
A two-stage method for the capacitated multi-facility location-allocation problem

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Abstract: This paper examines the capacitated planar multi-facility location-allocation problem, where the number of facilities to be located is specified and each of which has a capacity constraint. A two-stage method is put forward to deal with the problem where in the first stage a technique that discretises continuous space into discrete cells is used to generate a relatively good initial facility configurations. In stage 2, a variable neighbourhood search (VNS) is implemented to improve the quality of solution obtained by the previous stage. The performance of the proposed method is evaluated using benchmark datasets from the literature. The numerical experiments show that the proposed method yields competitive results when compared to the best known results from the literature. In addition, some future research avenues are also suggested.

Keywords: capacitated; continuous location problem; heuristics; variable neighbourhood search; VNS.

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1 Introduction

In this study, we investigate the planar multi-facility location-allocation problem, known as the multi-source Weber problem (MSWP). In this problem, we are provided with a set of customers, located at n fixed points, with their respective demands. We need to locate M facilities in continuous space to serve these n customers, and to find the allocation of these customers to these M facilities in order to minimise the sum of the weighted total Euclidean distances to serve these customers. If the facilities are assumed to have unlimited capacities, the customers are served by the nearest facility (for examples, see Brimberg et al., 2008, 2014). However, in practice, this may not be true as facilities have capacity restrictions and as a result some customers may have to be served by a farther facility as the nearest facility cannot satisfy their demands. This study focuses on the MSWP in the presents of capacity constraints. This problem is referred to as capacitated multi-source Weber problem (CMSWP).

The earliest work on the CMSWP was conducted by Cooper (1972) who designed the well-known alternating transportation-location (ATL) heuristic. The ATL is an enhancement of the alternate location-allocation (ALA) method initially proposed by Cooper (1964) to solve the classical location-allocation problem. The basic idea of ATL is that the location-allocation problem and the transportation problem (TP) are alternately applied until no epsilon (ϵ) improvement in total cost is found. Sherali and Shetty (1977) developed a convergent cutting plane algorithm to deal with the rectilinear distance CMSWP.

In 1992, the CMSWP was restudied by Sherali and Tuncbilek (1992). They proposed a branch and bound algorithm to compute strong upper bounds via a Lagrangean relaxation scheme and a partitioning approach. Sherali et al. (1994) put forward a reformulation-linearisation technique (RLT) to solve the rectilinear distance CMSWP and reformulated the problem as a linear mixed-integer programming. Sherali et al. (2002) investigated the capacitated Euclidean and ℓ_p distances MSWP by creating a branch and bound based on a partitioning of the allocation space to construct global optimisation procedures. Zainuddin and Salhi (2007) tackled the CMSWP by designing a perturbation-based heuristic, which considered borderline customers whose locations were situated approximately half-way between their nearest and their second nearest facilities.

Aras et al. (2007a) put forward three heuristic methods which adopted Lagrangean heuristic, the p -median method of Hansen et al. (1998), and the cellular heuristic of Gamal and Salhi (2003) to solve the CMSWP with Euclidean, squared Euclidean, and ℓ_p distances. Aras et al. (2007b) applied simulated annealing, threshold accepting, and genetic algorithms to tackle the CMSWP with rectilinear, Euclidean, squared Euclidean, and ℓ_p distances. In a subsequent research, Aras et al. (2008) used their earlier approaches to tackle the CMSWP with rectilinear distance.

Luis et al. (2009) studied the CMSWP by proposing the concept of region-rejection which restricted some locations to be sited too close to the previously chosen locations. A discretisation technique of converting a plane into a discrete space was also put forward. Mohammadi et al. (2010) developed genetic algorithms to solve the CMSWP. Luis et al. (2011) designed a novel guided reactive greedy randomised adaptive search which integrated adaptive learning with the concept of region-rejection. Akyüz et al. (2014) put forward two forms of branch and bound algorithms where the first is based on the allocation space whereas the second on the partition of the location space to tackle the CMSWP.

