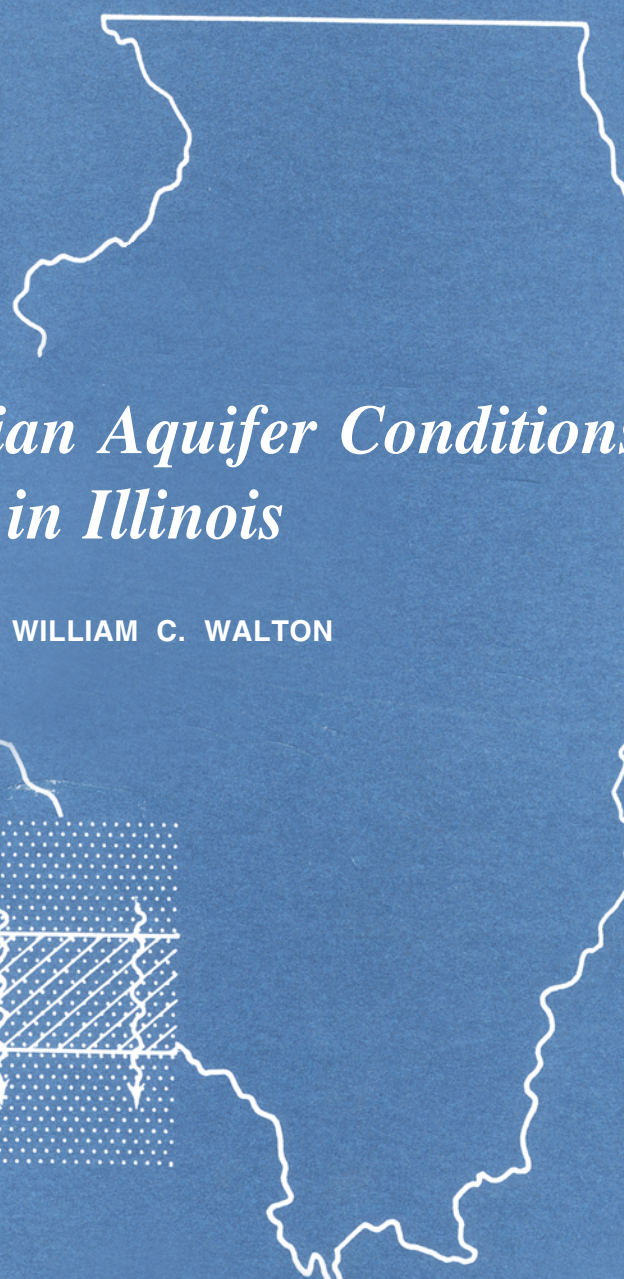


REPORT OF INVESTIGATION 39

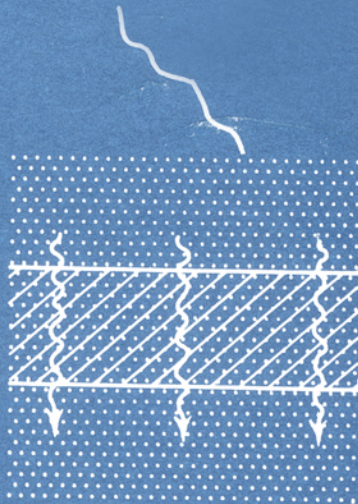
STATE OF ILLINOIS  
WILLIAM G. STRATTON, Governor

DEPARTMENT OF REGISTRATION AND EDUCATION  
VERA M. BINKS, Director



*Leaky Artesian Aquifer Conditions  
in Illinois*

by WILLIAM C. WALTON



ILLINOIS STATE WATER SURVEY  
WILLIAM C. ACKERMANN, Chief

URBANA  
1960



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# LEAKY ARTESIAN AQUIFER CONDITIONS IN ILLINOIS

By

WILLIAM C. WALTON

Illinois State Water Survey

## Abstract

Leaky artesian conditions exist in many parts of Illinois where aquifers are overlain by deposits or confining beds which impede or retard the vertical movement of ground water. Under leaky artesian conditions, the cone of depression developed by a pumping well is influenced by the vertical permeability of the confining bed in addition to the hydraulic properties and geohydrologic boundaries of the aquifer.

The vertical permeability of a confining bed often can be determined from the results of pumping tests by using the nonsteady-state leaky artesian aquifer equation derived by Hantush and Jacob (1955). A time-drawdown type curve method for analyzing pumping test data under nonsteady-state conditions is described in detail. A distance-drawdown type curve method for analyzing pumping test data under steady-state conditions devised by Jacob (1946) is also described. These two methods are applied to available pumping test data for Illinois. The results of a test made near the village of Dieterich in Effingham County are presented to illustrate the analysis of data. A summary of the leaky artesian test data collected to date indicates that the vertical permeability of glacial drift deposits in the southern half of Illinois ranges between 0.08 and 1.6 gallons per day (gpd) per square foot.

Effects of leakage closely resemble the effects of a recharge boundary if the effects of partial penetration are excluded. The data for the Dieterich pumping test are used to show that recognition of leaky artesian conditions is critically important in predicting the water supply potential of wells and aquifers.

A form of Darcy's law is applied to data on the piezometric surface of the Cambrian-Ordovician Aquifer to determine the order of magnitude of the vertical permeability of the Maquoketa Formation. The Maquoketa Formation has a maximum thickness of about 250 feet, consists largely of beds of dolomitic shale, and confines water in the Cambrian-Ordovician Aquifer under artesian pressure. The Cambrian-Ordovician Aquifer is encountered at an average depth of 500 feet below the surface at Chicago, has an average thickness of 1000 feet, consists mainly of beds of sandstone and dolomite, and is the most highly developed source of large ground-water supplies in northeastern Illinois. Computations indicate that the average vertical permeability of the Maquoketa Formation in northeastern Illinois is about 0.00005 gpd per square foot. Leakage in 1958 through the Maquoketa Formation in northeastern Illinois is estimated to be about 8,400,000 gpd or about 11 per cent of the water pumped from deep wells.

## Introduction

In Illinois, ground water is obtained for municipal, institutional, commercial, and industrial supplies largely from (1) thick and extensive Iron-ton-Galesville and Mt. Simon Sandstones of Cambrian age and the Glenwood-St. Peter Sandstone of Ordovician age; (2) sand and gravel deposits of Pleistocene age which in comparison to the sandstones mentioned above are thin and limited in areal extent; and (3) thick and extensive dolomites of Silurian age and the Galena-Platteville Dolomites of Ordovician age. Minor amounts of ground water are derived from sandstones and limestones of Mississippian, Devonian, and Pennsylvanian age. In all of these aquifers, ground water occurs under leaky artesian conditions at many places.

Leaky artesian conditions exist where an artesian aquifer is overlain or/and underlain by deposits (confining bed) which impede or retard the vertical movement of ground water. If a well is drilled through the confining bed and into the aquifer, the water in the well will rise above the top of the aquifer. Water may or may not flow over the top of the well. The surface to which water will rise under leaky artesian conditions, as defined by

water levels in a number of wells, is the piezometric surface. When the pressure head, and hence, the piezometric surface is lowered by the pumping or free flow of wells, the aquifer is not dewatered but is still completely full. The water discharged from the well is derived by the compaction of the aquifer and associated beds, by the expansion of the water itself, and by vertical leakage through the confining bed into the aquifer.

Artesian conditions differ from leaky artesian conditions in that under artesian conditions the confining bed overlying and/or underlying an aquifer is assumed to prevent the movement of water and the confining bed is referred to as an *aquiclude*. In the majority of cases, geologic deposits are capable only of impeding the movement of ground water rather than preventing it.

Chamberlin (1885, pp. 131-173) made the following pertinent statements concerning confining beds.

"No stratum is entirely impervious. It is scarcely too strong to assert that no rock is absolutely impenetrable to water . . . . But in the study of artesian wells we are not dealing with absolutes but with avail-

ables. A stratum that successfully restrains the (sic) most of the water, and thus aids in yielding a flow, is serviceably impervious. It may be penetrated by considerable quantities of water, so that the leakage is quite appreciable and yet be an available confining stratum. The nearest approach to an entirely impervious bed is furnished by a thick layer of fine, unhardened [sic] clay. In this case solidifying permits the formation of fissures and the clay rocks are less impervious than the original clay beds. The clayey shales rank next as confining strata, after which follow in uncertain order shaly (sic) limestones, shaly sandstones, the various crystalline rocks, and even compact sandstones."

Hall, Meinzer, and Fuller (1911, p. 52) described leaky artesian conditions in Minnesota. Pertinent remarks made by these writers are:

"In localities where the water from the deeper beds rises to a level below that of the surficial ground-water table, the two bodies of water are not in equilibrium, and if the material separating them is at any point not entirely impervious water will pass from the surficial layer into the deeper beds. This relation is the general one throughout southern Minnesota . . . . Confining layers of till are not sufficiently impenetrable to prevent the escape of waters upward from the confined beds when the pressure is outward. Neither can they prevent the passage of water downward into these beds in locali-

ties where the balance of pressure favors movement in this direction."

Most if not all of the so called artesian aquifers in Illinois are actually leaky artesian aquifers. If the permeability of the confining bed is very low, vertical leakage may be difficult to measure within the average period (8 to 24 hours) of pumping tests. However, since the cone of depression created by pumping a well tapping a leaky artesian aquifer continues to expand until discharge is balanced by the amount of induced leakage, it does not follow that vertical leakage is of small importance over extended periods of time. As the cone of depression grows in extent and depth, the area of leakage and the vertical hydraulic gradient become large. Accordingly then, with long periods of pumping, contribution by leakage through a confining bed may be appreciable even though the vertical permeability is very low. If a source is available to replenish continuously the confining bed, the cone of depression developed by a well pumping for long extended periods will be influenced by the vertical permeability of the confining bed in addition to the hydraulic properties and geohydrologic boundaries of the main aquifer. Any long-range forecast of well or aquifer yield must include the important effects of leakage through the confining bed. The vertical permeability of a confining bed often can be determined from the results of pumping tests as described in the following section.

## Analysis of Pumping Test Data for Leaky Artesian Aquifers

### Theory

Hantush and Jacob (1955, pp. 95-100) derived an equation describing the nonsteady-state drawdown distribution in an infinite leaky artesian aquifer which can be expressed by the relation given below:

The formula was developed on the basis of the follow-

$$s = (Q/4\pi T) \int_u^{\infty} \left(\frac{1}{y}\right) \exp(-y - r^2/4B^2y) dy \quad (1)$$

or, evaluating the integral,

$$s = (Q/4\pi T) \left[ 2K_0(r/B) - I_0(r/B) \right] \left[ -Ei\left(-\frac{r^2}{4B^2u}\right) \right] \\ + \left[ \exp\left(-\frac{r^2}{4B^2u}\right) \right] \left\{ 0.5772 + \ln u + \left[ -Ei(-u) \right] - u \right. \\ \left. + u \left[ I_0\left(\frac{r}{B}\right) - 1 \right] / \left(\frac{r^2}{4B^2}\right) - u^2 \sum_{n=1}^{\infty} \sum_{m=1}^n \frac{(-1)^{n+m} (n-m+1)!}{(n+2)!^2} \left(\frac{r^2}{4B^2}\right)^m u^{n-m} \right\}$$

$$\text{where } u = \frac{r^2 s}{4 T t} \quad \text{and } B = \sqrt{T/(P'/m')}$$

$K_0\left(\frac{r}{B}\right)$  = modified Bessel function of the second kind and zero order,  $I_0\left(\frac{r}{B}\right)$  = modified Bessel function of

the first kind and zero order,  $s$  = drawdown,  $Q$  = discharge,  $T$  = coefficient of transmissibility,  $S$  = coefficient of storage,  $r$  = distance from pumped well,  $t$  = time after pumping started,  $P'$  = vertical permeability of confining bed,  $m'$  = thickness of confining bed through which leakage occurs.

ing assumptions : That the aquifer is infinite in areal extent and is of the same thickness throughout; that it is homogeneous and isotropic; that it is confined between an impermeable bed and a bed through which leakage can occur; that the coefficient of storage is constant; that water is released from storage instantaneously with a decline in head; that the well has an infinitesimal diameter and

penetrates the entire thickness of the formation; that leakage through the confining bed into the aquifer is vertical and proportional to the drawdown; and that the hydraulic head in the deposits supplying leakage remains more or less uniform.

