# When Costly Voting is Beneficial*

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August 2018

**Abstract**

We present a costly voting model in which each voter has a private valuation for their preferred outcome of a vote. When there is a zero cost to voting, all voters vote and hence all values are counted equally regardless of how high they may be. By having a cost to voting, only those with high enough values would choose to incur this cost. We show that, by adding this cost, welfare may be enhanced even when the cost of voting is wasteful. Such an effect occurs when there is both a large enough density of voters with low values and the expected value of voters is high enough.

**JEL codes**: C70, D72.

**Keywords**: costly voting, externalities.

## 1 Introduction

> "The object of our deliberations is to promote the good purposes for which elections have been instituted, and to prevent their inconveniences." (Edmund Burke as cited in Lakeman and Lambert, 1959, p. 19)

Groups within society often have to make collective decisions. In order to reach correct social decisions, the valuations of all those affected by the decision should be aggregated. By leaving some

*We wish to thank Slava Lis, Alan Miller and seminar participants at the International Meeting of the Society for Social Choice and Welfare, the Association for Public Economic Theory Conference, the Society for the Advancement of Economic Theory Conference, ISI-Delhi Conference, University of Exeter and the University of Haifa for helpful comments.

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out, a group may reach an incorrect decision. For example, take a committee that must decide an issue at a meeting. Each member has a certain private value to the results of the decision reached by the committee. The committee's social value of the decision is the sum of the individual private values and, hence, aggregation is necessary to reach the correct decision. This scenario fits many decision problems such as public good provision where aggregation is the basis of cost-benefit tests or many of the issues that are decided by the California ballot propositions.

A common method to reach a decision is to have a majority vote.[1] Since each member of the committee has information that is relevant to the decision, we would normally think that ensuring all participate in voting would improve the final outcome. In fact, many countries (including Argentina, Australia, Belgium, and Greece) have compulsory voting to ensure inclusion.[2] There is, however, significant difference between aggregating all private values and using a vote while ensuring full participation of all voters.

In social valuation, the strength of preference counts. In majority voting, the options for expressing preference for any particular alternative are limited to either voting for it, or not voting for it (that is, vote for an alternative or abstain).[3] This means with majority voting it is not possible to demonstrate intensity of preferences.[4] One voter mildly in favor of an alternative exactly offsets another voter who is strongly opposed. The addition of voters that do not have strong preferences can distort election outcomes. Indeed, in Australia where voting is mandatory, "donkey votes" (those that simply were cast by order of a ballot) give a 1% edge to those listed first (see Orr, 2002, and King and Leigh, 2009).

Several voting mechanisms to replace simple majority voting have been proposed to ameliorate this problem. One method is to combine the voting on several issues that are decided sequentially by allowing voters to store votes (or use future votes) to indicate intensity (see Casella, 2005, Casella and Gelman, 2008, Casella, Gelman, and Palfrey, 2006, Casella, 2011, Jackson and Sonnenschein, 2007, Engelmann and Grimm, 2012). One may also give voters extra votes that can be used on

[1] See Drexl and Klein (2013) and Gershkov et al. (2013).

[2] Enforcement ranges from fines (Australia) to disenfranchisement (Belgium) or making it difficult to obtain a passport or driver's license (see The Guardian, July 4, 2005).

[3] Among the rare exceptions are reality TV shows such as Pop Idol where individuals can vote more than once (and pay for each vote).

[4] As mentioned by Mueller (2003, page 104): "Majority rule records only these ordinal preferences for each individual on the issue pair. The condition for the Pareto optimality of the supply of the public goods requires information on the relative intensity of individual preferences."

issues that are more important to them (see Hortala-Vallve, 2012). Vote trading can also be allowed (see Casella, Palfrey, and Turban, 2014), or votes can be bought or sold on the open market (see Casella, Llorente-Saguer, and Palfrey, 2012, Lalley and Weyl, 2015).[5] Finally, Bognar, Börgers, and Meyer-ter-Vehn (2015) find the optimal dynamic voting mechanism given that votes are costly.

While these variations may improve welfare, for good reason, there may be resistance to moving away from simple majority voting for making decisions.[6] In this paper, we suggest that welfare can also improved while keeping the mechanism of majority voting by ensuring that voters who have only mild feelings about the alternatives are excluded via costly voting.[7] We do see such issues as relevant in elections. In the UK it is common for parties to charge for voting on the choice of party leader. In 2015, the UK Labour party lowered the cost of voting to £3 (∼\$4) and streamlined the process for registering. This may have helped Corbyn to victory. Ironically, in 2016, for party leadership the cost of voting was raised to £25 (∼\$33) with a less convenient sign-up period. Eaton (2016) claimed that this would more likely eliminate less passionate centrists.

We can see how costly voting can shift the outcome to socially efficient one by deterring voters in the following example. There are two options: A and B. If one voter values option A at 20 utils and option B at 0 and two voters value option B at 6 and option A at 0, then costless voting will result in option B with total utility of 12. However, imposing a voting cost of 6 will deter those preferring option B from voting. The welfare from costly voting will be 20 utils minus a cost of 6 utils (since there will be only one voter) resulting in an overall improvement. However, it is possible that the outcome of the vote is shifted to being socially efficient, but the cost involved in voting is too high to improve welfare. This can happen if each voter that prefers option B has a value of 9 for B, instead of 6. To deter voting for option B, the cost would have to be 4.5 or higher. (Each voter either moves the outcome from a tie to win or a loss to a tie, which is worth half the value.) This would more than offset the gain from shifting the winning option from B to A in overall surplus of 2.

Our paper determines the properties of the distribution of values (when unknown), that ensure

[5] If it is known who has better information or likely to care more about an issue, then votes can be given different weights as in Budescu and Chen (2014).

[6] Britian's 2011 attempt to move from the first-past-the-post to alternative-vote was soundly defeated (see BBC news, May 7, 2011).