Other variants on the Weber problems includes the works of Jamalian and Salahi (2014) who studied the uncapacitated multi-facility Weber problem with uncertain location of demand points and Euclidean and ℓ_p distances for both interval and ellipsoidal uncertainty sets. Fernandes et al. (2014) designed a global optimisation algorithm to solve the single facility Weber problem with limited distances and in the presences of minimum and maximum numbers of customers served. Uno et al. (2015) reformulated the Weber problem by considering random and fuzzy demands, known as the extension of the weighted Weber problems. Drezner (2015) modified the well-known Weiszfeld algorithm based on a parabolic approximation and optimality test of demand points to deal with the single facility Weber problem. Farham et al. (2015) suggested a hybrid evolutionary algorithm to solve the single facility Weber problem in the presence of congested regions where locating a facility is restricted but travelling is allowed. Hosseini-zhad et al. (2015) applied a cross entropy metaheuristic to tackle the CMSWP by introducing production and installation costs.

Brimberg et al. (2008, 2014) provide a comprehensive review on the MSWP and in their works; a variable neighbourhood search (VNS) was also pointed out to be applied to solve efficiently the uncapacitated case. To the best of our knowledge, this is the first time that the VNS is adapted to tackle this hard variant of the Weber problem with capacity constraints.

The remainder of this paper is structured as follows: in the next section, the mathematical formulations of the CMSWP are presented. The section thereafter discusses the proposed two-stage method. Then, a section on computational results is reported. The last section provides some conclusions and highlights some potential future research directions.

2 The CMSWP formulations

The CMSWP is defined as there are a set of customers, located at n fixed points, with their respective demands. The aim is to locate M facilities in a continuous space with their capacities b , and to find the allocation of these n customers to these M facilities without violating the capacity of any of the facilities while minimising the total sum of transportation costs. First, we present the notations and the decision variables then followed by the mathematical formulation for the CMSWP.

- *Notations:*

M	the number of facilities to be located
x_{ij}	the allocation quantities sent from facility i to customer j , $i = 1, \dots, M$; $j = 1, \dots, n$
$d(X_i, a_j)$	is the Euclidean distance between facility i and customer j
$a_j = (a_j^1, a_j^2) \in \mathfrak{R}^2$	is the location of customer j
w_j	the demand, or weight, of customer j , where $j = 1, \dots, n$
b_i	a capacity of facility i , where $b \in N$
N	a set of facilities' capacities.

- *Decision variables*

$X_i = (X_i^1, X_i^2) \in \mathfrak{R}^2$ are coordinates of facility i .

The CMSWP can be mathematically formulated as:

$$\text{Minimise } \sum_{i=1}^M \sum_{j=1}^n x_{ij} d(X_i, a_j) \quad (1)$$

Subject to

$$\sum_{i=1}^M x_{ij} = w_j, \quad j = 1, \dots, n \quad (2)$$

$$\sum_{j=1}^n x_{ij} \leq b_i, \quad i = 1, \dots, M \quad (3)$$

$$x_{ij} \geq 0, \quad i = 1, \dots, M, j = 1, \dots, n \quad (4)$$

The objective function (1) is to minimise the sum of the transportation costs using Euclidean distances. Constraint (2) ensures that the total demand of each customer is met.

Constraint (3) guarantees that capacity constraints of the facilities are not violated. Constraint (4) is non-negativity constraints, which imply that the decision variables x_{ij} are positive flows. It is observed that once the M facilities are located, the problem becomes the generalised assignment problem. In addition, if the size of the located facilities is known, the problem reduces to the classical TP. The problem is considered to have at

least one feasible solution by assuming that $\sum_{j=1}^n w_j \leq \sum_{i=1}^M b_i$.

3 Solution method

In this work, a two-stage method is introduced where the first stage aims to generate a relatively good solution whereas in the second stage, a metaheuristics technique is implemented to improve the solution. In the first stage, a technique called the Basic Cell Approach (BCA) is used to generate a relatively good initial facility configuration. This approach is adapted from Irawan and Salhi (2015), which was originally designed by Gamal and Salhi (2003) to solve the MSWP and it was also modified by Luis et al. (2009) to tackle the CMSWP. The BCA discretises continuous space into discrete cells based on the location of customers. This approach determines the promising cells (areas) that many customers are located. Then, initial facility locations are generated pseudo randomly within these cells and a local search based on the well-known on Cooper's ATL heuristic (Cooper, 1972) is applied to improve the facility locations.

