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The Dynamic Model of Double Auction Market

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Abstract

Most financial markets operate as double auction markets in which buyers and sellers submit limit and market orders. In this case the traders have to decide firstly whether they want to submit a buy or sell order and then secondly what the limit price of this order is. In this thesis I develop further a theoretical model based on Chatterjee and Samuelson (1983) in which two traders trade with each other in a double auction market. Assuming that both traders assign a private value to the asset they are trading, which is known only to them but not their trading partner, I determine whether the traders should submit a buy or sell order and what the optimal limit price should be. I develop a single-period model in which traders only trade once and thus cannot learn each other's private values from trading as well as a multi-period model that allows to infer to some degree the other trader's private value from their order submission behavior.

Using this theoretical model as a benchmark, I then conducted experiments with students to evaluate whether the actual behavior of students fits the theory developed. Although we find that in general the behavior of traders is consistent with the proposed theory, there are some significant differences. Most notably traders seem to underreact to differences in their own private value, i.e. do not adjust their limit price to the extend suggested by theory. I evaluate these outcomes in light of results established results in behavioral finance.

Dedication

To my mother,
Thank you so much for your love, help and support

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Introduction

1.1 Limit order markets

Limit order markets are one of the most popular markets around the world. Most stock markets are organized either as pure limit order markets, such as Nasdaq and Tokyo or allowing limit order trading, such as NYSE. The prevalence of limit order markets benefits from the development of the electronic trading system. Meanwhile, the growth of ECN limit order markets also takes market share away from deal markets. Limit order markets facilitate the trading of financial products, decrease the transaction costs by increasing the competition among financial firms and increase the transparency of the financial markets by allowing traders to access more trading information (Swan and Westerholm, 2006). In limit order markets, traders are facing the choice to submit market orders or limit orders. Market orders are orders to buy or sell a specified quantity at the best price currently available and limit orders are orders to buy a specified quantity at a price not exceeding some specified maximum, or to sell a specified quantity at a price not less than some specified minimum (Luckock, 2001). Unlike market orders which are executed immediately, limit orders are stored in limit order book at first. And then, the incoming market orders would be matched with the best offer in this book. Limit order markets are different from other trading systems, such as dealer markets and call auction markets, because, in limit order markets, there are no intermediaries to clear markets. The execution of limit orders is subject to the price priority rule: the first, market orders are executed; the second, only after all market orders have been executed, the limit order with the best price (the lowest price for a buy order and the highest price for a sell order) would be executed first, then the second one, the third one... When prices are equal, the execution follows the time priority rule that the orders submitted earlier would be executed prior to the orders submitted later. Traders who submit limit buy (sell) orders with the price lower (higher) than ask (bid) ones would gain less execution costs when these limit orders are filled. Furthermore, limit orders also provide price-contingent executions when traders are unable or unwilling to monitor the market continuously. Therefore, traders could use limit orders to improve the execution price and take advantage of time priority. However, the execution probability of limit orders is uncertain, which depends on market conditions, competition among traders, and so on. In addition, when limit orders are matched with the market orders submitted by informed traders after information announcement, they are picked off, which called the 'winner's curse'. Overall, traders who submit limit orders face the trade-off among the price improvements, the execution probability and the 'winner's curse'.

1.2 The gap between limit order markets and double auction markets

Nowadays, more and more researchers explore the theories of limit order markets. They are interested in the efficiency of limit order markets, the liquidity provision of limit order markets and the submission strategies of traders. In my research, I investigate the optimal dynamic order submission strategies of traders in limit order markets. In limit order markets, automated trade execution systems use continuous double auction mechanism. So, it is possible for us to apply auction theories to research of limit order markets. To investigate the traders' behavior in limit order markets, it is important to understand traders' strategies in double auction markets. In most of the research on limit order markets, the execution probability is treated as an exogenic factor without considering the game-theoretical problems among traders. With auction theory, it is easier to solve the game-theoretical problems faced by traders when they compete with others to obtain profits. It is also convenient to observe how individuals' behavior to affect the formation of transaction price. I hope to develop a dynamic model to investigate the order placement strategies of traders when they comprehend the game-theoretical problem and the price formation process under competition among traders. In most double auction models, traders only act as buyers or sellers in double auction market. In my model, I allow informed traders to choose the orders side according to their own status and market condition. As we all know, in limit order markets, the informed traders place limit orders on either the bid or the ask side depending on their expected profit. To allow traders to choose the order side makes my research closer to the essence of limit order markets. So far, the research on auction theory and the ones on limit order markets were separated before. Considering their advantages and disadvantages, my research bridges between auction markets and limit order markets and throws light on the research of the traders' behavior in limit order markets.

1.3 Research aim

My research aim is to investigate the informed traders' submission strategies in a dynamic double auction market and the efficiency of the market to disseminate private information. This research will focus on the informed traders' submission strategies in double auction markets. In limit order markets, the traders' behavior is affected by various factors including their private information, the public information about others and assets. I will investigate the relationship among these variables. To better understand the traders' behaviors in double auction markets, it is important to clarify the game-theoretical problems among traders during the process of trading. Meanwhile, I would investigate the process of private information disclosure and the change of transaction prices in the continuous trading game. These implications would reveal the process of belief update of traders in the context of game theory.

1.4 The dynamic model of double auction market

The first static model describes the trading game between two players in a simple double auction market with incomplete information. This model is based on the classic bargaining model of Chatterjee and Samuelson (1983). Different from other bargaining models, I allow traders to choose the order side. λ denotes the probability of traders to submit a buy/sell order. With this parameter, I could observe the optimal traders' behavior and the effects of their strategies on the equilibrium price and other variables in the market. The outcome of this model is consistent with previous research. The trader would like to pay more when his reservation price is higher and ask more compensation when he places a sell order. Besides his private information, the information of the other one also affects his submission strategies. As the trader would increase/decrease his offers to match the orders of the other one, the mean value of the other one's reservation value distribution has a positive relationship with the offer prices of the trader. However the variances of the other one's reservation value distribution have different effects on the traders' optimal behavior. The variance has negative effect on the traders' buy offers and positive effect on the trader's sell offers. Considering the traders' valuations follow normal distributions, the higher variance means a fatter tail of normal distribution. The buyer need to increase his offer price to achieve the same probability of execution while the seller could improve his price with same execution probability. I also find that the parameter λ equals 0.5 when every aspect of these two traders are the same. I then extend this simple one period model to a multi-period one. The extension is necessary because the trader's strategy behavior would be different under the condition of multi-trading periods. Under this condition, to draw a clearer picture of the valuation distribution of the other one, they would consider not only the behavior of the other one, but also the information from the previous trading. This multi-period model is also important to the research on the market efficiency of double auction markets. In this market, the information asymmetry should be reduced quickly in the process of trading. In my multi-period model, the variables have similar effects on the traders' submission strategies as ones in the single-period model have. This multi-period model also reveals that the traders could increase the execution probability by updating their information on the other one. During the process of multi-period trading, the execution probability increases to make trade happen and the traders' private information come to be disclosed.

