Complex dynamics of drill-strings: Theory and experiments

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Abstract. We investigate complex drill-string dynamics in a downhole drilling where strong nonlinear interactions between various types of vibration take place. First, we present a low dimensional model of the downhole drilling where a drill-bit cutting a rock formation has a strong coupling between torsional and axial oscillations. The model can be used to study drilling stability as an example results are given. Then we introduce a new experimental rig developed by the Centre for Applied Dynamics Research at the University of Aberdeen, capable of reproducing all major types of drill-string vibration. One of the most important features of this versatile experimental rig is the fact that commercial drill-bits, employed in the drilling industry, and real rock-samples are used. The rig operate in different configurations, which enables the experimental study of various phenomena, such as stick-slip oscillations, whirling and drill-bit bounce. It also allows to determine mechanical characteristics of the drill-bits, which are used to calibrate mathematical models.

1 Introduction

Dynamics of drill-strings belongs to one of the most challenging modelling problems and has been the subject of intensive research for many years. Highly nonlinear nature of the interactions between a drill-string and a bottom hole assembly (BHA) and their borehole, generates complex coupled vibration with its axial, torsional and lateral components. This dynamic behaviour being a result of the borehole creation and various frictional and geometrical effects along a drill-string and a BHA, makes the problem difficult to study in a comprehensive manner. The literature on drill-string vibration is vast and many surveys have been written including [1, 2]. Again, drill-strings vibrate axially, laterally and torsionally leading to bit-bounce, forward and backward whirling, and torsional oscillations respectively. The last type associated with so–called stick-slip vibration has attracted most of the attention as it has severe consequences on drilling effectiveness.

Drill-string vibration has been modelled using lumped-mass models, Cosserat-continuum models, and finite elements. In this paper, we focus our attention on axial and torsional vibration via a fully coupled two degrees-of-freedom model with a state-dependent time delay. The axial and rotary degrees-of-freedom are to mimic effective motions of the drill-

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bit in axial and rotational directions respectively. The time delay is predominantly a result of the drill-bit geometry and its rotational velocity. Contrary to majority of published work, the dynamic loading of the drill-bit reflect both the cutting and frictional effects occurring during a wellbore creation process. This process is controlled by many parameters including Weigh On Bit (WOB) and rotary speed.

In this study we will focus on modelling and analysis of the coupled axial-rotary vibration of a two-degrees-of-freedom model of the dynamic interactions between a drill-bit and the drilled formation. The paper is structured as follows. Section 2 outlines the physical and mathematical models developed in [3, 4]. In Section 3, we describe the Aberdeen Drill-String Dynamics experimental rig. Finally, in Section 4, some closing remarks are made summarising the work and giving an outlook for the future work.

2 Physical and mathematical model [3, 4]

The physical and mathematical model of the two degrees-of-freedom has been derived in [3, 4]. The model in the axial direction idealizes the BHA and the drill-string as a lumped mass M neglecting the axial compliance of the drill-pipes. A constant vertical hook load H_0 (which is related to the applied WOB) is applied at the top of the mass M and the W arising from the bit-rock interaction is applied on the mass as shown in Fig. 1(a). The position of the drill-bit is measured by U from a fixed reference. The axial force and torque exerted by the rock on the bit is decomposed into the cutting and friction components denoted by appropriate subscripts:

$$T = T_c + T_f,\ W = W_c + W_{f,} \tag{1}$$

where

$$T_c = \frac{a^2 \epsilon d}{2},\ W_c = \xi \epsilon a d, \tag{2}$$

here a represents the radius of the bit, d represents the instantaneous depth of cut, ϵ represents the intrinsic specific energy of the rock and ξ characterizes the inclination of the cutting forces on the cutting face.

The friction components of the WOB and TOB are given by

$$w_f = \sigma a l,\ \ T_f = \frac{\mu \gamma a W_f}{2}, \tag{3}$$

where the parameter γ is related to the orientation of the cutters on the bit, σ refers to the maximum contact pressure at the bit-rock interface, and l refers to the total wear-flat length in all the blades. Two consecutive blades of such a drag bit are shown in Fig. 1(b). The instantaneous depth of cut removed by each blade is the difference between the present axial position of the bit, and the axial position occupied by the bit $1/n$-th revolution ago. Thus the instantaneous depth of cut faced by a single blade (say 2, in Fig. 1(b)) is given by

$$d_n = U(t) - U(t - t_n). \tag{4}$$

Here t_n is the time taken by the bit to rotate by an angle equal to $\frac{2\pi}{n}$. Thus the time delay $t_n(t)$ satisfies the following equation

$$\Phi(t) - \Phi(t - t_n) = \frac{2\pi}{n}. \tag{5}$$

The total depth of cut per revolution for n blades is

$$d = n(U(t) - U(t - t_n)).$$

In the absence of torsional oscillations, the bit would rotate at a constant angular velocity Ω_0, and then the time delay $t_n = \frac{2\pi}{n\Omega_0}$ will be a constant. However, when the drill-string undergoes torsional vibrations the time delay itself is governed by the system dynamics through Eq. (5).

