

TRACKING OPTIMISATION  
AND THE  
MEASUREMENT  
OF  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$   
AT NA62

Andrew Sturgess

*Thesis submitted to the University of Birmingham  
for the degree of Doctor of Philosophy*



Supervisors:  
**Prof. Cristina Lazzeroni**  
**Dr. Chris Parkinson**

Particle Physics Group  
School of Physics and Astronomy  
University of Birmingham  
Birmingham, B15 2TT

*October 20, 2018*

UNIVERSITY OF  
BIRMINGHAM

**University of Birmingham Research Archive**

**e-theses repository**

This unpublished thesis/dissertation is copyright of the author and/or third parties. The intellectual property rights of the author or third parties in respect of this work are as defined by The Copyright Designs and Patents Act 1988 or as modified by any successor legislation.

Any use made of information contained in this thesis/dissertation must be in accordance with that legislation and must be properly acknowledged. Further distribution or reproduction in any format is prohibited without the permission of the copyright holder.

## Abstract

The NA62 experiment at CERN is designed to perform a precision measurement of the ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching fraction. The experiment relies on a kinematic background rejection using the missing mass-squared signatures of the contributing background components. As the signal is of the  $\mathcal{O}(10^{-10})$  smaller than that of the most common kaon decays, the tracking performance of the detector must be exceptional in order to achieve a large signal to background ratio of  $S/B \approx 10$ . In this thesis, two tools to aid with the tracking procedure have been developed. Firstly, a field map which describes the fringe field of the spectrometer magnet has been created, tested and implemented in the NA62 Monte-Carlo software. Additionally, an analysis procedure to improve the precision and accuracy of track momentum in data was developed.

Such tools are essential, not only for the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay, but also for all decays containing tracks. In particular, they have been used to aid the measurement of the branching fraction and the form factor for the Standard Model decay  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  which is described here. The measurement has been performed using the data collected by the NA62 experiment in 2016 and 2017. The event selection, trigger efficiency determination and the analysis techniques used for the measurement are presented. The analyses of the two data sets are provided separately to demonstrate the progressing sensitivity of the detector. The final results are compatible with literature, and the 2017 measurement is currently the world's most precise for this channel. This demonstrates the future potential of NA62 for when larger data sets become available in the next few years.

## Declaration of author's contribution

The design, construction and operation of the NA62 experiment at the Super Proton Synchrotron is the result of the hard work and collaboration of hundreds of people and institutes around the world. Without this effort, the study of fundamental high-energy phenomena would not be possible. In this thesis, the analyzed data sample was collected by the NA62 experiment in 2016 and 2017. This data was used to measure the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  branching fraction and form factor which is described in Chapters 6 to 9. The analysis is the result of my own work. The theoretical motivation and descriptions of the detectors provided in Chapter 2 and 3 are included to provide a complete picture of the analysis. Whilst I was not involved in the design or construction of the experiment, I took part in the data-taking operation between 2015 and 2017.

Except for the measurements used to build the simulation, the development of the fringe-field map described in Chapter 4 and the spectrometer calibration tool explained in Chapter 5 are both the result of my own work except where explicitly stated. No other qualifications have been achieved with the work reported in this thesis.

## Acknowledgements

I would like to express my appreciation to my supervisors Cristina Lazzeroni and Chris Parkinson, as without their expertise, guidance and suggestions, the research presented in this thesis would not have been possible. I would also like to thank John Fry for his exceptionally useful critiques of this thesis. I am also grateful to Ľuboš Bičian for the hours of discussions and meetings we shared. Furthermore, I wish to express gratitude to the University of Birmingham for providing me with funding and a place to develop my skills as a researcher, programmer and teacher.

To the NA62 group, I thank you for the enlightening experience and for all of the good times. A special mention goes to the inhabitants of W313 and W327 with whom I spent a vast deal of time with over the last four years; thanks for being my own personal Stack Exchange. To all of my friends outside of physics, especially to Jane and Noel, cheers for pretending to be interested in my work! Furthermore, thanks goes to Ado and the rest of the BB for all of the San Marino challenges over the years, and to Kyle Lafferty for ending the dream of World Cup glory prematurely every time. I wish to thank my parents for their unwavering support, cups of tea and unbelievable sense of humour, and to my siblings for reminding me that they are still the intelligent ones. Finally, to Milly, as without her unbelievable excellence, I would have given in a long time ago.

*For Billy, because he's my guy.*

# Contents

<b>1 Introduction</b>	20
<b>2 Theoretical motivation</b>	23
2.1 The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay	23
2.1.1 CKM framework	23
2.1.2 Unitarity	25
2.1.3 Theoretical prediction of $K \rightarrow \pi \nu \bar{\nu}$	28
2.1.4 Experimental status	31
2.2 The $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay	33
2.2.1 Anomalies in flavour physics	33
2.2.2 Lepton flavour universality	35
2.2.3 The form factor	36
2.2.4 Experimental Status	39
<b>3 The NA62 experiment</b>	44
3.1 Experimental strategy	44
3.2 NA62 Beam-line	47
3.3 Detectors at NA62	49
3.3.1 Detector overview	49
3.3.2 Tracking	51
3.3.2.1 GigaTracker (GTK)	51
3.3.2.2 Straw spectrometer (STRAW)	52
3.3.3 Particle Identification	53
3.3.3.1 Kaon Tagger (KTAG)	53
3.3.3.2 Ring Imaging Cherenkov (RICH)	55
3.3.4 Hodoscopes	56

3.3.4.1	NA48 Hodoscope (CHOD)	56
3.3.4.2	NewCHOD	57
3.3.5	Photon Veto	58
3.3.5.1	Large Angle Veto (LAV)	58
3.3.5.2	Liquid Krypton calorimeter (LKr)	58
3.3.5.3	Small Angle Veto (SAV)	60
3.3.6	Muon Veto	61
3.3.6.1	Muon Veto (MUV) <sub>1,2</sub>	61
3.3.6.2	MUV3	62
3.4	Trigger and data acquisition (TDAQ)	63
3.4.1	The trigger logic	64
3.4.2	L <sub>0</sub> primitives	65
3.4.3	L <sub>1</sub> algorithms	66
3.5	NA62 Framework	67
<b>4</b>	<b>Simulation of the MNP33 fringe field</b>	<b>69</b>
4.1	Motivation for mapping fields	69
4.2	Measurement sample	72
4.2.1	Blue-tube planes	73
4.2.2	Grey-tube planes	75
4.3	Building the field-map	78
4.3.1	Measurement deletion	78
4.3.2	Plane interpolation	81
4.3.3	Linking the fringe map with MNP33	85
4.3.4	Fringe field downstream of MNP33	90
4.4	Overall Performance	90
4.4.1	Fringe field systematics	90
4.4.2	Contribution to field integral	93
<b>5</b>	<b>Development of a spectrometer calibration procedure</b>	<b>95</b>
5.1	Motivation	95
5.2	Correction Model	96
5.2.1	The $\alpha$ parameter	97



5.2.2	The $\beta$ parameter	98
5.3	Calibration Procedure	99
5.3.1	Exploiting $K_{3\pi}$ decays	99
5.4	Calibration algorithm	101
5.4.1	Tuning the fit	104
5.5	Results	107
5.5.1	In 2016	107
5.5.2	In 2017	110
5.5.3	Simulation	112
5.5.3.1	Misalignment in the spectrometer	113
5.5.3.2	Scaling the MNP33 field	114
5.5.4	Monitoring	114
<b>6</b>	<b>Event selection</b>	<b>117</b>
6.1	Strategy	117
6.2	Data samples	119
6.2.1	Recorded data samples	119
6.2.2	MC samples	121
6.2.3	Tuning the Monte Carlo (MC)	122
6.2.3.1	Sub-detector efficiency	123
6.2.3.2	Three-track reconstruction efficiency	125
6.2.3.3	Applying detector efficiencies	127
6.3	Base-line selection	128
6.3.1	Base-line vertex and track requirements	128
6.3.2	Base-line particle identification	130
6.3.3	Base-line spectra	131
6.4	Developing the NA62 selection	133
6.4.1	Tuning vertex and track criteria	133
6.4.2	Tuning PID and including precision timing	135
6.5	The final selection	139
6.5.1	Track and vertex requirements	139
6.5.2	Kaon identification	140

6.5.3	Decay identification	140
6.5.4	The final spectra	141
<b>7</b>	<b>Measuring trigger efficiency</b>	<b>144</b>
7.1	Strategy	145
7.2	KTAG	146
7.3	RICH	148
7.4	STRAW <sub>e</sub>	150
7.5	Q <sub>X</sub> (New Charged Hodoscope (NewCHOD))	153
7.5.1	Flaw in the NewCHOD firmware	155
7.5.2	Q <sub>X</sub> emulator	157
7.5.3	Determining $K_{\pi\mu\mu}$ Q <sub>X</sub> efficiency	158
7.6	!LAV	160
7.6.1	Flaw with !LAV algorithm	161
7.6.2	!LAV emulation	163
7.6.3	Determining $K_{\pi\mu\mu}$ !LAV efficiency	163
7.7	MO <sub>2</sub> (MUV3)	165
7.8	Full trigger efficiencies	168
<b>8</b>	<b>The <math>K^+ \rightarrow \pi^+ \mu^+ \mu^-</math> analysis</b>	<b>171</b>
8.1	Analysis strategy	171
8.2	Reconstructing $d\Gamma/dz$	173
8.2.1	Number of candidates	174
8.2.1.1	$N_i$ and $N_i^B$	174
8.2.1.2	$N_{3\pi}$	179
8.2.2	Acceptances	180
8.2.3	Results	182
8.2.3.1	Extracting the LECs	182
8.2.4	Systematics	184
<b>9</b>	<b>Future prospects</b>	<b>189</b>
9.1	Strategy	189
9.2	Event sample	190

<b>9.3 Trigger efficiency</b> . . . . .	193
<b>9.4 Comparison to 2016</b> . . . . .	193
<b>9.4.1 Emulator performance</b> . . . . .	194
<b>9.5 Result</b> . . . . .	196
<b>10 Conclusions</b>	199
<b>Appendix A <math>K_{3\pi}, K_{\pi\mu\mu}</math> data-MC comparison</b>	211

---

# List of Figures

2.1 The unitarity triangles . . . . .	26
2.2 Constraints on the unitarity triangle in the $(\bar{\rho}, \bar{\eta})$ plane . . . . .	28
2.3 $Z$ -penguin and box contributions to the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay . . . . .	29
2.4 The E949 detector . . . . .	32
2.5 Observed $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ candidates at BNL experiments . . . . .	32
2.6 $Z, \gamma$ -penguin and box contributions to the $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay . . . . .	36
2.7 The two-pion loop contribution to the $\chi$ PT form factor . . . . .	37
2.8 Predicted $d\Gamma/dz$ spectrum with respect to $z$ for each component of form factor $W(z)$ . . . . .	38
2.9 The E865 detector . . . . .	40
2.10 The NA48/2 beam-line and detector . . . . .	41
2.11 The three invariant mass spectra for the two $ee$ one $\mu\mu$ measurements from E865 and NA48/2 . . . . .	42
3.1 The $K_{\pi\nu\nu}$ kinematics; only the $K^+$ and $\pi^+$ are observable. . . . .	45
3.2 The missing mass <sup>2</sup> distribution for the main decay modes of the $K^+$ . . . . .	46
3.3 Optics and layout of the K12 beam in the vertical view . . . . .	47
3.4 Downstream beam-line layout post TRIM5 . . . . .	48
3.5 The NA62 beam-line and detector. . . . .	50
3.6 Left: the $YZ$ view and right: the $XZ$ view of the GTK . . . . .	51
3.7 The STRAW detector . . . . .	52
3.8 STRAW chamber views and tube staggering . . . . .	53
3.9 Schematic drawing of the CEDAR and KTAG . . . . .	54
3.10 Schematic drawing of the RICH . . . . .	55
3.11 Schematic drawing of the NA48 CHOD . . . . .	56
3.12 Schematic diagram of the NewCHOD . . . . .	57

3.13 Photograph and schematic drawing of a single LAV station . . . . .	59
3.14 Schematic diagram of a quadrant of the LKr . . . . .	60
3.15 Photograph of the IRC and SAC . . . . .	61
3.16 A schematic of the full MUV system . . . . .	62
3.17 The MUV3 tile rate for nominal beam conditions . . . . .	63
3.18 The general flow of the TDAQ for every sub-detector . . . . .	64
4.1 The three field-map regions . . . . .	70
4.2 The vacuum tube measurement grid structure . . . . .	73
4.3 $Z$ -dependence of the blue-tube field . . . . .	74
4.4 Kinematic variation in measurable observables due to residual mag- netic fields . . . . .	75
4.5 $Z$ -dependence of the grey-tube field . . . . .	76
4.6 Field surfaces for the start, middle and end of the grey-tube region . .	77
4.7 Data/Fit residuals for specific $Z$ -planes . . . . .	79
4.8 Difference in 2D $B_Y$ fit after a significant point deletion . . . . .	80
4.9 The residual impact on the $B_X$ field integral with suspicious planes removed . . . . .	81
4.10 The surface-parameter $Z$ -dependence fits for $B_X$ . . . . .	83
4.11 The surface-parameter $Z$ -dependence fits for $B_Y$ . . . . .	84
4.12 $Z$ -dependence of the grey-tube field with map overlay . . . . .	85
4.13 Difference in fringe field and MNP33 maps at the cross-over boundary	86
4.14 fringe field scale-factor plotted against the field difference residual . .	87
4.15 Difference in fringe field and MNP33 post scaling . . . . .	87
4.16 Displaying the buffer region by comparing both fringe field and MNP33 maps to the raw measurements . . . . .	89
4.17 Final map deviation from measurements used to estimate map system- atic error . . . . .	91
4.18 Difference between raw field and fringe map integral for the full grey- tube at each measurement position . . . . .	92
4.19 Fringe and MNP33 total field integrals . . . . .	94
5.1 Estimating the size of spectrometer calibration parameter $\alpha$ . . . . .	97

5.2	Period A $K_{3\pi}$ from K3piSelection Analyzer	100
5.3	Algorithm output for Run 6610 for an example $(\alpha, \beta)$ trial.	102
5.4	$\chi^2$ fit for Run 6610	104
5.5	One-dimensional projections of the $(\alpha, \beta)$ fit	105
5.6	Response of $\alpha$ in 1D with the mass window	107
5.7	Time-dependence of $\alpha$ in 2016A	108
5.8	Time-dependence of $\beta$ in 2016A	108
5.9	Correlation between $(\alpha, \beta)$ parameters in 2016A	109
5.10	The change in $K_{3\pi}$ mass post $(\alpha, \beta)$ corrections in 2016A.	110
5.11	Time-dependence of $\alpha$ in 2017A	110
5.12	Time-dependence of $\beta$ in 2017A	111
5.13	Correlation between $(\alpha, \beta)$ parameters in 2017A	111
5.14	The change in $K_{3\pi}$ mass post $(\alpha, \beta)$ corrections in 2017A.	112
5.15	Response in $\alpha$ after deliberate spectrometer misalignment	113
5.16	Response in $\beta$ after scaling the MNP33 field	114
5.17	Data-processing graphical report for 2016 Run 6610.	115
6.1	The 2016A and 2017A intensity spectrum	120
6.2	Magnitude of $\Omega_C$ in Dalitz-space for $K_{\pi\mu\mu}$ and $K_{3\pi}$	122
6.3	The MUV3 and NewCHOD detector efficiencies in 2016A	124
6.4	Three-track reconstruction efficiency for Data and MC	126
6.5	Three-track reconstruction efficiency at the first STRAW chamber	127
6.6	The $K_{\pi\mu\mu}$ invariant mass distribution in 2016A for the base-line selection	131
6.7	The $K_{\pi\mu\mu}$ invariant mass distribution from the NA48/2 analysis	132
6.8	The 2016A invariant mass vs $Z$ -Vertex and $\mu^-$ opening angle	133
6.9	Comparison of opening angles in the Lab frame	134
6.10	Invariant mass distribution in 2016A for base-line selection including TRIM5 suppression	135
6.11	Supporting diagrams for the MUV3 related logic.	137
6.12	Time-residuals of KTAG candidates to vertex and trigger	138
6.13	Invariant mass distribution in 2016A for base-line selection including TRIM5 suppression and improved PID	138

6.14 The final normalisation $K_{3\pi}$ spectrum for 2016A . . . . .	142
6.15 The final $K_{\pi\mu\mu}$ spectra for 2016A. . . . .	142
7.1 The KTAG algorithm time and intensity dependence in 2016A . . . . .	147
7.2 The RICH primitive efficiency time and $\pi$ momentum dependence in 2016A . . . . .	148
7.3 Comparing online and emulated RICH primitive efficiency in 2016A .	149
7.4 The RICH Efficiency vs $z$ in 2016A . . . . .	150
7.5 The STRAW <sub>e</sub> algorithm efficiency with respect to time and intensity in 2016A . . . . .	151
7.6 The STRAW <sub>e</sub> momentum dependence in 2016A . . . . .	151
7.7 The STRAW <sub>e</sub> Efficiency vs $z$ in 2016A . . . . .	152
7.8 The NewCHOD Illumination for $K_{3\pi}$ and $K_{\pi\mu\mu}$ MC samples. . . . .	153
7.9 The MC firing probability for NewCHOD quadrants . . . . .	154
7.10 The $Q_X$ primitive efficiency with respect to time and intensity in 2016A	155
7.11 An example flow diagram of the NewCHOD firmware flaw . . . . .	156
7.12 Comparing primitive and emulated $L_0$ efficiencies for $Q_X$ in 2016A . .	157
7.13 The two-dimensional emulated $Q_X$ grid. . . . .	159
7.14 Reconstructing the Monte Carlo + Accidentals (MCA) $Q_X$ efficiency for $K_{3\pi}$ and $K_{\pi\mu\mu}$ . . . . .	160
7.15 The !LAV algorithm efficiency with respect to time and intensity in 2016A	161
7.16 Comparing !LAV algorithm and emulated $L_1$ efficiencies in 2016A. . .	163
7.17 The $\delta T$ distribution for LAV12 . . . . .	164
7.18 Reconstructing the MCA !LAV efficiency for $K_{3\pi}$ and $K_{\pi\mu\mu}$ . . . . .	165
7.19 The normalised MUV3 illumination for $K_{3\pi \rightarrow \pi\mu\mu}$ data and $K_{\pi\mu\mu}$ MC samples. . . . .	166
7.20 The $MO_2$ primitive efficiency with respect to time and intensity in 2016A	167
7.21 The $MO_2$ Efficiency vs $z$ in 2016A . . . . .	167
7.22 The overall $K_{\pi\mu\mu}$ efficiency spectra vs $z$ in 2016A . . . . .	169
8.1 Fitting TRIM5 background to extract the overall slope . . . . .	175
8.2 The RooFit PDF spectrum. . . . .	176
8.3 The 2016A $z$ -spectra in log and linear scale. . . . .	177

8.4	The $z_{\mu\mu}$ -resolution for $K_{\pi\mu\mu}$ MC.	178
8.5	The number of observed normalisation $K_{3\pi}$ .	179
8.6	The two-dimensional acceptance grid.	181
8.7	The evolution of the $K_{\pi\mu\mu}$ acceptance.	181
8.8	The reconstructed $d\Gamma/dz$ vs. $z$ spectrum for 2016A	183
8.9	The two-dimensional representation of the form factor extraction	184
9.1	The final $K_{3\pi}$ and $K_{\pi\mu\mu}$ spectra for 2017A.	191
9.2	The 2017A RooFit $M_{\pi\mu\mu}$ and $z_{\mu\mu}$ spectra	192
9.3	The 2017A $z_{\mu\mu}$ spectra	193
9.4	The $L_0$ and $L_1$ trigger efficiencies in 2017.	193
9.5	Comparing 2017 RICH and $Q_X$ primitive and emulated efficiencies.	194
9.6	Reconstructing the MCA $Q_X$ efficiency for $K_{3\pi}$ and $K_{\pi\mu\mu}$ for 2017A	195
9.7	The overall $K_{\pi\mu\mu}$ efficiency spectra vs $z_{\mu\mu}$ in 2017A	196
9.8	The reconstructed $d\Gamma/dz$ vs. $z$ spectrum for 2017A	197
A.1	Data-MC comparison for $K_{3\pi}$ signal for $Z$ -Vertex and $P$ -Vertex	211
A.2	Data-MC comparison for $K_{3\pi}$ signal for vertex-beam CDA and trans- verse momentum $P_T$	211
A.3	Data-MC comparison for $K_{3\pi}$ signal for $\pi^+$ track momentum and open- ing angle $\Theta_R$	212
A.4	Data-MC comparison for $K_{3\pi}$ signal for $\pi^-$ track momentum and open- ing angle $\Theta_R$	212
A.5	Data-MC comparison for $K_{\pi\mu\mu}$ signal for $Z$ -Vertex and $P$ -Vertex	213
A.6	Data-MC comparison for $K_{\pi\mu\mu}$ signal for vertex-beam CDA and trans- verse momentum $P_T$	213
A.7	Data-MC comparison for $K_{\pi\mu\mu}$ signal for $\pi^+$ track momentum and open- ing angle $\Theta_R$	214
A.8	Data-MC comparison for $K_{\pi\mu\mu}$ signal for $\mu^-$ track momentum and open- ing angle $\Theta_R$	214



# List of Tables

2.1	Current experimental measurements of the LECs	39
3.1	The main $K^+$ decay processes and the methods to suppress their signatures	45
3.2	The nominal properties of the K12 secondary beam	49
3.3	A short summary of the NA62 detector systems and their purposes.	50
3.4	The magnet systems important for the analysis in this thesis. Both magnets are dipoles.	51
3.5	Brief description of the detectors which contribute to the $L_0$ and $L_1$ , including the type of criteria implemented.	65
4.1	Summary of measurement deletion algorithm	80
4.2	Comparing the raw and field map integrals for central grid position (0 cm, 0 cm)	88
5.1	Details of the scan configuration for the $(\alpha, \beta, P_{\pi^-})$ computation	102
5.2	Comparison of fit performance for several tuning configurations	105
5.3	Comparing the goodness of fit versus the mass window	106
5.4	Comparing the result of the correction for 2016A and 2017A.	112
6.1	Summary of the current NA62 data samples for 2016 and 2017	119
6.2	Trigger streams in 2016 and 2017 for $K_{3\pi}$ and $K_{\pi\mu\mu}$	120
6.3	Summary of the generated NA62MC samples	123
7.1	Emulated MC efficiencies for the $Q_X$ components	154
7.2	Comparison of the trigger efficiency components for $K_{3\pi}$ and $K_{\pi\mu\mu}$ in 2016A.	169
8.1	Roofit summary for 2016A.	176

8.2 The LECs measurements in 2016A	184
8.3 Systematic contributions to the 2016A model-independent branching fraction measurement	187
8.4 Systematic contributions to the 2016A positive solution	188
8.5 Systematic contributions to the 2016A negative solution	188
9.1 Comparing 2016 and 2017 $L_0$ and $L_1$ efficiencies	194
9.2 The LECs in 2017A	197

# Acronyms

$\chi$ PT Chiral Perturbation Theory. [37](#)

$\mathcal{CP}$  Charge-Parity. [24](#), [25](#), [27](#), [30](#), [40](#)

AGS Alternating Gradient Synchrotron. [40](#)

BNL Brookhaven National Laboratory. [29](#), [31](#), [32](#), [39](#), [40](#)

C.L Confidence Level. [28](#), [31](#), [33](#), [168](#)

CDA Closest-Distance of Approach. [124](#), [129](#), [140](#)

CEDAR Cherenkov Differential counter. [48](#), [54](#)

CHOD NA48 Charged Hodoscope. [56-58](#), [65](#), [100](#), [130](#)

CKM Cabibbo-Kobayashi-Maskawa. [24](#), [25](#), [27](#), [28](#)

CREAM Calorimeter Readout Module. [59](#), [65](#)

FCNC Flavour-Changing Neutral Current. [24](#), [29](#), [33-36](#)

GIM Glashow, Iliopoulos and Maiani. [24](#), [29](#), [30](#), [35](#), [36](#)

GTK GigaTracker. [51](#), [72](#), [125](#), [158](#), [164](#)

J-PARC Japan Proton Accelerator Research Complex. [31](#)

KOTO  $K^0$  at Tokai. [31](#)

KTAG Kaon Tagger. [53](#), [54](#), [65](#), [67](#), [120](#), [137](#), [138](#), [140](#)

**L0TP** Level 0 Trigger Processor. [64](#), [65](#)

**LAV** Large Angle Veto. [58](#), [59](#), [65](#), [160](#)–[165](#), [168](#), [169](#)

**LECs** Low-Energy Constants. [37](#)–[39](#), [43](#), [117](#), [121](#), [144](#), [171](#)–[173](#), [182](#)–[184](#), [187](#), [190](#), [197](#), [199](#), [200](#)

**LFU** Lepton Flavour Universality. [23](#), [35](#)

**LFUV** Lepton Flavour Universality Violation. [33](#), [34](#), [38](#)

**LFV** Lepton Flavour Violating. [40](#)

**LKr** Liquid Krypton Calorimeter. [49](#), [50](#), [58](#)–[61](#), [64](#), [65](#), [129](#)–[131](#), [139](#), [140](#)

**LNV** Lepton Number Violating. [130](#)

**MC** Monte Carlo. [7](#), [69](#), [70](#), [85](#), [86](#), [96](#), [100](#), [101](#), [107](#), [112](#), [114](#), [118](#), [119](#), [121](#), [122](#), [125](#)–[128](#), [131](#), [133](#)–[136](#), [142](#), [146](#), [149](#), [162](#)

**MC** Monte Carlo. [113](#)

**MCA** Monte Carlo + Accidentals. [13](#), [146](#), [158](#)–[160](#), [162](#)–[165](#), [185](#), [195](#)

**MFV** Minimal Flavour Violation. [38](#)

**MUV** Muon Veto. [6](#), [8](#), [48](#)–[50](#), [61](#)–[63](#), [65](#), [66](#), [121](#), [123](#), [124](#), [128](#)–[131](#), [136](#), [137](#), [139](#)–[141](#), [165](#), [166](#), [181](#), [182](#), [186](#)

**NA62FW** NA62 Framework. [67](#), [107](#)

**NA62MC** NA62 Monte Carlo. [67](#), [69](#), [71](#), [72](#), [74](#), [75](#), [85](#), [95](#), [112](#), [123](#)

**NewCHOD** New Charged Hodoscope. [8](#), [57](#), [58](#), [62](#), [65](#), [66](#), [100](#), [123](#)–[125](#), [128](#)–[130](#), [140](#), [153](#)–[158](#), [166](#), [181](#), [182](#)

**NP** New Physics. [23](#), [30](#), [31](#), [34](#)–[36](#), [38](#), [39](#), [42](#)–[44](#)

**PDF** Probability Density Function. [174](#)–[176](#), [182](#), [183](#), [192](#), [196](#)

**PMT** Photomultiplier Tubes. [54](#), [55](#), [57](#), [58](#), [60](#), [61](#), [63](#), [65](#), [66](#), [148](#)

**RICH** Ring Imaging Cherenkov. [5](#), [50](#), [55](#), [56](#), [65](#), [66](#), [120](#), [148](#)

**SAV** Small Angle Veto. [60](#)

**SES** Single Event Sensitivity. [33](#)

**SLAC** Stanford Linear Accelerator Centre. [29](#)

**SM** Standard Model. [23-25](#), [27-29](#), [31](#), [33-36](#), [38](#), [44](#)

**SPS** Super Proton-Synchrotron. [47](#)

**SR** Signal Region. [32](#), [42](#), [46](#), [95](#), [100](#), [101](#), [118](#), [128](#), [132](#), [133](#), [166](#), [174](#), [176](#)

**TDAQ** Trigger and Data Acquisition. [63](#), [64](#), [119](#), [120](#), [125](#), [144](#), [145](#), [155](#), [189](#), [193](#),  
[196](#)

**TDC** Time to Digital Converter. [54](#), [65](#)

**UT** Unitarity Triangle. [26-28](#), [33](#)

# 1

## Introduction

For the last half-century, the Standard Model (SM) of particle physics has developed into one of the most accomplished theories in history. The most recent verification is the discovery of the Higgs-like resonance at the LHC by both the ATLAS and CMS experiments. The resonance, with mass  $M_H = (125.18 \pm 0.16) \text{ GeV}/c^2$ , has solved the puzzle of the last unconfirmed particle in the SM [1][2]. The unprecedented success of the theory demands extreme scrutiny. If the SM is the principal description of the fundamental interactions in the universe, then it is the responsibility of humanity to truly understand its implications. As an approximate theory, there are a large number of open questions that are yet to be understood: why are there so many arbitrary parameters, and why are there three generations of fundamental particles? It is through the tireless collaboration of thousands of physicists around the world that a plethora of wide-ranging experiments are attempting to answer these questions. While LHC physics continues to probe higher and higher energy scales, intensity-frontier experiments are concentrating on measuring large statistics of rare decays in the hope of observing deviations from SM predictions. Both approaches are complementary and are necessary to truly test the validity of the SM.

The topic of this thesis is an analysis performed by the intensity-frontier experiment NA62. The design, construction and operation of the experiment represents the efforts of over two hundred physicists, without whom this thesis would not be possible. Situated at the CERN Super Proton Synchrotron (SPS), NA62 is a kaon fac-

tory. The kaon has always played a pivotal role in experimental flavour physics. For example, the large discrepancy between the kaon production and decay time-scale led to the postulation of the strangeness quantum number and the expansion to a second generation of the quark family. With the strange and charm quark came the explanation as to why  $K_L \rightarrow \mu^+ \mu^-$  is highly suppressed with respect to  $K^+ \rightarrow \mu^+ \nu_\mu$ . The neutral kaon sector also provided the first evidence of both direct and indirect Charge-Parity ( $\mathcal{CP}$ ) violation [3][4], which relates to one of the big questions in physics: why is there such a large asymmetry between matter and antimatter in the universe?

NA62 is optimised to measure the ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay, which is yet another probe of the SM. With a branching fraction of  $\approx 10^{-10}$ , the decay is considered a golden-channel because of a precise SM prediction; any deviation from expectation by an experimental measurement could hint to New Physics (NP). After only one year of data acquisition, NA62 has reached the SM sensitivity for the process, and the paper is soon to be submitted to Physics Letters B. Furthermore, the demanding nature of the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  measurement means that NA62 is ideal for the measurement of other rare processes. Decays of the form  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  offer a kaon-physics approach to measure observables which have analogous counterparts in other flavour sectors; it is some of these observables which have displayed tantalising hints of NP being a preferable solution over the SM prediction. In this thesis, the least well-measured  $\pi^+ \ell^+ \ell^-$  channel  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  is investigated. Whilst the branching fraction is large ( $\approx 10^{-8}$ ) in comparison to the flagship measurement, it can be used to test Lepton Flavour Universality (LFU) in a way which is complementary to current  $B$ -physics analyses.

The first chapter outlines the theoretical motivation for these rare kaon measurements. Importantly, the experimental technique to observe and measure both  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  relies on high-precision tracking. In Chapter 3 the beam and sub-detector systems of NA62 are described, while Chapters 4 and 5 provides the development of the MNP33 fringe-field map and a procedure to calibrate the spectrometer momentum determination, both vital for precision analyses involving

charged tracks. Using 2016 NA62 data, a selection for the rare decay  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  is developed in Chapter 6, whilst the understanding of the trigger efficiency, a highly important aspect of the data analysis, is provided in Chapter 7. Both the branching fraction and form factor for the decay are extracted in Chapter 8, including a study of the relative systematic errors. Notably, the work in this chapter suggests that the 2016 data set suffers from a series of complicated systematic issues; in Chapter 9, the 2017 data set is studied to learn about the future prospects of the measurement. A paper on the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  analysis presented in this thesis is in preparation for submission in the near future.



# 2

## Theoretical motivation

In the last century, the kaon has played a pivotal role in developing the understanding of electroweak interactions in the [Standard Model \(SM\)](#). The NA62 experiment is the next in a long line of fixed target kaon experiments at CERN. The primary aim is to perform a precise measurement of the ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching fraction in order to search for [New Physics \(NP\)](#). Whilst NA62 is optimised for this measurement, the experiment is well equipped to study other rare  $K^+$  processes because of the demanding nature of the principal decay. An example is the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  channel which is the subject of the analysis in Chapter [8](#), and can be used to test [Lepton Flavour Universality \(LFU\)](#). In this chapter, the theoretical motivation to study both of these decays is explored, including the current experimental status.

### 2.1 The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay

#### 2.1.1 CKM framework

In the [SM](#), kaons are the lightest mesons to exhibit ‘strangeness’, a historical term first used in the early 1950’s to describe a new set of particles with peculiar features. One unusual property is the relative discrepancy in the timescales of production and decay; particles are produced in  $\approx 10^{-23}$  s but survive for a substantially longer time of up to  $\approx 10^{-10}$  s [\[5\]](#)[\[6\]](#). Additionally, strange particles are only ever produced in even numbers, suggesting the conservation of some unknown property. In 1953, and in an effort to adequately describe these processes, Gell-Mann and Nishijima

introduced the strangeness quantum number [7][8]. Strangeness was postulated to be conserved by all interactions *apart* from the nuclear weak force, in an effort to explain the long lifetime of these strange particles. Strangeness  $S = -(n_s - n_{\bar{s}})$  is the difference in the number of strange  $n_s$  and anti-strange  $n_{\bar{s}}$  particles in a process.

In the modern day [SM] strangeness enters because of the strange quark, a constituent component of the kaon and all strange hadrons. The strange quark is a member of the second generation of the three quark families described by the Cabibbo-Kobayashi-Maskawa (CKM) unitary matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.1)$$

The CKM matrix  $V_{CKM}$  relates the flavour eigenstates  $d, s, b$  to the weak interaction eigenstates  $d', s', b'$  which couple with the  $u, c, t$  quarks. The magnitude of each element  $V_{iq}$  ( $i = u, c, t, q = d, s, b$ ) contains the strength of the flavour-changing charged current interaction between generations, where  $i \rightarrow q$  describes the quark transition; a Flavour-Changing Neutral Current (FCNC) is forbidden at tree-level by the Glashow, Iliopoulos and Maiani (GIM) mechanism (see Section 2.1.3). For a unitary  $n \times n$  matrix (such as  $V_{CKM}$ ), there are  $N = n^2$  free parameters. With the addition of the invariance of the CKM matrix under reparameterisation, the number of parameters is reduced to just  $N = (n - 1)^2$ . The presence of only three quark families in the SM leads to four parameters, of which three are real and free and one is a complex phase; the presence of the complex phase is responsible for Charge-Parity (CP) violation. To display the parameters more clearly, the CKM matrix can be decomposed using the Wolfenstein parameterisation [9]:

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda(\bar{\rho} - i\bar{\eta}) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\bar{\rho} + i\bar{\eta})] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (\bar{\rho} + i\bar{\eta})] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\bar{\rho} + i\bar{\eta})] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}, \quad (2.2)$$

which is a power-series expansion of parameter  $\lambda$  up to the  $\mathcal{O}(\lambda^6)$ , where  $A$ ,  $\lambda$ ,  $\bar{\rho}$  are the real parameters and the imaginary quantity  $i\bar{\eta}$  represents the complex phase. The parameters are related by [10],

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad (2.3)$$

$$A^2\lambda^2 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad (2.4)$$

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \quad (2.5)$$

The location of the phase  $i\bar{\eta}$  is not significant physically, as any number of different parameterisations will lead to the same result provided that there is a single complex phase that cannot be transformed away. This is quantified by the Jarlskog invariant, which measures  $\mathcal{CP}$  violation,

$$J_{\mathcal{CP}} \equiv |\text{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*)| = A^2\lambda^6\bar{\eta}, \quad (i \neq k, j \neq l). \quad (2.6)$$

without any sum over  $i, j, k, l$  [11]. As long as  $\bar{\eta}$  is non-zero,  $\mathcal{CP}$  violation will be present in the SM and directly proportional to  $J_{\mathcal{CP}}$ .

## 2.1.2 Unitarity

Possibly the most natural representation of the four CKM parameters can be extracted by exploiting the unitarity of the CKM matrix,

$$\left(V_{CKM}^\dagger V_{CKM}\right)_{i,j} = \sum_q V_{qi}^* V_{qj} = \delta_{ij}. \quad (2.7)$$

In general, the diagonal elements ( $i = j$ ) are real, whereas the off-diagonal elements ( $i \neq j$ ) are complex, making them suitable for measuring  $\mathcal{CP}$  violation. Of the  $2n^2 = 18$  relations, the off-diagonal terms can be represented in the complex plane as triangles. The most common representations are the triangles where each 'side' is the same order of magnitude  $\mathcal{O}(\lambda^3)$ ,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (2.8)$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0, \quad (2.9)$$

which is displayed in the complex plane as seen in Figure 2.1. The first triangle

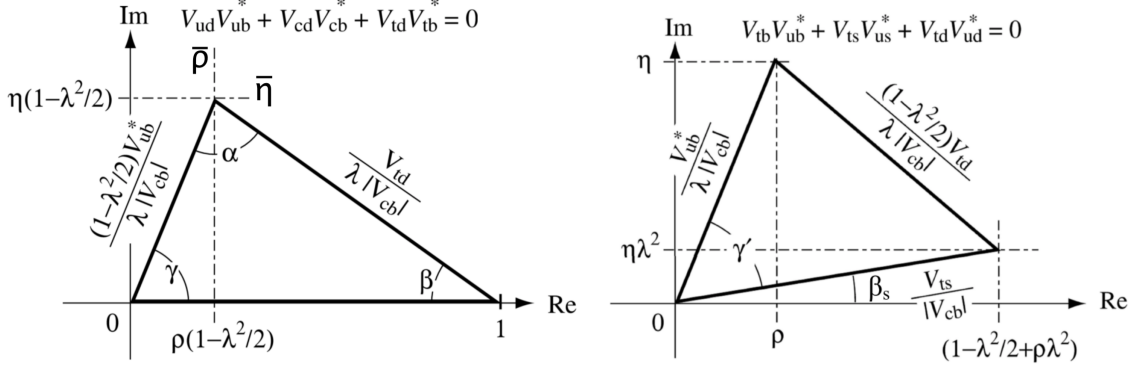


Figure 2.1: The two same-order complex triangles obtained using Equations 2.8 and 2.9, with the sides re-scaled by  $V_{cd}V_{cb}^*$  in the first instance and  $V_{us}V_{cb}^*$  in the second. The area of both triangles is equivalent to  $J_{CP}/2$  [10].

is often referred to as the **Unitarity Triangle (UT)**. The coordinates of the vertex in the complex plane are  $(\bar{\rho}, \bar{\eta})$ , while the two remaining vertices occupy only the real dimension with coordinates  $(0, 0)$  and  $(1, 0)$ . The magnitude of the sides  $S_u$  and  $S_t$ , corresponding to  $(0, 0) \rightarrow (\bar{\rho}, \bar{\eta})$  and  $(\bar{\rho}, \bar{\eta}) \rightarrow (0, 1)$ , are given in Ref. [10] as,

$$S_u \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| \equiv \frac{(1 - \frac{\lambda^2}{2})V_{ub}^*}{\lambda|V_{cb}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}, \quad (2.10)$$

$$S_t \equiv \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| \equiv \frac{V_{td}}{\lambda|V_{cb}|} = \sqrt{(1 - \bar{\rho}^2) + \bar{\eta}^2}. \quad (2.11)$$

and the angles  $\alpha$ ,  $\beta$  and  $\gamma$ :

$$\alpha \equiv \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) = \arg \left( -\frac{1 - \bar{\rho} - i\bar{\eta}}{\bar{\rho} + i\bar{\eta}} \right), \quad (2.12)$$

$$\beta \equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) = \arg \left( \frac{1}{1 - \bar{\rho} - i\bar{\eta}} \right), \quad (2.13)$$

$$\gamma \equiv \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) = \arg(\bar{\rho} + i\bar{\eta}). \quad (2.14)$$

The second triangle is tilted from the horizontal axis by the angle  $\beta_s$  defined as,

$$\beta_s \equiv \left( -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right) = \lambda^2\bar{\eta} + \mathcal{O}(\lambda^4), \quad (2.15)$$

which is extracted from  $B_s^0$  measurements such as  $B_s^0 \rightarrow J/\psi K^+ K^-$  [10][12]. A considerable effort is placed in measuring the sides and angles of the **UT** in modern day

experimental flavour physics. Examples of direct constraints are the measurements of the **CKM** matrix elements **[13]**:

$$|V_{CKM}| = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00500 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.04110 \pm 0.00130 \\ 0.00875^{+0.00032}_{-0.00033} & 0.04030 \pm 0.00130 & 0.99915 \pm 0.00005 \end{pmatrix}. \quad (2.16)$$

The most precise value of  $|V_{ud}|$  is extracted from nuclear physics experiments via super-allowed  $\beta$  decays **[14]**. For the elements  $|V_{ui}|$  and  $|V_{ci}|$  ( $i = d, s, b$ ), most measurements are constrained via leptonic and semileptonic  $K$ ,  $D$  or  $B$  decays. For example,  $|V_{us}|$  can be extracted using the semi-leptonic  $K^+ \rightarrow \pi^0 \ell^+ \nu_\ell (K_{\ell 3})$  decays, processes which are common at NA62. Parameters involving the top-quark ( $|V_{ti}|$ ) are typically more difficult to constrain due to the energy scale of  $m_t \approx 175 \text{ GeV}/c^2$ . The parameter  $|V_{tb}|$  can be extracted based on single-top production cross-sections, but the determination is much less accurate compared to limits imposed by the constraint of unitarity. The matrix elements  $|V_{td}|$  and  $|V_{ts}|$  are most accurately obtained from rare loop-mediated  $B$  or  $K$  decays. Combining the measurements from the matrix elements with the determination of the **CKM** parameters leads to the unitarity triangle in  $(\bar{\rho}, \bar{\eta})$  space, displayed in Figure **[2.2]**. The globally preferred fit parameters and corresponding **UT** angles are determined to be **[13]**:

$$A = 0.8110 \pm 0.026, \quad (2.17)$$

$$\lambda = 0.22506 \pm 0.00050, \quad (2.18)$$

$$\bar{\rho} = 0.124^{+0.019}_{-0.018}, \quad (2.19)$$

$$\bar{\eta} = 0.356 \pm 0.011, \quad (2.20)$$

$$\alpha = (87.6^{+3.5}_{-3.3})^\circ, \quad (2.21)$$

$$\beta = (21.9^{+0.7}_{-0.7})^\circ, \quad (2.22)$$

$$\gamma = (73.2^{+6.3}_{-7.0})^\circ. \quad (2.23)$$

The Jarlskog invariant  $J_{\mathcal{CP}}$  is therefore  $(3.04^{+0.31}_{-0.20}) \times 10^{-5}$ , corresponding to twice the area of the **UT** and equivalent to the amount of **CP** violation in the **SM** **[13]**.

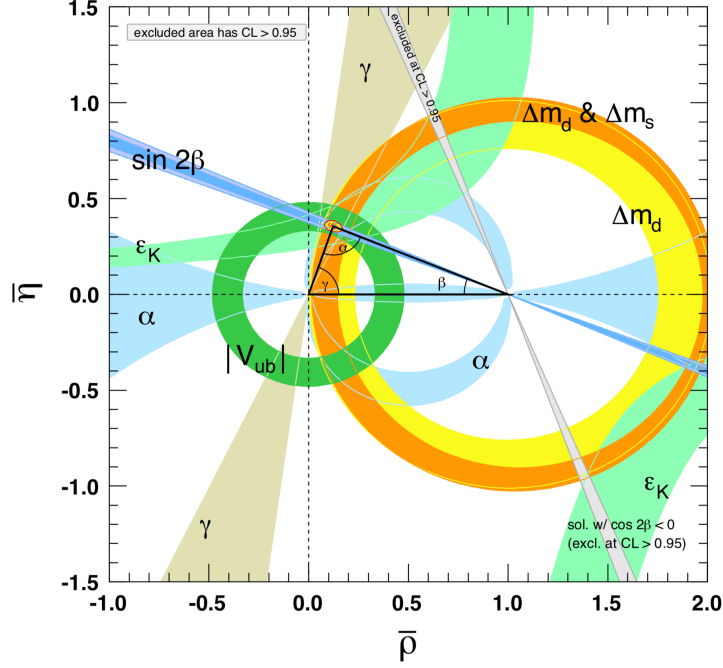


Figure 2.2: Constrains on the unitarity triangle in the  $(\bar{\rho}, \bar{\eta})$  plane, where shaded areas have a 95% **Confidence Level (C.L.)** [13].

Importantly, the current measurements are in agreement with unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9996 \pm 0.0005, \quad (2.24)$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.0400 \pm 0.0320, \quad (2.25)$$

$$\alpha + \beta + \gamma = (183_{-8}^{+7})^\circ, \quad (2.26)$$

where the Equations 2.24 and 2.25 correspond to the first and second rows of the **CKM** matrix and Equation 2.26 refers to the **UT** angles [13]. As the sensitivity and precision of experimental measurement improves, if there is a deviation from unitarity or the parameters disagree significantly for different processes, the validity of the **SM** can be questioned and modifications due to NP could be considered.

### 2.1.3 Theoretical prediction of $K \rightarrow \pi \nu \bar{\nu}$

The kaon sector has the potential to play an even more vital role in experimental flavour physics. The decays  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are ultra-rare and considered as golden channels because of their clean theoretical prediction. By simply measuring the branching fraction of the processes, the validity of the **SM** can

be tested. The decays are ultra-rare because of the **GIM** mechanism. Unlike the charged weak current, changing quark flavour via the neutral current is forbidden at the tree-level in the **SM**. This is a consequence of the existence of the charm quark, which was postulated by Glashow, Iliopoulos and Maiani in 1970, before it was discovered later via the  $J/\psi$  meson at both the **Stanford Linear Accelerator Centre (SLAC)** and **Brookhaven National Laboratory (BNL)** [15] [16]. In their work to unify both the electromagnetic and weak interaction, it was discovered that the existence of a fourth new "charm" quark was needed, in addition to  $u, d, s$ , to explain the rarity of the **FCNC** process  $K_L \rightarrow \mu^+ \mu^-$  with respect to the common leptonic  $K^+ \rightarrow \mu^+ \nu_\mu$  decay. The **GIM** mechanism leads to two distinct features: **FCNC** processes are forbidden at tree-level by the structure of the third component of the weak isospin; but can proceed at higher order loops which are subsequently suppressed based on ratio of the quark mass to the  $W^\pm$  boson [17]. Figure 2.3 contains the dominant short-distance Feynman diagrams for the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay. Here, three diagrams

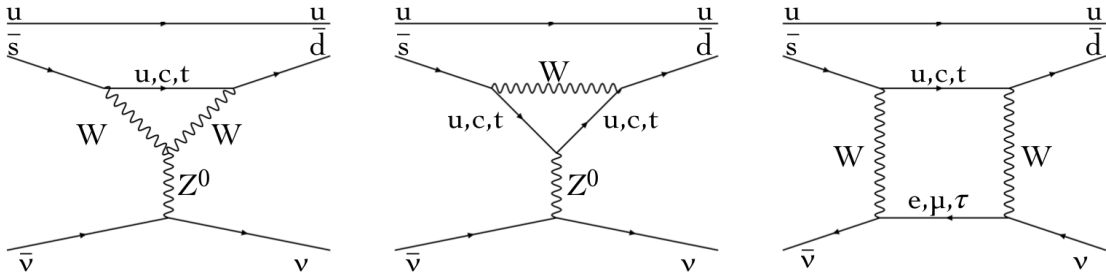


Figure 2.3: The dominant Feynman diagrams for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay;  $Z$ -penguin and box loop diagrams. Each diagram is an example of the short-distance contribution, where the  $|V_{td}|$  dependence enters in the loop.

contribute to the process, with two  $Z$ -penguins and a box loop. The amplitude of the decay is of the form [18],

$$A(s \rightarrow d\nu\bar{\nu}) \approx \sum_{q=u,c,t} \lambda_q X_{SM}(x_q), \quad (2.27)$$

$$\approx \frac{m_t^2}{M_W^2} \lambda_t + \frac{m_c^2}{M_W^2} \ln\left(\frac{M_W}{m_c}\right) \lambda_c + \frac{\Lambda_{\text{QCD}}^2}{M_W^2} \lambda_u, \quad (2.28)$$

where  $\lambda_q = V_{qd}V_{qs}^*$  ( $q = u, c, t$ ),  $x_q = m_q^2/M_W^2$  (where  $m_q$  is the quark mass and  $M_W$  is the  $W^\pm$  mass),  $X(x_q)$  is a loop function for the quark-contributions and  $\Lambda_{\text{QCD}} \approx 1$  GeV is a constant corresponding to the energy-scale where non-perturbative QCD

is dominant. This is a power-like (hard) GIM mechanism, dominated by the first term involving the top-quark mass. As a result, the decay is suitable to measure the matrix element  $|V_{td}|$  (assuming no NP contributions) and is dominated by only short-distance QCD effects, reducing the systematic uncertainties in the theoretical prediction. The SM branching fraction for the two  $K \rightarrow \pi\nu\bar{\nu}$  decays is,

$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) = \kappa_L \left( \frac{\text{Im } \lambda_t}{\lambda^5} X \right)^2 (1 - \delta_\epsilon), \quad (2.29)$$

$$\begin{aligned} \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = \kappa_+ (1 + \Delta_{\text{EM}}) \left\{ \left( \frac{\text{Im } \lambda_t}{\lambda^5} X \right)^2 \right. \\ \left. + \left( \frac{\text{Re } \lambda_t}{\lambda^5} X + \frac{\text{Re } \lambda_c}{\lambda} (P_c + \delta P_{c,u}) \right)^2 \right\}, \quad (2.30) \end{aligned}$$

where  $\delta_\epsilon = \sqrt{2}|\epsilon|[1 + P_c/(A^2\lambda)]/\eta$ ,  $P_{c,u}(X)$  is the loop-function for the  $c, u$  quarks ( $\delta P_{c,u}$  is the difference between the two) and  $\Delta_{\text{EM}}$  includes the electromagnetic corrections [19]. The largest theoretical uncertainty originates in the  $\kappa_{+,L}$  terms, which correspond to the long-distance effects. Ultimately, both terms depend on the hadronic matrix elements which are constrained by experiment,

$$\kappa_L = (2.231 \pm 0.013) \times 10^{-10} (\lambda/0.225)^8, \quad (2.31)$$

$$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11} (\lambda/0.225)^8, \quad (2.32)$$

and are extracted via semi-leptonic  $K_{\ell 3}$  decays [20]. The  $K^+$  process has a CP conserving real and a CP violating imaginary component, whilst the  $K_L$  decay is purely imaginary and therefore is maximally CP violating. The numerical prediction is:

$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) = (3.00 \pm 0.30) \times 10^{-11}, \quad (2.33)$$

$$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (8.40 \pm 1.00) \times 10^{-11}, \quad (2.34)$$

corresponding to a precision to within 10% [21]. Of this, only  $\approx 4\%$  is due to the true theoretical uncertainty; the largest component is the parametric error entering from the  $\kappa$  terms (Equations 2.31 and 2.32) due to the CKM elements. The channels are seen as golden because of this small theoretical uncertainty; if an experimental measurement is similarly as precise and there is a notable deviation between the values,



it can be expected that [NP](#) is contributing within the loop at a similar magnitude to the [SM](#).

### 2.1.4 Experimental status

The NA62 and [K<sup>0</sup> at Tokai \(KOTO\)](#) experiments aim to measure the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  branching fractions to a precision better than 10% in order to test the [SM](#). The current experimental limits on the measurements are,

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (17.3_{-10.5}^{+11.5}) \times 10^{-11}, \quad \text{[22]} \quad (2.35)$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 3.0 \times 10^{-9} \quad (90\% \text{ C.L.}) \quad \text{[23]}. \quad (2.36)$$

The  $K_L$  channel has not yet been observed. The best upper limit has been set by the first-stage of the [KOTO](#) experiment at the [Japan Proton Accelerator Research Complex \(J-PARC\) Proton Synchrotron](#) [22]. At [KOTO](#), protons with central momentum 30 GeV/c are directed onto a 66 mm long gold target to produce a secondary neutral  $K_L$  beam which is offset by 16° with respect to the primary beam axis. The detector is a hermetic particle veto, with the Caesium-Iodide electromagnetic calorimeter (CsI) used to detect the secondary  $\pi^0 \rightarrow \gamma\gamma$  decay in the absence of any other signals [24]. Equation 2.36 was achieved using data from 2015, and is expected to improve using the full 2015-2018 data set.

The  $K^+$  channel was measured with the E787/E949 experiments at the [BNL](#) [23]. Both experiments were performed using stopped kaon decays, where E949 was the final upgraded stage of the experiment. A secondary  $K^+$  beam with momentum centred at 710 MeV/c was slowed using an active scintillator fibre target, and the decay products measured using the apparatus displayed in Figure 2.4. The detector was immersed in a 1T solenoidal magnetic field, with the hermetic argon-ethane drift chamber occupying the radial regions around the target. A range-stack followed the drift-chamber, and was designed to measure the position and energy of the secondary charged particles. The stack contained multiple layers of plastic scintillators coupled to multi-wire proportional chambers for accurate position determination. The outer-layer of the detector is the photon-veto barrel, useful for

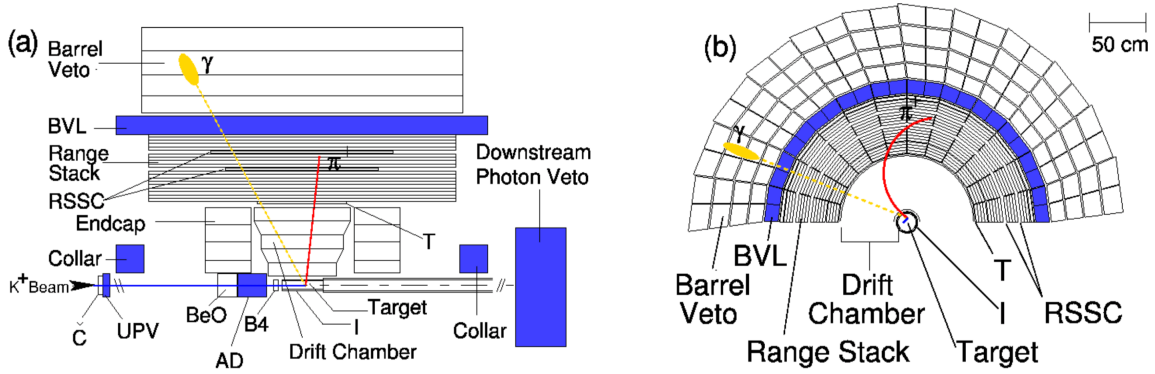


Figure 2.4: The E949 detector, the final form of E787 experiment at [BNL](#) [\[25\]](#).

suppressing the background decays containing  $\pi^0$  such as  $K^+ \rightarrow \pi^+\pi^0$ . The total number of  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  candidates recorded by both collaborations was 5+2 (E787 + E949), as displayed in [Figure 2.5](#). Two types of [Signal Region \(SR\)](#) were necessary

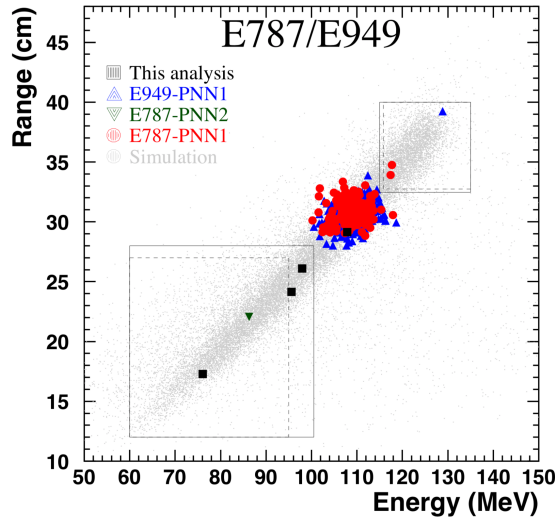


Figure 2.5: The seven observed  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  candidates at the two [BNL](#) experiments. Two Signal Regions (SRs) were used which divide the phase-space, as the irreducible  $K^+ \rightarrow \pi^+\pi^0$  background enters at the centre of the region. The range refers to the radial distance from the beam-pipe and measured with the range-stack [\[23\]](#).

due to the irreducible  $K^+ \rightarrow \pi^+\pi^0$  background at the centre of the  $K \rightarrow \pi^+\nu\bar{\nu}$  phase-space.

The NA62 experiment uses an alternative decay in-flight technique, which greatly improves photon-detection efficiency in order to suppress backgrounds such as  $K^+ \rightarrow$

$\pi^+\pi^0$ . After a year of data-taking and the observation of a single  $K \rightarrow \pi^+\nu\bar{\nu}$  candidate, NA62 has already reached the **SM Single Event Sensitivity (SES)** [26]:

$$SES = (3.15 \pm 0.01_{\text{stat}} \pm 0.2_{\text{syst}}) \times 10^{-10}, \quad (2.37)$$

which corresponds to a limit on the branching fraction of,

$$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) < 11 \times 10^{-10} \text{ (90\% C.L.)}. \quad (2.38)$$

NA62 aims to improve the precision in the  $K^+$  channel by collecting of the  $\mathcal{O}(100)$  candidates over the lifetime of the experiment.

## 2.2 The $K^+ \rightarrow \pi^+\mu^+\mu^-$ decay

### 2.2.1 Anomalies in flavour physics

Despite the success of the **SM** especially in terms of the **UT** and the discovery of the Higgs-like resonance, there have been a notable number of high-profile anomalies appearing indirectly after Run 1 at the LHC. The LHCb experiment in particular has observed three notable examples relating to the  $B$ -physics sector [27]:

- **$P'_5$  deviation:** The observable  $P'_5$  is one of five angular variables used to describe four-body **FCNC**  $B$ -decays [28]. A tension has been discovered with respect to the **SM** for the  $B \rightarrow K^*\mu^+\mu^-$  decay for this variable of between  $2\text{--}3\sigma$  [29] [30].
- $B_s \rightarrow \phi\mu^+\mu^-$ : By combining the results of the CDF experiment with LHCb, the differential branching fraction of the  $B_s \rightarrow \phi\mu^+\mu^-$  decay deviates by  $3.5\sigma$  from the lattice QCD prediction in the region  $14.0 < q^2 < 19.0 \text{ GeV}^2/c^4$  [31] [32] [33].
- **Lepton flavour universality:** LHCb has observed a tension suggesting **Lepton Flavour Universality Violation (LFUV)** for  $b \rightarrow s$  transitions. The most prominent example is for the  $B \rightarrow K\ell^+\ell^-$  decays ( $\ell = e, \mu$ ) in the phase-space region between  $1 < m_{\ell\ell}^2 < 6 \text{ GeV}^2/c^4$ , where  $m_{\ell\ell}^2$  is the dilepton invariant mass squared.

The ratio,

$$R(K\ell\ell) = \frac{\mathcal{B}(B \rightarrow K\mu^+\mu^-)}{\mathcal{B}(B \rightarrow Ke^+e^-)} = 0.745_{-0.074}^{+0.090} \pm 0.036, \quad (2.39)$$

deviates by  $2.6\sigma$  from the theoretically clean **SM** prediction of  $R_{SM}(K) = 1.0003 \pm 0.0001$  [34] [35]. The first observation of a **LFUV** discrepancy came from BaBar, which has subsequently been remeasured by LHCb [36], for the ratio of the tree-level processes,

$$R(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\nu_\ell)} = 0.322 \pm 0.018 \pm 0.012. \quad (2.40)$$

However, the prediction is  $R_{SM}(D^*) = 0.252 \pm 0.003$ , a discrepancy of over  $3\sigma$  [37] [38].

By combining the discrepancies for the loop mediated  $b \rightarrow s$  transitions, the global fits find that the **(NP + SM)** solution is preferred to the **SM** by close to  $5\sigma$  [39] [40]. The effective Hamiltonian for **FCNC**  $b \rightarrow s$  ( $|\Delta B| = 1$ ) transitions is,

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_i C_i^B(\mu)Q_i^B(\mu) + (h.c), \quad (2.41)$$

where  $G_F$  is the Fermi coupling constant, and  $\{Q_i^B\}$  is a set of local composite operators with Wilson coefficients  $C_i^B$  [27]. As all **FCNC** decays are forbidden at tree-level, only the loop semi-leptonic operators with non-zero Wilson terms contribute in the form of penguin or box diagrams. The two of specific interest are,

$$Q_9^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1 - \gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell], \quad (2.42)$$

$$Q_{10}^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1 - \gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell]. \quad (2.43)$$

It is through these processes that **NP** is expected to enter, which can be interpreted as a contribution on top of the **SM** component to  $C_{9,10}^B$  [27].

## 2.2.2 Lepton flavour universality

Although some of the most interesting discrepancies currently originate in  $B$ -physics,  $K$ -physics can play a highly complementary role to **NP** searches. The most obvious examples are the  $K \rightarrow \pi\nu\bar{\nu}$  decays, explained previously in Section 2.1. Aside from the measurement of the golden channels, one of the most directly accessible phenomena to study is **LFU**. In the **SM**, the coupling of leptons to the electroweak gauge bosons is independent of flavour; the difference in the branching fractions for weak processes involving different generations of leptons depends only on the mass difference between them. However, as with lepton and baryon number, lepton flavour universality is not a fundamental symmetry but is inferred through precise experimentation. The evidence provided by the  $R(K\ell\ell)$  and  $R(D^*)$  measurements hint that the **SM** might not be the complete picture.

The rare **FCNC** processes  $K^+ \rightarrow \pi^+\ell^+\ell^-$  ( $\ell = e, \mu$ ) are directly analogous to the decays used to measure  $R(K\ell\ell)$ , as they both yield related observables. In a similar fashion to  $b \rightarrow s$  transitions,  $s \rightarrow d$  ( $|\Delta S| = 1$ ) processes are described using the effective Lagrangian,

$$\mathcal{L}_{eff}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu) + (h.c.), \quad (2.44)$$

where again,  $\{Q_i\}$  are a set of local composite operators with corresponding Wilson coefficients  $C_i$  [27]. Figure 2.6 contains the contributing Feynman diagrams to the process; the  $K^+ \rightarrow \pi^+\ell^+\ell^-$  amplitude depends on the logarithm-like (soft) **GIM** mechanism, as the  $\gamma$ -penguin also contributes. Therefore, these decays are only rare, unlike the ultra-rare  $K \rightarrow \pi\nu\bar{\nu}$  [41], and the related theoretical uncertainties are larger. Like with  $B$  decays, only the loop diagrams contribute because they generate the only non-zero Wilson coefficients for the semileptonic operators  $Q_{7V}$  and  $Q_{7A}$ ,

$$Q_{7V} \equiv Q_{11} = [\bar{s}\gamma^\mu(1 - \gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell], \quad (2.45)$$

$$Q_{7A} \equiv Q_{12} = [\bar{s}\gamma^\mu(1 - \gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell]. \quad (2.46)$$

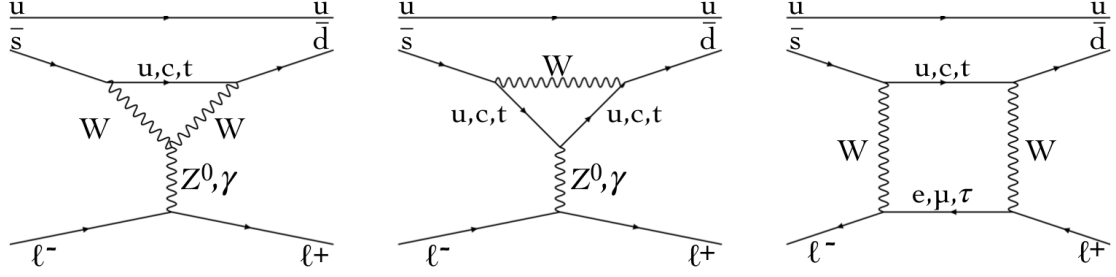


Figure 2.6:  $Z, \gamma$ -penguin and box contributions to the  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  decay, with an amplitude governed by the logarithm-like (soft) **GIM** mechanism.

Again, **NP** processes can be interpreted as deviations from the **SM** Wilson coefficients  $C_{7V,7A}$  [27].

### 2.2.3 The form factor

The semi-leptonic **FCNC** processes  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  ( $\ell = e, \mu$ ) are described by single-photon exchange at low energies (i.e  $K \rightarrow \pi \gamma^* \rightarrow \pi \ell^+ \ell^-$ ) [42]. The amplitude for the process is determined using an electroweak transition form factor  $W(z)$ , such that the differential decay rate is given by,

$$\frac{d\Gamma_{\pi\ell\ell}}{dz} = \frac{\Omega_C \alpha^2 M_K}{12\pi(4\pi)^4} \lambda^{3/2}(1, z, r_\pi^2) \sqrt{1 - 4\frac{r_\ell^2}{z}} \left(1 + 2\frac{r_\ell^2}{z}\right) |W(z)|^2, \quad (2.47)$$

where  $\alpha$  is the fine structure constant, and,

$$r_{\pi,\ell} = M_{\pi,\ell}/M_K, \quad (2.48)$$

$$z = (q/M_K)^2 \equiv (m_{\ell\ell}/M_K)^2, \quad (2.49)$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \quad (2.50)$$

The term  $\Omega_C$  contains the Coulomb interaction, described by,

$$\Omega_C(\beta_{ij}) = \prod_{i,j=1,2,3;i<j} \frac{2\pi\alpha Q_i Q_j}{\beta_{ij}} \left( e^{\frac{2\pi\alpha Q_i Q_j}{\beta_{ij}}} - 1 \right)^{-1}, \quad (2.51)$$

where  $\beta_{ij}$  is the relative velocity of any two daughters in the decay and  $Q_i = \pm 1$  are the associated charges [43]. The decay is constrained to the phase-space between  $4r_\ell^2 \leq z \leq (1 - r_\pi^2)$ . The form factor  $W(z)$  is an analytical function to describe

the single-photon exchange. The form factor also has a non-zero contribution from intermediate dispersion states. At small dilepton invariant masses, a  $\pi^+\pi^-$  loop enters into the transition and is the dominant dispersion relation, whereas the higher masses can be approximated with a high degree of accuracy by a low-order polynomial with respect to  $z$  [42][44]. The form factor therefore contains two terms,

$$W(z) = G_F M_{K^+}^2 W^{pol}(z) + W^{\pi\pi}(z), \quad (2.52)$$

where  $W^{\pi\pi}(z)$  is the contribution from the two-pion state and  $W^{pol}(z)$  describes the low-order polynomial. The  $W^{\pi\pi}(z)$  term can be calculated via Figure 2.7 by ex-

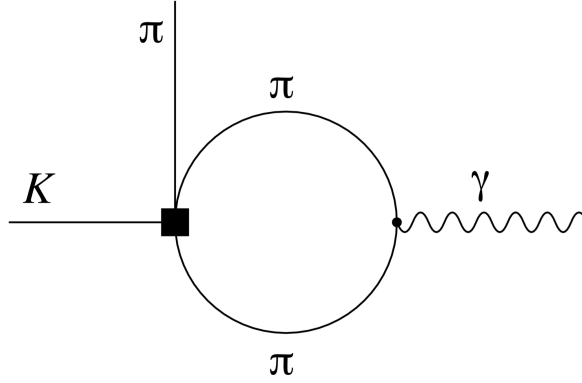


Figure 2.7: The two-pion loop contribution to the [Chiral Perturbation Theory \( \$\chi\$ PT\)](#) form factor, the dominant dispersion relation at low  $m_{\ell\ell}$ . Other dispersion states can occur at high  $m_{\ell\ell}$ , such as the  $K$ -loop [42][44].

panding the  $K \rightarrow 3\pi$  amplitude up to  $\mathcal{O}(p^4)$ . The  $W^{pol}(z)$  term takes the simplest form [45],

$$W^{pol}(z) = (a_+ + b_+ z), \quad (2.53)$$

where  $a_+$  and  $b_+$  are free [Low-Energy Constants \(LECs\)](#). The main assumption is that this polynomial describes all contributions from intermediate states except the two-pion loop. Figure 2.8 contains the theoretical  $d\Gamma/dz$  shape for the  $\mu\mu$  channel, which is the subject of the thesis, with each component of the form factor  $W(z)$  displayed. The [LECs](#) have been taken from the current best measurements (Table 2.1). The  $W^{\pi\pi}(z)$  term is small in comparison to  $W^{pol}(z)$  for the current-best [LECs](#). However, the overall  $d\Gamma/dz$  magnitude is larger than for just  $W(z) = W^{pol}(z)$  due to the

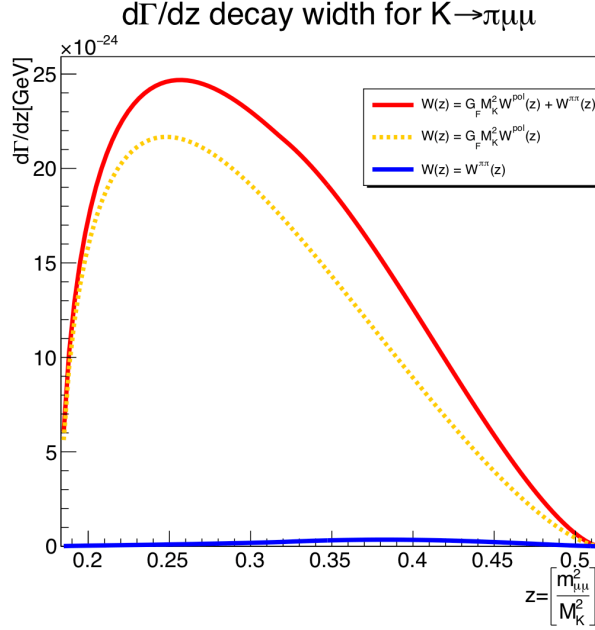


Figure 2.8: Predicted  $d\Gamma/dz \sim |W(z)|^2$  spectrum with respect to  $z$  for several versions of the form factor  $W(z)$ . The overall spectrum is dominated by  $W^{pol}(z)$ .

cross term between the two components. Importantly, for LFU to hold true, the **LECs** must be identical for both  $\mu\mu$  and  $ee$  channels of  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ ; any deviation between the parameters  $a_+^{\mu\mu} - a_+^{ee}$  would be a sign of short distance **NP** [40]. As shown in Ref. [27], the difference in the parameters  $a_+^{\text{NP}}$  can be represented as a dependence on the Wilson coefficient  $C_{7V}$ , by projecting the  $SU(3)_L$  chiral current onto the  $\Delta S = 1$  sector [46], leading to,

$$a_+^{\text{NP}} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* C_{7V}^{\text{NP}}, \quad (2.54)$$

and therefore, **NP** will enter as a difference in the Wilson coefficients of the two channels [40],

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2}V_{ud}V_{us}^*}. \quad (2.55)$$

The crucial aspect of this measurement is that it is an entirely complementary technique to the  $B$ -physics searches, but with different systematics. If **LFUV** is observed, the simplest extension to the **SM** is **Minimal Flavour Violation (MFV)**, which if assumed can directly relate the Wilson coefficients of loop processes in  $K$ -physics to



$B$ -physics, specifically  $C_{7V}$  to  $C_9^B$ ,

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2}V_{td}V_{ts}^*}, \quad (2.56)$$

as the coefficients of the two-sectors are correlated in the presence of **NP**. The quality of the bounds is therefore restricted by only the experimental determination of the **LECs** for both  $ee$  and  $\mu\mu$  channels.