I designed experiments to test the results from my theoretical models. Subjects are asked to trade in pairs as rational informed traders. They are provided private signal on the reservation price and the common value distribution. Although there are some bias between the experimental results and the implication of theoretical models, most of the experimental results are consistent with my theoretical models. The bias can be explained by the heuristic bias of subjects, such as overconfidence, and the irrational behaviors of the subjects. As I selected the students with basic knowledge of financial markets as subjects, they are difficult to act as good as professional rational traders that are assumed in the theoretical model. The experimental results prove the efficiency of my theoretical model and help us to better understand the traders' behavior in double auction markets. It also provides some implications for future research by reducing the gap between my theoretical model and real financial markets.

The remainder of this thesis is organized as follows. Section 2 presents the related literature and theories on double auction markets. Section 3 concerns the theory of limit order markets. Section 4 describes the single-period model and section 5 develops the multi-period model. Section 6 introduce the process and results of the experiments. And conclusions in section 7.

2

Auction markets

Auction is one of trading systems for exchange, in which assets or bids are allocated to the winners by competition. The auction mechanism is important because of its vital role and long-historic influence on many aspects of the economy. Many markets are organized as some forms of auction markets in which goods or service such as antique, treasury bills, operating rights, and etc., are traded. Properties, antiquities and precious art objects are sold under the hammer in auction markets, which are familiar to most of people. Service, rights and licences are also auctioned off to enterprisers. The range of commodities that trade in auction markets is from flowers, tobacco to securities, treasures, and etc. In recent years, with the emergence of e-markets via internet, more and more resource is allocated by auction markets. Because of growing demand to transfer the assets between different sectors, there is tremendous growth of auction markets all over the world. Therefore, the properties of auction mechanism always draw lots of attention.

Auction theory focuses on the people's behavior in auction markets and the properties of auction mechanism. The competition among participants in auction markets raises game-theoretical problems. The application of the game theory in auction markets is one of efficient methods to research auction markets, especially semi-efficient markets with incomplete information. Previous researchers focus on not only investigate the game-theoretical problems of the traders in auction markets, but also try to improve the efficiency of auction markets. The context of the auction theory includes negotiation between buyers and sellers, optimal auctions, price formation, and etc. Furthermore, auction theory, which links to other theories, such as market efficiency, is used as an essential instrumental to solve many economic puzzles. In order to improve market efficiency, much work bridges auction theory and market competitive theory. Some researchers investigate the trading behavior of traders by combining the auction theory and bargain theory (a part of game theory). For either research, the experimental method provides good supports. In recent years, experimental work on auctions is increasing.

There are four different types of basic auction markets: first-price sealed auction, second-price sealed auction, English Auction (open-ascending auction), Dutch Auction (open-descending auction). In these auctions, English auction is the oldest and the most common auction in use. Based on this four basic auctions, more and more new auction forms have been developed in the context of theory or in practice, such as CDA (continuous double auction) and Anglo-Dutch auction. CDA is one of the important forms of auctions applied in financial markets with electronic systems.

2.1 Introduction to auction markets

2.1.1 Main types of auctions

According to the numbers of different types of participants, auction markets are classified as standard bid auction, procurement auction, and double auction. Bid auctions deal under the situation that there is one seller facing many buyers. In procurement auctions, many sellers compete to trade with one buyer. Most research work on auction markets use the perspective of normal auction. It is not necessary to specify that the auction is bid auction since there is not much difference between bid or sell auction. Double auction is the auction market that many buyers face many sellers to trade more than one unit of objects simultaneously. According to the offer submission procedure, double auction could be classified as open and close auction markets. In open auction markets, traders are free to resubmit their offers according to the information of other traders offers. In close auction markets, traders submit sealed offers without informed of the offers of others. There are four types of standard auctions in practice: English auction, Dutch auction, first price sealed auction and second price sealed auction (Krishna, 2002).

The *English auction*, also referred to as ascending auction, is the auction market in which bid price increases from low price called out by an auctioneer until only one bidder accepts the price. It is very common in many traditional markets, such as antique and property markets. It has been using since 1674. In this auction market, the auctioneer is important to start with low price and promote the participants to call out their offers. As the price goes up, bidders quit the competition one by one once the price reaches his own reservation price. The auction ends until there is just one bidder left in the market. The object is allocated to the winner with the highest offer price. When the last bidder with highest signal on the object calls out the price that equals to the reservation price of the one with the second-highest signal, the second person quits and the last bidder gets the object with the second-highest price which is lower than his own reservation price. Therefore, the English auction is also called second-price open auction. Normally, the auctioneer sets 'reserve' price before the start of auction. If the reservation prices of the buyers are all lower than this price, the object would remain unsold. Because of the repeated bidding, the procedure of English auction is longer than other auctions. The English auction is widely used in markets of arts, antiques, real estate, and etc.

The *Dutch auction*, also referred as descending auction, is auction market in which the auctioneer starts with high price, then decreases the price gradually until one bidder accepts the price. The buyer who first calls out to accept the price is the one who successfully get the object in the auction and pay the price when he hits the offer. The procedure of the auction is longer than the first-price sealed auction but shorter than the English auction. The Dutch auction is used in the market of tobacco, wholesale cut flower (Dutch tulip auctions), and some other farm products. The descending auction is also called the first-price open auction because the trading price is also the highest price provided by the bidders.

The *first-price sealed auction* is the auction market in which the bidders are asked to submit their own offers simultaneously without knowing the offers of other participants. The object would be allocated to the bidder with the highest offer price and the transaction price is equal to the bid offer of the winner. The procedure for bidding is very quick because all the traders submit their offer simultaneously and the bids could only be submitted once. The auction for industrial real estate is a good example of first-price sealed auction. Buyers provide their sealed offer according to their own information about the value of the estate. The one who will pay the highest price for the estate will get the asset on his offer price. Other participants get nothing without any payment. This auction is also used to sell securities. Similarly, the auction to buy service is the ask auction where the winner is the one who submits the lowest sell price. In the competition for building construction contract, the monopolist buyer seeks the company that provides the lowest price for their service.

The *second-price sealed auction* refers to the auction markets in which all the bidders submit their sealed offer together, and the object will be sold to the buyer who provides the highest offer. The submitting rules are same as the first-price sealed auction's. The successful bidder only pays the second highest price submitted by the bidders. This auction is also called Vickery auction, which is proposed by Vickrey (1961). In practice, the Vickery auction is rare. There are some auction markets similar to the Vickery auction. One is the auction market for Treasury bills in which the bills are sold at one price, not the highest bid. In the auction markets on the internet such as eBay, the winner just pays the price with an increment on the second price instead of his bid.

There are other types of auctions. All-pay auction asks all the participants to pay their offer at the end of the auction while the object only goes to the one with highest offer price. The model lobbying is an example of this auction. French auction is also known as Walrasian auction, in which the buyers provide their price as well as the quantities that they want to buy. The price is the optimal price that reaches the highest quantity being traded. It is used to price gold, bonds, and etc. There are some hybrids of basic auction or market design with more complex market rules.