(a) (b)

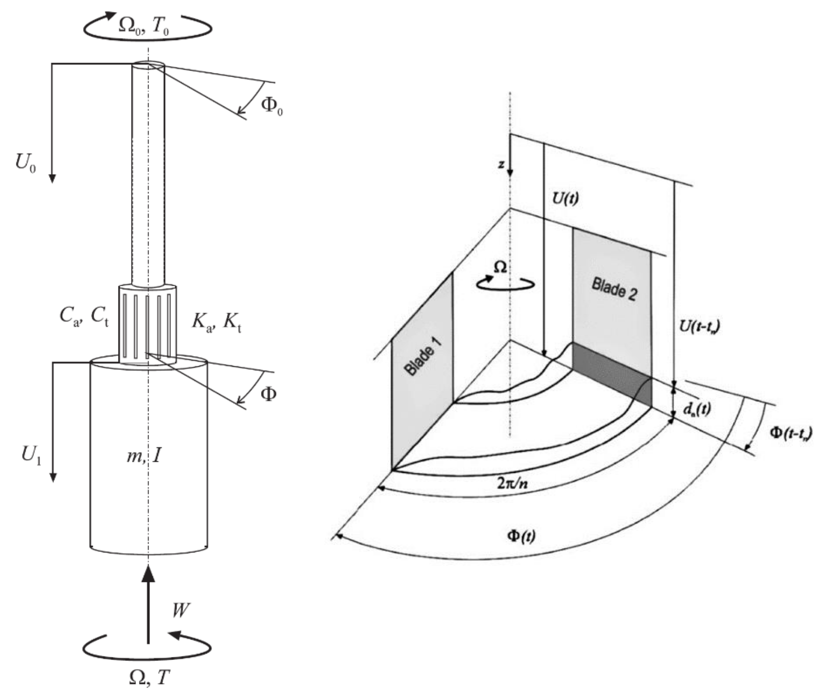

Figure 1. (a) Schematic of the drill-string model with a lumped mass approximating the BHA (*m, I*), and a flexible cylinder modelling the stiffness (K_a, K_t) and damping (C_a, C_t). (b) Schematic of a drag bit illustrating the definition of the instantaneous depth of cut. Two successive blades of the bit are shown. The instantaneous depth of cut faced by a blade (say 2) equals the difference between the present axial position of the bit and its position at an earlier instant when the immediately preceding blade (say 1) was occupying the current angular position of blade 2.

Thus the governing differential equations of the model presented in Figure 1(a) are [3, 4]

$$M\ddot{U} + C_a\dot{U} + K_a(U - V_0 t) = W_0 - \xi a \epsilon d\, H(\dot{\Phi})H(d) - W_f H(\dot{U}), \tag{6}$$

$$I\ddot{\Phi} + C_t\dot{\Phi} + K_t(\Phi - \Omega_0 t) = -\frac{a^2\epsilon d}{2}\, H(d)H(\dot{\Phi}) - \frac{\mu\gamma a W_f}{2}\operatorname{sgn}(\dot{\Phi})H(\dot{U}), \tag{7}$$

here H(·) and sgn(·) are the Heaviside and sign functions respectively.

Equations (6) - (7) constitute a set of two coupled delay differential equations with a state-dependent delay, along with the discontinuities associated with cutting and friction forces. The dynamics of the full nonlinear system is complex and analytical treatment is difficult. In [3] we have performed the steady drilling stability analysis at a constant rate of penetration, which provides some useful indications on the stability boundaries. Here, we include an example stability map of the fully nonlinear system computed by brute force numerical simulation of Eqs (6) and (7), which is shown in Figure 2.