In order to obtain both the **LECs** and the corresponding branching fraction, the differential decay rate  $d\Gamma/dz$  spectrum must be reconstructed using experimental data. In this thesis, NA62 data is used to perform this task. By integrating the spectrum and normalising to the  $K^+$  decay-width, the branching fraction can be measured in a model-independent fashion. By fitting the spectrum with the theoretical prediction of  $d\Gamma/dz$  (Equation **2.47**), the **LECs** can be extracted.

## 2.2.4 Experimental Status

The current and most precise determinations of the  $K \rightarrow \pi^+ \ell^+ \ell^-$  branching fractions are,

$$\mathcal{B}(K^+ \rightarrow \pi^+ e^+ e^-) = (3.11 \pm 0.04_{\text{stat}} \pm 0.12_{\text{syst}}) \times 10^{-7}, \quad (2.57)$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = (9.62 \pm 0.21_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-8}. \quad (2.58)$$

performed by the NA48/2 experiment **[47]** **[48]**. The **LECs** were also extracted, contained in Table **2.1**. There have been two measurements of the  $ee$  channel **LECs**, firstly by the E865 experimental at **BNL** and secondly by the NA48/2 experiment at CERN. Only NA48/2 has measured the **LECs** for the  $\mu\mu$  channel. Notably, the existing literature misses the measurement of the second minima in the positive  $(a_+, b_+)$  space.

Table 2.1: Current experimental measurements of the **LECs** for both  $ee$  and  $\mu\mu$  channels.

Channel	$a_+$	$b_+$	Experiment
$ee$	$-0.587 \pm 0.010$	$-0.655 \pm 0.044$	E865 <b>[49]</b>
$ee$	$-0.578 \pm 0.016$	$-0.779 \pm 0.066$	NA48/2 <b>[47]</b>
$\mu\mu$	$-0.575 \pm 0.039$	$-0.813 \pm 0.145$	NA48/2 <b>[48]</b>

The E865 experiment was performed at the [BNL Alternating Gradient Synchrotron \(AGS\)](#), constructed specifically to search for the [Lepton Flavour Violating \(LFV\)](#) decay of  $K^+ \rightarrow \pi^+ \mu^+ e^-$  [\[50\]](#). [Figure 2.9](#) contains the schematic of the apparatus, where an unseparated 6 GeV/c beam with a 2.5%  $K^+$  component is allowed to decay upstream of a series of sub-detectors. The kaons decay in an evacuated volume 5 m

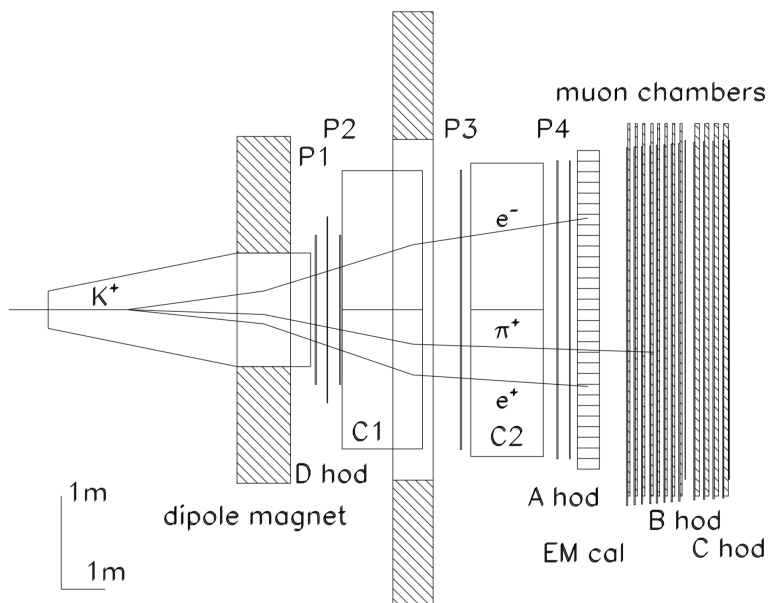


Figure 2.9: The E865 experiment;  $K^+$  decay in-flight, steered by two sets of dipole magnets to separate pions and electrons [\[49\]](#).

upstream of a charge-separating dipole magnet. Tracking was performed in the four  $P1 \rightarrow 4$  proportional chambers, interspaced by a spectrometer dipole magnet with a 0.833 Tm field integral. Two Cherenkov counters  $C_{1,2}$  filled with  $H_2/CH_4$  were used for particle-identification, while a Shashlik electromagnetic calorimeter was installed for energy measurements, containing 600  $11.4 \times 11.4 \text{ cm}^2$  modules equivalent to 15 radiation lengths in depth. The final detector system is the muon range stack, corresponding to iron-planes segmented between 24 regions of proportional tubes.

The NA48/2 experiment at CERN was proposed to search for direct [CP](#) violation in  $K^\pm$  decays, redesigned from the neutral kaon experiment NA48 which occupied the same experimental cavern [\[51\]](#). [Figure 2.10](#) displays the drawing of the NA48/2 beam-line and detector. NA48/2 also used an unseparated beam, but con-

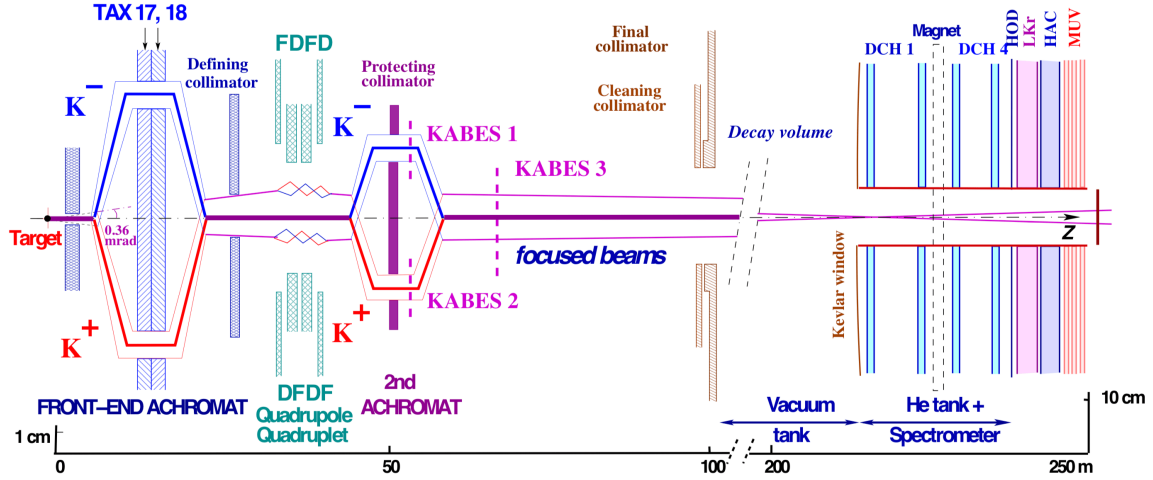


Figure 2.10: The NA48/2 beam-line and detector. Two kaon beams  $K^\pm$  were propagated to the decay volume [47].

tained both  $K^\pm$  which followed similar paths due to symmetric beam-optics. The secondary beam was tuned to have momentum  $(60 \pm 3)$  GeV/c, and only 5.7% of particles in the beam were  $K^\pm$ . The evacuated decay-volume is followed by four  $DCH1 \rightarrow 4$  drift-chambers, housed in a helium tank, where the spectrometer magnet with field integral  $\approx 0.4$  Tm was sandwiched by the second and third chamber. The spectrometer was followed by the Charged Hodoscope (CHOD), consisting of two planes (one vertical and horizontal) of plastic scintillator strips, used for fast triggering. The Liquid Krypton Calorimeter (LKr) is a homogeneous electromagnetic calorimeter, using a  $7 \text{ m}^3$  active volume of liquid krypton and 13248 transversely segmented cells, corresponding to a depth of 27 radiation lengths. Finally, the muon detector (MUV) was located furthest downstream, comprised of three planes of 2.7 m long plastic scintillator strips. The ends of each strip were coupled to PMTs, which were aligned horizontally in the first and third planes and vertically in the middle plane.

The invariant mass spectra for the current  $K^+ \rightarrow \pi^+ \ell^+ \ell^-$  measurements are shown in Figure 2.11. Here, (a) corresponds to the two-dimensional reconstructed  $K_{\pi e e}$  invariant mass versus dielectron mass for E865, with the top-left plot displaying the MC agreement in one-dimension. Both (b) and (c) are from NA48/2, corresponding

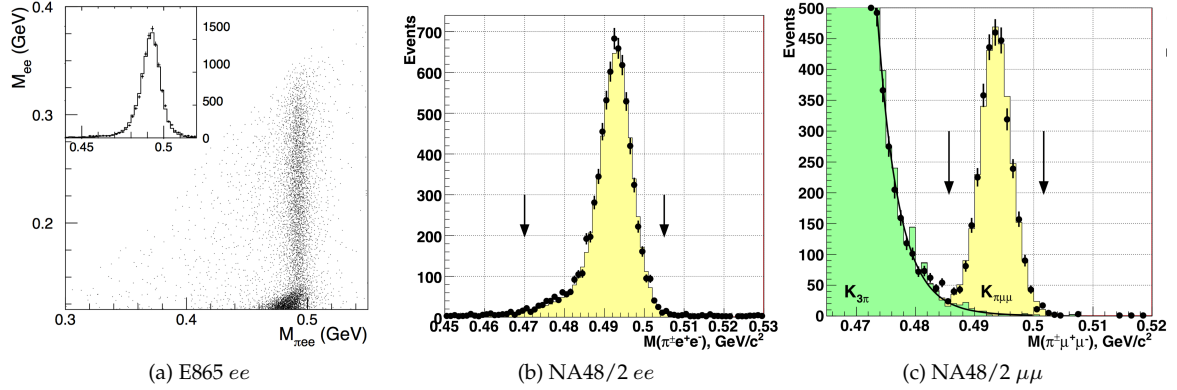


Figure 2.11: The three invariant mass spectra where (a) is the  $ee$  channel from E865, (b) and (c) are the  $ee$  and  $\mu\mu$  channels from NA48/2 experiments. The  $\text{SR}$ 's are denoted by the arrows [49] [47] [48].

to the  $K_{\pi\ell\ell}$  invariant mass including MC agreement for the  $ee$  and  $\mu\mu$  channels respectively. In all analyses, the selection signature was the identification of a single pion track and opposite sign lepton pair originating from a three-track vertex in the decay volume. The E865 sample contains 10300 candidates with background contamination of 1.2%, mostly due to  $K^+ \rightarrow \pi^+ e^+ e^- \gamma$  decays where the photon was missed. The  $\text{SR}$  was defined using the dielectron invariant mass of  $> 150 \text{ MeV}/c^2$ . The sample of 7235  $K_{\pi ee}$  candidates from NA48/2 was collected using a  $\text{SR}$  defined as  $470 < M_{\pi ee} < 505 \text{ MeV}/c^2$  (slightly asymmetric with respect to  $M_K$  due to the radiative tail) with a background contamination of  $(1.0 \pm 0.1_{\text{stat}})\%$ . The 3120  $K_{\pi\mu\mu}$  candidates were collected in  $\text{SR}$   $|M_{\pi\mu\mu} - M_K| < 8 \text{ MeV}/c^2$ . The sample suffers from a relatively large background contamination at  $(3.3 \pm 0.5_{\text{stat}} \pm 0.5_{\text{syst}})\%$ , where the tail-end of secondary  $\pi \rightarrow \mu\nu_\mu$  decays of the common  $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$  process enters directly into the  $\text{SR}$ . Due to the kinematic similarity of the signal and background decays, the separation between the two samples is small and can only be increased with improved tracking and a larger dipole field-integral. At NA62 both larger and significantly cleaner samples of the  $ee$  and  $\mu\mu$  channels are expected to be collected over the lifetime of the experiment because of vast increases in instantaneous rate, improved tracking and larger field-integrals (see Chapters 3 and 4).

Using the current available measurements, the constraint on the  $\text{NP}$  contribution to

the Wilson coefficient  $C_9^B$  from the perspective of  $K$ -physics is,

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2}V_{td}V_{ts}^*} = -19 \pm 79, \quad (2.59)$$

where  $a_+^{ee}$  is the average of the two results in Table 2.1 [40]. The determination of this value is consistent with zero, but the test is limited by the large error present in the  $\mu\mu$  channel. The required improvement in precision is of the  $\mathcal{O}(10)$  to probe NP, which requires samples of the  $\mathcal{O}(100)$  larger. In this thesis, an analysis is performed to extract the LECs for the  $K^+ \rightarrow \pi^+\mu^+\mu^-$  decay, using the 2016 and 2017 NA62 data (Chapters 6 to 9). This is a proof of concept to perform the most precise  $K^+ \rightarrow \pi^+\mu^+\mu^-$  measurement at the newest  $K$  experiment at CERN.

# 3

## The NA62 experiment

The NA62 experiment is designed to collect  $\approx 100$  ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  ( $K_{\pi\nu\nu}$ ) decays in a time period of three years [52] [53]. By performing a precise measurement of the branching fraction, a comparison with the SM prediction can be made in order to search for any discrepancy that could be attributed to NP. In this chapter, the main experimental strategy is discussed and a description of the full NA62 detecting apparatus is provided. As the  $K_{\pi\nu\nu}$  decay is experimentally challenging, NA62 is well suited to perform other rare analyses such as the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  decay, which is the subject of this thesis. Special emphasis is placed on systems critical to the tracking optimisation and analysis performed in the following chapters.

### 3.1 Experimental strategy

The ultra-rare  $K_{\pi\nu\nu}$  decay is an experimentally challenging process to measure. In principle, the characteristic signature is simple: A positively charged kaon, tagged in the beam, and a single charged pion track in the absence of any other signals in the downstream of the experiment. However, in an extreme high-intensity environment like the one at NA62, any decay or process that leads to this same experimental signature can contribute directly as background.

At NA62, most of the main sources of background are the other decay modes of the  $K^+$ , described in Table 3.1. Here, a strategy to suppress these backgrounds is in-

Table 3.1: The main  $K^+$  decay processes and the methods to suppress their signatures, followed by the two rare decays discussed in this thesis and their selection strategies [54] [55].

Decay Channel	Label	Branching Fraction(%)	Reduction (Selection) Strategy
$K^+ \rightarrow \mu^+ \nu_\mu$	$K_{\mu 2}$	$63.56 \pm 0.11$	$\mu$ veto, two-body kinematics
$K^+ \rightarrow \pi^+ \pi^0$	$K_{2\pi}$	$20.67 \pm 0.08$	$\gamma$ veto, two-body kinematics
$K^+ \rightarrow \pi^+ \pi^+ \pi^-$	$K_{3\pi}$	$5.58 \pm 0.02$	Track multiplicity, three-body kinematics
$K^+ \rightarrow \pi^0 e^+ \nu_e$	$K_{e3}$	$5.07 \pm 0.04$	$\gamma$ veto, $E/p$
$K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	$K_{\mu 3}$	$3.34 \pm 0.03$	$\gamma, \mu$ veto
$K^+ \rightarrow \pi^+ \pi^0 \pi^0$	$K_{3\pi}^0$	$1.76 \pm 0.02$	$\gamma$ veto, three-body kinematics
$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$K_{\pi\mu\mu}$	$(9.4 \pm 0.6) \times 10^{-8}$	Three-track vertex, two $\mu$ veto signals
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$K_{\pi\nu\nu}$	$(1.7 \pm 1.1) \times 10^{-10}$	One $\pi^+$ track only

cluded, highlighting the variety of different methods that are required. Notably, the current experimental determination of  $K_{\pi\nu\nu}$  suggests that the branching fraction of the more common modes is of the  $\mathcal{O}(10^{10})$  larger, requiring a background suppression of the  $\mathcal{O}(10^{11})$  to achieve a reasonable signal to background ratio  $S/B \approx 10$ .

The NA62 experiment uses a 75 GeV/c high-momentum  $K^+$  secondary beam-line, allowing  $K^+$  to decay in-flight. The  $K_{\pi\nu\nu}$  kinematics are displayed in Figure 3.1: As

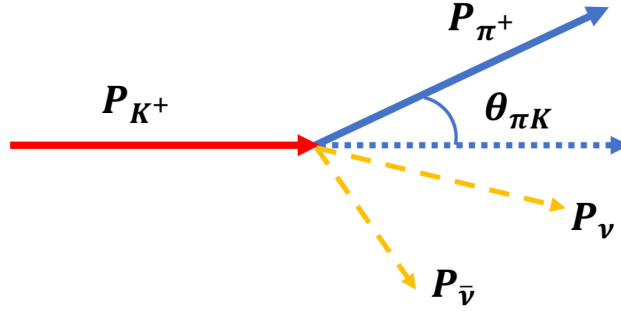


Figure 3.1: The  $K_{\pi\nu\nu}$  kinematics; only the  $K^+$  and  $\pi^+$  are observable.

the  $\nu\bar{\nu}$  pair is invisible to the experiment, the only measurable observables are the  $K^+$  and  $\pi^+$  momenta and the angle  $\theta_{\pi K}$  between the two. As a fraction of the energy is 'missing', the most suitable description of the kinematics is the squared missing invariant-mass  $m_{miss}^2$ , on the assumption that the observed charged track is a pion:

$$m_{miss}^2 = (P_K - P_\pi)^2, \quad (3.1)$$

$$= M_K^2 + M_\pi^2 - 2E_K E_\pi + 2|P_K||P_\pi| \cos \theta_{\pi K}, \quad (3.2)$$

where  $E_{K,\pi} = \sqrt{M_{K,\pi}^2 + P_{K,\pi}^2}$ . The unconstrained missing mass distribution is displayed in Figure 3.2, normalised to the total decay width. The signal decay is multiplied by a factor of  $10^{10}$ . As  $K_{\pi\nu\nu}$  is a three-body decay, the  $m_{miss}^2$  distribution is

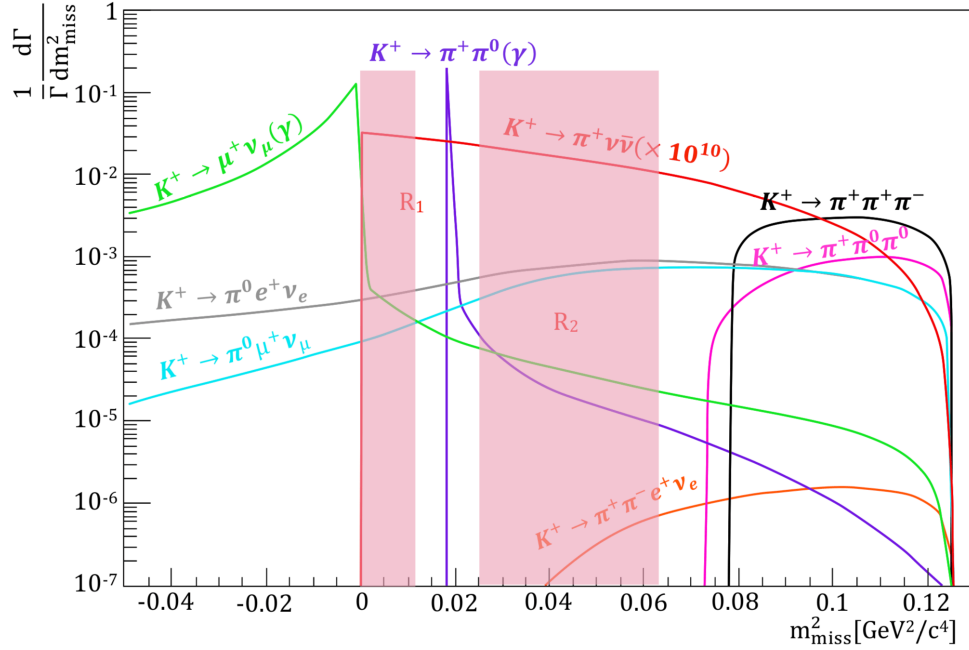


Figure 3.2: The missing mass<sup>2</sup> distribution for the main decay modes of the  $K^+$ , including the decay  $K_{\pi\nu\nu}$  multiplied by a factor of  $10^{10}$ . The  $K_{2\pi}$  peak leads to the natural choice of two signal regions on either side,  $R_1$  and  $R_2$  [54].

continuous with upper and lower limits. The two-body  $K^+ \rightarrow \pi^+\pi^0$  peaks sharply inside of the  $K_{\pi\nu\nu}$  spectrum because a pion is correctly assigned, but has a radiative tail due to  $K^+ \rightarrow \pi^+\pi^0\gamma$  decays. The most common decay  $K^+ \rightarrow \mu^+\nu_\mu$  is always negative as  $m_\pi > m_\mu$ . Similar to the signal, the  $K^+ \rightarrow \pi^+\pi^+\pi^-$  distribution is a constrained continuous distribution, but as two pions are undetected, the distribution is situated at large positive values of  $m_{miss}^2$ .

Two Signal Regions (SRs) are defined as follows:

- **Region I:** Between 0 and the sharp peak from  $K^+ \rightarrow \pi^+\pi^0$
- **Region II:** Between the sharp  $K^+ \rightarrow \pi^+\pi^0$  peak and before the low  $K_{3\pi^0}$  threshold.

These regions were defined to reject the backgrounds for the majority of  $K^+$  decays.



A notable feature of Figure 3.2 is that both signal regions still have non-zero contributions from background decays which feature a  $\pi^0$  or  $\gamma$  (for example,  $K^+ \rightarrow \pi^0 \ell^+ \nu_\ell$  and  $K^+ \rightarrow \pi^+ \pi^0 \gamma$ ). Powerful hermetic photon rejection of the  $\mathcal{O}(10^8)$  is required to reject either the primary photons or the secondary  $\pi^0 \rightarrow \gamma\gamma$  decays. A major motivation to use a high-momentum  $K^+$  beam is the huge gain in photon detection efficiency, a feature that was a large limiting factor for background suppression in previous stopped kaon experiments such as E787 and E949 [23] [25] [52].

## 3.2 NA62 Beam-line

The NA62 experiment is situated in the ECN3 cavern in the North Area of CERN. The primary 400 GeV/c proton beam is delivered by the Super Proton-Synchrotron (SPS) in 30 s super-cycles with two five-second flat-top spills (also known as bursts). The extracted proton beam is directed onto the 400 mm long 4 mm radius T10 beryllium target at the start of the experimental cavern ( $Z = 0$  m), from which the K12 high-intensity secondary hadron beam is derived. Before the decay volume at  $Z = 102.4$  m, a series of magnet and collimator systems are used to shape and steer the K12 beam. Figure 3.3 contains a schematic of the vertical optics of the beam-steering in this region. A fraction of the secondary beam at an angle to maximise

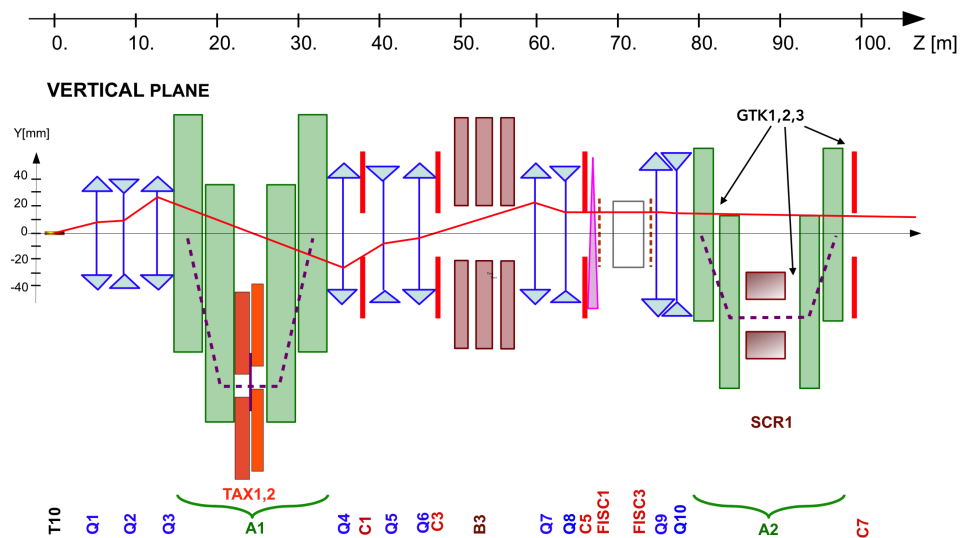


Figure 3.3: Optics and layout of the K12 beam in the vertical view. The solid line is the nominal trajectory of  $K^+$  particle at 75 GeV/c from the target [53].

the kaon component passes through a 15 mm bore collimator towards the small aperture quadrupole triplet (Q1, Q2, Q3) which focus the secondary beam to a central momentum of roughly 75 GeV/c. Next the A1 front-end achromat, which consists of four dipole magnets, is used to remove off-momentum protons. The first two dipoles steer the beam vertically in order to sweep muons away and then traverse TAX1, a narrow bore collimator to absorb protons and other beam-halo particles. Before TAX2, a  $1.3X_0$  tungsten radiator is placed, specifically chosen to cause positrons to lose energy via bremsstrahlung, while leaving the rest of the beam relatively untouched. Following A1, a series of quadrupoles (Q4, Q5, Q6) and collimators (C1, C2) are used to refocus the beam and remove unwanted beam particles; at this stage, the degraded positrons from the radiator and particles outside of the  $(75 \pm 0.8)$  GeV/c range are swept away by the C3 collimator, while surviving muons are removed by three 2 m long B3 dipoles. The Cherenkov Differential

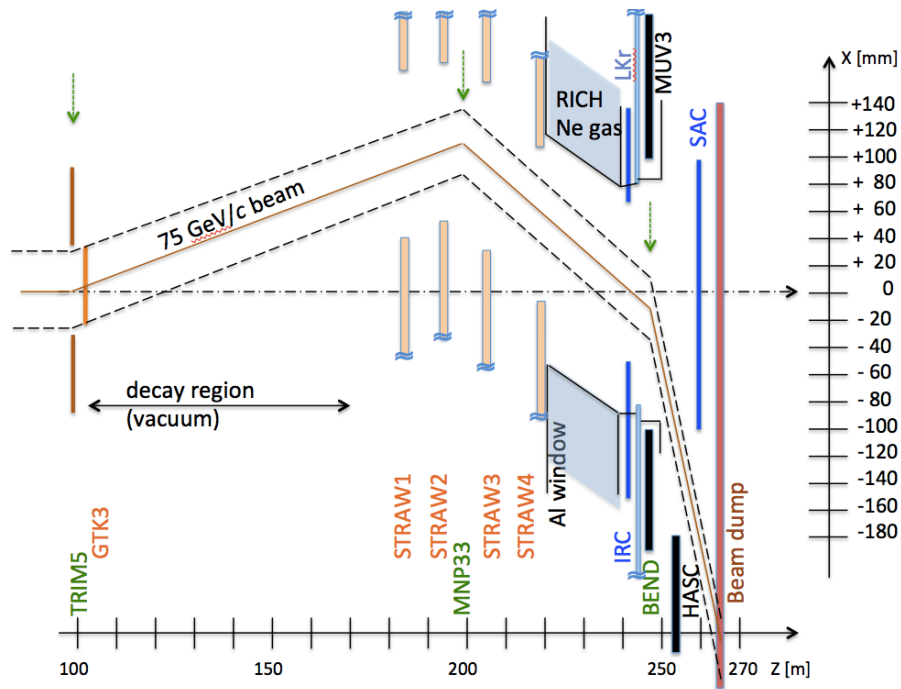


Figure 3.4: Downstream beam-line layout post TRIM5. The TRIM5 steers the beam by 1.2 mrad in the  $X$ -dimension up to the MNP33 dipole which then deflects the beam back by -3.6 mrad to match with the central aperture of the LKr which is on the central axis. The beam dump is placed after the SAC, and the beam is deflected by -13.2 mrad just after the MUV3 [53].

counter (CEDAR) tank is situated at  $Z = 69$  m. For high-efficiency kaon tagging, the beam must be reshaped to be parallel and aligned with the beam-axis, performed

by two quadrupoles (Q7, Q8) and cleaning collimators (C4, C5) which absorb the beam tail-ends. The beam is steered up to the GTK tracking-system, which contains three silicon micro-pixel stations. The second A2 achromat is placed between GTK1 and GTK3, which has four vertically deflecting C-shaped dipoles. The achromat defocuses any remaining muons using the scraper magnets whilst shaping the secondary beam, with the final collimators absorbing any deflected particles. At the end of GTK3 at 102 m, the TRIM5 dipole magnet is used to deflect the beam by an angle of 1.2 mrad in the positive  $X$ -dimension. Figure 3.4 displays the  $X$ -direction of the beam with respect to  $Z$ . The TRIM5 deflection is corrected by -3.6 mrad at the MNP33 large-aperture dipole magnet which is situated inside of the STRAW spectrometer, to match the central aperture of the Liquid Krypton Calorimeter (LKr) calorimeter whose face is aligned with the central axis. The final stage is to dump the beam, which is deflected by -13.2 mrad just after the MUV3 detector. Table 3.2 contains the overall properties of the K12 beam as it enters the decay-volume. No-

Table 3.2: The nominal properties of the K12 secondary beam, including the instantaneous rates of the various beam components.

Momentum	Mean Value $\Delta P/P$ (RMS)	75 GeV/c 1.0%
Instantaneous Rate	$p$	173 MHz (23%)
	$K^+$	45 MHz (6%)
	$\pi^+$	525 MHz (70%)
	$\mu^+$	$\approx$ 5 MHz (<1%)
	Total	750 MHz

tably, only 6% of the 75 GeV/c beam are kaons. As all downstream sub-detectors have a central beam-hole, the vast majority of the K12 beam particles continue along the beam-axis without being observed downstream.

### 3.3 Detectors at NA62

#### 3.3.1 Detector overview

A schematic drawing of the NA62 experiment is displayed in Figure 3.5. Here,  $K^+$  decays from the K12 beam which occur in the fiducial volume between 102.4 m <  $Z$  < 180.0 m will be fully reconstructed. As the beam is at high-momentum,

the sub-detector architecture is hermetic for particles up to 50 mrad with respect to the  $Z$ -axis. A short summary of the sub-detector systems is provided in Table 3.3.

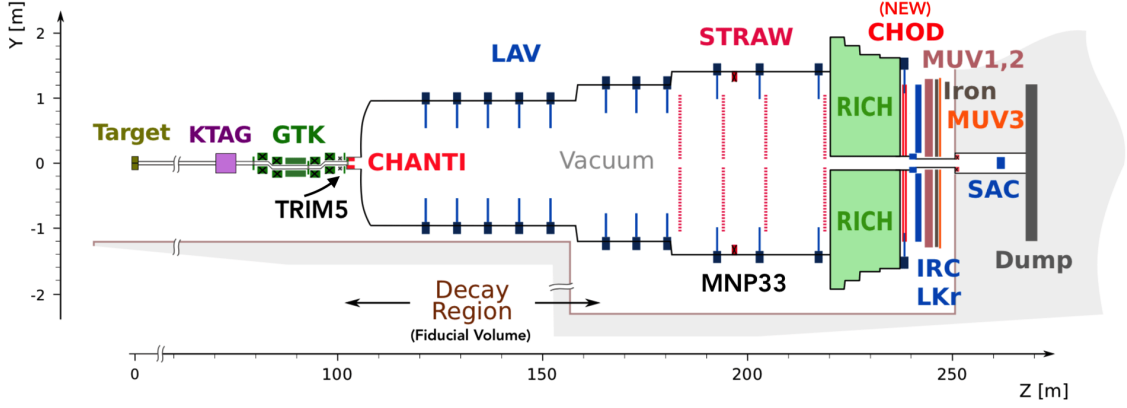


Figure 3.5: The NA62 beam-line and detector. The length is approximately 270 m, with  $K^+$  decays occurring in the fiducial decay region [53].

The sub-detectors denoted as ‘Other’ are small systems crucial for the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Table 3.3: A short summary of the NA62 detector systems and their purposes.

System	Detector	Purpose
Tracking	GigaTracker	Precise $K^+$ momentum, time and position measurement
	STRAW	Reconstruct charged tracks from $K^+$ decays
Particle ID	KTAG	Uses Cherenkov light to time-stamp $K^+$
	RICH	Cherenkov detector to provide $\pi$ and $\mu$ separation
Hodoscopes	NA48 CHOD	Fast minimum bias triggers for charged tracks
	NewCHOD	More complex kinematic triggers for charged tracks
Photon Veto	LAV	Large angle photon veto $8.5 \text{ mrad} < \theta < 50 \text{ mrad}$
	LKr	Intermediate photon veto between $1.5 \text{ mrad} < \theta < 8.5 \text{ mrad}$
	SAC	IRC and SAC, reject photons parallel to beam $1.5 \text{ mrad} < \theta$ .
Muon Veto	MUV1,2	Hadronic calorimeters for $\pi$ and $\mu$ separation
	MUV3	Behind an iron wall to detect $\mu$ only
Other	CHANTI	Rejects inelastic beam interactions from the GTK stations
	MUV0	Rejects large angle $\pi^-$ from $K_{3\pi}$ downstream of RICH
	HASC	Rejects beam-parallel $\pi^+$ from $K_{3\pi}$ at the beam-dump

measurement only; the CHANTI is used to veto events containing inelastic  $K^+$  interactions, whereas both the MUV0 and HASC are used to detect specific topologies of  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  decays which contribute specifically as single track background. Additionally, Table 3.4 contains a description of the magnet systems which are critical for physics analyses at NA62. The TRIM5 steering dipole is especially important for understanding the type of background appears for the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  analysis. More detail can be found in Chapter 6. As the MNP33 is the spectrometer dipole,

Table 3.4: The magnet systems important for the analysis in this thesis. Both magnets are dipoles.

Magnet	Z-Position [m]	X-deflection [mrad]	Purpose
TRIM5	102.000	+1.2	Used to align the beam with the STRAW
MNP33	197.645	-3.6	Used for track momentum determination

it plays a vital role in many tracking calibration analyses described in this thesis. Explicit detail of the MNP33 can be found in Chapter 4. For the remainder of this chapter, only the most crucial detector systems to the analyses presented in the following chapters are described in detail.

### 3.3.2 Tracking

#### 3.3.2.1 GigaTracker (GTK)

The **GigaTracker (GTK)** is designed to provide precision measurements of the  $K^+$  momentum, time and position within the K12 beam. The **GTK** is situated upstream of the fiducial decay volume and consists of three hybrid silicon micro-pixel stations, offset vertically around the A2 achromat. Figure 3.6 contains a schematic drawing of the **GTK** system. The momentum measurements are extracted via the vertical

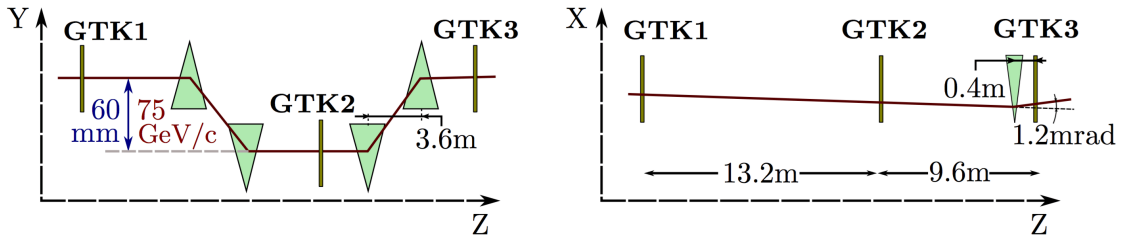


Figure 3.6: Left: the  $YZ$  view and right: the  $XZ$  view of the GTK. The red-line denotes the optimal path of a  $75 \text{ GeV}/c$   $K^+$  beam particle [53].

displacement of the kaon path between **GTK**1 and 2. The momentum precision is  $\sigma(P) = 1.5 \text{ MeV}/c$ , whilst the angular resolution on the beam-slope determination ( $dX/dZ, dY/dZ$ ) is  $16 \mu\text{rad}$ . As the **GTK** is subjected to an instantaneous rate of up to  $750 \text{ MHz}$ , the time resolution of the detector is of the  $\mathcal{O}(200 \text{ ps})$  in order to differentiate between the pile-up of kaons. Each station receives approximately  $4 \times 10^{14}$  neutrons per  $\text{cm}^2$  in a year of data taking. The stations must be radiation hard to last a year of data acquisition, while only corresponding to a small amount of

material ( $0.005X_0$ ). Each micro-pixel detector consists of  $1.8 \times 10^4$  pixels of  $300 \times 300 \mu\text{m}^2$  area, arranged in an array of  $200 \times 90$  elements. Ten application-specific integrated circuits (ASIC) chips are used per station, used to read out  $40 \times 45$  pixels each.

### 3.3.2.2 Straw spectrometer (STRAW)

The Straw spectrometer (STRAW) is used to measure the momentum and trajectory of the  $K^+$  decay products after the fiducial decay volume. Figure 3.7 contains a drawing of the end of the vacuum region where the STRAW is located. The spec-

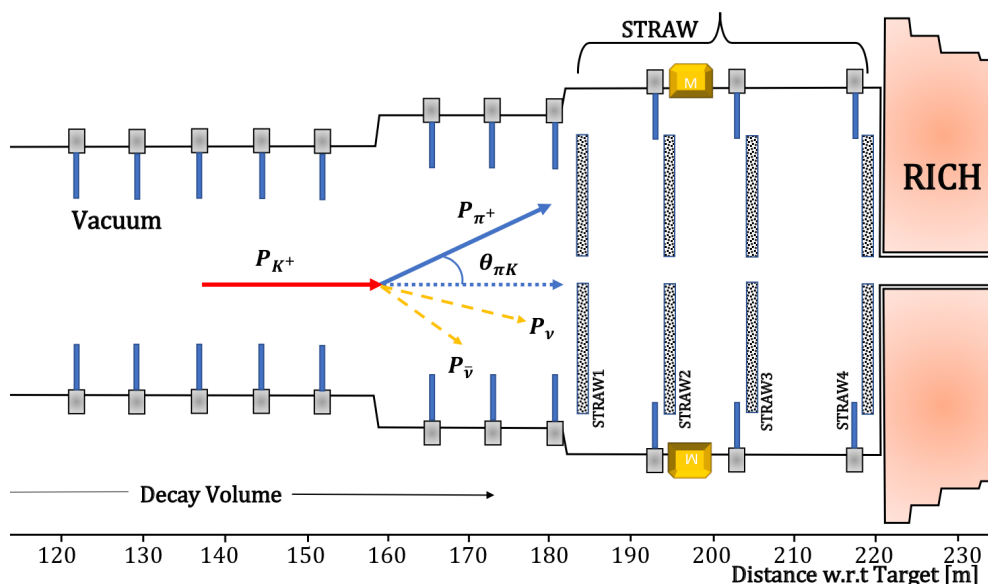


Figure 3.7: The STRAW; four chambers, separated by 10 m (15 m for chamber four), interspaced by the MNP33 dipole magnet.

trometer consists of four straw chambers and the MNP33 large-aperture dipole magnet. The magnet is positioned between the second and third chambers, and delivers a field integral of  $\approx 0.9 \text{ Tm}$  in the positive  $X$ -dimension (equivalent to a  $+270 \text{ MeV}/c$  kick). The chambers are built using two modules, each containing two views. The first module contains the  $(X, Y) = (0^\circ, 90^\circ)$  views, and the second is rotated by  $-45^\circ$  relatively, such that  $(U, V) = (-45^\circ, 45^\circ)$ . Each view consists of 448 straw tubes, divided into four layers and staggered to guarantee at least two hits per view. Figure 3.8 contains a schematic of the module orientation in the chambers, including the straw tube staggering. Each chamber has a diameter of 2.1 m and a 12 cm gap

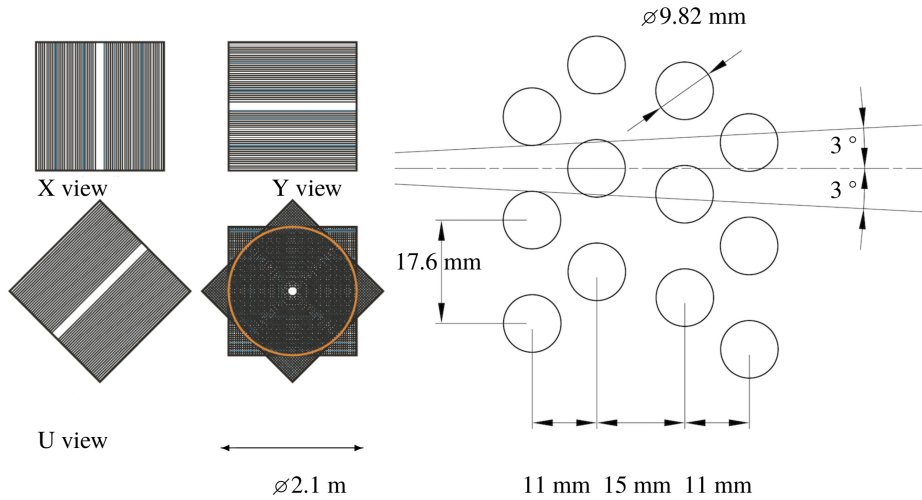


Figure 3.8: Left: how a full STRAW chamber is constructed using four ( $U, V, X, Y$ ) views and right: the staggering of the four-layers of straw tubes to guarantee at least two hits for a track per view [53][56].

in each view at the centre for the beam-pipe. The straw tubes are constructed out of  $36 \mu\text{m}$  thick polyethylene terephthalate (PET), and are  $9.82 \text{ mm}$  in diameter and  $2160 \text{ mm}$  in length. The tubes are gas-filled with a combination of 70% Ar and 30%  $\text{CO}_2$  operated at atmospheric pressure. The straws are coated with  $50 \text{ nm}$  copper, and lined with  $20 \text{ nm}$  of gold, containing a tensioned gold-plated tungsten wire  $30 \mu\text{m}$  in diameter. The combined material budget for the entire STRAW is ultimately very low, at  $0.02X_0$ , in order to reduce multiple-scattering. Each STRAW chamber uses custom front-end electronics to read-out data, where sets of 16 straw tubes are controlled by a FPGA served by two CARIOCA chips, developed for the LHCb muon read-out [57]. After evaluation, the track momentum resolution is consistent with design at  $\sigma(P)/P = 0.3\% \oplus 0.005\% \cdot P$ , whilst the angular resolution is  $\sigma(\theta) = 60 \mu\text{rad}$  for tracks with momentum  $P = 10 \text{ GeV}/c$  and only  $20 \mu\text{rad}$  at  $P = 50 \text{ GeV}/c$  [53].

### 3.3.3 Particle Identification

#### 3.3.3.1 Kaon Tagger (KTAG)

The role of the Kaon Tagger (KTAG) is to identify (i.e time-stamp) the 6% component of the K12 hadron beam which are  $K^+$ . The kaon tagging system comprises of

two components; a differential CEDAR and a photomultiplier octagon (KTAG). The CEDAR is a nitrogen ( $N_2$ ) 0.94 m<sup>3</sup> filled tank which is placed directly in the beam-line at  $Z = 69$  m after the T10 target. Figure 3.9 contains a schematic diagram of the CEDAR, including the Cherenkov light-path of a  $K^+$  candidate. By controlling the

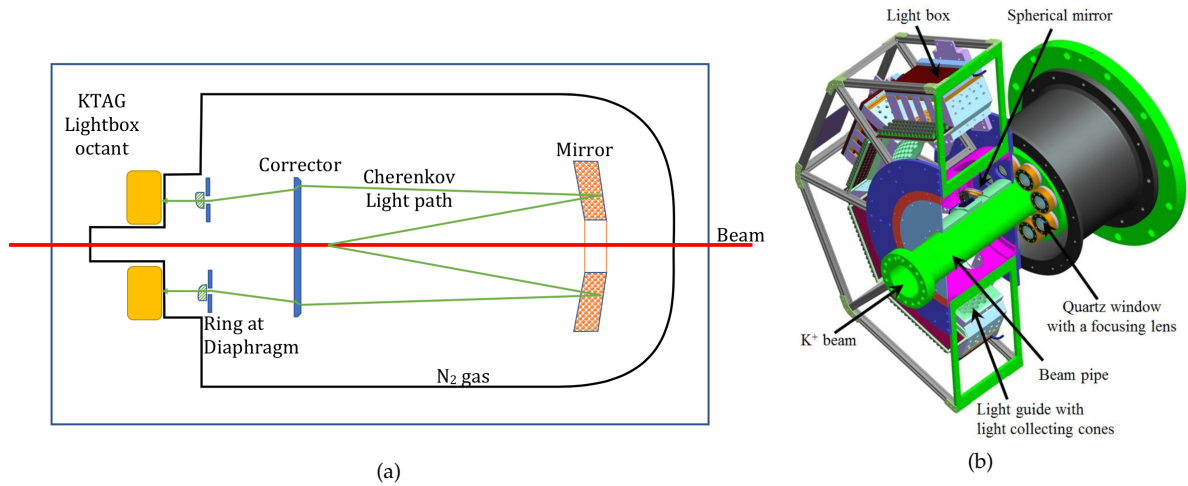


Figure 3.9: Schematic drawing of the CEDAR and KTAG. The tank is shown in (a), including the tracing of the  $K^+$  Cherenkov light, whereas (b) displays the KTAG photo-multiplier octant which directs the Cherenkov light to eight light boxes [53] [58].

pressure in the tank (1.7 bar for  $K^+$  operation), the opening angle of the Cherenkov light-cone can be controlled such that only  $K^+$  candidates generate rings which can pass through the narrow slit diaphragm and into the KTAG. Therefore, the KTAG is tuned with both a pressure and alignment scan to maximise detection efficiency. The light exits the diaphragm through eight quartz windows into an octagon of spherical mirrors which reflect the light radially towards the photo-multiplier 'light' boxes. Each light box (also known as a sector) contains 48 Photomultiplier Tubes (PMT), which are read-out using 128-channel Time to Digital Converter (TDC) boards. The coincidence of signals in multiple sectors is used to time-stamp each beam kaon with a time-resolution  $\sigma(t) = 70$  ps, as the photons generated from both protons and pions will not register coincidental signals in more than five light boxes due to the diaphragm. Naturally, the KTAG system is highly sensitive to beam conditions, and so the beam must be shaped to be flat and parallel with the beam-axis as it traverses the CEDAR tank.



### 3.3.3.2 RICH

The RICH is the downstream particle identification detector situated at  $Z = 223$  m, used primarily to separate pions, muons and electrons in the momentum range between  $15 \text{ GeV}/c < P < 35 \text{ GeV}/c$ . The RICH is a 17.5 m long ferro-pearlitic steel cylinder filled with neon-gas at atmospheric pressure, corresponding to a refractive index  $(n - 1) = 62 \times 10^{-6}$ . Figure 3.10 contains a schematic drawing of the RICH. The tank gradually reduces in diameter, with the upstream section the largest at 4.2

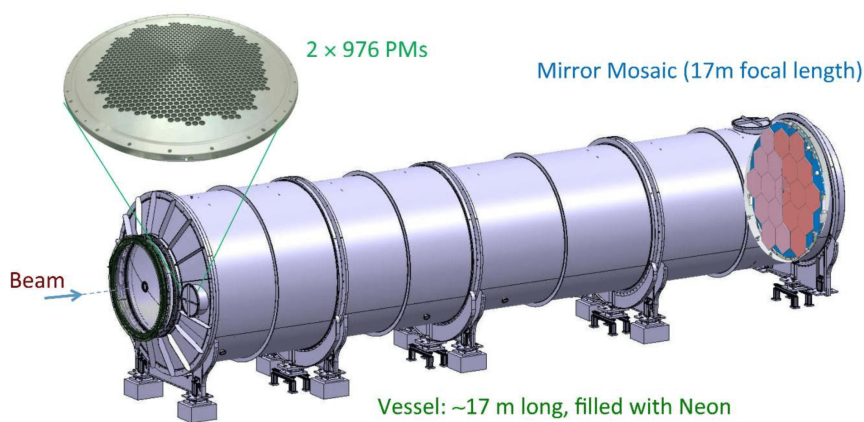


Figure 3.10: Schematic drawing of the RICH: the beam traverses the centre of the detector, with particles above threshold producing Cherenkov light which is reflected by the two halves of the RICH mirror (denoted by different colours) back to the two PMT arrays [53].

m to house the two honeycomb structured photo-multiplier matrices, each containing 976 PMTs residing either side of the beam-pipe. Any particle with mass  $m$  and momentum  $P$  above the threshold  $P_{\text{threshold}} = m/\sqrt{n^2 - 1}$  will produce Cherenkov light. The light travels towards the RICH mirror, a mosaic constructed using 20 hexagonally-shaped spherical mirrors. The two vertical halves of mosaic have different centres of curvature, in order to reflect the Cherenkov light to either the left or right PMT matrix. This is to avoid light intercepting the beam-pipe. The Cherenkov cone with angle,

$$\cos(\theta_c) = \frac{1}{n\beta} = \frac{1}{n} \sqrt{1 + \frac{m^2}{P^2}} \quad (3.3)$$

will form a ring on the PMT matrix with radius  $r = f \tan(\theta_c)$ , where  $f = 17$  m is the focal length. By approximation that  $n - 1 \ll 1$  and that the mass  $m$  is far smaller than

the momentum  $P$ , the radius can be collapsed using the small angle approximation to be,

$$r = f \tan \theta_c \approx f \sqrt{\frac{2(n-1)}{n} - \frac{m^2}{nP^2}}. \quad (3.4)$$

Therefore, the **RICH** can be used in combination with the spectrometer; the momentum measurement can assign the particle type of a track based on the radius.

### 3.3.4 Hodoscopes

#### 3.3.4.1 NA48 Hodoscope (CHOD)

The **NA48 Charged Hodoscope (CHOD)** is a device used to obtain a fast response to the charged products of  $K^+$  decays. Figure 3.11 displays a schematic drawing of the detector.

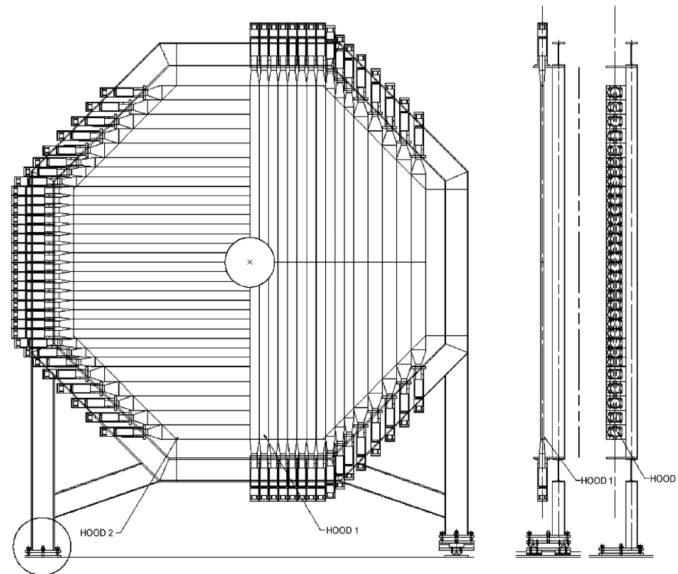


Figure 3.11: Schematic drawing of the NA48 **CHOD**. The first plane is built using vertical slabs whilst the second contains horizontal slabs [53].

The **CHOD** is comprised of two consecutive octagonal planes separated by 40 cm, situated at  $Z = 239$  m downstream of the T10 target. Each plane consists of 64 plastic scintillator slabs, with the first aligned vertically and the second horizontally. Each slab is 20 mm thick, corresponding to  $0.1X_0$ , but vary in length (between 600 mm for outer counters to 1210 mm for the inner) depending on their position relative to the

beam-hole. The slabs are read-out at one end by fast **PMTs**; for a charged particle traversing the detector, two time measurements are performed corresponding to a single vertical and horizontal slab measurement. The time resolution of the **CHOD** is  $\sigma(t) = 400$  ps, compatible with the NA48 experiment despite the far higher instantaneous rate, providing a suitable source for fast minimum bias triggers for decays with at least a single charged track.

### 3.3.4.2 NewCHOD

The **NewCHOD** is a charged hodoscope designed specifically for NA62, situated just before the NA48 **CHOD** at  $Z = 238$  m. Figure 3.12 contains a schematic diagram of the face of the **NewCHOD**, including the positioning of the 152 plastic scintillator tiles.

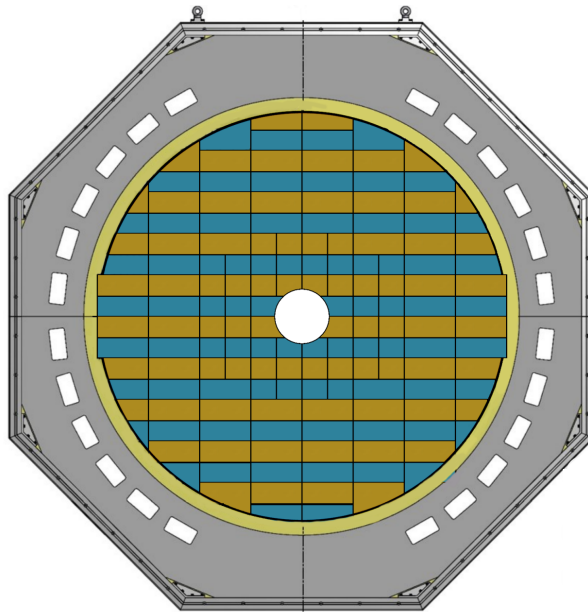


Figure 3.12: Schematic drawing of the **NewCHOD**. The single plane structure uses 152 tiles, with unique sizes and shapes at the edges and at the beam-hole of the detector [59].

Each tile is 30 mm thick, covering a region between  $140 \text{ mm} < R < 1070 \text{ mm}$ . The tile centres are spaced by 107 mm such that they overlap by 1 mm in the  $Y$ -dimension; this was achieved by staggering the tiles longitudinally around a 3 mm central support foil. Each tile is 108 mm in height, and they are either 134 or 286 mm wide depending on the position relative to the beam-pipe. Only the tiles on the outside edge of the detector are unique sizes. The resulting material thickness of the detector

is  $0.13X_0$ . The scintillation light from each tile is collected by wavelength-shifting fibres and detected by a pair of Silicon **PMTs** (SiPM). The 304 SiPM pairs are read out by a single 512-channel TEL62 board [60]. The time resolution is slightly larger than for the NA48 **CHOD**, at  $\sigma(t) = 1$  ns. The **NewCHOD** is used to define more complicated geometrically based firmware algorithms for use in the low-level trigger. Most of the criteria use **NewCHOD** quadrants, defined by dividing the face of the detector into four quarters of equal size. These quadrant triggers are used for decays containing either single or multiple charged tracks (see Section 3.4).

### 3.3.5 Photon Veto

#### 3.3.5.1 Large Angle Veto (LAV)

The **Large Angle Veto (LAV)** is a system of twelve independent ring-shaped detectors placed at longitudinal positions beginning at the start of the decay volume and finishing just before the **CHOD** (Figure 3.5). As the **LAV** spans a large longitudinal range, the inner and outer ring radii increase with  $Z$  as the vacuum tank size increases. The **LAV** covers the angular region between  $8.5 \text{ mrad} < \theta_\gamma < 50 \text{ mrad}$  with respect to the  $Z$ -dimension, designed to detect wide-angle photon emission originating from the  $\pi^0 \rightarrow \gamma\gamma$  in the  $K_{2\pi}$  decay. Figure 3.13 displays a single **LAV** station, most of which contain either four or five layers of lead-glass crystal blocks. The rings are staggered with respect to the first layer so that the gaps between the crystals are covered, with each block corresponding to  $10 \times 37 \text{ cm}^2$  of surface area relative to the  $Z$ -axis. High-energy photons which interact with the glass blocks generate electromagnetic showers which produce Cherenkov light in the material; the resulting photons are detected by the **PMTs** coupled to the base of each block. To ensure a  $10^8$  rejection power for the photon veto overall, the **LAV** has achieved the design inefficiency of better than  $10^{-4}$ . This level is due to low-energy  $E_\gamma \leq 200$  MeV photons which are difficult to distinguish from noise in the detector.

#### 3.3.5.2 Liquid Krypton calorimeter (LKr)

The Liquid Krypton calorimeter (**LKr**) is the same detector used in the NA48 experiment. Placed at  $Z = 241$  m downstream of the target, the **LKr** is an electromagnetic,

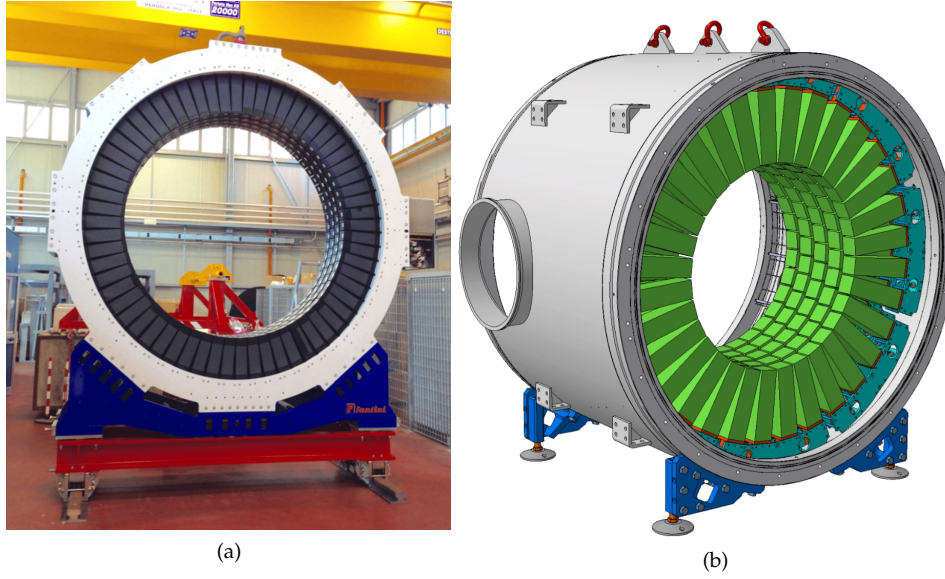


Figure 3.13: A LAV station, where (a) is a photograph of LAV12 before insertion into the beam and (b) is a schematic drawing with five lead-glass layers [53] [61].

quasi-homogeneous calorimeter which uses  $7 \text{ m}^3$  of liquid krypton housed inside a cryostat at 120 K. At NA62, its principle purpose is to veto photons in the angular region between  $1.0 \text{ mrad} < \theta_\gamma < 8.5 \text{ mrad}$  with respect to the  $Z$ -dimension. The face of the calorimeter is octagonal, covering a region between  $80 \text{ mm} < R < 1280 \text{ mm}$ , and it is 1270 mm in depth which corresponds to  $27X_0$ . Figure 3.14 displays a quadrant of the LKr including the  $2 \times 2 \text{ cm}^2$  Cu-Be electrode cells. In total, 13248 cells cover the transverse face of the detector and each span the full depth. To handle the increased rates relative to NA48, a brand-new Calorimeter Readout Module (CREAM) was developed. The energy resolution of the LKr as measured by NA62 was estimated based on simulated data,

$$\frac{\sigma(E)}{E} = \frac{4.8\%}{\sqrt{E}} \oplus \frac{11\%}{E} \oplus 0.9\%, \quad (3.5)$$

which is in agreement with the NA48/2 resolution [53]. With the energy measured in GeV, the first term is shower-dependent, the second due to noise and the final due to residual cell miscalibration.

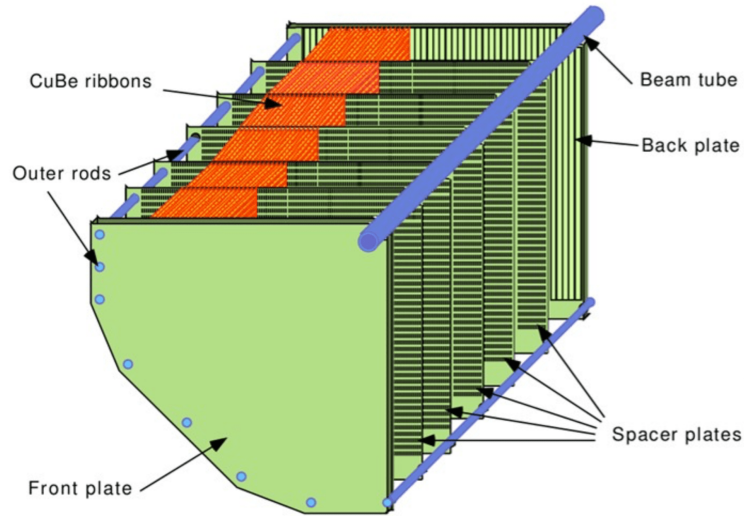
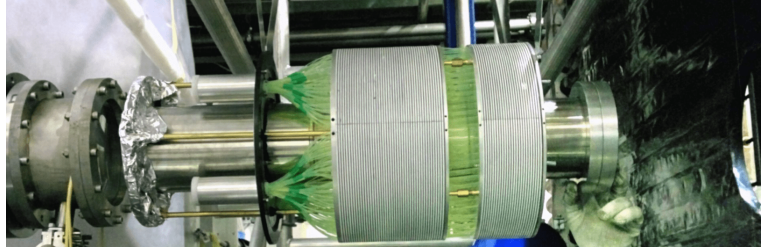


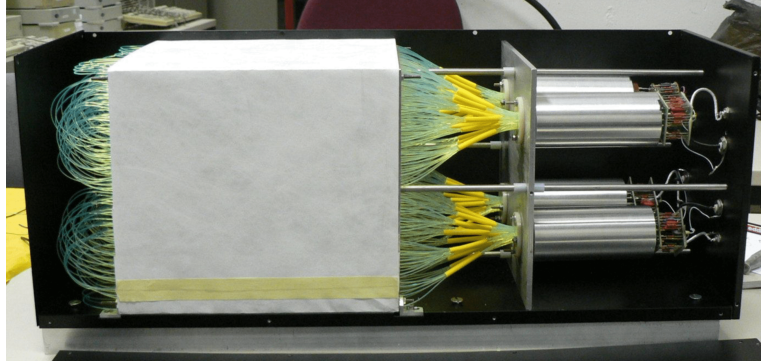
Figure 3.14: Schematic drawing of a quadrant of the **LKr**. The Cu-Be cells traverse the depth of the detector, kept separate by spacer panels [53].

### 3.3.5.3 Small Angle Veto (SAV)

The **Small Angle Veto (SAV)** consists of two systems, the Intermediate Ring Calorimeter (IRC) and the Small Angle Calorimeter (SAC). The IRC sits just before the **LKr** and wraps directly around the beam-pipe. The SAC is placed at  $Z = 261$  m directly in the beam-line, after the K12 beam has been deflected to the dump. Both are used to detect photons which are emitted parallel to the beam with angles below 1.5 mrad. Figure 3.15 contains a photograph of both detectors. Both the IRC and SAC are Shashlik calorimeters, using plastic scintillator as the active volume and lead as the absorber. The IRC covers the transverse region between  $120 \text{ mm} < R < 290 \text{ mm}$ , slightly offset in positive  $X$ . The IRC contains two separate modules, with the first containing 25 layers and the second 45, where each layer is 3 mm thick corresponding to lead absorber and plastic scintillator. The depth is equivalent to  $19X_0$ . The SAC is similarly designed but is contained to a single volume, consisting of 70 layers with dimensions  $205 \times 205 \text{ mm}^2$  and also corresponds to  $19X_0$ . Four **PMTs** are used to read-out the scintillation light.



(a)



(b)

Figure 3.15: Photograph of (a) the IRC and (b) the SAC. The IRC wraps around the beam-line, whereas the SAC is inserted directly into the beam-line [53].

### 3.3.6 Muon Veto

#### 3.3.6.1 MUV1,2

The MUV1,2 are both iron and scintillator hadronic sampling calorimeters. The purpose of these calorimeters is to provide energy-based separation for pions and muons. In conjunction with the LKr, pions are expected to have deposited their energy in at least one of the detectors such that the value  $E_{\text{Tot}} = E_{\text{LKr}} + E_{\text{MUV1}} + E_{\text{MUV2}}$  can be used as a particle identification discriminant. Figure 3.16 contains a schematic of the full MUV system. The MUV1(2) has 24(22) layers, equivalent to 4.1(3.7) interaction lengths. For the MUV1, 22 of the 24 layers cover a transverse region of  $2700 \times 2600 \text{ mm}^2$ , whilst the first and last layers extend to slightly larger dimensions to shield the electronics. Each iron plate is separated by a plane of 9 mm thick, 60 mm wide scintillator strips, which are arranged in both the horizontal and vertical direction and span the full transverse size of the detector. Strips in the same consecutive position are read out to a pair of PMTs (one for each end of the strip) by wavelength-shifting fibres. With 44 strips per plane, there are  $44 \times 2$  vertical and hor-

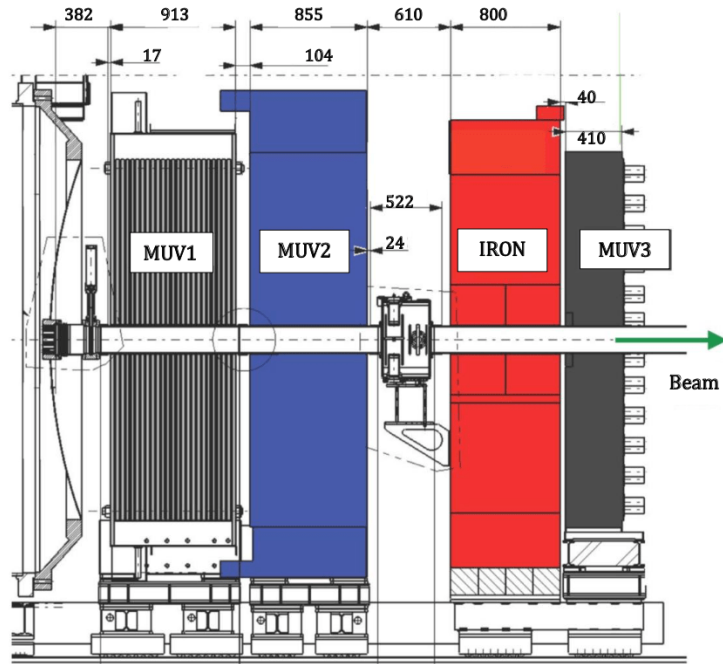


Figure 3.16: A schematic of the full **MUV** system. **MUV1** and **MUV2** are both sampling hadronic calorimeters used to separate pions from muons, whereas the **MUV3** is used to detect/veto muons which traverse the iron wall [54] [53].

horizontal channels, equivalent to 176 channels overall. The **MUV2** is the refurbished front module of the NA48 hadron calorimeter (HAC) but with reversed longitudinal orientation. Each **MUV2** scintillator plane contains 22 strips, with each spanning 1300 mm, equivalent to half of the transverse size of the detector. The same read-out strategy is used as **MUV1**, with  $22 \times 2$  vertical and horizontal channels, resulting in 88 channels overall.