2.1.2 Double auction market

Different from standard auctions where one monopolist has absolute power to decide the price and faces multiple bidders on the other side of the market, double auction markets operate standardized goods where both buyers and sellers could submit offers to trade specified quantities at specified prices and also initiate trades by accepting the other side of offers. Furthermore, the continuous double auction market refers to the double auction markets with two-side traders operating continuously to provide the condition for bid or ask matching. The double auction mechanism is used in the majority of stock exchanges, such as New York Stock Exchange and Tokyo Stock exchange. Normally, there is a book or list to store the orders that have been submitted. In this order book, the highest buy offer matches the lowest sell order with the condition that the bid price is higher than sell price. The large volume order will be splitted into small parts to match with the first highest existing opponent offer and then the second best offer, then the third... The orders, which are not executed, are stored in the book to wait for trading until they have fulfilled the trade condition or expire. Double auction market is linked to the bargain theory with incomplete information. The main difference between double auction and standard auction is that buyers and sellers are treated symmetrically in double auction while there is a monopolist in standard auction.

2.2 The main results in single auction markets

2.2.1 *The revenue equilibrium*

Various types of auction markets operate under different circumstance of economy around the world. They bring up different questions for the participates in the markets, which will be solved from their own prospective. The regulars are concerned with how to exert the tax on the monopolists without the information of their cost. They also seek more efficient way to simulate the monopolists by adopting the proper auction mechanisms which allocate the resource more efficiently. The monopolist choose the optimal auction design to extract more revenues, reduce the probability of collusion and attract more bidders. The bidders choose the optimal submission strategies which provide him the maximum surplus. One of the most important theories about the optimal auction is the Revenue Equilibrium. Early in 1961, Vickrey (1961) first addressed some important problems of auction markets including introducing the special case of Revenue Equivalence Theorem. Following his work, some other researches (John, 1989; Myerson Roger, 1981; Riley and Samuelson, 1981)proved that the *Revenue Equivalence Theorem* could be applied in various auction markets generously.

Although the Revenue Equivalence Theorem are constructed based on a series of restricted assumptions, it provides the essential theory for further research on auction markets. The assumptions for the Revenue Equivalence Theorem are:

1. The participants in the auction market are risk-neutral.
2. Every bidder has a private signal independently drawn from a common increasing distribution, which is not convex heavily.
3. Every bidder bids for one unit of items.
4. The bidder with a higher evaluation on the objective will submit a higher offer.

In standard auction markets, one single seller faces N buyers. With the assumption that the bidders are symmetric, all the valuations of bidders come from the same function. Also, with the assumption that higher valuation leads to higher offer, makes that the object would be allocated to the bidder with the highest valuation. It is not difficult to prove that the bidder with a higher valuation has a higher probability of winning. In the Vickery auction, the optimal strategy of the buyer is to reveal his reservation price. The surplus for a bidder from the auction equals the amount of his reservation price times the probability of winning, then minus his expected payment. The surplus for the bidder with the lowest valuation will be zero. Both of these two markets have the same surplus function and the function of probability to win. Then these two markets will provide same revenue to the auctioneer because the expected payment from the bidder is same. Therefore any market design contributes the same revenue to the monopolist. This theory is generously applied in not only standard auctions but also other auction forms.

The *Revenue Equivalence Theorem* also fulfills the private value case or common-value case. These two value types are main value types in auction research. As we know, in auction market, the information is not complete. The sellers do not know the valuation of the buyers. Every buyer has his own private information. The situation for private value is simple. There is no actual value for the object. Each bidder sets his reservation price according to his private information about the actual value. In the common-value case, there is a common sense about the actual value of asset. However, everyone has a private signal for the true value and this signal will change when he considers signals of others to update his evaluation on the object. Although considering the coefficient among individuals' signals, it is more complex to seek the resolution for the common-value model. The *Revenue Equivalence Theorem* still provides some support for the research on common-value model. Furthermore, some research finds that the *Revenue Equivalence Theorem* can be extended to the case of κ unit object - κ is larger than 1.

2.2.2 Optimal auctions on the perspective of margin revenue vs. margin cost

Another important basic theory for auction markets is optimal auctions on the perspective of margin revenues vs. margin cost. There are some analogies between the auction theory and the price theory. The basic economic theory of demand and supply curve presents that the highest revenue for a company is achieved when the margin revenue equals zero. Bulow and Roberts (1989) applied the margin revenue theory in the research on optimal auctions. From the perspective of the demand and supply curve, the auctioneer acts as a company that faces a demand curve for its product. Because the reservation price of bidders is derived independently from an increasing distribution, the margin revenue of the seller to certain bidder actually equals the actual value of the bidder. According to the *Revenue Equivalence Theorem*, in any form of auction markets, the object is allocated to the bidder with the highest signal. Then the monopolist extracts the revenue that exactly equals to the expected margin revenue of the bidder with the highest signal. The margin cost for the seller is his reservation price. Therefore, the monopolist will not sell the object to the bidder whose margin revenue is lower than his margin cost. This theory states the fact that any type of auction markets could achieve the optimal status to get the highest margin revenue by allocating the object to the bidder with the highest signal when the monopolist sets his reservation price properly.

2.2.3 Risk-averse traders

It is an important assumption for the *Revenue Equilibrium Theorem* that bidders and monopolist are risk-neutral. Some research relaxes this assumption and considers the situation of the risk-averse bidders or risk-averse traders in auction markets. First, we address the situation that bidders are risk-averse. Because the small amount of increase on the bidder's offer will increase the probability of winning, which leads to dramatically higher expected profit, the risk-averse bidder acts more aggressively in the first-price auction. Although the aggressive submission behavior will reduce the expected surplus of bidders, bidders could get the higher utility with their high risk-averse parameter in the first-price auction. Therefore, the risk-averse bidders prefer the first-price auction to second-price auction. If the seller is risk-neutral, the risk-averse bidders could provide more revenue for him in the first-price auction than the second-price auction. This result is different from the one of equilibrium revenue. In the papers written by Matthews (1987)and Maskin and Riley (1984), these results about the risk-averse bidders are presented.

The situation that the risk-averse monopolist faces many risk-neutral bidders is different from the one above. Waehrer et al. (1998) address this situation in their research. According to the *Revenue equilibrium theorem*, the seller would receive the same expected revenue because the bidder with the highest signal will bid the price with expected revenue. In the first-price auction, the price that the bidder needs to pay is fixed. And in the second-price auction, the price is random which adds more risk to the seller. Therefore, the risk-aversion seller prefers the first-price auction to the second-price auction.