[7] There is a widely held belief that costly voting is detrimental since it deters voting (and is a cost to those that do vote). The fact that a cost doesn't seem to deter everyone in practice leads to the literature explaining the paradox of why people vote (see Dhillon and Peralta, 2002, for an overview).

the imposition of a wasteful cost of voting will ex-ante improve welfare. We use a model with a continuous distribution of values both when there is a fixed number of supporters for each outcome and when there is aggregate supporter uncertainty, more specifically, where each supporter randomly (iid) supports each outcome. We find that whether costly voting is superior to costless voting requires that the expected value of a voter times the density of the lowest voters be larger than one for fixed supporters (or 1/2 with aggregate supporter uncertainty). Intuitively, the density of the lowest voters determines how many voters will be eliminated by the cost while the expected value is the benefit due to leaving out a voter that has a low value of the outcome (increasing the likelihood that someone with a higher value will win). We also show that a government would never want to have mandatory voting by imposing fines or subsidizing voting but would wish to implement a poll tax (a charge for voting) if it is politically practical. Finally, we show that for a fixed level of equilibrium voting, the utilitarian outcome is more likely with a uniform cost of voting than with a random cost.

Krishna and Morgan (2015) also recognize that costly voting deters those with low values more than those with high values. In their model, majority voting with costly voting leads to the utilitarian outcome being more likely to being chosen. Whether or not having a cost to voting is overall beneficial is not addressed.[8] Börgers (2000, 2004) asks a question in the spirit of our analysis. Namely, whether a reduction of the costs of voting can be damaging.[9] While he graphically shows such a possibility, in his model a sufficient reduction would always be beneficial, since while there is uncertainty for which alternative a voter prefers, there is no difference in intensity of preference for a particular alternative.[10]

Our paper also relates to the public good provision literature. Palfrey and Ledyard (1994, 1999,

[8] Krishna and Morgan (2011) have a version of Krishna and Morgan (2015) but with a common element (compentency). With large numbers of voluntary voters, the welfare optimizing candidate is elected. We and this literature also build upon Bulkley et al. (2001) and Osborne et al. (2000) that establish that when voting is costly the outcome of the voting game will have an equilibrium in which only voters with high values (from the extremes) will participate.

[9] Krasa and Polborn (2009) vary the Börgers model by allowing for ex-ante asymmetry of preferences over alternatives.

[10] It is possible to see how Börgers (2000) works with a simple numerical example: There are two voters, $V_1$ and $V_2$, and two candidates, $A$ and $B$. Each voter has a 50% chance of preferring each candidate and values their candidate winning at 1 (and the other at 0). If the cost of $V_1$ voting is $0.25 - \epsilon$ and $V_2$ voting is $0.25 - 2\epsilon$, then both will vote and the total surplus will be $1.5 - 0.5 + 3\epsilon$. Note the expected value when both vote is the average of when they agree, value of 2, and when they disagree, value of 1. Also, note that the expected benefit for a voter voting (given the other voter votes) is 0.25. This is since half the the other voter agrees with a voters choice and the other half, it moves from a loss to a tie. If costs increase such that the cost of $V_1$ voting is $0.25 + \epsilon$ and $V_2$ voting is $0.25 - \epsilon$, then only $V_2$ would vote yielding a surplus of $1.5 - 0.25 + \epsilon$. Of course, if voting costs drop to zero, both will vote and surplus will be 1.5.

2002) look at mechanisms including simple voting schemes for providing public goods. In Palfrey and Ledyard, the voting options are to provide or not provide a public good where provision could have a loss for those that have little value for it and have to pay for it. In this paper we have two options each with non-negative value and voting can potentially have a cost.

While less related, the Condorcet Jury literature models voting by a group of individuals with a common value over two alternatives (see Young, 1988). Krishna and Morgan (2012) show that as the cost of voting goes to zero, voluntary voting is the optimal mechanism. Ghoshal and Lockwood (2009) have combined the common value in the Condorcet Jury literature with the private value of alternatives and comparisons in Börgers (2004). They find that if the voters put a high weight on personal preferences then there is an inefficiently high voter turnout and in the case voters care more about the common aspect then there is an inefficiently low voter turnout. Information acquisition has been studied in the Condorcet Jury literature by allowing voters to buy information about the common feature of the alternatives (see Persico, 2004, Gerardi and Yariv, 2005).

The lobbying literature models a similar problem (see Austen-Smith and Wright, 1992, Baye, Kovenock and de Vries, 1993, Che and Gale, 1998, 2006, Kaplan and Wettstein, 2006). The method of reaching a group decision is by allowing would-be voters to send a signal of how much they care: by lobbying. With this method, we would expect that voters with strong preferences or special interest groups to have greater influence on the outcome than with voting. This is due to the ability of voters with more extreme preferences to send a stronger signal. Such undue influence is not necessarily harmful; lobbying may be welfare enhancing over voting since under voting the outcome can be determined by a large number of voters that do not strongly care about the outcome or vote without any information about the specific issues. In Chakravarty and Kaplan (2010), we determine under which conditions, a purely wasteful signal (which we call shouting) will lead to a more efficient solution than voting. In Chakravarty and Kaplan (2013), for purely private goods, we find the optimal allocation mechanism when only wasteful signals can be used and determine under which conditions making use of these signals is useful. The environment also fits the classic approach of using a Vickrey-Clarke-Groves mechanism, but such a mechanism of allowing the rich to directly buy the outcome that they desire could be considered morally repugnant (see Leider and Roth, 2010).

In the next section, we examine our model when there is an equal number of voters supporting each option and the uncertainty in the model is in the strength of each voter's support for the option. Then in section 3, we analyze the case where, in addition to the uncertainty of the strength, there is uncertainty as to which alternative a voter supports. In section 4, we consider the possibility where each voter can have a random cost of voting. We conclude in section 5.

# 2 Aggregate Supporter Certainty

## 2.1 Model

Here we model a committee making a binary decision such as in which of two districts, $A$ or $B$, to build a casino. There are an equal number of representatives from each district, and each representative has a private value for it being built in his district and a common cost for showing up to vote. This cost is wasted, such as the time used to stand in the queue to vote or the monetary transportation costs of going to the polling station. We call this the Aggregate Supporter Certainty (ASC) model, since it is known how many potential voters support each option.

