VSB – TECHNICAL UNIVERSITY OF OSTRAVA

FACULTY OF ECONOMICS

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Determinace kreditního rizika u portfolia dluhových aktiv

Determination of Credit Risk for Debt Assets Portfolio

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Description:

1. Introduction
2. Description of the Financial Risk
3. Description of the Credit Risk Management and Models
4. Determination of Credit Risk by Selected Models
5. Conclusion

Bibliography
List of Abbreviations
Declaration of Utilisation of Results from the Diploma Thesis
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References:

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Content

1. Introduction

For managers, it is important to manage the risk in banks. Risk management include identify and prioritization of risks, then adjust the risks to make it minimize. The aim of risk management is to assure uncertainty does not influence the company's business goal. It has many kinds of risks, but the most important risk is credit risk.

The aim of this thesis is to determine the capital requirement for unexpected losses from credit risk of the portfolio under Basel agreement include Basel I, II, III and use CreditMetric™ model to determine the economic capital of the portfolio.

There are three main chapter in this thesis, in chapter two and chapter three are both theory part. In chapter two is the description of the financial risk. It's include credit risk, market risk, liquidity risk, operational risk and other risks. In chapter three is the introduction of some models can used to calculate credit risk, the main model is CreditMetric™ model and Basel agreement. In CreditMetric™ we can manage the risk by using some steps include in this model.

Chapter four is the most important chapter, it is the calculation part in this thesis, we find ten bonds that trade on Frankfurt stock exchange, the credit risk of these ten bonds can be calculated. The time horizon of the portfolio is from 15.03.2017 to 15.03.2018 and the nominal value is 10 million euro. We can use the standard approach and foundation internal rating-based approach include in Basel agreement to determine the capital requirement from unexpected loss from credit risk. And use CreditMetric™ model to calculate the economic capital of ten bonds portfolio. After calculate all these values, we compare the capital requirement from different model to know how different it is.

2. Description of Financial Risk

In this chapter, we mainly talk about four types of banking risk, they are credit risk, market risk, liquidity risk and operational risk. And there also have some other risk, we will also talk about that. And about our thesis, we need to have the calculation about credit risk, so we will introduce more detail about that. The credit risk is important for both individual and business, all entities need to pay attritional on it.

Risk happens when the produce propose and labors gains are uncertain, it has profit uncertain and cost uncertain. And in banks the potential loss will be attentional.

When the business runs, they will cause risk. To face this risk they need to monitor, manage, and measure these risks. For bank, risk management is really important, it used to measuring the risk of its current portfolio of assets and other exposures, communicating the risk profile of the bank to other bank. The main goal of risk management is to maximize shareholders' value, if the risk happened, the shareholders will be influenced. Lots of risk are happened because of default.

Figure 2.1. Banking risks

2.1. Description of credit risk

As we know, credit risk is important for banks, here the definition of credit risk is the potential that a bank borrower or counterparts will fail to meet its obligations in accordance with agrees term. It the largest risk that banks face, it always default risk in banks. Credit risk is a decline in the credit-standing, the credit-standing is valued by interest rate charged on bonds or other debt issues, change in the value of share and also rating by the rating company. The high expected return caused high credit risk, low expected return caused low credit risk.

Here in Tab 2.1. We have the risk-weighted assets and capital requirement about the Bank Of China. And in the Tab we also have the percentage of each risk-weighted asset and total risk-weighted assets.

Tab 2.1. The risk weighted assets and capital requirement in BOC in2016 (In millions of RMB)

	Risk-weighted assets	Capital requirements	%
Credit risk-weighted assets	9,116,728	945,368	80.8%
Market risk weighted assets	221,791	68,546	1.97%
Operational risk-weighted assets	1,039,457	155,271	9.22%
Additional risk-weighted assets	891,636	63,523	7.91%
Total	11,269,592	1232808	100%

Source: Bank of China

2.1.1. Types of credit risk

In this part we will introduce the types of credit risks, three types of credit risk will be introduced:

- default risk,
- credit spread risk,
- downgrade risk.

The default risk is the possibility that the company or individual will not be able to pay the required amount of debt. The lenders and investors face default risk in almost all forms of credit extension. In order to mitigate the impact of default risk, lenders often charge a rate corresponding to higher demand returns.

Credit spread is the rate of return between two securities, the first is the bonds from the company and the second is the risk free bonds. These two securities need to have the same maturity, cash flow structure. It means credit spread risk is risk asset return minus risk free rate return. In simple, it is the risk caused by investors risk aversion change, because it will influence credit spread.

The downgrade risk means the the rating of the company decline, the rating agencies rating the company again after the company's bonds has been issued, and the rating of the company is lower then before, the investors and debtors will meet the higher risk.

2.1.2. Factors affecting the credit risk

There are lots of factors influence credit risk, for example the economic cycle, this is the macroeconomic factor. If the economic expansion, the credit risk will reduce. And sometimes the borrower is unable to repay in full, the special event of the company, economic environment will also influence credit risk happened. But as we know, here has some main factors:

- probability of default,
- exposure at default,
- loss given default,
- time horizon.

Probability of default - refers to the possibility that the borrower will not able to repay the bank loan. Probability of default is calculate expected losses of loan, loan pricing and the foundation of the credit portfolio management, so how to accurately calculate the probability of default of commercial banks credit risk is important.

About probability of default, we can get it if we have the credit rating of the company. Credit rating is an evaluation of the credit risk of a prospective debtor, it can forecast the ability to pay back the debt and the debtor defaulting. The credit rating is developed in American first, and the credit rating agency is evaluation of the qualitative and quantitative information for the debtor.

The internal rating approach is determines the risk weight of a credit based on factors such as probability default, loss given default. According to the internal rating approach, bank can classify the risks into corporate business risk, national risk, interbank risk and so on. Bank use the parameters and the internal predict to determine the risk factor and calculate the risk they will face.

External rating approach is rating by rating agency, is agency evaluate the credit which is creditworthiness of the borrowers, this approach focus on both borrowers and issuers.

Here in Tab 2.2. We have long-term rating matrix about investment grade rating and non-investing grade rating by two different agency. We can see in this table reflect the investment grade rating is AAA to BBB, non-investment rating is from BB to C, there also have rating grade D is default.

Tab 2.2. Long-term rating matrix

Investment grade rating		Non-investment rating	
Moody's	Standard & Poor	Moody's	Standard & Poor
Aaa	AAA	Ba1	BB+
Aa1	AA+	Ba2	BB
Aa2	AA	Ba3	BB-
Aa3	AA-	B1	B+
A1	A+	B2	B
A2	A	B3	B-
A3	A-	Caa1	CCC+
Baa1	BBB+	Caa2	CCC
Baa2	BBB	Caa3	CCC-
Baa3	BBB-	Ca	CC
		C	C
		C	D

Exposure at default - is the amount of expected loss that exposure when the lender default. Exposure at default measures actual exposure, potential exposure and total exposure, that total exposure equal to actual exposure plus potential exposure. And we mention expected loss is the average of the probability distribution of future losses. We can calculate expected loss by formula:

$$EL = PD \cdot LGD \cdot EAD. \tag{2.1}$$

Where PD is probability of default, EAD is exposure at default and LGD is loss given default.

Loss rate given default - is the loss rate experience by a lender on a credit exposure if the borrower defaults. The formula will be:

$$Loss\ given\ default = 1 - recoverage\ rate \tag{2.2}$$

Where RR is recovery rate, that can be taken at the value from 0% to 100%.

The LGD is never known when a new loan is issued, nor it is perfectly known even when the default occurs, in secondary market the LGD and RR can be estimated based on the market price after default. Here are two methods to estimate the recovery rates, market LGD and default LGD. Market LGD is use the prices of defaulted exposures as an estimate of the recovery rate. Default LGD is use market data surly are a very objective and up-to-date source of information for LGD estimation. The average recovery rate of the four major empirical studies based on corporate bond default data was reported. All empirical studies confirm that the recovery rate increases with the safety of the default bonds and decrease with the degree of subordination. Here we will have table 2.3, recovery rates on the defaulted bonds.

Tab 2.3. Recovery rate of defaulted bond.

	Carty & Lieberman [96a]			Altman & Kishore [96]		
	Number	Average	Std.dev.	Number	Average	Std.dev.
Senior Secured	115	$53.80	$26.86	85	$57.89	$22.99
Senior Unsecured	278	$51.13	$25.45	221	$47.65	$26.71
Senior Subordinated	196	$38.52	$23.81	177	$34.38	$25.08
Subordinated	226	$32.74	$20.18	214	$31.34	$22.42
Junior Subordinated	9	$17.09	$10.90	-	-	-

Source: Carty & Lieberman [96a] – Moody's Investors Service

In Tab 2.3, we can see different recovery rates like in senior secured, senior unsecured, senior subordinated, subordinated and junior subordinated.

Time horizon - is also called plan scope, which is the fixed point in the future and at this point certain processes will be evaluated or assumed to be over. In an accounting, financial or risk management system, it is necessary to allocate such a fixed time range so that the performance of alternative solution can be evaluated as the same time. Constant time horizon are more use for banks and hold-to-maturity time horizon is Jed's by institution

2.1.3. Ratio indicators of credit risk

Credit risk ratio is the potential loss that the bank would incur if the borrower fails to meet its obligations. The equation that has relationship with credit risk ratio are as follow:

Non-performing loans (NPL) is the sum of borrowed money upon which the debtor has not made his schedule payment for at least 90 days. The non-performing loans is any loan that can reasonably be expected on renter default.

Coverage ratio (CR) measure the ability of banks to absorb potential losses from non-performing loans. If the banks can absorb more potential losses from non-performing loans, that means the ability of banks is great.

Other ratio about credit risk ratio is charge-offs ratio (COR) that the net charge-offs loans and leases, is the contractual committed periodic interest and principal payment on lease and debt. And the loan to deposit ratio (LTD) used to calculate the ability of lenders.

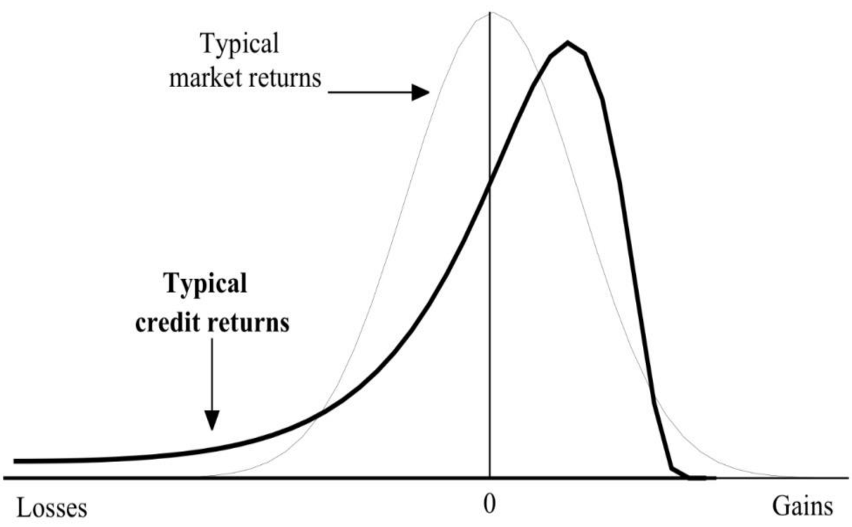
The capital adequacy ratio (CAR) Is a measure of a bank's capital. It is expressed as a percentage of ban's risk weighted credit exposures.

And we also have following ratios to calculate is the bank has credit risk. Tier 1, tier 2, and tier 3 can be the different tire in capital, tier capital is the measure of the financial strength of a bank because it is composed of core capital.

2.1.4. Different between credit risk and market risk

The comparison of distribution of credit returns and market returns will be shown in figure 2.2, it also the different between credit risk and market risk.

Figure 2.2. The different between credit risk and market risk.



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 7p.

Here in Figure 2.2, we can see the skewed of credit returns are highly and it has fat tailed, we need more than just the mean and standard deviation to fully understand a credit portfolios' distribution. And we also have estimating the credit at the confidence level of 95%. The reason of the long downside tail of the distribution of the credit returns is default in credit risks, so measuring credit risk of the portfolio is difficult, and we have lots of methods will be mentioned.

2.2. Description of market Risk

Here in this part we will talk about market risk, that market risk is the risk of bank loos due to changes in market prices, and the market price are changes by interest rates, exchange rate, also equity and commodity prices. Market risk refers to the investment risk that may be faced due to market fluctuations. The risk is that the investment value will drop. Also known as systemic risk.

2.2.1. Types of market risk

Here are four types of market risk:

- interest rate risk,
- equity risk,
- foreign exchange risk,
- commodity risk.

Interest risk - is the risk that arise when interest rate fluctuating, it means that the interest rate increase or decrease will influence the risk or not. The interest rate rise, the long-term assets falls more than short-term liabilities, so the bank's equity reducing. In the other side, if the interest rate risk, the bank will pay higher interest rate on its deposits. As the consult, the value of a bond will increase if interest rates decrease and decrease if interest rates increase.

Equity risk - is the risk in finance when holding equity in an investment, it's often in the equity in companies when purchase the stocks, but equity risk doesn't refers to risk in paying real estate or holding equity in properties. When the price of stock change, the equity risk will happened, because stock has the shares or equity, if the bank purchase ownership in other companies, and makes then to face the changes of the stock price.

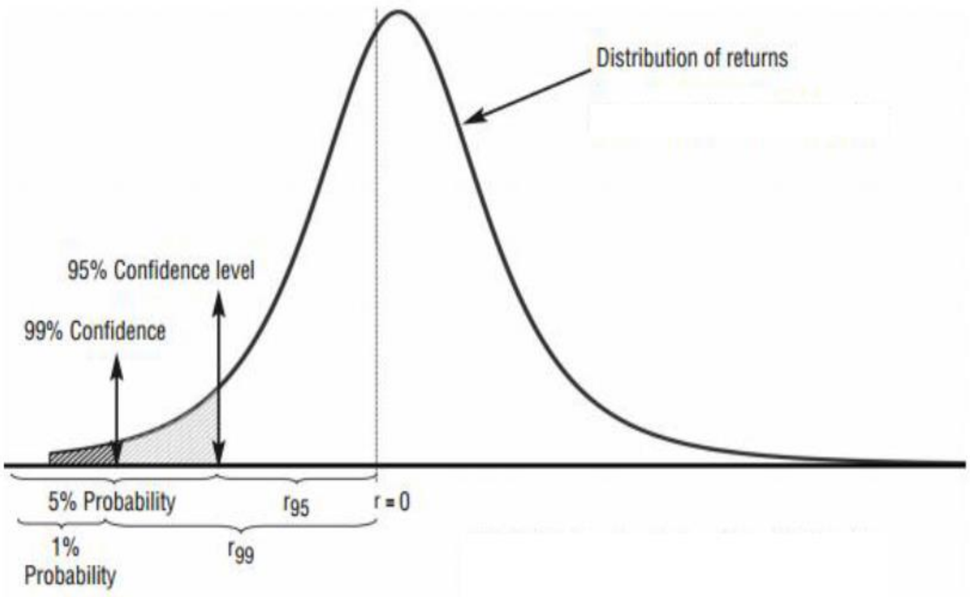
Foreign exchange risk - is the risk when the currency exchange rate change influence bank's assets or liabilities change. When the foreign subsidiary of the company prepares financial statements in a currency other than the reporting currency of the consolidated entity, foreign exchange risk also exists. The risk is that before the transaction is completed, there may be an adverse change in the currency of the face value currency relative to the base currency.

Commodity risk - is happen when commodity price change then make some loss. It used in commodity positions and derivative commodity positions, like futures contracts. And in commodity risk includes agricultural commodities industrial commodities and energy commodities, it will changes when demand and supply has changes.

2.2.2. Value-at-risk

Value-at-risk (VaR) was bank used to determine how much market risk in their portfolios, it's help banks know the potential loss in future time period. The predicted loss in VaR is defined at the specific confidence level at 95% over a given period of time. If the VaR estimates the loss level at 99% of the time, it means the actual loss level will be less than that number. In measuring VaR, the distribution of possible return will be seeing in the period that predicted. Here in Figure 2.3 will show the distribution of returns for a portfolio and also interpretation of VaR.

Figure 2.3. Graphical interpretation of value-at-risk.



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 170p.

Here horizontal X-axis is the possible gains and losses. The losses to the left and the profits to the right. The area under the curve will be sum at one. The height of the curve is the distribution of return loss or gains.

2.3. Description of operational risk

Operational risk is the risk of loss due to insufficient internal processes or failures. People and systems or external events. This definition includes legal risks but does not include strategic and reputational risks. We will introduce some operational risk.

2.3.1. Types of operational risk

- internal process risk,
- people risk,
- systems risk,
- external risk,
- legal risk.

Internal process risk - for example failure of the bank's processes and procedures and inadequate control environment. Internal process risk is the risk associated with the failure of a bank's process, it includes lack of controls, marketing errors, money laundering, documentation or reporting, transaction error and internal fraud. All these can be take in banks operations every day and the errors are in business practices. If we want to reduce internal process risk, we need to analyzing procedures and processes.

People risk -the example of people risk is employee errors and fraud, so the defined of people risk is the loss when employee error or fraud. This risk occurs because of high staff turnover which frequent changes in staffing means new people do not have the required background, poor management practices where employees report different risk event, poor staff training and overreliance on key staff.

Systematic risk - is the risk of a collapse of the entire financial system or of the entire market and is not a risk associated with any single entity, group or component of the system that can be included without compromising the overall system. It refers to the risks posed by interconnectedness and interdependence in a system or market, and the failure of a single entity or group of entities can lead to chain failures that can lead to bankruptcy or paralysis of the entire system or market. The systems risk include data corruption, inadequate project control or programming errors and others.

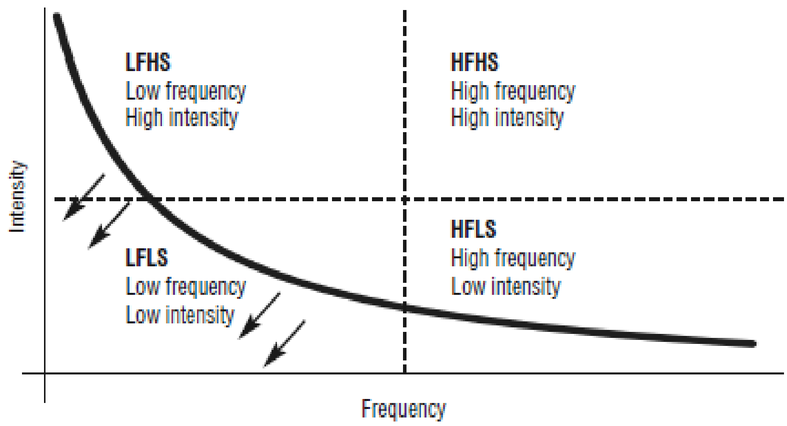
External risk - is the risk that the events out of t control of the bank. External risk events are usually infrequent, but it happens it will influence bank's business and operation. Here other banks event will influence the bank, theft and terrorist attacks will also cause external risk.

Legal risk - is the risk of loss to an institution, the loss caused by a defective transaction or change in law, failing to take appropriate measures to protect assets also caused legal risk. Legal risk is the risk of financial or reputational loss resulting from the lack of awareness or misunderstandings in it's relationships, processes, products and services.

2.3.2. Operational loss events

Operational loss event can be classified in two ways, the frequency of the event or the potential loss of the event. Operational risk management focus on the event occurs often but has less influence or event is not often occurs and not serious but has the large influence.

Figure 2.4. loss intensity and frequency chart of operational risk event



Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 170p.

The cost of managing and monitoring high-frequency/low-impact events is higher than the cost of these incidents, while low-frequency/high-impact events mean that poorly managed banks are doomed to failure. In Figure 2.4, is the loss intensity and frequency chart of operational risk event.

In Figure 2.4, we can see high-frequency/low-impact (HFLI) event, that loss from HFLI operational risk may be minor and this kind of event is important to bank's business decision-making processes. Low-frequency/high-impact operational risk is a challenge for risk managers, it's need managers to make new method to solve it.

2.4. Description of liquidity risk

Liquidity risk is when certain financial assets, securities or commodities can be not traded fast enough in the market in a period of time, will not influence market price

The liquidity coverage ratio refers to highly liquid assets held by financial institutions to meet short-term obligations. The ratio is a generic stress test that aims to anticipate market-wide shocks. There has some risk indicators which means it will create more incomes, but it still can't be too higher, between 70% and 80% this index is better, when this index reach 80%, the bank have small buffer that they need.

Here are two types of liquidity risk:

Day-to day liquidity risk is relates to daily works in the bank such as withdrawals, it occurs when depositors withdraw money, the operation is easy and normal for bank to manage daily. But sometimes the institutions run out of cash, they need to borrowing funds from other banks, then the liquidity risk will occurs.

A liquidity crisis occurs, it occurs when depositors need a large amount of cash, the bank doesn't have enough money, so they need to borrow funds at an elevated interest rate higher than market rate that other banks are paying for similar borrowings.

2.5. Other risk types

This part we will introduce some different kinds of risks such as regulatory risk, reputational risk and business risk.

Regulatory risk is the risk of regulatory and legal changes that may affect the industry or business. This change in regulations can make the industry framework happen and the cost structure changes. In the post-financial crisis environment, the regulatory environment is inherently more complex. Supervision and law enforcement are more confrontational, intensive and intrusive. Regulators are making judgments about the soundness of the business models of regulated companies and the suitability of the products they are selling. If they see or foresee problems, they will immediately intervene.

Reputation risk is the potential loss of the bank's position in the public opinion. Recovering from real or perceived reputation issues is not easy. Organizations lose important business and there is no other reason than public loss of confidence in public relations, resulting in public relations issues, even with relatively solid systems, processes and finances.

Business risk is caused by lack of profit due to uncertainty or even the possibility of loss companies face various risks, some of which may result in serious loss of profits or even bankruptcy. And all large companies have extensive "risk management" departments

3. Description of Credit Risk Management And Models

In this chapter, we will describe how to manage the credit risk, it is important to manage credit risk when the bank realize the borrowers fails to meet the obligations as the agreement. The model to manage credit risk are scoring model-Altman's Z-score, rating system and portfolio models. The main part that will be describe is CreditMetrics^{TM} model, in this part include risk measurement framework, credit quantity correlation and applications of the model. Then the description of Basel I, II, III in regulation of capital requirements will be in this chapter.

3.1. Models of credit risk management

About credit risk management the principles for the assessment are built in Basel committee. Credit risk models have become an important part of financial institution risk management systems in the past few years. The models that manage credit risk are scoring model-Altman's Z-score, rating system and portfolio models. It always has evaluation methods for a credit risk model to evaluation expected losses and unexpected losses.

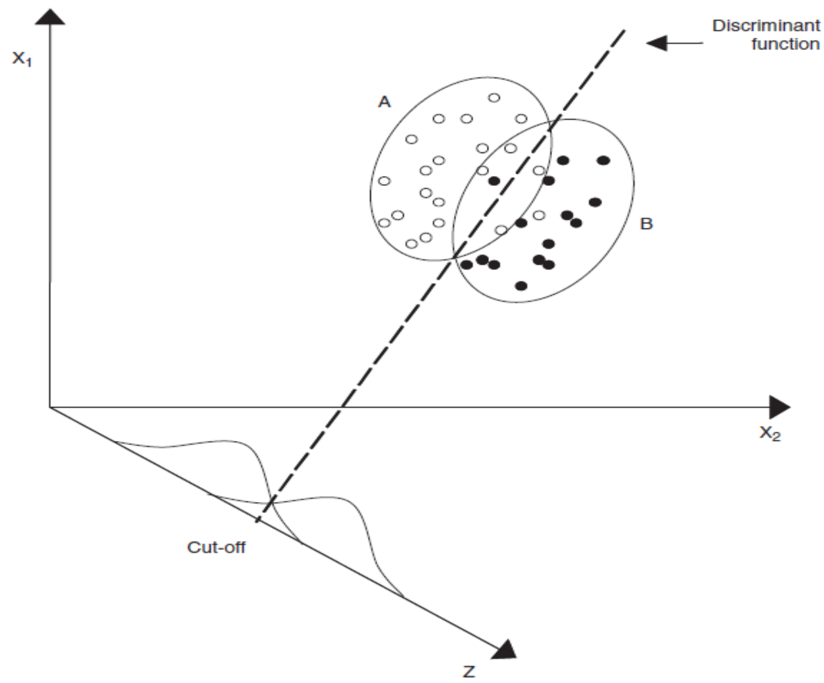
3.1.1. Scoring model-Altman's Z-score

The scoring model used in manage credit risk to forecast a company's default, with the company's key economic and financial indicators as inputs, and importance in predicting defaults. Credit-scoring model has lots of analysis, the linear discriminant analysis, regression models and some recent heuristic inductive models.

