USING MODELS
TO DEVELOP FRACTION CONCEPTS

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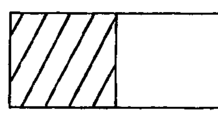
Students frequently have difficulty performing computation involving fractions. These difficulties are due, in part, to students' lack of understanding fraction notation and computational algorithms. Using concrete materials can help children develop the understanding necessary for working efficiently with fractions.

Fortunately, concrete models of fractions need not be expensive. They can be made by teachers and by students. When students make the materials, they can learn much about fractions while making the models, as well as while using them later.

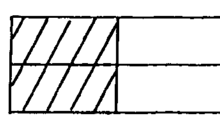
Two such models will be discussed here. One model involves paper folding to illustrate fraction concepts. The other model uses colored paper of different sizes to represent fractional parts.

Paper Folding

Begin by having students fold a sheet of paper in half and shade one of the halves. Discuss the fact that 1 of 2 equal parts or 1/2 of the paper is shaded.



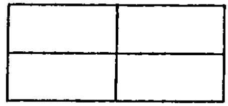
Have students fold the paper in half lengthwise to create four equal parts. Now, how many parts are shaded? [2] So 2 of 4 equal parts or 2/4 of the paper is shaded. This illustrates that 1/2 and 2/4 are two different names for the same portion of the sheet of paper. This motivates the concept of equivalent fractions.



An additional example of equivalent fractions can be generated by folding the paper again to create 8 equal parts. Now 4 of 8 equal parts will be shaded.

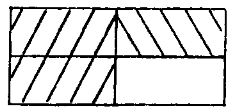
Other examples can be created by beginning with folding the paper in thirds.

Addition of fractions can also be illustrated through paper folding. Begin with a sheet of paper folded in fourths as shown. Have students use this to solve the following problem:

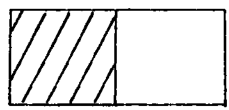


> Two cakes were baked for a party. After the party, one-half of the chocolate cake was left, and one-fourth of the lemon cake was left. What part of a whole cake was left in all?

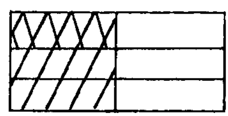
By shading 1/2 and 1/4 of the paper, students can see that 3/4 of a cake was left.



Multiplication problems can also be shown. In order to do so, students must realize that 1/3 x 1/2, for example, means 1/3 of 1/2. To show this we first need to show 1/2.



Now we can find 1/3 of the 1/2 by folding the paper in thirds lengthwise. Students can see that 1/3 of 1/2 is 1/6.



<u>Colored Paper Fraction Strips</u>

Another model of fractions can be made by folding and then tearing or cutting paper of different colors to represent different fractional parts. Strips are of the same width, they vary in length according to the fraction represented. The following color scheme might be used:

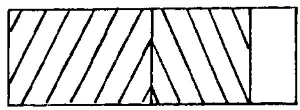
White:	1
Blue:	1/2
Pink:	1/3
Gold:	1/4
Green:	1/6
Yellow:	1/12

Students can make these materials themselves. Interesting discussions may arise when students try to determine how to fold the paper into different numbers of equal parts. They discover, for example, that folding the paper into fourths will not lead them to sixths.

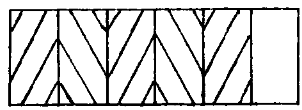
After students have made these fractions strips, have students place a blue strip on the white to cover 1/2. Have students find different ways to cover the blue part with other colors without having gaps or overlaps. Students will discover that 2 gold strips, 3 green strips, and 6 yellow strips cover the blue strip. They will also discover many different combinations such as 1 pink, 1 green and 1 gold, 1 green, 1 yellow. This will review equivalent fractions (probably better introduced with paper folding) and motivate using these strips to represent addition combinations.

Problems like the following might be used to introduce addition.

> A recipe calls for 1/2 cup milk and 1/3 cup water. How much liquid does the recipe require?



This situation can be represented by a blue strip and a pink strip. The problem is to cover this with strips of the same size (strips of the same color). A bit of experimentation will show that both the blue and pink strips can be covered with green strips.



If we record what has been done with the materials, it might look like this: ⟶ The 1/2 has been covered by 3/6, and the 1/3 has been covered by 2/6. By solving many problems like this, students can see that covering the problem strips with ones of the same color produces fractions with the same denominator. This helps students see the need for common denominators.

$$\begin{array}{rcl} \frac{1}{2} & = & \frac{3}{6} \\ + \ \frac{1}{3} & = & \frac{2}{6} \\ \hline & & \frac{5}{6} \end{array}$$

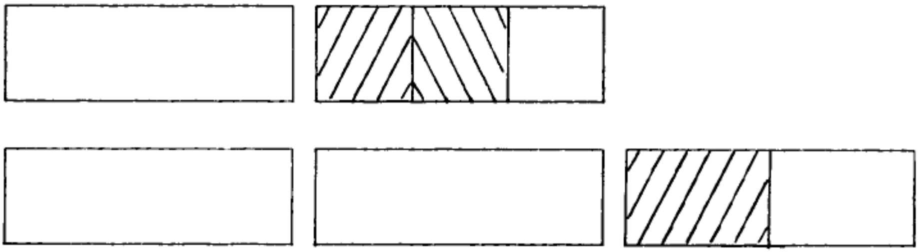
Other problems which illustrate this point well are:

