DE LA RECHERCHE À L'INDUSTRIE

INFLUENCE OF MULTIPLE LIGHT SCATTERING ON PDV MEASUREMENTS IN PRESENCE OF EJECTA

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OVERVIEW

➢ PDV RESPONSE OF AN EJECTA CLOUD IN VACUUM

- PDV spectrum: 1D model
- Parametric studies
- Statistical estimation and uncertainties

➢ EXPERIMENTAL RESULTS

- Collimated *vs* diverging PDV lens
- Estimation of the areal mass of ejecta M_s
- Comparison with Asay probe results

➢ MULTIPLE LIGHT SCATTERING IN THE EJECTA

- Doppler Monte Carlo-based approach
- Light interaction between free surface and ejected particles
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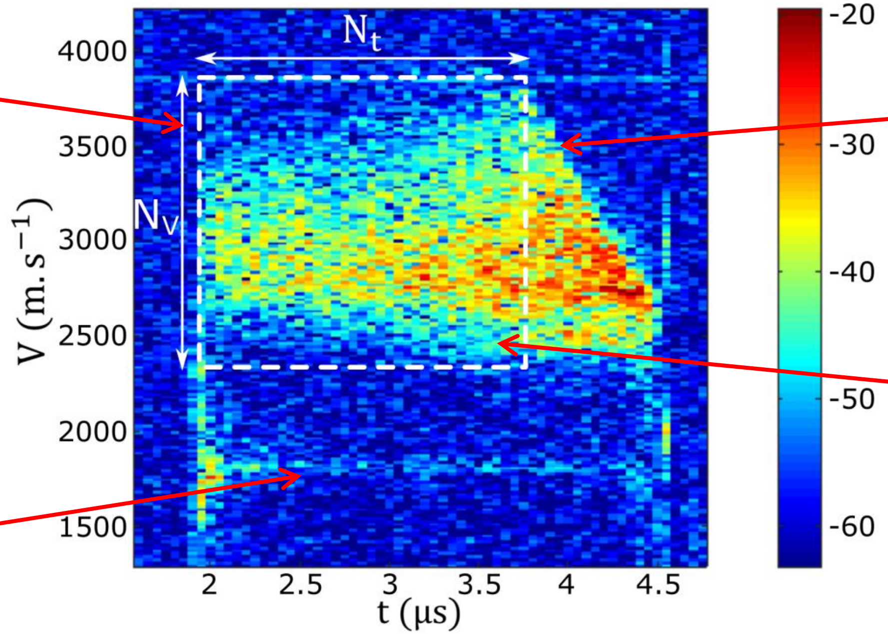
A TYPICAL PARTICLE CLOUD PDV MEASUREMENT

PDV spectrogram

(Sn 60x8 µm, HE 29 GPa).



Multiple ejecta Velocities.

Collisions with the lens.

Slower and faster Velocities appear at late times?

Free surface: not so obvious…

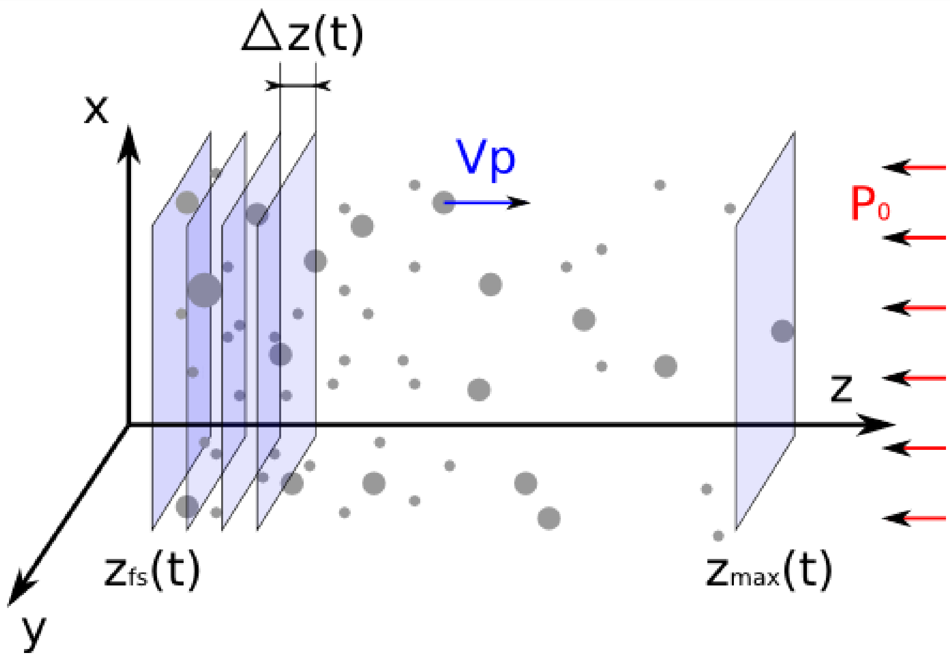
In this talk, we try to answer the following questions:

- How are the properties of the cloud related to the PDV response?
- How can we process the data?
- Is there a possible way to estimate an areal mass from PDV?
- Are there any artifacts on the spectrum due to multiple light scattering?

PDV RESPONSE OF AN EJECTA CLOUD: 1D MODEL

Description of the cloud:

At a given time of motion, a particle cloud is discretized into *N* slabs.
$z_{fs}(t)$: free surface; $z_{max}(t)$: head of the cloud.



Collimated light beam (PDV probe).

- ➢ Velocities are collinear to the z-axis, between $V_{min} = V_{fs}$ and V_{max}.
- ➢ In vacuum, we assume uncorrelated particle sizes and velocities and invariant cloud properties along x and y.

CLOUD PARAMETERS DEFINITIONS

Some parameters need to be defined:

- $\boldsymbol{M_s}$ $(\mathrm{kg/m^2})$: areal mass of ejecta,
- $\langle \boldsymbol{\sigma_{ext}} \rangle$ $(\mathrm{m^2})$: average extinction cross section,
- $\langle \boldsymbol{V_p} \rangle$ $(\mathrm{m^3})$: average particle volume,
- $\boldsymbol{\rho}_{\mathbf{p}}$ $(\mathrm{kg/m^3})$: metal density,
- $\boldsymbol{\beta}$: slope of the cumulated areal mass-velocity function $M(V)$,
- $\boldsymbol{\alpha}$: critical exponent of the size distribution (if power law),
- **S** $(\mathrm{m^2})$: surface of ejection,
- $\mathbf{d_{min}}$ & $\mathbf{d_{max}}$, (m); $\mathbf{V_{min}}$ & $\mathbf{V_{max}}$ $(\mathrm{m/s})$: lower and upper bounds of particle diameters and velocities.

A 1D model describing the PDV response of an ejecta cloud can be proposed.

PDV RESPONSE OF AN EJECTA CLOUD: 1D MODEL

Cumulated areal mass-velocity function of ejecta M(V):

$$M(V) \simeq \frac{\rho_p \langle V_p \rangle}{S} \int_V^{V_{max}} f(V)dV = M_s \cdot \exp\left[-\beta\left(\frac{V}{V_{fs}} - 1\right)\right].$$

Distribution of sizes and velocities (resp. $f(V)$ and $f_d(d_p)$) :

Exponential behavior:

$$f(V) = \frac{\beta M_s S}{\rho_\mathrm{p} \langle V_\mathrm{p} \rangle V_{fs}} \cdot \exp\left[-\beta\left(\frac{V}{V_{fs}} - 1\right)\right].$$

$f_d \propto d_p^{-\alpha}$ or $f_d \sim \log N\,(\mu, \sigma)$ (power law or lognormal scaled).

PDV SPECTRUM

PDV spectrum:

We assume that particles are randomly arranged in each slab $\left[V-\frac{\delta V}{2}, V+\frac{\delta V}{2}\right]$, i.e., no relation between optical phases (uniformly distributed between $-\pi$ and π). A first order theory of scattering gives the average PDV spectrum between $V-\frac{\delta V}{2}$ and $V+\frac{\delta V}{2}$:

$$\langle\Phi(V)\rangle = \langle P\rangle \delta V f(V) \times \exp\left[\frac{-2\langle\sigma_{ext}\rangle}{S} \cdot \int_{V}^{V_{max}} f(V)dV\right].$$

- This model takes into account <u>some</u> of the multiple scattering (attenuation).
- $\langle P\rangle$: average collected power per particle.