### 3.3.6.2 MUV3

The **MUV3** detector is designed to be a fast muon veto. It is located behind an 80 cm deep iron-wall at the end of the **MUV** system, where the wall is used to stop any surviving hadrons (Figure 3.16). The **MUV3** is a large square-faced detector with transverse size  $2640 \times 2640 \text{ mm}^2$ . Like the **NewCHOD**, the **MUV3** consists of plastic scintillator tiles, with 140 regular  $220 \times 220 \text{ mm}^2$  tiles which cover the majority of the face, and eight smaller tiles ( $4/9$  of the regular area) which are placed around the beam-hole as the rate in that region is substantially higher. Figure 3.17 contains the expected rates for the **MUV3** tiles for nominal beam conditions. Notably, the



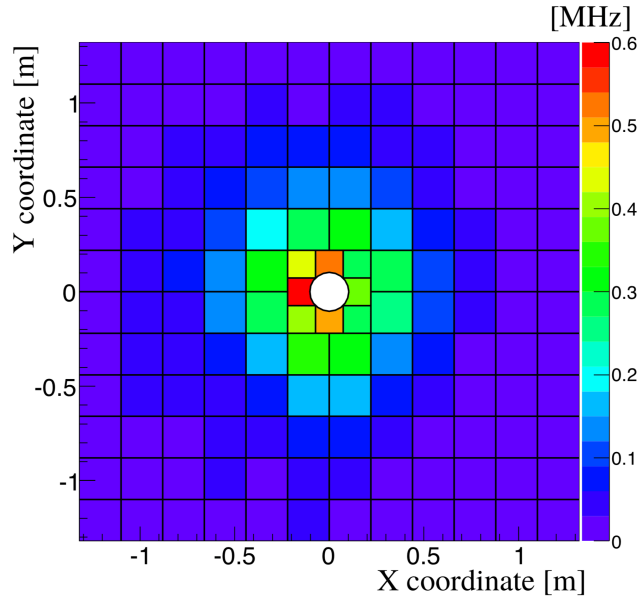


Figure 3.17: The **MUV3** tile rate for nominal beam conditions. A single 'hot-tile' is present in the inner tiles, with rate 3.2 MHz which is higher than the colour-scale [53].

rate increases as the tiles approach the beam-pipe, with one 'hot-tile' which counts at 3.2 MHz. This is due to the decay of beam pions to muons. Two **PMTs** are used to read-out each tile, placed directly behind the scintillator. The signals are fed to a constant-fraction discriminator (CFD) and are then sent to a TEL62 read-out board. In total, the **MUV3** has 296 read-out channels. The time-resolution on the individual **MUV3** channels is  $\sigma(t) \approx 600$  ps. With the average reconstruction efficiency above 99% (see Figure 6.3), the **MUV3** is the primary muon identification system at NA62.

### 3.4 Trigger and data acquisition (TDAQ)

One of the most important aspects of the experiment is the **Trigger and Data Acquisition (TDAQ)** system. The **TDAQ** refers to the series of processes which collect data and record it to disk for further analysis. In this section, an overview of the NA62 **TDAQ** is provided, including greater emphasis on the aspects that are heavily involved in the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  analysis described between Chapters 6 to 9.

### 3.4.1 The trigger logic

The NA62 experiment adopts a multi-level triggering strategy to ultimately reduce the downstream particle flux (10 MHz) to a rate limited by the available bandwidth for storing the data to tape ( $\approx 100$  KHz). A minimum bias approach is not feasible at this instantaneous rate, requiring a complicated **TDAQ** strategy to achieve stable data-acquisition [62]. The trigger levels are as follows:

- $L_0$ : A hardware-based trigger, evaluating sub-detector digitised signals before passing the decision to the **Level 0 Trigger Processor (L0TP)**. Each trigger 'primitive' can then be used to build a trigger 'mask', where the **L0TP** returns a decision based on the timing coincidence of multiple primitives. The maximum  $L_0$  hardware trigger rate is 1 MHz.
- $L_1$ : A trigger implemented in software, requiring loose selection criteria for specific sub-detectors based upon the signal decay of interest. Each 'algorithm' is applied sequentially, aiming to reduce the trigger rate to below 100 KHz.

The first two stages of the trigger were in operation for both 2016 and 2017. Figure 3.18 displays the flow of the **TDAQ** for each NA62 sub-detector. Only five de-

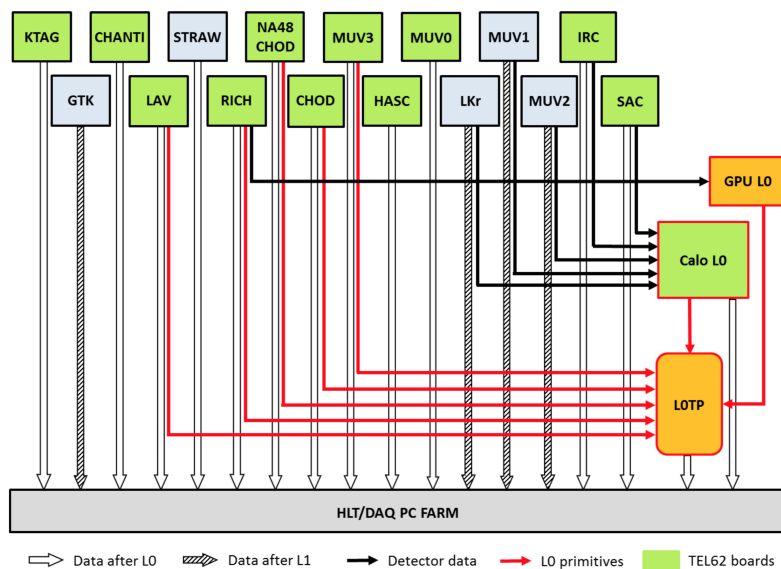


Figure 3.18: The general flow of the **TDAQ** for every sub-detector. The red arrows denote detectors which generate  $L_0$  primitives, whilst the black contribute to the  $L_0$  Calo initially. Detectors in green use TEL62 boards [63].

tectors which use TEL62 boards contribute to the  $L_0$ , whilst all calorimeters (**LKr**),

MUV1, MUV2) pass through the L<sub>0</sub> Calo. As the L<sub>1</sub> trigger is applied at the software level, it is possible to utilise any of the detectors. A brief description of the detectors used in the trigger in 2016 and 2017 is provided in Table 3.5.

Table 3.5: Brief description of the detectors which contribute to the L<sub>0</sub> and L<sub>1</sub>, including the type of criteria implemented.

Trigger Level	Detector	Type of Criteria
L <sub>0</sub>	RICH	Loose PMT multiplicity for charged tracks
	NewCHOD	Geometric and tile based multiplicity for charged tracks
	CHOD	Minimum bias multiplicity for charged tracks
	LAV	Fast $\gamma$ veto using LAV12
	Calorimeters	Total energy using LKr and MUV1,2
L <sub>1</sub>	KTAG	Loose $K^+$ time-stamping
	LAV	$\gamma$ veto using all LAV stations
	STRAW	Partial reconstruction of charged tracks

### 3.4.2 L<sub>0</sub> primitives

The L<sub>0</sub> hardware trigger is designed to use a small set of the fastest sub-detector signals to reduce the input rate from 10 MHz to below 1 MHz. Each sub-detector generates primitives, a time-stamped data-packet containing information on whether specific criteria were met according to the trigger firmware for each sub-detector. The majority of the detectors involved in the L<sub>0</sub> generate primitives using TEL62 boards equipped with TDCs, whilst the calorimeter primitives are produced using the CREAM readout system (Figure 3.18) [60]. Only the TEL62-generated primitives are important for the analysis presented in Chapter 8. Each TDC primitive consists of a 64-bit data block, which is segmented into 12.5(6.25) ns time slots in 2016(2017). If multiple primitives are generated at times within the resolution of the sub-detector, the primitives are merged. As all TDC triggers are based on hit multiplicity in the same event, hits must be sorted to build the time-clusters which produce the primitives. This is a feature which has become important when interpreting specific inefficiencies (see Chapter 7).

In order to build a L<sub>0</sub> 'mask', the L0TP identifies which primitives were simultaneously true within a 10 ns time-window, which is the reason for the time-sorting. If any of the primitive coincidences match with any of user-requested trigger masks,

the  $L_0$  trigger is fired and the event is passed to the  $L_1$  software stage. Two masks were important for this thesis, defined as the following combination of primitives:

- **Multi-track trigger:**  $\text{RICH} \cdot Q_X$ ,
- **Dimuon trigger:**  $\text{RICH} \cdot Q_X \cdot \text{MO}_2$ .

The primitives are described below,

- **RICH:** Low threshold-multiplicity requirement of at least two or more **RICH** **PMT** supercells (defined as 8-adjacent **PMT**s) fired within a 12.5(10) ns coincidence for 2016(2017). This is expected to be efficient for events containing positively charged tracks.
- **$Q_X$ :** The requirement of diagonally-opposite **NewCHOD** quadrants to contain hits within a 10 ns time-window. This primitive is designed to collect three-track decays.
- **$\text{MO}_2$ :** The requirement of signals in at least two or more outer **MUV3** pads that are within 10 ns of each other. This is designed to collect dimuon final states.

To perform rare three-track decay measurements, one must collect a normalisation sample of an abundant, well-understood decay with three-track topology, i.e  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  ( $K_{3\pi}$ ) through the Multi-track mask. The Dimuon mask is used to collect a sample of rare  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  ( $K_{\pi\mu\mu}$ ) decays, using the same logic apart from the additional  $\text{MO}_2$  condition. By using close to the same logic, most systematic effects can be vastly reduced by normalisation.

### 3.4.3 $L_1$ algorithms

After an event has passed the  $L_0$  stage, the  $L_1$  software trigger is put into effect on the PC farm. The algorithms are applied sequentially, meaning the rate is subsequently reduced at each stage. Again, there are three relevant algorithms which are applied at the  $L_1$  stage for both Multi-track and Dimuon triggers. All of the possible  $L_1$  algorithms in 2016 and 2017 data are described below,

- **KTAG:** Searches for at least five out of eight **KTAG** sectors to be in time-coincidence and within 10 ns of the  $L_0$  trigger time. This is to positively identify a beam kaon candidate consistent with the event time.
- **!LAV:** Used as a veto, the algorithm searches all twelve LAV stations for at least two hits with leading and trailing signal edges; the time of the leading signal edge of the hit must also be within 10 ns of the  $L_0$  trigger time. The motivation is to suppress the abundant  $K^+ \rightarrow \pi^+\pi^0$  background by detecting at least one photon from  $\pi^0 \rightarrow \gamma\gamma$ .
- **STRAW<sub>e</sub>:** Denoted 'e' for 'exotic', this algorithm uses a fast two-dimensional Hough transform and simple momentum evaluation to perform a crude track-reconstruction [64]. The trigger accepts events containing a negative-track. Negative tracks appear only in three-track decays and are typically a signature in exotic decay searches.

For the analysis described between Chapters [6] to [9], either all or at least a combination of the above  $L_1$  algorithms were implemented.

### 3.5 NA62 Framework

**NA62 Framework (NA62FW)** is a dedicated software framework which was developed specifically for the NA62 experiment. The framework is comprised of three independent packages:

- **NA62MC:** The **NA62 Monte Carlo (NA62MC)** framework is a full-detector simulation and is based on the Geant4 package [65]. NA62MC is used to produce large simulated event samples of the individual  $K^+$  modes. The properties of the decays are contained within event generators, which are implemented based on the most recent experimental or theoretical calculations. For each sample, the persistency of hits for each sub-detector is stored having traced the decay-products through the full geometry of the detector. This includes the simulation electromagnetic and hadronic interactions, plus multiple-scattering and the effect of magnetic fields. It is designed to be as close to real-life data-taking conditions as possible.

- **NA62Reconstruction:** The reconstruction package is based on ROOT and is modularised to reconstruct events and candidates from raw data or NA62MC for each sub-detector system [66]. NA62Reconstruction is also used to create an online monitor for data-acquisition, and is particularly useful during the run to spot systematic issues with any of the sub-detectors.
- **NA62Analysis:** Like NA62Reconstruction, NA62Analysis is based on ROOT and contained a series of programs and tools to perform analysis. For example, a NA62 'Analyzer' is a general tool written in C++ which can read in a list of events (the output of reconstruction) and select events based on their properties. This is the general method to implement a selection for NA62. In addition, a large number of NA62 specific tools have been developed to aid in data-analysis. Typical examples include the vertex builder (which searches for track-vertices in an event) and the downstream track container (which links together all sub-detector associations).

In this thesis, contributions to both the NA62MC and NA62Analysis frameworks have been made. For the analysis described from Chapter 6 onwards, all three packages were used extensively. The full documentation can be found in Ref. [67].

# 4

## Simulation of the MNP33 fringe field

In this chapter, the simulation of the MNP33 magnetic fringe field is described. Using a database of field measurements recorded in 2013, a three-dimensional map was created by fitting measurement planes with smooth two-dimensional polynomial surfaces. By extrapolating the parameters of the surfaces longitudinally, a map covering the full three-dimensional geometric acceptance of the vacuum tank was created. The accuracy of the map was calculated by direct comparison with raw measurements, and the systematic errors estimated to ensure that the map performs to within the same tolerances set by the tracking system for precision momentum measurements.

### 4.1 Motivation for mapping fields

The implementation of a highly accurate three-dimensional field map for NA62MC is mutually beneficial to all aspects of data-analysis at NA62. If the simulation provided only a simplistic representation of the field present in the detector (for example, a fixed momentum kick at the spectrometer magnet), the accuracy of the MC productions would be severely compromised with respect to the raw data in the following categories:

- Track/vertex trajectory and quality.
- Sub-detector illuminations downstream.

- Accuracy of signal and background acceptances.

If the tracking trajectory is inherently different between data and simulation, the illumination of the sub-detector systems downstream of the spectrometer will differ between **MC** and reality. As simulated samples are used to estimate signal acceptance, the discrepancy renders the calculation completely unreliable. This is especially detrimental for the precision measurement of rare or ultra-rare processes such as  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  or  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ . To avoid this, the simulated map must be three-dimensional to cover every possible trajectory through the field.

Three-distinct regions based upon the longitudinal  $Z$  coordinate are displayed in Figure 4.1. The regions are designated based on the nature of the field approaching

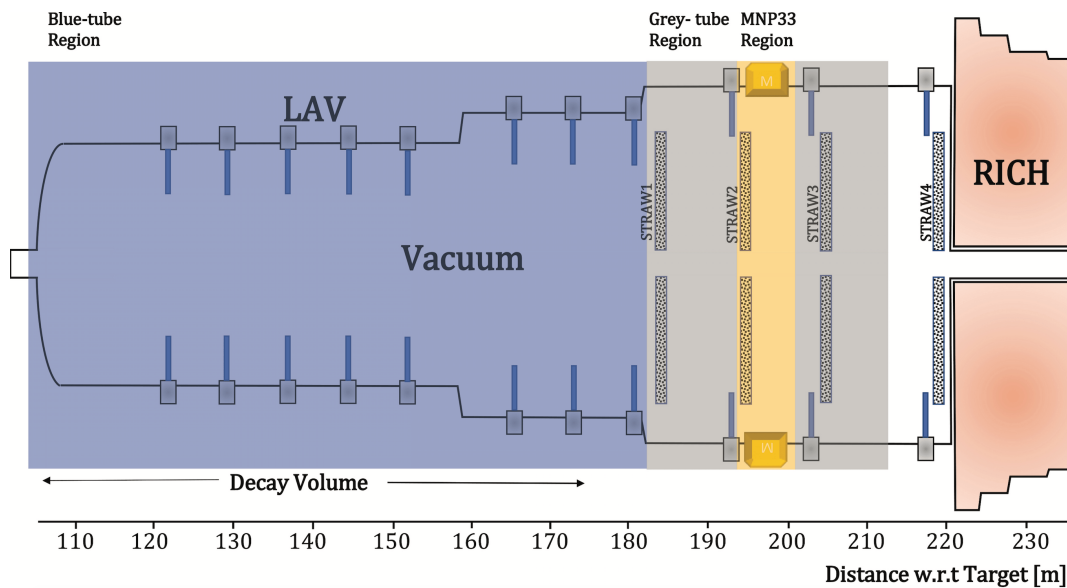


Figure 4.1: The NA62 decay volume, divided into the three natural field-map regions. The blue-tube region makes up the majority of the vacuum tank and contains LAV1-8. The grey-tube region sits symmetrically around the MNP33 magnet region, and houses the spectrometer stations.

before and after the MNP33 magnet, as explained below:

- **The blue-tube:** The region between  $104.458 \leq Z[\text{m}] < 181.175$  contains the majority of the vacuum tank, consisting of the first eight LAV stations interspaced by twelve 6 m long steel 'blue' tubes. The blue-tube is the first region that charged tracks traverse before reaching the spectrometer. The field is typically small in this region of  $\mathcal{O}(10 \mu\text{T})$  and is due to a combination of two indepen-



dent sources. Firstly, the steel tubes are reworked from the NA48/2 experiment, where residual magnetisation occurs as a result of work-hardening. A contribution of similar size is also expected due to the non-zero Earth field, known to be  $\approx 40 \mu\text{T}$  at the geographic location of the experiment [68].

- **The grey-tube:** The regions between  $181.175 \leq Z[\text{m}] < 193.363$  and  $200.907 < Z[\text{m}] \leq 212.200$  contain the grey-tube segments, situated either side of the MNP33 magnet. In this region, the fringe field of the MNP33 magnet is dominant. The field increases rapidly to values approaching  $\approx 1 \text{ mT}$ , and is maximal for  $Z$ -positions which are closest to the magnet.
- **MNP33 Dipole Magnet:** The region between  $193.363 \leq Z[\text{m}] \leq 200.907$  contains the MNP33 large aperture dipole magnet. The magnet is situated in between the two central spectrometer chambers, and was substantially rebuilt since the NA48/2 experiment. The magnet has a large rectangular cross-section of 4.4 wide by 4 m high. The physical dipole is 1.3 m long, centred at 196.995 m, and the iron yokes open by 2.4 m vertically and 3.2 m horizontally. The primary field points in the negative  $Y$ -dimension, and is substantially larger than the other components. Over the MNP33 volume, the integrated vertical field delivers  $I_{B_Y} = \int B_Y \cdot dZ \approx 0.9 \text{ Tm}$ , corresponding to a transverse momentum kick in the  $X$ -dimension by  $\approx 270 \text{ MeV}/c$ . The field strength at the centre of the magnet is 0.38 T.

At the time of this analysis, both the blue-tube and MNP33 field maps were developed and implemented into NA62MC [67]. The missing component is the fringe field of the grey-tubes, which is developed here. Defined using Euclidean geometry, the strategy to develop the field map is to use a large sample of field measurements at different  $(X, Y, Z)$  coordinates to build a three-dimensional field map, as follows:

- Fit each  $(X, Y)$  measurement  $Z$ -plane with a smooth two-dimensional curve which accurately describes the surface for each field orientation.
- Develop a criterion for identifying and removing rogue measurements, to improve the quality of the fit.

- Extract the parameters in each plane, and then fit the longitudinal dependence of each parameter with a suitable function.
- Link the map to the blue-tube and MNP33 field-maps already in place for [NA62MC](#), ensuring a slow and physical transition at the cross-over boundary.

The fringe field integral must be known to within the following precision:

$$\sigma_{I_B} = \Delta \int B \cdot dZ < 3 \times 10^{-4} \text{ Tm}, \quad (4.1)$$

as outlined in Ref [\[69\]](#), in order to have a negligible effect on the measurement of kinematic variables by the spectrometer. Notably, processes containing charged tracks are insensitive to the  $B_Z$  component of the magnetic field, as the NA62 beam-direction is roughly parallel to the  $Z$ -axis [\[70\]](#). Additionally, the MNP33 is anti-symmetric about the centre of the magnet resulting in the complete cancellation of the upstream and downstream  $B_Z$  components.

A description of the measurement procedure for the vacuum tubes is found in Section [4.2](#) followed by the construction of the field map in Section [4.3](#), where the surface fits, point deletion and map-linking is performed. Finally, the systematic errors of the map are estimated in Section [4.4](#).

## 4.2 Measurement sample

The field measurements were performed in the space of a single week in September 2013. A sensor comprised of three orthogonal Siemens KSY44 Hall probes was used to measure the field strength in each dimension, explained fully in Ref [\[69\]](#). In the vacuum tubes, measurements were performed to cover the forward decay ‘cone’ with half-angle of 20 mrad, ensuring complete coverage of the secondary  $K^+$  decay products between the [GTK3](#) and STRAW2. This was implemented using the grid structure displayed in Figure [4.2](#). The sensor was placed at each  $(X,Y)$  position in the grid-mount and a full set of measurements were recorded at specific values of  $Z$  in order to create a series of measurement planes. For the blue-tube, the spacing was typically 2 m between planes, with fifteen planes before 120 m recorded using only

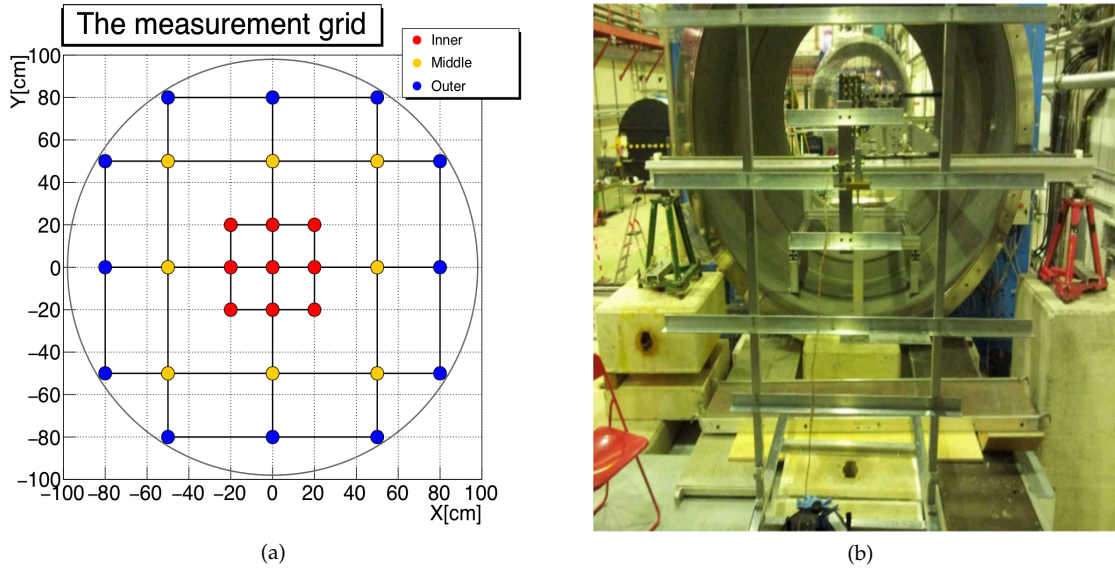


Figure 4.2: Magnetic Field measurement grids. (a) is a schematic of the grid used for the vacuum tube measurements where each coloured marker corresponds to a sensor measurement position. (b) is a photograph of the real mount [69].

the inner grid mount and twenty-five planes using both inner and middle grids for the  $Z$ -values after. Therefore, the number of blue-tube measurements (if a single set is recorded per plane), corresponds to  $N = 15 \cdot 9 + 25 \cdot (9 + 8) = 135 + 425 = 560 B_i$  components in total. The grey-tube region is measured similarly, creating measurement planes spaced by 30 cm and using the full grid structure. The smaller separation is to account for rapidly increasing field components. With fifteen additional  $Z$ -planes, the grey-tube consists of  $N = 15 \cdot (9 + 8 + 12) = 435 B_i$  components. The error associated to each measurement is  $5.6 \mu\text{T}$ , based on the precision of the hall-probes. Only the region upstream of the MNP33 is measured, due to technical limitations, with the intention that the field can be reflected symmetrically around the magnet centre for the downstream region.

### 4.2.1 Blue-tube planes

As the blue-tubes use the same measurement strategy, understanding the field composition in this region is beneficial for the fringe field map development. Figure 4.3 displays the  $Z$ -dependence of the  $B_X$  and  $B_Y$  components for the central grid position (0 cm, 0 cm) for the blue-tube, including the evaluation from the field map

developed in Ref. [71]. The most notable features are for  $B_X$ , where the field is ex-

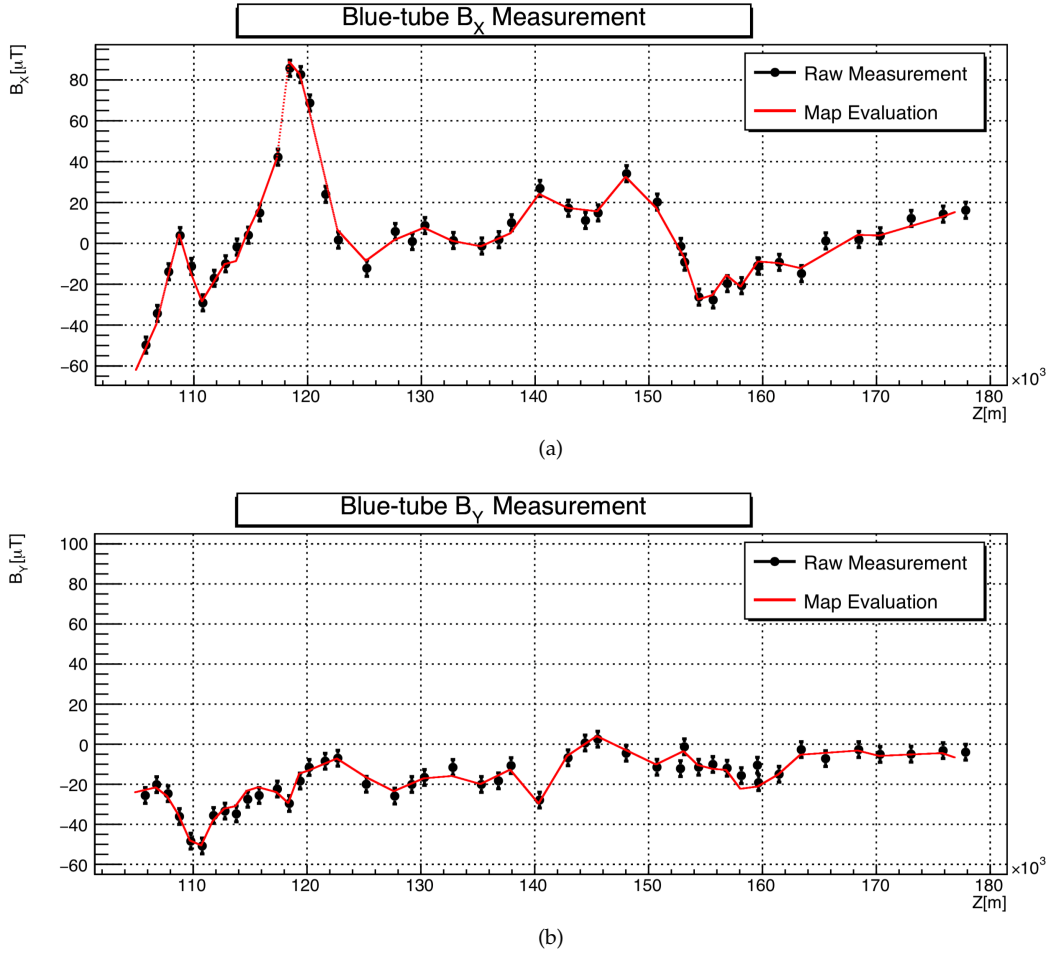


Figure 4.3:  $Z$ -dependence of the blue-tube field for central grid position (0 cm, 0 cm) with respect to (a)  $B_X$  and (b)  $B_Y$ . The red-line displays the blue-tube field-map evaluation of the field [69].

tremely complex.  $B_X$  increases to  $\approx 80 \mu\text{T}$  at  $Z = 120 \text{ m}$ , as the residual field is affected by the presence of man-hole covers and flanges where work-hardening significantly alters the magnetisation of dense material. Despite the low magnitude of the field components with respect to the MNP33, the considerable length of the tank leads to significant trajectory modification for charged tracks. For example, the central position raw field-integrals are  $\int B_X \cdot dZ \approx 411 \mu\text{Tm}$  and  $\int B_Y \cdot dZ \approx -1057 \mu\text{Tm}$ , which result in certain kinematic properties as displayed in Figure 4.4. The resolution on the observable invariant mass for  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  ( $K_{3\pi}$ ) is altered significantly when including the field map into NA62MC compared to the corrected case, equivalent to the map being turned off. A tool was developed as described in Ref. [71] to correct for the field modification. The corrections applied improve the

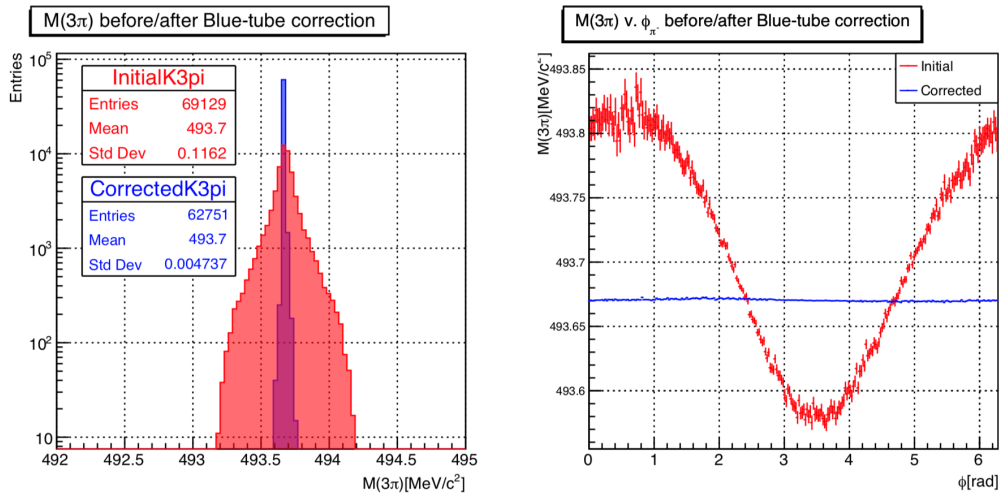


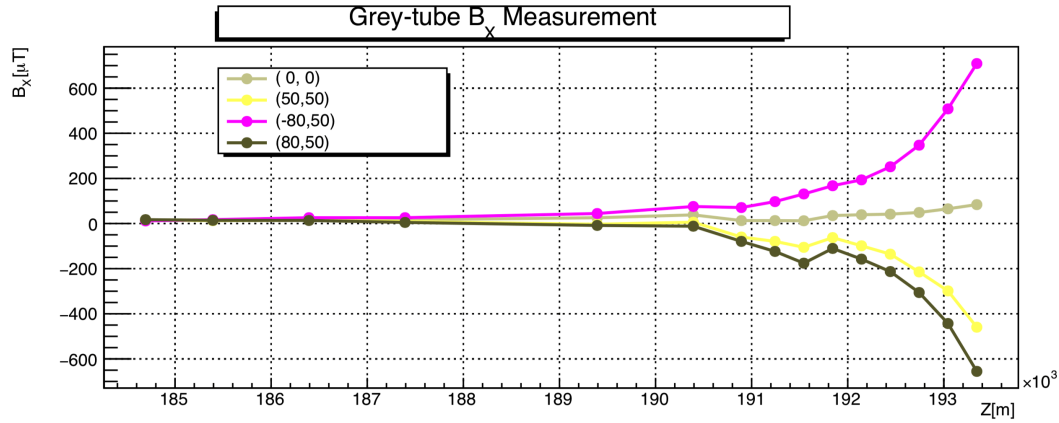
Figure 4.4: Left: The  $(K_{3\pi})$  invariant mass simulated in **NA62MC** in the presence of the blue-tube field. Right: the variation in the invariant mass with respect to the azimuthal angle  $\phi \approx P_X/P_Y$  of the  $\pi^-$ . The distributions seen in red correspond to the effect from the blue-tube field, whereas blue is after correcting the field using the tool.

resolution and the azimuthal angle variation of the mass to far below the experimental requirements.

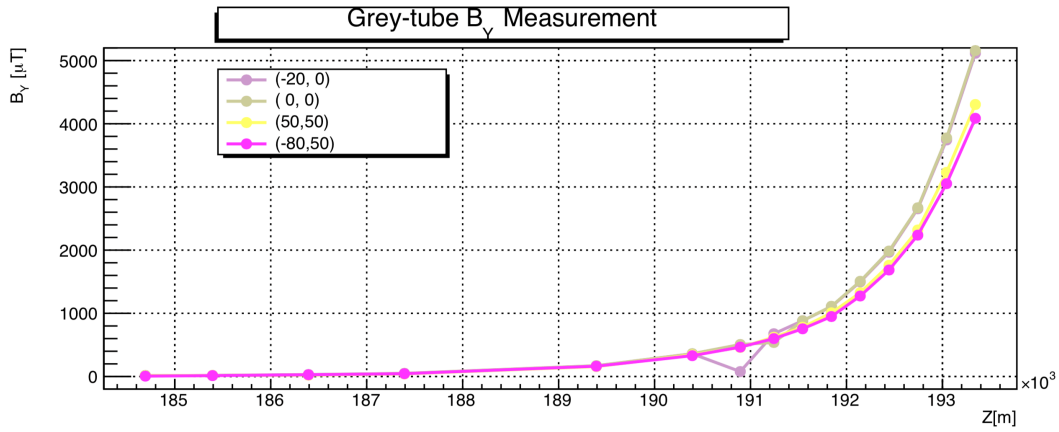
## 4.2.2 Grey-tube planes

Figure [4.5](#) displays the  $Z$ -dependence of the  $B_X$  and  $B_Y$  components for a few specifically chosen grid-positions in the grey-tube region. As expected, most grid-positions display a rapid increase in field magnitude with  $Z$ . As  $B_Y$  is the primary field-component of the MNP33, all grid-positions increase up to  $\approx 5000 \mu\text{T}$  with the largest increase for positions at the centre of the grid. The  $B_X$  component is a second-order field, where central grid-positions do not increase rapidly unlike those on the edge of the grid. Measurements for  $B_X$  are within  $\pm 800 \mu\text{T}$  at the final  $Z$ -plane where  $(X, Y) = (+, +)$  and  $(-, -)$  grid-positions have negative magnitudes and  $(X, Y) = (+, -), (-, +)$  positive. For comparison with the blue-tube, the raw field integrals in the grey-tube for the central position are  $\int B_X \cdot dZ \approx 224 \mu\text{Tm}$  and  $\int B_Y \cdot dZ \approx 5438 \mu\text{Tm}$ , noting that the maximal contribution for  $B_X$  occurs at the corners of the grid.

The  $Z$ -dependence in Figure [4.5](#) shows many interesting features. In  $B_Y$ , one grid



(a)



(b)

Figure 4.5:  $Z$ -dependence of the grey-tube field for multiple grid positions  $(X[\text{cm}], Y[\text{cm}])$  with respect to (a)  $B_X$  and (b)  $B_Y$ .

measurement at  $Z = 190.863$  m is a clear anomaly. In  $B_X$ , several grid positions show a distinct non-uniformity in the region between  $191 < Z[\text{m}] < 192$  m of  $\approx 50$   $\mu\text{T}$ . There are three possibilities for these effects,

- **Hysteresis:** Several MNP33 trips occurred during the measurement procedure. If the MNP33 magnet trips, the current supplied to the coils must be ramped slowly to approach back to the design field-strength. This can cause the magnet to realign and become magnetised differently from the previous conditions and the field to lag as the current reaches the nominal value. As hysteresis such as this is expected to be at the level of  $1/1000$ , the effect is of the  $\mathcal{O}(1 \mu\text{T})$  for the fringe field and is therefore unlikely to be the cause.
- **Magnetisation:** In the blue-tube region, the magnetisation of reworked mate-

rial led to clear non-uniformities in the field map at certain positions in  $Z$ . In this regions, the residual field changed by up to  $\mathcal{O}(100 \mu\text{T})$ , a similar magnitude to the distortion in Figure 4.5. This suggests that magnetisation due to reworking is also a major contribution for the grey-tubes.

- **Human error:** The single anomaly observed in  $B_Y$  is expected to be due to human error, as the magnitude is a factor  $\approx 10$  smaller than measurements for close-grid positions at the same  $Z$ .

These issues highlight the need for some quality control when constructing the field map. The measurement plane surfaces can also be plotted, as displayed in Figure 4.6. Here, both  $B_X$  and  $B_Y$  are displayed for three out of the fifteen planes

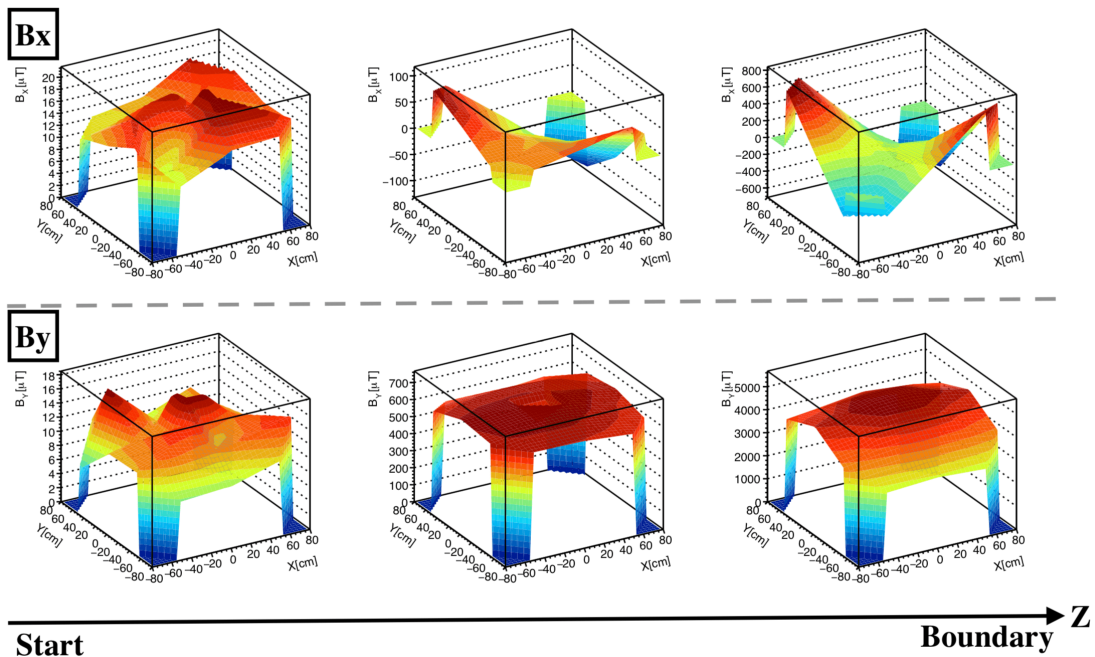


Figure 4.6: The field surfaces for planes at the start, middle and end of the grey-tube region. The top row is for  $B_X$  and bottom for  $B_Y$ . The shape in both cases transitions to physical shapes at high  $Z$  from the blue-tube-like fluctuations observed at low  $Z$ .

corresponding to the start, middle and end of the grey-tube measurement region. The typical magnitudes at the beginning are low and blue-tube-like, but as the field increases rapidly with  $Z$ , the shape transforms into real physical shapes. By the end of the grey-tube,  $B_X$  is distinctly saddle-shaped and  $B_Y$  appears to be a two-dimensional elliptical paraboloid, as the stray fringe field is now completely dominant with respect to the residual field.

### 4.3 Building the field-map

The measurements are now used to build the three-dimensional fringe field map. Each plane is fitted with the following two-dimensional polynomial,

$$B(X, Y) = B_0 + \alpha_x X + \alpha_y Y + \alpha_{xx} X^2 + \alpha_{yy} Y^2 + \alpha_{xy} XY, \quad (4.2)$$

where  $B(X, Y)$  is the evaluated field at a given  $(X, Y)$  position,  $B_0$  is the constant term of the field fit, and the  $\alpha$  parameters correspond to the constants of the linear or quadratic terms with respect to the coordinate space. The lowest-order polynomial which returned a reasonable description was chosen, by fitting each  $z$ -plane with polynomials of increasing order to find the best agreement overall. The shapes displayed in Figure 4.6 display paraboloidal surfaces which can be described mathematically with two-dimensional polynomials. As a set of parameters is obtained per measurement plane, the plane-parameter dependence with  $Z$  can be modelled. The three-dimensional field is therefore,

$$B(X, Y, Z) = B_0(Z) + \alpha_x(Z)X + \alpha_y(Z)Y + \alpha_{xx}(Z)X^2 + \alpha_{yy}(Z)Y^2 + \alpha_{xy}(Z)XY, \quad (4.3)$$

where each of parameters of Equation 4.2 is replaced by a  $Z$ -dependent smooth curve extracted by fitting. With this, the field can be known anywhere and the map becomes three-dimensional. To improve the accuracy of the map, care must be taken to handle points which are statistically rogue; i.e systematically shifted from expectation. The field must also then be linked to the blue-tube and MNP33 maps. This is easier for the blue-tube which uses the same measurement technique, whereas the MNP33 uses a factor  $\approx 10$  more measurements and an entirely different grid structure.

#### 4.3.1 Measurement deletion

Using Equation 4.2, each of the fifteen measurement planes is fitted to create a smooth surface to describe the field-shape which can then be used for interpolation. Figure 4.7 displays the two-dimensional  $(X, Y)$  data-fit residual for three specifically



chosen planes.

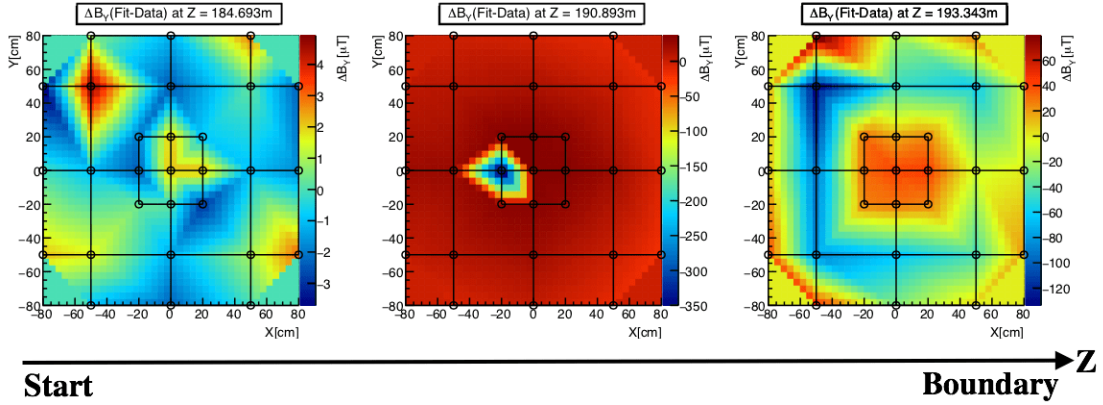


Figure 4.7:  $B_Y$  data-fit residuals for  $Z$ -planes at the start, middle and end of the grey-tube. The measurement grid structure has been drawn for perspective. The central plot is the  $B_Y$  plane where a clear anomaly was observed in Figure 4.5

At the start of the grey-tube,  $\Delta B_Y$  is small across the grid and is consistent with the  $5.6 \mu\text{T}$  error assigned from the sensor. By the end, the magnitude of the field increases by a factor  $\approx 100$  and so the differences are far greater than the measurement error. The central case of Figure 4.7 corresponds to the anomaly observed in Figure 4.5, suspected to be due to human error. The point pulls the fit and worsens the agreement for the other points in the grid. An algorithm was designed to handle systematically suspicious points as follows:

- Fit measurement plane with Equation 4.2.
- Delete any grid-position where the data-fit residual  $\Delta B_i$  deviates by more than  $\pm 100 \mu\text{T}$ , and re-perform the fit.

The algorithm uses residuals instead of pull distributions because the error assigned to each point is identical. The value of  $100 \mu\text{T}$  is chosen close to double the magnitude of the deviation observed due to magnetisation in the blue-tubes. The algorithm is limited to delete only one-point per plane to keep the measurement sample as large as possible. Regardless, point deletion does not drastically affect the overall fit. Figure 4.8 displays the difference between the  $Z$ -plane fit before and after the deletion of the rogue measurement (denoted by an 'X') from the central plane displayed in Figure 4.7. The maximal change of the fit occurred closest to the point that

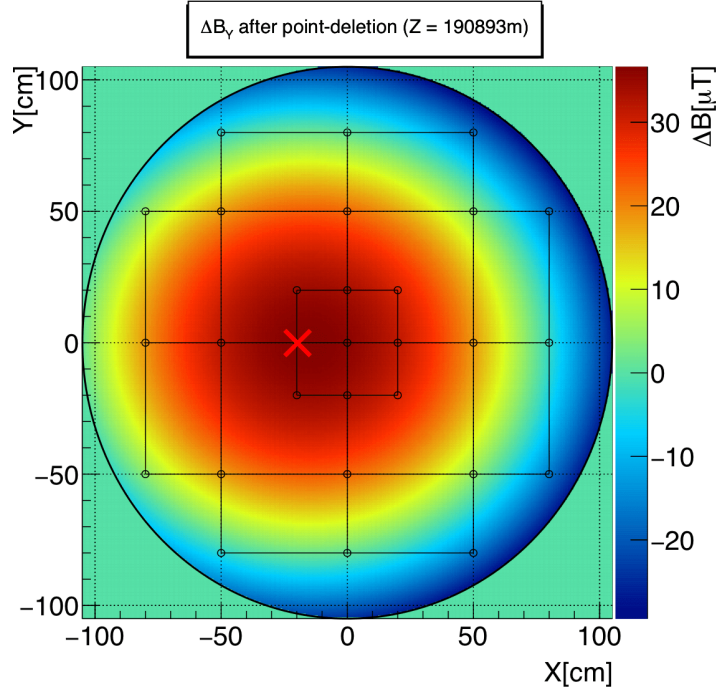


Figure 4.8: Difference in the 2D  $B_Y$  fit before and after deletion of measurement point (-20 cm, 0 cm) for  $Z$ -plane 190.893 m. The outline of the plot corresponds to the grey-tube inner radius.

was deleted, but even in this extreme case (the largest deviation by any point in the sample), the typical  $\Delta B \approx 30 \mu\text{T}$  is small with respect to the average field magnitude of the plane ( $500 \mu\text{T}$ ). Table 4.1 displays the four measurements which were deleted by the procedure, including one at the final measurement plane for both  $B_X$  and  $B_Y$ .

Table 4.1: Summary of the measurement-deletion algorithm. Four points were deleted in total.

Field	$Z$ -Plane[m]	Coordinate (X[cm],Y[cm])	Fit-Data $\delta B[\mu\text{T}]$
$B_X$	193.343	(-50,50)	103.2
$B_Y$	190.893	(-20,0)	392.7
$B_Y$	191.243	(0,0)	121.9
$B_Y$	193.343	(-50,50)	133.6

Additionally, each measurement plane as a whole was investigated for systematic effects. As observed in Figure 4.5 of Section 4.2, several grid-positions show a distortion in  $B_X$  between  $191 < Z[\text{m}] < 192$ . The cause is suspected to be due to magnetisation and creates a kink in the otherwise smooth  $Z$ -dependence. To observe the effect of this kink with respect to a smooth curve, the raw field-integrals for  $B_X$  were calculated for the entire fringe field region with and without the two planes

for each grid position. The difference between the grid-position integrals is calculated and the residual fitted with a Gaussian curve in Figure 4.9. In general, the

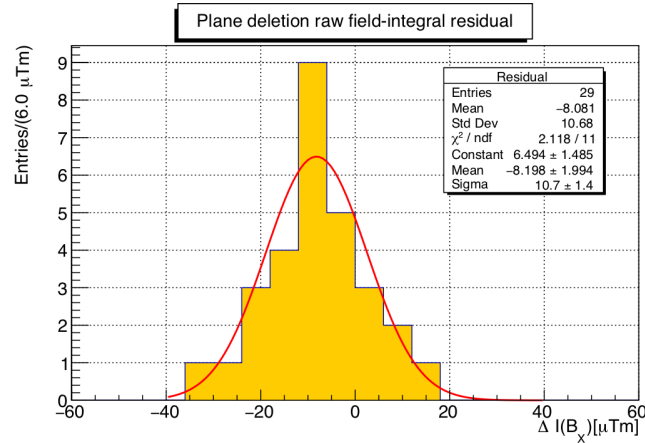


Figure 4.9: The residual of the field integral for  $B_X$  at each grid-position before and after deleting two planes, fitted with a Gaussian distribution. The impact of the deletion is relatively small.

larger deviations come from positions furthest from the centre. The deletion of the two planes has a small effect on the raw  $B_X$  field-integral, shifting the value by  $\approx -8 \mu\text{Tm}$  on average for all grid positions. This corresponds to less than 3% of the total field-integral for  $B_X$  at the central-grid where the field magnitude is smallest. The shift is small because the first plane 191.123 m and second 191.153 m deviate in opposite directions, leading to a partial cancellation. For simplicity, the two planes were permanently removed for  $B_X$  as the impact on the field-integral is minuscule with respect to the performance requirements.

### 4.3.2 Plane interpolation

After the quality-control measures in the previous section, each  $B_i$  measurement plane is fitted for the final time with the two-dimensional polynomial described by Equation 4.2. In order to have a three-dimensional field-map, the  $Z$ -dependence of each of the six parameters must be modelled, performed using polynomial fits of the form,

$$f_z = p_0 + \sum_{n=1}^b p_n (z - z_0)^n \quad (4.4)$$

where  $p_0$  is a constant parameter of the polynomial,  $z_0$  is the  $Z$ -plane value at the end of the blue-tube region (183.331 m), and  $b$  is the smallest number required to obtain an adequate fit, in order to keep the number of terms as small as possible. Due the rapidly-increasing nature of the fringe field, for some of parameters it was necessary to modify Equation 4.4 to include an exponential term,

$$f_z = p_0 + \gamma(z - z_0)e^{\delta(z_E - z)} + \sum_{n=1}^b p_n(z - z_0)^n \quad (4.5)$$

where  $\gamma, \delta$  are additional fit parameters, and  $z_E = 193.343$  m is the  $Z$ -plane at the end of the grey-tube. By multiplying non-constant terms by  $(z - z_0)$ , the fringe field map can be easily matched to the blue-tube. The blue-tube map is implemented with a similar method, by fitting measurement planes with two-dimensional polynomial and interpolating the parameters. Simply, the constant terms in Equation 4.4 and 4.5 can be fixed to match those from the blue-tube map  $p_0^{BT}$ , leading to a seamless transition at the boundary  $z_0$ .

Figure 4.10 and 4.11 displays the six  $\alpha$  surface parameter distributions with respect to  $Z$  for the  $B_X$  and  $B_Y$  field. The lower magnitude of the  $B_X$  field with respect to the  $5.6 \mu\text{T}$  measurement error leads to a better fit performance than for  $B_Y$ , and clearly the parameter dependence with  $Z$  is not always smooth. The region around 191 m, which was the focus of measurement quality previously, displays some features that are hard to describe fully with polynomial curves. However, the regions of rapid increase are well described by the exponential-polynomial fits in all parameters, such that the field evaluation is expected to be reliable. Figure 4.12 contains the  $Z$ -dependence of the multiple grid-positions displayed in Figure 4.5 but now containing the fringe field simulation. The agreement with the data is excellent. The field integrals at the central grid position (0 cm, 0 cm) for the simulation are  $\int B_X \cdot dZ \approx 220 \mu\text{Tm}$  and  $\int B_Y \cdot dZ \approx 5448 \mu\text{Tm}$ , which differs by less than 2%(1%) for  $B_X(B_Y)$  from the raw field measurements. The comparison can be made for all grid positions with the average deviation calculated to be about the same value.

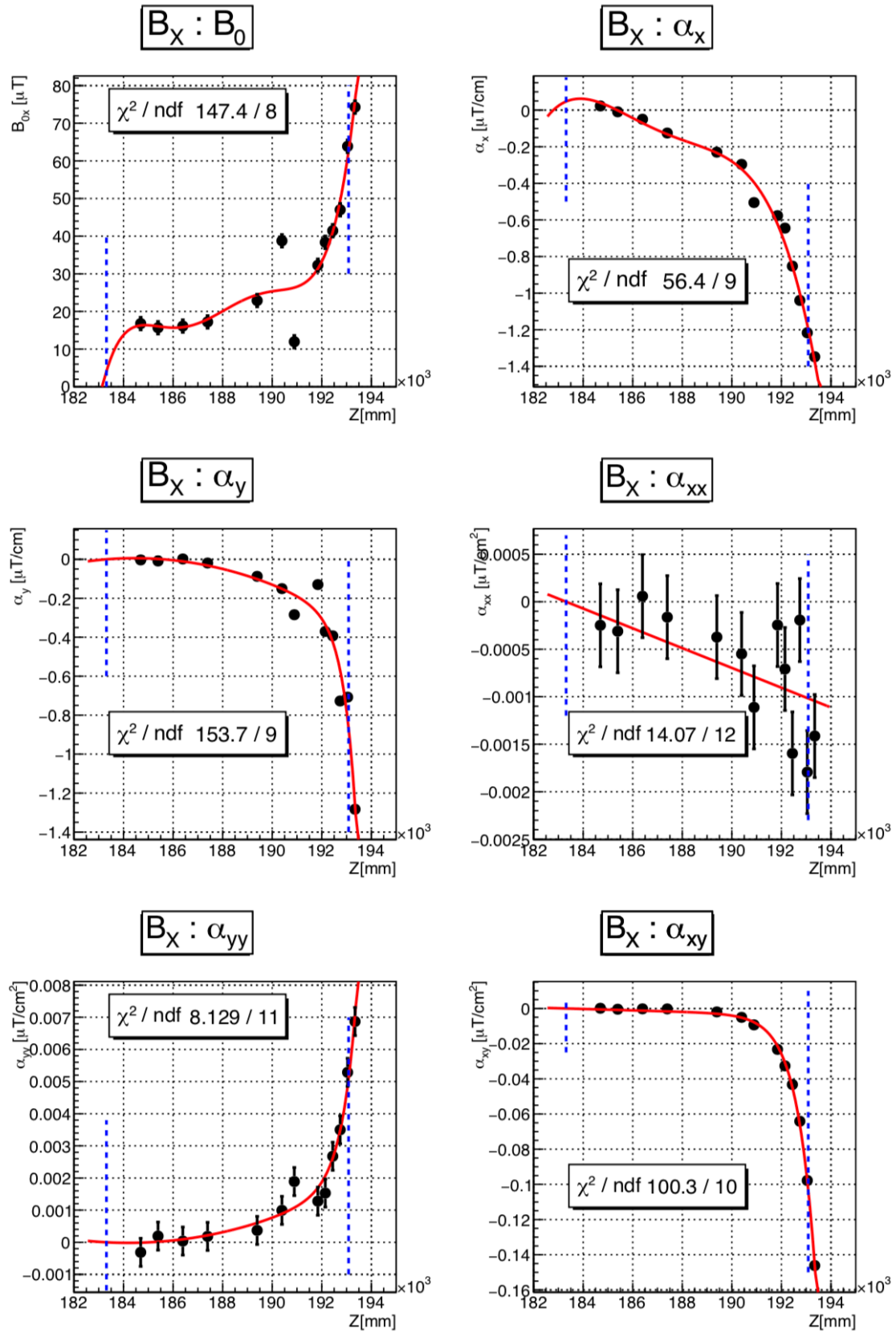


Figure 4.10: The six surface-parameter  $Z$ -dependence fits for  $B_X$ . The blue-lines denote the start and end of the fringe field regions, the black dots correspond to the values of the parameter at each  $z$  coordinate and the red line is the final fit.

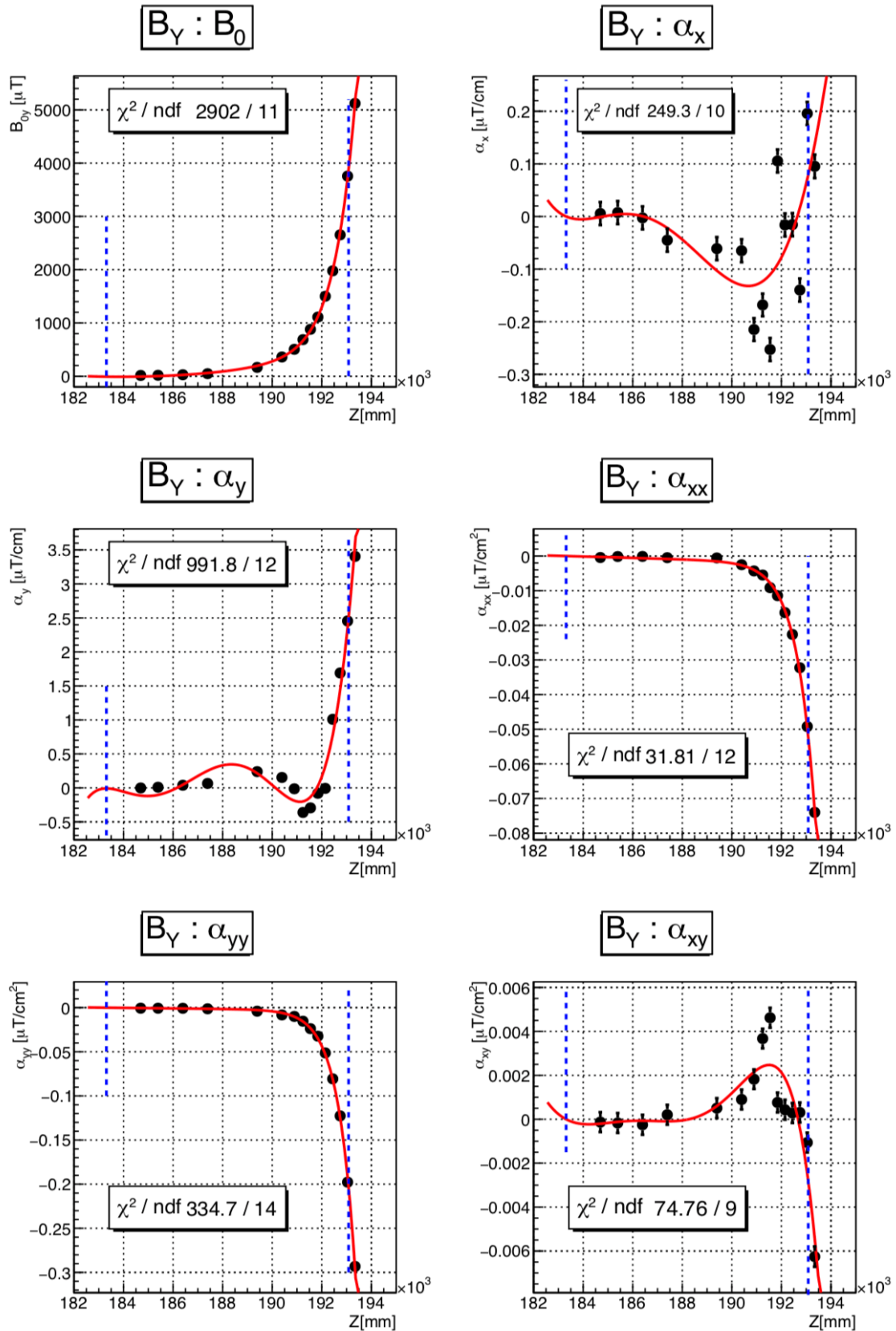


Figure 4.11: The six surface-parameter  $Z$ -dependence fits for  $B_Y$ . The blue-lines denote the start and end of the fringe field regions, the black dots correspond to the values of the parameter at each  $z$  coordinate and the red line is the final fit.

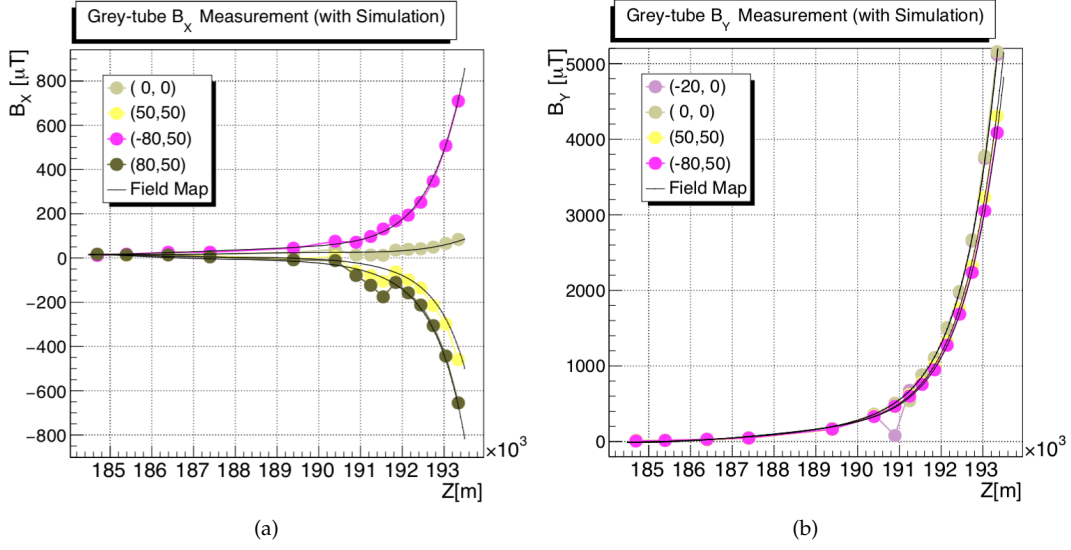


Figure 4.12:  $Z$ -dependence of the grey-tube field now including the field-map simulation for multiple grid positions  $(X[\text{cm}], Y[\text{cm}])$  with respect to (a)  $B_x$  and (b)  $B_y$ .

### 4.3.3 Linking the fringe map with MNP33

As described by Ref. [72], the MNP33 field was measured using a far more granular grid-structure with respect to the vacuum tubes, with ultra-fine 80 mm steps in  $Z$  to build up over 3000 measurements. The map implemented in NA62MC is also constructed differently, using a direct trilinear interpolation approach on the measurement sample instead of any smooth fitting techniques. The high-accuracy of this approach comes at the cost of CPU performance, with the MNP33 map responsible for up to 20% of the total CPU time per event in the MC generation [73].

Though there is no simple method to connect the maps, the final blue-line marked at  $Z = 193.083$  m in the parameter fits of Figure 4.10 and 4.11 corresponds to the start of the MNP33 field-map region; this occurs within the grey-tube volume. In this region, measurements were performed using both the vacuum tube and MNP33 grid structures. Figure 4.13 contains the residual difference between the fringe field and MNP33 maps at the boundary for both  $B_x$  and  $B_y$ . There is a clear discrepancy for the  $B_y$  component between the two field maps, where the fringe field is far larger by up to 500  $\mu\text{T}$  at the boundary. As the MNP33 measurement procedure was far more extensive than for the grey-tube, the MNP33 map is considered to

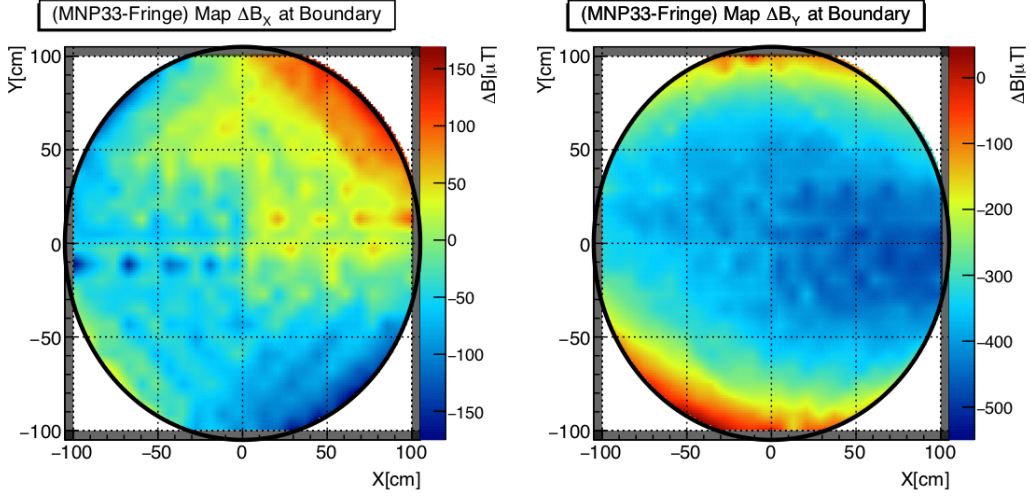


Figure 4.13: Difference in fringe field and MNP33 maps at the cross-over boundary for left,  $B_X$  and right,  $B_Y$ . The grey-border corresponds to the MNP33  $(X, Y)$  region, simulated between  $(\pm 100 \text{ cm}, \pm 100 \text{ cm})$ , and the black-line the inner radius of the grey-tube. A clear discrepancy can be observed in  $B_Y$ , with a difference in the central region up to  $-500 \mu\text{T}$ .

be more reliable, and so the decision was taken to globally re-scale the fringe field map in order to match with the MNP33. To determine the optimal scale fraction  $f_{FF}$ , the difference between the fringe field and MNP33 field maps was evaluated at all MNP33 grid positions. By multiplying the fringe field evaluation by  $f_{FF}$  and performing a fine-scan, the ideal scale-fraction was found when the mean of the residual difference was equivalent to zero for  $B_Y$ . Figure 4.14 displays the result of the fine-scan in  $f_{FF}$ . The scaling constant is therefore,

$$f_{FF} = 0.897. \quad (4.6)$$

which will shrink the field integrals by 10%. To apply the scale-factor, a simple modification is made to the fits described by Equations 4.4 and 4.5. All terms are multiplied by  $f_{FF}$ , except for the constant terms as to preserve the blue-tube matching. The scaling feature is desirable, as the MNP33 magnet can change current, such that the magnitude (but not shape) of the field can alter depending on the data-taking configuration of NA62. The MC scale-factor applied to the field is therefore made in factors of  $f_{FF}$ . Figure 4.15 shows the differences between the two maps at the boundary post-scaling. While the agreement improves and residual difference in  $B_Y$  is now more symmetric in  $(X, Y)$ , a smooth transition across the entire face



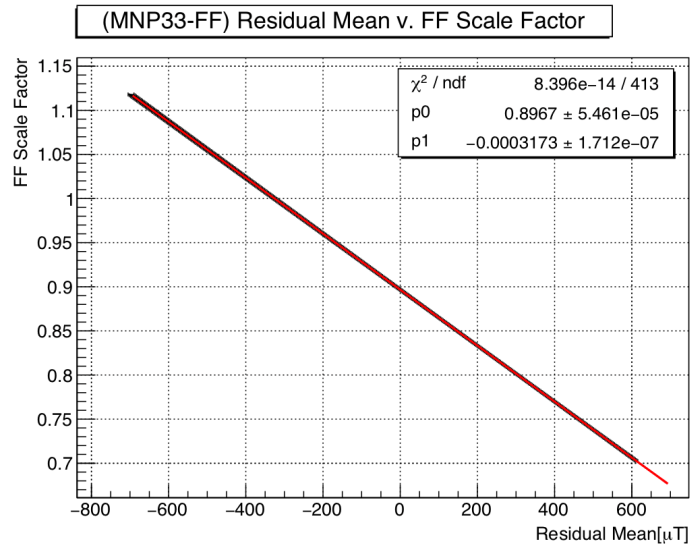


Figure 4.14: The fringe field scale-factor fine-scan plotted against the residual mean of the MNP33/Fringe field. The ideal scale-fraction is situated at zero on the  $X$ -axis

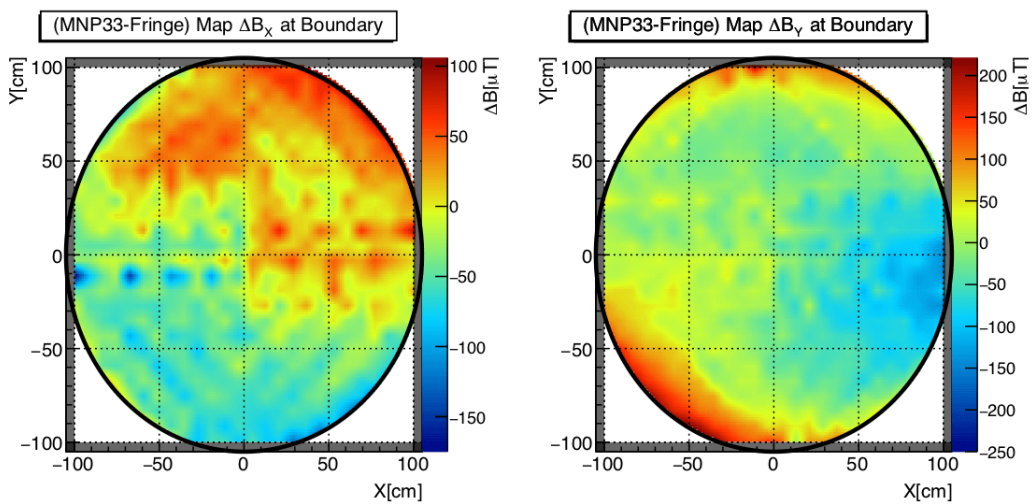


Figure 4.15: Difference in fringe field and MNP33 maps at the cross-over boundary for left,  $B_X$  and right,  $B_Y$  after scaling by Equation [4.6](#). The difference is now more symmetrical, but is not smooth or exact.

is impossible with only a re-scaling. In order to avoid field discontinuities at the boundary, a buffer region was introduced to slowly interpolate between the maps. The region is implemented in the following fashion,

- Define the start of the buffer region  $Z_B = 192.683$  m, 0.4 m before the MNP33 map begins. This is motivated by the typical gap between measurements planes in the grey-tube region.
- The field evaluated at  $Z \leq Z_B$  uses the standard fringe field map.
- In the region  $Z_B < Z < Z_{\text{MNP33}}$ , the field values are interpolated linearly between the fringe field map at  $Z_B$  and the MNP33 map at  $Z_{\text{MNP33}}$ .
- The field evaluated at  $Z > Z_{\text{MNP33}}$  uses the standard MNP33 field map.