In stage 2, a powerful metaheuristics called VNS is implemented to improve the quality of solution obtained by stage 1. In this method, Cooper's ATL is also incorporated as its local search. VNS was formally designed and proposed by Brimberg and Mladenović (1996) and Hansen and Mladenović (1997) to deal with facility location problems. VNS has been widely used to solve various hard combinatorial optimisation problems including travelling salesman problems, vehicle routing problems, location problems, scheduling, and time tabling problems, among others. A comprehensive review on variants and applications of VNS is provided by Hansen and Mladenović (2001), Hansen et al. (2010) and Brimberg et al. (2014). Figure 1 presents the main steps of the proposed two-stage method to tackle the CMSWP.

Figure 1 The main steps of the proposed two-stage method to solve the CMSWP

Stage 1

- Build $m = 2 * M$ equal squares covering all demand points by using The Basic Cell Approach (BCA). The BCA also select M points (X) pseudo randomly based on cumulative probability distribution.
- Improve the solution (X) by implementing Cooper's ATL.

Stage 2

- Implement the proposed VNS starting from X . Note that Cooper's ATL is also used in our proposed VNS as the local search.
- Record the best solution as the final result.

In the following subsections, the well-known Cooper's ATL is first briefly presented as this method is used in the proposed method. This is followed by a subsection explaining the main steps of the BCA and the proposed VNS algorithm.

3.1 Cooper's ATL heuristic

The reasoning behind the ATL is to construct M open facilities which are randomly selected from the customer locations, then the TP, using these M open facilities, is applied to find the allocation for this capacitated problem. At this point, the output is the M independent set of allocations, each subset consisting of n_i customer locations where

$i = 1, 2, \dots, M$ and $\sum_{i=1}^M n_i \geq n$. It is worth noting that we used ' \geq ' instead of '=' as some

customers may have their demand split between different facilities during the allocation stage. The Weiszfeld equations, see (5), is iteratively carried out to get the new location of these M facilities ($i = 1, 2, \dots, M$).

Note that the demand of some customers may have been split as a result of the solution of the TP, such as $w_{j_i} \leq w_j$. It can also be observed that some customers may be utilised more than once in (5) but every time, it is only a portion w_{j_i} of their demand utilised. The location-allocation problem and the TP are alternately applied until no improvement in total cost can be obtained. The step by step of Cooper's ATL heuristic is illustrated in Figure 2.

$$X_i^{1(k)} = \frac{\sum_{j_i=1}^{n_i} \frac{w_{j_i} a_{j_i}^1}{d(X_i^{(k-1)}, a_{j_i})}}{\sum_{j_i=1}^{n_i} \frac{w_{j_i}}{d(X_i^{(k-1)}, a_{j_i})}} \quad \text{and} \quad X_i^{2(k)} = \frac{\sum_{j_i=1}^{n_i} \frac{w_{j_i} a_{j_i}^2}{d(X_i^{(k-1)}, a_{j_i})}}{\sum_{j_i=1}^{n_i} \frac{w_{j_i}}{d(X_i^{(k-1)}, a_{j_i})}} \quad (5)$$

where the superscript k refers to the iteration number within the Weiszfeld iterative procedure; $(a_{j_i}^1, a_{j_i}^2)$ is the location of the set of n_i customer points, $j_i = 1, 2, \dots, n_i$; $(X_i^{1(k)}, X_i^{2(k)})$ indicates the new found location of the i^{th} facility at iteration k ($i = 1, 2, \dots, M$) and w_{j_i} designates to all or a fraction of the j^{th} customer demand that is served by facility i .

Figure 2 The main steps of cooper's ATL heuristic

Step 1	Generate M initial starting locations at random from customers' locations.
Step 2	Find the corresponding allocation by solving the TP.
Step 3	Find the new location using (5).
Step 4	Solve the TP to obtain the new corresponding allocation based on the new locations and its cost.
Step 5	Iterate steps 3 and 4 until there are no further changes in cost, within some tolerance, in two successive iterations.

3.2 The BCA

An illustration of the BCA is shown in Figure 3. Firstly, square cells are constructed which will cover all demand points with a side δ and then empty cells (cells with no customers) are deleted. If the number of non-empty cells is not in the range of a prescribed number of aggregated spatial unit (ASU) then the value of δ is revised and this process step is repeated. Once the specified number of the non-empty cells, ρ , is reached,

a point or a potential location is chosen randomly from each cell to represent the aggregated point (ASU) within that cell. Finally, to increase the diversity of the solutions, the remaining $(m - \rho)$ ASUs are generated at random. The detailed algorithm of the BCA is given in Figure 4.

