



RICE UNIVERSITY

SYSTEMATIC APPROACHES FOR RETAIL SERVICE  
LOCATION DECISIONS

by

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A handwritten signature in cursive script, reading "Peter G. Howe", written over a horizontal line.

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ABSTRACT: SYSTEMATIC APPROACHES FOR RETAIL SERVICE LOCATION DECISIONS

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This thesis investigates systems applications to community facility planning by focussing on the use of models in locating retail facilities.

This approach was taken because a number of major concepts employed in retail location are directly transferrable to most types of urban services where consumers may choose to utilize a number of different locations.

A general decision making process for locating retail services is described. Review of the types of information needed by a retail location planner finds that the central issue he faces is estimating the sales volume of a proposed site. The effect of other competing locations, consumer preferences and accessibility make this task difficult without some type of systematic approach. This could be called the classical "problem" of retail location.

An extensive search was made of the work of others related to this problem. A number of approaches were found which attempted to represent the inter-related elements of consumers, access, and retailers which constitute a retail system. No dominant theory has been developed in the area; instead, a number of individual lines of inquiry were found with similarities between.

Several selected location models are then reviewed in application to specific problems. The major criticism provided focusses on the degree of difficulty model authors have in representing consumer - retailer behavior and the type of information required to support the modelling.

It was found that no one type of model can be regarded as superior since each may have been developed for different planning applications which vary in type of retail service and geographic area represented.

There are other steps in retail location decision making where further applications of systems approaches may be valuable. These include population and income forecasting for a small area and economic evaluation of location alternatives once gross sales have been estimated. Further development of these areas in conjunction with the retail models described is suggested.

Finally, a number of concepts found in various approaches to retail location may have direct benefit in the successful application of planning standards commonly used by architects and urban designers. Insight gained through certain theoretical approaches to retail location imply that increased care should be taken in the derivation and application of meaningful planning standards.

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## SYSTEMATIC APPROACHES FOR RETAIL SERVICE LOCATION DECISIONS

### I. INTRODUCTION

While a review of current practices in community facilities planning finds some use of systems approaches, generally, applications seem limited. It was not clear whether few approaches existed or whether most planners had not adopted use of them. Perhaps the planning process is too poorly understood to allow for the use of a systems approach.

A systems approach may be defined as a representation of a set of interrelated elements in some state of relative equilibrium with each other. This representation usually occurs in the form of either a theoretical or operational model of service user and provider interaction.

A wide range of types of community services were observed in terms of the current state-of-the-art in location planning as reflected in the literature. These services include health, educational, welfare, retail commercial, public recreational, emergency medical, police and fire. Over the last fifty years, retail commercial services shows extensive development including the consideration of several different disciplines such as geographic, economic and those dealing with commerce and psychology. This extensive research was believed to be a fertile approach into understanding most areas of community services planning since many systems approaches were believed to be broadly transferrable.

Therefore, improved community facilities planning may be achieved through successful integration of 1) a well defined decision making process; 2) simple techniques or models, which may or may not be computer based, which can be used at strategic points in the decision making process; and 3) a well developed information base.

Thus, the objectives of this thesis are to:

1. Review retail facility planning experiences in order to formulate a general process for decision making.
2. Research past theories related to the treatment of systems in locating retail facilities.
3. Critical analysis of application of selected models developed for retail location.
4. Discuss how these models may be integrated into an overall planning, decision making process including identification of gaps where further development of techniques is needed.
5. Evaluate the applicability of the retail location decision making with its accompanying models to other types of public and private community services.
6. Suggest where further research and development in the area may be warranted.

### Background

This investigation was undertaken because of a concern both for the types of methods and information used by architects, planners and urban designers in making decisions in programming community facilities. It is a personal observation that the use of "standards",



which has widespread acceptance among these professionals, can be misleading and in most cases likely to provide information which is not adequate for successful facility location. Typically these standards may be used to estimate the future amount of service or facility required as a linear function of the standard and a future population. Within these standards, measures of the effect of distance as a measure of convenience is implicit and little if any consideration is made for the economic or behavioral equilibrium which exists in a community. Consumer expenditures, taxes, public budgets, and developer cash flows are rarely considered in an explicit manner as alternative facility locations are examined.

Using standards may be an acceptable approach to planning community facilities as long as a better set of decision rules may be defined which describe how standards are to be developed and then applied. Such rules would have to be reflective of the basic nature of a retail system which includes the interaction of the consumer of a good or service, providers of the good or service, and the means of connection between.

Another area of the personal concern is the lack of rigorous treatment of the use of information in the facility planning decision making process. A personal conclusion is that the decision making process is under-defined, therefore not allowing for the successful integration of information to support decision making and at the same time not providing insight into where and how further development of decision assistance routines or models would be beneficial. Thus,

integration of systems approaches are suggested herein through inclusion in a "decision assistance framework" where in appropriate information would be available to the planner at each step in the decision making process.

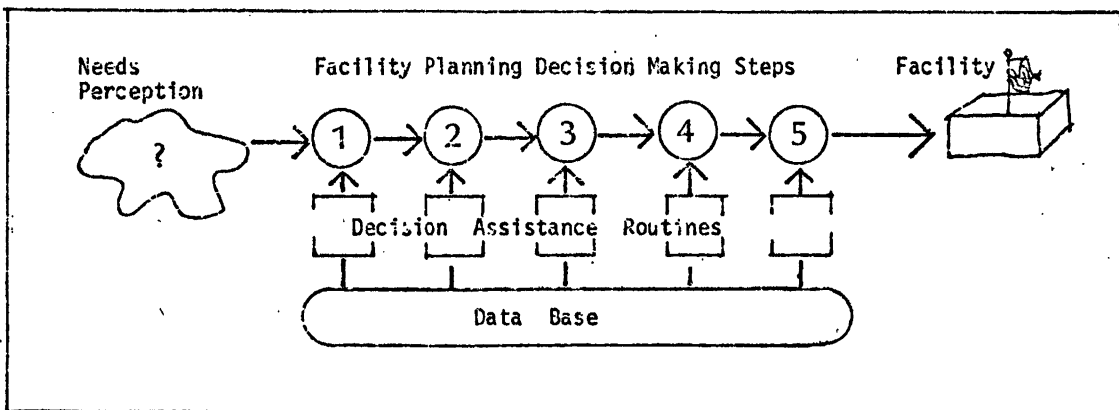


Figure 1-1 A Facility Planning Decision Assistance Framework

It is, therefore, an assumption of this work that providing a decision information base as well as a means for using a number of economic and consumer behavior oriented computational and simulation procedures is a needed and valid direction for facility planning research to follow. Although the focus of the investigation is retailing, this concept and the procedures included may be applied to other types of public community services so long as it is recognized that the objectives on which decisions are based will differ since some measure of public utility, not profit, is the rudimentary concern of facility providers.

## Approach

Chapter II following will review past examples and propose a general decision making process for locating retail facilities. Significant requirements for information and sources which support each step in this process are detailed.

Chapter III will focus on one principal step in this decision making process: that of estimating potential retail sales at a given location. Review of most past work found this to be the primary area of consideration. Discussions will focus on the theoretical aspects of a number of systems approaches which have been developed over time and then identify similarities between the various approaches.

Chapter IV's focus will be on critical analysis of five selected models which have been applied to real planning applications. These are but a few of many different models examined in the research for this thesis. Brief descriptions of other models appear as Appendix I to this work.

An examination of the integration of models into the overall planning decision making process will be made in Chapter V along with an identification and discussion of other areas where further development of techniques or models might further assist the decision maker. Thus, major elements of a decision assistance framework are presented.

Chapter VI provides conclusions, extension of findings for retail facilities to other types of services, and suggestions for areas where further research may be beneficial.

## II. LOCATION DECISION MAKING FOR RETAIL SERVICES

### A. The Retail Location Decision

As cities in the United States have grown, retail firms have also extended their services to meet this growth. Early market research issues focussed on which street corner of central cities were optimal locations for highly traffic oriented businesses. As time passed, the pedestrian focus shifted to automobiles in terms of primary access, and evaluating alternative retail locations became significantly more complex since the consumer could now shop in a larger geographic area. The decision, however, has not changed - a retailer must choose the optimum location for a retail facility so that the likelihood of consumer shopping and consequent sales will be maximized.

The conditions which underlie retail location decisions reinforce the accuracy of the results to the retailer. Location implies fixed capital investment which will require a number of years to amortize either directly through a mortgage or indirectly through a lease. Secondly, the trend has moved increasing toward major capital investment in larger scale facilities such as regional shopping malls, thus, increasing further the relative importance of the location decision.

A review of literature in city planning, economics and commerce-marketing finds that a number of principles have been suggested to guide the location analyst. At the same time several extensive analytical approaches have been explicitly developed. These will

form the focus of discussions herein.

Retail location decisions are faced by a wide range of businesses. The individual store owner-operator is equally concerned by the appropriateness of this location decision as is the large private consortium that is developing a multi-million square foot shopping mall. Retail locations are selected in all communities without respect to size. Community sizes may however, have an influence on which types of facilities are located. Regardless of the specific situation, it is the contention here that a retail location decision making process can be described; and within, specific steps may be discussed.

B. General Principles of Store Location

Nelson (1958) discusses a number of principles which describe the functions and relationships of retail facilities. First he classifies locations as GENERATIVE or SUSCIPIENT<sup>1</sup> depending upon the customer's direct or coincidental attraction from place of residence. A third type, SHARED BUSINESS, is secured by a retail store as the result of the generative power of its neighbor.

Others, Copeland (1923), Holton (1958), and Bucklin (1962), have attempted to define the often used CONVENIENCE, SHOPPING, and SPECIALTY GOODS classification.<sup>2</sup> For the individual consumer, convenience goods are considered to have little price difference between sellers, thus justifying the minimum amount of investment of time in shopping while shopping goods are the opposite, meriting the investment in shopping time and cost for a potential gain in

lower price. Specialty goods, however, generally require special effort by the consumer in shopping and purchasing due to a limited market for the good. The consumer may buy the good because of necessity, although he willingly expends the time in shopping for the most preferred item. These definitions provide an understanding of various patterns of consumer behavior which also can be summarized.

Nelson (1958) attempted to understand and document consumer and retail merchant behavior in 8 basic principles which underlie any process for approaching the retail location problem:

1. Adequacy of present trading area potential: By understanding the population and income of an area, it is possible to determine the maximum amount of money which is likely to be spent on a consumer item.
2. Accessibility of site to trading area: A chief reason for a retailer selecting a site is to have maximum accessibility to potential business or at least to minimize inaccessibility. For generative business the consuming public must have knowledge of the type and variety of merchandise available. Shared business will use the same principle, but will achieve this through the drawing power of generative business. Immediate, visible and convenient access is the prime determinant of susceptible businesses' successful attraction of customers in the area for other purposes.
3. Growth potential: A retailer will prefer to locate in an area with growth potential since it will minimize the risk of declining profit ratios in future.

4. Business interception: A retailer should establish a new outlet between the consumer and his traditional habitual source of goods. "It is much easier to stop them en route, as it were, than to pull them off or away from a beaten path."<sup>3</sup>
5. Cumulative attraction: The synergistic effect of two or more similar or compatible stores located together having greater customer attraction than at a single or individual location.
6. Compatibility: Maximum retail business potential is located where there is no interruption in shopper traffic and customer interchange is at a maximum. Shopping centers and districts receive their essence through this principle.
7. Minimizing competitive hazard: Site location should account for competition of similar firms. Projections and plans should minimize competitive sites nearby, control sites or make sure sites are in a non-intercepting position.
8. Site Economics: A retail site should rationalize the relation between its cost and its productivity. Size, shape, topography, load-bearing capacities, existing improvements, as well as adjacent amenities such as street lighting, availability of utilities, side walk and street conditions, and land uses all affect the potential costs and productivity of a site.

Underlying each of Nelson's principles are the forces which motivate both retail firm and individual consumer behavior. For a retailer the maximization of profits and the minimization of the risk of losses for both individual stores and for the company as a whole is the major objective. While for the consumer, a number of market

researchers and economists have conceptualized him as a rational being who considers not only the price of a purchase item but the cost of obtaining the item, too.

Baumol and Ide (1956) introduced the concept of the cost of a customer's perceived risk of not finding what he or she wants on a shopping trip, i.e. an unsuccessful shopping trip. This results in the customer choosing a store where he believes the greatest number of items carried will minimize the risk of the trip being unsuccessful. Bucklin (1967) looked at the meaning of mass (store size) in consumer preferences and hypothesized that customers think that store size, i.e. variety, will reduce time spent on searching costs. Others, Aspinwall (1962), Doyle and Fenwick (1974) and Stanley and Sewall (1976), have focussed on the importance of store image: relative prices, reputation for quality, layout, parking and service to the customers perceived risk or cost in store choice. Finally, Reilly (1931) and a host of others including Converse (1948), Huff (1963), and Lakshmanan and Hansen (1965) have hypothesized that the most important factor to the consumer is the travel cost which shopping entails. In all research the common element is the fact that a consumer wishes to minimize all costs which shopping entails.<sup>4</sup>

### C. A Store Location Decision Process

Nelson's principles provide a logical context to develop a process for approaching decisions about retail facility location. He discusses in detail the process one uses to locate specific stores as well as shopping centers. The most systematic outgrowth of his work is a detailed "checklist" by which the analyst may rate alternative



candidate sites. By ranking of trading area potential, accessibility, growth potential, business interception, cumulative attraction potential, compatibility, competitive-hazards and site economics (Nelson's Eight Principles), one may narrow the list of potential sites before computation of an estimated sales volume. Although the checklist is long, it gives guidance to the analyst through a highly detailed search of most factors which could affect store location. The analyst in the evaluation will base his decisions on detailed data and well informed judgment.

Applebaum (1968) provides a more directed step-by-step outline for approaching the study of location of a retail facility. His sixteen steps for a supermarket study in an area in which one's firm is represented are:

1. define the objective
2. analyze the economic base
3. study the population and its characteristics
4. ascertain environmental conditions
5. make an inventory of competition
6. appraise competition
7. study consumer attitudes
8. study your own company's market coverage and penetration
9. analyze your own store's performance
10. appraise your own store facilities and locations
11. study areas of underpenetration
12. consider competitor's likely location moves
13. develop a store location strategy plan

14. calculate your own company's future position in the area
15. project investment requirements, profits, and return on investment
16. prepare a written report

For studying a new market area, steps 1-7 and 12-16 only would need to be covered. Such a work process will provide information which will be highly responsive to a firm's needs.

Also this process is close to a general process for retail facility location. The analyst must account for the type of retail facility under consideration and the manner in which consumer shopping patterns will be affected. For instance, ladies apparel shops and grocery stores relate to their customers in a different way.

Nelson's concept of suscipience and compatibility is an important consideration for a ladies specialty shop, while convenience may have a far more profound impact in the grocery location. With either store type, however, the basic decision making process is similar.

Based on the principles of Nelson and the work of Applebaum and others,<sup>5</sup> a retail location decision making process may be described by the following steps:

1. Identify decision to be made.
2. Define objectives. Specify the type of facility, size, timing, market objectives, and specific financial performance objectives required for the new facility. How do these objectives relate to the overall objectives of the firm?

3. Define trade area boundaries. Locate the neighborhood, community, and city in which the facility is to be constructed. Through the use of past experience, is it possible to determine the general market area the facility will probably serve? Are there natural, political or other boundaries which will affect the service area of the facility?
4. Describe trade area. Within the likely trade area describe the resident population, households, income, and socio-economic status. Furthermore, describe other land use patterns and the nature and functioning of roadways and public transportation systems. What are significant geographical features? Are there climatic factors which will affect operation of the retail facility? Finally, are there natural or publically imposed environmental constraints which will impact a facility location decision?
5. Evaluation of gross demand for merchandise sold. Analyst should focus research on an estimate of the total sales volume for the retail trade area for the current and targeted time period of facility development. Research necessary to provide this will include review of the general economic outlook of the region, current and future population, households, household income, income available for purchases and shopping habits by socio-economic status. Sales volume should be estimated for market area as a whole and by workable geographic subdivisions as well. The scale of analysis will determine whether subdivisions are sub-regional sectors, census tracts, block groups, or blocks.

6. Describe existing pattern of facilities. For facility type under consideration, locate all existing outlets. Estimate current sales volumes, customer preferences and image. Evaluate the strength: financial, reputation and otherwise, of the owner/operator firms for each outlet. This should include one's own firm.
7. Describe consumer attitudes and shopping patterns. What is mode of access for customers who shop at facility type in question? What are customer attitudes towards outlets, brands, stores, shopping areas? Describe current pattern of resident shopping at existing facilities.
8. Identify facility gaps or areas of market underpenetration. Compare available sales at residence or work location with current pattern of sales. Find areas of net sales surplus or market underpenetration. If none are found, check for efficacy of intercept locations or terminate location problem with a recommendation for no new sites.
9. Find potential site locations in areas of underpenetration. Decide on store entry strategy - image, size, orientation to market.
10. Estimate economic performance of each site. For each site, test for site related constraints. Will constraints prohibit or significantly alter costs of development strategy? If so, then proceed with next alternative site. What will competitor's response to alternative strategy be? How will this affect his (known and anticipated) strategies for facilities in the area?

What are estimated sales for each existing and proposed outlet?

11. Evaluate economic performance. If acceptable alternatives are found, estimate site acquisition or lease costs. Estimate other costs including utilities, insurance, store overhead, amortization of structure, if new structure is considered. Carry out proforma analysis. Compute rate of return on investment.
12. Test objectives. Does return on any of the sites meet objectives? If not, modify assumptions and repeat or select a new entry strategy step 8.
13. Select a site. Select site which most adequately meets objectives or other reasons for selection.
14. Detail strategy, document. Detail site availability, terms, specifics of market strategy. Document location exercise and significant recommendations.

The major underlying elements of this decision making process include the setting of objectives, definition of a trade area, description of the existing consumer expenditure patterns, identification of possible growth potential, and formulation and testing of strategies for new facility location and development. This illustrated in Figure 2-1.

Time and money will dictate the degree of specificity allowable in using this process. When carried out with a concern for accuracy and details, this decision process requires both extensive effort and a massive volume of information about the trade area.

Many steps are oriented to manual treatment due to amount of judgment required on the part of the analyst. Moreover, several steps stand out as being not only tedious but difficult to define from a conceptual standpoint.

These include: The definition of a trade area; the prediction of future consumer expenditure patterns at a proposed facility; and the detailed, time sensitive, economic analysis of each potential facility site location.

A review of location research literature finds that application of systems approaches have focussed on these areas. Without these approaches any location decision making process may be characterized as being:

1. Pedestrian, time consumptive and expensive;
2. Wrong headed in a macro-sense - while the analyst may use a great deal of judgment in each step, the overall complexity of the process may negate the opportunities for testing a suitable number of alternative for assumptions which are unclear or uncertain to the analyst;
3. Subject to inconsistency - the analyst may make decisions judgments in an inconsistent manner if the process is repeated. Decisions are highly sensitive to an individual analyst's knowledge and skills;
4. Unresponsive to demands for rapid input into firm decisions.

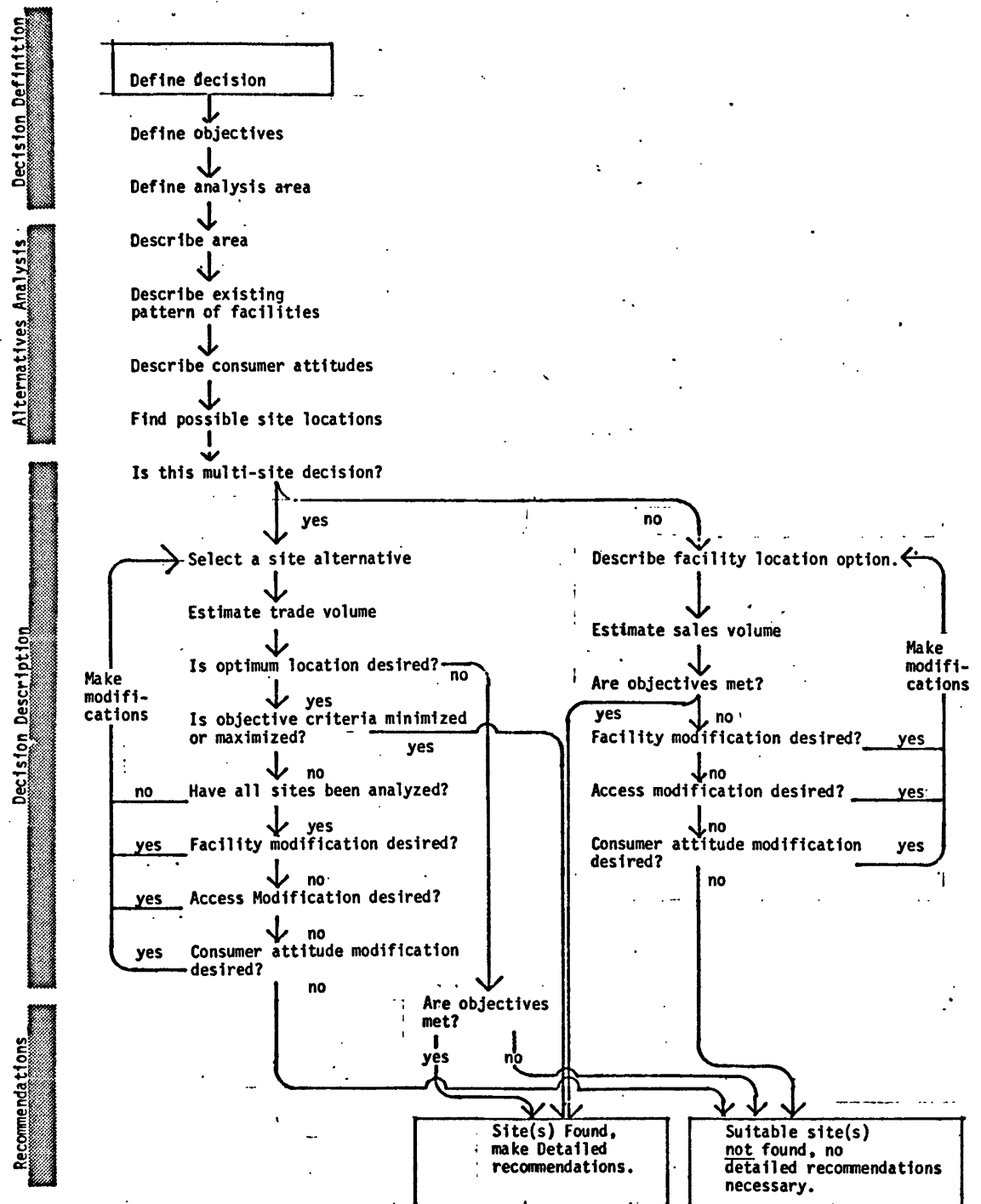


Figure 2-1 A Facility Location Decision Making Process

Chapter III will focus on systems applications in the primary problem area in retail location: estimating sales volume anticipated at any selected location, given some level of information about consumers and other retailers in the area.

