RENTAL EQUIVALENCES: AN ALGEBRAIC APPLICATION

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Frequently in agricultural states pieces of cropland are owned by one person and farmed by another. The renter must then supply a return to the landlord. Sometimes this is done by specifying a flat dollar amount: another way is to "share the crop" in some way. In some states, the following two methods are considered as equivalent in the long run.

Method 1: The landlord and renter split the proceeds and all expenses equally.

Method 2: The landlord is paid 1/3 of the proceeds; the renter then pays all expenses from the 2/3 share.

Question: Under what relationship between proceeds and expenses are these two methods equivalent?

Let P = proceeds and E = expenses. By method 1, the landlord's return is $\frac{P}{2} - \frac{E}{2}$ or $\frac{P - E}{2}$. By method 2, the landlord's return is $\frac{P}{3}$. For the two methods to be equivalent, $\frac{P - E}{2} = \frac{P}{3}$.

Solving: $\frac{P-E}{2} = \frac{P}{3}$

$$3(P - E) = 2P$$

$$3P - 3E = 2P$$

$$P = 3E$$

$$E = \frac{P}{3} \text{ or } \frac{1}{3} \cdot P$$

In other words, expenses total 1/3 of the proceeds. For example, suppose that a crop worth \$3000 can be raised for \$1000 in expenses. By method 1, the landlord receives \$1500 proceeds and pays \$500 in expenses. He has a return of \$1000. By method 2, the landlord receives $\frac{\$3000}{3}$ or \$1000. The renter pays the \$1,000 expenses from \$2,000 renter share of the proceeds.

What happens if E = rP(or $r = \frac{E}{P}$) for other values of r? For instance, assume $r = \frac{1}{2}$; expenses are then half the

proceeds. What "method 2" division would then be equivalent to method 1? In method 2, let q be the fraction of the proceeds paid directly to the landlord, with the renter paying all expenses from his/her share. For method 1 to be equivalent to method 2 in this new case,

$$\frac{P - \frac{1}{2}P}{} = q \cdot P$$

landlord's share method 1		landlord's share method 2
$\frac{1}{2} \cdot P$	=	$2q \cdot P$
$\frac{1}{2}$	=	$2q$
$\frac{1}{4}$	=	q

Thus in method 2, the landlord would be paid 1/4 of the proceeds. This share would equal what was left for the renter when the expenses were paid from his/her 3/4 share. Verify this with numerical data.

In general, if $E = r \cdot P$, the value of q can be found as follows.

$$\frac{P - rP}{} = qP$$

landlord's share method 1		landlord's share method 2
$P(1 - r)$	=	$2qP$
$1 - r$	=	$2q$
$\frac{1-r}{2}$	=	q

Table 1 reports pairs of values for q and r which make methods 1 and 2 equivalent.

Table 1

$r(\frac{\text{Proceeds}}{\text{Expenses}})$	q(Fraction of proceeds paid to landlord in method 2)
0	$\frac{1}{2}$ (no expenses; landlord receives half the proceeds by either method)
.1	.45
.2	.40
.3	.35
$(\frac{1}{3})$	$(\frac{1}{3})$ original case
.4	.30
.5	.25
.6	.20
.7	.15
.8	.10
.9	.05
1.0	0 (expenses equal proceeds; no profit to share)

Which values of r and q are appropriate for the region in which you live?

Find settings other than renting farm land in which this type of problem may occur. In general, look at situations in which income producing property or franchises are being negotiated.