Figure 3 The illustration of the BCA (see online version for colours)

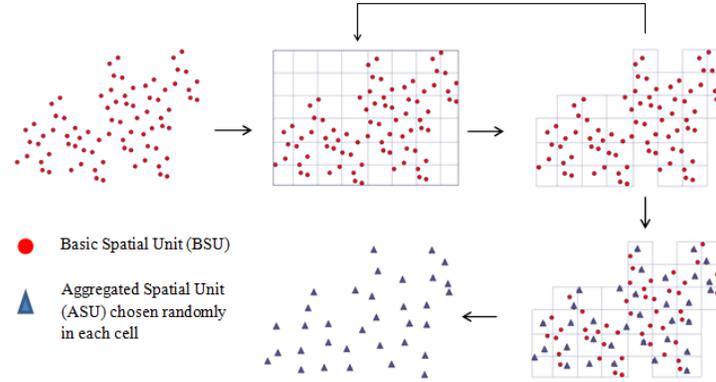


Figure 4 The main step by step of the BCA

Step 1	Define the parameters the number of ASUs (m), γ , and λ .
Step 2	Initialise the length of the side of the cell δ as follows: $\delta = (x_{\max} - x_{\min}) / \sqrt{m \left(\frac{x_{\max} - x_{\min}}{y_{\max} - y_{\min}} \right)}$ where x_{\max} and x_{\min} refer to the maximum and the minimum x coordinates of the potential points, respectively. y_{\max} and y_{\min} are defined similarly.
Step 2	Let ρ denote the number of non-empty cells, where $\rho \in [m\gamma(1 - \lambda), m\gamma]$.
Step 3	Construct square cells of length δ which cover all demand points where cell 1 has its bottom-left corner at (x_{\min}, y_{\min}) . If $\rho \in [m\gamma(1 - \lambda), m\gamma]$ then go to step 8.
Step 4	Let δ_L and δ_U be lower and upper bounds of the length of the side of the cell. Set $\delta_U = \delta$ and $\delta_L = \delta_U / 2$.
Step 5	Construct square cells of length δ_L . If $\rho < m\gamma(1 - \lambda)$ then set $\delta_U = \delta_L$ and $\delta_L = \delta_U / 2$ and repeat this step, otherwise conduct the bisection method as in step 6.
Step 6	Calculate $\delta = (\delta_U + \delta_L) / 2$.
Step 7	Construct square cells of length δ . If $\rho \geq m\gamma(1 - \lambda)$ and $\rho \leq m\gamma$ then go to step 8, otherwise, if $\rho < m\gamma(1 - \lambda)$ then $\delta_U = \delta$ else $\delta_L = \delta$ and go back to step 6.
Step 8	Set count = 0.
Step 9	Generate randomly $\beta \in (0, 1)$ and choose \tilde{z} st $\tilde{z} = F_{(z)}^{-1}(\beta)$ with $F_{(z)} = \sum_{v=1}^z P_v$.
Step 10	Choose randomly a demand point k in the cell \tilde{z} .
Step 11	Set $X = X \cup \{k\}$. If $ X = M$ stop the search. Otherwise set count = count + 1 and return to step 9.

3.3 The proposed VNS algorithm

In stage 2, the implementation of the metaheuristic VNS is adopted to enhance the solutions obtained from the first stage. The aim is to reduce the risk of being trapped at poor local minima. Figure 5 presents the main steps of the proposed VNS to tackle the CMSWP. Let c_{\max} denote the number of cycles (times) for executing the VNS. In the VNS, the neighbourhood search is conducted by ‘shaking’ the current solution. In our method, the shaking process is done by swapping a randomly chosen facility with a randomly facility chosen to diversify the search even more. In the local search, the Cooper’s ATL heuristic is implemented.

Figure 5 The proposed VNS algorithm to solve the CMSWP

<p>Step 1 Repeat c_{\max} times the following steps:</p> <p>Step 1.1 Set counter $k = 1$</p> <p>Step 1.2 Shaking Do the following step k times.</p> <ul style="list-style-type: none"> • Remove randomly 1 facility from X and add 1 facility randomly as well. The new facility is chosen at random from one of customer locations. <p>Step 1.3 Local search</p> <ul style="list-style-type: none"> • Apply local search (Cooper’s ATL heuristic) starting from X. <p>Step 1.4 Move or not If there is an improvement, update the solutions and set $k = 1$ else $k = k + 1$.</p> <p>Step 1.5 If $k \leq c_{\max}$ then return to step 1.2, else go to step 2</p> <p>Step 2 Keep the best solution as the final result.</p>

4 Computational experiments

To evaluate the performance of the proposed two-stage method, the experiments were conducted using the well-known datasets taken from Brimberg et al. (2000). These datasets were originally used for the case of MSWP. Initially, there were four datasets in Brimberg et al. (2000), which consist of 50, 287, 654, and 1,060 customer points, respectively. In this experiment, we only use three datasets to represent medium and large problems, i.e., 287, 654, and 1,060 customer points.