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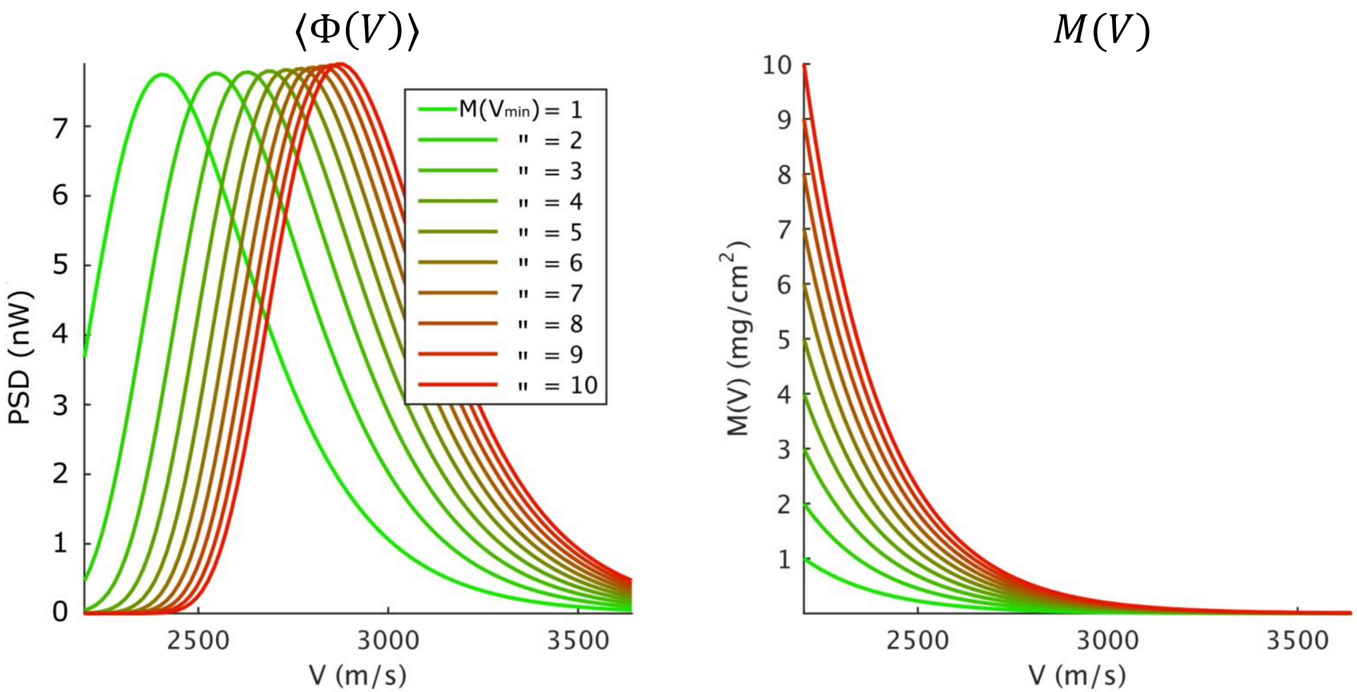
PARAMETRIC STUDIES

We can study the way how the parametric dependencies of the ejecta cloud influence the PDV response (see Ref. [1]):*

➢ $V_{min} = 2200\ m/s$, $V_{max} = 3650\ m/s$, $d_{min} = 1\ \mu m, d_{max} = 10\ \mu m, \beta = 10.8$, $\alpha = 4$ (power scaled size distribution), areal mass between 1 and 10 $\mathrm{mg/cm^2}$.



*[1] Franzkowiak et al., "[PDV-based estimation of ejecta particles' mass-velocity function from shock-loaded tin experiment]()", Rev. Sci. Instrum., **89** (2018).*

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STATISTICAL ESTIMATION AND UNCERTAINTIES

Each point $\left[V - \frac{\delta V}{2}, V + \frac{\delta V}{2}\right] \times \left[t - \frac{\delta T}{2}, t + \frac{\delta T}{2}\right]$ ($[\delta V, \delta T]$ is the sampling in the time-velocity spectrogram) follows a speckle statistics, coming from the continuous evolutions of optical phases (i.e., relative distances) between particles.

Probability density of the PDV spectrum:

For an additive average background noise $\langle B_\phi \rangle$ in the whole bandwidth:

$$P\big(\Phi(V)\big) = \frac{1}{\langle B_\Phi \rangle + \langle \Phi(V) \rangle} \cdot \exp\left[-\frac{\Phi(V)}{\langle B_\Phi \rangle + \langle \Phi(V) \rangle}\right].$$

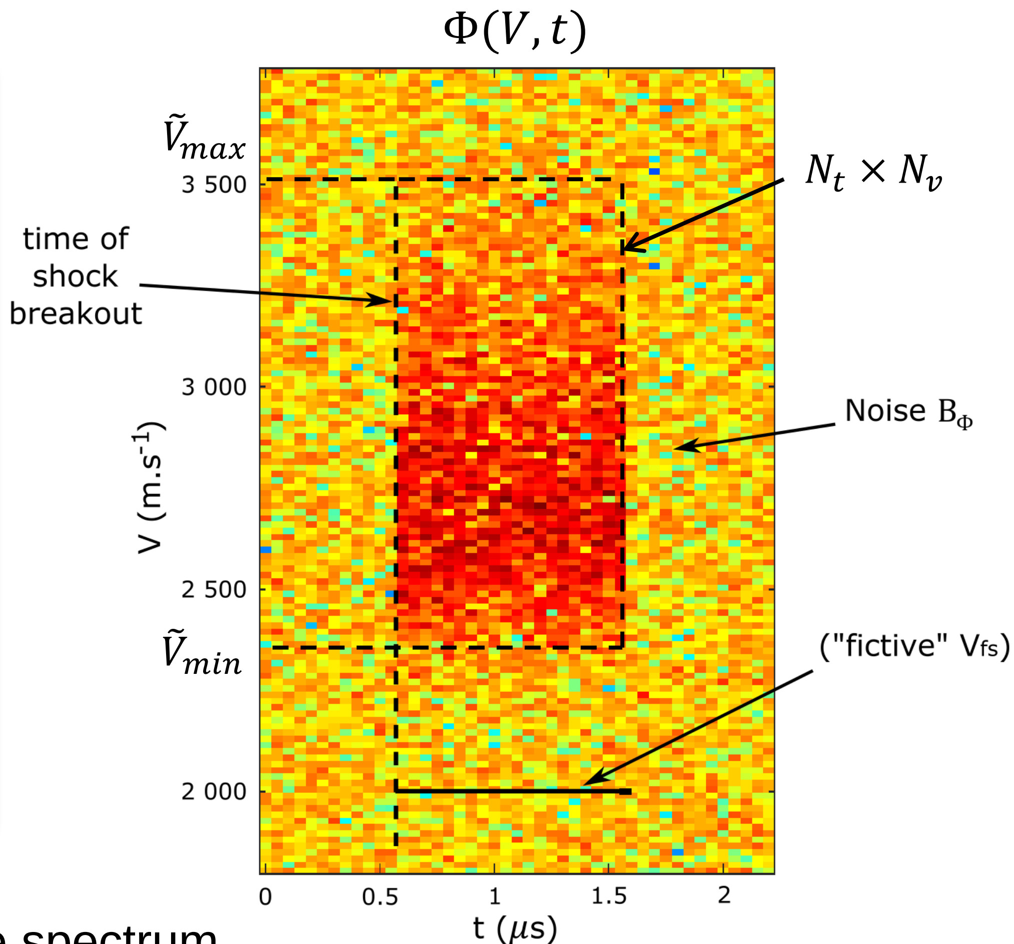
- B_Φ follows also a speckle statistics.
- Statistical estimations using the model $\langle \Phi(V) \rangle$ and $P(\Phi(V))$ can be performed.

PDV SIMULATION EXAMPLE

Simulated PDV spectrum, using model $\langle \Phi(V) \rangle$ and $P(\Phi(V))$:

- $V_{fs} = 2000\, m/s$, $V_{max} = 3600\, m/s$, $M_s = 12$ mg/cm^2, $\beta = 11$, $\delta T = 50$ ns, $d_{min} = 1$ µm, $d_{max} = 10$ µm.

- $RSB = \max\langle \Phi(V) \rangle / B_\Phi$.
- $\tilde{V}_{min}$ & $\tilde{V}_{max}$: lowest (largest) observable velocities from the cloud.



- In a vacuum, the underlying average spectrum $\langle \Phi(V) \rangle$ at each time step, and for a collimated beam, does not depend on time.
- A PDV measurement is inherently noisy.

Which parameters of interest $\hat{\zeta}$ are the most likely to have generated the data?

A Maximum Likelihood approach is presented to estimate the underlying parameters.