2.2.4 Asymmetric signals

Some research seek the resolution for the problem in which the private signals are drawn from different distribution. First, we consider the private-value model. Imagine there are two demand curves with same shape but different shift. It is not difficult to see that the margin revenue of the bidder with lower distribution curve is higher than the one of the bidder with higher distribution curve. Therefore, the monopolist prefers to sell the object to the weak bidder rather than the strong one. The situation of optimal auction mechanism will be different from the one of asymmetrical signal. Bulow et al. (1999) address the asymmetric situation in the first-price auction. In the second-price auction, the dominant strategy for the bidder is to tell his true value. While in the first-price auction with symmetrical signal, the bidder from a lower distribution acts more aggressively with lower surplus. The bidder's expected revenue in the first-price auction that the seller could extract is probably higher than the expected revenue in the second-price auction. Therefore, the seller prefers the first-price auction to the second-price auction when he faces asymmetric bidders.

Imagine that the distribution curves are different in the shape but have the same support, the bidder with flatter distribution acts more aggressively in the open market than the sealed market, which contributes more expected revenue to the seller. So the open auction is the optimal mechanism for the monopolist whose trading purpose is to maximize his expected revenue. Maskin and Riley (1985) state the optimal mechanism for the seller when he faces the bidders with different distributions.

The results above are based on the private-value model. In the context of common-value model, the bidder will face the problem of the "winner's curse": if he does not include the bad signal of other bidders and increases his evaluation based on the accumulated information of the good signal, he is likely to pay more on the object in the auction he wins. Klemperer (1998) describes the situation of the buyer with small advantage on the reservation price in the common-value model in ascending auction markets. The buyer prefers to adopt less aggressive strategies to avoid the winer's curse problem. His opponent will also act less aggressively if he thinks that the others will adopt less aggressive strategies. Therefore, in ascending markets, traders will act less aggressively if the valuations have common-value components.

2.3 The main results in double auction markets

2.3.1 κ-double auction

2.3.1.1 Two person bargain model

As a widely used trading mechanism, double auction markets are one of the hottest research topics that has been investigating from various prospectives. Different from the standard auction in which the monopolist has absolute bargain power in double auction markets, both buyers and sellers have the power to decide the trading rule. From the function of κ-double auction: $p = \kappa b + (1 + \kappa)s$, we could see that the trading price is decided by the key factor κ. If the κ equals 0.5, the buyer and seller has equal weight to decide the trading price.

Chatterjee and Samuelson (1983) first introduce the $\kappa-$double auction model about trader behavior in double auction markets. They construct a two-person bargaining model with incomplete information in single stage. Their research bridges the gap between the double auction theory and the bargaining theory. In their model, they introduce the game-theoretical resolution into the double auction model to investigate the optimal behavior of traders. They also find the boundary conditions for traders' behavior in the equilibrium. Their results present that traders have the incentive to conceal their true value in the κdouble auction markets with incomplete information.

This basic model is constructed under some important assumptions: there are just two rational risk-neutral traders standing on different sides of the market. They submit their offer simultaneously to trade one unit of asset. Traders have their own reservation price and assess the possible strategy of the other one according to his information about the distribution of the opponent's reservation price. The opponent is also aware of his adversary's reservation distribution and knows his adversary's assessment on his own reservation distribution. This situation continues and influences the traders' submission strategies. v_i and v_{3-i} are the reservation price of the buyer and the seller on the asset separately. $M_i(v_{3-i})$ denote the reservation distribution for the seller regarded by the buyer. Then we have $F_i(v_{3-i})$ as the offer distribution of the seller induced by the underlying value v_{3-i}. Similarly, the offer distribution for the buyer regarded by the seller is $G_{3-i}(v_i)$ with underlying value v_i. Moreover, the bid/ask offer by buyer/seller is denoted by $b/\ s$. The expected profit for the buyer is affected by the strategies of the seller, this expected profit will be:

$$\pi_i(b, v_i) = \int_{\underline{s}}^{b} (v_i - P) g_i(s) ds \qquad b \geq \underline{s}, \tag{2.1}$$

$$= 0 \qquad b < \underline{s}. \tag{2.2}$$

The buyer could obtain profits only if his bid offer is higher than the ask offer. g_i is the density function of G_i. Similarly, the profit of the seller only occurs under the condition that his sell offer is lower than the highest buy offer. The function for the seller's expected profit is given:

$$\pi_{3-i}(s, v_{3-i}) = \int_{s}^{\bar{b}} (P - v_{3-i}) f_{3-i}(b) db \qquad s \leq \bar{b}, \tag{2.3}$$

$$= 0 \qquad s > \bar{b}. \tag{2.4}$$

In Nash equilibrium, the trader's optimal strategy of responding to the other one is the strategy that the trader could not increase his profit without changing the strategy of the other one. With these two equations, they prove that the optimal submitting offers of the traders increase with the reservation price. In other words, the buyer provides a more generous offer when he has a higher reservation price and the seller asks more compensation with a higher underlying value. They also derive the bounded conditions for strategies of traders when their reservation prices have been changed. When the reservation price of buyer v_i is lower than the optimal strategy offer of the seller with the lowest reservation price $\tilde{S}(\underline{v}_{3-i})$, no trade will happen. With the increase of the reservation price, at $v_i = \tilde{S}(\underline{v}_{3-i})$, the buyer submits the bid that equals to $\tilde{S}(\underline{v}_{3-i})$. When the optimal strategy of the buyer with the highest underlying value $\tilde{B}(\bar{v}_i)$is higher than the optimal strategy of the seller with the highest reservation price $\tilde{S}(\bar{v}_{3-i})$, the probability of the trade is higher than zero but lower than 1. If the buyer adopts the strategy $b = \tilde{S}(\bar{v}_{3-i})$, the trade must happen with probability 1. Therefore, it is impossible that the buyer submits the offer higher than $\tilde{S}(\bar{v}_{3-i})$. $\tilde{S}(\bar{v}_{3-i})$ is the up bounded condition for the buyer's bid offer. Similar situation happens on the offer submission strategies of the seller. The up and below bounded condition is $\min[\tilde{S}(\bar{v}_{3-i}), \tilde{B}(\bar{v}_i)]$ and $\max[\tilde{S}(\underline{v}_{3-i}), \tilde{B}(\underline{v}_i)]$. In this range, the traders' offers increase with the increase of reservation price. Their results reflect optimal strategies of the traders in double auction markets. In this simple two-person double auction market with single stage, the market is not efficient to reveal the true reservation price of assets under the behaviors of the traders to cover their true valuation of assets. It is true that the buyer will provide an offer lower than his reservation price and the seller raises his offer above his reservation price in the situation that either of these two stands symmetric or asymmetric. The parameter κ has great effect on the behavior of both sides and, furthermore, their profit. When κ increases, the buyer has more weight to decide the transaction price, which makes the buyer reduce his offer far more from his reservation price. On the other side, the strategy of the seller is closer to his reservation price. He has the tendency to tell his true value when he stands in a weak position in deciding the price. Their model gives insightful observations in to the game-theoretical problems among participates in double auction markets. However, this model just describes a simple two person market. Some further research is needed to extend it to multi-traders.

