that the required return on land is given by $r_t^L = r_t^S + \Delta$. This holds for both the fundamental land price and a land price bubble. Asset market clearing requires the risk-free rate on savings r^S in steady state to satisfy the No-Arbitrage condition

$$r^S = \frac{\dot{q}_t}{q_t} + \frac{z}{q_t} - \Delta = f'(k) - \delta = \begin{cases} \dfrac{\omega'(M^S/P)}{u'(f(k) - \delta k + z)} - \mu \ \text{ if } \ L_t = 1 \ , \\ \\ \gamma \ \text{ if } \ L_t < 1 \ . \end{cases} \tag{1.53}$$

[33] This conclusion is opposite to Kocherlakota (2013), who shows how land price declines worsen stagnation in an overlapping generations model. However, land yields utility rather than income in his setting so that changes in z do not affect consumption directly.

Using the same argument as in Proposition 1.2, the real return on land cannot become negative. The risk-premium then implies that the short-term real rate cannot fall below the premium. Specifically, it has to hold that

$$r^S > -\Delta \ \text{ and } \ k < k_\Delta \ , \tag{1.54}$$

where k_Δ is defined by $f'(k_\Delta) = \delta - \Delta$. It follows that the risk-free rate can indeed become negative once a risk premium on land is included. The return on land can never be a restrictive factor for the return on savings if the risk premium is sufficiently high. Hence, I will assume $0 < \Delta < \delta$. For simplicity, I abstract from the possibility of bubbles and assume a stationary land price in steady state, which is equal to the real rent z capitalized at the required return from land $r^S + \Delta$.[34]

The risk premium affects the feasibility of full employment with both standard wealth preferences and insatiable wealth preferences. Full employment requires the return on savings in (1.53) to match the natural rate of interest. Specifically, it requires

$$f'(k) - \delta = \rho - \frac{v'(\tilde{a})}{u'\left[f(k) - \delta k + z\right]} > \max\{-\mu, -\Delta\} \ , \tag{1.55}$$

where $v(\tilde{a}) = v(a)$ includes the real money stock or is net of money for $v(\tilde{a}) = v(a - m^S)$. The binding constraint on the return on savings results from the risk premium on land or the money growth rate, whichever is lower in absolute terms. Proposition 1.7 is modified as follows.

Proposition 1.9 *In the monetary economy with $\beta = 0$ and $\Delta > 0$, the full employment steady state in (1.55) is the unique equilibrium if the real money stock affects the wealth preference, i.e. if $v(\tilde{a}) = v(a)$. The secular stagnation steady state cannot exist for $v(\tilde{a}) = v(a)$.*

For the case of $v(\tilde{a}) = v(a - m^S)$, the full employment steady state is not feasible if

$$\mu < -r^*_{NC}(\Delta) < \Delta \tag{1.56}$$

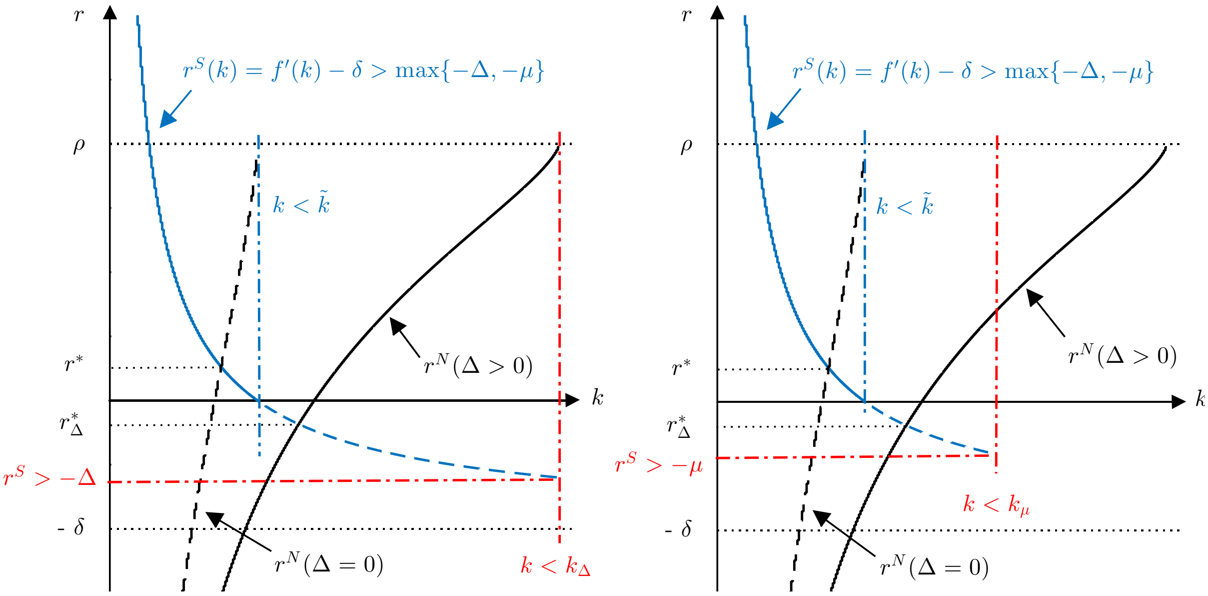
where $r^*_{NC}(\Delta) \in (-\Delta, \rho)$ denotes the steady state real interest rate in the neoclassical model with land and a risk premium $\Delta > 0$. It holds that $r'_{NC}(\Delta) < 0$.

This steady state is unique under mild restrictions and the dynamic system exhibits saddle-point stability similar to the case of $\Delta = 0$ in Proposition 1.7.

The proof is equivalent to the case of $\Delta = 0$ in Appendix A.5 and graphically illustrated in Figure 1.6, which shows the different returns in the presence of a risk premium on land.

[34] The risk premium affects the conditions for rational bubbles. Each bubble has to grow with $r^S + \Delta$ to be feasible. This is not possible with standard wealth preferences due to the transversality condition since $r \to \rho$. With $\beta > 0$, rational bubbles are possible if $\Delta < \beta/u'(c^*)$, where c^* is consumption in steady state.

Figure 1.6: The Steady State of the Model with Risk Premium for $\beta = 0$



Note: This figure shows the natural rate r^N for the model with (solid line) and without risk premium (dotted line) as well as the return on savings r^S as a function of the capital stock k. The left graph assumes an increase in Δ, whereas the right graph shows an increase in μ. The restrictions $r > -\Delta$ and $r > -\mu$ are represented by the dotted vertical and horizontal lines. k_Δ and k_μ are defined by $f'(k_\Delta) - \delta = -\Delta$ and $f'(k_\mu) - \delta = -\mu$.

The introduction of the risk-premium lowers the real return r^*_{NC} in the steady state of the neoclassical model with land. For a given k, a higher risk premium reduces the land price, which lowers household wealth. The marginal utility from wealth increases and the natural interest rate declines. As a consequence, the capital stock is reduced and so is the return on savings, which might become negative as well. This is illustrated in the left graph of Figure 1.6.

Full employment is feasible despite the negative interest rate if the real money stock is part of the wealth preference, i.e. $v(\tilde{a}) = v(a)$, for the same reason as before. Whenever the real rate approaches its lower bound, the associated increases in the land price or the real money stock stimulate the natural rate into positive territory via the wealth preference. Hence, r^N approaches ρ whenever k approaches k_Δ or k_μ in the left graph.

However, if the real money stock does not affect household wealth, the money growth rate can become a binding constraint on the real return on savings for $\Delta > \mu$. Hence, secular stagnation can occur as the result of an increase in the risk premium together with a strict inflation target. This is illustrated in the right graph of Figure 1.6, where Δ increases while $\mu = 0$. As the natural rate declines following the increase in the risk premium, a low money growth rate prevents the return on savings from falling sufficiently. Since the real money stock does not stimulate the natural rate via the wealth preference for $v(\tilde{a}) = v(a - m^S)$, full employment cannot be realized if $\mu < -r^*_{NC}(\Delta > 0)$, which is condition (1.56) in Proposition 1.9.

In this case, full employment can be restored by both an increase in the money growth rate μ or a reduction in the risk premium on land Δ. The first measure directly lowers the return on money and allows the real return on savings to match the lower natural rate. The second measure increases the land price and therefore stimulates the natural rate r^N. Hence, a higher money growth rate and a reduction in the risk premium can restore full employment with standard wealth preferences. Once $\Delta = 0$, full employment is always feasible unless the economy is deflationary and $v(\tilde{a}) = v(a - m^S)$.

Consider now the case of insatiable wealth preferences, which is described in Proposition 1.8 for $\Delta = 0$. With a risk premium on land, the modified version is as follows.

Proposition 1.10 *In the monetary economy with $\beta > 0$ and a risk premium $\Delta > 0$ on land investment, full employment is not feasible if*

$$\begin{aligned} &\beta > (\rho + \Delta)u'\left(c(k_\Delta)\right) && \text{for } \mu \geq \Delta \ , \\ &\beta > (\rho + \mu)u'\left(c(k_\mu)\right) && \text{for } \Delta > \mu \text{ and } v(\tilde{a}) = v(a) \ , \\ &\mu < -r^*_{NC}(\Delta) && \text{for } \Delta > \mu \text{ and } v(\tilde{a}) = v(a - m^S) \ , \end{aligned} \tag{1.57}$$

where k_Δ is given by $f'(k_\Delta) = \delta - \Delta$ and k_μ by $f'(k_\mu) = \delta - \mu$. In addition, there exists a secular stagnation steady state with $0 < L < 1$ if

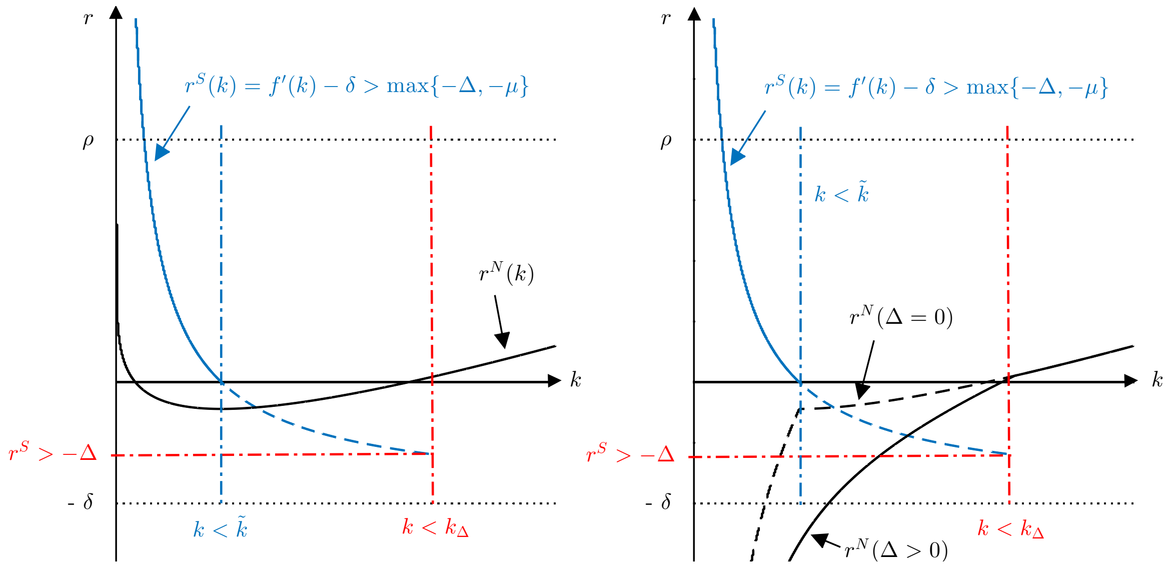
$$\begin{aligned} &\frac{\beta}{u'\left(f(k^*) - \delta k^* + z\right)} > \rho - \gamma > \frac{\beta}{u'(z)} && \text{for } v(\tilde{a}) = v(a) \ , \\ &\frac{v'\left(k^* + \frac{z}{\gamma + \Delta}\right)}{u'\left(f(k^*) - \delta k^* + z\right)} > \rho - \gamma > \frac{v'\left(\frac{z}{\gamma + \Delta}\right)}{u'(z)} && \text{for } \Delta > \mu \text{ and } v(\tilde{a}) = v(a - m^S) \ , \end{aligned} \tag{1.58}$$

where k^* is given by $f'(k^*) = \delta + \gamma$. Moreover, conditions (1.47) and (1.48) of Proposition 1.8 continue to apply. The full employment and secular stagnation steady states are unique under mild restrictions and the dynamic systems satisfy saddle-point stability.

The proof is equivalent to the case of $\Delta = 0$ in Appendix A.6 and graphically illustrated in Figure 1.7. Propositions 1.9 and 1.10 reduce to Propositions 1.7 and 1.8 for $\Delta = 0$.

With a positive lower bound on the marginal utility from wealth, increases in the land price or the real money stock cease to stimulate the natural rate at some point, which remains permanently below the time preference rate and potentially below the return on savings. Then full employment is not feasible and stagnation occurs. For a sufficiently low risk premium, it is the return on land that constitutes a lower bound on the return on savings r^S, while the money growth rate provides the lower bound and stagnation is a monetary phenomenon for $\mu < \Delta$.

Figure 1.7: The Steady State of the Model with Risk Premium for $\beta > 0$



Note: This figure shows the natural rate r^N as well as the return on savings r^S as a function of the capital stock k. In the left graph, $v'(a) = \beta$, while $v(\tilde{a}) = v(a - m^S)$ is shown in the right graph. Both graphs assume an increase in Δ. The vertical and horizontal lines represent the lower bounds on the real rate imposed by land.

If stagnation is a monetary phenomenon, increases in the money growth rate can restore full employment by lowering the return on money. This is the same effect as in the case of $\beta = 0$ and $v(\tilde{a}) = v(a - m^S)$. However, they cease to be effective for $\mu \geq \Delta$ as stagnation becomes a real phenomenon due to the existence of land. To see this, note that condition (1.57) is independent of the money growth rate for $\mu \geq \Delta$.

With insatiable wealth preferences, the effects of a reduction in the risk premium Δ are in stark contrast to the case of standard preferences, where lowering the risk premium helps to restore full employment, similarly to an increase in the inflation target. With insatiable wealth preferences, a reduction in the risk premium makes full employment actually harder to sustain and can result in secular stagnation.

A lower risk premium increases the minimum return on savings r^S. However, the associated increase in the land price ceases to stimulate the natural rate sufficiently to be compatible with full employment. With $v'(a) = \beta$, the natural rate is unaffected by the risk premium, which is illustrated in the left panel of Figure 1.7. The steady state interest rate under full employment is negative, which is only sustainable if the inflation rate μ and the risk premium Δ are sufficiently high. Then, lowering the risk premium can make full employment unfeasible. The same effect holds with $v(\tilde{a}) = v(a - m^S)$ and $v''(a) < 0$, which is illustrated in the right graph. Although a lower risk premium stimulates the natural rate, the effect on the minimum return on savings always dominates and full employment is not feasible for a low risk premium under (1.57).

The risk premium also affects the steady state properties. Under full employment, a reduction in the risk premium reduces the capital stock as it increases the natural rate of interest. As a consequence, consumption falls if the capital stock is below the Golden Rule level or expands if the capital stock is above the Golden Rule level.

In the secular stagnation steady state, the risk premium does not affect the capital stock per worker, which is determined by (1.49).[35] In addition, consumption and output are unaffected for $v'(a) = \beta$ as changes in wealth no longer affect the desire for savings. Then variations in the risk premium simply result in changes in asset prices that do not feed back into the real economy. It is only for the case of $v(\tilde{a}) = v(a - m^S)$ that a lower risk premium increases the realized labor supply and hence consumption under stagnation. However, the reduction in the risk premium can at the same time prevent a return to full employment.

To summarize, the conclusions of the previous sections hold in the general setting with a positive risk premium. With standard wealth preferences, full employment is feasible if the risk premium on land is below the money growth rate or if the real money stock yields utility to households in steady state. If $v(\tilde{a}) = v(a - m^S)$, full employment is not feasible if the risk premium is high, but can be restored by a higher money growth rate.

In contrast, secular stagnation occurs as the unique steady state of the model if wealth preferences are insatiable and the risk premium or the inflation rate is sufficiently low. Then an increase in the risk premium can actually restore full employment, while a higher money growth rate is only effective if money is the restrictive factor. Lowering the risk premium can potentially stimulate consumption under stagnation, but it also makes full employment less sustainable.

1.6 Conclusion

Many advanced economies struggle to maintain full employment despite historically low nominal interest rates. Unconventional monetary policies have narrowed interest rate spreads and contributed to higher prices of stocks, land and other assets, but failed to stimulate household consumption and investment demand sufficiently.

I have presented a theory of secular stagnation in a monetary economy with land that can rationalize these observations. Persistent stagnation can occur in steady state as a consequence of a permanent oversupply of savings. Households are not willing to consume at full employment because holding money or investing in land is more attractive. The return on these assets might be too high for full employment to be feasible.

[35]Note that the model can be extended to include a risk premium on the return on capital as well. Then equation (1.49) is affected by variations in the risk premium on capital. A higher spread decreases the capital stock under stagnation.

In a monetary economy, the rate of inflation constrains the real return from savings and the feasibility of full employment effectively becomes a policy choice. In contrast, the availability of land implies that the real interest rate cannot fall below zero or the risk premium on land.

With standard wealth preferences, the introduction of land substantially weakens the case for secular stagnation as any excess savings are invested in land and the associated increase in the fundamental value of land stimulates household consumption via the wealth preference. Full employment is not feasible for a sufficiently low inflation rate, but only if monetary wealth does not affect the utility from wealth. Then, both an increase in the money growth rate or a reduction in the risk premium on land can restore full employment.

In contrast, the case for secular stagnation becomes stronger with insatiable wealth preferences. Then, increases in wealth at some point cease to stimulate consumption and secular stagnation occurs as the unique equilibrium. It is the very existence of land that accounts for the failure of the economy to operate at full employment since the possibility of land investment prevents the real return on savings from falling sufficiently.

Stagnation in an economy with land is therefore a real phenomenon and cannot always be cured by monetary policy. In particular, higher inflation targets cease to be effective in restoring full employment when spreads are low. In addition, policies to reduce spreads in financial markets might have unintended negative consequence and prevent full employment from being feasible if the risk premium on land becomes too low. Therefore, fiscal policy seems to be more appropriate, particularly with respect to tax policy as discussed in detail in Michau (2017).

Chapter 2

Credit Booms, Debt Overhang and Secular Stagnation*†

Why do economies fall into prolonged periods of economic stagnation, particularly in the aftermath of credit booms? We study the interactions between debt, liquidity and asset prices in a model of persistent demand shortage. We show that financially more deregulated economies are more likely to experience persistent stagnation. Credit booms mask this structural aggregate demand deficiency. However, the resulting debt overhang permanently depresses spending in the long run since deleveraging becomes self-defeating because of deflation. These findings are in line with the macroeconomic developments in Japan during its lost decades and other advanced economies before and during the Great Recession.

The structure of this chapter is as follows. The first section introduces and motivates our research question and gives a broader overview of the chapter. In section 2.2, we present stylized facts on private sector debt and asset prices and discuss our main assumption about liquidity preferences. Section 2.3 introduces a model of economic stagnation, which is analyzed in section 2.4. We discuss the role of leverage for economic stagnation in section 2.5 and some extensions of the model as well as policy recommendations in section 2.6. The final section concludes.

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2.1 Introduction

Many advanced economies suffer from insufficient aggregate demand in the aftermath of the global financial crisis despite unconventional monetary policy actions of unprecedented scales. In addition, the experience of Japan shows that economic stagnation and deflationary tendencies can prevail for decades without any natural recovery. Hence, worries that economies might permanently fail to operate at full employment are widespread and expressed in the "secular stagnation" hypothesis.[1] Proponents of this view emphasize the importance of asset prices, credit availability and private sector debt. This is for at least two reasons.

On the one hand, credit booms or asset price booms are seen as a means to temporarily stimulate a stagnating economy. In particular, Summers (2014a,b) argues that the credit boom in the United States in the early 2000s was masking the underlying lack of aggregate demand by initiating unsustainable consumption spending of households. Similar effects were at play during the stock market boom of the 1990s. Therefore, Summers (2014a) concludes that "the difficulty that has arisen in recent years in achieving adequate growth has been present for a long time but has been masked by unsustainable finances" and "it has been close to 20 years since the American economy grew at a healthy pace supported by sustainable finance".[2]

On the other hand, the resulting indebtedness of the private sector is considered a major impediment to economic recovery because it depresses consumption spending and investment demand. For instance, Eggertsson and Krugman (2012) emphasize the reduction in private demand due to debt overhang during balance sheet recessions.[3] In addition, housing wealth and leverage are main determinants of economic activity in the United States (cf. Mian, Rao, and Sufi, 2013; Mian and Sufi, 2011) and Japan (cf. Ogawa, 2003; Ogawa and Wan, 2007). Credit growth is also a strong indicator for financial crises (cf. Borio and Lowe, 2002; Shin, 2013). These crises are associated with substantially higher output losses than normal recessions despite forceful monetary policy actions because of the decoupling of monetary aggregates and the volume of credit (cf. Jordà, Schularick, and Taylor, 2015; Schularick and Taylor, 2012).

It follows that interactions of asset prices, credit availability and private sector debt are important factors for the emergence and the severity of economic stagnation. But how does an economy fall into stagnation and what is the role of these factors?

[1]The term "secular stagnation" itself goes back to Hansen (1938, 1939) and was taken up by Larry Summers (2013). Yet, Keynes (1936) in Chapter 17 of the General Theory already argues that permanent demand shortage can exist as a steady state phenomenon in a monetary economy.

[2]A related argument is made by Krugman (2013): "In other words, you can argue that our economy has been trying to get into the liquidity trap for a number of years, and that it only avoided the trap for a while thanks to successive bubbles."

[3]Other theoretical treatments of deleveraging shocks at the zero lower bound include Eggertsson, Mehrotra, and Robbins (2017) among others.

We develop a stylized dynamic macroeconomic model to analyze the interactions between asset prices, leverage and economic stagnation. The model features three types of assets and two types of infinitely-lived households: Borrowers obtain funds from savers, but their borrowing ability is limited by the value of collateral that is endogenously determined in the housing market following Iacoviello (2005). Households gain utility from consumption, housing services and real money balances. The last follows Sidrauski (1967) and reflects, among other things, the demand for liquidity.

We follow the research line initiated by Ono (1994, 2001) and assume insatiable liquidity preferences: The marginal utility of money stays strictly positive even for very large money holdings, which prevents consumption of the saver from increasing as potential output rises. This in turn creates stagnation if consumption of the borrower is sufficiently restricted as is the case when the economy suffers from debt overhang. Hence, economies with a higher leverage are more prone to suffering from insufficient aggregate demand.

Our setting implies that asset price or credit booms can temporarily stimulate an economy that would otherwise suffer from demand deficiency. A credit or asset price boom, which is triggered by financial liberalization, enables borrowers to temporarily increase their consumption spending, stimulating aggregate demand and inflation. In addition, housing demand is stimulated and the real house price increases, thereby reinforcing the initial credit boom as the value of collateral increases. However, in the new steady state, borrowers' consumption is depressed by higher real interest payments to savers. Savers however do not increase their consumption accordingly as they prefer to hoard money because of strong liquidity preferences. As a consequence, aggregate demand falls permanently short of potential output and the economy experiences persistent deflation.

In this steady state, borrowers do actually delever but these efforts are self-defeating due to deflation. Their real debt burden remains unaffected and permanently depresses spending so that the economy does not recover. Monetary policy becomes ineffective since injections of liquidity are held as cash by savers and do not stimulate spending.[4] Hence, there is a temptation for policymakers to stimulate sluggish growth by initiating lending booms that come at the cost of greater damage in the long run.

These findings are in line with the macroeconomic developments in Japan before and during its lost decades. They also help to shed light on the situation of other advanced economies before and during the Great Recession.

[4] This is consistent with the apparent ineffectiveness of the Bank of Japan's unconventional monetary policy actions in the late 1990s and early 2000s in stimulating inflation and output despite improving financial market indicators (see Ueda, 2012b; Ugai, 2007).

2.2 Money, Credit and Stagnation

Which empirical patterns should a model of secular stagnation be able to replicate? A look at the transition of Japan from a high growth to a stagnating economy allows us to derive stylized facts on asset prices, credit, money and economic stagnation. We also show that recent developments in other advanced economies are reminiscent of the Japanese situation. In addition, we argue that our assumption of insatiable liquidity preferences is able to capture these patterns and relate our approach to the existing literature.

Stylized Facts: Consider the macroeconomic development of Japan over the last three decades as illustrated in Figure 2.1. The transformation from a high growth to a stagnating economy is apparent in panels (a) and (b) which show real GDP growth and inflation: From 1980 to 1991, the Japanese economy grew at an average rate of 4.4% in real terms with an annual inflation rate of 1.9%. In contrast, real GDP grew at only 0.9% on average in the period since 1992 with inflation falling into negative territory.[5]

A distinguishing feature of the high growth and the stagnation period is the behavior of asset prices and credit. As shown in panels (c) and (d), Japan experienced a credit and asset price boom during its high growth period. Within ten years, the outstanding amount of private credit to the non-financial sector relative to GDP increased from 1.4 in 1980 to 2.2 in 1992 primarily driven by bank lending to small and medium-sized corporations and declining lending standards (see Posen, 2003). Credit to the private sector grew on average by 7.9% in real terms during the period from 1980 to 1991 while residential property prices (as a proxy for collateralizeable assets) increased by 5.1% in real terms.[6] Credit expansion and asset price inflation in terms of stock, land and housing prices were at the core of Japan's Bubble Economy.[7]

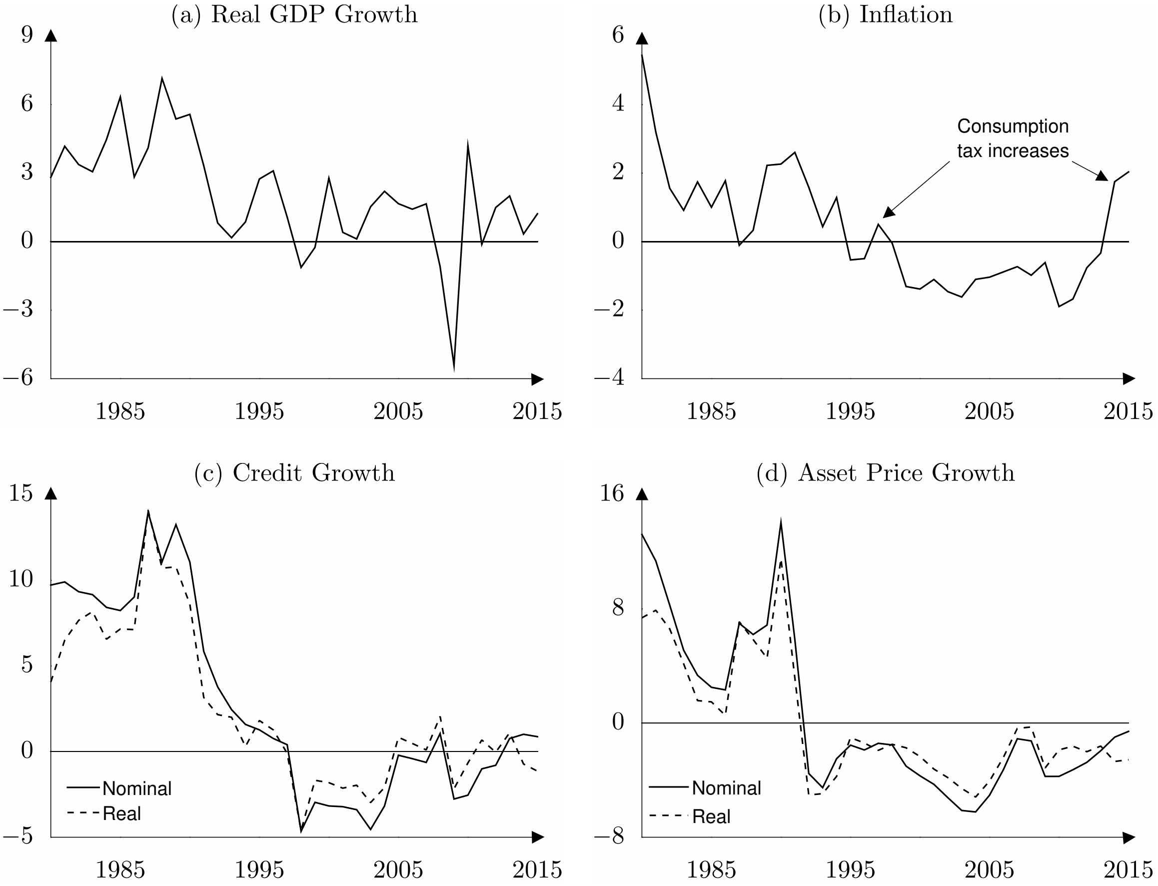
In contrast, asset prices declined and the private sector disencumbered in the stagnation period following the asset price crash of the early 1990s: Credit to the private sector declined by 0.8% on average each year after 1991 while the real amount of credit stagnated. Credit as

[5] We measure inflation by the GDP Deflator. The pattern is the same for CPI inflation at 2.6% (1980-1991) and 0.25% (1992-2015). The tendency is the same when we exclude the financial crisis episode since 2008. Then real growth is slightly higher at 1.2% (1992-2007) but still substantially below the pre-1992 average. Similar developments hold for other measures of economic activity, like real consumption expenditure growth, which declined from 4.0% to 1.0%. Note that the recent increase in inflation in panel (b), as well as the spike in 1997, can be explained by increases in the consumption tax in April 1997 and 2014. Apart from the tax effect, there is no indication of a persistent increase in inflation.

[6] Property price increases were higher for commercial property (6.0%) and in the six major cities (12.1%). Similarly, the subsequent decline was stronger for commercial property (-5.6%) and in cities (-4.8%). All housing price data comes from the Bank for International Settlement's (BIS) "Long series on nominal residential property prices" database; Sources: National sources, BIS Residential Property Price database, www.bis.org/statistics/pp.htm

[7] For further discussion of the Japanese experience and the similarities with the recent developments in the United States, see Tsuruta (1999), Shimizu and Watanabe (2010), Ueda (2012a), among others.

Figure 2.1: Macroeconomic Developments in Japan, 1980-2015

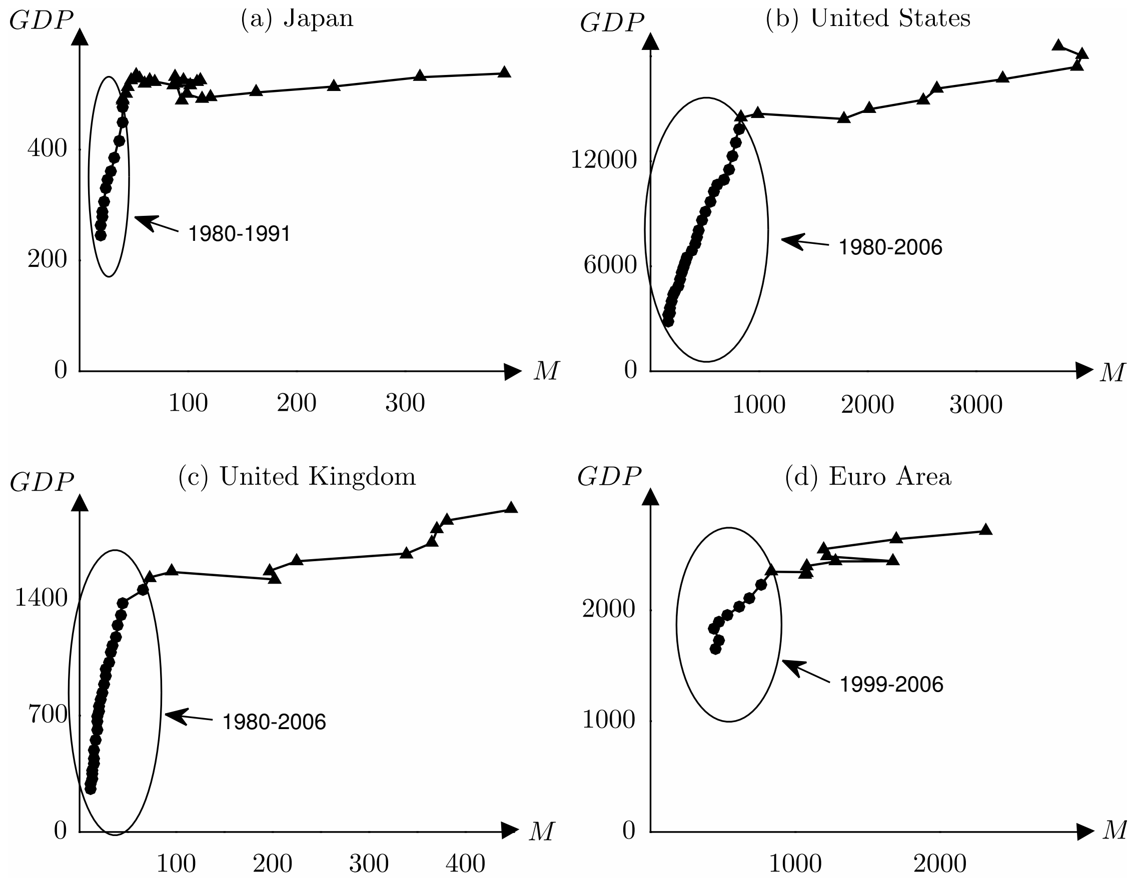


Note: Data on real GDP and inflation is from the World Bank. Inflation is measured by the GDP Deflator, in percent. Credit measures credit to the private non-financial sector and asset prices refer to the residential property price index. Both series are from the Bank for International Settlements (Sources: National sources, BIS Residential Property Price database, www.bis.org/statistics/pp.htm). Real variables are deflated with the GDP Deflator.

a share of GDP declined by almost 20% from 2.2 in 1992 to a level of 1.6 in 2015. At the same time, nominal property prices decreased substantially by 3.1% per year on average, while price decreases were somewhat smaller in real terms due to deflation. The decline in asset prices continued throughout the stagnation period without any indication of a sustained recovery.

In addition, monetary policy became ineffective in stimulating output after 1991. Panel (a) of Figure 2.2 illustrates the relationship between base money and nominal GDP since 1980 for Japan. The neoclassical quantity equation seems to fit the data quite well during the 1980s as increases in the monetary base were associated with an increase in nominal spending. Yet, there clearly is a structural break in the early 1990s associated with the transition to economic stagnation. Subsequent substantial increases in the money supply - particularly during quantitative easing in the early 2000s and in the context of "Abenomics" - did not translate into higher nominal spending but simply resulted in a decline in the circulation velocity of money.

Figure 2.2: Breakdown of the Quantity Equation

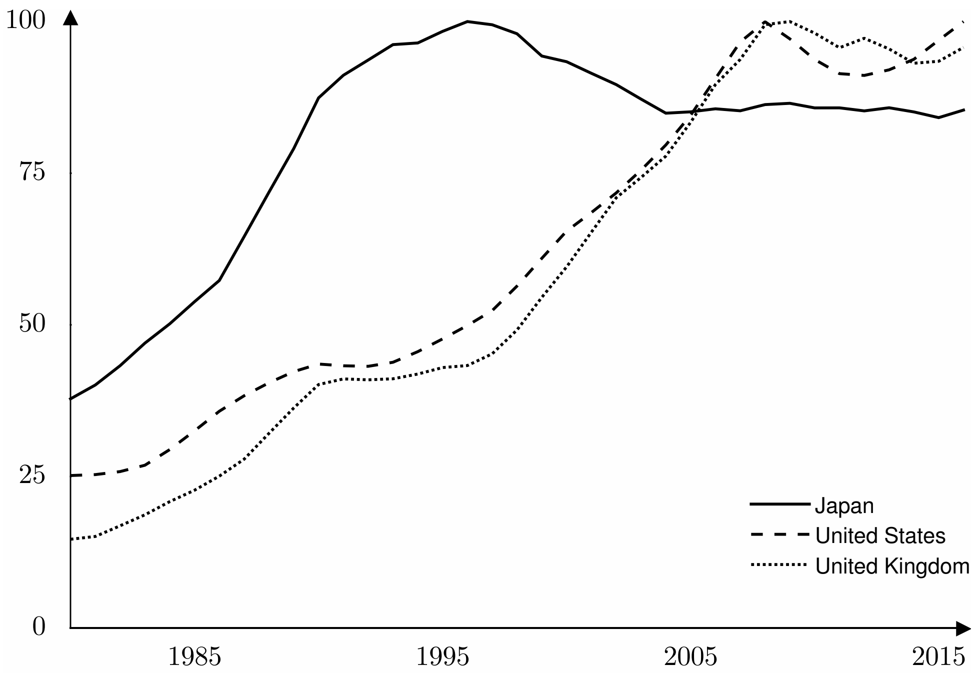


Note: This figure shows the monetary base and nominal GDP for each country. Units are trillion of Yen (Japan), billion of US-Dollar (United States), billion of Pound Sterling (United Kingdom) and billion of Euro (ECB). Data is from the Federal Reserve Bank of St. Louis (FRED) Database (US and UK), the Bank of Japan and the World Bank (Japan), Eurostat and the ECB. Data on the Euro Area includes other liquidity providing operations by national central banks.

Many advanced countries have experienced similar developments during the Great Recession. Panels (b) to (d) in Figure 2.2 show that the formerly stable relationship between base money and nominal spending substantially changed in the United States, the United Kingdom and the Euro Area as expansions in the money supply ceased to stimulate nominal spending. This structural change is associated with a prolonged period of depressed spending because of persistent debt overhang in the aftermath of a credit boom. The extent of the credit expansion and the subsequent debt overhang are reminiscent of Japan's experience as illustrated in Figure 2.3.

To sum up, we observe persistent economic stagnation despite unprecedented expansions of the monetary base. The emergence of stagnation is associated with the end of a credit boom that results in substantial debt overhang of the private sector. In the following sections, we present a dynamic macroeconomic model of aggregate demand shortage that theoretically explains these phenomena.

Figure 2.3: Credit Booms and Debt Overhang



Note: This figure shows real credit to the private non-financial sector. Data is from the Bank for International Settlements (BIS). Nominal credit is deflated by the GDP Deflator and normalized to 100 in the year of its peak: 1996 (Japan), 2008 (US) and 2009 (UK).

Related Literature: We contribute to the literature on persistent aggregate demand shortage based on the insatiability of liquidity or wealth preferences. This literature was initiated by Ono (1994, 2001) and substantially extended and microfounded by Ono and Ishida (2014). The main modification in these models is the idea that the marginal utility from holding real money balances has a strictly positive lower bound:

$$\lim_{m \to \infty} v'(m) = \beta > 0$$

As a consequence, increases in money holdings or wealth at some point cease to stimulate consumption spending as agents prefer to hoard money or wealth instead.

The idea of a causal relationship between aggregate demand shortage and the insatiability of liquidity preferences goes back as far as Chapter 17 in Keynes (1936) as described and analyzed in detail by Ono (2001). Moreover, Murota and Ono (2011) provide an explanation of this feature based on behavioral economics. Specifically, they show that this property can be linked to relative status preferences with respect to money. From an empirical point of view, Ono, Ogawa, and Yoshida (2004) offer support for the insatiability of liquidity preferences based on quarterly data in Japan using parametric and non-parametric methods.

Ono (1994) introduces heterogeneous households into this framework and redistributive policies are analyzed by Matsuzaki (2003) for consumption taxes and Hashimoto (2004) for intergenerational transfers.[8] In these models, financial markets are assumed to be perfect and agents are heterogeneous only with respect to their initial wealth.

We extend this framework to feature financial market imperfections and introduce heterogeneity in time preference rates to motivate borrowing. This allows us to analyze interactions of collateral, asset prices and aggregate demand. We implement these via a borrowing constraint in the spirit of Kiyotaki and Moore (1997) and Iacoviello (2005) such that the value of collateral is endogenously determined in the housing market.[9]

Research interest in models of secular stagnation has increased substantially in the aftermath of the financial crisis. Michaillat and Saez (2014) and Michau (2017) develop similar models of stagnation that build upon a constant marginal utility of wealth. Eggertsson, Mehrotra, and Robbins (2017) analyze stagnation and the effects of deleveraging in an OLG framework. Eggertsson, Mehrotra, and Summers (2016) extend this setup to the open economy. In addition, some recent contributions analyze the effects of (the burst of) asset price bubbles on economic growth (cf. Biswas, Hanson, and Phan, 2017; Boullot, 2017; Hanson and Phan, 2017).

Our work is linked to the idea of balance sheet recessions. In our setup, households do continuously pay off debt. Yet, deflation makes these efforts self-defeating: As contracts are in nominal terms, deflation increases the real value of debt. The real debt burden remains unaffected and continues to impede the recovery. Balance sheets are not restored, even in the long run. Deleveraging shocks are also discussed in a borrower-saver framework by Eggertsson and Krugman (2012). However, the authors do not model persistent but only temporary stagnation.

Our approach differs from the liquidity trap literature.[10] This view explains aggregate demand shortages as the consequence of negative shocks in combination with a lower bound on the nominal interest rate. Stagnation is a temporary phenomenon. This is in stark contrast to the experience of Japan where deflationary forces already prevail for more than two decades. It is difficult to make the case for the prevalence of price rigidities over such a long period. In our model, stagnation occurs in steady state despite the possibility of continuous price adjustment.