The justification for the buffer region is contained in Figure 4.16, using the ( $\pm 20$  cm,  $\pm 20$  cm) grid positions that are common to both the vacuum tube and MNP33 measurement approaches. The disagreement between the fringe field and MNP33 raw measurements can be seen most clearly for  $B_X$  where the magnitudes are small. If a buffer region was not used, the field would be discontinuous at the boundary. To understand the impact of the buffer region on the field integral, the calculation must be modified slightly. As the MNP33 map starts at 193.083 m, the final grey-tube measurement plane at 193.383 m is bypassed and is transferred to the MNP33 field integral instead, leading to a 25% decrease overall for central  $B_Y$  values. The scaling of the measurements and map by  $f_{FF}$  also decreases the overall field integral by 10%. Table 4.2 compares the raw measurement integral to the field map integrals with and without the buffer zone for central grid position (0 cm, 0 cm). The level of

Table 4.2: Comparing the raw and field map integrals for central grid position (0 cm, 0 cm) for the final fringe-field map.

Integral Type	$\int B_X \cdot dZ$ [ $\mu\text{Tm}$ ]	$\int B_Y \cdot dZ$ [ $\mu\text{Tm}$ ]
Raw Measurements	181	3676
Field Map	182	3691
Field Map (Buffer)	184	3695

difference between the field map integral with and without the buffer zone is similar for all grid positions, with the buffer case 2(3)  $\mu\text{Tm}$  larger for  $B_X(B_Y)$  on average.

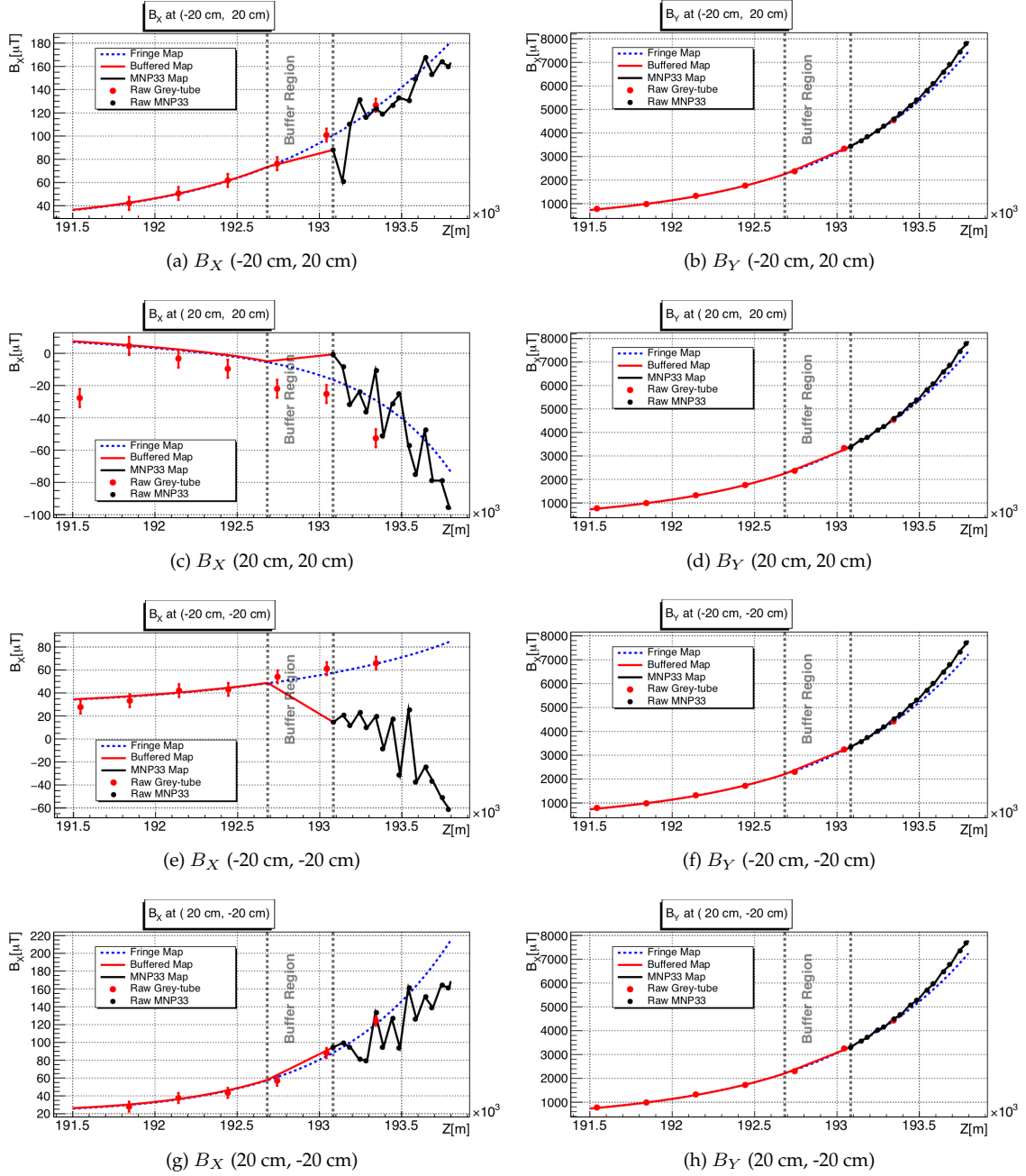


Figure 4.16: Displaying the buffer region by comparing both fringe field and MNP33 maps to the raw measurements for four grid positions. The left column contains  $B_X$  and right  $B_Y$ . The red line is the buffered fringe field map, including blue-line which is the version without the buffer. The black line is the MNP33 map, whilst the black points are the raw MNP33 measurements. The agreement with the raw data is excellent after scaling by  $f_{FF}$  for all fringe field cases. The buffer region is most clearly seen for the low magnitude  $B_X$  fields, where the boundary would be discontinuous if using the blue-line.

### 4.3.4 Fringe field downstream of MNP33

The fringe field data set was only produced upstream of the MNP33, relying on the observation that the field is highly symmetric around the centre of the magnet in the  $Z$ -axis. In addition, it was not technically possible to install the measurement grid downstream after the magnet had been commissioned. Therefore, the fringe field map had to be reflected about the centre of the MNP33 magnet at  $Z_m = 196.995$  m and matched to the MNP33 field. The only real difference for the downstream fringe field is the introduction of a final buffer-zone at  $Z = 210.679$  m to slowly reduce the field components to zero over 1.4 m. The choice of zero instead of the earth field, for example, is entirely arbitrary and the difference between choosing either approach to the magnitude of the downstream field integral is  $\approx 5 \mu\text{Tm}$ , less than 1% of the downstream fringe-field for both  $B_X$  and  $B_Y$ .

## 4.4 Overall Performance

### 4.4.1 Fringe field systematics

To estimate the systematic errors of the field map, we can first consider the measurement errors provided by the authors of Ref. [69]. Here, the systematics are calculated for measurement and sampling errors, where they note that the largest systematic uncertainty for the fringe field is due to the mixing of the  $B_Y$  component of the field into  $B_X$  via sensor miscalibration. By considering all sources for the important transverse components only, the total data measurement systematic is given as

$$\sigma_{I_{B_X}}^{\text{Data}} = 120 \mu\text{Tm}, \quad (4.7)$$

$$\sigma_{I_{B_Y}}^{\text{Data}} = 40 \mu\text{Tm}. \quad (4.8)$$

For the simulated map developed here, the contribution to the field integrals due to the difference between the field map and the measured data must be considered. This is estimated by computing the difference between the raw measurements and the field map evaluation for each  $Z$ -plane, with a single value per grid position. Figure 4.17 displays the mean deviation for  $B_X$  and  $B_Y$  in the fringe field region, ex-

tracted by fitting the residual for each plane with a log-likelihood Gaussian fit and noting that the data values were scaled by  $f_{FF}$  to account for the overall disagreement between the fringe field and MNP33 maps.

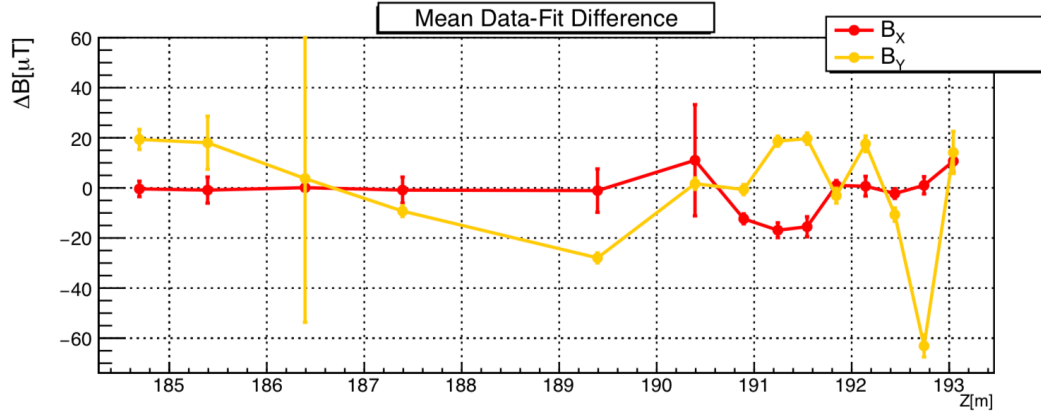


Figure 4.17: The average field deviation between the final field map and the measurements at each  $Z$ -plane. The larger magnitude of  $B_Y$  on average leads to larger deviations overall.

The total error in the field map components is estimated as the sum in quadrature of each plane deviation, ignoring the final fringe field measurement plane at 193.331 m which was not used in the map. This leads to the overall field systematic for the fringe map,

$$\sigma_{B_X}^{\text{FF}} = 30 \mu\text{T}, \quad (4.9)$$

$$\sigma_{B_Y}^{\text{FF}} = 83 \mu\text{T}, \quad (4.10)$$

To translate these errors into an overall field integral systematic, the following method was adopted:

- Create a modified field evaluation: sum the average field deviation and the field-map evaluation for all grid-positions at each  $Z$ -plane (extracted from Figure 4.17).
- In order estimate the systematic using a trapezium rule integration, define the following interval:

$$I_{\text{interval}} = \left( \frac{B_{P_s} + B_{P_e}}{2} \right) \Delta Z_{se}. \quad (4.11)$$

Here  $B_{P_{s,e}}$  is the evaluated field for the start and end planes of the interval

and  $\Delta_{Z_{se}}$  is the distance in  $Z$  between the two interval planes. For fourteen grey-tube planes, there are thirteen intervals that must be computed in order to calculate the total field integral.

- Evaluate the difference in the interval integral for the modified and standard fields. The systematic error is the sum in quadrature of this difference for all thirteen intervals.

The overall error is multiplied by a factor of two as the fringe field has an upstream and downstream component,

$$\sigma_{I_{B_X}}^{\text{FF}} = 16 \mu\text{Tm}, \quad (4.12)$$

$$\sigma_{I_{B_Y}}^{\text{FF}} = 70 \mu\text{Tm}. \quad (4.13)$$

As a sanity check, the difference between the raw field and field map integrals were instead estimated for each grid-position for the whole grey-tube. The result is displayed for both  $B_X$  and  $B_Y$  in Figure 4.18. The average field integral deviation

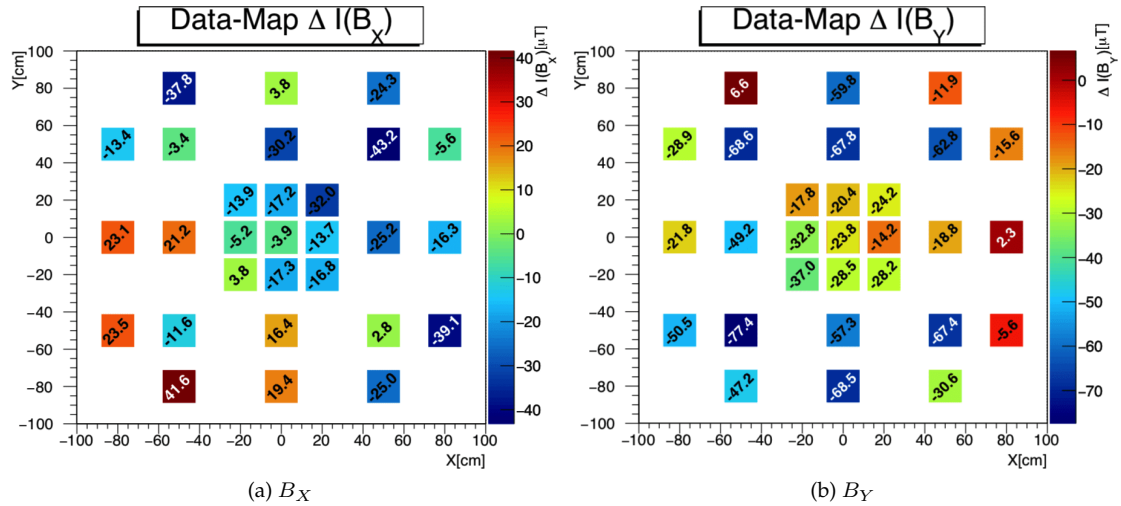


Figure 4.18: Difference between raw field and fringe map integral for the full grey-tube at each measurement position. A cross-check for the systematic calculation can be used by taking the average deviation across the surface for both  $B_X$  and  $B_Y$ .

across the surfaces is in agreement with the artificial modification approach, with the raw measurement being slightly lower on average in both cases at  $\approx -8 \mu\text{Tm}$  for  $B_X$  and  $\approx -35 \mu\text{Tm}$  for  $B_Y$ . The systematics for the field map are different in nature from the measurement systematics where  $\sigma_{B_X}$  is larger than  $\sigma_{B_Y}$ . The  $B_Y$  systematic

is dominated by the large deviation at plane 192.743 m, but this region sits inside the buffer region and so the agreement depends on accuracy of the MNP33 field map. Combining the errors in quadrature with the remainder of the decay-region systematics leads to only a small modification to Equations [4.7](#) and [4.8](#).

$$\sigma_{I_{B_X}}^{\text{FF-Total}} = 121 \mu\text{Tm}, \quad (4.14)$$

$$\sigma_{I_{B_Y}}^{\text{FF-Total}} = 81 \mu\text{Tm}, \quad (4.15)$$

remaining well within the performance requirements of  $3 < 10^{-4}$  Tm.

#### 4.4.2 Contribution to field integral

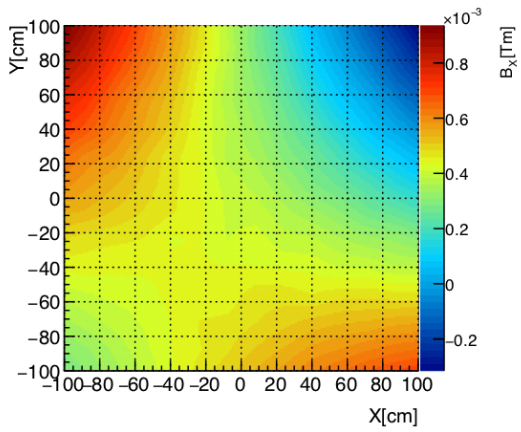
The final field integrals for  $B_X$  and  $B_Y$  are displayed in [Figure 4.19](#), containing both fringe field and MNP33 maps in  $(X, Y)$  space. The fringe field contributes less than 1% of the full field integral for both  $B_X$  and  $B_Y$  components. The fringe field integrals for (0 cm, 0 cm) are,

$$I_{B_X} = \int B_X \cdot dZ = (368 \pm 121) \mu\text{Tm}, \quad (4.16)$$

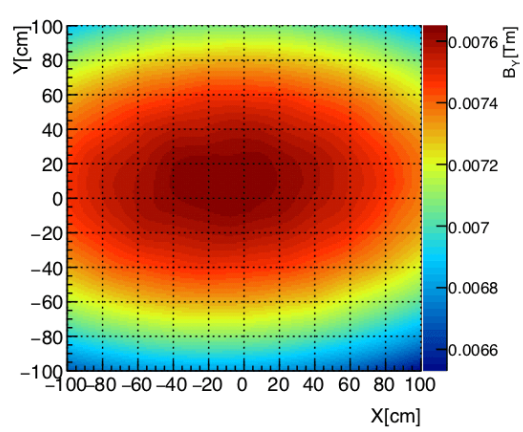
$$I_{B_Y} = \int B_Y \cdot dZ = (7390 \pm 81) \mu\text{Tm}, \quad (4.17)$$

which is roughly the average value across  $(X, Y)$  for  $B_X$  and close to the maximal contribution for  $B_Y$ . The  $B_X$  field is fairly asymmetric, contributing negatively in the (+,+) corner and positively in the (+,-) and (-,-) regions, with (-,-) close to the average value. The  $B_Y$  field is much less varied and changes by only  $\approx 10\%$  across the full  $(X, Y)$  space.

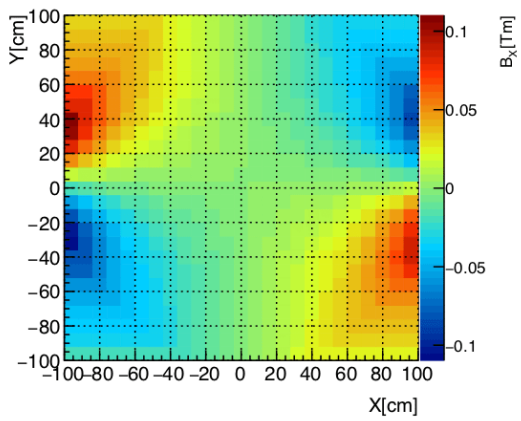
$B_x$  integral for the full Fringe Field



$B_y$  integral for the full Fringe Field



$B_x$  integral at end of the MNP33



$B_y$  integral at end of the MNP33

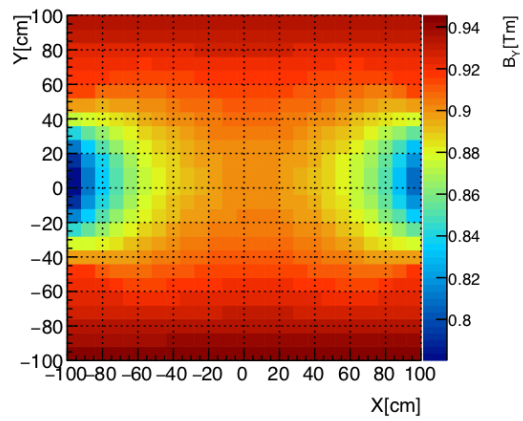


Figure 4.19: The total field integrals in Tm for the fringe (top row) and MNP33 (bottom row) field maps.  $B_x$  is contained in the first column and  $B_y$  the second, with the upstream and downstream fringe fields summed together.



# 5

## Development of a spectrometer calibration procedure

In this chapter, the development of a spectrometer momentum calibration procedure is presented. The correction model is explained, and the implementation of an algorithm is provided after a data-driven tuning strategy. Additionally, the time-dependence of the calibration parameters is interpreted in both 2016 and 2017 NA62 data, including the effect of the correction on the reconstructed kaon mass. Finally, the performance is verified further, using simulated samples after deliberate alteration of `NA62MC` and `NA62Reconstruction` to mimic the effects that the procedure is designed to correct for.

### 5.1 Motivation

For the measurement of the ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay, the most fundamentally important sub-detector system is the spectrometer. As described in Section [3.1](#), the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  analysis relies on kinematic background rejection, using the missing-mass signatures of the main  $K^+$  decay modes; a systematically wrong momentum determination can result in decay misidentification, diminishing the precision of the analysis. The same is true for other analyses, such as the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  decay (Chapter [8](#)) where both the mean value and resolution of the invariant mass defines the Signal Region ([SR](#)).

There are two aspects to consider for ensuring high track quality:

- a high performance track reconstruction, including spacial alignment.
- a complete and accurate description of the magnetic fields in the decay volume and spectrometer.

The latter case of these criteria is described in great detail in Chapter 4, whilst the former is beyond the scope of this thesis. The purpose of this chapter is, however, to describe the development of an analysis tool which improves the determination of track momentum without any prior knowledge of field accuracy or reconstruction performance. A tool is required that uses a data-driven method to correct for any residual miscalibrations in the tracking system, regardless of their origin or magnitude.

Section 5.2 contains the correction model used, including a prediction of the calibration constants, whilst Sections 5.3 and 5.4 describe the procedure developed to accurately extract the constants from data. Finally, the performance of the calibration is measured for both 2016 and 2017 data sets and verified further using MC samples in Section 5.5.

## 5.2 Correction Model

The adopted momentum correction model was implemented previously for NA48/2 data analysis [74]. The tracking system was similar, with four spectrometer drift chambers interspaced by the same MNP33 magnet in operation at NA62, but with a smaller field integral [75]. The model is simple, as displayed in Equation 5.1,

$$P_{nom} = P_{obs}(1 + \beta)(1 + \alpha q P_{obs}), \quad (5.1)$$

where  $P_{nom}$  is the nominal (corrected) momentum,  $P_{obs}$  is the reconstructed momentum and  $q$  is sign of the track charge [76]. Here, parameters  $\alpha$  and  $\beta$  are calibration constants used to address either field miscalibration or spectrometer misalignment, with the definition (or derivation) of each provided in this section. Whilst the model itself is over-simplified, the possibility of using a more complicated approach is lim-

ited by the requirements of the data-processing; the model must be simple, make sense physically and use only a small amount of CPU time for each event.

### 5.2.1 The $\alpha$ parameter

The  $\alpha$  parameter, in principle, can correct for any residual spectrometer chamber misalignment. The effect of a chamber misalignment can be understood using the example provided by Figure 5.1. Here, it is assumed that only the final spectrometer chamber is offset in the  $X$ -direction by some unknown size  $\Delta X$ .

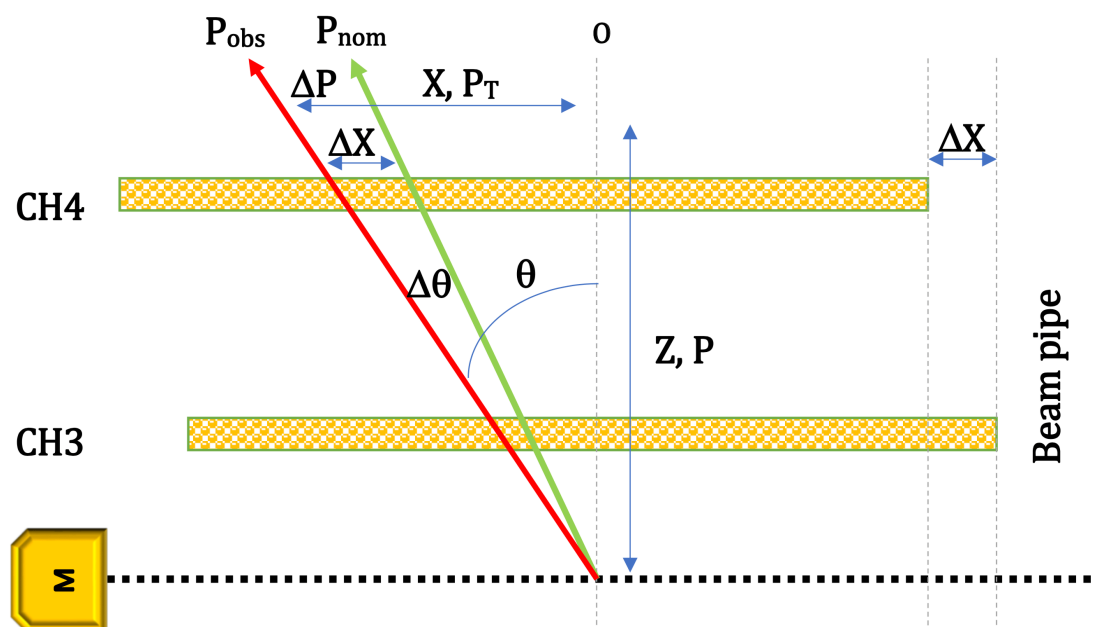


Figure 5.1: A diagram to estimate the size of calibration parameter  $\alpha$ , by considering a shift  $\Delta X$  of the final spectrometer chamber in the  $X$ -direction relative to the nominal position; 'M' is the MNP33 magnet.

In Figure 5.1, the observed track with momentum  $P_{obs}$  is deviated from the true-track with momentum  $P_{nom}$  because of the  $\Delta X$  misalignment, where  $P_{nom} = P_{obs} + \Delta P$ . The misalignment also leads to a small deviation in the angle  $\theta$ , which can be used to estimate  $\Delta P$  based on,

$$\theta = \frac{P_T}{P_{obs}} = \frac{X}{Z}. \quad (5.2)$$

where  $P_T$  is the transverse kick of the MNP33 magnet equal to 270 MeV/c,  $X$  is the transverse distance from the origin  $O$  and  $Z$  is the longitudinal distance between

the magnet centre and chamber four. Using a Taylor expansion for Equation [5.2](#), the expression for  $\Delta\theta$  follows naturally,

$$\Delta\theta = -\frac{P_T\Delta P}{P_{obs}^2} = \frac{\Delta X}{Z}. \quad (5.3)$$

Finally, rearranging the second half of Equation [5.3](#) for  $\Delta P$  leads to the definition of  $\alpha$ ,

$$\Delta P = -\left(\frac{\Delta X}{ZP_T}\right)P_{obs}^2 = \alpha P_{obs}^2. \quad (5.4)$$

This is the origin of the  $\alpha$  dependent term in the correction model as  $P_{nom} = P_{obs} + \Delta P = P_{obs}(1 + \alpha q P_{obs})$ , with  $q$  used to handle the sign of the track charge. For example, a misalignment of  $\Delta X = 1$  mm corresponds to a prediction of,

$$\alpha = \frac{\Delta X}{P_T Z} \approx \frac{1 \text{ mm}}{5.7 \times 10^6 \text{ MeVc}^{-1}\text{mm}} \approx 1.8 \times 10^{-7} [\text{MeV/c}]^{-1}. \quad (5.5)$$

In principle, the real origin of the magnitude of  $\alpha$  is far more complicated, but the spectrometer reconstruction has built in spatial alignment for both individual STRAW tubes and chambers [\[77\]](#). Therefore, we expect to observe values of  $\alpha$  far smaller than the prediction in Equation [5.5](#), as the alignment is typically better than 100  $\mu\text{m}$ . Additionally, as  $\alpha$  is in inverse units of momentum, the effect of the calibration results in a shape change in the momentum spectra.

### 5.2.2 The $\beta$ parameter

The calibration parameter  $\beta$  is designed to correct for any miscalibration of the total magnetic field integral. As seen in Chapter [4](#), the precision of the field integral is directly related to the track momentum. Therefore,  $\beta$  is defined as,

$$\beta = \frac{\Delta B}{B} = \frac{\Delta P}{P} \approx 10^{-3}, \quad (5.6)$$

where the maximum possible size of  $\beta$  is the magnitude of the permitted systematic uncertainty in the total MNP33 field integral provided by Ref. [\[69\]](#). Unlike the 'shape-changing' parameter  $\alpha$ , the parameter  $\beta$  is dimensionless and not dependent on the track charge, expected to alter only the magnitude of momenta.

## 5.3 Calibration Procedure

### 5.3.1 Exploiting $K_{3\pi}$ decays

In order to obtain the calibration parameters  $(\alpha, \beta)$ , a data driven approach is essential. The clear experimental signature of the three-track decay  $K^+ \rightarrow \pi^+\pi^+\pi^-$  ( $K_{3\pi}$ ) is a perfect channel for multiple reasons; the decay is not only abundant, but also contains three observable tracks. As the pion four-vectors are directly accessible, the kaon invariant mass can be calculated accurately. Therefore, the calibration procedure can exploit  $K_{3\pi}$  to find  $(\alpha, \beta)$  parameters which tune the momentum such that the  $K_{3\pi}$  invariant mass closely matches the PDG combined fit value of  $M_{K^+} = 493.677 \text{ MeV}/c^2$ , a value that has been measured extensively in numerous experiments [55].

The selection used to collect  $K_{3\pi}$  candidates is implemented into NA62FW as the general analyzer `K3piSelection` [67]. The analyzer is used by multiple analysis tools to perform several tasks, such as,

- Calculation of the beam-parameters (Momentum, slope, position).
- Calculation of kaon flux and the Number of Protons on Target (POT).
- Determination of efficiencies for three-track trigger topologies.

In order to remain consistent with the beam-parameters calculation, the standard selection was adopted, which requires at least one good three-track vertex consistent with the  $K_{3\pi}$  hypothesis of the kaon mass. Vertices are reconstructed using a five-dimensional fit based on track parameters (slopes, positions and momentum) at the first STRAW chamber. The fit corrects for residual magnetic fields (using a Kalman filter) and multiple scattering [67] [78]. The  $K_{3\pi}$  selection is provided below, requiring exactly one good three-track vertex built in the event, defined as:

- The vertex charge is  $Q = +1$ .
- The vertex fit  $\chi^2$  is less than 25.
- The vertex longitudinal position ( $Z$ -Vertex) is within (102; 180) m

- The vertex total momentum ( $P$ -Vertex) is within (72; 78) GeV/ $c$ .

For the tracks in the three-track vertex:

- The track is within the geometric acceptance of the four STRAW chambers and the `NewCHOD`.
- At least one of the tracks must have a spatially associated `CHOD` candidate. The CHOD candidate time is used to build a vertex time which is the average of  $n \leq 3$  tracks.
- The vertex time must be within 10 ns of the trigger time responsible for collecting the event.

Finally, the invariant mass of the three tracks in the  $K_{3\pi}$  hypothesis lies in the range of  $490 \leq M_{3\pi} \leq 497 \text{ MeV}/c^2$ .

Additionally, the tool will only read data that has been pre-selected to contain three-track decays. Using the `FilterRestrictedThreeTrackVertex` tool, the time taken to read any data sample can be dramatically reduced by building a more compact data set containing only suitable decays [67]. The filter reduces the raw data size by a factor of  $\approx 30$ , and applies a looser version of all vertex related cuts above [79]. Figure 5.2 displays the  $K_{3\pi}$  invariant mass spectrum for the full 2016A filtered data set, compared to signal MC. Fitting the distribution with a Gaussian,

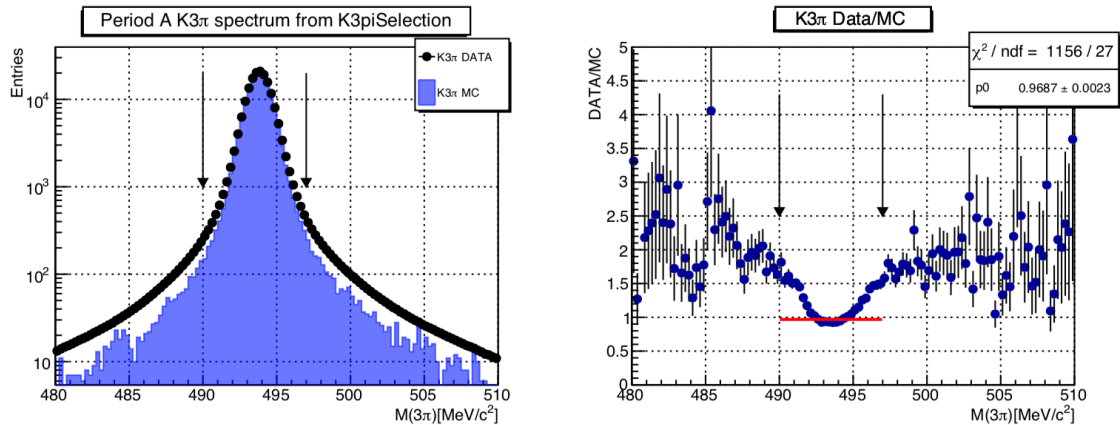


Figure 5.2: Left: The  $K_{3\pi}$  invariant mass spectrum for the full 2016 Period A data set, with the `SR` denoted by the arrows. Right: The data/`MC` ratio, with flat line fit in the `SR`.

the mean invariant mass is  $493.807 \text{ MeV}/c^2$  and the resolution  $\sigma_M = 0.870 \text{ MeV}/c^2$ .

In principle, the application of a suitable  $(\alpha, \beta)$  correction will improve the  $\approx 0.2$  MeV/c<sup>2</sup> discrepancy between the data and nominal value. In general, the data/MC agreement is reasonable within the SR, whereas the mass side-bands suffer due to the lack of coincidental kaon decays in the MC. Agreement in this region can be improved with a more stringent three-track selection (see Chapter 6).

## 5.4 Calibration algorithm

To determine calibration constants  $(\alpha, \beta)$ , a suitable algorithm is required. The nature of the task leads to two logical stages:

### Stage A: The event level, "DATA MODE"

1. Collect  $K_{3\pi}$  candidates by calling `K3piSelection`.
2. Calculate the pion four-vectors for  $n_\alpha \times n_\beta$  trial values of  $(\alpha, \beta)$ .
3. Reconstruct the kaon mass at each trial, saving the results in histograms in  $n_{P_{\pi^-}}$  bins of the negative pion momentum  $P_{\pi^-}$ .

### Stage B: The end of run level, "HISTOGRAM MODE"

1. Process a large number of events via Stage A.
2. Fit the invariant mass distribution for each trial  $(\alpha, \beta, P_{\pi^-})$  with a Gaussian curve and extract the fit parameters.
3. Build a test-statistic  $\chi^2$  described in Equation 5.7 and create a  $(\alpha, \beta)$  vs.  $\chi^2$  distribution from the parameter scan.
4. Fit the distribution using a two-dimensional polynomial, extracting the ideal calibration constants at the coordinates of the minima of the fit.

For an NA62Analysis analyzer, Stage A will be run first with the reconstructed data to obtain the result of the scanning procedure in the form of histograms. Stage B will then use the histogram output to perform the calibration calculation. Stage B can process the histogram-output from events collected from a burst, or more ordinarily, a data-taking run (typically  $10^3$  bursts).

As an alternative to saving  $\approx 10^5$  histograms from Stage A, the information is condensed into a single 2D histogram, with a  $Y$ -axis corresponding to three-dimensional index for each  $(\alpha, \beta, P_{\pi^-})$  trial and the  $X$ -axis as the invariant mass of the  $K_{3\pi}$  candidates. Table 5.1 contains the chosen configuration for the scan in the calibration parameters. The scan range is slightly larger than the predictions made in Sections 5.2.1 and 5.2.2 and the binning is different in each dimension due to the initial sensitivities of the parameters (See Figure 5.4). The number of histograms needed

Table 5.1: Details of the scan configuration for the  $(\alpha, \beta, P_{\pi^-})$  computation, motivated by the estimation in Sections 5.2.1 and 5.2.2

Parameter	Range	Step Size	Number of Bins
$\alpha$ [MeV/c] <sup>-1</sup>	$\pm 2 \times 10^{-7}$	$4 \times 10^{-9}$	101
$\beta$	$\pm 6 \times 10^{-3}$	$2 \times 10^{-4}$	61
$P_{\pi^-}$ [GeV]	10, 45	2	18

ordinarily is  $N_H = n_\alpha \times n_\beta \times n_{P_{\pi^-}} = 110898$ , which instead denotes the number of bins needed in the  $Y$ -dimension. The histogram is filled in a three-dimensional loop over the scan parameters, where Figure 5.3 displays an example of the output from each stage of the algorithm. Due to the vast size of the scan histogram, a reduced

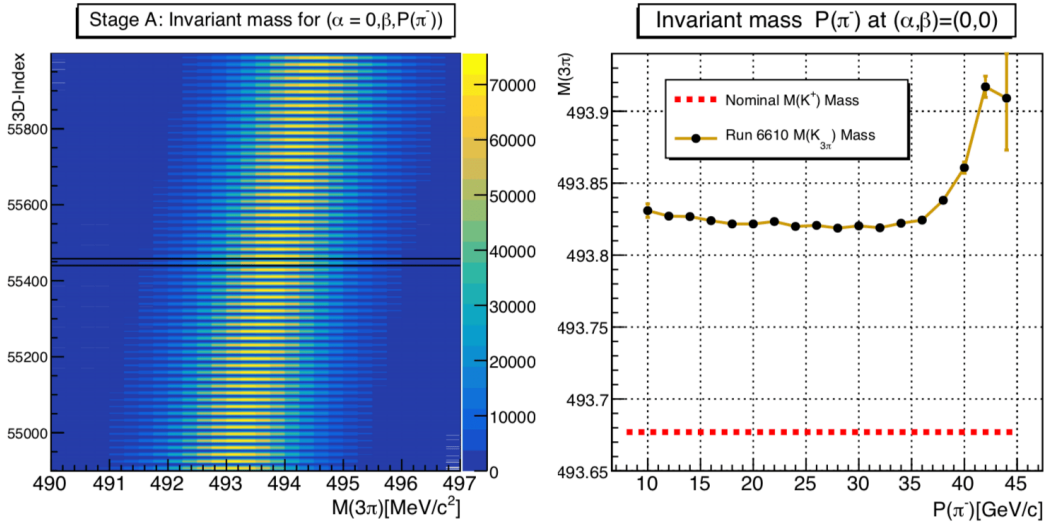


Figure 5.3: Left: A reduced-range example of the histogram output from Stage A for Run 6610; black lines denote the invariant mass distributions vs momentum for  $(\alpha, \beta) = (0, 0)$ . Right: Comparing the invariant mass vs. momentum distributions for  $(\alpha, \beta) = (0, 0)$  compared to nominal.

range is shown for only  $(\alpha = 0, \beta, P_{\pi^-})$ , containing  $N_H/n_\alpha = 1098$  bins. The black-lines denote a region with  $n_{P_{\pi^-}}$  bins where  $(\alpha, \beta) = (0, 0)$  as an example. Stage B



is initiated by extracting and fitting each  $P_{\pi^-}$  slice with a Gaussian, used to create a mass versus momentum graph for each  $(\alpha, \beta)$  trial, displayed in the right-hand plot in Figure 5.3. This leads to the creation of  $n_\alpha \times n_\beta = 6161$  graphs in total, where each is used to build a  $\chi^2$  test-statistic, defined as,

$$\chi^2 = \sum_{i=1}^{n_{P_{\pi^-}}} \frac{(M_{3\pi,i} - M_{K^+}^{PDG})^2}{(\sigma_{3\pi,i})^2}, \quad (5.7)$$

where  $M_{3\pi,i}$  is the fitted invariant mass and  $\sigma_{3\pi,i}$  is the uncertainty at momentum bin  $i$  for some trial  $(\alpha, \beta)$ . In principle, the calibration parameters which correct the raw distribution to the nominal case (denoted by the dashed red line) will minimise the statistic. As  $\alpha$  is momentum dependent, it is expected to distort the shape of the distribution and  $\beta$  to control the overall scale. To obtain the constants, the creation of an  $(\alpha, \beta)$  against  $\chi^2$  distribution can be used to find the minima, by fitting with a two-dimensional polynomial with cross-term (in case of parameter correlation),

$$\chi^2(\alpha, \beta) = p_0 + p_1\lambda\alpha + p_2\beta + p_3\lambda^2\alpha^2 + p_5\beta^2 + p_6\lambda\alpha\beta. \quad (5.8)$$

Here,  $p_{1;6}$  are fitting constants and  $\lambda \approx (\beta/\alpha)_{est} = 10^4$  is a scale factor based on the estimates of the  $(\alpha, \beta)$  parameters from Section 5.2. It is used for the  $\alpha$  terms so that each component of Equation 5.8 is the same order of magnitude; this allows the fit to converge to sensible answers first time without any prior configuration. Figure 5.4 displays the  $\chi^2$  distribution for an arbitrary run in 2016. The  $\chi^2$  distribution has been normalised by the number of degrees of freedom (NDF) at each  $(\alpha, \beta)$  bin; as the NDF is flat, the effect only changes the rate of increase of the  $Z$ -axis and not the position of the minima. The change in  $\chi^2/\text{NDF}$  is more drastic in the  $\beta$ -dimension than in  $\alpha$  (without feature normalisation) motivating the choice of the different number of step sizes in each axis. The right-hand plot is the effect of several  $(\alpha, \beta)$  parameters on the mass against momentum distribution, including the found minimal case. The best result shifts the spectrum toward the nominal kaon mass at the most populated region of the plot (between 20-35 GeV), but with a notable change in shape. It is also clear from the  $(\alpha, \beta) = (0, \beta_{min})$  distribution that a trivial  $\beta$  correction could have just been applied instead of a two-dimensional approach.

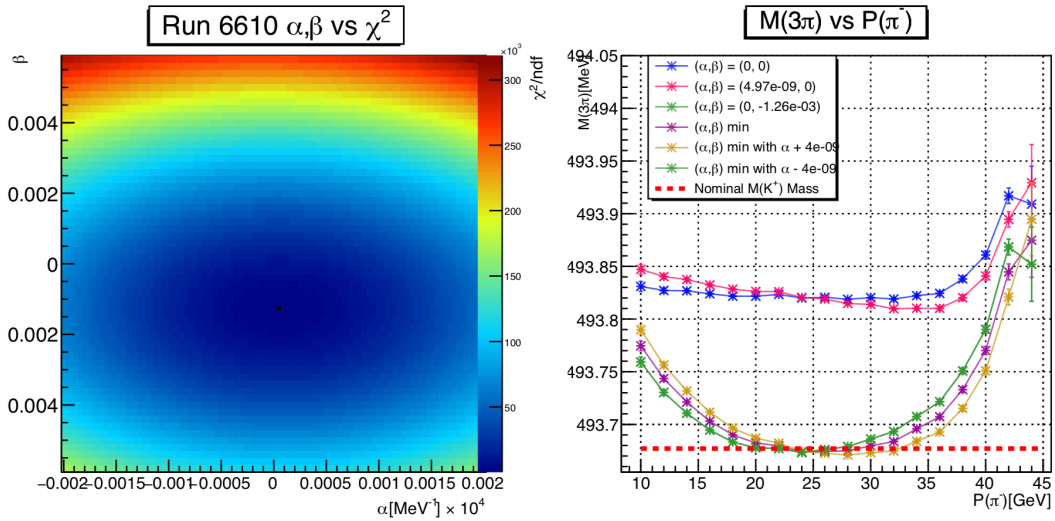


Figure 5.4: Left:  $(\alpha, \beta)$  vs.  $\chi^2/\text{NDF}$  distribution, with polynomial minima is denoted by the star. Right: the resulting mass vs. momentum distributions from the correction with several examples for comparison, such as the minimum  $\pm$  a step in  $\alpha$ .

However, by additionally obtaining  $\alpha$  constants, we can both fine-tune the momentum determination and also obtain some knowledge of the chamber misalignment for each run. The distributions which are the  $(\alpha_{min} \pm \alpha_{step}, \beta_{min})$  show that  $\alpha$  only slightly modifies the standard distribution; it is clear that there is no solution with this simple model that can shift the blue line to the nominal kaon mass without a shape distortion. Notably, the distribution which is just to set  $(\alpha, \beta) = (0, \beta_{min})$  also results in a shape change;  $\beta$  does not just scale the distribution, but also affects the shape.

### 5.4.1 Tuning the fit

The description previously focused on building an algorithm to determine calibration constants. In order to guarantee the effectiveness of the algorithm, the fitting performance at each stage requires special attention. The top-down view of Figure 5.4 provides only limited information; for a more complete picture, Figure 5.5 contains one-dimension projections to show the agreement side on. As the  $(\alpha, \beta)$  fit is performed with the weights in each bin set to unity (as statistical bin errors are meaningless for the test-statistic), the case of an unconstrained fit is dominated by the large  $\chi^2/\text{NDF}$  bins. The minimum is approximately in the correct place, but the shape is not well described. This can be improved either by cutting bins of

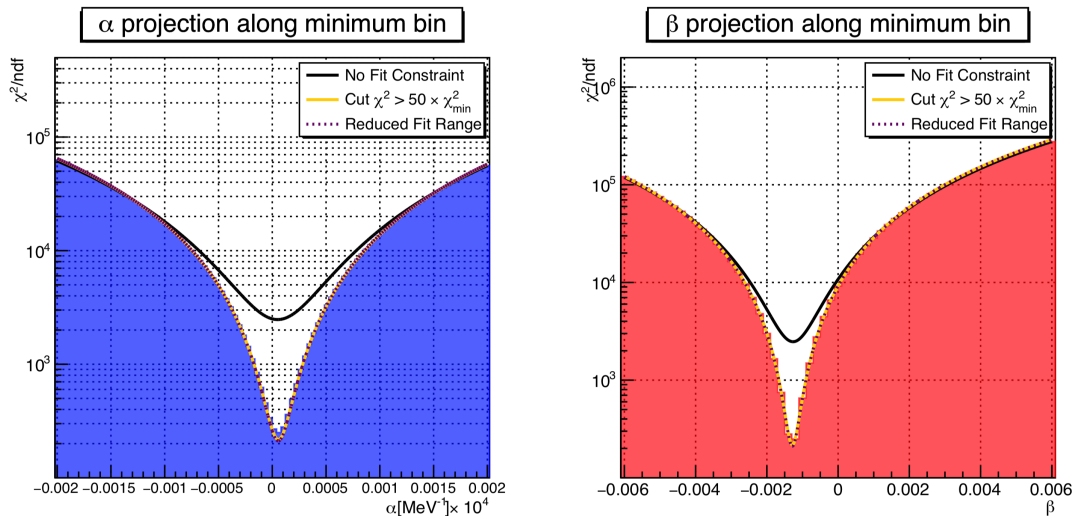


Figure 5.5: One-dimensional projections of the fit with left:  $\alpha$  and right:  $\beta$ . The black line denotes the standard fit, whereas the orange and magenta lines show where (a)  $\chi^2$  bins larger than the  $50 \times \chi_{min}^2$  have been removed and (b) the fit was reduced in range to  $\pm 10(5) \alpha(\beta)$  scan steps around the minimum bin.

large magnitude or reducing the fit to within a smaller range around the minimum bin. This results in a large improvement in the shape agreement, summarised in Table 5.2, where the errors assigned to  $(\alpha, \beta)$  are computed by finding where the  $\chi^2$  distribution increases by 2.3 from  $\chi_{min}^2$  [80]. The standard approach returns  $(\alpha, \beta)$

Table 5.2: Summary of the  $(\alpha, \beta)$  results for several tuned  $\chi^2/\text{NDF}$  fits. Without some constraint on the minima, the standard approach returns an answer deviated from the tuned cases by more than  $1\sigma$  for both parameters.

Method	$\alpha$ [MeV/c <sup>-1</sup> ] ( $10^{-8}$ )	$\beta$ ( $10^{-3}$ )	$(\alpha, \beta)$ fit $\chi^2/\text{NDF}$ [ $10^3$ ]
Standard (No constraint)	$0.50 \pm 0.03$	$-1.26 \pm 0.01$	2999.5
Cut $\chi^2 > 50 \times \chi_{min}^2$	$0.56 \pm 0.03$	$-1.28 \pm 0.01$	0.3
Reduced Fit Range	$0.56 \pm 0.03$	$-1.28 \pm 0.01$	0.1

which are deviated by  $\approx 2\sigma$  from the fits which performed well. For this reason, the  $\chi^2$  cut approach was adopted as it is automatically tuned to the minimum of the distribution. In addition to the final-stage fitting improvements, the mass spectra used to build the components of the test-statistic require additional scrutiny. By fitting the invariant mass spectra at each momentum bin, the  $\chi^2/\text{NDF}$  distribution is highly dependent on the mass fit quality. During the invariant-mass building Stage A, improvements could be considered:

- Request trigger information to remove events which intrinsically distort the

spectrum.

- Include cuts to remove beam-related background.
- Increase the granularity of the  $(\alpha, \beta)$  scan.

An example of a distortion would be from the dimuon trigger line which requires two MUV3 candidates in the final state. For  $K_{3\pi}$  to fire such a condition requires two secondary pion decays, which if occurring in the spectrometer volume would affect the invariant mass. Cuts to remove beam-related background would improve the tails in Figure 5.2 by increasing the purity of the sample. However, both approaches would deviate from `K3piSelection`. The final point, which is to increase the scan granularity is also undesirable as it will increase both the CPU time required and the memory size of the output files that are stored during reprocessing.

Regardless, the actual Gaussian mass fits are free to be tuned appropriately. In the default case, the centre of the maximum bin  $b_{max}^c$  is found for each mass spectrum and used to define a fit range based on the mass resolution  $\sigma_M$ ,

$$(b_{max}^c \pm n\sigma_M) \tag{5.9}$$

where  $n$  is an integer between (1; 5) and rounded  $\sigma_M = 0.90 \text{ MeV}/c^2$ . Table 5.3 displays the response of both the mass and  $(\alpha, \beta)$  fits with  $n$ . For  $n = 1$ , only

Table 5.3: Comparing the goodness of fit versus the size of the mass-range in the Gaussian fit. This is limited to  $n = 4$  by the size of the mass-window in the selection.

n	Gaussian fit $\chi^2/ndf$	$(\alpha, \beta)$ fit $\chi^2/ndf$ ( $10^3$ )	$\alpha[\text{MeV}/c^{-1}]$ ( $10^{-8}$ )	$\beta(10^{-3})$
1	1.3	3.4	$0.41 \pm 0.05$	$-1.20 \pm 0.01$
2	112.5	8.3	$0.50 \pm 0.03$	$-1.28 \pm 0.01$
3	356.4	1.3	$0.55 \pm 0.03$	$-1.28 \pm 0.00$
4	529.3	0.2	$0.56 \pm 0.03$	$-1.28 \pm 0.00$

the core of the distribution is fitted, whilst  $n = 4$  accounts for the full selection window. As the mass fit extends in range and the goodness of fit worsens, the  $(\alpha, \beta)$  fit improves and the parameters stabilise. This can be seen in Figure 5.6. The shape of the  $(\alpha, \beta)$  vs.  $\chi^2/\text{NDF}$  becomes less fluctuated as the tails of the distribution enter into the mass fit. As we rely purely on a good calibration result,  $n = 4$  is the optimal

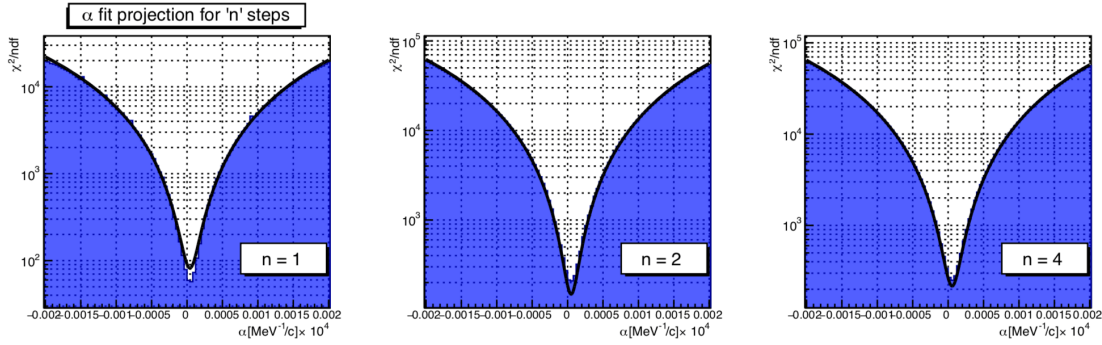


Figure 5.6: Response of the fit shown in one-dimension for  $\alpha$  with a mass-window where left:  $n = 1$ , centre:  $n = 2$ , and right:  $n = 4$ . The first case has a less stable shape in the distribution and over-estimates the minimum, whereas the centre case underestimates. By  $n = 4$ , the minima is adequately described.

solution.

## 5.5 Results

The analyzer `AlphaBetaComputation` was implemented into the `NA62FW`, designed to be used at the initial stage of the data processing [67]. In ideal operation, the computation is used in two stages to mirror Stage A and B described previously. The analyzer is used directly after the event filtering stage to produce the scanning histograms, and then again (on the output of the first stage) to find the optimal  $(\alpha, \beta)$  parameters. As the reprocessing is performed run-by-run, the calibration parameters are determined and subsequently used in all standard analysis tools during the later stages of reprocessing. In this section, the results of the computation are presented for both the 2016 and 2017 data sets (see Section 6.2 for a full description). In addition to the final parameters, the effect of the correction to the measured kaon mass is provided. Finally, a study exploring the response of the calibration for deliberately miscalibrated `MC` is presented to understand the effectiveness of the tool.

### 5.5.1 In 2016

The 2016A data set corresponds to the sample recorded after the full commissioning of the NA62 detector between September and November 2016 (see Section 6.2). The full sample is equivalent to 169 runs ( $\approx 8 \times 10^4$  SPS bursts) which successfully

passed the full reconstruction and data-reprocessing procedure. For each run, a set of calibration constants was determined. Figure 5.7 shows that for the latter stages

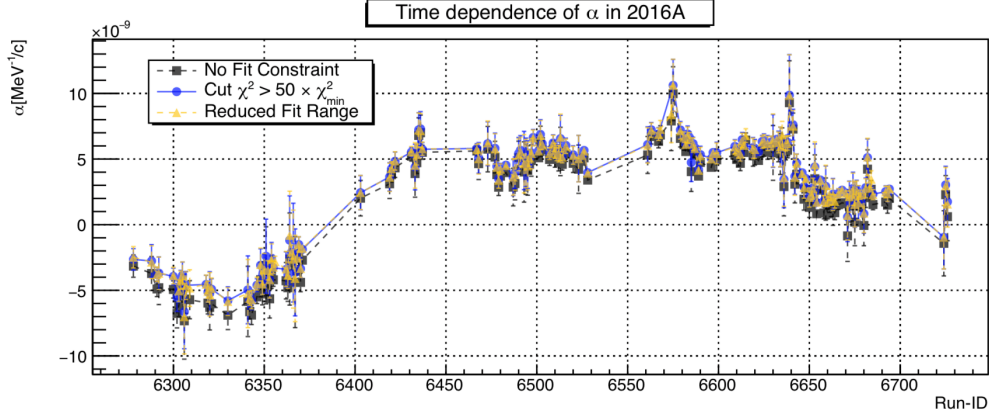


Figure 5.7: Time-dependence of  $\alpha$  in 2016A. The blue dots correspond to the chosen fit technique.

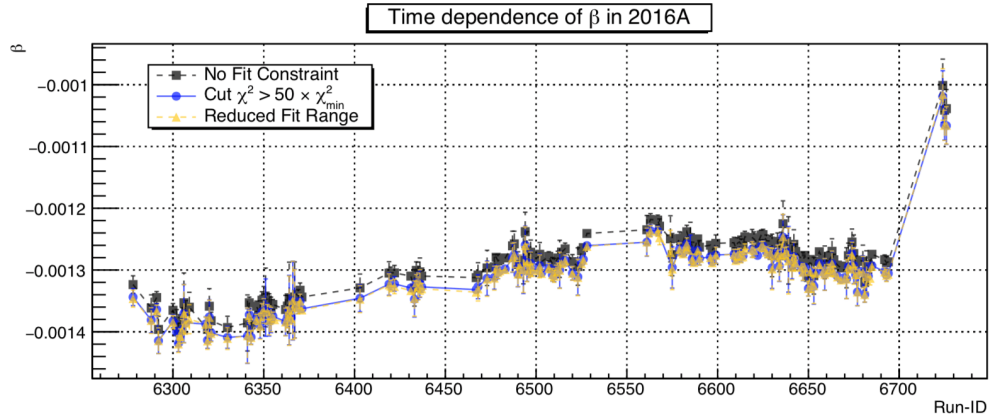


Figure 5.8: Time-dependence of  $\beta$  in 2016A. The blue dots correspond to the chosen fit technique.

of 2016,  $\alpha$  was stable at  $\approx 5 \times 10^{-9} \text{ MeV}^{-1}\text{c}$  after transitioning from  $\approx -5 \times 10^{-9} \text{ MeV}^{-1}\text{c}$  in the early part of the run. Using the conversion from Equation 5.5, this corresponds to  $\approx 10 \mu\text{m}$  in misalignment. A possible reason for the transition could be temperature related. For example, the  $\Delta\alpha$  in 2016 is equivalent to  $\approx 10^{-8} \text{ MeV}^{-1}\text{c}$  which corresponds to a  $57 \mu\text{m}$  change in the misalignment. For a metre long polyethylene STRAW tube, the temperature change required to achieve such an expansion is of the  $\mathcal{O}(1 \text{ K})$ , which is entirely feasible for a detector operated at high voltages.

The parameter  $\beta$  shown in Figure 5.8 appears to be more stable at  $\approx -1.35 \times 10^{-3}$ , only becoming slightly smaller over time. The only significant feature can be found

at the end of 2016, where  $\beta$  jumps by  $\approx 0.5 \times 10^{-3}$ . The jump is understood to be a residual effect of the MNP33 magnet being deactivated so that a series of non- $K^+$  runs could be recorded for the exotic programme. The final three runs correspond to the return of the standard  $K^+$  beam operation. The only difference between the constrained and unconstrained fit approaches appears to be a systematic shift by less than a step-size of the scan for both  $(\alpha, \beta)$  parameters, which was observed previously in Table 5.2

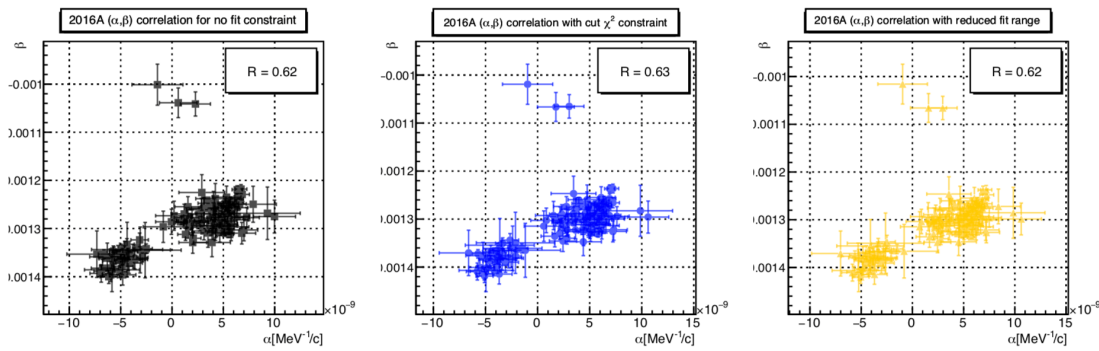


Figure 5.9: Correlation between  $(\alpha, \beta)$  parameters in 2016A for Left: no fit constraint, Centre: with a  $\chi^2/\text{NDF}$  cut and Right: the reduced fit range.  $R$  is the correlation factor i.e Pearson's regression.

Additionally, Figure 5.9 contains the regression  $R$  between the calibration parameters in 2016A for the three fit approaches. Each method is consistent, suggesting that both parameters are moderately correlated, yielding  $R \approx 0.6$ . However, the stable conditions do not yield dramatic changes in the parameters. For example, the three largely deviated points correspond to the final three runs where  $\beta$  was expected to change. Finally, the effect of the correction on the  $K_{3\pi}$  mass is presented in Figure 5.10. While the mass resolution remains the same after the correction ( $\sigma_M \approx 0.86 \text{ MeV}/c^2$ ), the invariant mass displays a large improvement, having shifted the invariant mass by  $-0.14 \text{ MeV}/c^2$  to leave the systematic discrepancy between  $M_{3\pi}$  and  $M_{K^+}$  to  $\approx +6 \text{ KeV}/c^2$ . In addition, the corrected values remain stable over the full period, providing evidence that the computation is consistent regardless of the starting values.

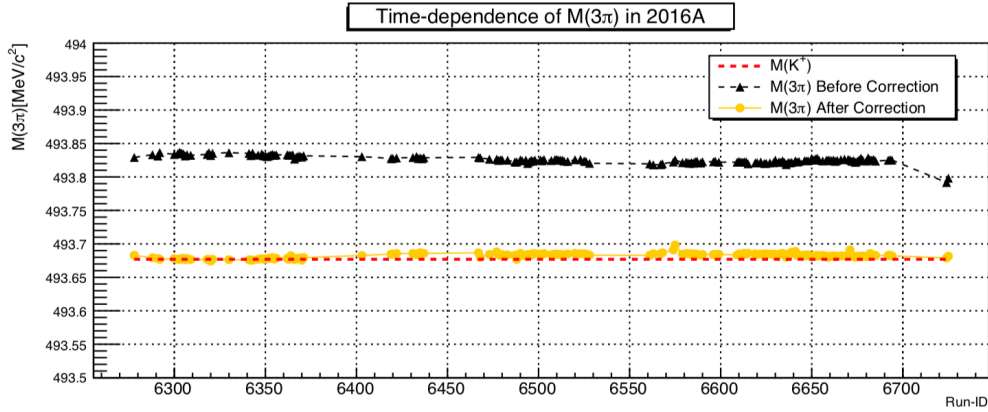


Figure 5.10: Comparison of the time-dependence of the  $K_{3\pi}$  mass post  $(\alpha, \beta)$  corrections in 2016A.

## 5.5.2 In 2017

The 2017A data set corresponds to the sample recorded between September and October 2017 (see Section 6.2). The full sample corresponds to 79 runs ( $8 \times 10^4$  SPS bursts) and is a similar size to 2016A. Again, a set of calibration constants were extracted for each run. Figure 5.11 displays that  $\alpha$  was an order of magnitude larger

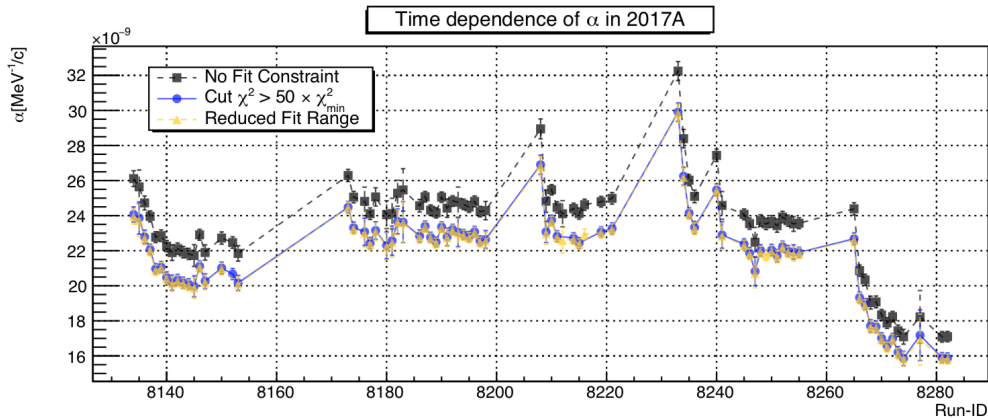


Figure 5.11: Time-dependence of  $\alpha$  in 2017A. The blue dots correspond to the chosen fit technique.

in 2017, suggesting misalignment of the order  $\approx 100 \mu\text{m}$ . In general,  $\alpha$  is stable except from the start and end of the run-period, with some 'spikes' at certain runs in between. In Figure 5.12, the parameter  $\beta$  is perhaps more interesting due to frequent MNP33 magnet trips that were a notable feature during the data-taking process. The issues are reflected by +15% change in  $\beta$  between Runs (8220; 8270), before returning to values similar to those from before the start of the issues. The peak  $\beta$  value occurs after the MNP33 was turned off altogether, which was seen similarly in 2016.



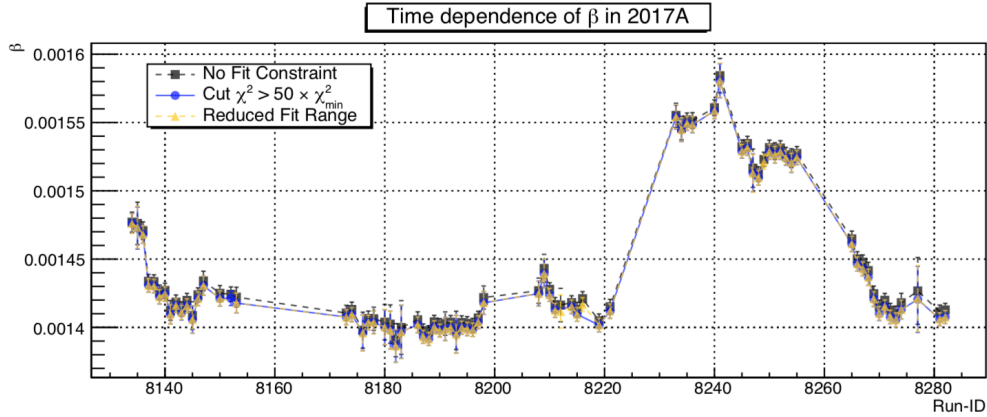


Figure 5.12: Time-dependence of  $\beta$  in 2017A. The blue dots correspond to the chosen fit technique.

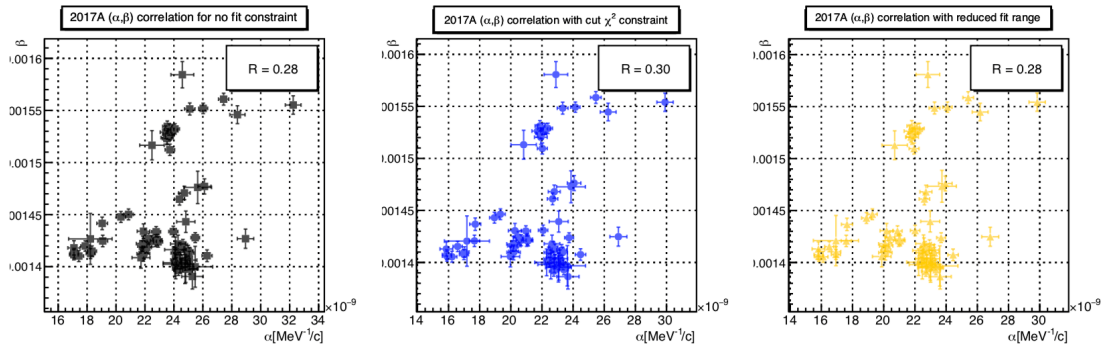


Figure 5.13: Correlation between  $(\alpha, \beta)$  parameters in 2017A for Left: no fit constraint, Centre: with a  $\chi^2/\text{NDF}$  cut and Right: the reduced fit range.  $R$  is the correlation factor i.e Pearson's regression.

As seen previously, constraining the fit systematically shifts the parameters found by the computation by a fixed value within a step-size of  $(\alpha, \beta)$ .

Again, both  $\alpha$  and  $\beta$  were plotted in order to determine the regression. Unlike 2016, the typical regression  $R = 0.28$  suggests that the parameters are even less correlated. This suggests that the magnet miscalibration only dramatically affects  $\beta$  and not  $\alpha$ , as the points where the MNP33 is known to have tripped resides in a similar area of the plot to the 2016 cases where the MNP33 had changed current. As  $\approx 20\%$  the data experienced this issue, it is likely the  $R$  is smaller as a result as the magnetic field changes are not reflected in  $\alpha$ . Like 2016, the effect of the correction on the  $K_{3\pi}$  is presented in Figure 5.14. In general, the invariant mass before correction is different in 2017 compared to 2016. Table 5.4 contains the main differences between the two samples and the magnitude of the corrections. As in 2016, the mass resolution remains the same post correction (at  $\sigma_M \approx 0.88 \text{ MeV}/c^2$ ), with the invariant mass

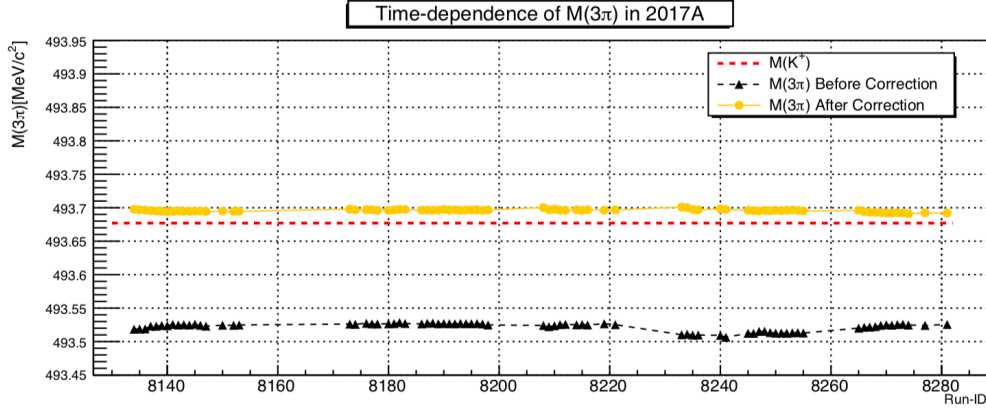


Figure 5.14: Comparison of the time-dependence of the  $K_{3\pi}$  mass post  $(\alpha, \beta)$  corrections in 2017A.