The linear discriminant analysis is the analysis based on variables identification, uses data obtained from a sample of companies to draw a boundary that separates the group of reliable one from the group of insolvent ones.

In figure 3.1. is the graphic representation of linear discriminant analysis, and it shows the discriminant function. This model is Fisher model in the case that the reliable group is A and insolvent group is B, they described by two variables χ_1 and χ_2, the score is the combining of two variables shown in *z* axis. So the discriminant analysis in simplest version is *z* score as a linear combination by some variables.

Figure 3.1. Graphic representation of linear discriminant analysis.



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 288p.

In Figure 3.1, we have two variables and then input the *z* score, but if we have *n* variables, and also *i*th company, the score can be:

$$z_i = \sum_{j=1}^{n} \gamma_j x_j, \qquad (3.1)$$

$$z_i = \sum_{j=1}^{n} \gamma_j x_{i,j}. \qquad (3.2)$$

In these two formulas, the γ_j in linear combination is to obtain score z which discriminates between abnormal and healthy companies. To get z_i value obtain must be such as to maximize the distance between the means of two groups of abnormal and healthy companies. In the case, z_i value is expected as healthy companies to be as similar as possible to one another. It can be shown that this condition is satisfied if the vector of the gamma coefficients calculate by follow formula:

$$\gamma = \sum -1 \, (x_1 - x_2). \tag{3.3}$$

In discriminant analysis, the best discriminant score to credit risk is Altman's Z-score, use the Z-score formula for predicting bankruptcy and companies default, this is a measure to calculate the financial distress status of companies, always use in company income and balance sheet to measure the financial health of the company. In Altman's Z-score, it has five independent variables and can be shown as the formula:

$$z_i = 1.2x_{i,1} + 1.4x_{i,2} + 3.3x_{i,3} + 0{,}6x_{i.4} + 1.0x_{i,5}, \tag{3.4}$$

where x_1 is working capital/total assets, x_2 is retained profit/total assets, x_3 is earnings before interest and tax/total assets, x_4 is market value of equity/book value of total liabilities, x_5 is turnover/total assets

Here if the *z* score is higher, the probability of default is lower, in discriminant analysis has cut-off point, Altman set the cut-off point at a value of 1,81, if the value is lower than 1.81 it means the *z* score of the company is too risky. The cut-off value can be calculated as the average between mean *z* score from healthy companies and mean *z* score from insolvent companies.

3.1.2. Rating system

This part will be rating system, that evaluate the credit of the borrower, nowadays we rating the credit by international credit rating agencies as we mention before is Moody's and Standard&Poor's. rating system. About rating system, we have some steps, the rating assignment step is simple introduce how the main rating agencies process and the internal system that created by banks. The rating quantification step that focus on estimate the probability of default with different rating grades. The rating validation step is used to evaluate the quality of a rating system. The steps are:

- rating assignment,
- rating quantification,
- rating validation.

Rating assignment - first, we will know what is the different between the internal rating and agency rating, here has three main factors are if the borrowers being evaluated, the information that borrower can get from bank and agency are different, the bank and agency has different targets.

The assignment of agency rating like the Standard Statistics assigned its first rating to a corporate bond in 1916, agency rating can be positive, negative or stable in the future as it developed. All the company wants to get the higher rating, it means the probability of default is lower when it compared with others, because the assignment of an issuer to a given rating grade is estimate of the probability of the default.

Rating assessment in bank internal rating system depend on market segment that they are developed, the assignment of a bank's PD rating sometimes involves several stages, implying the production of "partial" ratings.

Rating quantification - for rating quantification, there has three approaches. The statistical approach is the individual probability of default is calculated for each borrower based on the score obtained with a credit-scoring model, this approach is simple to specific the probability of default, but in discriminant analysis score the input of variables is normal makes the statistical approach used seldom. The mapping approach used because public data for the default rates of agency rated companies exist, many banks find it useful to establish a link between their internal ratings and those by Moody's or Standard & Poor's. But the mapping can be unstable and unreliable.

The approach that always use is actuarial approach, is based on actual default frequencies, this approach requires that past default rates recorded in various ratings be used as an estimate of the borrower's probability of default in the future. Rating agencies often use this method to regularly publish statistical data on defaults recorded in previous years and decades.

As the international rating agencies began to publish default rate, they have some procedures used to estimate, the procedure involves the computation of marginal and cumulative default rate. We have the formula to get the marginal default rate:

$$d_t^{'} = \frac{D_t}{N_t}, \tag{3.5}$$

in formula (3.5), where D_t is the number of default recorded in year t, and N_t is the number of issuers present at the start of year t.

The marginal survival rate in year t can be:

$$s_t' = \frac{N_t - D_t}{N_t} = 1 - d_t', \tag{3.6}$$

the cumulative default rate for the period between 0 and T, the formula is:

$$d_T = \frac{\sum_{t=1}^{T} D_t}{N_1}, \tag{3.7}$$

the cumulative survival rate between 0 and T is given by:

$$s_T = 1 - p_T = \frac{N_1 - \sum_{t=1}^{T} D_t}{N_1}, \tag{3.8}$$

and in the definition, $N_{t+1} = N_t - D_t$, we can get:

$$s_T = \prod_{t=1}^{T} s_t', \tag{3.9}$$

$$d_T = 1 - s_T = 1 - \prod_{t=1}^{T} (1 - d_t'). \tag{3.10}$$

Rating validation - The rating system should be regularly reviewed to assess its effectiveness, so we have rating validation. Also some quantitative criteria for validating rating assignments, some more methods have been proposed to verify the appropriateness of the rating assignment process, for example contingency tables.

Tab 3.1. Example of a Contingency Table.

		Performing	Defaulting
Rating by model	Low-risk (“pass”)	Correct valuation (N_1 cases)	Type I errors (N_2 cases)
	High-risk (“fail”)	Type II errors (N_3 cases)	Correct evaluations (N_4 cases)

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 288p.

Here the representation of the number: The number *N*1 of companies correctly rated as "healthy" by the model, the number *N*2 of companies incorrectly rated as healthy, the number *N*3 of companies incorrectly rated as being too risky, the number *N*4 of companies correctly rated as high-risk.

3.1.3. Portfolio models

As the possible losses on credit can be expected loss and unexpected loss, in this part is the ways to quantify unexpected loss, use some approach to measure the value at risk. As same as the method to get the value at risk in market risk, to estimate the value at risk on credit risk is to determine the maximum loss a credit portfolio can face during a predetermined time horizon with a certain confidence level. Here has four portfolio models to describe:

- CreditMetrics™
- CreditPortfolioView™
- CreditRisk+™
- PortfolioManager™

In these models we need to select time horizon and confidence level before analysis. the time horizon need to be selected because the credit portfolio's value depends on the distribution of losses that may occur in the future, it is necessary to specify the future time interval we wish to mention, for example the distribution of loss in the next three years is obviously more uncertain than the distribution in the next three days. As for many reasons we always set the risk horizon at one year.

For the choice of the confidence level, the level of confidence used by all banks' business areas should be uniform for all types of risk.

CreditPortfolioView^{TM} - this model developed by the consulting firm McKinsey, the model told us when the migration rate changes the default are different, for example the migration rates is lower the default decline and the migration rates is higher the default will be more. The model is based on the observation that credit cycles depend on the economic cycle, and some macroeconomic variables also has the association with the migration and default.

To estimate the probabilities of default, we consider the reaction the probability of default when one or a group of companies that responds to changes in the economic cycle consistently at the time t. That we can get the model's function:

$$p_{jt} = \frac{1}{1 + e^{-y_{j,t}}}, \tag{3.11}$$

where $y_{j,t}$ is the value at time t of a "health index" of the segment j based on macroeconomic factors.

When $y_{j,t}$ is a linear combination of several macroeconomic variables x_{j1}, $x_{j2}, \dots x_{jn}$, the function will be different as :

$$y_{j,t} = \beta_{j,0} + \beta_{j,1}x_{j,1,t} + \beta_{j,2}x_{j,2,t} + \beta_{j,3}x_{j,3,t} + v_{j,t}, \tag{3.12}$$

where β_{j1}, $\beta_{j2}, \dots \beta_{jn}$ is estimated based on historical experience, $v_{j,t}$ is random error.

We also use the model to condition the entire transition probability matrix, the mean long-term transition matrix is adjusted to reflect the expected default probabilities for the subsequent year. In Table 3.2 is the CreditProtfolioView model of the economic cycle and transition matrix.

Tab 3.2. CreditProtfolioView: economic cycle and transition matrix.

Relationship	Economic cycle phase	Default probability	Downgrade probability	Upgrade probability
$\frac{SDP_t}{\mu_{SDP}} > 1$	Recession	Increase	Increase	Decrease
$\frac{SDP_t}{\mu_{SDP}} < 1$	Expansion	Decrease	Decrease	Increase

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 427p.

In this table, where μ_{SDP} is historical average. If the value of the probabilities in year t is greater than μ_{SDP}, the phase of the economic cycle is unfavourable.

CreditRisk+^{TM} - this model built in 1997 by Credit Suisse Financial Product, the model using methods from the insurance business, it applies credit risk to some typical insurance math tools. But the important we need to know the model can only focus on default risk not migration risk, that we can see the default risk and recovery rate expose can be seen as determined factor, so the exposure risk and recovery rate can be not estimated.

The CreditRisk+^{TM} model is highly effective in estimating the risk of portfolio with a large number of position., it always manages some traditional banking portfolio.

If we already had a risk horizon, the model can describe the probability distribution of the number of future defaults. The formula can be:

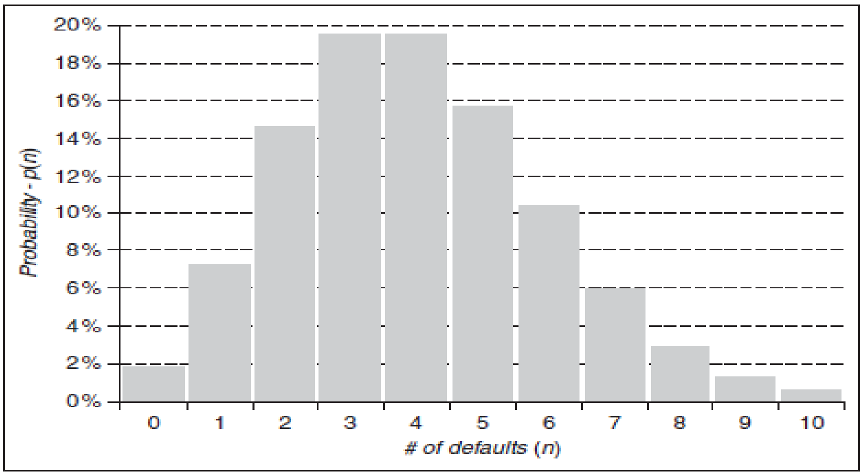
$$p(n) = \frac{e^{-\mu}\mu^n}{n!}, \tag{3.13}$$

where μ is the expected number of default represents the sum of all the probability of defaults of the customers in the portfolio. Here we will have an example, if a bank with 400 clients, each with a probability of default is 1% and the value of μ is 4. The probability that no defaults occur can be:

$$p(0) = \frac{e^{-4}4^0}{0!} = 1.83\%. \quad (3.14)$$

In Figure 3.2 is calculate by the formula before to get $p(n)$ when n is 0 to 10, in this situation the probability distribution is different with normal one, skewed to right. When n is higher, the probability decreased to zero.

Figure 3.2. An example of passion distribution.



PortfolioManager™ - the PortfolioManager™ model is built by a California-based firm KMV acquired by Moody's Investor Services, so the model also called KMV model. The model started from when the equity equal to the value of call option, the maturity equal to the residual life of debt, strike price equal to nominal repayment value of debt. In Tab 3.3 shows how the two positions have the same result when it produce.

Tab 3.3. Matrix of payoffs as a shareholder or for the purchase of a call option on asset value with a strike price of F.

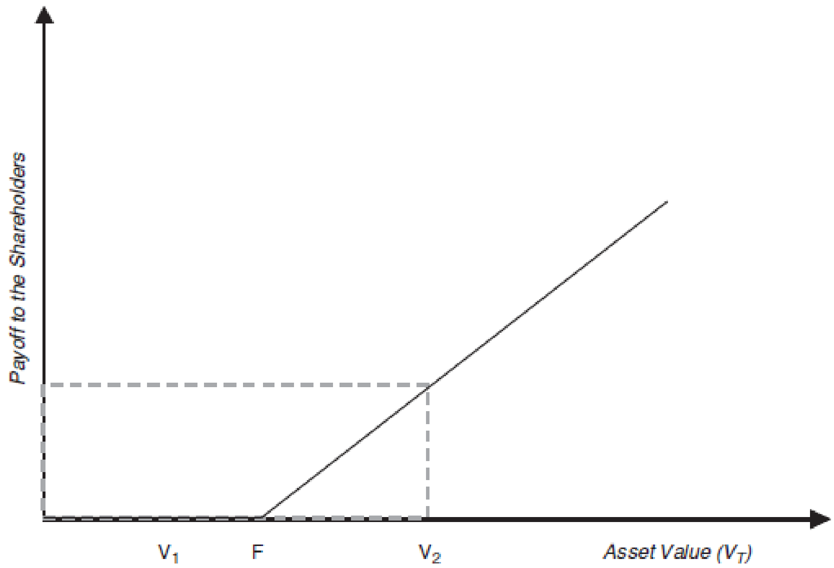
	Payoff at time 0	Payoff at T	
		If V_T<F	If V_T>F
Shareholder	$-E_0$	0	$(V_T$-F)
Purchase of a call option	$-C_0$	0	$(V_T$-F)

Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 332p.

We can see the payoff at T when the V_T<F means company face insolvent and use the remaining assets to pay the debt that they had before. This makes the shareholders lose all the initial investment. In the other hand, if V_T>F the shareholders can get the value at V_T-F, this is the gains at long-term call option.

In the model and some other analysis, if the time horizon is longer, more the curve of marginal PDs declines, in Figure 3.3 is the decline PDs excuse the decreasing term structure of the spreads.

Figure 3.3. Shareholders payoff profile.



Source: ANDREA, S. and ANDREA, R. Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies. Wiley Finance, 2007. 330p.

In KMV model use the default point equal to short-term debt plus 50% of long-term debt. DP is default point, STD is short-term debt and LTD is long-term debt.

$$DP = STD + \frac{1}{2}LTD, \tag{3.15}$$

then, we will turn to a formal definition of distance to default.

$$DD = \frac{V_0 - DP}{V_0 \sigma_V}, \tag{3.16}$$

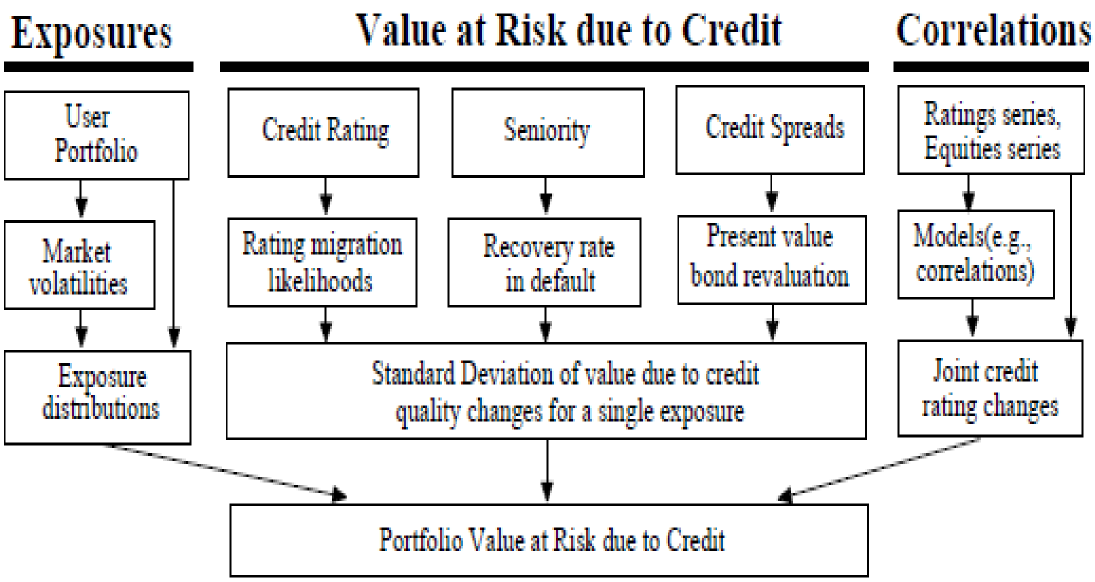
for using KMV model, there are lots of benefits and limitations.

3.2. Description of CreditMetric™

In this part we describe CreditMetric™ model, this is the most important model to manage credit risk. It has risk measurement framework, credit quantity correlation and the application of model output. The model is to assess the risk of changes in the value of debt due to the change in the value of debt because of changes in the quality of its obligations. It is not only include changes in the value of potential default events, but also upgrades and downgrades of credit quality, the VaR's fluctuations in value not just expected losses.

And in the model we have some provision for different exposure types of credit risk shows in Figure 3.4.

Figure 3.4. "rode map" of the analytics within CreditMetrics.



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 41p.

Value-at-risk (VaR) was bank used to determine how much market risk in their portfolios, it's help banks know the potential loss in future time period. The predicted loss in VaR is defined at the specific confidence level at 95% over a given period of time. And we have two ways to get the value at risk:

The first way is losses from the portfolio of debt assets set at a significance level of α, which is greater than the predetermined value losses:

$$\Pr\left(-\Delta\widetilde{\prod} \geq VaR\right) = \alpha, \tag{3.17}$$

the second way is the profit from the portfolio of debt assets ($\Delta\widetilde{\prod}$) that is less than the predetermined value gains at the significance level α shown as:

$$\Pr(\Delta\widetilde{\prod} \geq -VaR) = \alpha, \tag{3.18}$$

value at risk be also from Merton's model, which uses stock prices as input and seeks to determine the equilibrium bond spread to estimating probability distribution, the probability distribution of the increase in the value of the portfolio assets can be:

$$\Delta\widetilde{\prod} = \tilde{V}_P^T - V_P^t = \sum_n \tilde{V}_{n,J,T} x_n - \sum_n V_{n,j,t}\, x_n, \tag{3.19}$$

where $\tilde{V}_P^T$ is the default value of the portfolio, V_P^t is the predicted value of the portfolio, $\tilde{V}_{n,J,T}$ is the value of n-th asset with i-th rating category in the portfolio at the end of time horizon, and $V_{n,j,t}$ is the value of n-th asset with i-th rating category in the portfolio, x_n is the amount of the portfolio.

Economic capital for financial services firm is the amount of the risk capital, and estimate it under the real basic, the firm need to cover the risk that they already face. The economic capital also the cash flow that determine the firm to live under the bad situation. So we need to know how to calculate the economic capital for a firm, the formula will be:

$$Economic\ capital = VaR_\alpha - E(-\Delta\widetilde{\prod}), \tag{3.20}$$

where VaR_α is when simulated values of portfolio's profits get the order, the value VaR at a particular level of significance is equal to n-th worst, $E(-\Delta\tilde{\prod})$ is the means value of the portfolio gains.

3.2.1. Risk measurement framework

We use CreditMetric^{TM} model to estimate credit risk and has risk measurement framework, it has many steps in the model:

- Step 1: Credit rating migration,
- Step 2: Valuation,
- Step 3: Credit risk estimation,

Step 1: Credit rating migration

In Tab 3.4, is the one-year rating transition matrix for 2016 by Standard & Poor's, we can refer the probability of default in this table.

Tab 3.4. One year transition matrix (%) in 2016

Initial rating	Rating at year-end(%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	87.05	9.03	0.53	0.05	0.08	0.03	0.05	0.00
AA	0.52	86.82	8.00	0.51	0.05	0.07	0.02	0.02
A	0.03	1.77	87.79	5.33	0.32	0.13	0.02	0.06
BBB	0.01	0.10	3.51	85.56	3.79	0.51	0.12	0.18
BB	0.01	0.03	0.12	4.97	76.98	6.92	0.61	0.72
B	0.00	0.03	0.09	0.19	5.51	74.26	4.46	3.76
CCC/C	0.00	0.00	0.13	0.19	0.63	12.91	43.97	26.78

Source: Standard & Poor's CreditWeek..

In Tab 3.4 we have been mention the credit rating migration, is the credit quality migration likelihoods for obligation currently rated. To estimate both likelihood of default and chance of migrating to is important and it need to estimate to any possible credit quality state at the risk horizon. From the Tab 3.4 the transition matrix for one year, we know how migrating from CCC to AAA within one year and so on. As we mention before, we can set a transition matrix to any system of grouping similar credits, these groups called rating category. But we must specify the default likelihood of each category and the likelihood that a certain category of company may migrate to other categories, because there is no rating category and how many categories there are.

Step 2: Valuation

Valuation of the bond - This step is to estimate the value at risk horizon, because of migration of risk, the value is calculate once for each migration, as for that it has eight revaluation for one bond. To value bond by two ways, the first is in the event of a default we estimate the recovery rate based on seniority class. The second way in the event of up(down) grades, the end is change in credit based rating migration.

First way: valuation in the state of default. In Tab 3.5 is the recovery rates based on seniority class.

Tab 3.5. Recovery rate by seniority class.

Seniority Class	**Mean (%)**	**Standard Deviation (%)**
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] – Moody's Investors Services

In Tab 3.5 is the mean of the recovery rate and the standard deviation of the recovery rate, to describe the table the senior unsecured represent BBB bond and the mean value of the default is 51.13%, the standard deviation of the recovery rate is 25.45%.

In Tab.3.6 is the one year forward zero curves by credit rating category.

Tab 3.6. One-year forward zero curves by credit rating category (%)

Category	Year 1	Year 2	Year 3	Year 4
AAA	3.60	4.17	4.73	5.12
AA	3.65	4.22	4.78	5.17
A	3.72	4.32	4.93	5.32
BBB	4.10	4.67	5.25	5.63
BB	5.55	6.02	6.78	7.27
B	6.05	7.02	8.03	8.52
CCC	15.05	15.02	14.03	13.52

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 27p.

Second way: Valuation in the state of up(down) grade. And it has two steps in this way, to obtain the forward zero curve by credit rating category and next is using these zero curve to revalue the bond's cash flow now.

As to give the example to estimate bonds, we assume the bond pays an annual coupon rate at 7%, face value of the bond is $100, the bond pays $7 each at the end of the next four years. The maturity of the bond is five years, so at the end of the fifth year the bond pays a cash flow of face value plus coupon at $107.

The example that we have is the rating of BBB, we assume the BBB bond upgrades to A, the formula to calculate the bond value at the end of one year can be:

$$\mathrm{V} = \mathrm{c} + \frac{c}{(1+i)} + \frac{c}{(1+i)^2} + \frac{c}{(1+i)^3} + \cdots + \frac{c+M}{(1+i)^n}, \tag{3.21}$$

in our case the value at the end of one year is:

$$\mathrm{V} = 7 + \frac{7}{(1+3.72\%)} + \frac{7}{(1+4.32\%)^2} + \frac{7}{(1+4.93\%)^3} + \frac{7+100}{(1+5.32\%)^4} = 113.20.$$

And we calculate the bond at the end of one year for each credit rating category in Tab 3.7.

Tab 3.7. Possible one year forward value for a BBB bond.

Year-end rating	Value ($)
AAA	113.93
AA	113.74
A	113.20
BBB	112.07
BB	106.42
B	95.87
CCC	87.53
Default	51.13

Valuation of discount rate - To calculate discount rate is the interest rate used in discounted cash flow analysis to determine the present value of future cash flows, we need to calculate it in commercial banks. To estimate it, we have two steps to deriving risky yield curve from the transition matrix.