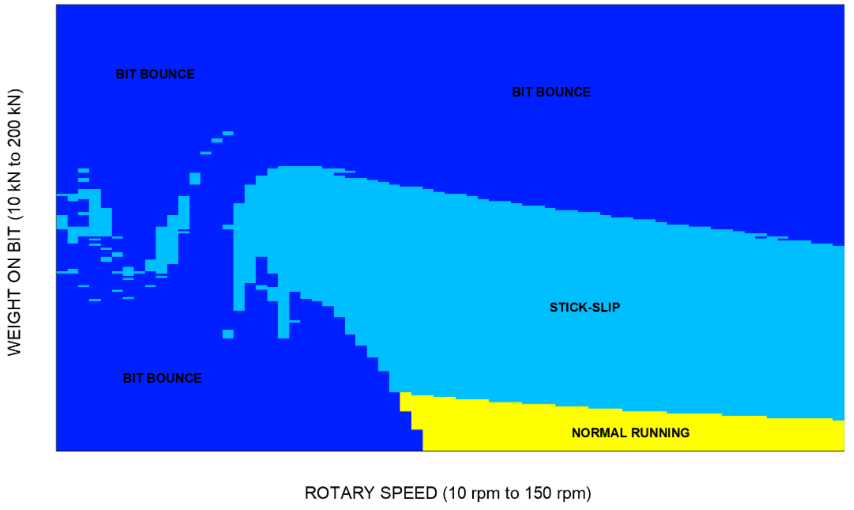

Figure 2. Stability map of stable (normal running) and unstable responses (stick-slip and bit bouce) computed for the fully nonlinear system. The system parameters were chosen the same as given in Table 1 of [4].

The stability map shown in Figure 2 and given in the co-ordinates used in the drilling industry, namely, WOB versus rotary speed, provides more insight than the results from the linear stability analysis presented in 4. Firstly, it is able to distinguish between two different types of instability, stick-slip and bit bounce. Secondly, it narrows down the area of the stable operation. Thirdly, it indicates a new stable region for low rotational speed and relatively high WOB, which was not reported previously.

3 Aberdeen drill-string dynamics experimental rig

In this section we describe briefly the experimental rig developed at the University of Aberdeen [5], which has been utilized to study different types of drill-string vibrations. This includes the experimental setup presented in Fig. 3, which has been described in detail in [5]. The most important feature of the experimental apparatus is its versatility, which means that depending on a chosen configuration, different types of drilling phenomena can be observed including stick-slip, bit-bounce and whirling. Moreover, the drilling rig is unique in using (i) flexible shafts to replicate torsional and bending properties of the drill-string, (ii) real commercial drill-bits (both PDC and roller-cone types) and real rock samples. The main components of the experimental setup, depicted in Fig. 3 can be grouped in three categories

- drill-string composed of flexible/rigid shaft, BHA, WOB disks, and the drill-bit,
- rock samples and cutting fluid circulation system and
- sensors, instrumentation and Data Acquisition System (DAQ).