Table 5.4: Comparing the result of the correction for 2016A and 2017A. The values are in  $\text{MeV}/c^2$

Data set	$M_{3\pi}$ Initial	$M_{3\pi}$ post-correction	$\Delta M_{3\pi}$	Difference from Nominal	$\sigma_M$
2016A	493.825	493.683	-0.142	+0.006	0.860
2017A	493.522	493.696	+0.174	+0.018	0.881

displaying a large improvement, shifting on average by  $+0.174 \text{ MeV}/c^2$  to leave the systematic discrepancy between the  $M_{3\pi}$  and  $M_{K^+}$  at  $\approx +18 \text{ KeV}/c^2$  for 2017A. The difference here is  $\approx 10 \text{ KeV}/c^2$  larger than for 2016, but the result of the procedure is highly dependent on the initial data-taking conditions. For example, the  $(\alpha, \beta)$  correction to the momentum re-aligns the invariant mass in different directions for the two data sets.

### 5.5.3 Simulation

In addition to producing  $(\alpha, \beta)$  constants in data, the tool can be applied to signal `MC`. This can provide verification that the computation behaves as expected; NA62MC and NA62Reconstruction can be modified to apply the effects that the tool is designed to correct for. For example,

- In NA62Reconstruction, deliberately misalign a spectrometer chamber by shifting the alignment constants to find a response in  $\alpha$ .
- In `NA62MC`, change the field scale factor  $M_{sf} = B/B_{nom}$  of the MNP33 to study the response in  $\beta$ .

To understand the response of the computation,  $10^5$  signal `MC`  $K_{3\pi}$  were generated,

corresponding to  $\approx 10^4$  Monte Carlo (MC) candidates post-selection.

### 5.5.3.1 Misalignment in the spectrometer

For this study, the spectrometer reconstruction was modified to include the alignment procedure. In standard operation, the alignment is only performed for real data where the positions of the spectrometer chambers may not be nominal. By allowing the simulation to use the alignment, systematic chamber shifts can be applied. The alignment is performed by offsetting the four ( $U, V, X, Y$ ) coordinates of each STRAW chamber with constants determined using a muon run, where the hit positions of the STRAW were very accurately known. By allowing the simulation to use the alignment procedure, a shift can be applied in any direction by simply changing the alignment file. Only the  $X$ -dimension of the fourth chamber was misaligned in order to compare with the simplistic case outlined in Section 5.2.1. Figure 5.15 displays the change in the  $\alpha$  result when shifting away from nominal alignment values.

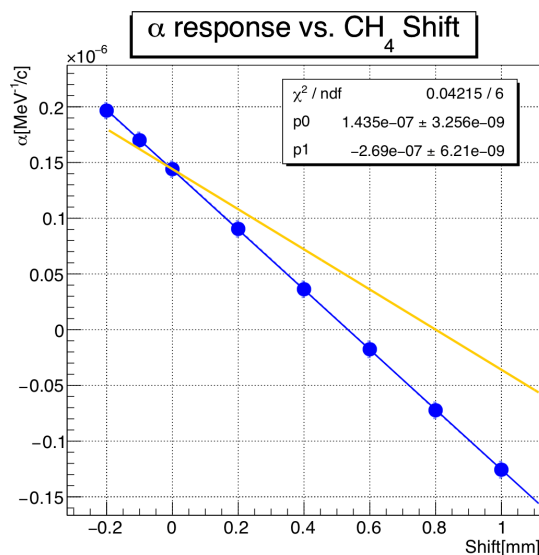


Figure 5.15: Response in  $\alpha$  after the deliberate misalignment of spectrometer chamber 4. The  $X$ -axis is the shift from the nominal constant. The yellow line depicts the simplistic expectation.

The effect is large and strongly linear for  $\alpha$ , where a 1 mm misalignment almost covers the full  $\alpha$  scan range. Clearly the response, whilst slightly different from the simplistic expectation, shows the a linear trend as expected for the computation to correct for the effect.

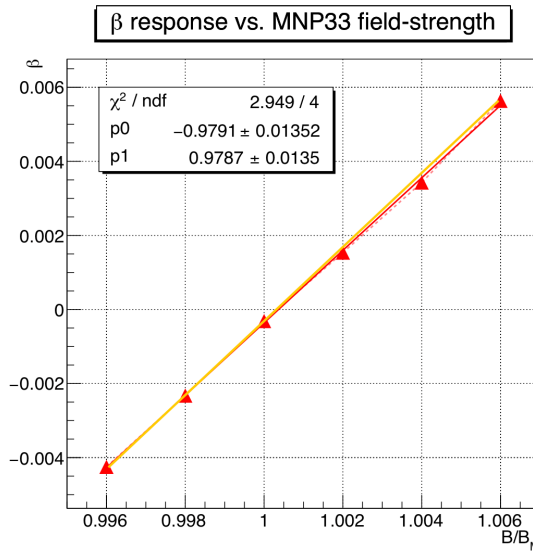


Figure 5.16: Response in  $\beta$  after the re-scaling of the MNP33 magnet. The  $X$ -axis is the ratio of the field to the nominal value. The yellow line depicts the simplistic expectation.

### 5.5.3.2 Scaling the MNP33 field

In the case of  $\beta$ , alternative sets of MC samples were generated for several  $M_{sf}$  values between (0.996; 1.006) and reconstructed with the default chamber alignment constants. Figure 5.16 displays the change in  $\beta$  for each of these samples. With respect to the first test, magnetic field scaling is the simplest of the two mechanisms; as track momenta relies on the measurement of the precise kick of the MNP33 field, the relationship between the field and  $\beta$  is linear, precisely what is demonstrated. Again, the algorithm handles the discrepancy of a miscalibration in the field integral in the appropriate manner. Additionally, this reinforces what is seen in 2017A for runs with particular magnetic field problems; a significant change in  $\beta$ .

## 5.5.4 Monitoring

So that the data processing experts at NA62 can monitor the spectrometer-calibration performance, a set of histograms were produced in the form of a post-processing graphical report, created at the end of Stage B for each run. Figure 5.17 displays a typical post-processing graphical report, using an arbitrary run in 2016. The top left-hand corner is the print-out information of the Run number,  $(\alpha \pm \sigma\alpha, \beta \pm \sigma\beta)$  and the goodness of fit  $\chi^2/\text{NDF}$ . The remaining two plots on the top column are the

## Run 6610

$$\alpha = (0.57 \pm 0.03) \times 10^{-8} \text{ MeV}^{-1}$$

$$\beta = (-1.28 \pm 0.00) \times 10^{-3}$$

$$\frac{\chi^2}{\text{NDF}} = 279.85$$

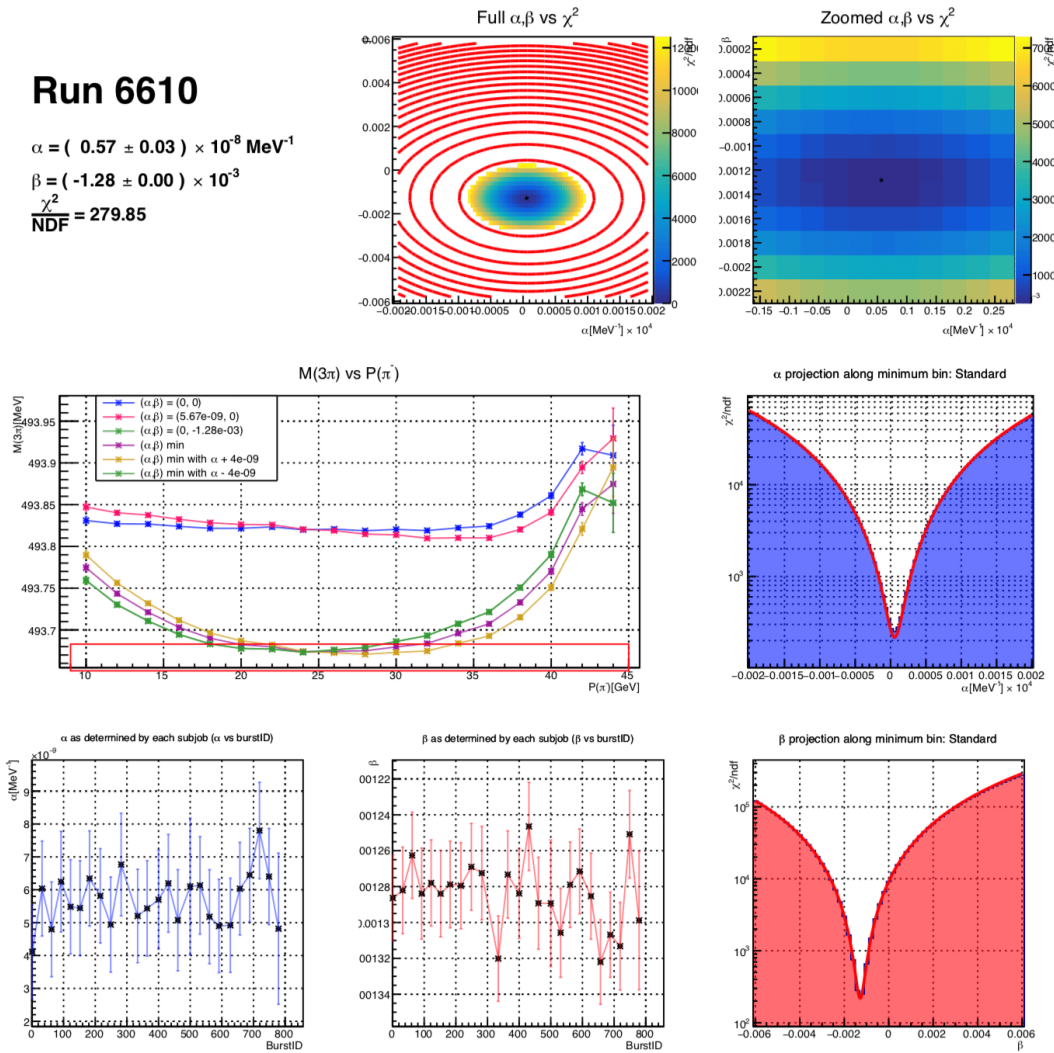


Figure 5.17: Data-processing graphical report from 2016 Run 6610.

standard  $(\alpha, \beta)$  vs.  $\chi^2/\text{NDF}$  plot, along with a version which is focused on a few bins around the minimum. This row is intended to provide immediate information about the fit minima. The central graph is the invariant mass plotted against the negative pion momentum before and after the correction, including a few interesting cases. The blue and red plots to the right of the canvas, as displayed previously in this thesis, are the one-dimensional projections of the fit, used to check for clear disagreement between the fit and the data. The final plots at the bottom are used to check the stability during a run. Typically, the runs are split into smaller groups of bursts to be submitted as parallel jobs on the lplus batch service. As explained in Section 5.4, the Stage B can also determine constants for a group of bursts. The results of each smaller job enter directly into this plot to show the change in the

parameters during the run. Naturally, as the statistics are smaller in each job, the precision is worse than for the overall run which leads to larger errors for bursts. Regardless, the stability can be useful especially for runs with interesting features, such as a magnet trip, as  $\beta$  will ramp up or down. In the past, the computation aided understanding of runs which were missing the spectrometer alignment constants in reconstruction, observable via a large shift in  $\alpha$ .

# 6

## Event selection

In this chapter, the development of a selection for the rare  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  ( $K_{\pi\mu\mu}$ ) decay is presented. As described in Chapter 2, the current largest sample of  $K_{\pi\mu\mu}$  decays ( $N_{\pi\mu\mu} = 3120$ ) was collected by the NA48/2 experiment between 2003 and 2004. However, the sample is contaminated to the level of  $(3.3 \pm 0.7)\%$  due to the tails of the irreducible  $K_{3\pi}$  background [48]. Using data recorded in 2016 by the new NA62 detector, the selection developed below is used to collect a sample of decays where the background contribution is suppressed to a negligible level. In addition to the signal, a selection to collect a sample of normalisation  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  ( $K_{3\pi}$ ) decays is described, developed by extracting the relevant criteria of the final  $K_{\pi\mu\mu}$  selection. This is to maximise the symmetry between the two samples. As presented in the later chapters, the sample of decays collected here is used to reconstruct the  $d\Gamma/dz$  spectrum which is utilised to extract the model-independent branching fraction and form factor LECs.

### 6.1 Strategy

The main motivation to collect a background-free sample of  $K_{\pi\mu\mu}$  decays is the suppression of the associated systematics effects. In this analysis, the spectrum of the kinematic variable  $z_{\mu\mu} = (M_{\pi\mu\mu}/M_{K^+})^2$  is used to extract both the branching fraction and the LECs of the form factor. If a large background component is present in the sample and corresponds to a spectrum that is difficult to predict, the associated sys-

tematic effects will be hard to estimate accurately and will subsequently reduce the precision of the analysis. At NA62, the experimental setup offers the possibility to collect a  $K_{\pi\mu\mu}$  sample with a negligible background. In the previous measurements, the dominant background was  $K_{3\pi}$  decays where two of the pions decayed in flight to muons, mimicking the signal final-state. In the NA48/2 measurement, the corresponding uncertainties were the largest systematic terms in the analysis [48].

The similarity of the two decays is both useful and a difficulty. For example, the  $K_{\pi\mu\mu}$  decay is characterised by one beam-originating three-track vertex, from which two reconstructed opposite sign muons and one positively charged pion originate. Similarly, the abundant  $K_{3\pi}$  decay is characterised by many of the same features with the exclusion of two final-state muons. This allows for the collection of a large normalisation sample with many similar features to the signal, but at the cost that  $K_{3\pi}$  can contribute as background in the SR if the pions decay in flight. Only with a highly-tuned selection which exploits the unique features of NA62 can a sensible balance be achieved.

The collection of both a  $K_{\pi\mu\mu}$  signal sample with negligible background and a well-symmetrised normalisation  $K_{3\pi}$  sample involves exploiting the following types of criteria:

- Vertex and track criteria, relating to kinematics and topology.
- Particle identification, relating to unique sub-detector responses.
- Precision timing between sub-detector signals.

Firstly, using the data and MC samples described in Section 6.2, a base-line NA48/2-like selection was implemented in order to compare the performance of NA62 with the previous experiment and is provided in Section 6.3. The  $K_{\pi\mu\mu}$  selection was finalised by maximising the number of observed signal candidates whilst rejecting the high-mass sideband events in 2016A, which was achieved by exploiting unique features of the NA62 design. The final selections (signal, normalisation) are presented in Section 6.5 including the invariant mass spectra for both signal  $K_{\pi\mu\mu}$  and



normalisation  $K_{3\pi}$ . Both selections are applied equally to both data and **MC** samples in the analysis.

## 6.2 Data samples

### 6.2.1 Recorded data samples

The selection was developed using a data-driven approach, using the very first data set produced by the NA62 experiment after the full detector commissioning in September 2016. At the time of this thesis, two samples were available for analysis, corresponding to data taken in the fall of 2016 and 2017. Both samples were reprocessed and reconstructed using the same NA62FW revision, and their features are summarised in Table **6.1**. The main focus of this thesis is on the 2016A data set.

Table 6.1: Summary of the current NA62 data samples for 2016 and 2017, including some key features of the data-acquisition **[81]**.

Sample	$N_{\text{Runs}}$	$N_{\text{Bursts}}$	Sample Features
<b>2016A</b>	169	89482	<ul style="list-style-type: none"> <li>- Minimum bias stream: Control trigger.</li> <li>- <math>K_{3\pi}</math> Normalisation: 2016A Multi-track trigger</li> <li>- <math>K_{\pi\mu\mu}</math> Signal: 2016A Dimuon trigger.</li> </ul>
<b>2017A</b>	79	82510	<ul style="list-style-type: none"> <li>- Minimum bias stream: Control trigger.</li> <li>- <math>K_{3\pi}</math> Normalisation: 2017A Multi-track trigger</li> <li>- <math>K_{\pi\mu\mu}</math> Signal: 2017A Dimuon trigger.</li> </ul>

The label 'A' is used to denote the period of data taking where both the conditions of the **TDAQ** were consistent over time and the sub-detector systems were performing nominally. Additional samples are expected to be available from summer 2018, which will increase the statistics in 2016 by a factor  $\approx 2$  and 2017 by a factor  $\approx 5$ . The two samples were collected with different trigger configurations, presented in Table **6.2**. For 2016A, both the signal and normalisation trigger streams were very similar, different by only the  $L_0$  condition  $MO_2$  used for collecting a Dimuon sample. The strategy changed for 2017A based on the data quality observed in 2016, where large inefficiencies were observed due to both the  $Q_X$  primitive and !LAV algorithm, as discussed in detail in Chapter **7**. In 2017A, the  $Q_X$  issue was fixed and the !LAV

Table 6.2: The trigger streams for normalisation  $K_{3\pi}$  and signal  $K_{\pi\mu\mu}$  in 2016 and 2017. The label  $D$  denotes the downscale on the trigger, and the primitives and algorithms below are described at the end of Chapter 3.

Sample	Decay	L <sub>0</sub> Primitive	L <sub>1</sub> Algorithm	$D$
2016A	$K_{3\pi}$	RICH · Q <sub>X</sub>	KTAG → !LAV → STRAW <sub>e</sub>	(25; 150)
	$K_{\pi\mu\mu}$	RICH · Q <sub>X</sub> · MO <sub>2</sub>	KTAG → !LAV → STRAW <sub>e</sub>	1
2017A	$K_{3\pi}$	RICH · Q <sub>X</sub>	KTAG → STRAW <sub>e</sub>	100
	$K_{\pi\mu\mu}$	RICH · Q <sub>X</sub> · MO <sub>2</sub>	STRAW <sub>e</sub>	2

removed all together from both streams. Additionally, the KTAG condition was removed from the Dimuon trigger so that exotic (forbidden) decays could be studied using the same stream. Apart from the differences in the trigger configurations, the 2017A data set was recorded at higher instantaneous beam intensity with respect to 2016A which directly affects the TDAQ performance. Figure 6.1 displays the relative intensity spectra for the samples as recorded by events collected with the Dimuon trigger. A detailed discussion of the 2017A data set is the topic of Chapter 9.

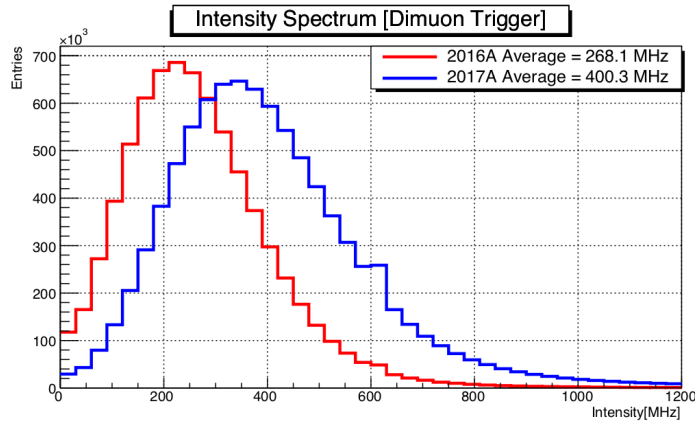


Figure 6.1: Comparison of the instantaneous intensity spectrum for 2016A and 2017A as observed by Dimuon triggered events. The average intensity is  $\approx 130$  MHz larger in 2017.

After the processing of both data sets, the filter `FilterDimuonThreeTrackVertex` was used to select only events that are considered significant for the analysis, so that the time taken to read the full data set is drastically reduced [67]. The tool is based on the filter used by the computation of  $(\alpha, \beta)$  described in Section 5.3 and offers a reduction factor of  $\approx 190$ . The selection is designed to collect both normalisation and signal, depending on the trigger fired for the event. The criteria are summarised

below, and they again focus on finding at least one good three-track vertex:

- The event must have fired either the Control, Multi-track or Dimuon trigger.
- At least one good three-track vertex built in the event, defined as:
  - The vertex fit  $\chi^2$  is less than 40.
  - The vertex longitudinal position ( $Z$ -Vertex) is greater than 102 m.
  - The vertex total momentum ( $P$ -Vertex) is less than 90 GeV/c.
  - All tracks in the vertex are within the geometric acceptance of the four STRAW chambers.
- **For Dimuon triggered events only:** At least two vertex tracks have a spatially associated MUV3 candidate.

All conditions used in the filter are implemented again in the selection but with tighter constraints. Additionally, bursts or events with low trigger counts or with sub-detector responses systematically shifted from the expectation are removed automatically by the standard bad burst mechanism, where all trigger related detectors are checked for issues [67].

## 6.2.2 MC samples

Large generations of MC events were generated specifically for this analysis. The two signal MC samples for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  are generated in the standard fiducial volume, between  $102.425 < Z < 180$  m downstream of the target. The  $K_{\pi\mu\mu}$  generator uses the form factor described in Section 2.2.3 for the transition, using LECs  $(a_+, b_+) = (-0.575, -0.700)$  which are the average of the E865 and NA48/2  $K_{\pi\mu\mu}$  measurements [55]. Additionally, the effect of the Coulomb interaction term  $\Omega_C$  between the final state particles is simulated for both signal and normalisation samples. Figure 6.2 contains  $\Omega_C$  for both  $K_{\pi\mu\mu}$  and  $K_{3\pi}$ ; the size is dependent only on the invariant masses of the pairs of daughters in the decay. Whilst the average  $\Omega_C$  is close to unity in both cases, hot-spots occur corresponding to where the daughter pair is at rest, such that the Coulomb interaction becomes involved by either attracting or repelling the particles in the pair. For  $K_{\pi\mu\mu}$ , the largest effect occurs where the

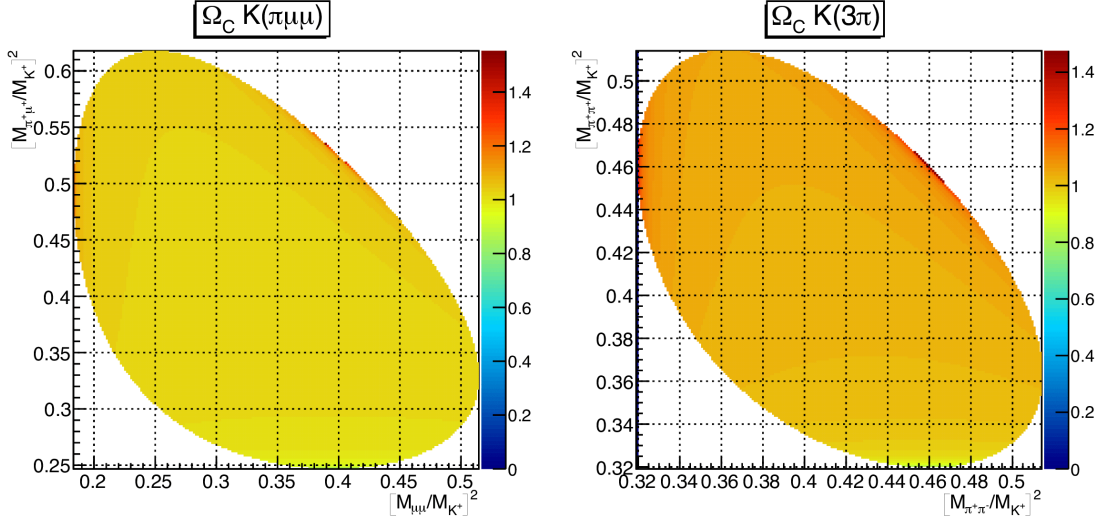


Figure 6.2: Magnitude of  $\Omega_C$  in Dalitz-space for  $K_{\pi\mu\mu}$  and  $K_{3\pi}$ . The term is on average is  $\approx 1.03$  for both decays, though the Dalitz space is symmetric in the case of  $K_{3\pi}$ . Three hot-spots are present which deviate largely from the average. The  $X$ -axis in right corresponds to the kinematic variable  $z_{\mu\mu}$ .

dimuon pair is at rest (i.e  $z_{\mu\mu} = 4r_\mu^2$ ), where  $Q_C$  rapidly increases. However, the hot-spots corresponding to where  $\pi^+\mu^+$  or  $\pi^+\mu^-$  are at rest occur approximately at the same point in  $z_{\mu\mu}$ , cancelling the effect when integrating over the  $M_{\pi^+\mu^+}$  dimension. The  $K_{3\pi}$  case displays the exact same features, but the Dalitz space is symmetric as all three daughters have the same mass.

Samples corresponding to processes that could contribute as backgrounds were also generated. The uncommon semi-leptonic  $K^+ \rightarrow \pi^+\pi^-\mu^+\nu_\mu$  ( $K_{\mu 4}$ ) decay with a branching fraction of  $(1.4 \pm 0.9) \times 10^{-5}$ , can contribute via decay-in-flight of the  $\pi^-$  track. In addition,  $K_{3\pi}$  decays occurring within the volume of the TRIM5 magnet are expected to be important; the magnet distorts the momentum of the tracks, resulting in a warped invariant mass spectrum. A summary of the generated samples used in the analysis can be found in Table [6.3](#), including the statistics and the volume downstream of the target used.

### 6.2.3 Tuning the MC

Efforts were made to ensure that the simulation is as close to real-life data-taking conditions as possible. Firstly, the efficiencies of important sub-detectors were mea-

Table 6.3: Summary of the generated NA62MC samples. The generated volume is with respect to the distance to the target, with  $N_{\text{Gen}}$  corresponding to the number generated.

MC Sample	$N_{\text{Gen}}$	Generated volume downstream
$K_{3\pi}$	$2.6 \times 10^7$	(102.425; 180.000) m
$K_{\pi\mu\mu}$	$10^7$	(102.425; 180.000) m
$K_{\mu 4}$	$2 \times 10^7$	(102.425, 180.000) m
$K_{3\pi}^{\text{TRIM5}}$	$10^8$	(96.950; 102.425) m

sured in data and included in the analysis framework. The second consideration is to measure the differences in the three-track reconstruction efficiency between data and simulation. Below, the extraction and implementation of these efficiencies is described.

### 6.2.3.1 Sub-detector efficiency

In this analysis, sub-detector efficiencies must be understood to produce a trustworthy acceptance measurement for both signal and normalisation. Although the bad-burst mechanism removes bursts and events based on expected detector responses, the detector inefficiency is expected to be a subtle yet systematic effect in all data sets. Fortunately, only a small number of sub-detector systems are used extensively in the analysis. In order of importance,

- STRAW: Track and vertex reconstruction.
- MUV3:  $\mu^\pm$  particle-identification.
- NewCHOD: precision timing.

The STRAW spectrometer is naturally the most important for all charged-track analyses, and is the subject of Section 6.2.3.2. The MUV3 detector is used for muon identification; the efficiency of the detector directly affects the  $K_{\pi\mu\mu}$  signal acceptance but not the  $K_{3\pi}$  normalisation. The NewCHOD efficiency is important for both samples, as it is present in the trigger and is also used for precision timing.

Two standard tools, `MUV3Efficiency` and `NewCHODEfficiency` [67], were implemented to determine both the MUV3 and NewCHOD detector efficiencies. Their methodologies are discussed below:

- **MUV3 Efficiency:** the analyzer selects a sample of halo muons using spectrometer and MUV 1/2 energy information. Halo muons are typically at large Closest-Distance of Approach (CDA) with respect to the beam and illuminate the full MUV3 face unlike  $K^+$  decays. By direct extrapolation of the selected tracks to the MUV3, the tool uses the standard MUV3 spatial search-radii to find whether a candidate was reconstructed as expected. The tool provides the tile-by-tile efficiencies.
- **NewCHOD Efficiency:** this analyzer uses all available one-track candidates to ensure a complete NewCHOD illumination. Like the MUV3 version, tracks are extrapolated and the standard NewCHOD spatial search-radii are used to find whether a hit was reconstructed as expected. Again, the tile-by-tile efficiencies are returned.

As both analyzers are designed to use minimum bias data, the efficiencies for both detectors were measured using the 2016A samples filtered for control-triggered events [67]. Figure 6.3 contains the tile-by-tile efficiency for both detectors, On average, the tile-

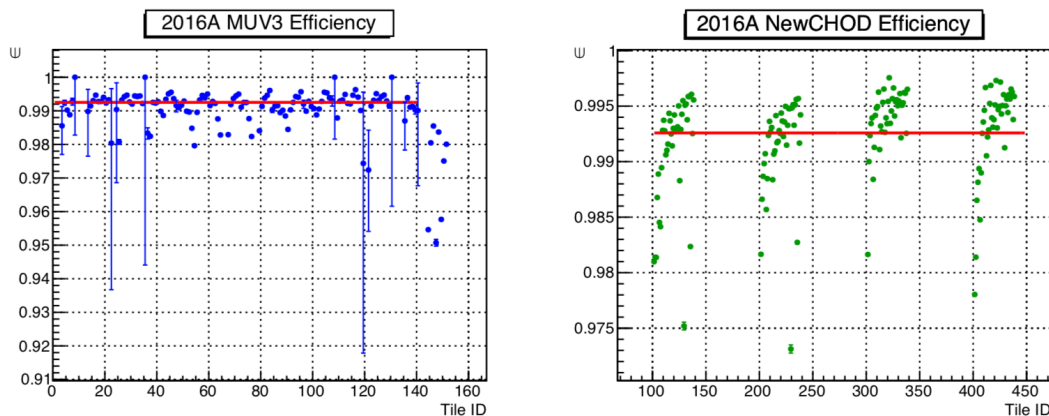


Figure 6.3: The MUV3 and NewCHOD detector efficiencies in 2016A, versus Tile-ID. Tiles with ID >140 are the inner MUV3 tiles, whilst the lower efficiency tiles at the start of each NewCHOD 'block' are also closest to the beam-line. The red lines denote the 'average' efficiencies overall, which are very similar (note the different Y-scales).

efficiency for both sub-detectors is very high at  $\approx 99.3\%$ . The MUV3 tiles with large errors all appear at the far edges of the detector where the particle rate is extremely low. Additionally, the lowest efficiency tiles all originate from regions closest to the beam-line for both sub-detectors. The numbering scheme of the MUV3 means that

the last nine tiles are the inner pads, whilst the first tiles at the start of each `New-CHOD` quadrant (four regions are clear in the plot) correspond to the inner-most regions. This is expected as the rate in these regions is the highest, which can lead to unavoidable losses because of radiation-related failures of the `TDAQ`.

### 6.2.3.2 Three-track reconstruction efficiency

The spectrometer track reconstruction efficiency is the most important detector-related efficiency to consider, as tracks are used for all kinematic calculations and sub-detector associations. If there is a fundamental difference in the reconstruction performance for data and `MC`, then the simulation is not a good description of real-life. Any residual differences must be taken into account.

A dedicated analyzer was developed based upon a similar study for single-track decays [82]. The procedure is to measure the efficiency using  $K_{3\pi}$  decays in a completely unbiased way, as follows,

- Build all two-track vertices using `SpectrometerVertexBuilder`.
- Assign  $\pi^\pm$  masses to the vertex tracks, and build a probe-track based on the missing four-momentum between the nominal kaon and the two-track vertex.
- Select events where the probe-track is consistent with the  $\pi^\pm$  hypothesis to within 10 MeV/c.
- Apply vertex and track-quality cuts based upon the  $K_{\pi\mu\mu}$  selection.
- Search for a real non-vertex reconstructed track consistent with the probe-track.

An event is considered efficient if a  $K_{3\pi}$  candidate can be fully reconstructed. This requires the probe-track to have a reconstructed match that was not used to build the vertex. The vertex and track-quality cuts used match those described in the selection development in Section 6.5.1, in order to measure reconstruction efficiencies that are the most relevant to the  $K_{\pi\mu\mu}$  analysis. In this preliminary tool, the beam-parameter determined kaon is used to build the probe track. In principle, this definition of the kaon is fixed per run and the precision could be improved by using matching `GTK`

candidates to measure the kaon properties on an event-by-event basis.

The analyzer is designed to be used identically for both data and **MC** in order to perform a direct comparison. Using the min-bias control-filtered data and the standard  $K_{3\pi}$  **MC** samples, the reconstruction efficiency can be plotted versus multiple variables. A directly relevant example is probe-momentum, as seen in Figure **6.4**. The agreement in (a) for data and **MC** is excellent, with only a small divergence

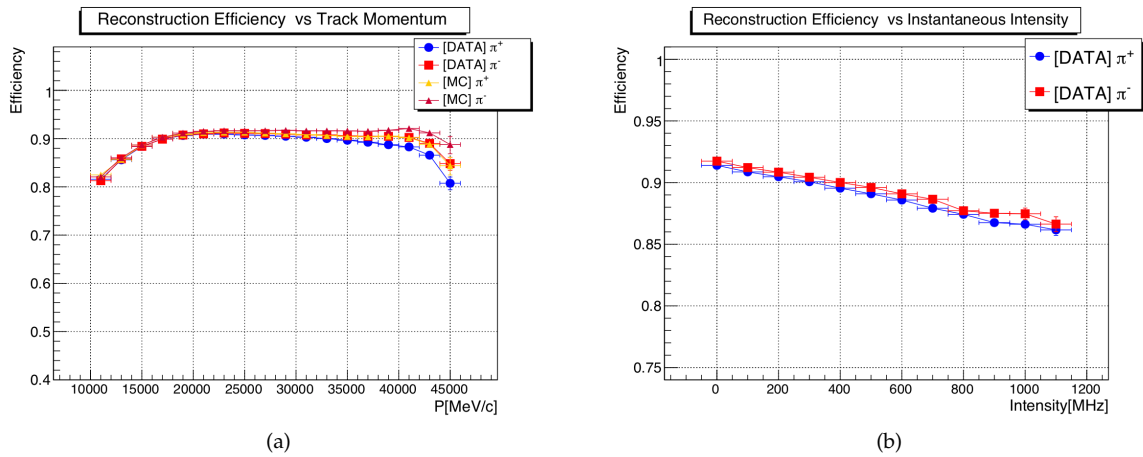


Figure 6.4: Three-track reconstruction efficiency versus (a) track-momentum for both data and **MC** and (b) for data versus instantaneous beam intensity. The first case shows excellent agreement but with a small divergence at high momentum. The reconstruction is also intensity-dependent, with a small difference between the efficiency as determined by the charge of the probe-track.

at high-momentum. The efficiency is also slightly dependent in (b) on the instantaneous beam-intensity, with a small but notable difference between the efficiency for three-track events depending on the charge of the probe that is created. This is quantified by Figure **6.5**, which is the reconstruction efficiency versus  $(X, Y)$  at the first STRAW chamber. The larger inefficiency occurs around the beam-hole and the outermost edges. Tracks in the inner regions are often reconstructed in fewer views than those at intermediate  $(X, Y)$ , whereas those which occur close to the beam-line or the outer-edges are prone to acceptance related effects. The radius of the illumination is under half the diameter of a STRAW chamber, where positive-probe tracks ( $\pi^+$ ) illuminate the chamber relatively evenly. Ultimately, by the fourth chamber, tracks on the outer edges can be swept out of acceptance as a result. In the case of negative probes ( $\pi^-$ ), a low-efficiency tail is evident to the left of the beam-



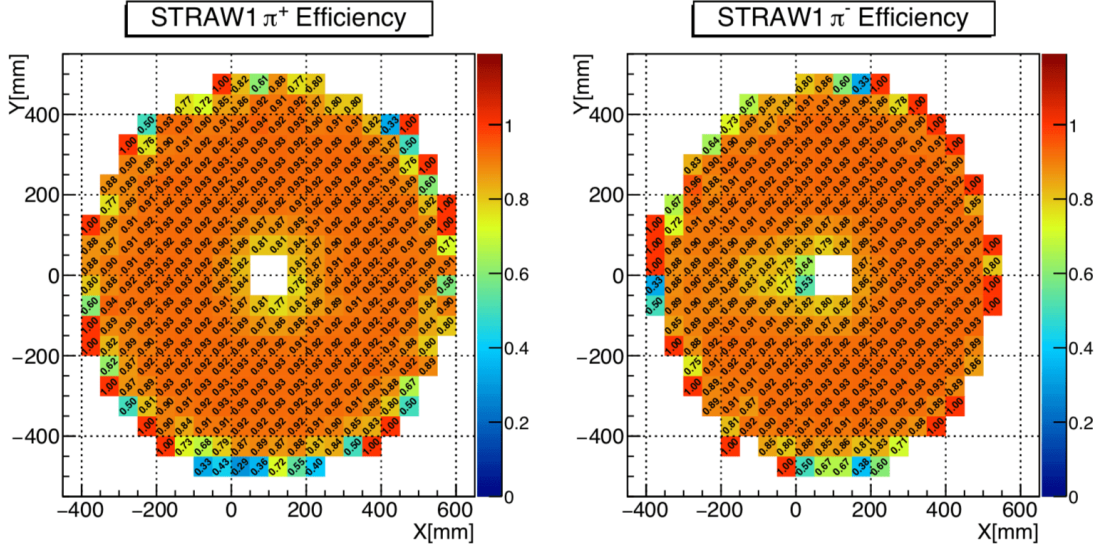


Figure 6.5: Three-track reconstruction efficiency at the first STRAW chamber, for left: the positive probe and right: the negative probe. The lower-efficiency of negative tracks can be seen to the left of the chamber hole, as they are swept by the MNP33 magnet towards the hole of the chambers downstream, whereas the effect for positive tracks is most dramatic on the outer edges. Overall, the negative case is of a slightly higher efficiency.

hole where the MNP33 magnet sweeps tracks inside of the beam-hole in the later chambers. The overall efficiency for both types of probe is essentially equivalent, at  $\approx 91\%$ . Despite the preliminary nature of this measurement, the analysis is in agreement with similar studies performed based on `MC`-only methods. In Ref. [83], an alternative `MC` Truth-Reco matching is performed, recording efficiencies for  $K_{3\pi}$  events of  $\approx 89\%$ . By loosening the  $K_{\pi\mu\mu}$  tuning, the method described here provides a high-level of agreement with these values.

### 6.2.3.3 Applying detector efficiencies

A pre-analyzer `ApplyDetectorEfficiency` was developed to remove reconstructed candidates based upon the detector and track-reconstruction efficiencies for simulated samples. This is used before the stage of selection, so that both data and `MC` are as close to equivalent as possible before the proper analysis. In the case of the track-reconstruction, it is the ratio between the data and `MC` response that is important. Using the ratio, the `MC` simulation can be artificially corrected to match the real data distribution. The additional complication is that the data response is intensity dependent. To handle all of the above, the following method is

implemented:

- **MUV3, NewCHOD**: Tile-by-tile efficiencies are read by the pre-analyzer. The MC MUV3/NewCHOD candidates, which each have an associated tile, are removed if a flat random number generation between (0; 1) is greater than the corresponding tile-efficiency as measured in data.
- **Track reconstruction**: the momentum-dependent reconstruction efficiencies are read by the pre-analyzer for both data and MC samples. The ratio of the data and MC efficiency is created with respect to the track momentum. Simulated track candidates with momentum  $p$  are removed if a flat random number generation between (0; 1) is greater than the data/MC efficiency ratio distribution at the corresponding momentum bin. Additionally, the ratio can be produced for any intensity between  $\mathcal{I} = (0; 1200)$  MHz based on the response in data.

The signal and normalisation acceptances which are determined using MC samples are therefore intensity-dependent, and must be combined using weighted averages based on the residual intensity spectrum.

## 6.3 Base-line selection

The NA62 experiment is designed to be technically superior with respect to the previous NA48/2 experiment. The more sophisticated tracking system has an improved design resolution, allowing for smaller Signal Regions (SRs). Additionally, the implementation of a larger magnet current is expected to increase separation between signal and the dominant irreducible background from  $K_{3\pi}$  decays observed in the previous analysis. In order to observe the situation for NA62 in comparison, a NA48/2-like base-line selection was adopted, after tuning specific kinematic cuts to suit the NA62 experimental setup.

### 6.3.1 Base-line vertex and track requirements

Require exactly one good three-track vertex present in the event, defined as:

- The vertex charge is  $Q = +1$ .

- The vertex fit  $\chi^2$  is less than 20.
- The vertex longitudinal position ( $Z$ -Vertex) is within (102; 180) m
- The vertex total momentum ( $P$ -Vertex) is within (73; 77) GeV/c.
- The vertex transverse momentum  $P_T$  is less than 30 MeV/c.
- The vertex-beam **CDA** is less than 50 mm.

For the tracks in the three-track vertex:

- Each track must be within the geometric acceptance of the four STRAW chambers, **NewCHOD**, **LKr** and **MUV3**.
- Each of the tracks must be separated from one another by at least 20 cm at the **LKr** face.
- At least two of the tracks must have a spatially associated **NewCHOD** candidate. The NewCHOD candidate time is used to build a vertex time which is the average of  $2 \leq n \leq 3$  tracks.
- The vertex time must be within 10 ns of the trigger time responsible for collecting the event, and self-consistent to within 10 ns of each of the vertex tracks.

The selection searches for good quality (low  $\chi^2$ ), beam-originating three-track vertices. The cut in longitudinal  $Z$ -Vertex corresponds to the fiducial decay volume defined by the geometrical region of the vacuum tank, whilst the  $P$ -Vertex,  $P_T$  and CDA conditions assure consistency with the  $K^+$  beam momenta and position. The requirement for the tracks to be in the acceptance of multiple sub-detectors is for several reasons,

- **STRAW**: Improves the track quality and momenta.
- **NewCHOD**: Improves vertex time by increasing association likelihood.
- **LKr**: Improves the  $E/P$  calculation.
- **MUV3**: Reduces the chance of mis-identification.

The association of tracks to detector candidates is performed spatially using search-radii along the line of the extrapolation. Search-radii are required due to multiple scattering in the STRAW tubes. In general, the radii are different at each sub-detector and are inversely dependent on track momenta. For the `NewCHOD` case, the radius is defined by the width of the truth-reconstructed residual of extrapolated track positions for a  $K_{\mu 2}$  sample, where the resolution is  $\sigma_{x,y} = (0.07 + 47.3\text{GeV}/p)$  mm [67].

Timing is only requested to check consistency with the trigger. `NewCHOD` candidates are used to build the vertex time as the CHOD is unreliable for the current available version of the data set. Whilst the `CHOD` has the best time resolution, hits in the top half of the detector experienced a flaw which applied the T0 timing corrections multiple times depending on the number of hits in coincidental horizontal counters [84]. With the next revision, `CHOD` candidates could be re-adopted.

### 6.3.2 Base-line particle identification

Build a particle-identification (PID) defined as:

- Exactly two vertex tracks with at least one spatially associated `MUV3` candidate ( $\mu$  identification) and one with none ( $\pi$  identification.)
- $\mu$  tracks must have a `LKr`  $E/P$  less than 0.2.
- $\pi$  tracks must have a `LKr`  $E/P$  less than 0.9.

Requiring two tracks to have a spatially associated `MUV3` candidate is the simplest form of  $K_{\pi\mu\mu}$  identification. The spatial association radii for `LKr` and `MUV3` are slightly different compared to the `NewCHOD` [67],

- `LKr` uses a search radius of  $(50 + 5)$  mm around track extrapolations to handle residual misalignment.
- `MUV3` uses a similar definition to the `NewCHOD`, where the resolution was determined to be  $\sigma_{x,y} = (2120 \text{ GeV}/p)$  mm.

The `LKr`  $E/P$  criterion was inspired by a study performed for the search for the `Lepton Number Violating (LNV)` decay  $K^\pm \rightarrow \pi^\mp \mu^\pm \mu^\pm$  at NA48/2 [74], where the

same **LKr** detector was in operation. By collecting a pure data sample of  $\pi^\pm$ ,  $\mu^\pm$  and  $e^\pm$  in data, the probability to have muons with  $E/P < 0.2$  was found to be  $(99.74 \pm 0.01)\%$ , whilst electrons can be suppressed by  $\approx 10^2$  using  $E/P < 0.9$ .

### 6.3.3 Base-line spectra

The base-line spectrum is built with dimuon-triggered candidates that survived the full base-line selection. Normalisation candidates are collected from multi-track-trigger events which pass only the vertex and track requirements. The  $K_{\pi\mu\mu}$  invariant mass spectrum is displayed in Figure 6.6, including the **MC** overlays of the main contributions for the 2016A data sample. For comparison, the distributions from the NA48/2 paper are displayed in Figure 6.7.

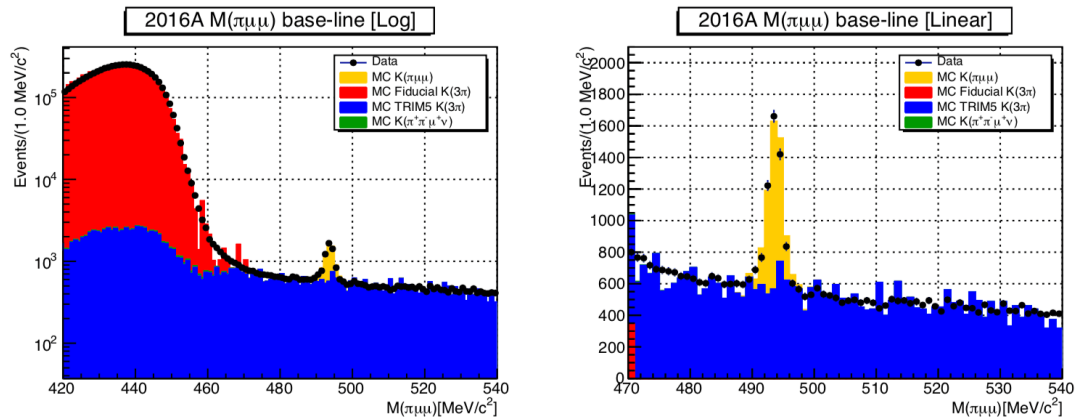


Figure 6.6: The  $K_{\pi\mu\mu}$  invariant mass distribution in 2016A for the base-line selection with right: with log-scale and left, linear in the region near  $M_{K^+}$ . Orange denotes the  $K_{\pi\mu\mu}$  **MC** expectation.

The spectrum contains a number of interesting features, indicating some obvious differences between the two data samples collected at the NA48/2 and NA62 experiments:

- **Separation:** A clear separation for NA62 data between the signal  $K_{\pi\mu\mu}$  (orange) and the low-mass backgrounds  $K_{3\pi}$  (red) and  $K_{\mu 4}$  (green). Both backgrounds enter due to secondary in-flight pion decays to muons, or via accidental in-time **MUV3** associations.
- **Resolution:** Improved signal resolution  $\sigma_M \approx 1.1 \text{ MeV}/c^2$ , over a factor two smaller than NA48/2.

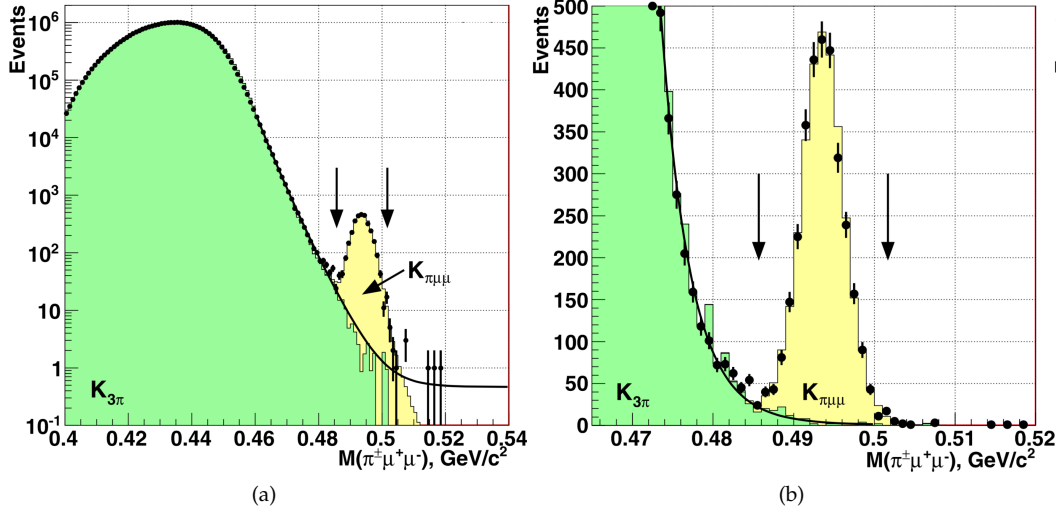


Figure 6.7: The  $K_{\pi\mu\mu}$  invariant mass distribution from the NA48/2 analysis. (a) is in log-scale and (b) in linear. The fiducial  $K_{3\pi}$  background tails directly into the **SR** (denoted by arrows) and the  $K_{\pi\mu\mu}$  mass resolution  $\sigma_M$  is equal to 2.5 MeV/c<sup>2</sup> [48].

- **Tail:** Significant exponential tail above 480 MeV/c<sup>2</sup> for NA62 data which was not observed at NA48/2. Hypothesised to be upstream  $K_{3\pi}$  decays.

The first two points are expected as a consequence of a more sophisticated tracking system. The better  $\sigma_M$  resolution allows for a smaller **SR** definition,

$$|M_{\pi\mu\mu,3\pi} - M_{K^+}| \leq 5 \text{ MeV}/c^2. \quad (6.1)$$

which is approximately five standard deviations for both cases. Despite the improvements, a large-exponential tail can be seen, a feature not observed by NA48/2. This component is well-described by the  $K_{3\pi}^{\text{TRIM5}}$  simulation represented by the blue-distribution, where  $K_{3\pi}$  candidates have been forced to decay upstream ( $Z = 102$  m) in the TRIM5 magnet region equivalent to  $\approx 0.5\%$  of the fiducial decay volume. The acceptance for these decays is  $\approx 4$  times greater than for the fiducial counterparts, as tracks are less likely to escape down the beam-pipe as they originate upstream. The TRIM5 steering magnet has the additional affect of distorting track momentum, leading to  $K_{3\pi}$  candidates with two secondary pion decays appearing in the long tails into and beyond the **SR**. To achieve the aim of a negligible background component, a selection which fully suppresses the TRIM5 component is required.

## 6.4 Developing the NA62 selection

In this section, the development of a more stringent  $K_{\pi\mu\mu}$  selection is described. A data-driven approach was used to obtain a final selection that removes all background candidates  $N_B$  in the high-mass sideband of the  $K_{\pi\mu\mu}$  **SR**, defined as  $M_{\pi\mu\mu} > 505 \text{ MeV}/c^2$ .

### 6.4.1 Tuning vertex and track criteria

The observation that the long exponential background is well described by TRIM5  $K_{3\pi}$  **MC** provides a hint on how it can be suppressed. Firstly, TRIM5 decays occur 'upstream' of the usual fiducial volume, and secondly the track momentum of the daughters is therefore distorted as the decay happens before the beam has been fully steered. In data, the invariant mass spectra  $M_{\pi\mu\mu}$  can be plotted against vertex/track-related variables as seen in Figure **6.8**. The left-hand distribution dis-

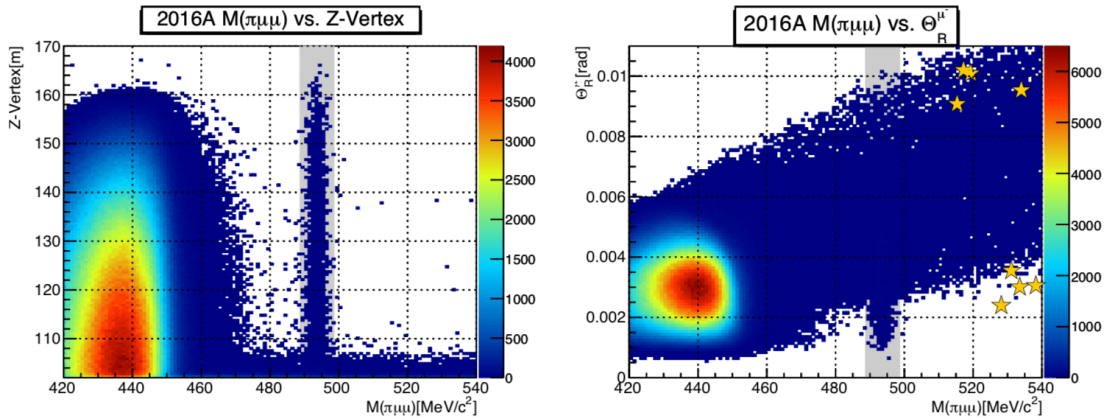


Figure 6.8: The 2016A invariant mass distributions plotted against left,  $Z$ -Vertex and right, the  $\mu^-$  opening angle  $\Theta_R$  with respect to the lab frame. The **SR** is in grey and unconstrained in either  $Y$ -dimension, and the stars correspond to candidates surviving the selection after a  $Z$ -Vertex  $> 110 \text{ m}$  cut is applied.

plays that the vast majority of sideband events occur at low  $Z$ -Vertex, strengthening the TRIM5 hypothesis. Though the TRIM5 magnet is only 40 cm long and centred at 102 m, distortion of all three vertex tracks can modify the determined decay coordinate, boosting the  $Z$ -Vertex well into the fiducial volume.

Additionally, the steering distortion can be observed using the lab-frame opening

angles  $\Theta_R$  of the daughter tracks. The range of opening angles available to each daughter track is fixed kinematically depending on the available phase-space and daughter mass. Pions in  $K_{3\pi}$  are limited to smaller opening angles compared to the products in  $K_{\pi\mu\mu}$  as a consequence. The right-hand-side of Figure 6.8 displays that low-mass candidates (mostly  $K_{3\pi}$  background) are at smaller opening angles, whilst the tail has a distinctive shape. The orange stars correspond to candidates surviving the selection after cutting away events with a  $Z$ -Vertex below 110 m, motivating additional criteria based on  $\Theta_R$ . Figure 6.9 contains the opening angles for the relevant processes for both positive and negative tracks. The fiducial MC generations show

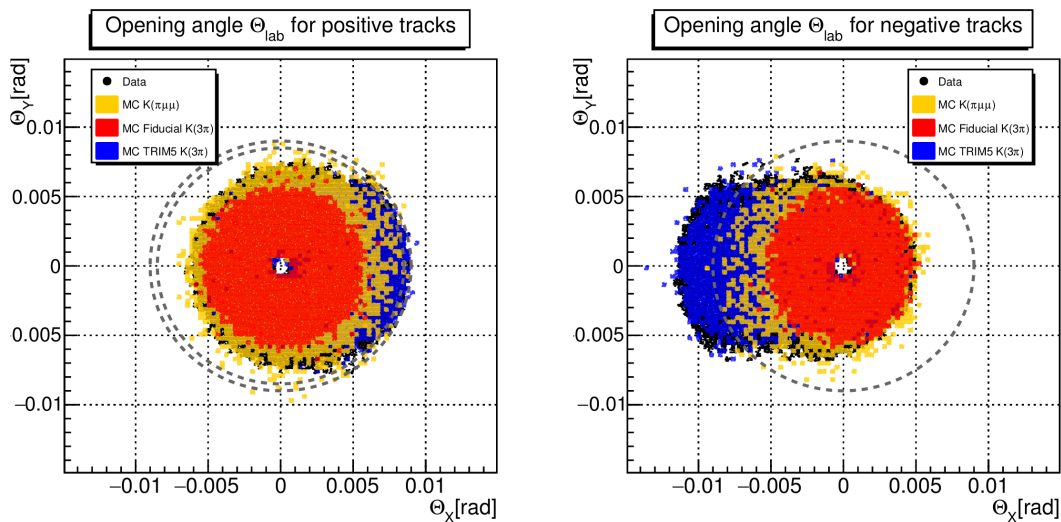


Figure 6.9: Comparison of opening angles for positive and negative tracks for the data set, fiducial MC  $K_{3\pi}$ , TRIM5  $K_{3\pi}$  and  $K_{\pi\mu\mu}$ . Each of the MC samples are scaled to match the expected entries. The selection criterion for each track is displays by the dashed circles, with the smaller radii case in the left-hand-side for the  $\pi^+$  track.

circular distributions, where  $K_{3\pi}$  sits inside of the available opening angles for  $K_{\pi\mu\mu}$ . The most indicative effect of the TRIM5 distortion is seen in the negative track distribution, where tracks follow an oval shape, spraying into the negative  $X$ -dimension beyond the  $K_{\pi\mu\mu}$  tails. The large difference with respect to positive tracks is because the steering is tuned for the positively charged kaons, so the positive products are steered similarly, whilst the negative tracks are deflected more dramatically.

The strategy to suppress the majority of the background is straightforward:

- The vertex longitudinal position ( $Z$ -Vertex) is within (110; 165) m



- The lab-frame opening angle  $\Theta_R$  for each track in the vertex must be:
  - for  $\mu$  tracks less than 9.0 mrad.
  - for  $\pi$  tracks less than 8.5 mrad.

The choice of opening angle cut is displayed on both sides of Figure 6.9, and applied to all three tracks for symmetry. Figure 6.10 displays the change in the invariant mass spectrum after applying the new cuts. The TRIM5 component predicted using

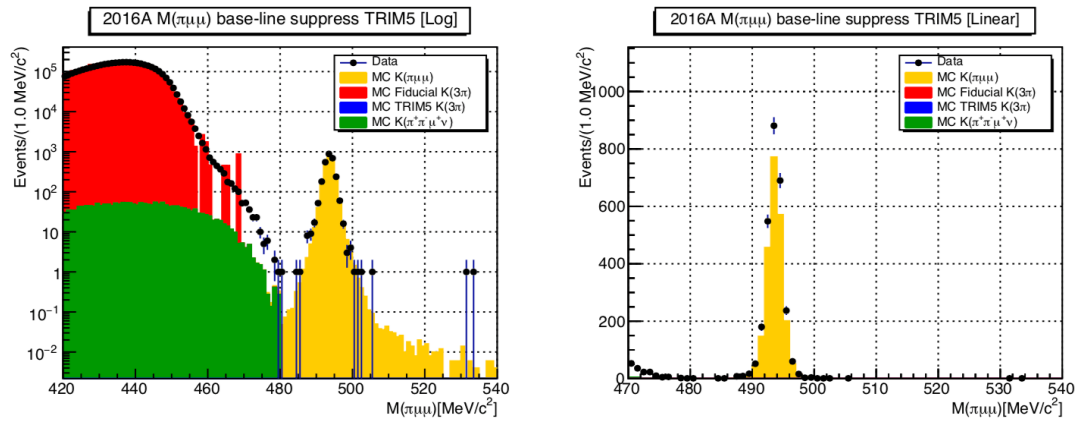


Figure 6.10: Invariant mass distribution in 2016A for base-line selection after including TRIM5 suppression cuts. Left, log-scale and right, high-mass linear. The majority of high-mass sideband candidates have been suppressed.

$N_{3\pi}$  and the  $K_{3\pi}$  generation is now fully suppressed, leaving only two candidates in the high-mass sideband. The linear scale shows that the prediction of the number of  $K_{\pi\mu\mu}$  candidates is  $\approx 10\%$  lower than the number observed. There are a number of possible reasons, such as a disagreement between data and  $K_{\pi\mu\mu}$  or that the trigger criteria respond differently depending on the decay entering the plot.

## 6.4.2 Tuning PID and including precision timing

To achieve a truly negligible background contribution, a more stringent particle identification is required to suppress the various components observed in Figure 6.10. Each possible background is discussed below,

- $K^+ \rightarrow \pi^+\pi^+\pi^- \rightarrow \pi^+\mu^+\mu^-$ : Whilst the mass-separation from  $K_{\pi\mu\mu}$  is vastly improved with respect to NA48/2,  $K_{3\pi}$  with two secondary pion decays in-flight is a large component. Here, pions are assigned muon masses, appearing

in the  $M_{\pi\mu\mu}$  spectrum at low values. Unfortunately, the lack of [MC](#) statistics for this process makes it difficult to compare with data in the region  $470 < M_{\pi\mu\mu} < 485 \text{ MeV}/c^2$ .

- $K^+ \rightarrow \pi^+\pi^-\mu^+\nu_\mu \rightarrow \pi^+\mu^+\mu^-\nu_\mu$ : Whilst occurring at a rate  $\approx 10^3$  lower than  $K_{3\pi}$ , only one secondary pion decay in-flight is required to enter as background. As the decay is four-body and a real muon is present in the intermediate stage, a broader invariant mass spectrum that approaches the  $K_{\pi\mu\mu}$  peak is predicted in [MC](#). An additional complication is due to the possible  $\pi^+\mu^+$  'swaps', where the pion is mis-identified as the muon and vice-versa.
- $K^+ \rightarrow \mu^+\pi^+\mu^- \rightarrow \pi^+\mu^+\mu^-$ : The signal process  $K_{\pi\mu\mu}$  can appear as a background to itself due to the possible  $\pi^+\mu^+$  swaps which also affect  $K_{\mu 4}$ . Assigning the wrong mass hypothesis to the tracks results in the prediction of flat tails around the peak in the [MC](#), contributing at the level of  $10^{-2}$  in the spectrum.
- Accidentals: The high-intensity environment of NA62 leads to a significant number of accidental candidates in the downstream sub-detectors. With more possible associations for vertex tracks, the probability for mis-identification is larger than that predicted in the [MC](#). Therefore, the  $\pi^+\mu^+$  swapping mechanism could be far larger in reality.

Therefore, a more stringent set of particle-identification criteria was introduced based upon the identification of the dimuon pair  $\mu^+\mu^-$ . The requirement of exactly two tracks with at least one spatially associated [MUV3](#) candidate is expanded, including the following conditions:

- The  $\mu$  tracks are opposite sign ( $\mu^+\mu^-$ ).
- Both  $\mu$  tracks are associated to at least one outer [MUV3](#) tile.
- Neither  $\mu$  track can be associated to the same [MUV3](#) candidate, provided that both tracks have only one association.
- Both  $\mu$  tracks are associated to [MUV3](#) candidates within 5 ns of the vertex time.

The first condition assures that only a SM-allowed sample is collected. This requirement also suppresses the contribution of secondary decays in-flight of  $K^+ \rightarrow$

$\pi^+\pi^+\pi^- \rightarrow \mu^+\mu^+\pi^-$ , which will enter directly into the spectrum provided there is no requirement on the muon charge configuration. Additionally, accidentals for this process are easy to obtain due to the beam-halo and the dominant  $K_{\mu 2}$  rate. The requirement for outer **MUV3** associations is to align with the trigger logic  $MO_2$ , which requires two outer **MUV3** pads to be fired within 10 ns. As a consequence, both criteria ensure a larger separation of the muon pair at the **MUV3** face. The final criteria are motivated by Figure 6.11. The  $\pi^+\mu^+$  swaps can occur where the real  $\mu^+$  track

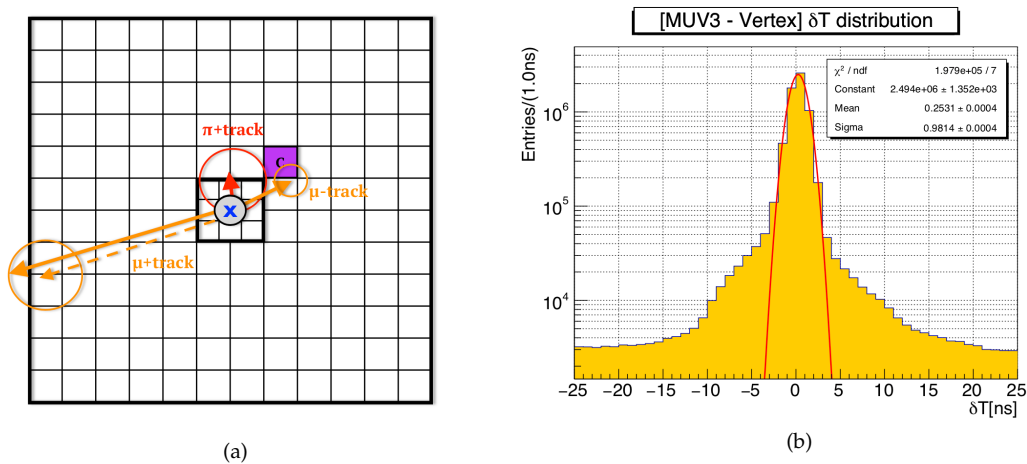


Figure 6.11: Supporting diagrams for the **MUV3** related logic. a) A schematic of a  $K_{\pi\mu\mu}$  decay originating upstream from the beam 'X', where the real  $\mu^+$  misses the detector and the  $\pi^+$  is accidentally associated to the same candidate as the  $\mu^-$ . b) the timing resolution between vertex tracks and **MUV3** candidates for a  $K_{\pi\mu\mu}$ -like selection.

misses the detector and the  $\pi^+$  is associated spatially to the same candidate as the  $\mu^-$  track. The example shows that even a single **MUV3** candidate could be misidentified as  $K_{\pi\mu\mu}$  in the circumstance of the true muon track missing the detector. Accidental rate also contributes to mis-identification. By applying a precise timing constraint, most of the accidental associations can be suppressed. The right-hand plot shows the timing residual between the vertex times and associated **MUV3** candidates, which has a 1 ns resolution.

The final advantage unique to NA62 is the **KTAG** detector. The **KTAG** time-stamps  $K^+$  candidates with a resolution to better than a few hundred ps. As both  $K_{\pi\mu\mu}$  and  $K_{3\pi}$  originate from kaon decays, checking if the three-track vertex has a **KTAG** candidate consistent in time is a definite advantage over the previous analyses. We

can introduce a requirement to find at least one good **KTAG** candidate consistent with the signal decay, defined as,

- At least four **KTAG** sectors fired,
- Candidate time within 10 ns of the trigger time,
- Candidate time within 10 ns of the vertex time.

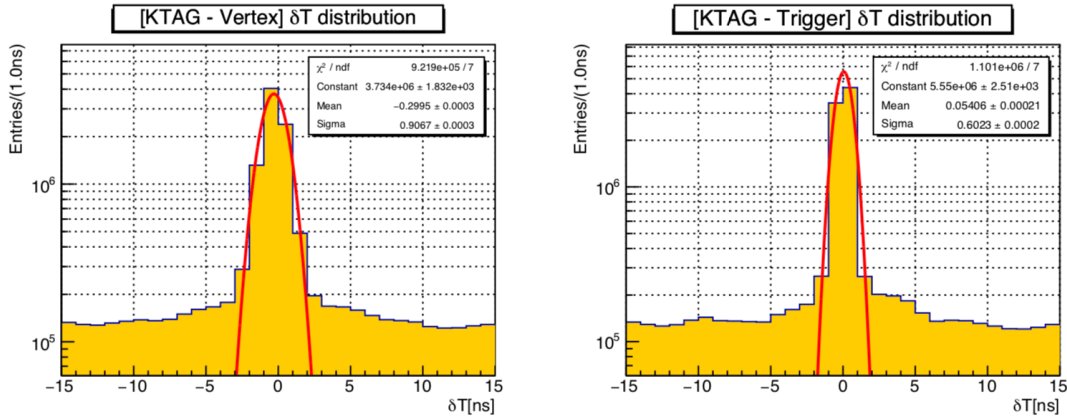


Figure 6.12: The timing residuals of left, **KTAG** candidates to three-track vertex times, and right, to the Dimuon trigger time. The resolution of both distributions is below 1 ns.

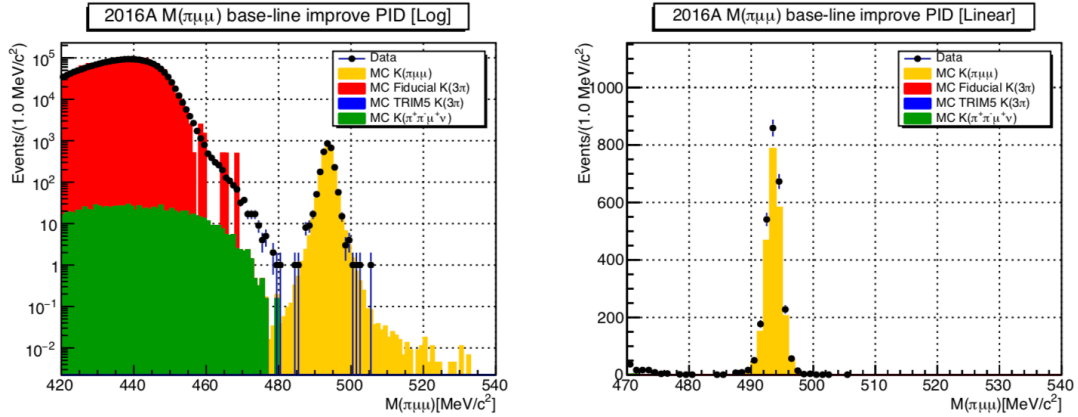


Figure 6.13: Invariant mass distribution in 2016A for base-line selection after TRIM5 suppression and the improved PID criteria. Left, log-scale and right, high-mass linear. All sideband candidates have been suppressed, with an improved separation between low-mass and the signal peak.