The integral

$$\int_u^{\infty} \left(\frac{1}{y}\right) \exp(-y - r^2/4B^2y) dy$$

was written by Hantush (1956, p. 702) symbolically as  $W(u, r/B)$  and was termed the "well function for leaky artesian aquifers." Equation 1 can be rewritten in abbreviated form, in the gallon-day-foot system of units as

$$s = \frac{114.6 Q}{T} W(u, r/B) \quad (2)$$

$$\text{where } u = \frac{2693 r^2 S}{Tt} \quad (3) \quad \text{and } \frac{r}{B} = \frac{r}{\sqrt{T/(P'/m')}} \quad (4)$$

$s$  = drawdown in observation well, in feet

$r$  = distance from pumped well to observation well, in feet

$Q$  = discharge, in gallons per minute

$t$  = time after pumping started, in minutes

$T$  = coefficient of transmissibility, in gallons per day per foot

$S$  = coefficient of storage of aquifer

$P'$  = vertical permeability of confining bed, in gallons per day per square foot

$m'$  = thickness of confining bed through which leakage occurs, in feet

### Nonsteady-State Time-Drawdown Type Curve Method

Hantush (1956, pp. 707-711) gave values of  $W(u, r/B)$  in terms of the practical range of  $u$  and  $r/B$ . Values of  $W(u, r/B)$  were plotted against values of  $\frac{1}{u}$  on logarithmic paper and a family of leaky artesian type curves was constructed as shown in figure 1.

Equations 2 and 3 can be rewritten as:

$$\log W(u, r/B) = \log \left[ \frac{T}{114.6 Q} \right] + \log s \quad (5)$$

$$\text{and } \log \frac{1}{u} = \log \left[ \frac{T}{2693 r^2 S} \right] + \log t \quad (6)$$

The terms within the brackets are constant for a given aquifer, observation well, and pumping rate. Equations 5 and 6 are similar and  $s$  is related to  $t$  in the same manner that  $W(u, r/B)$  is related to  $\frac{1}{u}$ . Values of  $s$  plotted on logarithmic paper against values of  $t$  describe a time-drawdown field data curve that is analogous to one of the family of leaky artesian type curves shown in figure 1.

The time-drawdown field data curve, plotted on logarithmic paper of the same scale as the type curves, is superposed on the family of leaky artesian type curves, keeping the  $W(u, r/B)$  axis parallel with the  $s$  axis and

the  $\frac{1}{u}$  axis parallel with the  $t$  axis. In the matched position the vertical scales of the graphs are displaced with respect to one another by the amount  $\left[ \log \frac{T}{114.6 Q} \right]$  and the horizontal scales are displaced by the amount  $\left[ \log \frac{T}{2693 r^2 S} \right]$ . A point at the intersection of the major axis of the leaky artesian type curve is selected and marked on the time-drawdown field data curve (the point also may be selected anywhere on the type curve). The coordinates of this common point (match point),  $s$ ,  $\frac{1}{u}$ ,  $W(u, r/B)$  and  $t$  are substituted in equations 2 and 3 to determine the coefficients or transmissibility and storage.  $T$  is calculated using equation 2 with the  $W(u, r/B)$  and  $s$  coordinates.  $S$  is determined using equation 3, the calculated value of  $T$ , and the  $\frac{1}{u}$  and  $t$  coordinates of the match point. The value of  $P'$  is determined by noting the value of  $r/B$  used to construct the particular leaky artesian type curve found to be analogous to the time-drawdown field data curve. The value of  $r/B$  of the selected type curve is substituted in equation 4 and  $P'$  is computed. The method described above is a modification of the type curve graphical method devised by Theis and described by Jacob (1940, p. 582).

If leakage through the confining bed into the aquifer is very small  $B \rightarrow \infty$  and equation 1 becomes

$$s = \frac{114.6 Q}{T} \int_u^{\infty} \frac{e^{-u}}{u} du \quad (7)$$

which is the nonequilibrium formula introduced by Theis (1935, pp. 519-524.). Thus, if leakage is not measurable during the pumping test, the time-drawdown field data curve will in the matching position coincide with the nonequilibrium (nonleaky) type curve described by the Theis nonequilibrium formula, which is shown in figure 1 as the outside curve of the family of leaky artesian type curves. However, if leakage is appreciable during the pumping test, the time-drawdown field data curve will follow the nonequilibrium type curve until the effects of leakage are felt at the observation well. At that time, the time-drawdown field data curve will deviate from the nonequilibrium type curve and follow one of the leaky artesian type curves depending upon the value of  $r/B$ .

### Steady-State Distance-Drawdown Type Curve Method

Hantush (1956, p. 702) pointed out that the steady-state distribution of drawdown caused by pumping a well at a constant rate from a leaky artesian aquifer is obtained from equation 1 by letting  $t$  approach infinity. The first detailed solution of the steady-state leaky artesian problem was developed by DeGlee (1930) and later verified by Jacob (1946, pp. 198-205). The steady-state equation in the gallon-day-foot system of units is as follows:

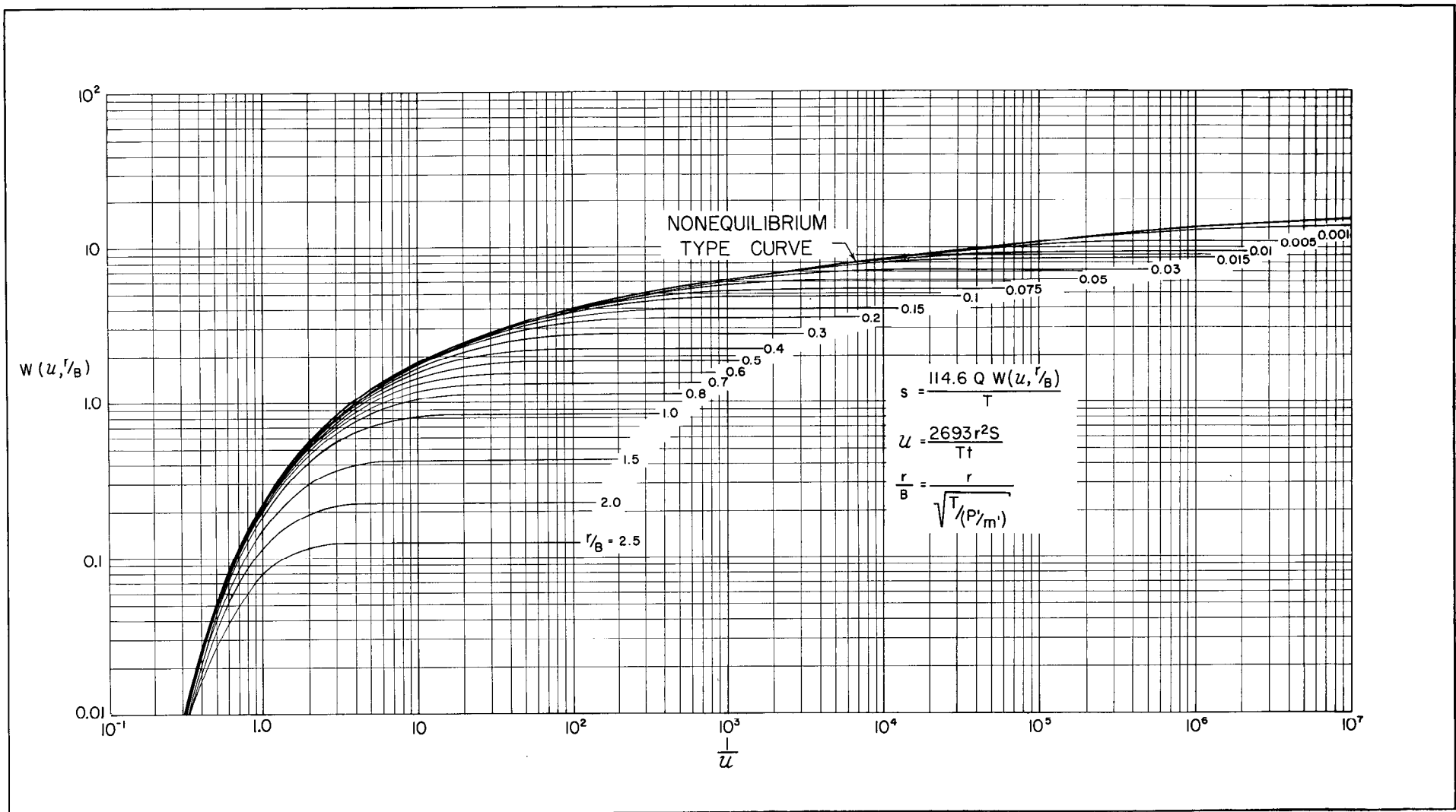


FIGURE 1 NONSTEADY-STATE LEAKY ARTESIAN TYPE CURVES



$$s = \frac{229 Q K_o(r/B)}{T} \quad (8)$$

$$\frac{r}{B} = \frac{r}{\sqrt{T/(P'm')}}}$$

where  $s$  = drawdown in observation well, in feet;  $r$  = distance from pumped well to observation well, in feet;  $Q$  = discharge, in gallons per minute;  $T$  = coefficient of transmissibility, in gallons per day per foot;  $P'$  = coefficient of vertical permeability of confining bed, in gallons per day per square foot;  $m'$  = thickness of confining bed through which leakage occurs, in feet;  $K_o(r/B)$  = modified Bessel function of the second kind of zero order.

Jacob (1946, p. 204) devised the following graphical method for determining values of the parameters  $T$  and  $P'$  under steady-state conditions. A steady-state leaky artesian type curve is prepared by plotting values of  $K_o(r/B)$  against values of  $r/B$  on logarithmic paper as shown in figure 2. Pumping test data collected under steady-state conditions are plotted on logarithmic paper with  $r$  as the abscissa and  $s$  as the ordinate to describe a distance-drawdown field data curve. A match of the two curves is obtained by superposing the steady-state leaky artesian type curve over the distance-drawdown field data curve, keeping the axes of the two graphs parallel. Match point coordinates,  $K_o(r/B)$ ,  $r/B$ ,  $s$  and  $r$  are substituted into equations 8 and 4 to determine  $T$  and  $P'$ . This procedure cannot be used unless the time is sufficiently long to give reasonable assurance of a steady-state flow, in which case the drawdowns used in making computations must be measured at the same time. The coefficient of storage cannot be computed by use of the steady-state leaky artesian type curve because under such conditions of flow, the entire yield of the well is derived from leakage sources only.

#### Data for Pumping Test Near Dieterich, Illinois

Several controlled pumping tests, involving one or more observation wells, were made under leaky artesian conditions in glacial drift aquifers in the southern part of Illinois. The results of a test, made near the village of Dieterich in Effingham County, are presented in detail to illustrate the analysis of data. The results of other tests are summarized later in this report.

