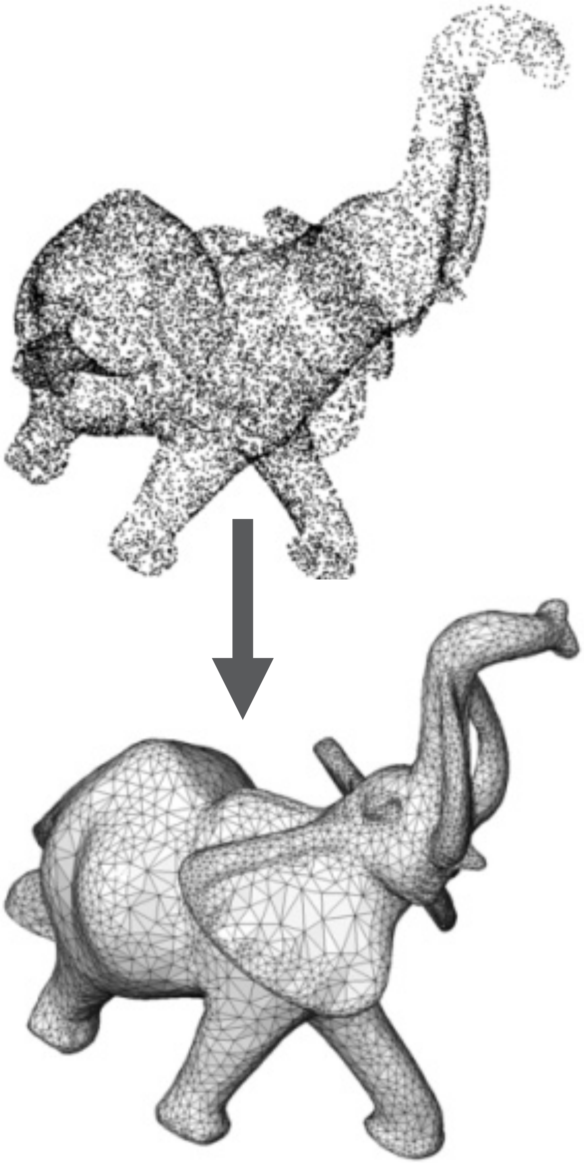
Statistical Interpolation for Surface Reconstruction of PDV and BLR data



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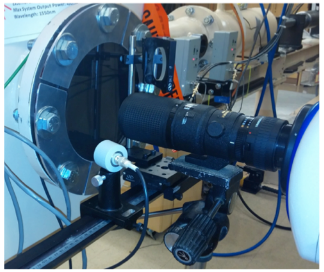
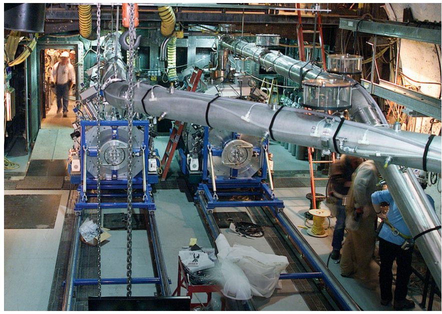
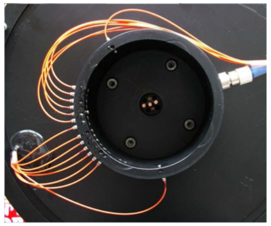
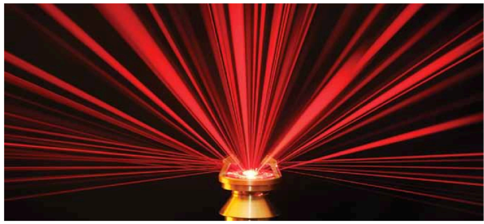
Motivation

- Data collection is becoming more and more dense in dynamic experiments with a variety of diagnostics

  - Radiography
  - Velocimetry
  - Ranging
  - Fiber Bragg
  - Pyrometry
  - Holography
  - Mie scattering / extinction
  - Assay foils
  - Surface Imaging
  - High speed photography

- How can we unify information from **multiple diagnostics** and **incorporate uncertainties** to tell a coherent story about an experiment in a meaningful way?