Statistical estimation from PDV data:

The likelihood function $\mathcal{L}$ is maximised (eq., minimization of –log $\mathcal{L}$):

$$\hat{\zeta} = \left[\widehat{M}\left(\tilde{V}_{min}\right), \hat{\beta}, \hat{\kappa}\right] = \text{argmax} \prod_{k=1}^{N_v} \prod_{m=1}^{N_t} P\left(\Phi_{\text{exp}}(V_k, t_m)\right).$$

- $M\left(\tilde{V}_{min}\right)$: areal mass between $\tilde{V}_{max}$ and the lowest detected velocity from the cloud $\tilde{V}_{min}$; β: slope of $M(V)$; $\hat{\kappa}$: nuisance parameter (amplitude).
- This method is <u>optimal in the case of exponential probability density functions:</u> if the Maximum Likelihood estimator is unbiased, it will have minimal variance.

OPTICAL VISIBILITY OF THE CLOUD

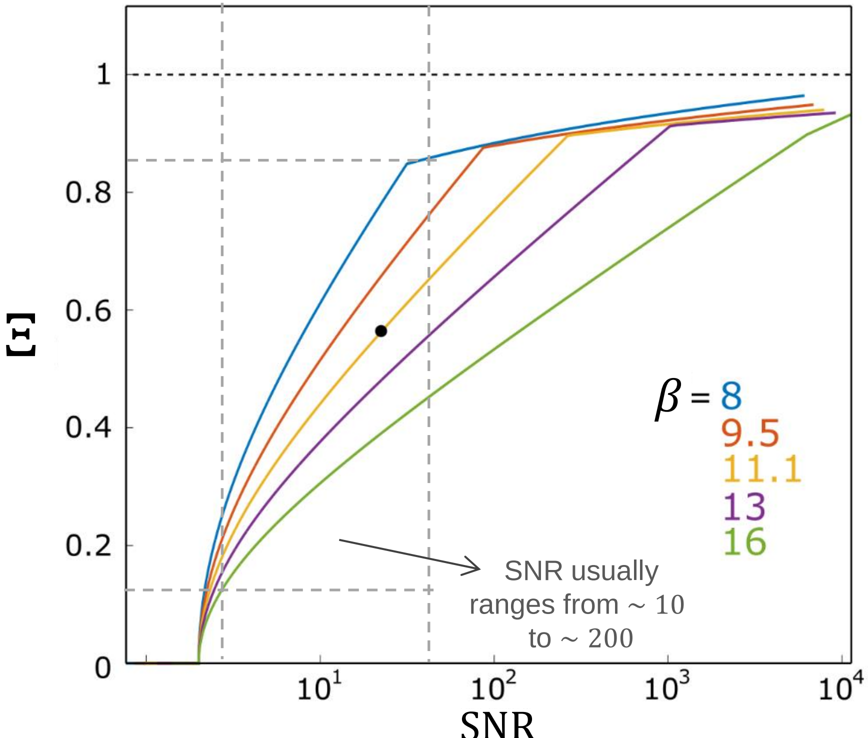
Optical visibility of the cloud in the spectrum:

$$\Xi = \frac{\tilde{V}_{max} - \tilde{V}_{min}}{V_{max} - V_{fs}}.$$

Ξ corresponds to the velocity domain observed in the PDV spectrum.

The optical visibility of the cloud:

- barely depends on M_s,
- increases with increasing SNR,
- is strongly dependent on β,
- reaches 1 as SNR tends to infinity.



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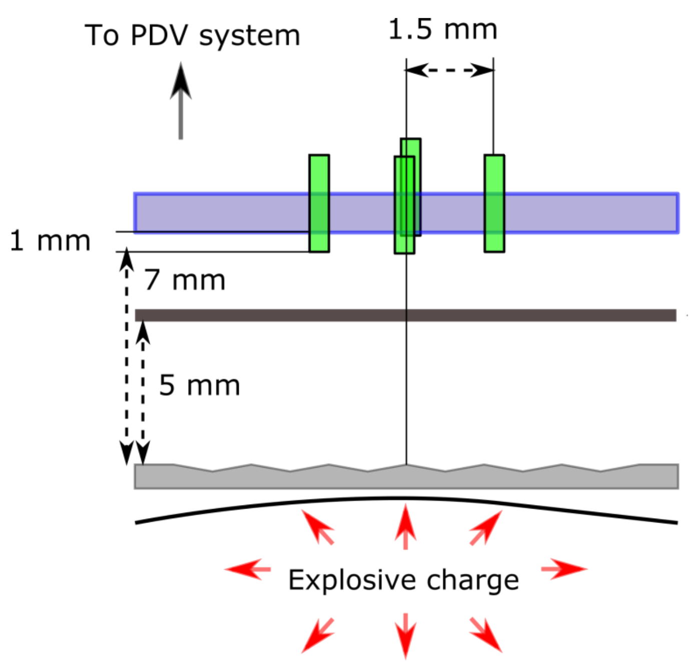
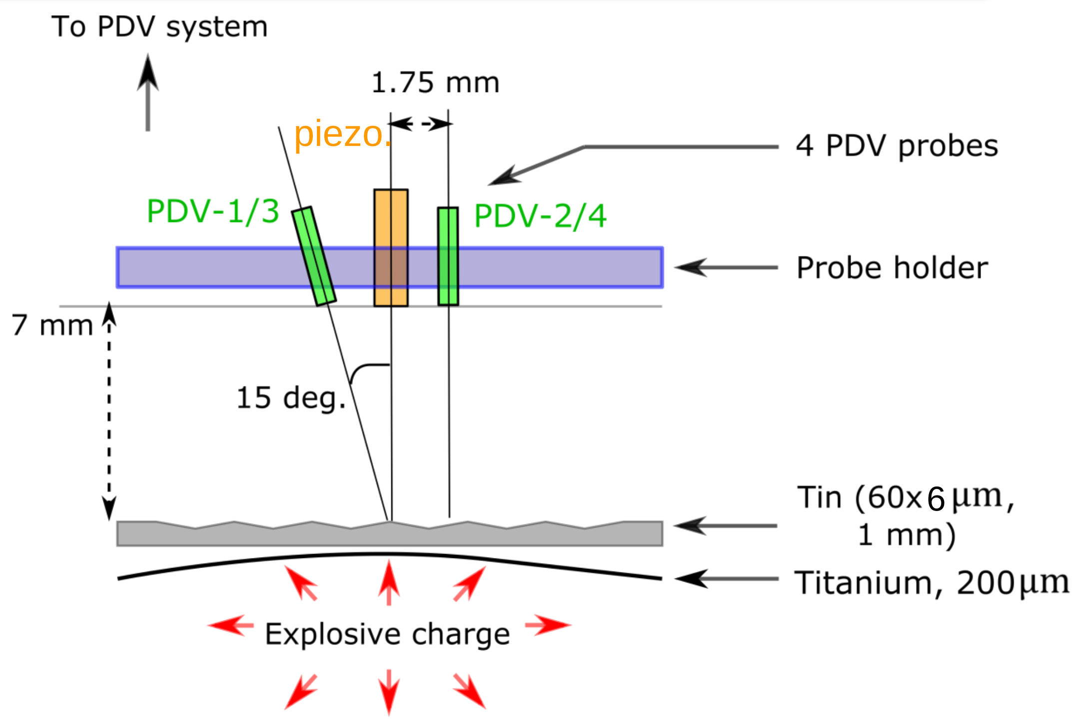
EXPERIMENTAL SETUP

Experimental setup:

A detonator (HE) with a slapper is used to induce a shock wave in a 1 mm-thick tin (Sn) plate. Unsteady peak breakout pressure is $P_{SB} \simeq 29$ GPa.



Asay foil experiments $(200\ \mu m, steel)$.

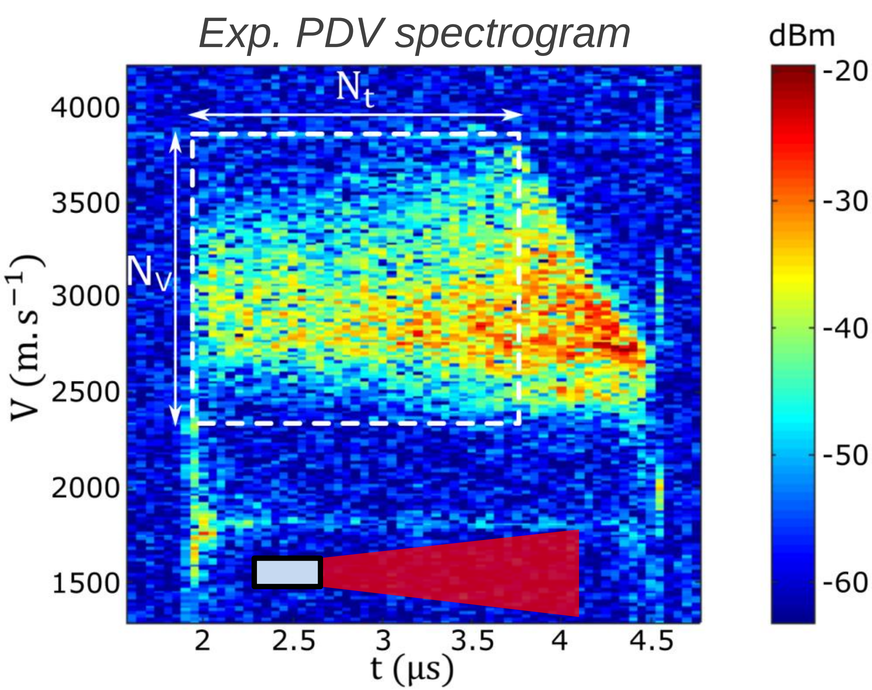


PDV / piezo. (LN) experiments.