[8] Further extensions include Ono (2006, 2014) and Hashimoto and Johdo (2009) who model persistent stagnation in a two-country framework to analyze the role of FDI and international spillovers of various policies as well as Rodríguez-Arana (2007) who analyzes fiscal deficits under stagnation. Moreover, Murota and Ono (2012) explain zero nominal interest rates and excess reserve holdings of commercial banks in a setup with preferences for deposit holdings. In addition, Ono (2015) applies this framework to the situation of Japan to explain the transition from high growth to secular stagnation based solely on the insatiability of wealth preferences.

[9] We do not explicitly model the asymmetric information problems that give rise to the financial friction, but refer to Townsend (1979) or Stiglitz and Weiss (1981) for a microfoundation.

[10] The modern treatment of this problem started with the seminal paper of Krugman (1998), which initiated an extensive literature, particularly since the global financial crisis.

2.3 The Model Economy

We use a continuous time model with money-in-the-utility that features competitive firms, two types of households and a central bank but abstracts from taxation or government expenditures. Agents have perfect foresight and there is no uncertainty in the model. We build on Ono (1994, 2001) for the idea of permanent demand shortage based on insatiable liquidity preferences and Iacoviello (2005) for modeling endogenous borrowing constraints with durable assets as collateral to introduce private sector debt.

2.3.1 Firms

The supply side is modeled as a Lucas tree. Firms are price takers and produce the amount $\bar{y}$ of the consumption good without any inputs or costs. This constitutes the economy's production capacity or a measure of potential output. Yet, actual sales are determined by aggregate demand C_t so that actual income y_t falls short of potential output in case of aggregate demand shortage. Firm sales are hence given by

$$y_t = \min\{C_t, \bar{y}\} \ . \tag{2.1}$$

Nominal firm profits simply equal $P_t y_t$ as production is costless. P_t denotes the price level. These are distributed equally across households and show up as exogenous income in the budget constraints. When falling short of potential output $\bar{y}$, aggregate demand determines firm profits and household income. As a consequence, there are feedback loops between spending and income.

In addition, we abstract from the labor market and the wage-setting process and instead introduce a reduced-form Phillips curve for the inflation rate π_t. Specifically, the price level dynamics under full employment differ from those in the presence of aggregate demand shortage as follows:

$$\pi_t = \frac{\dot{P}_t}{P_t} = \begin{cases} \mu \ \text{ if } \ C_t = \bar{y} \ , \\ \alpha\left(\dfrac{C_t}{\bar{y}} - 1\right) \ \text{ if } \ C_t < \bar{y} \ . \end{cases} \tag{2.2}$$

Under full employment, the dynamics of the price level are similar to the standard MIU framework. The price level adjusts to clear the money market and the inflation rate is determined by the growth rate of the money supply μ, such that the quantity equation holds and money is neutral. The central bank can perfectly control inflation in this case. In contrast, the output gap determines inflation in case of aggregate demand shortage, where the parameter $\alpha > 0$ governs the speed of price adjustment. A negative output gap will result in deflation. If the output gap persists in steady state, deflation will persist.

Similar relations are derived in standard macroeconomic models with a labor market based on downward nominal wage rigidity.[11] Specifically, Ono and Ishida (2014) and Ono (2015) provide the following microfoundation for equation (2.2) based on fairness concerns in the wage setting process:[12] In their model, the productivity of workers depends on their perception of being treated in a fair way. In particular, workers withhold effort when they are not remunerated at least with a "fair wage". Under full employment, competition among firms for workers determines the wage offer. Therefore, the dynamics of the price level determine the wage dynamics. The former are in turn dependent on the money supply growth. In contrast, firms have bargaining power when there is unemployment. However, the fair wage provides a lower bound on wage offers to prevent shirking. As a consequence, it is the dynamics of the fair wage that determine the wage and hence the price dynamics. These are in turn related to the level of unemployment or the output gap. Taken together, inflation is governed by an expression similar to equation (2.2), where α can be interpreted as the (exogenous) job separation rate faced by workers.

In addition, the Phillips curve in equation (2.2) is formally equivalent to wage setting frictions as in Eggertsson, Mehrotra, and Robbins (2017), Michau (2017) or Schmitt-Grohé and Uribe (2016, 2017). All of these contributions introduce some form of downward nominal wage rigidity that becomes binding in case of unemployment. Eggertsson, Mehrotra, and Robbins (2017) assume that wages cannot fall below a "wage norm", which is a combination of past wages and the marginal product of labor. In Michau (2017), wage demands of workers are guided by a reference rate of inflation, which creates an asymmetry in the wage dynamics. Finally, Schmitt-Grohé and Uribe (2016, 2017) introduce an exogenous lower bound on the growth rate of the nominal wage that becomes binding in case of unemployment. The same mechanism is used by Hanson and Phan (2017) and Biswas, Hanson, and Phan (2017).

It is worth pointing out that our conclusions on the role of asset prices and private sector debt for economic stagnation continue to hold in the presence of a richer modeling of the labor market. Specifically, the introduction of a production function and wage setting frictions in the spirit of Ono and Ishida (2014) does not alter our results qualitatively but comes at the cost of computational complexity. It is for this reason that we decided to rely on the reduced-form expression for inflation introduced above.

[11] As argued by Schmitt-Grohé and Uribe (2016): "There is abundant empirical evidence on downward nominal wage rigidity stemming mostly from developed countries." An overview of the empirical evidence is presented in section 8 of their paper.

[12] In these models, the representative household has a fixed labor endowment. In equilibrium, competitive firms make zero profits, which is why the real wage equals labor productivity, which is constant due to the linear production function. Firm sales $P_t y_t$ then are not a lump-sum transfer of profits but equal labor wages and the deflation gap is related to the labor market instead of the commodity market. Yet, the implications for the emergence of stagnation are only modestly affected.

2.3.2 Households

There is a mass one of infinitely-lived households. Each household is one of two types based on his time preference rate ρ_i: A fraction n of households are savers ($i = 1$) whereas the remaining fraction $1 - n$ are borrowers ($i = 2$) in the sense that $\rho_1 < \rho_2$.[13] This setting will endogenously result in differences in wealth levels and we will hence model an economy in which the "rich" (savers) lend to the "poor" (borrowers).[14]

Households have three means of savings: money $M_{i,t}$, credit contracts $B_{i,t}$ and real assets in the form of housing $h_{i,t}$. Money yields an interest rate of $R_M = 0$ whereas loans are contracted at the non-negative nominal interest rate $R_t > 0$. Let $B_{i,t} > 0$ denote savings in the form of loans issued and $B_{i,t} < 0$ debt in the form of credit. Let Q_t denote the nominal house price in period t. The return on housing depends on the resale value of the house in the following period and therefore can include potential capital gains or losses.

Then total nominal wealth $A_{i,t}$ is given by the sum of the household's money holdings, bond holdings and the value of its housing investment: $A_{i,t} = B_{i,t} + M_{i,t} + Q_t h_{i,t}$. In real terms, wealth is given by

$$a_{i,t} = b_{i,t} + m_{i,t} + q_t h_{i,t} \ , \tag{2.3}$$

where lowercase letters denote the respective variables in real terms such that q_t denotes the real house price defined as $Q_t = P_t q_t$. Households are the owners of firms and receive firm profits $P_t y_t$, where y_t is defined in (2.1). These profits are distributed equally across both types and considered exogenous by the households. In addition, households receive all income from seignorage in a lump-sum transfer $Z_{i,t}$. For the moment, this transfer is not important. Later, we will assume that $\mu = 0$ and hence $Z_{i,t} = 0$. Yet, it becomes relevant for the discussion of $\mu > 0$ in section 2.6. In real terms, the flow of funds constraint is given by[15]

$$\dot{a}_{i,t} = r_t a_{i,t} - R_t m_{i,t} - (r_t q_t - \dot{q}_t) h_{i,t} - c_{i,t} + y_t + z_{i,t} \ . \tag{2.4}$$

[13] The borrower-saver separation based on differences in time preference rates is a standard method to introduce borrowing incentives in macroeconomic models, see Sufi (2012). Since these differences are permanent, the roles of lenders and borrowers are static. Alternative ways of modeling include idiosyncratic income shocks or an uneven life-cycle income distribution.

[14] Alternatively, we could assume that agents differ in their initial wealth $a_{i,0}$ such that $a_{1,0} >> a_{2,0}$. We will use the terms "savers" and "rich households" interchangeably throughout the analysis as well as the terms "borrowers" and "poor households".

[15] Equation (2.4) is based on the following expressions for the evolution of nominal and real wealth where we use the composition of household assets to substitute for B_t:

$$\dot{A}_t = R_t B_t + \dot{Q}_t h_t - P_t c_t + P_t y_t = R_t A_t - R_t M_t - R_t Q_t h_t + \dot{Q}_t h_t - P_t c_t + P_t y_t + Z_{i,t}$$

$$\dot{Q}_t = P_t \dot{q}_t + q_t \dot{P}_t$$

$$\dot{a}_t = \left(\frac{\dot{A}_t}{P_t} \right) = \frac{\dot{A}_t}{P_t} - \frac{A_t}{P_t} \frac{\dot{P}_t}{P_t} = (R_t - \pi_t) a_t - R_t m_t - (R_t q_t - \pi_t q_t - \dot{q}_t) h_t - c_t + y_t + z_{i,t}$$

The household incurs opportunity costs when holding money because of the foregone interest income that would be associated with lending. Similar costs arise when investing in housing. Yet, housing investment involves the possibility of capital gains (or losses) associated with changes in the real house price captured by $\dot{q}_t$.

Impatient households have a strong motive to borrow. However, lenders require sufficient collateral in the form of housing because of problems of asymmetric information in the credit market. As a consequence, savers will only lend up to a fraction θ of the value of the borrower's collateralizeable assets.[16] In real terms, the associated borrowing constraint takes the form:

$$b_{2,t} \geq -\theta q_t h_{2,t} \ . \tag{2.5}$$

Housing is the only durable asset that serves as collateral. In contrast, money is not collateralizable because it is too fungible to be effectively seized by lenders in case of missed repayment.[17]

Apart from differences in time preference, households have identical preferences. They choose consumption, real money holdings and housing to maximize their lifetime utility function

$$U_0 = \int_0^\infty \left[u(c_{i,t}) + v(m_{i,t}) + w(h_{i,t})\right] e^{-\rho_i t} dt \ , \tag{2.6}$$

where $\rho_i > 0$ denotes the subjective discount rate of the household of type i. Utility from consumption and housing satisfies the Inada conditions. For simplicity, we make the following functional form assumptions on these instantaneous utility functions:

$$u(c_{i,t}) = ln(c_{i,t}) \ ; \quad w(h_{i,t}) = \gamma ln(h_{i,t}) \ ,$$

where $\gamma > 0$ is a positive constant. In contrast, the Inada conditions do not hold for the utility from real money balances. As discussed in the previous section and following Ono (2001), we deviate from the neoclassical assumptions and introduce insatiable liquidity preferences. Formally, the marginal utility of real money holdings does not converge to zero but approaches a strictly positive constant value:

$$\lim_{m \to \infty} v'(m) = \beta > 0 \ .$$

We will explain the consequences of this assumption in the following sections.

[16]We refer to the parameter θ as the loan-to-value ratio. Throughout this paper, we choose parameters to ensure that the borrowing constraint is always binding.

[17]It is easy to introduce a collateral value for money. Then, (2.5) becomes $b_{2,t} \geq -\theta_1 q_t h_{2,t} - \theta_2 m_{2,t}$, where θ_2 determines the collateralizability of money. For $\theta_1 = \theta_2 = \theta$, this formulation, together with (2.3), implies that (2.5) becomes a pure wealth constraint where the composition of wealth is irrelevant, i.e. $b_{2,t} \geq -\theta(1-\theta)^{-1} a_{2,t}$. Our main results are unchanged (and even stronger) when using this formulation.

Rich Households (Savers): Savers maximize lifetime utility (2.6) subject to the wealth composition (2.3) and the flow budget constraint (2.4). From the Hamiltonian function:

$$H_1 = u(c_{1,t}) + v(m_{1,t}) + w(h_{1,t}) + \lambda_{1,t}(r_t a_{1,t} - c_{1,t} - R_t m_{1,t} - (r_t q_t - \dot{q}_t)h_{1,t} + y_t) \ ,$$

we obtain the optimality conditions

$$\frac{1}{c_{1,t}} = \lambda_{1,t} \ , \tag{2.7}$$

$$\lambda_{1,t} R_t = v'(m_{1,t}) \ , \tag{2.8}$$

$$\frac{\gamma}{h_{1,t}} = \lambda_{1,t}(r_t q_t - \dot{q}_t) \ , \tag{2.9}$$

$$\dot{\lambda}_{1,t} = (\rho_1 - r_t)\lambda_{1,t} \ , \tag{2.10}$$

$$\lim_{t\to\infty} \lambda_{1,t} a_{1,t} e^{-\rho_1 t} = 0 \ . \tag{2.11}$$

Equations (2.7) to (2.11) describe optimal consumption, money demand, housing investment and borrowing of the rich agent as well as the transversality condition for real wealth. For the saver, the nominal interest rate R_t governs both the intertemporal allocation of consumption via (2.7) and (2.10) as well as the intra-temporal trade-off between money and consumption according to (2.7) and (2.8). This yields the following expression:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} + \rho_1 + \pi_t = R_t = v'(m_{1,t})c_{1,t} \ . \tag{2.12}$$

In optimum, the saver equates the marginal rate of substitution between present and future consumption to the marginal rate of substitution between present consumption and money holdings, i.e. the liquidity premium, which also equals the nominal interest rate that constitutes the opportunity cost of holding money. Under neoclassical assumptions, the liquidity premium is declining in $m_{1,t}$, all else equal, thereby stimulating consumption or decreasing the interest rate.

In contrast, with insatiable liquidity preferences, the marginal utility of real money holdings will reach the positive lower bound if wealth of the patient households is sufficiently high, i.e. $v'(m_1) = \beta$. Then the liquidity premium no longer declines with additional money holdings and $R_t = R_t(c_{1,t})$. As a consequence, consumption of the rich household is unaffected by changes in his money holdings for a given nominal interest rate. For that reason monetary policy becomes ineffective in single agent models such as Ono (2001): Additional money is stored as cash and does no longer stimulate consumption. The economy is trapped in a deflationary steady state despite an infinite expansion of the real money stock.

Poor Households (Borrowers): Borrowers maximize lifetime utility (2.6) subject to the wealth composition (2.3), the flow budget constraint (2.4) and the borrowing constraint (2.5). Therefore, their Hamiltonian function is given by:

$$H_2 = u(c_{2,t}) + v(m_{2,t}) + w(h_{2,t}) + \lambda_{2,t}(r_t a_{2,t} - c_{2,t} - R_t m_{2,t} - (r_t q_t - \dot{q}_t)h_{2,t} + y_t)$$

$$+\varphi_t(a_{2,t} - m_{2,t} - (1-\theta)q_t h_{2,t}) \ ,$$

from which the following optimality conditions are obtained:

$$\frac{1}{c_{2,t}} = \lambda_{2,t} \ , \tag{2.13}$$

$$\lambda_{2,t} R_t + \varphi_t = v'(m_{2,t}) \ , \tag{2.14}$$

$$\frac{\gamma}{h_{2,t}} = \lambda_{2,t}(r_t q_t - \dot{q}_t) + \varphi_t(1-\theta)q_t \ , \tag{2.15}$$

$$\dot{\lambda}_{2,t} = (\rho_2 - r_t)\lambda_{2,t} - \varphi_t \ , \tag{2.16}$$

$$\lim_{t\to\infty} \lambda_{2,t} a_{2,t} e^{-\rho_1 t} = 0 \ . \tag{2.17}$$

Equations (2.13) to (2.17) describe optimal consumption, money demand, housing investment and borrowing of the poor agent as well as the transversality condition. The borrower also equates the marginal rate of substitution between present and future consumption to the liquidity premium. This results from (2.13), (2.14) and (2.16) and gives

$$\frac{\dot{c}_{2,t}}{c_{2,t}} + \rho_2 + \pi_t = R_t + \varphi_t c_{2,t} = v'(m_{2,t})c_{2,t} \ . \tag{2.18}$$

The borrowing friction affects optimal money demand and the evolution of consumption. Impatience creates a strong motive to borrow funds for current consumption so that current funds have a higher value to the borrowers than to the savers. When these funds are used to increase liquidity instead of consumption, the household incurs an implicit cost of φ_t due to the borrowing constraint facing in fact a higher implicit interest rate than the saver. As a consequence, optimal money demand is reduced relative to the case without borrowing frictions.

Under neoclassical assumptions, the liquidity premium decreases with money holdings for the borrower, i.e. $v''(m_2) < 0$. In contrast, with insatiable liquidity preferences, $v'(m_2) = \beta > 0$ if the borrower becomes sufficiently wealthy. As a consequence, our model features different regions depending on the behavior of $v'(m_1)$ and $v'(m_2)$.

2.3.3 Asset Prices, Borrowing and Leverage

What determines the dynamics of the real house price? Households incur opportunity costs when investing in housing because of the opportunity loss of real interest income that is associated with bond savings. Yet, agents gain utility from housing which is captured by the user cost, i.e. the marginal rate of substitution between consumption and housing. For the saver, this follows from (2.7) and (2.9). For the borrower, housing investment comes at a higher cost, his implicit interest rate being higher. Yet, since housing serves as collateral, the associated borrowing costs are lower than those for money at $(1-\theta)\varphi_t$ which can be seen in (2.13) and (2.15).

Moreover, changes in the real house price affect the costs and benefits of housing due to valuation effects. In optimum, the real house price adjusts such that agents are indifferent between investing in an additional unit of housing and alternative uses. Hence, the real house price has to appreciate if the opportunity costs from housing exceed the user costs to compensate housing investors for the higher costs with capital gains. Similarly, the real house price has to depreciate if the benefits of housing exceed the opportunity costs resulting in capital losses for house owners. From equations (2.7), (2.9), (2.13) and (2.15), the dynamics of the real house price can be expressed as the difference between opportunity costs and housing benefits for both agents as

$$\dot{q}_t = r_t q_t - \frac{\gamma c_{1,t}}{h_{1,t}} = r_t q_t - \frac{\gamma c_{2,t}}{h_{2,t}} + \varphi_t(1-\theta)q_t c_{2,t} \ . \tag{2.19}$$

Throughout the analysis, we consider the case of a strictly binding borrowing constraint, i.e. $\varphi_t > 0$. Then, the borrower always takes on loans up to the maximum given by (2.5). It follows from (2.3) and (2.5) that total real assets of the borrower consist of his money holdings and housing investment, a fraction θ of which serves as collateral:

$$a_{2,t} = m_{2,t} + (1-\theta)q_t h_{2,t} \ . \tag{2.20}$$

Similarly, assets of the saver include loans to the borrower. From (2.18) and (2.19), the consumption value of borrowing $\varphi_t c_{2,t}$ - or equivalently, the consumption cost of debt-financed money holdings or housing investment - equals the difference in the liquidity premia and is proportional to the difference in the user cost of housing:

$$v'(m_{2,t})c_{2,t} - v'(m_{1,t})c_{1,t} = \varphi_t c_{2,t} = \frac{1}{1-\theta}\left(\frac{\gamma c_{2,t}}{q_t h_{2,t}} - \frac{\gamma c_{1,t}}{q_t h_{1,t}}\right) \ . \tag{2.21}$$

Hence, a binding borrowing constraint implies that it is more costly (in terms of consumption) for the borrower to hold money or invest in housing than for the saver.

2.3.4 Market Equilibrium Conditions

Aggregate demand C_t consists of the consumption demand of savers and borrowers as

$$C_t = nc_{1,t} + (1-n)c_{2,t} \ . \tag{2.22}$$

Aggregate demand relative to potential output determines the output gap which in turn is related to inflation via (2.2). In addition, aggregate demand determines firm profits and household income y_t in the flow budget constraints (2.4).

The central bank perfectly controls the nominal money supply M_t which grows at an exogenous rate μ. Hence, the real money supply m_t evolves as

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t \ . \tag{2.23}$$

In contrast, the nominal interest rate R_t is determined endogenously in the money market. It is related to inflation and the real interest rate via the Fisher Equation

$$R_t = r_t + \pi_t \ . \tag{2.24}$$

Money market clearing requires that the real money demand of savers and borrowers equals the real money supply m_t:

$$m_t = \frac{M_t}{P_t} = nm_{1,t} + (1-n)m_{2,t} \ . \tag{2.25}$$

Loans are financial claims among households. Hence, they are in zero net supply with

$$nb_{1,t} + (1-n)b_{2,t} = 0 \ . \tag{2.26}$$

In contrast, housing is a real asset. Following Iacoviello (2005), we assume a fixed supply of houses H and abstract from depreciation and construction.[18] Market clearing in the housing market then requires

$$nh_{1,t} + (1-n)h_{2,t} = H \ . \tag{2.27}$$

Equations (2.1) to (2.27) fully describe the model economy. The dynamics are summarized by a system of differential equations given by (2.4), (2.12), (2.18), (2.19) and (2.23) where (2.4) applies to both types. All other variables are derived from the solution to this system.

[18] This assumption seems reasonable for an economy like Japan that is characterized by land scarcity and a low price elasticity of the housing supply. A study by Shimizu and Watanabe (2010) concludes that the housing supply was very price inelastic during the Japanese housing boom of the late 1980s, partly due to the incentives given by the tax system as well as regulations on land utilization.

2.4 Analysis of the Model Economy

In the following analysis, we focus on the special case of a constant nominal money supply,

$$\mu = 0,$$

for simplicity. This implies a zero inflation rate under full employment as is evident from (2.2).[19] Yet, the qualitative conclusions of our analysis can be generalized and hold for various levels of μ as we will discuss in section 2.6 in greater detail.

Our model framework features three regions depending on the behavior of the marginal utilities $v'(m_1)$ and $v'(m_2)$. This is in turn related to the production capacity $\bar{y}$:

1. For low levels of potential output $\bar{y}$, the economy behaves as in the standard *neoclassical case*. The marginal utility of money is decreasing in money holdings for both households, i.e. $v''(m_i) < 0$, and aggregate demand equals potential output. The price level is constant and changes proportionally with the money supply.

2. For higher levels of $\bar{y}$, there is an *asymmetric* steady state under stagnation. In this region, the patient household's marginal utility of money is constant while the impatient household's liquidity premium still declines with additional money holdings, i.e. $v''(m_1) = 0$ and $v''(m_2) < 0$. Aggregate demand falls short of potential output and deflation occurs.

3. For very high levels of potential output, the *symmetric* steady state under stagnation might occur. In this region, the marginal utility of money has reached its lower bound for savers and borrowers, i.e. $v''(m_i) = 0$.

Among the three mentioned above, we focus on the asymmetric steady state under stagnation for several reasons. First, this steady state features economic stagnation unlike the neoclassical case. Secondly, indebtedness and asset prices play an important role in affecting the severity of stagnation.[20] Thirdly, it is more in conformity with what has occurred in the Japanese economy as discussed in the introduction. The asymmetric steady state is defined as follows:

> *Asymmetric Steady State*: The real and nominal interest rates are constant, the price level is declining at a constant rate, the real consumption level of each household is constant as is the real house price, and the borrower's asset level is constant while the saver's wealth expands infinitely:

$$\dot{r} = 0 \ , \ \dot{R} = 0 \ , \ \pi < 0 \ , \ \dot{c}_1 = 0 \ , \ \dot{c}_2 = 0 \ , \ \dot{q} = 0 \ , \ \dot{a}_1 > 0 \ , \ \dot{a}_2 = 0 \ . \tag{2.28}$$

[19] This parameterization also allows us to derive some expressions in closed-form that do not depend on $v(m)$.

[20] In contrast, changes in leverage cease to influence aggregate demand in the symmetric steady state and simply affect asset prices and the distribution of the housing stock.

2.4.1 The Occurrence of Persistent Stagnation

Intuitively, aggregate demand shortage occurs if potential output is so high that households are no longer willing to consume the available amount of $\bar{y}$ due to the insatiability of liquidity preferences of the saver.[21] For lower levels of potential output, the economy attains full employment at zero inflation (as $\mu = 0$) and the price level adjusts to clear the money market for a given nominal money supply in (2.25). We define the full employment steady state as follows:

> *Neoclassical Equilibrium*: The real and nominal interest rates are constant, the price level is constant, the real house price is constant and the consumption and wealth levels of all households are constant:
>
> $$\dot{r} = 0 \ , \ \dot{R} = 0 \ , \ \pi = 0 \ , \ \dot{c}_1 = 0 \ , \ \dot{c}_2 = 0 \ , \ \dot{q} = 0 \ , \ \dot{a}_1 = 0 \ , \ \dot{a}_2 = 0 \ . \tag{2.29}$$

Consider first the case of homogeneous agents: Suppose there are only patient households. From (2.12) with $\dot{c}_1 = 0$, the economy attains full employment, i.e. $c_1 = \bar{y}$, and zero inflation as long as the marginal utility of money can adjust such that $v'(m_{1,t})\bar{y} = \rho_1$. With insatiable liquidity preferences, there is a lower bound β of the marginal utility of real money holdings. Once the production capacity $\bar{y}$ exceeds the level of $\rho_1\beta^{-1}$, there is no longer a solution to (2.12) that is compatible with $\pi = 0$ and $c_1 = \bar{y}$. This is because households are no longer willing to consume the available output but prefer to accumulate money instead. As a consequence, stagnation and deflation occur in equilibrium.

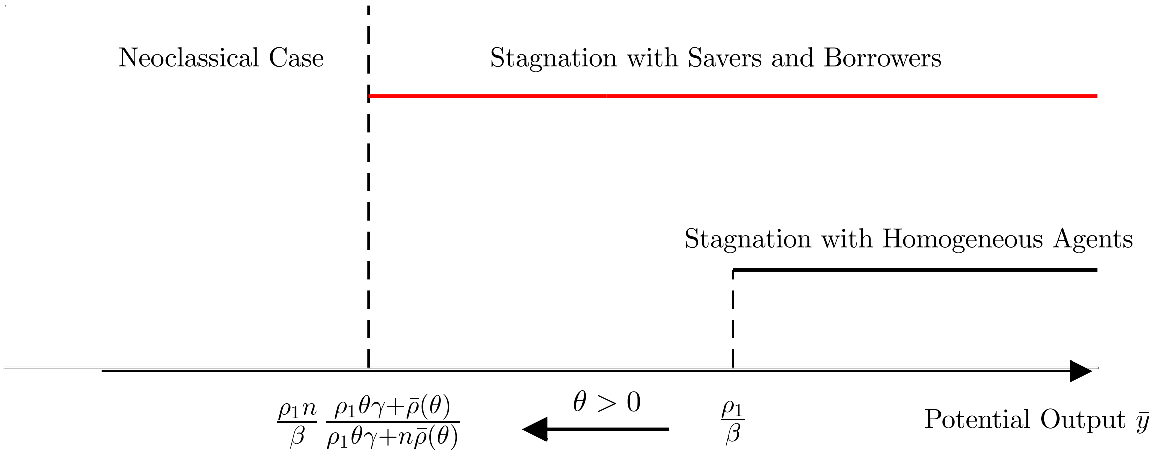
Similarly, suppose there were only impatient households (and $\varphi_t = 0$ in this case). From (2.18) with $\dot{c}_2 = 0$, the economy attains full employment as long as higher spending can be accommodated at zero inflation such that $v'(m_2)\bar{y} = \rho_2$. There is no solution to (2.18) consistent with full employment once $\bar{y}$ is above $\rho_2\beta^{-1}$. Taken together, the relevant condition for homogeneous agent models is given by $\bar{y} > \rho_i\beta^{-1}$ where ρ_i refers to the representative household. This condition is illustrated by the lower line in Figure 2.4.

In an economy with n savers and $1 - n$ borrowers, the distribution of spending under full employment determines the occurrence of stagnation. Since $\pi = 0$, we have from (2.12) and (2.24) that $R = r = \rho_1$. Then, consumption levels are derived from the budget constraints (2.4). The borrower consumes his income net of interest payments on debt. Income in turn depends on aggregate demand which equals potential output. His consumption in the neoclassical steady state is given by

$$c_2^{NC} = \frac{\bar{\rho}_\theta}{\bar{\rho}_\theta + \theta\rho_1\gamma}\bar{y} \ , \quad \text{where} \ \ \bar{\rho}_\theta \equiv \theta\rho_1 + (1-\theta)\rho_2 \ . \tag{2.30}$$

[21] In addition, the liquidity premium of the borrower must still decline with additional money holdings for asymmetric (instead of symmetric) stagnation to occur.

Figure 2.4: Occurrence of Persistent Stagnation



Note: This figure shows the equilibrium of the model for different values of $\bar{y}$ compared to homogeneous agent models. In particular, note that in the heterogeneous agent framework economic stagnation occurs for smaller levels of potential output.

Note that $\bar{\rho}_\theta$ is the debt-weighted average discount rate. This follows from (2.4), (2.20) and the requirements $\pi = 0$, $\dot{q} = 0$ and $\dot{a}_2 = 0$. The saver behaves similarly, but receives interest income on its lending. Hence, steady state consumption of the saver exceeds consumption of the borrower in the neoclassical steady state due to the redistribution associated with ownership of financial assets:

$$c_1^{NC} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta + n\theta\rho_1\gamma}\bar{y} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta} c_2^{NC} > c_2^{NC} \; . \tag{2.31}$$

It is easy to see from these expressions that aggregate demand equals potential output. Yet, it follows from (2.12) and (2.18) with $\dot{c}_i = 0$ that the consumption levels of both agents in (2.30) and (2.31) are consistent with zero inflation only if the marginal utility of money falls sufficiently. In particular, for the neoclassical case to exist it has to hold that

$$v'(m_{1,t}) = \frac{\rho_1}{c_1^{NC}} \quad \text{and} \quad v'(m_{2,t}) = \frac{\rho_2}{c_2^{NC}} \; . \tag{2.32}$$

With insatiable liquidity preferences, there exists a lower bound on the marginal utility of money such that $v'(m_{i,t}) \geq \beta$. Hence, the neoclassical case is not feasible once $\beta c_i^{NC} > \rho_i$. So with rising levels of potential output $\bar{y}$, at some threshold the stagnation steady state will occur. Since $\rho_1 < \rho_2$ and $c_1^{NC} > c_2^{NC}$, it is the saver's marginal utility of money that will reach its lower bound first for rising levels of $\bar{y}$. Then, aggregate demand falls short of the production capacity and the economy enters the stagnation steady state. We derive the following proposition from combining (2.31) and (2.32):

Proposition 2.1 *The neoclassical equilibrium with full employment and zero inflation cannot be attained once potential output exceeds the following threshold:*

$$\widetilde{y} \equiv \frac{\rho_1 n}{\beta} \frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} < \frac{\rho_1}{\beta} \ . \tag{2.33}$$

The threshold $\widetilde{y}$ *is affected by the model parameters as follows (see Appendix B.1):*

$$\frac{\partial\widetilde{y}}{\partial\beta} < 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\rho_1} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\rho_2} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\theta} < 0 \ , \quad \frac{\partial\widetilde{y}}{\partial n} > 0 \ , \quad \frac{\partial\widetilde{y}}{\partial\gamma} < 0 \ .$$

Once potential output exceeds $\widetilde{y}$, the economy is in the asymmetric steady state under stagnation defined in (2.28) and suffers from insufficient demand and deflation. Additional income does no longer stimulate consumption of the saver who chooses to accumulate wealth instead. This is represented by the upper line in Figure 2.4. The lower the insatiability parameter β, the higher potential output needs to be for the economy to enter stagnation. Similarly, increases in the time preference rate of the saver ρ_1 or in their fraction of the population n also increase the income threshold. The same holds for a higher time preference rate ρ_2 of the borrower.

What we add is the insight that financially more developed countries, i.e. countries with higher leverage, drift into stagnation already at a lower level of potential output. This is because the higher debt is associated with lower steady state consumption demand from the borrower. To see this, note from (2.30) that if financial markets are closed and no borrowing is possible, i.e. if $\theta = 0$ or $\gamma = 0$, the consumption levels of both households are equal and given by $c_i = \bar{y}$ under full employment. Once we allow for borrowing, housing investment is associated with an increase in indebtedness of the borrower. This in turn results in a higher real interest burden on poor households and reduces their affordable consumption. This gives rise to a more unequal income distribution but does not affect aggregate demand as long as the rich households expand their consumption accordingly. If they invest in liquidity holdings instead, aggregate demand falls short of the economy's production capacity and stagnation occurs.[22]

Let us contrast this condition with the existence condition in single-agent models without lending and borrowing as in Ono (2001), which was discussed above. Condition (2.33) is reduced to this expression if we abstract from housing ($\gamma = 0$), if we do not allow for borrowing ($\theta = 0$) or if there are only rich households ($n = 1$). In all other cases, $\widetilde{y}$ is below the threshold of the single-agent model. Hence, the economy enters stagnation in an earlier stage, which is illustrated in Figure 2.4. The reason is that consumption of the saver is higher due to additional income associated with interest payments on loans.

[22]The same effect arises for a higher value of γ. Economies that invest more heavily in assets in fixed supply are hence more prone to stagnation. Also note that the effects of θ and γ are mutually reinforcing.

2.4.2 The Asymmetric Steady State under Persistent Stagnation

Under asymmetric stagnation, higher consumption spending of the borrower stimulates consumption spending of the saver. This follows from (2.12) with $\dot{c}_{1,t} = 0$ and (2.2) to substitute for π_t. The reasoning is as follows: An increase in consumption of the borrower expands aggregate demand and mitigates deflation. Less deflation in turn increases the nominal interest rate via (2.24) since the real rate equals the saver's time preference rate ρ_1 as obtained from (2.10) with $\dot{\lambda}_{1,t} = 0$. The nominal rate has to equal the marginal rate of substitution between consumption and money holdings for the saver and since the marginal utility of money is constant due to insatiability, consumption c_1^* has to increase. This is the same relation as in Ono (1994) and Matsuzaki (2003) and results from the insatiability of liquidity preferences in combination with sluggish price adjustment as manifested in the Phillips curve relation in (2.2):[23]

$$c_1^* = \frac{(\rho_1 - \alpha)\bar{y}}{\beta\bar{y} - \alpha n} + \frac{\alpha(1-n)}{\beta\bar{y} - \alpha n} c_2^* \; . \tag{2.34}$$

Spillovers from aggregate demand are stronger the higher the share of spending constrained households $(1-n)$ and the higher the speed of price adjustment α. In particular, steady state consumption c_1^* is not directly affected by the borrowing decision or asset composition of the impatient household. Yet, there are indirect effects via c_2^*.

In contrast, consumption spending of the borrower is affected by his money holdings. This follows from optimality condition (2.18) with $\dot{c}_{2,t} = 0$ and (2.2) and (2.34) to substitute for π^* and c_1^* and implies

$$c_2^* = \frac{\chi}{v'(m_2^*)(\beta\bar{y} - \alpha n) - \beta\alpha(1-n)} \; , \tag{2.35}$$

$$\text{where } \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) \; .$$

More money induces more consumption of the borrower. This is necessary to equalize the liquidity premium to the nominal interest rate which is itself a function of c_2^* via (2.12) and (2.34). Note that this channel does not exist for the saver who accumulates money without expanding consumption.

Importantly, we require parameter restrictions to guarantee positive consumption levels c_1^* and c_2^* in steady state. From (2.34) and the denominator of (2.35), since $v'(m_2^*) > \beta$, we make the following assumptions throughout this paper:

$$\text{(i) } \rho_1 > \alpha \quad \text{and} \quad \text{(ii) } \beta\bar{y} > \alpha \; . \tag{2.36}$$

[23] Steady state values of the respective variables will be marked with a * for notational convenience.

In addition, the shadow value of borrowing is determined by the difference in discount rates. From (2.12) and (2.18) with $\dot{c}_{1,t} = \dot{c}_{2,t} = 0$, the Lagrange parameter on the borrowing constraint is given by

$$\varphi^* = \frac{\rho_2 - \rho_1}{c_2^*} > 0 \ . \tag{2.37}$$

Hence, the borrowing constraint is binding in steady state. Higher money holdings of the borrower reduce the value of additional funds since c_2^* increases in m_2^* as discussed above.

Total wealth of the borrower consists of money and housing investment net of loans via (2.20). The steady state value of housing of each agent is proportional to his consumption and given by

$$q^* h_1^* = \frac{\gamma}{\rho_1} c_1^* \ \text{ and } \ q^* h_2^* = \frac{\gamma}{\bar{\rho}_\theta} c_2^* \ , \quad \text{where } \ \bar{\rho}_\theta \equiv \theta\rho_1 + (1-\theta)\rho_2 \ . \tag{2.38}$$

This follows from (2.16) with $\dot{\lambda}_{2,t} = 0$ and from (2.19). Increases in θ provide higher incentives for borrowers to invest in housing because of its role as collateral. Note that (2.5) and (2.38) imply that the real level of debt is constant. Hence, borrowers do delever in nominal terms, which however is self-defeating due to deflation. Substituting (2.35) solved for m_2^* and (2.38) into (2.20) implies the following expression for the real wealth of the borrower in steady state:

$$a_2^* = \frac{(1-\theta)\gamma}{\bar{\rho}_\theta} c_2^* + m_2^* \ , \tag{2.39}$$

where m_2^* is a function of c_2^* given by (2.35). It follows that real wealth and consumption demand are positively related for the borrower. An increase in consumption induces both higher money holdings and higher housing investment, and hence higher real wealth.

Higher consumption demand implies higher housing demand, because they are substitutes. This is true for both types of households. Market clearing then requires a higher house price in response to an increase in aggregate demand. This follows from condition (2.27) in combination with steady state housing demands. The real house price, obtained from (2.27) and (2.38), is given by

$$q^* = \frac{\gamma}{H} \left[\frac{n}{\rho_1} c_1^* + \frac{1-n}{\bar{\rho}_\theta} c_2^* \right] \ . \tag{2.40}$$

From (2.34), $c_1^* = c_1^*(c_2^*)$ and hence $q^* = q^*(c_2^*)$. An increase in the borrower's consumption increases the real house price in steady state. Note that q^* increases in γ and decreases in H, all else equal. The borrower's real assets are constant in the asymmetric steady state. From the budget constraint (2.4) with $\dot{a}_{2,t} = 0$ and (2.20), (2.22) and (2.38), we get

$$n c_1^* + (1-n) c_2^* + (\rho_1 - \beta c_1^*) m_2^* = \left(\frac{\theta \rho_1 \gamma}{\bar{\rho}_\theta} + 1 \right) c_2^* \ , \tag{2.41}$$

where $c_1^* = c_1^*(c_2^*)$ from (2.34) and $m_2^* = m_2^*(c_2^*)$ from (2.35). The borrower obtains real income from two sources: First, households receive firm profits which are determined by aggregate demand. This is reflected in the term $nc_1^* + (1-n)c_2^*$ of (2.41). Secondly, inflation affects the real return on money. Since money does not pay interest, the real return is given by the rate of *deflation*, i.e. $-\pi^* = \rho_1 - \beta c_1^*$. This income is used to finance consumption expenditures c_2^* and to pay interest on debt. These interest payments depend on the household's borrowing capacity which is determined by the value of housing collateral via (2.5). The collateral value in turn is related to consumption via (2.38). In steady state, real interest payments are a fraction $\theta\rho_1\gamma\bar{\rho}_\theta^{-1}$ of consumption and increase with θ, γ and ρ_1 but decrease with ρ_2, because $\bar{\rho}_\theta = \theta\rho_1 + (1-\theta)\rho_2$.

Finally, it follows from (2.25) that the real money stock becomes infinitely high because of the effects of deflation. The increase in the real money supply exclusively benefits the saver in the asymmetric steady state. Hence, his real wealth expands at the rate of deflation. However, this does not violate the transversality condition (2.13) since it holds that $\rho_1 > \alpha > -\pi^*$. In contrast, increases in the real money stock accrue to both agents in the symmetric steady state.

Equations (2.34) to (2.41) define the asymmetric steady state under persistent stagnation. Once m_2^*, c_1^* and c_2^* are jointly determined from the combination of (2.34), (2.35) and (2.41), all other variables are derived from these values.

Model Dynamics under Asymmetric Stagnation: The model dynamics are represented by a system of six differential equations for consumption and real wealth of savers and borrowers, the real house price and the real money supply. All other variables can be derived from this system: From (2.2) and (2.22), it follows that $\pi_t = \pi(c_{1,t}, c_{2,t})$. From (2.12) with $v'(m_{1,t}) = \beta$, we have $R_t = R(c_{1,t})$ and hence $r_t = r(c_{1,t}, c_{2,t})$ from (2.24). Given $c_{1,t}, c_{2,t}, q_t$ and $a_{2,t}$, equations (2.20), (2.21) with $v'(m_{1,t}) = \beta$ and (2.27) determine $m_{2,t}$, $h_{1,t}$ and $h_{2,t}$. From (2.21), it follows that the implicit cost due to the borrowing constraint, φ_t, is a function of the same four variables.