2.3.1.2 Multi-trader double auction model

Wilson extends the two-person bargain model to the multi-trader model to investigate market efficiency. As we state above, traders with incomplete information in double auction markets have the incentive to misreport their true value. Therefore, the market mechanism in a two-person double auction market is not efficient in concealing the information to achieve market efficiency. This result is not applicable in the multi-traders double auction markets. Wilson supposes that there are numerous buyers and sellers who have independent private valuation drawn from the identical distribution. Everyone just submits one offer to trade one unit of the asset. Their model prove that the double auction mechanism is incentive efficient if the number of traders is sufficiently large. The double auction markets are independent of the trader's information distribution and its efficiency is caused from its rule rather than the type of participants. Rustichini et al. (1994) investigate the extent of the misreport behavior of traders in double auction markets. They prove that the misreports for a buyer/ seller is small - $O(1/m)/O(1/n)$ and the corresponding inefficiency is even smaller - $O(1/m^2)$ $/O(1/n)$(m is the total number of buyers and n is the total number of sellers). Traders tend to report the true value about the asset with increase of market size. The dominant strategy in double auction markets is to tell the true valuation, which means the double auction market is efficient in revealing the information. McAfee (1992)explains the reason of the efficiency of double auction markets with multi-traders from another aspect. In his model, he supposes that the buyer buys at the next highest value and the seller sells at the next lowest value. This situation is similar to the one in the second-price sealed auction in which the best strategy of the traders is to report their true valuations. The efficiency of double auction markets has been attained.

2.3.1.3 Buyer's bid double auction

As a special case of the κ double auction, the buyer's bid double auction is the double auction market in which the $\kappa = 1$. The transaction price is determined by the offers of buyers, while the sellers have no power to influence the trading price. Therefore, the dominant trading strategy for the seller is to submit the offer on his reservation price. The buyers have the incentive to misreport their true valuation because their strategic behavior may have effects on the transaction price. Williams (1991) gives insight into the strategic behavior of the buyers in the buyer's bid double auction with different numbers of buyers and sellers ($m \neq n$). The main result is that this double auction market will converge to become efficient as the number of buyer increasing. The departure rate of the buyer's offer from his reservation price is $O(1/m)$. The similar function for the buyer is also attained from their model. This rate is very small and the market tends to become efficient when the number of the buyer becomes large.

2.4 Experimental research on Auction theory

Experiments are widely used on the research of auction markets. Experimental research is efficient to identify the people's behavior in the auction market and capture the implication of the auction theory. There is plenty of experimental work on single auction market to investigate various theoretical models, such as the private-value model, common-value model, asymmetric information model etc. Most experimental research on double auction markets focus on the equilibrium formation in the double auction market. Some experiments on double auction markets try to explain the double auction theory from the viewpoint of demand and supply curves. However, the difference between the experiment results and theoretical models is not trivial and leaves many questions for further research.

2.4.1 Experiments on standard auction

On the aspect of private-value model, the experimental results do not follow the *Revenue Equilibrium Theorem.* The experiments of Coppinger and Titus (1980) show that the bid price in first-price auction is higher than Dutch auction. Furthermore the higher bids in second-price auction than English auction is also found by J.H. Kagel and Levin (1987). The difference between the laboratorial results and auction theories is explained by the subjects' behaviors and preconization. As the common-value model is more complex than private-value model, the subjects' performance is worse in the experiments of common-value auction. Evidence shows that bidders in sealed bid common value auctions suffer 'winner's curse' heavily (Dyer, 1989). Because of the effects of 'winners curse', public information reduces revenues in the experiment, which increase the revenues predicted by theory. The 'winner's curse' is found in other experiments on English auction and first-price auction as well.

2.4.2 Experiments on double auction

Difference from the research of standard auction that focuses on auction theory, the experimental research on double auction markets put more efforts to investigate the available double auction models. Cason and Friedman (1993) investigate three double auction markets concerned with price formation: Wilson (1987) Dutch auction model, in which the transactions always happen between the highest buyer and lowest seller; Fridman's double auction market, in which traders ignore the feedback of the other one and follow the Bayesian game to update their order prices; Gode and Sunder (1993) ZI model, in which traders submit their orders randomly. Comparing with the other two models, the model of Wilson's is more close to the experiment results. Kagel and Vogt (1993) investigate the buyer's bid double auction model by experiment. Their results show that the buyers bid more as the ones in standard first-price auction. The efficiency does not increase as fast as the implication of theory with the increasing of the number of traders.

Under most of the situation, the results of the experiments show that the subjects' behavior is better than the ones in ZI model. However, the auction model with rational traders assumption can not explain the implication of the experiments. Although there is no compatible experimental results for my experimental results, the previous experimental research indicate that people's behaviors can partially explain the difference between the theoretical model and experimental results.

2.5 Investor psychology and behavioral finance

The research on auction markets in the extent of auction theory focuses on the traders' behavior from the point of game theory. As a separate branch of economic and financial theories, behavioral finance analyzes the investors' behavior from aspect of human psychology. By investigating the cognitive process or heuristic traits of human, researchers can better understand the behavior of investors or consumers, the price formation and the equilibrium in financial markets. The assumption of rational traders is an important assumption in many financial theoretical models. This assumption implicates that the individual errors are canceled out systematically. In recent years, more and more researchers found that traders are biased systematically . The financial theories on aspects of market efficiency only partially explain some phenomena in financial markets. It is important to introduce behavioral finance to solve some economic puzzles from the point of psychology. In the model of behavioral finance, the subjects are supposed to be irrational and their specific actions are the results of their cognitive bias under certain circumstances, for example, the temporary demand and supply imbalance. Behavioral finance theory is divided into two parts: cognitive psychology and limits to arbitrage. The main part is cognitive psychology, which is concerned with the cognitive process of investors. Hirshleifer (2001) classifies the deriving of the cognitive bias into three types: heuristic simplification, self-deception and emotional loss of control. The heuristic simplification includes the imprecise assessment of information about himself or outside environment, the selective problem on the samples, and etc. The researchers find that people are prone to pay attention to on the salient, concrete, short-term and irrelevant information and ignore the abstract, statistical, long-term, information. People are found to overreact or underreact to certain types of the information, or optimize their expectation. Furthermore, the overconfidence of investors can be reduced by repeating or training, but cannot be eliminated. The influence of the overconfidence is complex. It increases the cost of investors and decreases their expected utility, even decreases the efficiency of the market (Odean, 1998). Some claim positive effects of overconfidence in financial markets (Daniel et al., 1998). Overall, the effects of overconfidence depends on the type of investors and the circumstance they are involved.