Notes: Chapter II

1. Nelson (1958) defines these terms as follows:  
Generative: Store to which consumer is directly attracted from his home.  
Suscipient: Store to which customer is impulsively or coincidentally attracted while away from his place of residence.
2. The Committee on Definitions of the American Marketing Association provided the following classification system in 1948:  
Convenience Goods: Those consumers' goods which the customer purchases frequently, infrequently, and with a minimum of effort.  
Shopping Goods: Those consumers' goods which the customer in the process of selection and purchase characteristically compares on such basis as suitability, quality, price and style.  
Specialty Goods: Those consumer goods on which a significant group of buyers are habitually willing to make a special purchasing effort.
3. Nelson (1958), p. 54.
4. Baumol and Ide (1956) did note that for some for which shopping is a form of recreation the cost may even be negative while shopping may actually cost the consumer, he may be willing to pay for this cost with that portion of personal income spent on recreation.
5. Thompson (1965) provides a similar step-by-step approach.



### III. REVIEW OF SYSTEMS APPROACHES TO RETAIL LOCATION

The most difficult task in any process of retail location decision making is that of estimating gross sales distribution among facilities given some information about 1.) the size and distribution of a local residential or employment population, and 2.) characteristics of the transport system. Difficulty arises as a result of trying to describe the interaction of the parts of this complex system. This chapter will review past work which has dealt with describing this system in attempts to provide some method for deriving a pattern of gross sales.

In an ideal sense, the elements of the retail system include spatially distributed establishments operated by retail merchants, potential customer sales at location of residence or work and all modes of interface between, including phone, pedestrian, automobile, and mass transportation. At any point in time, within a given geographic area, there exists an equilibrium between competing establishments selling a good in terms of the portion of the total potential consumer sales each provides. This may be described by the relative attractiveness and location of each establishment as perceived by all potential customers.

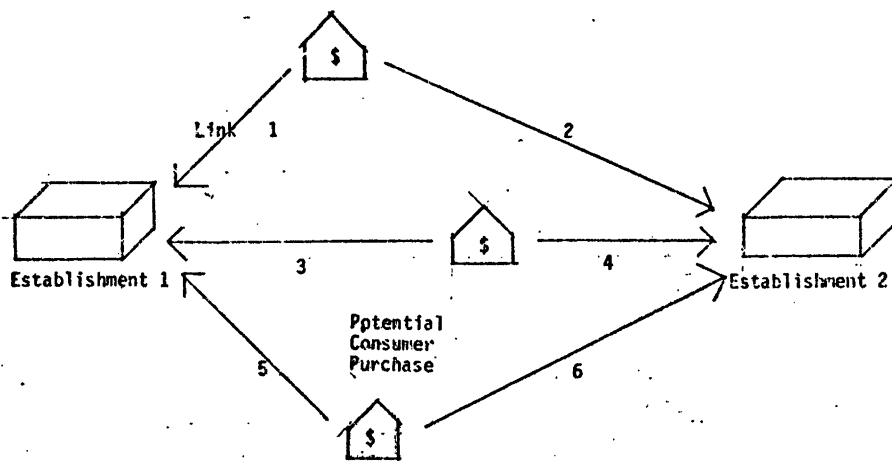


Figure 3-1 Elements of an Idealized Retail System

Most approaches researched attempt to provide a description of this system focussing primarily on the interaction between consumers and retail outlets. In general no one theory was found which described this phenomenon. Instead, a number of differing lines of inquiry were found, with a great deal of similarity between. These will be pointed out as central place theory, rent theory, the gravity concept of human interaction, facility location models, and various behavioral models are discussed.

#### A. Central Place Theory

Although this approach was originally conceived to describe the location of all types of service activities, primary emphasis by scholars over time has been placed on retail establishments. Using an assumption that consumers are equally distributed in space and will always choose to utilize the closest outlet, central place theory treats the consumer and retailer behavior in a highly idealized manner.

The German geographer Walter Christaller and economist August Losch are responsible for its initiation. Christaller (1933) postulates that there is a level of aggregate consumer demand below which a good will not be offered for sale, or in, turn below which a certain type of business will not be profitable and, therefore, will not exist. Dependent upon the volume of sales required to sustain a business, the geographic area necessary for its support will be of direct relation. Thus, within a region there will be as many central places as are necessary to support a population. These central places will be hierarchially distributed from the highest to the lowest order of business. Lower orders of business will be able to locate between as well as within higher order centers.

Christaller added that for every center of the highest order, there are on the average at least three trade areas of the next lowest size and two places of the next lowest order which lay on the hexagonal trade area boundary of the next higher level. This network has a rule of progression such that the number of trade areas increases as another level of trade area is added by a multiple of 3, a  $K = 3$  network. Figure 3-1 illustrates the hexagonal urban hierarchy. Although Christaller hypothesized other hierarchical arrangements, e.g.  $K = 4$  and  $K = 7$ , for any arrangement the  $K$  value remains constant.

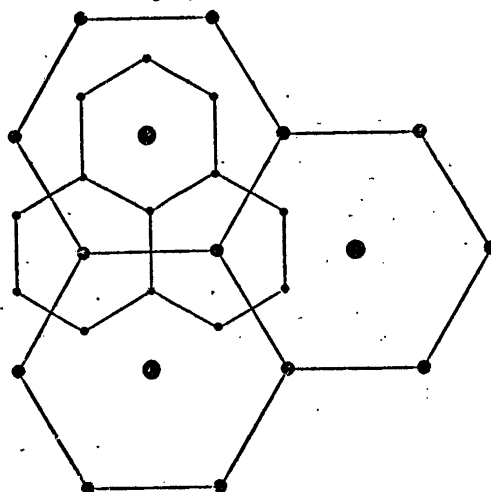


Figure 3-2 A Hierarchy According to Christaller's Marketing Principles.  
from: Shopping Capacity Sub-committee (1970).

Clark and Rushton (1970) argue that this hierarchical structure breaks down when one observes consumer behavior and the competitive interaction of each center. Higher level centers must contain lower level functions in order for the hierarchy to be maintained, yet higher level centers will for the same good be more attractive because of offering higher order goods, and, therefore, will have a proportionally larger market area.

August Lösch (1941a), viewed the urban spatial structuring problem in a manner similar to Christaller, yet since he based his hierarchical structuring on the economics of the firm, his hierarchy is structured from the smallest unit to the largest. Lösch maintained that the market area for any product will be sized such that economies of scale in production are balanced with the costs of transportation. There is a point on the production curve for a firm where the marginal increase of profits from economies of scale becomes less than the marginal increase of costs due to transportation. At that point the boundary of the firm's market area must be drawn. Since this point is

different for each size of firm, Lösch points out that a region contains many overlapping market areas. Lösch, too, hypothesizes that the hexagon is the most economical shape for a trading area. The ordering of these hexagonal trading areas may be random, but Lösch like Christaller, maintains that there are more economical patterns for the arrangement of these areas. By superimposing a number of different sized hexagons on a center and rotating the net to cause maximum coincidence, Lösch was able to provide an arrangement which "...does not deprive any place of its access to every product, and at the same time provides for the best lines of transportation."<sup>1</sup> This approach will result in nearly a continuous sequence of centers rather than distinct tiers like in Christaller's formulation. However, it appears that Christaller's hierarchy may be a more suitable typology for approaching retail location.

A number of researchers have attempted, through geographic case studies, to verify the theories of Christaller and Lösch. For example, Berry, Barnum, and Tennant (1926) found in their studies of rural Iowa that hierarchies could be identified yet at the same time certain linear relationships were seen. They felt that any conclusions from central place studies would be affected by the amount of aggregation and spatial network abstraction permitted. Therefore, definitions of retail functions which differentiate the types of centers are based on types of retail goods ranging from daily-convenience goods: lowest order centers to specialty goods: major urban centers (CBD's in most studies). In most studies an index of centrality of sorts is developed and used to identify hierarchies. These are

usually based on center size and trade mix with some sort of index of trade area size. More complex approaches have also included retail employment, land values, rent, non-food turnover, food turnover, rateable values, number of retail outlets, front footage, ground floor area, land values and floorspace. These studies attempt to identify the discrete hierarchy in patterns of shopping centers which, when viewed collectively, result in a central place system.

Berry (1962a) concludes that at an aggregative level the major characteristic of central places systems is one of macroscopic negentropy<sup>2</sup> represented by:

$$LP = a_1 + b_1 CF$$

where : LP = the logarithm of the population of a central place;

CF = the number of central functions of that place;

$a_1$  and  $b_1$  = empirically determined parameters.

For the outer limit or range, D, of a center, Berry also found:

$$LD = a_4 + b_4 CF \text{ and } K_1 D = A$$

where : A = the trade area of the center in units of area;

LD = the logarithm of the range D;

$K_1$  = a constant;

$a_4$  and  $b_4$  = empirically determined parameters.

Additionally, where  $P_{ex}$  is the number of people residing within the maximum service area defined by D and  $P_{tot}$  is the total population ( $P + P_{ex}$ ) residing within A, a series of limits was observed by Berry:

$$LA \text{ (villages) does not exceed } 10.4 - 2.67 LP_{tot}$$

LA (towns) does not exceed  $9.265 - 2.067 LP_{tot}$

LA (cities) does not exceed  $22.25 - 4.75 LP_{tot}$

where : LA = the logarithm of the area ,

These were based on a series of studies by Berry and others in South Dakota, Iowa, urban and suburban Chicago and are illustrated by Figure 3-2.

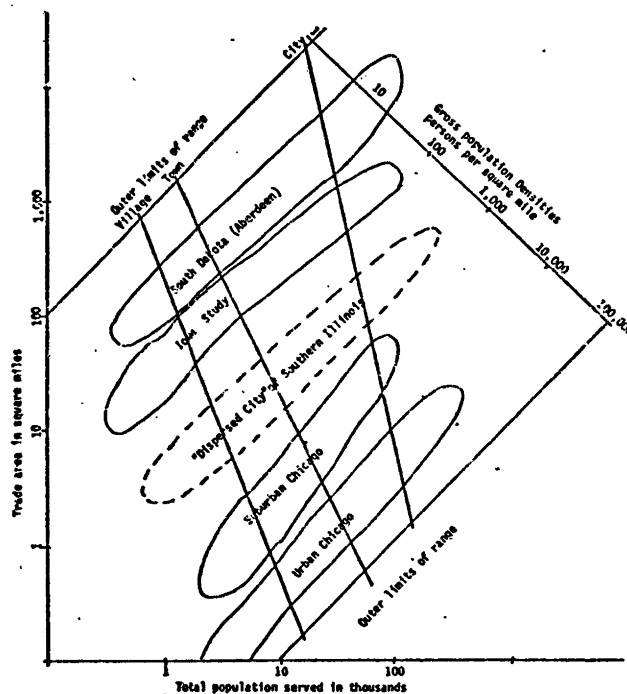


Figure 3-3 Central Place Hierarchies From Berry (1962).

Clark and Rushton (1970) observed that the central place behavioral premise was an inadequate description of spatial behavior in their study of intra-urban consumer behavior of Christ Church, New Zealand. They felt that conceptualization of an "indifference zone" might be useful in trying to establish an attractiveness index. This was based on an observation of consumer indifference to distance in the decision to patronize one place rather than its most competitive alternative. A conclusion was made that spatial models should

contain elements for preferences and spatial opportunities and a probabilistic approach would allow for the representation of diversity while still providing a strong postulate for central place theory.

Murdie (1965) studied the cross cultural differences in consumer behavior in a central place and concluded that culture had a significant effect on structuring of the central place system. Old order Mennonites in Ontario were found to exhibit different purchasing patterns because of both type of goods purchased and their mode of travel.

The main concepts from central place theory which influence systematic approach of the shopping problem are summarized by Cordley-Hayes (1968) as:

- a. There exists a threshold of demand below which a good cannot be offered for sale.
- b. The size of this population (and hence trade area) depends on the type or class of good.
- c. Centers are classified into hierarchial groups according to the size of the trade area; or equivalently classified by the type of goods offered at the center.
- d. Each high ranked center contains the goods offered by lower ranked centers.
- e. Free entry of business produces a contraction of trade areas to their minimum size.
- f. The close packing of circular or hexagonal trade areas generates a set of inter-leaving (nested) hexagonal lattices.

When comparing these concepts with our idealized concept of a retail system, it is notable that the implicit assumption of central place theory is that consumers will utilize the closest center. Thus, central place theorists have focussed on describing trade areas for each type facility and have described competitive interaction only

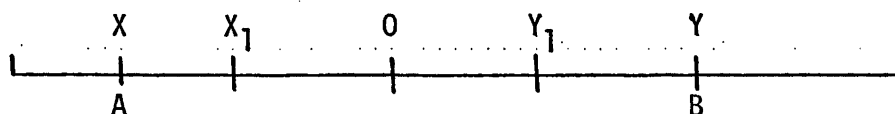


in terms of hierarchies. Because of this, the major contribution of central place theory has been in providing a framework for understanding consumer-retailer behavior on a regional or interregional basis where description of overall requisite support for specific retail functions is needed.

#### B. Classical Price and Rent Theory

Economists have attempted to discuss urban location through the location decision of the individual firm. As in central place theory, one assumes that a firm will locate in the urban setting in a manner that will maximize profits. However, economists have noted that the economy and convenience of agglomeration is an important ingredient of the location decision. This varies, dependent upon the type of retail firm. To describe this, Parker (1960) has used as an analogy the biological equilibrium of competitive, complementary and parasitic species. All shops of the same kind compete, while in some cases, proximal location is complementary to the position of both through provision of better shopping opportunities. Finally, some stores are parasitic in nature and must locate where there are larger volume stores with the intention of "capturing" some of this shopping volume.

The beach problem of Hotelling (1929) is the classical example of agglomeration. If demand is constant along a beach and retailers A and B locate themselves at positions X and Y, they respectively



capture fifty percent of the market. However, to maximize the likelihood of capturing more sales they move closer to the center to new positions at  $X_1$  and  $Y_1$ , each hoping to edge the other out of more business. Finally both sellers converge at point 0 where each has equal probability of capturing up to 100% of the convenience oriented customer trade.

In classical rent theory, the land market has as its center a dense, highly accessible central city and all land is assumed to gain its value due to physical proximity to this center. In the general case, specific means of access is not an issue. Alonso (1964) attempted to numerically describe the structure of rents based on the competitive allocation of sites to land uses which pay the highest price for each parcel. Assuming the market as the center city, there exists for each land use a rent gradient or negative progression of rents varying with the use's sensitivity for proximity to the market. Thus, at any point in the urban place there exists a land use or uses which will bid the highest price for a parcel of land with its associated access attributes. As one proceeds from the center city outward the price or rent for land will be represented by bid price curve which represents the aggregation of the individual use's highest bids for parcels in specific locations.

Alonso represented an individual's bid curve with a utility function:

$$Y = PX + P(r)q + k(t)$$

where :  $Y$  = income;

$P$  = price of the composite good (all other expenditures);

$Z$  = the quantity of the composite good;

$P(t)$  = the price of land at distance  $t$  from center city;

$q$  = the quantity of land;

$k(t)$  = the commuting costs from center city to distance  $t$ .

Thus at a given level of utility,  $U_0$ , for an individual a bid price may be defined by

$$\text{Price} = P_i(t) [U_0]$$

and subject to the constraints:

- a.  $Z$ ,  $q$ , and  $t$  lie on the curve representing the level of satisfaction  $U_0$ .
- b. An individual should dispose all of his income on land, commuting, or the composite good.
- c. The prices paid by an individual for land and the composite should only describe the utility he receives from them.

A similar function can be made for the firm as well, so that both sets of curves may be used to construct the bid price curve for the urban place. Alonso, then, further constrained his model with the assumption that all land in the urban context should be allocated to some sort of individual or firm use and, moreover, that the amount of land to be allocated is fixed.

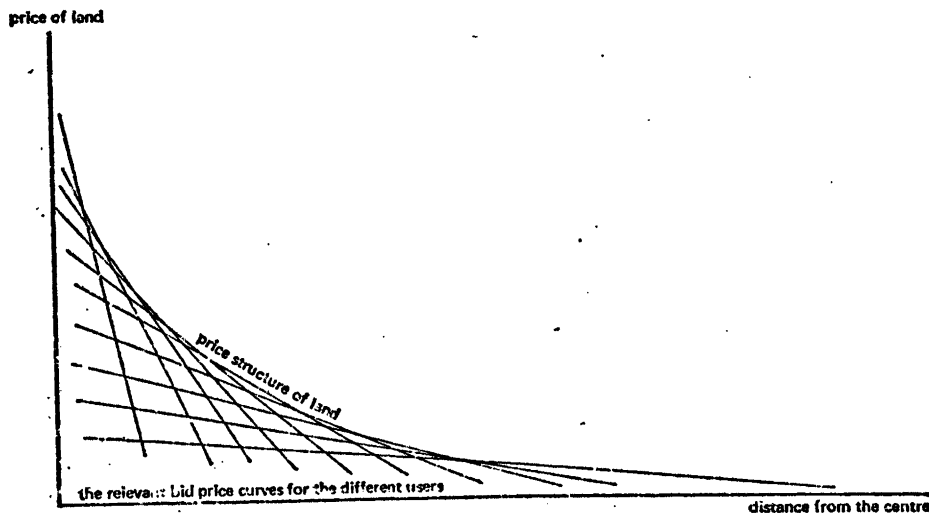


Figure 3-4 Bid Price Curves Similarly Ranked for Steepness Throughout  
from: Shopping Capacity Subcommittee (1970)

Alonso's model uses as its central concept the idea that firms (and individuals) locate to minimize their accessibility costs. The notable contribution of this approach, therefore, is that accessibility has a cost associated with it and that this cost plays a role in firm location. This may be contrasted to the implicit manner in which accessibility is treated in trade area size in central place theory.

As an approach for describing an idealized retail system, most price and rent theories focus on the elements of access and interaction between firms as key determinants of location. In a macro-sense these theories are useful in understanding firm location behavior without specifically focussing on the nature of consumer behavior. Consumers are treated in an aggregate manner, e.g. the "market" in Alonso's model.

### C. Gravity Concept of Human Interaction

This frequently used concept was developed originally from an analogy with Newtonian physics of matter. Molecular behavior is not normally

predictable on an individual basis, yet if large numbers are observed, then certain conclusions may be drawn about their activity. Carrothers (1956) concludes that the same may be true for human behavior. Group behavior may not resemble individual human behavior of a group member. Thus, Carrothers warns about the necessity for judging the threshold where human individual decision making capacities affects intended results from the use of an analogy such as the gravity concept. Carrothers states that the gravity concept of human interaction postulates that an attracting force of interaction between two areas of human activity is created by the population masses of the two areas, and a friction against interaction is caused by the intervening space over which the interaction must take place. Two centers interact as a direct function of their population and as an inverse of the distance between. Carrothers uses the mathematical form:

$$I_{ij} = \frac{f(P_i, P_j)}{f(D_{ij})}$$

where :  $I_{ij}$  = the interaction between center i and center j;

$P_i, P_j$  = the population of areas i and j;

$D_{ij}$  = the distance between center i and center j.

The explicit treatment of mass, e.g. population, may be compared with the central place approach where the number of retail functions in a center at j,  $n_j$ , in a trade area i where treated as a function of the population of i:

$$n_j = f(P_i)$$

Distance in central place theory is implicitly assumed in the size of the trade area i. Use of the gravity analogy has been considered by

social scientists for over a century.<sup>3</sup> H. C. Carey (1858) reasoned that social and physical phenomena may be described by the same law:

"Man, the molecule of society, is the subject of Social Science... The great law of Molecular Gravitation (is) the indispensable condition of the existence of being known as man... The greater the number collected in a given space, the greater is the attractive force that is exerted... Gravitation is here, as everywhere, in direct ratio of the mass and the inverse one of distance.<sup>4</sup>

E. G. Ravenstein (1885) used a similar concept in describing the pattern of urban migratory movement as being towards cities of large population and decreasing as the distance between the city and hinterland increases. Similar empirical evidence was explained by E. C. Young (1924). The volume of migration to a city is directly proportional to the attractiveness of the city and inversely proportional to the square of the distance between the point of origin and the city.

The first retail application of the gravity concept came in the late 1920's when W. J. Reilly attempted to find an inexpensive means to describe the nature of market areas for specialty goods in cities. Based on observation, Reilly (1931) found that population and distance were acceptable indices which could be used for determining market areas. The "Law" which he formulated said:

Two cities attract retail trade from any intermediate city or town in the vicinity of the breaking point approximately in direct proportion to the population of the two cities and in inverse proportion to the square of the distances from these two cities to the intermediate town.<sup>5</sup>

The mathematical expression of this statement is:

$$\frac{B_a}{B_b} = \left( \frac{P_a}{P_b} \right) \left( \frac{D_b}{D_a} \right)^2$$

where :  $B_a$  = the proportion of retail trade from the intermediate town attracted by city a;

$B_b$  = the proportion attracted by b;

$P_a$  = the population of a;

$P_b$  = the population of b;

$D_a$  = the distance from the intermediate town to a;

$D_b$  = the distance from the intermediate town to b

Reilly and later P. D. Converse (1946) eventually developed five more laws of retail gravitation. The second by Converse was used to measure the movement of shopping goods trade "breaking point" between two cities:

$$D_b = \frac{D_a + D_b}{1 + \sqrt{\frac{P_a}{P_b}}}$$

The third law as developed by Converse stated:

A trading center and a town in or near its trade area divide the trade of the town approximately in direct proportion to the populations of the two towns and inversely as the squares of the distance factors, using 4 as the distance factor<sup>6</sup> of the home town.<sup>7</sup>

Mathematically, this may be expressed as:

$$\frac{B_a}{B_b} = \left( \frac{P_a}{H_b} \right) \left( \frac{4}{d} \right)^2$$

where :  $H_b$  = the population of the home town

$d$  = the distance to the outside town

4 = an inertia factor

Laws four and five were developed by Converse to be used in situations where the trading center is more than 20 times the size of the intermediate town.

