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Vibration isolation of a carbon nanotube filled with a mass chain

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Abstract—Vibration isolation is one of the most prominent characteristics of elastic metamaterial with negative effective mass. In this work, we show that a carbon nanotube filled with a linear mass chain can behave like an elastic metamaterial with negative effective mass density and exhibit remarkable vibration isolation phenomena within a certain frequency range.

Keywords - carbon nanotube; metamaterial; negative effective mass; mass chain; vibration isolation.

I. INTRODUCTION

Elastic metamaterials characterized by negative effective mass and/or negative effective modulus have attracted considerable attention [1-3]. It has been shown that the negative effective mass/modulus of elastic metamaterials are essentially attributed to their internal degrees of freedom, particularly so-called "locally resonant" microstructure [4, 5]. Inspired by an idea that a carbon nanotube (CNT) filled with a mass chain [6-8] could offer an already fabricated and available metamaterial with locally resonant microstructure, the present work aims to investigate filled carbon nanotubes as a potential new kind of elastic metamaterials. Unlike the known elastic metamaterials proposed in literature which are all based on often complicated artificial design of their microstructure, the carbon nanotube-based metamaterials proposed in the present work can exhibit negative effective mass and achieve desirable metamaterial dynamic properties even without any artificial design of their microstructure.

II. CONTINUUM MODELING

Fig.1 shows the schematic of a single-walled CNT filled with a linear C-chain. The CNT is modeled as an elastic beam of length L, diameter d, bending rigidity EI, mass density ρ_1 and cross section area A_1. The inserted C-chain is treated as an elastic string of mass density ρ_2 and cross section area A_2 but with no meaningful bending rigidity. Thus, the coupled governing equations for the filled CNT can be written as [7]

$$EI\frac{\partial^4 w_1}{\partial x^4}+\rho_1 A_1\frac{\partial^2 w_1}{\partial t^2}=c(w_2-w_1) \tag{1}$$

$$\rho_2 A_2\frac{\partial^2 w_2}{\partial t^2}=c(w_1-w_2) \tag{2}$$

where w_1 and w_2 are the transverse deflection of the CNT and the C-chain, respectively, t is the time and c is the van der Waals interaction coefficient (per unit length) between the mass chain and the CNT.

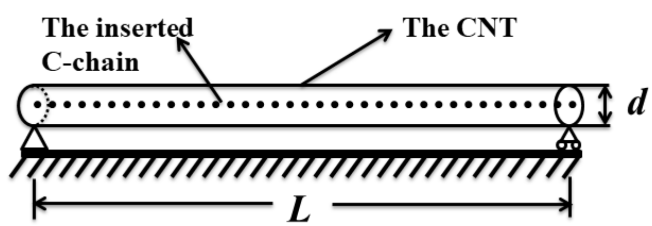

Figure 1 schematic of a single-walled CNT filled with a linear C-chain

Substituting Eq. (1) into Eq. (2), the governing equation of the filled CNT for the deflection w_1 can be obtained as

$$EI\left(1+\frac{\rho_2 A_2}{c}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^4 w_1}{\partial x^4}+(\rho_1 A_1+\rho_2 A_2)\times\left[1+\frac{\rho_1\rho_2 A_1 A_2}{c(\rho_1 A_1+\rho_2 A_2)}\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2 w_1}{\partial t^2}=0 \tag{3}$$

which is of the standard form of dynamic equation for a metamaterial elastic beam with negative effective mass density within a certain range of frequencies. To see this, let us consider a periodic harmonic motion $w_1(x, t)=f(x)\exp(i\omega t)$, where $f(x)$ is the mode shape function, i denotes the imaginary unit and ω represents the frequency. Substituting w_1 into Eq.