In auction markets, traders set up order price according to the information available to them. They need to obtain, analyze and update information before and during the trading. Rational investors in the theoretical model could act correctly to maximize their profit or utility by appropriate assessment of information. Whereas, the irrational investors have heuristic bias and have problem understanding and assessing the available information (Odean, 1998). In my theoretical model, I assume that traders are rational and risk-neutral to simplify my model. Much research finds that many investors are not only risk averse, but they also have cognitive bias such as overconfidence or underconfidence. I design the experiments to research the traders' submission strategies in the laboratory environment following the trading rules of my theoretical model. The subjects are asked to trade under certain trading rules as in the theoretical model. Comparing with rational traders in the theoretical model, these unprofessional students are prone to act with heuristic bias. They have problems understanding the information, are overconfident or underconfident on their judgement or miscalibrate their ability. We could consider that rationality of individuals is limited by their computational ability to analysis the information of the other one and the finite amount of time they have to make decisions. Therefore, it is necessary to introduce the behavioral finance theory into my research framework to explain the difference between the results of experiments and theoretical model and better understand the behavior of investors in the simple double auction market. Furthermore, behavioral finance helps us to construct the link between my theoretical model and the real financial market.

In addition, besides the modeling with rational traders, researchers explain the diverge of the subjects' behaviors by boundary rational subjects and ZI model etc. Boundary rational traders have limited ability to predict the behaviors of their opponents, which is different from the assumption of rational traders. Furthermore, the feelings of traders affect the their decision marking and result difference outcome according to the research of behavior finance. The researches on these topics provide useful theory to understand and analyze the experimental results of my research.

2.5.1 Concept of overconfidence

As a well-documented bias, overconfidence exists widely in people's behavior. Overconfidence refers to people who optimize their expectation or trust their own ability to make a decision. Normally, overconfidence can be miscalibrate, positive illusion, and etc. Miscalibrate describes the distance between the accuracy rate and probability assigned. Under this conception, the overconfident people are bias on accuracy of their knowledge to make the right decision. Miscalibrated people who are overconfident in their judgement, actually have less accuracy rate based on their previous experience. The miscalibrated behaviors in financial markets are observed as underestimating the uncertainty of the circumstance, the bias expectation to the information precision, the optimism in predictions of price variations. Slovic et al. (1977) found that the people' confidence increases when they are asked more difficult questions, which shows they miscalibrate their ability. Most research report that professional people perform better calibration behavior compared with non-professional people. Brenner et al. (2005)also points out that, although experts can improve their resolution, it does not ensure good calibration. However, even with more experienced or repeated tasks, miscalculated behaviors could not be eliminated entirely.

Another kind of overconfidence is positive illusions. Positive illusion includes better-than average effect, unrealistic optimism and illusion of control. Positive illusion also prevails to be used to explain the phenomena in financial markets and economy. The better-than average effect refers to people believe that their abilities are higher than others and their positive view is higher than others'. People are found to assign more responsibility for success and less for failure to themselves (Taylor and Brown, 1988). de Bondt (1998) found that investors are overly optimistic about the performance of their own stocks and ignore the performance of the index because of the aspects of the better-than-average effect. The optimistic bias means that people tend to give more positive views to the present than the past and they also give more positive views to the future.

Actually, most the research work which applies the psychology theory to financial markets does not distinguish the types of overconfidence, while it focuses on the effects of overconfidence in financial markets. In sum, overconfidence widely exists and influences the judgement of people in financial markets and is not eliminated by reputations or increase of experience.

2.5.2 Overconfidence in financial markets

Most researchers find that investors usually are overconfident in financial markets, especially non-professional investors. The overconfidence of investors is useful for interpreting some of phenomena in financial markets. For example, the overconfident traders prefer to sell winners short and hold losers long (Odean, 1998). Also, the December effect can be explained by the behaviors of overconfident traders. The abnormal return of the stocks of small firms is caused by the trading behaviors of the individual investors who are more overconfident than institutions and like to trade small stocks(Hirshleifer, 2001). Overconfidence also affects the variables in financial markets. For example, overconfident traders trade more frequently than the others in financial markets (Barber and Odean, 2001). Overconfident traders increase the market depth, underreact the information from the rational traders, and reduce the efficiency of financial markets. They also increase volatility because of their bias in interpreting information.

Because traders are overconfident on their ability to assess the information, the bias behaviors of traders, who could not assess the strength and weight of information accurately, are regarded as one types of miscalibration. Harris and Raviv (1993) investigate the judgement of the risk-neutral traders on the public signal and claim that the reactions of different types of traders to information are different. Furthermore, Kandel and Pearson (1995) also find that risk-averse traders have different interpretation to the mean and the variance of a public signal. One of types of overconfidence is that the traders are overconfident in the precision of their own information (Kyle and Wang, 1997). In the research of Daniel et al. (1998), the risk-neutral traders overreact to private information and properly weigh public information. The result of these behaviors is that they have positive return. On the other side, overconfident traders undervalue the information of the other one. Overconfident investors underweigh abstract, statistical and highly relevant information and overweigh salient, anecdotal and extreme information (Odean, 1998). They are prone to overreact to the concrete, salient information. For example, the information likely to be underweighed are price change, the recommend of certain analyst, and the issue of IPO; the information likely to be overweighted are the dividends, the issue of SEO and the offer price of other rational traders. Besides the research of the existance of overconfidence, Gervais and California (2001) model the development of overconfidence. They point out the people attribute too much of his success to his own ability, which leads to overconfidence. With the increasing of experience, people reassess themselves and better judgement his own ability.

On my research, the subjects' behaviors in experiments are irrational because of their heuristics bias. My theoretical models are constructed under the assumption that the traders are risk-neutral and rational. However, the students of different departments only have some basic knowledge of financial market. Their submission behaviors are difficult to close to the behaviors of experienced and professional traders, who could minimize their bias actions. I observed the bias behaviors such as overconfidence, conservatism, overreaction, and etc. Therefore, it is important to use the behaviorial finance theory in my research to interpret the diverge of the experimental results from the theoretical ones.

2.5.3 Bounded rationality and related theory in behavior finance

In many financial models, there is an important assumption that traders are rational. However, the empirical research has found that some of the games reach the expected equilibrium while the others do not fit the equilibrium. The term of boundary rational was first mentioned by Herbert Simon. He points out that most of the people are partially rational whose ability to achieve optimize strategy are limited by their own ability or resource. His research presents that because of the cost of information, their limited ability or other complex situation, people prefer to choose the most satisfactory solution other than the optimize solution. Following his research, many research approach to model the boundary rationality. Beauty Contest game is one of the popular method to research the boundary rationality of the subjects. Most of the research (Grosskopf and Nagel (2001),Guth and Sutter (2002), Sbriglia (2004) etc.)find that the average bids are higher than zero - the theoretical equilibrium number, which shows the existence of the boundary rationality. Fairchild (2007) designs a beauty contest game which starts from multi-player and then switch to a two-player game. In his experiment, the rational boundary exists in the multi-player stage and in the two player stage as well. However, he also finds the evidence that the subjects have the ability to learn rapidly. Therefore, in the two player stage, by the final rounds, their bids are close to zero. Camerer Colin et al. (2004) provide a model to observe the depth of reasoning. Their research find that the 1.5-step people' data fits most of the games. They also find that all professional people suffer from biases.