The choice of criteria is inspired by the  $L_1$  trigger algorithm which is present in both dimuon and multi-track trigger streams. The 10 ns time window is relatively broad with respect to the 1 ns resolution observed in the residuals, as displayed in Figure 6.12. By requesting a good **KTAG** candidate time-consistent with the vertex,

accidental contributions at the **MUV3** stage are also improved as three detectors are required to be in timing coincidence. The invariant mass spectrum after including the new PID criteria is contained in Figure **6.13**, where all high-mass candidates have been successfully removed and the signal and background separation is optimal.

## 6.5 The final selection

The selection was finalised by maximising the number of observed  $K_{\pi\mu\mu}$  candidates  $N_S$  whilst fully suppressing the sideband components  $N_B$ . A simplistic iteration procedure was used,

1. Remove each cut independently, and build a selection of the criteria which increase  $N_B$  when removed.
2. Continue to reintroduce cuts until  $N_B = 0$ , starting with the conditions which decrease  $N_S$  the least.

This resulted in the removal of a small number of cuts, specifically the  $E/P$  requirement for muons and the separation of the tracks at the **LKr**. The procedure was also used to rule out the use of the LAV in the analysis, based on the large amount of signal that would be lost due to accidental activity.

The final selection is used to collect both normalisation  $K_{3\pi}$  and signal  $K_{\pi\mu\mu}$ , by requiring vertex quality requirements and kaon identification for both signal and normalisation, then  $\mu^\pm$  identification for  $K_{\pi\mu\mu}$  signal alone. The use of extremely similar selections is to assure a high-level of symmetry between the two samples in order to cancel the selection-related systematic effects to first order. In addition to the criteria discussed previously, a track-momentum cut was included because the track-reconstruction efficiency could only be probed in the region between  $8 < P < 48$  GeV/c.

### 6.5.1 Track and vertex requirements

Exactly one good three-track vertex built in the event, defined as:

- The vertex charge is  $Q = +1$ .

- The vertex  $\chi^2$  fit is less than 20.
- The vertex longitudinal position ( $Z$ -Vertex) is within (110; 165) m.
- The vertex total momentum ( $P$ -Vertex) is within (73; 77) GeV/c.
- The vertex transverse momentum  $P_T$  is less than 30 MeV/c.
- The vertex-beam **CDA** is less than 50 mm.

For each track in the three-track vertex:

- Each track is within the geometric acceptance of the four STRAW chambers, **NewCHOD**, **LKr** and **MUV3**.
- At least two of the tracks must have a spatially associated **NewCHOD** candidate. The NewCHOD candidate time is used to build a vertex time which is the average of  $2 \leq n \leq 3$  tracks.
- The vertex time must be within 10 ns of the trigger time responsible for collecting the event, and self-consistent to within 10 ns of each of the vertex tracks.
- Momentum within (8; 48) GeV/c, for higher trigger and reconstruction efficiency, discussed in Chapter **7** and Section **8.2.2**.

### 6.5.2 Kaon identification

At least one good **KTAG** candidate, defined as

- At least four KTAG sectors fired.
- Candidate time is within 10 ns of the trigger time.
- Candidate time is within 10 ns of the vertex time.

### 6.5.3 Decay identification

For events collected with the dimuon trigger only,  $(\mu^+\mu^-)$  identification is applied, defined as:

- Exactly two tracks in the vertex with at least one geometrically associated **MUV3** candidate.

- Require  $\mu$  tracks to be opposite sign.
- Require lab-frame opening angle  $\Theta_R$  for  $\mu$  tracks to be less than 9.0 mrad.
- Require  $\mu$  tracks to be associated to at least one outer **MUV3** pad.
- Require associated **MUV3** candidate time to be within 5 ns of vertex.
- Require for no repeated **MUV3** pad associations for  $\mu$  track pairs that have only one association each.

Pion ( $\pi^+$ ) identification, defined as:

- Exactly one track with no **MUV3** candidate.
- Require lab-frame opening angle  $\Theta_R$  for  $\pi$  track to be less than 8.5 mrad.

Electron ( $e^\pm$ ) rejection, defined as:

- $E/P < 0.9$ .

All Dimuon triggered events surviving the selection are saved, as the candidates outside the signal region are used to estimate the background level in the signal region.

Independently, the following is applied for the Multi-track triggered events,

- At least one  $\pi^+$  track with no spatially associated **MUV3** candidates.

The cut above is to symmetrise with the  $\pi^+$  identification in the  $K_{\pi\mu\mu}$  selection and drastically reduce any effects of accidental **MUV3** association on the acceptance calculation.

#### 6.5.4 The final spectra

The final  $K_{3\pi}$  normalisation spectrum is contained in Figure **6.14**. The sample is very clean, with a background contamination of  $(0.0010 \pm 0.0001_{\text{stat}} \pm 0.0007_{\text{syst}})\%$ , where the error is dominated by the systematic uncertainty on  $\mathcal{B}(K_{\mu 4})$ . The final  $K_{\pi\mu\mu}$  spectrum is contained in Figure **6.15**, where the contribution from high-mass candidates has been suppressed and the separation between the low-mass tail and the  $K_{\pi\mu\mu}$  is

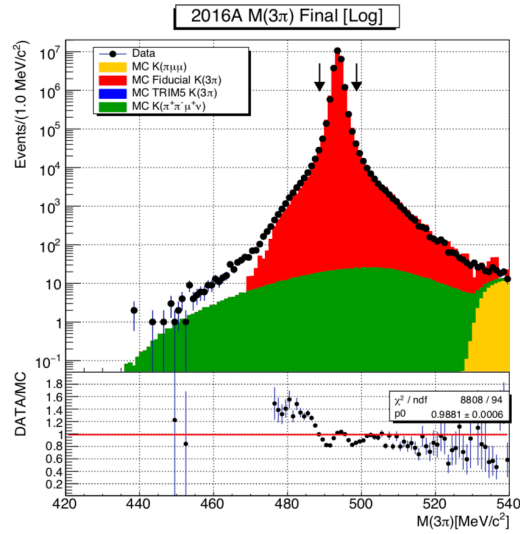


Figure 6.14: The final normalisation  $K_{3\pi}$  spectrum for 2016A. The signal region is denoted by arrows, using a  $\pm 5$  MeV window around the nominal kaon mass, and the bottom distribution is the data sample divided by the MC sample to show agreement.

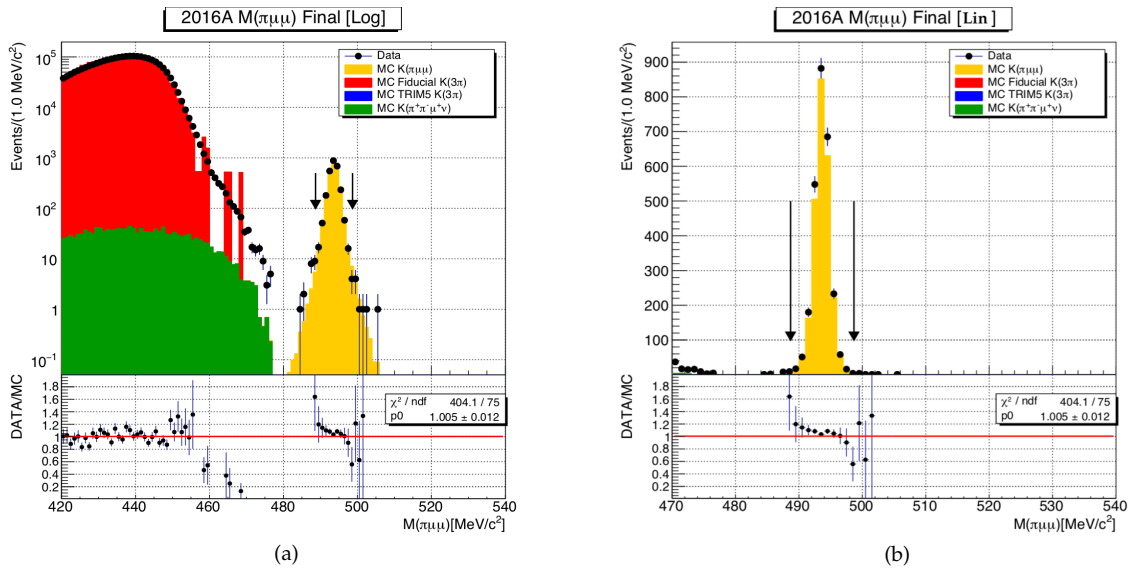


Figure 6.15: The  $K_{\pi\mu\mu\mu}$  spectra for 2016A in (a) log-scale and (b) linear. The signal regions are denoted by arrows, using a  $\pm 5$  MeV window around the nominal kaon mass, and the bottom distributions is the data sample divided by the MC sample to show agreement.



notable to below  $10^{-1}$ . Unlike  $K_{3\pi}$ , the number of background candidates has been estimated using RooFit to be below one, as explained in more detail in Section [8.2.1](#) in analysis Chapter [8](#). The data-MC discrepancy is reduced by a few percent in the final selection, and is explored further in the  $d\Gamma/dz$  reconstruction procedure.

Finally, a data-MC comparison procedure was performed for both the  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  samples. Multiple distributions can be found in Appendix [A](#) for the most important cuts implemented in the analysis. The only large shape-discrepancy is for the description of the vertex momentum, but as the disagreement is present for both normalisation and signal, the effect is expected to cancel out to first order with the normalisation procedure.

# 7

## Measuring trigger efficiency

In this chapter, the trigger efficiencies for the  $K_{3\pi}$  normalisation and  $K_{\pi\mu\mu}$  signal samples are measured. The samples were collected using the following trigger streams:

- $K_{3\pi}$ : RICH·Q<sub>X</sub> →KTAG→!LAV→STRAW<sub>e</sub> (Multi-track trigger)
- $K_{\pi\mu\mu}$ : RICH·Q<sub>X</sub>·MO<sub>2</sub> →KTAG→!LAV→STRAW<sub>e</sub> (Dimuon trigger)

The similarity of the streams is deliberate, so that any systematic effects of the data acquisition process will cancel via a normalisation procedure. For the data recorded in 2016, a series of flaws were discovered with several aspects of the [TDAQ](#), imposing the need for an extreme level of care when determining the efficiencies. As the flaws relate to both signal kinematics and beam intensity, a complete cancellation of the systematic components is highly implausible. Additionally, the abundance of  $K_{3\pi}$  decays means that each normalisation efficiency component can be easily measured using minimum-bias streams, but as the ratio  $\mathcal{B}(K_{\pi\mu\mu})/\mathcal{B}(K_{3\pi}) \approx 10^{-6}$ , alternative approaches must be developed to obtain high-precision efficiency measurements for  $K_{\pi\mu\mu}$ . The final and most important requirement for the  $K_{\pi\mu\mu}$  efficiency measurement is that it must be performed in bins of  $z_{\mu\mu} = (M_{\mu\mu}/M_{K^+})^2$ ; the extraction of both the branching fraction and form factor [LECs](#) relies on an accurate reconstruction of the underlying  $d\Gamma/dz$  versus the  $z_{\mu\mu}$  spectrum. The strategy to perform these measurements is explained in Section [7.1](#), with the computation for each common trigger component provided in order of difficulty between Section [7.2](#)-[7.6](#).

and followed by the  $\text{MO}_2$  calculation unique to the Dimuon trigger in Section 7.7. The final trigger efficiencies are provided in Section 7.8.

## 7.1 Strategy

For the complicated TDAQ implemented at NA62, it is important to measure the trigger efficiencies with respect to multiple quantities, in order to determine whether the data acquisition biases the collected data in any particular fashion. For example,

- The stability of the trigger-efficiency with respect to time.
- Kinematic dependence, observed with respect to track momentum  $P$ , the ratio of the variable  $z_{\pi^+\pi^-} = (M_{\pi^+\pi^-}/M_{K^+})^2$  for  $K_{3\pi}$  or the equivalent  $z_{\mu\mu} = (M_{\mu\mu}/M_{K^+})^2$  for  $K_{\pi\mu\mu}$ .
- The effect of variable beam-intensity on performance.

The number of observed candidates in any given physics analysis must be corrected for the trigger efficiency to infer the true number of candidates that were present in the detector. If the trigger is kinematically dependent, the  $z_{\mu\mu}$  spectrum of the collected  $K_{\pi\mu\mu}$  sample could be distorted compared with the true distribution. Variable beam intensity is also expected to affect the trigger performance as increased rate leads to more accidental candidates alongside the signal. To account for any of these effects accordingly, the following strategy was adopted to determine the normalisation efficiencies  $\epsilon_{3\pi}$ , and from that obtain the required signal efficiency spectrum  $\epsilon_{\pi\mu\mu}(z_{\mu\mu})$ .

- Determine the normalisation efficiencies  $\epsilon_{3\pi}$  for each primitive/algorithm using a sample of  $K_{3\pi}$  from either,
  - $L_0$ : Minimum-bias control trigger.
  - $L_1$ : Autopassed from Multi-track trigger (2% of  $L_1$  yield)

The dependence of the efficiency is measured with respect to time, a relevant kinematic variable ( $P, z_{\pi^+\pi^-}$ ) and the instantaneous beam intensity.

- For components which display no clear discrepancies with expectation, the normalisation efficiency  $\epsilon_{3\pi}$  can be converted to a binned-distribution of  $\epsilon_{\pi\mu\mu}(z_{\mu\mu})$  using the following formula,

$$\epsilon_{\pi\mu\mu}^{\text{DATA}}(z_{\mu\mu}) = \epsilon_{3\pi}^{\text{DATA}} \left( \frac{\epsilon_{\pi\mu\mu}^{\text{MC}}(z_{\mu\mu})}{\epsilon_{3\pi}^{\text{MC}}} \right), \quad (7.1)$$

where the **MC** values are extracted from a "simple" emulation of the trigger-component response in simulation for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$ .

- For components with known flaws, a series of trigger emulators were developed which replicate the L<sub>0</sub> firmware or L<sub>1</sub> algorithm. The  $\epsilon_{\pi\mu\mu}(z_{\mu\mu})$  is then obtained by measuring the efficiency directly using MC samples, after injecting accidentals into the simulation as extracted from data. This MC + Accidentals (**MCA**) method is verified after tuning the procedure using the result for  $K_{3\pi}$  compared to the online primitive or algorithm.

The selection used to collect the  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  samples is extracted from the final version described in Chapter **6**. The idea is to use the accurately measured normalisation efficiencies from the data and convert them using the response in MC. After the full 2016 data set was collected, both the Q<sub>X</sub> and !LAV components of the trigger were discovered to be flawed. This is the motivation for the final bullet point above; the flaws were found to be both kinematic and intensity dependent. Therefore, the **MCA** approach was implemented to tackle these measurements, especially as neither issue is truly factorisable. The MO<sub>2</sub> criterion is unique to the Dimuon trigger and the strategy is slightly different. The  $z_{\mu\mu}$  dependence is extracted directly from data using a sample of the low mass background  $K^+ \rightarrow \pi^+ \pi^+ \pi^- \rightarrow \pi^+ \mu^+ \mu^-$ , where two of the pions decayed in-flight.

## 7.2 KTAG

The L<sub>1</sub> KTAG algorithm is designed to positively identify a beam-kaon candidate consistent with the L<sub>0</sub> event time. The algorithm is considered the simplest to handle because the efficiency depends purely on the presence of the kaon and not the

decay products downstream. Therefore, the efficiency is considered to be identical for normalisation  $K_{3\pi}$  and signal  $K_{\pi\mu\mu}$  and will completely cancel in the ratio when normalising. Regardless, the efficiency is expected to be high for kaon decays, and the time-stability and beam-intensity dependence of the KTAG algorithm is displayed in Figure 7.1, measured directly using the Multi-track autopassed  $K_{3\pi}$ .

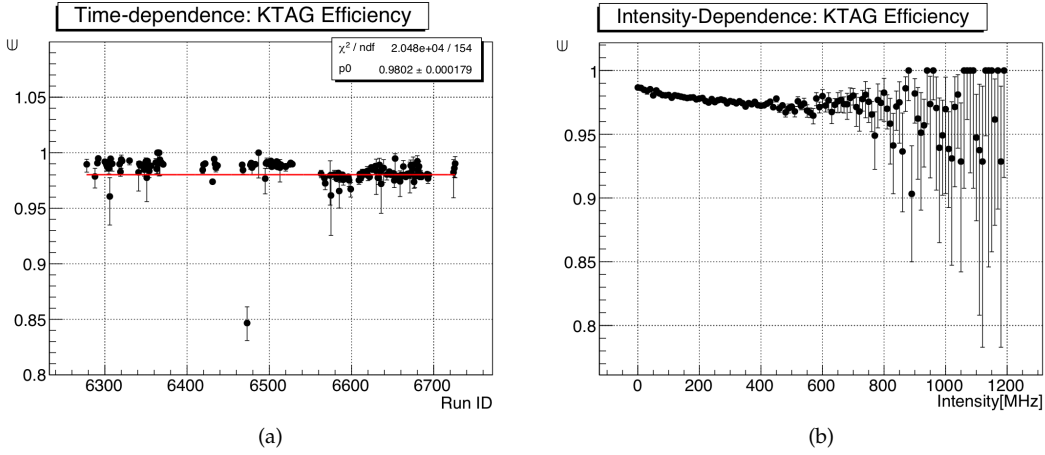


Figure 7.1: The KTAG algorithm efficiency (a) with respect to time and (b) versus beam-intensity. The red-line in (a) is the average efficiency across the period, stable except for one low-statistics run with ID = 6470.

As the KTAG algorithm is close to identical to the criterion implemented in the particle-identification of the selection, "simple" emulation yields an expected MC efficiency of  $1.0000 \pm 0.0000_{\text{stat}}$  for both samples. The equivalent measurement in data is,

$$\epsilon_{3\pi}^{\text{KTAG}} = \epsilon_{\pi\mu\mu}^{\text{KTAG}} = 0.9802 \pm 0.0002_{\text{stat}}, \quad (7.2)$$

where the value is the overall average extracted from (a) of Figure 7.1 and  $\epsilon_{\pi\mu\mu}^{\text{KTAG}}$  is assumed to be flat with respect to  $z_{\mu\mu}$ . The statistical error corresponds to the full 2016A data sample and is therefore small. The left-hand side of Figure 7.1 displays some variation in the efficiency with respect to Run ID. This is a consequence of the beam intensity which was constantly changing during the data acquisition process. Overall, the KTAG algorithm depends fairly weakly on the beam-intensity (as seen in the right-hand side of Figure 7.1) decreasing by  $\approx 1\%$  across the full spectrum.

The average overall efficiency is a reflection of this relationship.

### 7.3 RICH

The  $L_0$  RICH primitive is implemented to collect events with signals in at least two **RICH** **PMT** supercells. As  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  contain three charged tracks, the high beam momentum guarantees that at least one track is above the Cherenkov threshold required to generate hits in the RICH. Figure 7.2 contains the RICH efficiency with respect to time and track momentum  $P$  for  $K_{3\pi}$  minimum bias candidates. The ef-

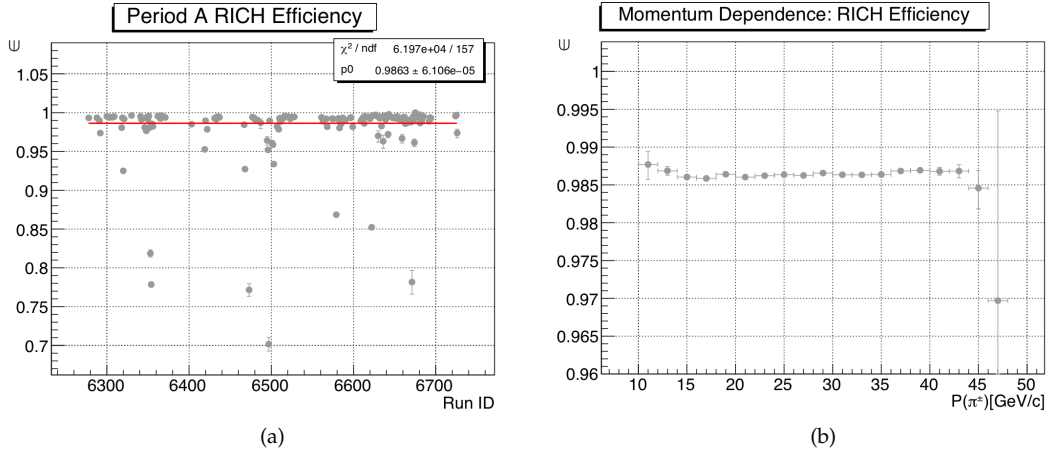


Figure 7.2: The RICH primitive efficiency (a) with respect to time and (b) versus  $\pi$  momentum. The red-line in (a) is the average efficiency across the period and is in general stable, except for  $\approx 10$  lower efficiency runs. The momentum dependence is generally weak across the available phase-space.

iciency is stable in-time aside from  $\approx 10$  low efficiency runs which have limited statistics. The distribution also displays little-to-no momentum dependence as is expected for a three-track process. The average RICH normalisation efficiency provided by the flat-line in (a) is measured to be,

$$\epsilon_{3\pi}^{\text{RICH}} = 0.9863 \pm 0.0001_{\text{stat}}, \quad (7.3)$$

where the small statistical error reflects the size of the 2016A data set. As explained in the strategy, to determine the  $\epsilon_{\pi\mu\mu}^{\text{RICH}}$  efficiency,  $\epsilon_{3\pi}^{\text{RICH}}$  is converted using the MC ratio method provided by Equation 7.1. To determine the MC components for both  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  required by the ratio, the  $L_0$  RICH emulator was used which replicates the online primitive generation at the analysis level offline [67]. The emulator

is fully flexible and can be used for both data and MC, with the former useful to test the emulator performance. Figure 7.3 compares the online-primitive intensity dependence with the emulated efficiency for  $K_{3\pi}$  minimum-bias data. The RICH emu-

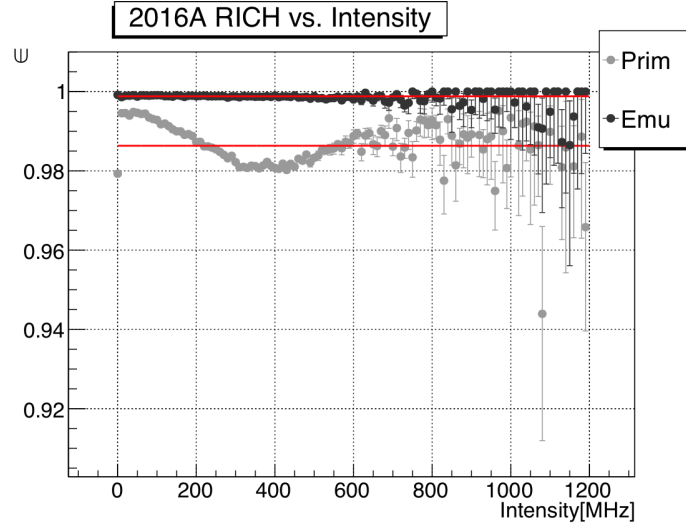


Figure 7.3: Comparing online and emulated RICH primitive efficiency in 2016A for  $K_{3\pi}$ . The online distribution displays a distinct shape-deviation near 300 MHz not reproduced by the emulator. The red-line is the overall average for the primitive efficiency.

lator measures the efficiency a few percent higher than the online primitive, without the distinct shape distortion in the region of intensity  $\mathcal{I} = 300$  MHz. The small number of low-efficiency runs that were observed in (a) of Figure 7.2 are expected to be the cause of this shape. Though an obvious discrepancy with the emulator, the same behaviour is expected for the  $K_{\pi\mu\mu}$  efficiency and is therefore contained in the value of  $\epsilon_{3\pi}^{\text{RICH}}$  when converting. The final  $\epsilon_{\pi\mu\mu}^{\text{RICH}}(z_{\mu\mu})$  is the result of the procedure presented in Figure 7.4. The response of the emulator returns as almost 100% efficient for both MC samples and the measurement is dominated by the actual  $K_{3\pi}$  efficiency. The average  $K_{\pi\mu\mu}$  RICH efficiency is determined to be,

$$\epsilon_{\pi\mu\mu}^{\text{RICH}} = 0.9864 \pm 0.0000_{\text{stat}} \pm 0.0060_{\text{syst}}. \quad (7.4)$$

The accuracy of this approach relies on the performance of the emulator with respect to the instantaneous beam-intensity compared with the primitive dependence. The statistical term reflects the size of the MC sample used, whereas the systematic error corresponds to the difference between the mean efficiency denoted by the red line

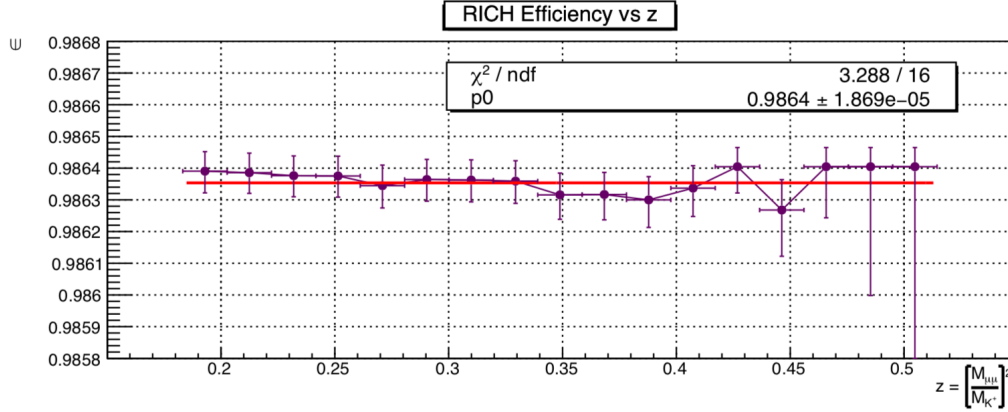


Figure 7.4: The RICH Efficiency vs  $z$  in 2016A. The essentially-flat shape is because the emulator returns efficiency 1.0 for almost all bins in both MC samples, where the actual scale is set by the value of  $\epsilon_{3\pi}^{\text{DATA}}$ . The red-line denotes the average efficiency.

in Figure 7.3 and the efficiency of the primitives at the average intensity at  $\mathcal{I} \approx 300$  MHz, equivalent to  $\approx 0.6\%$ , taking into account the discrepancy from low-efficiency runs.

## 7.4 STRAW<sub>e</sub>

The L<sub>1</sub> STRAW<sub>e</sub> algorithm uses a fast-reconstruction to search for events containing negative tracks. The algorithm is implemented in both streams as  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  contain either a  $\pi^-$  or  $\mu^-$ . Figure 7.5 contains the time and intensity dependence of the algorithm. The efficiency is stable with time, with a slow intensity fall-off across the spectrum. The normalisation efficiency provided by the flat line in (a) is measured to be,

$$\epsilon_{3\pi}^{\text{STRAW}_e} = 0.9588 \pm 0.0003_{\text{stat}}. \quad (7.5)$$

To determine the  $\epsilon_{\pi\mu\mu}^{\text{STRAW}_e}$  efficiency, the  $\epsilon_{3\pi}^{\text{STRAW}_e}$  is converted using the MC ratio approach used by the RICH. However, the STRAW<sub>e</sub> algorithm has no suitable emulation method to apply in the simulation as the algorithm uses digitised hits before the reconstruction level. Naturally, all events collected by the selection contain a negative track and so the ratio can only be unity for MC. Instead, Figure 7.6 contains the  $\pi^-$  momentum dependence of the algorithm for autopass  $K_{3\pi}$  candidates. Fitting the distribution with a first-order polynomial, the momentum-dependence



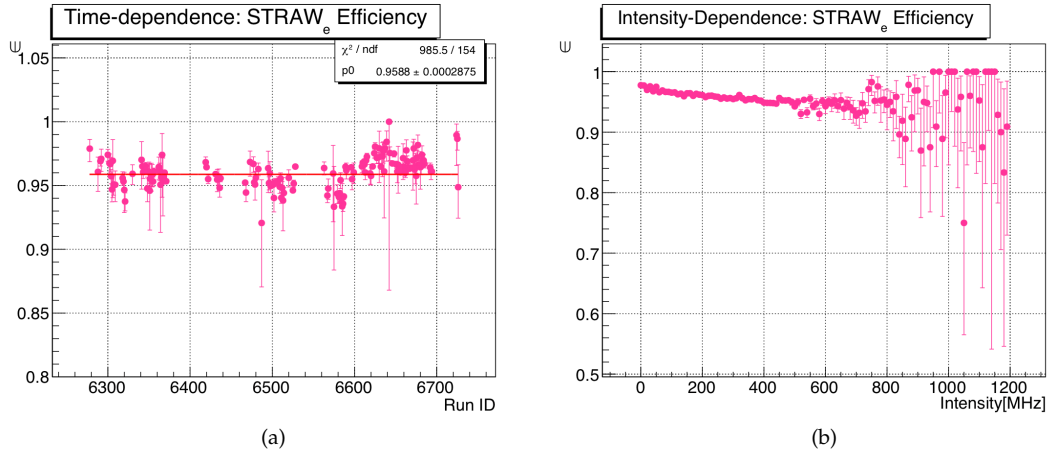


Figure 7.5: The  $\text{STRAW}_e$  algorithm efficiency (a) with respect to time and (b) versus intensity. The red-line in (a) is the average efficiency across the period and is in general stable. The algorithm also displays a gentle intensity dependence, decreasing by 1% across the spectrum.

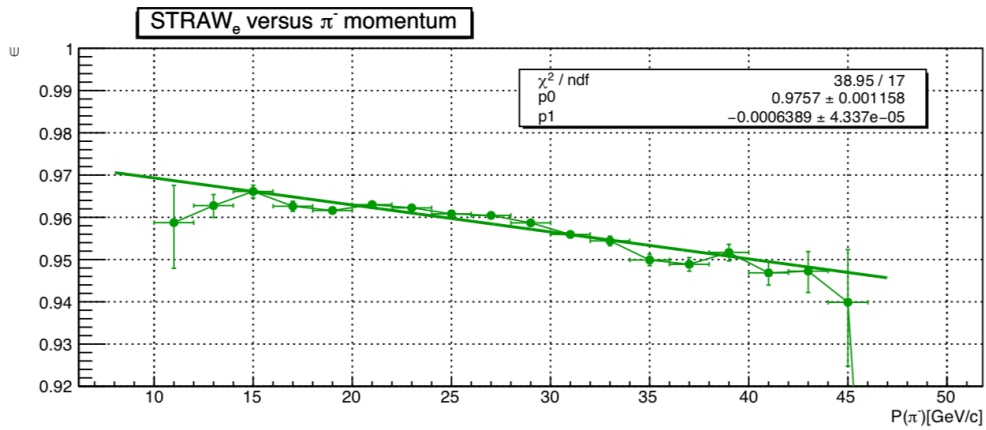


Figure 7.6: The  $\text{STRAW}_e$  momentum dependence in 2016A, measured using autopass  $K_{3\pi}$  candidates. The fit corresponds is a first-order polynomial, which decreases with  $\pi^-$  momentum.

function is determined to be,

$$\epsilon_{3\pi}^{\text{STRAW}_e}(P_{\pi^-}) = \epsilon_0 + \epsilon_1 P_{\pi^-} = 0.9757 - 0.0006 P_{\pi^-}, \quad (7.6)$$

which is the simplest polynomial that can adequately model the dependence. As both the signal  $K_{\pi\mu\mu}$  and normalisation  $K_{3\pi}$  have intrinsically different track-momentum spectra, the efficiency  $\epsilon_{\pi\mu\mu}^{\text{STRAW}_e}$  cannot be taken to be equivalent to the  $K_{3\pi}$  case. We assume that the functional form of Equation 7.6 is the algorithm's universal negative track momentum dependence, as previous studies suggest that the reconstruction efficiency for pions and muons is equivalent [85]. The overall efficiency in data

$\epsilon_{3\pi}^{\text{STRAW}_e}$  is then converted to a  $\epsilon_{\pi\mu\mu}^{\text{STRAW}_e}(z_{\mu\mu})$  distribution using the usual conversion method of Equation 7.1. The MC ratio is determined by evaluating the momentum-dependence of Equation 7.6 for MC events, based on the negative track momentum for each event. Figure 7.7 contains the final  $\text{STRAW}_e$   $z_{\mu\mu}$ -distribution for  $K_{\pi\mu\mu}$ . The

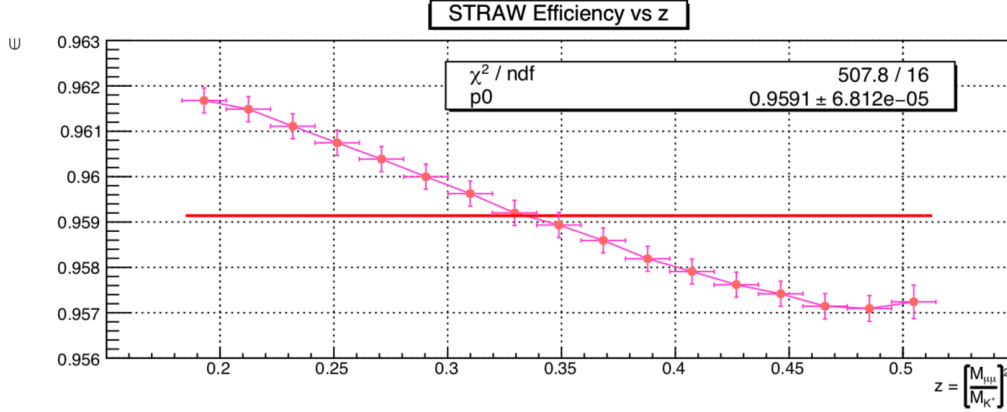


Figure 7.7: The  $\text{STRAW}_e$  Efficiency vs  $z$  in 2016A. The slope falls linearly across the phase-space, decreasing by  $\approx 0.5\%$  from start to finish. The red-line is to denote the overall average of the efficiency.

slope falls almost linearly, with the red line providing the average efficiency across the full phase-space,

$$\epsilon_{\pi\mu\mu}^{\text{STRAW}_e} = 0.9591 \pm 0.0001_{\text{stat}} \pm 0.0050_{\text{syst}}, \quad (7.7)$$

but with only a small decrease from start to finish of  $\approx 0.5\%$ . The precision of the approach relies on the accuracy of the momentum-efficiency model. The  $\chi^2/\text{NDF}$  fit to determine Equation 7.6 is  $\approx 2.5$ , meaning that the model could be optimised. Without a proper physics motivation to do so, we take a systematic error based on the maximum significant deviation from the line of 0.5% at  $P(\pi^-) = 35 \text{ GeV}/c$ . In addition, other contributions to the systematic term are considered, such as:

- **Fit error:** The effect of evaluating Equation 7.6 with a Gaussian smear on the fit parameters  $\epsilon_0$  and  $\epsilon_1$  based on their precision is included in the estimate.
- **Fit extrapolation:** Additionally, the momentum spectrum for  $K_{3\pi}$  is not as broad as for  $K_{\pi\mu\mu}$ , and so the efficiency for candidates with momentum outside of  $(10 \leq P^- \leq 46) \text{ GeV}/c$  is inferred from extrapolation. As the form is unknown outside of this region, the effect of using a flat extrapolation is con-

sidered instead, with the variation taken as a systematic.

However, both errors are negligible with respect to the model-dependent systematic.

## 7.5 $Q_X$ (NewCHOD)

The  $Q_X$  condition is one of two components of the trigger which were found to be flawed. At  $L_0$ ,  $Q_X$  is designed to collect events with hits in at least two diagonally opposed NewCHOD quadrants. The primitive is implemented in both trigger streams as  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  both contain three charged tracks. Requesting NewCHOD quadrants to satisfy  $Q_X$  for MC events yields an efficiency  $\epsilon_{MC}^{Q_X} \approx 0.9990$  for both samples, despite the fact that the illumination of the detector is different as displayed in the channel map of Figure 7.8. As muons are lighter than pions, the  $K_{\pi\mu\mu}$

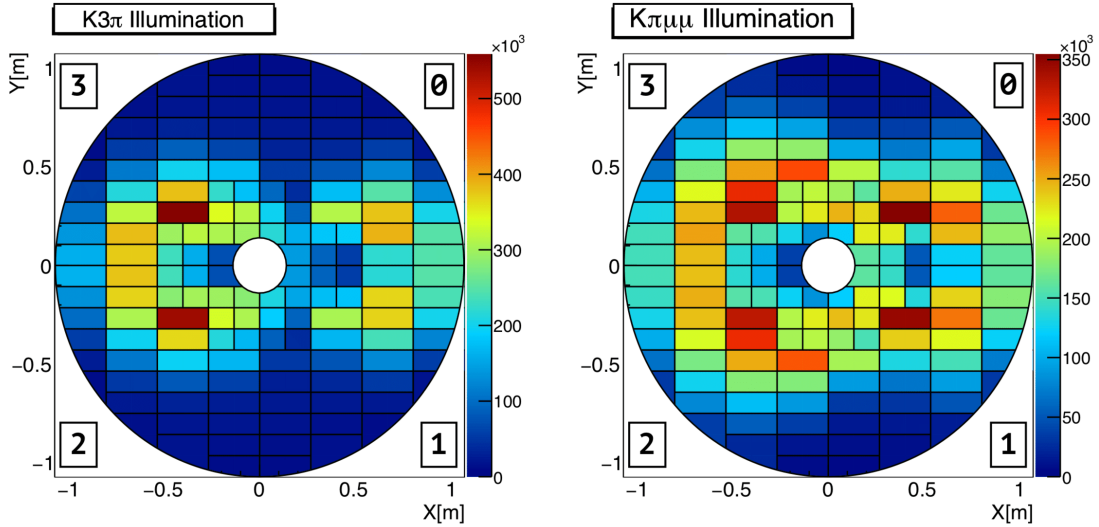
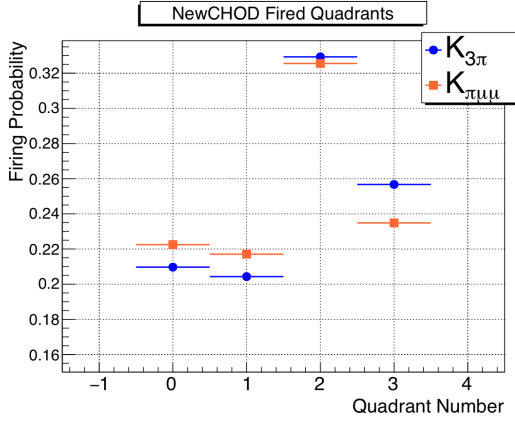


Figure 7.8: The normalised NewCHOD illumination for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  MC. The difference is purely kinematic as the opening angles for  $K_{3\pi}$  is limited with respect to  $K_{\pi\mu\mu}$  due to the difference in phase-space. The numbered corners refer to the quadrant number in the readout.

opening angle is larger on average than for  $K_{3\pi}$  leading to a wider spread of hits at the NewCHOD. Figure 7.9 contains the probability of firing each NewCHOD quadrant for both  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  MC, different on account of the contrasting illumination. In general,  $K_{\pi\mu\mu}$  is more likely to fire quadrants on the right-side of the detector, while  $K_{3\pi}$  the left. The  $Q_X$  condition can be split into three exclusive parts, such



	$\epsilon(K_{3\pi})$	$\epsilon(K_{\pi\mu\mu})$
$Q_X$	$0.9989 \pm 0.0001$	$0.9983 \pm 0.0001$
$Q_{02}^{\text{EX}}$	$0.4596 \pm 0.0015$	$0.4798 \pm 0.0014$
$Q_{13}^{\text{EX}}$	$0.4519 \pm 0.0015$	$0.4751 \pm 0.0014$
$Q_4$	$0.0879 \pm 0.009$	$0.0435 \pm 0.0006$

Figure 7.9 & Table 7.1: The firing probability of **NewCHOD** quadrants for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  MC, accompanied by the exclusive component efficiencies of  $Q_X$  in tabular form. The difference in illumination of the samples lead to contrasting quadrant probabilities. The result is a set of trigger-components that are distinct, but which sum to similar  $Q_X$  efficiencies overall. The errors are statistical only.

that  $Q_X = (Q_{02}^{\text{EX}} \vee Q_{13}^{\text{EX}}) \vee Q_4$ , where  $Q_4$  corresponds to hits in all four quadrants. Table 7.1 displays the "simple" emulated MC efficiencies for each component. In general, the larger opening angles of  $K_{\pi\mu\mu}$  leads to  $\approx 4\%$  higher likelihood for the exclusively diagonal components of  $Q_X$  to fire, whereas  $K_{3\pi}$  is twice as likely to fire all four quadrants  $Q_4$  compared with  $K_{\pi\mu\mu}$ . Despite this kinematic dependence, the components sum to the same  $Q_X$  efficiencies overall.

Figure 7.10 contains the time and intensity dependence of the primitive measured with minimum bias  $K_{3\pi}$  decays. The normalisation efficiency provided by the flat-line in (a) is measured to be,

$$\epsilon_{3\pi}^{Q_X} = 0.8740 \pm 0.0002_{\text{stat}}, \quad (7.8)$$

which is low in comparison to expectation, indicating the presence of a flaw. The primitive is also highly dependent on the beam-intensity, decreasing by  $\approx 20\%$  across the available range. A clear indication of the effect can be seen as some runs with  $\text{ID} > 6600$  are known to be at low-intensity. All examples of this have an efficiency substantially better than the average at  $\approx 95\%$ .

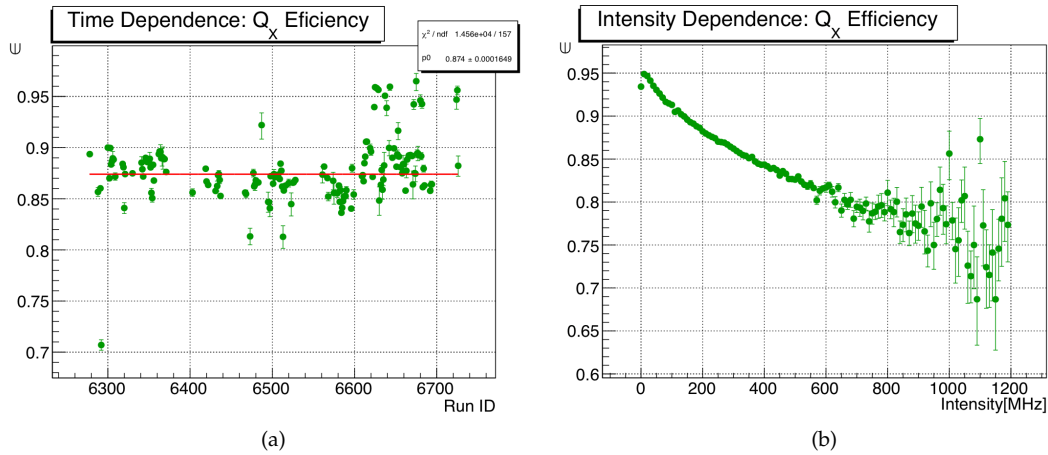


Figure 7.10: The  $Q_X$  primitive efficiency (a) with respect to time and (b) versus intensity. The red-line in (a) is the average efficiency across the period. The primitive displays a large intensity dependence, decreasing by  $\approx 20\%$  across the spectrum. Some runs with ID  $> 6600$  display a higher-efficiency than average, all of which correspond to a lower beam-intensity.

### 7.5.1 Flaw in the **NewCHOD** firmware

After discussion with **TDAQ** experts, the flaw was discovered with the **NewCHOD** primitive-generating firmware. The flaw occurs at the time-cluster merging stage, and can be understood using the flow-diagram presented in Figure 7.11. The flow is,

- Hits from the readout are sent to an array of buffers in the sorting modules. Each buffer, or 'split', contains hits related to a 100 ns time interval, with the earliest hits in the first split and the latest in the final. The hits in each split are all within 100 ns of each other, but are random and remain unsorted with respect to time.
- Each split is read from first-to-last to begin the clustering process. The very first hit is converted into the cluster format (containing all useful information) and passed to the distributor module.
- As the second hit is read and converted to a cluster, it is compared to the distributor cluster. The following can happen:
  - If the time of the two clusters is consistent, the clusters are merged in the distributor module and remain to be compared with the next hit.

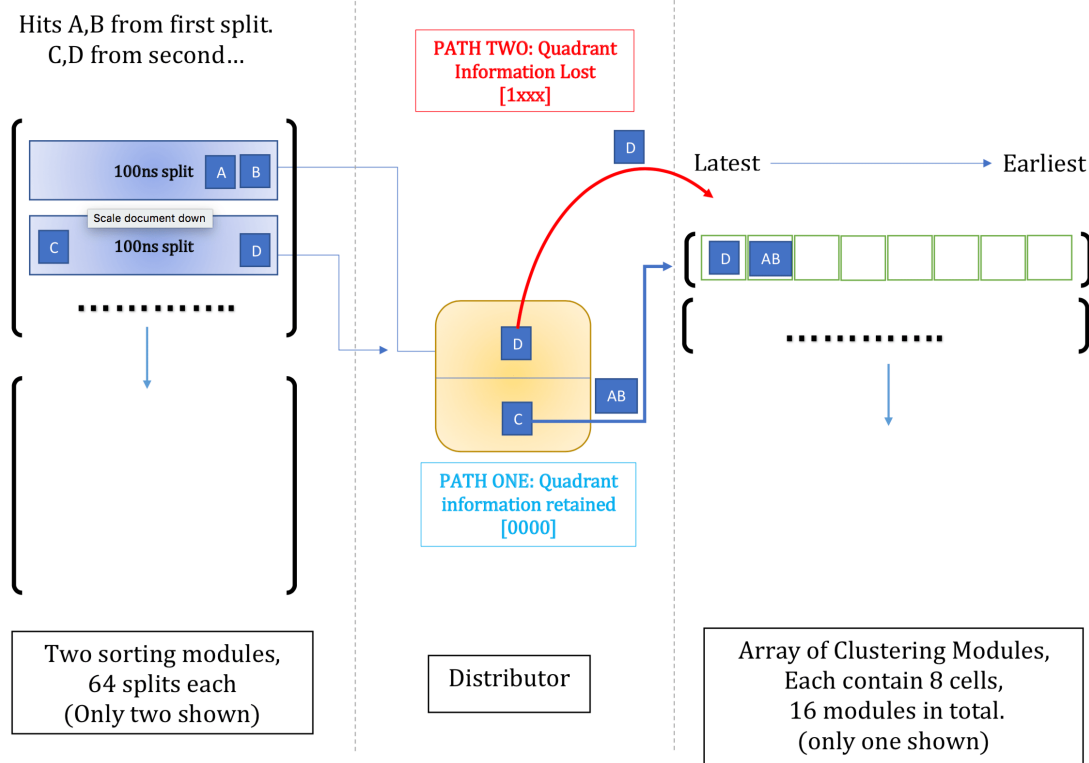


Figure 7.11: A flow diagram of the **NewCHOD** firmware flaw. Here hits *A* and *B* are from the first split and are time-consistent, so are merged at the distributor. Following this, the first hit read from the second split *C* actually has the largest time of the hits in the buffer. All other hits read from this split, such as *D*, jump straight to the clustering module and the quadrant information for the hit is lost.

- If the time is not consistent but the second hit is later in time, the distributor cluster is sent to the clustering array and the second hit becomes the distributor cluster used for the next comparison.
- Otherwise if later, the hit is converted to a cluster and pushed directly to the clustering array leaving the distributor cluster untouched.

In Figure **7.11**, a specific example is shown using four hits *A*, *B*, *C* and *D*. Hit *A* has the largest time in the first split, and resides at the distributor as the second hit *B* arrives. Here, the hits are consistent in time and are merged to form cluster *AB*. As the second split is read, the first hit *C* is found to be later than cluster *AB* and so *AB* is pushed to the clustering module via path one, and *C* now resides at the distributor. In the case that hit *C* has the latest time in the second split, all subsequent hits such as *D* are earlier and pushed directly into the clustering module via path two.

The flaw enters for this type of flow. In this path, the information for quadrants 1→3 is lost and  $Q_X$  cannot be satisfied.

As the information for certain quadrants is dropped, the kinematic difference between  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  becomes a bigger problem as the inefficiency depends on the **NewCHOD** illumination for each event. The randomised nature of the hits also becomes problematic with higher beam-intensity as an increased number of hits can enter each split, and the quadrant information is lost any time the distributor cluster is later than the hit being read. A simple conversion from  $K_{3\pi}$  is not a possibility as the two issues are not factorisable.

### 7.5.2 $Q_X$ emulator

In order to handle the complicated nature of the inefficiency, a  $L_0$  trigger emulator was developed which attempts to replicate the behaviour of the **NewCHOD** primitive generating firmware, including any flaws present [67]. The emulator can be used at the analysis level to compare the performance with the online primitives. Figure 7.12 contains the emulator and online efficiencies with respect to time and intensity. The excellent agreement is a result of a tuning procedure. As the ineffi-

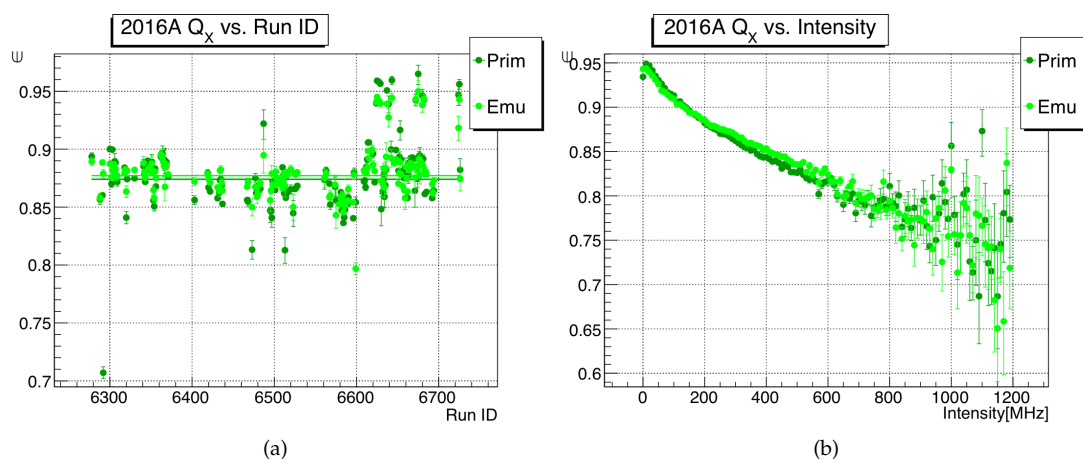


Figure 7.12: Comparing primitive and emulated  $L_0$  efficiencies for  $Q_X$  in 2016A, with (a) against time and (b) versus intensity. The emulator was tuned so that the result agrees excellently with the online primitive efficiency.

ciency depends largely on the random hit ordering in the 100 ns splits, the behaviour

was replicated by adding a partial-hit-randomisation algorithm. The effect is controllable by the variable `L0NewCHODEmulator::ShuffleFraction`, a fraction defined between (0; 1). If the fraction is zero, the hits are completely time-ordered and the flaw is not present (all hits in the splits follow the uncorrupted path one), leading to extremely high efficiencies. Likewise, a fraction of one corresponds to maximal randomness, using a Mersenne twister to shuffle the hits. The optimal fraction was found to be 0.73, a value representing a partial ordering. On average, primitive and emulated efficiencies agree to better than 0.1% overall.

### 7.5.3 Determining $K_{\pi\mu\mu}$ $Q_X$ efficiency

As the flaw leads to a geometric dependence caused by time-ordering of hits, a simple conversion from  $K_{3\pi}$  using MC ratios to get the  $K_{\pi\mu\mu}$   $Q_X$  efficiency as a function of  $z_{\mu\mu}$  is unreliable. Instead, a MC + Accidentals (MCA) approach is adopted, where the  $Q_X$  efficiency is extracted directly from the simulation. To inject accidentals into the simulation, the number of accidental `NewCHOD` hits were obtained using the following method:

- Take number of `NewCHOD` hits in the lower timing side-band of the  $\delta T = T_{\text{trig}} - T_{\text{NewCHOD}}$  distribution. An offset region +74 ns of  $T_{\text{trig}}$  is used, with a window of 28 ns, a region where the distribution is flat and best represents the accidental rate.
- The number of accidental hits of this region is sampled in bins of 4 ns, and plotted versus the instantaneous beam-intensity from the `GTK` between (0; 1200) MHz. The choice of many bins is to sample `NewCHOD` hits more smoothly.
- The information is fed to the `L0NewCHODEmulator`, so that the correct number of accidental hits are added to the algorithm for any given intensity.

With accidentals added, the emulator can be used at any intensity. To obtain the actual overall efficiency as measured by the MCA method, the results at each intensity must be combined using a weighted-average, where the weights are the number of entries at each intensity bin observed in the residual intensity spectrum.



To verify that the approach provides sensible results, both the  $\epsilon_{\pi\mu\mu}^{\text{Q}_X}(z_{\mu\mu})$  and  $\epsilon_{3\pi}^{\text{Q}_X}(z_{\pi+\pi-})$  spectra are created, as the latter can be measured directly in data to compare. To handle the effects of both intensity and kinematic dependence, a two-dimensional grid is constructed of  $z$  versus intensity, with the  $K_{3\pi}$  example displayed in Figure 7.13. The average  $\text{Q}_X$  efficiency is calculated at each  $z$ -bin by constructing the weighted-

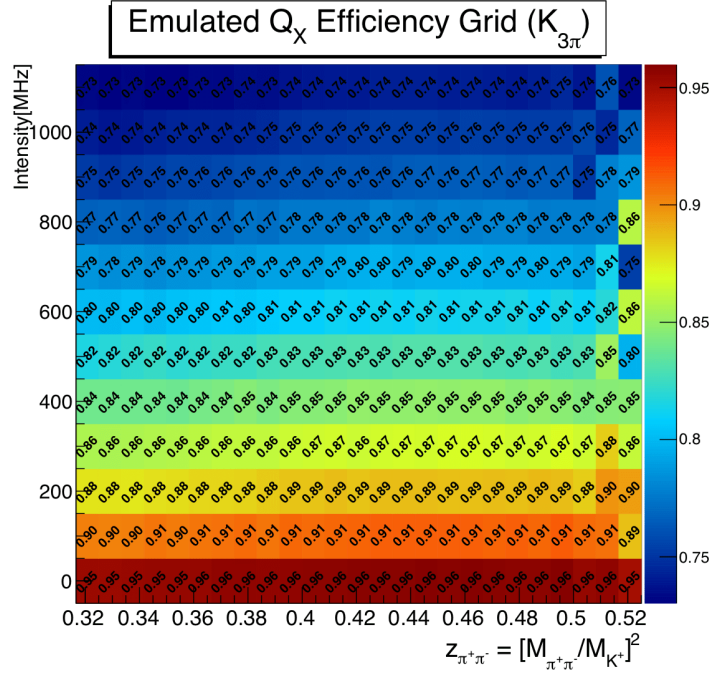


Figure 7.13: The two-dimensional **MCA**  $\text{Q}_X$  grid for  $K_{3\pi}$  decays. The diagonal labels correspond to the  $\epsilon_{\text{Q}_X}$  at each bin

average over the twelve 100 MHz intensity bins used in the grid,

$$\epsilon_z^{\text{Q}_X} = \frac{\sum_{\mathcal{I}=1}^{12} w_{\mathcal{I}} \epsilon_{z,\mathcal{I}}^{\text{Q}_X}}{\sum_{\mathcal{I}=1}^{12} w_{\mathcal{I}}} \quad (7.9)$$

where  $\mathcal{I}$  is the intensity-bin index,  $\epsilon_{z,\mathcal{I}}^{\text{Q}_X}$  is the emulated  $\text{Q}_X$  efficiency at the  $(z, \mathcal{I})$  bin and  $w_{\mathcal{I}}$  is the number of candidates in the  $\mathcal{I}^{\text{th}}$  bin of the intensity-spectrum. Finally, the weighted-average at each  $z$ -bin can be plotted in one dimension. This is displayed in (a) of Figure 7.14 where a direct comparison to the online primitive efficiency can be made, whereas (b) corresponds to the **MCA** reconstructed  $\epsilon_{\pi\mu\mu}^{\text{Q}_X}(z_{\mu\mu})$  spectrum which is used in the analysis to reconstruct  $d\Gamma/dz$ . The **MCA** and online primitive efficiencies in (a) agree to within 0.1% on average, after re-tuning the shuf-

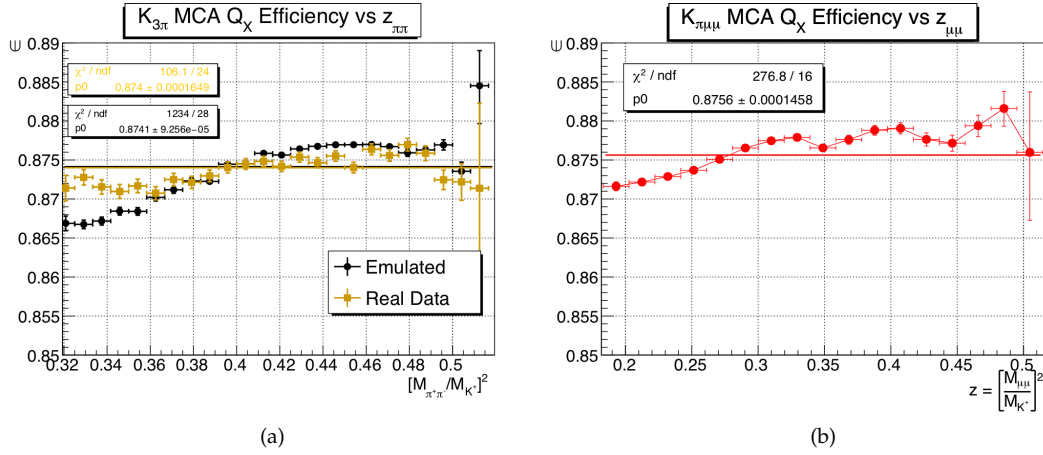


Figure 7.14: Reconstructing the **MCA**  $Q_X$  efficiency for (a)  $K_{3\pi}$  versus  $z_{\pi^+\pi^-}$  where it can be compared directly to the online primitive distribution, and (b)  $K_{\pi\mu\mu}$  versus  $z_{\mu\mu}$  which is used directly in the analysis.

the fraction to get a close agreement. The new shuffle fraction is 0.62, corresponding to increased time-ordering with respect to previously. The requirement to re-tune is unsurprising, as the overall efficiency depends on the time ordering of the hits; the emulator generates hits which are fundamentally different to the online counterpart. The shape of the efficiency spectrum for  $K_{3\pi}$  is flatter with  $z_{\pi^+\pi^-}$  compared to the reconstructed **MCA** approach. A systematic error of 0.5% is assigned to the  $K_{\pi\mu\mu}$  efficiency that can only be measured using this approach, corresponding to the deviation at low  $z_{\pi^+\pi^-}$  bins. The average  $K_{\pi\mu\mu}$  is therefore measured to be,

$$\epsilon_{\pi\mu\mu}^{Q_X} = 0.8756 \pm 0.0001_{\text{stat}} \pm 0.0050_{\text{sys}}, \quad (7.10)$$

extracted from the flat-line in (b) of Figure 7.14. The shape is roughly the same for  $K_{\pi\mu\mu}$  as  $K_{3\pi}$ , but 0.2% higher on average.

## 7.6 !LAV

The second component found to be flawed is the L<sub>1</sub> **!LAV** algorithm, designed to veto events containing at least two **!LAV** hits with leading and trailing edges that are consistent to within 10 ns of the L<sub>0</sub> trigger time. The algorithm is applied in both trigger streams to veto soft-photons from the large accidental  $K_{2\pi}$  background. As the algorithm criteria can easily be reproduced at the analysis level, simulating **!LAV**

for MC yields two very different efficiencies for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$ ,

$$\epsilon_{MC_{3\pi}}^{!LAV} = 0.9026 \pm 0.0001_{\text{stat}}, \quad (7.11)$$

$$\epsilon_{MC_{\pi\mu\mu}}^{!LAV} = 0.9562 \pm 0.0001_{\text{stat}}. \quad (7.12)$$

For comparison, the MC  $K_{3\pi}$  value can be compared directly to the data. Figure 7.15 contains the time and intensity dependence of the algorithm for autopass  $K_{3\pi}$  candidates. The normalisation efficiency provided by the flat-line in (a) is measured to

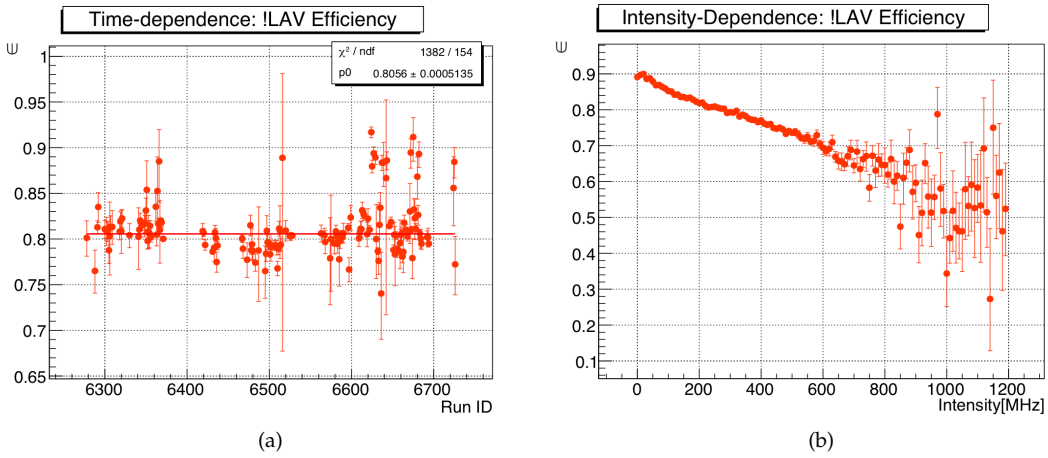


Figure 7.15: The !LAV algorithm efficiency (a) with respect to time and (b) versus intensity. The red-line in (a) is the average efficiency across the period. Like  $Q_X$ , the algorithm displays a large intensity dependence, decreasing by  $\approx 40\%$  across the spectrum. Some runs at low-intensity with ID  $> 6600$  display a far higher efficiency than average.

be,

$$\epsilon_{3\pi}^{!LAV} = 0.8056 \pm 0.0005_{\text{stat}}, \quad (7.13)$$

which is 10% lower than the MC prediction and highlights the presence of a flaw. The inefficiency is also driven by accidentals at higher beam intensity. As was observed with  $Q_X$ , the low-intensity runs present at run ID  $> 6600$  display a  $\approx 10\%$  higher-efficiency compared with the average.

### 7.6.1 Flaw with !LAV algorithm

Unlike  $Q_X$ , there is no technical limitation creating the inefficiency for the signal. The !LAV algorithm works exactly as intended, but the flaw is that the algorithm is

unsuitable for collecting unbiased samples of three-track decays. This is because the inefficiency has three components:

- A self-component, due to inelastic collisions and hadronic interactions with material from the daughters in the decay.
- Accidental component, due to in time large-angle photons or other non-decay originating pions interacting with material.
- Combinatorial component, where hits from the self or accidental component combine to fire !LAV if neither mechanism generated enough hits individually.

The self-component is dependent on the decay composition. Hadrons like pions have a chance to interact inelastically with material downstream of the detector, leading to a high flux of particles at large opening angles. A study performed using MC displayed that the dense RICH mirror is a hot-spot for inelastic pion collisions, in which the subsequent hadronic shower creates hits in the LAV12 station, causing !LAV to fail and the event to be vetoed [86]. This component can be described by the GEANT4 simulation of hadronic showers in NA62MC, which leads to the difference of prediction of the MC efficiencies in Equations 7.11 and 7.12. For  $K_{3\pi}$  decays, the self-component is maximal whereas  $K_{\pi\mu\mu}$  contains only one pion, and so the self-component is  $\approx 2.2$  times smaller in comparison.

The intensity-dependent accidental component is of a similar size to the  $K_{3\pi}$  self-component based upon the magnitude of the average normalisation efficiency. This component enters due to hadronic showers of accidental beam pions and the large rate of  $K_{2\pi}$  decays, but importantly, it will enter with an identical magnitude for both normalisation  $K_{3\pi}$  and signal  $K_{\pi\mu\mu}$ . The added complication is the combinatorial component. Though expected to be small, the magnitude depends on the number of hits in the tail due to the self-component, for cases where neither the self or accidental components could veto !LAV individually. As both of the final two components are absent from simulation, a simple MC conversion from  $K_{3\pi}$  is unreliable and a similar MCA approach to  $Q_X$  must be adopted.

## 7.6.2 !LAV emulation

Unlike  $Q_X$ , the !LAV algorithm is software-based and can be simply reproduced offline at the analysis stage. The offline algorithm can therefore be used on data to get a comparison with the online efficiency. Figure 7.16 compares the emulated and online algorithm efficiencies with respect to time and intensity as measured with autopass  $K_{3\pi}$  candidates. As there is no hit ordering dependence, the agreement

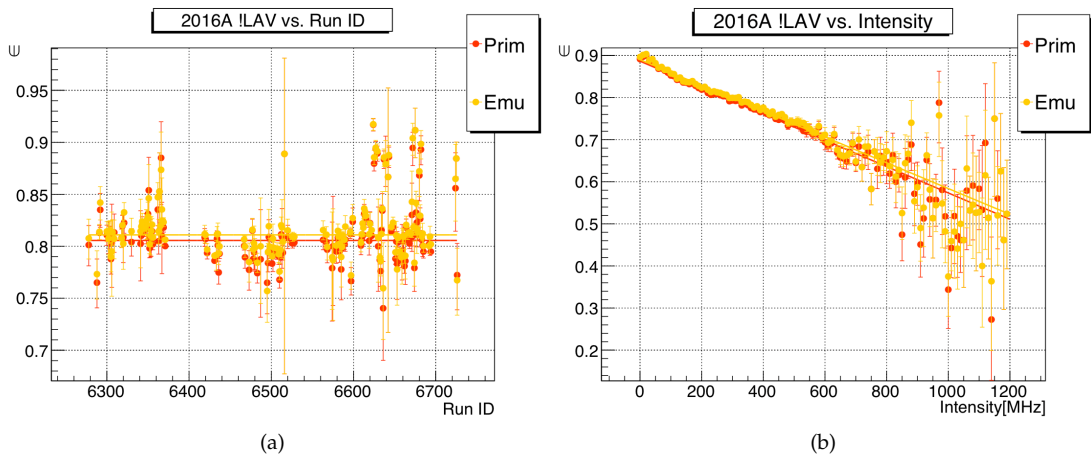


Figure 7.16: Comparing !LAV algorithm and emulated  $L_1$  efficiencies in 2016A, where (a) is with respect to Run ID and (b) is versus intensity. The offline emulation provides excellent agreement.

cannot be tuned. However, the agreement is excellent overall, with the emulated efficiency entering as  $\approx 0.5\%$  higher than the online-algorithm, but with very similar responses for both time and intensity distributions.

## 7.6.3 Determining $K_{\pi\mu\mu}$ !LAV efficiency

In order to measure the  $K_{\pi\mu\mu}$  efficiency adequately, all three components of the inefficiency need to be accounted for properly. The simplest approach is identical to the  $Q_X$  strategy; perform a MCA measurement. To do this and account for all components, accidentals must be extracted from data and injected accordingly. Figure 7.17 contains the  $\delta T = T_{\text{trig}} - T_{\text{LAV}}$  spectrum for LAV12, containing only hits with both leading and trailing edges. The peak at zero corresponds to the self-component with flat accidental component sitting beneath, with two regions indicated for accidental extraction as they are flat. The method, similar to  $Q_X$ , follows as,

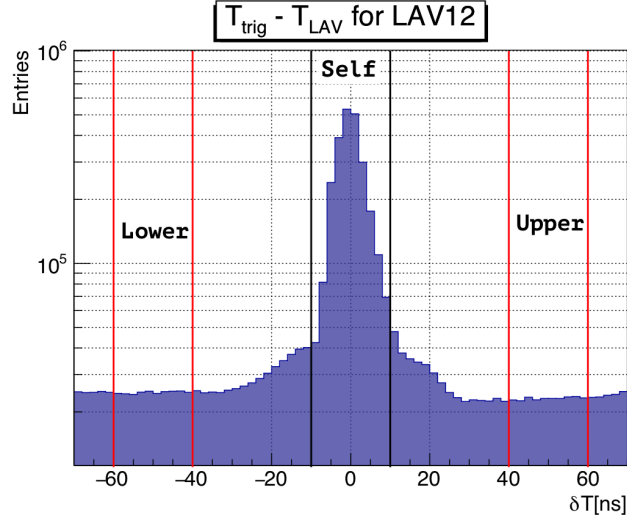


Figure 7.17: The  $\delta T$  distribution for LAV12. The large peak corresponds to the self-interactions, residing on top of the flat-background component. Two regions for accidental extraction are labelled, whereas the self region corresponds to the online window.