The number of open facilities (M) is set to be from 5 to 50 with an increment of 5. Since these datasets were originally used for the uncapacitated cases, hence Zainuddin and Salhi (2007) suggested that the capacity of facility i (b_i) is computed as the average

demand of each facility, such that $b_i = \left\lceil \frac{\sum_{j=1}^n w_j}{M} \right\rceil$, where $\lceil x \rceil$ is the smallest integer

$\geq x$. This setting is also adopted by Luis et al. (2009, 2011). It is also observed that there would be cases where the total capacity of the facilities (i.e., Mb_i) would be larger than

the total customer demand $\left(\sum_{j=1}^n w_j \right)$. In this case, such as $Mb_i > \sum_{j=1}^n w_j$, a dummy

customer with a 0 transportation cost and a demand given by $\left(Mb_i - \sum_{j=1}^n w_j \right)$ will be introduced. This dummy customer will only contribute at the TP stage and will be ignored during the location and the allocation processes.

The proposed VNS algorithm was implemented in C++ .Net 2012 and run on a PC with an Intel Core i5 CPU @ 3.20 GHz processor, 8.00 GB of RAM and under Windows 7. Some parameters were used during the execution of the proposed VNS algorithm, i.e., $k_{\max} = M$, $c_{\max} = M$, $m = 2 * M$, $\gamma = 1 - 0.1$, and $\lambda = 0.05$. The figures of these parameters were found based on our preliminary experiments.

The solutions of the MSWP provided by Brimberg et al. (2000) are used as lower bounds for the CMSWP. The percentage deviation is computed based on these lower bounds as follows:

$$dev(\%) = \frac{F_{best} - F_{LB}}{F_{LB}} \times 100 \tag{6}$$

where F_{best} designates to the best solution obtained by the proposed VNS algorithm and F_{LB} refers to the lower bound or ‘best’ cost for the MSWP.

Table 1 Comparison for all the results for the 287-customers problem

M	Lower bound	ZS		LSN		GLSN		The proposed VNS algorithm	
		Dev. (%)	CPU (sec)	Dev. (%)	CPU (sec)	Dev. (%)	CPU (sec)	Dev. (%)	CPU (sec)
5	9,715.63	7.90	56.57	7.92	11	7.92	14	7.94	110,662
10	6,705.04	25.77	539.11	25.80	17	25.80	41	25.80	487,580
15	5,224.70	33.82	1,837.17	33.89	25	33.99	118	33.87	1,250,507
20	4,148.84	38.99	2,532.21	38.48	38	38.48	251	38.44	1,586,951
25	3,348.71	44.37	3,442.54	44.33	64	44.31	478	44.27	1,537,624
30	2,716.91	54.23	6,265.81	54.15	85	54.14	792	54.10	1,657,516
35	2,238.18	69.27	8,310.36	69.24	125	69.23	1,202	69.17	3,126,617
40	1,900.84	84.96	10,948.99	84.33	151	84.33	1,688	84.26	2,115,969
45	1,630.31	94.82	12,169.82	94.92	219	94.82	2,530	94.89	2,731,977
50	1,402.58	115.94	19,814.90	116.06	312	115.93	3,815	115.86	3,974,198
OAV		57.01	6,601.76	56.91	104.70	56.89	1,092.90	56.86	