Our Approach

- A statistically informed interpolation for building a surface representation of a dynamic experiment

<u>Inputs</u>

Multi-diagnostic data products
- Experiment
- Simulation

Metrology data

→

<u>Model</u>

Statistical interpolation to create a surface representation

- Incorporates measured uncertainties
- Allows for data dropouts

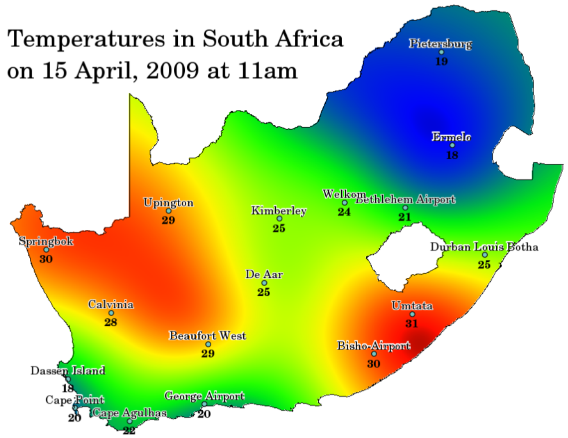
→

<u>Capabilities</u>

- Quality assurance for metrology
- Experiment design
  - Sensor optimization
- Quantitative analysis
- Experiment vs. Simulation comparison
- Diagnostic comparison
- Data visualization

Statistical Interpolation: Kriging

- Kriging is an interpolation method where interpolated points are modeled through a data-informed covariance structure and Gaussian process
  - Developed in 1960 for geostatistics by French mathematician Georges Matheron, as based on work by Danie G. Krige
  - Distance-weighted average, where weights are determined through a **covariance structure**
  - Best linear unbiased estimator: **minimizing the variance (uncertainty) in predicted values**.
  - **Incorporates uncertainties** in “data” samples
  - Allows for **uneven data sampling** (non-gridded, noncomplete)



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Statistical Interpolation: Kriging

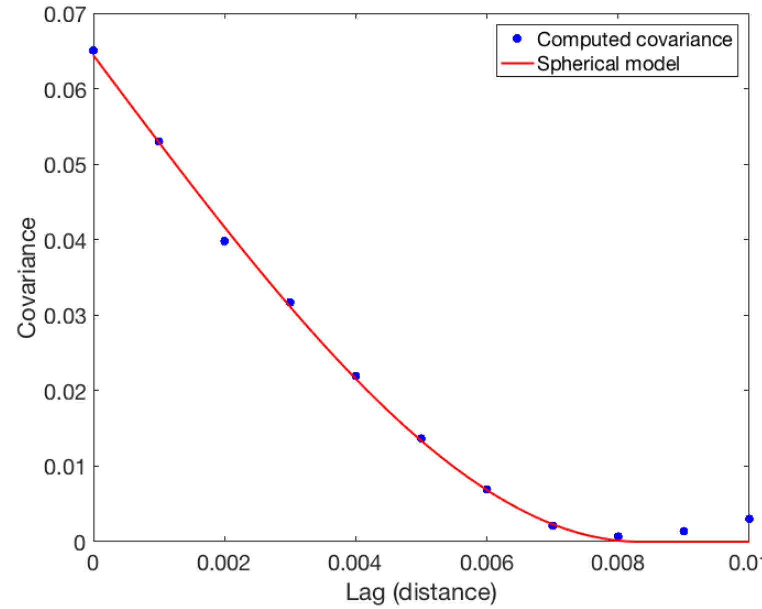
1. Compute and model covariance spatial structure

$$\hat{\gamma}(h) = \frac{1}{2n(h)} \sum_{(i,j)|h_{ij}=h} (y_i - y_j)^2$$

- Intuitively, data values are likely more strongly correlated when located closer together

