School mathematics and university outcomes

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Abstract

There is concern that, as participation of non-traditional entrants widens, many university students do not have the mathematical preparation required to learn skills vital for professional work. The purpose of this paper is to examine the relationship between mathematical attainment at secondary school and the outcomes of university study in quantitative disciplines.

An 'engagement' theory of higher-education study is used to investigate academic performance and progression among students who gained entry on the basis of Scottish Higher examinations to a university that has embraced widening participation. Within this environment there is considerable diversity. For example, although most students were 18 on entry, students were aged from 16 to 38. While pre-entry preparation in mathematics was not extensive, this varied. At the university, assistance with mathematical skills is embedded in programmes and is discipline specific.

Students with better pre-entry attainments in mathematics had better average marks, maintained greater study loads and were more likely to progress. However, non-traditional university students with poorer mathematical backgrounds were able to attain comparable outcomes.

Introduction

Hawkes and Savage (1999, ii), concluded that there is a decline in student 'mastery of basic mathematical skills and levels of preparation for mathematics-based degrees'. Further, 'the decline in skills and the increased variability within intakes are causing acute problems for those teaching mathematics-based modules across the full range of universities' (Hawkes & Savage 1999, iii). This decline has been associated with the drive to widen participation in higher education, which has increased participation among those over 21, those from disadvantaged socio-economic groups and those from postcodes where the proportion in higher education is low (Randall 2005; Houston, Knox & Rimmer 2007).

While those charged with lecturing mathematics-based content are in little doubt about declining mathematical skills among entrants and the need to accommodate this in day-to-day teaching, the overall effect is not so clear. Yorke (2002) studied changes in the proportions of good degrees (firsts and upper seconds) over a five year period in the 1990s. He noted that mathematical sciences was one in which an upward drift was apparent across the university sector. Simonite (2003) associated this with increasingly better grades at school.

Using data on two universities, researchers have formulated and tested an engagement model of higher-education study that links academic performance, study effort and progression (Houston & Rimmer 2005; Houston, et al 2007; Donnelly, McCormack & Rimmer 2007). In this paper the model is applied to entrants admitted in 2000 on the basis of Scottish Higher examinations to the University of the West of Scotland (UWS). Data were available on 276 students who enrolled in first-level programmes where the normal full-time load involved the study of more than four modules with quantitative or scientific elements. These programmes are referred to as 'quantitative programmes'. With this data, the links between school mathematics and university outcomes can be investigated in a population which exceeds benchmarks on widening participation.

Method

The approach is underpinned by the observations that: students choose or decide how much effort to apply to study; in general, grades improve with effort (Szafran 2001); better grades in turn induce increased effort; and greater effort increases the probability of progression (Houston & Rimmer 2005; Houston et al 2007). With UWS data, effort was observed in the form of 'load', the number of modules in which at least one assessment was attempted. The link from load to performance is given by:

$$\log(ave) = \alpha_0 + \alpha_1 load + \mathrm{A}^t X_1, \qquad \text{(Equation 1)}$$

where: *ave* is a performance measure[1]; α_0, α_1 are coefficients to be estimated; and A^t is a column of coefficients for the row X_1 of factors (other than *ave*) that influence university performance. The reverse link is:

$$load = F(\log(ave), X_2), \qquad \text{(Equation 2)}$$

where X_2 consists of variables (other than *ave*) that influence *load*.[2] Because the values of *load* range over the 'truncated counts' 1, 2, 3, …, 8 (see Table 1), the method of estimation involves nonlinear regression (Greene 2003) and so *F* is nonlinear in a constant β_0, the coefficient β_1 of *ave* and the coefficients B^t of other variables. The equations form a simultaneous system involving feedback between performance and load.

Progression is defined as being re-enrolled in the next level of study one month after the commencement of the next academic session, 2001-02. Obviously, some students do not satisfy progression rules. However at UWS, many students who could have progressed chose not to do so (Houston, et al 2007). Progression is related to load as follows:

$$\log(Odds) = \gamma_0 + \gamma_1 \log(load) + \gamma_2 age + \mathrm{H}^t X \,, \qquad \text{(Equation 3)}$$

where $Odds = \dfrac{\Pr(progression)}{1 - \Pr(progression)}$,

$\Pr(progression)$ is the probability of progressing; X denotes a row vector containing all of the variables included in X_1 and X_2 and H^t is a column of coefficients.