The first step is he derivation of n-year transition matrix. To consider transition probability from default to all those probabilities will be zero, or the default probability for a borrower who already is in financial difficulties will be 100%, in this situation the transition matrix can be:

$$\mathrm{T} = \begin{bmatrix} T_v & t_d \\ 0 & 1 \end{bmatrix}, \tag{3.22}$$

next, two-year transition matrix:

$$T^2 = \mathrm{T} \cdot \mathrm{T} = \begin{bmatrix} T_v^2 & (1 + T_v)t_d \\ 0 & 1 \end{bmatrix}, \tag{3.23}$$

the transition matrix for n-year:

the second step is use default probabilities to calculate risk-adjected yield curve, the one-year interest rate charge to the borrower can be:

$$T^n = \begin{bmatrix} T_v & \sum_{t=0}^{n-1} T_v^i t_d \\ 0 & 1 \end{bmatrix}, \tag{3.24}$$

$$\left(1 + r_1^i\right)\left(1 - p_1^i\right) + p_1^i R = 1 + r_1^F, \tag{3.25}$$

where R is the expected recovery rate if the borrower default, r_1^i is one year interest rate that has an i-rated, r_1^F is the one-year risk-free rate.

The risky loan can be compute as the expected amount of the riskless assets as:

$$r_1^i = \frac{r_1^i + p_1^i(1 - R)}{1 - p_1^i}, \tag{3.26}$$

the two-year interest rate as the relationship can be:

$$p_1^i R \frac{(1 + r_2^F)^2}{(1 + r_1^F)} + \left(p_2^i - p_1^i\right)R + \left(1 + r_2^i\right)^2\left(1 - p_2^i\right) = (1 + r_2^F)^2, \tag{3.27}$$

to calculate two-year interest rate, the formula can be:

$$r_2^i = \sqrt{\frac{(1 + r_2^F)^2 - p_1^i R \frac{(1 + r_2^F)^2}{(1 + r_1^F)} - \left(p_2^i - p_1^i\right)R}{\left(1 - p_2^i\right)}} - 1, \tag{3.28}$$

as similar as the formula of two year interest rate, n-year interest rate can be :

$$r_n^i = (1 + r_n^F)\left\{\frac{1 - R\sum_{j=1}^{n} \frac{p_j^i - p_{j-1}^i}{(1 + r_j^{F^j})}}{\left(1 - p_n^i\right)}\right\} - 1. \tag{3.29}$$

Step 3: Credit risk estimation

We now have all the information we need to assess the volatility of value caused by changes in credit quality, which are independent. That is the possibility of all possible outcomes we know. the steps before help use to get the values that we need, we will show then in Tab 3.7.

Tab 3.8. calculating volatility in value due to credit quality changes.

Year-end rating	Probability of state (%)	New bond value plus coupon ($)	Probability weighted value ($)	Difference of value from mean ($)	Probability weighted difference squared
AAA	0.02	113.93	0.02	2.42	0.0012
AA	0.33	113.74	0.38	2.23	0.0163
A	5.95	113.2	6.74	1.69	0.1689
BBB	86.93	112.07	97.42	0.56	0.2678
BB	5.3	106.42	5.64	-5.09	1.3758
B	1.17	95.87	1.12	-15.64	2.8638
CCC	0.12	87.53	0.11	-23.98	0.6903
Default	0.18	51.13	0.09	-60.38	6.5634
	Mean =	$111.51		Variance =	11.9476
				Standard deviation =	$3.46

To calculate the standard deviation as a measure of credit risk, we used a recovery value of $51.13 for case of default, in Tab3.7 is the mean and standard deviation and we calculate by the following formula:

$$\mu_{Total} = \sum_{i=1}^{S} p_i \mu_i , \tag{3.30}$$

$$\sigma_{Total} = \sqrt{\sum_{i=1}^{S} p_i \mu_i^2 - \mu_{Total}^2}, \tag{3.31}$$

where p_i is the probability of being in any given state, μ_i is the value within each state, μ is the mean and σ is the standard deviation.

We measure of credit risk by standard deviation, and also can measure it by percentile level, the significate level can be 95% or 99%. For this measure method, we need to have an ascending order to rewrite the probability weighted value, and the order of probabilities of adjacent states should be changed accordingly. We will show the ascending order in table:

Tab 3.9. Value and cumulative probabilities.

Year-end rating	Difference of value from mean ($)	Probability of state (%)	Cumulative probability (%)	New bond value plus coupon ($)
Default	-60.38	0.18	0.18	51.13
CCC	-23.98	0.12	0.30	87.53
B	-15.64	1.17	1.47	95.87
BB	-5.09	5.30	6.77	106.42
BBB	0.56	86.93	93.70	112.07
A	1.69	5.95	99.65	113.20
AA	2.23	0.33	99.98	113.74
AAA	2.42	0.02	100.00	113.93

In Tab 3.9, for the rating CCC we can see by separating at least 1% of the worst case, one can find a loss value of VaR at -15.64 at the 99% confidence level because the cumulative probability in this line is 0.30%, it is obviously less than 1%. But in the rating B the cumulative probability is 1.47% which is higher than 1%. Then, the value of the bond is $95.87 at the confidence level is 99%. If the confidence level is 95%, the value is equal to $106.42, both of them are lower than the mean value.

Tab 3.10. Joint migration probabilities with zero correlation (%).

Obligor 1 (BBB)		**Obligor 2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	D
		0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
AAA	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	0.04	0.29	0.00	0.00	0.00	0.00	0.00
A	5.95	0.02	0.39	5.44	0.08	0.01	0.00	0.00	0.00
BBB	86.93	0.07	1.81	79.69	4.55	0.57	0.19	0.01	0.04
BB	5.30	0.00	0.02	4.47	0.64	0.11	0.04	0.00	0.01
B	1.17	0.00	0.00	0.92	0.18	0.04	0.02	0.00	0.00
CCC	0.12	0.00	0.00	0.09	0.02	0.00	0.00	0.00	0.00
Default	0.18	0.00	0.00	0.13	0.04	0.01	0.00	0.00	0.00

Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 38p.

All the value we calculate before is one bond value, if we have a portfolio that have two bonds and we need to measure the credit risk of two bonds, let's assume the first bonds is BBB rating bond with the 7% annual coupon rate for five-year maturity and the second bond is A rating bond with the 6% annual coupon rate for three-year maturity, in this case where the two obligation credit rating changes are statistically independent. And in these two bonds the correlation of them is 0.30, the probability of the two obligators joint transition are in Tab 3.10.

We calculate A-rated bonds plus coupon like we calculate BBB-rated bonds. In Tab 3.11 is the value of two bonds we assumed.

Tab. 3.11. All possible values for two-bond portfolio($)

Obligor 1 (BBB)		**Obligor 2 (single-A)**							
		AAA	AA	A	BBB	BB	B	CCC	D
		103.70	103.61	103.42	102.77	100.31	98.58	86.09	51.13
AAA	113.9	217.63	217.54	217.35	216.70	214.24	212.51	200.02	165.06
AA	113.7	217.44	217.35	217.16	216.51	214.05	212.32	199.83	164.87
A	113.2	216.90	216.81	216.62	215.97	213.51	211.78	199.29	164.33
BBB	112.1	215.77	215.68	215.49	214.84	212.38	210.65	198.16	163.20
BB	106.4	210.12	210.03	209.84	209.19	206.73	205.00	192.51	157.55
B	95.87	199.57	199.48	199.29	198.64	196.18	194.45	181.96	147.00
CCC	87.53	191.23	191.14	190.95	190.30	187.84	186.11	173.62	138.66
D	51.13	154.83	154.74	154.55	153.90	151.44	149.71	137.22	102.26

In the same way for calculate the bonds mean and standard deviation, we have the mean and standard deviation of two-bonds portfolio, the formula for calculate that is:

$$\mu == \sum_{i=1}^{S=64} p_i \mu_i = 215.11,$$

$$\sigma = \sqrt{\sum_{i=1}^{S=64} p_i \mu_i^2 - \mu^2} = 3.70.$$

To calculate credit risk by this method and get the two bonds value in the percentile level and the confidence level is 99%, the likelihoods of all the values should be less than the sum to 1%. In table 3.10 we can see the number of $199.29 is the BBB-rated to A-rated, that is

$15.82 below the mean value.

Marginal risk - In subchapter 3.2.1. is the measurement of credit risk for an individual bond on a stands alone basis, but at the end the decision to hold a bond or not is likely to be made within the context of some existing portfolios. the calculation of marginal increase to the portfolio risk that would be created by adding a new bond to it.

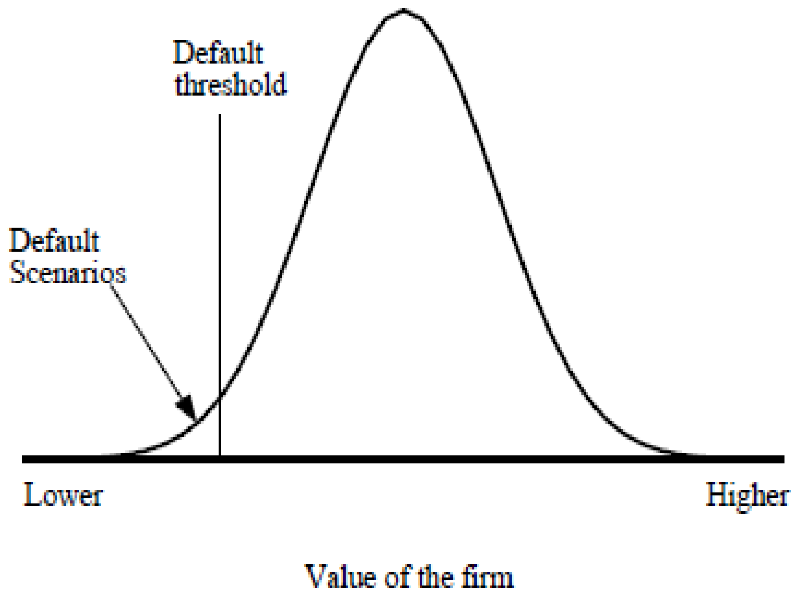
First is the calculation of marginal risk by using the standard deviation. In table 3.7 is the standard deviation of BBB-rated bond on a stands alone basis, the data we get is $3.46. The standard deviation of A-rated portfolio is $3.70, the increase of the standard deviation is $0.24 it represents marginal standard deviation. The value is lower than the stand-alone standard deviation which is $1.47, it means that the diversification effect caused by the individual bonds are not perfectly correlated.

Next is to extend the marginal risk calculation to percentile levels and confidence level. In table 3.7 we get the BBB-rated bond had a mean value of $111.51 and at the 99% percentile level value of $95.87. And the-A rated bond is added, the two-bond portfolio has a mean of $215.11 and a 99% percentile level of $199.29. The marginal risk between BBB bond and two-bond portfolio is represent by 15.64 between 15.82, that is equal to 0.18.

3.2.2. Credit quantity correlation

We are thinking the framework of using default as a function of the potential value, if the value of the asset just falls to a value lower than the amount of the outstanding liability, then it is impossible for the enterprise to fulfill its obligations and cause some default.

Figure 3.5. Model of firm value and its default threshold.

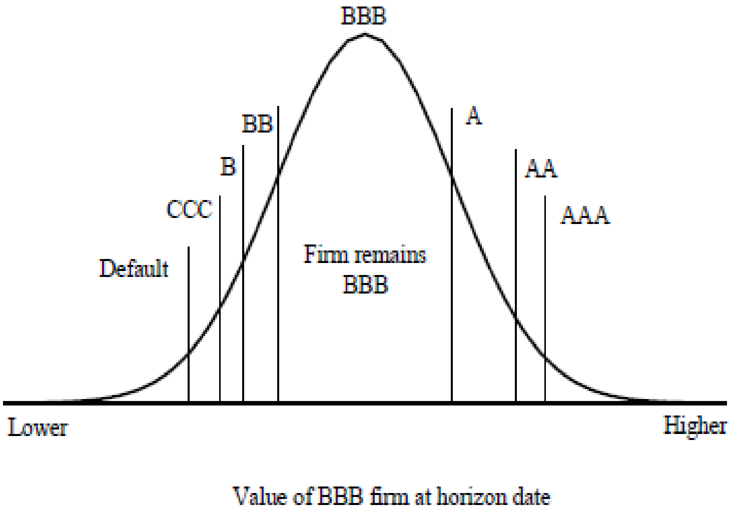


Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 37p.

In Tab 3.7, the asset correlation is 0.30 and we need to estimate the credit quality correlation parameters. Here in Figure 3.5 is the default as a function of the underlying value of the firm, and we can see that if the value of company decrease, the company will be default and may makes some unexpected losses

In Merton model it is easily to extended to rating change, it state the default threshold also grade thresholds. In Figure 3.6 is the model of firm value and generalized the firm's asset value relative to these thresholds determines its future rating, upgrade and downgrade are both showing:

Figure 3.6. Model of firm value and generalized



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 37p.

The simplest function to help we to calculate and then get credit quantity correlation. Here discrete returns of shares will be calculated as:

$$R_i = \frac{P_t - P_{t-1}}{P_{t-1}}, \quad (3.32)$$

where R_i represent the return on asset, P_t represent the value of the asset at time t, P_{t-1} represent the value of the asset on time t-1.

The expected return can be calculated as:

$$E(R_i) = \frac{1}{T}\sum_{t=1}^{T} R_i, \quad (3.33)$$

where $E(R_i)$ represent mean value of returns of assets, T is the number of observations.

The expected return can be also calculated as:

$$E(R_P) = \sum_{i=1}^{N} E(R_i)w_i = \overrightarrow{w^T}E(\overrightarrow{R}), \quad (3.34)$$

where all the parameters are represented the parameters in portfolio, and w_i is the weight of the i-th asset, N is the number of assets, $\overrightarrow{w^T}$ is the transposed vector variables, $E(\overrightarrow{R})$ is the vector of expected return.

The variance of the assets can be calculated as:

$$\sigma^2(R_i) = \frac{1}{T}\sum_{t=1}^{T}[R_{i.t} - R(R_i)]^2, \tag{3.35}$$

and the variance of the overall portfolio can be expressed as:

$$\sigma^2(R_P) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(R_i R_j) = \overrightarrow{w^T} C\overrightarrow{w}, \tag{3.36}$$

where w_i is the weight of i-th asset, w_j is the weight of j-th asset, $\sigma_{i,j}$ is the covariance of the returns of two assets, and C is the covariance matrix.

The correlation coefficient of the returns of two assets can be calculated as:

$$\sigma_{ij} = E\big[R_{i,t} - E(R_i)\big]\ [R_{j.t} - E\big(R_j\big)$$

$$= \frac{1}{T}\sum_{t=1}^{T}\big[R_{i,t} - E(R_i)\big]\big[R_{j,t} - E\big(R_j\big)\big], \tag{3.37}$$

the correlation between two assets can be calculated as:

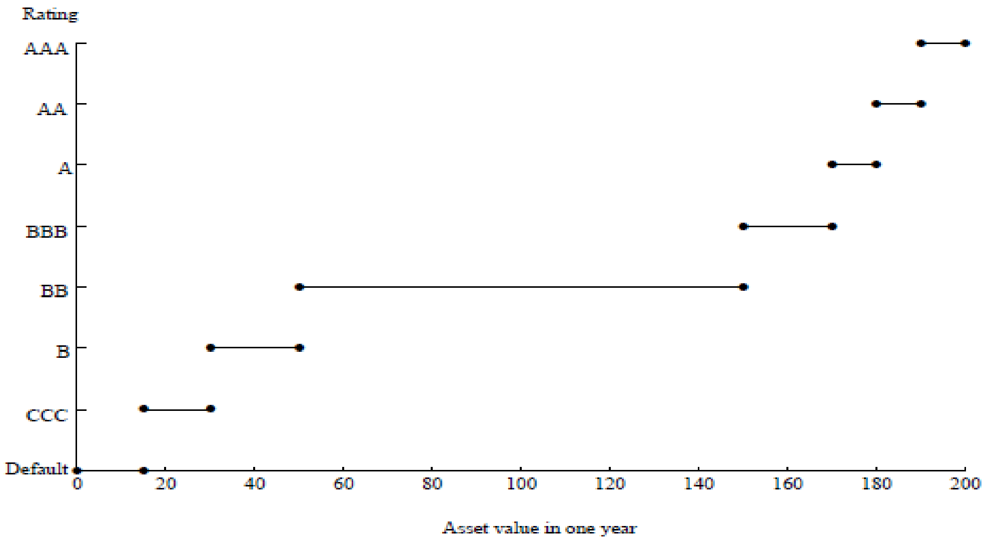
$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}, \tag{3.38}$$

also compute the correlation matrix for matrix C(m+n;m+n):

$$= \begin{bmatrix} \sigma^2(X_1) & \sigma^2(X_1;X_2)\ldots & \sigma^2(X_1;X_n) \\ \sigma^2(X_2;X_1) & \sigma^2(X_2)\ldots & \sigma^2(X_2;X_n) \\ \vdots & \vdots\cdots & \vdots \\ \sigma^2(X_m;X_1) & \sigma^2(X_m;X_2)\ldots & \sigma^2(X_n) \end{bmatrix}. \tag{3.39}$$

Asset value model - We recommend that the company's asset value is the process of promoting changes in its credit rating and default. This model is essentially Merton's option theory model, and the model association with the changes in assets value to credit rating changes

Figure 3.7. Credit rating migration driven by BB company asset value.



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 86p.

To describe company's credit rating evolution, is it necessary to model the company's change in asset value, the changes in asset value represent by percentage they are normally distributed and the parameter can be represented by mean and standard deviation. About asset thresholds, the assets return thresholds are Z_{Def}, Z_{CCC}, Z_{BBB} and so on, if $R<Z_{Def}$ it means the obligator will get in to default, if $Z_{Def}<R<Z_{CCC}$ means the obligator downgrade to CCC, here the Z_{Def} means the percentage the obligator would lead in the default. The probability of these events occur can be calculated if the R is normally distributed. The formula can be:

$$P_r\{Default\} = P_r\{R < Z_{Def}\} = \Phi(Z_{Def}/\sigma), \tag{3.40}$$

$$P_r\{CCC\} = P_r\{Z_{Def} < R < Z_{CCC}\} = \Phi(Z_{CCC}/\sigma) - \Phi(Z_{Def}/\sigma), \tag{3.41}$$

where Φ is the cumulative distribution for the standard normal distribution.

We calculated the probabilities of a BB-rated company for each rating in Tab 3.12.

Tab.3.12. Transition probabilities and thresholds for a BB-rated company.

Rating	**Probability from the transition matrix (%)**	**Cumulative Probability (%)**
Default	1.06	$1 - \Phi(Z_{AA}/\sigma)$
CCC	1.00	$\Phi(Z_{AA}/\sigma) - \Phi(Z_{A}/\sigma)$
B	8.84	$\Phi(Z_{A}/\sigma) - \Phi(Z_{BBB}/\sigma)$
BB	80.53	$\Phi(Z_{BBB}/\sigma) - \Phi(Z_{BB}/\sigma)$
BBB	7.73	$\Phi(Z_{BB}/\sigma) - \Phi(Z_{B}/\sigma)$
A	0.67	$\Phi(Z_{B}/\sigma) - \Phi(Z_{XXX}/\sigma)$
AA	0.14	$\Phi(Z_{XXX}/\sigma) - \Phi(Z_{Def}/\sigma)$
AAA	0.03	$\Phi(Z_{Def}/\sigma)$

Let's consider for AA-rated threshold we can get the value as the formula:

$$P_r\{AA\} = P_r\{R < Z_{AA}\} = \Phi(Z_{AA}/\sigma) = 0.14\%.$$

Then we need to solve Z_{AA} as the formula:

$$Z_{rating} = \Phi^{-1}(p)\sigma, \tag{3.42}$$

$$Z_{AA} = \Phi^{-1}(0.14\%)\sigma = 3.43\sigma.$$

Tab 3.13. Threshold values for asset return for a BBB rated obligor.

Threshold	Value
Z_{AA}	3.34σ
Z_{A}	2.93σ
Z_{BBB}	2.39σ
Z_{BB}	1.37σ
Z_{B}	-1.23σ
Z_{CCC}	-2.04σ
Z_{Def}	-2.30σ

Next we need to consider a second obligor A rated, and R' represent obligator's asset return, σ' represent the standard deviation of asset returns for this obligor, Z'_{rating} represent asset return thresholds. We show them in Tab 3.14.

Tab 3.14. Transition probabilities and asset return thresholds for A rating

Rating	Probability from the transition matrix (%)	Cumulative Probability (%)	Threshold
AAA	0.09		
AA	2.27	Z'_{AA}	$3.12\sigma'$
A	91.05	Z'_{A}	$1.98\sigma'$
BBB	5.52	Z'_{BBB}	$-1.51\sigma'$
BB	0.74	Z'_{BB}	$-2.30\sigma'$
B	0.26	Z'_{B}	$-2.72\sigma'$
CCC	0.01	Z'_{CCC}	$-3.19\sigma'$
Default	0.06	Z'_{Def}	$-3{,}24\sigma'$

To jointly describe the evolution of these two credit ratings, we assume that the two asset returns are correlated and normally distributed, and only specify the correlation between the two asset returns. The covariance matrix for the bivariate normal distribution can be:

$$\sum = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma' \\ \rho\sigma\sigma' & \sigma^2 \end{pmatrix}, \qquad (3.43)$$

if ρ is not zero, we can get it by following formula:

$$P_r\{Z_B < R < Z_{BB}, Z'_{BBB} < R' < Z'_A\} = \int_{Z_B}^{Z_{BB}} \int_{Z'_{BBB}}^{Z'_A} f(r, r'; \sum)(dr')dr, \tag{3.44}$$

where $f(r, r'; \sum)$ is the density function for the bivariate normal distribution, the covariance matrix is $(\sum)$, r and r' is the values that the two assets return may take on in the specific intervals.

Monte Carlo simulations - The Monte Carlo simulations is based on the production of random data, but through more sophisticated mechanisms. They involve estimating parameters of a particular probability distribution from historical samples and then extracting N simulated values from the probability distribution as risk factors.

We have two variables A and B the covariance matrix can be:

$$\sum = \begin{bmatrix} \sigma_A^2 & \sigma_{A,B}^2 \\ \sigma_B^2 & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} \sigma_A & 0 \\ \frac{\sigma_{A,B}^2}{\sigma_A} & \sqrt{\sigma_B^2 \left(\frac{\sigma_{A,B}^2}{\sigma_A}\right)^2} \end{bmatrix} \begin{bmatrix} \sigma_A & \frac{\sigma_{A,B}^2}{\sigma_A} 0 \\ 0 & \sqrt{\sigma_B^2 \left(\frac{\sigma_{A,B}^2}{\sigma_A}\right)^2} \end{bmatrix} = AA', \tag{3.45}$$

and the correlation matrix can be:

$$\sum = \begin{vmatrix} 1 & \rho \\ \rho & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ \rho & (1-\rho^2)^{1/2} \end{vmatrix} \begin{vmatrix} 1 & \rho \\ 0 & (1-\rho^2)^{1/2} \end{vmatrix}, \tag{3.46}$$

to calculate individual elements of the matrix:

$$p_{ii}\left(\sigma_{ii} - \sum_{k=1}^{i-1} p_{ki}^2\right)^{1/2}, \qquad for\ i = 1,2, \dots, N, \tag{3.47}$$

$$p_{ii} = 0, \qquad for\ i > j: i, j = 1,2, \dots, N. \tag{3.48}$$

Before we see how to use matrix A and its transposition, we need to know if we use the factor of the process examined m: the first step will generate m random values from 0 to 1 (p1, p2, ...

pm); the second step converts them to many values of normal values (v1, v2, v3); the third step uses formula to adjust them to produce x1, x2, ... xm to reflect their true mean and variance values. The covariance between different xi values is also essentially zero because they are generated in parallel but are independent.