The evolution of the saver's consumption is determined by (2.12) with $v'(m_{1,t}) = \beta$:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \beta c_{1,t} - \pi_t - \rho_1 \ . \tag{2.42}$$

Since $\pi_t = \pi(c_{1,t}, c_{2,t})$, it follows that $\dot{c}_{1,t} = \dot{c}_1(c_{1,t}, c_{2,t})$. Similarly, the evolution of the borrower's consumption is determined by (2.18) with $v'(m_{2,t}) > \beta$:

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = v'(m_{2,t})c_{2,t} - \pi_t - \rho_2 \ . \tag{2.43}$$

Since $\pi_t = \pi(c_{1,t}, c_{2,t})$ and $m_{2,t} = m_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, we have $\dot{c}_{2,t} = \dot{c}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$. The evolution of the real house price is determined by (2.19):

$$\frac{\dot{q}_t}{q_t} = r_t - \frac{\gamma c_{1,t}}{q_t h_{1,t}} \ . \tag{2.44}$$

With $r_t = r(c_{1,t}, c_{2,t})$ and $h_{1,t} = h_1(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, we have $\dot{q}_t = \dot{q}(c_{1,t}, c_{2,t}, q_t, a_{2,t})$. The evolution of the real wealth of both agents is determined by (2.4) where we use (2.19) to substitute for $\dot{q}_t - r_t q_t$, (2.18) for φ_t in (2.19) and (2.22) for y_t:

$$\dot{a}_{1,t} = -\pi_t a_{1,t} + \beta c_{1,t}\left[\theta n^{-1} q_t H + (1-\theta) q_t h_{1,t}\right] - (1-n+\gamma)c_{1,t} + (1-n)c_{2,t} \ , \tag{2.45}$$

$$\dot{a}_{2,t} = -\pi_t a_{2,t} + v'(m_{2,t})c_{2,t}(1-\theta)q_t h_{2,t} - (n+\gamma)c_{2,t} + n c_{1,t} \ . \tag{2.46}$$

It is easy to see that $\dot{a}_{1,t} = \dot{a}_1(c_{1,t}, c_{2,t}, q_t, a_{1,t}, a_{2,t})$ and $\dot{a}_{2,t} = \dot{a}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$. Finally, the real money supply decreases with the inflation rate as is clear from (2.25) with $\mu = 0$:

$$\frac{\dot{m}_t}{m_1} = -\pi_t \ . \tag{2.47}$$

Therefore, it holds that $\dot{m}_t = \dot{m}(c_{1,t}, c_{2,t}, m_t)$. Equations (2.42) to (2.47) fully describe the economy together with the initial asset levels $a_{1,0}$ and $a_{2,0}$. Given paths for these variables, we can derive the associated paths of all other variables. This system satisfies saddle-point stability around the asymmetric steady state as shown in Appendix B.2. In the next section, we analyze the dynamic and static properties of this steady state.

2.5 Asset Prices and Leverage Under Stagnation

As shown above, private sector debt affects the occurrence of persistent stagnation. In addition, leverage and asset prices also affect aggregate demand under stagnation. Specifically, we show that credit booms can temporarily mask aggregate demand insufficiency. However, this comes at the cost of more severe stagnation in the new steady state.

2.5.1 Credit and Asset Price Booms under Stagnation

We have argued that an economy can enter an equilibrium of persistent stagnation as a consequence of the debt burden of some households. However, an expansion of debt via financial liberalization can in the short run mask aggregate demand shortage by creating a temporary credit and asset price boom. Specifically, consider an economy that is suffering from insufficient aggregate demand. Suppose that lending standards loosen such that borrowers can take on more loans per unit of housing net worth. This setup is in line with the claims of Larry Summers about the U.S. economy during the early 2000s and also mirrors several features of the situation of Japan in the late 1980s that we described in the introduction.[24]

Figure 2.5 shows the associated model dynamics as deviations from the initial steady state for two values of the housing preference parameter. The increase in the loan-to-value ratio triggers a substantial credit boom. Borrowers can acquire new funds for a given collateral value some of which they consume and some of which they hold as money or invest in new housing. These funds are provided by savers and financed by their money holdings and the sale of houses. What follows is a temporary boom in both the real economy and the housing market.

The credit boom stimulates aggregate demand as borrowers increase their consumption. This creates inflation which lowers the real interest rate and stimulates consumption of the savers as well. As a consequence, the nominal interest rate increases. If the credit boom is sufficiently strong, the economy can temporarily return to full employment with aggregate spending being constrained by potential output.

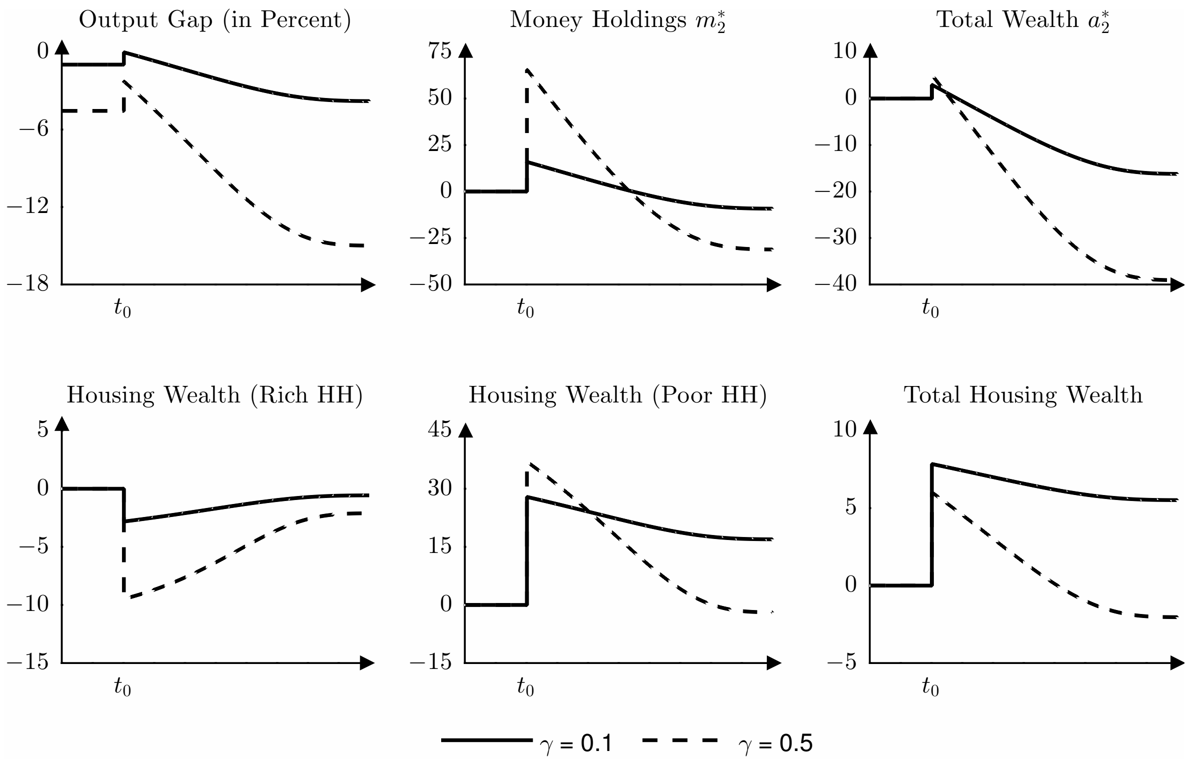
In addition, an asset price boom ensues since the real house price surges as housing demand of impatient households increases. The initial jump in the house price has a positive valuation effect on the housing holdings of both agents, which increases their real wealth. A feedback loop sets in with higher house prices increasing the collateral value of borrowers which in turn enhances their borrowing ability thereby reinforcing the initial credit boom. The housing allocation shifts in favor of the impatient households, which further strengthens the value of their collateral.[25]

The allocation of new funds among consumption, money and housing investment is guided by the parameters in the utility function of the borrower. Higher impatience implies a stronger increase in consumption and hence aggregate demand and inflation. In contrast, higher preferences for housing imply that more of the newly available funds are spent on purchasing fixed supply assets. As a consequence, higher preferences for housing imply a stronger amplification of the dynamics because of more pronounced collateral effects. In fact, aggregate consumption might actually fall during the credit boom for very high values of γ. Figure 2.5 illustrates the dependency of the dynamic responses on γ.

[24]Note that we proxy the credit boom by variations in θ but do not make explicit claims about the origin of this variation. The sources of the Japanese credit boom are still up to debate. Yet, Posen (2003) argues that both partial deregulation in corporate finance and a relaxation of lending standards in the mortgage market with mortgage limits rising from 65% of the home value on average to 100% played a major role for the Japanese credit boom. According to Posen (2003), "there is a consensus view among economists on how partial financial deregulation in Japan in the 1980s led to a lending boom". The effects of deregulation and financial liberalization are also well-documented in Tsuruta (1999).

[25]This is the same propagation mechanism as described in detail in Kiyotaki and Moore (1997) and Iacoviello (2005) among others, which creates amplification and persistence of shocks. Note that we only consider the case of a binding borrowing constraint. For the formal analysis of occasionally binding constraints, see Guerrieri and Iacoviello (2017).

Figure 2.5: Dynamic Effects of Credit Booms under Stagnation



Note: This figure shows the dynamics associated with a permanent increase in the loan-to-value ratio from $\theta = 0.15$ to $\theta = 0.5$. The output gap is given in percentage points. All other variables are depicted as deviations from the *initial* steady state in percent. We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta ln(m_{2,t})$. The figure is based on the following calibration: $\beta = 0.0005, y = 100, \rho_1 = 0.05, \rho_2 = 0.1, \alpha = 0.01, n = 0.5, H = 1$ and $\delta = 0.1$. Simulations are based on a modification of the relaxation algorithm of Trimborn, Koch, and Steger (2008).

Over time, some of the newly acquired assets are sold by the borrower to smooth its consumption and to make interest payments. Therefore, the allocation of the housing stock reverts in favor of the saver and aggregate demand remains above its new equilibrium for a prolonged period, thereby masking the underlying demand deficiency. Yet, eventually the resulting debt overhang pushes the economy into persistent stagnation, which is worse than before the credit boom, as we will show in the next subsection.

Also note that a house price boom, which is typically modeled by an increase in γ (see for example Iacoviello, 2005), can temporarily stimulate the stagnating economy though at the cost of more severe stagnation in the long run. The argument is similar: Higher housing demand creates an immediate increase in the real house price resulting in valuation gains for both households. In addition, the value of collateral that borrowers can pledge for funds increases, which initiates a credit boom. These funds are used to increase consumption, money holdings and housing investment, the last of which feeds back into the value of borrowers' collateral. The dynamics are similar to those in Figure 2.5.

2.5.2 Debt Overhang and Stagnation

While increases in θ and γ can temporarily stimulate aggregate demand by initiating a credit boom, they also affect the steady state properties. The former represents financial liberalization - or the degree of sustainable finance - whereas the latter is a proxy for the level of asset prices. Higher leverage θ and a higher house price worsen economic stagnation in steady state. This is summarized in the following proposition (see Appendix B.3 for the proof):

Proposition 2.2 *In the asymmetric steady state under stagnation, an increase in the loan-to-value ratio reduces aggregate demand and worsens deflation. It holds that*

$$\frac{dC^*}{d\theta} < 0 \ , \quad \frac{dc_1^*}{d\theta} < 0 \ , \quad \frac{dc_2^*}{d\theta} < 0 \ , \quad \frac{dm_2^*}{d\theta} < 0 \ , \quad \frac{da_2^*}{d\theta} < 0 \ , \quad \frac{d\pi^*}{d\theta} < 0 \ .$$

The same effects arise from an increase in the housing preference γ. It holds that

$$\frac{dC^*}{d\gamma} < 0 \ , \quad \frac{dc_1^*}{d\gamma} < 0 \ , \quad \frac{dc_2^*}{d\gamma} < 0 \ , \quad \frac{dm_2^*}{d\gamma} < 0 \ , \quad \frac{da_2^*}{d\gamma} < 0 \ , \quad \frac{d\pi^*}{d\gamma} < 0 \ .$$

Consider intuitively the effects of an increase in θ. Initially, the borrowing constraint (2.5) is relaxed allowing the borrower to acquire new funds, as described above. However, the new steady state is associated with higher debt and hence higher real interest payments as the real interest rate is not affected. These payments are a fraction $\theta\rho_1\gamma\bar{\rho}_\theta^{-1}$ of the borrower's consumption spending where $\bar{\rho}_\theta$ is decreasing in θ. This can be seen from (2.41). Therefore, higher leverage is associated with higher interest costs per unit of consumption which implies that the borrower's income is not sufficient to cover expenditures for a given c_2^* once θ increases. As this would violate the lifetime budget constraint of the borrower, his consumption spending has to decline.

This implies that the expenditures of the borrower are reduced ("spending effect"), raising disposable income. However, the lower spending reduces the borrower's income since aggregate demand declines ("demand effect"). This partially offsets the first effect. In addition, the real return on money holdings is affected ("capital gains effect"): Higher deflation increases the return on money. Yet, lower consumption discourages money holdings. The first effect is stronger, the higher money holdings, but the net effect is always negative (see Appendix B.2). This implies that a decrease in consumption reduces expenditures. Hence, consumption must decline in response to an increase in θ. These effects can be seen from the total differential of (2.41):

$$\underbrace{\left(n\frac{dc_1^*}{dc_2^*} + 1 - n\right)\frac{dc_2^*}{d\theta}}_{\text{Demand Effect}} + \underbrace{\left(-\pi^*\frac{dm_2^*}{dc_2^*} - m_2^*\frac{d\pi^*}{dc_2^*}\right)\frac{dc_2^*}{d\theta}}_{\text{Capital Gains Effect}} - \underbrace{\left(1 + \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\right)\frac{dc_2^*}{d\theta}}_{\text{Spending Effect}} = \underbrace{\frac{\rho_1\rho_2\gamma}{\bar{\rho}_\theta^2}c_2^*}_{\text{Interest Cost}} \ . \tag{2.48}$$

The decrease in consumption of the borrower feeds back into the other variables of the model. Aggregate demand decreases, which aggravates deflation via (2.2). Deflation in turn reduces the nominal interest rate via (2.24) since the real rate is determined by ρ_1. This reduces consumption of the saver, which can be seen from (2.34). In addition, money demand of the borrower declines, as is clear from (2.35), as does the borrower's real wealth in (2.39).

The effects of a higher θ on the real house price and the distribution of the housing stock are ambiguous because of two opposing effects on q^*: On the one hand, investment in housing becomes more attractive for a given level of consumption c_2^* since housing becomes more collateralizeable. Higher housing demand bids up the house price. On the other hand, there is an indirect effect on the house price because lower consumption spending decreases housing demand of both agents which in turn lowers the real house price. This can be seen from (2.40).[26]

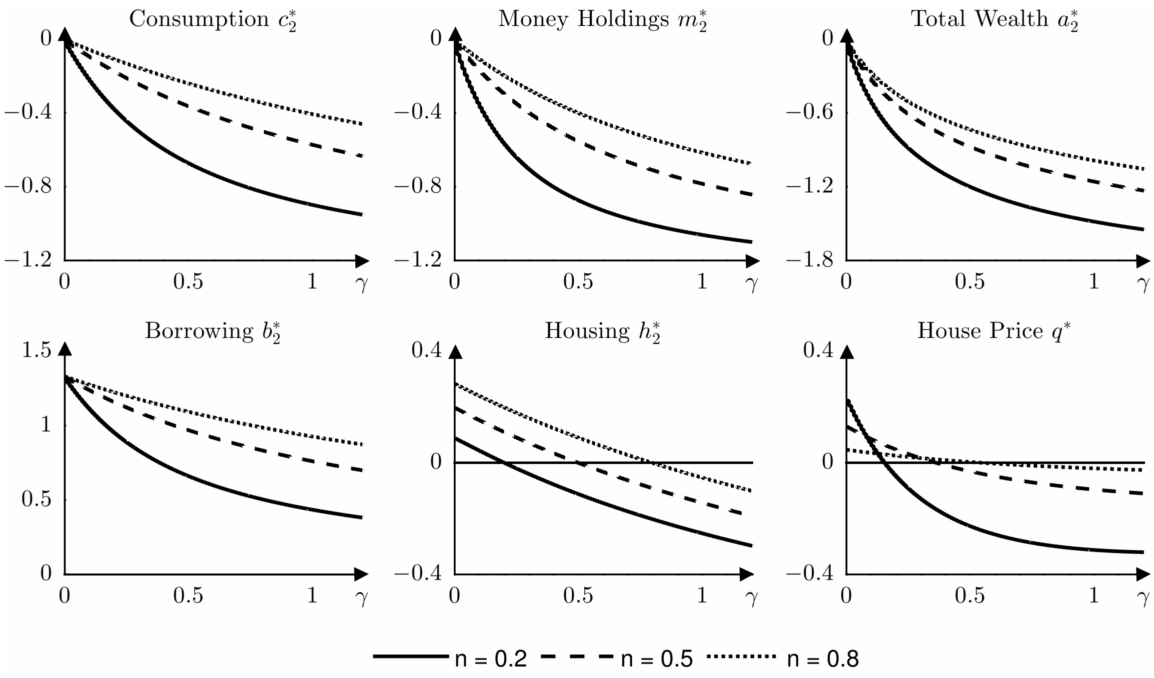
The preference for housing γ determines the relative strength of these effects. The higher γ, the weaker the effects associated with the higher collateral value relative to the negative effect on consumption. If γ is sufficiently high, the indebtedness of the borrower might actually *decline* in response to financial liberalization since housing is reallocated to the saver. The reason is that higher levels of γ are associated with a higher collateral value and hence higher household debt. A given change in θ needs to be balanced by a stronger decline in consumption and hence a stronger reduction in housing demand.

Figure 2.6 illustrates the effects of a rise in the loan-to-value ratio θ on the steady state. Each subplot shows the elasticity of the respective variable to a rise in θ as a function of the housing preference parameter γ for three different values of n. We set parameters such that the economy is at full employment for $\theta = 0$. In particular, note the negative effect on the borrower's housing investment and the real house price for large values of γ. This in turn implies that financial liberalization is associated with a substantial decrease in the real wealth of the poor household. Also, the responses of consumption and asset prices are stronger the higher the share of poor households.

In the literature, γ is typically calibrated to match empirical observations on the housing market. In a similar framework, Iacoviello (2005) chooses a value of $\gamma = 0.1$ to match the value of residential housing to output in the United States. Guerrieri and Iacoviello (2017) follow a related approach in a model with endogenous housing supply and select a value of $\gamma = 0.04$. When we apply the same criterion, the implied value of γ ranges between 0.08 and 0.1 and is substantially below unity. This implies the dominance of the collateral channel and hence financial liberalization raises asset prices and credit-financed housing investment.

[26] What is clear from (2.38), is that the equilibrium value of the saver's housing investment declines with θ while the effect on the value of the borrower's housing investment is unclear.

Figure 2.6: Elasticities with Respect to θ



Note: This figure shows the elasticities of the model variables with respect to the loan-to-value ratio θ as a function of the housing preference γ and for three different values of n. The y-axis shows the %−change in each variable in response to a 1% increase in θ in steady state. We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta ln(m_{2,t})$. The calibration is as follows: $\beta = 0.0005, \bar{y} = 100, \rho_1 = 0.05, \rho_2 = 0.1, \alpha = 0.01, \delta = 0.1, \theta = 0.5$ and $H = 1$.

These effects are in stark contrast to the standard neoclassical case with $v''(m_i) < 0$ for both types of households. From (2.30), it is clear that an increase in indebtedness reduces the consumption demand of the borrower in the neoclassical steady state since this agent faces higher real interest payments. Yet, aggregate demand is unaffected by variations in θ or γ because the saver increases his consumption level accordingly as long as his liquidity premium is decreasing in real money holdings, which can be seen from (2.31). As a consequence, changes in these parameters do result in a redistribution of available income and hence of consumption spending and housing investment. However, they do not trigger deviations from full employment because aggregate demand is not affected by these changes. In addition, the price level will adjust to clear the money market, which can be inferred from (2.25).

Similarly, aggregate demand is no longer affected by variations in these parameters once the model economy is in the symmetric steady state under stagnation. Then, variations in θ or γ cease to affect the consumption spending of both agents and simply lead to a redistribution of the housing stock and changes in the real house price. This case as well as other extensions of the model will be discussed in the next section.

2.6 Model Extensions and Discussion

In this section, we analyze two extensions of the model that have been turned off so far in order to focus on the core mechanism. In addition, we discuss policy recommendations.

2.6.1 Asymmetric and Symmetric Stagnation

From Proposition 2.1, we know that stagnation does not occur for sufficiently low levels of potential output. In addition, it is clear from (2.32) and the discussion in the previous section that the borrower will also eventually choose to accumulate money holdings if his consumption level is sufficiently high. More specifically, it follows from (2.35) that symmetric stagnation will occur once the borrower's consumption level has reached the critical threshold of $\chi/\beta(\beta\bar{y} - \alpha)$. We first derive a sufficient condition for asymmetric stagnation to prevail and then give an intuition for the occurrence of the symmetric case.

Under symmetric stagnation, $v'(m_{1,t}) = v'(m_{2,t}) = \beta$ and both households accumulate wealth infinitely. Consumption of neither type is stimulated by additional money. Formally, the symmetric steady state is defined as follows:

> *Symmetric Steady State*: The real and nominal interest rates are constant, the price level is declining at a constant rate, the real house price is constant, the real consumption levels are constant but the wealth of each household expands infinitely:

$$\dot{r} = 0\ ,\ \dot{R} = 0\ ,\ \pi < 0\ ,\ \dot{c}_1 = 0\ ,\ \dot{c}_2 = 0\ ,\ \dot{q} = 0\ ,\ \dot{a}_1 > 0\ ,\ \dot{a}_2 > 0\ . \tag{2.49}$$

The economy enters stagnation once potential output exceeds $\widetilde{y}$ defined in (2.33). Then, $v'(m_{1,t}) = \beta$ and there is deflation and demand shortage, i.e. $\pi < 0$ and $C < \bar{y}$ from (2.2) and (2.22). Consider the population-weighed average of (2.12) and (2.18) with $\dot{c}_1 = \dot{c}_2 = 0$:

$$n\rho_1 + (1-n)\rho_2 = \beta n c_1 + v'(m_2)(1-n)c_2 - \pi\ . \tag{2.50}$$

Symmetric stagnation cannot occur if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. To see this, suppose we have $v'(m_{2,t}) = \beta$ and $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Then from (2.2), (2.22) and (2.50), we get

$$\beta\bar{y} - \alpha < n\rho_1 + (1-n)\rho_2 - \alpha = \beta[nc_1 + (1-n)c_2] - \pi - \alpha = (\beta\bar{y} - \alpha)\frac{C}{\bar{y}}\ .$$

This only holds for $C > \bar{y}$ which cannot be the case. Hence, we always have $v'(m_{2,t}) > \beta$ for $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Together with Proposition 2.1 and Condition (2.36), this yields the following proposition for the existence of the asymmetric steady state:

Proposition 2.3 *Given the parameter restrictions $\rho_1 > \alpha$ and $\beta\bar{y} > \alpha$, the following condition is sufficient for the asymmetric steady state under stagnation to occur:*

$$n\rho_1 \left(\frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} \right) < \beta\bar{y} < n\rho_1 + (1-n)\rho_2 \ . \tag{2.51}$$

The first inequality in (2.51) follows from (2.33) and ensures that aggregate demand falls short of potential output and the second inequality ensures asymmetry. Intuitively, the second condition requires that the time preference rate ρ_2 is sufficiently high so that borrowers still strive for higher consumption. Yet, note that an increase in ρ_2 also tightens the first inequality, which is clear from Proposition 2.1.

Importantly, (2.51) is a sufficient condition for the existence of the asymmetric steady state but not a necessary condition. Under certain conditions, the asymmetric stagnation case prevails for higher values of potential output. This is the case when further increases in $\bar{y}$ do not stimulate the borrower's consumption to exceed the threshold discussed above. We discuss the necessary existence condition in detail in Appendix B.2 and only provide some intuition here.

Intuitively, the borrower's consumption depends on two factors as can be seen from (2.41): Income from firm profits which are determined by aggregate demand and capital gains on money holdings which depend on the rate of deflation. Under stagnation, an increase in the economy's production capacity worsens deflation which has two effects on the borrower's income. On the one hand, deflation reduces the consumption incentives of the saver. This reduces the income of the borrower since aggregate demand declines ("aggregate demand effect"). On the other hand, the purchasing power of money holdings rises which stimulates the borrower's consumption ("capital gains effect"). The second effect is stronger the higher his money holdings. If the capital gains effect dominates, the borrower's consumption increases with a higher production capacity as do his money holdings. Then, the marginal utility of money eventually reaches the lower bound and symmetric stagnation occurs.[27] But the asymmetric case may persist even for high levels of potential output $\bar{y}$ as long as the capital gains effect is weak or negative.

To summarize, our model features three regions depending on $\bar{y}$: If potential output is below the threshold $\widetilde{y}$ given by (2.33), the neoclassical case applies and there is no demand shortage. In contrast, stagnation occurs for $\bar{y} > \widetilde{y}$ because of the insatiability of liquidity preferences. The asymmetric case always occurs if condition (2.51) holds and might prevail for even higher values of potential output. Finally, the symmetric case occurs if consumption of the borrower under stagnation becomes sufficiently high.

[27]Thus, there will be an implicit threshold $\hat{y}$ such that there is symmetric stagnation for $\bar{y} > \hat{y}$. This threshold depends on the model parameters, particularly on those affecting equilibrium money holdings of the borrower. These in turn depend on the shape of the utility function $v(m)$. Therefore, we cannot give a closed-form expression.

2.6.2 Stagnation with Positive Money Growth

So far, we have focused on the case of zero trend inflation as a result of a constant money supply, i.e. $\mu = 0$, under full employment. Two considerations need to be taken into account when considering the case of $\mu > 0$ that affect the occurrence of stagnation as well as the existence of the stagnation steady state. For the general conditions and proofs, we refer to Ono and Ishida (2014) for the case of homogeneous households. Here, we will provide an intuitive discussion of the effects of $\mu > 0$ for the case of heterogeneous agents in the borrower-saver framework.

First, as argued above, stagnation occurs once one of the households is no longer willing to consume the amount consistent with full employment because of his insatiable desire for holding liquidity, i.e. once the following threshold is reached for any household:

$$\tilde{c}_i^{NC} > \frac{\rho_i + \mu}{\beta} \ . \tag{2.52}$$

This is a generalization of condition (2.32) for the case of positive money growth. Two effects emerge relative to the case of $\mu = 0$ that has been discussed so far.

Positive nominal money growth raises the nominal interest rate under full employment, due to the Fisher equation (2.24). This increases the opportunity cost of holding money for both agents, which stimulates their consumption, thereby increasing the liquidity premium. As a consequence, full employment can be sustained for higher levels of potential output and stagnation occurs at a later stage. In fact, for every level of potential output $\bar{y}$ there exists a nominal money growth rate μ such that full employment prevails. However, this comes at the cost of higher inflation.

In addition, there is a more subtle effect as positive money growth might affect both households' consumption levels $\tilde{c}_i^{NC}$ under full employment. This crucially depends on the assumption about the distribution of seignorage profits $z_t = \mu m_t$. If these are distributed in proportion to each agents money holdings, there is no effect on the full employment levels of consumption, given by (2.30) and (2.31).[28] However, if seignorage income is distributed equally across households, the household with lower money holdings benefits at the expense of the household with higher money holdings. For reasonable parameter specifications, it will be the saver whose consumption will be lowered by this effect, while the borrower benefits. This further increases the income threshold for stagnation.[29]

[28] The intuition is simple: Each household incurs implicit costs of money holdings due to inflation. In turn, the household benefits from inflation via the seignorage profits. If profits are distributed in proportion to money holdings, these effects exactly offset each other.

[29] It could even be the case that the saver's consumption is *lower* under full employment than the borrower's consumption level because of the redistributive effect of inflation. Yet, note that this effect only occurs for a very restrictive parameterization. Specifically, both the difference in discount rates and the money growth rate need to be sufficiently high and the loan-to-value ratio or the housing preference γ need to be sufficiently low.

Secondly, the existence condition of the asymmetric steady state is affected. Because of persistent deflation, the money supply expands indefinitely and so does wealth of the saver. With $\mu = 0$, the rate of expansion is given by the rate of deflation as is clear from (2.23). Since the deflation rate is below the real interest rate, as we assume $\rho_1 > \alpha$, the transversality condition (2.11) holds despite this expansion. With positive nominal money growth, however, the expansion of the real money supply increases to $\mu - \pi$ as does the growth rate of household wealth. For the transversality condition to hold, we need to require that this rate of expansion is below the time preference rate of the saver. Specifically, for a steady state to exist, it has to hold that

$$0 > \frac{\dot{m}_t}{m_t} - \rho_1 = \mu - \beta \tilde{c}_1^* \ , \tag{2.53}$$

where $\tilde{c}_1^*$ denotes the saver's consumption in the asymmetric steady state with $\mu > 0$.

On top of that, the occurrence condition of the symmetric stagnation steady state is affected by introducing positive money growth. The effects depend again on the assumption on the distribution of seignorage income. If this income is distributed in proportion to each household's money holdings, then there are no effects as the borrower's consumption under asymmetric stagnation is not affected. In contrast, if this income is distributed equally across households, the borrower's consumption will be stimulated under asymmetric stagnation. As the money supply expands, so does his exogenous income, which allows for higher consumption. Then, the symmetric stagnation case will eventually occur if condition (2.53) holds.

In conclusion, the equilibrium of the economy is conditional on the money growth rate. A sufficiently high rate of money growth may help to restore full employment. Since this comes at the cost of high inflation, policymakers are likely to be inclined to prefer a scenario of persistent stagnation and take measures to improve aggregate demand within that equilibrium. However, for any rate of money growth μ, there exists a level of potential output above which stagnation occurs. Sustainable full employment will hence require an ever-increasing expansion of the money growth rate. Even worse, the interplay of conditions (2.52) and (2.53) also implies that multiple equilibria can emerge with both stagnation and full employment as steady state equilibria for the same parameterization. It might also be the case that no equilibrium exists at all. So once the economy has reached stagnation, it will be very hard and costly in terms of high inflation to move towards the full employment steady state. For that reason, our analysis has focused primarily on the stagnation case with $\mu = 0$. Note, however, that the conclusions also hold for a low inflation scenario which requires sufficiently low levels of monetary growth.[30]

[30]This is similar to the assumptions of Michaillat and Saez (2014) and Michau (2017) that the central bank follows a sufficiently low inflation target.

2.6.3 Policy Recommendations and the Nature of the Friction

Two features in our model prevent the economy from reaching full employment - insatiable liquidity preferences and debt overhang. Insatiable liquidity preferences imply that stagnation always occurs for sufficiently high levels of potential output, even in the absence of financial frictions. The reason is that agents prefer to hold excessive money instead of consumption. This implies that expansionary monetary policy is ineffective in the stagnation steady state.[31] In fact, the deflationary steady state is characterized by an infinite expansion of the real money stock.

In contrast, the case for fiscal policy as a potential cure to stagnation is straightforward. The government is not constrained by the same liquidity motives as the private sector and can expand its spending.[32] Redistributive policies work by transferring resources from rich agents to poor ones. The latter expand their consumption while spending of the former is not directly affected (unless at the margin). Therefore, targeted redistributive interventions can help to stimulate the economy. In reality, targeted transfers might not be feasible though. Yet, Matsuzaki (2003) shows in a similar setting that lump-sum transfers financed by a consumption tax can increase aggregate demand if the fraction of poor households is sufficiently small.[33]

Private debt overhang is another factor that depresses aggregate demand since indebted households reduce their consumption spending. In fact, borrowers do delever in nominal terms in the deflationary steady state. But this is self-defeating because of the effects of debt deflation on the real value of their obligations. Hence, policies that limit household indebtedness and help to repair balance sheets of spending-constrained households are another option to expand aggregate demand. Yet, they include a potentially costly adjustment process in the short run.

Similar conclusions hold when we impose the borrowing constraint on the supply side. Although this analysis is beyond the scope of this paper, the following thought experiment clarifies this point: Suppose the collateralizeable asset is a factor of production and producers are constrained in their borrowing ability. As above, financial liberalization is associated with higher equilibrium collateral holdings by the borrower under certain parameter constellations. These in turn imply a higher equilibrium production capacity. Therefore, financial liberalization may improve equilibrium output under neoclassical assumptions. However, the economy is demand-constrained in our model because of the insatiability of liquidity preferences so that the implied

[31]Yet, a sufficiently large expansion of the money supply might restore the full employment case though at the cost of inflation as discussed before.

[32]Note that the expansionary effect of government spending has nothing to do with deficit-budget financing or balanced-budget financing. It works through a direct creation of demand. We refer to Ono (1994, 2001) for an explicit modeling of government spending. We abstract from public debt in our framework since we focus primarily on private debt.

[33]Note that this discussion uses aggregate demand as the relevant policy criterion rather than a welfare function based on individual utilities. The latter is complicated by household heterogeneity and the infinite expansion of the saver's money holdings in steady state.

improvements in the supply side actually worsen the output gap and deflation. An increase in indebtedness hence deteriorates equilibrium income for reasonable parameter ranges, irrespective of the modeling of the borrowing friction on the demand side or supply side.

Finally, our results continue to hold with insatiable wealth preferences instead of liquidity preferences. Unlike the latter, wealth preferences affect the equilibrium real interest rate by encouraging household savings (cf. Kumhof, Rancière, and Winant, 2015; Ono, 2015). As a consequence, the natural real rate of interest can turn negative in steady state (cf. Michau, 2017). In our setting, this would imply a redistribution from savers to borrowers as the real cost of debt becomes negative. However, the very existence of housing as a durable asset without depreciation prevents the real rate from turning negative in our setup. This can be easily seen from (2.19), which is unaffected by the introduction of wealth preferences. Housing yields a positive "dividend" stream in the form of the user cost of housing while the cost of housing investment are given by the real opportunity cost, since there is no depreciation. The real house price adjusts to make agents indifferent between housing investment and other uses of funds. Hence, from (2.19) a negative real rate of interest would require a decline of the real house price in steady state:

$$r^* < 0 \quad \Leftrightarrow \quad \frac{\dot{q}}{q} < -\frac{\gamma c_1^*}{h_1^*} < 0 \; .$$

This is not consistent with a stationary steady state. Moreover, it would imply that the real house price eventually converges to zero and hence that the current asset price itself is not well-defined. We can therefore exclude the possibility of a negative real rate in our model under wealth preferences. Hence, there cannot be a redistribution from savers to borrowers via negative interest cost of debt in steady state.

2.7 Conclusion

Many developed countries, particularly Japan, the Euro Area and the United States, have been suffering from persistent stagnation of aggregate demand under which some households do not increase consumption and keep wealth while others do not increase consumption because they are severely indebted. In all these cases, stagnation occurred after a credit and asset price boom. To analyze this phenomenon, we have introduced private indebtedness into a model with two types of households that have different time patience rates and insatiable preferences for money holding.

The less patient households borrow funds from the more patient ones but face a borrowing constraint that depends on the value of their collateral in the form of housing. Therefore their consumption is restricted by this constraint. The more patient households earn interests from lending and hence can expand consumption, but in fact do not because of high preference for money holding. Thus, aggregate demand shortages arise and deflation occurs as a steady state phenomenon. The deflation makes it more advantageous for the lenders to reduce consumption and hold money. It in turn expands the real value of debt of the borrowers and decreases their consumption because they have to pay high interests to the lenders.

If the borrowers could consume more, deflation would mitigate and stimulate the lenders' consumption as well, leading to an expansion of total income. Thus, a government that faces this situation may be tempted to ease the borrowing constraint. It will indeed enable the borrowers to consume more and mitigate deflation, which also stimulates the lenders' consumption by lowering the advantage of holding money. Moreover, easing the borrowing constraint makes the borrowers think housing investment to be more valuable because an increase in the value of housing enables them to borrow more for consumption. Thus, it triggers a house price boom.

However, those positive effects occur only in the short run. In the long run the borrowers are more indebted so that they have to reduce consumption, which worsens deflation and makes the lenders to decrease consumption and save more because money holding is more profitable. The decrease in total consumption stops the house price boom. The economy eventually falls into secular stagnation of aggregate demand. Thus, direct transfers from the richer to the poorer, which does not create debt overhang, will be more promising.

Chapter 3

The Seniority Structure of Sovereign Debt[*][†]

The difficulties associated with enforcing international contracts in the face of sovereign immunity imply that sovereigns have substantial discretionary power to prioritize the claims of one creditor over another. This chapter provides the first systematic empirical study of the implicit seniority structure of sovereign debt. Specifically, we analyze two situations in which differential treatment of creditors is observable: sovereign defaults, which we define as missing contractual payments, and sovereign debt restructurings, meaning the renegotiations that produce creditor losses. The data reveals a clear pecking order of sovereign debt repayment and default that is confirmed after controlling for a country's economic fundamentals.

This chapter is structured as follows. The first section introduces our research question and gives an overview of the paper. In section 3.2, we review the existing evidence on seniority in sovereign debt markets. In section 3.3, we analyze creditor seniority based on a new dataset on external sovereign debt arrears and establish stylized facts of creditor seniority. Section 3.4 compares haircuts on official and private sovereign debt restructurings and discusses the patterns of arrears during these restructurings. Section 3.5 concludes.

[*]This chapter is based on joint work with Christoph Trebesch (Kiel Institute for the World Economy and Kiel University) and Mark L. J. Wright (Federal Reserve Bank of Minneapolis). Parts of this chapter appeared as a column on VoxEU on August 11, 2015, entitled "Sovereign debt repayments: Evidence on seniority".

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3.1 Introduction

A borrower typically has multiple creditors. Hence, when a borrower is in distress, decisions must be made as to how to prioritize the claims of one creditor over another. For corporate and household borrowers, the sequencing of default and repayment is predetermined by national bankruptcy laws, such as Chapter 11 in the United States, and the terms of the different lending contracts. For sovereign borrowers, the absence of an international bankruptcy regime and the difficulties associated with enforcing international contracts in the face of sovereign immunity means that repayment priorities are more a matter of convention. And recently, these conventions have been under threat. In Greece, for example, the seniority of official creditors has been threatened as other Eurozone governments have been pressured to accept losses similar to the one accepted by private bondholders in 2012, while, in June 2015, Greece missed a payment to the IMF but decided to fully repay a yen-denominated bond held by private investors.

Have official debts always been de facto senior to the debts owed to private banks and bondholders? Have there been changes in the implicit seniority of different types of debt as policies have changed and debt contracts have evolved? What do the decisions of sovereigns to differentially treat their creditors tell us about the costs of default? Should theorists amend their models to allow for multiple types of creditors that are treated differentially? And if so, in what directions should their models be modified? Towards answers to these questions, this paper provides the first systematic empirical study of the implicit seniority structure of sovereign debt. Specifically, we analyze two situations in which differential treatment of creditors is observable: sovereign defaults, which we define as missing contractual payments, and sovereign debt restructurings, meaning the renegotiations that produce creditor losses. Throughout the analysis we focus on external creditors, since representative data on domestic arrears and domestic sovereign debt restructuring outcomes are not available.

We begin by studying episodes of missed payments, using a unique dataset on payment arrears to six external creditor groups: The International Monetary Fund (IMF), other multilateral creditors (in particular the World Bank), bilateral official creditors, bondholders, commercial banks, and trade creditors. For all but the IMF, we make use of proprietary information from the World Bank's Debtor Reporting System covering 127 low and middle income countries for the period 1979-2006. For the IMF, we use publicly available data on arrears from the IMF Finance Department. We then propose a new metric to capture seniority and discrimination across these six creditor groups: the *Relative Percentage in Arrears.* Intuitively, this measure defines seniority as the difference in default rates across creditor groups, measured by arrears per unit borrowed.

We use this metric to estimate the scope and determinants of creditor discrimination and the seniority structure of sovereign debt using descriptive statistics and panel regression techniques.

The results show a clear pecking order of sovereign debt repayments and default that is robust across countries and time, and which holds after controlling for macroeconomic fundamentals and fixed effects. As expected, the IMF is always the most senior creditor. Next in line are other multilateral official creditors and sovereign bondholders. On the lower end of the pecking order are loans from banks and loans from trade creditors. Somewhat surprisingly, loans from other governments seem to be slightly junior to bondholders as they are more likely to see arrears at earlier points in time and in larger amounts (as shares of debt outstanding to that creditor group). This pattern is especially prevalent after the end of the 1980s debt crises.

In addition, we move beyond arrears and study the outcome of sovereign debt renegotiations. Specifically, we shed light on implicit creditor seniority by comparing creditor losses (haircuts) suffered by official creditors in past sovereign debt restructurings to those suffered by private creditors such as banks and bondholders. To do so, we use the haircut estimates for private restructurings by Cruces and Trebesch (2013) and compare these to a new dataset on haircut estimates with official creditors in 414 sovereign restructurings between 1978 and 2015 that comes from Reinhart and Trebesch (2016b) and Tolvaisaite (2010) and is updated here. The resulting haircut database covers all sovereign debt restructurings with foreign private creditors and bilateral (Paris Club) creditors since 1978. These data show that, on average, private creditors are subjected to systematically lower haircuts than official creditors. This finding is robust to controlling for economic fundamentals as well as time effects. Although the results are clear-cut and robust, our findings on restructuring outcomes should be interpreted with some care since our haircut estimates for official creditors are based on strong assumptions that are hard to confirm due to limited data disclosure by the Paris Club.