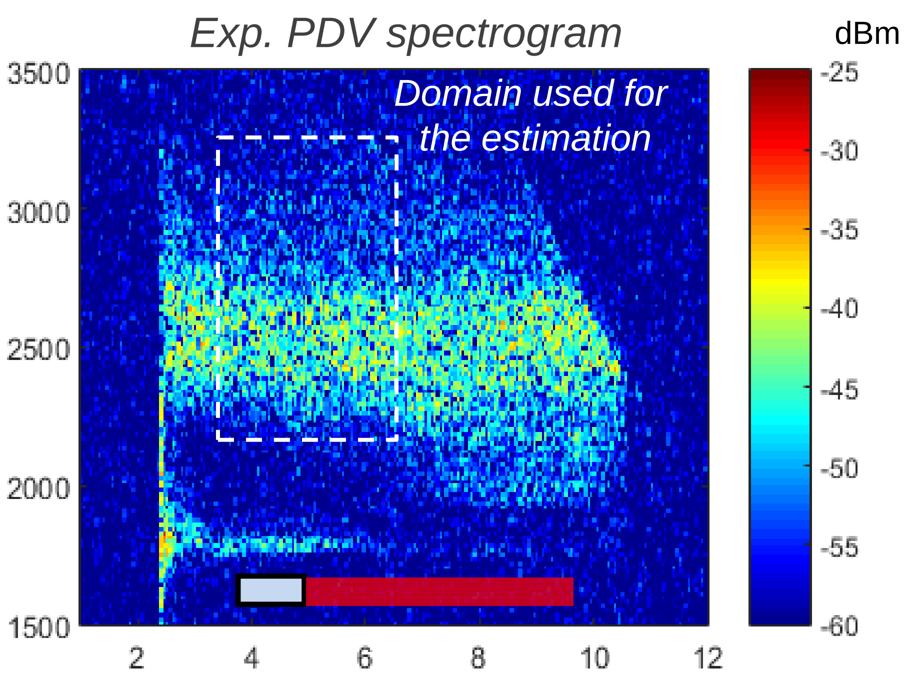
COLLIMATED VS DIVERGING PROBE LENS

Different PDV responses:

If the probe delivers a diverging beam, the optical coupling efficiency can be calculated and integrated in the Likelihood function (without modifying the model).



GRIN PDV lense (diverging beam)
distance plate-probe = 7 mm.



PDV pigtailed collimator (collimated beam)
distance plate-probe = 20 mm.

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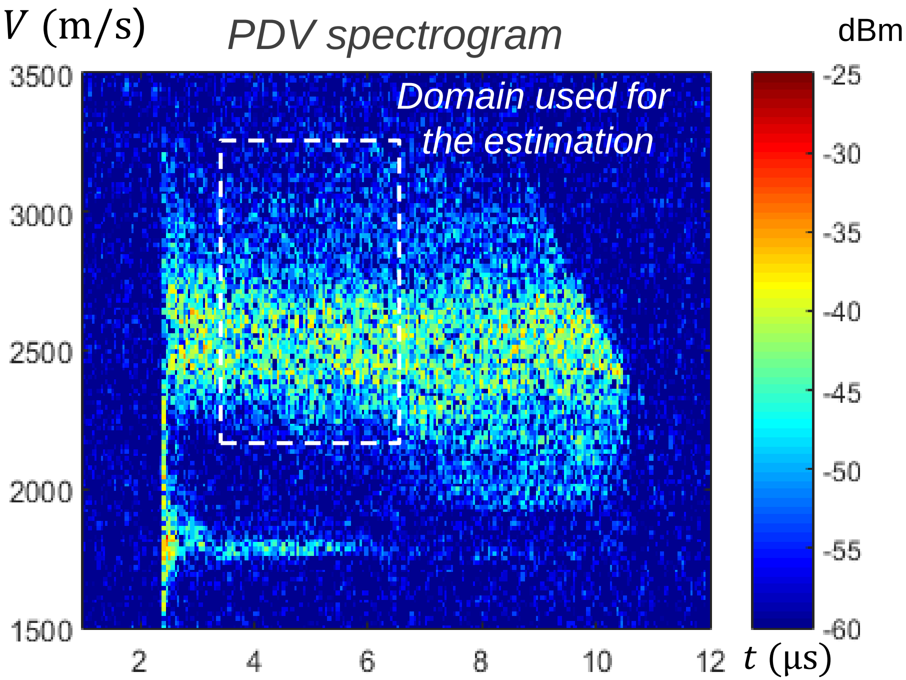
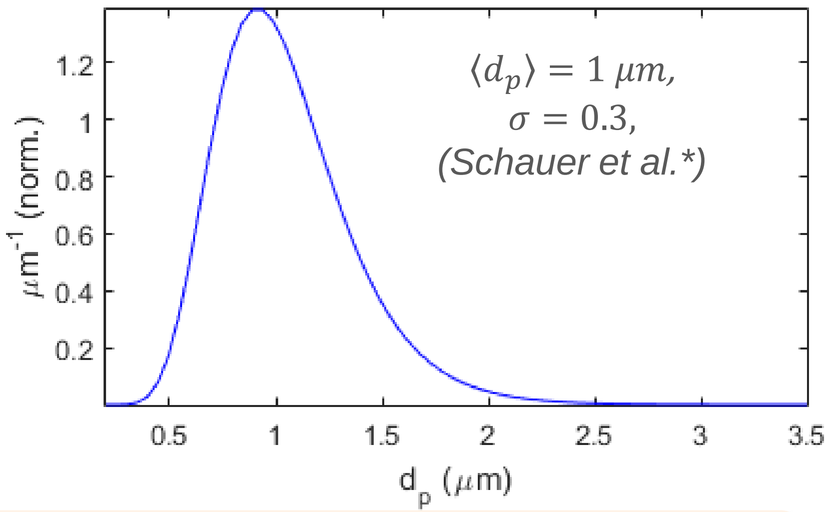
ESTIMATION OF THE AREAL MASS OF EJECTA



We have to assume a given particle size distribution to perform the estimation:



Estimated areal mass between V_{max} and $V_{fs} = 1960$ m/s (V_{fs} is determined from other HE-driven polished tin surfaces experiments):

$$\widehat{M}_s = 7{,}2 \pm 1{,}0\ \mathrm{mg/cm^2}.$$

- 45 time slices of the spectrum are used for the estimation. Invariant statistical properties of the spectrum with time are assumed (as expected theoretically).

**[3] Schauer et al., "Constraining ejecta particle size distributions with light scattering", LANL, Los Alamos, NM (United States), (2018).*