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(3), the effective mass density of the filled CNT can be obtained as the following form:

$$\rho_{eff} = \frac{1-\dfrac{\rho_1\rho_2 A_1 A_2}{c(\rho_1 A_1+\rho_2 A_2)}\omega^2}{1-\dfrac{\rho_2 A_2}{c}\omega^2}\rho_{avg} \tag{4}$$

It is clearly seen that the effective mass density become negative when the frequency is within the bandgap

$$\sqrt{\frac{c}{\rho_2 A_2}} = \omega_0 < \omega < \omega_0\sqrt{1+\frac{\rho_2 A_2}{\rho_1 A_1}} \tag{5}$$

It is noticed that the values and order of magnitude of this "metamaterial" bandgap are determined by the van der Waals interaction coefficient c and the mass of the inserted mass chain, and the bandgap can be wider if the mass of inserted chain is not very small as compared to the mass of the CNT.

III. FORCED VIBRATION DRIVEN BY PERIODICALLY VIBRATING ENDS

Consider forced vibration of a hinged filled CNT driven by periodically vibrating ends: $w_1=\delta\sin\omega t$ at $x=0$ and $x=L$, where δ is the amplitude of the vibrating ends. The stimulated steady state forced vibration of the filled CNT is of the form

$$w_1(x) = \delta\left(1+\sum_{k=1}^{\infty} a_k \sin\frac{k\pi x}{L}\right)\sin\omega t \tag{6}$$

where a_k (k=1,2,3,$\cdots$) are some real constants. Substituting Eq. (6) into Eq. (3), and using the Fourier series expansion

$$1 = \sum_{k=1}^{\infty}\frac{2[1-\cos(k\pi)]}{k\pi}\sin\frac{k\pi x}{L},\quad 0\le x\le L \tag{7}$$

where the coefficients a_k are to be determined. Thus, forced vibration of the hinged filled CNT driven by two periodically vibrating ends can be evaluated. To demonstrate this, a (5, 5) CNT filled with a linear C-chain is considered. The material and geometrical properties of the CNT are taken as: Young's modulus E=1TPa, mass density ρ_1=2200 kg/m^3, and single-walled thickness h=0.34nm.

The bandgap of the C-chain filled (5, 5) CNT is (ω_0, 1.05ω_0), where ω_0 is the lower-edge frequency=3.09THz. What plotted in Fig. 2 is forced vibrational modes of a C-chain filled-CNT driven by two periodically vibrating ends for excitation frequencies within the bandgap. It is seen that when the excitation frequency is within the bandgap, the forced vibrational mode is highly localized near the two vibrating ends but is vanishingly small in all other parts of the filled CNT, in sharp contrast to other excitation frequencies out of the bandgap for which the forced vibrational mode always spreads into the entire filled CNT. Therefore, a filled CNT indeed exhibits remarkable vibration isolation when it is excited by its periodically vibrating ends.

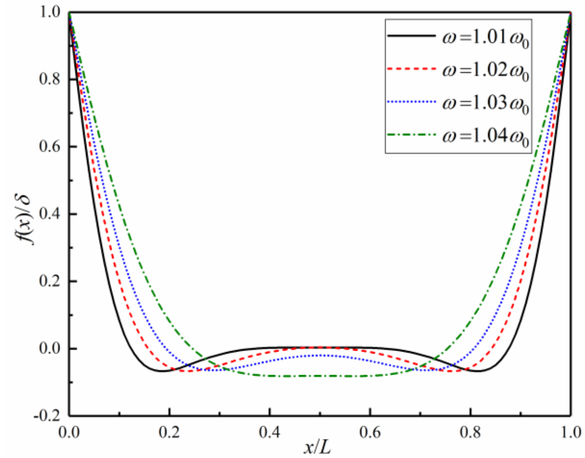

Figure 2 Forced vibrational mode shape of a filled CNT under different excitation frequencies (L=10d).

IV. CONCLUSIONS

A carbon nanotube filled with a mass chain can behave like a metamaterial with negative effective mass and exhibit remarkable vibration isolation within a certain frequency range without any artificial design of their microstructure. When the excitation frequency is within the bandgap, the forced vibrational mode of a filled carbon nanotube driven by periodically vibrating ends is highly localized near the two vibrating ends but is vanishingly small in all other parts of the filled CNT.

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