The system defined by Equations 1 to 3 contains dynamic elements. First, students enrol and work out a form of engagement with study, which results in their load and academic performance for the session; and second, once this is worked through, students decide whether to progress. Both elements are assumed to be influenced by student- and institutional factors. The data available on these are summarised in Table 1. The main focus in the current paper is school performance, measured by Scottish Highers, which are awarded as letter grades A, B or C. These were converted to numeric scores via the mapping A $\rightarrow$ 3, B $\rightarrow$ 2 and C $\rightarrow$ 1. To obtain an overall Higher score, the best three were summed. This is 'score over best three Highers' in Table 1. Restricting attention to three is consistent with research elsewhere (Houston et al 2007).

'Best three Highers' was not used directly, but two of its components were: one for the Higher 'mathematics score', which was assigned value zero if mathematics had not been attempted; and one for the total over 'non-quantitative Highers', that is, Highers other than quantitative subjects, such as science and IT. On average, the UWS mathematics score was modest (Table 1), due to the 41% who did not attempt Higher mathematics and because most who did were awarded a C grade. Next, ordinary least squares was used to regress the two scores on Higher score less mathematics score. Predicted and residual components were retained for use in estimating Equations 1 to 3. The residuals are interpreted as being associated with particular skills in mathematics or non-quantitative areas that are not associated with general or overall academic ability.

Results

The results of estimations are shown in Tables 2 and 3. In Table 2, the overall Higher score (excluding mathematics) has little correlation with mathematics score (p = 0.307), but is correlated with the non-quantitative score (p = 0.000). That the non-significant coefficient of the overall score was negative in the mathematics estimation can be attributed to this Higher being studied generally in isolation from other quantitative or scientific Highers.[3] Note that gender and age have influences (p = 0.001 and p = 0.086) on the non-quantitative score, while gender affects the mathematics score (p = 0.001).

The results of estimating Equations 1 to 3 are shown in Table 3. The coefficients of load in the performance estimation ($p = 0.000$), of performance in the load estimation ($p = 0.000$) and of load in the progression equation ($p = 0.000$) are different to zero. In estimating the simultaneous system consisting of Equations 1 and 2, each possible combination of predicted and residual components for the two Higher scores was tried. The results shown are for the combination that had the greatest impact on performance, assessed using goodness-of-fit and *t* statistics. There was no support for including combinations of Higher components in estimating load. Residual mathematics and predicted non-quantitative Higher score had effects on performance ($p = 0.000$ and $p = 0.000$). Residual mathematics is associated with progression ($p = 0.013$), but predicted non-quantitative score is not ($p = 0.490$).

Age and gender influence load ($p = 0.003$ and $p = 0.088$), which because of the feedback mechanism provides one means for these characteristics to affect performance. Another avenue is via Higher outcomes (Table 2). Neither age nor being female have significant effects on progression ($p = 0.291$ and $p = 0.249$), which might be suspected is due to colinearity. However, dropping load did not produce significance at conventional levels for either age or gender, while substantially reducing the explanatory power of the model. Being white was not included in the estimations for load and performance (as it had little influence), but it affects progression ($p = 0.007$).

Only one institutional factor is included in the estimations – faculty of enrolment. The students on quantitative programmes were enrolled in either: Paisley Business School (PBS); Communications, Engineering and Science; or Education and Media (E&M). On average, PBS students (who were studying accounting, economics, finance or land economics) incur a penalty relative to CES and E&M students.

Discussion

Features of the regressions are demonstrated in Figure 1 and Table 4. In the figure, graphs are given for Equations 1 and 2 in the simultaneous system for load and performance. The schedules are for white males, aged 18, admitted to quantitative programmes in CES or E&M with a Higher score of six. Two schedules are shown for Equation 1. The dashed one is for students as described, but whose best-three Highers did not include mathematics; the solid schedule is for students whose Higher points included a B in mathematics. The latter is further to the right, as these students have better average marks at any effort level. Only one curve is shown for Equation 2, as Higher scores have no direct influence.