Formally, there are two types of voters and $n$ voters of each type, so overall there are $2n$ voters. Each voter has cost $c \geq 0$. Assume that each voter $i$ has a privately observed value $v_i \geq 0$ that is independently drawn according to the non-atomic cumulative distribution $F$ with support $[0, \overline{v}]$, where $\overline{v} > 2c$.[11] We also assume that $F'(0)$ is finite. If $1 \leq i \leq n$, voter $i$ is a type $A$ voter who values a win by $A$ at $v_i$ and a win by $B$ at 0. If $n+1 \leq i \leq 2n$, voter $i$ is a type $B$ voter who values a win by $B$ at $v_i$ and a win by $A$ at 0. Choice $A$ wins if the number of votes it receives, denoted by $\#_A$, is strictly greater the number of votes choice $B$ receives, denoted by $\#_B$. Choice $B$ wins if $\#_B > \#_A$. If there is a tie, $\#_B = \#_A$, then the winner is determined randomly with equal probability of each winning.[12]

## 2.2 Equilibrium and social surplus

Denote $v^*(c)$ as a cutoff strategy such that a voter $i$ votes if his value is above $v^*(c)$ and doesn't vote if his value is below $v^*(c)$. For simplicity, we will sometimes write $v^*$ instead of $v^*(c)$. While we are implicitly looking at type-symmetric equilibria, for small values of $c$, all equilibria will be

[11] Without the assumption that $\overline{v} > 2c$, there exists a trivial equilibrium where nobody votes.

[12] Note that the voter's preference for platform $A$ or $B$ is modelled in the style of Palfrey and Rosenthal (1983, 1985).

type symmetric (see Myerson, 1998, and Krishna and Morgan, 2015). We denote the derivative of $v^*(c)$ w.r.t. $c$ as $v_c^*(c)$ or $v_c^*$. Also, we denote $\Pr_{-i}(event|v^*)$ as the probability that *event* occurs given that all voters except voter $i$ follows cutoff strategy $v^*$ and voter $i$ does not vote.

A voter $i$ where $1 \le i \le n$ (a type $A$) and with value $v_i$ will vote if

$$v_i \left[ \frac{1}{2} \Pr_{-i}(\#_A = \#_B | v^*) + \frac{1}{2} \Pr_{-i}(\#_A = \#_B - 1 | v^*) \right] > c.$$

The expression $\Pr_{-i}(\#_A = \#_B|v^*)$ represents the case when all other votes are tied. Hence, voter $i$ is pivotal since by $i$ voting, the outcome will change the vote from a tie to a win by voting. The expression $\Pr_{-i}(\#_A = \#_B - 1|v^*)$ represents the case when voter $i$ will change the outcome of a vote from losing to a tie. The gains in either instance is half the value, $\frac{v_i}{2}$. In the following two lemmas we describe the equilibrium and social surplus to voting. Note that all proofs are in the Appendix.

**Lemma 1** *A cutoff $v^*(c)$ forms a Bayes-Nash equilibrium if*

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} F(v^*)^{2(n-i-1)} (1 - F(v^*))^{2i} \left[ 1 + \frac{2i - n + 1}{n - i} \cdot F(v^*) \right] = 2c. \tag{1}$$

**Lemma 2** *The social surplus to voting is the expected value of the winner minus the costs of voting:*

$$\begin{aligned} SSV(c) &= \sum_{a=0}^{n} \sum_{b=0}^{n} \binom{n}{a} \binom{n}{b} F(v^*)^{2n-a-b} (1 - F(v^*))^{a+b} \left[ \begin{array}{c} (n - \max\{a,b\}) E[v|v < v^*] + \\ \max\{a,b\} E[v|v > v^*] \end{array} \right] \\ &\quad -2(1 - F(v^*)) n \cdot c. \end{aligned} \tag{2}$$

The expected value of the winner is computed by going through the possible number of voters for each candidate where $a$ is the votes for candidate $A$ and $b$ is the votes for candidate $B$. The probability of each case is calculated and multiplied by the expected value of the winner, which is calculated by the expected value of those that voted for the winner plus the expected value of those that wanted the winner to win but nonetheless didn't vote for him. The expression $2n(1 - F(v^*))c$ is the expected cost of the voters voting since $1 - F(v^*)$ is the probability of each voter voting and there are $n$ voters of each type.

## 2.3 When is it optimal to have a voting cost?

In the following proposition, we derive the conditions under which the optimal cost is strictly positive.

**Proposition 1** *Under ASC, if* $E[v] \cdot F'(0) > 1$, *then the optimal cost is strictly positive.*

The condition, $E[v] \cdot F'(0) > 1$, has two components that depend upon the distribution of $v$: the density at zero and the expected value. The combination of these two components must be large enough. Too low a value of the density at zero would mean that increasing cost does not eliminate enough low value votes. Too low an expected value would mean that the benefit to eliminating these low-value voters is not large enough. This condition is equivalent to $\lim_{c \to 0} \frac{dE[v|v>v^*(c)]}{dc} > 2$. This implies that the expected value of those voting is increasing in cost by a sufficient amount, namely 2. In other words, if one increases cost marginally by a dollar (at zero), then the expected value of those voting should go up by 2 in order for costly voting to be beneficial. This condition is also equivalent to $\lim_{\widetilde{v} \to 0} \frac{dE[v|v>\widetilde{v}]}{d\widetilde{v}} > 1$. This states that the mean-residual-lifetime function (MRL) of $F$ is strictly increasing at zero. It is satisfied by all strict log-convexity distributions (see Proposition 2 of Heckman and Honore, 1990). Similar conditions (such as having a monotone MRL) are used in a variety of economic applications (see Bagnoli and Bergstrom, 2005). These include McAfee and Miller (2012) who show that an increasing MRL at zero implies allocating appointments (or objects) by reservations is inefficient for low transportation costs.

The following example illustrates Proposition 1.

**Example 1** $F(v) = v^\alpha$ *where* $\alpha > 0$, $c < 1/2$, $n = 1$.

We have $\lim_{v \to 0} vF'(v) = \lim_{v \to 0} \alpha v^\alpha = 0$. From (1), $v^*(c) = 2c$. We can then write equation (2) as

$$
\begin{aligned}
(2c)^{2\alpha} E[v|v &< 2c] + (1 - (2c)^{2\alpha})E[v|v > 2c] - 2(1 - (2c)^\alpha)c \\
&= \frac{\alpha - 2c(1+\alpha) + (2c)^\alpha(\alpha + 2c)}{1+\alpha}.
\end{aligned}
$$

The slope of the surplus w.r.t. $c$ is

$$
SSV'(c) = -2 + \frac{2^\alpha c^{\alpha-1}(\alpha^2 + 2(1+\alpha)c)}{1+\alpha}.
$$

For $\alpha > 1$ , $\lim_{c\to 0} SSV'(c) = -2$. For $\alpha = 1$, $\lim_{c\to 0} SSV'(c) = -1$. For $0 < \alpha < 1$, $\lim_{c\to 0} SSV'(c) = \infty$. Hence, when $0 < \alpha < 1$, the surplus improves by increasing the cost. When $\alpha \geq 1$, the surplus is at the highest when cost is zero. (When $\alpha \geq 1$, $SSV'(c)$ is strictly increasing in $c$ for all $c > 0$, hence $SSV'(c)$ can equal zero only once. Since $SSV(0) = SSV(1/2)$, no one votes in either case, and $SSV'(0) < 0$, the point at which $SSV'(c) = 0$ must be a minimum.)