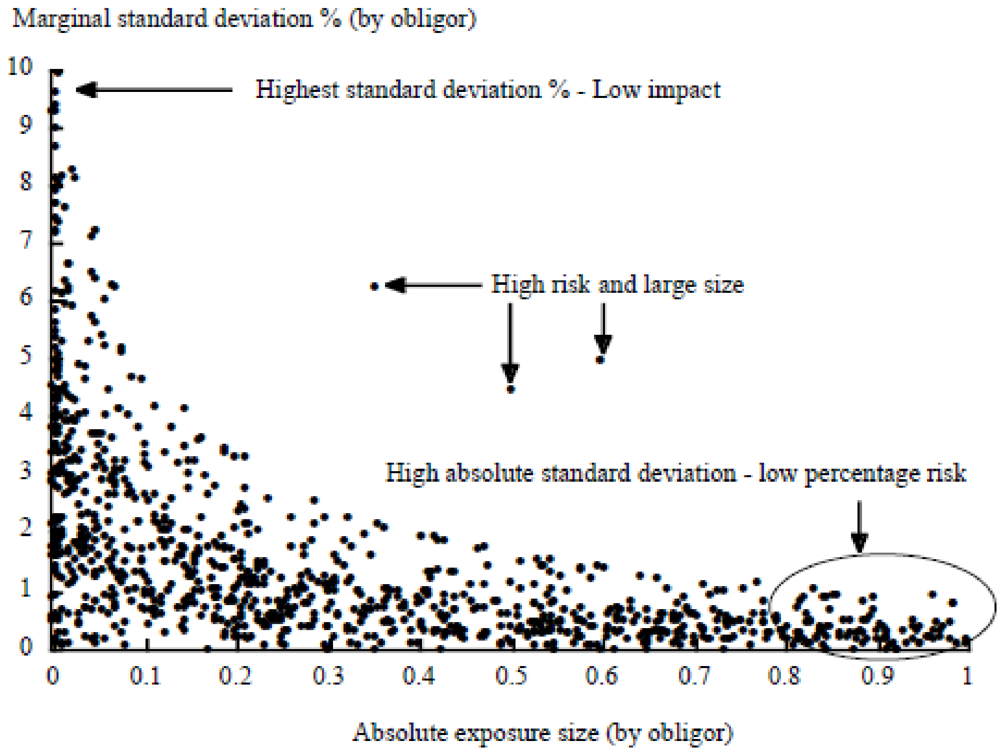
3.2.3. Applications

In this part we will have the applications of model outputs, the model we use is CreditMetrics, and in the past subchapter we calculate credit risk in one-bond and two-bonds portfolio by using standard deviation. In order to optimize the risk returns we receive, it is necessary to measure the risks we take, this is why we use CreditMetrics. The measure of credit risk have many application, in application we have some point to mention, like set priorities for reducing portfolio risk, measure and compare credit risk, so that institutions best allocate scarce risk resources by limiting concentration. To reduce risk we have two ways, the first way is prioritizing risk reduction actions and the second way is credit risk limits.

Prioritizing risk reduction actions - actions means actions in solving risk, if the risks are worth reducing it will absolute exposure size and statistical risk level, the risk default exposures can be shown in following figure.

In Figure 3.8, we have some information to get that will be: Reassessing the absolute largest debtors (In Figure 3.7 lower right corner), the single default would have the biggest impact, reassess the highest risk percentage of the debtor (In Figure 3.7 upper left corner), the debtor who reassessed the maximum absolute risk (In Figure 3.7 upper right corner) considered it to be the single largest contributor to portfolio risk.

Figure 3.8. Risk versus of exposures within a typical credit portfolio.



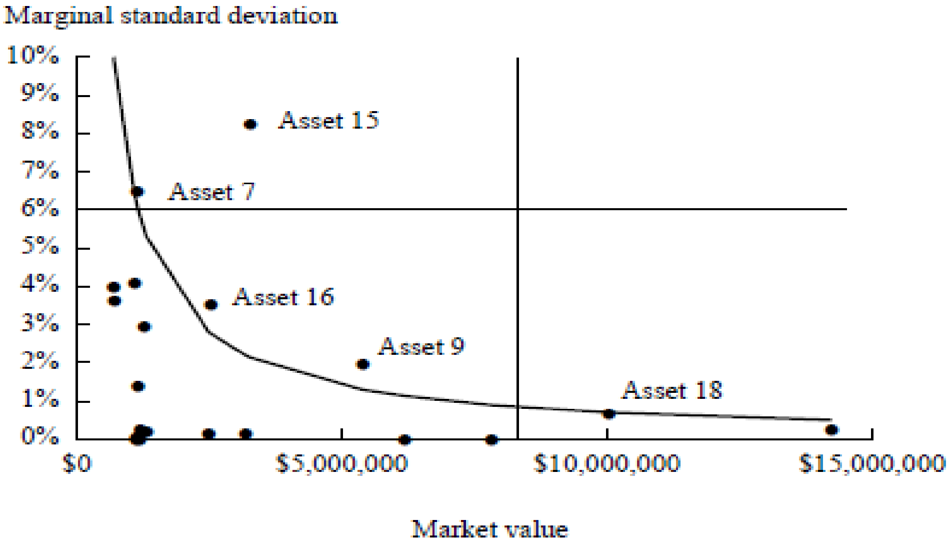
Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 134p.

The last information told us the obligator have the maximum risk, this is reassessed aimed at high risk and is the top priority of the debtor, when their credit rating is better, their exposure to risk is very big, but as a result of recent downgrade, now has a higher percentage of risk. When the risks were high, they had a higher percentage of credit ratings downgrades.

Credit limits

We can use not only prioritizing risk reduction actions to reduce risk but also credit limits to reduce risk, next we will introduce what kinds of risk measurement need to use and what policies use in limits manage. The CreditMetrics has three main possible in credit limits will be mentioned. And in Figure 3.9 is the risk limits that will happened in the portfolio.

Figure 3.9. Possible risk limits for an example portfolio.



Source: CUPTON, G. M., C. C., FINGER, and M., BHATIA. CreditMetrics Technical Document. New York: J. P. Morgan, 1997. 135p.

In previous part we consider that it is necessary to measure the higher risk that has been exposures, it has the largest influence in portfolio. And also use credit limits is useful to measure the credit risk that has been exposures.

3.3. Regulation of capital requirements

Capital requirement is how much the bank and financial institution hold as a required by the financial regulator, the bank and financial need these requirement because of ensure they will not go to insolvent due to excess financial leverage. Why we need to regulation of capital requirements is to ensure that companies operating in this industry are carefully managed. The internationally established rules for determining capital requirements are mainly the Basel Committee. That helps banks and depository institutions calculate their capital. We have Basel I in 1988, introduce how a capital measurement system for banks and financial institutions. Basel II agreement was reached in June 2004, it has a complex capital adequacy structure. The Basel III set because of financial crisis in 2007-08.

3.3.1. Basel I

Basel I is set in 1988's Basel Accord, the Basel committee on banking supervision in Basel, Switzerland, issued a set of minimum capital requirements for Banks. Then came a new set of rules, called Basel II, to replace the Basel accord. Allowing Banks to take on other types of risk is considered one of the reasons for the us subprime crisis that began in 2008.In fact, American bank regulators have adopted a more consensual approach to Banks by requiring them to comply. And the two important tiers are:

Tier 1 equals to common stock and surplus plus noncumulative perpetual preferred stock plus minority interest in the equity accounts of consolidated subsidiaries plus selected identifiable intangible assets and minus goodwill and other intangible assets.

Tier 2 equals the allowance for loan and lease loans plus subordinated debt capital instruments plus mandatory convertible debt plus intermediate-term preferred stock plus cumulative perpetual preferred stock with unpaid dividends plus other long-term hybrid capital instruments.

To calculate Tier 1 ratio and total capital ratio we have the formula:

$$Tier\ 1\ ratio = \frac{Tier\ 1\ capital}{RWA} \geq 4\%, \tag{3.49}$$

$$Capital\ adequacy\ ratio = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA} \geq 8\%. \tag{3.50}$$

Banks must also report off-balance-sheet items such as letters of credit, unused commitments and derivatives, these are all risk-weighted assets. It is mainly pay attention to credit risk and risk weighted assets, and in Basel I credit risk take part bank's assets in five parts, the risk weighted assets can be:

0% (cash, bullion, home country debt like Treasury bonds),
20% (short-term securities like mortgage-backed securities with AAA-rated),
50% (municipal revenue bonds, residential mortgages),

100% (most corporate debt) and some assets that has no rating.

The international bank need hold the capital equal to 8% risk weighted.

The risk-weighted assets for N items can use the formula to calculate:

$$RWA = \sum_{i=1}^{N} w_i \, EAD_i, \tag{3.51}$$

where w_i is the risk weight of the i-th asset, EAD_i is the exposure at default of i-th asset.

Basel I focus on credit risk and risk weighted assets, we need the capital adequacy ratio(CAR) can be calculated as:

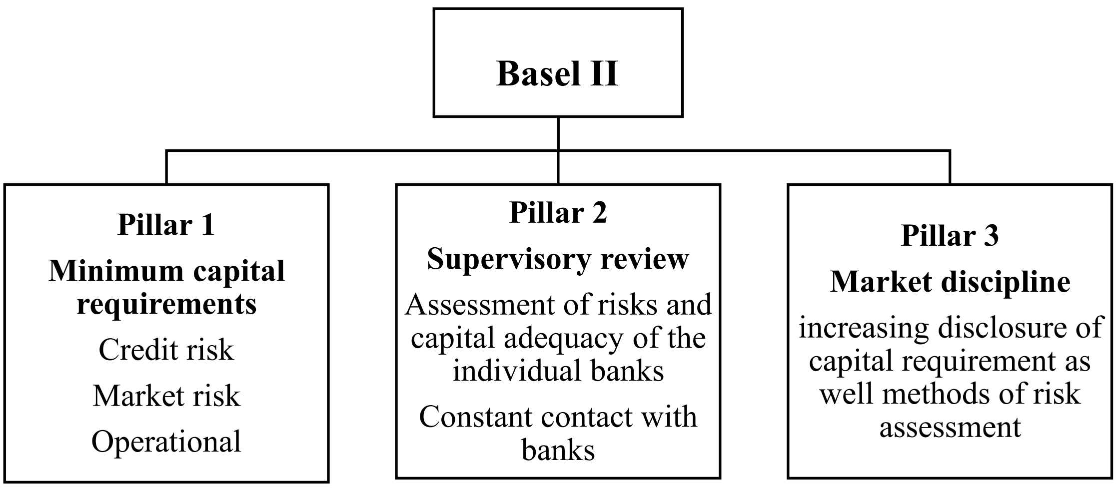
$$CAR = \frac{Tier\ 1\ capital + Tier\ 2\ capital}{RWA + (12.5CR_m)} \geq 8\%, \tag{3.52}$$

where CR_m is the capital requirement for market risk.

3.3.2. Basel II

As the use of Basel I for few years, the Basel committee consider that there has some problem by using Basel I, there is no distinction between asset classes and a rough analysis the potential risk of risk management practices. Basel II, which was set up by the Basel committee in June 2004. The main objective of Basel II is to promote adequate capitalization of Banks and to encourage the improvement of risk management practices to enhance the stability of the financial system.

Figure 3.10. The details of the three pillars in Basel II.



Pillar 1: Minimum capital requirement

In pillar 1 to calculate capital requirement, this is the main propose of Basel, and in the figure above we have three pillars, first we will describe pillar 1, which use the function of credit risk, market risk and operation risk to set capital requirement. And we have some approaches to measure these kinds of risk, we will show it in following table.

Tab 3.15. Method for calculating capital according Basel II.

	Credit Risk	**Market Risk**	**Operational Risk**
Approaches	• Standardized Approach • Foundation Internal Ratings-Based (IRB) Approach • Advanced IRB Approach	• Standardized Approach • Internal Models Approach	• Basic Indicator Approach • Standardized Approach • Advanced Measurement Approach
Result	Risk-weighted asset value for credit risk	Market risk capital charge	Operational risk capital charge

Source: APOSTOLIK, R., CH. DONOHUE and P. WENT. Foundations of Banking Risk: An Overview of Banking, Banking Risks, and Risk-Based Banking Regulation. Wiley Finance, 2009. 203p..

Bank need to get some value to calculate credit risk, market risk and operational risk by the approach above. The first value we need to get is total risk-weighted assets for the bank:

$$RWA_T = RWA_C + 12.5(CR_m CR_o), \quad (3.53)$$

where CR_m is market risk capital requirement, CR_o is the operational risk capital requirement.

The eligible regulatory capital (RC) is calculated by:

$$RC = Tier\ 1\ Capital + Tier\ 2\ Capital - Deductions, \quad (3.54)$$

the Basel II minimum capital requirement is:

$$CAR = \frac{RC}{RWA_T} = \frac{RC}{RWA_C + 12.5(CR_m + CR_o} \geq 8\%. \qquad (3.55)$$

To get the value to calculate risks, we didn't use Tier 3 capital which represent by the subordinated debts with the maturity higher than two years. For credit risk, Tire 1, Tire 2 capital for credit risk need to be higher than $8\% RWA_C$, for the market risk, Tire 1, Tier 2, and Tier 2 capital for market risk must be higher than CR_m.

We have been mention to calculate credit risk, we can use standardized approach, foundational internal ratings-based approach and advanced, and the result of these approach is the risk-weighted asset for credit risk. Next, we describe the standard approach to get credit risk.

Standard approach - By using standard approach to calculate credit risk, the banks use the rating from External Credit Rating Agencies to get the quantity required capital for credit risk. To summary the risk weighting in this approach will be in Tab 3.16.

Better rating are relevant to lower weights in calculation the risk-weighted assets, and in Tab 3.16, we can get the information that government has the risk weight exposure from 0% to 100%, public sector, banks and corporations has the risk weight exposure from 20% to 150%. And if the bank rated between BB to B, the risk-weighs under Basel II can be 100%.

Tab.3.16. Capital requirement risk weights under Basel II.

	Government	Public sector	Banks	Corporations
AAA to AA	0%	20%	20%	20%
A+ to A-	20%	50%	50%	50%
BBB+ to BBB-	50%	100%	100%	100%
BB+ to B-	100%	100%	100%	100%
B+ to B-	100%	150%	150%	150%
Below B-	100%	150%	150%	150%
Unrated	100%	100%	100%	100%

Source: BIS.

As the data of risk weights shows in above table, for example if a loan for $100 to a nonfinancial company with AAA-rated transition to $20 of risk-weighted assets leading to a capital requirement of 20*8%=$1.6, it is means non-weighted exposure.

Foundation internal rating-based (IRB) approach - The bank which use foundation internal rating-based approach, partly or whole responsible to estimate the degree of risk stand-alone loan and portfolio bonds. Some risk drivers they are default risk, recovery risk and exposure risk and maturity need to be known. In internal rating-based approach, we have two approaches from it, first is foundation approach and second is advanced approach. Foundation approach is estimate the obligator's PD using banks internal methods, also LGD and EAD will be valued. Advanced approach measures the four risks by using bank's internal method.

In foundation approach, the senior unsecured and unsubordinated in the bank to get the LGD at 45%, and subordinated debts are 75% LGD, the maturity can be 2.5. in both foundation and advance approach we need to calculate following values.

Risk-weighted assets can be compute as:

$$RWA = CR \cdot 12.5 \cdot EAD, \tag{3.56}$$

capital requirement(CR) can be calculated as:

$$CR = \left[LGD \cdot N\left(\sqrt{\frac{1}{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right) - pd \cdot LGD\right] \frac{1+(M-2.5)\cdot b}{1-1.5\cdot b}, \tag{3.57}$$

$$LGD^* = max\left[0; LGD \cdot \left(\frac{E^*}{E}\right)\right], \tag{3.58}$$

where E is the value exposure, E^* is the value after hedging, when the standard normal N(0,1) is exist N(x) is represent the nominal distribution. G(z) is when N(x)=z the distribution function for it.

The correlation can be compute as:

$$\mathrm{C} = 0.12 \cdot \frac{[1-\exp(-50 \cdot PD)]}{[1-\exp(-50)]} + 0.24\left[1 - \frac{[1-\exp(-50 \cdot PD)]}{[1-\exp(-50)]}\right], \tag{3.59}$$

and after formula (3.60), the correlation for commercial bank is similar, can be computed as:

$$R' = R + 0.04\left[1 - \left(\frac{S-5}{45}\right)\right], \tag{3.60}$$

$$\text{Maturity adjustment(b)} = (0.11852 - 0.054778 \cdot \ln(\text{PD})^2. \tag{3.61}$$

Pillar 2 Supervisory review

To have supervisory review, the bank need to ensure the minimum capital requirement that we mention in pillar 1 and in pillar 2 the process of supervisory review is the evaluate a bank's capital adequacy. In pillar 2 there are four key principles of supervisory review, the first principle is banks need to have a process to assess its overall capital adequacy ratio and its risk profile, they also need to know how to maintain its capital level strategy. Second principle is the supervisory need to review and evaluate internal capital adequacy in the band to ensure it is useful to supervisory the capital ratio. Third principle is the bank should operate more than minimum capital ratio, that means the bank's capital is higher than regulatory and the fourth principle is the supervisory prevent the capital below the minimum standards required.

Pillar 3 Market discipline

Pillar 3 focus describe minimum capital requirement and supervisory review, and focus on capital information to solve the company's problem. Pillar 3 also need the information substantial significance to the operation of the company and the evaluation of the operation of the company by investors.

3.3.3. Basel III

In Basel III is the bank capital adequacy ratio that have the stress testing and market liquidity risk of regulatory framework. The Basel Committee admit in 2010-11 that Basel III measure in 2013, it is important to 2007 financial crisis to set Basel III. It is mainly strengthens minimum capital requirement ratio more than Basel II, it also have the requirement about how much liquidity asset hold and stable of assets, to get the lower risk in banks operation. Here in Tab 3.17 is the capital ratio in Basel III, and we will get some information from that.

Tab 3.17. Capital ratio in Basel III.

	Phases	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
Capital	Leverage ratio			a			b	
	Minimum CR	3.5%	4.0%	4.5%				**4.5%**
	Capital conservation buffer				0.625%	1.25%	1.875%	**2.50%**
	Minimum capital buffer	3.5%	4.0%	4.5%	5.125%	5.75%	6.375%	**7.0%**
	CET1		20%	40%	60%	80%	100%	**100%**
	Minimum Tier 1 capital	4.5%	5.5%	6.0%				**6.0%**
	Minimum total capital		8.0%					**8.0%**
	Minimum total capital buffer		8.0%		8.625%	9.25%	9.875%	**10.5%**
	Capital instruments		Phased out over 10 years horizon beginning 2013					
Liquidit	Minimum CR			60%	70%	80%	90%	**100%**
	Net stable funding ratio						c	

Here in Basel II to require minimum common equity Tier 1 ratio is 4.5% need to be maintained all the time in bank. And there are two capital buffers in Basel III, the first is capital conservation buffer, equal to 2.5% of risk weighted assets, in table the minimum common equity plus capital conservation buffer equal to 7% in 2009, because of common equity Tier 1 ratio is 4.5%. the second buffer under Basel III is the discretionary counter-cyclical buffer, the level ranges between 0% to 2.5% of risk weighted asset and met by common equity Tier 1 capital.

And also some other regular value in Basel III, like the minimum level of total capital must be 8% all the time, minimum leverage ratio that is Tier 1 capital divided total exposure are expected to maintain in excess 3%, and the liquidity requirement that shows by liquidity coverage ratio is higher than 100%.

4. Determination of credit risk by selected models

By the theory part in chapter 3, the credit risk and other kinds of risk have been mention, and all the risks need to estimate like expected losses and unexpected loss caused by risk.

In this chapter, is to calculate capital requirement and economic, we calculate the capital requirement by Basel we use standard approach and foundation internal rating-based approach, and use CreditMetric^{TM} model to calculate economic capital. The CreditMetric^{TM} model have many steps, and get value of portfolio through increase the value of each bonds, final result will be compute by using credit risk characteristic.

We select ten different bonds from ten companies, the time horizon is the calculation cover the unexpected losses one year from January first, 2018. First ,we input data that we need to use to calculate, and after calculate the credit risk under Basel and by CreditMetric^{TM} model, the result will be compared.

4.1. Input data

The data we use to calculate capital requirement under credit risk under unexpected loss are from ten different bonds issued on Frankfurt Stock Exchange, the nominal value of each bond is one million euro, the total nominal value of bonds is 10 million euro. Here we find ten corporate bonds that issued by different company, all bonds issued in euro.

The main value will be shown in following Tab.4.1, the rating of the company can help us to calculate capital requirement by using Basel agreement, all the ratings are from Standard&Poor agency, other values like coupon in each bonds, and also nominal value, maturity, market price and pcs will in Tab 4.1.

Tab.4.1. Basic information about bonds portfolio.

	Rating	**Coupon**	**Nominal value**	**Maturity**	**Market price**	**pcs.**
Allianz	AA	0.25%	2,000€	06/2023	99.31%	500
Adidas	AA-	1.52%	1,000€	10/2026	107.25%	1,000
Daimler	A	2.375%	200,000€	12/2021	104.18%	5
Apotheke	AA-	0.75%	5,000€	10/2027	100.39%	200
Tesco	BB+	6.13%	2,000€	2/2022	114.49%	500
Nestel	AA-	4.25%	1,000€	03/2020	104.13%	1,000
Air France	BB-	3.75%	2,000€	10/2022	107.70%	500
Oracle	AA-	3.13%	1,000€	07/2025	116.68%	1,000
H-L	BBB-	6.75%	1,000€	02/2022	107.30%	1,000
PE	BB	2.00%	100,000€	03/2024	101.89%	10

Source: Frankfurt Stock Exchange (FSE).

In Tab 4.1, we can see the rating is from AA to BB-, rating AA in Allianz company is the highest one, it means the company has good information. The bonds nominal value are from 1000 euro to 200000 euro and the coupon of them are different. The market price we get are both from January first, 2018 in Frankfurt Stock Exchange.

Tab.4.2. Probability of default.

Rating	**PD**	**Rating**	**PD**
AAA	0.0007%	**BBB-**	0.2747%
AA+	0.0022%	**BB+**	0.7117%
AA	0.0024%	**BB**	1.2581%
AA-	0.0044%	**BB-**	4.1917%
A+	0.0142%	**B+**	8.8480%
A	0.1075%	**B**	24.4180%
A-	0.2020%	**B-**	48.6187%
BBB+	0.2045%	**CCC**	
BBB	0.2730%		

Source: Standard & Poor's.

In Tab 4.2, to calculate credit risk under Basel, we need to know the probability of default for different rating bonds, if the rating is higher the probability of default is lower, one the other hand, the lower rating caused higher probability of default. The probability of default has the transition matrix for European companies.

For each Senior Unsecured bonds we have been selected, which is a priority over debts owed by issuer, the rating of each bonds have been mention in above table. So the recovery rate that we get from Cath & Lieberman is 51.31%, and the loss given default is calculate by formula (2.2) is 48.87%.

4.2. Calculate the credit risk under Basel

In this part we use standard approach and internal rating-based approach of Basel I, II, III. After get the basis information, capital requirement can be calculated under Basel I, II, III, in subchapter 3.3 is the theory part of Basel, Basel Committee helps banks and depository institutions calculate their capital requirement under risk.

4.2.1. Under Basel I

In Tab.4.3 is the capital requirement under Basel I, the nominal value of each bonds is 1 million euro, the risk weights are shows in Tab 3.15 for each different rating, the risk weighted assets and capital requirement is calculated by formula (3.15) and formula (3.56).

Tab.4.3. Capital requirement under Base I.

Basel I	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
Allianz	AA	1,000,000€	20%	200,000€	16,000€
Adidas	AA-	1,000,000€	100%	1,000,000€	80,000€
Daimler	A	1,000,000€	100%	1,000,000€	80,000€
Apotheke	AA-	1,000,000€	100%	1,000,000€	80,000€
Tesco	BB+	1,000,000€	100%	1,000,000€	80,000€
Nestel	AA-	1,000,000€	100%	1,000,000€	80,000€
Air France	BB-	1,000,000€	100%	1,000,000€	80,000€
Oracle	AA-	1,000,000€	100%	1,000,000€	80,000€
H-L	B+	1,000,000€	100%	1,000,000€	80,000€
PE	BB	1,000,000€	100%	1,000,000€	80,000€
Total	-	-	-	**9,200,000€**	**736,000€**

After calculation of capital requirement under Basel I, some bonds has different risk weights because of rating, here Allianz company is a financial institution, so the risk weights is 20%, and the capital requirement is 16000 euro. The total risk weighted assets is 9.2 million and the total capital requirements are 736 thousand euro.

4.2.2. Under Basel II

By calculate the capital requirement under Basel we use standard approach and internal rating-based approach, first we will show the capital requirement under standard approach, the approach has some things different with Basel I because of risk weights are different. The result will be in following table.

Tab.4.4. Capital requirement under Basel II-SA.

Basel II-SA	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
Allianz	AA	1,000,000€	20%	200,000€	16,000€
Adidas	AA-	1,000,000€	20%	200,000€	16,000€
Daimler	A	1,000,000€	50%	500,000€	40,000€
Apotheke	AA-	1,000,000€	20%	200,000€	16,000€
Tesco	BB+	1,000,000€	100%	1,000,000€	80,000€
Nestel	AA-	1,000,000€	20%	200,000€	16,000€
Air France	BB-	1,000,000€	100%	1,000,000-€	80,000€
Oracle	AA-	1,000,000€	20%	200,000€	16,000€
H-L	BBB-	1,000,000€	100%	1,000,000€	80,000€
PE	BB	1,000,000€	100%	1,000,000€	80,000€
Total	-			**5,500,000€**	**440,000€**

In Tab 4.4 is the standard approach under Basel II, after calculation the risk weighted assets is 6 million which decrease 1.6 million by compared with Basel I, the capital requirement under this approach is 440 thousand euro which is decrease 40.22%. These changes are caused by rating under Basel II is different with Basel I. As we can see Hapag-Lloyd AG (H-L) company's rating is BBB-, so the risk weights is 100% that is higher, is means the bond is not good and have higher capital requirement. Also the absolutely change of capital requirement is 296 thousands euro, means for this portfolio have lower capital requirement under Basel II.