This contribution is, to the best of our knowledge, the first systematic empirical study of creditor seniority in external sovereign debt markets. Previous work, such as seminal work of Borchard (1951), eschewed careful statistical analyses in favor of an impressionistic summary of the evidence, while other authors, such as Kaletsky (1985), simply asserted seniority conventions without any evidence whatsoever. Both these, and other authors, claimed that official creditors, as well as private trade creditors, were implicitly senior to other private creditors. In contrast, we show, by using a novel dataset, that trade creditors are consistently the least senior creditors, and that bilateral official creditors are typically less senior than private bondholders. This is especially provocative in light of policy discussions of seniority in the context of crisis lending (e.g. Borensztein et al., 2005; Gelpern, 2004; Roubini and Setser, 2003; Steinkamp and Westermann, 2014), and bailouts (e.g. Boz, 2011; Corsetti, Guimarães, and Roubini, 2006; Fink and Scholl, 2016; Roch and Uhlig, 2016), that typically assume the seniority of official lending.

The lack of formal analyses of seniority in a sovereign debt context contrasts sharply with the large and well-established corporate finance literature on priority rules and seniority in bankruptcy (see Bebchuk, 2002; Bolton and Oehmke, 2015; Bolton and Scharfstein, 1996; Franks and Torous, 1989; Gilson, John, and Lang, 1990; Hart and Moore, 1995; Weiss, 1990, to name just a few).[1] In contrast to this literature, we find that trade creditors are relatively junior creditors in a sovereign context.

Motivated by the obviously different political and legal enforcement constraints, a small sovereign debt literature has examined the relative treatment of domestic as opposed to international creditors both theoretically (in particular Broner, Erce, et al., 2014; Broner, Martin, and Ventura, 2010; Guembel and Sussman, 2009) and empirically. On the empirical side, Sturzenegger and Zettelmeyer (2007), Erce and Diaz-Cassou (2010), Kohlscheen (2010), and Erce (2012) all provide evidence on discrimination between domestic and external creditors, either by conducting case studies or by using Standard & Poor's simple binary indicator of external and domestic defaults. Our paper complements this literature and finds discrimination between different classes of external creditors despite the fact that they face substantially similar sets of limitations on the enforcement of their claims. Likewise, our paper complements studies of seniority *within the same group of creditors*, typically bondholders, as in the study of debt dilution through the issuance of new short-term bonds (see Bolton and Jeanne, 2009; Chatterjee and Eyigungor, 2015; Hatchondo, Martinez, and Sosa-Padilla, 2016).

Lastly, whereas the vast bulk of the sovereign debt literature focuses on the occurrence of defaults on debts owed to private external creditors (see Aguiar and Amador, 2014; Panizza, Sturzenegger, and Zettelmeyer, 2009), as well as the extent of private creditor losses in the event of a restructuring (see e.g. Benjamin and Wright, 2009; Cruces and Trebesch, 2013; Sturzenegger and Zettelmeyer, 2008), we study private *and* official defaults and creditor losses simultaneously. This is a surprisingly neglected area of research in light of the fact that debts owed to official creditors often account for the lion's share of sovereign claims, e.g. in Greece or Portugal today and in many countries in the developing world. Our work thus complements the small literature examining debts owed to both official and private creditors from both a theoretical perspective, as in Dellas and Niepelt (2016), and empirical perspectives, as in Aylward and Thorne (1998), Chauvin and Kraay (2005), Cheng, Diaz-Cassou, and Erce (2016) and Reinhart and Trebesch (2016a, 2016b, 2016c).

[1] See Andersson et al. (2013) for an analysis on the pecking order of repayment in consumer bankruptcy.

We proceed as follows. The next section gives an overview on the existing views and evidence on seniority in sovereign debt markets. In section 3.3, we analyze creditor seniority in case of missed payments based on a new dataset on external sovereign debt arrears. We establish stylized facts of debt repayment and arrears by running panel regressions and using counterfactual decomposition techniques. Section 3.4 compares haircuts on official and private external creditors during restructurings. We present the procedure for estimating haircuts for Paris Club restructurings as well as an empirical framework to assess discriminatory treatment of creditors. We also analyze the behavior of arrears during restructurings, thereby providing a synthesis with the previous section. The final section concludes and gives an outlook for future research.

3.2 An Overview on Seniority in Sovereign Debt Markets

The concept of seniority in the context of sovereign lending lacks a clear, commensurable definition. It captures the idea that sovereigns typically borrow from different creditors, both domestic and external ones, in the form of various debt contracts and that they have in principle discretionary power to prioritize repayments of these claims. In the absence of an internationally enforceable insolvency scheme, their sovereignty allows governments to provide preferential treatment to some creditors, while discriminating against others. While there exists no de jure seniority structure in the sovereign context, a set of rules and conventions are considered to guide these government decisions resulting in a de facto seniority structure in sovereign debt markets. This section reviews the existing body of knowledge on this implicit structure.

The possibility of differential treatment necessitates the prevalence of heterogeneity among a sovereign's lenders. On the most basic level, a distinction is made between domestic debt and external debt.[2] As data coverage on domestic debt is far from complete, empirical evidence in most cases relies on selected case studies of sovereign debt restructurings with both domestic and external creditors (cf. Erce, 2012; Erce and Diaz-Cassou, 2010; Kohlscheen, 2010; Sturzenegger and Zettelmeyer, 2007). In particular, Erce and Diaz-Cassou (2010) provide evidence that domestic factors, such as the robustness of the banking system and the reliance of corporations on international capital markets, affect the decision to discriminate against foreign creditors. In contrast, Broner, Erce, et al. (2014) assume that domestic creditors tend to be favored in general and express this in a higher probability of repayment relative to external ones.[3]

[2] The relevant criterion is the residency of the creditor. Domestic debt is owed to residents, while external debt is owed by residents to non-residents of a specific country, see World Bank (2015). Other definitions are related to the currency denomination or the governing law under which the debt is issued.

[3] Guembel and Sussman (2009) argue that the existence of domestic debt is a means to sustain external debt in the absence of default penalties. External creditors are assumed to be able to sell their debt claims to domestic ones. As a consequence, sovereigns cannot effectively discriminate against creditors in their model.

In the following, we will exclusively focus on external debt, or more specifically, on public and publicly guaranteed, long-term external debt.[4] Following the definition of the World Bank, we distinguish six groups of external creditors as follows: (i) *bilateral* creditors are official agencies that make loans on behalf of one government to another government or to public and publicly guaranteed borrowers in another country, (ii) *multilateral* loans are those made by official agencies owned or governed by more than one country that provide loan financing. They include international financial institutions such as the World Bank, regional development banks, and other intergovernmental agencies, but not the IMF, (iii) sovereign *bonds* are debt instruments issued on capital markets by public and publicly guaranteed debtors with durations of one year or longer, (iv) *bank* creditors are private banks that provide loans and other financial services, often in the form of syndicated lending, (v) *trade credits* to the public sector include credits from manufacturers, exporters, and other suppliers of goods, (vi) the *International Monetary Fund (IMF)*, which is a multilateral creditor but treated as a separate category because of its unique position in the international financial system.

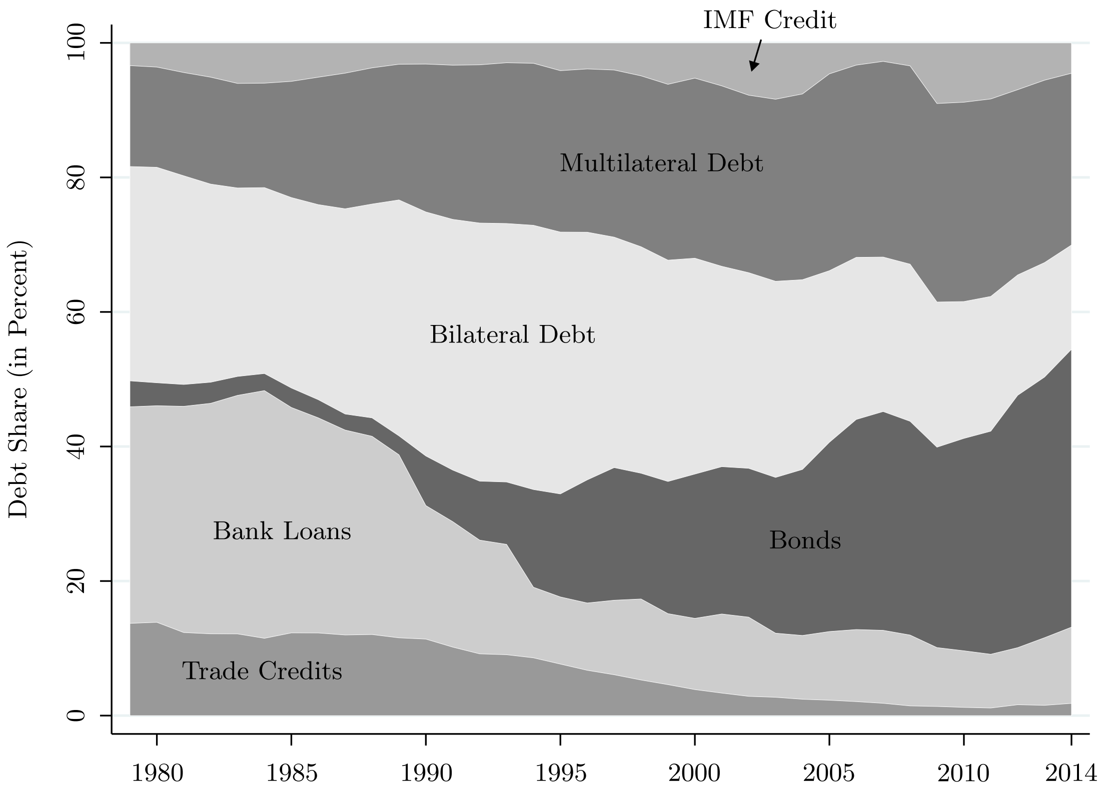
The relative importance of these creditor groups has substantially fluctuated over time. Figure 3.1 shows the share of each group's outstanding debt in the total amount of public and publicly guaranteed external debt over time for the sample of 127 developing and emerging economies, which we will also use in the analysis of arrears in section 3.3.

Two observations stand out. First, official debt accounts for a substantial share of total sovereign debt, particularly in developing countries, but also in Greece or Portugal for example. In light of this observation, there is surprisingly little research on relative losses of official versus private creditors. The vast bulk of the sovereign debt literature focuses on the occurrence of defaults on debts owed to private external creditors (see Aguiar and Amador, 2014; Panizza, Sturzenegger, and Zettelmeyer, 2009), as well as the extent of private creditor losses in the event of a restructuring (see e.g. Benjamin and Wright, 2009; Cruces and Trebesch, 2013; Sturzenegger and Zettelmeyer, 2008).[5] In contrast, there is only some small literature examining debts owed to both official and private creditors from both a theoretical perspective, as in Dellas and Niepelt (2016), and an empirical perspective, as in Aylward and Thorne (1998), Chauvin and Kraay (2005), Cheng, Diaz-Cassou, and Erce (2016) and Reinhart and Trebesch (2016a, 2016b, 2016c). We complement this literature by providing the first systematic study of both private and official defaults and creditor losses.

[4]Long-term debt includes all debt titles with an original maturity of more than one year, see the appendix of World Bank (2015) for a detailed documentation.

[5]In addition, the standard default measures by Moody's or Standard & Poor's purely rely on missed payments to private creditors, hence ignoring roughly half of the other claims in international sovereign debt markets. We therefore consider them an insufficient proxy for studying seniority.

Figure 3.1: Debt Shares over Time by Creditor Group



Note: The figure shows the share of each creditor group's debt in the total stock of public and publicly guaranteed long-term external debt over a sample of 127 developing and emerging economies. Data is from the World Bank's World Development Indicators (WDI) dataset. For IMF debts, we use the variable "use of IMF credit" from the WDI.

Secondly, the relative importance of creditor groups shifts over time. In particular, the share of bond debt relative to bank loans and trade credits increased drastically in the early 1990s following the Brady debt exchange deals.[6] In addition, there seems to be a slight decline in the share of bilateral debt relative to lending by multilateral creditors. In contrast, the shares of official debt and private debt themselves have remained relatively stable.

What does seniority mean in the sovereign debt context? In the corporate world, seniority is typically defined in absolute terms. In case of a corporate bankruptcy, claims of a senior creditor are served first and fully (to the extent possible) from the available bankruptcy assets. Junior creditors are only paid once these claims are fully served. In contrast, most authors agree that seniority is a relative concept in sovereign debt markets guided by a set of informal rules and conventions (see Roubini and Setser, 2003). Various authors employ different working definitions.

[6]Bolton and Jeanne (2009) interpret this shift from bank lending to bond contracts as the result of differences in seniority across these instruments. Specifically, they argue that lenders endogenously shift their funds into instruments that are more difficult to restructure, such as bonds, and hence enjoy a higher seniority in the event of sovereign defaults. Although we do not explicitly examine this hypothesis, our findings can be interpreted as in line with this view.

For instance, in Chatterjee and Eyigungor (2015) seniority means that "in the event of default, a creditor who lent earlier must be paid in full before a later creditor can be paid anything at all". Steinkamp and Westermann (2014) imply that seniority means that "the preferred lender gets his money back first, in case of insolvency". In contrast, Broner, Erce, et al. (2014) interpret seniority as a higher probability of repayment and Bolton and Jeanne (2009) relate it to the relative bargaining power of different creditor groups. While the exact definition varies by author, the concept ultimately comes down to the order of repayment in case of financial distress or default. Two questions are particularly relevant: When are debts (not) repaid? And how severe is the write-down if claims are eventually restructured? Both questions are typically (though not necessarily) interrelated, but matter separately, for example because of regulatory reasons.

Before discussing these questions in turn, it is to be noted that differential treatment might arise along two dimensions: Discrimination across debt titles within the same creditor group or discrimination across creditor groups. While much has been written on the former aspect - particularly in the context of debt dilution of existing longer-term claims because of the issuance of new short-term debt instruments (see Bolton and Jeanne, 2009; Chatterjee and Eyigungor, 2015; Hatchondo, Martinez, and Sosa-Padilla, 2016) - the latter topic has to our knowledge not been treated in a systematic way. Therefore, we will focus on *seniority across creditor groups* and thereby complement the above-mentioned literature on seniority within a group.

Creditor seniority can implicitly be inferred from two observable events: Repayment patterns and restructurings outcomes. While the first one involves the magnitude and duration of missed contractual payments that are accumulated mostly in times of financial distress, the second one deals with creditor losses during sovereign debt restructurings. Much attention has been devoted to the latter issue. Creditor losses during these events, as measured by haircuts, have been analyzed predominantly for private debt (see e.g. Benjamin and Wright, 2009; Cruces and Trebesch, 2013; Sturzenegger and Zettelmeyer, 2008) and to a smaller extent for official debt (see e.g. Das, Papaioannou, and Trebesch, 2012; Reinhart and Trebesch, 2016b; Tolvaisaite, 2010). In addition, these events have been used to study seniority in the context of domestic versus external debt as outlined above. Haircuts are considered the consensus measure of creditor losses during restructurings. And even though the underlying assumptions are subject to heated debate, particularly with respect to the proper interest rates and discount rates to value future claims, using this measure in principle allows for establishing a ranking across various creditors and instruments. Yet, there has been no analysis comparing creditor losses among these groups so far. We provide a first systematic comparison in section 3.4 of this paper for the most important subset of official debt restructurings, namely those with the Paris Club.

In contrast to the vast literature on sovereign debt restructurings, much less attention has been devoted to the case of arrears in the sovereign debt context, presumably due to limited data availability on the creditor or loan level. While there are some studies examining individual creditors - see Aylward and Thorne (1998) for the International Monetary Fund - or selected cases, there is no systematic comparative study of the arrears behavior across a sovereign's diverse creditor base.[7] This is surprising as these missed payments are sizable and tend to occur quite frequently, particularly in developing countries. They typically accumulate in the run-up of restructurings but are neither a pre-condition for these - see Asonuma and Trebesch (2016) on preemptive restructurings in the absence of arrears - nor are they fully resolved by these. In addition, traditional measures of default do not capture the dynamics of these missed payments (see Appendix C.1 for a discussion). The patterns of arrears are diverse and vary considerable across countries, which allows for new insights into the seniority structure of sovereign debt. To illustrate this diversity and the interdependence between arrears and restructurings, Figures C.5 to C.10 in Appendix C.3 show official and private arrears (relative to GNI) for a few selected countries together with dates of official debt restructurings with the Paris Club or private debt restructurings.

Some countries accumulate arrears primarily on official debt (like Ethiopia or Nigeria), while others fail to pay their private creditors (like Argentina or Ecuador). Others in turn run arrears on both official and private lenders simultaneously (like Morocco or Peru). Arrears can be short-lived or protracted. They can continue to increase after restructurings or can be fully resolved as in Argentina or Ecuador in the 1990s.

This heterogeneity already suggests that the patterns of arrears and restructurings, while interrelated, differ in their characteristics and hence deserve to be studied separately. Furthermore, although the problem of accumulated arrears is typically resolved at some point (usually in the wake of a restructuring), the protracted nature of arrears can make them a substantial burden for the individual creditor. For example, missed payments have been a significant problem for the IMF in the late 1980s when more than 14 percent of the IMF's outstanding loans have been in arrears, threatening to impede its role as lender of last resort (cf. Boughton, 2012). In addition, arrears are, from a regulatory point of view, costly as banks have to set aside capital if a loan has been in arrears for a while, even if they think they will eventually be made whole in a restructuring. For these reasons, several authors suggest a wider definition of sovereign default than currently captured by the standard binary default measures or by the occurrence of debt restructurings (see Arellano, Mateos-Planas, and Ríos-Rull, 2013; Easton and Rockerbie, 1999).

[7]In contrast, the dynamics of arrears have been analyzed in other fields. For example, Andersson et al. (2013) study the pecking order of consumer loans using arrears on credit card debt and mortgages.

While creditor losses in restructurings are commonly measured via haircuts, it is not entirely obvious how seniority is measured via arrears.[8] Most of the analysis on arrears has been conducted in terms of Probit or Logit regressions on the occurrence of missed payments (cf. Andersson et al., 2013; Aylward and Thorne, 1998). Yet, this does not allow for a comparative treatment across creditors. In addition, the exposure of each creditor group, i.e. its lending volume, has to be taken into account. Hence, in section 3.3, we suggest a new measure, the *Relative Percentage in Arrears*, that allows for a comparative analysis of arrears while taking differences in lending behavior into account. To our knowledge, we are the first to construct such a discrimination measure in the sovereign debt context. We borrow parts of the analytic framework from the literature on wage discrimination. The two main approaches to identify unjustified wage differences among treatment groups, which differ by race, sex or age, include dummy variable regressions and counterfactual decomposition techniques in the spirit of Oaxaca (1973) and Blinder (1973). We apply similar frameworks to the analysis of creditor discrimination in sovereign debt markets.

Despite the lack of a comprehensive formal treatment of seniority, various papers, policy institutions and experts in financial markets have expressed different views on the informal rules supposedly guiding the repayment behavior of sovereigns as well as on the underlying rationale for the associated pecking order.

Specifically, seniority is frequently related to the relative enforceability of debts. This can have a legal basis, but also a practical one. While legal provisions might suffer from the same enforceability problem that gives rise to discriminatory power of governments in the first place, differences in the bargaining power of creditor groups can result in a de facto seniority structure. For example, Bolton and Jeanne (2009) argue that international bonds are senior to bank loans as they are more difficult to restructure because of the dispersion within the creditor group. It is less costly for a sovereign to renegotiate terms with a few big banks instead of a large number of individual bond holders. As a consequence, sovereigns might be more inclined to default on bank loans rather than bond payments. Since creditors anticipate this behavior, the result is a shift from bank to bond finance as seen in Figure 3.1. Similarly, Roubini and Setser (2003) point towards the lower risk of litigation that countries face vis-à-vis the Paris Club, which gives incentives for strategic defaults on Paris Club loans, making official bilateral debt de facto junior to private debt. The related argument is made that trade creditors are senior creditors as default on them results in an immediate and costly cut-off from trade.

[8] Note that there is still considerable debate about the measurement of restructuring outcomes, in particular with respect to discount rates of future claims, interest rates and the specification of the haircut measure. These problems become more severe in the context of official debt restructurings due to a lack of transparency in the restructuring process of the Paris Club. We will discuss these issues in section 3.4.

It is also the institutional environment that supposedly gives the International Monetary Fund de facto seniority among external creditors. In particular, the IMF's status as "preferred creditor" can be interpreted as the result of the positive externality it provides to other market participants (see Roubini and Setser, 2003). Other creditors benefit from the Fund's role as lender of last resort in times of crisis and are in return willing to accept its senior status making crisis lending possible in the first place. In addition, defaulting on the IMF results, from a sovereign's point of view, in denied access to future emergency lending, therefore making defaults less likely. However, note that the IMF is not de jure a preferred creditor. In fact, seniority of the IMF has been repeatedly under threat.[9] Similar to the IMF, other multilateral creditors and international financial institutions (IFI) tend to be considered relatively senior in the literature (cf. Borensztein et al., 2005; Boz, 2011; Corsetti, Guimarães, and Roubini, 2006; Steinkamp and Westermann, 2014). We confirm this view in our empirical analysis, both with respect to the occurrence and severity of arrears and the relative size of haircuts.

Interestingly, bilateral official creditors are also perceived senior by many contributors, going back as far as Kaletsky (1985). This is particularly true in the context of crisis lending in Europe. Specifically, loans given by bilateral official creditors are considered de facto senior in surveys like the World Economic Survey, as argued by Steinkamp and Westermann (2014). Also, the Paris Club's comparability of treatment clause constitutes a legal provision, which intends to ensure at least equal footing of official creditors. Yet this view might be flawed as argued by Roubini and Setser (2003) and pointed out above in detail. Above that, theory does not offer a convincing rationalization for the relative status of bilateral creditors.

To summarize, there are various views and perceptions of seniority among a sovereign's external creditors, both in theoretical and applied work, without a solid and comprehensive empirical analysis. Our paper attempts to fill this gap and provides, to our knowledge, the first systematic empirical study of creditor seniority in external sovereign debt markets.

3.3 Creditor Seniority: Evidence from Missed Payments

To measure implicit seniority in sovereign debt markets we compile a unique new dataset on arrears by sovereign debtors towards six creditor groups. The creditor groups are defined with respect to the debt contract under which they provide funds. Three of them are official creditors and three of them are private ones.

[9] As mentioned above, protracted arrears on the IMF have been a pressing concern in the late 1980s. In addition, the recent developments during the Euro crisis, particularly Greece's refusal to repay an IMF loan while honoring its commitment to bondholders, have put the perception of the Fund's seniority under question.

3.3.1 The Dataset: Arrears across Six Creditor Groups

Our arrears dataset is constructed from two sources. First and foremost, we obtain detailed data on payment arrears on government and government-guaranteed external debt towards five creditor groups from the World Bank's Debtor Reporting System (DRS), which is the database underlying the Global Development Finance (GDF) and International Debt Statistics (IDS) publications. The DRS database includes loan-level information on all loan and credit agreements by all debtor countries that borrow from the World Bank. The reporting of these debt statistics is mandatory for World Bank clients, including detailed information on arrears. However, for reasons of confidentiality, the World Bank only publishes aggregated data on the country level and there is no arrears data by type of debtor. As a result, the arrears series that are publicly available do not distinguish between arrears by different types of private or public debtors. In contrast, our data allows us to measure payment arrears incurred by a country's public sector only, and broken down towards five external creditor groups. In addition, we obtain payment arrears on the IMF from the IMF Finances database (on the IMF website).

Public sector arrears in the DRS database are defined as late payments as of end-of-year, or more precisely as principal and interest payments due but not paid on long-term external debt obligations of public debtors or guaranteed by a public entity, thus including government debt and government-guaranteed (quasi-sovereign) debt.[10] Long-term external debt in the DRS is defined as debt that has a maturity of more than one year and that is owed to nonresidents and is repayable in foreign currency. Arrears are measured in current US-Dollar at the end of each year on a cumulative basis. Arrears towards the IMF are measured in Special Drawing Rights (SDRs) and converted in US-Dollars using December averages of the SDR-Dollar exchange rate from the IMF website.[11]

The definition of creditor groups follows the one by the World Bank as described in the previous section and outlined in greater detail in World Bank (2015). Specifically, we distinguish (i) *bilateral* creditors, (ii) *multilateral* creditors, which include international financial institutions such as the World Bank, regional development banks, and other intergovernmental agencies, but not the IMF, which is included as a separate category, (iii) sovereign *bonds*, (iv) *bank* creditors that provide loans and other financial services and (v) *trade credits*, which include credits from manufacturers, exporters and other suppliers of goods. In addition, the (vi) *International Monetary Fund (IMF)* is included in a separate category due to its unique role in the international financial system.

[10]For an overview of the definitions of variables and creditors, see World Bank (2015) for example.

[11]To ensure comparability of the data, we restrict IMF arrears to payments that are six or more months overdue and use end-of-year IMF data.

Importantly, our database allows us to assess defaults via missed payments towards the entire spectrum of a government's external creditors. This contrasts earlier work relying on conventional measures of sovereign default by rating agencies such as Standard & Poor's or Moody's or by Reinhart and Rogoff (2009). These datasets measure defaults towards private creditors only, particularly towards bondholders and banks. However, they do not capture missed payments and restructurings towards official creditors, as emphasized by Reinhart and Trebesch (2016c).[12] Hence, the arrears data enables us to provide a more complete picture of creditor seniority.

To measure the share of arrears per unit of lending, we match the arrears data with disaggregated debt stock data of the face values of public and publicly guaranteed debt by creditor group, which we also obtain from the DRS dataset.[13] For the debt owed to the IMF, we use the "use of IMF credit" stock variable from the World Bank, also measured in current US-Dollars. This IMF data is available for all but 5 countries for which we have data on payment arrears from DRS.[14] In addition, we choose to drop two countries (Afghanistan and Montenegro) that have observations on debt stocks for only two years.[15] In line with the DRS documentation, we code arrears as zero whenever debt stock data is available and arrears are not reported.

The result is an unbalanced panel of sovereign arrears towards six external creditor groups covering 127 countries and up to 28 years. The time coverage by country ranges from 8 to 28 years. Data for all years is available for roughly 75% of the sample countries.

The data shows that sovereign arrears are widespread and sizable. On average, 8.0% of the stock of public and publicly guaranteed external debt is in arrears. However, there is a large heterogeneity across creditor groups, with some creditors being particularly likely to suffer from arrears, as we will show below. The data also shows that defaults are partial, as emphasized by Arellano, Mateos-Planas, and Ríos-Rull (2013). Countries usually continue to make partial debt service payments at the same time that they accumulate arrears. Also symbolic token payments are frequent. Indeed, and quite remarkably, there are only three observations in the entire sample with a full payment suspension, i.e. with zero debt service payments and complete arrears accumulation.

[12] In Appendix C.1, we compare our arrears data with the default data by Standard and Poor's and show that the S&P default dummy is indeed a poor indicator of missed payments towards bilateral creditors.

[13] There are 31 countries in our sample for which we have disaggregated data on arrears but not on debt stocks. For these countries, we substitute the aggregate debt stock data from the World Bank's WDI dataset which is available on the creditor level. The comparison of the debt stock data for countries for which both data is available shows that there are no significant inconsistencies between the two sources. This is to be expected since the disaggregated data is the raw data underlying the WDI.

[14] Data on IMF debt stocks is not available for Croatia, Iraq, Latvia, Russia and the Slovak Republic. Yet, these omissions are not likely to alter the results much because arrears are quantitatively small (less than 1% of GNI on average, except for Russia) and the pecking order as measured by the share of creditor-specific arrears in total arrears does not qualitatively differ from the results presented here.

[15] Afghanistan accumulates arrears to the IMF, bilateral creditors and trade creditors. Montenegro concentrates all arrears on multilateral creditors.

3.3.2 Measuring Seniority from Arrears: Methods and Stylized Facts

We first describe our measurement approach and then present new stylized facts on the implicit creditor seniority in sovereign debt markets. We show results for the entire sample and then refine the analysis by looking at specific subsamples and at changes over time.

To capture the heterogeneous treatment of creditors we propose two alternative measures of seniority based on the arrears data. The first captures the scope of arrears per unit of lending of each creditor group, irrespective of the scope of lending and arrears to other creditors. The second measure is more sophisticated and captures the relative distribution of arrears across creditor groups, controlling for the debt composition of each sovereign. For each measure, we compare the results for the six creditor groups for the entire sample and various subsamples.

The first measure is simple and quantifies the absolute scope of arrears per unit of lending. The "arrears to debt ratio" of creditor group k at time t in country i is defined as:

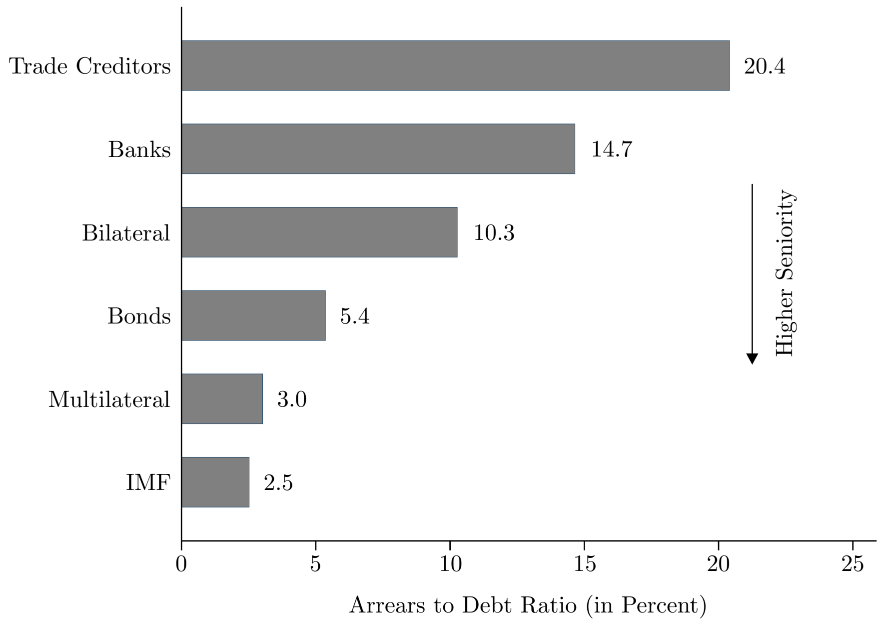
$$ATD_{i,t,k} = \frac{arrears_{i,t,k}}{debt_{i,t,k} + arrears_{i,t,k}} \tag{3.1}$$

This measure normalizes arrears by the nominal stock of public and publicly guaranteed debt of each creditor group. Debt stocks are adjusted for the level of arrears so that $ATDs$ are bounded below by 0 and bounded above by 1. This also allows us to interpret the denominator as the total amount of payments due in case debts can be accelerated, i.e. if creditors are legally entitled to receive the entire face value of their claim upon default (as is customary for sovereign bonds).

The pecking order of external sovereign debt implied by the ATD measure is illustrated in Figure 3.2 which shows the unweighted average of the ATD in the full sample. There is a clear seniority structure: bonds, IMF credit and multilateral loans are senior whereas bilateral loans, bank loans and trade credits are junior. Arrears to debt ratios are lowest for the IMF (2.5%, on average) and highest for exporters and suppliers (more than 20% of their outstanding claims). These averages are however biased by outliers. For the IMF, for example, the occurrence of arrears is rare, but once arrears occur they tend to accumulate fast, resulting in a high ATD.

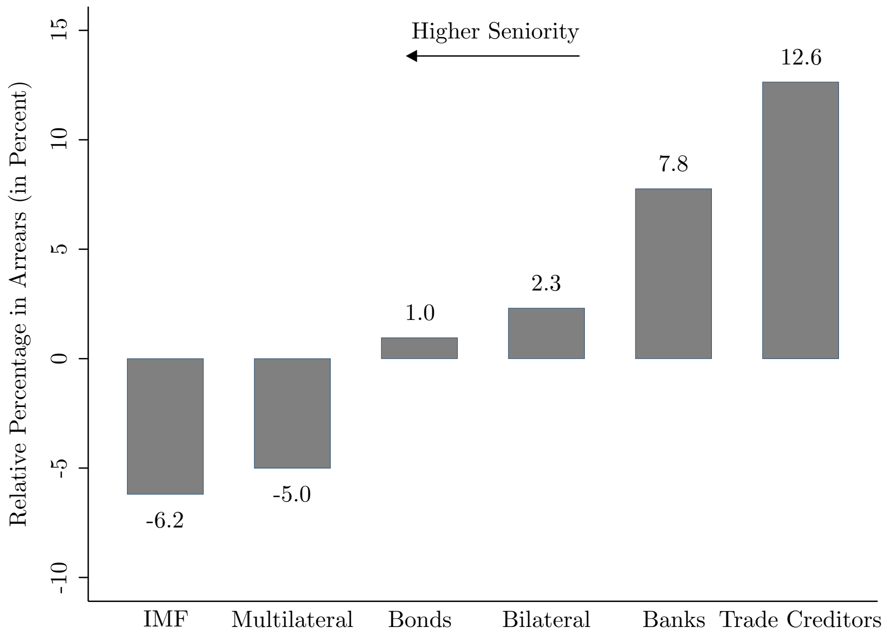
The ATD measure is simple and intuitive, but it has an important disadvantage: it does not capture the relative arrears burden across creditor groups, which depends on the overall stock and on the composition of sovereign arrears in each country-year spell. To see this, take a low-income country with no access to private capital markets or to IMF credit. In a crisis, such a country is likely to accumulate arrears mostly towards bilateral lenders and trade creditors. If this country is also more likely to suffer a financial crisis, then the high observed arrears burden to official and trade creditors in Figure 3.2 should not be interpreted as evidence of discrimination. Instead,

Figure 3.2: Arrears to Debt Ratios by Creditor Group



Note: The bars show the average ATD ratio over the period 1979-2006 for each creditor group given by equation (3.1). Averages are calculated as unweighted means of country averages.

Figure 3.3: Relative Percentage in Arrears by Creditor Group



Note: The bars show the average RPIA ratio over the period 1979-2006 for each creditor group given by equation (3.2). Averages are calculated as unweighted means of country averages.

the higher $ATDs$ could merely reflect the characteristics of countries that are most prone to accumulate arrears, in particular their (in-)ability to access private capital markets abroad.

As an alternative, we therefore propose a measure that captures the relative distribution of arrears controlling for a country's debt composition and, thus, for differences in external borrowing patterns. The Relative Percentage in Arrears ($RPIA$) captures the difference between the arrears to debt ratio of a specific creditor group k and the total arrears to debt ratio of a country:

$$RPIA_{i,t,k} = \frac{arrears_{i,t,k}}{debt_{i,t,k} + arrears_{i,t,k}} - \frac{\sum_k arrears_{i,t,k}}{\sum_k (debt_{i,t,k} + arrears_{i,t,k})} \tag{3.2}$$

Intuitively, in Equation (3.2), the average arrears ratio (as percent of total sovereign debt) serves as a benchmark to evaluate discrimination. Creditors are favored if they face lower arrears per unit of debt than the average creditor. Conversely, a creditor group is discriminated against if it faces higher arrears per unit of debt than the average creditor. The $RPIA$ is bounded in $[-1, 1]$ with a positive sign indicating discrimination and a negative sign indicating favoritism. A value of zero indicates fair treatment.

More specifically, the $RPIA$ of a creditor group k is closely related to a weighted average of the arrears to debt ratios of all creditor groups via the following expression:

$$RPIA_{i,t,k} = ATD_{i,t,k} - \sum_k ds_{i,t,k} \cdot ATD_{i,t,k} \tag{3.3}$$

where $ds_{i,t,k}$ is the debt share of group k, i.e. the fraction of creditor group k's debt stock (adjusted for arrears) in the total debt stock of country i at time t. Hence, by construction the $RPIA$ will always be zero if a country borrows from one group only, irrespective of the level of the arrears to this group.

In addition, the $RPIA$ formula assigns a higher weight to country-year observations with higher arrears to debt ratios. This is relevant when aggregating $RPIAs$ over countries or time. For two countries with an identical arrears distribution across creditors, those creditors which face a higher arrears burden are also given a greater weight in the aggregate.

For illustration, consider a country with two creditors that chooses to default only towards creditor 1 while creditor 2 is fully paid. Let x denote the share in total debt and arrears of creditor 1. Then, the $RPIA$ of creditor 1 equals $(1 - x) \cdot ATD_1 > 0$ wheres $RPIA_2$ is simply given by the negative of the total arrears to debt ratio. In the limit when all funds are provided by creditor 1, $RPIA_1$ approaches zero indicating a fair treatment despite the concentration of arrears. Similarly, when all funds are provided by creditor 2, $RPIA_1$ approaches the arrears to debt ratio of creditor 1.

Figure 3.3 summarizes $RPIAs$ for the six creditor groups for the full sample. The results confirm the pecking order observed above. Bilateral creditors, banks and trade creditors have positive $RPIAs$, indicating discrimination, while multilateral creditors and the IMF have negative ratios. Bondholders have the lowest $RPIA$ within the group of private creditors. Exporters and suppliers are most heavily discriminated against: they face a 12.6 percentage point higher ATD than the representative creditor. The IMF is most favored with a 6.2 percentage point lower ATD compared to the average creditor.

The pecking order we observe in Figures 3.2 and 3.3 holds when we weight each country by the size of its debt stock so as to mimic a world portfolio of debt (and arrears). It also holds when we use arrears to debt ratios without adjusting the debt stock for the stock of arrears, although we get higher arrears ratios on average.

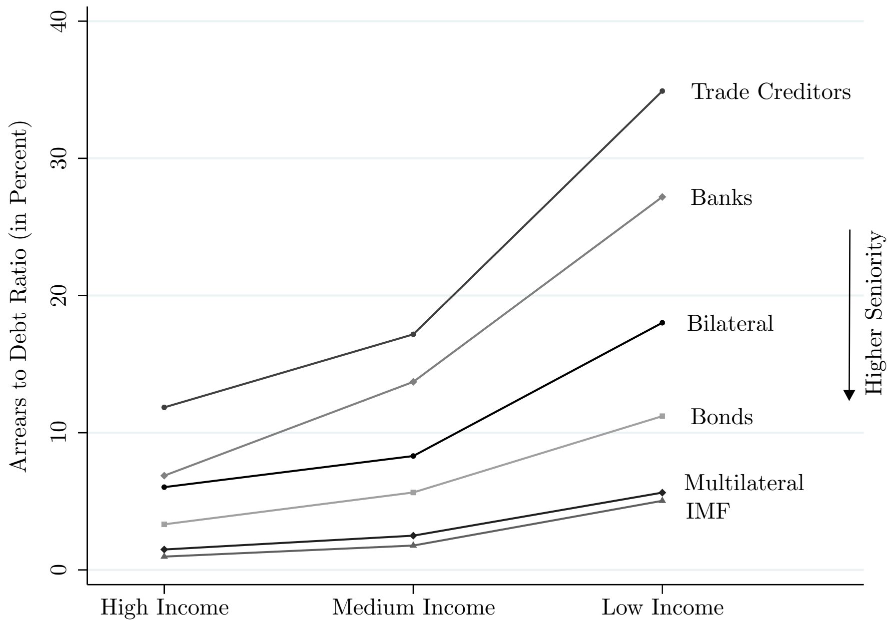
The $ATDs$ vary notably across countries and sub-samples. Figure 3.4 shows the average arrears to debt ratios for different income groups based on the classification of the World Bank though the ordering is similar when we use other GDP per capita criteria. As expected, $ATDs$ are highest for countries with low income and lowest for countries in the high income group. This holds for all creditor groups. Yet, the pecking order is qualitatively similar in each sub-sample we look at. Bonds, IMF credit and multilateral loans remain senior in all income groups whereas bilateral loans, bank loans and trade credits are junior.

The ordering is similar in a break-down by world regions, using the regional classification of the World Bank. The level of arrears is particularly high in Sub-Saharan Africa which includes most of the low income countries in the sample. However, the implied pecking order is again largely unaffected. Bonds, IMF credit and multilateral loans are senior in all regions whereas bilateral loans, bank loans and trade credits are junior.[16]

This pecking order is also unaffected if we cut the sample by political regimes, by countries with and without access to the capital market or by type of exchange rate regime. Table C.1 in the appendix summarizes average $RPIAs$ for different country groups. Bilateral creditors are discriminated against in all subsets except for the Middle East and North Africa region. Similarly, the average $RPIA$ is positive for banks and trade creditors in all subsets except for banks in East Asia. In contrast, multilateral creditors and the IMF are always favored. Similarly, bondholders are senior to other private creditors in all subsamples. Taken together, the pecking order is remarkably stable although the extent of seniority and the level of arrears differ markedly across subsets of the sample.

