MATHEMATICS CONTEST CORNER
What Do Pythagorean Triples Have To Do With It?

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Part of the *fun* in devising mathematics contest problems is rigging them to work with integers and so they have nice solutions. Consider the following problem:

> Two intersecting circles are drawn with radii 13*cm* and 20*cm*. The length of their common chord is 24*cm*. Find two possible values for the distance between the centers of the circles.

As with many problems, the best solution can be seen if one begins with a sketch of the scenario. One such sketch (with center A outside of the larger circle) is shown in Figure 1.

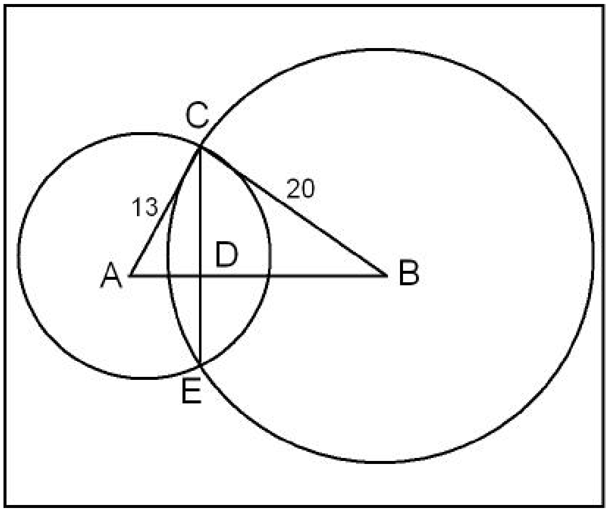

Figure 1

Since chord CE is 24cm, segment CD must be 12cm. Thus, by the Pythagorean theorem, $\overline{AD} = 5cm$ and $\overline{DB} = 16cm$. Therefore, one possible distance between the centers of the circles is $\overline{AD} + \overline{DB} = 5cm + 16cm = 21cm.$

A second sketch for this problem (with center A inside the larger circle) is shown in Figure 2.

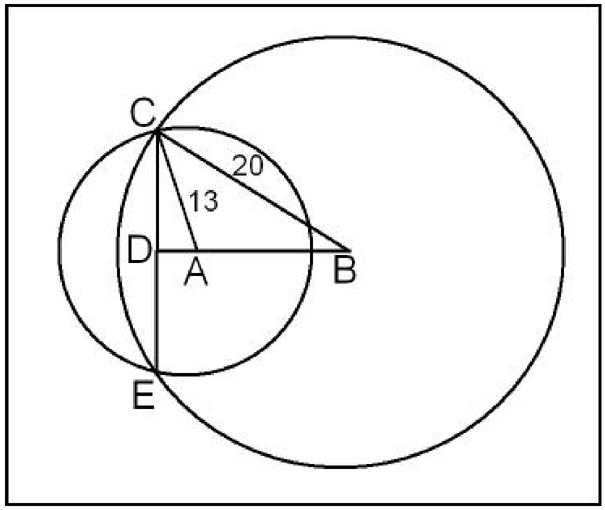

Figure 2

Since $\overline{AD} = 5cm$ and $\overline{DB} = 16cm$, the second possible distance between the centers is $\overline{DB} - \overline{AD} = 16cm - 5cm = 11cm.$

Thus, the solution to the given problem is 21*cm* and 11 *cm*.

Although this problem is interesting in and of itself, some discussion of how it was devised is beneficial. Note that right triangles ADC and BDC share a common side. Therefore, the use of Pythagorean triples was employed to *rig* the problem with an integer solution.

A Pythagorean triple follows the classic right triangle relationship: $a^2 + b^2 = c^2$. Note that a and b are the lengths of the legs of the right triangle and c is the length of the hypotenuse. Sets of (a, b, c) where all three lengths are integers are called Pythagorean triples. A non-trivial Pythagorean triple is not a multiple of any other such triple. For example, 10 non-trivial Pythagorean triples are:

(3, 4, 5)
(5, 12, 13)
(7, 24, 25)
(8, 15, 17)
(9, 40, 41)
(11, 60, 61)
(12, 35, 37)
(13, 84, 85)
(15, 112, 113)
(16, 63, 65)

A trivial Pythagorean triple might be (12, 16, 20) because it is a multiple of the non-trivial triple (3, 4, 5) – the multiplier being 4. Setting up this problem required two legs to have a length of 12*cm* since the common chord had a length of 24*cm*. Thus the non-trivial triple (5, 12, 13) and the trivial triple (12, 16, 20) were used. The problem could have been setup many ways. For example, two non-trivial triples (5, 12, 13) and (12, 35, 37) could have also been used since each contains a leg of length 12. Of course with the second set, the radii of the two circles would have to be 13 and 37 respectively.

It is beneficial to share problem construction techniques with the students in a mathematics club or a class using contests as a teaching technique. This helps to deepen student understanding of mathematics. Further, asking students to generate problems and solutions can be a great source of practice problems -- and you will find that the quality of the problems they generate increases as you discuss why existing problems are setup and stated as they are.

A Teaching Tip

Try giving our initial problem to your class or mathematics club. After you have discussed its solution, ask the students to generate a similar problem with different circle radii. Tell the students that you would like all of the solutions to be integers. As solutions are proposed, you should be able to introduce the Pythagorean triple concepts.

Give it a try and you will be pleased with the results!

Quote:

"So, about 100 seventh graders, all doing poorly in math, were randomly assigned to workshops on good study skills. One workshop gave lessons on how to study well. The other taught about the expanding nature of intelligence and the brain.

The students in the latter group "learned that the brain actually forms new connections every time you learn something new, and that over time, this makes you smarter."

Basically, the students were given a mini-neuroscience course on how the brain works. By the end of the semester, the group of kids who had been taught that the brain can grow smarter, had significantly better math grades than the other group."

From National Public Radio - Morning Edition, Thursday, February 15, 2007. See http://www.npr.org/templates/story/story.php?storyId=7406521 .
Students' View of Intelligence Can Help Grades, By Michelle Trudeau