Emotions influence the people's decision making and give different outcome rather than the expectation of theory. For example, the regret and disappointment of traders when they experience a negative outcome is an importance factor to affect their decision making (Loomes and Sugden, 1982). Loewenstein et al. (2001) point out that the emotions that people experience during the process of decision making influence their eventual decision. Their risk-as-feeling model explain the process of the influence of feelings in the decision making of people. Furthermore, the research of Forgas (1995) find that the degree of feelings to influence the decision making depends on how risky, uncertain and abstract the decision is. High affect infusion strategies are applied when the decision is complex. In another word, feeling influence people's decision more when they face complex and unfamiliar situation. Therefore, the subjects in my experiment may suffer the effects of their feelings such as regret, depress etc. , therefore their behaviors diverge from the optimal behaviors in theory.

With the effects of emotions, the boundary rational traders seek the satisfactory decision rather than optimal decision. The optimal decision does not mean the best decision because of the effects of other unmodeling factors. Maybe the choice of boundary rational traders is the better choice to compete with their irrational opponents since they recognize that their opponents are irrational.

There is another feeling that lead to bias behaviors of people: regret aversion. Regret-aversion refers to that people take bias behaviors when they afraid that their decision would be sub-optimal. Zeelenberg et al. (1996) find the evidence from their experiments that subjects prefer to choice the regret-minimizing choice. The results are different from the previous research which claims that the regret-aversion people choose the risk-minimizing decision. Their risk-seeking or risk-aversion depends on the feedback they received. In my experiments, in the single-stage game, traders are asked to trade several times with different parameters. Although each round of trade is supposed to be independent, the subjects who are regret-aversion may adjust their behaviors according to the feedback of previous trading experience.

In my theoretical model, with the assumption of rational traders, traders are anticipated to submit optimal order prices. While, in the experiments, the subjects are expected to be boundary rational because of their limited knowledge of financial markets, their limited ability to calculate the optimal order price, and their concerning for the irrational behavior of the other one. All these factors hold back the subjects to be act as a rational traders. Therefore, the difference between the theoretical values and the experimental data is considered as the result of the boundary traders in the experiments.

2.5.4 Behaviorial finance in the belief updating process

In continuous financial markets, the traders' behaviors to update information is complex. Their behaviors depend on the combined effects of various determinants. From the points of behavioral finance, the bias behaviors, such as conservatism, overconfidence, overreaction, and etc, are possible to be observed the markets. Conservatism causes investors to underreact to short-term information and overreact to long-term information. The overconfident traders in the financial market take bad bets without aware of their information disadvantage and they act more aggressively, which results a high trading volume (Shefrin, 2002). Meanwhile, the costs of information decreases the possibility of investors to pursue new information. The belief updating of investors in the dynamic environment is important to explain the price formation in the multi-period trading and the equilibrium in financial markets. Therefore, my research on the traders' behaviors in the multi-period trading links the cognitive process of the traders to the price formation and market efficiency.

Overconfidence always act an important role in many situations in financial markets. Overconfident traders overweight their own information and underweight their own information of the others. Batchelor and Dua (1992) find that overconfidence forecasters put little weight on the information of other forecasters. Richardson et al. (1999) finds that traders are overoptimistic at long time horizons and pessimistic at short horizons. Cost of the information also influence the process of belief updating simultaneously. The cost of information decreases the activities of investors to obtain necessary information, which in turn influences the decision making of investors. It is proved that people prefer to include the information that is easily accessible and interpreted with less cost to update their decision.

Conservatism in financial markets refers to people undervaluing or are slow to react to new information in markets, or overweigh pervious information to construct their belief on the prices. They are biased on the importance between the new information and the previous information. Edwards (1968) finds that people do not change their beliefs as much as would a rational Bayesian regarding the new information. Griffin and Tversky (1992) claim that conservatism arises when people pay attention to the strength of the information and pay less attention to the weight of the information.

Shefrin (2002) interpret the conservation as a combine effect of overconfidence and anchoring-and-adjustment, which makes the investors are inefficient to analysis the new information.

2.5.5 Framing and anchoring

To understand and interpret the traders' behaviors in the experiments, researches on framing and anchoring provide some useful theories from the points of behavior emotion. Framing is interpreted as the way of people thought is under a series of mental emotional filters that builded by them. Amos and Daniel (1981) said that framing can influence the choice of decision problems. Their research develops the prospect theory, which models the real-life decision, rather than optimal-decision. Anchoring refers to the cognitive bias that in the process of decision making, people rely heavily on one aspect of even or one information to make the choice and adjust their values to it. Because people overestimate one or two factors, their evaluation must diverge from the true value. Either framing or anchoring can develop bias in people's decision making process.

Prospect theory is developed by Amos and Kahneman in 1979. It addresses the question of people's decision when they need evaluate the gains and losses. Comparing with expected utility theory, the prospect theory tries to explain the traders' expectation of profits in term of psychology. The value is assigned to gain and loss and the decision weight substitute the probability (Daniel Kahneman, 1979). Prospect theory can interpret some economic behavior, such as frame, risk-aversion/risk-seeking, disposition effect etc.. The disposition effect describes the phenomena that traders hold the assets with loss and sell the assets with gain. The explanation is that people like to recognize their profit and do not will to recognize their loss.

In sum, people's behavior in finance suffer from various biases, which drives their decision outcomes away from the prospects of theoretical model. The behavior finance theory such as, overconfident, boundary rationality, and regret-aversion etc. can partially explain the results of my experiments. However, my experiments are not designed to test any behavior financial theory. The purpose of the experiments is to test the efficiency of the double auction models. It lefts the possibility to extent my experiments in double auction market to test the behavior bias in the financial market in the future.

3

Limit order markets

The double auction mechanism exists in many different financial markets in practice. One important market that is constructed according to the trading rules of double auction markets is the limit order markets. Limit order markets are developing dramatically with the development of the electronic trading systems. The majority of Exchanges around the world operate as Limit order markets which adopt the continuous double auction mechanism. In limit order markets, normally, traders are allowed to submit limit orders and market orders to buy or sell. Limit order is the order that has an upper limit price for buy order and a lower limit price for sell order, which are specified by traders. Compared with other markets, limit order markets are more efficient and attractive because they increase the probability of decreasing the cost for traders. Meanwhile, they also increase the uncertainty of the profits to traders and put traders into the dilemma as to whether to increase the profit or decrease the execution probability. As one part of microstructure theory, limit order markets have attracted a lot of interesting. Researchers investigate the price formation process, the efficiency and equilibrium, and the investors behaviors in limit order markets. Because limit order markets are more complex than standard auction markets and dealer markets, there are not too many theoretical models to investigate the order submission and order book, compared with the abundant empirical work on this market. Some theoretical work (Harris, 1998; Parlour, 1998; Rosu, 2008)are of note for us to better understand limit order markets.