In Tab 4.5 is capital requirement under internal rating-based approach, the formulas we use are (3.57), formula (3.58) and formula (3.59), both risk weighted assets and capital requirement are shown.

Tab.4.5. Capital requirement under Basel II-FIRB.

Basel II - FIRB	**Rating**	**RWA**	**CR**
Allianz	AA	39,314€	3,145€
Adidas	AA-	52,235€	4,179€
Daimler	A	335,879€	26,870€
Apotheke	AA-	52,235€	4,179€
Tesco	BB+	880,169€	70,413€
Nestel	AA-	52,235€	4,179€
Air France	BB-	1,537,729€	123,018€
Oracle	AA-	52,235€	4,179€
H-L	BBB-	564,392€	45,151€
PE	BB	1,084,662€	86,773€
Total	**-**	**4,651,083€**	**372,087€**

The foundation internal rating-based approach is estimate the obligator's PD using banks internal methods, we have LGD and EAD as we have been mentioned, after we get the probability of default in Tab 4.2. The risk weighted assets is 4,651,083 euro and the capital requirement is 372,087 euro, because of some important value used in this approach. To compared with standard approach under Basel II, in foundation internal rating-based approach the risk weighted assets is increase about 90 thousand. The absolute change of capital requirement is 67,913 euro and the relative change of capital requirement is 15.43%. The lowest relative change is PE company is 8.47%, the capital requirement of this company is increase from 80 thousand euro to around 86 thousand euro.

4.2.3. Under Basel III

By calculate capital requirement and risk weighted assets under Basel III, wo also have same method as Basel II, the standard approach and foundation internal rating-based approach. In standard approach, the capital adequacy ratio is 13% include countercyclical buffer, but we didn't use this buffer so the minimum capital adequacy ratio is 10.5% to be used. The risk weighted assets and capital requirement are in Tab. 4.6.

Tab. 4.6. Capital requirements under Basel III-SA.

Basel III - SA	**Rating**	**Nominal value**	**w**	**RWA**	**CR**
Allianz	AA	1,000,000€	20%	200,000€	21,000€
Adidas	AA-	1,000,000€	20%	200,000€	21,000€
Daimler	A	1,000,000€	50%	500,000€	52,500€
Apotheke	AA-	1,000,000€	20%	200,000€	21,000€
Tesco	BB+	1,000,000€	100%	1,000,000€	105,000€
Nestel	AA-	1,000,000€	20%	200,000€	21,000€
Air France	BB-	1,000,000€	100%	1,000,000€	105,000€
Oracle	AA-	1,000,000€	20%	200,000€	21,000€
H-L	BBB-	1,000,000€	100%	1,000,000€	105,000€
PE	BB	1,000,000€	100%	1,000,000€	105,000€
Total	-	-	-	**5,500,000€**	**577,500€**

In Tab.4.6, the risk weighted assets is 5.5 million and the capital requirement is 577 thousand. To compared with result in Basel I shown in Tab 4.3, the relative change of risk weighted assets is 40.22% and the absolute change is 3.2 million, the relative change of capital requirement is 21.54%, both absolute change and relative change are decrease in Basel III-SA. And compared with the result in Basel II-SA shown in Tab 4.4, the absolute change if capital requirement is 130 thousand and the relative change of it is 31.25%, is increase in Basel III.

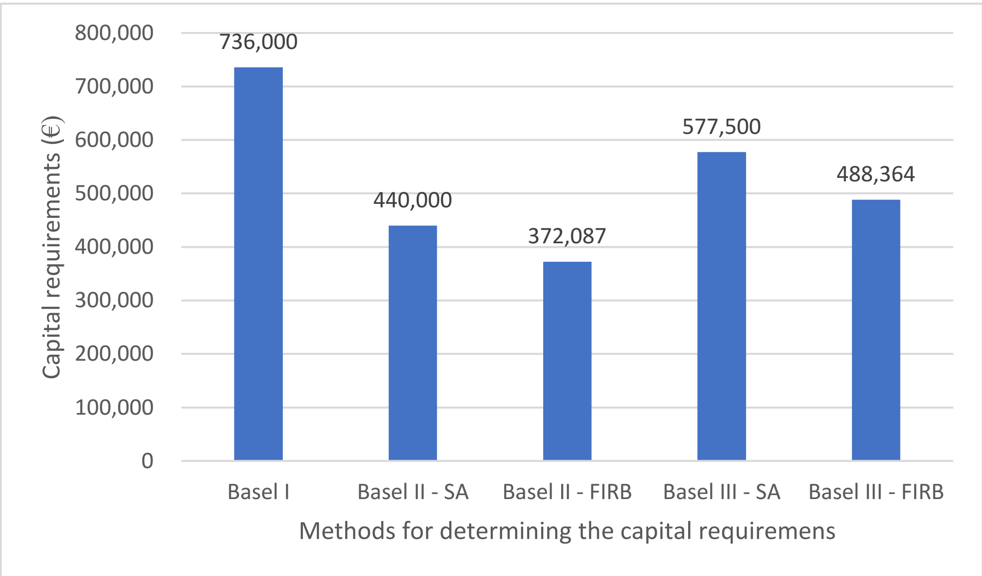
Next is the foundation internal rating-based approach in Basel III, is estimate the obligator's PD using banks internal methods, also LGD and EAD will be valued the minimum capital adequacy ratio is 10.5%, this is not same as Basel II. After calculated by Basel III-FIRB, we compared the value of the capital requirement in credit risk from unexpected loss. The results are in Tab 4.7.

Tab.4.7. Capital requirement under Basel III-FIRB.

Basel III - FIRB	Rating	RWA	CR
Allianz	AA	51,600€	4,128€
Adidas	AA-	68,558€	5,485€
Daimler	A	440,841€	35,267€
Apotheke	AA-	68,558€	5,485€
Tesco	BB+	1,155,221€	92,418€
Nestel	AA-	68,558€	5,485€
Air France	BB-	2,018,270€	161,462€
Oracle	AA-	68,558€	5,485€
H-L	BBB-	740,765€	59,261€
PE	BB	1,423,619€	113,890€
Total	-	**6,104,547€**	**488,364€**

In Tab 4.7, the result of Basel III-FIRB, the total risk weighted assets is around 6 million and total capital requirement is around 488 thousand. To compared with Basel III-SA, the absolute change of capital requirement is 89,136 euro and the relative change of it is 18.25%. The risk weighted assets also increase around 0.6 million. To compared with Basel II-FIRB, the relative change of capital requirement is 31%, both risk weighted asset and capital requirement are increase.

Figure 4.1. Capital requirement under Basel I, II, III.



In Figure 4.1 is the result of capital requirement in Basel I, II, III both standard approach and foundation internal rating-based approach in Basel II, III. Different approach can get different according to bank risk weighted and some value that based on Basel Committee. The capital requirement from Basel II, III is lower than the result from Basel I, it means in these few years, Basel Committee has some important change in their laws and adjust the risk weighted assets and capital adequacy ratio what will influence the result of capital requirement.

4.3. Calculate the credit risk by CreditMetric™

CreditMetric™ model, we have been mentioned in Chapter 3 has risk measurement framework, credit quantity correlation and the application of model output.

The calculation of credit risk by using this method will be in this chapter, all the data are the result that we need. To use this model, first we need to calculate correlation matrix and covariance matrix to get the yield from them. Second is make sure the value of each rating of bonds, to use transition matrix for each different rating we can put out the long-term yield curve. Third is use Monte Carlo model to compute 25000 random variables of each different bonds, the random variables will use in Cholesky decomposition matrix, and through the value of the yield curve to get the sum of random variables and Cholesky decomposition matrix. At the end, we will get value of portfolio through increase the value of each bonds, final result will be compute by using credit risk characteristic.

4.3.1. The correlation among bonds

As we mention before, the first step by CreditMetric™ model is to get the correlation matrix, the correlation is the between market price of ten companies stock traded on Frankfurt stock exchange from 15.03.2017 to 15.03.2018, all the market prices are shown in Annex 2.

Tab 4.8. Correlation matrix between shares.

	Allianz	Adidas	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
Allianz	1.00	0.45	0.48	0.02	0.04	0.28	0.20	0.10	0.02	0.33
Adidas	0.45	1.00	0.29	0.03	0.03	0.19	0.01	0.02	-0.03	0.10
Daimler	0.48	0.29	1.00	0.10	-0.03	0.28	0.11	0.07	0.04	0.37
Apotheke	0.02	0.03	0.10	1.00	-0.06	0.00	-0.06	0.07	-0.03	0.07
Tesco	0.04	0.03	-0.03	-0.06	1.00	0.16	-0.02	0.00	0.54	0.01
Nestel	0.28	0.19	0.28	0.00	0.16	1.00	0.06	0.04	0.03	0.11
Air F	0.20	0.01	0.11	-0.06	-0.02	0.06	1.00	0.03	0.00	0.07
Oracle	0.10	0.02	0.07	0.07	0.00	0.04	0.03	1.00	0.00	0.03
H-L	0.02	-0.03	0.04	-0.03	0.54	0.03	0.00	0.00	1.00	-0.01
PE	0.33	0.10	0.37	0.07	0.01	0.11	0.07	0.03	-0.01	1.00

In this part we get the correlation matrix between these ten companies, it reflects the relate in two companies. We use MS Excel Data-Data analysis-correlation to get the correlation matrix which has relationship with market price of these ten companies shares in the days we selected. Not only correlation matrix, but also covariance matrix we need to use, both of them are shown in Annex 3. And in Tab 4.8 the correlation between shares inform us which two companies are similar in their produce and which of them are no relationship. The highest correlation is 0.54, the relationship between Tesco company and Hapag-Lloys AG company, it means the operation of these two companies are similar. And the lowest one is -0.06, the relationship between Air France and Shop Apotheke.

4.3.2. Calculation of the value of bonds

The second step of CreditMetric^{TM} model is get the yield curve that is based on the present value of these ten bonds, the model we use is asset value model. The transition matrix should be find on Standard&Poor agency, it will be shown in Annex 1. And by the original transition matrix we can get the transition matrix in each year from 2018 to 2027, the matrix will be shown in Annex 4. It also has some important value like risk free rate, probability of default and recovery rate can be known, the recovery rate of Senior Unsecured bonds is 51.31% in Carty&Liberman, the risk-free rate is find on Erste Group online website from 2018 to 2027 and the risk-free rate are influence by interest rate swap, the forward rate will also be shown in

Tab 4.9 by calculate by formula.

Tab.4.9. Spot rate (IRS) and forward rates (f_n^F) from 2018 to 2027 (%).

Year	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
	1	2	3	4	5	6	7	8	9	10
IRS	-0.12	-0.03	0.08	0.20	0.32	0.45	0.56	0.68	0.76	0.88
f_n^F	-0.12	0.05	0.29	0.57	0.78	1.12	1.26	1.46	1.44	1.97

Source: Erste Group

By using IRS and forward rate in Tab 4.9 and using formula (3.29), we can calculate n years forward yield of these bonds we have selected from 2018 to 2027, the yield curve of forward rate will be shown in Annex 5. If we got the value of forward yield curve, next is necessary to calculate the present value of these ten company's bonds, we calculate present value by formula (3.21), in Tab 4.10 is the present value under different rating.

Tab.4.10. Present values of bonds (in euro).

Bond	Allianz	Adi das	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
AAA	1,900	1,009	214,364	4,465	2,527	1,118	2,293	1,130	1,294	105,185
AA+	1,900	1,008	214,346	4,463	2,527	1,118	2,293	1,130	1,294	105,162
AA	1,900	1,008	214,347	4,463	2,527	1,118	2,293	1,130	1,294	105,161
AA-	1,900	1,008	214,325	4,461	2,527	1,118	2,292	1,129	1,294	105,135
A+	1,899	1,007	214,267	4,458	2,526	1,118	2,292	1,129	1,294	105,081
A	1,895	1,005	213,915	4,446	2,521	1,116	2,288	1,126	1,291	104,853
A-	1,895	1,005	214,018	4,445	2,522	1,117	2,288	1,126	1,292	104,873
BBB+	1,891	1,001	213,657	4,429	2,517	1,116	2,284	1,123	1,289	104,587
BBB	1,886	999	213,357	4,415	2,513	1,114	2,280	1,120	1,287	104,341
BBB-	1,877	993	212,477	4,387	2,502	1,110	2,270	1,114	1,282	103,783
BB+	1,878	992	212,773	4,384	2,505	1,113	2,272	1,114	1,283	103,797
BB	1,857	981	210,857	4,331	2,481	1,104	2,250	1,101	1,271	102,658
BB-	1,806	949	206,571	4,184	2,425	1,086	2,198	1,069	1,242	99,707
B+	1,770	930	202,957	4,101	2,381	1,070	2,157	1,048	1,220	97,714
B	1,690	889	194,331	3,922	2,279	1,028	2,063	1,002	1,168	93,376
B-	1,444	759	168,133	3,344	1,967	901	1,777	858	1,009	79,861
CCC	1,164	615	133,748	2,713	1,571	709	1,421	692	805	64,417
D	509	255	50,900	1,273	509	255	509	255	255	25,450

The grid we callout with yellow is the present value of each rating bonds, the value of bonds can be calculated by the yield and default risk. For example. The default value of Allianz is

1900 euro.

4.3.3. Simulation of the value of the portfolio

In this part is the third step of calculate credit risk under CreditMetric^{TM} model, to simulation of value of the portfolio by using Monte Carlo simulation. We have 25000 random variables to be use, the random variable is compute by MS Excel-Date-Data analysis-Random Number Generator, the standard normal distribution is N(0,1), we can find all random variables in Annex 6.

After we get the correlation matrix, we compute Cholesky decomposition matrix because of the bonds are independent, we should consider these correlations when simulating the rate of return. The Cholesky decomposition matrix is shown in Tab 4.11.

Tab.4.11. Cholesky decomposition matrix.

	Allianz	Adidas	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
Allianz	1	0	0	0	0	0	0	0	0	0
Adidas	0.454	0.891	0	0	0	0	0	0	0	0
Daimler	0.477	0.086	0.875	0	0	0	0	0	0	0
AP	0.018	0.028	0.104	0.994	0	0	0	0	0	0
Tesco	0.042	0.013	-0.060	-0.060	0.996	0	0	0	0	0
Nestel	0.284	0.067	0.157	-0.025	0.152	0.931	0	0	0	0
Air F	0.205	-0.091	0.020	-0.068	-0.032	0.004	0.972	0	0	0
Oracle	0.098	-0.028	0.030	0.063	0.003	0.012	0.014	0.992	0	0
H-L	0.022	-0.050	0.036	-0.031	0.539	-0.068	0.003	-0.007	0.837	0
PE	0.328	-0.054	0.254	0.039	0.017	-0.025	-0.001	-0.010	-0.043	0.906

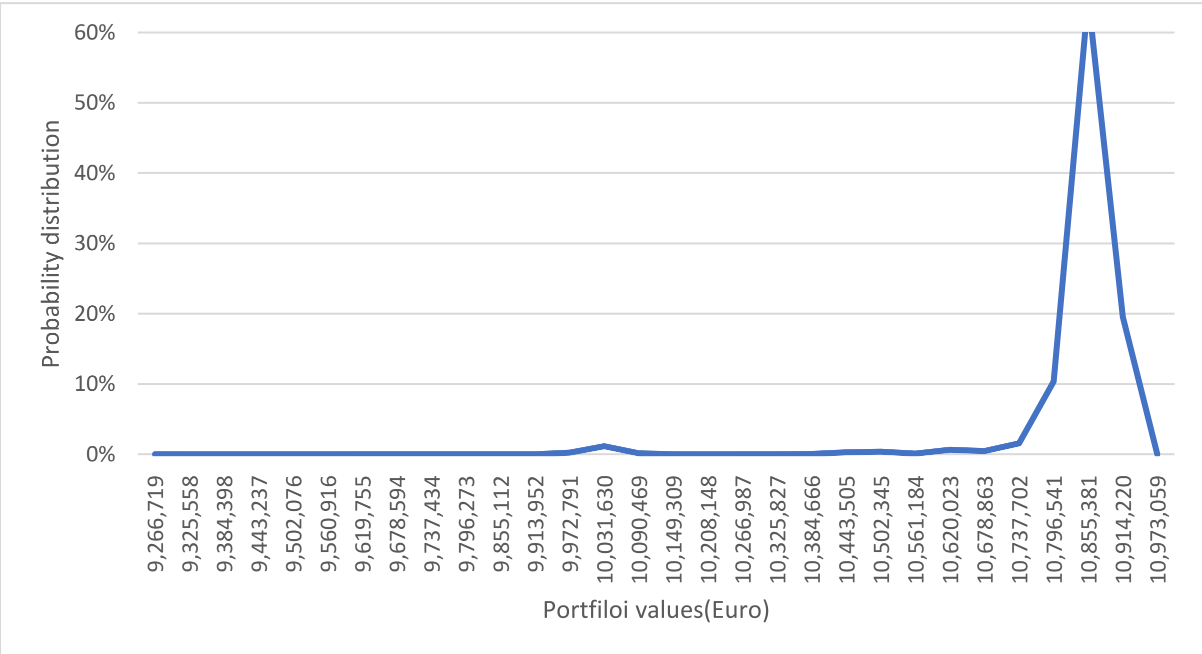
By using random variables and standard normal distribution N(0,1), the Cholesky decomposition matrix can be calculated by formula (3.47) and formula (3.48), the variables that reflects the relative degree of correlation between individual issuer yields. The matrix of correlation random variables are in Annex 7.

The correlation yield can be given a rating, the transition matrix between each rating classification will be shown in Annex 8. And the correlation yield rating assignment for the bonds that we selected can be computed by MS-Excel-IF function, the rating assignment will in Annex 9. The present value in previous part are shown, and after rating assignment, the value of individual bonds can be known in Annex 10. To multiply these values and the number of portfolio to obtain the total value of individual bonds. The total value under each different situation, the total value of the portfolio can be obtained.

4.3.4. Calculation of credit risk

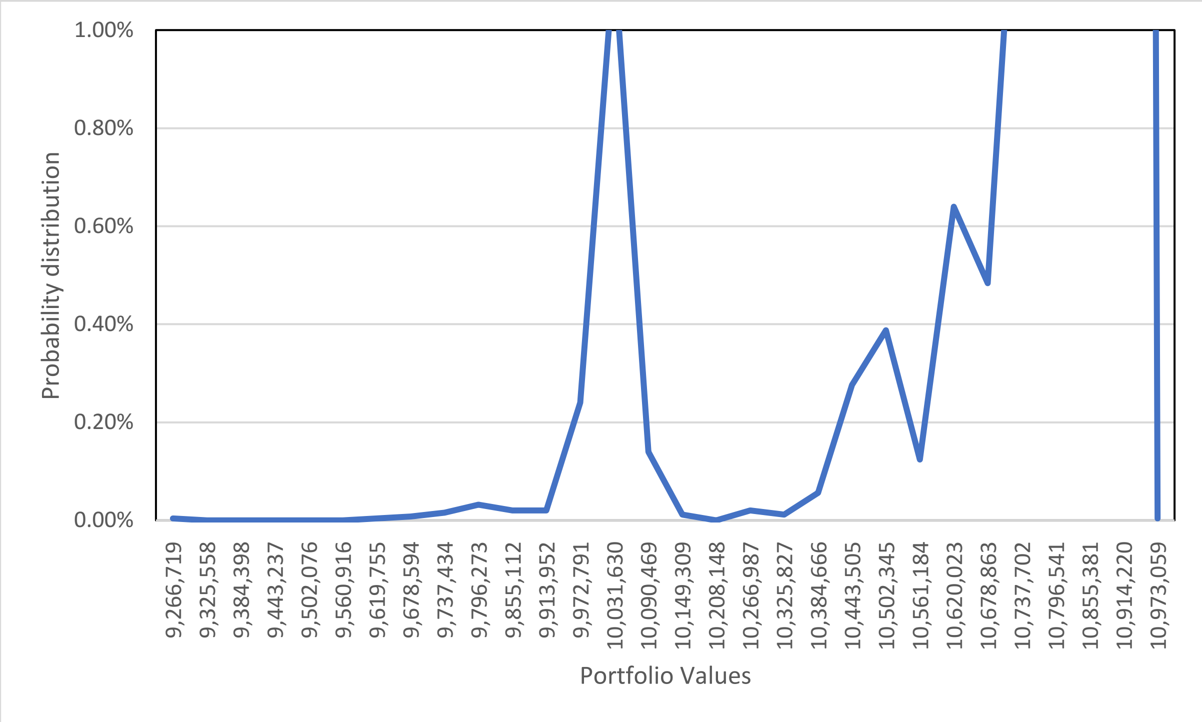
the last step of the model by calculate the credit risk is the calculation, the value of the portfolio has been calculate by the previous part, the probability distribution of the portfolio value will be shown in Annex 11, we use figure to know the trend of the probability distribution.

Figure 4.2. Probability distribution of portfolio values.



In figure 4.2 is the probability distribution of the portfolio values, the minimum value is 10,794,592 and the maximum value is 10,920,024, the probability is 39.4%. the trend of the probability is not obvious, if we want to see the trend of the probability obvious, and what is the minimum value portfolio in figure 4.3 can be shown.

Figure 4.3. Probability distribution of the portfolio values – adjusted.



Next is the value of these bonds at initial rating, the expected value is calculated, and the expected loss is the expected value minus value at initial rating. All the value are shown in Tab.4.12.

Tab.4.12. Results of the portfolio value(Euro)

	Value at initial rating	**Expected value**	**Expected loss**
Allianz	949,988	949,763	225
Adidas	1,008,052	1,007,899	153
Daimler	1,069,577	1,069,717	-140
Apotheke	892,249	892,044	205
Tesco	1,252,395	1,248,345	4,051
Nestel	1,118,239	1,118,158	81
Air France	1,098,834	1,088,121	10,714
Oracle	1,129,272	1,129,056	216
H-L	1,281,686	1,281,710	-25
PE	1,026,577	1,028,927	-2,350
Portfolio	**10,826,869**	**10,813,741**	**13,129**

In Tab.4.12, the total portfolio value at initial rating is 10,826,869 euro, the expected value is 10,813,869 euro and the total expected loss of the portfolio is 13,129 euro. From the expected loss, the highest expected loss is H-L company that represent 86.60% of the total expected loss, it means this company is not health at their bond. The lowest expected loss is PE company, has -2.350 euro expected loss means the company gains a lot in their bonds. The percentage between the expected loss and value at initial rating is 0.12%, it means the portfolio has less loss and have the low correlation between these two values.

The standard deviation and marginal standard deviation are two parameter in the portfolio, the standard deviation is measure that is used to quantify the amount of variation or dispersion of a set of data values. Marginal standard deviation is necessary to estimate which assets are helpful in the portfolio and which are not helpful, the parameter also can analysis the influence of each bonds to total risk.

