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ISOSCELES TRIANGLE PATTERNS

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Teachers of mathematics at all levels of instruction are interested in practice activities which can be used by their students to develop skills or to maintain skills previously learned. Since mathematics is, in part, a study of patterns, many teachers are interested in activities which involve their students in conjecturing, investigating, and verifying. It is a serendipitous occurrence when activities can be found which at the same time lend themselves to the maintenance of computational skills and the discovery of patterns.

An example of this type of pattern involves drawing geometric designs on familiar number arrangements. Table 1 displays a sequence of nested isosceles triangles drawn on the interior of an extended addition table. We shall perform two activities on this sequence. (Continued on the next page)

Table 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34

Activity 1

Add the 3 vertex numbers of each isosceles triangle:

Triangle A: 19 + 19 + 21 = 59

Triangle B: 18 + 18 + 24 = 60

Triangle C: 17 + 17 + 27 = 61

Triangle D: 16 + 16 + 30 = 62

Observe that these sums differ by 1. Table 2 displays another sequence of nested isosceles triangles on the interior of the extended addition table.

Table 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33

The sums of the 3 vertex numbers for the triangles are:

A : 47

B : 48

C : 49

D : 50

Again, note that the consecutive sums differ by 1.

<u>Activity II</u>

Add the numbers which lie on the perimeter of each of the triangles of Table 1. These sums and the resulting differences are:

```
A :  79
          > 164
B : 243         > 8
          > 172
C : 415         > 8
          > 180
D : 595
```

If the same activity is performed on Table 2, the sums and the resulting differences are:

```
A :  63
          > 132
B : 195         > 8
          > 140
C : 335         > 8
          > 148
D : 483
```

Again, the second differences are 8. Draw other sequences of nested isosceles triangles with the same orientation as those on Tables 1 and 2, but positioned elsewhere on the table. Observe that the patterns of activities 1 and 2 hold.

Table 3 shows a sequence of nested isosceles triangles drawn on the interior of an extended subtraction table. (Continued on the next page)

DATES TO REMEMBER

March 20 - 22, 1986	OCTM Annual Meeting Akron, Ohio
April 2 - 5, 1986	NCTM 64th Annual Meeting Washington, D.C.
April 25 - 26, 1986	Ohio Section Spring Meeting John Carroll University
April 25 - 27, 1986	Ohio Academy of Science University of Toledo
July 16 - 18, 1986	Ohio Section Short Course History of Calculus Ashland College
April 8 - 11, 1987	NCTM 65th Annual Meeting Anaheim, California

Table 3

0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14
2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13
3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12
4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11
5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7
9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5
11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4
12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3
13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2
14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1
15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

The sums of the 3 vertex numbers are:

A : 11

B : 12

C : 13

D : 14

The sums of the numbers which lie on the perimeter and their consecutive differences are:

```
A :  15
          > 36
B :  51        > 8
          > 44
C :  95        > 8
          > 52
D : 147
```

Again, note that the consecutive vertex sums differ by 1, while the consecutive perimeter sums have a 2nd difference of 8. Verify that these two patterns hold if other sequences of nested pentagons are drawn with the same orientation, but positioned elsewhere on the extended subtraction table.

Challenge: Draw nested isosceles triangles with the same orientation as those of Tables 1 through 3 on an extended multiplication table. Do similar patterns hold?

RECOMMENDATIONS FROM A LEADERSHIP CONFERENCE* ON MIDDLE SCHOOL MATHEMATICS

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Introduction:

The performance of students in mathematics programs in Ohio can be improved. Presently about 45% of the graduates of Ohio secondary schools progress on to some form of higher education and about 45% of those students must take some form of remedial mathematics. Improvements can be made at many levels in the school, but an arena of particular concern is the middle school program. Only about one-sixth of the school systems in Ohio retain the traditional junior high school administrative pattern. In the shift to different curricular and administrative arrangements, new issues and problems in making mathematics teaching and learning effective have developed. A leadership conference was convened to address these issues and concerns as a pre-session

* The Leadership Conference on Middle School Mathematics was funded by The Ohio State University, College of Education and College of Mathematical and Physical Sciences in conjunction with the Ohio Department of Education.