We also see that the conditions of Proposition 1 are satisfied for $1/2 \leq \alpha < 1$, however, $\lim_{v\to 0} F'(v)F(v) = 0$ is not satisfied for $0 < \alpha < 1/2$. Thus, we demonstrate that Proposition 1's conditions are sufficient but not necessary.

# 3 Aggregate supporter uncertainty.

## 3.1 Model and Initial Results

In the previous section, we assumed that there were an equal number of supporters for either A or B. Here we assume there are $n$ voters and each voter has an equal and independent chance of desiring each outcome. This leads to aggregate supporter uncertainty (ASU). Again, each voter has a level of support for his desired outcome drawn according to the non-atomic cumulative distribution function $F$ with support $[0, \overline{v}]$, where $\overline{v} > 2c$ and $F'(0)$ is finite. The voter preferences over platforms is similar in style to that in Börgers (2000, 2004), we will discuss the differences later.

In the following lemma, we show there is a unique equilibrium and determine its cutoff condition.

**Lemma 3** *There is a unique Bayes-Nash equilibrium with cutoff $v^*(c)$ that satisfies*

$$v^* \sum_{a=0}^{n-1} \binom{n-1}{a} F(v^*)^{n-1-a} (1 - F(v^*))^a \begin{cases} \binom{a}{a/2} \left(\frac{1}{2}\right)^a & \text{if } a \text{ is even,} \\ \binom{a}{(a-1)/2} \left(\frac{1}{2}\right)^a & \text{if } a \text{ is odd.} \end{cases} = 2c. \tag{3}$$

This leads to a similar condition to ASC on the expected value and density of the low types for which the optimal cost of voting is positive.

**Proposition 2** *Under ASU, if $n$ is even and $E[v] \cdot F'(0) > \frac{1}{2}$, then the optimal cost is strictly positive.*

Notice that the condition with aggregate supporter uncertainty $E[v] \cdot F'(0) > \frac{1}{2}$ and an even number of voters is weaker than that when there is certainty $E[v] \cdot F'(0) > 1$ (which by our assumptions also has an even number of total voters). Thus, for all distributions where it would be worthwhile to have voting costs without uncertainty in number of supporters for each outcome, it would also be worthwhile to have positive voting costs with such supporter uncertainty. We note that we are unable to use our methods to find a result when $n$ is odd.

## 3.2 What is the optimal level of voting?

We saw in the previous subsection that voting, even if it is costly, can have benefits to social surplus. However, in the previous subsections, who voted was determined by equilibrium conditions. In this subsection, we wish to ask what should be the correct level of voting for society. This is the equivalent of asking if a social planner can decide a critical level of value only above which people should vote, what should it be?

Finding this level allows us to compare the optimal level to the equilibrium level. We can then ask what policy recommendations we can give to induce this level – having penalties for not voting or adding a poll tax. These penalties and taxes are just transfers and thus do not affect overall welfare.[13] Remember, in contrast, a cost of voting yields no direct benefit to anyone and given the same voting outcome is a waste to society. In the following proposition, we can compare the equilibrium with the optimal level of voting.

**Proposition 3** *Under ASU, (i) there is an excess of voting, (ii) there should be no fines for failing to vote (no mandatory voting), (iii) there should be a poll tax to discourage voting.*

We can see the intuition of the Proposition by the following. A voter's vote will be pivotal in two instances. Case (A): when there is a tie in votes without his vote. Case (B): when the other

[13] Another method would be to employ some criteria for voting that reflects values. For instance, Jefferson felt only the educated should vote (Padover 1952, page 43). In our model, they would have better information and hence higher values for certain candidates. For instance, with the Brexit vote, some voters might be hurt by a Brexit but not know it (such as seniors wanting to retire in Spain) or gain from a Brexit and not know it (such as workers in uncompetitive industries). With lack of knowledge, these voters could have been close to indifferent to voting between either option. When Brexit won the vote many voters claimed to have not been aware of all the consequences of the vote and now felt strongly in favor of remain.

There have also been literacy and property ownership as requirements. While literacy might have been used to disenfranchise certain minority groups, a property ownership requirement for the most part was to restrict voting to groups that had a stake in the country (high values).

candidate leads by 1 without his vote. In case (A), there will be no externality imposed since the other voters balance each other out. For case (B), the net externality imposed by a voter on others by voting is $-E[v|v > v^*]$. This externality is negative and a voter doesn't take this into account. Thus, in equilibrium there is a higher level of voting compared to the optimal level.

Börgers (2000, 2004) develops a model with costly voting and shows (in Börgers, 2004) that the unique equilibrium is superior to both mandatory voting still with voting costs (or equivalently bribing people to vote) and random choice (with no one voting). He also graphically shows (in Börgers, 2000) that the unique equilibrium may be superior to one with lower voting costs, but importantly did not show that it is superior to one with zero voting costs. With the technique used in the above proposition, it becomes apparent that Börgers (2004) also has an excess of voting.[14] The main difference of our model to that of Börgers (2000, 2004) is that we allow for different intensities of preferences while Börgers (2000, 2004) has different voting costs. In both models, it is always optimal to charge a poll tax when there is a positive cost of voting; however, only in our model is it worthwhile to have a poll tax when the cost of voting is zero. This is because when the marginal voter has a value of 0 and when pivotal (moving from a loss to a tie) is replacing a voter with a higher value. In Börgers (2000, 2004), it is optimal to have all voters voting when there is zero voting costs, since everyone has the same valuation (in absolute terms) and thus this will maximize the information aggregated. Krasa and Polborn (2009) vary the Börgers model by allowing for ex-ante asymmetry of preferences over alternatives. They find that for a large enough number of voters, it is optimal to move towards mandatory voting from voluntary voting (by a penalty for not voting or a subsidy to voting). There are no restrictions on $F$ for a poll tax to be optimal. The tax is a transfer unlike a wasteful cost of voting so is a superior alternative. It is not clear that a poll tax is politically viable. This then leads to our previous sub-section where it may be worthwhile to maintain (or even induce) a cost of voting. Doing so eliminates voters with little intensity for their preferred candidate.