A pumping test was made by G. E. Neher of the State Water Survey in cooperation with Marbry & Johnson, Inc., Consulting Engineers, and E. C. Baker & Sons, Well Contractor, on July 2 and 3, 1951. A group of wells (figure 3) located about one mile southwest of the corporate limits of the village of Dieterich in sec. 22, T. 7 N., R. 7 E. was used. The generalized graphic logs of the wells are given in figure 4. The effects of pumping well 18 were measured in observation wells 15, 16, and 19. Pumping was started at 2:10 P.M. on July 2 and was continued for a period of about 20 hours at a constant rate of 25 gpm until 10:00 A.M. on July 3.

Drawdowns in the pumped well and observation wells were determined by comparing the extrapolated graphs

of water levels measured before pumping started with the graphs of water levels measured during pumping. Drawdowns were plotted against time on logarithmic paper. The time-drawdown field data graph for observation well 19 is given, as an example, in figure 5.

The time-drawdown field data graph was superposed on the family of leaky artesian type curves. The time-drawdown field data curve closely follows but falls slightly below the  $r/B = 0.2$  type curve. By interpolation, a  $r/B = 0.22$  type curve was selected as analogous to the time-drawdown field data curve. Match point coordinates and a  $r/B$  value of 0.22 were substituted into equations 2, 3, and 4 to compute coefficients of transmissibility and storage of the aquifer, and the vertical permeability of the confining bed. Computations for well 19 are given in figure 5.

Drawdowns in observation wells 15, 16, and 19 at the end of the test when steady-state conditions prevailed were plotted on logarithmic paper against the distances, from the respective observation wells to the pumped well, to describe a distance-drawdown field data curve (a portion of a profile of the cone of depression). The steady-state type curve was matched to the distance-drawdown field data curve and match point coordinates were substituted in equations 8 and 4 for computation of coefficients of transmissibility and vertical permeability as shown in figure 6.

The average values of  $T$ ,  $S$ , and  $P'$  computed by using time-drawdown and distance-drawdown data are 1500 gpd per foot, 0.0002, and 0.10 gpd per square foot respectively. As indicated by the distance-drawdown curve shown in figure 6, the 20-hour test sampled an area of the sand and gravel aquifer having a radius of roughly 2000 feet. The coefficients computed from the results of the test represent the average hydraulic properties of the aquifer and confining bed within that cone of depression.

#### *Application of Results of Pumping Test Near Dieterich, Illinois*

The test at Dieterich was chosen as an example for demonstration of analysis of data under leaky artesian conditions partly because, as shown in figure 3, the wells are near a possible source of recharge (Dieterich Creek). Two interpretations of the test data are therefore possible if the effects of partial penetration are excluded. The decrease in the time-rate of drawdown can be attributed either to the effects of leakage through the confining bed or to the effects of induced infiltration of surface water (recharge boundary). Serious errors in long-term predictions of the water-supply potential of the wells near Dieterich may be made by misinterpretation of the pumping test data.

If Dieterich Creek is assumed to be a recharge boundary, then the water pumped will be continuously replenished during dry periods by the induced infiltration of surface water. The cone of depression would be greatly limited in its growth. Suppose that three 6-inch diameter wells, 25 feet deep, with 5 feet of screen and spaced