The dynamics implied in Equations 1 and 2 can be demonstrated in the figure. The mechanism is discussed in greater detail in Houston and Rimmer (2007). Consider two students, with the different Higher mathematics attainments, who were working towards loads of six modules. One way this arose at UWS was that students attempted assessments in three modules per semester. Suppose the students have information, such as first-semester results, that suggest they will attain one of the average marks at A. If they revise load on the basis of information on performance to date, then they would increase load to B (as shown by the arrows emanating from A in the figure), because they expect that performance will increase to C. For the student with a B in mathematics among the best-three Highers, load becomes 7.1 and average mark becomes 45.5; for the other student, load increases to 6.9 and average mark becomes 39.0. On average, the student with the better Higher mathematics is simulated to have a passing average; the other student fails at least some modules in the overall load.

At UWS all modules include a coursework component. Hence students may receive feedback frequently and so may revise expected study outcomes frequently. This implies that the dynamic adjustments involve more than two iterations, which converge on the stable attractor at E_1 (Houston & Rimmer 2007). Similarly, a student intending to pursue a load less than four and behaving as assumed above, would follow a trajectory in which average marks and load decline along the dashed arrows away from E_2.

Values at the intersection E_1 of Equations 1 and 2 are shown in Table 4 for a range of students. The values for males with a mathematics B among the best-three Highers (the comparator group in the table) are load = 7.7 and average = 57.4. On the other hand, the values at E_1 for a male without a mathematic Higher among the best three are load = 7.6 and average = 53.7. That is, the two students have similar loads, but averages that differ by 3.7. Together, the first four rows demonstrate that greater attainment in Higher mathematics is associated with better average marks in quantitative programmes (see also Simonite 2003). Further, it is clear that greater attainment in Higher mathematics is associated with increased likelihood of progression. Note that the probability of progression is 0.73 for a student whose best-three Highers did not include mathematics. That is, the probability that such a student would not progress is estimated to exceed one quarter. It is only about one in 11 for the comparator group.

Also shown in Table 4 are the outcomes for students who would be in the comparator group, except that they differ on one pre-entry characteristic. Four things are notable. First, women with the same school mathematics achievement attain higher average marks and are more likely to progress in the study of quantitative programmes. Second, non-white students take the same loads as comparable white students, have about the same average mark, but have substantially lower progression probability. This demonstrates that progression does not depend solely on academic performance (Houston et al 2007). Third, older students attempt greater loads, have better average marks and are more likely to progress than students in the comparator group.

Fourth, students studying quantitative programmes in the Business School are at a disadvantage, even though they have the same Higher grade of B in mathematics. One explanation of this is that assessment standards are more severe in PBS than in other schools. This is consistent with other evidence (Yorke 2002; Houston et al 2007). However, in addition it may be that different teaching and learning cultures pervade university schools, as dealing with deficiencies in mathematical skills are handled differently within each discipline.

At the bottom of Table 4, two rows of outcomes are shown for women with low attainments in Higher mathematics. Recall from Table 2 that, on average, females incurred a penalty in the estimation of mathematics Higher score. This is because women admitted to quantitative programmes were less likely to have studied Higher mathematics and they were less likely to attain grades of A or B. However, women with no Higher mathematics or a grade of C, attain average marks and progression probabilities that are about the same as, or exceed those of, males who had a Higher mathematics grade of B. Thus in the case of females, poorer mathematics preparation has not severely constrained university performance, even though it might be argued that they could have done even better with comparable outcomes for study of Higher mathematics. It is possible that the efforts women exert once at university lead to substantial pay-offs, including overcoming any shortcomings in mathematical background.

External examiners at UWS have not suggested that academic standards are compromised to allow students with poor preparations in mathematics to pass and progress. The reverse is the case, with externals remarking that standards are high. Thus, allowing standards to slip is unlikely to explain the results.