[14] The externality imposed on others by voting is absent in case (A) and equal to the value in case (B). For the same reason as the proof of Proposition 2, there is an excess of voting in the Börgers model as well.

## 4 Random Cost of Voting

In our two models, we have assumed that all voters have the same cost of voting. Imposing a cost may not uniformly affect the voters. In this section, we assume that each voter in addition to their privately known value will also have a privately known cost. It is conceivable that a designer may have control over not only the cost but the type of cost.

For example, a government can make it difficult to obtain an absentee ballot which would lead to a uniform cost of those expatriates trying to vote. Alternatively, the government could require voters to fly back in order to vote, as is required in Israel. These latter costs could mirror random costs since they would vary by the distance needed to travel. Similarly, a government could make it difficult to register to vote in general for a uniform cost or, for random costs, make it difficult for voters to change voting location and force them to return to their hometowns, as was in the case of the abortion referendum in Ireland. Not having the polls open for very long without giving people time off from work will make the cost of voting random. Finally, for voting in committees moving to an app-based voting may impose a higher cost on those less technologically able.

We analyze the possibility that a designer can choose for voters to have a random cost of voting according to the cumulative distribution $G(c)$ in addition to their value drawn from the cumulative $F(v)$. Note that we are assuming that $v$ and $c$ are independent. There is an equilibrium $v(c)$ where a voter votes if $v \geq v(c)$ and does not vote if $v < v(c)$. We further note that this $v(c)$ must be linear since a voter's decision is voting if the probability of being pivotal times the benefit is larger than the costs $P(pivatol) \cdot v/2 \geq c$. This requires $v/c$ to be larger than a certain ratio. We are able to show that replacing a random cost with a uniform cost that maintains the same proportion of those who vote will inherently increase the expected value of those that vote. Since the winning coalition has a disproportionately higher amount of voters, this will increase the expected value of the winning coalition.

**Proposition 4** *For any interior cutoff equilibrium $v(c)$ with costs distributed by $G(c)$ and values distributed by $F(v)$, for the same value distribution, there exists an equilibrium with a higher expected value of the winning option with uniform costs $\widehat{c}$, where $1 - F(v(\widehat{c})) = \int_0^1 (1 - F(v(c)))dG$.*

Note that this analysis ignores the cost side of voting. If we look at the total surplus of voting, then social surplus can decrease by replacing random costs with a uniform cost. This can happen if there is little variance in the value of voting. There could be an equilibrium where the cutoff cost to voting in most cases is high but there is a significant chance of a low cost. In order to maintain the same level of voting, it would be necessary to have a high uniform cost. For instance, say a voter is always pivotal and the value of voting is approximately 2 and costs are randomly distributed according to the uniform distribution on $[0, 2]$. In equilibrium, half the voters will vote and pay an expected cost of $1/2$. In order to maintain this with a uniform cost, the cost would have to be 1, which is a worse outcome via social surplus.

While one cannot always improve surplus by replacing a random cost with a uniform cost while maintaining the expected number of voters, we conjecture that there is always a surplus superior equilibrium with a uniform cost and a higher number of voters. For instance in the above example, a cost of voting of 0 would be superior.

We finally note that if the cost could be associated with preference intensity (as in Chakravarty and Kaplan 2013), then following the intuition there, it is likely that a negative relationship could improve welfare since it would be easier for those with high values to indicate their preference than those with lower values.

## 5 Conclusion

Since the nineteenth century, political scientists have been in agreement that increasing the franchise will be beneficial to the society (Lakeman and Lambert, 1959, page 19). So over the last century in democracies, the right to vote has been given to most of the adult society and the requirements of registration to vote such as property qualifications have been removed. Social scientists have further asked the question whether or not it makes sense to require people to vote. In addition this requirement to vote also gets around the paradox that since each individual may find his vote negligible will choose not to vote if there is a cost to voting. We show that not only should one not require people to vote but there is a distinct benefit to having some people not vote and increasing the (wasteful) cost to voting may paradoxically be beneficial to society. (Note increasing a wasteful cost of voting may be politically more viable than imposing a poll tax.) For instance, if a committee has an important vote, scheduling the meeting at an inconvenient time may improve the outcome.

This also shows that allowing absentee ballots or internet voting can be damaging.[15]

It may be possible that with repeated play (such as with committees) voters may voluntarily withhold votes without cost to voting. However, Engelmann and Grimm (2012) experimentally find that in a repeated game when preferences are privately known, without imposed budget constraints on votes, voters will not cooperate and withhold votes (even with two players).[16]

There are many directions of future research where one can extend our results by expanding the environment. One direction is to increase the number of alternatives on the ballot to more than two. With committee voting this seems quite logical. Furthermore, once this is done, one can introduce approval voting to ameliorate strategic voting (see Brams and Fishburn, 1978). Another direction is to introduce a common value element in addition to a private value as in Osborne and Turner (2010).

# 6 Appendix

**Proof of Lemma 1**

The cutoff will be such that the value for voting equals the cost.

$$v^* \cdot \left[\frac{1}{2}\Pr_{-i}(\#_A = \#_B | v^*) + \frac{1}{2}\Pr_{-i}(\#_A = \#_B - 1 | v^*)\right] = c.$$

When the voter $i$ prefers $A$ ($i \leq n$), we can rewrite the probabilities in this equation as follows:

$$\begin{aligned} &\Pr_{-i}(\#_A = \#_B | v^*) \\ &\quad = \Pr_{-i}(\#_A = \#_B = 0 | v^*) + \Pr_{-i}(\#_A = \#_B = 1 | v^*) + ... + \Pr_{-i}(\#_A = \#_B = n-1 | v^*) \\ &\quad = \sum_{i=0}^{n-1} \binom{n-1}{i}\binom{n}{i} (1 - F(v^*))^{2i} F(v^*)^{2(n-i)-1} \end{aligned}$$

[15] In Israel, absentee ballots are not allowed. Voters fly back to Israel specifically for the election (Los Angeles Times, May 9, 1999). Our results show that this waste may actually be socially efficient.