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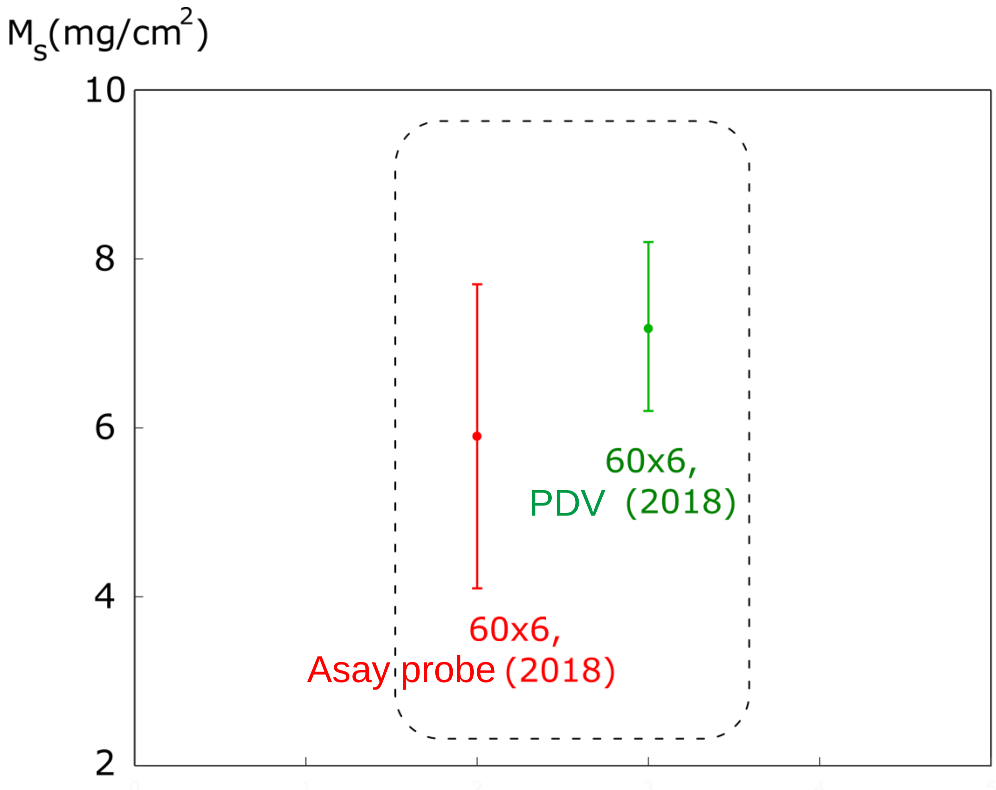
COMPARISON WITH ASAY PROBE RESULTS

Comparison between PDV and Asay probe estimations of M_S:

Independent estimations of the areal mass of ejecta can be compared.



Bidimensional triangular-shaped Sn surface (1 mm-thick, HE drive, 29 GPa, vacuum).

When the PDV spectrum can be analyzed (good SNR and if possible time-invariant behavior):

- Very good agreement between independent PDV and Asay probe results ($2h = 6\ \mu m, \lambda = 60\ \mu m$).
- Sensitivity of the PDV results to a change in size distribution.

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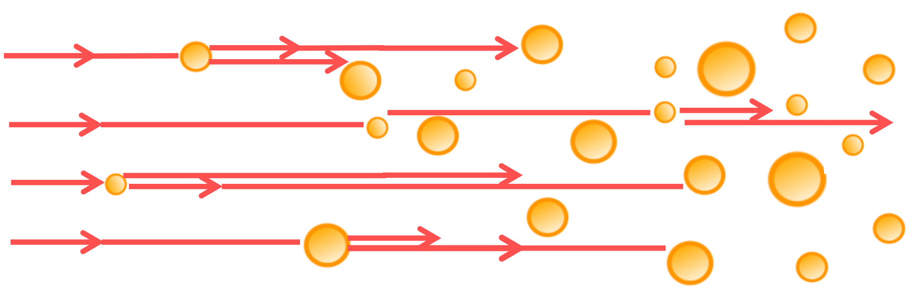
DOPPLER MONTE CARLO (MC) – BASED APPROACH*

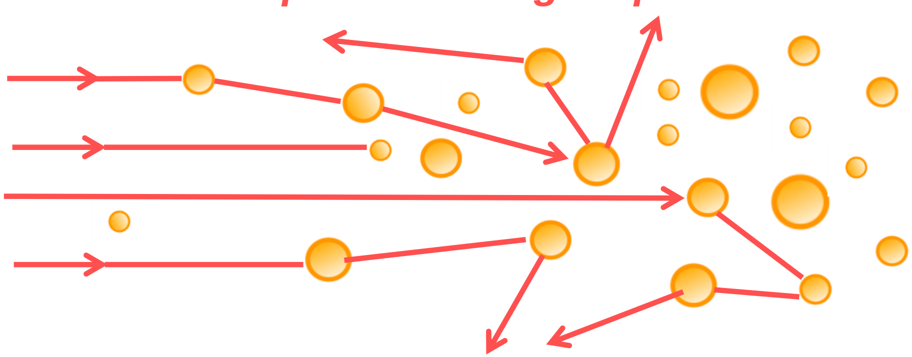
Multiple light scattering in the ejecta:

How does the multiple light scattering in the ejecta modify PDV results?

Some of the multiple scattering sequences:



All multiple scattering sequences:



- *First order theory scattering (1^{st} part):* scattered – induced attenuation of the coherent field. Does not account for multiple sequences between particles.

- ***Doppler MC – approach:*** stochastic approach to the time-dependent vector radiative transfer equation.

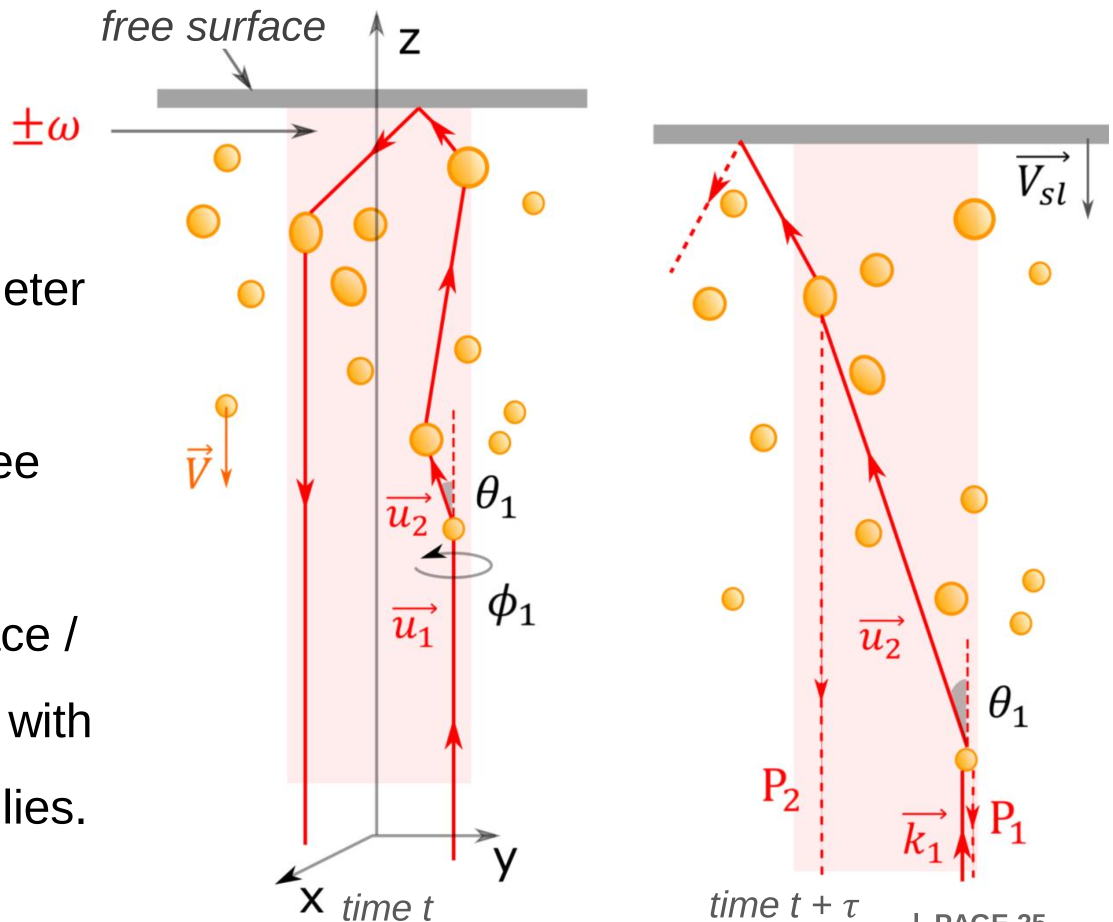
*[2] Franzkowiak et al., "Multiple light scattering in metallic ejecta produced under intense shockwave compression", Appl. Opt., **57**, 2766-2773 (2018).

DOPPLER MC – BASED APPROACH

Photon transport in the ejecta:

Multiple light scattering photon paths in the ejecta.

- Limited field of view of the probe $\pm\omega$ = size of the beam.
- Each particle has a given diameter and velocity.
- Possible light paths: ejecta / free surface / ejecta.
- *Probabilistic approach:* 3D space / time discretization of the cloud with light – particle collision probabilies.