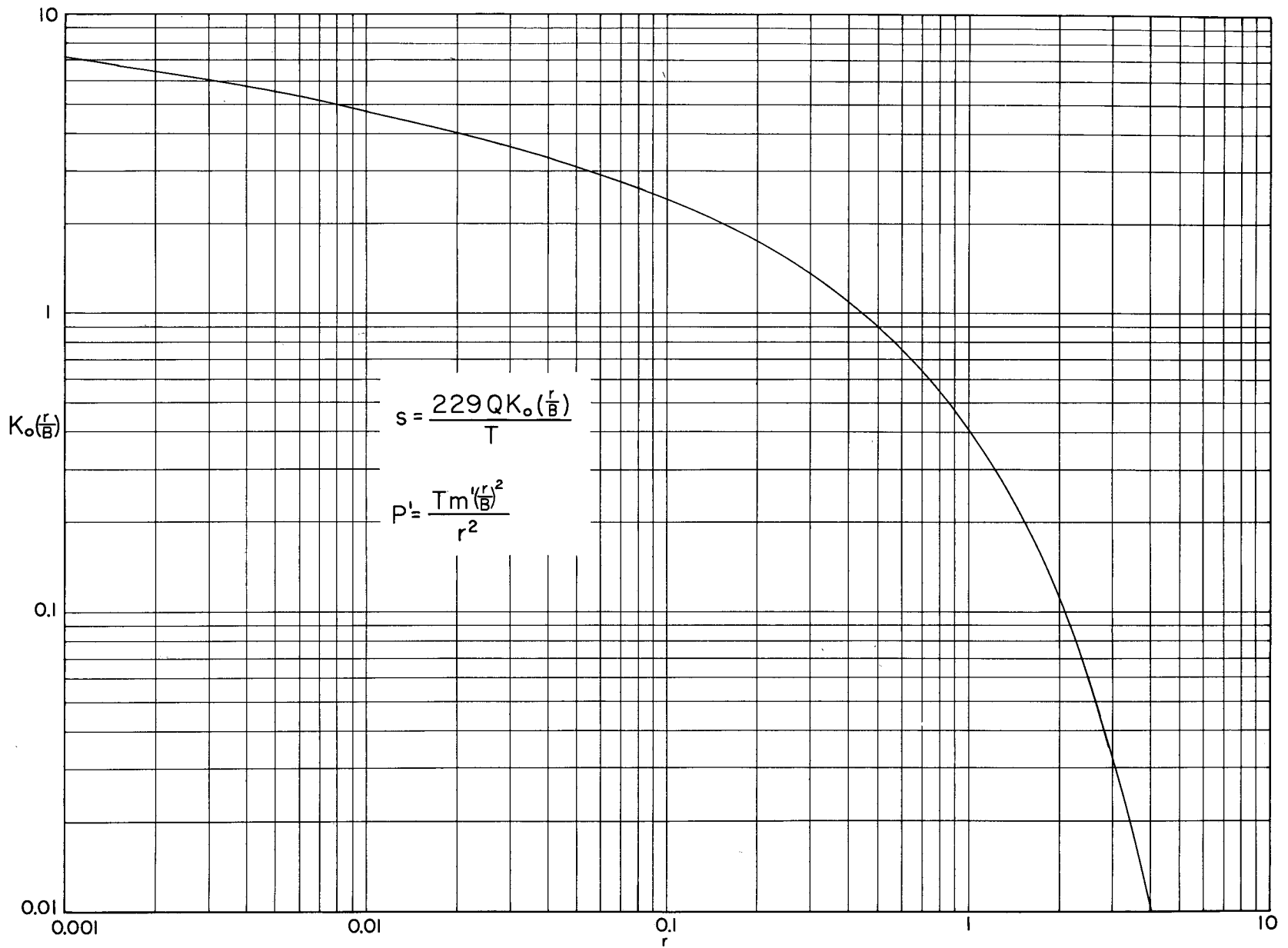


FIGURE 2 STEADY-STATE LEAKY ARTESIAN TYPE CURVE

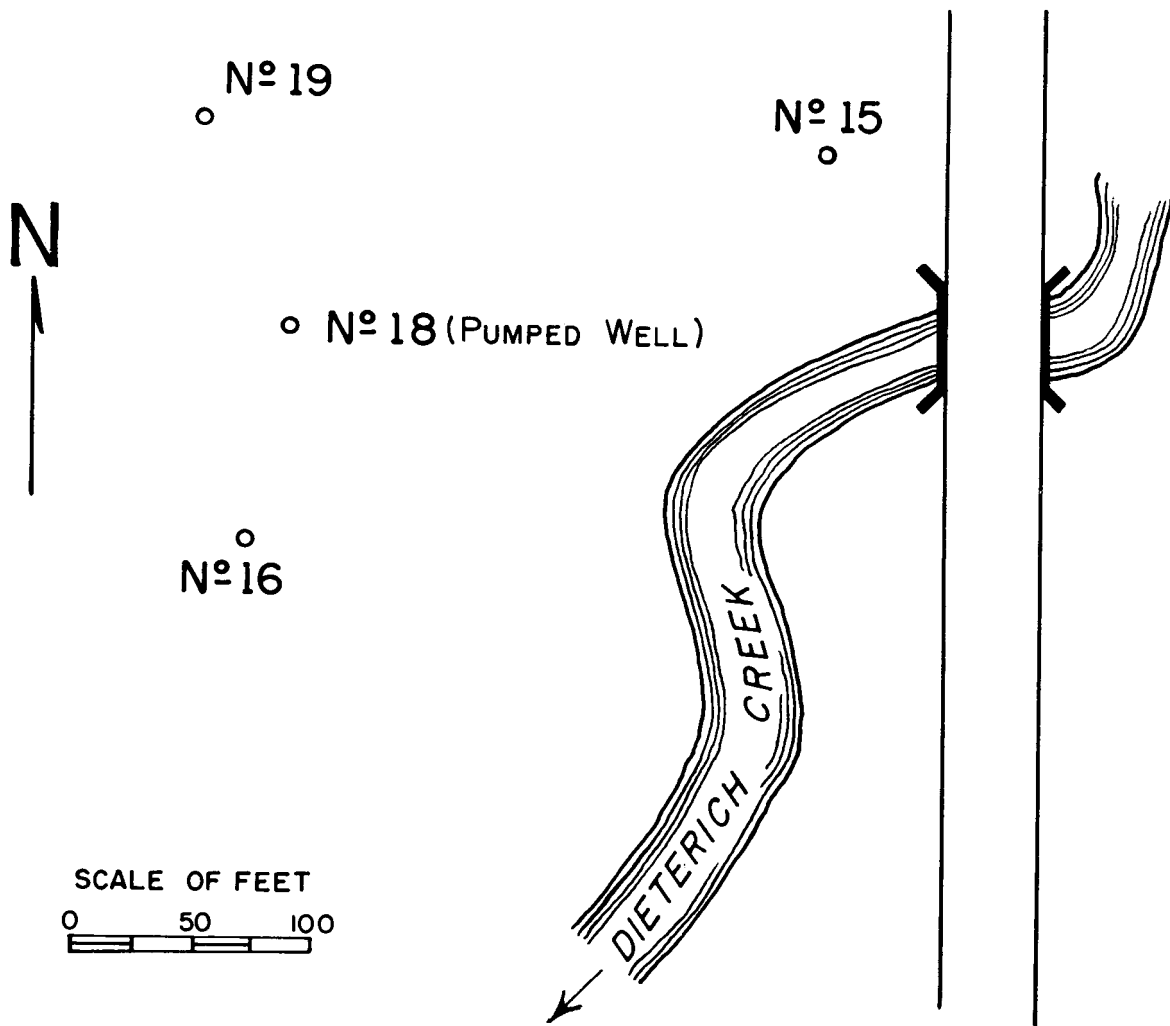


FIGURE 3 MAP SHOWING LOCATION OF WELLS USED IN TEST NEAR DIETERICH, ILLINOIS

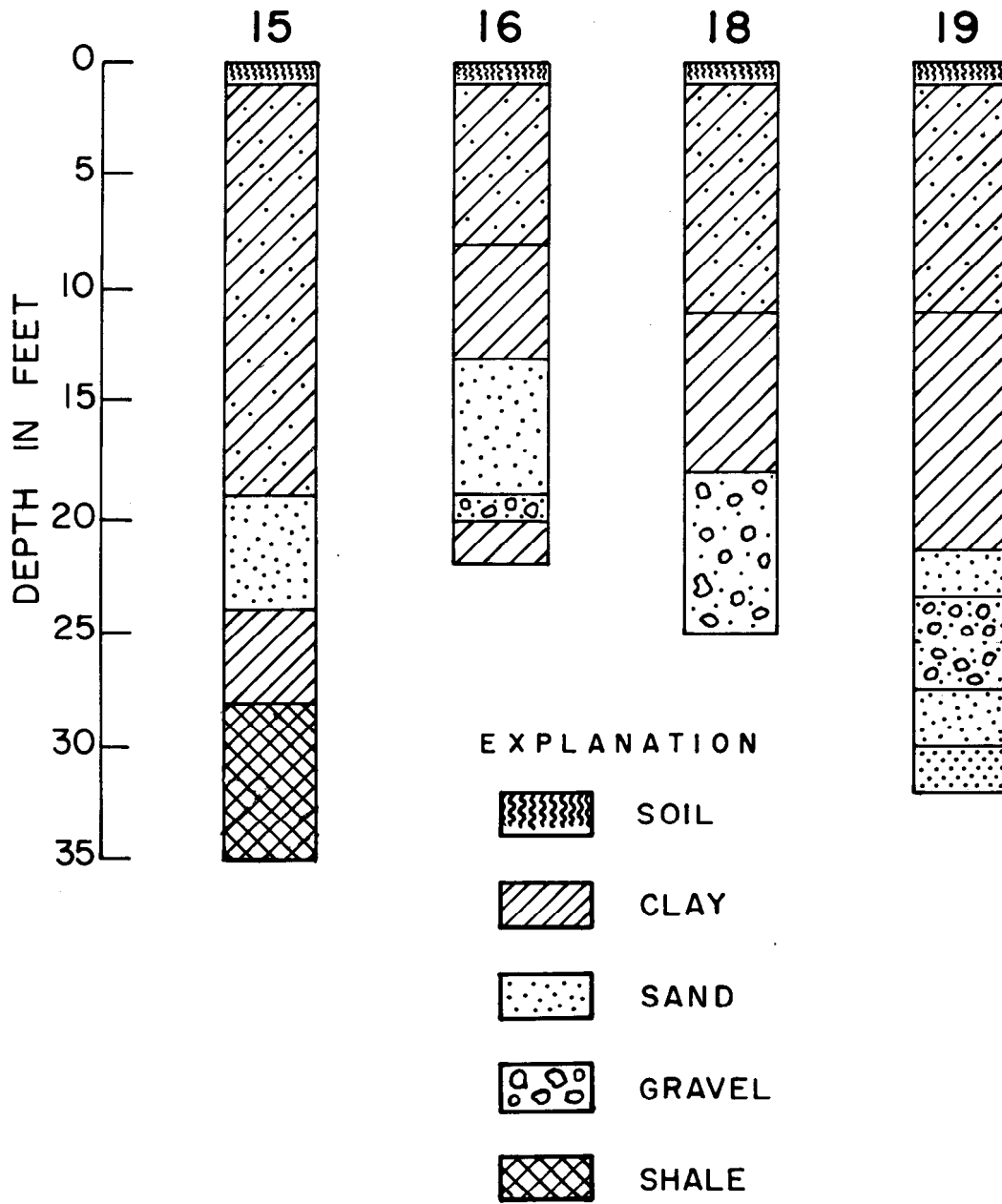


FIGURE 4 GENERALIZED GRAPHIC LOGS OF WELLS USED IN TEST NEAR DIETERICH, ILLINOIS

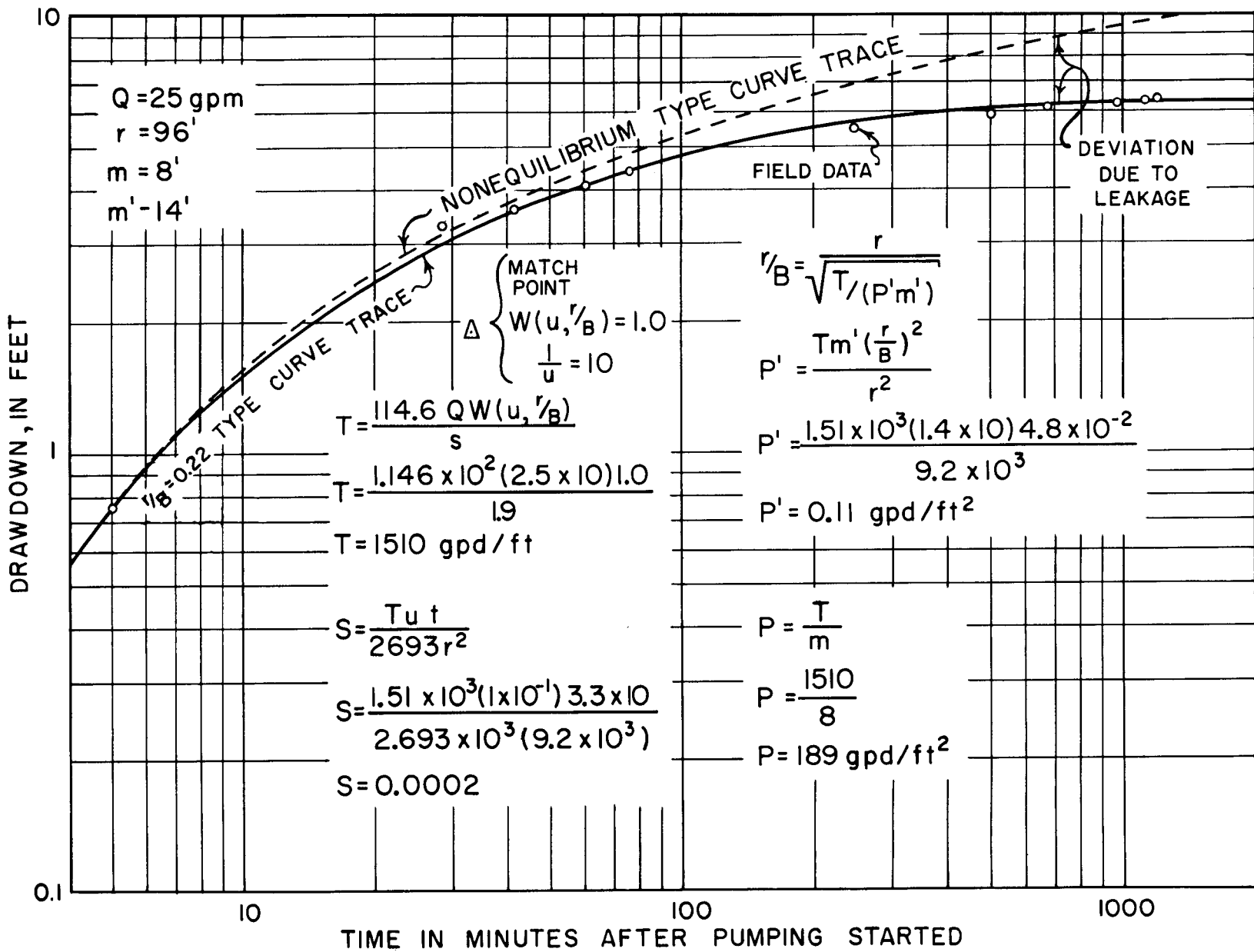


FIGURE 5 TIME-DRAWDOWN GRAPH FOR WELL 19 NEARDIETICH ILLINOIS



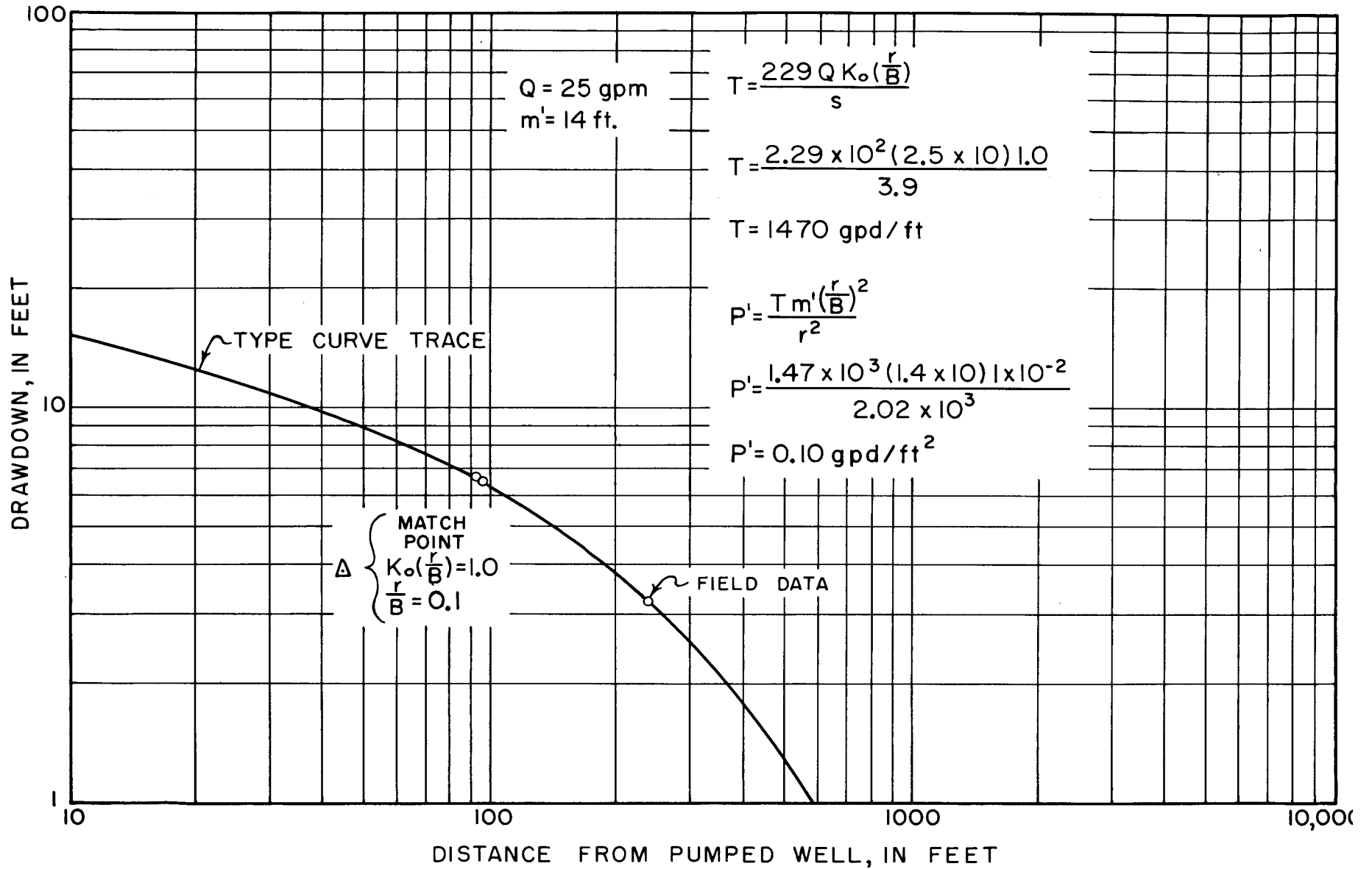


FIGURE 6 DISTANCE-DRAWDOWN GRAPH FOR TEST NEAR DIETERICH, ILLINOIS

250 feet apart, are drilled on a line through well 18. The sustained yield of the three-well system, as computed by a method proposed by Rorabaugh (1948, p. 63), would be about 40 gpm based on a recharge-boundary analysis of pumping test data. The sustained yield of the three-well system is defined as the rate at which the wells can be continuously pumped without eventually dewatering the aquifer below the top of screens.

Available geohydrologic data indicate that Dieterich Creek is not a recharge boundary. The stream bed rests on clayey materials and has not cut into the aquifer. In addition, the stream bed is silted and is relatively impermeable. The stream bed is only a few feet wide and stream flow (135 gpm at the time of the test) during the summer and fall months is low. Because of the small area of infiltration, low stream flow, and the presence of relatively thick clayey materials beneath the stream bed, very little recharge from Dieterich Creek can be expected during dry periods. Thus, it is concluded that the recharge-boundary analysis is incorrect. Recharge to the aquifer will occur as leakage through the confining bed and it will be derived largely from precipitation which reaches the water table.