[16] Note that Kaplan and Ruffle (2012) do find cooperation in a similar flavored repeated entry game, but have a higher negative externality of entry than exists with voting.

and

$$\Pr_{-i}(\#_A = \#_B - 1|v^*)$$
$$= \Pr_{-i}(\#_A = 0, \#_B = 1|v^*) + \Pr_{-i}(\#_A = 1, \#_B = 2|v^*) + ... + \Pr_{-i}(\#_A = n-1, \#_B = n|v^*)$$
$$= \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} (1 - F(v^*))^{2i+1} F(v^*)^{2(n-i-1)}.$$

Substituting these expressions into the cutoff value equation yields:

$$v^* \sum_{i=0}^{n-1} \left[ \binom{n-1}{i} \binom{n}{i} (1 - F(v^*))^{2i} F(v^*)^{2(n-i)-1} + \binom{n-1}{i} \binom{n}{i+1} (1 - F(v^*))^{2i+1} F(v^*)^{2(n-i-1)} \right] = 2c. \tag{4}$$

Since the equilibrium entails the probability of being pivotal equalling twice costs over benefits, the above equation resembles the equilibrium condition in Palfrey and Rosenthal (1983, 1985). This equation can then be simplified as

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} F(v^*)^{2(n-i-1)} (1 - F(v^*))^{2i} \left[ 1 + \frac{2i - n + 1}{n - i} \cdot F(v^*) \right] = 2c$$

The same equation holds for voters preferring $B$ ($i > n$). ■

**Proof of Proposition 1**

We proceed by establishing two properties of the equilibrium. (i) $v^*(0) = 0$. This follows directly from equation (1): when there is a zero cost of voting, everyone votes. (ii) If $n = 1$ or $\lim_{v \to 0} vF'(v) = 0$, then $\lim_{c \to 0} v_c^*(c) = 2$. To establish this observe that equation (1) must hold for all $c$, so we can take the derivative w.r.t. $c$ and take the limit as $c \to 0$. As $c \to 0$, we have $v^* \to 0$, thus we can replace $\lim_{c \to 0}$ with $\lim_{v^* \to 0}$. Hence, we have $\lim_{c \to 0} 1 - F = 1$ and $\lim_{c \to 0} F = 0$. Notice that from these limits, the only potential remaining term from the derivative of the LHS of (1) is when $i = n - 1$. (For $n > 1$, the rest vanish.) This yields:

$$\left[ 1 - (n-1) \lim_{v^* \to 0} v^* \cdot F'(v^*) \right] = \frac{2}{v_c^*(0)}. \tag{A1}$$

Hence, $v_c^*(0) = 2$ when $n = 1$ or $\lim_{v \to 0} vF'(v) = 0$.

We can now prove the main result using these two properties. To show that the optimal cost is strictly positive, it is sufficient to show that $\lim_{c\to 0} SSV'(c) > 0$. We will prove this by showing that the derivative of the expected value of the voters that prefer the winning candidate is higher than the derivative of the expected costs.

By assumption, $F(0) = 0$ and $F'(0)$ is finite. Hence, (iii) $\lim_{v\to 0} F'(v)v = 0$ and (iv) $\lim_{v\to 0} F'(v)F(v) = 0$. Given (iii) and (iv), the derivative of $SSV$ as $c$ goes to 0 can be determined as follows. Using the product rule, the derivative is equal to the sum of the values times the derivative of the probabilities plus the sum of the probabilities times the derivative of the values. The probability of $a$ voters voting for $A$ and $b$ voters voting for $B$ is

$$\binom{n}{a}\binom{n}{b} F\left(v^*\right)^{2n-a-b}\left(1-F\left(v^*\right)\right)^{a+b}. \tag{A2}$$

The limit of (A2) as $c \to 0$ is zero if $a+b < 2n$ and 1 if $a+b = 2n$. If $a+b = 0$, then the derivative of (A2) is $2nF^{2n-1}F'v_c$ which goes to 0 since $\lim_{v\to 0} F'(v)F(v) = 0$ and $\lim_{c\to 0} v_c^* = 2$ by the above limits. If $a+b = 2n$, the derivative is $-2n\left(1-F\right)^{2n-1}F'v_c$ which goes to $-2nF'v_c$. Otherwise, the derivative of (A2) is

$$\binom{n}{a}\binom{n}{b}\left[\left(2n-a-b\right)F\left(v^*\right)^{2n-a-b-1}\left(1-F\left(v^*\right)\right)^{a+b} - (a+b)F\left(v^*\right)^{2n-a-b}\left(1-F\left(v^*\right)\right)^{a+b-1}\right]F'v_c$$

which goes to zero unless $a+b = 2n-1$. When $a+b = 2n-1$, the derivative goes to $\binom{n}{a}\binom{n}{b}F'v_c$. Note that $a+b = 2n-1$ when either $a = n$ and $b = n-1$ or vice-versa. In each case, $\binom{n}{a}\binom{n}{b} = n$.

Let us now look at the sum of the values times the derivative of the probabilities. When $a+b = n$, we have $(n - \max\{a,b\})E[v_i|v_i < v^*] + \max\{a,b\}E[v_i|v_i > v^*] = nE[v_i|v_i > v^*]$. When $a+b = 2n-1$, we also have $(n-\max\{a,b\})E[v_i|v_i < v^*] + \max\{a,b\}E[v_i|v_i > v^*] = nE[v_i|v_i > v^*]$. Thus, when multiplied by the derivative of the probabilities and summed over the possible values for $a$ and $b$, the limit goes to 0.

We are left with the sum of the probabilities times the derivative of the values. As we saw above, as $c$ goes to 0 the only time the probabilities are non-zero is when $a = b = n$. Thus,

$$\lim_{c\to 0} SSV'(c) = n\frac{dE[v_i|v_i > v^*]}{dc} - 2n + \lim_{c\to 0} 4n \cdot c \cdot F'(v^*).$$

Since $E[v|v>v^*] = \frac{\int_{v^*}^{\infty} v dF(v)}{1-F(v^*)}$, we have

$$\frac{dE[v|v>v^*]}{dc} = \frac{-v_c^* \cdot v^* F'(v^*)}{1-F(v^*)} + \frac{F'(v^*) v_c^* \int_{v^*}^{\infty} v dF(v)}{(1-F(v^*))^2}.$$

Hence, $\lim_{c\to 0} \frac{dE[v|v>v^*]}{dc} = \lim_{c\to 0} 2F'(v^*) \int_0^{\infty} v dF(v) = \lim_{c\to 0} 2E[v]F'(v^*)$. Thus, since $E[v]F'(0) > 1$, $SSV'(0) > 0$ and it is optimal to increase costs.