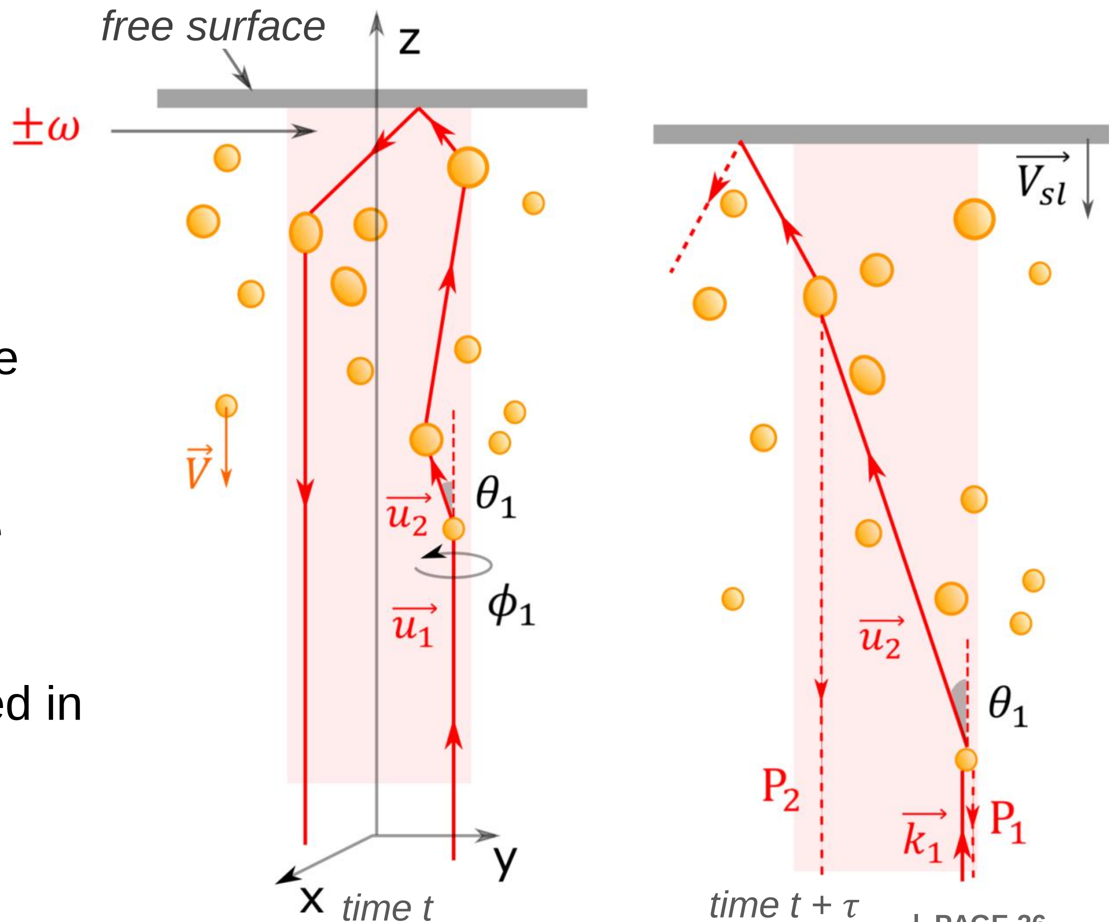
DOPPLER MC – BASED APPROACH

Photon transport in the ejecta:

Multiple light scattering photon paths in the ejecta.

The following assumptions are used:

- Particle sizes and velocities are uncorrelated.
- The properties of the cloud are invariant along x and y.
- Particles are randomly arranged in each slab (V to $V + \delta V$).



DOPPLER MC-BASED APPROACH: DIAMETER SAMPLING

As a photon propagates in the ejecta, at each interaction site, the particle diameter is sampled from the size distribution.

Particle diameter sampling:

For a power – law distribution and a random deviate $\eta \in [0,1]$:

$$d_{\mathrm{p}} = \left[\left(d_{max}^{1-\alpha} - d_{min}^{1-\alpha}\right) \cdot \eta + d_{min}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}.$$

For a lognormal distribution and a random deviate $\eta \in [0,1]$:

$$d_{\mathrm{p}} = \exp\left[\log\mu + \sqrt{2}\sigma \cdot \mathrm{erf}^{-1}\left[\eta\left[\mathrm{erf}(D_{max}) - \mathrm{erf}(D_{min})\right] + \mathrm{erf}(D_{min})\right]\right],$$

with:

$$D_{\{max,min\}} = \frac{\log d_{p,\{max,min\}} - \log\mu}{\sqrt{2}\sigma}.$$

➢ The partition functions are easily inverted for these two laws.

DOPPLER MC-BASED APPROACH: PEEL-OFF TECHNIQUE (VARIANCE REDUCTION)

A local estimation is used to speed up the calculation.

Peel-off (or local estimation) technique:

At each light – particle collision (scattering), the probability of backscattering to the PDV probe is calculated. A photon has undergone n_e scatterings in the ejecta. $\forall\, k \in [\![1, \cdots, n_e]\!]$:

$$P_k = \prod_{l=1}^{k}\left(1 - \frac{Q_{abs,\,l}}{Q_{ext,l}}\right) \cdot \exp\left[-\sum_{i=1}^{j_k} \mu_i \delta z\right] \cdot p_k(\boldsymbol{u_k}, -\boldsymbol{u_z})\,.$$

- Q_{abs}/Q_{ext}: absorption / extinction of light.
- Negative exponential: probability of not being scattered in the return path.
- p_k: probability of backscattering to the probe ($-\boldsymbol{u_z}$) with an incident direction $\boldsymbol{u_k}$ on the particle.

DOPPLER MC-BASED APPROACH: CONSTRUCTION OF THE PDV SPECTRUM

n_{ph} photons being propagated in the ejecta, the average PDV spectrum can be estimated.

Doppler spectrum:

From $V_{d,k}$ and P_k, we get:

$$\Phi_{mc}(V) = \frac{1}{n_{ph}} \sum_{i=1}^{n_{ph}} \sum_{j=1}^{n_e(i)} P_j \, \delta\left(V - V_{d,\,j}\right) \xrightarrow[n_{ph} \to +\infty]{} \langle \Phi(V) \rangle \, .$$

- A Monte Carlo calculation can be performed to estimate the average Doppler spectrum.
- Convergence of the calculation: $\sqrt{n_{ph}}$.
- Intensites are summed rather than amplitudes (random particle arrangement).

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LIGHT INTERACTION BETWEEN THE FREE SURFACE AND EJECTED PARTICLES

We present an example of a Doppler MC calculation.

Example (tin):

$M_s = 0.5\ \mathrm{mg/cm^2}$, $\beta = 8.5, V_{fs} = 2000$ m/s, $V_{max} = 3600$ m/s, $f_d \propto d_p^{-5,2}$,

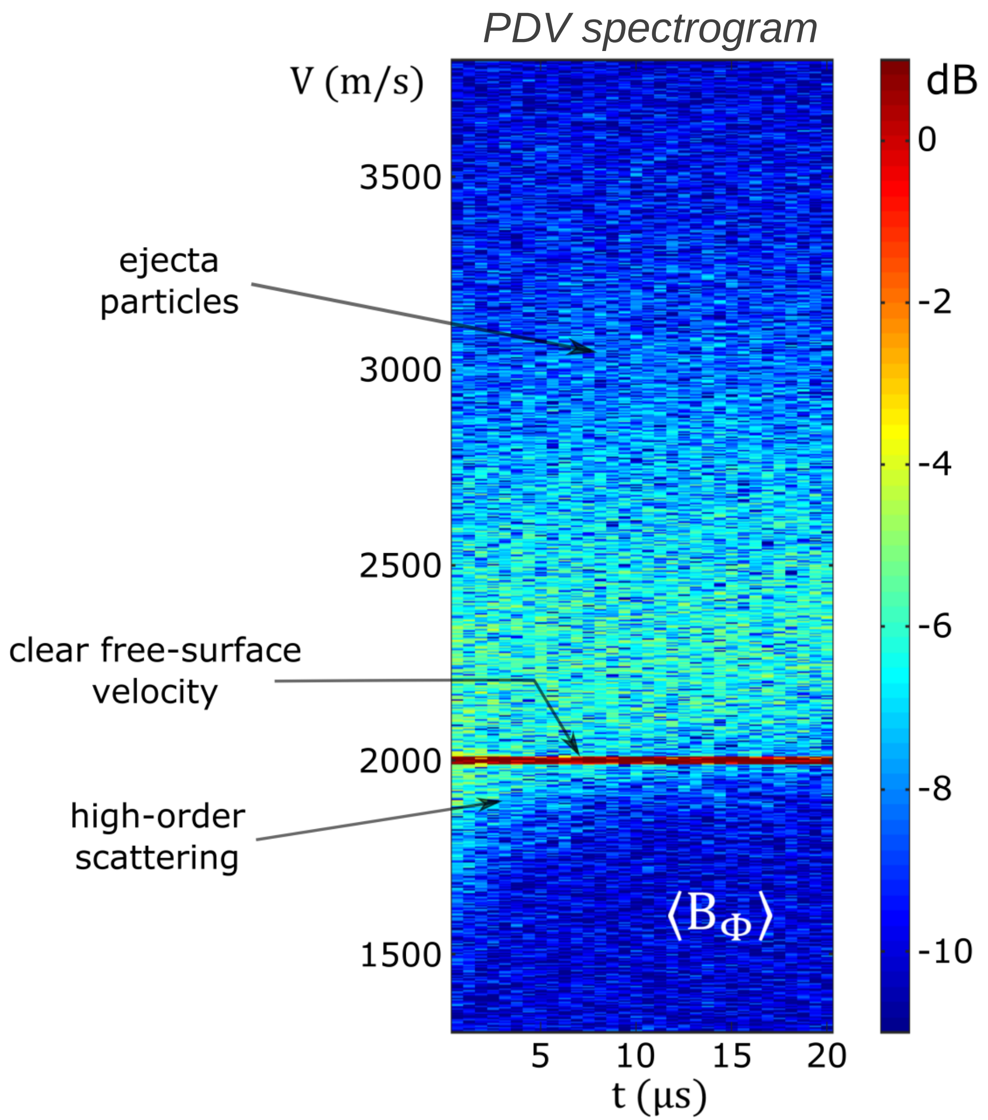
$d_{min} = 1\ \mu m, d_{max} = 10\ \mu m$. PDV probe: collimated beam, $\omega = 250\ \mu m$.