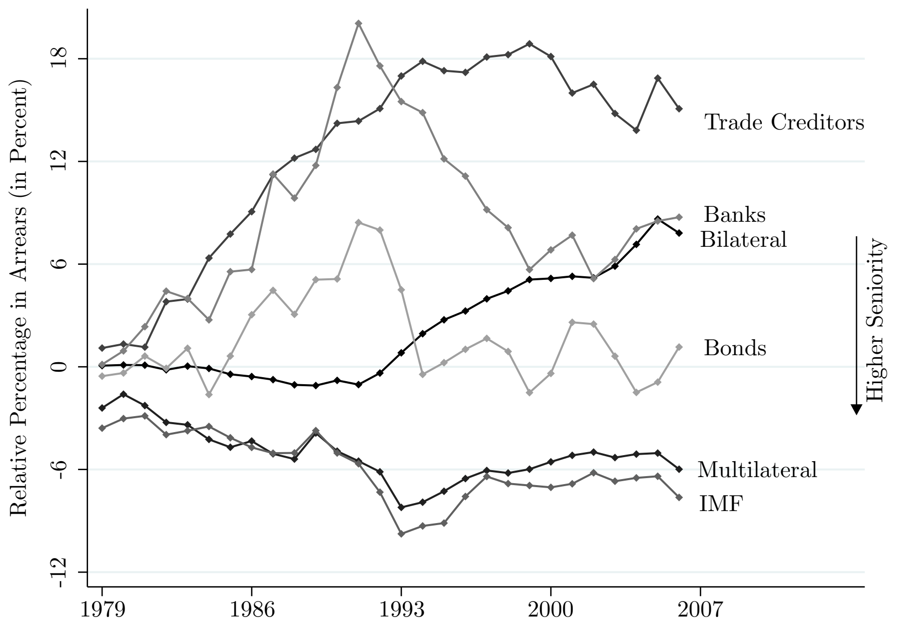
[16]East Asia is an exception since the IMF faces higher average arrears than banks, mainly because of two outliers: Cambodia and Vietnam. There are also smaller shifts in the relative ranking within the set of junior and senior creditors.

Figure 3.4: Arrears to Debt Ratios by Income Groups



Note: This figure shows average ATD ratios by debtor country income group. Income groups are based on the classification of the World Bank, where high income refers to "high income" or "upper middle income" countries here. Medium income countries are those classified as "lower middle income" by the World Bank. Averages are calculated as unweighted means of country averages.

Figure 3.5: Relative Percentage in Arrears over Time



Note: This figure shows the average RPIA ratios for each year from 1979 to 2006. Averages are calculated as unweighted means over all observations in each year.

The patterns we observe are also relatively stable across time. Between 1979 and 2006 the seniority order (as measured by $RPIAs$) has remained fairly similar, as show in Figure 3.5. The relative seniority of multilateral lenders and the IMF can be observed in each decade. Similarly, the discrimination against banks and trade creditors has occurred in all years, with banks faring worst in the late 1980s and early 1990s, just prior to the Brady debt exchanges that resolved the debt crises in developing countries. The most notable qualitative change in seniority can be observed for bilateral lenders, which were slightly favored until the early 1990s but have been increasingly discriminated against since then.

3.3.3 Determinants of Seniority: Fundamentals, Country and Time Effects

The established pecking order might be explained by observable characteristics of countries and creditors. For example, bilateral lenders might face higher arrears simply because they lend more extensively to low-income countries compared to bondholders or the IMF. To account for such a possibility, we control for the income level of countries and the scope of bilateral lending. If the different treatment cannot be explained away with macroeconomic and political variables, then we have a more convincing case that discrimination is at work. Thus, we can interpret the unexplained part of the ATD and $RPIA$ ratios as creditor seniority in a narrow sense.

We rely on two alternative empirical strategies to separate the role of observable from unobservable characteristics. First, a plain-vanilla reduced form pooled regression and, second, the Oaxaca-Blinder counterfactual decomposition technique, which has been widely used to identify discrimination in labor markets. In both parts the $RPIA$ ratios, our favorite metric of creditor discrimination, will be used as dependent variable.

To select control variables, we follow earlier work on the determinants of default and debt arrears, in particular Detragiache and Spilimbergo (2001) and Manasse and Roubini (2009), and consider a broad set of macroeconomic and financial variables that are potential drivers of arrears. We first account for the scale and composition of sovereign debt by including the debt to GNI ratio towards each creditor group (adjusted for arrears). This is the only group-specific control variable. Since our discrimination measure is already adjusted for creditor-specific debt stocks, including this variable will capture potentially over-proportional effects of debts on arrears. Moreover, we include the ratio of total external debt to GNI as emphasized by Manasse and Roubini (2009). We also account for the level of development, which is important for the default behavior as indicated by Figure 3.4. Specifically, we include real GDP per capita as well as current and lagged real GDP growth as reported in the World Bank's WDI dataset. We account for financial crises by including dummy variables for systemic banking crises and for currency crises from Laeven and Valencia (2012) and we control for political turmoil by including a dummy for ongoing external or civil wars, obtained from the Correlates of War dataset (see Sarkees and Wayman, 2010). Finally, the ability of countries to borrow in private capital markets is a potential determinant of arrears. Therefore, we include a dummy variable that proxies access to private markets. Specifically, we use a dummy that indicates whether a country is eligible for the IMF's Poverty Reduction and Growth Facility as proposed by Allen (2008).

We consider further control variables in our full specification though this comes at the cost of fewer observations. We aim to account for the level of financial development. For this purpose, we include the share of private debt in total external debt as well as a proxy of financial openness, which is defined as the sum of financial assets and liabilities as a fraction of GNI, using the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). We control for the broader macroeconomic situation by including a country's trade openness, defined as the sum of exports and imports to GNI, the gross investment to GNI ratio as well as CPI inflation, again using WDI data. Finally, we include the ratio of short-term external debt to GNI, a country's foreign reserves as a fraction of total external debt as well as debt service payments on PPG debt and the IMF relative to exports to account for liquidity considerations. Data is again obtained from the WDI. We summarize all variables used in the regressions in Table C.2 in Appendix C.3. To mitigate concerns of endogeneity we lag all explanatory variables by one period, except for real GDP growth which is included in contemporary and lagged form.

We start with a pooled OLS regression that uses creditor-specific ATDs and RPIAs as dependent variable, so that we include up to six observations for each country-year spell. To detect discrimination, we use dummies for each creditor group. The intuition behind this approach is simple: we assume that sovereigns should, in principle, respond to economic and political fundamentals equally across creditor groups. During a crisis, arrears should increase at an equal rate towards each group. However, by including creditor-specific dummy variables, we allow countries to discriminate on an ad-hoc basis, irrespective of the economic conditions faced at the time. The effect of observable fundamentals will be captured by the control variables, while (unexplained) creditor-specific effects will be picked up by the creditor dummies. One caveat with this approach is that we cannot use a fixed-effects regression framework since these dummies are constant on the country-creditor level. Instead, we later add country and year dummies to account for country-fixed and year-fixed effects.

Specifically, we use the following regression framework:

$$y_{i,t,k} = \boldsymbol{X}_{i,t,k}\boldsymbol{\beta} + \sum_{k=1}^{K-1} \alpha_k I_k + u_{i,t,k} \tag{3.4}$$

where $y_{i,t,k}$ is our discrimination measure and $\boldsymbol{X}_{i,t,k}$ is the vector of control variables that may include dummies for each country and for each year to capture country-fixed and year-fixed effects. I_k are creditor-specific indicator variables for each group k where $k = K$ refers to the benchmark category, which will be the group of bilateral creditors. $u_{i,t,k}$ are standard errors clustered on the country level.

In this setting, we identify creditor seniority by differences in intercepts among creditors, under the assumption of equal sensitivities towards $X_{i,t,k}$. The sign, significance and size of the I_k coefficients will indicate the seniority structure of debt after controlling for fundamentals. Table 3.1 shows the key results, focusing on the creditor group dummies. The full regression results are shown in Table C.3 in Appendix C.3.

The pecking order described in the last section is confirmed in Table 3.1. In column 1, we use the $RPIA$ as dependent variable and control for our basic set of fundamentals. All creditor dummies have the expected sign and are statistically significant at the 1% level except for bonds. The coefficients for multilateral creditors and the IMF are negative, indicating that these groups face significantly lower arrears per unit of debt and a lower $RPIA$ than the benchmark category (bilateral creditors). In contrast, banks and trade creditors face significantly higher arrears per unit of lending and a higher $RPIA$. In columns 2-4 we add country-fixed effects, time-fixed effects, as well as a richer set of control variables, respectively. Adding these explanatory variables clearly improves the fit of the regression, but barely changes the results for the discrimination dummies.

One concern with these results is that the discrimination measures show substantial persistence, as the $RPIAs$ are strongly correlated over time. We account for this by including a one-period lag of the dependent variable in column 5. The resulting coefficient is significant and sizable. Including it considerably improves the fit of the regression and decreases the size of the discrimination dummies. However, they still show statistically significant differences across creditor groups and imply the same seniority structure.

In a last step, we estimate Equation (3.4) using the arrears to debt ratios as dependent variable. Column 6 shows results for the pooled OLS estimation (with country and year dummies), while column 7 shows coefficients from a Tobit regression, to account for the truncated nature of the data as arrears and $ATDs$ cannot be negative. These estimates are qualitatively similar to our previous findings. The Tobit coefficients for IMF and bondholder arrears differ, however, reflecting the fact that arrears to these creditors generally occur less often than to other groups. Bondholders are significantly favored relative to bilateral creditors once we take into account that arrears occur far less frequently.

Table 3.1: Creditor Dummy Regressions with Arrears Measures

	(1) RPIA	(2) RPIA	(3) RPIA	(4) RPIA	(5) RPIA	(6) ATD	(7) ATD
Multilateral	-8.09***	-8.08***	-8.08***	-8.34***	-1.42***	-8.09***	-14.45***
	[1.03]	[1.03]	[1.04]	[1.20]	[0.20]	[1.04]	[0.78]
IMF	-10.39***	-10.56***	-10.70***	-10.92***	-1.68***	-10.74***	-54.70***
	[1.24]	[1.26]	[1.28]	[1.46]	[0.25]	[1.24]	[1.40]
Bondholders	0.07	0.35	0.54	-0.02	0.57	0.42	-11.91***
	[2.11]	[2.07]	[2.10]	[2.21]	[0.43]	[2.05]	[1.38]
Banks	7.17***	7.48***	7.59***	5.94***	1.50***	7.59***	5.76***
	[1.46]	[1.47]	[1.48]	[1.56]	[0.32]	[1.48]	[0.86]
Trade Creditors	10.70***	10.89***	10.98***	9.00***	1.93***	10.83***	12.04***
	[1.70]	[1.70]	[1.71]	[1.76]	[0.31]	[1.70]	[0.82]
Country Fixed Effects	No	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	No	No	Yes	Yes	Yes	Yes	Yes
Full Controls	No	No	No	Yes	No	No	No
Lagged RPIA	No	No	No	No	Yes	No	No
Adjusted R^2	0.195	0.242	0.249	0.249	0.786	0.437	0.133
Observations	11931	11931	11931	9176	11790	11931	11931

Standard errors in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Note: This table shows regressions results for equation (3.4). Columns 1-5 use the $RPIA$ as dependent variable, while columns 6-7 use $ATDs$. All regressions are estimated with pooled OLS except for column 7 which uses a Tobit regression of the ATD. All regressions include the set of basic controls listed in Table C.2 in the appendix. Column 1 shows the most parsimonious specification, column 2 adds country fixed effects, and column 3 also includes time fixed effects. This specification (column 3) is our benchmark and is also used in columns 6 and 7. Column 4 adds the full set of controls, while column 5 adds a lagged dependent variable.

The pooled regression framework is an intuitive way to separate the discriminatory component from the effects of economic fundamentals. Discrimination dummies offer a ranking of creditors which is easy to interpret - both qualitatively and quantitatively. However, there are also drawbacks. One limitation is that this approach assumes that sovereigns (should) behave equally across creditors in response to changing economic conditions. This is because the coefficients for all control variables are assumed to be the same for each creditor group, which is unlikely to be true. Indeed, when running regressions for the determinants of discrimination for each creditor group separately, we observe substantial differences in the estimated betas. Therefore, we relax the assumption of equal sensitivities across creditors in a next step.

Our second approach to identify creditor seniority is the Oaxaca-Blinder decomposition technique. The technique is based on Blinder (1973) and Oaxaca (1973) and is commonly used in the literature on labor market discrimination, for example, to estimate the gender wage gap. The basic idea is to decompose an outcome variable of interest into explained and unexplained variation using a set of control variables. More precisely, the technique attributes the difference in mean outcomes of two group to two sources: differences in observable characteristics and differences in factor sensitivities. For example, high arrears to bank creditors relative to bondholders could be the result of more intense borrowing from banks, e.g. because countries have no access to bond financing. Alternatively, sovereigns could simply be more inclined to accumulate arrears towards banks creditors rather than bondholders for a given amount of borrowing. The first explanation reflects fundamental differences, while the second one is interpreted as discriminatory behavior. The Oaxaca-Blinder decomposition technique allows us to separate these effects by controlling for differences in control variables across creditors. The unexplained component then allows us to back out the scope of creditor discrimination.

Formally, consider a standard linear regression model that relates the discrimination measure $y_{i,t}$, the $RPIAs$, to a set of fundamentals $\boldsymbol{X}_{i,t}$ and the error term $u_{i,t}$ for each creditor group k:

$$y_{i,t} = \boldsymbol{X}_{i,t}\boldsymbol{\beta}_k + u_{i,t} \tag{3.5}$$

Let $\hat{\boldsymbol{\beta}}_k$ denote the vector of estimated coefficients for creditor group k. We estimate this regression separately for each creditor group via pooled OLS including dummy variables for country-fixed and time-fixed effects in the vector of control variables.[17] The difference in mean $RPIAs$ between *two* creditor groups m and n can then be decomposed as:

$$\overline{RPIA}_m - \overline{RPIA}_n = (\overline{\mathbf{X}}_m - \overline{\mathbf{X}}_n)\hat{\boldsymbol{\beta}}^*_{m,n} + \overline{\mathbf{X}}_m(\hat{\boldsymbol{\beta}}_m - \hat{\boldsymbol{\beta}}^*_{m,n}) + \overline{\mathbf{X}}_n(\hat{\boldsymbol{\beta}}^*_{m,n} - \hat{\boldsymbol{\beta}}_n) \tag{3.6}$$

where $\overline{\mathbf{X}}$ refer to the sample means of the control variables. This decomposition assumes the existence of a fair pricing vector $\hat{\boldsymbol{\beta}}^*_{m,n}$. For example, $\hat{\boldsymbol{\beta}}^*_{m,n}$ would tell us by how much the $RPIA$ should increase if a creditor's debt to GNI ratio increases by one unit given fair treatment. However, this vector is not observable and hence needs to be approximated. In the literature on wage discrimination, it is commonly approximated by the coefficient vector of male workers who are assumed to face no wage discrimination. Since we do not know which creditor group faces fair treatment, we obtain $\hat{\boldsymbol{\beta}}^*_{m,n}$ by estimating (3.5) in a pooled regression for groups m and n. As a caveat, this approach only allows pairwise comparisons of creditor groups.

Using (3.6), we can attribute differences in $RPIAs$ to the two sources discussed above:

- The first term $(\overline{\mathbf{X}}_m - \overline{\mathbf{X}}_n)\hat{\boldsymbol{\beta}}^*_{m,n}$ measures differences in $RPIAs$ that result from differences in the control variables across creditor groups. For instance, higher arrears to bank creditors relative to bondholders as the result of more intense borrowing from banks are captured by this term.

[17] OLS is the standard approach for Oaxaca-Blinder in the literature and convenient to estimate.

Table 3.2: Oaxaca-Blinder Decomposition

	Difference in RPIA	Explained (fundamental)	Unexplained (discrimination)	Discrimination (in Percent)
Bilateral - Multilateral	7.99	0.08	7.91	99.00
Bilateral - IMF	9.10	3.00	6.10	67.07
Bilateral - Bondholders	0.86	1.47	-0.62	-71.63
Bilateral - Banks	-7.68	-2.52	-5.16	67.22
Bilateral - Trade Creditors	-11.59	-3.31	-8.28	71.45
Multilateral - IMF	1.11	1.11	0.00	-0.54
Multilateral - Bondholders	-7.13	-3.07	-4.06	56.94
Multilateral - Banks	-15.66	-4.10	-11.57	73.84
Multilateral - Trade Creditors	- 19.58	-3.99	-15.59	79.62
IMF - Bondholders	- 8.24	- 3.50	- 4.74	57.55
IMF - Banks	-16.77	-1.26	-15.51	92.49
IMF - Trade Creditors	-20.69	- 0.13	-20.55	99.37
Bondholders - Banks	-8.53	-5.16	-3.38	39.58
Bondholders - Trade Creditors	-12.45	-8.34	-4.10	32.96
Banks - Trade Creditors	-3.91	-1.92	-2.00	50.99

Note: The table shows the Oaxaca-Blinder decomposition for the mean difference in RPIAs across creditor groups. The mean difference is listed in the first column. The second and third columns show the part of the difference that is fundamentally justified and the part of the difference that is unexplained and thus indicative of creditor discrimination. The last column states the fraction of the mean difference that reflects discrimination in percent. Note that mean differences vary from the sample statistics because of data availability with respect to the explanatory variables.

- The second term $\overline{\mathbf{X}}_m(\hat{\boldsymbol{\beta}}_m - \hat{\boldsymbol{\beta}}^*_{m,n}) + \overline{\mathbf{X}}_n(\hat{\boldsymbol{\beta}}^*_{m,n} - \hat{\boldsymbol{\beta}}_n)$ is the difference in $RPIAs$ that results from differences in the estimated factor sensitivities for a given level of fundamentals. This term constitutes discrimination since these differences would prevail even if creditor groups had equal characteristics.

Importantly, decomposition (3.6) can only be used for pair-wise comparisons. We therefore show results for each creditor-group-pair separately, e.g. to tease out the explained and unexplained differences in arrears patterns towards banks versus bondholders or towards bilateral creditors versus the IMF. Table 3.2 summarizes these results.

We use the set of basic controls defined in Table C.2 as well as dummies for country-fixed and time-fixed effects. The first column shows mean differences in $RPIAs$ for each creditor-pair. This difference is decomposed into differences that are attributable to differences in fundamentals (Column 2) and differences that are unexplained, indicating discrimination (Column 3). The last column shows the share of the differences in $RPIAs$ that cannot be explained by the control variables, which can be interpreted as the intensity of discrimination between two creditor types.

Consider for example the aforementioned comparison of bank creditors and bondholders, which is illustrated in the third row from below in Table 3.2. On average, bondholders face a 8.5 percentage point lower $RPIA$ than banks, so they appear to be favored. Around 60% of this difference (5.2 pp) can be attributed to differences in observable characteristics. This might be because bondholders lend to richer countries or in more favorable time periods and hence face lower arrears. Yet, roughly 40% of the difference (3.4 pp) cannot be attributed to these factors and hence represents discriminatory treatment of banks or favoritism towards bondholders.

In general, the unexplained component is sizable, accounting on average for roughly 60% of the mean difference in $RPIAs$ across creditor groups. This is a strong indication for the existence of an implicit seniority structure. For three pairs - bilateral versus multilateral creditors, the IMF versus banks and the IMF versus trade creditors - almost all of the difference is based on the discriminatory component. Fundamentals can explain more than half of the difference only for multilateral creditors compared to the IMF, bondholders compared to bank creditors and bondholders relative to trade creditors. Again, we account for serial correlation by adding the lagged dependent variable as additional regressor. Table C.4 in Appendix C.3 shows the corresponding Oaxaca-Blinder decomposition. The discriminatory component averages around 12% which is substantially lower than before although still sizable.

To summarize, the regression and decomposition results confirm the stylized facts on the seniority structure of sovereign debt. There is a clear and robust pecking order of sovereign debt during defaults that holds even when controlling for a wide variety of macroeconomic and political factors that might affect discriminatory behavior of sovereigns.

3.4 Creditor Seniority in Sovereign Debt Restructurings

In this section, we analyze unequal treatment of creditors during restructurings by comparing the size of creditor losses (haircuts) on private and official debt.

Specifically, we use the estimates by Cruces and Trebesch (2013) for haircuts on private external creditors, which cover 187 restructurings of bank loans and bond debt since the late 1970s. For haircuts on official external creditors, we rely on an updated version of the dataset by Reinhart and Trebesch (2016b) and Tolvaisaite (2010), which provides haircut estimates for more than 400 Paris Club restructurings. The Paris Club is a club of creditor governments founded in 1956 and has been the main forum for renegotations of official debt over the past decades.[18] The regular members of the Paris Club include the most important creditor country governments, in

[18] There is also a number of debt restructurings with official creditors outside the Paris Club forum. However, systematic information and data availability on these deals is even more limited than on the Paris Club. Hence, we choose to focus on the Paris Club as the single most important subset of official debt renegotiations.

particular OECD countries. Its stated purpose is to provide assistance for debtor countries with payment difficulties based on the principles of unanimity and equal burden sharing among its members. Moreover, the Paris Club emphasizes the comparability of treatment principle, under which private creditors should get no better treatment than official ones. An overview on the history and operations of the Paris Club is provided by Cheng, Diaz-Cassou, and Erce (2016).

We follow the literature, in particular Sturzenegger and Zettelmeyer (2008), Benjamin and Wright (2009) and Cruces and Trebesch (2013), to compute haircuts for sovereign debt restructurings. Specifically, we apply the following formula to all restructurings since 1978 for which sufficient data is available:

$$\text{Haircut} = 1 - \frac{\text{Present Value of New Debt}}{\text{Face Value of Old Debt}} \tag{3.7}$$

We thus compare the face value of the old debt before the restructuring to the present value of the new debt after the restructuring. Using face values differs from Sturzenegger and Zettelmeyer (2008) and Cruces and Trebesch (2013), but is helpful for the context here.[19] From a practical point of view, fewer assumptions are necessary and face value data on the debt affected is provided on the Paris Club website. Moreover, using face values for the old debt can be justified conceptually by the fact that debt repayment can be accelerated in default situations so that the creditor is entitled to the immediate repayment of future payment streams. Whichever method, we are ultimately interested in the *relative* performance of private and official creditors during restructurings so that the choice of the haircut estimation methodology should not be crucial as long as it is consistent across groups.

3.4.1 Computing Haircuts on Official Sovereign Debt (Paris Club)

The haircut estimates for official sovereign debt restructurings, which were computed by Reinhart and Trebesch (2016b) and are updated here, are based on a number of assumptions regarding interest rates, debt payment profiles and discounting. This is because we lack information on the detailed loan-level restructuring terms due to the intransparency of the Paris Club. As a result, the estimated haircuts are to be taken with care, particularly for restructurings in the 1980s for which the Paris Club provides the least degree of details.

With these limitations in mind, the estimates are helpful for gaining an overview of trends and patterns of Paris Club debt relief, and suffice to make a rough comparison of haircuts of private versus official creditors that complements our analysis of arrears patterns. We also try to

[19] For a detailed discussion of present value versus face value haircut estimates, see the aforementioned contributions by Sturzenegger and Zettelmeyer (2008) and Cruces and Trebesch (2013).

be consistent within our approach and outline all assumptions and limitations as transparently as possible. Moreover, we discuss the robustness of our results to variations in our main assumptions. Future research might be able to come up with more accurate estimates, although a move towards higher transparency on the part of the Paris Club will be necessary.

At the center of each Paris Club debt restructuring is the final agreement, also called the "Agreed Minutes". The Agreed Minutes are agreed by all members and provide a guideline for the bilateral agreements with each creditor government, which formalize the debt relief legally (see Appendix C.2 for details).[20] The terms of the Agreed Minutes are for the most part standardized and have evolved over time. The 1980s mainly saw short- and medium-term reschedulings in the form of maturity extensions, but over the course of the 1990s and 2000s the terms became increasingly concessionary, granting more comprehensive debt relief and debt stock cancellations.[21] The Agreed Minutes contain details on the consolidation period, the cut-off date, the grace and maturity periods of the new debt, and on the amounts of restructured debt as well as potential debt write-offs. This information is the main input for the estimation of haircuts on Paris Club restructurings. We compare and complement the information from the Paris Club website with details from other sources. These include a survey of debt restructurings with official creditors from the Institute of International Finance (2001), information from the Global Development Finance reports as well as the IMF's Annual Reports on Official Financing for Developing Countries.[22] In case of inconsistencies, we use the Paris Club data.

Our final sample includes 414 Paris Club debt restructurings between 1978 and 2015.[23] These were particularly frequent during the heydays of the debt crises in the 1980s and 1990s but have declined substantially since the mid-2000s. Since the late 1980s, the Paris Club allows for differential treatment of official development aid (ODA) and market-related debt like export credits (non-ODA). In case of ODA debt the grace and maturity periods are more generous, while non-ODA debt can receive partial write-offs, particularly for low income countries.

Since face value data is provided by the Paris Club, most assumptions concern the calculation of present values in (3.7). These are calculated as follows:

$$\text{Present Value of the New Debt} = \sum_{j=t_0}^{T} \frac{\text{Debt Service}_j(r_{ds})}{(1 + r_{discount})^j} \tag{3.8}$$

[20]Unfortunately, the specific bilateral agreements that actually provide debt relief are not publicly available. Hence, we trust that the Agreed Minutes are a sufficiently reliable indicator for the bilateral terms.

[21]Martin and Vilanova (2001), Gueye et al. (2007) and Cheng, Diaz-Cassou, and Erce (2016) provide a more detailed overview of the different terms. Also see Appendix C.2 for a summary.

[22]In particular, we refer to the appendices of the 2002, 2003 and 2006 Global Development Finance reports.

[23]There is a small number of Paris Club deals before 1978, starting with Argentina in 1956. We do not include these deals due to limited data availability and also because we lack the counterfactual haircut estimates for private creditors. For an overview of early Paris Club deals, see Das, Papaioannou, and Trebesch (2012).

where t_0 is the date in which the first interest or principal payment is due (after a potential grace period), T is the maturity of the new debt, r_{ds} is the interest rate charged on the new debt and $r_{discount}$ is the relevant discount rate. In the following we will discuss our main assumptions regarding the interest rate r_{ds}, the discount rate $r_{discount}$, the debt repayment profile (including any potential write-offs) and the composition of restructured debt.

1. Interest Rate: The interest rate on restructured Paris Club debt is called the moratorium interest rate. According to the terms of the Paris Club, moratorium interest rates are either based on the original interest rate of the loan for ODA debt, the "appropriate market rate" (non-ODA debt, option A) or reduced interest rates (non-ODA debt, option B).[24] Unfortunately, the rates actually agreed on bilaterally are not publicly available and we do not know what share of the debt falls under each of these options. The Paris Club only provides a vague definition of the "appropriate market rate" from which reduced rates are derived. Importantly, none of these rates contains a country-risk premium.[25] Hence, we need to make assumptions with respect to the interest rate in our calculations. We assume that each rescheduling is based on the appropriate market rate. We proxy this rate by the average terms of borrowing (ATB) by official creditors from the World Bank's GDF dataset. This rate is representative for the average interest rate set by official creditors for a particular debtor.[26] Data coverage is almost comprehensive since the ATB is available for all but 6 restructurings. In addition, Reinhart and Trebesch (2016b) also use this rate to calculate debt service payments in their baseline estimates.[27]

2. Discount Rate: We consider two alternatives for this important variable:

- The baseline estimates rely on the Paris Club's own discounting approach, using the commercial interest reference rate (CIRR) for the US-Dollar prevailing in the year and month of the restructuring (see Mandeng, 2004). CIRR is essentially a risk-free rate that represents the funding cost of advanced countries. More specifically, CIRRs are currency-specific interest rates for major advanced countries defined as "minimum interest rates that shall apply to official financing support under the Arrangement on Officially Supported Export

[24] For an overview, see for example Martin and Vilanova (2001) or the website of the Paris Club.

[25] The Paris Club defines the Appropriate Market Rates as: "Interest rate defined in bilateral agreements implementing the Paris Club Agreed Minutes, based upon standard interest rates of the currency considered, plus a management fee. This rate may be fixed or variable and does not include a country-risk premium."

[26] This rate is defined as follows: "Average terms of borrowing on public and publicly guaranteed debt are given for all new loans contracted during the year and separately for loans from official and private creditors. Annual average interest rate is obtained by weighting by the amounts of the loans."

[27] An alternative proxy is the 6-month USD based Libor that prevailed in the year and month of the restructuring deal, using data from the IMF. Yet, unlike the average terms of borrowing, the Libor rate is not specific to the debtor country. As a robustness check, we also conduct the analysis with the Libor as an interest rate proxy. The results do not change qualitatively.

Credits" (see OECD website). Besides the Paris Club, also the World Bank and the IMF calculate official debt relief based on a weighted average of CIRRs for different currencies (see International Monetary Fund, 2014). In addition, CIRR data is widely available allowing us to compute haircuts for 397 out of 414 Paris Club deals. However, this rate is not country-specific and does not reflect the market assessment of country risks. We take the resulting haircut estimates as a lower bound.

- Alternatively, we use a market-based discounting approach, which allows to compare haircuts on private and official creditors based on the same discount rates. The idea is to run a fair horse-race between the two type of estimates and to view official debt through the lens of a risk-averse taxpayer who discounts future repayments with a commercial rate. Specifically, we follow the discounting approach by Cruces and Trebesch (2013) for private debt restructurings and use their computed "exit yields" that vary by country and month to compute haircuts for Paris Club restructurings. This is possible for 323 out of the 414 deals.[28] The exit yield is the interest rate that would have been used by the market participants at the exit of each restructuring if the holder of the restructured instrument wanted to sell it. Put differently, the exit yield is a proxy for future default risk right after the restructuring has been implemented. The resulting haircut estimates are taken as an upper bound.

3. Restructuring Options, Amortization Schedules, and Write-offs:

- We distinguish between ODA and non-ODA debt whenever information on the distribution is provided by the Paris Club. This is the case for most restructurings since the late 1990s, namely 75 Paris Club deals. In addition, for 178 restructurings under Classic terms we know that the Paris Club creditors made no distinction between ODA and non-ODA debts. For all other cases (around 40% of the sample), we assume that all debts are non-ODA debt when terms of middle income countries apply (34 restructurings under Houston terms) and ODA debt when terms of low income countries apply (112 restructurings under Toronto, London, Naples, Lyons and Cologne terms).

- Debtor countries can often choose between two rescheduling options for non-ODA debt. Under option A debtors receive a partial debt stock cancellation and the new debt is issued with market interest rates. Alternatively, they can choose option B without debt stock

[28] We lack imputed market rates for about 20% of the sample. This is mostly due to the fact that the poorest countries have little to no market information such as credit ratings. Indeed, 77 of the 91 missing cases are HIPCs. The smaller sample will result in a downward bias, since average haircuts are likely to be higher if these poorest countries were included.

cancellation but at a below-market interest rate and longer grace and maturity periods. Gueye et al. (2007) explain that before the HIPC Initiative around half of the Paris Club creditors chose each option. With the advance of the debt relief initiative, option A became the dominant one. Unfortunately, we have no information which option is actually chosen for around one fourth of the sample (112 out of 414 restructurings). However, our previous assumption implies that all debts are treated as ODA debt in these cases. This implies that we do not need to take cancellation rates into account (since these only apply to non-ODA debt), but only vary grace and maturity periods.

- We apply the amortization schedule outlined in the Paris Club terms whenever available (about 40% of all cases).[29] Otherwise, a linear amortization schedule is assumed, meaning that the principal is redeemed in equal amounts in each year between the end of the grace period and maturity. Furthermore, we assume that the rescheduling date equals the date when the Agreed Minutes were signed and that grace and maturity period start at the midpoint of the consolidation period. The latter assumption is in line with the methodology of the secretariat of the Paris Club as argued by Martin and Vilanova (2001).
- Finally, we ignore additional bilateral debt relief by creditor governments that is provided on top of Paris Club agreements.[30]

The resulting haircut estimates are illustrated in Figure 3.6 for the risk-free CIRR discount rate and Figure 3.7 for the market-based rate, where the circles represent the volume of restructured debt in real terms. Figure 3.8 is the equivalent for haircuts in private debt restructurings from Cruces and Trebesch (2013). In addition, Table C.5 in Appendix C.3 summarizes basic statistics for our haircut estimates both for the full sample and during different decades.

Our estimates depend strongly on the discount rate assumption. In general, haircuts are lower when we use the CIRR as a discount rate instead of the market exit rates. Specifically, the average Paris Club haircut is 59.9% with CIRR discounting and 78.6% with market exit rates.[31] This is to be expected since the CIRR serves as a lower bound on the range of possible discount rates while market rates represent an upper bound. However, the correlation between the resulting sets of estimates is very high with a correlation coefficient of 0.87.

[29] Standardized repayment schedules are available for restructurings under London, Naples, Lyons and Cologne terms as well as for a small number of recent restructurings under Classic terms, like Gabon in 2000 or the Dominican Republic in 2005. These account for more than 40% of all Paris Club restructurings. All of them have an increasing amortization schedule, as the fraction of the principal due for repayment increases over time.

[30] Bilateral creditors often expressed their willingness to grant debt relief in addition to their commitments under the HIPC Initiative. For instance, this resulted in full debt relief for Afghanistan in 2010. However, we restrict our analysis to the standard terms and ignore additional relief. Hence, realized haircuts for HIPC Exit Terms will be higher than the estimated ones once we take additional relief into account.

[31] The respective values are 52.9% for CIRR rates and 76.1% for market exit rates when using the Libor instead of the ATB as interest rate.

Figure 3.6: Haircuts on Official Debt (Risk-free Rate)

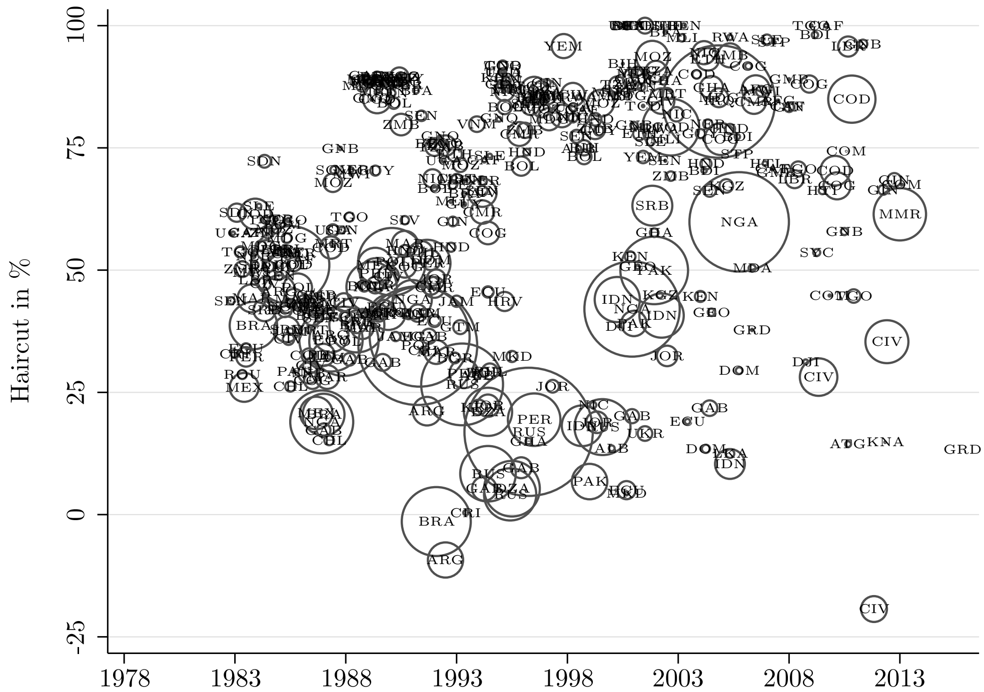
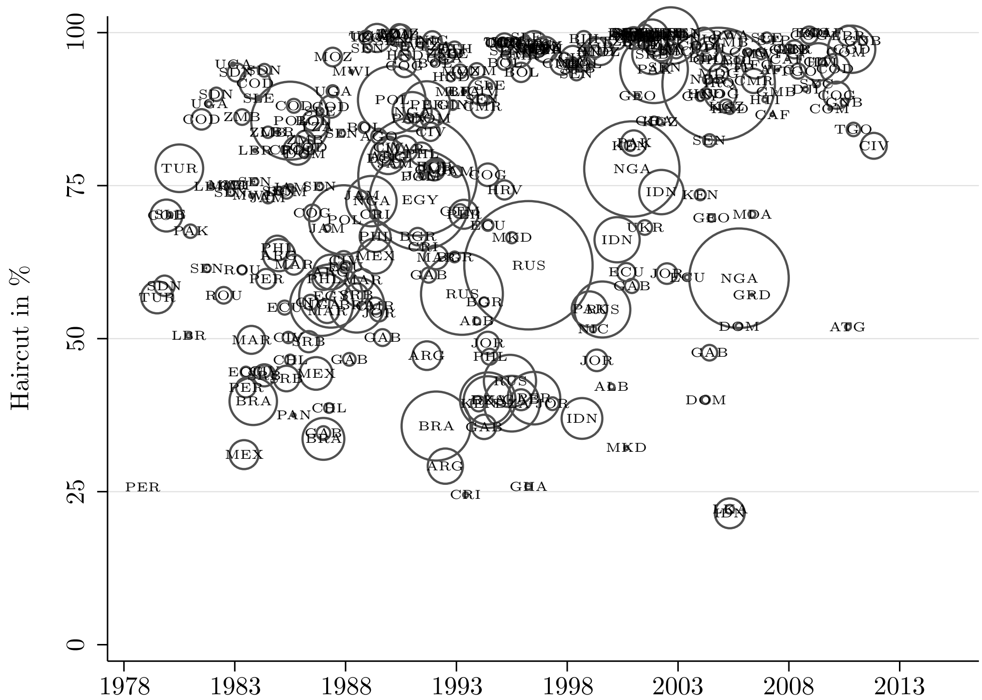
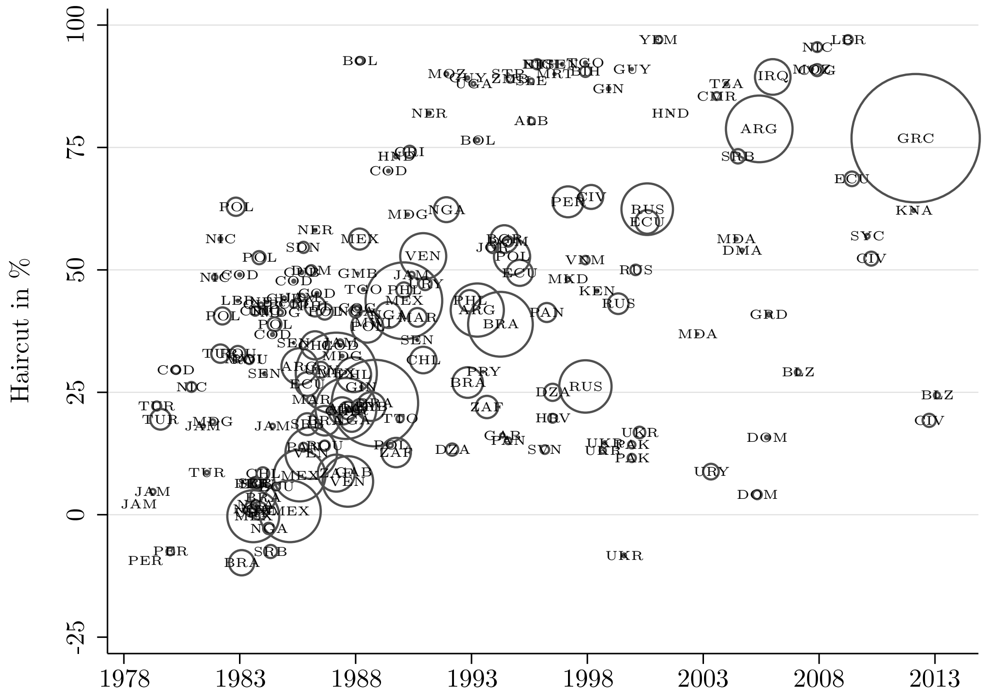


Figure 3.7: Haircuts on Official Debt (Market Rate)



Note: Circles represent the volume of restructured debt in real terms (i.e. inflated to 2015 US-Dollar using the US Consumer Price Index).

Figure 3.8: Haircuts on Private Debt



Note: Circles represent the volume of restructured debt in real terms (i.e. inflated to 2015 US-Dollar using the US Consumer Price Index).

3.4.2 Restructurings with Private and Official Creditors: Stylized Facts

Our dataset covers 601 restructurings of official and private external sovereign debt in 97 countries over the years 1978 to 2015. We use this data to establish stylized facts on creditor losses that shed light on the seniority structure of sovereign debt over the past 40 years. Importantly, our Paris Club haircut estimates should be treated with caution. In particular, the estimates are not suitable for a fine-grained comparison of haircuts on official and private debt in individual crisis cases (like Argentina 2001-2010). Put differently, our data do not allow for a systematic test of the "comparability of treatment" principle. Such an analysis would require more precise and more comprehensive data on haircuts on official debt. However, we can summarize the following stylized facts on restructurings:

- There have been more than twice as many Paris Club restructurings (414) as restructurings with private creditors (187) since 1978. In this group, restructurings of bank loans (165) have been far more frequent than bond restructurings (22). In total, 88 different countries have implemented at least one Paris Club deal whereas private debt obligations were restructured with a total of 71 different countries. This is illustrated in Figure C.3 in Appendix C.3.