These are:

$$\frac{B_a}{B_b} = \left( \frac{P_a}{P_b} \right) \left( \frac{D_b}{D_a} \right) \quad \text{and} \quad D_b = \frac{D_a + D_b}{1 + 3 \sqrt{\frac{P_a}{P_b}}}$$

Finally, Converse modified formula #3, after field tests to be:

$$\frac{B_a}{B_b} = \left( \frac{P_a}{H_b} \right) \left( \frac{1.5}{d} \right)^2$$

The development of these laws involved several national surveys which were not based on a probability sample. The results obtained were used to find the exponent of 2. Of the 255 cities sampled by Reilly, the distance exponent ranged from less than 1.5 to 12.5, but the greatest frequency was found between 1.5 and 2.5, hence the use of 2.

The laws were tested and found to be reasonably good predictors, yet none of the tests can imply the correctness of the laws. For both Reilly and Converse the primary objective was not so much to postulate a new "law" as to provide an economical method to the practitioner for determination of market areas.

Others at the same time looked at the gravity concept more seriously. J. W. Stewart (1949) and G. F. Zipf (1947) used the Newtonian physics formulation as set forth by Carey. They represented the force of interactions between two bodies of population as directly proportional to the product of the populations and inversely proportional to the square of the distance between them.

$$F_{ij} = \frac{P_i P_j}{D_{ij}^2}$$



Where :  $F_{ij}$  = the force of interaction between concentration  
i and j;

$P_i$  = the population of location i;

$P_j$  = the population of location j;

$D_{ij}$  = the distance between locations i and j.

When extended to the total interaction between a region i and all other regions this formula becomes:

$$F_i = k \sum_{j=1}^n \frac{P_i P_j}{D_{ij}^2}$$

Where : k = a constant.

Stewart attempted to develop a measure of the potential for the possibility for interaction of two populations. Given an individual at location i, the potential for possible interaction of i with population j is measured:

$$V_{ij} = k \frac{P_i}{D_{ij}}$$

Where :  $V_{ij}$  = potential at i of the population of area j.

The total possibility of interaction between the individual and all other areas may be expressed as

$$V_i = k \sum_{j=1}^n \frac{P_i}{D_{ij}}$$

Where  $V_i$  = total population potential at i.

While both Reilly and Zipf defined the distance factor as the square of the distance; Anderson (1953), Carrothers (1956), and Huff (1963) believed that the distance parameter should be subject to calibration.

Anderson modified the distance term with  $\alpha = f (1/P_j)$ ; thusly, inversely related to population size. Carrothers posed that the exponent was inversely related to distance itself, "friction of unit of distance against interaction caused by short distances is disproportionately greater than friction per unit of distance caused by larger distances."<sup>8</sup> This may be expressed as:

$$\alpha = f (1/D_{ij})$$

Stewart (1948) found that population, too, varied in its role in determining market areas as a function of the value of the molecular weight in his physical analogy.

In the late 1950's and 1960's, scores of applications of the gravity formulation were made. Various authors began to modify the definitions of the factors, but the concept of physical analogy remained the same.

Huff (1963) incorporated the concept of probability into the gravity concept and used floorspace of retail centers to replace population and travel time to replace distance. His model may be stated for each type of good as:

$$P_{ij} = \frac{F_j/t_{ij}^b}{\sum (F_j/t_{ij}^b)}$$

Where :  $P_{ij}$  = the probability of a consumer shopping in a center at  $j$ ;

$F_j$  = the floorspace at center  $j$ ;

$t_{ij}$  = the travel time from  $i$  to  $j$ ;

$b$  = an empirically estimated parameter.

Lakshmanan and Hansen (1965) put the gravity model to use in the Baltimore area in estimating total sales in zone j,  $S_j$ . A revised gravity formulation used was:

$$S_{ij} = (E_i) \left( \frac{F_j / t_{ij}^b}{\sum F_j / t_{ij}^b} \right)$$

Where :  $E_i$  = total sales at zone i;

$S_{ij}$  = sales from zone i to be spent in zone j;

$F_j$  = the floorspace at center j;

$t_{ij}$  = the travel time from i to j;

$b$  = an empirically estimated parameter.

The Haydock<sup>9</sup> shopping model and Lewisham<sup>10</sup> models employed in England during the same period were similar except for the empirically calibrated structural parameters a and b, where a was used with the floor area term and b distance.

Lewis and Traill (1968) consider it a weakness of the gravity model that competition is not properly considered. Shops provide opportunities to individuals who may claim them to shop therein. The opportunity - claimant approach described this behavioral pattern. At a given location there may be competition between claimants and this may be expressed as:

$$C_j = P_j + \sum_i \frac{P_i}{D_{ij}^b}$$

Where  $C_j$  = the total number of effective competitors at zone j;

$P_j$  = the population of claimants located in zone j;

$P_i$  = the same for zone i;

$D_{ij}$  = the distance from zone  $i$  to  $j$ ;  
 $b$  = an empirically determined exponent.

This will modify the standard gravity model to:

$$S_{ij} = (E_i) \left( \frac{F_j / C_i^a D_{ij}^b}{\sum F_j / C_i^a D_{ij}^b} \right)$$

Where :  $F_j$  = some measure of facilities available;  
 $a$  = an empirically determined parameter;  
 $S_{ij}$  = sales from zone  $i$  to be spent in zone  $j$ ;  
 $E_i$  = total sales at zone  $i$ ;  
 $C_j$  = the total number of effective competitors at zone  $j$ ;  
 $D_{ij}$  = the distance from zone  $i$  to zone  $j$ ;  
and  $b$  = an empirically determined parameter.

Schneider (1959), however, comments on the unappropriateness of applying the gravity analogy. There is not a clear relation between lines of force and lines of movement in the retail application; moreover, this approach does not deal with frequencies of behavior. An alternative approach was provided where the probability of a customer's trip finding a terminal at an element of a region is proportional to the number of terminal opportunities contained in the element. At the same time a customer prefers his trips to be as short as possible extending in length only when failing to find a terminal. Stouffer (1940) had already defined this variant of the gravity approach in his study of residential mobility. He too challenged that there was no necessary relationship between mobility and distance, that, instead: "The number of persons going a given distance is directly proportional to the number of opportunities at that distance and

inversely proportional to the number of intervening opportunities." <sup>11</sup> This may be expressed as:

$$\frac{\partial y}{\partial s} = \left(\frac{a}{x}\right) \left(\frac{\partial x}{\partial s}\right)$$

Where :  $y$  = the number of persons moving from an origin to a circular band of width  $as$ ;

$x$  = the number of intervening opportunities;

$a$  = a constant.

Schneider operationalized this concept in a shopping oriented model

$$\frac{\partial Q}{\partial V} = -L(Q)$$

Where :  $V$  = the cumulative number of nearer opportunities already passed by;

$Q$  = the proportion of trip makers continuing beyond opportunity  $V$ ; and

$L$  = a fixed rate of trip attenuation

and when  $V = 0$ ,  $Q = 1$

$$Q = e^{-LV}$$

Harris (1964) suggests that spatial separation is not a matter of indifference to a consumer, and thus, his approach allows for two separate parameters. Where  $M$  is the parameter which governs the attenuation of trips due to distance and  $N$  is the same for attenuation due to opportunities, Harris defines his intervening opportunities model as:

$$\frac{\partial Q}{\partial V} = -NQ \quad \text{and} \quad \frac{\partial Q}{\partial P} = -MQ$$

Where :  $Q$  = the proportion of trip makers continuing beyond opportunity  $V$ ;

V = the cumulative number of nearer opportunities  
already passed by;

N = a parameter which governs the attenuation of trips  
due to opportunities ;

M = a parameter which governs attenuation due to distance.

The development of the gravity model as a means of describing retail trade provides an interesting progression. Early formulations such as those of Reilly and Converse used the gravity concept to represent the normative behavior of customers and retailers. More recent approaches, however, reflect a more probabilistic approach, e.g. the work of Schneider (1959), Huff (1963), and Wilson (1968). Intervening opportunities was thought to be a more theoretically sound approach to the problem than the early gravity analogies. It should be noted that other analogies have been applied besides the gravity concept. Within the last 10 years a number of researchers have used an approach based on an electric circuit analog in attempt to describe community service systems including retailing.<sup>12</sup>

Of the approaches reviewed, thus far, the gravity concept provides a more explicit representation of a retail system than does central place or rent theory. Consumer: retailer interaction over distance is treated directly, unlike central place where trade area size and hierarchial pattern represent interaction. All factors, e.g. shopping center drawing power and distance to centers, being equal, a gravity formula will attribute equal likelihood of a shopper visiting any of several centers, while central place implicitly must assume

the customer to always go to one center, since residents are assumed not to shop outside of their own trade areas. Geographical units are, therefore, not as well defined in gravity approaches as they must be in central place.

Unlike rent theory, costs of location are not represented at all in gravity models. Yet accessibility is implicitly represented by 'location' in rent theory. The two approaches are, in fact, quite compatible since gravity models describe relative accessibility, while rent theory postulates that costs of a site increase as a location becomes more accessible. Rent theory capitalizes relative accessibility into the cost of a site.

#### D. Facility Location Assignment Routines

Another approach has been developed in operations research for warehouse or plant location, and may be a useful analog for retail location, also. In these operations research location models, the number, location and size of sources (in our case retail outlets), which will supply a given set of destinations (households at place of residence) with some commodity or service, must be determined.

Cooper (1963) describes the problem solved using such a model:

- Given:
1. the location of each destination;
  2. the requirements of each destination, and
  3. a set of shipping costs for the region of interest;

Determine:

1. the number of sources,
2. the location of each source, and
3. the capacity of each source.

Revelle, Marks, and Liebman (1970) describe the objective of the problem as minimizing the sum of facility and transportation costs, given the demand at a given number of areas and a number of alternative facility sites. The number, location, and size of facility to be built will be found subject to the objective.

In solving such a problem, transportation costs may be decreased only at the expense of increased facilities or vice versa. However, at some number of facilities in an optimal pattern of location, the total cost is assumed to reach a minimum. Marginal costs for increasing or decreasing facility numbers or changing location patterns will increase once this optimal pattern and number of facilities is found.

This reasoning is parallel to rent and location theories previously discussed. Alonso (1964) theorizes that location of an individual or firm is a function of both the costs of rent (or location price) and the benefits of accessibility. A firm will locate where marginal benefits of increased accessibility will exceed the relative marginal increase in cost of rent and the rent is the highest bid of all possible land uses. Lösch (1938) argues that the location of firms serve the smallest trade area which will support the economic operation of the firm; thus, minimizing travel costs. In rent theory, central place theory and operations research location models, costs to the consumer of travel are considered by a firm in its location decision.



The mathematical formulation for most operations research location models is:

$$\text{Minimize: } Z = \sum_{j=1}^n \sum_{i=1}^m d_{ij} (X_{ij}) + \sum_{i=1}^m F_i (Y_i)$$

$$\begin{aligned} \text{Subject to: } \quad & \sum_{j=1}^n X_{ij} = Y_i && i = 1, 2, \dots, m \\ & \sum_{i=1}^m X_{ij} = D_j && j = 1, 2, \dots, n \\ & X_{ij} \geq 0 && i = 1, 2, \dots, m \\ & && j = 1, 2, \dots, n \\ & Y_i \geq 0 && i = 1, 2, \dots, m \end{aligned}$$

Where :  $X_{ij}$  = amount shipped from facility  $i$  to demand area  $j$ ;

$Y_i$  = total amount shipped from facility  $i$ ;

$d_{ij} (X_{ij})$  = the cost of shipping quantity  $X_{ij}$  from  $i$  to  $j$ ;

$F_i(Y_i)$  = the cost of establishing and operating a facility at site  $i$ , where  $Y_i$  is being shipped from  $i$ ;

$D_j$  = the demand at area  $j$ ;

$n$  = the number of demand areas;

$m$  = the number of proposed facility sites.

The function  $F_i(Y_i)$  is often non-linear due to the economics of scale in facility construction and operation. Therefore, operations researchers have not been able to use linear programming successfully to solve the problem and other approaches have been devised to arrive at both optimal and sub-optimal solutions.<sup>13</sup>

Although the advancement of thought about this approach has increased

significantly during the last 15 years, the problem has been conceptualized in more simple terms for several centuries. Cooper (1963) cites that as early as 1647 Cavalieri considered the problem of finding the point, the sum of whose distances to three points is a minimum, and found that each side must have an angle of less than  $120^\circ$  with the minimum point.

Others, through time have developed this idea. Cooper points out that in 1834, Heinen noted that the minimum location in a  $120^\circ$  triangle was its apex. Fagnano in 1775 found that the minimum point connecting the vertices of a quadrilateral to be the intersection of the diagonals. Tedenat in 1810 and later in 1937 Steiner noted that for  $n$  points, the sum of the cosines of the angle between an arbitrary line and the line which connects the minimum point to all other points to be zero. Georg Pick in the appendix of Weber (1929) describes the least cost problem as:

$$K = a_1 r_1 + a_2 r_2 + a_3 r_3$$

Where :  $a_n$  = the tons of material to be transported from a to  
the place of production

$r_n$  = the distance between the place of production and  
the source.

$$n = 1, 2, 3$$

$K$  will be the least cost when each (a) (r) product is in physical equilibrium with the others.

Isard (1956) also considers the Weberian theory but adds multiple sources and places of production (in their own market areas with their own sources) and multiple products. This does not pose the problem

of multiple sources which are considered in warehouse location models. Revelle, et. al. (1970) have discussed a number of recent formulations which have been developed for the plant location problem. The reader is referred for further discussion of these approaches. The warehouse or plant location problem approach will allow for analysis of facility location where the interaction and costs of the location decisions of others is not important. Retail application of this approach may yield an optimum pattern of branch store locations for a distributor; however, it would have to be assumed that the retailer maintains a monopolistic position. Since most retailers don't, the effect of competition must also be considered.

A more sophisticated variant of the warehouse or plant location problem is the quadratic assignment problem where the net utility of a particular site for a particular activity depends not only on capital, operating, and transport costs but also on the interaction costs of other facilities' selected locations.

As observed by Hopkins (1975), economic theory of the market system may be represented by this model. Through the market system of allocation, prices for resources provide all the necessary information needed for optimal allocation and no a priori plan of future actions by others is required.

Hopkins has expressed the quadratic assignment formulation as:

$$\text{Maximize: } \sum_{i=1}^n \sum_{k=1}^n a_{ik}(X_{ik}) - \sum_{i=1}^n \sum_{k=1}^n \sum_{j=1}^n \sum_{h=1}^n C_{ijkh}(X_{jh})(X_{ik})$$

$$\text{Subject to: } \sum_{i=1}^n X_{ik} = 1, \sum_{k=1}^n X_{ik} = 1, X_{ik} = 0 \text{ or } 1$$

Where :  $C_{ijkh}$  = cost to activity  $i$  in location  $k$   
 when activity  $j$  locates at  $h$ , and  
 $a_{ik}$  = the profit for activity  $i$  in location  $k$  exclusive  
 of interactions and land.

For the individual locator, utility may be expressed as:

$$U_{ik} = a_{ik} - \sum_{j=1}^n \sum_{h=1}^n C_{ijkh} (X_{jh}) - P_k$$

Where :  $P_k$  is the price of the location;  
 $i$  = the activity number;  
 $k$  = activity location;  
 $j$  = competitor's activity; and  
 $h$  = competitor's location

Koopmans and Beckman (1957) attempted to examine whether an optimal assignment in the quadratic assignment problem can be sustained by a system of rents on locations and on the prices on intermediate commodities that depend on the location at which each commodity is quoted. They found that an individual activity cannot make a choice based only on its profitability and price. The value of an interaction term must also be known; this term is the result of the location decisions of others. Heffley (1972) and Hopkins (1975) found that price may be an appropriate means for arriving at an optimal allocation, if and only if difference in income due to location is sufficiently large to dominate the differences in interaction costs. The value of the quadratic assignment approach is that it can account for the cost of location decisions by others in finding the optimal location of a set of facilities. Therefore, if quadratic assignment

is used in describing a retail system, costs of interaction between competitive locations may be considered. This representation is not made with the same degree of explicitness in all other approaches reviewed.

#### E. Consumer Behavior Models

It is interesting to note that most approaches discussed up to this point attempt to explain interaction between consumers and outlets in a retail system through the effects of mass (number or size of retail outlets) and distance. Only in some later gravity applications are other factors such as attractiveness, consumer socio-economic status and the like considered. In general, all approaches consider consumer behavior in a highly aggregated manner and usually treat the consumer as a rational economic being. Central place, rent and plant location theories describe a retail system through the rationality of the firms which make location decisions. In contrast, this section will focus on a number of models describing consumer behavior which have been developed by economists and market researchers over the last 20 years.

Alderson (1957) describes the consumer as a rational problem solver sensitive to the market stimuli provided him. For example, informative and persuasive advertising was felt to be requisite in a successful retail marketing campaign. Alderson noted in his "functional approach" to marketing the extreme degree of heterogeneity which characterizes U.S. Market behavior. Consumer tastes are diverse and so, too, are thoughtful market strategies. Hence, a heterogenous

supply is needed to meet a heterogenous demand.

A number of authors including Baumol (1956), Dash/Schiffman (1976), and Bucklin (1966) note economic utility as perceived by the customer. This may be reflected in a number of behavioral patterns.

Consumers do generally attempt to minimize driving time so long as convenience shopping is concerned. In a study of the Cleveland area, Bruner and Mason (1968) observed that a shopping center's trading area is limited by driving time and that the significant driving dimension for most areas is 15 minutes. Ambrose (1968), in a study of the Sussex coast area of England notes that for most goods a linearly decreasing preference for shopping at a center may be observed within 2.0 to 2.5 miles of a center location. In addition, it was observed that additional "willingness" to travel excess distances was exhibited when shoppers made purchases of personal goods such as clothes rather than food. Shoppers apparently were tolerant of some additional travel costs to insure an adequate level of choice.

Bucklin (1966) studied shopping behavior in detail. He hypothesizes that the consumer is a profit-maximizing entity who equates the probable return from seeking a better price or superior quality, against the out of pocket costs of shopping and the wear and tear upon his capital and physical being. To test the hypothesis, a survey was made in the Oakland, California area to test three basic generalizations:

1. The customer will shop more extensively where the cost of shopping is low;

2. The customer will shop more extensively when he or she initially knows little about the product which is being bought and the store that sells it; and
3. The consumer will shop more extensively when the value of the product is high.

The results of his survey corroborated the generalizations made except that consumers with little or no information about product features did not shop more extensively than those with greater knowledge. Such findings support Alderson's and others' concept of rational consumer behavior.

William Baumol (1956) also based his hypotheses on variety in retailing outlets on the rational consumer's desire to maximize total utility. The costs of shopping to a customer include not only that of the goods purchased but the cost and trouble of transportation, searching for items in a store, the time and opportunity costs in finding the initiative to start to shop. Thus, a consumer in making shopping decisions will select a store where the probability of a successful shopping trip, i.e. that he will find the items shopped for, will be the greatest. This leads Baumol to the hypothesis that there is an optimum level of variety for a retailer to maintain which will allow the consumer adequate choice and not begin to diminish marginal returns due to inventory size. Other researchers such as Doyle and Fenwick (1974) have studied the importance of store image in the customer's store selection decision. Image encompasses such factors as relative prices, variety of goods sold, reputation for

quality, layout, and parking facilities. They observe that a consumer will maximize his total utility involved with goods purchases including secondary costs. Image may be an important part of these costs.

Dash, Schiffman and Berenson (1976) developed a store choice paradigm based on the customer's perception of risk. This was based on earlier hypotheses which identified risk as another secondary cost of goods purchases. It was found that purchase likelihood was inversely related to perceived risk. A direct relationship was also found between consumer brand loyalty and perceived risk. Based on product importance, perceived product risk, and general self-confidence of the consumer, the same product would be shopped for in a specialty store if the consumer was more self confident, perceived less product risk, and felt that the product was of more importance. On the other hand, a department store would be visited if the customer was less self-confident, perceived more product risk and less product importance.

Past experience is also a factor which affects store selection. Rao (1969) hypothesizes that the more recent a customer's experience is with a store and the more frequent visits occur to a store, the greater the likelihood exists that a consumer will repurchase a product in that store. This concept was originally developed by Kuehn (1962) in a "linear learning model" for brand switching. This hypothesized that the more experience a customer had with a brand, the greater the likelihood will be that the consumer will repurchase the brand.



In summary, market researchers and economists have provided a number of hypotheses related to the behavior of consumers.

1. Consumers are heterogeneous and general axioms of consumer behavior cannot be totally substantiated.
2. Consumers, in general, are rational in terms of maximization of utility associated with their purchases and purchasing activities.
3. Total shopping costs include not only primary purchase prices but secondary costs such as: travel time and distance, shopping time, product or shopping trip failure, possible product repair, opportunity costs for time used for shopping, and personal costs of fatigue due to the shopping exercise.
4. These are reflected in consumer store choice behavior. A consumer will select a retail facility based on his real or perceived cognition of travel distance, store image, shopping opportunities, risk of trip failure, risk of product failure, product price, and past experiences with shopping at the facility.

Behaviorial models have focussed on describing interactions in a retail system through the consumer's decision making process. This provides a useful contrast to central place, rent and plant location theories where the focus primarily on rational firm location. Many consumer behaviorial tendencies discussed have been included in various formulations of the gravity model. A good example may be found in Stanley and Sewall (1976).

This chapter has examined a number of approaches which describe the

interactions of consumers and retailers in what may be called a retail system. It should be noted that no one theory of retailing was observed. Instead were found a number of differing lines of investigation which attempt to explain the general interrelationships of a retail system.

Chapter IV will take a more detailed look at five selected models in application.