- Average PDV spectrum $\Phi_{mc}(V)$: MC simulation, 15 000 photons at each time step. Initial length of the cloud: 1 mm. $\delta T = 625$ ns. 32 time steps.

- Reconstruction of a typical measurement: estimated average PDV spectrum $\Phi_{mc}(V) \rightarrow \langle \Phi(V) \rangle$ + probability density of the spectrum $P(\Phi(V))$.

- Polarization transport of the scattered light is included.

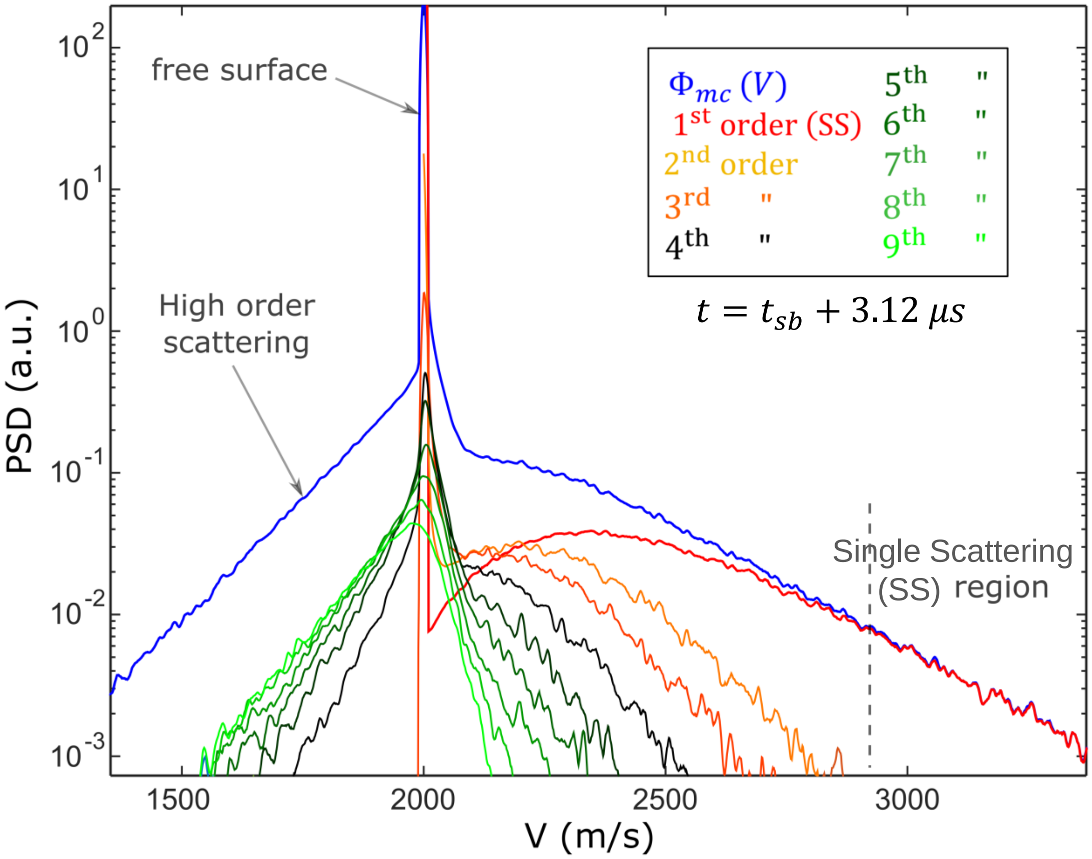
SIMULATION OF LIGHT INTERACTION BETWEEN FREE SURFACE AND EJECTED PARTICLES



- Multiple scattering sequences ejecta / surface / ejecta generate light with Doppler velocities $V < V_{fs}$.
- This feature disappears at later times due to:
  - the limited field of view,
  - increased scattering mean free paths.
- Better understanding of a PDV measurement in presence of ejecta.

SIMULATION OF LIGHT INTERACTION BETWEEN FREE SURFACE AND EJECTED PARTICLES

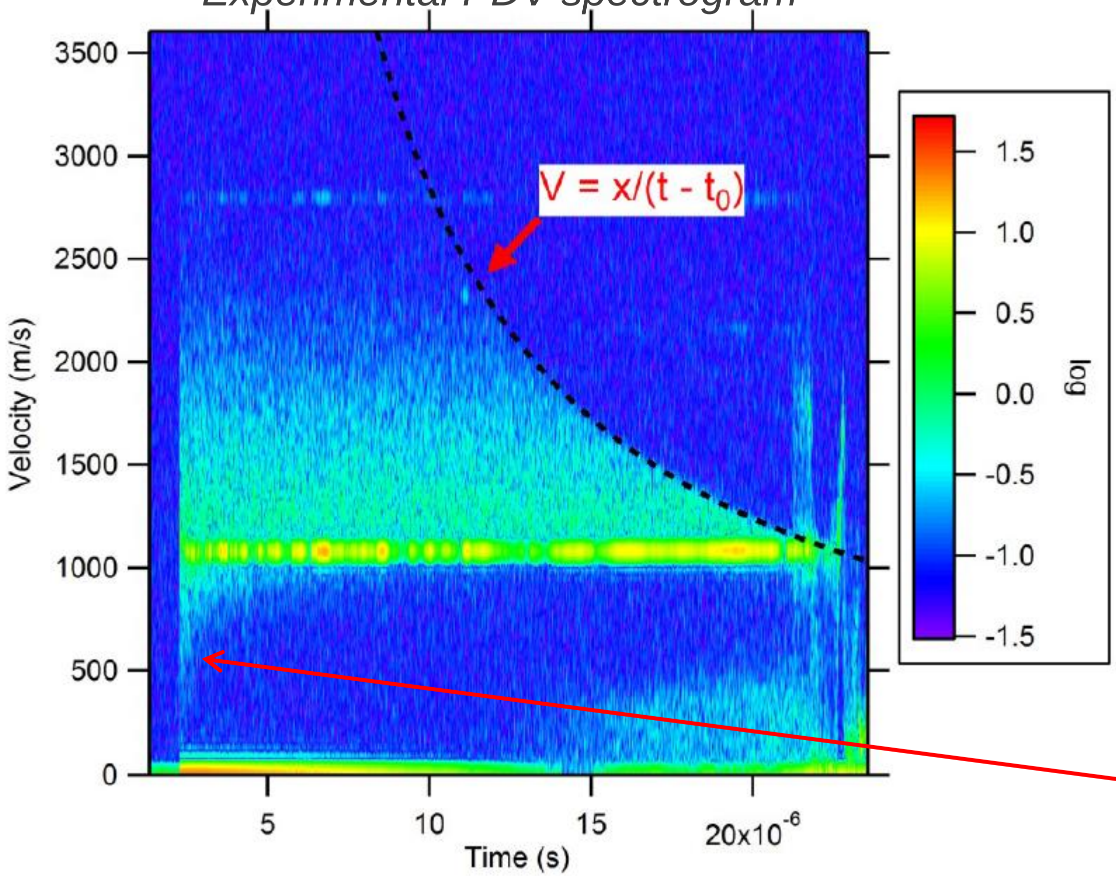
Different contributions to the Doppler spectrum



- Large amplitude at $V = V_{fs}$ (semi-transparent ejecta cloud).
- Different scattering orders contribute differently to the PSD.
- High-order scattering explains the typical behavior at $V < V_{fs}$.
- MC simulations can be performed for other values of M_s and β. The feature $V < V_{fs}$ disappears for larger M_s.