To the best of our knowledge, the results of Zainuddin and Salhi (2007), ZS for short, Luis et al. (2009), LSN for short, and Luis et al. (2011), GLSN for short are the only ones available in the literature for direct comparison. The summary results for the three datasets are presented in Tables 1 to 3, respectively. For completeness, the overall average (OAV) for each dataset is also given. The ital numbers in Tables 1 to 3 refer to the best new found solutions. Based on our results, the proposed method provides encouraging solutions when compared to the results from the literature. Our results are superior from the results given by Zainuddin and Salhi (2007) and Luis et al. (2009) and very competitive compared to Luis et al. (2011). For instance, in the case of the 287-customers problem, the proposed method found six new best solutions and the OAV

outperform the results of Zainuddin and Salhi (2007) and Luis et al. (2009, 2011) by 0.26%, 0.09%, and 0.05%. In the case of the 654-customers problem, the proposed method found one new best solutions. In the case of $n = 1,060$ customers, the proposed method gives better results by 1% and 0.50% from Zainuddin and Salhi (2007) and Luis et al. (2009) respectively and the proposed method found five new best solutions out of ten. However, based on the OAV, the proposed method still provides favourable results compared to the GLSN of Luis et al. (2011).

To diversify the search, our proposed two-stage method explores different neighbourhoods by swapping potential facility locations. However, as we introduce a multi-start approach within the search, the proposed method requires more computing time.

Table 2 Comparison for all the results for the 654-customers problem

<i>M</i>	<i>Lower bound</i>	<i>ZS</i>		<i>LSN</i>		<i>GLSN</i>		<i>The proposed VNS algorithm</i>	
		<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>
5	9,715.63	54.00	62.67	54.00	18	54.00	25	54.00	215,555
10	6,705.04	42.81	136.94	42.81	31	42.81	81	42.81	459,415
15	5,224.70	67.69	852.92	67.69	64	67.69	355	67.69	859,849
20	4,148.84	69.37	1,086.59	69.36	107	69.36	726	69.36	1,788,603
25	3,348.71	47.52	3,842.79	47.52	180	47.51	1,331	47.52	7,523,410
30	2,716.91	86.77	852.18	76.33	247	76.33	2,199	76.33	9,113,601
35	2,238.18	78.13	5,772.53	78.13	354	78.13	3,455	78.13	12,558,625
40	1,900.84	44.25	4,740.56	43.85	517	43.84	5,112	43.84	13,148,668
45	1,630.31	59.23	5,112.34	55.28	638	55.26	7,265	55.26	18,756,620
50	1,402.58	41.96	20,333.38	29.13	807	29.13	9,884	29.12	15,897,503
OAV		59.17	4,279.29	56.41	296.30	56.40	3,043.30	56.40	

Table 3 Comparison for all the results for the 1,060-customers problem

<i>M</i>	<i>Lower bound</i>	<i>ZS</i>		<i>LSN</i>		<i>GLSN</i>		<i>The proposed VNS algorithm</i>	
		<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>	<i>Dev. (%)</i>	<i>CPU (sec)</i>
5	1,851,879.88	1.06	370.85	1.06	60	1.06	72	1.06	206,564
10	1,249,564.75	3.11	1,167.35	3.11	101	3.11	264	3.11	1,330,936
15	980,132.13	1.63	4,063.94	1.63	255	1.63	845	1.63	5,424,609
20	828,802.00	3.42	5,116.90	3.35	436	3.35	1833	3.33	12,549,620
25	722,061.19	3.87	30,551.64	3.91	662	3.85	3261	3.87	14,588,140
30	638,263.00	3.92	45,191.83	3.95	1002	3.92	5311	3.93	17,450,855
35	577,526.63	3.35	21,713.60	3.36	1365	3.31	8253	3.31	20,181,102
40	529,866.19	6.02	47,253.61	6.16	1700	5.99	12754	6.04	24,628,143
45	489,650.00	7.83	12,0072.31	7.88	2204	7.74	17125	8.01	29,593,225
50	453,164.00	5.87	13,0753.06	5.50	2797	5.29	23854	5.40	37,597,025
OAV		4.01	40,625.51	3.99	1,058.00	3.92	7,357.20	3.97	16,355,021.90

5 Conclusions

This study has put forward the two-stage method to tackle the capacitated multisource Weber problem. In the first stage, initial facility configurations are generated using the BCA. In the second stage, the improvement is made through the implementation of the VNS. Our results found seven new best solutions out of 30 solutions. This has shown that our proposed two-stage method produce competitive results when compared to the existing results in the literature.

Potential future investigations based on the current study may include implementations on other metaheuristic techniques such as large neighbourhood search or evolutionary algorithms, see Gendreau and Potvin (2010) for a comprehensive review on metaheuristics. The inclusion of set up cost for constructing facilities in the objective function can be one of the new research areas.

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