### Notes: Chapter III

1. Lösch (1938), p. 74.
2. Berry makes a direct analogy to the information theory concept of entropy as an expression for uncertainty in information as the logarithm of the number of states a system may have, Shannon and Weaver (1949). Berry, (1962b) states:  
 Central functions are thus measures of the 'informational content' of central places. A condition of macroscopic negentropy exists in the distribution of central functions in the aggregate, and therefore so must an organizational hierarchy in which less ubiquitous central functions are related to more ubiquitous central functions in constant per cent ratios.  
 It is interesting to also note that the entropy concept is later used by Wilson (1968) a number of urban modelling applications including a shopping gravity model. See Cordley-Hayes (1968) for a discussion.
3. By comparison note that central place theory dates back to the 1930's and most modern price and rent theories to 1910, i.e. Weber (1909, translated: 1924).
4. Carrothers (1956), p. 94.
5. Swartz (1963), p. 11.
6. Converse (1946) later after testing found a lower factor, 1.5, to be more suitable for the Chicago area.
7. Swartz (1963), p. 14.
8. Carrothers (1956), p. 97.

9. Department of Town Planning, University of Manchester (1966).
10. Rhodes and Witaker (1967).
11. Carrothers (1956), p. 98.
12. See: Ellis (1966), White and Ellis (1971).
13. If all constraints in the formulation are linear, the warehouse location problem may be solved using linear programming to minimize costs. However,  $F_i(Y_i)$  is usually non-linear because of the large fixed investment in land, foundations, utilities, rail sidings, etc. before any amount of goods may be stored or manufactured. See Revelle, Marks, and Liebman (1970), p. 697.

#### IV. CRITICAL ANALYSIS OF SELECTED MODELS

##### A. A Framework for Analysis

The preceding chapter has examined a number of approaches which attempt to describe the interactions of consumers and retailers in what may be viewed as a retail system. Through these approaches a number of models have been developed which may have practical application in retail location analysis, primarily aiding in the task of evaluating sales volume potential at a given location. Practical application of these models usually results in some benefit to the researcher, however a number of problems may be in evidence. This chapter will, therefore, focus on criticism of selected models in application.

Five models from the various approaches of Chapter III have been selected because they appropriately represent the range of geographic application, theory and practical complexity found. The models are:

1. a formulation of central place theory by Berry (1965);
2. a refined gravity model following Huff (1963) by Stanley and Sewall (1976);
3. micro-analytic approach of consumer shopping behavior modelling by MacKay (1972);
4. a hypothetical application of a plant location model following Reville, et. al. (1970); and
5. a retail system model by White and Ellis (1971).

Criticism of each model in application will attempt to center around major advantages a retail facility planner may gain in using each

model as well as any weakness which may be inherent. The general discussion will respond to the following order of questioning.

1. What was the reason for the development of the model? What objective does it serve?
2. What are the underlying theoretical principles the author employed in developing the approach?
3. At what geographical scale was analysis made using the approach, and what type or types of facilities were dealt with? Are certain geographical or other restrictions or assumptions made so as to make application possible?
4. How is the model formulated?
5. What information is required?
6. What information for location decision making is provided by the model?
7. What are the strengths and weaknesses of the approach? How closely does the information resulting describe the real situation? Can such a determination be made? Is the model static or dynamic in terms of changing assumptions? Is time considered or is model cross sectional only? How economical is approach? Would it likely be used again and again? Why not?

For a detailed presentation of a number of other retail models reviewed the reader is encouraged to refer to Appendix I.

B. The Retail Component of the Urban Model ← B.J.L. Berry

Berry (1965) presented a series of equations which could be used in describing the commercial structure of the Chicago and Northeastern

Illinois area. The model is descriptive and cross-sectional, i.e. describing only the present structure of the retail markets. Moreover, Berry notes that its use may be safely extended to include short-run adaptive planning purposes where incremental growth needs to be considered.

The model relies on central place theory in its approach to the selection of analysis units. Variations of retail establishment mix, range, and size are described for each analysis zone, and Berry places emphasis on the need for 'ecological' considerations in the selection of trade zone boundaries, e.g. balanced supply and demand. Trade zones are large: 10 were used for the complete city of Chicago area, excluding the Central Business District.<sup>2</sup> The final definition used by Berry to define a trade zone was that "intensive" area within which a constant rate of accumulation of trips with distance holds.

Central place theory is implicit in a primary assumption employed by Berry that the retail system approximates an interdependent equilibrium between the numbers and kinds of activities offered by a center on the one hand and the size of the market area served on the other:

$$B = f (P)$$

$$D = f (B)$$

Where : B = the number of kinds of businesses offered in a center;

P = the population (of trade area) served;

D = the maximum distance that customers will travel to the center.

Sixteen variables were used by Berry in the development of his model:

- |                             |                           |
|-----------------------------|---------------------------|
| 1. Number of functions      | 9. Social class           |
| 2. Number of establishments | 10. Family class          |
| 3. Total center area        | 11. Total competition     |
| 4. Shopping center area     | 12. Planned competition   |
| 5. Ground floor area        | 13. Unplanned competition |
| 6. Population of trade area | 14. Ribbon competition    |
| 7. Area of trade area       | 15. Discount competition  |
| 8. Median income            | 16. Population density    |

For both planned and unplanned centers Berry used factor analysis to isolate collinear variables before deciding on what variables to include in the final linear model. Analysis suggested that functions of center and space demands were collinear with size and population of market area served, thus providing evidence of the place of the center in a central place hierarchy. By using least squares fitting Berry found a function for each of the center descriptive variables ( $Z$ ) based on the following dependent variables: population of market area ( $P$ ), population density ( $d$ ), social class ( $C_s$ ), and family class ( $C_F$ ). The function is linear in form:

$$Z = b_0 + b_1P + b_2d + b_3C_s + b_4C_F$$

Figure 4-1 shows the resulting parameters and fit statistics for Berry's model for the Chicago area.

Dependent Variable	Independent Variables				Intercept	R <sup>2</sup>	Type of Center
	P	d	C <sub>s</sub>	C <sub>r</sub>			
Establishments	0.655 (0.074)	-0.600 (0.089)	0.185 (0.045)	0.093 (0.040)	1.107	.93	Unplanned
Establishments	1.083 (0.239)	-0.393 (0.238)	0.113 (0.082)	-0.025 (0.095)	-2.657	.74	Planned
Total Area	0.645 (0.075)	-0.418 (0.091)	0.086 (0.046)	0.068 (0.040)	4.130	.90	Unplanned
Total Area	1.000 (0.223)	-0.568 (0.223)	0.127 (0.077)	0.154 (0.089)	3.265	.83	Planned
Shopping Center Area	0.728 (0.081)	-0.471 (0.097)	0.068 (0.043)	0.071 (0.050)	3.574	.91	Unplanned
Shopping Center Area	0.777 (0.204)	-0.436 (0.203)	0.058 (0.070)	0.184 (0.081)	3.406	.82	Planned
Ground Floor Area	0.734 (0.072)	-0.379 (0.087)	0.100 (0.044)	0.040 (0.038)	3.238	.93	Unplanned
Ground Floor Area	0.745 (0.192)	-0.378 (0.191)	0.077 (0.066)	0.193 (0.076)	3.094	.83	Planned
Total Establishments in Market Area	1.294 (0.111)	0.193 (0.135)	-0.048 (0.069)	0.035 (0.060)	-3.981	.96	Unplanned
Total Establishments in Market Area	0.922 (0.114)	-0.291 (0.114)	0.005 (0.039)	0.235 (0.045)	-0.424	.96	Planned

Figure 4-1 Parameters of Models Describing Retail Relationships  
for Centers in Northeastern Illinois from: Berry(1965)

Information required by the model is evident from the variables.

Socio-economic data was available through census sources while shopping center data, needed for model calibration, was collected through surveys. In turn, information provided by the model was descriptive of a number of center related features which could be used in short range planning.

In general Berry's model is descriptive of the retail structure in the Chicago area with more than 80% of the variance explained by the model in most cases. In the analysis of planned centers, market areas replaced areal units as the unit of observation, but the results in Figure 4-1 do not show marked improvements. In fact, unplanned centers appear to be more predictable given Berry's model.

Quality of the results of the model are closely related to the market area selected. The large size of Berry's analysis areas is partially responsible for the quality the results. Yet the large analysis areas make the model less useful.



Berry points out that by decreasing the size of trade areas, the predictive capacity (and the relevance in terms of central place hierarchies) will be lost. Definition of trade areas is dependent on adequacy of information. Pavement surveys may be needed to define current shopping patterns, hence trade areas.

The model cannot account for any exogenous factors which may effect a center within a trade area, either. Effects of zoning or government planning intervention may alter regional distribution of retail activities. As large planned centers continue to become more dominant, prediction of number of central place functions becomes less and less important. Retail area (square feet) will become a more important result. Changing ideas about shopping centers may alter the relative importance of central place hierarchies in general, and this should be considered in any predictive work where classification of retail activities is used.

To conclude, both the analytic approach taken by Berry and the resulting model are useful examples of application of central place theories. The model's major strength is analysis of current equilibrium between demand and supply and not prediction of future demand. The empirically determined parameters of the model would have to be reviewed closely before population forecasts of others may be used model forecasts. Berry does not recommend this use of the model.

C. Probabilistic Model of Huff as Modified by Stanley and Sewall

Chapter III discusses a large number of refinements on the retail gravitation approach of Reilly and Converse (1949). Instead of

attempting to forecast some distance which may be used to define a market area, Huff (1963) modified his approach to result in the probability of a shopper at zone  $i$  shopping in zone  $j$ , if the amount of shopping center in zone  $j$  and the locations of zones  $i$  and  $j$  are known.

Huff argues that this normative model is based on maximizing the consumer's utility in a potential store selection choice where the utility is directly proportional to the mass (or size) of the shopping center and inversely proportional to some root of distance to a center. While the result is a simple probability of a consumer shopping at a center, Huff demonstrated the addition of population and potential sales terms which allow the user to calculate directly a sales volume for a center at a location  $j$ .

The user must set the boundary of his analysis area which should be sufficiently large to contain most of the probable users of a center at site  $j$ . This area does not have to be construed as the market area for the center, and thus, does not need to be selected with the analytical rigor used in Berry's Chicago model.

The initial formulation of the model was:

$$P(C_{ij}) = \frac{S_i \cdot \delta}{T_{ij}^{\lambda}} \left( \frac{j}{\sum_{j=1}^n T_{ij}^{\lambda}} \right)$$

Where :  $P(C_{ij})$  = the probability of a consumer at a given point  $i$  of traveling to a given shopping center at point  $j$ ;

$S_j$  = the square footage of selling space devoted to the sale of a particular class of goods by shopping center  $j$ ;

$T_{ij}$  = the travel time involved in getting from  $i$  to  $j$ ; and

$\lambda$  = a parameter to be estimated empirically to reflect the effect of travel time on various kinds of shopping trips.

For the estimation of annual sales volume at a center at location  $j$  Huff added additional terms:

$$E(A_{ij}) = \frac{S_j}{T_{ij}^\lambda} \left( \frac{S_j}{T_{ij}} \right)^n (C_i)(B_{ik})$$

Where  $E(A_{ij})$  = expected annual sales in dollars at shopping center at  $j$  for product in class  $k$  from each  $i^{\text{th}}$  unit of population;

$C_i$  = the number of consumers in  $i^{\text{th}}$  statistical unit ;

$B_{ik}$  = the annual amount budgeted by consumers in the  $i^{\text{th}}$  statistical unit for product class  $k$ .

Stanley and Sewall (1976) have modified this basic model by Huff to include a factor which accounts for consumer's perception of the image of the shopping center at  $j$ . Based on survey results measured by a multiple dimensional scaling routine where the "distance" between and "ideal" facility and a given "facility" may be measured, Stanley and Sewall modified the Huff model:

$$E(A_{ij}) = \frac{S_j^{\lambda s} (D_{ij}^{\lambda d})}{T_{ij}^{\lambda t}} (C_j)(B_{ik}) \cdot \frac{\sum_{j=1}^n \frac{S_j^{\lambda s} D_{ij}^{\lambda d}}{T_{ij}^{\lambda t}}}{\sum_{j=1}^n \frac{S_j^{\lambda s} D_{ij}^{\lambda d}}{T_{ij}^{\lambda t}}}$$

Where :  $D_{ij}$  = the image distance of a retail outlet at  $j$  for consumer in  $i$ ;

$\lambda s$  = a parameter for calibration of the sensitivity in determination of sales volume to changes in store area;

$\lambda t$  = a parameter for travel time;

$\lambda d$  a parameter for image distances.

Commonly available data was required by the original Huff model; however, Stanley and Sewall's use of an image term requires that a survey be made of consumer preference for facilities as a function of various features. Huff anticipated more aggregated behavior on the part of groups towards purchase of each type of good. Huff worked with shopping centers in a larger geographical area than Stanley and Sewall's work in locating supermarkets. This implies that a range of scale in application is possible, but only with the need for inclusion of additional behavioral information as geographic scale increases.

Results of these gravity models are exceedingly useful since retail sales volume estimates are fundamental in making a decision on a retail location. Of more importance, however, is the fact that the results consider the effect of competition, too, a feature not

directly considered by the Berry approach.<sup>3</sup> Isolated market areas for a given store are not used and need not be defined a priori to the analysis. If two stores of equal attractiveness were to locate on adjacent sites, this model would give an equal share of the dollar volume to each facility. As one store becomes more attractive or moves its site in favor of a closer proximity to a consumer population, it will receive a greater share of the market.

The original Huff model could be used in an economical manner for store or center planning. However, due to difficulties in establishing the parameters, the model remained normative in nature. Work by Nakanishi and Cooper (1974) is estimating parameters allowed for the descriptive capacity of the Huff formulation to be increased. Stanley and Sewall's modelling approach also provided a substantial increase in the Huff approach's descriptive capacity. In a test of 12 supermarkets the authors found that driving time accounted for approximately one half of the store patronage variance. With the addition of the image factor, driving time remained as significant and image accounted for an additional 20% of the variance. Figure 4-2 presents some comparative results of Stanley and Sewall's test.

Chain	Store Location	Number of Patrons		
		Observed	Expected Based on Driving Time	Expected Based on Driving Time and Image
A	1	17	4.75	10.73
A	2	6	4.71	10.58
B	3	0	4.13	2.02
B	4	5	6.43	4.16
B	5	11	9.62	8.04
C	6	2	4.83	2.19
D	7	2	4.80	3.79
E	8	22	30.33	30.26
F	9	8	4.82	3.55
G	10	2	4.84	3.55
G	11	0	3.52	2.11
G	12	18	10.22	12.02
Total sample		93	93	93

Figure 4-2 Driving Time vs. Driving Time and Image Comparison of Observed and Expected Number from Neighborhood Who Shop for the Majority of Groceries at One of the Supermarkets from: Stanley and Sewall (1976)

The strengths of this approach are that it allows an economical means of fairly accurate estimation of the competitive influence of retail facilities based on store size, image and location. It is amenable to forecasting, although the sensitivity to the empirically set parameters must be observed. Results are directly useful in the locational decision making process, and the routine may be a part of a manual search for a sub-optimal site location. An excellent application of a search was made by Lakshamanan and Hansen (1965).

(See Appendix I for a summary of their approach.)

A possible weakness of these gravity models is the treatment of rationality in individual consumer behavior in store selection. While Huff, Stanley and Sewall, and others assume that consumers maximize their economic utility, there is evidence by others, e.g. Ambrose (1968), that some consumers shop for recreational reasons or as an antidote to boredom. Thus, variables considered and included in the

models may not explain enough of the attributes of human behavior. This condition becomes more symptomatic as geographic scale is increased, i.e. Stanley and Sewall may have felt more influence from this tendency than Huff. This illustrates another potential problem with the gravity approach. Although the gravity model appears to be applicable over a broad range of geographic scale, it is never clear to the researcher when assumptions about aggregate behavior begin to break down as scale increases. Inclusion of an empirically determined image term by Stanley and Sewall represents one way of accounting for this characteristic.

Another weakness of both the Huff and Stanley and Sewall models is the representation of modes of access. Most retail outlets have multi-modal access, yet the models use only one representation of consumer: retail center distance. Measurement of distance within a zone is a problem, too. There is little evidence of an appropriate measure of distance to use within an analysis zone, yet the greatest number of trips theoretically will come from within the zone of a center. At the other extreme, neither Huff or Stanley and Sewall have attempted to represent possible shopper trips which will originate outside the geographical area included in the modelling exercise. Some trips are likely to result as a function of regional or extra-regional attraction in the case of shopping centers. For groceries and other convenience establishments, through zone traffic may result in additional business. Other critics<sup>4</sup> note that a certain bias may result from representation of centers within the set of

zones used in an analysis, since central zones will always have the highest level of accessibility. This may be controlled, however, by using a proportionally large number of analysis zones and representing distance in terms of travel time. These problems of range of trade influence illustrate the analytical strength of central place trade areas as employed by Berry.

D. A Microanalytic Approach of MacKay

David MacKay (1972) provides an interesting approach to store location analysis which incorporates individual customer store choice and shopping behavior. Simulation of individual shopping behavior results in an estimate by facility of customer visits and sales volume. Moreover, unique to this model, is the consideration of the influence of the multi-stop shopping trip. Other approaches allow for evaluation of competition, but no other models investigated account for the impact of multi-stop shopping trips, hence the impact of compatible retail facilities as described by Nelson (1958).

Due to the complexity of the simulation process involved, MacKay's experimentation was carried out in Tinley Park, a suburb of Chicago with a residential population of over 12,000 and a grid street network. This was required to simplify distance computation.

MacKay contended that aggregate models of consumer behavior may misrepresent effective trade-offs of distance required in more micro-analytic analysis. In an example, it was reasoned that a consumer may prefer to travel slightly greater distances to gain the opportunity to shop at multiple location types. Thus, accounting, for



such behavior is dependent on the degree to which multi-stop shopping may be accounted for in modelling consumer behavior.

Mackay's approach uses both discriminant analysis<sup>5</sup> and Monte Carlo simulation to portray a three stage shopping decision process for a consumer. First, a decision is made (on a given day) to go shopping or not; then, a decision is made of how many stops to make. Finally, is decided for each stop made which establishments are to be visited.

For an individual shopper of known household characteristics such as family size, age, educational level, employment status, income and number of driver's licenses the simulation sequence goes as follows:

1. A Monte Carlo process samples a discriminant function determining whether the shopper will make a trip on a given day of the week or not. Independent variables for each discriminant function include household characteristics, attitudes toward stores, and shopping trip data. (See Figure 4-3) The discriminant functions are structured such that prior knowledge of the decision of a previous day is used in establishing the posterior probability of a trip on a given day.
2. Once the shopping function indicates the existence of a trip, another set of discriminant functions is used with Monte Carlo sampling to determine the number of stops to be made.
3. Then which establishments are to be visited at each trip stop is found in a similar manner. Three heuristics were conceived which could be used for establishment selection:

- a. Single stop - distance minimization: Consumer chooses establishment based on minimization of distance from each to home.
  - b. Modified sequential distance minimization: Consumer minimizes total distance travelled for small trips and reverts to a. for large trips.
  - c. Discriminant heuristic: Same as b., except discriminant function is used in selection of primary establishment for which analysis is being conducted.
4. Once stops are selected an estimate could be made using a regression equation of the dollar volume sales which occurred for the individual customer.

This process is repeated for a number of customers until confidence in the sample size is adequate to allow extension of the resulting shopping pattern to the total population of the analysis area.

Data requirements for this approach are significant due to the derivation of the discriminant functions. Besides population and demographic data; store attitude, shopping habits, mode of travel, accompaniment to store, and distance information are required. Figure 4-3 indicates which variables were used in each set of discriminant functions.

Variable set	Discriminant functions		
	To shop or not shop	Number of stops	Establishment selection <sup>b</sup>
Household characteristics	X	X	X
Attitude toward stores	X	X	X
Shopping trip variables			
Timing	X	X	X
Mode		X	X
Accompaniment		X	X
Distance			X
Establishment			X

<sup>a</sup> The variable set is an input for the related discriminant function.

<sup>b</sup> The establishment selection functions which determine the order of the first supermarket stop do not employ distance or establishment variables.

Figure 4-3 Variable Sets Used for Different Discriminant Functions from: MacKay (1972)

Results from the model may be produced at each step of the modelling process on an aggregate or individual consumer basis and are implicit in the discussions above. Estimated sales for all establishments is computed, even if the analyst may only require information for an establishment under consideration.

MacKay points out that due to the use of the discriminant functions, the model is descriptive in nature and cannot be used for long range forecasts. He did indicate, however, that for short run, simple changes to the retail network, the model could be used by providing locational coordinates for a new establishment and assumed information related to the perceived stimuli of the new facility.

By using a split sample, MacKay was able to test and operate the model using the same survey of over 300 Tinley Park residents. Overall the approach exhibited good descriptive capabilities. In a test, where sales volume was estimated in the model, Tinley

Park total food sales were estimated by the model at \$8.16 million while estimated actual sales were at \$8.36 million.

Mackay's approach to modelling shopping behavior provides a number of interesting contrasts with other models reviewed. It is micro-analytical and does not attempt to represent aggregate group behavior. Instead, a probabilistic view of individual consumer behavior is represented by the use of discriminant functions. Group behavior is represented as the sum of the individual choices in shopping trip planning and store choice decisions. This is a significant contribution of Mackay's model. Another strength of the model is the analytical capability it provides in understanding the effect of competitive and complementary retail establishments. The model allows the simultaneous evaluation of not just one proposed facility but a mix as well - say the establishments a developer may wish to locate in a neighborhood shopping center or a multiple outlet location decision problem. Changes in a city's infrastructure as well as changes in consumer behavior may also be evaluated using this approach.

Weaknesses are of a practical nature. Extensive survey data is required for derivation of the discriminant functions. Mackay's split sample was valuable in calibrating the model and use of such an approach should accompany any future application. Since Monte Carlo is used with some sixteen discriminant functions, computation time required in operating the model could be large. This however, is a direct function of the size of the sample of individuals simulated and may be comparable to computational requirements of most gravity models.

### E. Plant Facility Location Models

Revelle, Marks, and Leibman (1970) discuss eleven different facility location models, for which Leon Cooper (1963) and Efromymson and Ray (1966) are some of the authors cited. This discussion will not focus on a single specific application, but instead it will further describe the use of the plant facility location model in retail location.