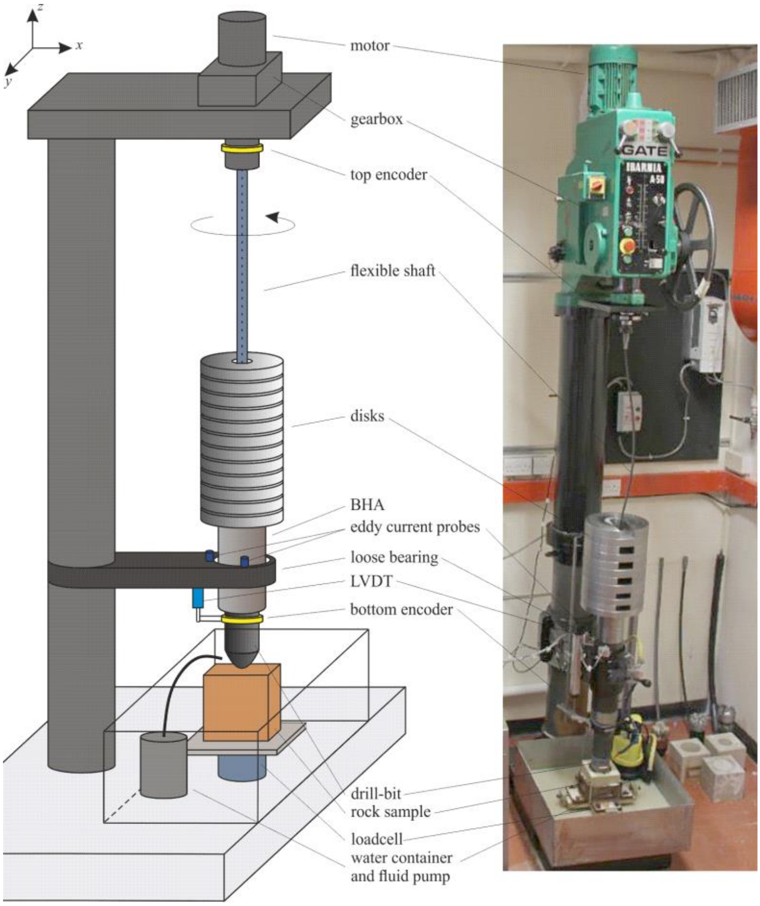

Figure 3. A schematic (left) and photograph (right) of the Aberdeen Drill-String Dynamics Experimental Rig showing its main components such as BHA, flexible shaft, WOB disks, drill-bit, rock sample and motor. The instrumentation include a 4D load-cell, LVDT, top and bottom encoders and eddy current probes.

As can be seen in Fig. 3, the drill-string is driven from the top using a modified pillar drilling machine, equipped with a 3kW electric motor that can provide up to 1032 rpm, depending on the chosen gearing configuration. In our experiments either a rigid or flexible shaft are driven from the top. The latter combines a high torsional rigidity with low bending stiffness to represent extreme slenderness of a drill-string. At the other end, the flexible shaft is connected to the BHA section made of a heavy steel shaft, which is held in transversal direction using a loose bearing. The drilling machine is equipped with the spindle allowing axial movement of the top of the drill-string in the range 0 to 220 mm, which is particularly useful when the helical bucking of a drill-string is studied [5]. An axial static force or a WOB is realized by placing steel disks on the top of the BHA providing WOB within the range 0.93 to 2.79 kN. At the end of the BHA, commercial drill-bits are attached and placed on the top of rock sample. In our studies we have used various rocks including sandstone, granite, and limestone. The versatility of the drilling rig allows to use different sizes of the drill-bits and rock samples. As can be seen in Fig. 3, the sample is placed inside the container, which has two levels to separate most of the cuttings from the water. We use a water based solution with anti-corrosive agent, which is pumped directly into the drill-bit rock interface. After

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debris are removed from the borehole, the cuttings and the drilling fluid are driven by gravity to the lower level of the tank.
A variety of different sensors are used in the experimental setup, which allows us to conduct detailed measurements of the most important parameters of the drilling process. These include two rotary encoders to measure top and bottom speed, two eddy current probes to measure position of the BHA inside the borehole and a Linear Variable Differential Transducer (LVDT) to observe Rate of Penetration (ROP). The most advanced sensor in our setup is the four component dynamometer (Kistler 9272), placed directly below the rock sample, which allows to measure the WOB (WOB), Torque On Bit (TOB), and two forces acting in transversal directions *x* and *y* as depicted in Fig. 4. The ranges of measurement for TOB and WOB are: 0-200 Nm and 0-20 kN respectively.