- Take the number of LAV hits in the lower timing side-band of the  $\delta T = T_{\text{trig}} - T_{\text{LAV}}$  distribution, defined as offset by +50 ns with a  $\pm 10$  ns window.
- The number of accidental hits in this region is plotted versus the instantaneous beam-intensity from the GTK between (0; 1200) MHz.
- The information is fed into the modified LAV algorithm emulator, and a number of accidentals are extracted at random for any given intensity.

The emulated LAV algorithm returns an efficiency based on all three components at any intensity. As was performed for  $Q_X$ , both the  $\epsilon_{3\pi}^{\text{LAV}}(z_{\pi^+\pi^-})$  and  $\epsilon_{\pi\mu\mu}^{\text{LAV}}(z_{\mu\mu})$  distributions were created using a two dimensional grid, and collapsed into one-dimension by calculating the weighted-average at each  $z$ -bin. The first case is used to verify the technique as the same spectrum can be extracted directly in data, whereas the second is used directly to reconstruct  $d\Gamma/dz$ . Both distributions are contained in Figure 7.18. On average, the emulated efficiency using the MCA approach for  $K_{3\pi}$  is 0.6% lower than for the observed value in data. As the shape agreement is excellent, this value is taken as a systematic for the  $K_{\pi\mu\mu}$  measurement. As the accidentals enter at the correct level in MC, the hadronic interactions appear to be over-predicted for events containing three pions. The simulation is therefore expected to be even better for  $K_{\pi\mu\mu}$  as only one pion is in the event, as the systematic effect could be up

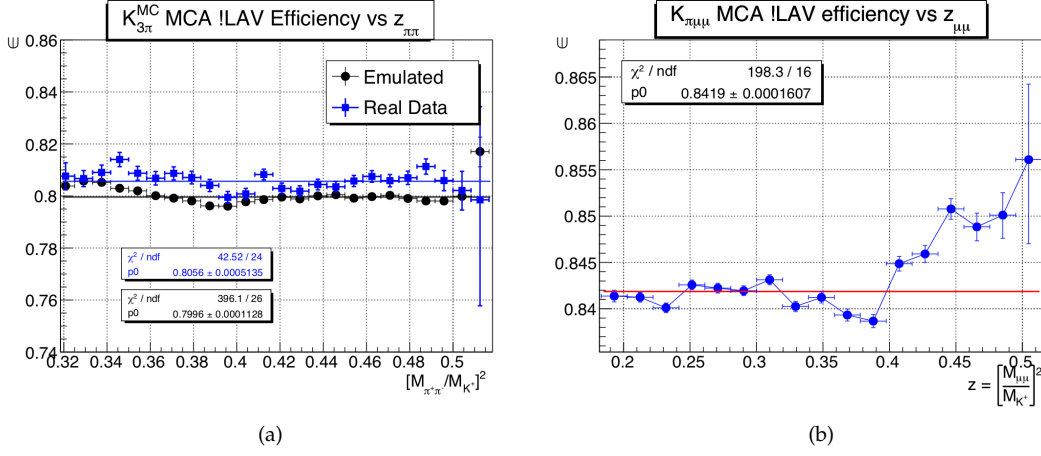


Figure 7.18: Reconstructing the **MCA** !LAV efficiency for (a)  $K_{3\pi}$  versus  $z_{\pi^+\pi^-}$  where it can be compared directly to the online algorithm distribution, and (b)  $K_{\pi\mu\mu}$  versus  $z_{\mu\mu}$  which is used directly in the analysis.

to three times smaller. The average  $K_{\pi\mu\mu}$  efficiency is therefore measured to be,

$$\epsilon_{\pi\mu\mu}^{\text{!LAV}} = 0.8419 \pm 0.0002_{\text{stat}} \pm 0.0060_{\text{syst}}, \quad (7.14)$$

extracted from the flat line in (b) of Figure 7.18. The shape is fairly different in comparison to  $K_{3\pi}$ , especially for high  $z_{\mu\mu}$  bins where the efficiency appears to increase linearly. Candidates in this region contain  $\pi^+$  with transverse momentum  $P_T$  typically close to zero, unlike elsewhere in the phase-space. This reduces the chance for direct  $\pi^+$  interaction, leading to **!LAV** being less likely to be fired. The behaviour is not observed for  $K_{3\pi}$  because all three daughters are pions.

## 7.7 MO<sub>2</sub> (**MUV3**)

The MO<sub>2</sub> component is unique to the Dimuon trigger, requiring at least two **MUV3** outer pads to be within a 10 ns coincidence. The primitive is used to collect candidates with two final state muons, a process which is relatively rare at NA62 except for accidental pile-up of decays containing muons. As  $K_{3\pi}$  is unsuitable, a different process was used. Based on the low-mass background observed in the final dimuon spectrum of Chapter 6, a sample of minimum-bias  $K^+ \rightarrow \pi^+\pi^+\pi^- \rightarrow \pi^+\mu^+\mu^-$  decays were collected using the following criteria:

- Event passes the final NA62-like  $K_{\pi\mu\mu}$  selection.
- Event is collected with the minimum bias control trigger.
- Candidate is consistent within the **SR**:  $|M_{3\pi} - M_{K^+}| \leq 5 \text{ MeV}/c$ .

The  $K_{3\pi}$  candidates surviving the  $K_{\pi\mu\mu}$  selection provide an ideal handle to measure the  $\text{MO}_2$  component. Figure 7.19 compares the normalised **MUV3** illuminations for  $K_{3\pi \rightarrow \pi\mu\mu}$  measured in data compared to  $K_{\pi\mu\mu}$  MC. Provided there is no geometric-

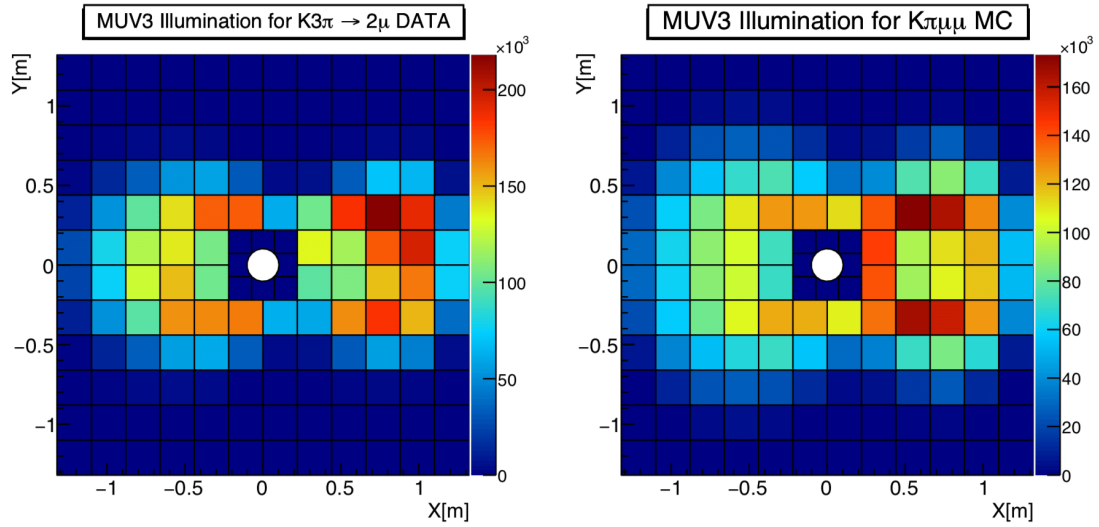


Figure 7.19: The **MUV3** Illumination for left,  $K_{3\pi \rightarrow \pi\mu\mu}$  data and right  $K_{\pi\mu\mu}$  in MC. Like the **New-CHOD**, the first case is fundamentally  $K_{3\pi}$ , with relatively lower opening angles. The absence of candidates in the inner pads is an artefact of the selection, as candidates with only in-time outer pad associations are selected, reducing the probability of association in this region.

based inefficiency, the illuminations in data are expected to be relatively similar. The sacrifice for using such a natural equivalent to the signal is statistics, where only  $\approx 10^4$  decays appear for control-triggered events in 2016A, whereas closer to  $10^7$  candidates were available for  $K_{3\pi}$  measurements. Figure 7.20 contains the time and intensity dependence of the efficiency for these candidates, re-binned with respect to previous distributions. The average  $\text{MO}_2$  efficiency for these decays as measured by the red line in (a) is,

$$\epsilon_{K_{3\pi \rightarrow \pi\mu\mu}}^{\text{MO}_2} = 0.9710 \pm 0.0019_{\text{stat}}. \quad (7.15)$$

To improve the precision of this measurement, the  $\text{MO}_2$  efficiency can instead be measured using the Multi-track trigger sample. As the Multi-track is identical to



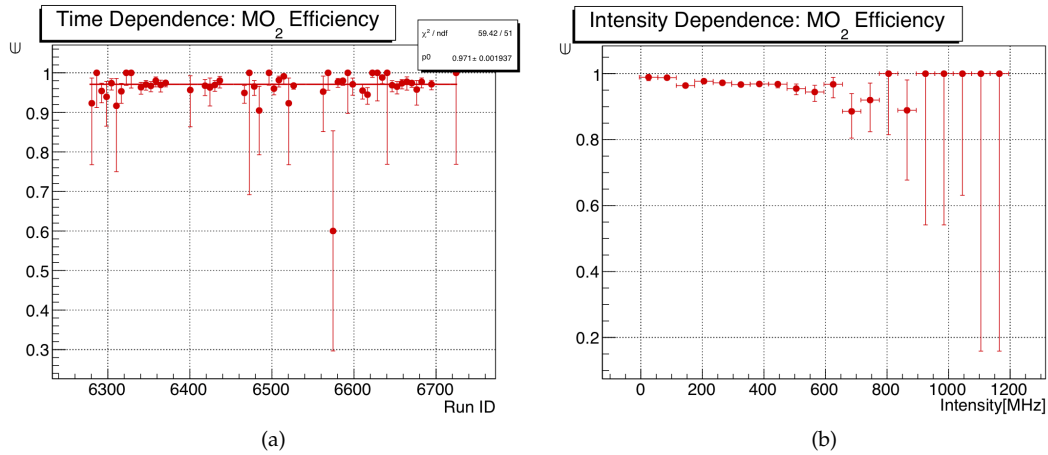


Figure 7.20: The  $MO_2$  primitive efficiency (a) with respect to time and (b) versus intensity for  $K_{3\pi} \rightarrow \pi^+ \mu^- \mu^-$  minimum bias decays. The red-line in (a) is the average efficiency across the period. The primitive displays a gentle intensity dependence which is limited by low-statistics.

the Dimuon trigger but without  $MO_2$ , the decays collected are far more suitable. Measuring the component using this stream also removes the double counting of inefficiency. For example, if events are inefficient for RICH or  $Q_X$ , they could be similarly inefficient for  $MO_2$ . However, the inefficiency would be counted twice when combining the results as the  $L_0$  criteria are not applied sequentially like  $L_1$ . Therefore, the Multi-track  $MO_2$  efficiency is expected to be higher and more likely to reflect the real  $K_{\pi\mu\mu}$  dependence. Additionally, the gain in statistics compared to minimum bias is a factor of  $\approx 10$ . Using this sample, the  $z_{\mu\mu}$  dependence can be plotted directly, as seen in Figure 7.21. The bins between the dashed grey lines

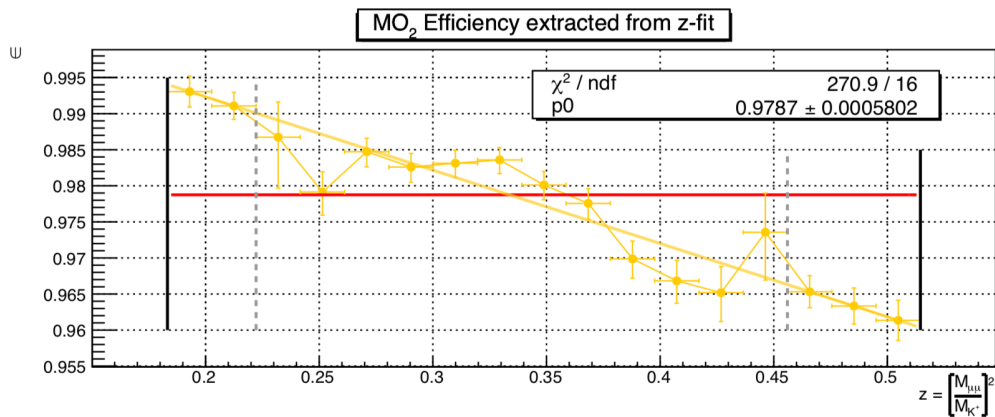


Figure 7.21: The  $MO_2$  Efficiency vs  $z$  in 2016A. The shape between the grey dashed lines represent the measurement in data, which is then extrapolated using a linear fit to the full phase-space. The variation across the region is  $\approx 3\%$ .

represent the data measurements, which do not occupy the full  $K_{\pi\mu\mu}$  phase-space denoted by the black lines. To extend the measurements for the full region, the data points were fitted with a first order polynomial and extrapolated as denoted by the orange line. This provides a measurement for the first two and last five bins. The errors assigned to the bins are based on the errors on the gradient, taken at a 68.3% **C.L.** As we are effectively blind to the true efficiencies in this region, the variation of the chosen bins here (for example, a flat extrapolation) is considered as a systematic error. The red-line denotes the average efficiency across the full phase-space, measuring the  $MO_2$  efficiency for  $K_{\pi\mu\mu}$  decays as.

$$\epsilon_{\pi\mu\mu}^{MO_2} = 0.9787 \pm 0.0006_{\text{stat}} \pm 0.0010_{\text{syst}}, \quad (7.16)$$

which is almost 1% higher than for the control trigger minimum-bias sample and with an error three times smaller. No clear shape change was observed with respect to  $z_{\mu\mu}$  between using minimum bias and Multi-track samples.

## 7.8 Full trigger efficiencies

The full trigger efficiency for  $K_{3\pi}$  Multi-track and  $K_{\pi\mu\mu}$  Dimuon events is obtained by calculating the product of each trigger component for each stream. Therefore, a single Dimuon efficiency distribution in terms of  $z_{\mu\mu}$  is produced using the components, as shown in Figure **7.22**. The scale of the trigger efficiency is dominated by both  $Q_X$  and **LAV**, whereas the shape is mostly due to the change in  $MO_2$  efficiency across the phase-space. The KTAG efficiency is included, taken as flat across the phase-space and extracted directly from the  $K_{3\pi}$  measurement. Table **7.2** contains the trigger efficiencies for each component of the Dimuon and Multi-track triggers for  $K_{\pi\mu\mu}$  signal and  $K_{3\pi}$  normalisation. The errors for  $K_{3\pi}$  are purely statistical and their magnitude reflects the large size of the sample. The same value is used for the KTAG in both.

The flat line in the  $K_{\pi\mu\mu}$  efficiency spectrum provides the overall signal trigger efficiency, whereas  $K_{3\pi}$  is extracted from the products in Table **7.2** with the final values

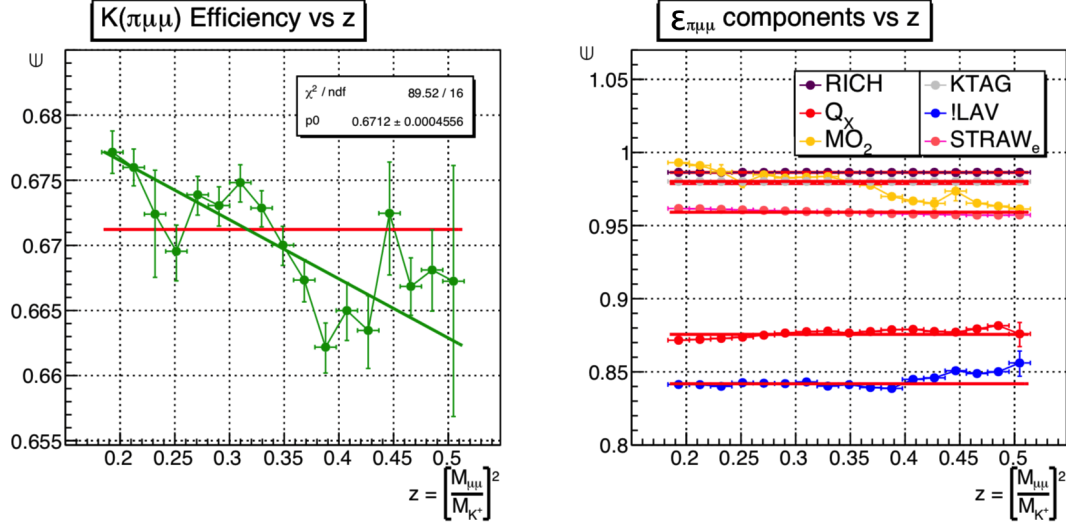


Figure 7.22: The overall  $K_{\pi\mu\mu}$  efficiency spectra vs  $z$  in 2016A. Left, the combined efficiencies; the red line is the overall average whilst the green provides the rough trend. Right is the individual components. The scale is dominated by  $Q_X$  and  $!LAV$ , decreasing linearly mostly due to the  $MO_2$  efficiency.

Table 7.2: Comparison of the trigger efficiency components for  $K_{3\pi}$  and  $K_{\pi\mu\mu}$  in 2016A. The  $MO_2$  mask is only present for the  $K_{\pi\mu\mu}$  signal and the largest magnitude difference is for the  $!LAV$  algorithm.

Component	$K_{3\pi}$	$K_{\pi\mu\mu}$
RICH	$0.9863 \pm 0.0001_{\text{stat}}$	$0.9864 \pm 0.0000_{\text{stat}} \pm 0.0060_{\text{syst}}$
$Q_X$	$0.8740 \pm 0.0002_{\text{stat}}$	$0.8756 \pm 0.0001_{\text{stat}} \pm 0.0050_{\text{syst}}$
$MO_2$	-	$0.9787 \pm 0.0006_{\text{stat}} \pm 0.0010_{\text{syst}}$
KTAG	$0.9802 \pm 0.0002_{\text{stat}}$	$0.9802 \pm 0.0002_{\text{stat}}$
$!LAV$	$0.8056 \pm 0.0005_{\text{stat}}$	$0.8419 \pm 0.0002_{\text{stat}} \pm 0.0060_{\text{syst}}$
$STRAW_e$	$0.9587 \pm 0.0003_{\text{stat}}$	$0.9591 \pm 0.0001_{\text{stat}} \pm 0.0050_{\text{syst}}$

as,

$$\epsilon_{\pi\mu\mu}^{\mathcal{F}} = 0.6712 \pm 0.0005_{\text{stat}} \pm 0.0137_{\text{syst}}, \quad (7.17)$$

$$\epsilon_{3\pi}^{\mathcal{F}} = 0.6526 \pm 0.0005_{\text{stat}} \pm 0.0080_{\text{syst}}. \quad (7.18)$$

The combination of the trigger efficiency components by multiplication introduces an additional systematic due to double-counting of inefficiency. As  $L_1$  is sequential, and  $MO_2$  for  $K_{\pi\mu\mu}$  measured using Multi-track events, the double-counting occurs only when combining the RICH and  $Q_X$  efficiencies at  $L_0$ . A discrepancy of 0.8% was found when comparing the product  $RICH \cdot Q_X$  with the direct measurement of the overall  $L_0$  Multi-track efficiency using events with positive CHOD primitives.

However, as the systematic error is present with the same magnitude for both signal and normalisation, it cancels directly in the ratio when normalising. With these final measurements (Figure 7.22, Table 7.2), the trigger efficiencies can be fully accounted for when reconstructing the  $d\Gamma/dz$  spectrum in bins of  $z_{\pi\mu\mu}$  as presented in Chapter 8.

# 8

## The $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ analysis

In this chapter, the measurement of the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  ( $K_{\pi\mu\mu}$ ) decay is presented. By reconstructing the  $d\Gamma/dz$  spectrum, both the model-independent branching fraction and the Low-Energy Constants (LECs) of the form factor are extracted from the 2016A NA62 data set. As discussed in Chapter 2, both of the measurements have been performed previously by the NA48/2 experiment, measuring  $\mathcal{B}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = (9.62 \pm 0.25) \times 10^{-8}$  and  $(a_+, b_+) = (-0.575 \pm 0.039, -0.813 \pm 0.145)$ . In general, the errors are dominated by low statistics and the contribution from the irreducible  $K_{3\pi}$  background [48]. The analysis performed here uses the sample of decays collected using the selection from Chapter 6 where the contribution from background is suppressed to a negligible level, taking into consideration the study performed on the trigger efficiency in Chapter 7. A discussion of the resulting systematic effects is also provided.

### 8.1 Analysis strategy

In theory, the decay is described as a single virtual photon exchange where the shape and magnitude of the spectra of variable  $z_{\mu\mu} = (M_{\mu\mu}/M_{K^+})^2$  is dependent on the transition form factor  $W(z_{\mu\mu})$ . The partial decay-rate is described in terms of  $z_{\mu\mu}$ , as

$$\frac{d\Gamma_{\pi\mu\mu}}{dz} = \Omega_C \frac{\alpha^2 M_{K^+}}{12\pi(4\pi)^4} \lambda^{3/2}(1, z_{\mu\mu}, r_\pi^2) \sqrt{1 - 4\frac{r_\mu^2}{z_{\mu\mu}}} \left(1 + 2\frac{r_\mu^2}{z_{\mu\mu}}\right) |W(z_{\mu\mu})|^2. \quad (8.1)$$

where  $\Omega_C$  is the Coulomb interaction term,  $r_{\pi,\mu} = M_{\pi,\mu}/M_{K^+}$  and  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ . In  $\chi$ PT, the form factor  $W(z_{\mu\mu})$  is described as,

$$W(z_{\mu\mu}) = G_F M_{K^+}^2 (a_+ + b_+ z_{\mu\mu}) + W^{\pi\pi}(z_{\mu\mu}), \quad (8.2)$$

where  $W^{\pi\pi}(z_{\mu\mu})$  is a dispersion term corresponding to the pion loop and the form factor parameters  $(a_+, b_+)$  are free low energy constants (LECs) which characterise all of the other possible transition loops [42][44]. In order to extract the LECs using experimental data, the partial decay-rate  $d\Gamma/dz$  must be reconstructed. The strategy of the analysis is therefore:

- Develop a  $K_{\pi\mu\mu}$  selection to collect candidates with a negligible contribution from all known backgrounds.
- Use a highly symmetrical selection to collect an abundant normalisation sample which has similar properties to the signal decay.
- Tune the MC simulation to account for important detector and track-reconstruction effects observed in the data.
- Compute the efficiencies and the effects of the trigger logic on both signal and normalisation samples.
- Compute  $d\Gamma/dz$  based on the acceptances, trigger efficiencies and the number of observed candidates for both signal and normalisation and extract both the branching fraction and low-energy constants.

The first four points of the strategy were already addressed in previous chapters. The development of a background-free selection for  $K_{\pi\mu\mu}$  was presented in Chapter 6, suppressing the systematic contribution of a large background  $z_{\mu\mu}$  spectrum. A highly symmetric selection for normalisation  $K_{3\pi}$  is also described, including the tuning of the simulation to improve the agreement with reality. The trigger efficiencies were studied and measured extensively in Chapter 7 in order to obtain a  $K_{\pi\mu\mu}$  efficiency spectrum with respect to  $z_{\mu\mu}$  and also the overall normalisation efficiency for  $K_{3\pi}$ .

The final piece of the strategy is to combine all of the information from previous chapters to reconstruct the  $d\Gamma/dz$  spectrum. The candidates collected from the selections are counted and presented in Section 8.2.1, including an estimate how negligible the background component of the  $K_{\pi\mu\mu}$  sample really is. The signal acceptances, calculated with the newly tuned simulation, are presented in Section 8.2.2 and include a comparison of the shape with respect to  $z_{\mu\mu}$  without the tuning. Finally, the  $d\Gamma/dz$  spectrum was reconstructed. The model-independent branching fraction was extracted by direct integration and the LECs were obtained by fitting the distribution with Equation 8.1, after correcting for the effects of the Coulomb interaction term  $\Omega_C$  and the non-linearity of the candidates in each bin. The propagation of systematic errors to both measurements is also performed, and the details are provided in Section 8.2.3.

## 8.2 Reconstructing $d\Gamma/dz$

In this section, the reconstruction of the  $d\Gamma/dz$  spectrum with respect to  $z_{\mu\mu}$  is described. The spectrum can be calculated in  $i$  bins of  $z$  following,

$$\left(\frac{d\Gamma_{\pi\mu\mu}}{dz}\right)_i = \left(\frac{N_i - N_i^B}{N_{3\pi}}\right) \cdot \left(\frac{A_{3\pi}}{A_i}\right) \cdot \left(\frac{\epsilon_{3\pi}}{\epsilon_i}\right) \cdot \left(\frac{1}{\Delta z_i}\right) \cdot \left(\frac{\hbar}{\tau_K}\right) \cdot \mathcal{B}(K_{3\pi}). \quad (8.3)$$

Each bracket is a combination of measurable quantities which must be known, with three major components,

- **Number of candidates:**  $N_i$  and  $N_i^B$  are the number of  $K_{\pi\mu\mu}$  and background candidates at the  $i^{\text{th}}$   $z_{\mu\mu}$ -bin.  $N_{3\pi}$  is the total number of collected  $K_{3\pi}$  normalisation candidates. This is calculated in Section 8.2.1 using the events passing the selection developed in Chapter 6.
- **Acceptances:**  $A_i$  is the  $K_{\pi\mu\mu}$  acceptance at the  $i^{\text{th}}$   $z_{\mu\mu}$ -bin, whereas  $A_{3\pi}$  is the total  $K_{3\pi}$  normalisation acceptance. This is calculated in Section 8.2.2 after the MC tuning described in Chapter 6.
- **Trigger Efficiencies:**  $\epsilon_i$  is the  $K_{\pi\mu\mu}$  trigger efficiency at  $i^{\text{th}}$   $z_{\mu\mu}$ -bin, whereas  $\epsilon_{3\pi}$  is the total normalisation trigger efficiency. Both components were calculated fully in Chapter 7 and are used here.

The remaining parameters are independent of the measured samples and are related to the width of the decay. The bin-width  $\Delta z_i$  is fixed to be the same size across the full decay phase-space, limited to  $4r_\mu^2 \leq z_{\mu\mu} \leq (1 - r_\pi)^2$ . The measurement also requires two external inputs, both the  $K_{3\pi}$  branching fraction and  $K^+$  lifetime  $\tau_K$  [55].

## 8.2.1 Number of candidates

The number of candidates enter in three places in Equation 8.3. Two quantities,  $N_i$  and  $N_i^B$  depend on the number of observed  $K_{\pi\mu\mu}$  candidates and expected background in the SR at each  $z$ -bin. The quantity  $N_{3\pi}$  is the integrated amount of  $K_{3\pi}$  decays in the full sample collected using the normalisation trigger. Whilst the  $K_{3\pi}$  sample statistically consistent with background-free at this sensitivity, a data-driven approach was performed to define a more reasonable  $K_{\pi\mu\mu}$  SR and predict the amount of background present in each  $z_{\mu\mu}$ -bin.

### 8.2.1.1 $N_i$ and $N_i^B$

To measure both  $N_i$  and predict  $N_i^B$ , the definition of the SR used to collect the  $K_{\pi\mu\mu}$  signal was tuned directly for the spectrum. The RooFit package was used to build both a signal and background Probability Density Function (PDF) with four components, two for signal and background respectively [87],

- A core Gaussian, to describe the  $K_{\pi\mu\mu}$  signal near  $M_{K^+}$ .
- A second wider Gaussian, to pick up the non-Gaussian tails of the signal.
- An exponential slope, measured directly on the tail-end of the low-mass events above 460 MeV.
- A second exponential, hypothesised as any surviving contribution from TRIM5 upstream background.

The first component provides a measurement of  $M_{K^+}$  as determined by the  $K_{\pi\mu\mu}$  candidates, useful for defining the SR, whilst the second is designed to better describe the non-Gaussian tails for the signal. The final two components, relating to the background, are used to determine the amount of background in the newly defined SR. The shape of the first exponential can be inferred directly by RooFit, but



the second requires examination of the invariant mass spectra for TRIM5 decays, which was suppressed in the selection development (see Section 6.4.1). As observed in the left-hand-side of Figure 8.1, a second exponential is the natural choice for these decays, instead of a generic flat component which was used by NA48/2 in this region. To determine the value of the slope to be inputted into RooFit, the  $Z$ -Vertex criterion for the selection was turned off and the region between (505; 540) MeV was fitted with an exponential curve. In order to check that the shape of surviving events did not change with respect to the cut on  $Z$ -Vertex, the criterion was re-added in steps of 1 m between (102; 110) m and the shape re-fitted, as described by the right-hand-side of Figure 8.1. As the linear slope of Figure 8.1 is consistent

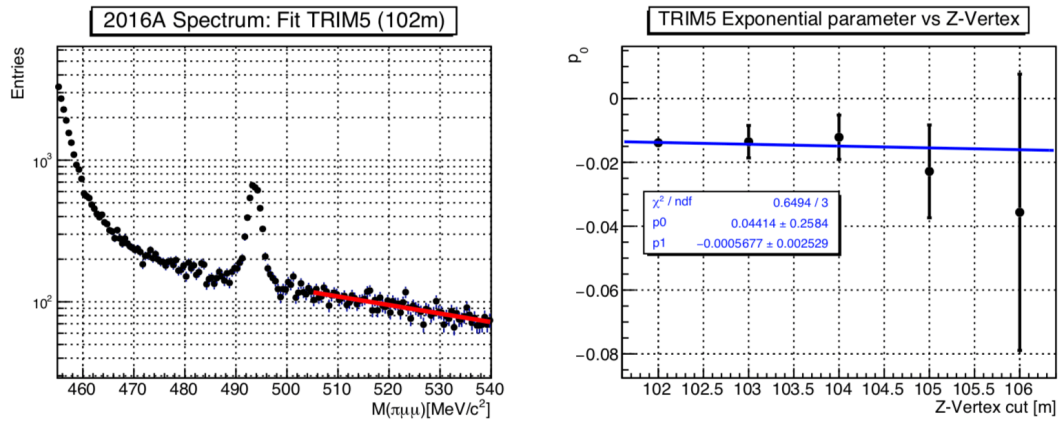


Figure 8.1: Fitting TRIM5 background to extract the overall slope. Left: The  $K_{\pi\mu\mu}$  invariant-mass spectra with a  $Z$ -Vertex cut at 102m (from Filter); the red line corresponds to the exponential fit across the high-mass region. Right: the TRIM5 slope parameter  $p_0$  after subsequently harder  $Z$ -Vertex cuts. Statistics begin to suffer, but the slope is consistent with being flat as the gradient of the first-order-polynomial fit is within one standard-deviation of zero. Only the points with adequate statistics are included.

to within a standard-deviation of being flat across the  $Z$ -Vertex cut space, the parameter was fixed and added into the background PDF. As TRIM5 is a hypothesis, the scale is free and is decided automatically by the number of candidates in the upper sideband of the signal PDF. The final result can be seen in Figure 8.2, where an extended maximum-likelihood fit has been performed [88]. The fit provides a number of useful measurements with respect to invariant mass, summarised in the first block of Table 8.1. The determined resolution  $\sigma_M$  is unchanged, but the SR is shifted to be centred at the mean value found by RooFit. Equation 6.1 is modified to

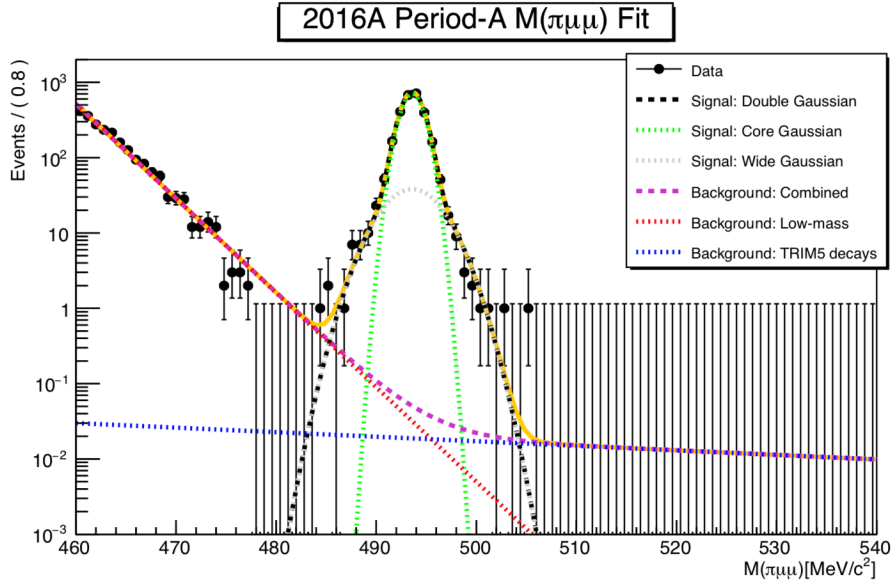


Figure 8.2: The RooFit PDF spectrum. The black-dots correspond to the data-set, whereas the full PDF is the combination of the four individual signal region and background functions. Only the shape of the TRIM5 component is inferred indirectly, and its scale is dependent on the observation of one event at  $\approx 505 \text{ MeV}/c^2$ .

become,

$$|M_{\pi\mu\mu} - 493.599| \leq 5 \text{ MeV}/c^2, \quad (8.4)$$

and is used to compute the number of signal candidates and expected background, contained in the second block. The fit expects  $N_{\pi\mu\mu}^S = 2677$  candidates, compared to

Table 8.1: The RooFit summary. The signal mass  $M_{\pi\mu\mu}$  and resolution  $\sigma_M$  are determined and used to define the modified SR in Equation 8.4. Using the modified SR the number of signal  $N_S$  and expected background candidates  $N_B$  is determined. The largest background contribution is from the low-mass tail, with less than one background event expected in the SR in total.

Quantity	Value
$M_{\pi\mu\mu}$	$(493.599 \pm 0.023) \text{ MeV}/c^2$
$\sigma_M$	$(1.081 \pm 0.032) \text{ MeV}/c^2$
$N_{\pi\mu\mu}^S$	$2676.98 \pm 51.47$
$N_{\text{Low-mass}}^B$	$0.54 \pm 0.01$
$N_{\text{TRIM5}}^B$	$0.24 \pm 0.01$
$N_{\text{Total}}^B$	$0.78 \pm 0.02$

2676 actual candidates observed. The largest background contribution is expected to be from the low-mass tail events, which is approximately twice the size of the TRIM5 contribution in the signal region. Regardless, less than one background event is ex-

pected in total such that the signal to background ratio  $S/B$  is  $\approx 3 \times 10^3$ . The lack of background is a significant improvement with respect to the NA48/2 analysis, where  $\approx 52$  background candidates were observed corresponding to a  $S/B$  of  $\approx 60$ . The improvement is mostly because of the implementation of a larger MNP33 current, increasing the separation between low-mass candidates and the signal region.

With the number of signal and an estimate of the number of background candidates determined, the  $z_{\mu\mu}$ -spectrum can be produced. Whilst the number of background is now predicted, the shape in  $z_{\mu\mu}$  is unknown. For the low-mass component, the  $z_{\mu\mu}$ -spectrum was observed directly, taking the shape from the distribution between (460; 475) MeV/c<sup>2</sup>. The TRIM5 component is extracted as the  $z_{\mu\mu}$ -spectrum from candidates between (505; 540) MeV/c<sup>2</sup> after a  $Z$ -Vertex cut of 102 m. Both spectra are appropriately scaled relative to the  $S/B$  ratio, and can be seen in Figure 8.3. In gen-

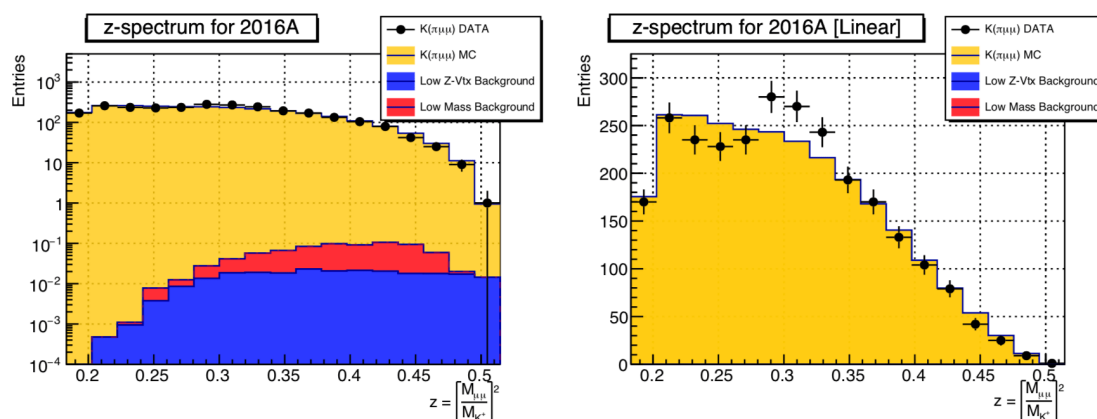


Figure 8.3: The 2016A  $z$ -spectra in left, log-scale and right, linear. The background contributes maximally for higher  $z_{\mu\mu}$ -values but many orders of magnitude below the signal. The linear-scale shows some discrepancy between MC and data for the six bins between  $0.21 \leq z_{\mu\mu} \leq 0.34$ .

eral, the background components are only visible in log-scale and are typically four orders of magnitude below the signal in each bin. The linear scale exposes a shape-discrepancy between MC and data for the six central bins between  $0.21 \leq z_{\mu\mu} \leq 0.34$ . The first three bins in this region sit approximately one standard-deviation below the MC expectation, whereas the next three are closer to two standard-deviations above expectation. Whilst the number of bins deviated is roughly in expectation for statistical fluctuations, the proximity of the bins with respect to each other is suspicious. Two possible causes are due to kinematic dependences,

- Distortion via the trigger stream.
- Differences between data-MC reconstruction efficiency.

However, the trigger efficiency varies by less than 2% across the sample as observed in Figure 7.22, and the reconstruction effects do not drastically alter the acceptance shape as observed in Figure 8.7. Without a valid explanation, the residual difference must be taken as a systematic effect and propagated into the error analysis.

The choice of bin size is the result of dividing the full phase-space equally by seventeen, leading to  $\Delta z_i = 0.0195$ . The bins are large with respect to the resolution  $\sigma_{z_{\mu\mu}} = 0.0011$ , determined using the  $10^7$   $K_{\pi\mu\mu}$  MC sample after fitting the residual of truth and reconstructed  $z_{\mu\mu}$  with a Gaussian. The resolution is stable across the phase-space, as seen in the right-hand-side of Figure 8.4. One of the striking feature

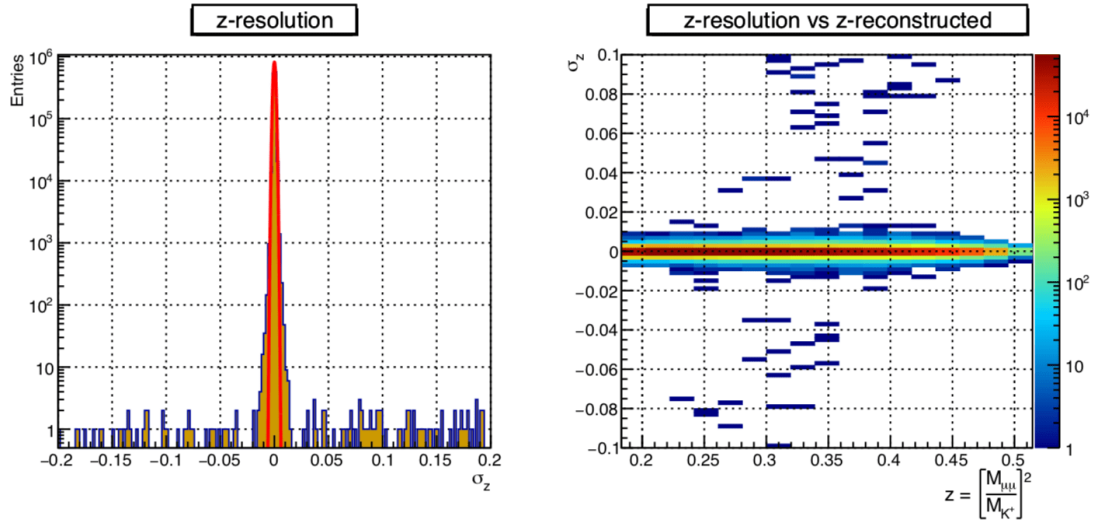


Figure 8.4: The  $z_{\mu\mu}$ -resolution for  $K_{\pi\mu\mu}$  MC. Left, the residual between truth and reconstructed values, where the resolution is extracted via a Gaussian fit. Right, the resolution plotted with respect to the reconstructed  $z$ .

of both distributions is a flat-component where the residual difference between truth and reconstructed  $z_{\mu\mu}$  is large. The effect, though altering only a small number of candidates, is due to the  $\pi^+\mu^+$  swaps (mis-identification of the  $\pi^+$  as  $\mu^+$ ), described previously. The swap-probability can be calculated using the spectrum, defined as:

$$P_{\text{swap}} = \frac{N(\text{Candidates} \geq \pm 5\sigma_z)}{N(\text{Total})} = 0.0026 \pm 0.0001. \quad (8.5)$$

The effect of the swaps is expected to be larger in data due to the accidental rate. As the  $z_{\mu\mu}$ -spectrum is distorted as a result, it must be considered as a systematic source.

### 8.2.1.2 $N_{3\pi}$

Unlike  $K_{\pi\mu\mu}$ , only the integrated number of  $K_{3\pi}$  candidates recorded in 2016A is required. The caveat is that the observed number does not reflect the total number as the normalisation sample is recorded with a downscaled trigger. The  $K_{3\pi}$  SR is defined as,

$$|M_{3\pi} - M_{K^+}| \leq 5 \text{ MeV}/c^2. \quad (8.6)$$

used to count events surviving the  $K_{3\pi}$  selection from the Multi-track trigger. Figure 8.5 contains the number of  $K_{3\pi}$  candidates across the full 2016A period, before and after correcting for the trigger downscale. The raw number of  $K_{3\pi}$  is found to

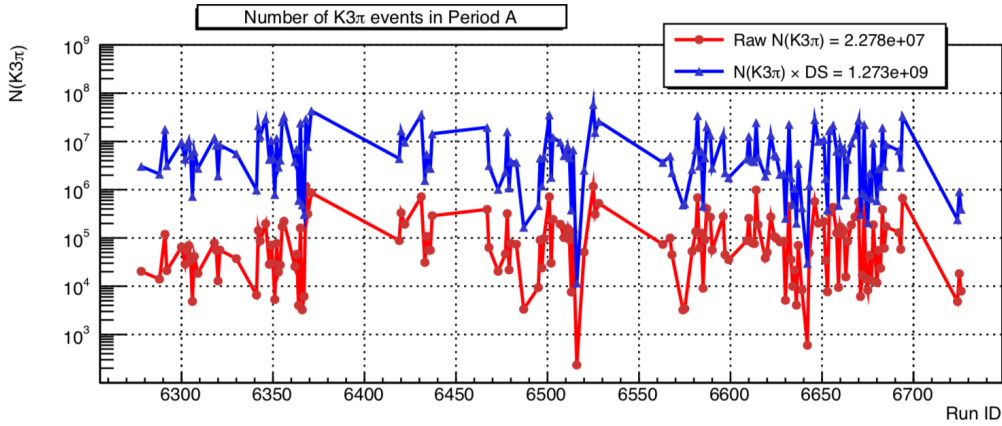


Figure 8.5: The number of observed normalisation  $K_{3\pi}$  candidates across 2016A. The red distribution accounts for the raw observed numbers, and the blue is after correcting for the normalisation trigger downscale.

be ,

$$N_{3\pi}^{\text{Raw}} = (2.278 \pm 0.001) \times 10^7 \quad (8.7)$$

and the average downscale equates to a factor of 56. The final inferred number of  $K_{3\pi}$  is,

$$N_{3\pi}^{\times D} = (1.273 \pm 0.001) \times 10^9. \quad (8.8)$$

## 8.2.2 Acceptances

The acceptance is a ratio defined as,

$$A = \frac{N(\text{Candidates passing selection})}{N(\text{Candidates originating in fiducial volume})}. \quad (8.9)$$

The ratio is the fraction of decays which originated in the fiducial volume between (102; 180) m that subsequently survived the full selection. It corresponds to the fraction of candidates that we can expect to observe. As acceptance can only be calculated using MC samples, the simulation was tuned in Chapter 6 to match as many of the data-taking conditions as possible, including sub-detector and intensity-dependent reconstruction effects. Additionally, a data-MC comparison has been performed in Appendix A which displays similarly good agreement for both  $K_{\pi\mu\mu}$  and  $K_{3\pi}$  samples.

As the acceptance for each sample depends on the residual intensity spectrum, the acceptance is calculated using a similar weighted-average technique as the trigger efficiencies in Chapter 7. A two-dimensional grid is reconstructed of  $z_{\mu\mu}$  against intensity, as displayed in Figure 8.6, where each bin corresponds to the acceptance at  $(z_{\mu\mu}, \mathcal{I})$ . The acceptance is then determined for each  $z_{\mu\mu}$ -bin by constructing the weighted average of the twelve 100 MHz intensity bins used in the grid,

$$A_{z_{\mu\mu}} = \frac{\sum_{\mathcal{I}=1}^{12} w_{\mathcal{I}} A_{z_{\mu\mu}, \mathcal{I}}}{\sum_{\mathcal{I}=1}^{12} w_{\mathcal{I}}}. \quad (8.10)$$

where  $\mathcal{I}$  is the intensity-bin index,  $A_{z_{\mu\mu}, \mathcal{I}}$  is the acceptance at the  $(z_{\mu\mu}, \mathcal{I})$  bin and  $w_{\mathcal{I}}$  is the number of candidates in the  $\mathcal{I}^{\text{th}}$  bin of the intensity-spectrum. Using the weighted average approach across the full grid results in Figure 8.7, containing  $A_i$ , and the evolution of the  $K_{\pi\mu\mu}$  acceptance from before the detector and reconstruction efficiencies were accounted for. The final corrected acceptances  $A^{\mathcal{F}\mathcal{C}}$  are:

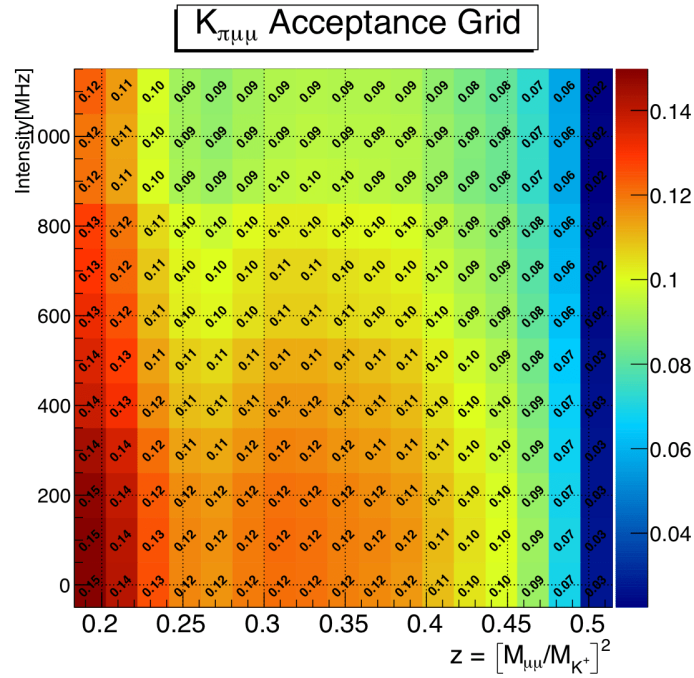


Figure 8.6: The two-dimensional  $K_{\pi\mu\mu}$  acceptance grid. The diagonal labels correspond to the acceptance for each bin.

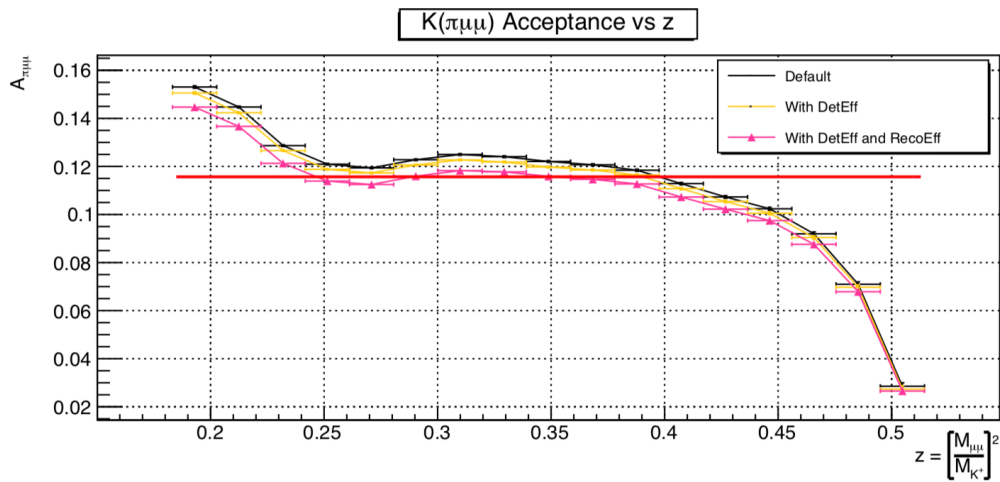


Figure 8.7: The evolution of the  $K_{\pi\mu\mu}$  acceptance plotted in bins of  $z_{\mu\mu}$ . The black line is the acceptance measured directly from the MC sample, the orange after adding the `NewCHOD` and `MUV3` efficiencies and the pink is the final result after adding in the reconstruction efficiency and performing the weighted average approach. No significant change in shape is observed.

$$A_{\pi\mu\mu}^{\mathcal{FC}} = (11.607 \pm 0.005)\%, \quad (8.11)$$

$$A_{3\pi}^{\mathcal{FC}} = (10.847 \pm 0.003)\%, \quad (8.12)$$

where  $A_{\pi\mu\mu}^{\mathcal{FC}}$  is the integrated value of  $A_i$  across the full phase-space, and  $A_{3\pi}^{\mathcal{FC}}$  is determined using the same weighted averaged approach but without the binning of a kinematic variable. The signal acceptance is largest at the first  $z_{\mu\mu}$ -bin at  $\approx 15\%$ , but falls off for high  $z_{\mu\mu}$ . The effect of the modifications due to the detector and reconstruction efficiencies is larger for  $K_{\pi\mu\mu}$ , as the stringent  $\mu^\pm$  particle-identification criterion leads to a sensitivity towards the `MUV3` detector efficiency, decreasing the acceptance by  $\approx 0.3\%$  overall. The `NewCHOD` efficiency changes the acceptance in both cases by less than  $0.01\%$ . The effect of the reconstruction efficiency decreases both acceptances by a similar magnitude to the `MUV3` case, by  $\approx 0.4\%$ . Importantly, adding detector and reconstruction efficiencies does not change the acceptance shape with respect to  $z_{\mu\mu}$ ; the distortion of the spectrum between  $0.21 \leq z_{\mu\mu} \leq 0.34$  observed in Figure `8.3` appears to be an independent effect.

## 8.2.3 Results

### 8.2.3.1 Extracting the LECs

The reconstructed  $d\Gamma/dz$  against  $z_{\mu\mu}$  spectrum is presented in Figure `8.8`. Both distributions are identical aside from the curves, which are the result of fitting with the theoretical  $d\Gamma/dz$  shape provided in Equation `8.1`. The left-hand plot corresponds to positive `LECs` whilst the right-hand are negative, as the form factor enters into the partial rate as  $|W(z)^2|$ . The spectrum has been corrected for the Coulomb factor  $\Omega_C$ , shifting the distribution down by approximately  $3\%$  across the phase-space, but mostly in the first  $z_{\mu\mu}$ -bin. Additionally, the effective  $z_i$  values where  $d\Gamma/dz$  is evaluated has been corrected to account for the effects of using large bin-widths. As the underlying `PDF` has a shape that can vary rapidly within a large bin, instead of placing  $z_i$  at the bin-centre or bin barycentre, we evaluate for each bin,

$$\mathcal{F}(z_{lw}) = \frac{1}{\Delta z} \int_{z_1}^{z_2} \mathcal{F}(z_i) dz, \quad (8.13)$$



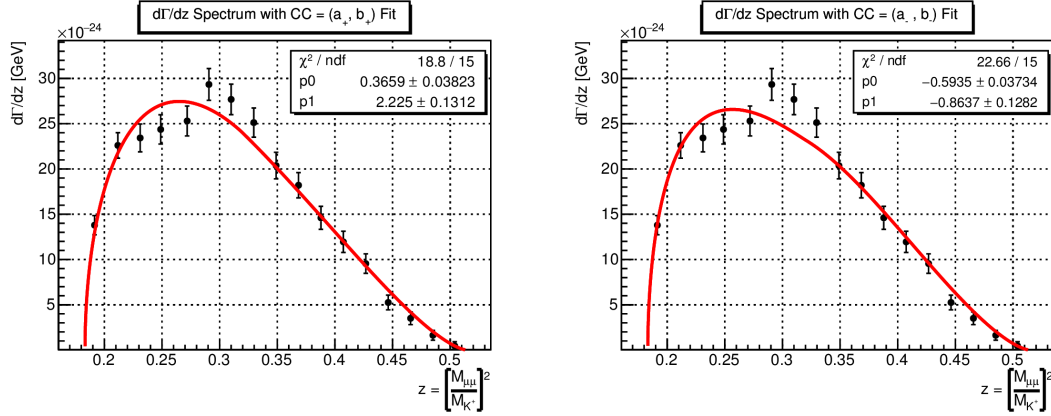


Figure 8.8: The reconstructed  $d\Gamma/dz$  vs.  $z_{\mu\mu}$  spectrum for 2016A. Left, the positive form factor solution and right, the negative form factor solution. The positive case is preferred according to the fit  $\chi^2$ .

where  $\mathcal{F}(z)$  is the prediction of the underlying PDF based on the NA48/2 measurement (i.e  $d\Gamma/dz$ ),  $\Delta z$  is the bin-width,  $z_{1,2}$  are the lower and upper bin edges and  $z_{lw}$  is the true data-point position for bins of large-width [89]. To extract the model-independent branching fraction, the  $d\Gamma/dz$  spectrum is integrated directly and normalised to the  $K^+$  decay width ( $\hbar/\tau_{K^+}$ ) without the Coulomb correction applied,

$$\mathcal{B}(K^+ \rightarrow \pi^+ \mu^- \mu^-) = (10.429 \pm 0.203_{\text{stat}} \pm 0.192_{\text{syst}} \pm 0.045_{\text{ext}}) \times 10^{-8}, \quad (8.14)$$

which is  $\approx 2\sigma$  larger than the NA48/2 and PDG-combined measurement. Despite an improved statistical error due to the suppression of the background component, the systematic error is almost twice the size of the previous measurement, dominated by the  $z_{\mu\mu}$ -spectrum distortion. This is discussed further in Section 8.2.4

To extract the LECs, the result can be presented in two-dimensional  $(a_+, b_+)$  space versus the fit  $\chi^2$ , as displayed in Figure 8.9. Here, the minima are represented by the yellow and grey stars, where yellow has the lowest  $\chi^2$ . Contours are displayed for both, representing three standard-deviations. This leads to two sets of LECs presented in Table 8.2 where the positive solution is preferred by  $\Delta\chi^2 = 3.9$ . The systematic errors dominate over the statistical, again largely due to the  $z_{\mu\mu}$ -distortion. Whilst the positive solution is preferred, only the negative can be compared with NA48/2 where the NA62 measurement is in agreement to one standard-deviation

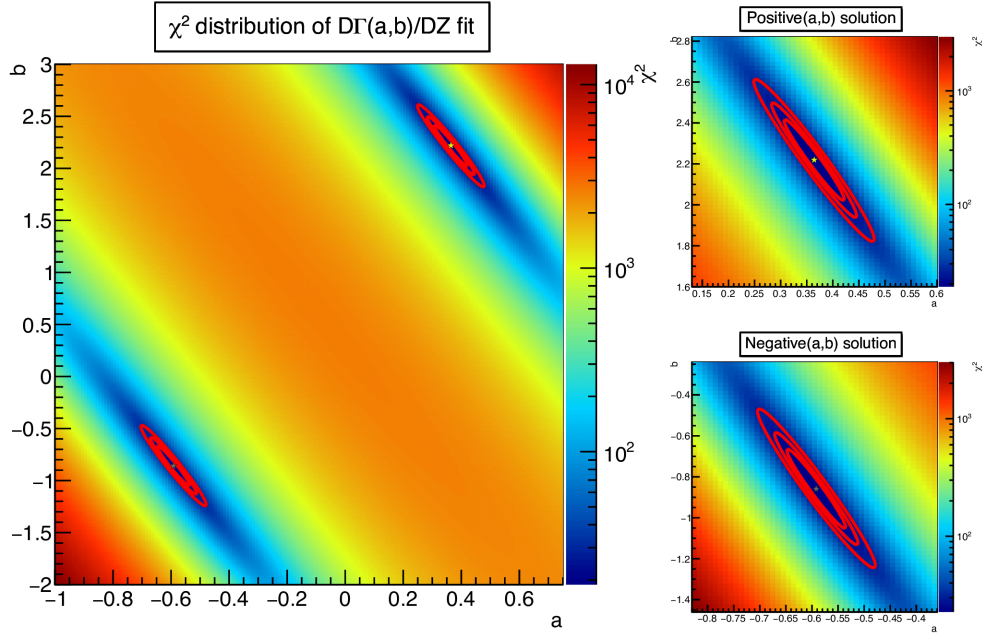


Figure 8.9: The two-dimensional representation of the form factor extraction of  $(a_+, b_+)$  versus fit  $\chi^2$ . The yellow and grey stars represent the minima, with the yellow case being the lowest  $\chi^2$ . The contours represent the first three standard-deviations from the minima.

Table 8.2: The **LECs** in 2016A, included statistical, systematic and external errors. The positive solution is statistically preferred by  $\Delta\chi^2 = 3.9$ .

Sign	LEC	Value	$\chi^2/\text{NDF}$
Positive	$a_+$	$0.366 \pm 0.038_{\text{stat}} \pm 0.050_{\text{syst}} \pm 0.001_{\text{ext}}$	18.8/15
	$b_+$	$2.220 \pm 0.131_{\text{stat}} \pm 0.184_{\text{syst}} \pm 0.004_{\text{ext}}$	
Negative	$a_+$	$-0.594 \pm 0.037_{\text{stat}} \pm 0.048_{\text{syst}} \pm 0.001_{\text{ext}}$	22.7/15
	$b_+$	$-0.864 \pm 0.128_{\text{stat}} \pm 0.175_{\text{syst}} \pm 0.004_{\text{ext}}$	

for both parameters.

## 8.2.4 Systematics

To compare the measurement with previous analyses, a detailed study of the systematic effects has been undertaken. The systematic and external errors presented in the final measurement above are described here. Any variable that enters into the reconstruction of  $d\Gamma/dz$  can be considered a source of systematic error and therefore, we break down the type of systematic into categories:

- **Trigger Efficiency:** As discussed in Chapter 7, the method used to obtain the  $K_{\pi\mu\mu}$  efficiencies leads to systematic terms,

- RICH: The largest effect here is the disagreement between the primitive and emulator efficiency. The systematic is taken from the maximum shape fluctuation of the primitive efficiency from the average.
  - $Q_X$ : The systematic difference between the measured primitive efficiency and the MCA emulated weighted-average approach was estimated by comparing the efficiency versus the  $z_{\pi^+\pi^-}$  spectra for  $K_{3\pi}$  candidates.
  - $MO_2$ : As the efficiency versus  $z_{\mu\mu}$  distribution is extracted from  $K_{3\pi \rightarrow \pi\mu\mu}$  decays, the efficiency distribution covers a small phase-space region than for signal  $K_{\pi\mu\mu}$ . The systematic is estimated by determining how different models of extrapolation (flat, linear) affect the final measurements.
  - !LAV: Similarly to  $Q_X$ , the systematic difference between the measured primitive efficiency and the MCA emulated weighted-average approach was estimated by comparing the efficiency versus the  $z_{\pi^+\pi^-}$  spectra for  $K_{3\pi}$  candidates. Note that hadronic simulations are the largest issue, and are expected to be a smaller effect for  $K_{\pi\mu\mu}$  than for  $K_{3\pi}$ .
  - $STRAW_e$ : The only non-negligible contribution is due to the limitation of the momentum-efficiency model. The systematic is taken from the maximum deviation between the data and fit at  $P(\pi^-) = 35 \text{ GeV}/c$ .
- Acceptances also have a number of sources,
    - The distortion observed in the  $z_{\mu\mu}$ -spectrum between  $0.21 \leq z_{\mu\mu} \leq 0.34$  was not explained by reconstruction, detector or trigger-efficiency effects and so a systematic term is assigned to the effect taken as a systematic. The magnitude is estimated by reweighting the acceptance distribution by the ratio of the data spectrum to MC in the region of the six suspicious bins. This is the largest systematic in the analysis.
    - The statistical error of the  $K_{3\pi}$  acceptance is treated as a systematic.
    - The errors on the detector tile-efficiencies and data/MC reconstruction ratio enter as a systematic effect, estimated by comparing the standard acceptance with one where the efficiencies were Gaussian-smear based on the statistical error at each tile or momentum bin.

- Systematics due to background,
  - RooFit provides the error in the expected number of background.
  - Additionally, as the background shape is inferred and not reliably known, we also estimate the effect of a completely flat-background contribution set at the scale of  $N_B$  instead of the prediction based on TRIM5 or low-mass tail events.
- Systematics due to bin properties,
  - The non-negligible resolution on the  $z_{\mu\mu}$  variable means that bin-migration is possible. The effect of this is simulated by Gaussian smearing the data sample by the resolution to calculate the systematic difference.
  - The effect of  $\mu^+$  mis-identification is also considered, which appear as  $\pi^+\mu^+$  swaps in the data sample. The swap-probability was determined based on the resolution spectrum (Equation 8.5), and scaled to match data by estimating the ratio of in-time MUV3 associations for three-track events between data and MC, found to be a 10% effect. Approximately 7 mis-identified candidates are predicted for this data sample. The effect was estimated by randomly selecting candidates (using the swap-probability) and assigning a random  $z_{\mu\mu}$  value instead of the measurement anywhere in the available phase-space. The difference observed in the calculation is taken as the systematic error.
- The effect of external measurements,
  - The measurement of the  $K_{3\pi}$  branching fraction is an external parameter. The error in this measurement has decreased by 3% since the NA48/2 analysis.
  - Form factor only: the  $K^+$  lifetime cancels for the branching fraction calculation (as the integral is divided by  $\hbar/\tau_{K^+}$ ), but not for the form factor fit. The precision of the lifetime is  $\approx 0.2\%$ .
  - Form factor only: the chosen data-point position in  $z_{\mu\mu}$ , corrected for the effect of large bin-widths, takes input from the NA48/2 measurement. The

error in the position is taken from the overall error in those parameters and is only available for one sign of solution, as the previous measurement omits the positive **LECs**.

The size of the systematic contribution to the branching fraction due to each source is presented in Table **8.3**. The largest systematic effect is due to the estimation of the  $z_{\mu\mu}$ -spectrum distortion, followed by the trigger efficiency systematics and the external measurement of the  $K_{3\pi}$  branching fraction. For the form factor, the systematic contributions to the positive solution **LECs** are presented in Table **8.4**, and the negative solution in Table **8.5**. Like for the branching fraction, the  $z_{\mu\mu}$ -distortion effect is the largest term and as it drastically affects the shape of the spectrum, it is the dominant error term overall. The effect is therefore likely to be the determining factor in which sign of solution is statistically preferred. Additionally, the positive solutions have slightly larger error terms due to the nature of the  $\chi^2$  shape, which increases more slowly than for the negative case.

Table 8.3: Systematic contributions to the 2016A  $K_{\pi\mu\mu}$  model-independent branching fraction measurement. The errors are split into categories and are factors of  $10^{-8}$ .

Type	Systematic	$\sigma\mathcal{B}_{\text{syst}} [10^{-8}]$	
		up	down
Trigger	RICH	0.063	-0.064
	$Q_X$	0.059	-0.060
	$MO_2$	0.005	-0.005
	!LAV	0.074	-0.075
	$STRAW_e$	0.054	-0.054
Acceptance	$A_{\pi\mu\mu}$ distortion	0.141	-0.141
	$A_{3\pi}$	0.003	-0.003
	Detector efficiency	0.033	-0.033
Background	$N_B$	0.000	0.000
	$N_B$ shape	0.000	0.000
Binning	Resolution ( $z$ )	0.003	-0.0003
	$\pi^+\mu^+$ swap	0.000	0.000
External	$\mathcal{B}(K_{3\pi})$	0.045	-0.045
Total		0.192	-0.192

Table 8.4: Systematic contributions to the 2016A positive solution. The errors are split into categories and measured for their effect on the measurement both up and down.

Type	Systematic	$\sigma a_{\text{syst}}$		$\sigma b_{\text{syst}}$	
		up	down	up	down
Trigger	RICH	0.0015	-0.0015	0.0047	-0.0047
	$Q_X$	0.0014	-0.0014	0.0044	-0.0044
	$\text{MO}_2$	0.0031	-0.0031	-0.0096	0.0096
	!LAV	0.0017	-0.0017	0.0055	-0.0055
	$\text{STRAW}_e$	0.0012	-0.0012	0.0041	-0.0041
Acceptance	$A_{\pi\mu\mu}$ distortion	0.0491	-0.0491	-0.1818	0.1818
	$A_{3\pi}$	0.0000	0.0000	0.0002	-0.0002
	Detector efficiency	0.0023	-0.0023	-0.0029	0.0029
Background	$N_B$	0.0000	0.0000	0.0001	0.0001
	$N_B$ shape	0.0003	-0.0003	-0.0012	0.0012
Binning	Resolution ( $z$ )	0.0065	-0.0065	-0.0211	0.0211
	$\pi^+\mu^+$ swap	0.0045	-0.0045	-0.0153	0.0153
External	$K^+$ Lifetime	0.0004	-0.0004	0.0013	-0.0013
	$\mathcal{B}(K_{3\pi})$	0.0010	-0.0010	0.0033	-0.0033
	Data point position	0.0005	-0.0005	0.0023	-0.0012
Total		0.0500	-0.0500	0.1840	-0.1840

Table 8.5: Systematic contributions to the 2016A negative solution. The errors are split into categories and measured for their effect on the measurement both up and down.

Type	Systematic	$\sigma a_{\text{syst}}$		$\sigma b_{\text{syst}}$	
		up	down	up	down
Trigger	RICH	0.0015	-0.0015	0.0047	-0.0047
	$Q_X$	0.0014	-0.0014	0.0043	-0.0043
	$\text{MO}_2$	0.0030	-0.0030	-0.0094	0.0094
	!LAV	0.0017	-0.0017	0.0055	-0.0054
	$\text{STRAW}_e$	-0.0012	0.0012	0.0041	-0.0041
Acceptance	$A_{\pi\mu\mu}$ distortion	0.0465	-0.0465	-0.1721	0.1721
	$A_{3\pi}$	0.0000	0.0000	0.0002	-0.0002
	Detector efficiency	0.0023	-0.0023	-0.0029	0.0029
Background	$N_B$	0.0000	0.0000	0.0000	0.0000
	$N_B$ shape	0.0003	-0.0003	-0.0011	0.0011
Binning	Resolution ( $z$ )	0.0062	-0.0062	-0.0203	0.0203
	$\pi^+\mu^+$ swap	0.0046	-0.0046	-0.0161	0.0161
External	$K^+$ Lifetime	0.0004	-0.0004	0.0013	-0.0013
	$\mathcal{B}(K_{3\pi})$	0.0010	-0.0010	0.0033	-0.0033
	Data point position	0.0004	-0.0004	0.0011	-0.0009
Total		0.0475	-0.0475	0.1750	-0.1750

# 9

## Future prospects

In this chapter, the future prospects of the  $K^+ \rightarrow \pi^+ \mu^+ \mu^- (K_{\pi\mu\mu})$  analysis at NA62 are explored. As explained in Chapter 7, a number of flaws were discovered in the Trigger and Data Acquisition (TDAQ) in the 2016 data. As a result, a number of large systematic errors reduced the precision of the measurements. With efforts made by experts to rectify these issues, both the form factor and branching fraction of the  $K_{\pi\mu\mu}$  decay are measured for 2017A, using an identical analysis approach to 2016A. By comparing the difference in the systematic effects, the prospects of performing the world's most precise measurement of the  $K_{\pi\mu\mu}$  are described below.