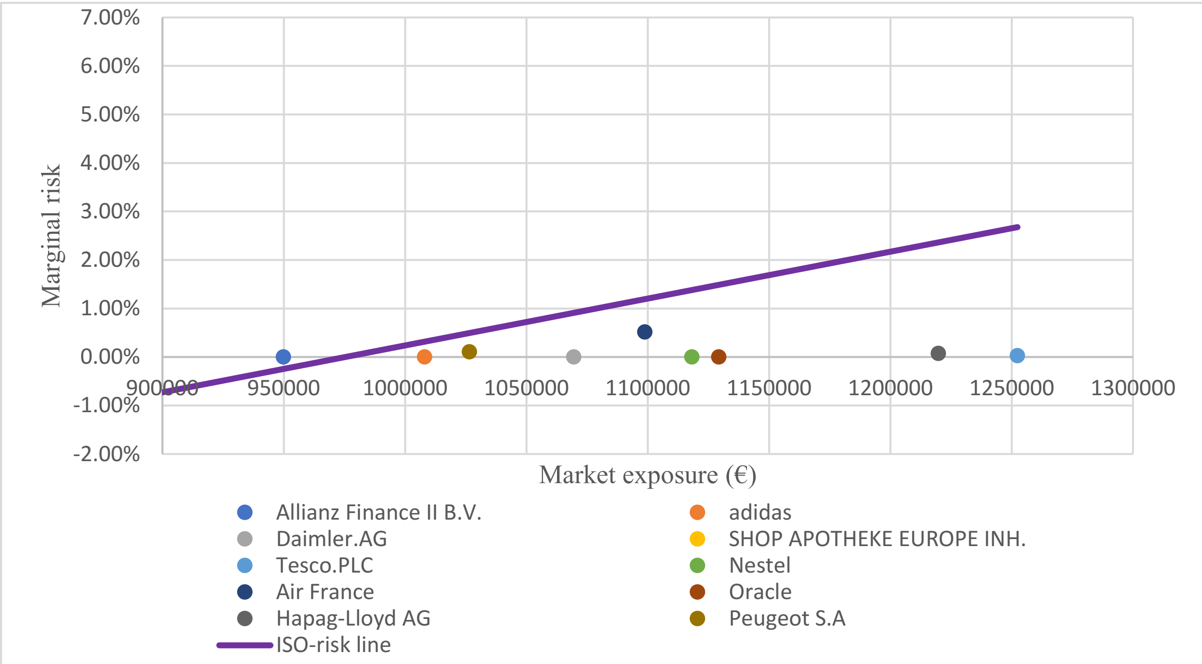
Tab.4.13. Parameter of the risk.

	Standard deviation		**Marginal standard deviation**	
	%	**Euro**	**%**	**Euro**
Allianz	0.0838%	796	0.0009%	95
Adidas	0.0735%	740	-0.0002%	-23
Daimler	0.1614%	1,727	0.0006%	62
Apotheke	0.1088%	970	-0.0002%	-17
Tesco	1.4683%	18,330	0.0180%	1,917
Nestel	0.0347%	388	0.0000%	-1
Air F	9.4753%	103,102	0.5434%	57,782
Oracle	0.0926%	1,046	0.0000%	1
H-L	2.2378%	28,682	0.0355%	3,773
PE	4.9799%	51,240	0.1111%	11,814
Portfolio	**1.1348%**	**120,668**		

The standard deviation reflects the risk of the portfolio, so if the standard deviation is lower the risk will also be lower. In Tab 4.13, the portfolio standard deviation is 1.1348% means the portfolio risk is low. The standard deviation of each bonds also be shown, the lowest one is Nestel company which is 0.0347%, it means is has highest initial rating, the highest one is 9.4753% from Air France company, it means the initial rating is lowest. The marginal standard deviation reflects the also the initial rating, the highest one also Air France company. The marginal standard deviation is lower than standard deviation, and the lowest is Nestel and Oracle company, it has the good initial rating in these two companies.

The marginal standard deviation can reflect marginal risk, in figure 4.5 is the summarize of marginal risk, all the point have same absolute marginal risk is called ISO-risk line, the absolute marginal risk is multiple by the exposure and the marginal standard deviation. The point under ISO-risk line means the bonds that the point reflects has lower risk, on the other side if the point above the ISO-risk line the bond has higher risk.

Figure 4.5. Marginal risks.



In above figure have one point above the ISO-risk line, it means Allianz company's bond are above the line, and the company has the high risk of their bond. And others bonds are all below the ISO-risk line, it means their bonds has low risk.

The portfolio value will be shown at last, before the previous steps we get some important value, the portfolio value and VaR is calculated by the confidence level at 99.9%, 99.5% and 99%, the significant level will be 0.1%, 0.5% and 1%.

Tab.4.14. Significant level and corresponding portfolio value and VaR. (Euro)

alpha	Portfolio value	VaR
0.1%	9,913,721	-913,149
0.5%	9,982,342	-844,527
1%	10,001,601	-825,268

Here under the significant level at 0.1%, we get the portfolio value is 9,913,721 euro and the VaR is -913,149 euro, this is the highest VaR that have been calculated. the economic capital it necessary to calculated by formula (3.20), the economic capital is the capital requirement under the unexpected loss of the risk. In Tab 4.15 is the final economic capitals under the significant level 0.1%, 0.5%, and 1%.

Tab.4.15. Economic capitals.

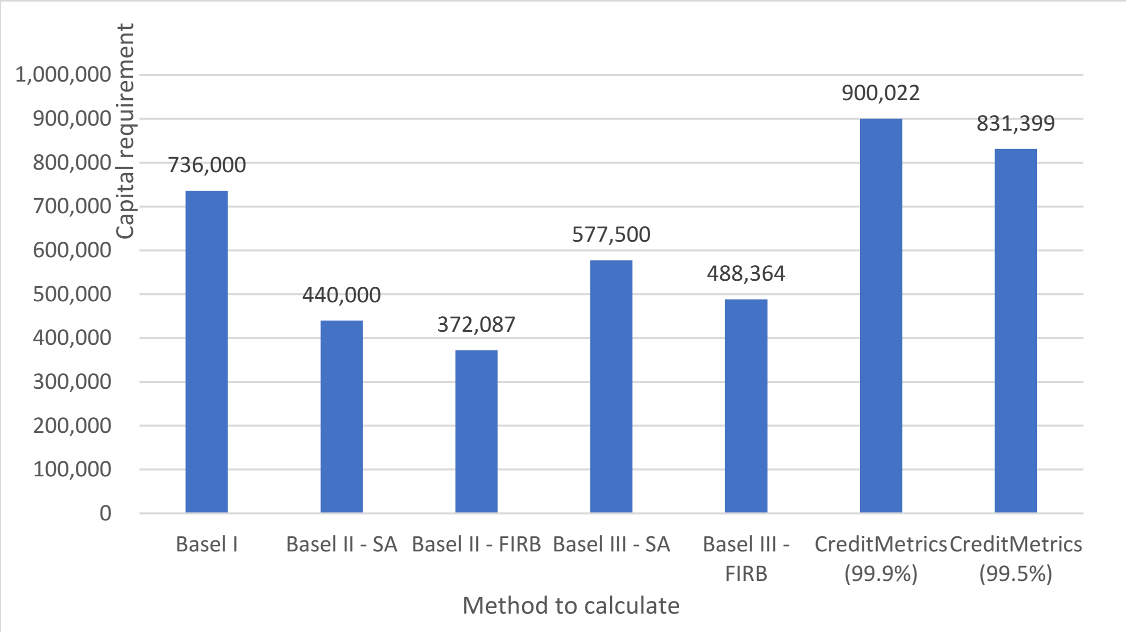
alpha	Economic capital (Euro)
0.1%	900,022
0.5%	831,399
1%	812,140

In Tab 4.15, the economic capital when the significant level change, if the significant level increase cause the economic capital decrease, here when the significant level is 0.1%, the economic capital is 900,022 euro and the significant level is 1%, the economic capital is 831,399 euro. It means the unexpected loss change because of credit risk change.

4.4. Evaluation of resuls

In this chapter is the calculation of the capital requirement by using Basel I, II, III, and the calculation of economic capital by using CreditMetric^{TM}, we use different to calculate capital requirement because each of them has their own parameter and variables. For example, CreditMetric^{TM} model use 25000 different variables to get the economic capital at the end, the credit quantity can be ensured. We make a figure of the result of the two methods, the capital requirement can be shown obviously in figure 4.7.

Figure 4.7. Capital requirement by using different model.



The Basel agreement calculate how much the bank and financial institution hold as a required by the financial regulator, the bank and financial need these requirement because of ensure they will not go to insolvent due to excess financial leverage. And the CreditMetric™ model get value of portfolio through increase the value of each bonds, final result will be compute by using credit risk characteristic. In figure 4.7 the capital requirement from Basel II-SA and FIRB has the similar value and Basel III-SA and FIRB has the similar value, but we use the Credit Metrics model has the different value when the confidence level is 99.9% and 99.5%.

But in standard approach in Basel II and III, the value is higher than the value in foundation internal rating-based approach. In Basel II to compare the standard approach and foundation internal rating-based approach, the absolute change is 67.913 euro, in Basel III the absolute change of these two approaches is 89,136 euro, which is higher than the absolute change in Basel II, because the capital adequacy ratio is increase 2.5%. The relative change of these two approaches in both Basel II and Basel III is 18.25%.

5. Conclusion

Credit risk is become common know, risks may come from a variety of sources, including financial market uncertainty, project failure threats, legal liability, credit risk, accidents, natural causes and disasters, deliberate attacks by opponents, or uncertain. Because of the unexpected loss of risk, it is important for companies to estimate the risk, the most important risk is credit risk, which always means the possibility that the company or individual will not be able to pay the required amount of debt.

The aim of this thesis was to determine the capital requirement for unexpected losses from credit risk of the portfolio under Basel agreement include Basel I, II, III and used CreditMetric^{TM} model to determine the economic capital of the portfolio.

There are three main chapter in this thesis, in chapter two and chapter three were both theory part. In chapter two was the description of the financial risk. It was include credit risk, market risk, liquidity risk, operational risk and other risks. In chapter three was the introduction of some models that can use to calculate credit risk, the main model is CreditMetric^{TM} model and Basel agreement.

Chapter four was the most important chapter, it was the calculation part in this thesis, we find ten bonds that trade on Frankfurt stock exchange with the nominal value of each bonds is 10 million euro. The capital requirement under the risk from unexpected loss, standard approach and foundation internal rating-based approach include in Basel agreement can be used. The economic capital of ten bonds portfolio was calculate by CreditMetric^{TM} model.

The capital requirement was compared at the end of chapter 4, the capital requirement is 736,000 euro in Basel I, it was higher than the result in Basel II and III. The result in Basel II, both standard approach and foundation internal rating-based approach has the similar result around 440,000 euro, and in Basel III the result in both two approaches was around 500,000 euro. And absolute change in Basel III is higher than the absolute change in Basel II, because the capital adequacy ratio is increase 2.5%. The relative change of these two approaches in both Basel II and Basel III is 18.25%.

The result of CreditMetrics^{TM} under the confidence level is 99.9% is 900,022 euro, it is higher than the result under the confidence level is 99.5%. It means the unexpected loss change because of credit risk change.

All the companies and individual need to have risk management as a planning, under risk management the unexpected loss can be prepared before the accident, make the loss and cost in the minimum value, all the management and how important it is we have been shown in this thesis.

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List of Abbreviations

BIS	Bank if international Settlements
BOC	Bank of China
CAR	Capital adequacy ratio
CR	Capital requirement
DD	Distance of default
EAD	Exposure at default
EDF	Expected default frequency
EL	Expected loss
FIRB	Foundation Internal Rating-based approach
FSE	Frankfurt Stock Exchange
IRS	Interest rate swap
LCR	Liquidity coverage ratio
LGD	Loss given default
LLA	Loan loss allowance
LTD	Long-term debt
NPL	Nonperforming loans
NSFR	Net stable funding ratio
OECD	Organization for Economic Co-operation and Development
PD	Probability of default
RC	Regulatory capital
RR	Recovery rate
RWA	Risk-weighted asset
STD	Short-term debt
UL	Unexpected loss
VaR	Value-at-Riak

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Wang Xinran

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List of Annexes

Annex 1: Probability matrix from Standard & Poor's

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	85.03%	6.72%	1.52%	0.87%	0.22%	0.43%	0.00%	0.00%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.09%	74.86%	15.03%	2.73%	0.82%	0.82%	0.55%	0.55%	0.00%	0.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.22%	1.20%	78.98%	8.50%	4.14%	1.31%	0.54%	0.22%	0.00%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.08%	0.08%	4.56%	74.98%	12.26%	2.73%	1.24%	0.17%	0.08%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.00%	0.07%	0.63%	5.51%	73.97%	10.89%	2.58%	0.49%	0.35%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A	0.00%	0.23%	0.17%	0.74%	4.69%	73.46%	11.21%	2.29%	1.14%	0.17%	0.06%	0.06%	0.06%	0.06%	0.00%	0.00%	0.00%	0.11%
A-	0.05%	0.00%	0.16%	0.16%	0.98%	7.22%	76.11%	7.93%	1.48%	0.82%	0.16%	0.05%	0.11%	0.00%	0.00%	0.00%	0.00%	0.05%
BBB+	0.00%	0.00%	0.00%	0.14%	0.29%	0.86%	7.43%	73.50%	8.71%	1.21%	0.36%	0.57%	0.21%	0.21%	0.07%	0.00%	0.14%	0.07%
BBB	0.00%	0.00%	0.10%	0.00%	0.19%	0.01	0.88%	7.89%	69.98%	7.89%	1.66%	1.07%	0.10%	0.10%	0.39%	0.10%	0.10%	0.10%
BBB-	0.00%	0.00%	0.16%	0.00%	0.16%	0.64%	0.48%	1.43%	8.90%	67.25%	6.52%	2.70%	0.79%	0.32%	0.32%	0.00%	0.32%	0.32%
BB+	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.30%	0.60%	0.90%	11.64%	58.81%	8.06%	2.39%	1.79%	0.30%	0.00%	0.30%	0.00%
BB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.50%	0.00%	1.75%	11.25%	56.75%	6.25%	2.75%	1.00%	0.00%	0.75%	0.50%
BB-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.25%	0.25%	8.89%	59.01%	12.84%	4.20%	0.49%	0.25%	1.48%
B+	0.00%	0.00%	0.00%	0.00%	0.00%	0.23%	0.00%	0.23%	0.00%	0.00%	0.23%	2.93%	8.80%	54.63%	8.35%	3.84%	1.35%	1.81%
B	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.38%	0.38%	0.38%	1.51%	12.08%	45.66%	8.30%	4.53%	4.15%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.63%	0.00%	0.00%	0.00%	0.00%	1.27%	6.33%	49.37%	15.82%	10.13%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.15%	3.46%	9.20%	25.29%	37.93%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Annex 2: Shares prices from March 26th, 2015 to March 16th, 2016 (€)

15/03/2017	Allianz	Adidas	Daimler	Apotheke	Tesco	Nestel	Air F	Oracle	H-L	PE
16/03/2017	169.743	179.98	71.186	27.09	2.1443	71.434	7.307	40.322	29.988	19.354
17/03/2017	171.399	183.2	71.42	27.21	2.146	71.999	7.528	42.767	30.5	19.21
20/03/2017	170.525	182.939	71.29	27.841	2.13	72.11	7.56	42.029	31.072	18.948
21/03/2017	170.184	181.436	71.176	27.275	2.13	72.259	7.605	42.35	31.15	18.803
22/03/2017	169.553	180.318	70.65	27.81	2.151	71.721	7.549	41.59	30.196	18.578
23/03/2017	169.304	179	69.9	27.73	2.97	71.75	7.427	41.541	28.1	18.623
24/03/2017	170.012	179.5	70.685	27.335	2.217	72.034	7.454	41.687	26.406	18.55
27/03/2017	169.078	178.59	70.615	27.815	2.181	72.2	7.493	41.971	28.93	18.453
28/03/2017	169.791	177.456	70.685	27.785	2.242	72.45	7.19	41.263	27.11	18.396
29/03/2017	171.15	177.312	71.887	27.75	2.185	72.001	7.135	41.209	27.54	18.479
30/03/2017	172.308	178.339	72.25	27.613	2.2	72.265	7.025	41.493	26	18.965
31/03/2017	172.771	177.435	69.584	27.285	2.188	72.261	6.994	41.649	25.75	18.954
03/04/2017	173.5	178.235	69.2	27.13	2.218	71.733	6.996	41.867	27.355	18.75
04/04/2017	171.993	175.761	68.451	27.275	2.154	71.805	7.084	41.95	27.339	18.587
05/04/2017	173.487	177.734	67.901	27.275	2.131	71.848	7.048	41.841	27.206	18.452
06/04/2017	172.608	176.15	67.18	27.685	2.199	71.693	7.149	42.105	27.27	18.435
07/04/2017	173.181	176.638	67.16	27.3	2.187	71.868	7.099	41.648	26.762	18.056
10/04/2017	172.39	178.266	67.01	26.945	2.233	71.979	6.957	41.609	27.582	18.068
11/04/2017	172.366	176.881	66.99	27.7	2.25	70.601	7.19	41.61	27.745	18.03
12/04/2017	170.891	176.085	66.556	27.29	2.249	71.182	7.157	41.416	27.266	17.9
13/04/2017	171.748	177.626	66.99	27.125	2.219	71.686	7.609	41.458	27.066	17.85
18/04/2017	171	177.755	66.236	26.91	2.187	71.36	7.176	41.739	26.717	17.6
19/04/2017	169.306	177.695	65.625	27.45	2.229	70.804	7.26	41.434	26.185	17.257
20/04/2017	169.254	177.309	65.749	27.05	2.128	70.559	7.525	41.438	26.124	17.667
21/04/2017	169.156	178.491	66.374	27.01	2.1	70.771	7.437	41.182	26.223	18.27
24/04/2017	170.717	180.713	66.304	27.042	2.085	70.75	7.563	41.927	26.328	18.15
25/04/2017	174.628	184.444	67.799	28	2.077	71.375	7.39	41.213	26.1	19.229
26/04/2017	175.999	185.948	68.608	28	2.109	71.136	7.93	40.831	27	18.938
27/04/2017	175.292	185.136	68.508	28.032	2.096	71.285	7.8	41.371	26.681	19.056
28/04/2017	174.699	185.448	68.049	28	2.133	71.834	7.927	41.21	26.764	18.878
02/05/2017	175.244	183.929	68.442	28.446	2.174	70.93	7.681	40.9	26.489	19.25
03/05/2017	176.499	183.329	68.056	30.6	2.107	71.279	7.643	41.484	26.854	19.212
04/05/2017	176.698	182.1	67.75	30.86	2.11	71.297	7.742	41.26	26.809	18.603
05/05/2017	173.457	183	67.806	32.5	2.082	72.443	7.596	41.576	26.814	18.926
08/05/2017	174.298	184.73	68.55	35	2.053	73.582	8.355	41.369	26.969	19.538
09/05/2017	172.912	182.425	68.653	35.45	2.175	74.038	8.464	41.701	27.121	19.203
10/05/2017	173.058	183.75	68.567	36.95	2.146	73.462	8.507	41.87	26.517	19.25

11/05/2017	172.865	181.5	68.767	35.7	2.173	73.806	8.62	41.703	26.44	19.613
12/05/2017	173	179.169	68.717	33.8	2.154	73.839	8.992	41.603	26.943	19.32
15/05/2017	172.621	176.209	68.968	32	2.113	74.299	9.289	41.148	27.45	19.48
16/05/2017	173.525	176.964	69.168	34.99	2.094	74.221	9.6	41.131	27.096	19.033
17/05/2017	172.832	177.319	68.732	34.81	2.081	75.076	9.325	40.846	27.905	19.221
18/05/2017	169.412	173.173	67.849	35.93	2.093	74.722	9.12	40.058	26.95	18.705
19/05/2017	169.917	174	67.671	35.545	2.146	74.616	8.997	39.686	26.9	18.56
22/05/2017	169.882	172.287	67.889	35.516	2.139	75.28	9.043	39.71	26.557	18.406
23/05/2017	170.831	171.938	67.751	36.4	2.13	76.048	9.289	39.443	26.46	18.606
24/05/2017	171.171	171.154	66.8	39.5	2.121	75.645	9.462	39.751	27.018	18.472
25/05/2017	171.25	169.5	65.633	40.8	2.125	75.388	9.55	39.829	29.15	18.289
26/05/2017	171.154	171.124	65.526	41.03	2.121	75.167	9.966	39.981	29.133	18.141
29/05/2017	170.58	172.09	65.377	43.5	2.15	76.05	9.909	40.517	27.998	17.863
30/05/2017	171.727	173.001	65.437	44	2.162	75.484	9.83	40.656	28.145	17.914
31/05/2017	170.49	174.187	65.197	44.5	2.179	75.49	9.85	40.65	28.101	17.902
01/06/2017	171.587	170.734	64.706	41.12	2.104	75.856	10	40.47	27.87	17.493
02/06/2017	171.545	171.2	65.367	41.489	2.091	75.999	10.214	40.478	27.446	17.871
06/06/2017	173.568	171.657	65.951	43.305	2.102	76.426	10.343	410.8	27.3	17.98
07/06/2017	171.028	169.285	65.103	41.5	2.1	76.861	10.233	40.536	26.978	17.781
08/06/2017	170.586	171.049	65.123	40.26	2.07	76.97	10.804	40.479	26.938	17.77
09/06/2017	172.291	168.88	65.163	38.435	2.043	74.649	11.014	40.651	25.885	17.85
12/06/2017	173.384	170.078	65.313	39.59	2.06	74.612	11.28	40.282	26.191	17.934
13/06/2017	173.295	172.5	65.722	39.101	2.039	74.073	11.039	39.268	26.843	18.104
14/06/2017	173.521	175.75	65.898	39.45	2.102	74.672	11.96	40.297	26.013	18.034
15/06/2017	174.24	177.681	65.699	40.844	2.116	74.222	11.2	39.991	27.354	17.787
16/06/2017	173.096	173.469	65.103	40.3	2.106	75.125	10.924	39.888	26.628	17.65
19/06/2017	174.1	174.056	65.347	40.12	2	76.59	10.875	40.218	26.045	17.881
20/06/2017	176.064	174.909	65.692	41.65	1.97	76.556	11.152	40.6	26.35	18.214
21/06/2017	175.8	172.025	65.618	41.905	2.937	77.237	11.05	41.385	26.151	18.187
22/06/2017	174.899	170.92	65.352	42.665	1.946	75.832	11	41.35	26.216	17.834
23/06/2017	174.761	169.627	65.501	43.59	1.9	75.827	11.059	45.107	25.988	18.331
26/06/2017	173.73	168.887	65.143	42.995	1.907	75.612	11.125	46.155	26.092	18.216
27/06/2017	174.744	168.986	65.372	42.348	1.904	78.595	11.589	45.714	25.89	18.2
28/06/2017	173.896	168.557	64.75	42.795	1.884	78.812	11.919	45.3	25.944	17.845
29/06/2017	174.503	168.282	65.085	42.753	1.942	78.473	12.129	44.789	25.709	17.722
30/06/2017	172.88	164.568	64.134	40.369	1.96	76.387	12.231	43.95	26.305	17.43
03/07/2017	173.259	168.73	63.545	39.495	1.928	76.398	12.438	44.055	25.25	17.504
04/07/2017	174.98	168.88	64.169	38.39	1.923	76.522	12.85	43.8	25.9	17.661
05/07/2017	176.8	168.543	64.196	38.503	1.905	75.87	12.599	43.61	26.592	18.111