- There are several options for functional forms of covariance

$$\gamma(h) = a + (s^2 - a)\left\{\frac{3|h|}{2r} - \frac{1}{2}\left(\frac{|h|}{r}\right)^3\right\}$$



2. Compute weights by minimizing mean squared prediction error

$$\min \sigma_\varepsilon^2 = Var(Y_0) + \sum_{i=1}^n \sum_{j=1}^n w_i w_j C_{ij} - 2\sum_{i=1}^n w_i C_{i0}$$

subject to $\sum_{i=1}^n w_i = 1$

- Weights are based on fitted covariance model and are independent of measured values

3. Predict at desired prediction sites (krige)

$$\hat{Y}(s_0) = \sum_{i=1}^n w_i y_i$$

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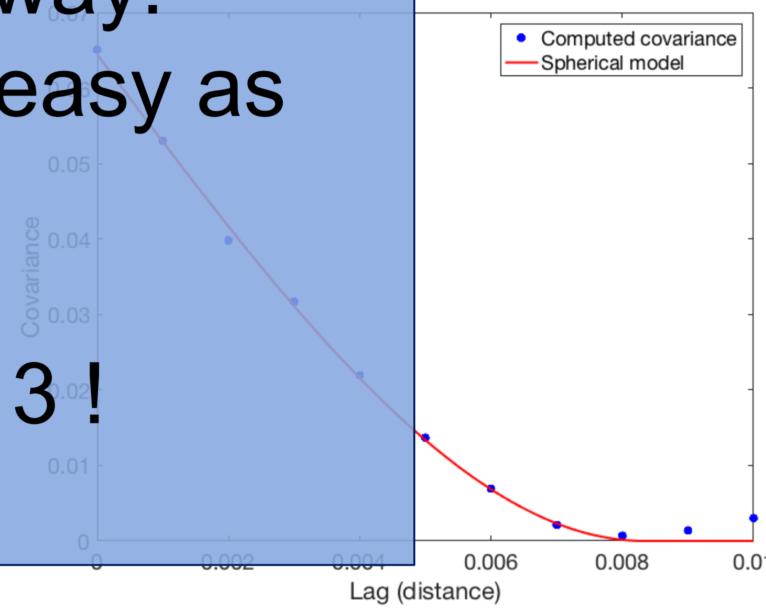

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The Takeaway:

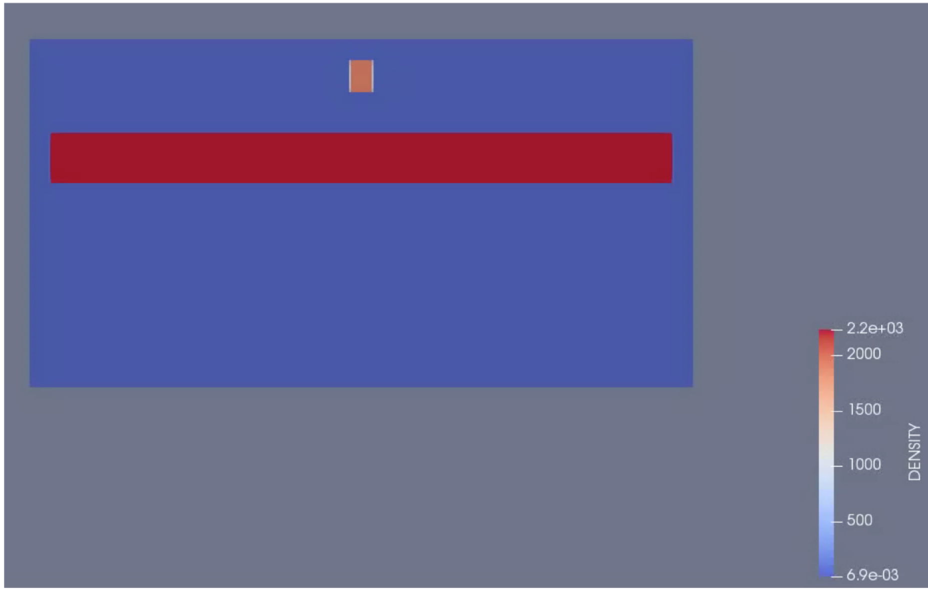
Kriging is as easy as

1 … 2 … 3 !