The results of the pumping test lend support to this interpretation. If Dieterich Creek were a recharge boundary, the cone of depression would be distorted, and distance-drawdown data would yield values of  $T$  and  $P'$

not in agreement with the values of  $T$  and  $P'$  computed from time-drawdown data. However, values of  $T$  and  $P'$  computed from both time-drawdown and distance-drawdown data agree indicating that Dieterich Creek is not a recharge boundary.

The pumping test method of analysis of leaky artesian conditions is adequate for short pumping periods but further adjustments are necessary for long periods of pumping. At Dieterich, leakage during the short-term test lowered the water table very little, and for practical purposes the confining bed was not drained. However, as a consequence of prolonged heavy pumping, during the summer and fall months when recharge to the water table is very small, the confining bed will be drained and leakage will not keep up with discharge as it did during the pumping test.

Computations made, taking into consideration the draining of the confining bed and a part of the aquifer, indicate that the sustained yield of the hypothetical three-well system is about 25 gpm.

This sustained yield of the three-well system, as computed by the leaky artesian analysis (25 gpm), is much less than when sustained yield is computed by the recharge boundary analysis (40 gpm). Thus, it is apparent that recognition of leaky artesian conditions can be critically important in predicting the yield of aquifers.

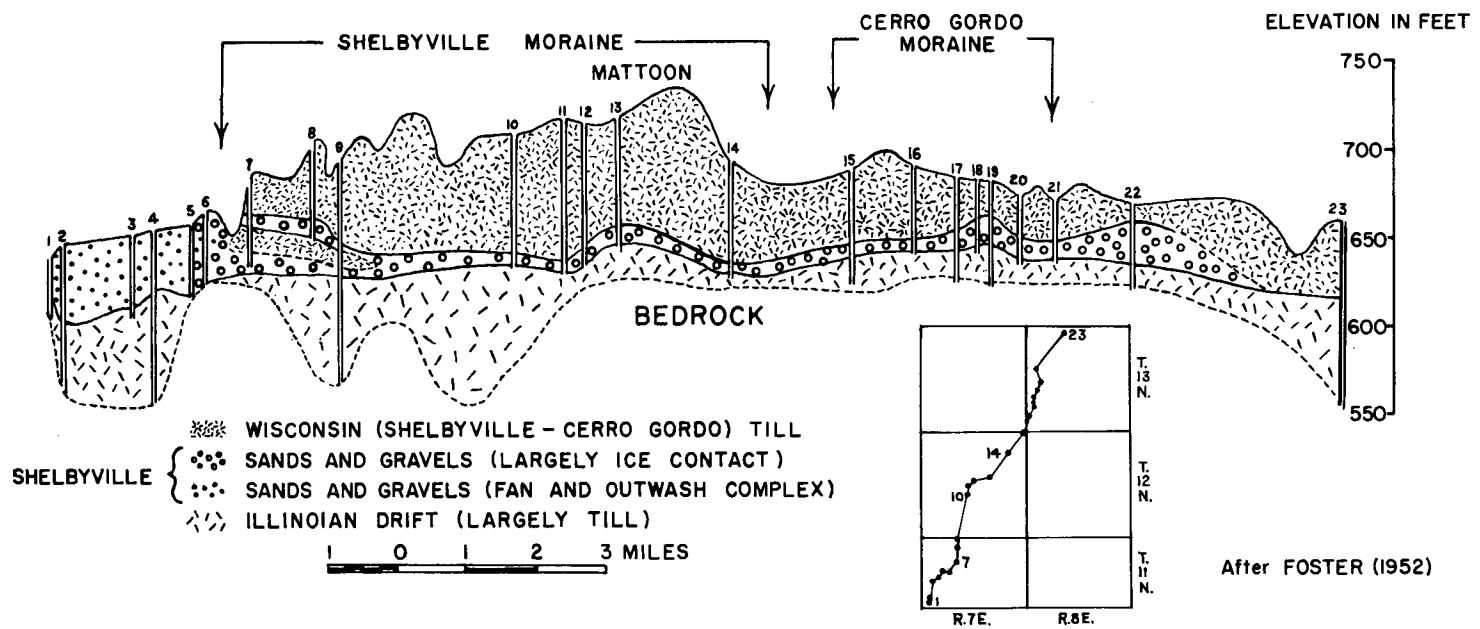


FIGURE 7 CROSS SECTION OF THE GLACIAL DRIFT IN THE VICINITY OF MATTOON, ILLINOIS

## Leakage Through Confining Beds in Illinois

### Glacial Drift

Confining beds of Pleistocene age are well known throughout most of central, eastern, and much of southern Illinois. Large areas in western, south central, and southern Illinois are covered by glacial drift of Illinoian age. The drift cover is relatively thin and seldom exceeds 75 feet in thickness. The bedrock beneath the drift is shale, sandstone, and limestone of Pennsylvanian age which yield only small amounts of water to wells. Large deposits of water-yielding sand and gravel are scarce in the glacial drift and they occur chiefly in existing or buried valleys and as lenticular and discontinuous layers. The sand and gravel aquifers are commonly overlain by deposits of till that contain a high percentage of silt and clay and have a low permeability. In many areas, recharge to the aquifers is derived from vertical leakage through the till.

In the area of the Wisconsinan glacial drift in the east central and northern parts of Illinois, drift is thicker and consequently may contain more aquifers. The glacial drift is several hundred feet thick in deeply buried preglacial valleys such as the Mahomet Valley in east central Illinois. The outwash sand and gravel deposits partly filling these ancient valleys exceed 100 feet in thickness at places. Permeable deposits are commonly interbedded and overlain by layers of till which greatly retard the vertical movement of water. Permeable glacial deposits also occur on bedrock uplands and are often covered with till.

A typical glacial drift aquifer and its confining bed near Mattoon in east central Illinois were described by Foster (1952, pp. 85-94.) The bedrock in the vicinity of the city of Mattoon is immediately overlain by Illinoian drift as shown in figure 7. Bedrock consists of shales, thin limestones, sandstones, and coals of Pennsylvanian age which yield small amounts of water to wells. The relief of the bedrock surface is not great except southwest of Mattoon where two bedrock valleys occur.

The Illinoian drift was described by Foster (1952, p. 89) as "a grey or grey-green calcareous clay till." The upper surface of the Illinoian drift is gently undulating; the thickness of the till is greatest in bedrock valleys where it is about 90 feet. At many places the top of the Illinoian till is marked by the Sangamon soil zone or peat deposits.

A widespread layer of permeable sand and gravel probably of ice-contact origin and Wisconsinan age overlies the Sangamon soil to points about five miles south of Mattoon. Here complex fan and outwash materials occur above the Sangamon zone south of the Shelbyville moraine that marks the limit of Wisconsinan glaciation. Post-Shelbyville deposits include the Cerro Gordo moraine. The Cerro Gordo moraine contains only scattered thin lenses of sand.

The aquifers in the Mattoon area are the ice-contact deposits of sand and gravel north of the limit of Wisconsinan glaciation and the fan and outwash deposits

south of the Shelbyville moraine. The textural composition of the ice-contact sand and gravel varies from place to place. Fine sand with a maximum thickness of 10 feet overlies the Sangamon soil at places and sometimes constitutes the entire aquifer. Coarse, clean gravels interbedded with silty sand exceeding 15 feet in thickness occur over wide areas. The fan and outwash deposits have thicknesses up to 65 feet and are in general more permeable than the ice-contact deposits.

The till deposits of the Shelbyville and Cerro Gordo moraines constitute the confining bed which overlies the ice-contact sand and gravel aquifer. The confining bed has an average saturated thickness of about 30 feet in the Mattoon area. The general textural composition of the confining bed and the aquifer below is described by the correlated driller's log of a well in sec. 30, T. 13 N., R. 7 E. given below:

<i>Formation</i>	<i>Depth (feet)</i>
Cerro Gordo-Shelbyville deposits (confining bed)	
Top soil .....	0-2
Yellow clay .....	2-12
Gravelly yellow clay.....	12-14
Gravelly blue clay .....	14-26
Shelbyville sand and gravel (ice contact)	
Medium sand .....	26-28
Clean coarse sand and gravel .....	28-39
Very dirty sand and gravel .....	39-45
Sangamon soil	
Peat .....	45

The nonpumping water level in the well was about seventeen feet below land surface in May, 1954.

The vertical permeability of the Cerro Gordo-Shelbyville deposits is estimated from pumping test data to be 0.63 gpd per square foot. The coefficients of transmissibility and permeability of the Shelbyville sand and gravel are estimated from pumping test data to be 25,600 gpd per foot and 1600 gpd per square foot respectively.

Prior to 1935 the water supply for the city of Mattoon was obtained from wells (Doran well field) penetrating the Shelbyville ice-contact sand and gravel aquifer in sec. 30, T. 13 N., R. 7 E. The sustained yield of the Doran well field was not great because of the following reasons: (1) the sand and gravel aquifer was relatively thin (average thickness in the Doran well field area is about 16 feet), (2) recharge to the aquifer was limited by the vertical permeability of the till, and (3) during extended dry periods recharge from precipitation was not sufficient to replenish the till bed continuously, and heavy pumping ultimately drained the till and part of the aquifer.

### *Summary of Test Data for Illinois*

A summary of the leaky artesian test data collected to date in Illinois and the coefficients of transmissibility, storage, and vertical permeability computed therefrom are given in table 1. Values of P' given in the table range between 0.08 and 1.6 gpd per square foot. The

least permeable glacial drift confining bed is the clayey material overlying the aquifer at Winchester. The most permeable confining bed is the sandy clay materials at Cowden. The coefficient of vertical permeability of the confining bed overlying the Doran well field at Mattoon is high compared to most of the values of  $P'$  given in table 1.

Most of the confining beds listed in table 1 are less than 15 feet thick. To facilitate the planning of future pumping tests in areas where confining beds exceed 15 feet in thickness, theoretical time-drawdown curves for several leaky artesian conditions thought to exist in Illinois are presented in figure 8. It is apparent from the curves that as the thickness of the confining bed increases, or the coefficient of transmissibility increases, or  $P'$  decreases, the time that it takes for leakage to affect drawdown data increases and the effects of leakage decrease. The influence of  $T$ ,  $P'$ , and  $m'$  on the cone of depression is shown by the theoretical distance-drawdown graphs in figure 9. As  $T$  decreases, the cone of depression deepens. As  $T$  increases, the virtual radius of the cone of depression decreases. Both the depth and virtual radius of the cone of depression increase as  $P'$  decreases or as  $m'$  increases.

#### Maquoketa Formation

The Maquoketa Formation overlies the Cambrian-Ordovician Aquifer in large parts of northeastern Illinois, including the Chicago region, and to great extent confines the water in the aquifer under artesian pressure. As described in a detailed report on the ground-water resources of the Chicago region (Suter, et al, 1959), the

Cambrian-Ordovician Aquifer is the most highly developed source of large ground-water supplies in north-eastern Illinois and consists in downward order of the Galena-Platteville Dolomite, Glenwood-St. Peter Sandstone, and Prairie du Chien Series of Ordovician Age; the Trempealeau Dolomite, Franconia Formation, and Ironton-Galesville Sandstone of Cambrian Age. The sequence, structure, and general characteristics of these rocks are shown in figure 10a. The Cambrian-Ordovician Aquifer is underlain by shale beds of the Eau Claire Formation which have a very low permeability. Available data indicate that on a regional basis, the entire sequence of strata, from the top of the Galena-Platteville to the top of the shale beds of the Eau Claire Formation, essentially behave hydraulically as one aquifer.