Note that even though there is a unique equilibrium at $c = 0$, when $c > 0$, there is a possibility of multiple type-symmetric equilibria (see Palfrey and Rosenthal, 1983, 1985). However, even if this is the case, under these conditions we have shown that all of the type-symmetric equilibria must yield higher social surplus than the equilibrium when $c = 0$. Since $c$ is small, all equilibria must be type symmetric (see Myerson, 1998, and Krishna and Morgan, 2015). ■

**Proof of Lemma 3**

There are $n$ voters overall. Take the decision of an individual voter. Consider each case where exactly $a$ other voters vote. This occurs with probability $\binom{n-1}{a} F(v^*)^{n-1-a} (1-F(v^*))^a$. If $a$ is even, this voter is pivotal only if there is a tie. This happens if exactly $a/2$ vote for each outcome, which occurs with probability $\binom{a}{a/2} \left(\frac{1}{2}\right)^a$. If $a$ is odd then the voter is pivotal if there is exactly one less voter that votes for his preferred outcome. This has $(a-1)/2$ voting for his outcome and $(a+1)/2$ voting for the other outcome. This occurs with probability $\binom{a}{(a-1)/2} \left(\frac{1}{2}\right)^a$. Using the above forms equation (3). The LHS of (3) is 0 when $v^* = 0$. The LHS equals $\overline{v}$ when $v^* = \overline{v}$ which is larger than 2c. Hence, since the LHS is continuous, there exists an interior solution to the equation. Finally, we want to show uniqueness. The probability of being pivotal given that $a$ other voters vote is decreasing in $a$. As $v^*$ increases the distribution of the number of other voters stochastically shifts downwards. Hence, as $v^*$ increases, the overall probability of being pivotal increases. Thus, the LHS of (3) is strictly increasing in $v^*$ and we have a unique solution. ■

**Proof of Proposition 2**

We begin by noticing that when there is a zero cost of voting, everyone votes, $v^*(0) = 0$.This follows directly from equation (3).We now look at $\lim_{c\to 0} v_c^*(c)$. Take the total derivative w.r.t. $c$ of the cutoff equation (3). Since $\lim_{v\to 0} F'(v)v = 0$, the only term remaining on the LHS is when

$a = n - 1$. Thus,

$$\begin{aligned}\lim_{c\to 0} v_c^*(c) &= 2/\begin{cases}\binom{n-1}{(n-1)/2}\left(\frac{1}{2}\right)^{n-1} & \text{if } n-1 \text{ is even,}\\ \binom{n-1}{(n-2)/2}\left(\frac{1}{2}\right)^{n-1} & \text{if } n-1 \text{ is odd.}\end{cases}\\ &= 2^n/\begin{cases}\binom{n-1}{(n-1)/2} & \text{if } n \text{ is odd,}\\ \binom{n-1}{n/2-1} & \text{if } n \text{ is even.}\end{cases}.\end{aligned}$$

For convenience, we look at the social surplus of the equilibrium above random allocation. Doing so does not affect the analysis since we are taking the derivative. The social surplus of the equilibrium (above random allocation) is the expected benefits (above random allocation) minus the costs of voting. The expected benefits depends only upon the number of voters that vote (in expectation all those that don't vote have the same expectation as a random allocation). If two vote for an option and two vote against it, the difference in surplus from random allocation is zero. If three vote for an option and two vote against it, the difference in surplus is $E[V_i|V_i > v^*]$. In general, the social surplus above random allocation is the number that voter for the winning option minus the number that vote for the losing option times $E[V_i|V_i > v^*]$. Given there is an equal chance that a voter that chooses to vote votes for either option, the number of votes for option A minus those for option B follows a one-dimensional random walk as voters vote. The social surplus given that a certain number of voters vote is the expected absolute value of this number times $E[V_i|V_i > v^*]$. If $a$ voters vote, then this expectation is $\begin{cases}\frac{(a-1)!!}{(a-2)!!} & \text{if } a \text{ is even,}\\ \frac{a!!}{(a-1)!!} & \text{if } a \text{ is odd.}\end{cases}$ (see Weisstein, 2010). Note that the double factorial, $n!!$, is either all strictly positive even numbers up to $n$ multiplied together or all strictly positive odd numbers up to $n$ multiplied together depending upon whether $n$ is even or odd. The social surplus above random allocation is given by

$$\begin{aligned}SSV(c) &= \sum_{a=0}^{n}\binom{n}{a}F\left(v^*(c)\right)^{n-a}\left(1-F\left(v^*(c)\right)\right)^a\left[E[v|v>v^*(c)]\cdot\begin{cases}\frac{(a-1)!!}{(a-2)!!} & \text{if } a \text{ is even,}\\ \frac{a!!}{(a-1)!!} & \text{if } a \text{ is odd.}\end{cases}\right]\\ &\quad -(1-F\left(v^*(c)\right))n\cdot c.\end{aligned}$$

Taking the limit as $c \to 0$ of the derivative yields:

$$\lim_{c\to 0} SSV'(c) = \lim_{c\to 0} v_c \left( \begin{array}{c} \left(\frac{dE[v|v>v^*(c)]}{dv^*} - nE[v|v > v^*(c)]F'(v^*(c))\right) \cdot \begin{cases} \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even,} \\ \frac{n!!}{(n-1)!!} & \text{if } n \text{ is odd.} \end{cases} \\ +nE[v|v > v^*(c)] \cdot F'(v^*(c)) \begin{cases} \frac{(n-2)!!}{(n-3)!!} & \text{if } n \text{ is odd,} \\ \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even.} \end{cases} \end{array} \right) + \lim_{c\to 0} F'((v^*(c)))n \cdot v_c \cdot c - n. \tag{5}$$

Note that $v_c \frac{dE[v|v>v^*(c)]}{dv^*} = \frac{dE[v|v>v^*(c)]}{dc^*}$ and $\lim_{c\to 0} \frac{dE[v|v>v^*(c)]}{dc^*} = F'(0)E[v] \lim_{c\to 0} v_c$, thus $\lim_{c\to 0} \frac{dE[v|v>v^*(c)]}{dv^*} = E[v] \cdot F'(0)$. Also note that $\lim_{c\to 0} F'((v^*(c))) \cdot v_c \cdot c = 0$ if $\lim_{v\to 0} F'(v)v = 0$ and $v_c$ is finite. Hence, we can simplify (5) to yield:

$$\lim_{c\to 0} SSV'(c) = \left(\lim_{c\to 0} v_c\right) E[v] \cdot F'(0) \left( n \begin{cases} \frac{(n-2)!!}{(n-3)!!} & \text{if } n \text{ is odd,} \\ \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even.} \end{cases} - (n-1) \begin{cases} \frac{(n-1)!!}{(n-2)!!} & \text{if } n \text{ is even,} \\ \frac{n!!}{(n-1)!!} & \text{if } n \text{ is odd.} \end{cases} \right) - n.$$