Data Demonstration

- Simulated data:
  - Lexan projectile impacting a glass surface
  - Chamber pressure: 4.23 Torr
  - Plate thickness: 2.45 cm

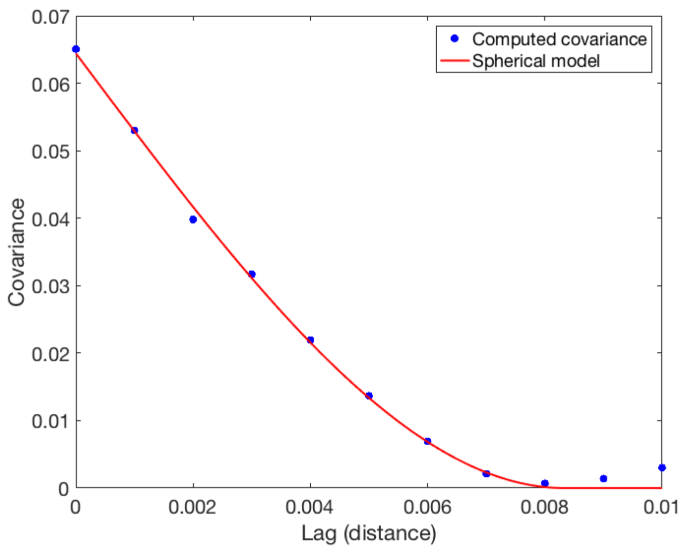
- Objectives:
  - Compute dense estimates of a single "PDV" trace given sparse simulation data
  - Compute dense surface reconstruction of a "line of PDV" traces