It may be assumed that the retail problem is a subset of general facility location problem, since a principle objective for the retailer is to minimize his total costs in locating in the retail system. For a distributor, costs include both warehousing and distribution, while for a retailer, it is assumed that costs include facility development and customer shopping - a function of customer volume and distance. This is true only if one assumes that the optimal location for a retailer minimizes the cost of shopping and at the same time has the lowest facility cost. As a retailer moves a facility to a location at a greater distance from the demand, shopping costs will increase and facility costs will decrease. Thus, the maximal profit solution will find that point where marginal gains in shopping costs will be offset by marginal reduction in facility costs.

A formal description of a retail application of the plant location model as discussed in Chapter III is:

$$\text{Minimize } Z = \sum_{j=1}^n \sum_{i=1}^m d_{ij} (X_{ij}) + \sum_{i=1}^m F_i(Y_i)$$

$$\text{Subject to: } \sum_{j=1}^n X_{ij} = Y_i \quad i = 1, 2, \dots, n$$

$$\sum_{i=1}^m X_{ij} = D_j \quad j = 1, 2, \dots, m$$

$$X_{ij} \geq 0 \quad i = 1, 2, \dots, m$$

$$j = 1, 2, \dots, n$$

$$Y_j \geq 0 \quad i = 1, 2, \dots, n$$

Where :  $X_{ij}$  = consumers shopping at facility  $i$  from demand area  $j$ ;

$Y_i$  = total shopping volume at facility  $i$ ;

$d_{ij}(X_{ij})$  = cost of shopping as a function of consumers  
from  $i$  shopping at  $j$ ;

$F_i(Y_i)$  = Capital and operational costs of a retail  
facility at location  $i$  as a function of  
shopping volume;

$D_j$  = total shopping demand area  $j$ ;

$n$  = the number of consumer demand area; and

$m$  = the number of proposed retail facility locations .

The problem is solved in one of several different ways which generally start with one facility, then add additional facilities at possible site locations until overall costs are minimized. Other approaches have started with all possible facilities and removed facilities one by one until a least cost solution is found. It should be pointed out that shopping trip assignment in this model is not defined.

Therefore, use of the model would have to incorporate some routine (e.g. gravity model), for assigning shopping trips to a given arrangement of a specified number of facilities. A discussion of more

refined solutions to the problem may be found in Revelle, et. al. (1970).

Data required by this approach is not specifically described since it has presented here in a general context. A typical application, however, would require some measure of demand: population, expendible dollars, or shopping trips; a measure of distance, which is often implicit in the coordinates of  $i$  and  $j$ ; and estimates of facility costs including site, leases, advertising, and other operational costs which may vary from one site location to the next. The model would find an optimal or suboptimal set of sites as well as the estimated trade volume, and total location costs.

The major strength of this approach is that it provides a method for evaluating economic considerations related to facility location. Other models reviewed, thus far, have not provided the framework which may be found in plant location models for evaluating the costs of highly accessible locations. Other models are descriptive in nature, thus attempting to describe consumer: retailer interaction for a given set of facility locations. Plant location models take a more normative approach by attempting to find the number and pattern of facilities required for optimum interaction within the retail system. Because of the explicit handling of costs a plant location model may account more directly for impacts of other factors such as construction costs, government interaction, utilities, rents, and advertising costs. It is important to observe that the plant location model may be used in conjunction with the application of central place models or gravity models and is not mutually exclusive of them.

However, competition is only indirectly considered through the use of this approach. The analyst should account for competitor's facilities in the number of facilities located, but the model has no means of fixing the location of existing facilities in the analysis area when finding optimal locations. Hence, the result provided is an optimal locational pattern for all retail facilities of a given type. The variant of this model, the quadratic assignment model, allows for only slightly better representation of competition through the use of interaction costs. For example, it is possible with quadradratic assignment to represent interaction costs between a facility  $i$  located at  $j$  and another (competitor's) facility  $l$  located at  $k$ . Measurement of this interaction term, however, is not easily conceptualized and computed.

Another characteristic is that for most applications, substantial computing resources are required, if the problem can be formulated, to start with. Past retailing examples are not available; thus, application would require substantial conceptualization of the process used to solve the problem. Yet, the comprehensiveness of its complementarity to the retail decision making process makes such an approach appealing and worth more intensive consideration as a retail model.

#### F. Discrete Physical System Analogy - White and Ellis

White and Ellis (1971) used an electrical engineering technique for modelling retail sales in the towns of Kitchener and Waterloo, Ontario. They hypothesized that retail shopping activity could be represented



as a series of flows and pressure changes in a large network representing the street pattern of a community.

The objective of their approach was to find, for a given set of retail locations, an estimate of sales potential. This allowed for one to determine the performance of a new store and its impact on the sales of all other stores. In addition, the approach would also allow estimates of sensitivity of store performance to changes in store characteristics or the traffic network.

Characteristics of every component are expressed as a functional relationship between  $X$  and  $Y$  as measured at the ends of the component. Flow variable or through variables are expressed as  $Y$  while pressure or across variables are expressed as  $X$ . In describing the topology of the network; each component may be defined as one or more directed line segments, which, when interconnected, will represent a linear graph of the total system.

Three types of components are defined: Origins are the areas of money generation, links - roads connecting origins to destinations, and destinations - the retail establishments. White and Ellis used the model to simulate sales in the community's 24 supermarkets.

For origins, yearly grocery money flowing from the origin ( $Y_i$ ) may be expressed as:

$$Y_i = 52 (C_i)(P_i)$$

Where :  $C_i$  = the food cost per week per capita;

$P_i$  = the population the origin area  $i$ .

Links are expressed as the resistance to flow and are measured as:

$$X_{ij} = R_{ij} (Y_{ij})$$

Where :  $Y_{ij}$  = the pressure to cross the link connecting node i to node j;

$R_{ij}$  = the resistance to flow through the link ij; and

$X_{ij}$  = the yearly net flow of shopping money through link ij.

In application, driving time across the link was used by White and Ellis for  $R_{ij}$ . Link length, type and speed should all be considered in determining a proportional measure for driving time.

Destinations must be represented in terms of their attraction,  $A_j$ . White and Ellis used regression analysis to estimate  $A_j$  as a function of floor area, number of check out lanes, relative price, specials and location.

The terminal equation for the destination is:

$$Y_j = A_j (X_j)$$

Where :  $Y_j$  = yearly sales at supermarket j;

$A_j$  = the attractiveness factor; and

$X_j$  = the propensity to shop at supermarket j.

The above set of terminal equations representing the network may be evaluated using the chord formulation technique as described by Koenig, et. al. (1967). This requires computer use. Figure 4-4 represents the network evaluated by White and Ellis for twenty-four grocery stores and 30 origin areas in the Kitchener-Waterloo area.

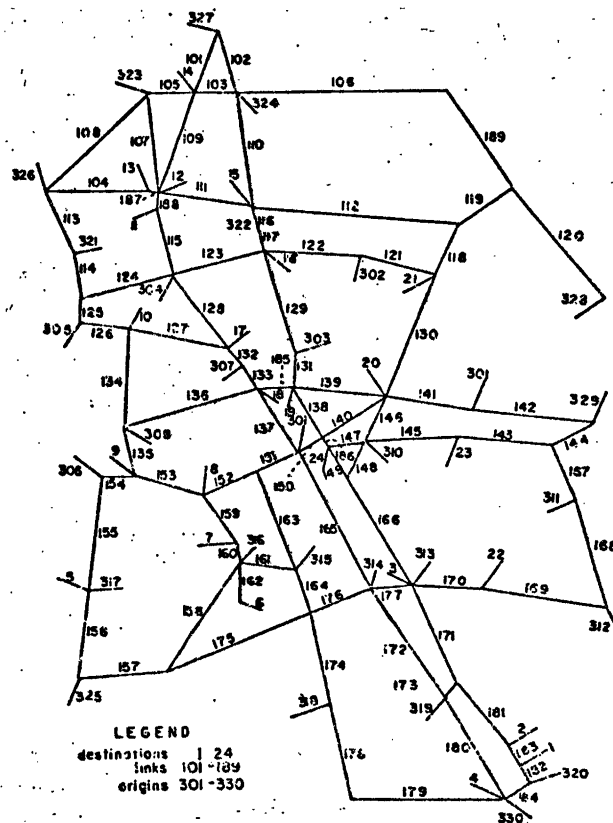


Figure 4-4 System Linear Graph for Kitchener and Waterloo, Ontario from: White and Ellis (1971)

Data requirements for the model are similar to those of the Huff models. Although not specifically stated, the centroid of some areal unit such as census tracts were used for population origins. Distance measures should include not only distance but speed and road quality considerations as well. Destination data requirements are most difficult since an investigation of the relative attractiveness across a number of measures must be made from site surveys of local establishments. Such an approach will provide a more robust variable in the model for attractiveness, similar to Stanley and Sewall's additional term in the Huff model.

The primary result of the White and Ellis model is an estimated annual sales volume in dollars for each retail establishment included.

In the test case made by its authors, the model provided usable results with an average error of 12.0% and a root mean square error of 16.5% for the 24 groceries.

The strength of this approach lies in the fact that it simulates the equilibrium which exists between consumer's available dollars and retail establishment dollar sales for a given good through a process where the network of access and attractiveness may be fairly well defined. It does not rely on the theoretical construct of gravity; yet it employs another physical analogy - that of physical systems. Comparison of these analogies is interesting. Gravity models assume the form of interaction that will take place between consumers and retailers, while the physical systems model assumes the form of all elements of the system and then computes the interaction which results.<sup>6</sup> Neither type of model provides an optimal result since they attempt to describe the existing state of a retail system. This may be contrasted to plant location models. Both gravity and physical systems models, however, may be easily used in sensitivity testing by changing an element in the system at each successive application. With such a method, a sub-optimal solution may be found; however, the process may be time consuming.

White and Ellis' use of an attractive term in simulating retail nodes is comparable to the work of Stanley and Sewall. This improves substantially the representation of drawing power of each node when compared with other alternatives such as use of a mass or sales volume term. In the Link model, use of a resistance term allows representa-

tion of current roadway conditions and congestion. This feature was not found in such an explicit manner in other models reviewed.

Overall, use of a physical system model requires a precise description of each of the elements which interact in a retail system. Once such a representation is made, the user may then have a straightforward method for estimating consumer-retailer interaction.

#### G. Summary

Five models for retail location have been presented with a criticism of the major strengths and weaknesses of each. Although, it is not possible to assess which approach is best, it is worthwhile to compare general attributes and potential limits for application each may have.

#### Scale of Application

Two extremes were exhibited. Brian Berry's model was developed for use at the regional level, using as a foundation central place theory. In contrast, David MacKay's approach, as its name implies, is used for micro-analysis of convenience outlets at a neighborhood level. The other models fall on a geographic spectrum somewhere between, with lesser restrictions placed on the sense of the geographic area used.

#### Theoretical Basis

Central place theory was used by Berry in his model of Chicago, while Huff's approach is a gravity formulation. MacKay attempts to describe individual consumer behavior through the use of his discriminant functions. White and Ellis have used a direct physical analogy with little discussion of its appropriateness, while plant location

algorithms are based on classical economic theory. Thus, no one theoretical base is common to all approaches.

A number of assumptions in the approach each model took are similar. All models assume the rationality of consumer and retailer location selection decisions.

In most models this rationality is expressed in terms of costs and utilities to both consumers and retailers. Finally, the market was assumed to be unconstrained in all models reviewed.

#### Information Requirements

The type of information required for each is similar, while, of course, analytical units may vary. It is possible to apply the Huff model or to develop a plant location problem without any descriptive statistical analysis. The other models required data collection by survey. Analysis of this information was found to significantly improve the meaning and quality of results obtainable from the model.

#### Optimality of Model Results

Only the plant location algorithm incorporates a search for a sub-optimal or optimal solution. With the other models, the analyst would be required to employ the model in an interactive manner to improve the relationship between the results and the objective of the planning effort.

#### Economy

It is difficult to assess the economies associated with each approach, since this is directly related to the quality and quantity of information the analyst has prior to commencing the application and the

type of application made. Survey usage and computer time will be significant factors related to economy of use. Computer usage is probably greatest in the plant location and MacKay's micro-analysis models because of the amount of interaction employed. Berry's linear model, of course, doesn't even necessarily require computer use.

#### Type of Information Provided

Retail sales at a retail establishment (or an aggregate of establishments) is obtainable using any of the approaches.

#### Major Factors Which Affect Results Obtainable

Distance was a factor of all models except Berry's. This model described utilization through market area selection. Distance was therefore an important criteria for market area selection. Population and personal wealth are common to each model. Representation of aggregate consumer behavior was found in the models of Huff, Stanley and Sewall, and White and Ellis. MacKay, in comparison, attempted to represent individual consumer behavior. Time was not explicitly considered in any approach. No model investigated the influence of multiple consumer types, e.g. work force vs. residential populations. Neither was representation of multi-modal access to shopping facilities in the development of distance terms used in any model. In most cases distance was represented by a simplified travel time or distance variable which took little if any account for roadway condition and traffic volumes. Only the White and Ellis model had such capability.

### Prediction

Due to the descriptive, statistically derived parameters used in all approaches, except the plant location model, reasonable application to long range forecasts cannot be made. In all cases assumptions for future periods are required when short range predictions are made. This allows for the testing of future facility locations, access system modifications, or structural changes in residence populations.

Figure 4-5 summarizes and compares major attributes of the five models considered in this chapter along with other models reviewed and documented in Appendix I. Chapter V will review major contributions provided to retail location decision making through the development of the models reviewed.



Approach	Theoretical Base	Scale of Application	Unit of Analysis	Service Type Firm, Facilities	Service Composition	Competition Considered	Forecast <sup>***</sup> Utility	Result Provided	Quantitative Method Used	Predictor Variables
B.J. Berry Retail Model (1965)	Central Place	Regional	Analysis Zones (300,000 ± pop.)	Specialty/Shopping Center	Aggregate of Establishments	No	Less	For zone, retail units, floor area	Factor/Regression Anal.	population, social class, family class, discount competition
Reilly & Converse (1945)	Gravitation	Intra-regional	Towns	Specialty/Shopping Goods (possibly convenience)	Individual good or establishment type	Between towns	Same	Trade area boundary between towns	Normative Model	population, distance
Lakshmanan and Hansen (1955)	Gravitation	Regional/Sub-regional	O-D Zones (similar to tracts)	Shopping Center	Aggregate Retail Space	Yes	Same	Shopping Center sales for specific locations	Normative model w/calibrated distance parameter	Consumer expenditures at place of residence, distance
Puff (1963) with Stanley & Sewall (1976) modifications	Gravitation with inertia variable	Sub-regional	Block group	Establishment/Convenience	Grocery	Yes	Less	Probability of consumer shopping @ individual service units (can be extended to \$ volume goods expenditure at center)	Normative w/empirically calibrated parameters	Sq. ft. @ each supermarket, driving time, store image measure based on survey results)
Haydock Shopping Model (1964)	Gravitation	Regional	Analysis Zones (14 sq. mi.)	Shopping Center Specialty/Shopping Goods	Aggregated	Yes	Same	For zone durable goods expenditure at center	Normative model calibrated actively	Personal expenditures, travel time, number of stores, department stores, chain stores, markets in center
Britton Harris (1964)**	Intervening Opportunities	Sub-regional	Analysis Zones (not specified)	Convenience Goods	Establishment or Goods type	Yes	Same	Amount of shopping visits	Normative model	Distance, supply of commodity, attractiveness measure, potential expenditures
Get's (1963)**	Map transformation	Sub-regional	Trade areas	Shopping Goods	Aggregated	No	Less	Trade areas for retail center of fixed site	Normative model-geographical, graphic approach	Population dot map, existing center size, sales volume, total population
White & Ellis (1971)	Physical Systems	Sub-regional	Tracts*	Convenience	Grocery	Yes	Same	\$ volume sales at outlet	Network analysis	Sales at origins, road system, characteristics, attractiveness factors for groceries
Mockay (1972)	Behavioral Simulation	Sub-regional Neighborhood	Block groups*	Convenience	Multiple Establishments	Yes	Less	Number of shopping visits - \$ volume sales	Discriminant functions - Monte Carlo simulation	Household characteristics, establishment attractiveness factors, shopping
Piant Location Model Reveille et. al. (1973)	Mathematical simulation - classical economic theory	Sub-regional	Tracts* or Block groups	Convenience*	Establishment and goods type	Yes	Same	Establishment location @ location	Iterative mathematical optimization model	Demand at tracts, \$ volume, distances, establishment number and possible sites
Micro-analytic Technique (1953)	Micro-economic	Sub-regional Neighborhood	Blocks	Convenience	Establishment	Yes	Same	Net sales of outlet @ location	Manual, decision process	Population, economic characteristics, social characteristics, attractability data, profile of existing establishment
Vacuum Technique <sup>***</sup> Haison (1956)	Micro-economic	Sub-regional Neighborhood	Block groups/Neighborhood Tracts	Convenience	Establishment	Yes	Same	Net sales available for outlet at any location	Manual, simple computation	(Same with less detail)

Notes:  
 \* Not specifically stated by author, but interpreted from empirical evidence presented.  
 \*\* Described in Appendix I  
 \*\*\* Describes comparative utility of predictive application vs. descriptive application for which model was developed.

Figure 4-5 Comparison of Retail Location Approaches

Notes: Chapter IV

1. Berry (1965) commented on the need care in selecting analytical units. Ecological units reflect a balance of retail supply and demand and the use of Chicago CRP "analysis zones" was felt to be slightly misrepresentative of these units because of the smaller size of the zones. Berry notes that use of census tracts would only make this problem worse.
2. The number of zones for Northeastern Illinois was not documented.
3. Competition could be considered only in terms of the definition of market areas by Berry.
4. Shopping capacity subcommittee (1920), p. 55.
5. Morrison (1969) states that the objective of discriminant analysis is to classify objects into one of two or more mutually exclusive and exhaustive categories, given a set of independent variables. An individual may be classified by a discriminant score formulated as:

$$Z_i = b_0 + b_1X_{1j} + b_2X_{2j} + \dots + b_nX_{nj}$$

Where :  $X_{ji}$  = the  $i^{\text{th}}$  individual's value of the  $j^{\text{th}}$  independent variable

$b_j$  = the discriminant coefficient for the  $j^{\text{th}}$  variable.

$Z_i$  = the  $i^{\text{th}}$  individual's discriminant score

$Z_{\text{crit}}$  = the critical value(s) of the discriminant score used to make the classification into two (or more) groups.

6. Ellis and Van Doren (1966), p. 61.

## V. INTEGRATION OF SYSTEMS APPROACHES INTO DECISION MAKING PROCESS

An overview of theoretical precedents and a discussion of several representative models of retail location have been presented in chapters three and four, respectively. Discussions have been limited to covering models developed for the purpose of solving the classical retail "problem" of estimating retail trade volume given information about potential customers, the retail facility of interest, and other retail facilities within the area. The purpose of Chapter V is to extend our understanding gained of these models to a discussion of development of a model for representing the decision making process presented in Chapter II. After drawing some conclusions about the models found in Chapter III and IV, this chapter will discuss use of models in other steps of the decision making process and describe a general decision making.

### A. Observations on Approaches

Several conclusions may be drawn about the range of past retail models, which may not only act as a referent for future application but also may help in the identification of gaps and future directions for research.

#### Importance of facility type

Figure 4-5 provides an indication of the wide range of applications systematic approaches have had in the past. A clear statement of the type of facility under location consideration is requisite not only for interpretation of which type of model can be used but also

the proper interpretation of the results provided. Following the principles of Nelson (1958) and others, it is important to consider the degree to which a shopping center will perform a specialty function within a region. Likewise, it is important to consider the generative or incipient character of a specific type of establishment to be located. The proper determination of market or trade area may be made if these characteristics are understood. For example, unless care is given for the development of an appropriate attractiveness term, using most forms of a gravity model would be inappropriate for a highly susceptible retail establishment such as a fast food restaurant or gasoline station. Specific site location is essential to the success of these retail ventures.

#### Treatment of Consumer Behavior

Consumer behavior is what most location models attempt to represent. However, most systems approaches reviewed were macro-models which treat individual consumer behavior on a highly aggregated basis. Reilly's laws are a most outstanding example. Aggregate human behavior is hypothesized to exhibit attractive tendencies related to the mass of population and space between the masses. In contrast, Mackay's micro-analytic simulation approach is directed at representing an individual's behavior in shopping for convenience goods within a neighborhood.

#### Competition

Some approaches have dealt more directly with competition than others. For example, gravity models have achieved because

of their treatment of multiple (or competitive) locations. Few approaches, however, actually discriminate between each facility location in a manner which would allow for the treatment of one location as one's own and another as a competitor's. The quadratic assignment procedure came the closest of any approach researched to exhibiting this characteristic. In a quadratic assignment problem statement, interaction cost (or benefits) between facilities at differing locations may be specified.

#### Shopping Behavior: Shopping Costs

A number of authors, Baumol (1956), Bucklin (1966), Dash, Schiffman, and Berenson (1976) to name a few, have attempted to describe customer behavior in terms of both real and perceived costs. This was found to be the fundamental criteria on which most systems approaches to retail location are conceived. Some approaches have been more explicit in the treatment of costs than others; and usually, the more robust the approach, the better each aspect of shopping cost is represented. For example, most approaches consider minimization of distance, travel time, or travel cost as a primary criteria for selection of a retail facility at  $j$  by consumer at  $i$ . More sophisticated approaches, however, will account for minimization of secondary costs such as parking and egress, prices, variety, store quality, brand or chain names and the like which may cause the customer to patronize a facility  $j'$  which could have greater travel costs associated than facility  $j$ .

### Knowns vs. Unknowns

Generally, all approaches deal with providing an estimate of a measure of the potential for retail facility utilization at a given location. As stated earlier, this is the fundamental retail location problem. Variants include central place and Reilly's formulations for defining trade area boundaries, a critical step in moving towards understanding location suitability. Requirements for what must be known for each approach are generally the same: Spatially located consumer population attributes, access characteristics, and the pattern and attributes of retail facilities of interest.

### Hierarchies

Several approaches attempted to deal with retail location in a broader context. The authors of the Shopping Capacities Subcommittee Report (1970) classified these approaches as step-by-step methods. Generally such an approach will have an algorithm which is followed manually in the analysis of a retail location. Such an approach was taken in the Haydock market study by the Department of Town Planning of the University of Manchester (1964). In the processural model used (Figure 5-1) information both gathered and developed supported decisions which were used in a final estimation of shopping demand in three center alternatives. In the model, concepts of central place theory and a gravity formulation by Reilly were employed.