Look at the case when $n$ is even. We have: $\lim_{c\to 0} SSV'(c) = 2^n / \binom{n-1}{n/2-1} \frac{(n-1)!!}{(n-2)!!} \cdot E[v]F'(0) - n = 2^n \cdot \frac{(n/2-1)!(n/2)!}{(n-1)!} \cdot \frac{(n-1)!!}{(n-2)!!} \cdot E[v]F'(0) - n$. Since $\frac{2^{n/2}}{n!} = \frac{1}{(n/2)!(n-1)!!}$ and $2^{n/2-1}(n/2-1)! = (n-2)!!$, we have $\lim_{c\to 0} SSV'(c) = (2E[v]F'(0) - 1) \cdot n$. Now this is strictly greater than zero if and only if $E[v] \cdot F'(0) > \frac{1}{2}$.

Note that if we try the same method when $n$ is odd, we have: $\lim_{c\to 0} SSV'(c) = \frac{2^n}{\binom{n-1}{(n-1)/2}} E[v] \cdot F'(0) \left(n\frac{(n-2)!!}{(n-3)!!} - \frac{n!!}{(n-3)!!}\right) - n = -n$. Now this is never greater than zero. Hence, in that case, using this method we are unable to determine when it is optimal to have a positive cost when $n$ is odd.

■

**Proof of Proposition 3**

The optimal cutoff (if interior) should then solve:

$$\sum_{a=0}^{n-1} \binom{n-1}{a} F(v^*)^{n-1-a} (1 - F(v^*))^a \begin{cases} \binom{a}{a/2} \left(\frac{1}{2}\right)^a v^* & \text{if } a \text{ is even,} \\ \binom{a}{(a-1)/2} \left(\frac{1}{2}\right)^a (v^* - E[v|v > v^*]) & \text{if } a \text{ is odd.} \end{cases} = 2c. \tag{6}$$

From this equation, the equilibrium cutoff will then be lower than the optimal cutoff. To see this the LHS is similar to the LHS of equation (3) but $(v^* - E[v|v > v^*])$ multiplied by one of the probabilities instead of $v^*$, which is then smaller. Since the LHS of (3) is increasing in $v^*$. The LHS of (3) will be smaller than $2c$ for all $v^*$ less than the equilibrium cutoff. Consequently, the LHS of (6) is smaller than $2c$ for all $v^*$ less than the equilibrium cutoff. Hence, the solution to (6) must be higher than the equilibrium cutoff. Since the optimal level of voting is lower than the equilibrium level, a government could charge for voting in order to implement the optimal cutoff. This would be change the $c$ in equation (3) such that the $v^*$ that solves that the solution of equation (3) matches the solution to the equation (6). Note that while (6) may have more than one solution, all solutions would be at a higher $v^*$, then the equilibrium. Also, the only possible non-interior optimal cutoff for $c > 0$ is where no one votes (it can never be socially optimal for a zero-valued voter to vote). Thus again, the optimal cutoff would be higher than the equilibrium. ■

**Proof of Proposition 4**

The fraction of those voting is

$$\int_0^1 \int_{v(c)}^1 dFdG = \int_0^1 (1 - F(v(c)))dG$$

If we define $\widehat{c}$ such that $1 - F(v(\widehat{c})) =.\int_0^1 (1 - F(v(c)))dG$.

We can then have a new regime were costs are uniform at $\widehat{c}$, and voting occurs whenever $v \geq \widehat{c}$. This insures that the probability of voting is the same in both regimes. Due to the mean value theorem, $\widehat{c}$ exists. We will now show that the surplus is higher under a regime where there is a uniform cost of $\widehat{c}$. Let us start by looking at the expected value of those that vote. With random costs, the expected value is $\int_0^1 \int_{v(c)}^1 vdFdG$. Under fixed $c$, the expected value is $G(\widehat{c}) \int_{v(c)}^1 vdF$. Area lost by moving to uniform costs $\widehat{c}$ is $\int_0^{\widehat{c}} \int_{v(c)}^{v(\widehat{c})} vdFdG$. Area gained by moving to uniform costs $\widehat{c}$ is $\int_{\widehat{c}}^1 \int_{v(\widehat{c})}^{v(c)} vdFdG$. By definition of $1 - F(v(\widehat{c}))$, we have $F(v(\widehat{c})) = \int_0^1 F(v(c))dG$. Thus,

$$\int_0^{\widehat{c}} [F(v(\widehat{c})) - F(v(c))]dG = \int_{\widehat{c}}^1 [F(v(c)) - F(v(\widehat{c}))]dG.$$

This then implies

$$\int_0^{\widehat{c}} \int_{v(c)}^{v(\widehat{c})} dFdG = \int_{\widehat{c}}^1 \int_{v(\widehat{c})}^{v(c)} dFdG.$$

Thus, the measure of values gained is the same as the measure of values lost. Also, since all values gained are above $v(\widehat{c})$ and all values lost are below $v(\widehat{c})$. In expectation $E[v|v > v(\widehat{c})] > E[V|V > v(c)]$. The winners have proportionally more voters than the losers. Hence, keeping the proportion of voters the same and increasing the expected value increases social surplus.

Denoting $p$ as the probability of voting. With aggregate certainty, the expected number of those voting in the winning coalition is

$$\sum_{a=0}^{n}\sum_{b=0}^{n}\binom{n}{a}\binom{n}{b}p^{2n-a-b}(1-p)^{a+b}[\max\{a,b\}]$$

which is larger than the expected number of those voting for a particular option, which can be written as

$$\sum_{a=0}^{n}\sum_{b=0}^{n}\binom{n}{a}\binom{n}{b}p^{2n-a-b}(1-p)^{a+b}a.$$

By maintaining the same proportion of voters but shifting those that vote to have higher values. Increasing $E[v|vote]$, keeping $p$ constant and holding $pE[v|\text{vote}] + (1-p)E[v|\text{not vote}] = E[v]$, we increase the expected value of the winning coalition. While this last analysis is done for the aggregate supporter certainty model, with similar reasoning it will hold for the aggregate supporter uncertainty model. Namely, the winning coalition will have on average more voters than any particular coalition. ■

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