As shown in figure 10b, the Maquoketa Formation has a maximum thickness of about 250 feet and thins to the north and west to less than 50 feet. The formation dips regionally to the east at a uniform rate of about 10 feet per mile. Bergstrom and Emrich (see Suter, et al, 1959, p. 33) divided the Maquoketa Formation into three units; lower, middle, and upper. As described by Bergstrom and Emrich,

“the lower unit, is normally a brittle, dark brown, occasionally gray or grayish brown, dolomite shale grading locally to dark brown, argillaceous dolomite. The middle unit is dominantly brown to gray, fine-to-coarse-grained, fossiliferous, argillaceous, speckled dolomite and limestone. It is commonly interbedded with a fossiliferous brownish gray to gray, dolomitic shale. The upper unit is a greenish gray, weak, silty, dolomitic

Table 1—Results of Leaky Artesian Pumping Tests in Illinois

#### Physical Data

Owner	Location	Date of test	Duration (hrs)	Pumping rate (gpm)	Lithology	
					Aquifer	Confining bed
(Driller's Log)						
Village of Beecher City	Effingham Co.	2/5-6/51	23	15	sand, fine & some gravel	clay, sandy
Village of Dieterich	Effingham Co.	7/2-3/51	20	25	sand, fine & some gravel	clay, sandy, hardpan
Village of Cowden	Shelby Co.	10/22/54	4	141	sand & gravel	sandy clay
City of Assumption	Christian Co.	5/29/58	3	32	sand, some gravel	clay, sandy
City of Mattoon	Coles Co.	2/24/54	4	156	sand & gravel	clay, gravelly
City of Barry	Pike Co.	7/4-3/56	21	207	sand & gravel	clay, sandy
City of Winchester	Scott Co.	11/9/49	8	100	sand & gravel	clay

#### Results

Owner	Location	Aquifer				Confining bed		
		T (gpd/ft)	m (feet)	P (gpd/sq.ft.)	S	P' (gpd/sq. ft.)	m' (feet)	(P'/m')
(gpd/cu. ft.)								
Village of Beecher City	Effingham Co.	1,220	7	175	0.0003	0.25	12	$2.1 \times 10^{-2}$
Village of Dieterich	Effingham Co.	1,500	8	188	0.0002	0.10	14	$7.1 \times 10^{-3}$
Village of Cowden	Shelby Co.	39,000	25	1,560	0.0080	1.60	7	$2.3 \times 10^{-1}$
City of Assumption	Christian Co.	4,900	12	408	.....	0.19	8	$2.4 \times 10^{-2}$
City of Mattoon	Coles Co.	25,600	16	1,600	0.0015	0.63	12	$5.2 \times 10^{-2}$
City of Barry	Pike Co.	119,000	36	3,300	0.0030	0.15	16	$9.4 \times 10^{-3}$
City of Winchester	Scott Co.	10,000	26	384	0.0003	0.08	16	$5.0 \times 10^{-3}$



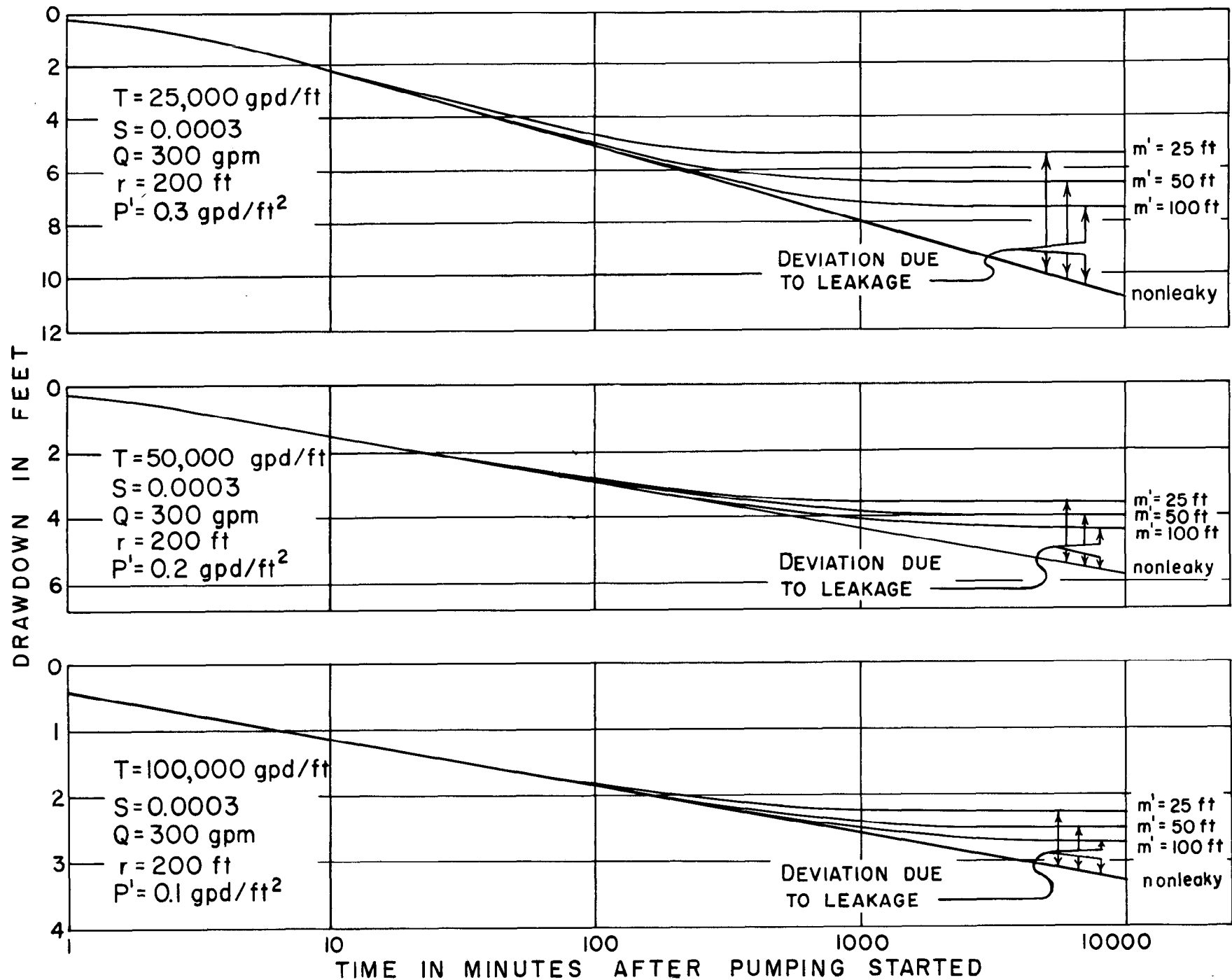


FIGURE 8 THEORETICAL TIME-DRAWDOWN GRAPHS FOR SELECTED LEAKY ARTESIAN CONDITIONS

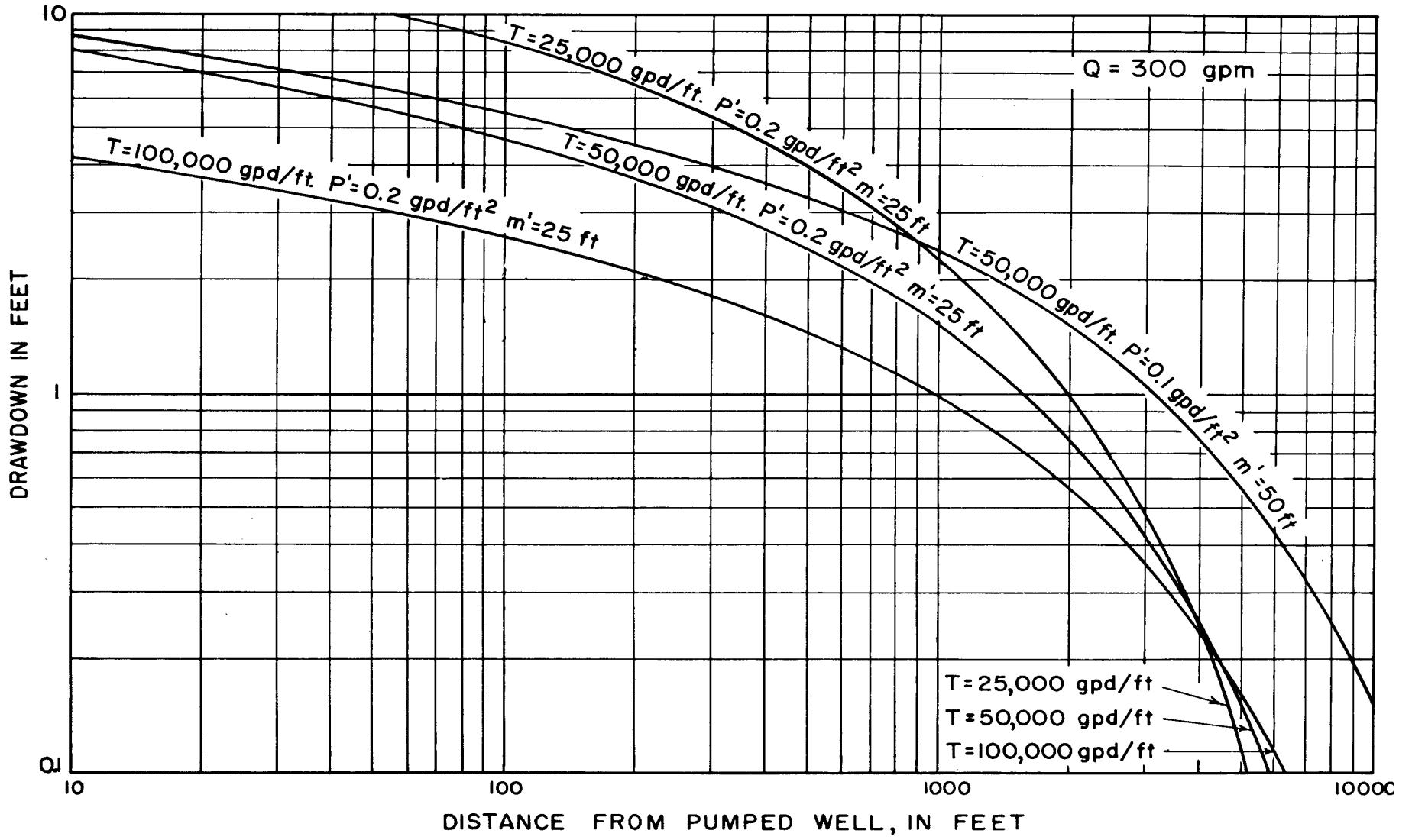
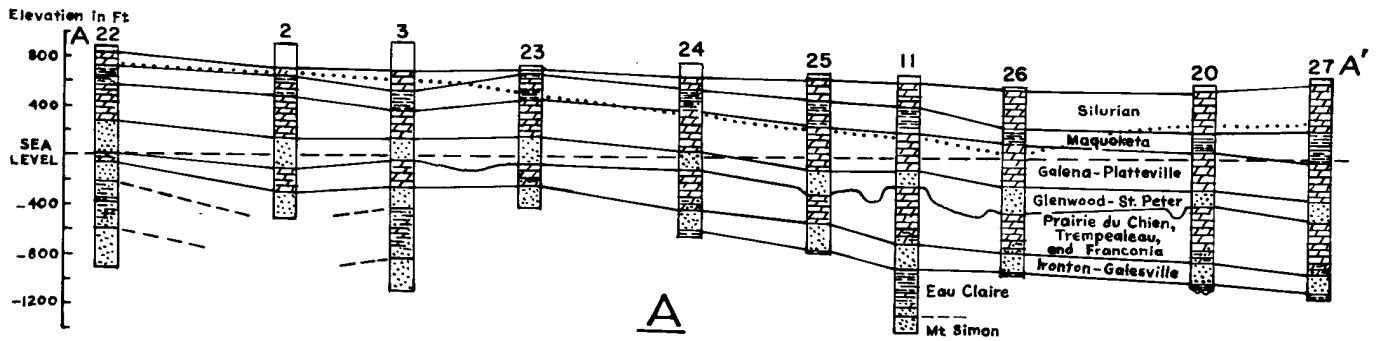


FIGURE 9 THEORETICAL DISTANCE-DRAWDOWN GRAPHS FOR SELECTED LEAKY ARTESIAN CONDITIONS



EXPLANATION

- Glacial Deposits
- Dolomite
- Shale
- Sandstone
- Piezometric profile 1958
- Thickness line, interval 50 feet
- Des Plaines Complex
- Pennsylvanian boundary
- Maquoketa boundary
- Silurian boundary

Thickness is not shown where the Maquoketa crops out or directly underlies the glacial drift.