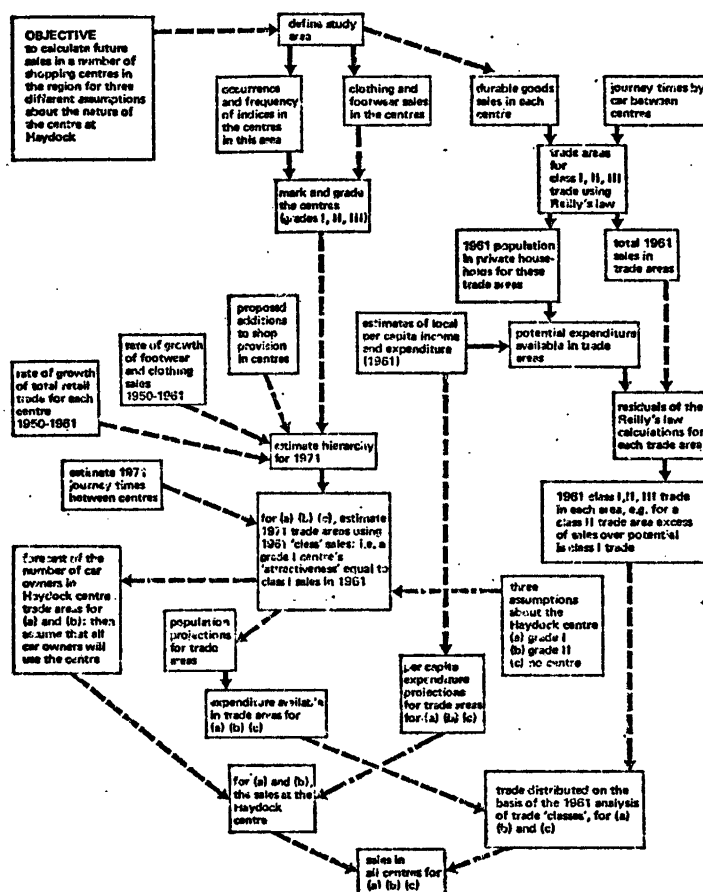


Figure 5-1 Scheme of Haydock Study Model  
from: Shopping Capacity Subcommittee (1970).

Such an approach is provocative, since it provides a more systematic view of how to go about analysis of retail potential. It is, of course, not new since it is the systematic extension of the work of Nelson (1958), Applebaum (1968) and others who have made substantial contributions to the retail location field.

### Selecting an Approach

Because of the broad range of retail location problems and objectives, it is my conclusion that there is no one optimal model which should be used in analysis of retail potential. Both the public and private sectors have an interest in retail location at a number of different

geographic scales. Thus, one must select the most suitable approach for a specific problem under consideration.

An attempt has been made to isolate key characteristics for selection of various processes, but selection of an approach is very much the function of the quality of information on hand or the resources available to obtain information. Generally, it can be concluded that better information will provide higher quality results from an approach selected. Also, as geographic scale increases so does the need for more detailed consumer and retailer information. Figure 5-2 shows a comparative range and scope of a number of approaches previously discussed.

Type of Information available	Scale and Type of Retail Activity (from Central Place Theory)		
	high order (regional)	medium order	low order (neighborhood)
General demographic,	← Reilly (1944) →		
Socio-economic, retail sales	← Berry (1965) →		
	← Lakshaman & Hansen (1965) →		
	← Haydock (1966) →		
Same w/more detailed retail establishment	← Getis (1963) →		
	← White & Ellis (1971) →		
	← Plant Location Models (1960's) →		
	← Nelson Vacuum (1968) →		
Same w/consumer attitudinal information	← Huff w/Stanley & Sewall (1976) →		
Same w/consumer shopping diary data.	← Micro-Analytic (1968) →		
	← MacKay (1972) →		
	(Macro-modelling)		(Micro-modelling)

Figure 5-2 Comparison of Scope of Retail Location Approaches



## B. Integration of Models into the Decision Making Process

It is the purpose of this thesis to focus on systems applications in the context of a retail decision making process. However, the central portion of discussion up to now has focussed on the theoretical development and application of models in analysis of potential utilization of retail facilities. Recalling the decision making process of Chapter II, these models assist the decision maker in several steps of a 14 step location planning process. This process may be summarized as follows:

- |                     |   |
|---------------------|---|
| Formulation:        | Identify Decisions to be made.              |
|                     | Set Objectives which must be met.           |
|                     | Set Geographic Boundaries of Analysis Area. |
| Description:        | Describe Analysis Area                      |
|                     | Find total retail potential                 |
|                     | Describe existing facility pattern          |
|                     | Describe consumer attitudes                 |
|                     | Find potential sites                        |
| Location Selection: | Propose site (or sites)                     |
|                     | Estimate Economic Characteristics           |
|                     | Evaluate Economic Performance               |
|                     | Test Objectives                             |
|                     | Select site (or sites)                      |
| Detail Conclusions: | Detail location strategy                    |

The models discussed in Chapter III and IV are most directly applicable in helping the decision maker to set geographic boundaries to

the planning area and estimating the potential sales volume or patronage of potential sites. Sales volume is a differentiating economic characteristic of alternative site locations.

Although many other steps in the planning process tend to be more pedestrian in nature, there are other steps besides those mentioned above where the application of systems approaches might be beneficial.

First, since retail facility planning is by definition future oriented, information accounting for growth and change in the community is important to the retail decision maker. He usually is dependant on forecasts by others of small area population growth or change yet reliable forecasts may not be available. Therefore, availability of small area population and income forecasting models may be of great use to the planner.

Second, a number of approaches for financial evaluation of retail property are available, e.g. see Cerf and Wendt (1968). Commonly referred to as "cash flow models", these routines describe net cash flows and returns for a real estate project given income such as net sales, leases, and other fees and expenses such as utilities, insurance, taxes, operational costs, maintenance costs, and amortization costs. In conjunction with retail models which describe potential sales, these cash flow models can provide a more complete picture of the financial performance of a possible site to the retail decision maker. What appears to be lacking in past work, however, is an overall decision model for retail location which incorporates the application of a number of the models or routines previously dis-

cussed. Such an approach would not be that dis-similar to the work of Nelson, Applebaum, and others. It would be more refined in terms of describing a data base to support decision making and routines such as those which forecast population, sales potential, or cash flow which would assist a planner in making a retail location decision.

To understand how such a decision making model would beneficially integrate various routines to aid the planner, an example is useful. Given estimated gross sales dollars at a site (found using one of any of a number of the retail models discussed above) and land and other costs, it would be helpful to turn to an economic evaluation procedure (such as a cash flow model) to find the return-on-investment. The site alternative may be evaluated in terms of the objective with this rate of return, not just dollar gross sales. This would allow for other considerations such as advertising, taxes, insurance, leases, and other costs to be included in evaluation of site location alternatives. Without the assistance of the two types of models, this evaluation process would be far too time consuming to be economically pursued. Given the capital intensity of most retail decisions, providing this increased analytical capability to the analyst to pursue a number of location alternatives may be economically quite attractive, since it may help him reduce substantially the amount of uncertainty he has about the location decision.

#### C. Systematic Approach to the Decision Making Process

Following, then, this line of reasoning, a model to aid in retail

decision making provides the framework in which a number of existing sub-models (our retail models, cash flow models, etc.) are easily employed. In addition, such a model would provide a data base which would support the use of the sub-models with the storage and transfer of information as it is generated. In general, the model may be described by the steps of the retail decision making process as described in Chapter II:

<u>Model Step</u>	<u>Useful Submodels or Data Base</u>
Identify location decision to be made.	(none)
Set objectives.	(none)
Set geographic boundaries.	Central place theory models Reilly's Laws (?)
Describe Analysis Area.	Geographic data base
Find total retail potential.	Demographic data base
Describe existing facility pattern.	Land Use data base (depending on specificity)
Describe Consumer Attitudes.	(Various analytical procedures which are dependent upon surveys.)
Find Potential Sites.	Geographic information handling procedure
Propose Site (or sites).	(none)
Estimate Economic characteristics	Most retail analysis models discussed previously - e.g. Huff model, White and Ellis, MacKay model, etc.
Evaluate Economic Performance.	Cash Flow Models and Supporting data bases.
Test Objectives.	(none)
Select Site or (sites).	(none)
Detailed Location Strategies.	(none)

The use of an overall model for the retail location process is not without precedent. The plant location model discussed in Chapter IV is a simple but elegant expression of such an approach. In a sense the Haydock Study (1964) could be considered such an approach due to the concentrated effort on description of process. Blumberg and Pennington (1973) developed the Advanced New Community System Retail Model which forecasts potential sales and then allocates sales to specific center locations in a region. The system allows for the generation and testing of a number of scenarios. Components include a data base, a zonal purchasing potential model, an economic model for forecasting population and personal expenditures and a gravity model for allocating retail sales. Although the modelling system was created for new communities, no major obstacles would bar its application in an existing urban area.

Given the examples of the past, relevant sub-models as suggested above may be discussed more thoroughly in terms of opportunities which may be provided.

#### Geographic Data Base

Distance is a key measure in all models reviewed for estimating retail sales potential. A data base with geographically located information about population, households, personal expenditures, travel networks, and retail establishments would be of significant benefit to the retail planner. In fact, each retail model reviewed required the development of such a data base. Major features for such a data base would include procedures for encoding of geographic

information, systematic storage, and retrieval. Data may be described geographically as residing in cells or through the use of dot probabilities. In either case, allowance should be made to allow for compatibility with application at several different geographic scales.

#### Data Manipulation Routines

These routines would allow the retail planner to manipulate as desired, for purposes of analysis, information in the geographic and other data bases. For example, this routine could be used to sum the total personal sales dollars available in a geographic area and then to find the net sales by subtracting existing sales at an establishment from total personal sales dollars available, i.e. Nelson's vacuum calculation.

#### Demographic and Retail Establishment Data

Given experiences related in the retail models reviewed, the number of data elements which could be considered here is large. At the most primary level, population, households, household income distribution, expenditure pattern, automobile availability, ethnic, education, age should be incorporated into a data base which may be part of or linked to the geographic data base. Retail facility information should include establishment type, floor area, sales statistics, and, possibly, several quality or attractiveness indices. Population and income forecast procedures may be required to allow forecasted sales to be made depending on the quality and availability this data from other sources.

With the exception of several purely extrapolative models, most small area growth models are complex and may require more resources to set up and operate than any of the retail models. A well known example is that of Lowry (1964) where the interaction of retailing was considered along with industry and residences in determining future patterns of urban development.

### Retail Analytical Approaches

These include most of the models examined in Chapter IV and Appendix I.

I. In general such a model would be represented by the function:

$$E = f (S, C_1, C_2)$$

Where : E = sales at facility under consideration

S = total sales in trade area

$C_1$  = relative primary shopping costs (access) of facility compared with others in trade area.

$C_2$  = Secondary shopping costs (price, image, variety, secondary access) of facility compared with others in trade area. (This is the attractiveness factor in most models)

### Financial Models

Costs of operating a retail facility will vary from location to location as well as over time. These include costs of land, construction, capital, management, maintenance, taxes, insurance, utilities, advertising, and leases. The retailer hopes to offset these costs through sales income and, if he leases retail space, rents. Since the timing of the flows of these revenues and expenses is critical

to the success of a project, discounted proforma cash flow analysis will be the most useful form of financial modelling. Income from rents and sales along with costs of initial development, debt service, and operations may be considered over a time stream with such models, and a discounted rate of return may be computed over a desired time frame.

Examples of these models are abundant and can easily be integrated into the framework of the retail location decision model. See Roulac (1974) and Cerf and Wendt (1968).

#### Overview of Decision Making Model

It is unlikely that total computerization of the decision making model as outlined would have significant value to the location decision maker since many steps are easily done manually. Use of the computer should be limited to data storage and retrieval as well as the evaluation sub-models since large computational efficiencies may be achieved. Emphasis should be placed on design of the form of information provided to the decision maker at each step as well as integration of the various computerized steps into the overall manual process of the model. Figure 5-3 presents the general sequence of retail location decision making and points where employment of specific sub-models would be beneficial to the decision maker.



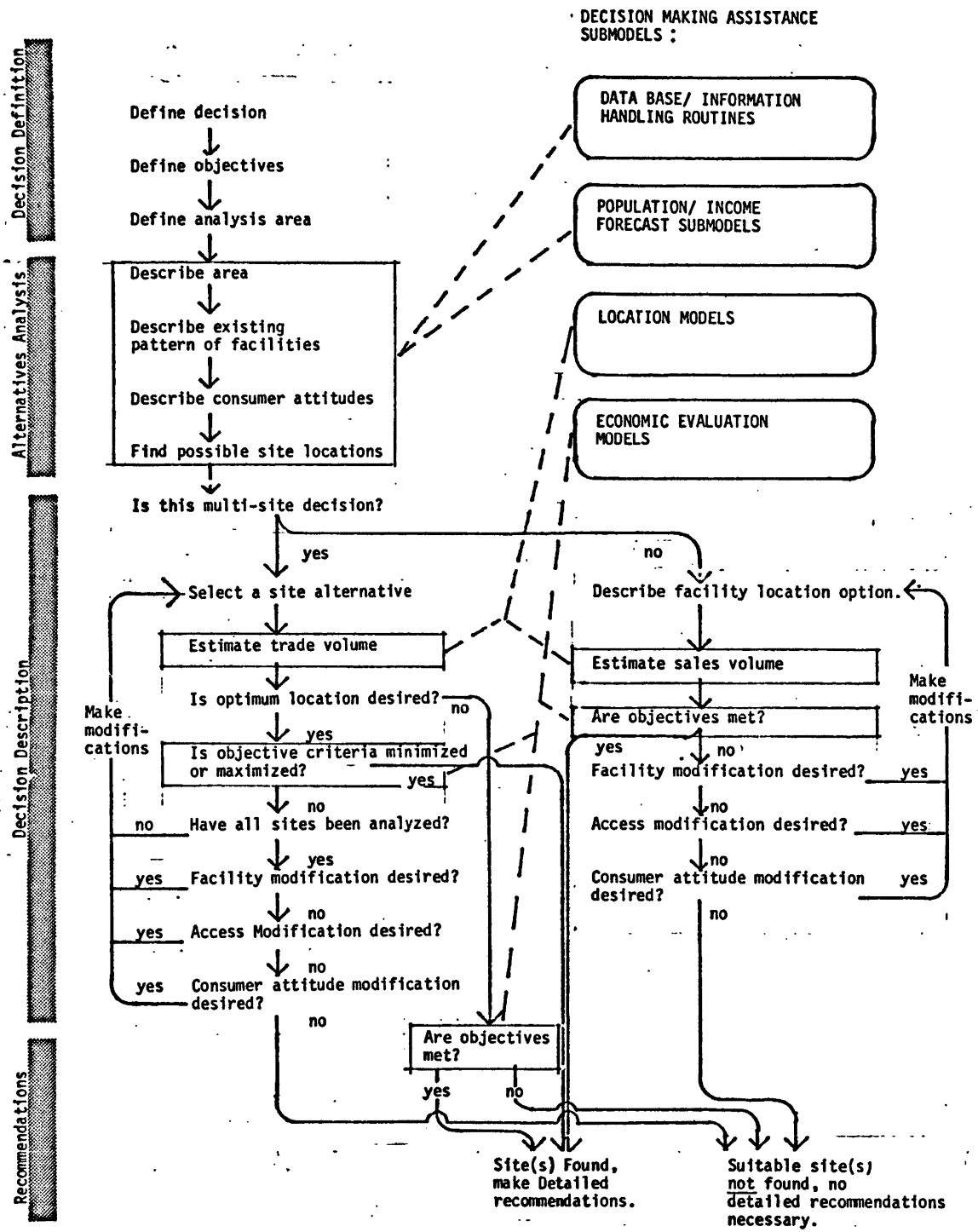


Figure 5-3 Decision Assistance Submodels within a Facility Location Decision Making Model

#### D. Summary

A framework for retail location decision making has been suggested based on: 1.) step by step location decision approaches used by retail location experts and 2.) review and criticism of retail shopping models developed over the last fifty years. Starting with a definition of objectives for the retail location decision to be made, this framework then calls for a description of all constraints, generation of alternative site locations, testing and selection of a most suitable alternative, and detailed recommendations to be made by the facility planner. Furthermore, in this Chapter, it has been concluded that description of this framework as a model for decision making is possible, using any of several retail location models reviewed in Chapter IV as components or sub-models within. Further description of this decision making model provides insight into where development and integration of other sub-models would enhance the model's utility to the retail location planner. These sub-models include small area population forecast and description routines; a geographic information and retrieval system, which is integrated to the population and retail shopping sub-models; and finally, a cash flow analysis routine for use in making economic evaluation of each potential site or retail facility program. While various sub-routines may make use of the computer, it is recommended that the overall location decision making model be treated manually. Use of judgment by the decision maker at various steps in the model suggest that this is an appropriate treatment of the modelling approach.

## VI. CONCLUSIONS AND EXTENSIONS

### A. Conclusions

Retail location has been a fertile line of inquiry into community facilities location because of the clarity of retail location objectives dealt with. In general it was found that consumers wish to minimize costs involved in obtaining goods while retailers require varying levels of agglomeration to support their merchandising. At some point, which may be expressed in geographic terms, equilibrium between consumers and merchants is experienced. Most retailing models, therefore, provide a concise description of the nature of interaction between consumers and competitive retailers in the shopping for and purchasing of retail goods.

Modelling applications in past retail market research have taken in a balanced manner both macro and micro approaches to the description of the interaction between consumers and retailers, and comparison between the two are interesting. Macro-models have represented aggregate consumer and retailer, and, thus, are often subject to severe constraints in adequately describing a given situation. Such a problem becomes more apparent as a researcher attempts to apply such a model to a smaller geographic area at a larger scale of analysis. On the other hand micro-approaches to describing retail interaction on the basis of individual decisions are complex and not easily transferrable. Results provided by such approaches were also subject to some variation in accuracy due to a number of exogenous factors which affected analysis made. The detail and complexity of

Mackay's microanalytic technique compared with Reilly's laws of Retail Gravitation and successive later applications attest to the contrast of approaches found in past work.

Analytical rigor has been viewed with associated choices which have to be made between testing of multiple location strategies and the depth in evaluation of any one strategy within a decision making framework. The ability to have both are required by the location decision maker: To the extent that they make this possible, systems approaches provide a significant contribution to retail decision making.

This implies that time and resources are constraints on the decision making process, an observation which was found to be true. The degree to which system approaches assist in decision making is a product of the reduction of analytical time requirements and economy in the handling of information. This becomes the determining factor of the economic utility of a general decision assistance model or component sub-models as posed herein in Chapter V. In addition, for the ultimate benefactor of the location decision, i.e. client, municipality, etc., such a model would increase the likelihood of consistency in decisions as they are made over time.

However, there are weaknesses which should be commented upon. Human nature may allow for haziness in the logic of using any of the models (or sub-models) described for evaluating facility location. These may yield wrong results, which may not be apparent. Geographic scale, facility types, and consumer attitudes all influence the proper

representation of consumer-retailer interaction. These should be adequately treated in the model employed. In some approaches, where iterative searches are required, e.g. a number of plant location routines, substantial computer resources may be needed or the size of the problem considered may be limited. Supporting data bases are not only expensive to compile, they require consistent maintenance to retain their utility over time. Such costs are a direct function of geographic detail and breadth of information contained and are justified only by the economic utility of application. However, for a facility planner faced with a frequent need to make location decisions, use of a decision making model with component population, location, and economic sub-models and supporting data bases is believed to provide substantial benefits which should outweigh associated costs.

Research of retail location theory and applications provided a number of insights to the use of normative planning standards commonly used by architects and urban planners. Central place hierarchies provide an important framework for the successful derivation and application of standards since they establish the thresholds at which each service can economically exist. This confirms the concept of a base population required for a given facility. Economic concepts of agglomeration are also important for the user of standards to understand. This implies that some services should exist in the focus with similar services to provide opportunity for shopping, e.g. fashion shoe stores should probably be located in groups, not individually dispersed. Nelson affirms this

with his concept of shared business. Furthermore, as Nelson describes in some stores, facility needs may not be individually forecasted and located by standards without regard to other facilities to which the facility may be economically associated. For example, certain specialty shops require adjacency to a department store for capture of sufficient shopping volume for survival. Distance has important implications as well. Several researchers found the overall size of trade areas as a function of the time a consumer was willing to invest into shopping travel. Application of planning standards within some logical market area should, therefore, be considered. Others such as Mackay have illustrated that distance minimization is not always a sufficient assumption either since many consumers are willing to travel further to avail themselves of more opportunity for shopping in various types of stores. This, too, has implications for facility planning through use of standards. Overall, the focus on factors, such as distance, consumer behavior, and store attributes, in many systems approaches provides insight for use of planning standards. Consideration must be given to these factors in applying standards, since these are the key descriptors of service user and service provider interaction.

#### B. Extension to Other Types of Services

It is interesting that many of the systems approaches discussed in this thesis have broader applications than retail facilities. An abbreviated review of recent literature in emergency, health, and recreational services finds this to be the case and provides some

notable comparisons. Before discussing these, it is important to clarify some general differences in planning for public and privately provided community services.

Many community services other than retail are typically publicly provided. Revelle, et. al. (1970) identifies the major issue in approaching public sector facility location as the difficulty of defining goals, objectives and constraints.

Dollars serve as a practical measure of costs and benefits for site location utility for the private sector. However, dollar costs and benefits to users and providers of public services is less useful if such measures can be defined at all. Typically, some surrogate measure of utility must be defined in the evaluation of public service location. For example, this may be the average trip distance per user of a facility; or it may be some measure of demand such as the number of potential users. In some cases, maximum distance from a user to a facility may serve as a measure of evaluation depending on the type of service to be located. Constraints upon the problem are usually expressed in dollar cost terms for the public sector. Total dollar available for facility development and operations or total number of service outlets are commonly used constraining measures. Problem formulation in the public sector is generally more difficult because it requires the selection of an appropriate measure for evaluation which should be related to the appropriate part of the population which forms the actual recipient population or beneficiaries of the service.

A brief review of systems approach applications in the area of emergency services was made. Major criteria for evaluation of emergency services are usually distance and response time. For example, Dormont, Hausner, and Walker (1975) have used both maximum and average response time as well as workload per site location in their firehouse site evaluation modelling work. Similar measures are used for police and ambulance services. An additional form of approach is taken by Larsen (1975) in the application of queing theory, which describes that a unit of service may be consumed at only one location at a time. Allocation of services, then, must be based on the optimal deployment of unique, spatially distributed services.