Conclusion

The aim in this paper was to examine the role of school mathematics in university outcomes at an institution that has widened participation, emphasising the loads full-time students choose to study, their average marks in quantitative programmes and whether they progress from first- to second level at the earliest opportunity. The approach involved a model of student engagement and the precedent in earlier research of using best-three school results. This allowed us to conclude that the findings are in line with earlier research. It also provided a dynamic mechanism which set this research into mathematical preparation within the context of student effort.

Within this context, it emerged that non-traditional entrants to quantitative programmes, notably women, can overcome weaker preparations in school mathematics to perform creditably in quantitative programmes. Further, provided engagement with study is strong, in the form of attempting near full loads, students are on pathways to enrolling again next session in the second level of their programmes. A notable exception to this is students who were classed as non-white in the research.

It is hoped that the approach of this paper is applied in other settings to explore the importance of pre-entry mathematics. Clearly, at institutions where school mathematics results are less modest and the incidence of studying other quantitative or science subjects is more widespread than at UWS, the findings may be different.

In the case of older Higher entrants, their experiences, after first leaving school and before doing Highers, may have further equipped them for quantitative programmes. This might go some way to explaining outcomes for those non-traditional UWS entrants who were over 21. Even if the finding of the current research on older students is associated with non-school experience, this does not invalidate the conclusion that many types of entrants to a widening-participation institution can succeed. Moreover, one purpose of widening participation is to provide opportunities to those who traditionally have not attended university. That some ultimately arrive with relevant experience reinforces the notion that alternative entry routes – other than arriving at university immediately after a single episode of schooling – are valid.

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	Average	Standard deviation	Range
Age	18.3	2.5	16 to 38
Load	7.5	1.4	1 to 8
Average mark	53.0	13.5	4.0 to 81.0
Score over best three Highers	4.4	1.6	1 to 9
Non-quantitative Higher score	2.7	1.7	0 to 8
Mathematics score	0.81	0.80	0 to 3
Distribution of mathematics attainments		*Per cent*	
None (Score = 0)		40.9	
C (Score = 1)		38.4	
B (Score = 2)		18.8	
A (Score = 3)		1.8	
Female		37.7	
White		92.8	
School of enrolment			
Paisley Business School		21.4	
Communication, Engineering & Science		77.9	
Education & Media		0.7	
Progressed to next level		75.7	
N		*276*	

Table 1 Summary statistics for students enrolled in first-level quantitative programmes at University of Paisley in 2000/01

	Mathematics Higher score	Non-quantitative Higher score
Score on best three Highers less mathematics Higher	-0.0301 (-1.02)	0.651 (12.00)*
Female	-0.335 (-3.37)*	0.500 (3.33)*
Age	-0.00662 (-0.39)	0.0390 (1.72)†
White	0.0685 (0.34)	0.140 (0.54)
Constant	1.12 (2.99)*	-0.871 (-1.83)†
Adjusted R^2	0.0363	0.461
F	3.59*	59.88*
N	276	276

t-statistics in parentheses. * and † denote significance at one and 10 per cent or better

Table 2 Explaining mathematics- and non-quantitative Higher scores

	Performance	Load	Progression
Load	0.387		1.60
	(2.89)*		(5.44)*
Performance		1.34	
		(6.52)*	
Residual mathematics	0.0437		0.655
Higher score	(3.86)*		(2.47)‡
Predicted non-quantitative	0.0252		0.120
Higher score	(5.95)*		(0.69)
PBS	-0.0737		-0.446
	(-4.33)*		(-1.01)
Female		0.176	0.492
		(1.70)†	(1.15)
Age		0.0847	0.110
		(2.94)*	(1.06)
White			1.55
			(2.72)*
Constant	0.846	-3.37	-14.57
	(2.39)‡	(-3.53)*	(-4.49)*
R^2	0.421	0.592	
System R^2 (McElroy)	0.939		
χ^2			110.1*
Hosmer-Lemeshow χ^2			8.75
McFadden R^2			0.360
Per cent correctly classified			85.9
N	276	276	276

t-statistics in parentheses. *, ‡ and † denote significance at one, five and 10 per cent or better.