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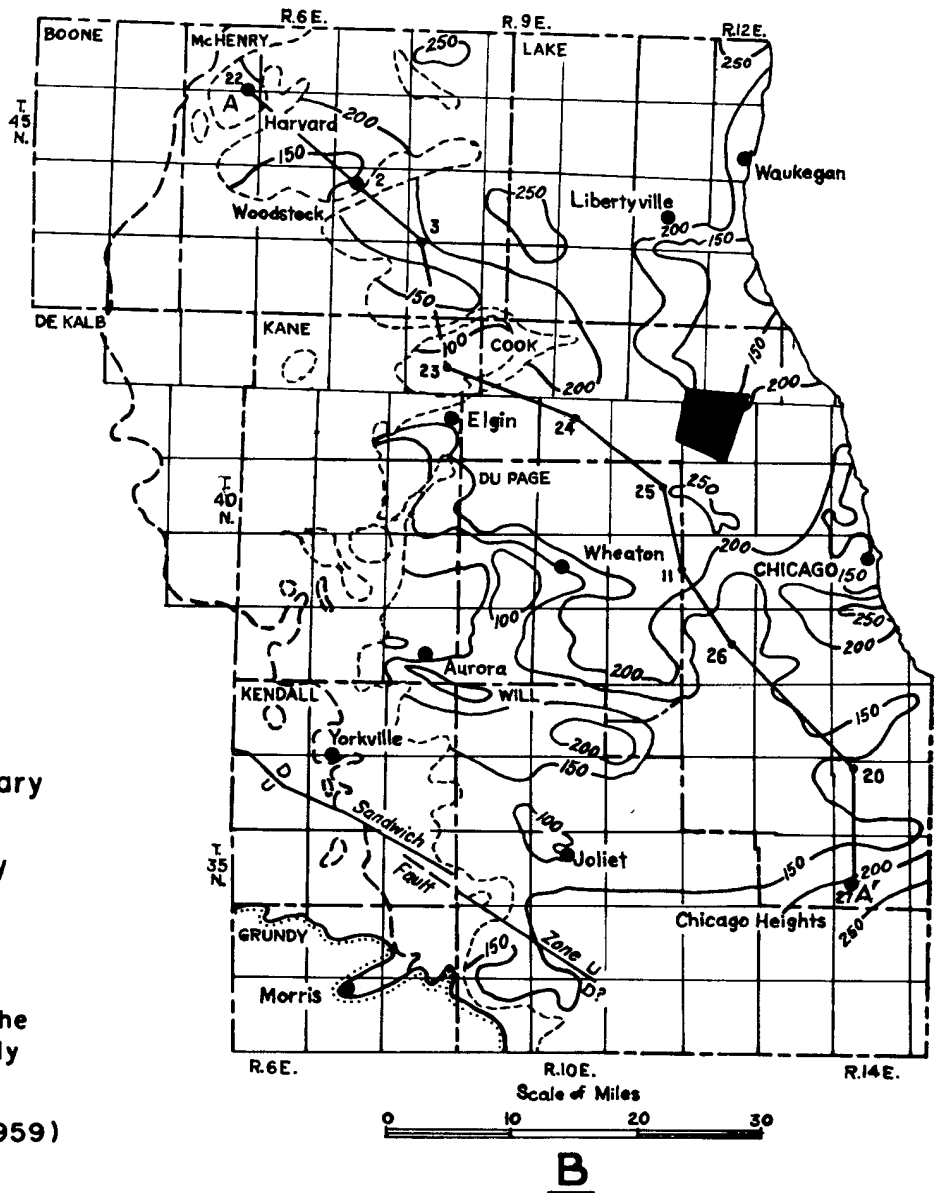
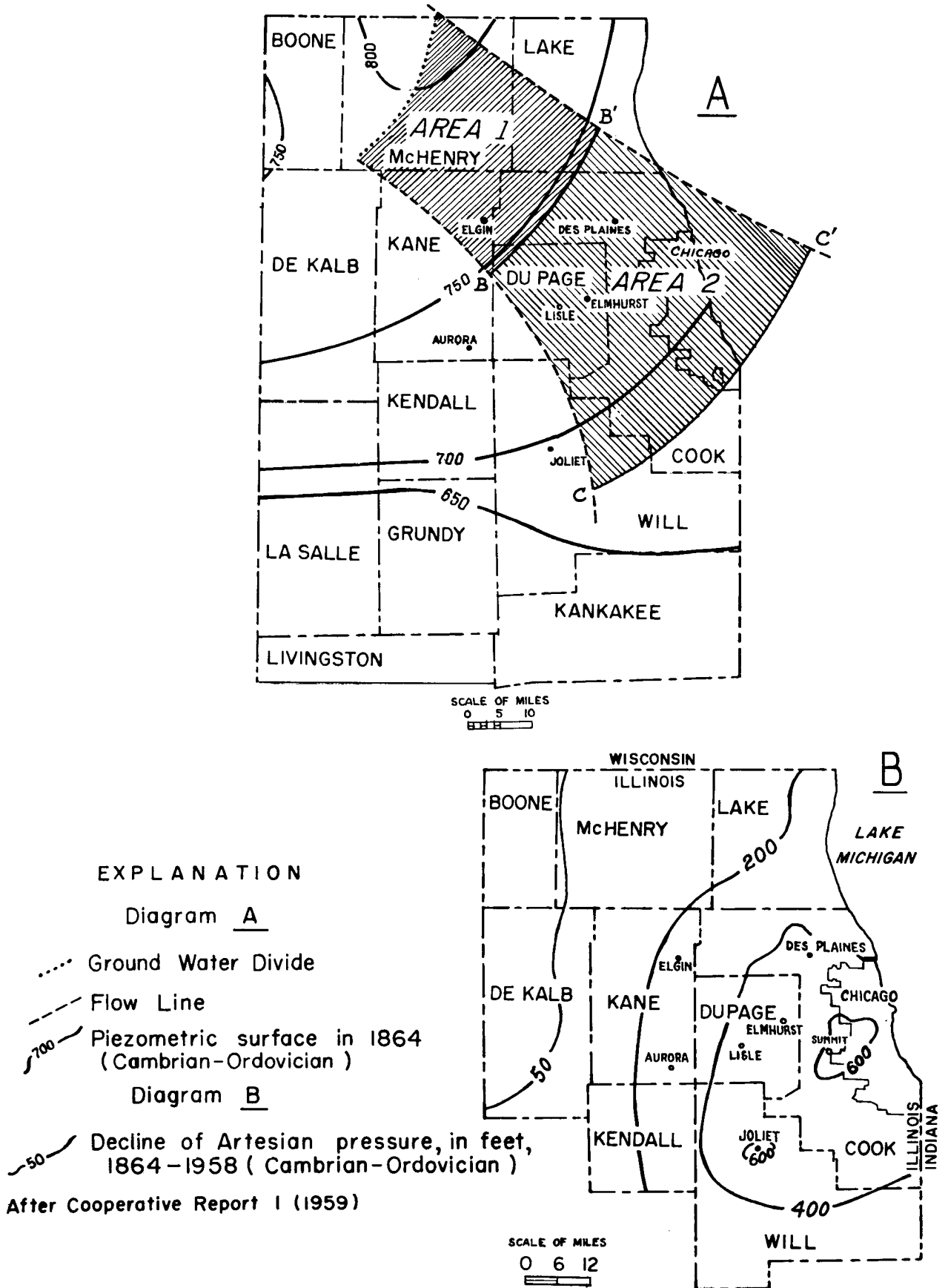


FIGURE 10 GEOHYDROLOGIC CROSS SECTION AND THICKNESS OF THE MAQUOKETA FORMATION IN NORTHEASTERN ILLINOIS



**FIGURE 11** PIEZOMETRIC SURFACE OF CAMBRIAN-ORDOVICIAN AQUIFER IN NORTHEASTERN ILLINOIS, ABOUT 1864, AND DECLINE IN ARTESIAN PRESSURE IN CAMBRIAN-ORDOVICIAN AQUIFER, 1864-1958

shale that grades into very argillaceous, greenish gray to gray dolomite. The lower unit is thicker in Cook and Will Counties where it exceeds 100 feet. It thins to the north and west to less than 50 feet. The middle unit is thicker to the west where it is more than 100 feet locally and thins to the east. The upper unit ranges in thickness from less than 50 feet in the west to more than 100 feet in parts of Cook and Will Counties. The lower dense shale unit is the most impermeable unit. Dolomite beds in the middle unit yield small quantities of ground water."

The Cambrian-Ordovician Aquifer beneath the Maquoketa Formation receives water from overlying glacial deposits mostly in areas west of the border of the Maquoketa Formation shown in figure 10b where the Galena-Platteville Dolomite, the uppermost unit of the aquifer, is directly overlain by glacial deposits. Recharge of the glacial deposits in turn is derived from precipitation that falls locally. The piezometric-surface map for the Cambrian-Ordovician Aquifer in the year 1864 (see figure 11a) indicates that under natural conditions water entering or recharging the aquifer was discharged in areas to the east and south by vertical leakage upward through the Maquoketa Formation and by leakage into the Illinois River valley.

The changes in artesian pressure produced by pumping since the days of early settlement have been pronounced and widespread. Pumpage from deep wells has increased from 200,000 gpd in 1864 to about 78 mgd in 1958. Figure 11b shows the decline of artesian pressure in the Cambrian-Ordovician Aquifer from 1864 to 1958 as the result of heavy pumping. The greatest declines, more than 600 feet, have occurred in areas of heavy pumpage west of Chicago, at Summit and at Joliet. In 1958, the piezometric surface of the Cambrian-Ordovician Aquifer was several hundred feet below the water table in most of northeastern Illinois, and downward movement of water through the Maquoketa Formation was appreciable under the influence of large differentials in head between shallow deposits and the Cambrian-Ordovician Aquifer. The vertical permeability of the Maquoketa Formation and the quantity of leakage through the confining bed in 1958 are discussed in the following sections.

#### *Vertical Permeability of Maquoketa Formation*

The quantity of leakage through a confining bed into an aquifer can be computed from the following form of Darcy's law :

$$Q_c = \left( \frac{P'}{m'} \right) \Delta h A_c \quad (9)$$

where :

$Q_c$  = leakage through confining bed, in gallons per day

$P'$  = vertical permeability of confining bed, in gallons per day per square foot

$m'$  = thickness of confining bed through which leakage occurs, in feet

$A_c$  = area of confining bed through which leakage occurs, in square feet

$h$  = difference between the head in the aquifer and in the source bed above the confining bed, in feet

The quantity ( $P'/m'$ ) was termed the leakage coefficient by Hantush (1956, p. 702). Values of the leakage coefficient determined from pumping test data are given in table 1.