### 9.1 Strategy

In order to probe the future prospects of the  $K_{\pi\mu\mu}$  measurement, the analysis must be repeated using the 2017A data set. In general, the 2017A data set can provide an insight to how the current selection strategy handles more challenging beam conditions, and how changes to the TDAQ relative to 2016A can reduce systematic errors. There are three important aspects to consider:

- **Signal to background:** the average instantaneous intensity for 2017 is 400.3 MHz, almost 130 MHz larger than the average for the 2016A data set. As a result, the accidentals contribution is expected to be larger, possibly leading to a non-negligible background component for the  $K_{\pi\mu\mu}$  sample.
- **Spectrum distortion:** A systematic distortion of six bins of kinematic  $z_{\mu\mu}$  spec-

trum in the region  $0.21 \leq z_{\mu\mu} \leq 0.34$  led to the largest systematic error in the whole of the 2016 analysis. The spectrum for 2017A must also be investigated.

- **Trigger efficiency:** Another large systematic contribution, much of the 2016A trigger was found to be flawed for the  $K_{\pi\mu\mu}$  analysis. In 2017A, many of the inefficiencies have been rectified and the impact of the improvements must be assessed.

Below, each of the three aspects is explored in detail. Section 9.2 contains the 2017A  $K_{\pi\mu\mu}$  and  $K_{3\pi}$  samples, including an estimate of the 2017A background contamination and an investigation of the  $z_{\mu\mu}$  spectrum. The trigger efficiency is evaluated in Section 9.3 and compared to 2016A values. Finally, the branching fraction and form factor LECs for the 2017A data set is provided in Section 9.5 focusing specifically on the same systematic errors that were relevant for the previous year.

## 9.2 Event sample

As described in Section 6.2, the 2017A sample is roughly the same size as 2016A, with  $\approx 83000$  reconstructed bursts available for analysis. The size of the sample is expected to increase by a factor  $\approx 5$  by the end of summer 2018 after the full data reprocessing is complete. The normalisation ( $K_{3\pi}$ ) and signal ( $K_{\pi\mu\mu}$ ) samples were collected with two different trigger streams that were reconfigured with respect to 2016:

- **Multi-track trigger (2017A):** RICH ·  $Q_X \rightarrow$  KTAG  $\rightarrow$  STRAW<sub>e</sub>,
- **Dimuon trigger (2017A):** RICH ·  $Q_X \cdot$  MO<sub>2</sub>  $\rightarrow$  STRAW<sub>e</sub>.

At L<sub>0</sub>, all primitive generating firmware was improved, including a fix for the quadrant-dependent flaw in the NewCHOD described in Chapter 7. At L<sub>1</sub>, the !LAV algorithm was removed from both triggers, and the KTAG algorithm removed from just the dimuon trigger so that exotic decays could be collected through the same stream. Due to the data-acquisition rates in 2017, the Multi-track trigger used a fixed down-scale factor of  $D = 100$  for all runs, whilst the Dimuon trigger was downscaled by a factor  $D = 2$ . Of the available  $\approx 83,000$  bursts, only  $\approx 68,000$  are used in this study,



chosen for where both the Multi-track and Dimuon triggers were performing nominally.

Using the final selection as presented in Section 6.5, the 2017A signal and normalisation spectra are displayed in Figure 9.1. For normalisation  $K_{3\pi}$ , the distribution is

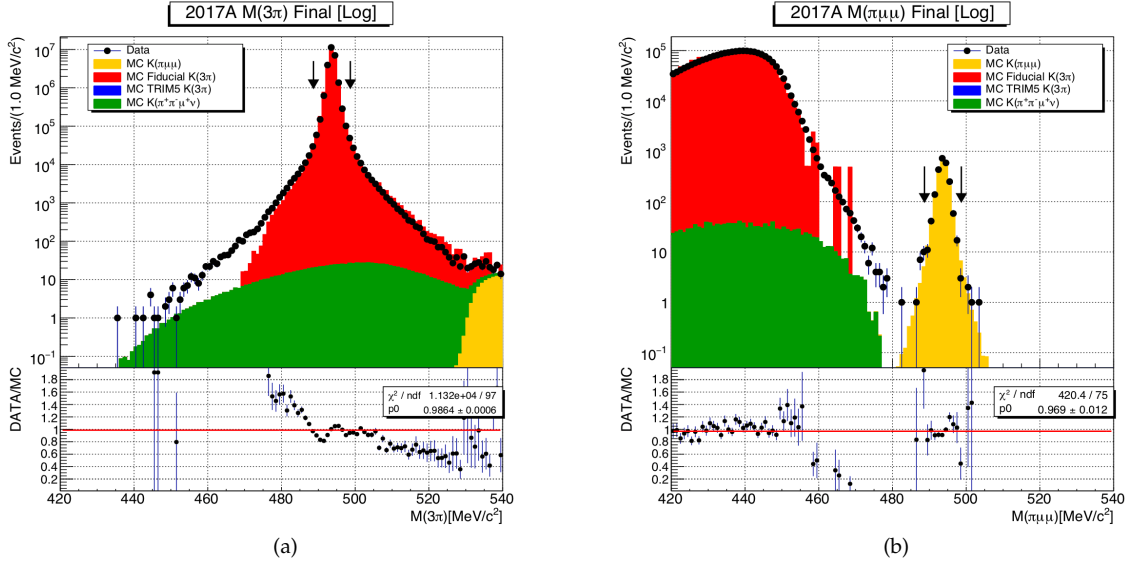


Figure 9.1: The (a)  $K_{3\pi}$  and (b)  $K_{\pi\mu\mu}$  spectra for 2017A in log-scale, with the SRs. The bottom pad shows the data-MC agreement based on the expected yields.

similarly as clean as 2016 (with background contamination  $\approx 0.1\%$ ). The size of the sample is equivalent to:

$$N_{3\pi}^{2017} = (2.433 \pm 0.001) \times 10^7 \quad (9.1)$$

which corresponds to  $2.433 \times 10^9$   $K_{3\pi}$  decays when accounting for the downscaling. The  $K_{\pi\mu\mu}$  sample is equally as promising, with no candidates in the upper mass sidebands and a reasonably well-separated signal to background peak. Using the RooFit package in an identical fashion to the 2016A analysis (see Section 8.2.1), (a) of Figure 9.2 displays the total PDF describing the spectrum. The total number of candidates collected in a SR defined as  $|M_{\pi\mu\mu} - 493.705| \leq 5 \text{ MeV}/c^2$  is,

$$N_{\pi\mu\mu}^{2017} = 2260. \quad (9.2)$$

This is equivalent to 4520 candidates when accounting for the downscale. No-

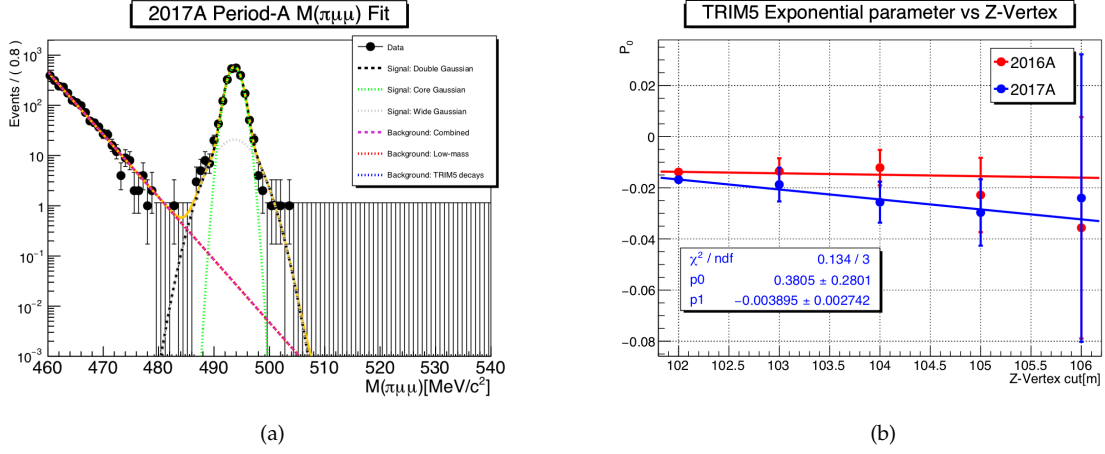


Figure 9.2: The 2017A RooFit PDF in (a) for the  $M_{\pi\mu\mu}$  spectrum and in (b), a comparison of the TRIM5 slope extraction between 2016 and 2017.

tably, the 2017A PDF is missing a high-mass component which was used in 2016 to describe the contribution from upstream decays. This component is predicted in RooFit to be entering at the  $\mathcal{O}(10^{-5})$  level, which is four orders of magnitude smaller compared to the previous year. This is mostly due to the smaller statistics, but also because the slope which is extracted from the data with relaxed  $Z$ -Vertex cuts is steeper. In (b) of Figure 9.2, the slope parameter  $p_0$  of this component versus increasing  $Z$ -Vertex cuts is presented for 2017A, with the 2016A data-points displayed for comparison. The 2017A slope is not consistent with zero, and so RooFit does not find a suitable solution for this component at this level of statistics.

The largest systematic error in 2016 was attributed to the apparent distortion of the kinematic  $z_{\mu\mu}$  variable of six bins between  $0.21 \leq z_{\mu\mu} \leq 0.34$ . Figure 9.3 displays the 2017A  $z_{\mu\mu}$  spectrum including a comparison to the MC expectation. Of the seventeen bins, six are deviated by  $1\sigma$  and one by  $2\sigma$  which is within expectation of statistical fluctuation. Importantly, the deviations are not in neighbouring bins, suggesting that this sample does not suffer from the effect seen in the previous year. However, this must be verified in the future by analysing the full 2017 data set when the reconstructed data is available.

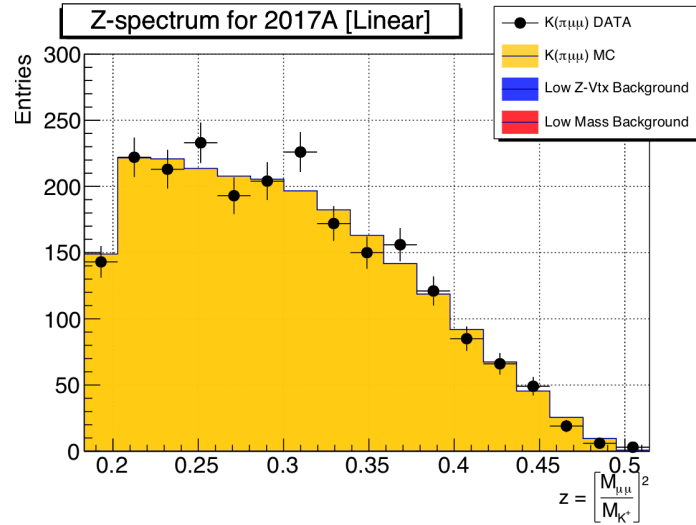


Figure 9.3: The 2017A reconstructed  $z_{\mu\mu}$  spectra compared to MC expectation.

### 9.3 Trigger efficiency

### 9.4 Comparison to 2016

In 2017, the NA62 TDAQ was re-optimised for multi-track decays. The improvements led to the observation of higher trigger efficiencies with respect to 2016. The difference is summarised quantitatively in Table 9.1 for  $K_{3\pi}$  decays, whilst Figure 9.4 displays the 2017A efficiencies with respect to time and instantaneous intensity. The

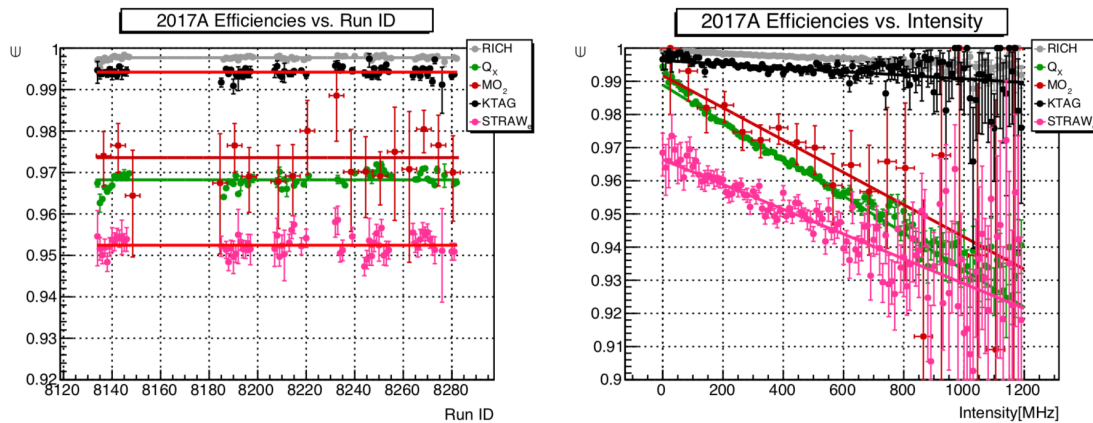


Figure 9.4: The  $L_0$  and  $L_1$  trigger efficiencies in 2017, with left with respect to time and right versus intensity.  $L_0$  are determined using the min-bias control trigger  $K_{3\pi}$  and  $L_1$  using multi-track auto-pass. The  $MO_2$  primitive measurement uses  $K_{3\pi} \rightarrow \pi^+ \mu^+ \mu^-$  as an equivalent to  $K_{\pi\mu\mu}$  signal.

most obvious improvement is the absence of !LAV, increasing the overall efficiency

Table 9.1: Comparing 2016 and 2017  $L_0$  and  $L_1$  efficiencies using  $K_{3\pi}$  decays. The  $MO_2$  primitive uses  $K_{3\pi} \rightarrow \pi^+\mu^+\mu^-$  as an equivalent to  $K_{\pi\mu\mu}$  signal. The  $Q_X$  efficiency has increased by almost 10%, while the !LAV is no longer present. The errors are statistical only; the magnitude reflects the large size of the 2017A sample.

Primitive/Algorithm	2016A	2017A
RICH	$0.9863 \pm 0.0002_{\text{stat}}$	$0.9977 \pm 0.0000_{\text{stat}}$
$Q_X$	$0.8740 \pm 0.0005_{\text{stat}}$	$0.9682 \pm 0.0001_{\text{stat}}$
$MO_2$	$0.9787 \pm 0.0006_{\text{stat}}$	$0.9850 \pm 0.0005_{\text{stat}}$
KTAG	$0.9802 \pm 0.0002_{\text{stat}}$	$0.9942 \pm 0.0002_{\text{stat}}$
!LAV	$0.8056 \pm 0.0005_{\text{stat}}$	-
$STRAW_e$	$0.9587 \pm 0.0003_{\text{stat}}$	$0.9525 \pm 0.0002_{\text{stat}}$

by 20%. The  $Q_X$  improvement is also large, with a 10% improvement. In general, each other trigger component (except  $STRAW_e$ ) is more efficient by  $\approx 1\%$ . Overall, for multi-track  $K_{3\pi}$ , the efficiency  $\epsilon_{3\pi}$  increases from  $\approx 65.9\%$  to  $91.5\%$  from 2016 to 2017.

### 9.4.1 Emulator performance

In terms of the measurement of  $K_{\pi\mu\mu}$ , both the RICH and  $Q_X$  trigger emulators are needed to convert the efficiency from  $K_{3\pi}$  decays. Figure 9.5 displays the difference between the emulated and the online primitive efficiencies for  $K_{3\pi}$  data for both trigger components. The RICH in (a) displays a vast improvement relatively

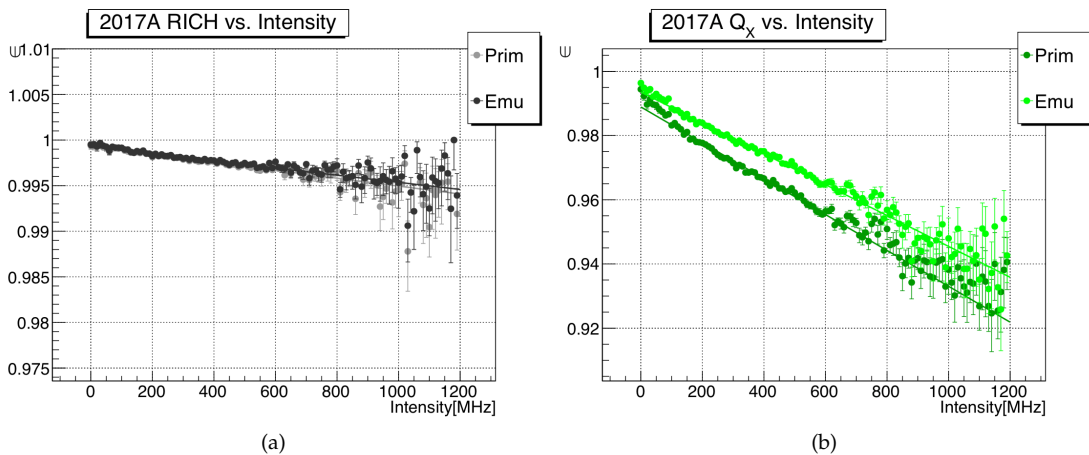


Figure 9.5: Comparing 2017A RICH and  $Q_X$  primitive and emulated efficiencies; (a) is for RICH and (b) for  $Q_X$ , where the latter shows an  $\approx 1\%$  disagreement.

from 2016. As the RICH firmware was more refined in 2017, the intensity distortion that was observed in the previous year is absent, leading to a good agreement be-

tween the online and emulated efficiencies. For  $Q_X$ , the largest geometric-related component of the inefficiency was fixed. However,  $\approx 1\%$  discrepancy is observed between the online and emulated primitives in (b). As the efficiency is no longer dependent on the hit-ordering, the emulator hits cannot be reshuffled as they were in 2016. From other studies, it is understood that the discrepancy is present for any primitive which requires more than one NewCHOD hit [90]. As a result, the **MCA** approach remains the most suitable for the  $K_{\pi\mu\mu}$  determination of  $Q_X$ . Figure 9.6 contains two distributions; (a) contains the validity of the approach, with the  $Q_X$  efficiency plotted versus kinematic  $z_{\pi^+\pi^-}$  for both the **MCA** approach and the online primitives. In (b), is the  $K_{\pi\mu\mu}$  equivalent, with the **MCA** efficiency versus  $z_{\mu\mu}$  which is used directly in the  $d\Gamma/dz$  reconstruction. The kinematic dependence of the on-

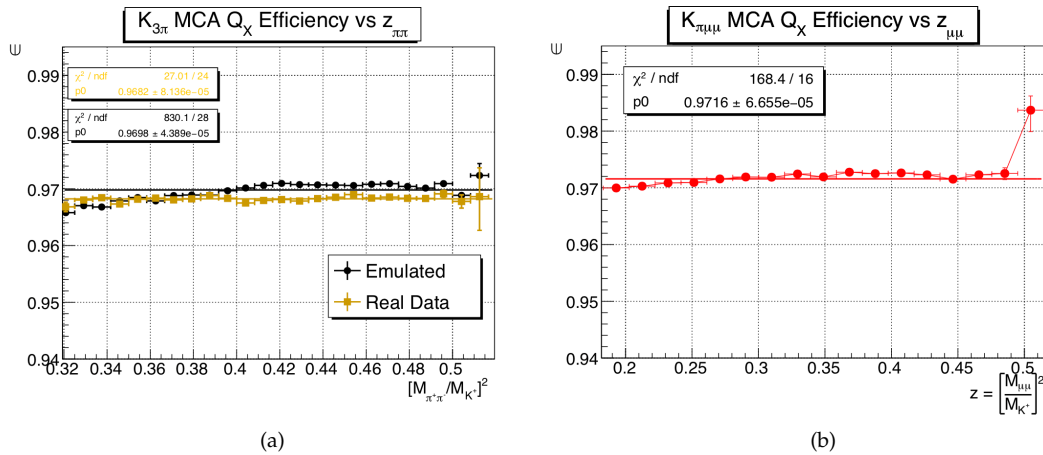


Figure 9.6: Reconstructing the 2017A **MCA**  $Q_X$  efficiency for (a)  $K_{3\pi}$  versus  $z_{\pi^+\pi^-}$  where it can be compared directly to the online primitive distribution, and (b)  $K_{\pi\mu\mu}$  versus  $z_{\mu\mu}$  which is used directly in the analysis. The overall efficiency is  $\approx 10\%$  higher with respect to 2016.

line primitive is flatter than the emulator. Despite the emulator disagreeing by  $\approx 1\%$  when used on the data, the **MCA** efficiency agrees to within 0.1%. As this is similar to 2016, a systematic of 0.5% is adopted. Additionally, the two kinematic distributions are very similar, with the  $K_{\pi\mu\mu}$  efficiency slightly higher on average than for  $K_{3\pi}$ .

In summary, the vast improvement in the RICH agreement and the lack of LAV removes or reduces two of the three largest systematic errors relating to the trigger efficiency for the rare-physics analysis, such that the 2017 data can be seen as the

superior data set from the perspective of the [TDAQ](#). The final 2017  $K_{\pi\mu\mu}$  efficiency is presented in the left side of [Figure 9.7](#) in bins of  $z_{\mu\mu}$ , which is the product the individual components displayed in the right hand side. The scale of the plot is

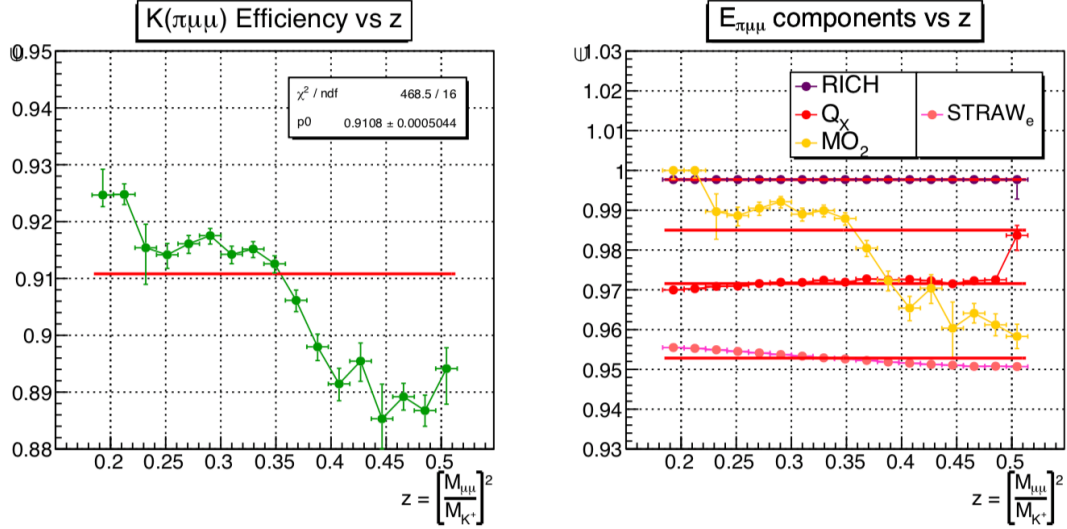


Figure 9.7: The overall  $K_{\pi\mu\mu}$  efficiency spectra vs  $z_{\mu\mu}$  in 2017A. Left, the combined efficiencies and right, the individual components. The scale is dominated by  $\text{STRAW}_e$  and the shape is dominated by the  $\text{MO}_2$  kinematic dependence, the same as observed in 2016.

dominated by the  $\text{STRAW}_e$  measurement, whilst the shape reflects the kinematic dependence of the  $\text{MO}_2$  primitive. The trend of the distribution is strikingly similar to 2016, but the average efficiency is  $\approx 91.1\%$ , which is 24% greater than for the previous year.

## 9.5 Result

Using the exact same methodology described in [Chapter 8](#), the  $d\Gamma/dz$  versus  $z_{\mu\mu}$  spectrum was reconstructed and fitted using the theoretical [PDF](#). [Figure 9.8](#) contains the result of the procedure. Again, the model-independent branching fraction is obtained by integrating the spectrum directly in the absence of the Coulomb corrections and normalising to the  $K^+$  decay-width:

$$\mathcal{B} = (9.637 \pm 0.206_{\text{stat}} \pm 0.074_{\text{syst}} \pm 0.041_{\text{ext}}) \times 10^{-8}, \quad (9.3)$$

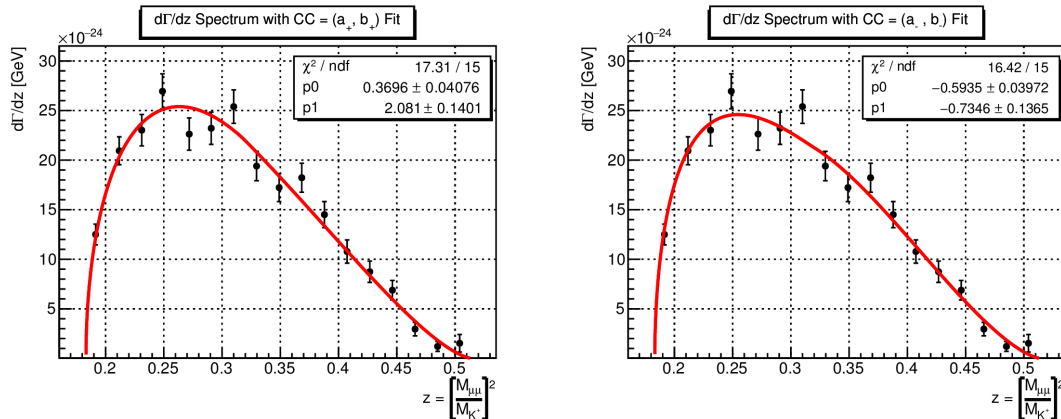


Figure 9.8: The reconstructed  $d\Gamma/dz$  vs.  $z_{\mu\mu}$  spectrum for 2017A. Left, the positive form factor solution and right, the negative form factor solution. The  $\chi^2/\text{NDF}$  is smaller than for 2016A, with the negative solution preferred by  $\Delta\chi^2 = 0.9$ .

which is within  $1\sigma$  of the both the NA48/2 measurement and the global fit [55]. Despite the low level of statistics, this is currently the most precise measurement of the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  decay. The 2017A LECs are extracted with the Coulomb corrected spectra and are contained in Table 9.2. Again, the negative LECs are within

Table 9.2: The LECs in 2017A, included statistical, systematic and external errors. The negative solution is statistically preferred by  $\Delta\chi^2 = 0.9$ .

Sign	LEC	Value	$\chi^2/\text{NDF}$
Positive	$a_+$	$0.370 \pm 0.041_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.001_{\text{ext}}$	17.3/15
	$b_+$	$2.080 \pm 0.140_{\text{stat}} \pm 0.017_{\text{syst}} \pm 0.005_{\text{ext}}$	
Negative	$a_+$	$-0.593 \pm 0.040_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.001_{\text{ext}}$	16.4/15
	$b_+$	$-0.735 \pm 0.137_{\text{stat}} \pm 0.018_{\text{syst}} \pm 0.005_{\text{ext}}$	

$1\sigma$  of the previous measurement. In 2017, the difference between the two solutions is far smaller, with the negative solution being preferred by  $\Delta\chi^2 = 0.9$ . Using the approach outlined in Section 8.2.4 for 2016, the same systematics have been estimated for 2017A to simply compare the effect of the improvements discussed in the above sections. The magnitude of the largest systematics is either drastically reduced or absent with respect to 2016, such that the statistical errors are by far the largest uncertainties in the measurement. Without the flaws in the trigger and the absence of a spectrum distortion, the largest systematic term for both measurements is the  $\text{MO}_2$  efficiency extrapolation which will be improved with larger data-set. Whilst a more careful approach is needed in the 2017A analysis because the considered sys-

tematics are now small with respect to the statistical error, they are at the correct order of magnitude required to compete with  $B$ -physics if a sample of the  $\mathcal{O}(10^5)$   $K_{\pi\mu\mu}$  decays is collected. It is expected that with the full 2017 and 2018 data-sets, the analysis is well-suited to begin to approach these targets.



# 10

## Conclusions

In this thesis, the 2016A data set from the NA62 experiment has been used to measure the branching fraction  $\mathcal{B}$  of the  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  decay and extract the low-energy constants (LECs) ( $a_+, b_+$ ) of the transition form factor  $W(z)$ . In total, 2676 suitable candidates were collected with a negligible background, obtaining:

$$\mathcal{B} = (10.429 \pm 0.203_{\text{stat}} \pm 0.192_{\text{syst}} \pm 0.045_{\text{ext}}) \times 10^{-8}, \quad (10.1)$$

$$a_+ = 0.366 \pm 0.038_{\text{stat}} \pm 0.050_{\text{syst}} \pm 0.001_{\text{ext}} \quad (10.2)$$

$$b_+ = 2.220 \pm 0.131_{\text{stat}} \pm 0.184_{\text{syst}} \pm 0.004_{\text{ext}} \quad (10.3)$$

where the branching fraction is 10% larger than the previous measurements, but consistent to within two standard deviations. The positive LECs are preferred over the negative solutions by  $\Delta\chi^2 = 3.9$ . The error in the branching fraction is dominated in equal measure by limited statistics and systematic errors. The systematics are large mostly notably because of the distorted  $z_{\mu\mu}$ -spectrum and the complicated nature of the trigger inefficiency. One of the major contributions of this thesis is the understanding of the 2016A trigger inefficiencies and how they can be handled in future analyses. In the case of the LECs, the  $z_{\mu\mu}$ -distortion is the largest systematic contribution and dominates instead of the statistical term.

In 2017, a new and improved data set was recorded, where most of the dominant systematic errors in the 2016A analysis were either removed or suppressed. In total,

2260 suitable background-free candidates were collected, obtaining:

$$\mathcal{B} = (9.637 \pm 0.206_{\text{stat}} \pm 0.074_{\text{syst}} \pm 0.041_{\text{ext}}) \times 10^{-8}, \quad (10.4)$$

$$a_+ = -0.593 \pm 0.040_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.001_{\text{ext}} \quad (10.5)$$

$$b_+ = -0.735 \pm 0.137_{\text{stat}} \pm 0.018_{\text{syst}} \pm 0.005_{\text{ext}} \quad (10.6)$$

which is consistent to within a standard deviation of previous measurements. This is currently the most precise measurement of this channel. In 2017, the negative **LECs** are the preferred solution with  $\Delta\chi^2 = 0.9$ , and with a smaller  $\chi^2/\text{NDF}$  overall. Now, the statistical errors are by far the most dominant and the largest systematic effect is from the extrapolation of the  $\text{MO}_2$  efficiency to cover the full phase-space of  $z_{\mu\mu}$ . It is clear from the difference between the 2016 and 2017 results that the measurement is extremely challenging. In order to truly determine the correct sign of the LECs, all of the available statistics must be used. As a data sample a factor of  $\approx 10$  larger is expected from 2017 and 2018, the most precise measurement of both the branching fraction and the **LECs** will be achieved in the near future, with the aim to begin approaching a precision of the  $\mathcal{O}(3)$  better.

To perform precision measurements of processes such as  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  or the ultra-rare  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , high performance tracking is required. Firstly, the MNP33 fringe field map was modelled and tested for its accuracy. As a result, the propagation of the charged tracks in the simulation is to within a high-level of agreement with reality. A computation which calibrates the spectrometer momentum calculation was also developed. By selecting a sample of  $K_{3\pi}$  decays and finding the ideal  $(\alpha, \beta)$  constants to achieve  $M_{3\pi} \approx M_K$ , the computation improves the stability and accuracy of the momentum determination regardless of the data-taking conditions of the experiment. The computation is used in the data-reprocessing, and is performed automatically for every new run at NA62. By introducing these two components, the accuracy of the momentum calculation in both simulation and data has increased, improving the prospects for nearly all future rare  $K^+$  analyses at NA62.

# Bibliography

- [1] ATLAS Collaboration, G. Aad *et al.*, “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” *Phys. Lett.* **B716** (2012) 1–29, [arXiv:1207.7214 \[hep-ex\]](#).
- [2] CMS Collaboration, S. Chatrchyan *et al.*, “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” *Phys. Lett.* **B716** (2012) 30–61, [arXiv:1207.7235 \[hep-ex\]](#).
- [3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, “Evidence for the  $2\pi$  Decay of the  $K_2^0$  Meson,” *Phys. Rev. Lett.* **13** (Jul, 1964) 138–140.  
<https://link.aps.org/doi/10.1103/PhysRevLett.13.138>.
- [4] NA48 Collaboration, A. Lai *et al.*, “A Precise measurement of the direct CP violation parameter  $\text{Re}(\epsilon'/\epsilon)$ ,” *Eur. Phys. J.* **C22** (2001) 231–254, [arXiv:hep-ex/0110019 \[hep-ex\]](#).
- [5] D. Griffiths, *Introduction to elementary particles*. Wiley-VCH, 2012.
- [6] G. D. Rochester and C. C. Butler, “Evidence for the Existence of New Unstable Elementary Particles,” *Nature* **160** (1947) 855–857.
- [7] K. Nishijima, “Charge Independence Theory of V Particles,” *Progress of Theoretical Physics* **13** (Mar., 1955) 285–304.
- [8] M. Gell-Mann, “The interpretation of the new particles as displaced charge multiplets,” *Il Nuovo Cimento* **4** (Apr., 1956) 848–866.
- [9] Z. Xing, “Wolfenstein parametrization reexamined,” *Phys. Rev. D* **51** (Apr, 1995) 3958–3959.  
<https://link.aps.org/doi/10.1103/PhysRevD.51.3958>.

- [10] S. Descotes-Genon and P. Koppenburg, “The CKM Parameters,” *Ann. Rev. Nucl. Part. Sci.* **67** (2017) 97–127, [arXiv:1702.08834](https://arxiv.org/abs/1702.08834) [hep-ex].
- [11] C. Jarlskog, “Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Nonconservation,” *Phys. Rev. Lett.* **55** (Sep, 1985) 1039–1042.  
<https://link.aps.org/doi/10.1103/PhysRevLett.55.1039>.
- [12] **LHCb** Collaboration, B. Aaij *et al.*, “Precision measurement of CP violation in  $B_s^0 \rightarrow J/\psi K^+ K^-$  decays,” *Phys. Rev. Lett.* **114** no. 4, (2015) 041801, [arXiv:1411.3104](https://arxiv.org/abs/1411.3104) [hep-ex].
- [13] **Particle Data Group** Collaboration, C. Patrignani *et al.*, “Review of Particle Physics,” *Chin. Phys.* **C40** no. 10, (2016) 100001.
- [14] J. C. Hardy and I. S. Towner, “Superaligned  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays: 2014 critical survey, with precise results for  $V_{ud}$  and CKM unitarity,” *Phys. Rev.* **C91** no. 2, (2015) 025501, [arXiv:1411.5987](https://arxiv.org/abs/1411.5987) [nucl-ex].
- [15] J. J. Aubert *et al.*, “Experimental observation of a heavy particle  $J$ ,” *Phys. Rev. Lett.* **33** (Dec, 1974) 1404–1406.  
<https://link.aps.org/doi/10.1103/PhysRevLett.33.1404>.
- [16] J. E. Augustin *et al.*, “Discovery of a Narrow Resonance in  $e^+e^-$  Annihilation,” *Phys. Rev. Lett.* **33** (Dec, 1974) 1406–1408.  
<https://link.aps.org/doi/10.1103/PhysRevLett.33.1406>.
- [17] L. Maiani, “The GIM Mechanism: origin, predictions and recent uses,” in *Proceedings, 48th Rencontres de Moriond on Electroweak Interactions and Unified Theories: La Thuile, Italy, March 2-9, 2013*, pp. 3–16. 2013. [arXiv:1303.6154](https://arxiv.org/abs/1303.6154) [hep-ph].
- [18] V. Cirigliano *et al.*, “Kaon Decays in the Standard Model,” *Rev. Mod. Phys.* **84** (2012) 399, [arXiv:1107.6001](https://arxiv.org/abs/1107.6001) [hep-ph].
- [19] G. Buchalla, J. Buras, A. and M. E. Lautenbacher, “Weak decays beyond leading logarithms,” *Rev. Mod. Phys.* **68** (1996) 1125–1144, [arXiv:hep-ph/9512380](https://arxiv.org/abs/hep-ph/9512380) [hep-ph].

- [20] F. Mescia and C. Smith, “Improved estimates of rare K decay matrix-elements from  $K_{\ell 3}$  decays,” *Phys. Rev.* **D76** (2007) 034017, [arXiv:0705.2025](#) [[hep-ph](#)].
- [21] A. J. Buras, D. Buttazzo, J. Girrbach-Noe, and R. Knegjens, “ $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in the Standard Model: status and perspectives,” *JHEP* **11** (2015) 033, [arXiv:1503.02693](#) [[hep-ph](#)].
- [22] **KOTO** Collaboration, K. Shiomi, “Search for the rare decay of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  at J-PARC,” *XXXIX International Conference on High Energy Physics* (2018). <https://indico.cern.ch/event/686555/>.
- [23] **E949** Collaboration, A. V. Artamonov *et al.*, “New measurement of the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching ratio,” *Phys. Rev. Lett.* **101** (2008) 191802, [arXiv:0808.2459](#) [[hep-ex](#)].
- [24] T. Masuda *et al.*, “Long-lived neutral-kaon flux measurement for the KOTO experiment,” *PTEP* **2016** no. 1, (2016) 013C03, [arXiv:1509.03386](#) [[physics.ins-det](#)].
- [25] **E949** Collaboration, S. Adler and *et al.*, “Measurement of the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching ratio,” *Phys. Rev. D* **77** (Mar, 2008) 052003. <https://link.aps.org/doi/10.1103/PhysRevD.77.052003>.
- [26] **NA62** Collaboration, R. Marchevski, “ $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ : first NA62 results,” *53rd Recontres de Moriond* (2018). <https://indico.in2p3.fr/event/16579/>.
- [27] A. Crivellin, G. D’Ambrosio, M. Hoferichter, and L. C. Tunstall, “Violation of lepton flavor and lepton flavor universality in rare kaon decays,” *Phys. Rev.* **D93** no. 7, (2016) 074038, [arXiv:1601.00970](#) [[hep-ph](#)].
- [28] S. Descotes-Genon *et al.*, “Optimizing the basis of  $B \rightarrow K^* \ell^+ \ell^-$  observables in the full kinematic range,” *Journal of High Energy Physics* **2013** no. 5, (May, 2013) 137. [https://doi.org/10.1007/JHEP05\(2013\)137](https://doi.org/10.1007/JHEP05(2013)137).
- [29] W. Altmannshofer and D. M. Straub, “New physics in  $b \rightarrow s$  transitions after LHC run 1,” *Eur. Phys. J.* **C75** no. 8, (2015) 382, [arXiv:1411.3161](#) [[hep-ph](#)].

- [30] S. Descotes-Genon *et al.*, “On the impact of power corrections in the prediction of  $B \rightarrow K^* \mu^+ \mu^-$  observables,” [JHEP 12 \(2014\) 125](#), [arXiv:1407.8526 \[hep-ph\]](#).
- [31] R. R. Horgan *et al.*, “Calculation of  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$  and  $B_s^0 \rightarrow \phi \mu^+ \mu^-$  observables using form factors from lattice QCD,” [Phys. Rev. Lett. 112 \(2014\) 212003](#), [arXiv:1310.3887 \[hep-ph\]](#).
- [32] **LHCb** Collaboration, R. Aaij *et al.*, “Differential branching fraction and angular analysis of the decay  $B_s^0 \rightarrow \phi \mu^+ \mu^-$ ,” [JHEP 07 \(2013\) 084](#), [arXiv:1305.2168 \[hep-ex\]](#).
- [33] **CDF** Collaboration, T. Aaltonen *et al.*, “Measurement of the Forward-Backward Asymmetry in the  $B \rightarrow K^{(*)} \mu^+ \mu^-$  Decay and First Observation of the  $B_s^0 \rightarrow \phi \mu^+ \mu^-$  Decay,” [Phys. Rev. Lett. 106 \(2011\) 161801](#), [arXiv:1101.1028 \[hep-ex\]](#).
- [34] **LHCb** Collaboration, R. Aaij *et al.*, “Test of lepton universality using  $B^+ \rightarrow K^+ \ell^+ \ell^-$  decays,” [Phys. Rev. Lett. 113 \(2014\) 151601](#), [arXiv:1406.6482 \[hep-ex\]](#).
- [35] C. Bobeth *et al.*, “Angular distributions of  $\bar{B} \rightarrow \bar{K} \ell^+ \ell^-$  decays,” [JHEP 12 \(2007\) 040](#), [arXiv:0709.4174 \[hep-ph\]](#).
- [36] **LHCb** Collaboration, R. e. a. Aaij, “Measurement of the ratio of branching fractions  $\mathcal{B}(\bar{b}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{b}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu)$ ,” [Phys. Rev. Lett. 115 \(Sep, 2015\) 111803](#), <https://link.aps.org/doi/10.1103/PhysRevLett.115.111803>.
- [37] **Heavy Flavor Averaging Group (HFAG)** Collaboration, Y. Amhis *et al.*, “Averages of  $b$ -hadron,  $c$ -hadron, and  $\tau$ -lepton properties as of summer 2014,” [arXiv:1412.7515 \[hep-ex\]](#).
- [38] S. Fajfer, J. F. Kamenik, and I. Nisandzic, “On the  $B \rightarrow D^* \tau \bar{\nu}_\tau$  Sensitivity to New Physics,” [Phys. Rev. D85 \(2012\) 094025](#), [arXiv:1203.2654 \[hep-ph\]](#).
- [39] W. Altmannshofer and D. M. Straub, “Implications of  $b \rightarrow s$  measurements,” in *Proceedings, 50th Rencontres de Moriond Electroweak Interactions and Unified*

*Theories: La Thuile, Italy, March 14-21, 2015*, pp. 333–338. 2015.

[arXiv:1503.06199](https://arxiv.org/abs/1503.06199) [hep-ph].

<https://inspirehep.net/record/1353682/files/arXiv:1503.06199.pdf>.

- [40] L. C. Tunstall, A. Crivellin, G. D’Ambrosio, and M. Hoferichter, “Probing lepton flavour (universality) violation at NA62 and future kaon experiments,” *J. Phys. Conf. Ser.* **800** no. 1, (2017) 012014, [arXiv:1611.00495](https://arxiv.org/abs/1611.00495) [hep-ph].
- [41] G. Buchalla, “Kaon and charm physics: Theory,” in *Flavor physics for the millennium. Proceedings, Theoretical Advanced Study Institute in elementary particle physics, TASI 2000, Boulder, USA, June 4-30, 2000*, pp. 143–205. 2001. [arXiv:hep-ph/0103166](https://arxiv.org/abs/hep-ph/0103166) [hep-ph].
- [42] G. D’Ambrosio, G. Ecker, G. Isidori, and J. Portoles, “The Decays  $K \rightarrow \pi \ell^+ \ell^-$  beyond Leading Order in the Chiral Expansion,” *JHEP* **08** no. hep-ph/9808289. FTUV-98-61. INFN-NA-IV-98-25. UWTHPH-1998-45, (Aug, 1998) 004. 18 p. <http://cds.cern.ch/record/362106>.
- [43] G. Isidori, “Soft-photon corrections in multi-body meson decays,” *Eur. Phys. J.* **C53** (2008) 567–571, [arXiv:0709.2439](https://arxiv.org/abs/0709.2439) [hep-ph].
- [44] G. D’Ambrosio, G. Isidori, A. Pugliese, and N. Paver, “Strong rescattering in  $K^+ \rightarrow 3\pi$  decays and low-energy meson dynamics,” *Phys. Rev.* **D50** (1994) 5767, [arXiv:hep-ph/9403235](https://arxiv.org/abs/hep-ph/9403235) [hep-ph].
- [45] G. Ecker, A. Pich, and E. de Rafael, “ $K \rightarrow \pi \ell^+ \ell^-$  decays in the effective chiral lagrangian of the standard model,” *Nuclear Physics B* **291** (1987) 692 – 719. <http://www.sciencedirect.com/science/article/pii/0550321387904913>.
- [46] G. Ecker, J. Kambor, and D. Wyler, “Resonances in the weak chiral lagrangian,” *Nuclear Physics B* **394** no. 1, (1993) 101 – 138. <http://www.sciencedirect.com/science/article/pii/055032139390103V>.

- [47] NA48/2 Collaboration, J. R. Batley *et al.*, “Precise measurement of the  $K^+ \rightarrow \pi^+ e^+ e^-$ ,” *Phys. Lett. B* **677** (2009) 246–254, [arXiv:0903.3130](#) [hep-ex].
- [48] NA48/2 Collaboration, J. R. Batley, “New measurement of the  $K^+$  to  $\pi^+ \mu^+ \mu^-$  decay,” *Physics Letters B* **697** no. 2, (2011) 107 – 115, <http://www.sciencedirect.com/science/article/pii/S0370269311000797>.
- [49] E865 Collaboration, R. Appel *et al.*, “A New measurement of the properties of the rare decay  $K^+ \rightarrow \pi^+ e^+ e^-$ ,” *Phys. Rev. Lett.* **83** (1999) 4482–4485, [arXiv:hep-ex/9907045](#) [hep-ex].
- [50] E865 Collaboration, R. Appel *et al.*, “Search for lepton flavor violation in  $K^+$  decays,” *Phys. Rev. Lett.* **85** (2000) 2877–2880, [arXiv:hep-ex/0006003](#) [hep-ex].
- [51] NA48/2 Collaboration, J. R. Batley *et al.*, “Search for direct CP violation in the decays  $K_{3\pi}$ ,” *Phys. Lett. B* **634** (2006) 474–482, [arXiv:hep-ex/0602014](#) [hep-ex].
- [52] G. Anelli *et al.*, “Proposal to Measure the Rare Decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  at the CERN SPS,” Tech. Rep. CERN-SPSC-2005-013. SPSC-P-326, CERN, Geneva, Apr, 2005. <http://cds.cern.ch/record/832885>.
- [53] NA62 Collaboration, E. Cortina Gil *et al.*, “The Beam and detector of the NA62 experiment at CERN,” *JINST* **12** no. 05, (2017) P05025, [arXiv:1703.08501](#) [physics.ins-det].
- [54] NA62 Collaboration, F. Hahn, “NA62: Technical Design Document,” Tech. Rep. NA62-10-07, CERN, Geneva, Dec, 2010. <https://cds.cern.ch/record/1404985>.
- [55] Particle Data Group Collaboration, K. A. Olive *et al.*, “Review of Particle Physics,” *Chin. Phys.* **C38** (2014) 090001.
- [56] NA62 Collaboration, V. Palladino, “First results from the NA62 STRAW spectrometer,” *PoS EPS-HEP2015* (2015) 250.



- [57] D. Moraes *et al.*, “The CARIOCA Front End Chip for the LHCb muon chambers,” Tech. Rep. LHCb-2003-009, CERN, Geneva, Jan, 2003.  
<http://cds.cern.ch/record/691709>.
- [58] E. Goudzovski *et al.*, “Development of the kaon tagging system for the NA62 experiment at CERN,” *Nucl. Instrum. Meth.* **A801** (2015) 86–94,  
[arXiv:1509.03773](https://arxiv.org/abs/1509.03773) [physics.ins-det].
- [59] NA62 Collaboration, S. Kholodenko, “NA62 Charged Particle Hodoscope. Design and performance in 2016 run,” *JINST* **12** no. 06, (2017) C06042,  
[arXiv:1705.05093](https://arxiv.org/abs/1705.05093) [physics.ins-det].
- [60] B. Angelucci, E. Pedreschi, M. Sozzi, and F. Spinella, “TEL62: an integrated trigger and data acquisition board,” *Journal of Instrumentation* **7** no. 02, (2012) C02046. <http://stacks.iop.org/1748-0221/7/i=02/a=C02046>.
- [61] F. Ambrosino, “The Large Angle Photon Veto System for the NA62 Experiment at CERN,” *Physics Procedia* **37** (2012) 293–300, [arXiv:1111.4075](https://arxiv.org/abs/1111.4075) [physics.ins-det].
- [62] NA62 Collaboration, “2016 NA62 Status Report to the CERN SPSC,” Tech. Rep. CERN-SPSC-2016-016. SPSC-SR-183, CERN, Geneva, Apr, 2016.  
<https://cds.cern.ch/record/2143208>.
- [63] M. Sozzi *et al.*, “The integrated low-level trigger and readout system of the CERN NA62 experiment,” *Submitted to Nuclear Inst. and Methods in Physics Research, A* (2017) .
- [64] J. Pinzino, “Studies of a STRAW Tracker L1 trigger for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and exotic decays,” *Internal NA62 Documentation* (May, 2017) . NA62-17-05.
- [65] S. A. et al., “Geant4 simulation toolkit,” *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **506** no. 3, (2003) 250 – 303. <http://www.sciencedirect.com/science/article/pii/S0168900203013688>.

- [66] R. Brun and F. Rademakers, "ROOT - An Object Orientated Data Analysis Framework," *Nucl. Inst. Meth. in Phys. Res. A* **389** (1997) 81–86.  
<http://root.cern.ch/>.
- [67] NA62 Collaboration, "NA62 Software," <https://na62-sw.web.cern.ch>.
- [68] N. C. for Environmental Information, "International Geomagnetic Reference Field," 2018. <https://www.ngdc.noaa.gov/geomag-web/#igrfwmm>.
- [69] J. R. Fry, G. Ruggiero, and F. Bergsma, "Precision magnetic field mapping for CERN experiment NA62," *Journal of Physics G: Nuclear and Particle Physics* **43** no. 12, (2016) 125004.  
<http://stacks.iop.org/0954-3899/43/i=12/a=125004>.
- [70] J. C. Maxwell, "A dynamical theory of the electromagnetic field," *Phil. Trans. R. Soc. Lond* **155** (1865) 459–512.
- [71] A. Chappuis and E. Goudzovski, "Magnetic field in the decay volume: simulation and tracking tool," *Internal NA62 Documentation* (Jul, 2015) . NA62-15-06.
- [72] J. R. Fry and G. Ruggiero, "The magnetic field of the NA62 dipole magnet," *Internal NA62 Documentation* (Oct, 2014) . NA62-14-09.
- [73] V. Duk, "Fast NA62MC simulation," *Internal NA62 Communication* (2018) . Lepton Flavour WG, NA62 Collaboration Meeting.
- [74] K. Massri, *Kaon identification and Search for Lepton Number Violation in  $K^\pm$  decay-in-flight experiments at CERN*. PhD thesis, University of Birmingham, 2015. <https://cds.cern.ch/record/2299436>.
- [75] NA48 Collaboration, V. Fanti *et al.*, "The beam and detector for the NA48 neutral kaon CP violation experiment at CERN," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **574** no. 3, (2007) 433 – 471. <http://www.sciencedirect.com/science/article/pii/S0168900207002719>.
- [76] B. Bloch-Devaux, "Alpha and Beta correction for NA48/2," *Internal NA48/2 Documentation* (May, 2005) . NA48-05-05.

- [77] R. Skogeby, *Resolution Improvements and Physical Modelling of a Straw Tracker*. PhD thesis, Umeå Universitet, 2017.
- [78] R. Frühwirth, “Application of Kalman filtering to track and vertex fitting,” *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **262** no. 2, (1987) 444 – 450. <http://www.sciencedirect.com/science/article/pii/S0168900287908874>.
- [79] E. Goudzovski, “Lepton Flavour WG: Filtering 06/04/2017,” *Internal NA62 Communication* (April, 2017) . Lepton Flavour WG.
- [80] K. Pearson, “X. on the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling,” *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* **50** no. 302, (1900) 157–175.
- [81] E. Goudzovski, “Lepton flavour WG 07/02/2018,” *Internal NA62 Communication* (Feb, 2018) . Lepton Flavour WG.
- [82] M. Koval, “Straw Spectrometer Status,” *Internal NA62 Communication* (2016) . Straw WG, NA62 Collaboration Meeting Washington.
- [83] M. Koval, “Spectrometer reconstruction efficiency for three track events,” *Internal NA62 Communication* (2018) . Lepton Flavour WG, NA62 Collaboration Meeting.
- [84] R. Lollini, “CHOD Data Quality,” *Internal NA62 Communication* (2018) . Data Quality and Processing WG.
- [85] E. Maurice and J. R. Fry, “Spectrometer efficiency measurement in 2015 minimum bias data,” *Internal NA62 Documentation* (August, 2016) . NA62-16-08.
- [86] C. Parkinson, “Why do  $K_{3\pi}$  events fail L1LAV?,” *Internal NA62 Communication* (2017) . Lepton Flavour WG, NA62 Collaboration Meeting.

- [87] W. Verkerke and D. Kirkby, "The RooFit toolkit for data modeling," *ArXiv Physics e-prints* (June, 2003), [physics/0306116](https://arxiv.org/abs/physics/0306116).
- [88] R. J. Barlow, "Extended maximum likelihood," *Nucl. Instrum. Meth.* **A297** (1990) 496–506.
- [89] G. Lafferty and T. Wyatt, "Where to stick your data points: The treatment of measurements within wide bins," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **355** no. 2, (1995) 541 – 547. <http://www.sciencedirect.com/science/article/pii/0168900294011125>.
- [90] C. Parkinson, A. Sturgess, *et al.*, "Status of the  $\pi\mu\mu$  analysis," *Internal NA62 Communication* (2018) . Lepton Flavour WG, NA62 Collaboration Meeting.

# Appendix A

## $K_{3\pi}, K_{\pi\mu\mu}$ data-MC comparison

The data/MC agreement is presented for  $K_{3\pi}$  (red) and  $K_{\pi\mu\mu}$  (orange) candidates collected with the final selection described in Chapter 6. The comparison is performed for vertex and track related variables.

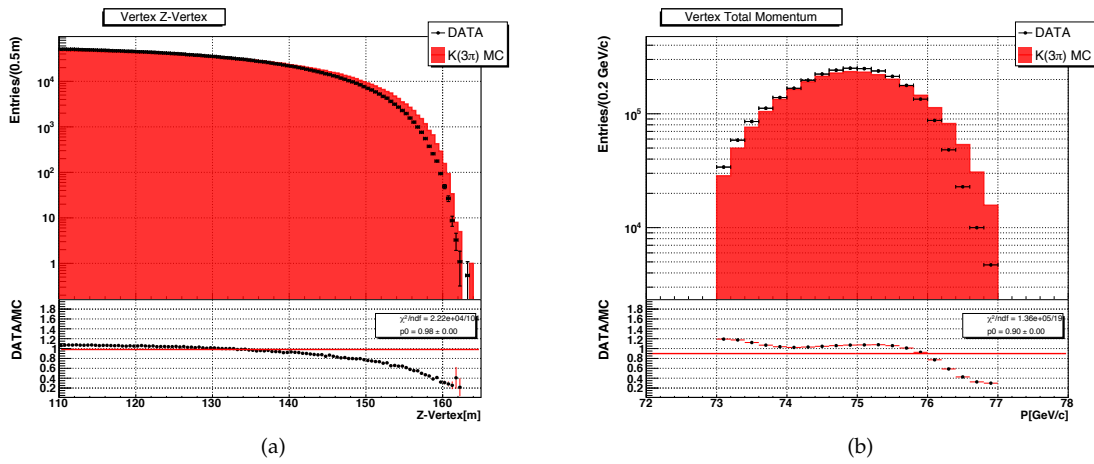


Figure A.1: Distributions of (a) the reconstructed longitudinal vertex position  $Z$ -Vertex and (b) the reconstructed vertex momentum  $P$ -Vertex for  $K_{3\pi}$  signal and MC. Cuts are applied between (110; 165)m for (a), and between (73; 77) GeV/c for (b). The largest discrepancy enters in (b), but is expected to cancel via normalisation with the  $K_{\pi\mu\mu}$  sample.

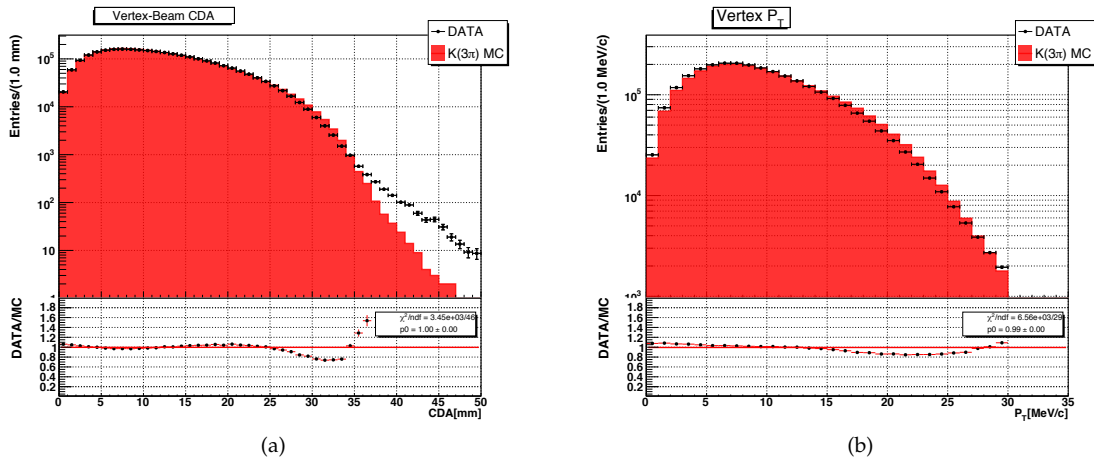


Figure A.2: Distributions of (a) the reconstructed vertex-nominal-beam CDA and (b) the reconstructed vertex transverse momentum  $P_T$  for  $K_{3\pi}$  signal and MC. Cuts are applied as  $<50$  mm for (a), and  $<30$  MeV/c for (b).

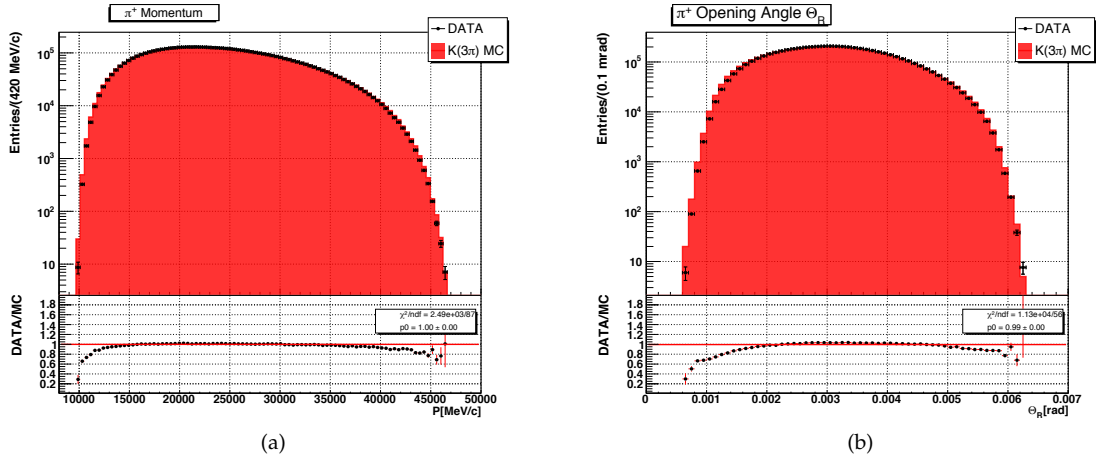


Figure A.3: Distributions of (a) the reconstructed  $\pi^+$  track momentum and (b) the reconstructed  $\pi^+$  opening angle  $\Theta_R$  for  $K_{3\pi}$  signal and MC. Cuts are applied between (8; 48) GeV/c for (a), while  $\Theta_R$  is used by the  $K_{\pi\mu\mu}$  PID only.

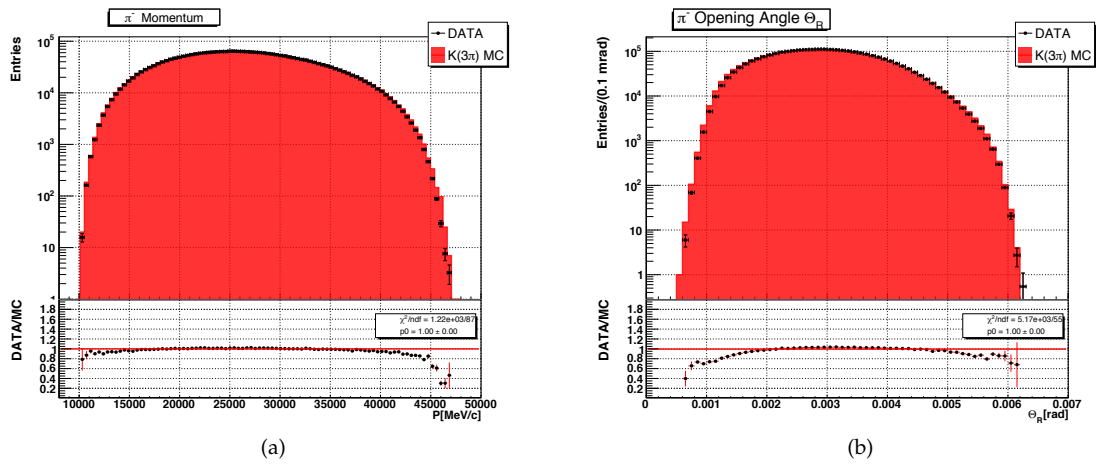


Figure A.4: Distributions of (a) the reconstructed  $\pi^-$  track momentum and (b) the reconstructed  $\pi^-$  opening angle  $\Theta_R$  for  $K_{3\pi}$  signal and MC. Cuts are applied between (8; 48) GeV/c for (a), while  $\Theta_R$  is used by the  $K_{\pi\mu\mu}$  PID only.

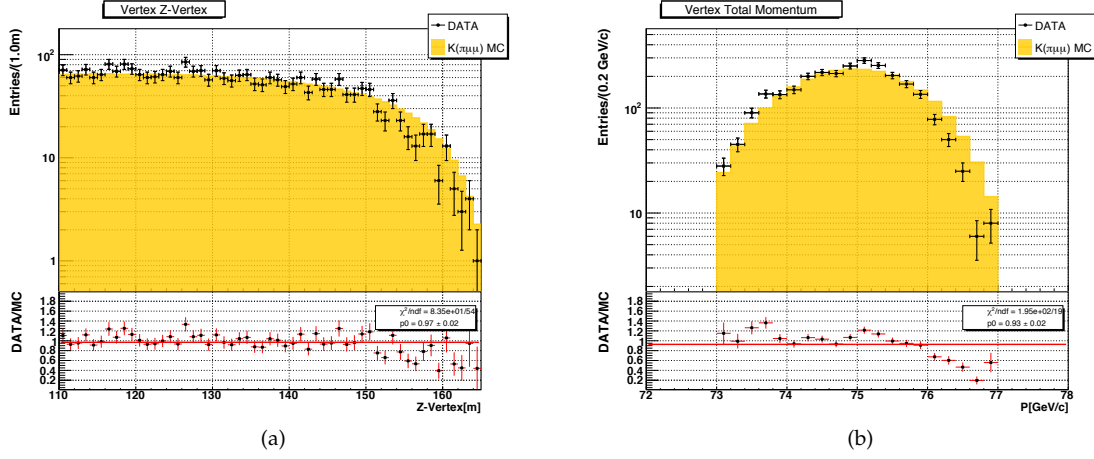


Figure A.5: Distributions of (a) the reconstructed longitudinal vertex position  $Z$ -Vertex and (b) the reconstructed vertex momentum  $P$ -Vertex for  $K_{\pi\mu\mu}$  signal and MC. Cuts are applied between (110; 165)m for (a), and between (73; 77) GeV/c for (b). The largest discrepancy enters in (b), but the shape is very similar to the normalisation sample in Figure [A.1](#)

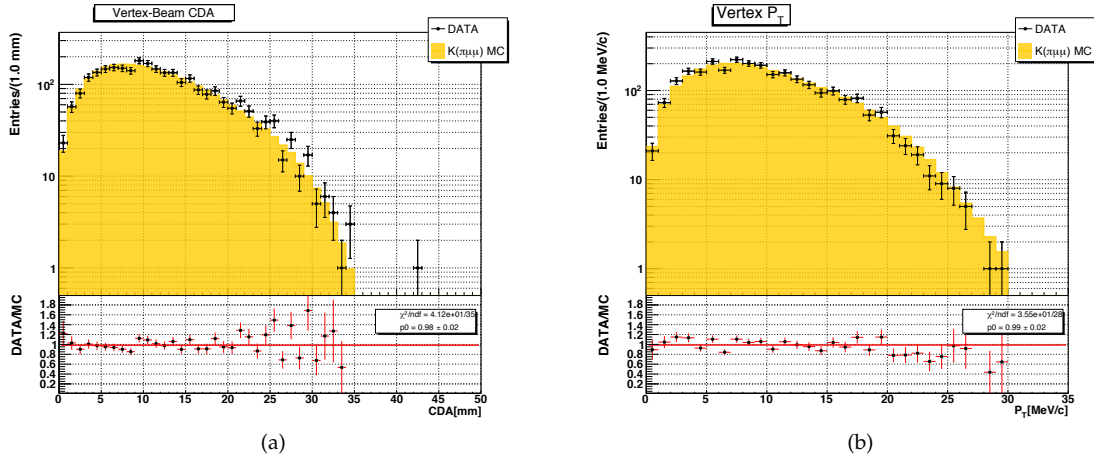


Figure A.6: Distributions of (a) the reconstructed vertex-nominal-beam CDA and (b) the reconstructed vertex transverse momentum  $P_T$  for  $K_{\pi\mu\mu}$  signal and MC. Cuts are applied as  $<50$  mm for (a), and  $<30$  MeV/c for (b).

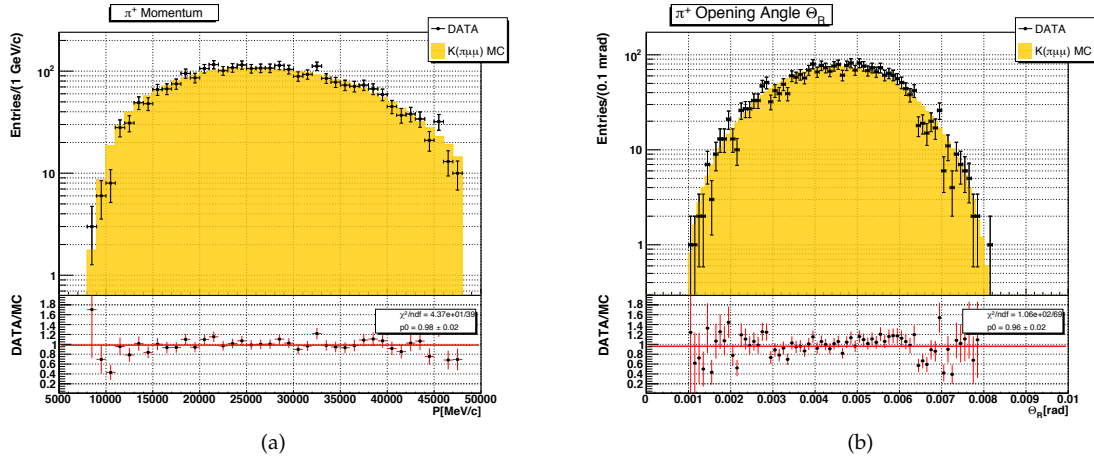


Figure A.7: Distributions of (a) the reconstructed  $\pi^+$  track momentum and (b) the reconstructed  $\pi^+$  opening angle  $\Theta_R$  for  $K_{\pi\mu\mu}$  signal and MC. Cuts are applied due between (8; 48) GeV/c for (a), and <8.5 mrad for (b).

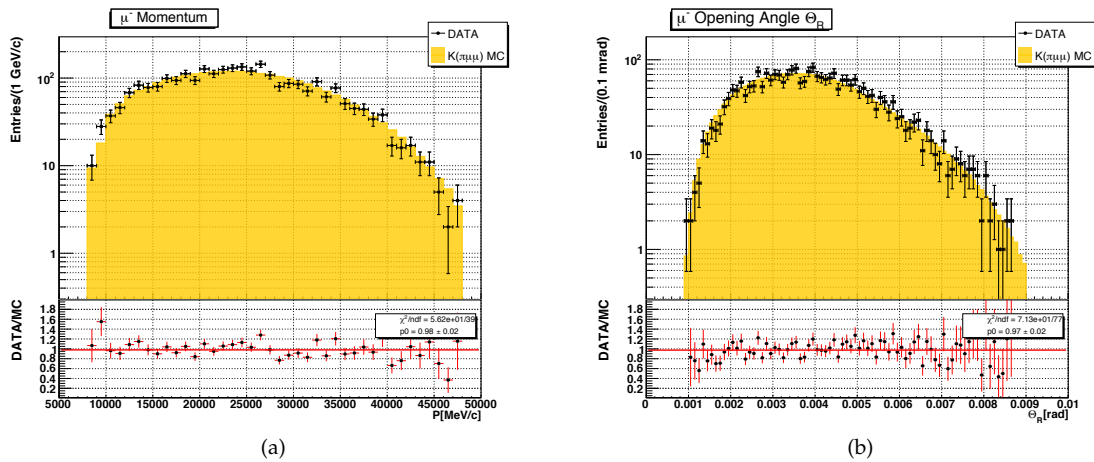


Figure A.8: Distributions of (a) the reconstructed  $\mu^-$  track momentum and (b) the reconstructed  $\mu^-$  opening angle  $\Theta_R$  for  $K_{\pi\mu\mu}$  signal and MC. Cuts are applied between (8; 48) GeV/c for (a), and <9.0 mrad for (b).