Table 3 Estimations for performance, load and progression among students enrolled in quantitative programmes

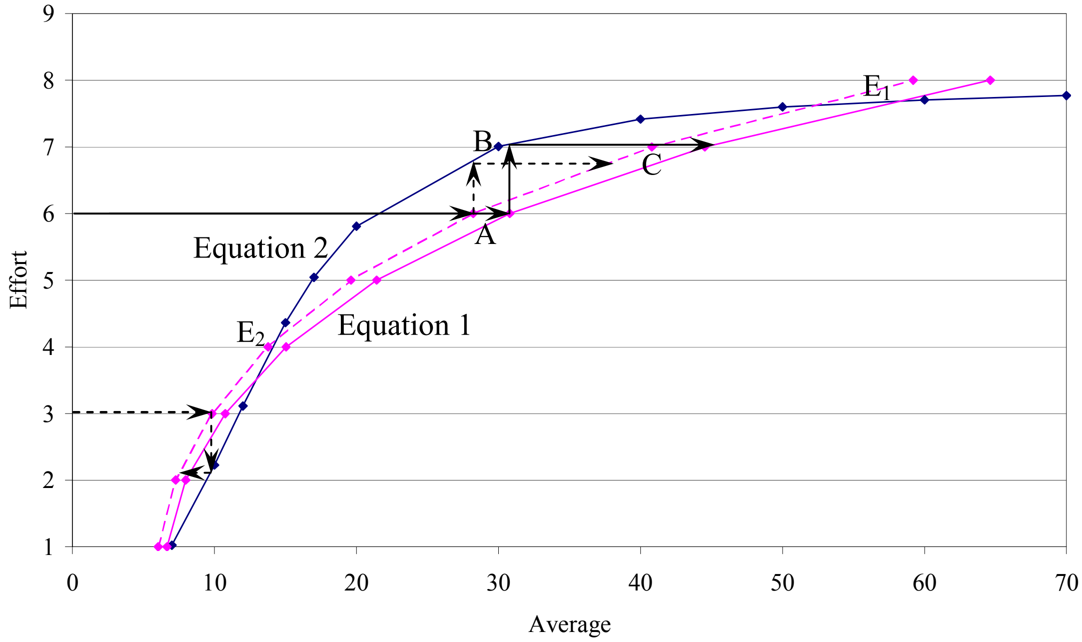

Figure 1 Simulating effort and average mark

	Load	Average	Progression
Comparator group [1]	7.7	57.4	0.91
Comparator group except			
no Higher in mathematics	7.6	53.7	0.73
mathematics Higher grade of C	7.7	55.5	0.85
mathematics Higher grade of A	7.7	59.2	0.94
female	7.8	60.8	0.96
not white	7.7	57.3	0.68
aged 25	7.9	61.8	0.97
enrolled in PBS	7.6	52.3	0.84
White female, aged 18, Higher score = 6 with			
no Higher in mathematics	7.7	57.2	0.91
mathematics Higher grade of C	7.8	59.0	0.93

[1] The comparator group consists of 18 year old, white males, who entered with a Higher score of 6, including a B in mathematics, and who were enrolled in CES or E & M.

Table 4 Load, average mark and progression for groups of first-year entrants

[1] For consistency with Donnelly, et al (2007) and Houston et al (2007), *ave* was taken to be (*total mark* + 1)/(*load* + 1). Thus the relationship between *ave* and the performance measure *average*, the quotient of *total mark* and *load*, is *ave* = (*average* × *load* + 1)/(*load* + 1). *Average* was used in constructing Tables 1 and 4 and Figure 1.

[2] X_1 and X_2 consist of different variables so that nonlinear, three-stage, least-squares can be applied to solve the system of equations (Greene 2003).

[3] In the UWS sample, many students did not have science or other-quantitative (non-mathematics) subjects among their best three, so that predicted and residual components of 'quantitative Higher score' had insignificant influences in estimating university performance, even in the absence of components of mathematics and non-quantitative Higher scores.