Community health and social services generally has been approached in a less systematic manner. Several studies, Sorenson (1946), Jenkins (1954), Stanley (1969), Rice Center (1975) have used a comparative analysis of "need" based on socio-economic characteristics of a spatially distributed population. This approach, however, makes no attempt at a prediction or optimization of user "demand" at a specific facility location based on access. Utilization cannot be an implicit assumption, since use is a strictly voluntary action.

Review of recreation services finds a number of systems approach applications similar to retail. However, in the mainstream of planning a general propensity towards using standards may be found. Gold (1973), makes a critical examination of years of utilization of normative parks and recreation planning standards and then proposes



a process of weighing standards with factors which are empirically developed through citizen's panels. Clawson and Knetsch (1966) used an economic approach in describing facility utilization. Users are assumed to give up a certain amount of income to travel to and use recreational facility. Marginal utility at various points on a price curve are examined to yield an estimate additional or fewer participants which are incurred as the "price" of a recreational activity is modified through changing its access or price characteristics. The same authors used a linear regression model in predicting future demand on an extrapolation of past data.

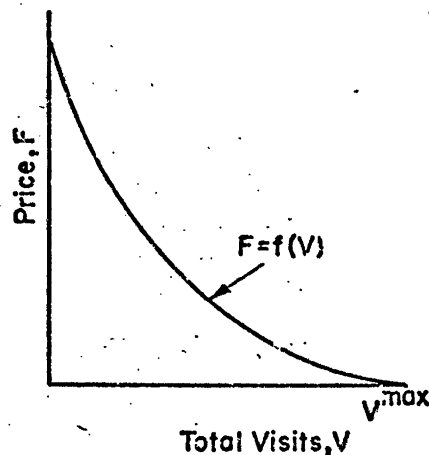


figure 6-1 Demand Function for Recreation Site  
from: Cesario (1969)

Ullman and Volk (1962), and Van Doren (1967) are two of several researchers who have used gravity models. Population and distance terms are similar to retailing applications, but attractiveness usually requires additional empirical analysis of a suitable index of type of facilities, natural attributes, and capacity. Wolfe (1972) uses a modified gravity or inertia model to represent the effect that distance has on the perception of distance. This was

to be especially useful related to urban spaces where walking was the primary mode of access and inter-regional activity such as state parks, where extensive driving was required. The inertia model developed is differentiated from the gravity formulation by a parameter which represents the "friction" of distance.

The physical systems model of White and Ellis (1971) in consumer research as discussed in Chapter IV, appears to have been initiated in the field of recreation research. Ellis (1965) made the initial analogy, while Cesario (1975) has extended the concept and simplified application by using a GPSS model.

In conclusion, recreational facilities planning research has followed (and in the case of Ellis preceded) retail location research in terms of adopting similar systems approaches, primarily due to the fact that distance was identified and consistently used as the major deterrent of an individual's access to a facility. It may therefore be observed that recreational facility location and commercial facility location may be treated as nearly identical location problems.

### C. Future Work

Based on the conclusions made and likely extensions to location of other service types, future work seems merited in three areas:

1. Further development of a general facility location decision making model is warranted because of its broad applicability to not only retail services but other services as well. The striking similarity of recreation to retail location exemplifies this fact. This development should include more extensive

investigation of the costs and benefits of a facilities planning data base and each sub-model which might be incorporated into such a system. At the same time, a refined description of a general context for making location decisions should be made. Such an effort would help the facility planner to sort out issues related to facility type, service range, and utilization characteristics.

2. Micro-level research into service utilization has been receiving increasing attention from retail researchers and should continue to do so. Results from such work provide a better framework for using macro-techniques which are in general more prolific in past research.
3. It is observed in this thesis that planners could benefit from a clearer understanding of the use of standards. Continued use of such an approach seems valid and economical, but only in a context which is truly reflective of the behavior of service providers and users. This may be exemplified in central place theory applications or the importance of distance in most retail interaction models. Use of standards should be reflective of these concepts, and a description in more direct terms of application would be beneficial to architects and community planners.

APPENDIX I - DESCRIPTION OF LOCATION APPROACHES  
REVIEWED

Selected Models Reviewed Include:

William J. Reilly, Law of Retail Gravitation, 1931.

Richard Nelson, Micro-analytic Technique, 1958.

Richard Nelson, Vacuum Calculation Technique, 1958.

Arthur Getis, Map Transformation Approach, 1963.

David L. Huff, Probabilistic Approach, 1963.

Britton Harris, Intervening Opportunities Approach, 1964.

Brian J. L. Berry, Retail Model for Chicago, 1965.

T. B. Lakshmanan and Walter G. Hansen, Gravity Model, 1965.

Haydock Shopping Model, 1966.

The Lewisham Model, 1967.

Charles Revelle, David Marks and John C. Liebman, Plant Location Analogy, 1970.

L. A. White and J. B. Ellis, Network Approach, 1971.

David B. MacKay, Micro-analytic Approach, 1972.

Thomas J. Stanley and Murphy A. Seawall, Modified Huff Model with Image Measures, 1976.

WILLIAM J. REILLY - LAW OF RETAIL GRAVITATION, 1931

Source: Reilly, William J. Law of Retail Gravitation. (New York; Reilly, 1931).

Objective:

To delineate the boundary or break point in the market area of two facilities.

Description:

$$\frac{B_a}{B_b} = \frac{P_a}{P_b} \frac{D_b^2}{D_a^2}$$

Where:  $B_a$  = proportion of trade from intermediate city attracted to City A.

$B_b$  = proportion to City B.

$P_a$  = population, City a.

$P_b$  = population, City b.

$D_a$  = distance from intermediate town to City A.

$D_b$  = distance from intermediate town to City B.

Nelson says: "Law says that people will normally go to the biggest place that they can get to easiest."

Huff (1963) says: "Hypothesis says that two cities attract retail trade from an intermediate city or town in the vicinity of the breaking point - approximately in direct proportion to the populations of the two cities and in inverse proportion to the square of the distance from the two cities to the intermediate form."

Restrictive Assumptions:

1. Population is surrogate for characteristics of retail market in city.
2. Distance is surrogate for travel time or travel cost.
3. Towns are used as analytical unit.

Data Requirements:

1. Populations - by town.
2. Distances - not well defined (highway distances were assumed).

Nature of Results:

Approach used to define trading areas for cities in a rural context - city and hinterlands. Result was simple ratio of distribution of trade to each city in question. Later Curtis Publishing (1947) revised formula to compute break point distance directly.

$$B_b = \frac{D_{ab}}{1 + \frac{P_a}{P_b}}$$

Where:  $B_b$  = the breaking point between City A and City B in miles from B.

$D_{ab}$  = Distance separating City A and B in miles.

$P_a$  = population City A.

$P_b$  = population City B.

Sensitivity to Data:

Overall, Reilly was impressed that the exponent of 2 was an acceptable parameter for his law. Past work has found this factor to vary significantly (from 1.5 to 3.0) - Relationship in Reilly's law is strictly a priori, not descriptive.

Treatment of Change:

Time was not considered by Reilly.

## RICHARD NELSON - MICROANALYTIC TECHNIQUE, 1958

(Nelson attributes development to Real Estate Research Corporation)

Source: Nelson, Richard L. The Selection of Retail Locations. (New York; McGraw-Hill, 1958).

### Objective:

Estimates the amount of sales volume expected at a proposed retail location is the purpose of this approach.

### Description:

Microanalysis technique is a step-by-step approach:

1. Establish trading area (based on a priori judgment about travel times and major barriers).
2. Divide trading area into small units (e.g. blocks).
3. Establish the amount to business available from each unit.
4. Distribute the business from each unit to various competing outlets including the proposed store. This is based on analyst's judgment, conditioned by local interviews, understanding of how many families live there, what their income is, what their spending habits are now both as to location and amount, the redistributive impact of the new facility.
5. Sum up unit by unit distributions.

### Restrictive Assumptions:

This method is most useful in location of convenience facilities such as grocery stores. Regional facilities have far too large a trading area to use such a fine grained technique.

Because of the grain and amount of human judgment used, there are comparatively few restrictive assumptions; however, this process is very expensive.

### Data Required:

Block level data needed is described above.

### Nature of Results:

Results will be estimated sales volumes for existing and new facilities in trade area.



Sensitivity to Data:

Overall, the sensitivity of this technique rests entirely in the judgment of the analyst. Nelson points out that a multiplicity of judgments does not result in errors that are cumulative - they are likely to cancel each other out.

Treatment of Change:

This is dependent upon the analysts familiarity with the trade area and possible future changes which may be experienced.

RICHARD NELSON, VACUUM CALCULATION TECHNIQUE, 1958

Source: Nelson, Richard L. The Selection of Retail Locations. (New York; McGraw-Hill, 1958).

Objective:

Technique used to estimate dollar volume sales in area available for possible new store location.

Description:

Steps are as follows:

1. Determine primary and secondary trading areas through a prior decision on travel times and geographic and other barriers.
2. Using population, income, and budget expenditure information of households, estimate total sales dollars available from primary and secondary trading areas for specific goods under consideration.
3. Make physical measurement of existing retail locations in area to determine current sales capacity.
4. Sum sales capacities for all existing retail locations.
5. Find the difference between total sales dollars available and the sum of existing sales capacities. If positive, this represents the potential for a new store in the area.

Restrictive Assumptions:

Primary and secondary trading areas are often ambiguous, thus biasing the accuracy of the vacuum calculations. Some business is very likely to go to and come from the world beyond. Such a technique would work best for a rural trading center.

Data Requirements:

By some geographic sub-unit: Number of households, expendible household income for retail line of interest, existing store sizes by location.

Nature of Results:

This is a relatively inexpensive technique for estimating sales volume potential for a trade area. It is subject to substantial error if unusual situations occur within the trade area.

Sensitivity to Data:

Results are directly related to sales information collected.

Treatment of Change:

Change not considered unless analyst happens to make allowances for future change in his calculations.

## ARTHUR GETIS - MAP TRANSFORMATION APPROACH, 1963

Source: Getis, Arthur. "The Determination of the Location of Retail Activities", Economic Geography, Vol. 43 (Jan., 1963) p. A-22.

### Objective:

Getis wished to provide a more rational method for delineation of trade area boundaries in which areas may be irregularly shaped, but theoretically sound.

### Description:

Steps used by Getis are as follows:

1. Using a population dot map of an urban area overlaid by a regular rectangular grid, transforms census tract based income data to obtain a total disposable income per cell.
2. Using personal consumption statistics, find total dollars per cell available for consumption of particular retail good of concern.
3. Compute total sales average per outlet existing for retail good of concern.
4. Distort grid system so that cell areas are proportion to retail sales dollars available.
5. Determine centroid of study area based on sales dollars availability.
6. Layout in equal sized hexagonal store market areas, starting with the center of the first store market area as concentric to the centroid of sales dollars availability is equal to the total sales potential divided by the average sales per store for area.
7. Centers of hexagonal market areas then may be transferred to original grid map for appropriate location.

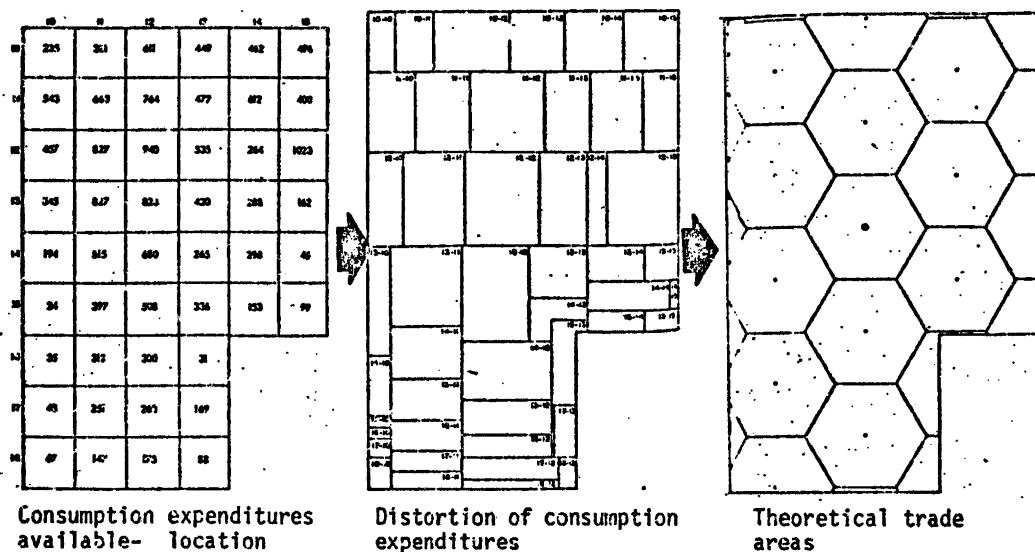


Figure- Getis' Map Transformation Process.

### Restrictive Assumptions:

Getis avails himself of this process after examining and rejecting the influence of a number of factors which can affect retail trade such as transportation network, competition between stores for convenience trade, land rent and influence of agglomerative location. Specifically, he makes the following assumptions:

1. Market areas exist for retail stores.
2. There is a minimum of (or no overlap between) market areas.
3. Consumption expenditure location has a direct bearing on the location of retail stores.
4. Travel time or cost is the same for any unit distance from place to place.
5. Rent or any other economic factor except consumption expenditures for goods supplied in retail stores has no bearing on the general location of the firm.

### Data Requirements:

Map transformation process requires population dot map of retail trade area of interest, current income data for geographic sub-units (census tracts), current sales information from local stores of type of interest, and optional location of existing retail facilities of interest.

### Nature of Results:

Although sales results for each retail unit are assumed to be equal, map transformation will provide approximate location and sales volume for a number of retail units to be determined.

### Sensitivity to Data:

Only key variables which will affect location to any extent are number and size of facilities assumed for location. Getis' simplifying assumptions rule out opportunity for variation in results due to changes in transportation patterns or the influence of competition. This process is normative not descriptive.

### Treatment of Change:

Getis concerned himself, in the Tacoma, Washington case study, with only the present situation. Forecast of future population patterns would be required to utilize this technique for some future time period. Time is not a factor which is addressed in this process.

DAVID L. HUFF, PROBABILISTIC APPROACH, 1963.

Source: Huff, David L., "A Probabilistic Analysis of Shopping Center Trade Areas", Land Economics. Vol. 39 (February, 1963), pp. 81-90.

Objective:

Huff's model was the result of an exercise to improve the formulation for delineating trade areas of proposed shopping centers.

Description:

A revised version of Reilly's gravity model is employed by Huff assuming that a customer's probability of shopping at a center is directly related to the number of items or types of goods carried by the center and inversely related to the travel time involved in getting there.

$$P(C_{ij}) = \frac{S_i}{\sum_{j=1}^n \frac{S_j}{T_{ij}^\lambda}} \quad \text{for all } i\text{'s}$$

Where  $P(C_i) =$

the probability of consumers from each of the  $i^{\text{th}}$  statistical units going to a specific shopping center  $j$ .

$\lambda$  = estimated parameter appropriate to a product class

$T_{ij}$  = travel time zone  $i$  to zone  $j$ .

$S_j$  = square footage of selling space devoted to the sale of a particular class of goods.

Restrictive Assumptions:

Huff admits that the utility of a shopping center to a consumer is based on a host of factors, far too difficult to appropriately weigh; thus, he limited scope to the two variables used. He cites that these have been found to be the leading factors; and, thus, disregards accounting for any others.

Huff conceptualizes travel time as consumer's opportunity cost. Other limitations include geographic area boundaries - Huff does not provide suggestion on definition of boundary, instead he suggests a real unit such as census tract or equivalent.

Empirical estimation of  $\lambda$  for various kinds of shopping trips is defined, but Huff does not detail a procedure for calibration.

Data Requirements:

1. Square footage of selling space devoted to sale of a particular class of goods in centers  $j$ ,  $j = 1, n$ .
2. Estimated travel times from consumer zones  $i$ ,  $i = 1, m$  to centers  $j$ ,  $j = 1, n$ .

Later Huff extends the utility of his model by including terms for number of consumers and expected annual sales. Additional data:

3. Annual amount budgeted by consumers in the  $i^{\text{th}}$  statistical unit for product class K.
4. The number of consumers in the  $i^{\text{th}}$  statistical unit.
5. The average number of shopping trips that customers make with respect to a given product purchase within a given time period.

Nature of Results:

Huff's original model provided the probability of consumer at a given point of origin  $i$  travelling to a given shopping center  $j$ .

$$P(C_{ij}) = \frac{S_i}{T_{ij}^\lambda} \left( \sum_{j=1}^n \left( \frac{S_j}{T_{ij}^\lambda} \right) \right)^{-1}$$

Modifications later included number of consumers  $c_i$ :

$$E(C_{ij}) = \frac{S_i}{T_{ij}^\lambda} (C_i) \quad \text{where: } E(C_{ij}) = \text{expected no. of customers @ particular shopping center.}$$

Expected annual sales:

$$E(A_{ij}) = \frac{S_j}{T_{ij}^\lambda} (C_i)(B_{ik}) \quad \text{where: } E(A_{ij}) = \text{expected annual sales}$$

$B_{ik} = \text{annual amount budgeted-prod. check k.}$



No. of Trips:

$$E(S_{ij}^m) = \frac{\frac{S_j}{T_i^\lambda}}{\sum_{j=1}^n \frac{S_i}{T_{ij}^\lambda}} (C_i)(S_m)$$

Where:  $E(S_{ij}^m)$  = expected number of shopping trips from neighborhood to center  $j$  in time period  $m$ .

$S_m$  = average no. of shopping trips customers make in product class with respect to product purchases in time period  $m$ .

Huff, however, states his major conclusion that his formula will allow to plot contours around statistical units which exhibit like probabilities of shopping at a shopping center. This will take form of contour map with contours representing various levels of probability.

Sensitivity to data:

1.  $\lambda$  - the parameter estimate - exponent is a critical determinant of the model's result.
2. Square footage of selling space and travel time each course linear variation in the model result.

Treatment of Change:

Huff does not deal with application to forecast work, it is assumed that model is descriptive of time frame for which it is given data.

## BRITTON HARRIS - INTERVEINING OPPORTUNITIES APPROACH, 1964

Source: Harris, Britton. A Model of Locational Equilibrium for Retail Trade. (Philadelphia: Institute for Urban Studies, University of Pennsylvania, unpublished manuscript, Oct. 1964).

Objective:

Harris' model describes the probability of trip terminations at a given location given other intervening opportunity for trip destinations between the given location on the origin of consumers.

Description:

For frequency of amount of shopping which goes beyond a district distance  $D$  away:

$$\text{Frequency} = (I + bw)^{-a} e^{-cw}$$

Where :  $W = V + BD$

$D$  = distance from origin to district of concern

$V$  = amount of shopping

$B$  = empirically determined constant - expressing the relative reluctance to travel greater distance as compared to the reluctance to seek out more opportunities.

Then, for trip generated of zone  $i$  being terminated at zone  $j$ :

$$P_{ij} = b^a (b + W_{ij})^{-a} e^{-cW_{ij}}$$

Where :  $W_{ij} = d V_{ij} + (I-d) D_{ij}$

$W/k$  arranged in order of access from  $i$ :

$$V_{ij} = \sum_{k=1}^j V_k Q_k e.$$

$a, b, c, d, e$  = parameters.

$D_{ij}$  = distance between centroids of zone  $i$  and zone  $j$ .

$V_k$  = supply of commodity offered at  $k$ .

$Q_k$  = measure of attractiveness of area  $k$ .

Restrictive Assumptions:

These follow those commonly found in other gravity models such as Lakshamana and Hansen (1965).

Data Requirements:

1. Distribution of potential expenditure at zone i.
2. Preliminary locations of retail facilities including facility to be located.
3. Supply of commodities offered in each shopping center.
4. A measure of distance between centers and origin zones.

Nature of Results:

1. Model computed hypothetical set of "arrivals" at zone j - ascertaining over and under conditions compared with available attraction.
2. New attraction factor of shopping zones was generated and substituted for original.
3. Process repeated until amount of improvement forecasted was not changed over a pre-specified amount.
4. Result is unique solution for problem.

Sensitivity to Data:

Harris' model like other more recent gravity models is very descriptive; thus requiring calibration of parameters for all independent variables used.

Treatment of Change:

Model is usable current description or short run predictive application.

## BRIAN J. L. BERRY - RETAIL MODEL FOR CHICAGO, 1965

Source: Berry, Brian, J. L. "The Retail Component of the Urban Model", Journal of American Institute of Planners., Vol. 31, No. 2 (May, 1965), pp. 150-155.

Objective:

Short run description of establishment requirements (by number) for problems of incremental growth or marginal decline. Berry suggests inclusion as part of larger urban model.

Description:

After using factor analysis to analyze the underlying patterns of 16 demographic and retail establishment data for both planned and un-planned urban shopping centers, Berry selected a linear descriptive model to use in forecasting establishments, total area, shopping center area, ground floor area, and total establishments in area.

Thus:

$$X = b_0 + b_1 P + b_2 d + C_s b_3 + b_4 C_F$$

Where:

X = a dependent variable listed above

P = population of market area

d = population density of market area

C<sub>s</sub> = social class of population

C<sub>F</sub> = family class of population

b<sub>0</sub>-b<sub>4</sub> = parameters found through multiple linear regression

Berry's model was descriptive of the Chicago area for 1958 and correlations were acceptably high.

Underlying hypothesis, therefore, is the Christaller-Losch geometric formulation of central place theory: "the retail system approximates an independent equilibrium between the numbers and winds of activities offered by a center on the one hand and the size of the market area served on the other."

B = f[P] where B = the number of kinds of businesses

P = population served

D = f [B] where D = the maximum distance that consumer will travel to the center.

and  $P = \pi D^2 d$ , where d is population density.

### Restrictive Assumptions:

Model was calibrated for time period of data and used for present forecasts, not long range forecasting. It describes the present state of the system. It used unplanned versus planned shopping centers. Market areas vary for each center and are determined a priori at time of data collection. Berry states: "Experimentation shows the most useful definition of the market area to be that "intensive" area within which a constant rate of accumulation of trips with distance holds."