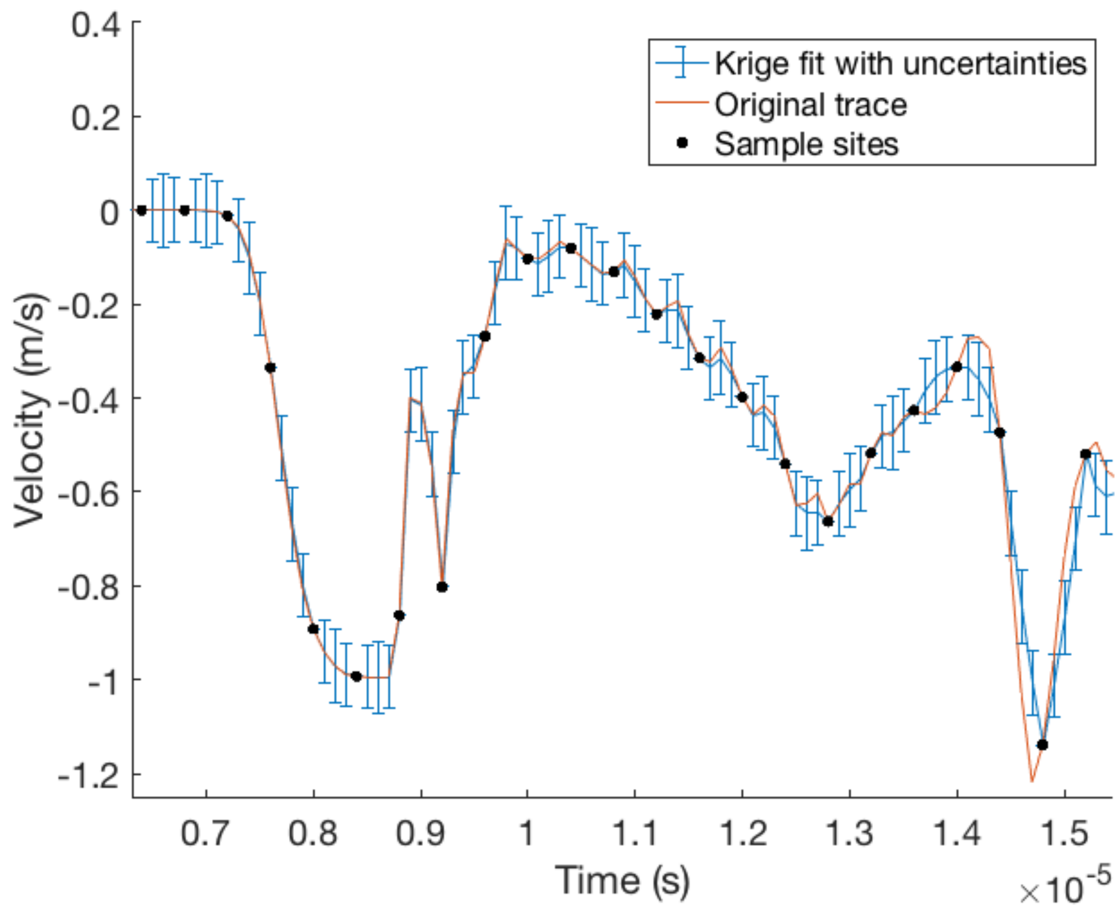


Simulation provided by B. T. Meehan

Data Demonstration

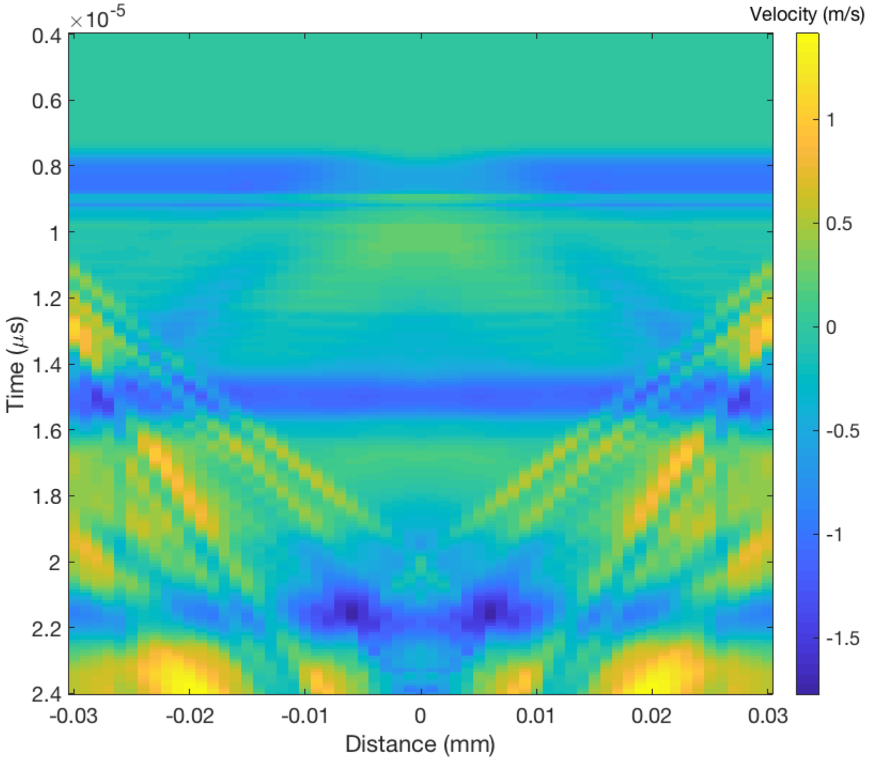
- Kriging on a single PDV trace



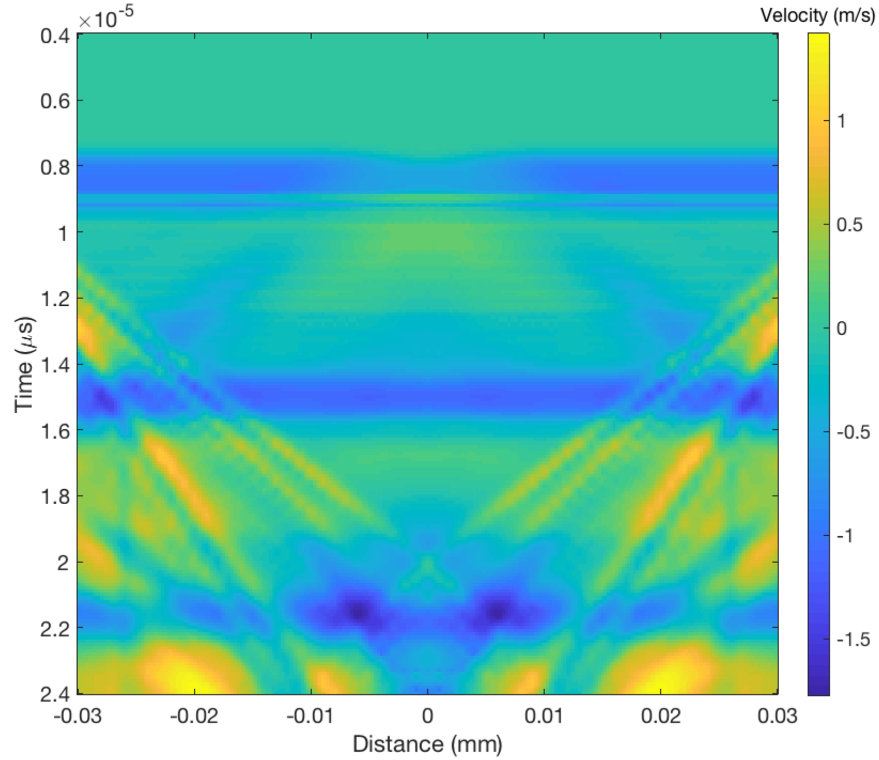


Data Demonstration

- Surface reconstruction map



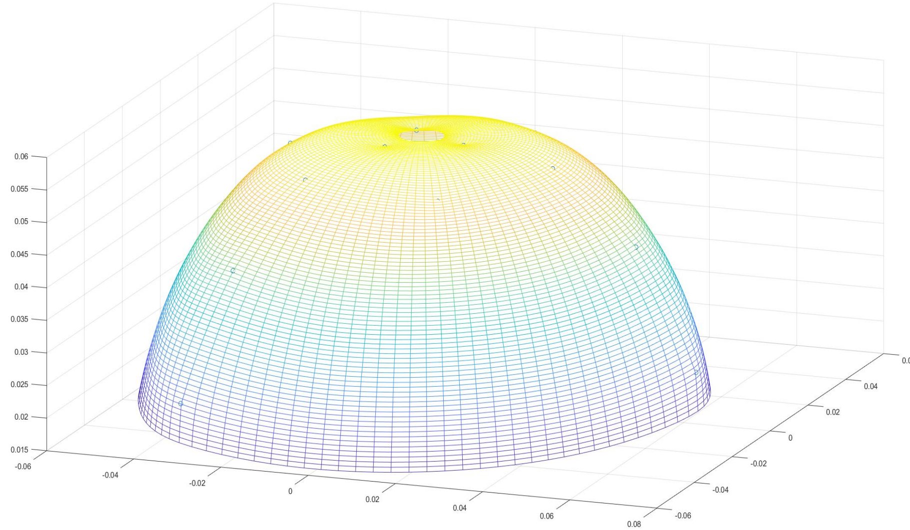
Original Simulation



Kriging Map

Future Outlook

Capabilities of the final surface reconstruction software:



- An interactive GUI for the user
- Integrate multiple diagnostics
- Experiment and simulation comparison
- Experimental design features
- Quality assurance for metrology
- Visualization interface
  - Visualize surface location, velocity, signal strength, data dropout/loss, ejecta, user-defined anomalies
- Diagnostic agreement