06/07/2017	176.704	176.862	63.581	38.33	2.002	75.414	12.145	43.84	26.26	18
07/07/2017	177.061	176.511	63.316	38.265	1.963	74.859	12.244	42.848	27.379	18.43
10/07/2017	178.332	176.163	63.575	39	1.936	74.5	12.626	43.198	27.37	18.179
11/07/2017	179.898	175.308	63.58	39.2	1.943	75.415	12.65	43.358	29.287	18.481
12/07/2017	179.95	175.05	64.234	39.168	1.048	74.729	12.795	43.547	32.327	18.762
13/07/2017	181.927	178.394	64.4	38.555	1.95	76.212	12.798	43.8	32.5	19.056
14/07/2017	183.849	180.503	64.696	39.875	1.919	76.301	13.161	44.164	32.25	18.906
17/07/2017	182.805	180.496	64.673	38.67	1.953	75.961	13.106	44.077	33.4	19.12
18/07/2017	181.919	182.256	64.584	38.398	1.98	75.83	13.682	43.962	32.109	19.14
19/07/2017	179.619	180.056	64.266	37.65	1.963	75.251	13.24	43.716	32.979	18.89
20/07/2017	180.466	179.244	64.236	38.275	2.023	75.924	13.253	44.144	34.614	18.844
21/07/2017	179.001	179.58	63.76	38.521	2.011	75.264	12.264	44.55	35	19.206
24/07/2017	176.768	177.043	62.423	38.371	1.944	74.871	11.554	43.612	34.152	18.481
25/07/2017	177.25	177.001	61.111	38.203	1.959	74.416	12.061	43.55	34.008	18.147
26/07/2017	179.458	176.74	61.484	37.799	1.97	74.2	11.979	43.809	33.974	18.25
27/07/2017	179.201	177.031	60.501	37.745	1.934	74.341	12.121	43.967	33.847	18.721
28/07/2017	181.722	183.51	60.149	38.469	1.97	72.3	12.398	43.2722	34	18.497
31/07/2017	180.2	192.876	59.682	38.05	1.94	71.941	11.569	42.892	34.28	18.254
01/08/2017	180.257	192.722	59.399	37.85	1.94	71.5	11.574	42.734	33.181	18.241
02/08/2017	182.427	191.9	60.039	37.678	1.936	72.104	11.403	42.27	33.945	18.337
03/08/2017	183.883	191.871	59.881	38.422	1.954	71.752	11.924	42	34.235	18.139
04/08/2017	183.328	193.198	59.909	38.19	2	71.94	11.861	42.105	33.91	17.976
07/08/2017	185.811	199.413	60.62	38.122	1.996	73.045	12.054	42.22	33.696	18.214
08/08/2017	185.846	199.26	60.45	38.78	1.979	72.437	12.461	42.3	33.816	18.11
09/08/2017	186.301	199.005	60.41	39.8	1.982	72.551	12.691	42.286	33.752	18.104
10/08/2017	184.249	193.45	60.109	39.83	1.989	72.663	12.4	41.773	33.305	18.029
11/08/2017	180.17	190.074	59.641	39.203	1.96	71.888	12.339	41.305	33.9	18.155
14/08/2017	179.85	191.923	59.874	37.774	1.95	71.353	12.025	40.855	33.75	17.8
15/08/2017	182.232	192.937	60.15	39.8	1.945	72.08	12.055	41.096	33.831	18.191
16/08/2017	183.296	194.469	60.34	39.2	1.937	71.592	12.761	41.697	34.167	18.179
17/08/2017	183.635	193.453	60.674	39.026	1.972	71.898	12.935	41.909	34.66	18.253
18/08/2017	182.518	191.462	60.41	40.31	1.964	71.792	12.73	41.8	34.863	18.248
21/08/2017	183.047	189.19	60.201	39.405	1.938	71.49	12.724	41.45	34.294	18.069
22/08/2017	180.948	187.502	60.2	39.752	1.916	71.643	12.859	41.33	36.294	17.969
23/08/2017	183.322	189.425	60.849	39.275	2	72.299	12.77	41.815	38.139	18.119
24/08/2017	181.762	188.605	60.704	39.125	2.942	71.95	12.681	41.689	37.742	18.175
25/08/2017	182.755	187.64	61.737	39.582	2.039	71.501	12.584	41.61	38.039	18.251
28/08/2017	182.609	187.9	62.29	39.685	2.014	70.874	12.521	41.513	37.88	18.2
29/08/2017	181.545	185.525	62.088	39.962	2.023	70.421	12.3	41	37	17.852

30/08/2017	179.038	182.805	61.542	39.4	1.961	70.766	12.364	41.144	36.669	17.792
31/08/2017	179.294	186.102	61.668	39.217	1.966	70.847	12.506	41.566	35.647	17.826
01/09/2017	180.099	188.757	61.257	42.525	1.98	71.137	12.9	42.222	35.953	17.711
04/09/2017	181.258	190.384	61.878	43.981	1.964	71.108	13.071	42.607	36.436	18.23
05/09/2017	181.02	189.608	61.968	41.817	2.03	70.5	12.981	42.467	36.015	18.236
06/09/2017	177.636	188.89	62.203	43.536	2.003	70.997	12.911	42.68	35.89	18.133
07/09/2017	180.304	192.096	64.97	43.99	2.02	70.721	12.834	42.804	35.85	18.547
08/09/2017	180.015	195.255	65	45.49	2.025	70.73	12.787	42.811	35.831	18.23
11/09/2017	179.809	194.883	64.917	43.662	2.07	70.843	13.396	43.03	36.152	17.857
12/09/2017	183.999	198.124	64.927	44.15	2.065	71.497	13.556	43.715	36.57	17.958
13/09/2017	183.8	198.007	65.201	44.5	2.077	70.992	13.846	44.033	36.443	18.249
14/09/2017	184.26	199.074	65.599	42.75	2.03	70.951	13.809	44.131	36.255	18.652
15/09/2017	184.617	197.787	65.757	41.7	2.036	70.634	13.61	44.256	36.158	18.769
18/09/2017	185.049	198.015	66.344	48.4	2.045	70.539	13.562	41.165	36.103	18.906
19/09/2017	185.126	196.881	66.494	47.305	2.064	70.759	13.474	40.308	36.562	18.984
20/09/2017	184.6	194.774	66.444	47.695	2.1	70.466	13.401	40	37.435	19.044
21/09/2017	184.275	193.29	66.509	48.66	2.074	70.227	13.401	40.112	37.916	19.216
22/09/2017	186.117	193.67	66.72	49.27	2.104	70	13.22	40.331	38.462	19.696
25/09/2017	186.999	192.001	66.9	48.015	2.071	69.639	13.081	39.9	39.78	19.561
26/09/2017	185.649	192.521	66.719	49.05	2.067	70.612	13.246	40.234	39.155	19.596
27/09/2017	185.717	187.681	66.829	48.49	2.117	72.354	13.134	40.672	37.5	19.79
28/09/2017	186.65	188.226	66.751	50.3	2.11	71.448	13.16	40.977	36.85	19.881
29/09/2017	187.466	188.58	66.651	52.75	2.129	70.884	13.339	40.843	36.8	19.922
02/10/2017	189.183	191.431	67.32	52.5	2.169	70.997	13.15	41	35.675	20.159
04/10/2017	191.451	195.951	67.494	51.49	2.115	71.999	13.534	41.592	35.7	20.45
05/10/2017	191.164	194.737	68.342	50.58	2.122	72.78	13.419	41.588	34.753	20.734
06/10/2017	192.903	193.913	68.572	50.601	2.086	72.673	13.455	41.45	35.63	20.846
09/10/2017	193.049	192.357	68.645	51.9	2.107	72.392	13.431	41.434	36.996	20.79
10/10/2017	193.734	194.858	68.518	61.75	2.16	72.513	13.301	41.254	36.864	20.593
11/10/2017	193.284	194.944	67.97	51.99	2.122	72.8	13.291	40.808	36.711	20.416
12/10/2017	193.831	193.856	68.191	52.8	2.075	72.517	13.211	40.771	37.968	20.301
13/10/2017	195.176	194.115	67.881	52.44	2.069	73.191	13.259	40.823	37.102	20.347
16/10/2017	195.114	193.191	67.809	53.5	2.099	73.66	13.15	40.867	36.641	20.301
17/10/2017	196.115	192.195	68.382	56.479	2.133	73.28	13.084	41.316	36.355	20.419
18/10/2017	195.794	190.95	68.817	58.53	1.132	73.074	13.284	41.684	3.589	20.39
19/10/2017	196.3	189.163	69.217	57.761	2.137	73.379	13.296	41.994	36.327	20.336
20/10/2017	195.824	186.755	68.88	57.1	2.078	72.94	12.789	41.647	35.346	20.123
23/10/2017	196.781	187.164	68.362	57.49	2.102	72.037	12.934	42.134	35.855	20.197
24/10/2017	196.412	187.555	68.658	60.06	2.107	71.703	12.951	42.158	35.565	20.121

25/10/2017	197.149	187.4	69.546	58.73	2.04	71.181	13.134	42.95	35.428	20.282
26/10/2017	196.857	186.95	69.253	58.97	2.064	69.79	13.566	42.582	35.87	20.055
27/10/2017	200.252	190.5	69.846	50.08	2.083	71.291	13.564	42.752	36.635	20.14
30/10/2017	199.532	191	70.829	59.23	2.096	71.51	13.446	43.7	36.342	20.211
01/11/2017	199	191.139	71.184	63.28	2.128	71.386	13.499	43.428	37.6	20.214
02/11/2017	203.009	187.695	72.929	62.833	2.05	72.029	14.036	43.579	36.551	20.451
03/11/2017	203.503	187.182	72.97	63.75	2.01	72.035	13.803	43.12	36.068	20.625
06/11/2017	202.765	187.377	73.141	72.782	2.008	73.065	12.844	43.235	34.985	20.731
07/11/2017	201.515	182.756	72.8	63.45	1.995	73.143	13.823	43.548	35.07	20.85
08/11/2017	200.56	183.418	71.751	60.799	2.008	71.953	12.15	43.703	34.677	20.561
09/11/2017	202.457	184.707	71.531	56.915	1.997	72.989	11.62	43.7	33.75	20.376
10/11/2017	199.385	175.999	70.452	55.97	1.984	72.398	11.2	42.194	32.6	19.672
13/11/2017	200.337	181.196	70.25	51.73	1.983	72.25	11.299	41.942	33.151	18.904
14/11/2017	198.711	184	70.367	56.8	1.977	72.804	11.216	42.237	31.163	18.848
15/11/2017	198.297	187.423	69.613	52.95	2.009	72.151	10.99	41.793	31.92	18.944
16/11/2017	196.101	186.172	69.26	50.973	2.095	71.7	11.028	41.449	35.105	18.764
17/11/2017	197.669	184.851	69.576	54.74	2.06	71.856	11.237	42.07	32.16	19.209
20/11/2017	197.43	185.626	69.056	54.03	2.061	72.368	11.289	42.599	32.615	18.766
21/11/2017	196.701	185.218	69.474	54.71	2.05	72.869	11.314	41.692	33.397	18.459
22/11/2017	199.441	187.549	70.9	55.1	2.144	73.204	11.45	41.374	32.938	18.646
23/12/2017	197.46	183.5	69.947	53.307	2.165	72.731	11.79	41.189	33.044	18.366
24/11/2017	196.283	181.682	70.087	54.22	2.162	73.06	11.35	41.2	33.05	18.299
27/11/2017	198.399	180.913	70.087	50.93	2.164	73.055	11.539	41.085	31.939	18.118
28/11/2017	196.751	179.054	70.027	51.95	2.16	73.014	11.46	41.082	31.998	17.97
29/11/2017	197.671	179.153	70.237	52.5	2.141	73.836	11.465	41.381	30.528	17.96
30/11/2017	199.129	177.604	69.9	51.88	2.234	72.82	11.535	40.835	32.105	17.495
01/12/2017	198.85	175.582	69.683	52.38	2.259	72.063	11.895	41.13	31.353	17.75
04/12/2017	196.8	175.779	68.875	52.35	2.195	72.3	12.193	41.481	31.56	17
05/12/2017	199.922	177.193	69.629	50.37	2.218	72.382	12.202	41.296	31.984	17.075
06/12/2017	198.4	177.391	69.933	52	2.309	72.568	12	40.91	31.438	17.217
07/12/2017	198.664	179.296	69.53	52.75	2.29	73.2	12.205	40.597	31.578	17.047
08/12/2017	199.359	180.699	69.937	54.5	2.303	73.177	12.521	41.205	32.85	17.122
11/12/2017	199	181.13	70.5	53.25	2.288	73.207	12.63	42.152	31.37	16.995
12/12/2017	198.273	175.741	70.173	53.709	2.334	72.74	12.63	42.703	31.16	17.055
13/12/2017	197.315	174	70.678	50	2.35	73.481	12.565	43.115	31.241	17.105
14/12/2017	196.2	168.31	70.899	45	2.341	73.485	12.74	42.723	30.889	17.17
15/12/2017	195.47	168.2	70.878	46.7	2.352	72.911	12.972	42.657	30.934	17.208
18/12/2017	197.352	172.371	70.922	45.785	2.326	72.205	13.03	40.703	30.619	17.015
19/12/2017	200.5	173.575	71.411	45.19	2.34	72.14	13.325	40.749	31.123	17.288
20/12/2017	199.5	172.965	71.082	46.739	2.313	71.671	13.44	40.681	31.276	17.102

21/12/2017	195.8	170.391	70.942	45.31	2.316	70.472	13.52	40.342	31.25	17.03
22/12/2017	196.109	169.846	71.368	46	2.344	71.252	13.53	40.216	30.953	17.23
27/12/2017	195.101	169.5	71.471	47.69	2.333	71.359	13.541	39.863	30.8	17.068
28/12/2017	194.639	169.894	71.299	45.65	2.32	71.455	13.512	39.63	32.014	17.162
29/12/2017	193.85	168.463	70.832	46.884	2.36	71.721	13.598	39.772	33.304	17.03
02/01/2018	192.179	167.35	70.726	47.045	2.343	71.611	13.585	39.5	33.43	16.955
03/01/2018	193.22	167.65	70.7	47.6	2.347	71.08	13.945	39.7	33.34	16.76
04/01/2018	193.38	168.7	71.25	47	2.358	71.1	14.32	39.8	33.08	17.175
05/01/2018	196.46	170.7	71.94	46.7	2.358	72.92	14.05	39.6	34.92	17.385
08/01/2018	198	172.15	72.86	45.9	2.363	71.14	14.2045	40.2	35.02	18.305
09/01/2018	198.98	173.1	73.71	46	2.362	71.58	14.185	40.8	35.32	18.06
10/01/2018	200.1	169.4	74.3	46.9	2.451	72	14.235	40.9	34.96	18.215
11/01/2018	200.2	167.9	74.02	45.6	2.435	71.2	14.115	40.8	34	17.715
12/01/2018	200.6	166.85	73.85	45.3	2.286	70.12	13.17	40.4	34.62	17.725
15/01/2018	202.05	168.55	74.27	45.4	2.307	70.14	13.345	40.7	35.72	17.93
16/01/2018	202.2	169.35	73.97	45.5	2.294	70.16	12.9	40.6	36.2	17.82
17/01/2018	201.25	168.05	74.1	42.3	2.39	71.04	13.005	40.6	34.92	17.11
18/01/2018	202.75	167.5	74.02	42.2	2.355	70.46	13.875	40.8	34.4	18.32
19/01/2018	202.85	171.3	74.49	44.8	2.396	70.42	12.99	41.1	34.42	18.28
22/01/2018	205.15	182.55	74.92	44.2	2.358	70.04	12.95	41.2	35.34	18.525
23/01/2018	205.35	183.65	75.35	45.7	2.356	70.24	12.93	41.3	34.6	19.17
24/01/2018	205.65	185.6	75.78	45.9	2.379	70.7	12.95	40.7	35.28	18.79
25/01/2018	204.55	184.55	75.41	44.9	2.41	70.36	12.56	41.8	34.84	18.695
26/01/2018	203.65	183.55	74.48	46	2.388	69.28	12.18	41.2	35.02	18.615
29/01/2018	204.1	184.85	74.77	46.8	2.42	69.72	12.185	42.3	34.08	18.355
30/01/2018	204.3	184.3	74.81	47.5	2.398	69.4	12.33	42.5	34.14	18.48
31/01/2018	203.95	184.85	74	46.6	2.374	69.64	12.31	41.36	33.3	18.185
01/02/2018	203.85	187.7	73.92	47.4	2.375	69.6	12.2	41.49	33.68	18.08
02/02/2018	201.45	184.4	72.45	46.5	2.374	69.74	12.425	41.3	33.34	18.59
05/02/2018	200	179.9	71.35	46	2.315	68.4	12.155	40.94	33.74	18.3
06/02/2018	197.16	177.9	70.33	44.7	2.252	66.96	11.91	40.61	33.28	18.3
07/02/2018	192.5	176.25	70.37	45.3	2.027	66.16	11.485	38.61	32.7	18.355
08/02/2018	195.5	183.3	70.8	34.2	2.29	67.54	11.635	40	32.58	18.595
09/02/2018	189.22	177.6	70.26	36.2	2.267	66.76	10.825	38.74	32.14	18.435
12/02/2018	182.92	174.8	69.51	36.2	2.327	66.18	10.67	37.96	31.16	17.265
13/02/2018	189.1	178.7	71.45	38.2	2.258	67.22	10.13	39.36	31.78	17.695
14/02/2018	187.4	176.25	70.8	37.8	2.258	66.56	10.175	39.13	31.68	17.62
15/02/2018	189.6	178.2	71.94	37.8	2.33	67.2	10.32	39.1	31.24	17.69
16/02/2018	190.54	177.7	71.9	38.5	2.284	65.9	10.7	40.14	30.86	17.86
19/02/2018	190.98	180.8	72.5	38.6	2.284	66	9.97	40.8	31.38	18.17

20/02/2018	191.04	180.2	70.66	38	2.301	64.88	10.025	41.79	32.76	18.185
21/02/2018	192.48	182.9	70.66	37.4	2.337	65	9.808	40.5	31.38	17.825
22/02/2018	192.46	180.8	70.42	37	2.331	64.9	10.135	40.77	31.2	17.735
23/02/2018	191.7	180.3	69.96	37	2.3	65.52	9.82	40.39	30.54	17.705
26/02/2018	191.7	181.9	73.33	36.6	2.33	66.1	9.902	40.48	31.92	18.03
27/02/2018	192.78	182	70.4	37.7	2.35	66.38	10.03	41.42	31.98	18.06
28/02/2018	192.56	181.35	70.35	40	2.34	66.02	10.085	41.56	32.84	18.075
01/03/2018	192.52	183.2	70.6	39.5	2.423	65.44	9.818	41.92	34.14	18.57
02/03/2018	187.8	175.5	68.92	39.3	2.361	64.06	9.856	41.37	33.18	19.32
05/03/2018	184.44	173.75	67.86	37.1	2.298	63.7	9.754	40.24	32.08	19.19
06/03/2018	189.02	176.2	67.74	38.3	2.263	64.98	9.7	41	31.9	19.245
07/03/2018	187.7	174.3	67.77	37.9	2.341	64.38	9.8	41.39	32.46	19.305
08/03/2018	188.46	174.1	67.88	38.8	2.369	63.5	10.06	41.42	31.64	19.075
09/03/2018	189.66	175.35	67.94	40	2.347	64.72	9.71	41.98	32	19.215
12/03/2018	189.92	174.1	67.86	41.2	2.343	65.5	9.396	42.79	31.52	19.395
13/03/2018	190.06	171.5	68.34	43.4	2.381	65.6	9.536	43.29	32.28	19.355
14/03/2018	185.36	171.65	67.62	40.3	2.374	65.2	9.51	42.7	31.3	19.29
15/03/2017	185.34	188.45	68.19	40.5	2.37	65.4	9.602	42.73	31.28	18.955

Annex 3: Covariance matrix

	Allianz	Adidas	Daimler	Apotheke	Tesco	Nestel	Air F	Oracle	H-L	PE
Allianz	0.000080	0.000060	0.000041	0.000007	0.000040	0.000024	0.000050	0.000507	0.000113	0.000047
Adidas	0.000060	0.000223	0.000042	0.000021	0.000050	0.000027	0.000005	0.000173	-0.000295	0.000024
Daimler	0.000041	0.000042	0.000093	0.000043	-0.000032	0.000025	0.000028	0.000394	0.000211	0.000057
Apotheke	0.000007	0.000021	0.000043	0.001880	-0.000302	-0.000001	-0.000075	0.001677	-0.000713	0.000048
Tesco	0.000040	0.000050	-0.000032	-0.000302	0.011699	0.000160	-0.000062	0.000102	0.033547	0.000021
Nestel	0.000024	0.000027	0.000025	-0.000001	0.000160	0.000090	0.000014	0.000224	0.000154	0.000016
Air F	0.000050	0.000005	0.000028	-0.000075	-0.000062	0.000014	0.000740	0.000516	-0.000035	0.000032
Oracle	0.000507	0.000173	0.000394	0.001677	0.000102	0.000224	0.000516	0.335807	-0.001005	0.000312
H-L	0.000113	-0.000295	0.000211	-0.000713	0.033547	0.000154	-0.000035	-0.001005	0.334323	-0.000071
PE	0.000047	0.000024	0.000057	0.000048	0.000021	0.000016	0.000032	0.000312	-0.000071	0.000254

Annex 4: Yield curves derived from the annual transition matrix

1st year 2018

From/To	AAA	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	72.38%	3.54%	1.72%	0.60%	0.81%	0.11%	0.07%	0.35%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA+	1.78%	23.27%	5.43%	2.23%	1.63%	1.10%	0.92%	0.10%	0.42%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.38%	62.98%	13.36%	7.45%	2.73%	1.22%	0.45%	0.07%	0.19%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA-	0.14%	7.12%	57.31%	18.59%	5.54%	2.54%	0.49%	0.24%	0.28%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
A+	0.01%	1.25%	8.35%	55.96%	16.41%	5.20%	1.22%	0.72%	0.18%	0.02%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
A	0.01%	0.38%	1.40%	7.13%	55.34%	17.08%	4.37%	2.03%	0.46%	0.14%	0.13%	0.11%	0.09%	0.01%	0.00%	0.01%	0.20%
A-	0.08%	0.28%	0.37%	1.86%	10.99%	59.37%	12.16%	3.01%	1.42%	0.33%	0.18%	0.18%	0.04%	0.02%	0.00%	0.02%	0.11%
BBB+	0.00%	0.03%	0.24%	0.58%	1.89%	11.31%	55.34%	12.73%	2.51%	0.78%	0.93%	0.37%	0.34%	0.16%	0.04%	0.16%	0.20%
BBB	0.00%	0.17%	0.04%	0.35%	1.04%	1.99%	11.53%	50.40%	11.15%	2.80%	1.76%	0.33%	0.29%	0.52%	0.17%	0.18%	0.27%
BBB-	0.00%	0.25%	0.03%	0.29%	1.02%	0.98%	2.82%	12.41%	46.76%	8.68%	4.06%	1.37%	0.74%	0.51%	0.08%	0.37%	0.71%
BB+	0.00%	0.02%	0.00%	0.03%	0.11%	0.53%	1.10%	2.26%	14.90%	36.28%	9.91%	3.58%	2.64%	0.70%	0.13%	0.40%	0.27%
BB	0.00%	0.00%	0.00%	0.01%	0.04%	0.41%	0.77%	0.32%	3.51%	13.14%	33.80%	7.78%	4.20%	1.58%	0.29%	0.75%	1.26%
BB-	0.00%	0.00%	0.00%	0.00%	0.03%	0.03%	0.10%	0.35%	0.54%	1.36%	10.71%	36.58%	15.36%	5.60%	1.40%	0.72%	2.95%
B+	0.00%	0.00%	0.00%	0.01%	0.30%	0.05%	0.32%	0.07%	0.13%	0.65%	4.10%	10.32%	32.13%	9.06%	4.85%	2.11%	4.19%
B	0.00%	0.00%	0.00%	0.00%	0.03%	0.00%	0.04%	0.09%	0.48%	0.50%	0.92%	2.68%	12.48%	22.61%	8.78%	4.70%	8.85%
B-	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.05%	0.75%	0.07%	0.04%	0.07%	0.21%	2.27%	6.67%	26.40%	12.12%	21.42%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.06%	0.01%	0.02%	0.05%	0.15%	1.45%	3.13%	7.20%	8.02%	48.62%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

10th year: 2027

From/To	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	17.63%	9.04%	11.12%	6.44%	5.11%	3.78%	2.54%	1.30%	0.81%	0.44%	0.12%	0.08%	0.04%	0.03%	0.02%	0.01%	0.01%	0.07%
AA+	1.75%	5.76%	15.60%	10.80%	9.89%	7.13%	5.31%	2.64%	1.36%	0.77%	0.23%	0.17%	0.09%	0.07%	0.04%	0.01%	0.02%	0.15%
AA	0.57%	1.43%	11.94%	11.17%	11.66%	8.85%	6.52%	2.94%	1.50%	0.74%	0.23%	0.16%	0.09%	0.07%	0.03%	0.01%	0.02%	0.16%
AA-	0.26%	0.61%	5.80%	10.61%	13.12%	11.34%	8.91%	4.11%	2.16%	1.00%	0.33%	0.24%	0.13%	0.10%	0.05%	0.02%	0.02%	0.25%
A+	0.11%	0.35%	2.61%	5.90%	10.61%	12.15%	10.96%	5.57%	2.99%	1.34%	0.46%	0.34%	0.19%	0.15%	0.08%	0.03%	0.04%	0.40%
A	0.09%	0.28%	1.25%	2.63%	5.76%	11.18%	13.30%	7.99%	4.48%	2.11%	0.78%	0.59%	0.33%	0.26%	0.14%	0.06%	0.06%	0.94%
A-	0.11%	0.16%	0.74%	1.49%	3.61%	8.63%	14.05%	9.99%	5.86%	2.91%	1.12%	0.84%	0.46%	0.37%	0.20%	0.09%	0.09%	0.99%
BBB+	0.04%	0.06%	0.37%	0.72%	1.79%	4.54%	8.95%	10.20%	7.42%	4.00%	1.66%	1.25%	0.67%	0.57%	0.32%	0.15%	0.14%	1.70%
BBB	0.02%	0.04%	0.28%	0.40%	0.98%	2.39%	4.64%	6.73%	7.14%	4.94%	2.22%	1.61%	0.88%	0.75%	0.41%	0.20%	0.17%	2.31%
BBB-	0.01%	0.03%	0.26%	0.30%	0.69%	1.58%	2.75%	4.09%	5.23%	5.08%	2.60%	1.94%	1.18%	1.02%	0.52%	0.27%	0.21%	3.54%
BB+	0.00%	0.01%	0.10%	0.11%	0.28%	0.73%	1.33%	2.07%	2.89%	3.59%	2.33%	1.94%	1.37%	1.25%	0.61%	0.34%	0.23%	3.66%
BB	0.00%	0.01%	0.05%	0.05%	0.14%	0.39%	0.73%	1.10%	1.48%	2.16%	1.76%	1.74%	1.45%	1.39%	0.68%	0.41%	0.25%	5.96%
BB-	0.00%	0.00%	0.02%	0.02%	0.06%	0.18%	0.30%	0.47%	0.64%	1.05%	1.15%	1.61%	1.86%	1.99%	1.03%	0.71%	0.39%	12.04%
B+	0.00%	0.00%	0.01%	0.02%	0.07%	0.18%	0.26%	0.34%	0.39%	0.59%	0.66%	1.01%	1.26%	1.47%	0.80%	0.62%	0.33%	15.34%
B	0.00%	0.00%	0.01%	0.01%	0.02%	0.08%	0.11%	0.18%	0.23%	0.32%	0.34%	0.53%	0.69%	0.89%	0.53%	0.46%	0.25%	22.09%
B-	0.00%	0.00%	0.00%	0.01%	0.02%	0.05%	0.08%	0.15%	0.19%	0.16%	0.13%	0.20%	0.27%	0.41%	0.29%	0.32%	0.17%	40.74%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.05%	0.05%	0.05%	0.08%	0.12%	0.17%	0.12%	0.12%	0.07%	56.98%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Annex 5: Forward yield curves from 2018 to 2027