### Data Requirements:

For market areas defined primary data includes current population, population density, social class (the factor score for the market area on the first dimension of a factor analysis of Census reported socio-economic data for the market area), and family class (the factor scores on a second dimension of social and economic structure). Other data used to calibrate the dependent variables include: Number of retail functions (number of different 4 digit sic codes for area), number of establishments, total site area of retail center, total site area less parking, ground floor area of businesses, total establishments in market area excluding establishments in center itself.

### Nature of Results:

Once the model was calibrated, the user may compute several pieces of retail center related information as reflected in data list above.

### Sensitivity:

Results are, of course, sensitive to independent variable input, the exact sensitivity determined by the user when he determines the parameters for the linear function itself.

### Treatment of Change:

In general, model fits a static context. Only slight variation over time as reflected by the independent variables would be reflected. Berry's model describes the present state of the system.

T. R. LAKSHMANAN, WALTER G. HANSEN, - GRAVITY MODEL, 1965

Source: Lakshmanan, T. R. and Hansen, W. G. "A Retail Market Potential Model", Journal of American Institute of Planners. Vol. 31, No. 2 (May, 1965) pp. 134-143.

Objective:

Model was developed to explore the "balanced distribution" of retail centers in the Baltimore metropolitan area through the testing of a number of intuitive alternatives for locational patterns. Results provided include expenditures in zone  $j$  spent at center  $F_j$  by residents of zone  $i$ , this may be summarized as total expenditures made at center  $F_j$ . Total driving time from zone  $i$  to center  $F_j$  is known and average driving time across the region may be computed.

Description:

Used gravity formulation where retail center in zone ( $F_j$ ) attracts consumer dollars ( $S_{ij}$ )

- in direct proportion to the consumer expenditures,  $C_i$
- in direct proportion to its size  $F_j$ .
- in inverse proportion to distance to the consumers ( $d_{ij}^a$ ) and
- in inverse proportion to competition  $\left( \sum_{k=1}^n \frac{F_k}{d_{ik}^a} \right)$

$$S_i = C_i \frac{\frac{F_j}{d_{ij}^a}}{\frac{F_i}{d_{ij}^a} + \frac{F}{d_{ij}^a} + \dots + \frac{F_n}{d_{in}^a}} = C_i \frac{\frac{F_j}{d_{ij}^a}}{\sum_{k=1}^n \frac{F_k}{d_{ik}^a}}$$

where:

$S_i$  = Consumer retail expenditures of population in zone  $i$ , spent at zone  $j$ .

$C_i$  = Total consumer retail expenditure of population in zone  $i$ .

$F_j$  = Size of retail center in zone  $j$ .

$d_i$  = Distance (in driving time) between zone  $i$  and zone  $j$ .

$a$  = An exponent applied to the distance variable.

Restrictive Assumptions:

1. User must provide location and size of future centers.
2. User must estimate changes in improvements to road networks which will affect access time, zone to zone.
3. Mode of transport is singular - auto was assumed.
4. Geographic units assumed user transportation study analysis zones - (probably census tract size or smaller).

Data Requirements:

By geographic zones:

1. Population - current and forecasted.
2. Per capita shopping goods expenditures - current and forecasted using a local econometric model.
3. Current and future shopping center locations:
  - a. Sales area.
  - b. Estimated travel time from zone of residence to center (current and future).

Model requires calibration of exponent applied to distance variable. Thus, optimal application would allow for use of survey data or shopper expenditure patterns in existing centers to be matched with model's predicted pattern. Lakshmanan and Hansen used sales-volume data on six major centers along with zone to zone comparison of origin destination survey data (shopping goods trips) to calibrate their model.

Nature of Results:

Model was used to evaluate 25 alternatives for future shopping center patterns for Baltimore. Testing these alternatives was made possible through the use of a set of decision rules regarding minimum, maximum floorspace per center, min.-max. sales volume per foot, maximum increase in driving time. These parameters were evaluated using the model's results:

1. Probable sales levels at each center - "existing" and "future".
2. The average trip length for shopping goods for the system as a whole.
3. The consumer dollars from each residential zone that are spent at each shopping center.

Sensitivity to Data:

Model does require calibrated parameter for distance. This factor effects influence of distance on overall results, i.e. "gravity" effect. Distance term is exponential - increases sensitivity. Also introduction of additional centers creates significant variation in results.

Treatment of Change:

Model is fairly sensitive to change - because of exogenous handling of future center locations and distance factors.



## HAYDOCK SHOPPING MODEL, 1966

Source: Shopping Capacity Subcommittee of The Development Committee for the Distributive Trades. Urban Models in Shopping Studies. (London: National Economic Development Office, 1970).

### Objective:

Model was required to calculate the future durable goods sales in existing and possible future major centers of the region for each set of alternative regional shopping strategies and assumptions about exogenous influences.

### Description:

$$S_{ij} = E_i \frac{F_j^a / t_{ij}^b}{\sum_j (F_j^a / t_{ij}^b)}$$

Where:  $S_{ij}$  = durable goods expenditure by residents of zone  $i$  in shopping center .

$E_i$  = expenditure available for shopping goods in zone  $i$

$F_j$  = given as an attractiveness factor:

$$F = (2V + 3D + C + m)$$

$V$  = no. of variety stores

$D$  = no. of department stores

$C$  = no. of chain stores

$M$  = no. of markets.

### Restrictive Assumptions:

Actual sales volume is not known; hence equation for  $F$ . Allowance made for competition without of market area, residential zones were approximately 14 miles each, required calibration using existing data - Parameter  $a$  ranged from 2.0 to 3.5 by 0.1 - Parameter  $b$  .50 to 5.0 by .05. Best results found through correlation analysis of actual and model's predicted.

### Data Required:

1. Current or forecasted expenditure availability in each zone.
2. Take off of zone to zone (or zone to center) distances.

3. Composition by store type of existing or proposed stores.

Nature of Results:

Interest of users was in evaluating program for Haydock Center given future development patterns of study area.

Sensitivity to Data:

1. a and b parameters had to be empirically set and have significant impact on model results.
2. Introduction of new centers would significantly alter results as one would expect.

Treatment of Change:

1. Required forecasted retail facility pattern.
2. Required forecasted residential distribution.

## THE LEWISHAM MODEL, 1967

Source: Shopping Capacity Sub Committee of The Development Committee for the Distributive Trades. Urban Models in Shopping Studies. (London: National Economic Development Office, 1970)

### Objective:

Model used to forecast sales volume by geographic area for a shopping center in area j.

### Description:

$$S_{ij} = \frac{F_j/d_{ij}^b}{\sum F_k/d_{ik}^b}$$

Where:  $F_j$  = total sales in a center j within study area.

$F_k$  = total sales in a numbered center k.

$d_{ij}$  = st. line dist. between centroid of zone i to center of zone within study area.

$d_{ik}$  = (same) for any shopping center

$b$  = empiracally determined parameters.

### Restrictive Assumptions:

Although geographical area of model was limited to southeast portion of London, it was expanded to include the impact of any center in the larger region as long as geographic coordinates were known. This makes this approach less restrictive than earlier gravity models.

### Data Requirements:

1. Sales data on existing centers by location.
2. Estimates of distances between centers. (Measured centroid to centroid within cells used.)
3. Empirical data about sales by location is needed for calibration of parameter b.

### Nature of Results:

Lewisham model results are similar to others from a gravity type model - estimated sales volume by center.

### Sensitivity to Data:

Like other models, the parameter is the critical determinant of model performance. It was assumed that this would have to be set through testing existing situation, hence survey was expected.

Feature of this version was freedom from constraint of study area; thus, results are not biased by size of analysis area. Some bias still exists since potential sales volume from customers outside of area is not considered. Assumed no customers entered study area to shop at facility - hence exact determination of study area boundary could have major impact on results. No differentiation is made for type of goods sold.

### Treatment of Change:

Model is indifferent to time period - user must supply pattern of centers, and pattern of potential sales, presumably for any time period.

REVELLE, MARKS, AND LIEBMAN, PLANT LOCATION ANALOGY, 1970.

Source: Revelle, Chas., Marks, David and Liebman, John C. "A Analysis of Private and Public Sector Location Models"., Management Science. Vol. 16, No. 11 (January, 1970) pp. 692-707.

Objective:

Authors summarize a number of plant location techniques which have been developed over the last 15 years. Generally, in such a problem, a number of demand areas for a good or service and a number of alternative plant location sites are given. The objective is to determine where plant facilities should be placed and which demand areas are to be served so that the sum of the transportation cost and amortized facility cost is minimized.

Description:

The general mathematical formulation of this problem is:

$$\text{Minimize } Z = \sum_{j=1}^n \sum_{i=1}^m d_{ij} (x_{ij}) + \sum_{i=1}^m F_i (y_i)$$

$$\text{Subject to: } \sum_{j=1}^n x_{ij} = y_i \quad i = 1, 2, \dots, m.$$

$$\sum_{i=1}^m x_{ij} = D_j \quad j = 1, 2, \dots, n.$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m.$$

$$j = 1, 2, \dots, n.$$

$$y_j \geq 0 \quad i = 1, 2, \dots, n.$$

Where:

$x_{ij}$  = amount shipped from warehouse  $i$  to demand area  $j$ ;

$y_i$  = total amount shipped from warehouse  $i$ ;

$d_{ij} (x_{ij})$  = cost of shipping quantity  $x_{ij}$  from  $i$  to  $j$ , dollars;

$F_j(y_j)$  = the cost of establishing and operating a warehouse at site  $i$ , where  $y_j$  is being shipped from  $i$ , dollars;

$D_j$  = the demand at area

$n$  = the number of demand areas; and

$m$  = the number of proposed warehouse sites.

There is a trade-off between facility establishment costs and costs of distributions, since as distribution costs are diminished total costs of facilities will increase. There should be some point at which the unit cost of the additional facility will exceed the decrease in transportation costs, at this point a minimum total cost will exist. Facility costs, due to economies of scale, are generally nonlinear. Thus, researchers have spent a large amount of time on methods of solving the numerical problems plant location models present.

Application of the plant location problem to retail facilities can be made, although direct examples were not found. Goods may flow either from sources (the areas  $j$ ,  $j=1, \dots, n$ ) to warehouse locations ( $i$ ,  $i=1, \dots, m$ ) or the other way.

In retail application one would be given:

1. Demand ( $D_j$ ) at area  $j$ . This is analogous to available consumer dollars in most shopping models.
2. The location of all  $j$ 's. (Same for shopping models.)
3. Cost of shipping quantity  $X_{ij}$  from  $i$  to  $j$ . This would be similar to the use of consumer travel time in most shopping models. Bucklin (1967) distinguishes the value of shopping and total travel time to most shoppers. A consumer not only wishes to minimize the home to store cost but search cost also - that inter-store travel the consumer must make to resolve satisfactorily his shopping objectives.

In a traditional plant location problem,

We would wish to find out:

1. The number of plants ( $m$ ). This would of course be store outlets. Most of the time this is known, unless one were attempting the location of many store outlets in a region simultaneously.
2. The location of each plant ( $j$ ). This is often known in a shopping model; however, when site alternatives are considered, the plant location problem is completely analogous.
3. The capacity of each plant ( $y_i$ ). Store size in shopping model.

In application of any plant location algorithm.

#### Restrictive Assumptions:

Plant location models usually are used to describe the optimal allocation of resources to a warehouse/distribution scheme for any sort of goods or services distribution from a central point.

It does not account for fixed competitive locations, store choice behavior, and concepts of store mass which are major determinants of successful retail outlet operation.

#### Data Required:

Adaption of any plant location algorithm for retail use would require the following data:

1. by location, available consumer expenditure dollars for specific goods or services under consideration.
2. current plus future available retail outlet locations.
3. estimates of link travel costs per unit of shopping demand (per available expendible dollars).

#### Nature of Results:

Plant location models are most seriously deficient in retail use because of the lack of a method for determining the amount of total demand of  $j$  ( $D_j$ ) which will be experienced at store at  $i$ . Accounting for relative importance of node  $j$ , solutions are based on minimization of distances from outlets to consumers as well as optimization of the number of available outlets.

#### Sensitivity of Results:

This is difficult to evaluate given the work reviewed in this research.

#### Treatment of Change:

Routines are not time sensitive. Little, if any, data used is descriptive of current situations, e.g., derived through statistical analysis of resident population surveyed.

L. A. WHITE and J. B. ELLIS, NETWORK APPROACH, 1921

Source: White, L. A. and Ellis, J. B., "System Construct for Evaluating Retail Market Locations", Journal of Market Research, Vol. VIII (Feb. 1971), pp. 43-46.

Objectives:

Model directed towards solving these problems:

1. An estimate of sales at a proposed store and how it would affect the business of other stores in the system.
2. An estimate of the sensitivity of sales to major changes in store characteristics and the traffic network.
3. A mathematical formula for describing the drowning power of a store in the system.

Description:

Draws on an analogy from electrical engineering network analysis to solve problem where system is formulated into 3 sets of components: origins (areas of money generation), links (roads connecting origins and destinations), destinations (supermarkets).

Each component is defined by a terminal equation.

For Origins:

$$Y_i = 52 (C_i) (P_i)$$

Where:  $Y_i$  = yearly flow of money from origin area  $i$  in dollars

$C_i$  = the cost of food per week per capita

$P_i$  = the population at origin area;

For Links:

$$X_{ij} = R_{ij} Y_{ij}$$

Where:  $X_{ij}$  = the pressure required to cross the link connected from node  $i$  to node  $j$ .

$R_{ij}$  = the resistance to flow through link  $ij$  (in most cases this would be driving time)

$Y_{ij}$  = the result from the origin terminal equation - yearly net flow of shopping money through link  $ij$  (White and Ellis used Link length, since most roads in their model had similar average speeds.)



For Destinations:

$$Y_j = A_j X_j$$

Where  $Y_j$  = the yearly sales of supermarket  $j$

$A_j$  = the attraction of supermarket  $j$

$X_j$  = the propensity to shop at supermarket  $j$

White and Ellis used a linear function found through multiple regression analysis to describe  $A_j$  as

$$1/A_j = 136925./(\text{floor area}) + 3.0099/(\text{number of checkouts}) + 84.4889 (\text{relative price}) + 0.2953/(\text{specials}) + 1.1303 (\text{location coefficient})$$

The model is solved using the computer by chord formulation.

Restrictive Assumptions:

Boundary of trade area must be defined, that is model does not account for expenditures or retail outlets outside the area defined by  $i$  and  $j$ .

Data Requirements:

Data needed includes:

1. household expenditure information
2. population
3. description of roadway network
4. descriptive information on existing and proposed retail units which allows for distinction of relative attractiveness of facilities.

Nature of Results:

This can vary depending upon what is known. It is assumed that primary result of concern would be sales volume annually in dollars at each supermarket  $j$ .

Sensitivity to Data:

An appealing feature of this approach is the flexibility it offers the analyst to test the sensitivity of the results to the data provided. White and Ellis did not describe, however, sensitivities of the model which they formulated.

Treatment of Change:

Again, the nature of this system allows for temporal changes to be analyzed through the replacement of one of any number of variables. One would have to examine the use of any descriptive data (i.e., store attractiveness) used in the system as changes over time are input. The example application in Kitchener/Waterloo, Ont. by the authors was strictly for the present time period only.

## DAVID B. MacKAY - MICROANALYTIC APPROACH, 1972

Source: MacKay, David B., "A Microanalytic Approach to Store Location Analysis", Journal of Marketing Research. Vol. IX (May, 1972).

### Objective:

MacKay begins discussion by challenging the validity of location models which do not account for multi-stop shopping trips. Location through a distance utility surrogate may be invalid when consumers may wish to satisfy several shopping needs on the same trip. Thus, his microanalytic approach is a spatially defined model of store selection that accounts for multi-stop shopping trips using discriminant analysis and Monte Carlo simulation.

### Description:

The framework of the model simulates the customer's shopping decision process and is characterized by three major steps:

1. The decision to go shopping or not to go.
2. The decision on how many stops to make.
3. The decision of which establishment to visit on each stop.

Total store visits are simulated by the model though generating a sample of individual customer behavior.

Each shopping trip is modelled in two phases:

#### Trip generation and trip structure.

##### Trip Generation:

Posterior probabilities are generated by the discriminant functions for each step in the trip generation process and then sampled by a Monte Carlo process to produce the outcome of the event.

1. The first set of functions determined whether or not the shopper will or will not make a shopping trip on a selected day.
2. Next, probability of making stops  $i, \dots, n$  are computed.
3. Then, which establishment types are visited at each stop are determined.

The result of a previous calculation are used in turn in the next decision step.

### Trip Structure:

Establishment types are replaced by locating specific stores in this step of the process through the use of movement heuristics:

Single-stop distance minimization: The consumer forms a list of stops which he plans to make but determines the establishments by picking those closest to his trip origin (home).

Modified sequential distance minimization: the consumer minimizes the total distance travelled for small trips but on larger trips reverts to a sequential distance minimization technique as the distances from the origin increase.

Discriminant heuristic: Same as modified sequential distance minimization except that when the consumer's itinerary indicates that a stop is about to be made at a principal establishment (the one focal to analysis), a discriminant function is used to determine which establishment is selected.

The rationale for the modified sequential distance minimization heuristic is that as consumers move further from home while shopping, the quality of their information set decreases and they are less able to minimize the total distance travelled. On stops farther from home, the consumer selects stores on a sequential distance minimization basis. For stops near home, where the consumer's information set is more complete, distances between multiples of stops are minimized.

### Restrictive Assumptions:

In test application, MacKay found it necessary to restrict the size of the community, degree of self-containment and network of streets to a grid. Compared to many other techniques, this approach requires for fewer a priori assumptions about consumer behavior patterns of shopping frequency, trip itinerary planning, and store choices.

### Data Requirements:

Major independent variables required for the discriminant functions include:

Household characteristics: demographic and socio-economic information such as family size, age group, education level, employment status, income, number of driver's licenses per household.

Attitudinal information: index of area resident's attitudes towards existing stores (supermarkets) which are a product of each store's characteristics (price, decor, etc.) and a relative weighting value.

Shopping trip variables: mode of travel, accompaniment to store, home to store distance, establishments visited, frequency of visits.

Information which is place specific must carry geographic code of some sort to allow use of trip structure heuristics.

Nature of Results:

Simulation results include the number stops to be made at each establishment and, through use of a regression equation, stops at specific establishments are converted into sales volumes.

Sensitivity to Data:

MacKay has developed a descriptive (simulation) model which through rigorous calibration of discriminant functions is capable of providing reasonable results describing an existing situation. Several tests were made using the alternative trip structuring heuristics. The discriminant heuristic was found to provide the closest description of actual consumer behavior surpassing more simple heuristics which resemble the one-step logic of traditional shortest distance routine.

Treatment of Change:

Although MacKay's approach is primarily descriptive, several applications representing the location of a new store facility were attempted. Because of the heavy reliance on empirical evidence, however, which is static temporally, one can challenge with reason the assumptions which surround the use of the model for forecast work.

## STANLEY AND SEWALL, MODIFIED HUFF MODEL WITH IMAGE MEASURES, 1976

Source: Stanley, Thos. J. and Sewall, Murphy A., "Image Inputs to a Probability Model: Predicting Retail Potential", Journal of Marketing. Vol. 40 (July 1976), pp. 48-53.

Objectives:

The authors argue that the Huff model would be substantially more usable if it could be refined to include a retailer image factor.

Description:

Huff formulated his model as:

$$P_{i\lambda} = \frac{S_j^{\lambda_s} T_{ij}^{\lambda_t}}{\sum_{j=1}^n S_j^{\lambda_s} T_{ij}^{\lambda_t}}$$

Where:  $P_{i\lambda}$  = the probability of a consumer in area  $i$  shopping at retail location

$S_j$  = the square feet of retail selling area of retail location .

$\lambda_s$  = the sensitivity of changes in shopping probability to changes in selling area.

$T_{ij}$  = driving time between area  $i$  and retail location  $j$

$\lambda_t$  = the sensitivity of changes in shopping probability to changes in driving time.

Huff found  $S_j$  as a proxy for assortment of merchandise although later critics have suggested that Huff's equation is of limited value in estimating sales of single stores since size per se has not been found to have great influence on drawing power.

Stanley and Seawall have modified the Huff model to include an image factor for forecasting grocery store sales:

$$P_{i\lambda} = \frac{S_j^{\lambda_s} T_{ij}^{\lambda_t} D_{ij}^{\lambda_d}}{\sum_{j=1}^n S_j^{\lambda_s} T_{ij}^{\lambda_t} D_{ij}^{\lambda_d}}$$

Where additionally:

$D_{ij}$  = the measure of image distance between an "ideal" supermarket

chain for consumers in area  $i$  and the chain represented in the market area by supermarket .

$\lambda d$  = the sensitivity of changes in shopping probability to changes in store image.

It was theorized that consumers perceive less utility in patronizing stores affiliated with a chain that is, in a relative sense, distant from their view of an ideal chain. Distance was measured from survey results using the INDSCAL multi dimensional scaling program which maps stimuli onto a set of dimensions, the distance between points representing the dissimilarity between stimuli.

#### Restrictive Assumptions:

The authors attempt to utilize an image factor was directed on reducing the normative characteristic of the Huff model. In their test case using twelve supermarkets, they achieved improved predictive results with the image factor.

Nevertheless, use of the "modified Huff model" requires restricting application to a specific market area to be determined by the user. No account is made for consumer patronage of facilities within the area from those who live outside the area.

#### Data Requirements:

The image factor was based on survey results of shopper's comparisons of store quality, cleanliness, location, prices, friendliness, and variety. Results of the INDSCAL-MDS program were used in a least squares estimate of the parameter  $d$ . Other information same as Huff (1963).

#### Nature of Results:

Result was probability of consumer patronage of store  $j$ . This could (per Huff's example) be extended to prediction of total sales dollars at store through addition of factor of total available consumer dollars (for groceries) at each resident zone  $i$ , where  $i = 1, \dots, m$ .

#### Sensitivity of Results:

In estimating parameters, the authors found that driving time still accounted for over 50% of the variation. Store image explained over 40% of the remaining variance.

#### Treatment of Change:

Contribution of authors was to offer an improved model for accurate estimates of potential store locations. Such a model can also

reflect changes in store chain image, too, with the resultant impact on consumer patronage. Model will remain useful so long as parameters validly represent consumer behavior regarding driving time and store image.



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