Equation 9 may be rewritten as:

$$P' = \frac{Q_c m'}{\Delta h A_c} \quad (10)$$

Thus, the vertical permeability of a confining bed may be determined if  $Q_c$ ,  $m'$ ,  $A_c$ , and  $h$  are known.

Equation 10 was used to determine the order of magnitude of the vertical permeability of the Maquoketa Formation. Figure 11a shows the piezometric surface of the Cambrian-Ordovician Aquifer before extensive development occurred in the Chicago region. Flow lines were drawn from the ground-water divide in McHenry County toward the northern and southern boundaries of Cook County at right angles to the estimated piezometric surface contours for 1864. The part of the aquifer (area 1) which is enclosed by the flow lines, the ground-water divide, and section B-B', was considered. In 1864 the piezometric surface was below the water table and downward leakage through the Maquoketa Formation into the aquifer was occurring in area 1. Leakage was equal to the quantity of water percolating through section B-B'. At section B-B' the hydraulic gradient of the piezometric surface was about two feet per mile, and the distance between limiting flow lines was about twenty-five miles. Based on data given by Suter, et al (1959, p. 50) the average coefficient of transmissibility of the Aquifer at section B-B' is about 19,000 gpd per foot.

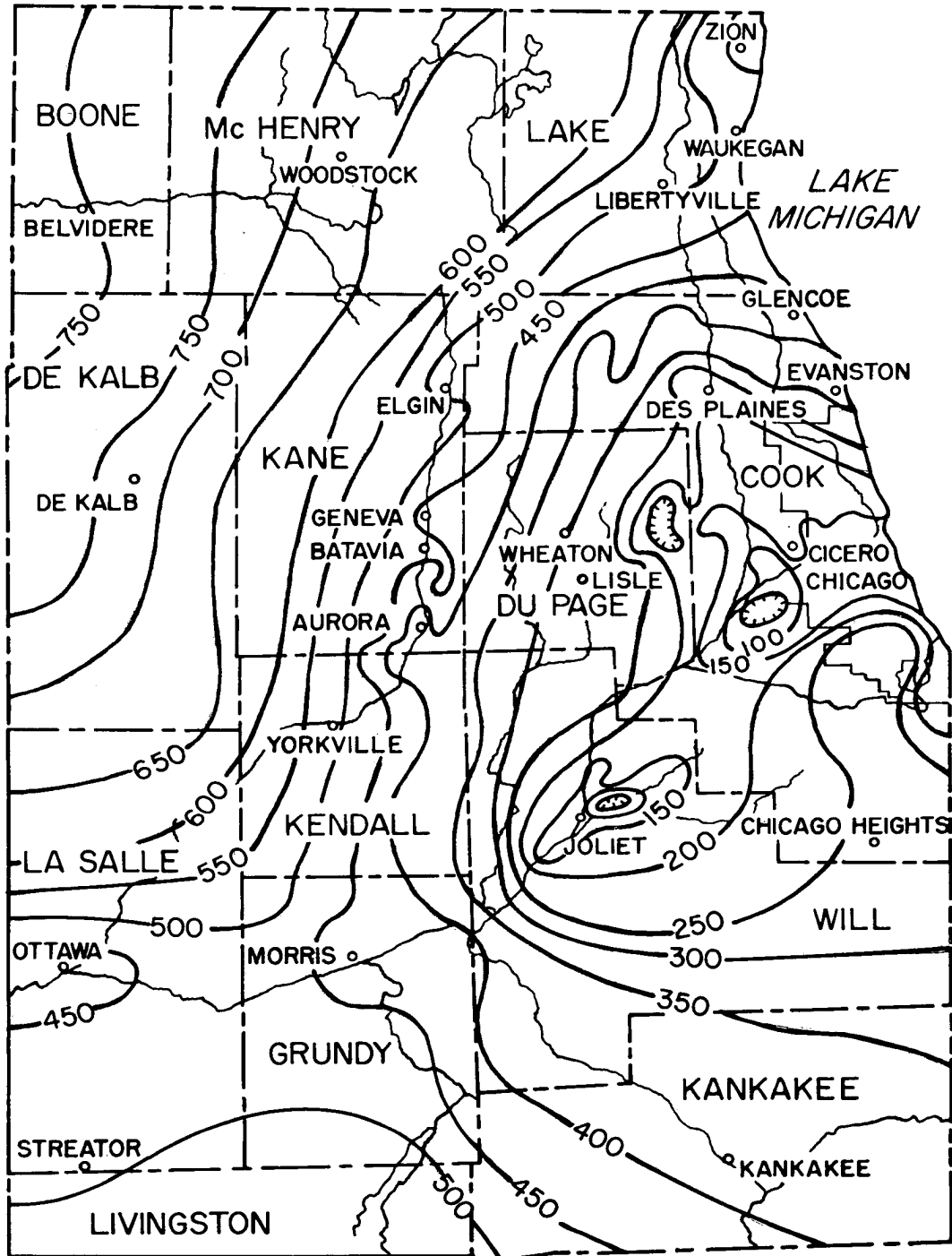
The quantity of water percolating through a given cross section of an aquifer is proportional to the hydraulic gradient (slope of the piezometric surface) and the coefficient of transmissibility. It can be computed by using the following modified form of the Darcy equation (see Ferris, 1959, p. 148) :

$$Q = TIL \quad (11)$$

in which  $Q$  is discharge, in gallons per day,  $T$  is coefficient of transmissibility, in gallons per day per foot,  $I$  is hydraulic gradient, in feet per mile, and  $L$  is width of cross section through which discharge occurs, in miles.

Using equation 11, the quantity of water moving southeastward through the aquifer at section B-B' was computed to be about 1,000,000 gpd. Leakage downward through the Maquoketa Formation in area 1 was therefore about 1,000,000 gpd in 1864. As measured from figure 11a, area 1 is about 750 square miles. The average  $h$  over area 1 was determined to be about 85 feet by comparing estimated elevations of the water table and the piezometric surface contours given in figure 11a. The average thickness of the Maquoketa Formation over area 1 from





From Cooperative Report 1 (1959)

SCALE OF MILES  
0 5 10

FIGURE 12 PIEZOMETRIC SURFACE OF CAMBRIAN-ORDOVICIAN AQUIFER IN NORTHEASTERN ILLINOIS IN 1958

figure 10b is about 175 feet. Substitution of these data in equation 10 indicates that the average vertical permeability of the Maquoketa Formation in area 1 is about 0.0001 gpd per square foot.

In 1864 the piezometric surface was above the water table southeast of section B-B', and the quantity of water entering the aquifer in area 1 was discharged by leakage up through the Maquoketa Formation in the areas between the limiting flow lines southeast of section B-B' in northeastern Illinois and northwestern Indiana.

Figure 11a indicates that appreciable flow was occurring through section C-C' near the Illinois-Indiana state line and that leakage between sections B-B' and C-C' was something less than 1,000,000 gpd. Therefore, the average vertical permeability of the Maquoketa Formation in area 2 is something less than that computed by substituting a value of 1,000,000 gpd for  $Q_c$  in equation 10. Area 2 was scaled from figure 11a and is about 1500 square miles. The average  $h$  over area 2 was determined to be about 70 feet by comparing estimated elevations of the water table and the piezometric surface contours given in figure 11a. The average thickness of the Maquoketa Formation over area 2 from figure 10b is about 200 feet. Substitution of these data in equation 10 indicates that the average vertical permeability of the Maquoketa Formation in area 2 is less than 0.00007 gpd per square foot.

Actually the leakage over area 2 is equal to the difference between the quantities of water moving through sections B-B' and C-C'. The quantity of water moving through section C-C' cannot be determined with any degree of accuracy because the location of the 650-foot piezometric surface contour is largely conjectural and the average coefficient of transmissibility of the aquifer at section C-C' is uncertain. On the basis of available data, the flow through section C-C' is estimated to be at least one third of the flow through section B-B' and

the average vertical permeability of the Maquoketa Formation in area 2 is estimated to be not less than 0.00002 gpd per square foot. A value of 0.00003 gpd per square foot was selected as the best estimate of  $P'$  for area 2.

Computations indicate that the average vertical permeability of the Maquoketa Formation increases to the north and west. Available geologic information supports this conclusion. The lower unit of the Maquoketa Formation, probably the least permeable of the three units (Bergstrom & Emrich, personal communication), thins to the west. In addition, the Maquoketa Formation is the uppermost bedrock formation below the glacial deposits in a large part of area 1 and locally may be completely removed by erosion.

A comparison of the average vertical permeability of the Maquoketa Formation with data in table 1 indicates that the glacial drift confining beds for which test data are available are at least 800 times as permeable as the Maquoketa Formation.

#### Leakage through Maquoketa Formation in 1958

Even though the vertical permeability is very low, leakage in 1958 through the Maquoketa Formation was appreciable. The area of the confining bed within the part of Illinois shown in figure 10b through which leakage occurred (4000 square miles) and the average head differential between the piezometric surface of the Cambrian-Ordovician Aquifer and the water table (300 feet) were great (figures 11a and 12). Computations made using the data given above, and assuming a  $m'$  of 200 feet and a  $P'$  of 0.00005 gpd per square foot, indicate that leakage through the Maquoketa Formation within the part of Illinois shown in figure 10b was about 8,400,000 gpd or about 11 per cent of the water pumped from deep wells in 1958.

### Conclusions

The vertical permeability of a confining bed often can be readily determined from pumping test data with the nonsteady-state leaky artesian equation derived by Hantush and Jacob. The solution of nonsteady drawdown distribution caused by pumping a well in an infinite leaky artesian aquifer is simplified by using a family of leaky artesian type curves. The vertical permeability of a confining bed and the quantity of leakage into an aquifer also can be determined from forms of Darcy's law by using data on the piezometric surface of the aquifer.

Although the vertical permeabilities of only seven glacial drift confining beds in southern Illinois have been computed from the results of pumping tests, they represent a good start in cataloging the vertical permeabilities of confining beds in the state. The values probably can be applied to geologically similar areas, at least in making rough quantitative investigations in the southern half of Illinois. Test data under leaky artesian conditions are not available for the northern half of Illinois. Controlled pumping tests should be made in the future in areas of

northern Illinois where leaky artesian conditions exist. The results of tests in many areas throughout Illinois will provide a means for detecting local and regional changes in the vertical permeability of glacial desposits and will aid greatly in the interpretation of the geology of Illinois as it relates to the quantitative appraisal of the State's water resources.

The vertical permeability of the Maquoketa Formation may vary greatly from place to place and locally other beds may serve to confine the Cambrian-Ordovician Aquifer, however, the computed average vertical permeability of the Maquoketa Formation indicates the order of magnitude of the parameter. No great accuracy is implied in the computations of the quantity of leakage through the Maquoketa Formation. The intent of quantitative studies is to better understand the relationship of the Maquoketa Formation to the Cambrian-Ordovician Aquifer and to estimate leakage with greater accuracy than mere expression of concept;

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Many former and present members of the State Water Survey and State Geological Survey participated in pumping tests, wrote earlier special reports which have been used as reference material, or aided the writer indirectly in preparing this report. Grateful acknowledgment is made, therefore, to the following engineers and geologists: Max Suter, W. H. Walker, G. B. Maxey, R. E. Bergstrom, R. R. Russell, R. J. Schicht, J. P. Dorr, G. E. Neher, R. T. Sasman, J. W. Foster, J. E. Hackett, G. H. Emrich, H. G. Rose, Jack Bruin, W. J. Roberts, R. E. Aten, and Otis D. Michels. Acknowledgment is made to consulting engineers, well drillers, and municipal officials who were most cooperative and helpful in making data available on pumping tests. H. F. Smith, Head Engineering Section, State Water Survey, encouraged the preparation of this report. J. W. Brother prepared illustrations.

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