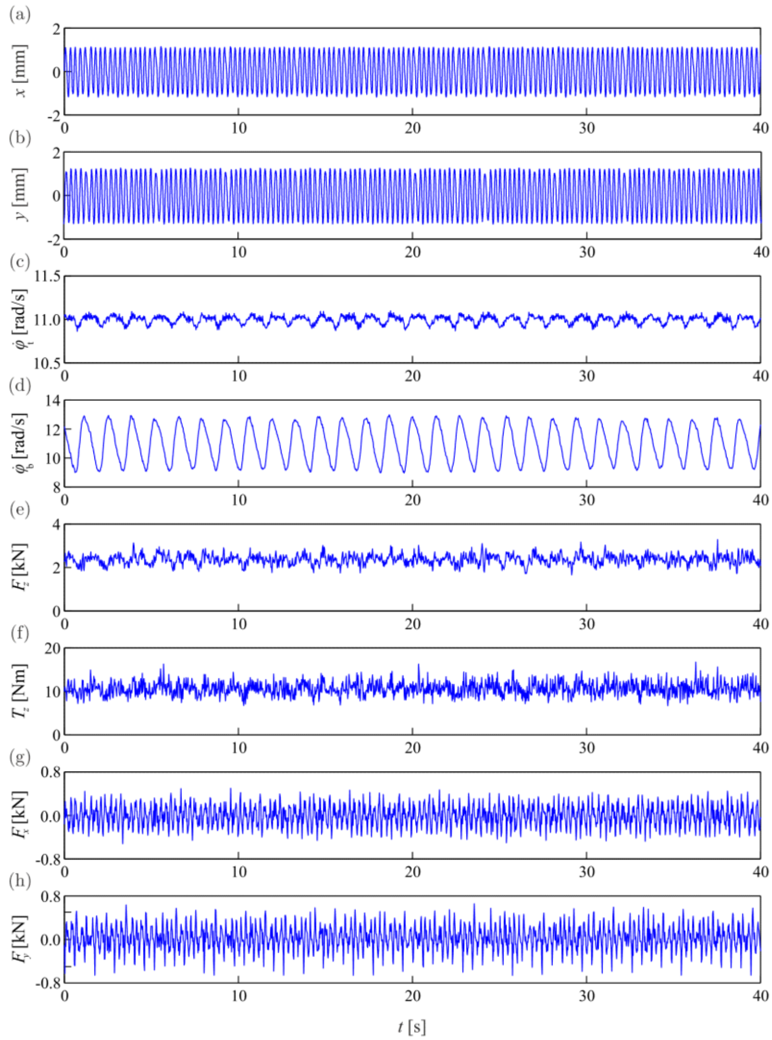

Figure 3. A sample of the recorded experimental time histories for configuration with 20 disks; (a) and (b) displacement of the BHA in the x and y direction respectively, (c) and (d) top and bottom speed, (e) F_z , (f) T_z, (g) and (h) transverse forces on *x* and *y* direction acting on the drill-bit.

For a given set of system parameters (drill-bit, rock sample, flexible shaft and WOB), the experiments are run at different top velocities and the dynamic responses of the BHA are recorded. This is performed by using LabVIEW based Data Acquisition System (DAQ), which allows to observe in real time readings from all the available sensors, as well as to save the data for subsequent data processing and analysis. In Fig. 4 we present an example family of time histories of a typical experiment for the configuration with 20 disks (each of mass 10.5 kg). The drill-string is driven from the top with an angular velocity of 11 rad/s, which is recorded and depicted in panel (c). Panels (a) and (b) depict the displacement of the BHA in transversal directions x and y. As can be seen in panel (c), the top speed, as provided by the motor, has oscillatory characteristics of small amplitude oscillations, which has a direct effect on the response of the angular speed of the BHA, presented in panel (d). We observe here oscillations of peak to peak amplitude of 6 rad/s, which is a direct consequence of the low stiffness of the flexible shaft. Note, that the lateral and torsional oscillations have different frequencies, as can be seen when comparing the time histories of $x(t)$, $y(t)$, $\dot{\varphi}_t(t)$, $\dot{\varphi}_b(t)$, shown in panels (a)-(d).

The experimental rig is equipped with accuracy load-cell (Kistler 9272) capable of high quality force measurements including the axial force and resistive torque, presented in panels (e) and (f) respectively. As a result of varying speed of the BHA, we observe small amplitude variation in both F_z and T_z, around their mean value. Additionally, the load-cell used in the experimental setup gives us information about forces in transverse directions F_x and F_y, shown in panels (g) and (h) respectively.

4 Closing remarks

In this paper we have discussed complex drill-string and BHA dynamics occurring in a downhole drilling. This is caused by strong nonlinear interactions between various types of vibration including whirling, axial and torsional oscillations. The latter two have been briefly presented here by looking at a low dimensional model of the downhole drilling, where a drill-bit cutting a rock formation has a strong coupling between torsional and axial oscillations. The model can be used to study drilling stability with regards to stick-slip and bit bounce. An illustrative example of the stability map was given for a fully nonlinear system. This compares well with the linear stability analysis undertaken for a steady drilling [3]. In the future work we will evaluate quantitatively the difference between linear and nonlinear stability analysis.

The Aberdeen Drill-String Dynamics Experimental Rig developed by the Centre for Applied Dynamics Research at the University of Aberdeen and discussed here, is one of the most advanced facilities of this type in the academic institutions. It is capable of mimicking of all major types of drill-string vibration, which can occur at the same time and can trigger each other. It is our understanding that a lack of an effective anti-vibration tool is a direct result of not sufficient insight how different type of vibration interact with each other. One of the most important features of this versatile experimental rig is the fact that commercial drill-bits, employed in the drilling industry, and real rock-samples are used. Hence the dynamics of the drill-sting and BHA is excited by forces generated during drilling and they are not simulated as in most experimental facilities. The rig can be used in different configurations, which allows experimental study of various phenomena, such as stick-slip oscillations, drill-bit bounce and whirling. The whirling has been provisionally studied in our group, where have seen experimentally and theoretically co-existence of forward and backward whirs [6, 7]. The rig also allows to determine mechanical characteristics of the drill-bits, which are used to calibrate mathematical models.

The future work will involve investigations of fully experimentally calibrated models to study the stability of drill-string and BHA, and to develop new methods of passive and active control to suppress or benefit from the generated vibration.

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