MISUSING MEDICAL PERCENTS

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Teachers are constantly looking for ways to assist their students to understand the concept of percent and to apply this concept to real world situations. The need for this instruction is evident when one hears many adults mangle percent information.

We recently heard a case of misusing percent on a local radio station. The announcer was describing a process for detecting a specific type of cancer. Medical researchers had become concerned that there was a significant number of "false negatives" resulting from the detection process previously used; that is, on occasion a person for whom the detecting process had identified no cancer would suddenly have symptoms of the cancer, leading researchers to suspect that the cancer had actually been present at the time of the original negative test. To address this problem, researchers developed a more comprehensive detection process. This new detection process appeared to be more successful, since the number of positive tests (apparent detection of cancer) increased by 25%.

So far, there is nothing wrong with the announcer's description. But he then said something that got our "mathematical attention". Specifically, he said, "in other words, 25% of the people who had negative tests with the previous detection process actually had this specific cancer." Is this statement correct?

To examine this statement let us make a preliminary assumption about the prevalence of this type of cancer. Let us suppose that the previous detection process yields positive results for 4% of the population tested; the new process would detect 25% more than this. A 25% increase would raise this 4% of the population testing positive to 5% testing positive. How many people would have negative tests with each process? (Recall that a negative test is one in which no cancer is detected.)

The original detection process identified 96% as having negative results. The new improved process identified 95% as having negative results. If this were applied to a typical group of 100 people, one person of the original 96 found to be negative by the original process is now identified by the new process as apparently having this specific cancer. This one person represents 1.04% of the 96, not 25% as the announcer thought.

We performed this computation with the assumption that 4% tested positive using the original process. How would our computations change if this 4% were altered?

To symbolize this, let us assume that x percent of the population tests positive for this cancer using the original process. The new process then yields positive tests for x + .25x or 1.25x percent of the population. The original process shows negative results for 100 − x percent of those tested, while the new test shows negative results for 100 − 1.25x percent of those tested. The percent decrease (in those testing negative) from 100 − x to 100 − 1.25x is

$$\frac{(100 - x) - (100 - 1.25x)}{100 - x} \cdot 100 = \frac{.25x}{100 - x} \cdot 100 = \frac{25x}{100 - x}$$

The following table displays the decreases in those testing negative from the original process to the newer process for various values of x.

The first column, x, is the percent testing positive for this cancer using the original test. The second column, $\frac{25x}{100 - x}$, is the percent of those testing negative using the original test who test positive using the new test.

x	$\frac{25x}{100 - x}$
1.00	.2524
2.00	.5102
3.00	.7732
4.00	1.0417
5.00	1.3158
6.00	1.5957
7.00	1.8817
8.00	2.1739
9.00	2.4725
10.00	2.7778
20.00	6.2500
30.00	10.7143
40.00	16.6667
50.00	25.0000
60.00	37.5000
70.00	58.3333
80.00	100.0000

The table cannot continue since 100% represents the entire population which tested negative by the original process.

The radio announcer would have been correct if 50% of the population tested positive for this cancer using the original process. This is extremely unlikely. The actual incidence of cancer is probably nearer the small values of x at the beginning of the table.

Could the value of x which yielded a second column value of 25% (as stated by the announcer) be found without building the entire table?

Write the equation: $\frac{25x}{100 - x} = 25$

Solve the equation:

$$25x = 25(100 - x)$$
$$25x = 2500 - 25x$$
$$50x = 2500$$
$$x = 50$$

Challenges for you and your class.

1) What would happen if the original increase in positive tests had been different from the 25% which began the problem?
2) Have your class find and report on other misuses and abuses of percent in the "real world".

SOFTWARE OFFER

Gerhard S. Plessinger, 4021 22nd St. NW, Canton, OH 44708 has written a computer program for Apple IIe which will create several systems of two or three linear equations all with the same solution set, or, if you prefer, with different solutions sets.

Gerhard is willing to share – please contact him directly to make your in-class practice more "systematic"!