	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
AAA	-0.12%	0.05%	0.29%	0.57%	0.79%	1.12%	1.26%	1.47%	1.45%	1.97%
AA+	-0.12%	0.05%	0.30%	0.57%	0.79%	1.12%	1.26%	1.47%	1.45%	1.97%
AA	-0.12%	0.05%	0.30%	0.57%	0.79%	1.12%	1.26%	1.47%	1.45%	1.98%
AA-	-0.12%	0.06%	0.30%	0.57%	0.79%	1.12%	1.27%	1.47%	1.45%	1.98%
A+	-0.11%	0.06%	0.30%	0.58%	0.80%	1.13%	1.27%	1.48%	1.46%	1.99%
A	-0.02%	0.12%	0.35%	0.62%	0.84%	1.17%	1.31%	1.51%	1.49%	2.02%
A-	-0.07%	0.10%	0.34%	0.61%	0.83%	1.16%	1.30%	1.51%	1.49%	2.02%
BBB+	-0.02%	0.14%	0.38%	0.65%	0.87%	1.20%	1.35%	1.55%	1.53%	2.06%
BBB	0.01%	0.17%	0.42%	0.69%	0.91%	1.24%	1.38%	1.59%	1.57%	2.09%
BBB-	0.23%	0.32%	0.54%	0.80%	1.00%	1.33%	1.46%	1.66%	1.64%	2.16%
BB+	0.01%	0.22%	0.48%	0.76%	0.99%	1.32%	1.46%	1.67%	1.64%	2.16%
BB	0.50%	0.55%	0.75%	1.00%	1.19%	1.51%	1.63%	1.82%	1.78%	2.29%
BB-	1.36%	1.19%	1.32%	1.53%	1.69%	1.98%	2.07%	2.23%	2.17%	2.65%
B+	2.01%	1.77%	1.84%	1.99%	2.10%	2.33%	2.38%	2.50%	2.40%	2.86%
B	4.62%	3.56%	3.24%	3.12%	3.04%	3.12%	3.05%	3.09%	2.92%	3.33%
B-	13.18%	9.59%	7.99%	7.00%	6.27%	5.87%	5.44%	5.18%	4.78%	5.00%
CCC	46.07%	24.21%	16.90%	13.26%	11.05%	9.74%	8.68%	7.98%	7.24%	7.21%

Annex 6: Random variables

Scenarios	Allianz	Adidas	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
1	0.694	0.214	-0.060	-0.509	1.310	0.132	0.379	0.320	-0.608	2.136
2	-0.737	0.973	-2.479	2.033	0.740	-0.651	0.692	-0.660	0.411	0.230
3	-0.531	-0.381	-1.096	-1.797	0.802	-1.818	0.571	-1.137	1.213	-1.823
4	0.207	0.467	0.123	0.820	1.999	0.663	0.724	-0.323	-1.199	-2.422
5	-1.818	-0.964	0.388	-0.038	0.985	0.240	-0.716	2.428	1.344	-0.219
6	-0.832	-0.734	0.214	0.919	1.172	1.507	2.112	2.459	0.281	1.767
7	-0.389	-0.782	1.935	0.714	-0.540	-0.585	-0.693	-1.270	-0.695	-0.645
8	0.360	-0.097	1.040	-0.283	-0.337	-2.055	0.490	-1.123	-2.410	0.309
9	1.794	-1.091	-0.611	0.344	0.173	-0.708	0.389	-1.211	-0.752	-1.151
10	-2.093	-1.007	-1.995	1.101	0.414	1.227	-1.224	1.548	0.972	1.662
11	1.037	1.434	-0.437	1.364	-0.879	-0.025	-1.264	0.067	0.117	1.132
12	-0.050	-0.827	2.181	-0.087	2.969	0.375	1.815	0.559	-0.283	-2.156
13	-0.147	1.366	0.636	0.169	0.258	-1.029	-0.337	-1.089	-2.053	1.666
14	-0.099	-0.238	0.011	1.095	1.650	-0.461	-0.187	0.242	0.623	0.316
15	0.870	1.184	0.541	0.961	1.673	0.571	0.461	0.362	1.256	-1.153
16	0.477	-1.198	-0.109	-1.150	0.596	0.192	-0.903	-1.812	-0.057	0.288
17	1.680	0.767	2.413	-1.242	0.075	-0.377	1.401	0.461	1.000	-1.222
18	-0.548	-0.097	-0.604	-2.349	-0.267	0.322	0.671	-0.023	0.692	0.072
19	1.211	-1.063	0.783	1.393	-2.472	0.857	-1.343	-1.285	-0.609	-0.669
20	-1.960	-0.117	0.665	-0.995	-0.114	-0.747	0.408	-0.121	-0.484	3.249
21	0.315	0.592	1.861	0.581	-1.041	1.187	1.357	0.649	-0.984	0.445
22	0.978	0.663	1.144	-0.775	0.236	1.749	0.620	-0.586	-1.272	0.210
23	0.760	-0.466	0.540	0.946	-1.069	-1.182	-1.876	-0.070	-1.947	2.962
24	1.113	-0.412	1.112	-0.253	-1.230	0.281	-1.648	1.341	-0.581	-0.351
25	1.226	-0.196	0.464	-0.631	-0.858	0.201	0.451	0.871	-0.913	-1.004
26	-0.899	-2.561	-2.165	0.106	1.118	1.529	0.804	1.740	-1.580	-1.430
27	-0.931	0.288	1.846	0.783	0.179	-0.277	-0.784	1.167	2.565	-0.574
28	-0.927	0.540	0.082	-0.891	-0.840	0.211	0.250	-0.718	-0.698	-1.539
29	0.860	-0.438	1.393	1.149	0.555	2.784	0.048	-2.247	-0.699	-0.624
30	-1.891	0.867	-1.570	0.518	0.414	-0.144	0.366	-0.752	1.630	0.832
31	-0.227	1.933	1.521	-2.124	0.249	0.497	-0.067	0.635	-0.718	2.096
32	-0.944	-0.382	1.813	1.864	-1.758	-0.733	2.404	0.122	-0.464	-0.476
33	1.778	1.320	0.247	1.774	-1.907	-0.164	0.378	-0.088	1.255	0.576
34	-1.176	-0.942	-1.133	0.625	0.120	0.191	0.005	1.568	0.602	-0.557
35	0.431	-0.281	2.471	-0.419	1.451	0.062	-0.148	0.381	-0.268	0.994
Etc	…	…	…	…	…	…	…	…	…	…

Annex 7: Correlated random variables

	Allianz	Adidas	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
1	1.642	0.069	0.374	-0.490	1.021	0.115	0.369	0.300	-0.601	1.935
2	-1.434	0.600	-2.036	1.901	0.839	-0.646	0.664	-0.660	0.334	0.209
3	-2.308	-0.576	-1.919	-2.007	1.124	-1.740	0.544	-1.119	1.094	-1.652
4	0.060	0.655	-0.475	0.552	1.381	0.759	0.697	-0.288	-0.898	-2.194
5	-1.912	-0.854	0.365	0.048	1.768	0.164	-0.657	2.402	1.134	-0.198
6	0.690	-0.863	1.024	0.878	1.518	1.376	2.087	2.421	0.158	1.601
7	-0.490	-0.391	1.466	0.720	-0.994	-0.499	-0.693	-1.249	-0.554	-0.585
8	0.247	-0.059	0.546	-0.225	-1.961	-1.769	0.452	-1.101	-2.030	0.280
9	0.387	-0.963	-0.968	0.226	-0.376	-0.592	0.359	-1.185	-0.579	-1.042
10	-2.648	-1.020	-0.985	1.253	1.194	1.048	-1.165	1.513	0.741	1.505
11	1.582	1.310	0.075	1.539	-0.757	-0.064	-1.227	0.055	0.049	1.025
12	0.557	-0.537	1.277	-0.436	2.769	0.436	1.772	0.578	-0.143	-1.953
13	0.822	1.282	0.706	0.263	-0.971	-0.875	-0.351	-1.083	-1.789	1.509
14	-0.141	-0.227	0.059	1.022	1.920	-0.477	-0.176	0.232	0.507	0.286
15	1.694	1.137	0.336	0.747	2.396	0.482	0.458	0.362	1.101	-1.045
16	-0.329	-0.969	-0.222	-1.224	0.619	0.151	-0.903	-1.801	-0.060	0.261
17	3.006	0.709	1.687	-1.375	0.494	-0.377	1.372	0.462	0.890	-1.107
18	-0.668	-0.284	-0.650	-2.393	0.136	0.253	0.653	-0.028	0.576	0.065
19	0.633	-0.593	0.854	1.514	-2.632	0.835	-1.324	-1.264	-0.480	-0.606
20	-0.804	-0.313	1.179	-0.857	-0.447	-0.744	0.390	-0.148	-0.546	2.943
21	2.241	0.653	2.061	0.607	-1.419	1.173	1.324	0.646	-0.842	0.403
22	2.427	0.799	1.182	-0.860	-0.203	1.704	0.590	-0.575	-1.073	0.190
23	0.981	-0.329	1.090	1.333	-2.184	-1.050	-1.832	-0.085	-1.757	2.684
24	1.147	-0.116	0.960	0.015	-1.444	0.320	-1.584	1.338	-0.471	-0.318
25	1.197	-0.116	0.171	-0.567	-1.344	0.287	0.448	0.880	-0.720	-0.910
26	-2.778	-2.312	-2.061	0.049	0.450	1.591	0.802	1.752	-1.259	-1.296
27	-0.155	0.363	1.608	0.798	1.538	-0.407	-0.736	1.146	2.171	-0.520
28	-1.174	0.582	-0.370	-0.941	-1.216	0.275	0.232	-0.692	-0.517	-1.394
29	1.729	0.082	1.491	0.891	0.580	2.628	0.013	-2.218	-0.558	-0.566
30	-1.951	0.510	-1.111	0.403	1.268	-0.274	0.350	-0.765	1.328	0.753
31	2.210	1.740	1.696	-1.990	-0.024	0.466	-0.059	0.615	-0.692	1.899
32	-0.163	-0.378	1.685	1.817	-2.195	-0.628	2.336	0.129	-0.368	-0.431
33	2.686	1.085	0.686	1.834	-1.249	-0.252	0.370	-0.102	1.025	0.522
34	-2.088	-0.948	-0.976	0.667	0.469	0.170	0.029	1.557	0.528	-0.505
35	1.880	-0.065	2.292	-0.423	1.332	0.054	-0.140	0.370	-0.268	0.901
Etc	…	…	…	…	…	…	…	…	…	…

Annex 8: Breakpoints

Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC
AAA	1.646	1.841	1.667	1.793													
AA+	-1.283	1.709	1.645	1.783	1.604	1.594											
AA	-1.844	-0.814	1.538	1.773	1.597	1.574	1.668										
AA-	-2.111	-1.577	-1.044	1.381	1.543	1.559	1.652	1.536									
A+	-2.378	-1.879	-1.528	-0.968	1.193	1.499	1.636	1.524	1.343	1.290							
A	-2.484	-2.016	-2.018	-1.707	-1.063	1.207	1.549	1.502	1.331	1.281				0.931			
A-		-2.206	-2.378	-2.130	-1.813	-1.030	1.113	1.438	1.297	1.245	1.040	0.906					
BBB+		-2.400	-2.716	-2.636	-2.362	-1.757	-1.248	1.039	1.247	1.220	1.027	0.896		0.922			
BBB				-2.807	-2.636	-2.130	-1.932	-1.198	0.896	1.147	1.002		1.170		0.751		
BBB-				-2.929	-3.195	-2.562	-2.260	-1.905	-1.200	0.791	0.966	0.878	1.158		0.751		
BB+						-2.697	-2.678	-2.137	-1.797	-1.211	0.572	0.815	1.146	0.913	0.739		
BB						-2.759	-2.863	-2.235	-2.062	-1.668	-1.134	0.468	1.134	0.904	0.726		
BB-						-2.834	-2.948	-2.457	-2.370	-2.040	-1.667	-1.213	0.781	0.799	0.714		0.740
B+						-2.929		-2.583	-2.414	-2.232	-1.979	-1.645	-0.868	0.524	0.666	0.951	0.740
B								-2.770	-2.462	-2.342	-2.512	-2.005	-1.520	-1.022	0.322	0.902	0.702
B-									-2.748				-2.010	-1.476	-0.955	0.685	0.595
CCC								-2.863	-2.878	-2.489		-2.241	-2.113	-1.858	-1.361	-0.645	0.338
								-3.195	-3.090	-2.727		-2.576	-2.175	-2.095	-1.734	-1.274	-0.307

Annex 9: Rating assignment

	Allianz	Adidas	Daimler	AP	Tesco	Nestel	Air F	Oracle	H-L	PE
Default	AA	AA-	A	AA-	BB+	AA-	BB-	AA-	BBB-	BB
Scenario										
1	AA+	AA-	A	AA-	BBB+	AA-	BB-	AA-	BBB-	A
2	AA-	AA-	BBB+	AAA	BBB-	AA-	BB-	AA-	BBB-	BBB+
3	A	AA-	BBB+	A	A	A	BB-	A+	BBB	B+
4	AA	AA-	A	AA-	A	AA-	BB-	AA-	BBB-	B-
5	A+	AA-	A	AA-	A	AA-	BB-	AAA	BBB	BB
6	AA	AA-	A	AA-	A	AA-	BBB+	AAA	BBB-	A
7	AA	AA-	A+	AA-	BB+	AA-	BB-	A+	BBB-	BB
8	AA	AA-	A	AA-	BB-	A	BB-	A+	BB	BBB+
9	AA	AA-	A	AA-	BB+	AA-	BB-	A+	BBB-	BB
10	A-	A+	A	AA-	A	AA-	B+	AA	BBB-	A
11	AA+	AA-	A	AA	BB+	AA-	B+	AA-	BBB-	A
12	AA	AA-	A+	AA-	A	AA-	BBB+	AA-	BBB-	B+
13	AA	AA-	A	AA-	BB+	AA-	BB-	A+	BB	A
14	AA	AA-	A	AA-	A	AA-	BB-	AA-	BBB-	BBB+
15	AAA	AA-	A	AA-	A	AA-	BB-	AA-	BBB	BB
16	AA	A+	A	A+	BBB-	AA-	B+	A	BBB-	BBB+
17	AAA	AA-	AAA	A+	BB+	AA-	BBB+	AA-	BBB	BB
18	AA	AA-	A	A-	BB+	AA-	BB-	AA-	BBB-	BBB+
19	AA	AA-	A	AA	B	AA-	B+	A+	BBB-	BB
20	AA	AA-	A	AA-	BB+	AA-	BB-	AA-	BBB-	A
21	AAA	AA-	AAA	AA-	BB	AA-	BBB+	AA-	BBB-	BBB+
22	AAA	AA-	A	AA-	BB+	AA	BB-	AA-	BBB-	BBB+
23	AA	AA-	A	AA-	B+	A+	B	AA-	BB	A
24	AA	AA-	A	AA-	BB	AA-	B	AA-	BBB-	BB
25	AA	AA-	A	AA-	BB	AA-	BB-	AA-	BBB-	BB
26	BBB+	A-	BBB+	AA-	BB+	AA	BB	AA	BB+	BB-
27	AA	AA-	AAA	AA-	A	AA-	BB-	AA-	AA-	BB
28	AA-	AA-	A	AA-	BB	AA-	BB-	AA-	BBB-	BB-
29	AAA	AA-	A+	AA-	BBB-	AAA	BB-	A-	BBB-	BB
30	A+	AA-	A-	AA-	A	AA-	BB-	AA-	AA-	BBB+
31	AAA	AA	AAA	A	BB+	AA-	BB-	AA-	BBB-	A
32	AA	AA-	AAA	AAA	B+	AA-	BBB+	AA-	BBB-	BB
33	AAA	AA-	A	AAA	BB	AA-	BB-	AA-	BBB	BBB+
34	A	AA-	A	AA-	BB+	AA-	BB-	AA	BBB-	BB
35	AAA	AA-	AAA	AA-	A	AA-	BB-	AA-	BBB-	A-
Etc	…	…	…	…	…	…	…	…	…	…

Annex 10: Values of bonds by rating and number of pieces.

	Allianz	Adidas	Daimler	Apotheke	Tesco	Nestel	Air F	Oracle	H-L	PE
	500	1000	5	200	500	1000	500	1000	1000	10
1	949,993	1,008,052	1,069,577	892,249	1,258,727	1,118,239	1,098,834	1,129,272	1,281,686	1,048,535
2	949,795	1,008,052	1,068,287	893,066	1,251,090	1,118,239	1,098,834	1,129,272	1,281,686	1,045,873
3	947,409	1,008,052	1,068,287	889,143	1,260,657	1,116,432	1,098,834	1,128,630	1,287,352	977,136
4	949,988	1,008,052	1,069,577	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,281,686	798,611
5	949,361	1,008,052	1,069,577	892,249	1,260,657	1,118,239	1,098,834	1,129,914	1,287,352	1,026,577
6	949,988	1,008,052	1,069,577	892,249	1,260,657	1,118,239	1,141,929	1,129,914	1,281,686	1,048,535
7	949,988	1,008,052	1,071,334	892,249	1,252,395	1,118,239	1,098,834	1,128,630	1,281,686	1,026,577
8	949,988	1,008,052	1,069,577	892,249	1,212,478	1,116,432	1,098,834	1,128,630	1,270,898	1,045,873
9	949,988	1,008,052	1,069,577	892,249	1,252,395	1,118,239	1,098,834	1,128,630	1,281,686	1,026,577
10	947,676	1,007,373	1,069,577	892,249	1,260,657	1,118,239	1,078,345	1,129,597	1,281,686	1,048,535
11	949,993	1,008,052	1,069,577	892,645	1,252,395	1,118,239	1,078,345	1,129,272	1,281,686	1,048,535
12	949,988	1,008,052	1,071,334	892,249	1,260,657	1,118,239	1,141,929	1,129,272	1,281,686	977,136
13	949,988	1,008,052	1,069,577	892,249	1,252,395	1,118,239	1,098,834	1,128,630	1,270,898	1,048,535
14	949,988	1,008,052	1,069,577	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,281,686	1,045,873
15	950,161	1,008,052	1,069,577	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,287,352	1,026,577
16	949,988	1,007,373	1,069,577	891,559	1,251,090	1,118,239	1,078,345	1,126,057	1,281,686	1,045,873
17	950,161	1,008,052	1,071,820	891,559	1,252,395	1,118,239	1,141,929	1,129,272	1,287,352	1,026,577
18	949,988	1,008,052	1,069,577	888,938	1,252,395	1,118,239	1,098,834	1,129,272	1,281,686	1,045,873
19	949,988	1,008,052	1,069,577	892,645	1,139,357	1,118,239	1,078,345	1,128,630	1,281,686	1,026,577
20	949,988	1,008,052	1,069,577	892,249	1,252,395	1,118,239	1,098,834	1,129,272	1,281,686	1,048,535
21	950,161	1,008,052	1,071,820	892,249	1,240,488	1,118,239	1,141,929	1,129,272	1,281,686	1,045,873
22	950,161	1,008,052	1,069,577	892,249	1,252,395	1,118,306	1,098,834	1,129,272	1,281,686	1,045,873
23	949,988	1,008,052	1,069,577	892,249	1,190,399	1,118,026	1,031,472	1,129,272	1,270,898	1,048,535
24	949,988	1,008,052	1,069,577	892,249	1,240,488	1,118,239	1,031,472	1,129,272	1,281,686	1,026,577
25	949,988	1,008,052	1,069,577	892,249	1,240,488	1,118,239	1,098,834	1,129,272	1,281,686	1,026,577
26	945,301	1,004,781	1,068,287	892,249	1,252,395	1,118,306	1,124,933	1,129,597	1,283,042	997,068
27	949,988	1,008,052	1,071,820	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,294,115	1,026,577
28	949,795	1,008,052	1,069,577	892,249	1,240,488	1,118,239	1,098,834	1,129,272	1,281,686	997,068
29	950,161	1,008,052	1,071,334	892,249	1,251,090	1,118,359	1,098,834	1,126,156	1,281,686	1,026,577
30	949,361	1,008,052	1,070,090	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,294,115	1,045,873
31	950,161	1,008,419	1,071,820	889,143	1,252,395	1,118,239	1,098,834	1,129,272	1,281,686	1,048,535
32	949,988	1,008,052	1,071,820	893,066	1,190,399	1,118,239	1,141,929	1,129,272	1,281,686	1,026,577
33	950,161	1,008,052	1,069,577	893,066	1,240,488	1,118,239	1,098,834	1,129,272	1,287,352	1,045,873
34	947,409	1,008,052	1,069,577	892,249	1,252,395	1,118,239	1,098,834	1,129,597	1,281,686	1,026,577
35	950,161	1,008,052	1,071,820	892,249	1,260,657	1,118,239	1,098,834	1,129,272	1,281,686	1,048,728
Et	…	…	…	…	…	…	…	…	…	…

Annex 11: Probability distribution of the portfolio value

Scenario	Values	Frequency	Cumulative frequency	R1	R2
1	-1,560,150	1	1	0.00%	0.00%
2	-1,501,311	0	1	0.00%	0.00%
3	-1,442,472	0	1	0.00%	0.00%
4	-1,383,632	0	1	0.00%	0.00%
5	-1,324,793	0	1	0.00%	0.00%
6	-1,265,954	0	1	0.00%	0.00%
7	-1,207,115	1	2	0.00%	0.01%
8	-1,148,275	2	4	0.01%	0.02%
9	-1,089,436	4	8	0.02%	0.03%
10	-1,030,597	8	16	0.03%	0.06%
11	-971,757	5	21	0.02%	0.08%
12	-912,918	5	26	0.02%	0.10%
13	-854,079	60	86	0.24%	0.34%
14	-795,239	286	372	1.14%	1.49%
15	-736,400	35	407	0.14%	1.63%
16	-677,561	3	410	0.01%	1.64%
17	-618,721	0	410	0.00%	1.64%
18	-559,882	5	415	0.02%	1.66%
19	-501,043	3	418	0.01%	1.67%
20	-442,203	14	432	0.06%	1.73%
21	-383,364	69	501	0.28%	2.00%
22	-324,525	97	598	0.39%	2.39%
23	-265,686	31	629	0.12%	2.52%
24	-206,846	160	789	0.64%	3.16%
25	-148,007	121	910	0.48%	3.64%
26	-89,168	388	1298	1.55%	5.19%
27	-30,328	2585	3883	10.34%	15.53%
28	28,511	16228	20111	64.91%	80.44%
29	87,350	4889	25000	19.56%	100.00%
30	146,190	0	25000	0.00%	100.00%