- Although debt restructurings with private creditors occur less frequently, they typically affect larger nominal debt volumes. This is mostly due to restructurings with middle income countries that involve larger amounts of private debt. However, the numbers are similar when scaled by the debtor's GNI. The average amount of debt affected in a Paris Club deal is 11.9% of GNI, while the respective number is 12.8% for private debt restructurings. Restructured amounts are shown in Figure C.4 in Appendix C.3.

- The number of private and official debt restructurings within a country is correlated, but there is considerable variation in the type of restructurings across countries. High-income countries are more likely to implement private debt restructurings, while countries with low income and no capital market access are more likely to restructure with the Paris Club. Serial restructurings frequently occur. For example, Senegal experienced 15 Paris Club deals, while Poland featured 8 restructurings with commercial creditors.

Moreover, we can summarize the following facts on the relative size of credit losses (haircuts) during these restructurings:

- Average haircuts on official creditors are substantially higher than those on commercial creditors. Panel (a) of Figure 3.9 compares the conservative Paris Club haircuts (lower-bound estimates with CIRR discount rates) and the haircuts for private creditors of Cruces and Trebesch (2013) (using market discount rates) over all restructurings in our sample. The difference is more than 20 percentage points on average, with Paris Club deals showing a mean haircut of 59.9%, compared to 40.4% and 40.5%, for bond and bank loan haircuts, respectively.

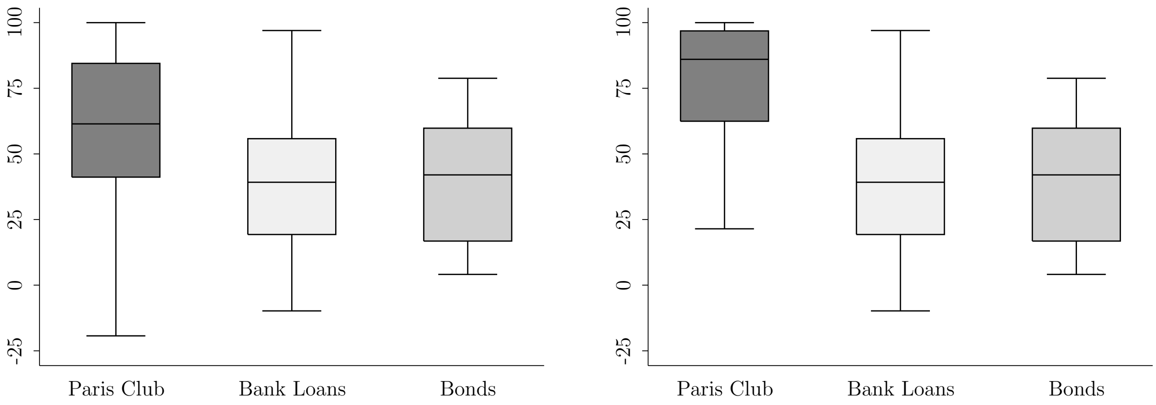
- The difference is even larger when we use the same discounting approach for both official and private restructurings, as shown in panel (b) of Figure 3.9. The average haircut for Paris Club deals then increases to 78.6%, again compared to 40% for private creditors. These are very large and statistically significant differences.

- The average haircut size difference between official and private creditors is largest for poor countries. Haircuts generally show a negative correlation with country income, for both official and private creditors. But Figure 3.10 shows that haircuts by the Paris Club are particularly high for lower and low-income countries. The average Paris Club haircut for low-income countries is 74.9% (using CIRR discounting), compared to 58.8% for private debt restructurings in this income group. These low-income countries have been granted increasingly concessional terms by official creditors.

Figure 3.9: Comparison of Haircuts for Different Discount Rates

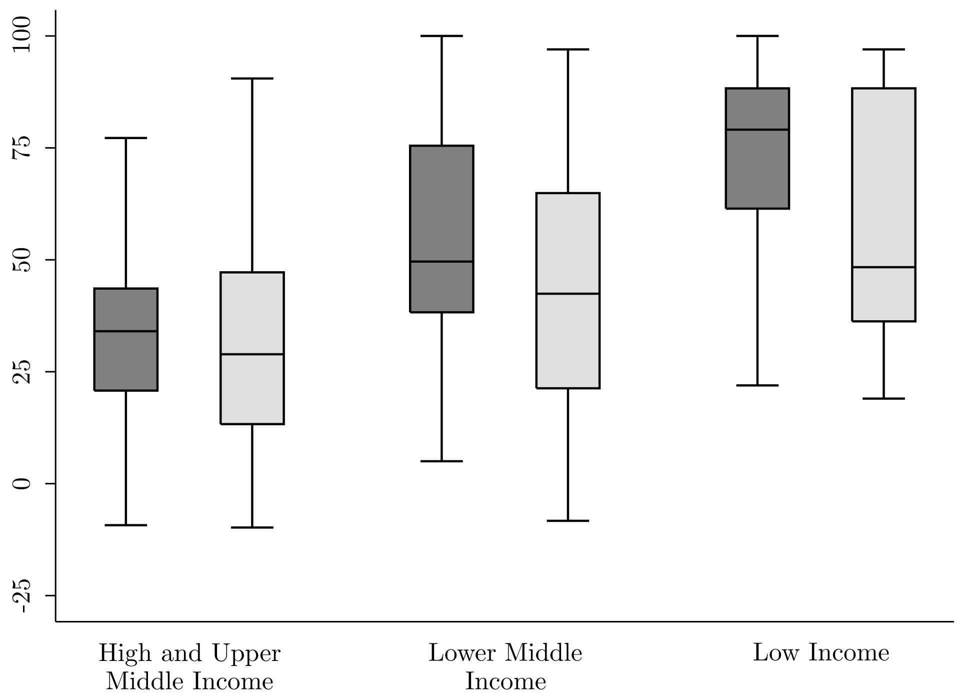
(a) Lower Bound Paris Club Haircuts

(b) Upper Bound Paris Club Haircuts



Note: The figures shows haircuts for official and private creditors in percent. The left panel shows lower bound estimates for the Paris Club, based on the risk-free (CIRR) discount rate. The right panel shows upper bound estimates using the same market discount rates (imputed exit yields) as in Cruces and Trebesch (2013). In both panels, the haircuts for bonds and banks are those of Cruces and Trebesch (2013).

Figure 3.10: Comparison of Haircuts by Income Group



Note: The left bars show Paris Club haircuts based on CIRR discount rates, while the right bars show haircuts for private creditors from Cruces and Trebesch (2013). Income groups are defined by the World Bank, as discussed in Figure 3.4 of the previous section.

3.4.3 Determinants of Haircuts: Fundamentals, Country and Time Effects

The comparison of haircuts suggests that private creditors fare better than official ones during restructurings. However, these differences might reflect debtor country characteristics rather than creditor discrimination. Hence, we need to control for factors that explain higher haircuts on official debt. In particular, many countries that restructure with the Paris Club have lower income and limited or no access to private capital markets. They also benefit from concessional lending in the form of official development aid. Furthermore, time effects might play a crucial role as global debt relief initiatives such as the HIPC Initiative are a main reason for the trend of increasing haircuts in the 1990s and early 2000s.

To control for country characteristics, we apply a standard OLS regression framework on the determinants of haircuts. Specifically, we pool the haircut estimates on private creditors and the Paris Club (with CIRR discounting) and regress these on dummy variables that capture the creditor type (private versus official) as well as a broad range of controls. This is conceptually the same as our approach in the previous section, where we include creditor dummies to study arrears in an OLS framework. The resulting regression can be written as follows:

$$HC_{i,t,k} = \boldsymbol{X}_{i,t,k}\boldsymbol{\beta} + \alpha_p I_p + u_{i,t,k} \tag{3.9}$$

where $HC_{i,t,k}$ the haircut on creditor group k in a restructurings with country i at time t, $\boldsymbol{X}_{i,t,k}$ is a vector of controls, I_p is an indicator variable for restructurings with private creditors and α_P is the associated coefficient of interest. Paris Club restructurings are the benchmark. The sign, significance and size of α_P indicates the ranking of creditors in restructurings after controlling for fundamentals. $u_{i,t,k}$ are standard errors clustered on the country level.

As controls, we choose the variables from section 3.3, but need to account for the different nature of restructurings by modifying our basic set of controls. First, we replace the amount of debt outstanding to each creditor group with a measure of debt restructured in each deal (relative to total external debt). In addition, we use the three-year moving average GDP growth rate prior to the restructuring instead of the contemporaneous and lagged GDP growth. Furthermore, the dummies on disturbances, banks and currency crisis are modified to indicate the existence of a respective crisis in the 3 years before a restructuring. All other controls remain the same. As before, we start with this set of basic controls and subsequently add time-fixed effects as well as further control variables with less data coverage. The latter are measures of trade and financial openness, the debt composition of a country, variables related to liquidity needs, in particular foreign reserves and debt service, as well as further macroeconomic controls.

Table 3.3: Creditor Dummy Regressions with Haircut Measures

	Lower Bound Estimates			Upper Bound Estimates		
	(1)	(2)	(3)	(4)	(5)	(6)
Private Creditors	-8.83***	-5.30**	-5.21*	-29.41***	-26.67***	-26.51***
	[2.46]	[2.42]	[2.88]	[2.34]	[2.25]	[2.65]
Restructured Debt (Share of External Debt)	0.24**	0.14*	0.10	0.21**	0.13	0.12
	[0.09]	[0.08]	[0.09]	[0.09]	[0.08]	[0.08]
External Debt to GNI	0.03***	0.03***	0.10***	0.05***	0.04***	0.08**
	[0.01]	[0.01]	[0.04]	[0.01]	[0.01]	[0.03]
Real GDP per Capita (in 1000 constant 2005 US$)	-3.70***	-3.60***	-3.27***	-3.50***	-3.27***	-2.71***
	[0.88]	[1.07]	[1.14]	[0.74]	[0.81]	[0.88]
GDP Growth (3-year MA)	1.51***	0.88***	0.78***	1.09***	0.61*	0.59
	[0.27]	[0.28]	[0.26]	[0.31]	[0.33]	[0.37]
War or Civilwar	1.49	0.10	-0.21	0.73	-1.14	-1.26
	[3.39]	[3.44]	[3.94]	[3.06]	[2.85]	[3.28]
Banking Crisis	0.35	-1.32	-0.85	-0.71	-3.81	-2.03
	[2.79]	[2.56]	[3.39]	[2.95]	[2.86]	[3.72]
Currency Crisis	-4.10	-2.82	-3.27	-2.67	-0.59	-1.22
	[2.64]	[2.33]	[2.82]	[2.43]	[2.37]	[2.78]
Lack of Market Access	23.54***	21.37***	17.94***	17.70***	16.94***	15.11***
	[3.89]	[4.56]	[4.92]	[3.88]	[4.24]	[4.89]
Time Fixed Effects	No	Yes	Yes	No	Yes	Yes
Full Set of Controls	No	No	Yes	No	No	Yes
Adjusted R^2	0.484	0.578	0.576	0.607	0.680	0.673
Observations	442	442	359	389	389	314

Clustered standard errors (on country level) in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Note: The table shows the regression results for equation (3.9). Columns 1-3 use haircuts with the risk-free (CIRR) discount rate as dependent variable, columns 4-6 use haircuts with market rates. All regressions are estimated with pooled OLS. All regressions include a set of basic controls. We subsequently add time fixed effects and a richer set of controls. Variables are defined in Table C.2 in Appendix C.3 and in the text.

Table 3.3 shows our main results on correlates of haircuts. Columns 1-3 show results using the lower-bound estimates for the Paris Club haircuts (CIRR discounting), while columns 4-6 use upper-bound estimates, when applying market discount rates for Paris Club deals. The full results, including the large set of control variables, are shown in Table C.6 in Appendix C.3.

The main variable of interest is the dummy for private creditors. The results confirm the descriptive findings. Haircuts on private debt tend to be significantly lower even when controlling for country fundamentals and time effects. As before, the difference is most pronounced when using the same discount rate for both Paris Club and private deals (columns 4-6), but the result also holds when we use the lower-bound Paris Club estimates. The size of the coefficients is large, with differences ranging between -5 and -30 percentage points depending on the specification.

The results in columns 1 and 3 are robust to a number of checks and extensions. First, we add time fixed effects. This does not change the overall picture, although the private restructurings dummy shows a smaller coefficient, probably because bond restructurings took place in a short sub-period in our sample (post-1998). Second, we add additional controls, as discussed, with no major effects on the coefficient. Standard error increase slightly, mainly because the number of observations decreases by almost one fifth.[32]

Furthermore, we control the robustness of these results to changes in our main assumptions. In particular, we use Libor instead of ATBs to compute debt service and make different assumptions on the debt type and restructuring choices for the Paris Club deals for which this is not known. Specifically, we assume that haircuts for all restructurings with partial debt stock cancellation equal the cancellation rate outlined in the Paris Club terms.[33] In each case, the overall finding is little affected. The case for lower haircuts for private creditors becomes weaker under the assumption of CIRR discount rates. However, private creditors continue to show significantly lower haircuts compared to official creditors under market exit rates. Moreover, we exclude all restructurings associated with the HIPC Initiative for both private and official creditors, in particular to avoid double counting associated with our assumption that there is no topping up. The results are again not affected.

We also use alternative estimation procedures. First, we run fractional response models, to account for the fact that the dependent variable is a share that is bound between 0 and 1. Second, we run a Tobit regression that allows us to simultaneously study the restructuring choice (determinants of restructuring) and the size of the haircut in that restructuring. Our main result is robust to the method applied.

Finally, we differentiate between final and intermediate restructurings since several countries implement multiple restructurings within the same debt crisis spell (see Reinhart and Trebesch, 2016a for a discussion). Following the approach of Cruces and Trebesch (2013), for private restructurings, we define a deal with the Paris Club as final if there has not occurred another restructuring of official debt within the following 4 years. The distribution of haircuts does not change substantially for official creditors when we consider only final restructurings and the average Paris Club haircut is hardly affected.[34]

[32]We also estimated (3.9) with two separate dummies for bank and bond restructurings instead of the more general one. For the case of market exit rates, both dummies are negative and significant at the 1% level in all specifications. For risk-free discount rates, these dummies are always negative but statistically significant (at least at the 10% level) only in the first two specifications.

[33]Since this cancellation rate applies to the net present value of the debt stock, we cannot apply it directly to the haircut estimates.

[34]Similarly, the average haircuts for bond restructurings is almost unaffected. However, haircuts on bank debt increase on average from 40.5% for all deals to 58.8% for final deals. This shift is mostly due to the high haircuts in the Brady deals of the 1990s, which put an end to the debt crises in Latin America and Africa.

3.4.4 Debt Restructurings and Arrears: A Synthesis

So far, we have analyzed the patterns of restructurings and arrears separately. We now extend the analysis to provide a concise synthesis of these interrelated events. Specifically, we investigate if there is a typical pattern of arrears before and after restructurings of private and official debt and how the behavior of arrears differs among them.

Figure 3.11 shows private arrears as a fraction of GNI during private debt restructurings while Figure 3.12 shows the dynamics of arrears on official debt during restructurings with the Paris Club. We condition the analysis on the initial level of private or official arrears (using a threshold of 1% of GNI as a benchmark) since the sample includes a non-negligible amount of observations with zero arrears.[35] For the same reason, we consider the behavior of both mean values and important percentiles.

Arrears on private debt exhibit a clear pattern: They are substantial before each restructuring averaging around 10% of GNI (although the median is considerably lower at around 3% of GNI) and tend to increase slightly in the years before the event. Arrears on private debt decrease in the course of the restructuring by around 7% of GNI on average. This is primarily driven by large decreases in arrears in high-arrears countries as illustrated by the behavior of the 75th percentile. In addition, arrears are completely reduced to zero in more than one quarter of all cases. This behavior is robust to the choice of the initial threshold. Note that there are no arrears prior to a private debt restructuring in 20% of all cases.

Official arrears are substantially higher prior to the restructuring both on average and for all percentiles. Roughly one third of Paris Club restructurings concerns cases where official debt arrears exceed 5% of GNI whereas the respective number is only about one fifth for private debt restructurings. This primarily reflects differences in debt composition: Countries that restructure with the Paris Club have in general lower income levels and a greater reliance on official debt than countries that restructure claims of private debtors. In addition, the decline in arrears during the restructuring year, though visible, is less comprehensive and less sustained compared to private debt restructurings.

Yet, it should be clear from the discussion above that the restructuring terms of the Paris Club evolved over time towards more comprehensive debt relief and differed between middle income and low income countries. Differentiating by restructuring terms, time periods or income levels is therefore necessary to provide a more conclusive view.

[35] Note that our data set on arrears is from 1979 to 2007, which implies that the analysis excludes the more recent restructurings. The total number of Paris Club deals is therefore reduced from 414 to 368, while the number of private debt restructurings declines to 165. Moreover, data availability on GNI as well as requirements on the sequencing of restructurings further reduce sample size.

Figure 3.11: Arrears on Private Debt during Private Debt Restructurings

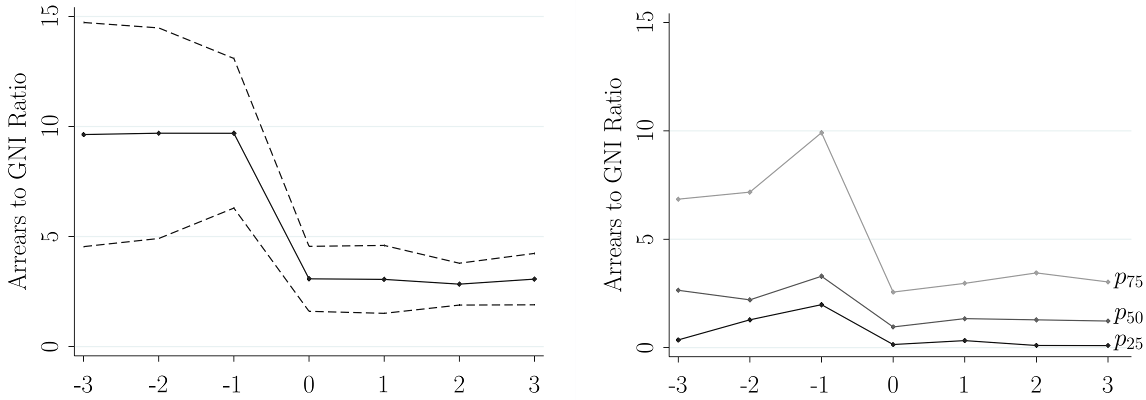
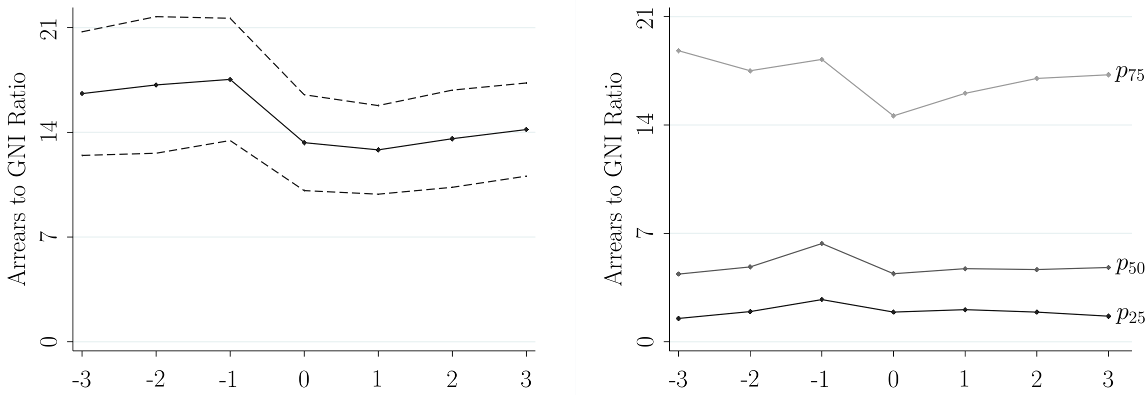


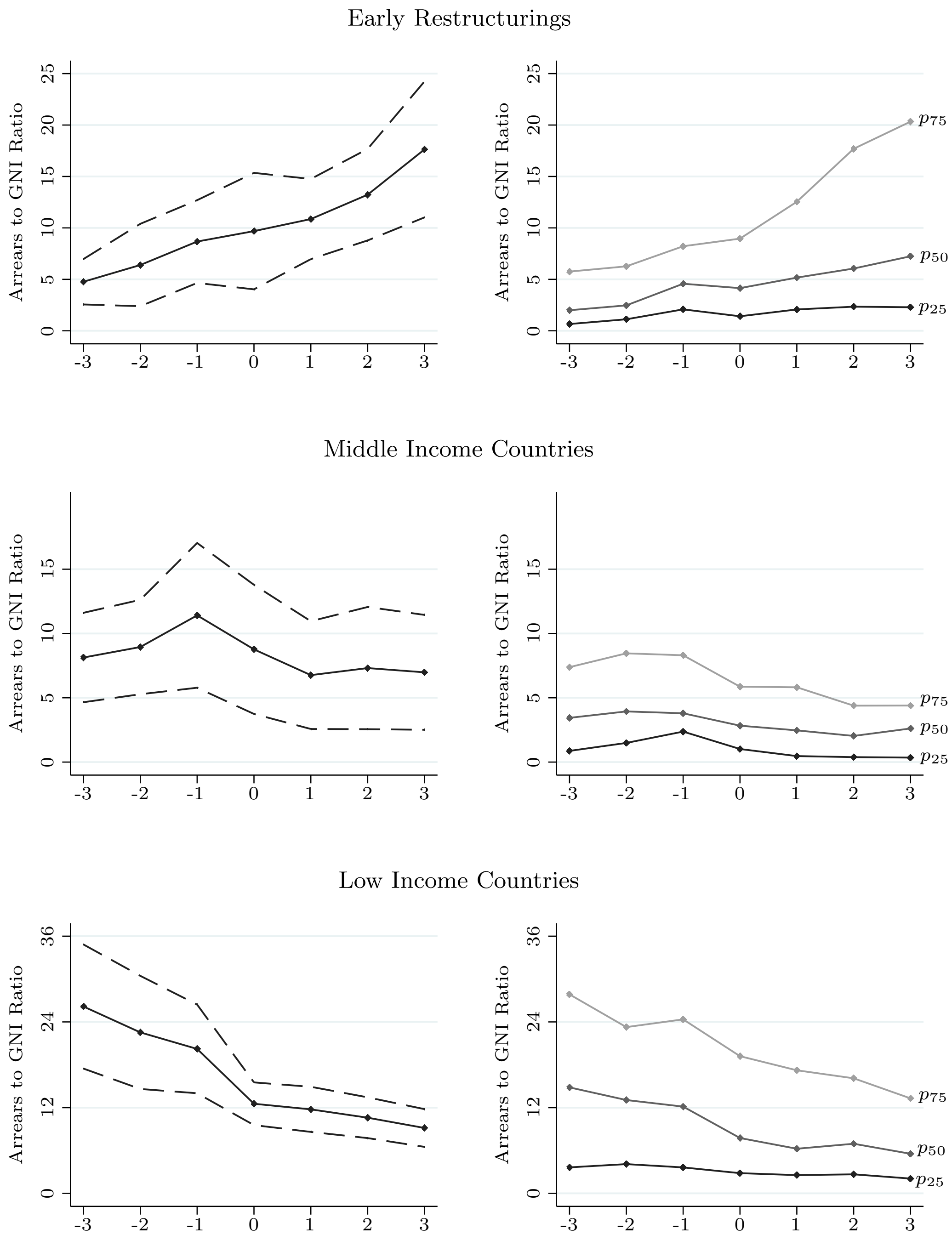
Figure 3.12: Arrears on Official Debt during Paris Club Restructurings



Note: The panels of each figure show the behavior of arrears for all restructurings for which arrears exceeded 1% of GNI one year prior to the restructuring, which takes place at time zero. This implies a total number of 74 private debt and 225 Paris Club deals. The panels on the left show the mean values and the 95% confidence intervals (two standard errors), whereas the panels on the right side illustrate the median as well as the 25th and 75th percentiles.

For this reason, we divide the sample of Paris Club restructurings into three subsets: (i) *Early Restructurings* include all restructurings prior to 1990, most of which occurred under Classic terms that did not include any debt cancellations and were primarily applied during the 1980s for both low and middle income countries to deal with temporary liquidity problems. (ii) *Middle Income Countries* subsumes all restructurings since 1990 with countries that were not part of the HIPC Initiative. In contrast to early restructurings, the Paris Club granted more concessional terms (including the possibility of partial debt cancellation) to middle income countries. (iii) *Low Income Countries* comprises all restructurings with low income countries since the launch of the HIPC Initiative in 1996, which correspond to coordinated and comprehensive debt relief efforts of various creditors.

Figure 3.13: Arrears on Official Debt for Subsets of Paris Club Restructurings



Note: The panels of each figure show the behavior of arrears on official debt for all restructurings for which arrears exceeded 1% of GNI one year prior to the restructuring, which takes place at time zero. The panels on the left show the mean values and the 95% confidence intervals (two standard errors), whereas the panels on the right side illustrate the median as well as the 25th and 75th percentiles. Sample size is as follows: Early restructurings include 133 deals, 57 of which feature arrears above 1%. For Middle Income Countries, 84 restructurings fulfill the criteria, of which we use 46 due to the 1% threshold. Low Income Countries include a total of 95 events most of which (79) fulfill the 1% criteria.

Figure 3.13 illustrates the dynamics of official arrears during Paris Club restructurings for these subsets. We again impose the threshold of 1% on the official arrears to GNI ratio for an observation to be included, though the results are qualitatively unchanged for lower thresholds.

The figure supports the common narrative: Primarily intended to deal with temporary liquidity problems, early Paris Club restructurings can be seen as passive or responsive. Countries usually begin to accumulate arrears before an early restructuring, which then provides temporary relief and stabilizes arrears. Yet, arrears continue to rise in the years following an early restructuring. In contrast, restructurings with the other two subsets are more active in the sense that they are an attempt to deal with substantial debt overhang as manifested in large arrears to GNI ratios (particularly for low income countries) before the restructuring. These arrears decline substantially during the year of the event. In addition, this decline appears to be sustainable since arrears do not increase again after the events.

3.5 Conclusion

In this paper, we have analyzed the implicit seniority structure in sovereign debt markets using a new dataset on the extent of missed payments (arrears) and the size of creditor losses (haircuts). We find a clear pecking order that is robust to controlling for a range of country specific variables: Multilateral official creditors and bondholders are senior lenders whereas bilateral official creditors, banks and trade creditors are junior. The fact that bilateral official creditors are junior to sovereign bondholders, and that this ranking has become more pronounced over time, is surprising given official creditors' historical insistence upon preferable treatment, and a number of policy and contractual reforms - such as allowing IMF lending into private arrears, and the adoption of collective action clauses into bond contracts - that should have reduced the bargaining position of bondholders. As such, our findings suggest that official creditors should rethink the process by which their debts are restructured under the auspices of the Paris Club.

The results also pose a challenge for the theoretical literature on sovereign debt which has typically ignored both the heterogeneity of sovereign debts as well as the heterogeneous treatment of creditors in the event of default. Perhaps most importantly, by examining the decisions of sovereigns to default and restructure debts differentially across creditors, our findings can shed light on the fundamental question of sovereign debt: what, precisely, are the costs of default to a sovereign country? For only with an answer to this question - that is, an understanding of the specific incentives constraining borrowing and default - can we deduce policy interventions that are both economically feasible and socially desirable.

Appendix A

Appendix to Chapter 1

A.1 Proof of Proposition 1.1

Let $\hat{k}$ be defined by $f(\hat{k}) - \delta\hat{k} + z = 0$ such that $c(\hat{k}) = 0$ in (1.21). Define $\tilde{k}$ by $f'(\tilde{k}) = \delta$ as the Golden Rule level of the capital stock. Define k^+ by $f'(k^+) = \delta + \rho$, such that the net marginal product and the return on savings at k^+ equal ρ. It holds that $k^+ < \tilde{k} < \hat{k}$.

Full employment condition (1.22) defines the function $F(k)$ as the difference between the return on savings and the natural interest rate as

$$F(k) \equiv f'(k) - \delta - \rho + \frac{v'(k)}{u'\left(f(k) - \delta k + z\right)} \; . \tag{A.1}$$

Existence: The full employment steady state exists if there is $\bar{k} > 0$ such that $F(\bar{k}) = 0$. $F(k)$ is a continuous function of k in $(0, \hat{k})$. It holds that $F(k) > 0$ for $k < k^+$. Also, $F(k^+) = v'(.)/u'(.) > 0$ and $F(\hat{k}) = f'(\hat{k}) - \delta - \rho = f'(\hat{k}) - f'(k^+) < 0$. It follows that there exists $\bar{k} \in (k^+, \hat{k})$ with $F(\bar{k}) = 0$, irrespective of $\beta > 0$ or $\beta = 0$.

Uniqueness: Wealth preferences can result in multiple equilibria, see Kurz (1968) and Bose (1971). Uniqueness requires restrictions on the third moments of $v(a)$ and $u(c)$ as in Kurz (1968) or restrictions on the model parameters, particularly on ρ, as in Bose (1971). $F'(\bar{k}) < 0$ for all $\bar{k}$ with $F(\bar{k}) = 0$ is a sufficient condition for uniqueness.[1] Specifically,

$$F'(\bar{k}) < 0 \quad \leftrightarrow \quad \left[\rho + \delta - f'(\bar{k})\right]\left[f'(\bar{k}) - \delta\right] < f''(\bar{k})\frac{u'(\bar{c})}{u''(\bar{c})} + \frac{v''(\bar{k})}{u''(\bar{c})} \; , \tag{A.2}$$

for all $\bar{k}$ with $F(\bar{k}) = 0$. This property has to hold locally at each $\bar{k}$. All terms on the RHS are weakly positive. Hence, uniqueness is guaranteed for $\rho = 0$. The same holds for ρ sufficiently close to zero, where sufficiently is dependent on the functional form of $u(c)$ and $f(k)$, particularly their third derivatives. This is also the conclusion of Kurz (1968).

[1]Since $F(k)$ is the difference between the return on savings r^S and the natural rate r^N, this condition requires the slope of the natural rate to exceed the slope of the return on savings at each intersection. Intuitively, the r^S curve has to intercept the r^N curve in Figure 1.1 from above. The same requirement guarantees uniqueness for all extensions of the benchmark model.

Stability: The dynamic system is given by the differential equations for consumption in (1.14) and the capital stock in (1.18) for a given initial value $k_0 > 0$:

$$\dot{c}_t = \frac{c_t}{\eta_c}\left[f'(k_t) - \delta - \rho + \frac{v'(k_t)}{u'(c_t)}\right] \ , \tag{A.3}$$

$$\dot{k}_t = f(k_t) - \delta k_t + z - c_t \ , \tag{A.4}$$

where I used (1.9) to substitute for the real rate in (1.14). Log-linearization around the steady state via a first-order Taylor approximation gives

$$\begin{pmatrix} \dot{c}_t \\ \dot{k}_t \end{pmatrix} = \begin{pmatrix} \rho + \delta - f'(\bar{k}) & -\frac{u'(\bar{c})}{u''(\bar{c})}\left[f''(\bar{k}) + \frac{v''(\bar{k})}{u'(\bar{c})}\right] \\ -1 & f'(\bar{k}) - \delta \end{pmatrix} \begin{pmatrix} c_t - \bar{c} \\ k_t - \bar{k} \end{pmatrix} \ ,$$

where I use the definition of η_c to simplify. The eigenvalues λ of the system solve

$$\begin{aligned}\Lambda(\lambda) &= \lambda^2 - \rho\lambda + \left[\rho + \delta - f'(\bar{k})\right]\left[f'(\bar{k}) - \delta\right] - \left[f''(\bar{k})\frac{u'(\bar{c})}{u''(\bar{c})} + \frac{v''(\bar{k})}{u''(\bar{c})}\right] \\ &= (\phi_1 - \lambda)(\phi_2 - \lambda) = \lambda^2 - (\phi_1 + \phi_2)\lambda + \phi_1\phi_2 = 0 \ ,\end{aligned}$$

where I rewrite the equation in terms of its solutions $\lambda_1 = \phi_1$ and $\lambda_2 = \phi_2$. Matching terms implies

$$\phi_1\phi_2 = \left[\rho + \delta - f'(\bar{k})\right]\left[f'(\bar{k}) - \delta\right] - \left[f''(\bar{k})\frac{u'(\bar{c})}{u''(\bar{c})} + \frac{v''(\bar{k})}{u''(\bar{c})}\right] < 0 \ , \tag{A.5}$$

which follows from (A.2). Hence, there is one negative and one positive eigenvalue and the system is saddle-point stable around the steady state. Any $k_0 > 0$ uniquely determines $c_0(k_0)$ and the system converges to the steady state along the saddle-path. Saddle-point stability holds globally, which can be seen in the phase diagram of (A.3) and (A.4).

A.2 Proof of Proposition 1.2

Start with the differential equation (1.2) for the land price, $\dot{q}_t = rq_t - z$. Let r denote the real rate in steady state. Define the integrating factor $\xi_s = e^{-\int r ds} = e^{-rs}$ to modify (1.2) as

$$\dot{[q_s\xi_s]} = \dot{q}_s\xi_s - q_s\dot{\xi}_s = [\dot{q}_s - rq_s]\xi_s = -z\xi_s$$

$$q_t\xi_t - A = -z\int_0^t \xi_s ds = -z\int_0^t e^{-rs}ds = -z\left[-\frac{1}{r}e^{-rt} + \frac{1}{r}\right]$$

$$q_t = \xi_t^{-1}A - \xi_t^{-1}z\left[-\frac{1}{r}e^{-rt} + \frac{1}{r}\right] = Ae^{rt} - ze^{rt}\left[-\frac{1}{r}e^{-rt} + \frac{1}{r}\right] = \left[A - \frac{z}{r}\right]e^{rt} + \frac{z}{r}$$

Let q_0 denote the steady state land price at $t = 0$. Then, $q_0 = A$ and the steady state price at time t is given by:

$$q_t = \left[q_0 - \frac{z}{r}\right] e^{rt} + \frac{z}{r} \tag{A.6}$$

Free disposal of land implies $q_t \geq 0$ for all t. From (A.6), $q_t \geq 0$ requires $r > 0$ as q_t approaches $z/r < 0$ if $r < 0$. For the special case of $r = 0$, equation (A.6) becomes

$$q_t = q_0 - zt \ , \tag{A.7}$$

which for any $q_0 > 0$ eventually becomes negative. Hence $r = 0$ is also not possible in steady state. We therefore must have $r > 0$.[2] Equation (A.6) with $r > 0$ rules out $\dot{q}_t < 0$ in steady state. For any $q_0 > 0$, this eventually results in $q_t < 0$, which is not possible.

A.3 Proof of Proposition 1.4

Define k^+ by $f'(k^+) = \delta + \rho$, $\tilde{k}$ as the Golden Rule level and $\hat{k}$ by $c(\hat{k}) = 0$ as in Appendix A.1 with $k^+ < \tilde{k} < \hat{k}$.

Full employment condition (1.23) defines the function $G(k)$ as the difference between the return on savings and the natural interest rate as

$$G(k) \equiv f'(k) - \delta - \rho + \frac{v'\left(k + \frac{z}{f'(k)-\delta}\right)}{u'(f(k) - \delta k + z)} \ . \tag{A.8}$$

Existence: The steady state exists if there is $k^*_{NC} > 0$ such that $G(k^*_{NC}) = 0$. $G(k)$ is a continuous function of $k \in (0, \tilde{k})$ with $G(k) > 0$ for $k < k^+$. Also $G(k^+) = v'(.)/u'(.) > 0$ and $G(\tilde{k}) = -\rho < 0$ as $v'(\infty) = 0$. Hence, there exists $k^*_{NC} \in (k^+, \tilde{k})$ with $G(k^*_{NC}) = 0$.

Uniqueness: The arguments of Proposition 1.1 apply accordingly. A sufficient condition for uniqueness is given by $G'(k^*_{NC}) < 0$ for all k^*_{NC} with $G(k^*_{NC}) = 0$ or equivalently

$$\left[\rho + \delta - f'(k^*_{NC})\right]\left[f'(k^*_{NC}) - \delta\right] < f''(k^*_{NC})\frac{u'(c^*_{NC})}{u''(c^*_{NC})} + \frac{v''(a^*_{NC})}{u''(c^*_{NC})}\left[1 + \frac{dq^*_{NC}}{dk^*_{NC}}\right] \ , \tag{A.9}$$

$$\text{where} \quad \frac{dq^*_{NC}}{dk^*_{NC}} = -\frac{zf''(k^*_{NC})}{\left[f'(k^*_{NC}) - \delta\right]^2} > 0$$

Note that condition (A.9) is less restrictive than condition (A.2) in the benchmark model as the additional term is strictly positive.

[2] If land depreciates at rate τ, we must have $r > -\tau$. See Michau, Ono, and Schlegl (2017) for the general case.

Effects on k and r: Let $\bar{r}$ and $\bar{k}$ denote the steady state real rate and capital stock in the benchmark model and r^*_{NC} and k^*_{NC} in the model with land. For $\bar{r} \leq 0$, it follows directly from Proposition 1.2 that $r^*_{NC} > 0 \geq \bar{r}$ and from (1.19) that $k^*_{NC} < \tilde{k} \leq \bar{k}$.

Let $\bar{r} > 0$ and $\bar{k} < \tilde{k}$. In the model with land, k^*_{NC} defines $q^*_{NC} = q(k^*_{NC}) > 0$. Consider the rise in q from zero to q^*_{NC} as exogenous and recover the effects on k from $\Omega(k, q) = 0$ in

$$\Omega(k,q) = f'(k) - \delta - \rho + \frac{v'(k+q)}{u'(f(k) + z - \delta k)} = 0 \ ,$$

which is equivalent to $F(k)$ in (A.1) except it includes q in $v(a)$. It follows that

$$\frac{dk}{dq} = -\frac{\partial\Omega/\partial q}{\partial\Omega/\partial k} = \frac{-v''(k+q)u'(c)}{F'(k)} < 0 \ , \tag{A.10}$$

as $F'(\bar{k}) < 0$ by (A.2). Therefore, $k^*_{NC} < \bar{k} < \tilde{k}$ and $r^*_{NC} > \bar{r} > 0$.

Stability: The dynamic system is given by (1.2) for the land price, (1.14) for consumption and (1.18) for the capital stock with $L_t = 1$ for a given initial $k_0 > 0$ as

$$\dot{c}_t = \frac{c_t}{\eta_c}\left[f'(k_t) - \delta - \rho + \frac{v'(k_t + q_t)}{u'(c_t)}\right] \ , \tag{A.11}$$

$$\dot{k}_t = f(k_t) - \delta k_t + z - c_t \ , \tag{A.12}$$

$$\dot{q}_t = \left[f'(k_t) - \delta\right] q_t - z \ , \tag{A.13}$$

where I used (1.5) and (1.9) to substitute for the real rate in (1.2) and (1.14). Log-linearization of this system around the steady state via a first-order Taylor approximation gives

$$\begin{pmatrix} \dot{c}_t \\ \dot{k}_t \\ \dot{q}_t \end{pmatrix} = \begin{pmatrix} \rho + \delta - f'(k^*_{NC}) & -\frac{u'(c^*_{NC})}{u''(c^*_{NC})}\left[f''(k^*_{NC}) + \frac{v''(a^*_{NC})}{u'(c^*_{NC})}\right] & -\frac{v''(a^*_{NC})}{u''(c^*_{NC})} \\ -1 & f'(k^*_{NC}) - \delta & 0 \\ 0 & \frac{zf''(k^*_{NC})}{f'(k^*_{NC})-\delta} & f'(k^*_{NC}) - \delta \end{pmatrix} \begin{pmatrix} c_t - c^*_{NC} \\ k_t - k^*_{NC} \\ q_t - q^*_{NC} \end{pmatrix} ,$$

where I use the definition of η_c. The eigenvalues of the transition matrix solve

$$\Lambda(\lambda) = -\lambda^3 + [\rho + f'(k^*_{NC}) - \delta]\lambda^2 - \Omega_1\lambda - \left[f'(k^*_{NC}) - \delta\right]\frac{u'(c^*_{NC})}{u''(c^*_{NC})}G'(k^*_{NC}) = 0 \ ,$$

$$\text{where } \Omega_1 \equiv \rho\left[f'(k^*_{NC}) - \delta\right] - \frac{u'(c^*_{NC})}{u''(c^*_{NC})}F'(k^*_{NC}) \ ,$$

with $F'(k^*_{NC}) < 0$ from (A.2) and $G'(k^*_{NC}) < 0$ from (A.9). Let φ_i denote the eigenvalues of the system.