COMPARISON WITH THE EXPERIMENTAL RESULTS OF LALONE ET AL.*

Experimental PDV spectrogram



- Gold ejecta produced under intense HE shock wave drive.
- Areal mass estimated using piezo. pins: $\sim 0.5\ \mathrm{mg/cm^2}$.
- Collisions between particles?

- We suggest another explanation to the experimental observation $V < V_{fs}$, due to multiple light scattering in the ejecta and the limited field of view of the probe.

*[4] Lalone et al., *"Spall strength and ejecta production of gold under explosively driven shock wave compression"*, National Security Technologies, LLC. (NSTec), Mercury, NV (United States), 2013.*

CONCLUSION

Estimation of the areal mass from PDV:

- A 1D model is presented (first order theory of multiple scattering)
- Good agreement is reached between PDV and Asay probe estimations
- Uncertainties on the estimation are determined, <u>but:</u>
- Knowledge of the size distribution is crucial for an improved estimation.

Doppler MC-based model:

- All multiple light scattering sequences are taken into account
- The average PDV response of an experiment in presence of ejecta can be estimated
- We suggest a new explanation to experimental artifacts observed

PERSPECTIVES

Perspectives:

➢ Much work remains to do in order to study the PDV response of ejecta particles:

- Influence of particles' non sphericity,
- Polarization issues may explain the diversity of contrasts observed in the PDV spectrograms,
- More comparisons between simulations and experiments.

References:

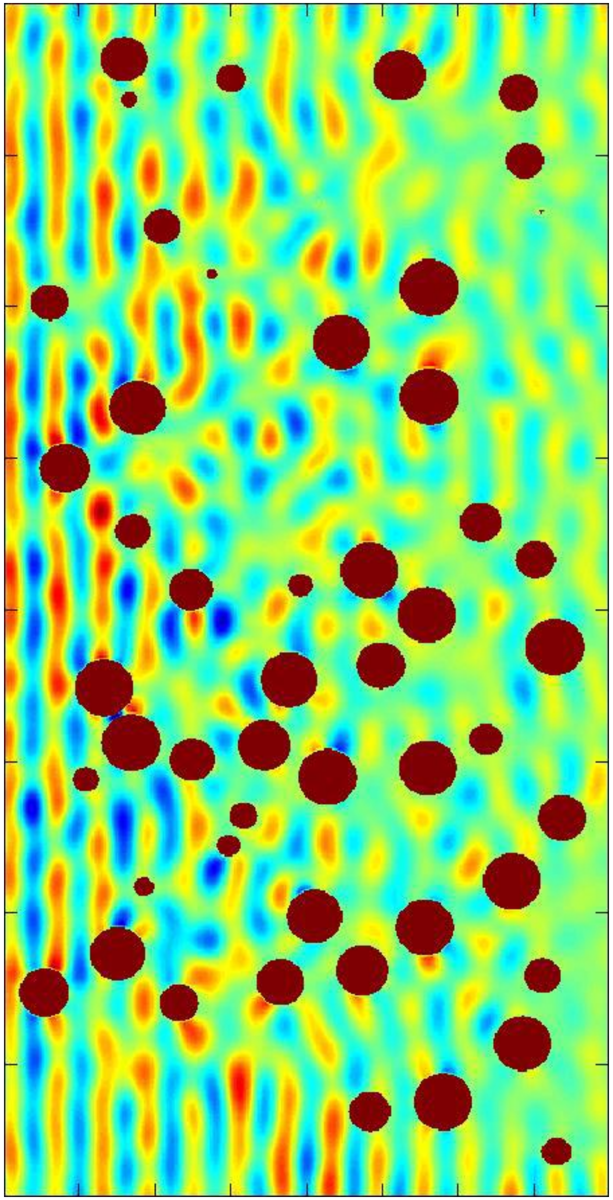
[1] Franzkowiak et al., "PDV-based estimation of ejecta particles' mass-velocity function from shock-loaded tin experiment", Rev. Sci. Instrum., 89 (2018).

[2] Franzkowiak et al., "Multiple light scattering in metallic ejecta produced under intense shockwave compression", Appl. Opt., 57 (2018).

[3] Schauer et al., "Constraining ejecta particle size distributions with light scattering", LANL, Los Alamos, NM (United States), (2018).

[4] Lalone et al., "Spall strength and ejecta production of gold under explosively driven shock wave compression", National Security Technologies, LLC. (NSTec), Mercury, NV (United States).

Propagation of a PDV beam in a metallic tin particle cloud.



THANK YOU FOR YOUR ATTENTION !

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BACKUP

STATISTICAL ESTIMATION AND UNCERTAINTIES

Uncertainties on the estimation are determined, either:

- By the Cramer-Rao bounds using the Fisher Information matrix:

Fisher Information matrix:

$$\forall k \in [\![1,3]\!] \quad I_{l,k} = \sum_{j=1}^{N_v} \sum_{j=1}^{N_t} \frac{\partial_k \langle \Phi \rangle \partial_l \langle \Phi \rangle \left(\frac{2\langle \Phi \rangle}{\langle \Phi \rangle + \langle B_\Phi \rangle} - 1 \right) + \langle B_\Phi \rangle \partial_l \partial_k \langle \Phi \rangle}{(\langle \Phi \rangle + \langle B_\Phi \rangle)^2},$$

$$\sigma^2{}_\zeta = \mathrm{diag}(I^{-1}).$$

- By a parabolic approximation of –log $\mathcal{L}$; $\mathcal{L}$ is a multivariate gaussian distribution and the covariance Γ^{-1} is estimated by the Hessian of –log $\mathcal{L}$.

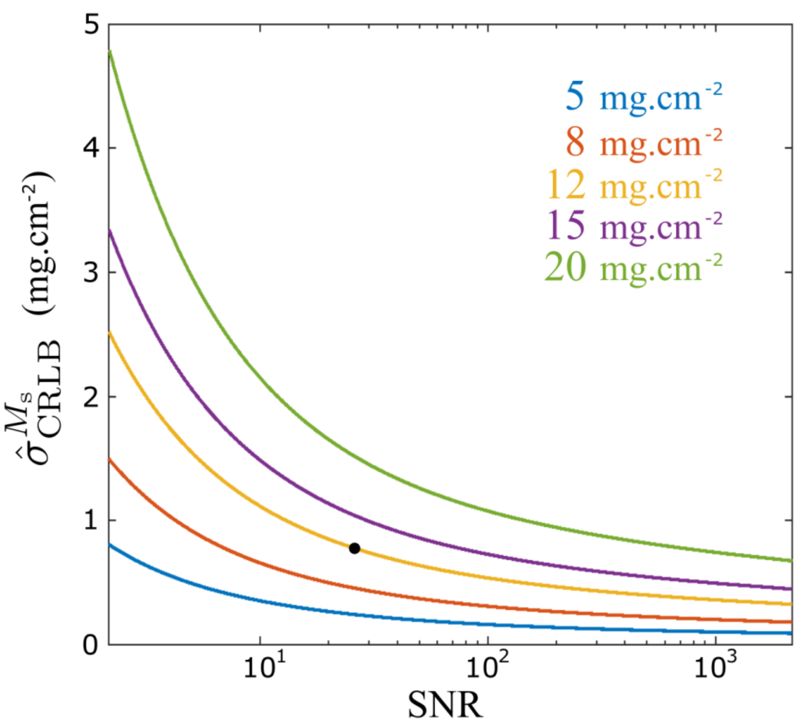
Parabolic approximation: $\forall \zeta$ in the vicinity of $\hat{\zeta}$:

$$\mathcal{L}(\zeta) = \frac{1}{(2\pi)^{3/2} |\Gamma|^{1/2}} \cdot \exp\left[-\frac{1}{2} (\zeta - \hat{\zeta})^T \Gamma^{-1} (\zeta - \hat{\zeta}) \right].$$

STATISTICAL ESTIMATION AND UNCERTAINTIES

Example:

The Cramer-Rao bounds for the areal mass estimation are calculated for different ejecta clouds: $V_{fs} = 2000$ m/s, $V_{max} = 3600$ m/s, $\beta = 11$, $\delta T = 50$ ns, $d_{min} = 1$ µm, $d_{max} = 10$ µm, $\alpha = 5.6$.



The uncertainty on the estimated areal mass:

- increases with increasing M_s,
- increases with decreasing SNR,
- is correlated to the estimation of β (non-zero non diagonal elements of the covariance Γ).