$1/2 + 1/6 =$ $1/3 + 1/4 =$ $1/4 + 1/6 =$
$2/3 + 1/6 =$ $3/4 + 1/6 =$ $5/6 + 1/12 =$

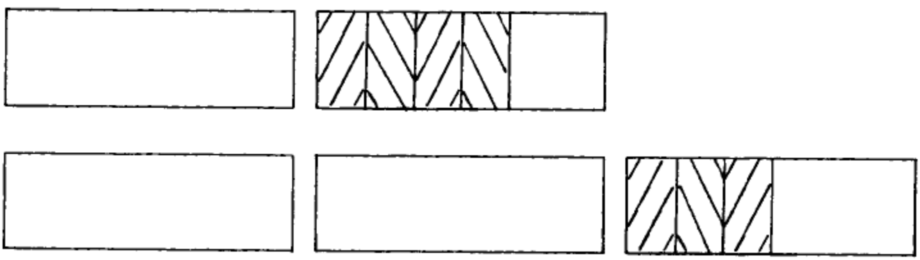
Addition problems with mixed numbers can also be solved with the fraction strips. Consider the following problem:

> I have two rugs which I would like to lay end to end in the hallway. One rug is 1 2/3 feet long; the other is 2 1/2 feet long. How long are the two rugs together?

This problem could be represented as follows:



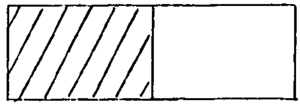
To solve the problem, we need to cover the fractional parts with strips of the same color and size.

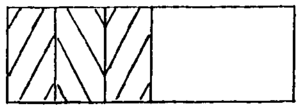


We have 3 7/6 feet. Students can move 2 of the 3 sixths to complete the top unit to find that this is 4 1/6 feet.

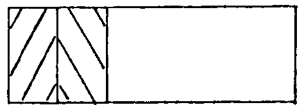
As students become better acquainted with the equivalent strips, they might be encouraged to replace strips with equivalent ones, rather than to cover strips. This may facilitate the problem above and will be needed to perform subtraction problems.

Consider, for example, 1/2 – 1/6. The task is to "take away" 1/6 from the 1/2. We must exchange to get sixths so that one of the sixths may be taken away.





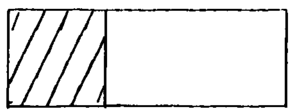
When 1/6 is removed, this remains:



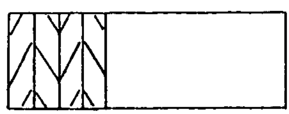
Again, the idea of common denominators is shown. Students can see that 1/2 must be exchanged for 3/6 in order to take away 1/6. Students may also notice that the result could be covered by or exchanged for 1/3. The work with the materials might be recorded as shown at the right.

$$\begin{array}{rcc} & \frac{1}{2} & = & \frac{3}{6} \\ - & \frac{1}{6} & = & \frac{1}{6} \\ \hline & \frac{2}{6} & = & \frac{1}{3} \end{array}$$

Some division problems also can be solved using the fraction strips. Consider the division exercise 1/3 ÷ 1/12. The question is: how many 1/12's are there in 1/3? We first need to show 1/3.



If we cover the third with twelfths, we find that four twelfths will cover the third.



Other division exercises can be shown with the fraction strips. The most appropriate exercises are those which have whole number answers, such as:

1/2 ÷ 1/6 =	1/3 ÷ 1/6 =
3/4 ÷ 1/12 =	2/3 ÷ 1/12 =

When the symbolic work with fractions is seen as simply a record of what has been done with concrete materials, students may gain an understanding of the computational algorithms. This understanding should allow students to be more efficient when working with fractions and to apply their knowledge to real-world situations.

MATH SWIFTIES

"I'm so happy that I can float," Tom declared buoyantly.
"This cream is bad," Tom said sourly.
"My toothpaste is all over the floor," Tom said crestfallenly.

Those of you who are GAMES magazine fans probably will recognize each of the above examples as a "Tom Swifty" – a line of dialogue ending with an especially appropriate adverb or verb. (We guess Tom Swift must have talked like that in his stories – or should have, if he didn't – can't seem to find the exact origin of the term, though.)

Well, of course we had to try our hand at some mathematical Swifties. Here are some of our attempts:

"2/3 is in lowest terms," Tom said properly.

"Multiplication is easy. Multiplication is easy." Tom repeated additionally.

"Subtracting integers is difficult for me," Tom said negatively.

"We just know that you (or your students) have some great math Swifties at the tip of your pencil," the editors said pointedly. Send them in and we'll include them in the Journal, so others can groan, too. "We'll give you full credit, of course," the editors promised authoritatively.

NEXT OCTM MEETING

The 1991 annual meeting will be in Cleveland, November 7-9, 1991. "Note the shift from spring to fall," they pointed out with conventional wisdom.