I rewrite the equation in terms of its solutions as $\Lambda(\lambda) = (\varphi_1 - \lambda)(\varphi_2 - \lambda)(\varphi_3 - \lambda)$ or

$$\Lambda(\lambda) = -\lambda^3 + (\varphi_1 + \varphi_2 + \varphi_3)\lambda^2 - (\varphi_1\varphi_2 + \varphi_1\varphi_3 + \varphi_2\varphi_3)\lambda + \varphi_1\varphi_2\varphi_3 = 0 \ .$$

Matching terms of these two equations defines the eigenvalues as

$$\varphi_1 + \varphi_2 + \varphi_3 = \rho + f'(k^*_{NC}) - \delta > 0 \ , \tag{A.14}$$

$$\varphi_1\varphi_2\varphi_3 = -\left[f'(k^*_{NC}) - \delta\right] \frac{u'(c^*_{NC})}{u''(c^*_{NC})} G'(k^*_{NC}) < 0 \ . \tag{A.15}$$

At least one eigenvalue is positive by (A.14), whereas (A.15) implies one or three negative eigenvalues. Hence, there is exactly one negative eigenvalue and the system exhibits saddle-point stability.

A.4 Proof of Proposition 1.6

Define k^+, $\tilde{k}$ and $\hat{k}$ as before with $k^+ < \tilde{k} < \hat{k}$. $G(k)$ is given in (A.8) with $v'(a) \geq \beta$ and represents conditions (1.26) in the bubbly and (1.27) in the fundamental steady state.

Existence and Uniqueness: The steady state exists if there exists $k^*_{NC} > 0$ such that $G(k^*_{NC}) = 0$ in (A.8). $G(k)$ is continuous in $k \in (0, \hat{k})$ since $v'(a) = \beta$ for $k \geq \tilde{k}$. However, $k^*_{NC} \in (0, \tilde{k})$ by Proposition 1.2. We have $G(k) > 0$ for $k < k^+$ and $G(k^+) = v'(.)/u'(.) > 0$. Existence requires $G(\tilde{k}) = \rho - \beta/u'(c(\tilde{k})) < 0$ or equivalently

$$G(\tilde{k}) < 0 \quad \leftrightarrow \quad \beta < \rho u'\left(c(\tilde{k})\right) \ . \tag{A.16}$$

This is condition (1.24) in Proposition 1.6.[3] Uniqueness requires $G'(k^*_{NC}) < 0$ for all k^*_{NC} with $G(k^*_{NC}) = 0$. Hence, uniqueness condition (A.9) applies accordingly.

Suppose instead $G(k^*_{NC}) = 0$ at $k^*_{NC} \in (\tilde{k}, \hat{k})$. Existence follows from Proposition 1.1 with $v'(a) = \beta$. Let

$$\Psi(k, L) \equiv f'(k) - \delta - \rho + \frac{\beta}{u'\left([f(k) - \delta k]L + z\right)} \ ,$$

with $L \in (0, 1)$ and $\Psi(k^*_{NC}, L) = 0$ in steady state. We have

$$\frac{dk}{dL} = -\frac{\partial\Psi/\partial L}{\partial\Psi/\partial k} = \frac{\beta u''(c^*_{NC})[f(k^*_{NC}) - \delta k^*_{NC}]}{[u'(c^*_{NC})]^2} \frac{1}{\partial\Psi/\partial k} > 0 \ , \tag{A.17}$$

[3]Intuitively, full employment requires that the natural rate r^N at $\tilde{k}$ is above the return on savings r^S, which is zero at $\tilde{k}$. Then an intersection in $k^+ < k^*_{NC} < \tilde{k}$ is ensured as both functions are continuous in this region.

as $k^*_{NC} < \hat{k}$ and $\frac{\partial \Psi}{\partial k} < 0$ from (A.2). Hence, a decline in L lowers k^*_{NC}. A stagnation steady state is defined for any $L \in (0,1)$ that satisfies

$$\Psi(\tilde{k}, L) < 0 \quad \leftrightarrow \quad \rho u'\left([f(\tilde{k}) - \delta\tilde{k}]L + z\right) > \beta \ , \tag{A.18}$$

and takes into account the non-negativity restriction $L > 0$. Taken together, this implies

$$\rho u'(z) > \beta \ . \tag{A.19}$$

Stability: In the fundamental steady state, the bubble component is always zero as it would otherwise grow infinitely as $r^*_{NC} > 0$. Hence, $\dot{q} = 0$ in steady state and real wealth is constant. The dynamic system is given by (A.11), (A.12) and (A.13) and the stability patterns are identical to the case of $\beta = 0$ in the previous proof.

In the bubbly steady state, we have $\dot{a}_t > 0$ as $\dot{q}^B_t > 0$ for given $q^B_0 > 0$ and $v'(a_t) = \beta$ in (A.11). Denote $x_t = 1/q^B_t$ with $x^*_{NC} = 0$ in steady state. The bubble equation (1.4) becomes

$$\dot{x}_t = -[f'(k_t) - \delta]x_t \ , \tag{A.20}$$

where I use (1.5) and (1.9) to substitute for r^B_t. The dynamic system is given by (A.11) with $v'(a_t) = \beta$, (A.12), (A.13) for the fundamental price q^F_t and (A.20) for the bubble. A first-order Taylor approximation around the bubbly steady state gives

$$\begin{pmatrix} \dot{c}_t \\ \dot{k}_t \\ \dot{q}^F_t \\ \dot{x}_t \end{pmatrix} = \begin{pmatrix} \rho + \delta - f'(k^*_{NC}) & -\frac{u'(c^*_{NC})}{u''(c^*_{NC})} f''(k^*_{NC}) & 0 & 0 \\ -1 & f'(k^*_{NC}) - \delta & 0 & 0 \\ 0 & \frac{z f''(k^*_{NC})}{f'(k^*_{NC}) - \delta} & f'(k^*_{NC}) - \delta & 0 \\ 0 & 0 & 0 & -f'(k^*_{NC} + \delta \end{pmatrix} \begin{pmatrix} c_t - c^*_{NC} \\ k_t - k^*_{NC} \\ q^F_t - q^{F*}_{NC} \\ x_t - x^*_{NC} \end{pmatrix} ,$$

where I use the definition of η_c and $v''(a) = 0$. The eigenvalues of this system solve

$$\Lambda(\lambda) = [-f'(k^*_{NC}) + \delta - \lambda][f'(k^*_{NC}) - \delta - \lambda] \cdot \begin{Vmatrix} \rho + \delta - f'(k^*_{NC}) - \lambda & -\frac{u'(c^*_{NC})}{u''(c^*_{NC})} f''(k^*_{NC}) \\ -1 & f'(k^*_{NC}) - \delta - \lambda \end{Vmatrix} = 0 \ ,$$

where $\left\|.\right\|$ denotes the determinant. From the proof of Proposition 1.1 with $v''(a) = 0$ it follows that this matrix has one positive and one negative eigenvalue. In addition, $\lambda_3 = f'(k^*_{NC}) - \delta > 0$ and $\lambda_4 = -[f'(k^*_{NC}) - \delta] < 0$. The bubbly equilibrium hence satisfies saddle-point stability. For given $k_0 > 0$ and $q^B_0 > 0$, which determine $c(k_0, q^B_0)$ and $q^F(k_0, q^B_0)$, the model converges to the steady state along the unique saddle path.

A.5 Proof of Proposition 1.7

Define k^+, $\tilde{k}$ and $\hat{k}$ as before with $k^+ < \tilde{k} < \hat{k}$. The steady state real money stock under full employment is defined by (1.38) and (1.40) as a function of the capital stock as

$$m(k) = \omega'^{-1}\left(\left[f'(k) - \delta + \mu\right] u'(f(k) - \delta k + z)\right) \ . \tag{A.21}$$

with $m'(k) > 0$ and $m(k) \to \infty$ for $k \to k_\mu$, where k_μ satisfies $f'(k_\mu) = \delta - \mu$. Hence, $k_\mu > \tilde{k}$ for $\mu > 0$ and vice versa.

Monetary Wealth Preferences: Consider first the case of money in $v(\tilde{a}) = v(a)$. Full employment condition (1.42) and (A.21) define $H(k)$ as

$$H(k) \equiv f'(k) - \delta - \rho + \frac{v'\left(k + \frac{z}{f'(k)-\delta} + m(k)\right)}{u'(f(k) - \delta k + z)} \ . \tag{A.22}$$

(i) Existence: The steady state exists if there exists $k^* > 0$ such that $H(k^*) = 0$. $H(k)$ is continuous in $k \in (0, \tilde{k})$ if $\mu \geq 0$ or $k \in (0, k_\mu)$ if $\mu < 0$. It holds that $H(k) > 0$ for $k < k^+$. Hence, we cannot have $k^* \in (0, k^+)$, which requires $k_\mu > k^+$ or $\mu > -\rho$. Also $H(k^+) = v'(.)/u'(.) > 0$. If $\mu \geq 0$, $H(\tilde{k}) = -\rho < 0$ and there exists k^* in $(0, \tilde{k})$ with $H(k^*) = 0$. If $\mu < 0$, $H(k_\mu) = -\rho < 0$ as $m(k) \to \infty$ and there exists k^* in $(0, k_\mu)$ with $H(k^*) = 0$ unless $\mu < -\rho$. Hence, existence requires $\mu < -\rho$ for $v(\tilde{a}) = v(a)$.

(ii) Uniqueness: A sufficient condition is $H'(k^*) < 0$ for all k^* with $H(k^*) = 0$ or

$$[\rho + \delta - f'(k^*)][f'(k^*) - \delta] < f''(k^*)\frac{u'(c^*)}{u''(c^*)} + \frac{v''(a^*)}{u''(c^*)}\left[1 + \frac{dq^*}{dk^*} + \frac{dm^*}{dk^*}\right], \tag{A.23}$$

$$\text{where } \ \frac{dq^*}{dk^*} = -\frac{zf''(k^*)}{[f'(k^*) - \delta]^2} > 0$$

$$\text{and } \ \frac{dm^*}{dk^*} = \frac{1}{\omega''(m^*)}\left[(f'(k^*) - \delta)(f'(k^*) - \delta + \mu)u''(c^*) + f''(k^*)u'(c^*)\right] > 0 \ ,$$

for all k^* with $H(k^*) = 0$. This condition is less restrictive than (A.2) or (A.9) since the additional term is positive.

(iii) Impossibility of Secular Stagnation: Under stagnation, we have $\pi = -\gamma < 0$ in steady state and therefore $\dot{m} > 0$ as $\mu + \gamma > 0$. Then $i = 0$ and $v'(a) = 0$. Secular stagnation condition (1.44) then requires $\gamma = \rho$. However, the steady state is not well-defined as K, L and c are indeterminate in this case and the transversality condition is violated for $\mu \geq 0$.

Non-monetary wealth preferences: With $v(\tilde{a}) = v(a - m^S)$, $H(k)$ is identical to $G(k)$ in (A.8). This implies $r^*_{NC} \in (0, \rho)$ and $k^*_{NC} \in (k^+, \tilde{k})$. As k^* is uniquely determined by $G(k)$ if (A.9) holds, P adjusts to clear the money market. From (1.41), define

$$J(P) = \frac{\omega'(M^S/P)}{u'(f(k^*) - \delta k^* + z)} - \mu - r^*_{NC} \ . \tag{A.24}$$

Full employment requires $J(P) = 0$. $J(P)$ is continuous and strictly increasing in P for a given M^S. It holds that $J(\infty) = \infty > 0$ and $J(0) = -\mu - r^*_{NC}$. Existence requires $J(0) < 0$ or equivalently $\mu > -r^*_{NC}$. Hence, we have $\mu > -r^*_{NC}$ in (1.44) for $v(\tilde{a}) = v(a - m^S)$. As $J'(P) > 0$, the full employment steady state is unique if (A.9) holds.

Stability: The dynamic system is given by (A.11), (A.12), (A.13) and (1.29), where m_t is potentially included in (A.11). There are no bubbles with $\beta = 0$. Using (1.30) to substitute for π_t, (1.38) for i_t and (1.9) for r_t, (1.29) is rewritten as

$$\dot{m}_t = \left[\mu - \frac{\omega'(m_t)}{u'(c_t)} + f'(k_t) - \delta\right] m_t \ . \tag{A.25}$$

A first-order Taylor approximation around the full employment steady state gives

$$\begin{pmatrix} \dot{c}_t \\ \dot{k}_t \\ \dot{q}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} \rho + \delta - f'(k^*) & -\frac{u'(c^*)f''(k^*)}{u''(c^*)} - \frac{v''(\tilde{a}^*)}{u''(c^*)} & -\frac{v''(\tilde{a}^*)}{u''(c^*)} & -\frac{v''(\tilde{a}^*)}{u''(c^*)} \\ -1 & f'(k^*) - \delta & 0 & 0 \\ 0 & \frac{zf''(k^*)}{f'(k^*)-\delta} & f'(k^*) - \delta & 0 \\ \frac{\omega'(m^*)u''(c^*)}{[u'(c^*)]^2} m^* & f''(k^*)m^* & 0 & -\frac{\omega''(m^*)}{u'(c^*)} m^* \end{pmatrix} \begin{pmatrix} c_t - c^* \\ k_t - k^* \\ q_t - q^* \\ m_t - m^* \end{pmatrix} ,$$

where I use the definition of η_c. The eigenvalues of the system solve

$$\left[-\frac{\omega''(m^*)}{u'(c^*)} m^* - \lambda\right] \cdot \left\| \begin{matrix} \rho + \delta - f'(k^*) - \lambda & -\frac{u'(c^*)}{u''(c^*)} \left[f''(k^*) + \frac{v''(\tilde{a}^*)}{u''(c^*)}\right] & -\frac{v''(\tilde{a}^*)}{u''(c^*)} \\ -1 & f'(k^*) - \delta - \lambda & 0 \\ 0 & \frac{zf''(k^*)}{f'(k^*)-\delta} & f'(k^*) - \delta - \lambda \end{matrix} \right\|$$

$$+ \frac{\omega'(m^*)u''(c^*)}{[u'(c^*)]^2} m^* \left[f'(k^*) - \delta - \lambda\right]^2 \frac{v''(\tilde{a}^*)}{u''(c^*)} + f''(k^*)m^* \frac{v''(\tilde{a}^*)}{u''(c^*)} \left[f'(k^*) - \delta - \lambda\right] = 0 \ ,$$

where $\left\|.\right\|$ denotes the determinant. The eigenvalues φ_i of the transition matrix then solve $\Lambda(\lambda) = (\varphi_1 - \lambda)(\varphi_2 - \lambda)(\varphi_3 - \lambda)(\varphi_4 - \lambda) = 0$, which is a quartic equation in λ. Matching terms implies that the eigenvalues are related as

$$\varphi_1\varphi_2\varphi_3\varphi_4 = -[f'(k^*) - \delta] \frac{\omega''(m^*)}{u'(c^*)} m^* H'(k^*) < 0 \ , \tag{A.26}$$

where $H(k)$ is defined in (A.22) and $H'(k^*) < 0$ follows from condition (A.23). Hence, the dynamic system has either one or three negative eigenvalues. In addition, we have

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 = -\frac{\omega''(m^*)}{u'(c^*)}m^* + 2[f'(k^*) - \delta] + [\rho + \delta - f'(k^*)] > 0 \ , \tag{A.27}$$

$$(\varphi_1 + \varphi_2)\varphi_3\varphi_4 + (\varphi_3 + \varphi_4)\varphi_1\varphi_2 = -[f'(k^*) - \delta]\frac{u'(c^*)}{u''(c^*)}G'(k^*) - \frac{\omega''(m^*)}{u'(c^*)}m^*\Omega_1$$
$$+ \frac{v''(\tilde{a}^*)}{u'(c^*)}m^*\left(2[f'(k^*) - \delta][f'(k^*) - \delta + \mu] + f''(k^*)\frac{u'(c^*)}{u''(c^*)}\right) < 0 \ , \tag{A.28}$$

where the negative sign follows from $G'(k^*) < 0$ in (A.9) and from the proof of Proposition 1.4. Suppose there are three negative and one positive eigenvalue, $\varphi_4 > 0$. Solving the two equations for φ_4 gives

$$-[\varphi_1 + \varphi_2 + \varphi_3] < \varphi_4 < -\frac{\varphi_1\varphi_2\varphi_3}{(\varphi_1 + \varphi_2)\varphi_3 + \varphi_1\varphi_2}$$

$$[\varphi_1 + \varphi_2 + \varphi_3](\varphi_1 + \varphi_2)\varphi_3 + [\varphi_1 + \varphi_2 + \varphi_3]\varphi_1\varphi_2 > \varphi_1\varphi_2\varphi_3$$

$$(\varphi_1 + \varphi_2)[(\varphi_1 + \varphi_2 + \varphi_3)\varphi_3 + \varphi_1\varphi_2] > 0 \ ,$$

which contradicts the assumption of three negative eigenvalues. Hence, the system has exactly one negative eigenvalue and is saddle-point stable around the steady state.

A.6 Proof of Proposition 1.8

Define k^+, $\tilde{k}$ and $\hat{k}$ as before with $k^+ < \tilde{k} < \hat{k}$. k_μ satisfies $f'(k_\mu) = \delta - \mu$ and $H(k)$ is defined in (A.22) as a continuous function of $k \in (0, \tilde{k})$ if $\mu \geq 0$ or $k \in (0, k_\mu)$ if $\mu < 0$.

Full Employment Steady State: Existence requires $H(k) = 0$ for some $k \in (0, \tilde{k})$ if $\mu \geq 0$ or $k \in (0, k_\mu)$ if $\mu < 0$. It holds that $H(k) > 0$ for $k < k^+$. Hence, we cannot have $k^* \in (0, k^+)$, which requires $k_\mu > k^+$ or equivalently $\mu > -\rho$. Also $H(k^+) = \tilde{v}'(.)/u'(.) > 0$. If $\mu \geq 0$, existence requires

$$H(\tilde{k}) < 0 \quad \leftrightarrow \quad \beta < \rho u'\left(c\left(\tilde{k}\right)\right) \ . \tag{A.29}$$

Then there exists at least one $k^* \in (0, \tilde{k})$ with $H(k^*) = 0$. If $\mu < 0$ and $v(\tilde{a}) = v(a)$, full employment requires

$$H(k_\mu) < 0 \quad \leftrightarrow \quad \beta < (\rho + \mu)u'\left(c\left(k_\mu\right)\right) \ . \tag{A.30}$$

Then, there exists at least one $k^* \in (0, k_\mu)$ with $H(k^*) = 0$ unless $\mu < -\rho$. For the case of $\mu < 0$ and $v(\tilde{a}) = v(a - m^S)$, the proof is identical to Proposition 1.7.

Uniqueness requires $H'(k^*) < 0$ for all k^* that satisfy $H(k^*) = 0$, which is expressed in conditions (A.9) and (A.23). In addition, the stability properties under full employment are identical to the case of $\beta = 0$ before.

Secular Stagnation Steady State: A secular stagnation steady state exists if (1.43) holds. Then, $k = k^*$, where k^* satisfies $f'(k^*) = \delta + \gamma$. Define $M(L)$ by (1.43) with $k = k^*$ as

$$M(L) = \gamma - \rho + \frac{v'(\tilde{a})}{u'([f(k^*) - \delta k^*]L + z)} . \tag{A.31}$$

$M(L)$ is continuous in L. The secular stagnation steady state exists if there is $0 < L < 1$ such that $M(L) = 0$.

If $v(\tilde{a}) = v(a)$ or if $q^B > 0$, $v'(\tilde{a}) = \beta$ in (A.31) and we have $M'(L) > 0$. We require $M(L = 0) < 0 < M(L = 1)$ for an equilibrium to exist. This implies:

$$\gamma - \rho + \frac{\beta}{u'(z)} < 0 < \gamma - \rho + \frac{\beta}{u'(f(k^*) - \delta k^* + z)} . \tag{A.32}$$

Uniqueness is guaranteed as $M'(L) > 0$. If $v(\tilde{a}) = v(a - m^S) = v(k^* L + q^S)$, then $v''(\tilde{a}) < 0$. Existence of the secular stagnation steady state then requires

$$\gamma - \rho + \frac{v'(z/\gamma))}{u'(z)} < 0 < \gamma - \rho + \frac{v'(k^* + z/\gamma)}{u'(f(k^*) - \delta k^* + z)} . \tag{A.33}$$

(A.32) and (A.33) are summarized in (1.46). Uniqueness requires $M'(L) > 0$ at $L = L^S$ or equivalently

$$M'(L) > 0 \quad \leftrightarrow \quad (\rho - \gamma)\frac{f(k^*) - \delta k^*}{k^*} > \frac{v''(a^S)}{u''(c^S)} , \tag{A.34}$$

which implies that a reduction in the labor input increases the natural interest rate.

Stability of the Stagnation Steady State: It holds that $L_t < 1$ and $k_t > K_t$. Inflation and the price level are determined by the nominal wage rigidity. Hence, m_t becomes pre-determined and evolves with $\mu - \pi_t$. Inflation is given by (1.33), (1.39) and (1.40) as

$$\pi_t = -\gamma - \frac{\dot{w}_t}{w_t} = -\gamma + \frac{f''(k_t)k_t}{f(k_t) - k_t f'(k_t)}\dot{k}_t = \frac{\omega'(m_t)}{u'(c_t)} - f'(k_t) + \delta$$

This determines the dynamics of the capital-labor ratio k_t and those of L_t together with $\dot{K}_t$ in (1.17). As $m_t \to \infty$, let $x_t = 1/m_t$ with $x_t \to 0$. For $v(\tilde{a}) = v(a)$, the dynamic system is given by

$$\dot{c}_t = \frac{c_t}{\eta_c}\left[f'(k_t) - \delta - \rho + \frac{v'(K_t + q_t + 1/x_t)}{u'(c_t)}\right] , \tag{A.35}$$

$$\dot{K}_t = \left[f(k_t) - \delta k_t\right] \frac{K_t}{k_t} + z - c_t \ , \tag{A.36}$$

$$\dot{k}_t = \frac{f(k_t) - k_t f'(k_t)}{f''(k_t) k_t} \left[\frac{\omega'(1/x_t)}{u'(c_t)} - f'(k_t) + \delta + \gamma\right] \tag{A.37}$$

$$\dot{q}_t = \left[f'(k_t) - \delta\right] q_t - z \ , \tag{A.38}$$

$$\dot{x}_t = \left[-\mu - f'(k_t) + \delta + \frac{\omega'(1/x_t)}{u'(c_t)}\right] x_t \ , \tag{A.39}$$

A first-order Taylor approximation around the stagnation steady state gives

$$\begin{pmatrix} \dot{c}_t \\ \dot{K}_t \\ \dot{k}_t \\ \dot{q}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} \rho-\gamma & 0 & -\frac{u'(c^*)f''(k^*)}{u''(c^*)} & 0 & -\frac{v''(\tilde{a}^*)}{u''(c^*)}\frac{1}{x^{*2}} \\ -1 & \frac{f(k^*)}{k^*}-\delta & \frac{f'(k^*)k^*-f(k^*)}{k^*}\frac{K^*}{k^*} & 0 & 0 \\ 0 & 0 & \frac{k^*f'(k^*)-f(k^*)}{k^*} & 0 & -\frac{f(k^*)-k^*f'(k^*)}{f''(k^*)k^*}\frac{\omega''(m^*)}{u'(c^*)}\frac{1}{x^{*2}} \\ 0 & 0 & f''(k^*)q^* & \gamma & 0 \\ 0 & 0 & 0 & 0 & -\gamma-\mu \end{pmatrix} \begin{pmatrix} c_t-c^* \\ K_t-K^* \\ k_t-k^* \\ q_t-q^* \\ x_t-x^* \end{pmatrix} ,$$

where I use $f'(k^*) - \delta = \gamma$, $v''(\tilde{a}) = 0$ and $\omega'(m) = 0$. The eigenvalues solve

$$(-\gamma-\mu-\lambda)\left(\frac{k^*f'(k^*)-f(k^*)}{k^*}-\lambda\right)(\gamma-\lambda)(\rho-\gamma-\lambda)\left(\frac{f(k^*)}{k^*}-\delta-\lambda\right) = 0 \ .$$

Hence, three eigenvalues, $\lambda_1 = \gamma$, $\lambda_2 = \frac{f(k^*)}{k^*} - \delta$ and $\lambda_3 = \rho - \gamma$, are positive, while two eigenvalues, $\lambda_4 = -\gamma - \mu$ and $\lambda_5 = \frac{k^*f'(k^*)-f(k^*)}{k^*}$, are negative. The system is saddle-path stable around the asymptotic steady state under persistent stagnation.

For $v(\tilde{a}) = v(a - m^S)$, we have $v''(\tilde{a}) < 0$. Then $\lambda_1 = \gamma > 0$, $\lambda_2 = \frac{f'(k^*)k^*-f(k^*)}{k^*} < 0$ and $\lambda_3 = -\gamma - \mu < 0$ are unchanged to above. The remaining two eigenvalues are recovered from

$$\lambda^2 - \left[\rho - \gamma + \frac{f(k^*)}{k^*} - \delta\right]\lambda + (\rho-\gamma)\frac{f(k^* - \delta k^*)}{k^*} - \frac{v''(\tilde{a}^*)}{u''(c^*)} = 0 \ ,$$

which can be rewritten as $\lambda^2 - [\lambda_4 + \lambda_5]\lambda + \lambda_4\lambda_5 = 0$. Matching terms implies that

$$\lambda_4\lambda_5 = (\rho-\gamma)\frac{f(k^* - \delta k^*)}{k^*} - \frac{v''(\tilde{a}^*)}{u''(c^*)} > 0 \ , \tag{A.40}$$

which follows from $M'(L) > 0$ in (A.34). Hence, $\lambda_4 > 0$ and $\lambda_5 > 0$. We have three positive and two negative eigenvalues and the stability patterns are identical to $v''(\tilde{a}) = 0$.

The proofs of Proposition 1.9 and 1.10 are equivalent to the proofs of Proposition 1.7 and 1.8 in Appendix A.7 and A.8 and not discussed in detail.

Appendix B

Appendix to Chapter 2

B.1 Proof of Proposition 2.1

The model parameters affect the stagnation threshold $\widetilde{y}$ in (2.33) as follows:

$$\frac{\partial \widetilde{y}}{\partial \beta} = -\frac{n\rho_1}{\beta^2}\frac{\gamma\theta\rho_1 + \bar{\rho}_\theta}{\gamma\theta\rho_1 + n\bar{\rho}_\theta} < 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \rho_1} = \frac{n[(1+\gamma)(\gamma+n)\theta^2\rho_1^2 + 2(1+\gamma)n\theta(1-\theta)\rho_1\rho_2 + (1-\theta)^2 n\rho_2^2]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \rho_2} = \frac{n\rho_1}{\beta}\frac{(1-n)\gamma\theta(1-\theta)\rho_1}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \theta} = -\frac{n\rho_1}{\beta}\frac{\rho_1\rho_2\gamma(1-n)}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial n} = \frac{\gamma\theta\rho_1^2[\gamma\theta\rho_1 + \bar{\rho}_\theta]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 \ ,$$

$$\frac{\partial \widetilde{y}}{\partial \gamma} = -\frac{n\rho_1}{\beta}\frac{(1-n)\rho_1\theta\bar{\rho}_\theta}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 \ .$$

B.2 Existence and Stability of the Asymmetric Steady State

Existence: Using (2.34) to substitute for c_1^* and (2.35) for c_2^*, we rewrite (2.41) as

$$F(m_2) \equiv \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2) - (1-n)\beta\rho_2]m_2 + n(\rho_1 - \alpha)\bar{y}[v'(m_2) - \beta] = A \ , \qquad \text{(B.1)}$$

$$\text{where} \ \ A \equiv \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) > 0 \ ,$$

$$\text{and} \ \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) > 0 \ .$$

The asymmetric steady state exists for $\bar{y} > \widetilde{y}$ if there exists a finite $m_2 > 0$ as a solution to this equation. Note that the RHS is a positive constant that is independent of m_2. In contrast, the LHS of this expression is a function of m_2. It holds that $\lim_{m_2 \to 0} F(m_2) = \infty$ and

$$\lim_{m_2 \to \infty} F(m_2) \;=\; \begin{cases} -\infty \;\text{ if }\; \beta\bar{y} < n\rho_1 + (1-n)\rho_2 \;, \\ \lim\limits_{m_2 \to \infty} \alpha(\beta\bar{y} - \rho_1 n)(v'(m_2) - \beta)m_2 \;\text{ if }\; \beta\bar{y} = n\rho_1 + (1-n)\rho_2 \;, \\ +\infty \;\text{ if }\; \beta\bar{y} > n\rho_1 + (1-n)\rho_2 \;. \end{cases}$$

Hence, there has to be a solution to (B.1) if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. This is the sufficient condition in Proposition 2.3. For higher values of $\bar{y}$, there may be two solutions, exactly one solution or no solution to (B.1). Existence then requires that the minimum or limit of $F(m_2)$ is smaller than the RHS:

$$\min_{m_2} F(m_2) < \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) \;. \tag{B.2}$$

This condition guarantees the existence of at least one solution to (B.1). In case of multiple solutions, we choose the solution that satisfies $F'(m_2^*) < 0$. This is for two reasons: First, it is consistent with continuous variations in $\bar{y}$. Second, this solution satisfies saddle-point stability, whereas the other solution is unstable. Therefore, a necessary condition for the asymmetric steady state is given by

$$\left.\frac{\partial F(m_2)}{\partial m_2}\right|_{m_2^*} < 0 \;. \tag{B.3}$$

Finally, if there is no finite value of m_2 that solves (B.1), we must have $\dot{a}_{2,t} > 0$ which implies that the economy is in the symmetric stagnation steady state.

To summarize: *The asymmetric steady state under stagnation exists for $\beta\bar{y} > n\rho_1 + (1-n)\rho_2$ if there exists a finite, positive value of m_2 that solves (B.1). Moreover, (B.2) is a sufficient condition for the existence of the asymmetric steady state given $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 \geq 0$. In addition, (B.3) is a necessary condition for the asymmetric steady state to occur.*

For illustration, consider the specific utility function $v(m_2) = \beta m_2 + \delta ln(m_2)$. Figure B.1 shows the behavior of the two sides of equation (B.1) as a function of m_2, which is given by

$$\alpha\beta(\beta\bar{y} - \rho_1 n - (1-n)\rho_2)m_2 + \delta\alpha(\beta\bar{y} - \rho_1 n) + \frac{\delta n(\rho_1 - \alpha)\bar{y}}{m_2} = \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) \;.$$

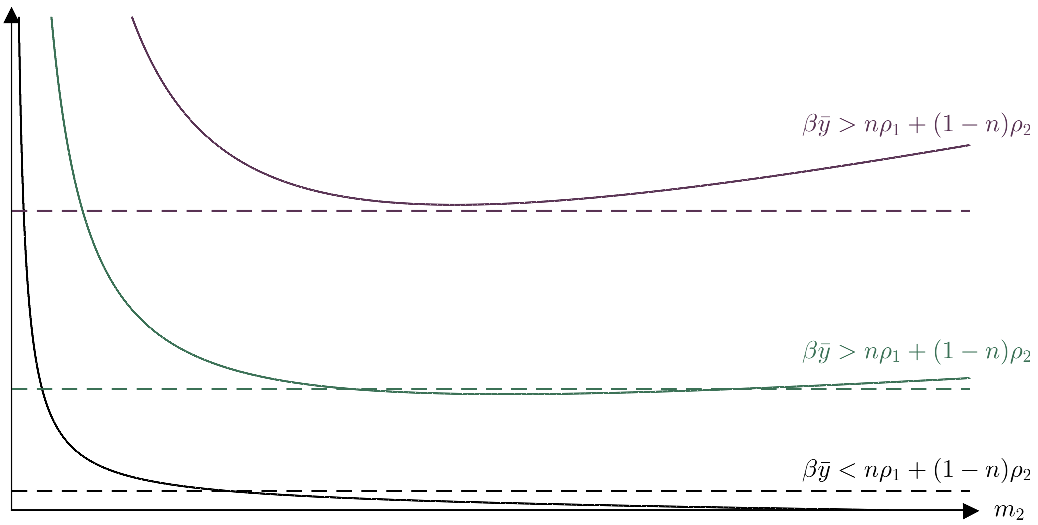
For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 = 0$, the existence of the asymmetric steady state requires a sufficiently low value of δ:

$$\delta < \bar{\delta} \equiv \frac{1}{\alpha(\beta\bar{y} - \rho_1 n)} \left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1)\right] \;. \tag{B.4}$$

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 > 0$, we require in addition that (B.2) holds which implies

$$\frac{\left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) - \alpha\delta(\beta\bar{y} - \rho_1 n)\right]^2}{4\alpha\beta n(\rho_1 - \alpha)\delta\bar{y}} > \beta\bar{y} - n\rho_1 - (1-n)\rho_2 \;. \tag{B.5}$$

Figure B.1: Existence of the Asymmetric Steady State under Stagnation



Note: This figure illustrates the LHS (solid line) and RHS (dotted line) of (B.1) for different values of potential output $\bar{y}$ and for the specific utility function $v(m_2) = \beta m_2 + \delta ln(m_2)$.

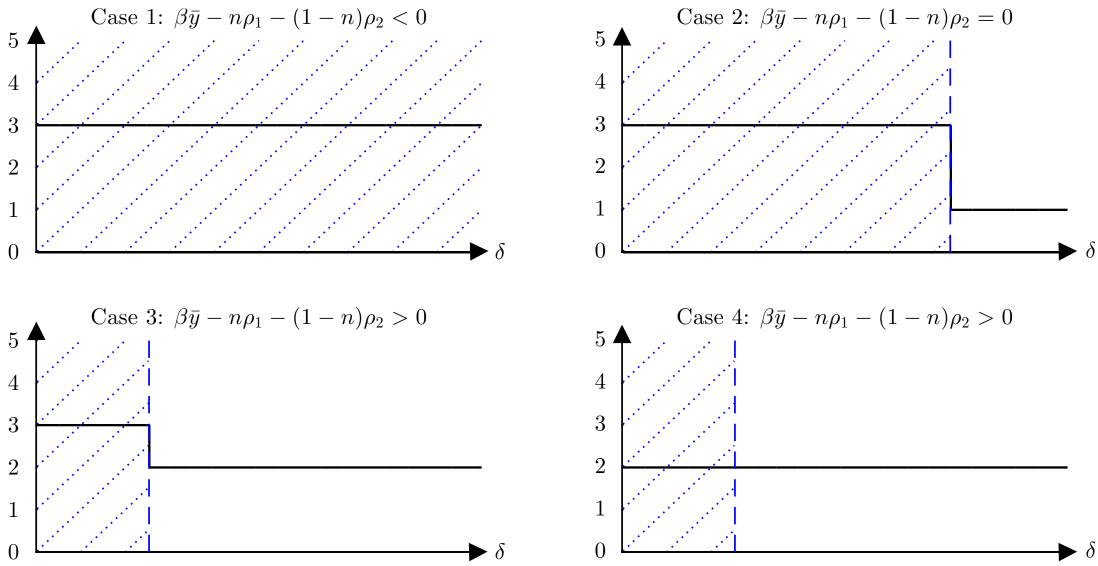
Stability: The dynamic system is characterized by six differential equations for c_1, c_2, q, a_1, a_2 and m given by (2.42), (2.43), (2.44), (2.46), (2.45) and (2.47) and by the static equations (2.20), (2.21) and (2.27) for m_2, h_1 and h_2. The asymmetric steady state under stagnation is characterized by a diverging real money supply and real assets of the saver. Define $z_{1,t} \equiv a_{1,t}^{-1}$ and $z_{2,t} \equiv m_t^{-1}$. Then the steady state of $\{c_{1,t}, c_{2,t}, q_t, a_{2,t}, z_{1,t}, z_{2,t}\}$ is given by $\{c_1^*, c_2^*, q^*, a_2^*, 0, 0\}$. We linearize the system around this steady state using a first-order Taylor approximation:

$$\begin{pmatrix} \dot{c}_{1,t} \\ \dot{c}_{2,t} \\ \dot{q}_t \\ \dot{a}_{2,t} \\ \dot{z}_{1,t} \\ \dot{z}_{2,t} \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} & 0 & 0 & 0 & 0 \\ v_{21} & v_{22} & v_{23} & v_{24} & 0 & 0 \\ v_{31} & v_{32} & v_{33} & v_{34} & 0 & 0 \\ v_{41} & v_{42} & v_{43} & v_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{66} \end{pmatrix} \begin{pmatrix} c_{1,t} - c_1^* \\ c_{2,t} - c_2^* \\ q_t - q^* \\ a_{2,t} - a_2^* \\ z_{1,t} - z_1^* \\ z_{2,t} - z_2^* \end{pmatrix} ,$$

where the entries v_{ij} in the transition matrix V refer to the respective terms in the linearized system. The eigenvalues ξ_i of V determine the stability of this system and solve

$$\left((v_{11} - \xi) \begin{Vmatrix} v_{22} - \xi & v_{23} & v_{24} \\ v_{32} & v_{33} - \xi & v_{34} \\ v_{42} & v_{43} & v_{44} - \xi \end{Vmatrix} - v_{12} \begin{Vmatrix} v_{21} & v_{23} & v_{24} \\ v_{31} & v_{33} - \xi & v_{34} \\ v_{41} & v_{43} & v_{44} - \xi \end{Vmatrix} \right) (v_{55} - \xi)(v_{66} - \xi) = 0 \ ,$$

Figure B.2: Stability of the Asymmetric Steady State under Stagnation



Notes: This figure shows the number of negative eigenvalues in V for the function $v(m_2) = \beta m_2 + \delta ln(m_2)$. Case 1 refers to existence condition (2.51) and cases 2 to 4 refer to conditions (B.4) and (B.5) which are represented by vertical lines. Variations in δ are shown on the x-axis. The calibration is as follows: $\beta = 0.0005; \rho_1 = 0.05; \rho_2 = 0.1; \alpha = 0.01; n = 0.5; H = 1$, $\theta = 0.5$ and $\bar{y} = 120$ (case 1), $\bar{y} = 150$ (case 2) and $\bar{y} = 200$ (cases 3 and 4). In the dashed areas, the existence conditions for the asymmetric steady state are fulfilled.

where $\left\|Q\right\|$ is the determinant of Q. Since only $c_{1,t}$, $c_{2,t}$ and q_t are jumpable, there must be three positive and three negative eigenvalues for the system to exhibit saddlepoint stability. $\xi_i = \beta c_1^* - \rho_1 = \pi^*$ is a solution and under stagnation $\pi^* < 0$. Thus, these two eigenvalues are negative. We use a numerical analysis for the other solutions.

Based on the functional form $v(m_2) = \beta m_2 + \delta ln(m_2)$, we simulate V for three cases determined by $\beta\bar{y} - n\rho_1 - (1-n)\rho_2$. For each case, we vary δ (and implicitly m_2^*), which determines the strength of the capital gains channel in (2.41). We then determine the number of negative eigenvalues. The results are summarized in Figure B.2 which also highlights the threshold parameter $\bar{\delta}$ in (B.4) or (B.5).

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 < 0$ (case 1), the system is saddlepoint-stable for all $\delta > 0$. This corresponds to condition (2.51). For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 = 0$ (case 2), the system is saddlepoint-stable for $0 < \delta < \bar{\delta}$. Hence, under existence condition (B.4) the steady state exhibits saddle-point stability. For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 > 0$, there are two solutions to (B.1) shown in cases 3 and 4. Both solutions require condition (B.5) to hold. Yet, only one of these solutions shows saddlepoint stability. This is the solution that fulfills condition (B.3). We therefore conclude that the model is saddlepoint-stable around the asymmetric stagnation steady state under conditions (B.1), (B.2) and (B.3).

B.3 Proof of Proposition 2.2

The effects of variations in the parameters on the asymmetric steady state are derived from the total differential of (B.1). Define $\Omega(m_2, x) \equiv 0$ where x is any parameter in the model as

$$\Omega(m_2, x) = \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2^*) - (1-n)\beta\rho_2]m_2^* + n(\rho_1 - \alpha)\bar{y}[v'(m_2^*) - \beta] - A \ ,$$

where A and χ are defined in (B.1). From this expression, we recover the effect on money demand of the borrower as

$$\frac{\partial\Omega(m_2, x)}{\partial\theta}dx + \frac{\partial\Omega(m_2, x)}{\partial m_2}dm_2 = 0 \ \Leftrightarrow \ \frac{dm_2}{dx} = -\frac{\partial\Omega(m_2, x)/\partial x}{\partial\Omega(m_2, x)/\partial m_2} \ . \tag{B.6}$$

where $\frac{\partial\Omega(m_2,x)}{\partial m_2} = F'(m_2) < 0$, which follows from (B.3) and the discussion in Appendix B.2.

Consider the effects of variations in the loan-to-value ratio θ and the housing preference parameter γ on the asymmetric steady state under stagnation:

$$\frac{\partial\Omega(m_2, \theta)}{\partial\theta} = -\frac{\rho_1\rho_2\gamma}{\bar{\rho}_\theta^2}\chi < 0 \ \text{ and } \ \frac{\partial\Omega(m_2, \gamma)}{\partial\gamma} = -\frac{\theta\rho_1}{\bar{\rho}_\theta}\chi < 0 \ . \tag{B.7}$$

It follows that

$$\frac{dm_2^*}{d\theta} < 0 \ , \ \ \frac{dm_2^*}{d\gamma} < 0 \ .$$

These results imply together with (2.35) that

$$\frac{dc_2^*}{d\theta} < 0 \ , \ \ \frac{dc_2^*}{d\gamma} < 0 \ .$$

The effects on the steady state value of the other variables can be derived from their relation with c_2^* and m_2^*. The effects on c_1^*, a_2^*, C^* and π^* are derived from (2.34), (2.39), (2.22) and (2.2) as

$$\frac{dc_1^*}{d\theta} < 0 \ , \ \ \frac{da_2^*}{d\theta} < 0 \ , \ \ \frac{dC^*}{d\theta} < 0 \ , \ \ \frac{d\pi^*}{d\theta} < 0 \ ,$$
$$\frac{dc_1^*}{d\gamma} < 0 \ , \ \ \frac{da_2^*}{d\gamma} < 0 \ , \ \ \frac{dC^*}{d\gamma} < 0 \ , \ \ \frac{d\pi^*}{d\gamma} < 0 \ .$$

Also note that the cross-derivative is strictly negative which implies mutually reinforcing effects of γ and θ as illustrated in Figure 2.6:

$$\frac{\partial^2\Omega(m_2, \theta)}{\partial\theta \ \partial\gamma} = -\frac{\rho_1\rho_2}{\bar{\rho}_\theta^2}\chi < 0 \ .$$

Appendix C

Appendix to Chapter 3

C.1 Creditor Seniority during S&P Default Episodes

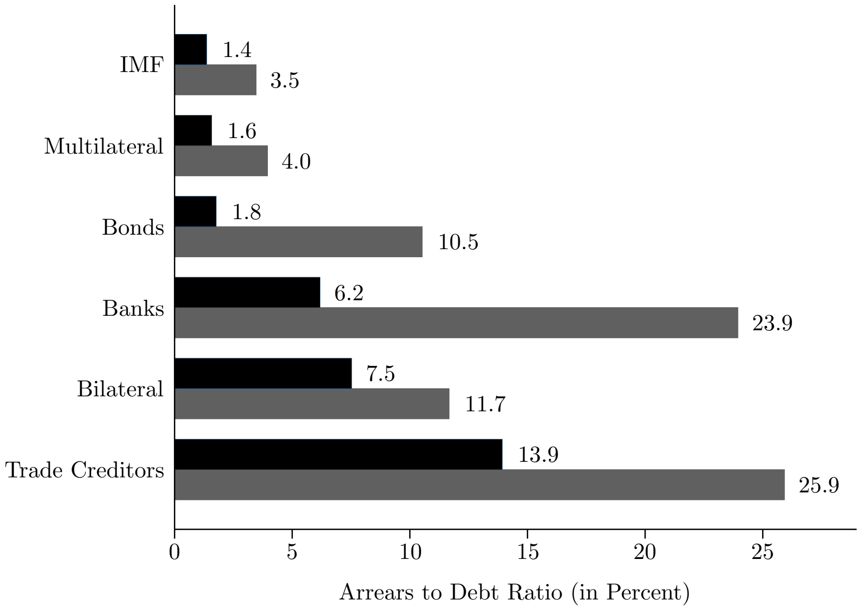
In section 3.3 of Chapter 3, we analyze the occurrence of default based on missed payments. Naturally, the question emerges how these episodes are related to conventional measures of sovereign defaults. To show this, we use the default indicator by Standard & Poor's, which captures missed payments and/or restructuring events by sovereigns towards external banks and bondholders. The S&P default measure is widely used in the literature and strongly correlates with other main databases, like Reinhart and Rogoff (2009). It is important to note, however, that the S&P default measures is not designed to capture default towards official creditors. Indeed, as pointed out by Reinhart and Trebesch (2016a), there is no readily available dataset on defaults towards official creditors. Nevertheless it is informative to check the scope of arrears in periods of S&P default.

Figure C.1 provides an overview of the average arrears to debt ratio for each creditor group during S&P default episodes in contrast to normal times. Arrears to debt ratios increase substantially for each creditor group during default episodes with the total ATD increasing from 4.9% in normal times to 10.7% during defaults. The extent of the increase differs across creditors and is concentrated on private lenders. Banks experience the biggest increase in absolute terms and bondholders face the largest relative increase.

Figure C.2 shows the same statistics for the $RPIA$, our preferred measure of creditor seniority. In terms of the relative arrears burden, the $RPIA$ highlights a less discriminatory stance of sovereigns towards bilateral lenders during defaults and a substantial increase in discrimination against banks and trade creditors. Multilateral lenders, bondholders and the IMF continue to be favored during defaults.

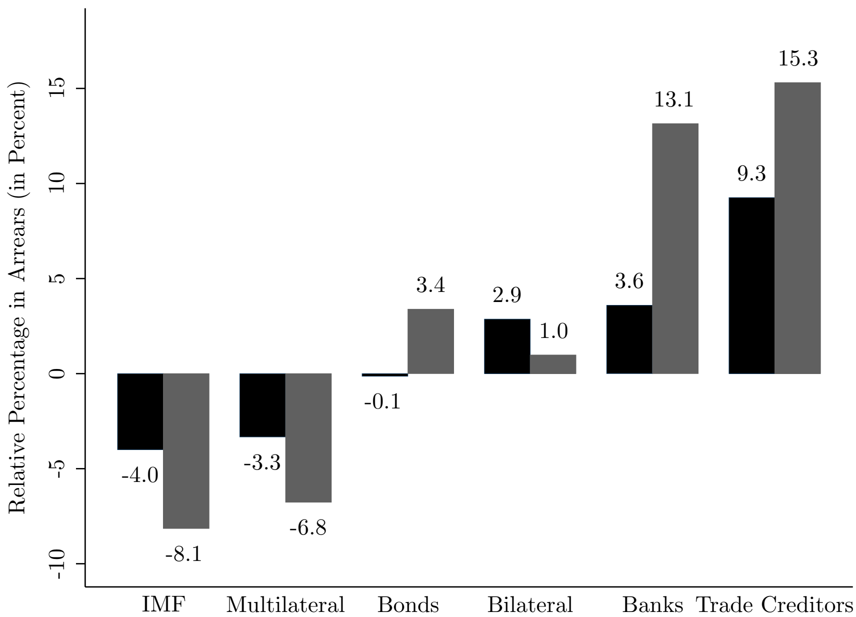
To summarize, the seniority structure of sovereign debt that we observe in Chapter 3 remains intact if we focus only on default episodes as defined by conventional measures. Moreover, the picture is consistent with the interpretation of the S&P dummy as a measure of default towards private creditors.

Figure C.1: Arrears to Debt Ratios during S&P Defaults



Note: This figure shows average ATDs during default episodes. The upper bars show average ATDs for normal times, the lower bars averages over 114 periods of defaults defined by S&P. Averages are calculated as unweighted means over all non-default observations and over all default periods respectively.

Figure C.2: Relative Percentage in Arrears during S&P Defaults



Note: This figure shows average RPIAs during default episodes. The left bars show average ATDs for normal times, the right bars averages over 114 periods of defaults defined by S&P. Averages are calculated as unweighted means over all non-default observations and over all default periods respectively.

C.2 Background on the Paris Club

Assembled for the first time in 1956, the Paris Club is an informal forum of the most important official creditor countries to deal with payment difficulties of debtors. Restructurings with the Paris Club were particularly relevant during the heydays of the debt crises in the 1980s and 1990s but have declined substantially since the mid-2000s. The Paris Club has reached 433 agreements with 90 debtor countries since 1956 (to 2015) restructuring over 580 billion US-Dollar.

The restructuring approach of the Paris Club has evolved over time. In the 1980s, negotiations took place on a case-by-case basis and focused on maturity extensions for debt service obligations to deal with short-term liquidity problems. During the 1990s and 2000s, restructurings became increasingly standardized and concessional including debt stock cancellations to tackle problems of long-term sustainability, particularly in low income countries.

The Paris Club invokes a number of principles, in particular those of unanimity and equal burden sharing among its members. In addition, the comparability of treatment clause requires countries to seek comparable debt relief from its other bilateral creditors as well as private ones.

A debtor country that wants to restructure its debts with the Paris Club has to demonstrate its inability to service its obligations and hence the need for debt relief. In addition, countries are expected to implement a structural adjustment program with the IMF. Once these criteria are met, negotiations with the group of creditor countries take place in Paris.[1] These result in a final agreement on the broad terms of restructurings - the "Agreed Minutes".

The Agreed Minutes are based on general terms that have evolved over time towards more comprehensive debt relief and concessional treatment, particularly for low income countries.[2] These general terms can be distinguished as follows:

- *Classic Terms* were the standard terms applied to restructurings with debtor countries until the late 1980s. Negotiations were on a case-by-case basis and the terms applied to both official development aid (ODA) and market-related debt like export credits (non-ODA).[3] The terms were modified in 2003 to provide more comprehensive and flexible treatment of unsustainable debt levels in non-HIPC countries including the possibility of debt stock cancellations ("Evian Approach").[4]

[1]In addition, several non-creditor representatives participate as observers as do representatives from various international organizations like the World Bank or the International Monetary Fund.

[2]Martin and Vilanova (2001), Gueye et al. (2007) and Cheng, Diaz-Cassou, and Erce (2016) provide a detailed overview of the different terms.

[3]Specifically, the Classic Terms are defined as follows: "Credits (whether ODA or non-ODA) are rescheduled at the appropriate market rate with a repayment profile negotiated on a case-by-case basis."

[4]Restructurings under the Evian Approach are also made on a case-by-case basis against the specific background of the debtor. Hence, we do not differentiate between these two approaches within the set of classic terms. Examples for the Evian Approach include Iraq (2004), Kenya (2004), Grenada (2006) or the Seychelles (2009).

- Restructuring terms for highly indebted middle income countries were enhanced in the *Houston Terms* of 1990. These terms allow for a differential treatment of non-ODA and ODA debt with a substantial extension of maturities and grace periods. Yet, there is no debt stock cancellation.

- Restructuring terms for highly indebted low income countries became increasingly concessional over time. In particular, the possibility of a partial debt stock cancellation of non-ODA debt was gradually extended from 33% of the eligible debt in 1988 (*Toronto Terms*) to 50% in 1991 (*London Terms*) and 66% in 1994 (*Naples Terms*). In the wake of the Heavily Indebted Poor Country Initiative, cancellation rates increased further to 80% in 1996 (*Lyons Terms*) and 90% in 1999 (*Cologne Terms*). Finally, debt relief at completion point under the HIPC Initiative is provided within the *HIPC Exit Terms*. All these terms allow for a differential treatment of ODA and non-ODA debt and give countries two options on how debt relief on non-ODA debt is delivered. In addition, maturity and grace periods are substantially extended, particular for ODA debt.[5]

The Agreed Minutes are not legally binding but serve as a benchmark for the following bilateral negotiations. Actual debt relief only results from the implementation of separate bilateral agreements, which are however not publicly available. The Agreed Minutes include guidelines for the agreed consolidation period, the cut-off date, the total amount of debt relief including the restructured debt stock and potential cancellation as well as the grace and maturity periods. Specifically, the Paris Club website explains that the Agreed Minutes document "states the commonly agreed debt treatment in writing. This is not a legally binding document but a recommendation by the heads of delegations of participating creditor countries and of the debtor country to their governments to sign a bilateral agreement implementing the debt treatment."

[5]Our estimated haircuts for official creditors vary considerably over Paris Club terms of agreement. Haircuts are in general lower for middle income countries that reschedule under Classic terms (average haircut of 40.4%) or Houston terms (38.5%) than for low income countries. For the latter, haircuts increase as Paris Club terms become increasingly concessional. Specifically, average haircuts increase from 75.7% under Toronto terms to 85.3% under HIPC Exit terms (not taking into account additional bilateral debt relief).

C.3 Additional Tables and Figures

The following tables and figures are included in this appendix:

- Tables: Table C.1 summarizes descriptive statistics of the $RPIA$ measure of creditor seniority for different subsamples. Table C.2 provides an overview of the control variables used in the regressions. Table C.3 shows the full regression results associated with Table 3.1. Table C.4 shows the Oaxaca-Blinder results when the lagged dependent variable is included in the regression. Table C.5 summarizes descriptive statistics of haircuts for different subsamples. Table C.6 shows the full regression results associated with Table 3.3.
- Figures: Figures C.3 and C.4 show the frequency of sovereign debt restructurings over time as well as the amounts of debt restructured by creditor type. Figures C.5 to C.10 show the behavior of arrears during restructurings for selected countries.

Table C.1: Sample Statistics: Relative Percentage in Arrears

	Bilateral Creditors	Multilateral Creditors	IMF	Bonds	Banks	Trade Creditors
Total Sample	2.31	-5.01	-6.20	0.95	7.76	12.64
High Income	0.46	-3.99	-4.32	-0.11	2.21	5.98
Medium Income	1.20	-4.58	-6.38	-0.13	7.36	10.25
Low Income	5.99	-6.84	-7.90	6.68	16.35	23.77
East Asia	1.24	-4.06	-2.28	-2.27	-0.75	6.89
Europe	0.76	-2.80	-3.63	-0.58	2.26	5.77
Latin America	0.51	-4.54	-5.45	2.31	9.54	12.10
Middle East	-0.79	-5.44	-10.01	-1.62	2.35	2.73
South Asia	0.13	-0.25	-0.24	-0.81	0.25	1.78
Sub-Saharan Africa	5.58	-7.16	-8.93	2.30	13.88	21.58
BLEND Countries	2.31	-2.82	-1.65	0.08	4.57	7.45
IBRD Countries	0.09	-4.13	-5.67	-0.72	2.44	5.77
IDA Countries	4.60	-6.68	-7.75	5.68	15.08	21.04
Autocracy	2.86	-6.92	-8.36	-1.07	8.09	13.69
Intermediate	2.87	-6.14	-6.76	4.61	11.13	17.21
Democracy	0.59	-2.44	-3.43	1.16	4.54	4.82

Note: Shown are unweighted means of country averages for the Relative Percentage in Arrears in percent. The definition of income groups (rows 2-4) is based on the classification of the World Bank and described in Figure 3.3. Regions (rows 5-10) and lending categories (rows 11-14) follow the definition of the World Bank as of 2015. Political regimes (rows 15-17) are based on the Polity II index which ranks from -9 to +9. Countries are defined as democracies (autocracies) if they have a score above 3 (below -3).

Table C.2: Overview of Control Variables

Basic Controls

Debt to GNI Ratio (creditor-specific): Public and publicly guaranteed long-term debt of each creditor group as a fraction of GNI in Percent. Source: WDI

External Debt to GNI: Total external debt is debt owed to nonresidents repayable in foreign currency, goods, or services. It is the sum of public, publicly guaranteed, and private nonguaranteed long-term debt, short-term debt, and use of IMF credit. Source: WDI

GDP per Capita: GDP per capita is gross domestic product (in current U.S. dollars) divided by midyear population. Source: WDI

GDP Growth: GDP growth is the growth rate (in percent) of nominal GDP (measured in current U.S. dollars). Source: WDI

GDP Growth (lag): GDP growth is the growth rate (in percent) of nominal GDP (measured in current U.S. dollars). Source: WDI

Disturbance Dummy: The disturbance dummy equals 1 if the country is engaged in a war or civilwar as indicated by the inter-state and intra-state war datasets by Sarkees and Wayman (2010). Source: Correlates of War dataset, Sarkees and Wayman (2010)

Capital Market Access Dummy: The market access dummy equals 1 if the country is eligible for the IMF's Poverty Reduction and Growth Facility (PRGF). Source: IMF, Allen (2008)

Dummy for Systemic Banking Crises: A banking crisis is defined as systemic if two conditions are met: (1) Significant signs of financial distress in the banking system (as indicated by significant bank runs, losses in the banking system, and/or bank liquidations) and (2) Significant banking policy intervention measures in response to significant losses in the banking system. Source: Laeven and Valencia (2012)

Dummy for Currency Crises: A currency crisis is defined as a depreciation of the currency relative to the U.S. dollar of at least 30 percent that is also at least 10 percentage points higher than the rate of depreciation in the year before. Source: Laeven and Valencia (2012)

Additional Controls

Private debt to external debt ratio: Private nonguaranteed external debt comprises long-term external obligations of private debtors that are not guaranteed for repayment by a public entity. Source: WDI

Financial openness: Financial openness is defined as the sum of external financial assets and liabilities as a fraction of GNI in percent. Source: Lane and Milesi-Ferretti (2007)

Trade openness: Trade openness is defined as the sum of a country's exports and imports as a fraction of GNI in percent. Source: WDI

Gross domestic investment to GNI: Gross domestic investment consists of outlays on additions to the fixed assets of the economy plus net changes in the level of inventories. Fixed assets include land improvements; plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. Inventories are stocks of goods held by firms to meet temporary or unexpected fluctuations in production or sales, and "work in progress." Net acquisitions of valuables are also considered capital formation. Source: WDI

CPI inflation: Inflation as measured by the consumer price index reflects the annual percentage change in the cost to the average consumer of acquiring a basket of goods and services that may be fixed or changed at specified intervals, such as yearly. The Laspeyres formula is generally used. Source: WDI

Short-term external debt to GNI: Short-term external debt is defined as debt that has an original maturity of one year or less. Available data permit no distinction between public and private nonguaranteed short-term debt. Source: WDI

Foreign reserves to external debt ratio: The ratio of a country's international reserves to total external debt stocks. Source: WDI

Debt service to exports: Debt service is the sum of principle repayments and interest actually paid in foreign currency, goods, or services. This variable covers long-term public and publicly guaranteed debt and repayments (repurchases and charges) to the IMF. Source: WDI

Table C.3: Creditor Dummy Regressions with Arrears Measures (Full Results)

	(1) RPIA	(2) RPIA	(3) RPIA	(4) RPIA	(5) RPIA	(6) ATD	(7) ATD
Multilateral	-8.09***	-8.08***	-8.08***	-8.34***	-1.42***	-8.09***	-14.45***
	[1.03]	[1.03]	[1.04]	[1.20]	[0.20]	[1.04]	[0.78]
IMF	-10.39***	-10.56***	-10.70***	-10.92***	-1.68***	-10.74***	-54.70***
	[1.24]	[1.26]	[1.28]	[1.46]	[0.25]	[1.24]	[1.40]
Bondholders	0.07	0.35	0.54	-0.02	0.57	0.42	-11.91***
	[2.11]	[2.07]	[2.10]	[2.21]	[0.43]	[2.05]	[1.38]
Banks	7.17***	7.48***	7.59***	5.94***	1.50***	7.59***	5.76***
	[1.46]	[1.47]	[1.48]	[1.56]	[0.32]	[1.48]	[0.86]
Trade Creditors	10.70***	10.89***	10.98***	9.00***	1.93***	10.83***	12.04***
	[1.70]	[1.70]	[1.71]	[1.76]	[0.31]	[1.70]	[0.82]
Debt to GNI	0.04**	0.04**	0.04**	0.05*	0.00	-0.04***	-0.03***
	[0.02]	[0.02]	[0.02]	[0.03]	[0.00]	[0.02]	[0.01]
Real GDP per Capita	0.00	-0.00	0.00	0.00**	0.00	-0.00***	-0.01***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Real GDP Growth	-0.23	-4.80	-2.52	-6.66*	-1.00	-10.56**	-17.56***
	[3.14]	[3.41]	[2.84]	[3.98]	[1.71]	[4.50]	[4.67]
Lagged Growth	-0.08	-2.92	-1.26	-1.24	1.60	1.46	-0.81
	[2.76]	[2.57]	[2.48]	[4.46]	[1.58]	[6.92]	[4.47]
War or Civilwar	1.62	-0.48	-0.67	-0.35	-0.23	1.24	2.28**
	[1.16]	[0.95]	[0.94]	[1.03]	[0.27]	[1.85]	[0.97]
Banking Crisis	-1.25	-1.06	-1.18	-0.48	-0.34	1.91**	4.18***
	[0.79]	[0.71]	[0.72]	[0.92]	[0.45]	[0.92]	[1.41]
Currency Crisis	-0.62	-0.63	-0.95*	-1.47**	0.12	1.83***	2.71**
	[0.60]	[0.52]	[0.51]	[0.59]	[0.30]	[0.70]	[1.17]
No Market Access	-5.12***	-6.76	-4.31	-0.14	-0.76	4.51	9.61***
	[1.11]	[4.19]	[4.43]	[4.47]	[0.87]	[5.98]	[2.68]
External Debt to GNI	-0.01**	-0.01*	-0.01	-0.04*	0.00	0.04***	0.04***
	[0.01]	[0.01]	[0.01]	[0.02]	[0.00]	[0.01]	[0.00]
Trade Openness				0.09***			
				[0.03]			
Debt Service to Exports				-0.03			
				[0.06]			
Reserves to Debt				0.00			
				[0.00]			
Short-term Debt to GNI				0.10*			
				[0.05]			
Share of Private Debt				-0.01			
				[0.04]			
CPI Inflation				-0.00			
				[0.00]			
Investment to GDP				-0.01			
				[0.05]			
Financial Openness				-0.01**			
				[0.00]			
Lagged RPIA					0.88***		
					[0.01]		
Constant	-0.27	1.08	1.55	-7.18**	0.47	2.64	13.63***
	[1.03]	[1.75]	[2.15]	[2.74]	[0.55]	[4.96]	[3.18]
Country Fixed Effects	No	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	No	No	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.195	0.242	0.249	0.249	0.786	0.437	0.133
Observations	11931	11931	11931	9176	11790	11931	11931

Standard errors in brackets; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table C.4: Oaxaca-Blinder Decomposition with Lagged Dependent Variable

	Difference in RPIA	Explained (fundamental)	Unexplained (discrimination)	Discrimination (in Percent)
Bilateral - Multilateral	7.99	6.87	1.12	14.07
Bilateral - IMF	9.08	8.20	0.88	9.70
Bilateral - Bondholders	1.08	1.45	-0.37	-34.22
Bilateral - Banks	-7.65	-6.11	-1.54	20.17
Bilateral - Trade Creditors	-11.69	-10.19	-1.51	12.84
Multilateral - IMF	1.09	1.18	-0.09	-8.49
Multilateral - Bondholders	-6.91	-6.03	-0.88	12.77
Multilateral - Banks	-15.64	-13.19	-2.45	15.65
Multilateral - Trade Creditors	-19.68	-17.60	-2.08	10.57
IMF - Bondholders	-8.00	-6.95	-1.05	13.12
IMF - Banks	-16.73	-13.82	-2.91	17.38
IMF - Trade Creditors	-20.77	-18.45	-2.32	11.18
Bondholders - Banks	-8.73	-8.13	-0.59	6.83
Bondholders - Trade Creditors	-12.77	-12.29	-0.48	3.76
Banks - Trade Creditors	-4.04	-3.55	-0.49	11.96

Note: The table shows the Oaxaca-Blinder decomposition for the mean difference in RPIAs across creditor groups. The mean difference is listed in the first column. The second and third columns show the part of the difference that is fundamentally justified and the part of the difference that is unexplained and thus reflects active creditor discrimination. The last column states the fraction of the mean difference that reflects discrimination in percent. Note that mean differences vary from the sample statistics because of data availability with respect to the explanatory variables.

Table C.5: Sample Statistics: Haircuts

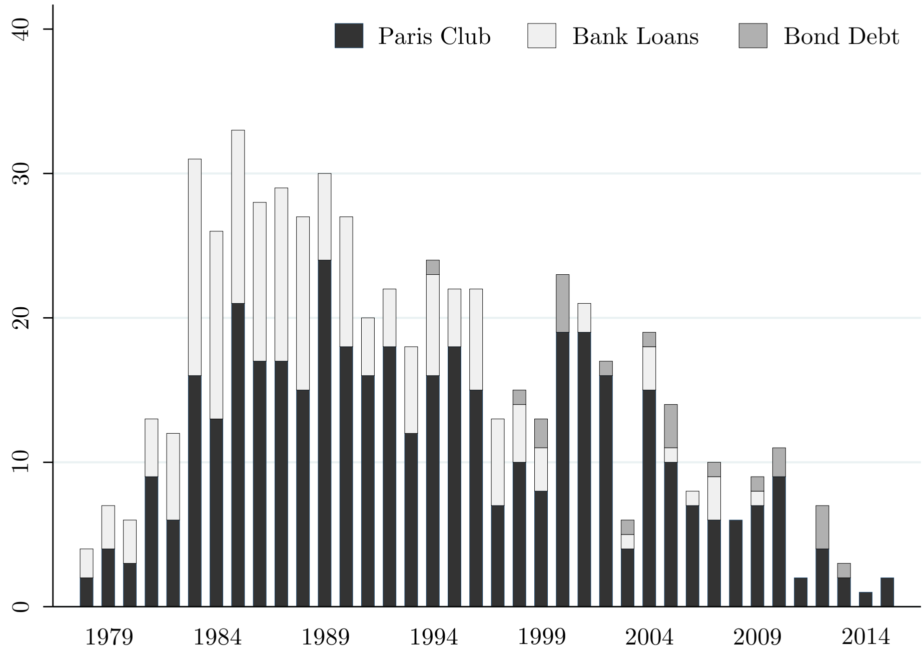
	Paris Club (Lower Bound)	Paris Club (Upper Bound)	Private Creditors (Market Rate)
Mean	59.9	78.6	40.5
Std. Deviation	25.8	20.8	27.7
Minimum	-19.3	21.5	-9.8
25th Percentile	41.2	62.5	19.2
50th Percentile	61.4	86.0	40.3
75th Percentile	84.5	96.9	56.3
Maximum	100	100	97.0
Observations	387	320	187
1978 - 1989			
Mean	51.0	68.7	27.8
Std. Deviation	18.6	17.8	19.9
Minimum	15.2	25.7	-9.8
25th Percentile	39.1	55.6	12.6
50th Percentile	47.6	70.4	28.8
75th Percentile	58.8	83.5	42.3
Maximum	89.8	99.5	92.7
Observations	124	102	99
1990 - 1999			
Mean	60.1	78.9	52.6
Std. Deviation	27.2	22.1	28.1
Minimum	-9.3	24.5	-8.3
25th Percentile	38.3	63.4	29.2
50th Percentile	68.7	89.1	49.2
75th Percentile	84.0	97.1	82.0
Maximum	95.8	99.9	92.3
Observations	137	109	58
2000 - 2014			
Mean	68.4	87.5	58.8
Std. Deviation	27.7	17.8	28.8
Minimum	-19.3	21.5	4.1
25th Percentile	50.4	85.6	36.9
50th Percentile	77.1	95.2	61.0
75th Percentile	88.7	99.1	85.5
Maximum	100	100	97.0
Observations	126	109	30

Table C.6: Creditor Dummy Regressions with Haircut Measures (Full Results)

	Lower Bound Estimates			Upper Bound Estimates		
	(1)	(2)	(3)	(4)	(5)	(6)
Private Creditors	-8.83***	-5.30**	-5.21*	-29.41***	-26.67***	-26.51***
	[2.46]	[2.42]	[2.88]	[2.34]	[2.25]	[2.65]
Restructured Debt	0.24**	0.14*	0.10	0.21**	0.13	0.12
(Share of External Debt)	[0.09]	[0.08]	[0.09]	[0.09]	[0.08]	[0.08]
External Debt to GNI	0.03***	0.03***	0.10***	0.05***	0.04***	0.08**
	[0.01]	[0.01]	[0.04]	[0.01]	[0.01]	[0.03]
Real GDP per Capita	-3.70***	-3.60***	-3.27***	-3.50***	-3.27***	-2.71***
(in 1000 constant 2005 US$)	[0.88]	[1.07]	[1.14]	[0.74]	[0.81]	[0.88]
GDP Growth (3-year MA)	1.51***	0.88***	0.78***	1.09***	0.61*	0.59
	[0.27]	[0.28]	[0.26]	[0.31]	[0.33]	[0.37]
War or Civilwar	1.49	0.10	-0.21	0.73	-1.14	-1.26
	[3.39]	[3.44]	[3.94]	[3.06]	[2.85]	[3.28]
Banking Crisis	0.35	-1.32	-0.85	-0.71	-3.81	-2.03
	[2.79]	[2.56]	[3.39]	[2.95]	[2.86]	[3.72]
Currency Crisis	-4.10	-2.82	-3.27	-2.67	-0.59	-1.22
	[2.64]	[2.33]	[2.82]	[2.43]	[2.37]	[2.78]
Lack of Market Access	23.54***	21.37***	17.94***	17.70***	16.94***	15.11***
	[3.89]	[4.56]	[4.92]	[3.88]	[4.24]	[4.89]
Trade Openness			-0.07			-0.02
			[0.05]			[0.05]
Debt Service to Exports			-0.11			0.05
			[0.11]			[0.12]
Reserves to Debt			0.09			-0.02
			[0.12]			[0.15]
Short-Term Debt to GNI			-0.35**			-0.16
			[0.16]			[0.15]
Share of Private Debt			-0.25			-0.29
			[0.26]			[0.23]
Inflation (CPI)			0.00			0.01**
			[0.00]			[0.00]
Investment to GNI			0.01			-0.09
			[0.20]			[0.20]
Financial Openness			0.00			-0.01
			[0.01]			[0.01]
Constant	39.14***	33.92***	39.30***	61.52***	52.85***	55.61**
	[3.45]	[4.45]	[10.10]	[3.70]	[6.38]	[23.56]
Time Fixed Effects	No	Yes	Yes	No	Yes	Yes
Adjusted R^2	0.484	0.578	0.576	0.607	0.680	0.673
Observations	442	442	359	389	389	314

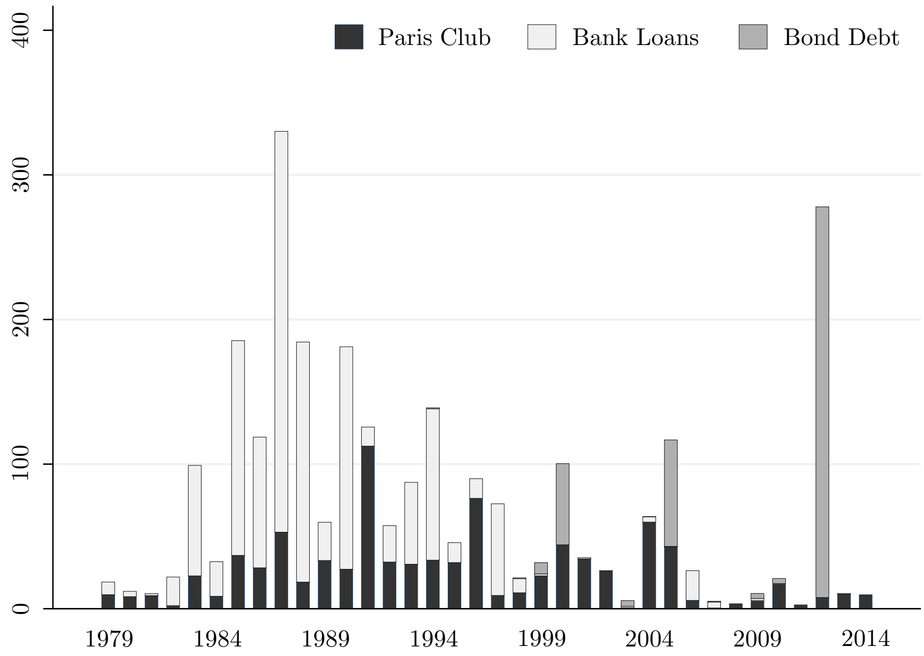
Clustered standard errors (on country level) in parentheses; * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Variables are defined in Table C.2 in Appendix C.3 and in the text of Chapter 3.

Figure C.3: Frequency of Restructurings by Creditor Group



Note: This figure shows the number of finalized restructurings per year for the Paris Club and for private creditors for the period 1978-2015.

Figure C.4: Amount of Restructured Debt by Creditor Group



Note: This figure shows the amount of restructured debt in real US-Dollars (billions) by year and creditor group for the period 1978-2015. Values are inflated to 2014 USD using US CPI inflation.

Figure C.5: Case Study: Argentina

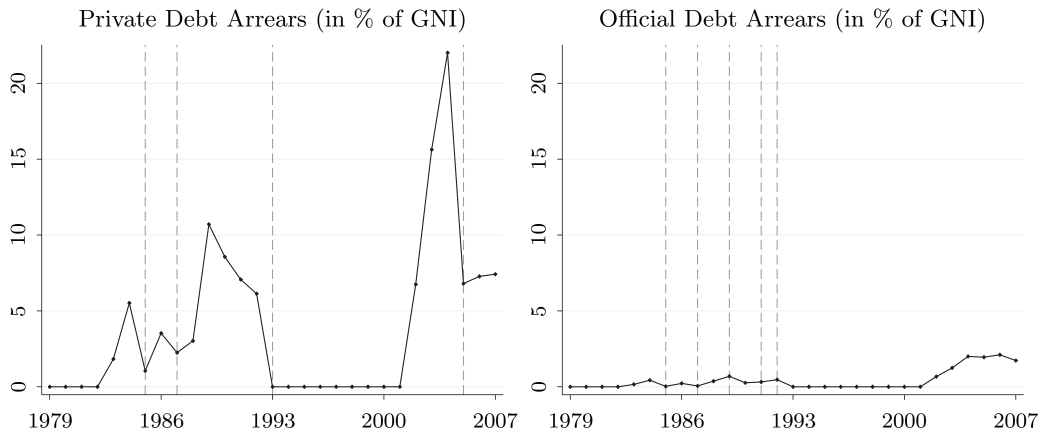


Figure C.6: Case Study: Ecuador

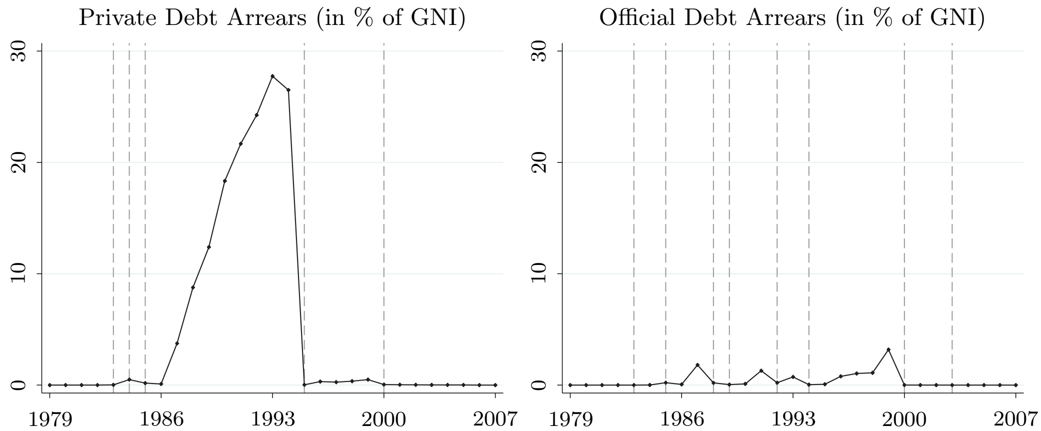
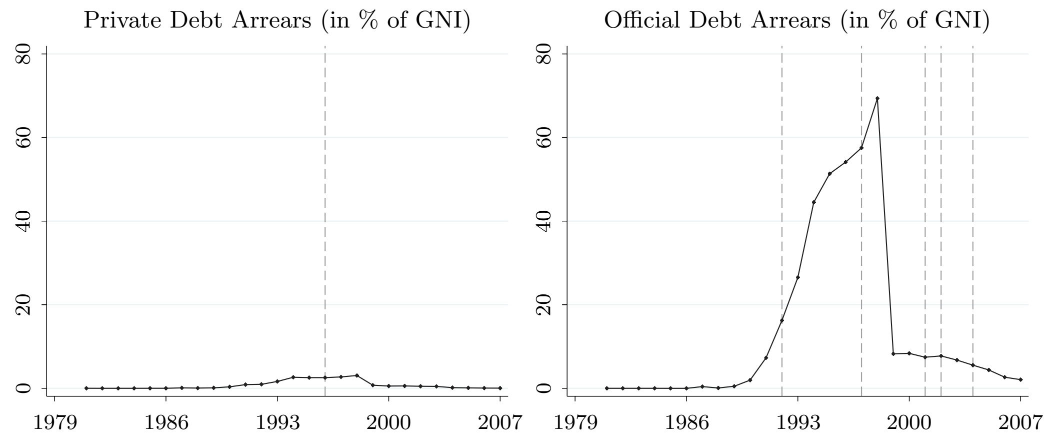


Figure C.7: Case Study: Ethiopia



Note: These figures show official and private arrears as a fraction of GNI. The vertical bars indicate restructurings of private debt (left) and official debt with the Paris Club (right).

Figure C.8: Case Study: Morocco

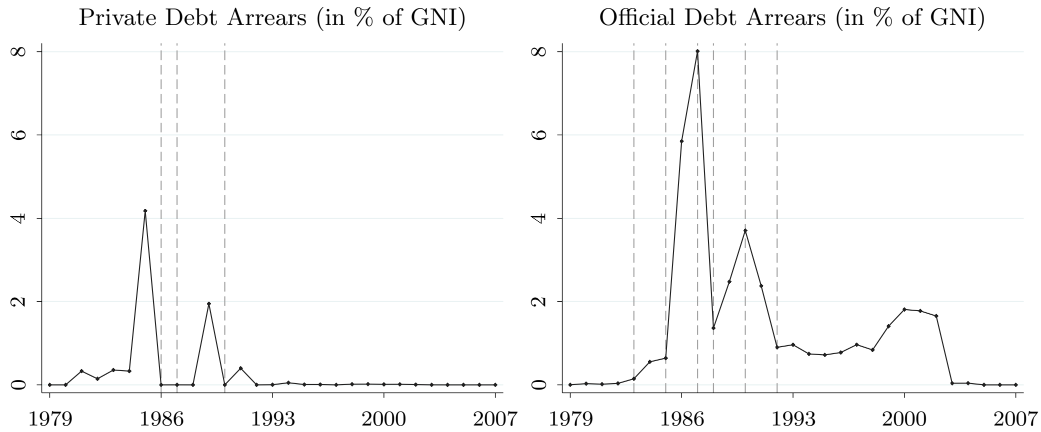


Figure C.9: Case Study: Nigeria

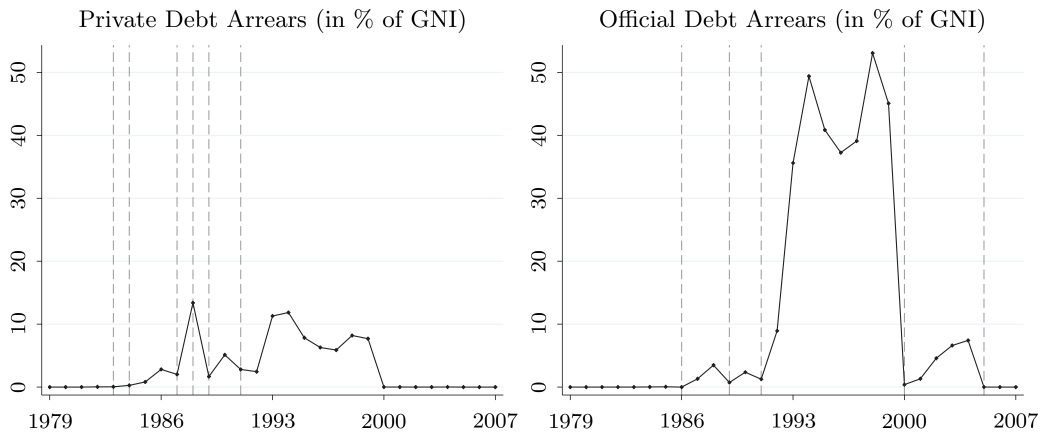
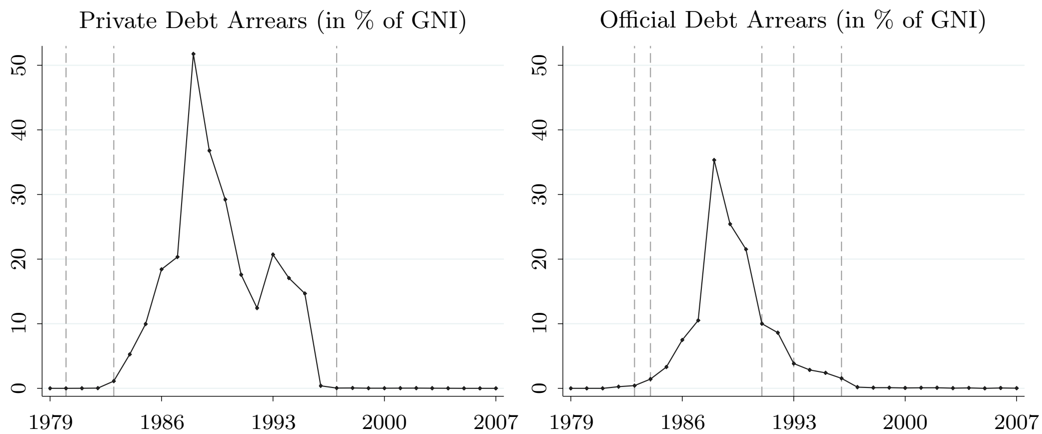


Figure C.10: Case Study: Peru



Note: These figures show official and private arrears as a fraction of GNI. The vertical bars indicate restructurings of private debt (left) and official debt with the Paris Club (right). For Nigeria, GNI data for 1999 is missing and replaced by the average of the 1998 and 2000 values for illustrative purposes.

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