3.1 Introduction to the limit order markets

As order-driven markets, which are different from price-driven markets, limit order markets are auction markets in which the prices are determined by the orders submitted by the participants. Order-driven markets are organized under certain trading rules, which include oral auctions, single price auctions, continuous electronic auctions and crossing networks. In limit order markets, limit buy/sell orders, which have the specified unilateral bounded prices, play the main role. Limit buy/sell orders cannot be executed with the price higher/lower than limit bid/ask. A limit order book acting as intermediator in limit order markets, listing limit buy/sell orders. Limit orders are stored in the order book on two sides according to their type and listed by their price sequence, and the orders with the same prices arranged by their time sequence. Different from auction markets which clearly has a uniform clear price, limit order markets match the highest buy order with the lowest sell order by pairs with different trading prices. The remaining part of the large orders are executed as market orders or become a limit order according to the trading rules in the different markets. Unexecuted limit orders can be cancelled by the traders or expire automatically at the end of the trading day.

Two principles are followed in limit order markets: price priority and time priority. Price priority means that orders are executed by the price sequence. According to this rule, market orders are executed first, then the limit orders with more aggressive price are executed before the execution of the less aggressive price limit orders. Time priority stipulate that limit orders first coming into the order book are executed before the latter orders, where the price of these orders are the same. According to these two priorities, limit orders provide investors with the probability to decrease the cost of buy or to increase the profit of sell. In particular, the informed traders have the chance to fix their earnings by submitting limit orders. However, the traders in limit order markets face the problem of the improvement of their offer prices and the decreasing of the execution probability of the orders at the same time. The uncertainty of execution arise by introducing limit orders. Market orders are guaranteed to be executed, while that does not happen to even the most aggressive limit order. Limit orders also solve the information disadvantage to limit order traders. Traders arriving in the market later incorporate the new information into their orders, and are obviously in a better position than the traders who submit limit orders early. Therefore, the submitting strategies of the new arriving traders are one of the important factors which affect the execution probability of the limit orders existing on the limit order book. The limit order traders also under the risk of "winner's curse": limit orders are executed when market price move to bad direction for these traders. Therefore, the submission decision is complex for limit order traders. They need consider not only the ex post information such as the status of the limit order books and transaction prices, but also the ex ante status of the order books. They also need monitor limit markets after the limit orders have been submitted and adjust their orders frequently to avoid the loss in the line of new information. Their costs for trading by limit orders are not only the submission fee but also the waiting cost.

Different types of limit markets have different regulation on market rules. On the aspect of order book, there are open book markets in which the information about the book is uncovered to all the traders, such as the Paris Bourse, and close book limit order markets in which the book information is not revealed. Some exchanges permit the existence of the "iceberg" orders which hide part of them from the public information. On the aspect of the participants, pure limit order book markets such as Tokyo Stock Exchange are the markets in which order book is the only intermediary for trading without involving the third part of participants except the buyers and sellers, comparing with some exchanges which are organized as hybrid system such as NYSE. On the aspect of the order type, Some exchanges allow the existence of the limit orders and market orders simultaneously. Some other exchanges treat any orders as limit orders.

3.2 Microstructure model

As an important section of the microstructure theory, the research topics in limit order markets include trader behavior, liquidity provision, price formation etc. One vital function of limit orders is to provide liquidity for markets. Therefore, limit order markets are believed to be one of the most efficient market to allocate resource. Therefore, most research work focus on the liquidity provision and efficiency of limit order markets. The traders' submission strategies which involve the game theory are less investigated. Most work focus on the empirical research for the limit order markets. The problem for theoretical model on limit order market is that the execution of the limit order is decided not only by the current status of limit order books, but also by the ex ante status of limit orders books. The profits of limit orders is a non-linear function (Parlour and Seppi, 2007) .

3.2.1 Static model on limit order book

the static model presents a repeated one period trading game in which the limit orders just exist for one stage on the book. The static model concentrates on the equilibrium of the order book and the liquidity provision of the limit orders. By the static model, we could figure out some characteristics of the limit order market, such as the competition between traders, the break-even condition of the limit order book and the effects of the costs for limit order traders.

Some static models have been constructed to investigate the equilibrium of the limit order book(Cohen et al., 1981; Glosten, 1994; Rock, 1996; Seppi, 1997). In the static model, the order submission strategies of the traders focus on the limit order price. The type of the trader has been predefined by his own property. The market order trader are impatient trader with private information. In the model of Glosten (1994), they are risk-averse informed trader who want to maximize their own utility function. These market orders match the limit orders on the order book. The large market order is executed by several limit orders of different prices. Therefore, the market order trader need decide the quantity of his order according to the existing limit orders on the order book and his own private value. The limit order traders are considered as the patient traders who provide liquidity and are compensated by trading with market order traders. They are risk-neutral traders who need make the decision on the price and quantity of the limit orders.

Limit order traders face risks in the limit order market. The main risk for them is the risk of unexecuted. The risk of non-execution is due to the uncertainty of the execution conditional on the status of the current limit order book and future order submission. The limit order will be executed only if when there is enough quantity of market orders arriving to match the limit orders with the same price priority. The other risk is the adverse section. Information disadvantage for the limit order trader is inevitable with the arriving of new market orders and with the new public information or private information. If the limit orders are picked off by the informed traders, the expected profits of the limit order trader become negative.

For the limit order trader, the transaction price is fixed in advance, which is his offer price. The uncertain part is the expected value of the assets in equilibrium. Their expected profits are also effected by the costs that happen in the process. The limit order traders face two parts of costs: the up-front costs include the decision marking cost , the order submission cost; the ex post order include the cost for monitoring the market and execution cost which relates to the execution probability of limit order. Because of the existing costs, the limit order traders could not make decision continuously. They need choose some decision points to monitor the market. Then limit order arriving rate follows Poisson stochastic process. Cohen et al. (1981) proves that if the market price is generated by the compound Poisson process, there is a jump on the execution probability of limit orders, that is the execution probability never equals one whatever how close the limit price is to the market price. Because of this "*gravitation pull*", there is a small bid-ask spread between two sides of limit order price.

The execution probability of limit order affects the utility function of the limit order trader. Then the optimal order placement strategies of limit buyers who want to maximize his utility function, is to place his limit order on the point between the market ask price and market bid price, on which the optimal limit price to maximize his utility function. Beyond this point, the high price decreases the utility function and is greater than the execution probability increase of the utility function. There is a jump of utility on the market ask price, because the trader could trade by market order with certainty. Below the optimal price, the decrease of execution probability becomes slowly, then anther peak of utility function below the market bid price.

The spread also has an effect on the utility function, that is the wide spread increase the utility function of the trader. When the spread is increasing, the traders prefer the limit order not the market order. While, with the increasing of limit orders, the competition between limit orders marks the spread decrease. Cohen also give the definition of equilibrium market spread: the probability of the bid-ask spread increasing is equal to the probability of the spread decreasing. This explains why the small stocks have larger spread while the big stocks have small spread.

The results from static model prove that the surplus of the limit orders will decrease to zero because of the competition between traders, which decrease the execution probability. The competition in the limit order market is similar to the competition between exchanges, therefor it is not necessary to introduce the third part into the limit order market. This form of competition also explain why the small traders prefer the hybrid limit order market which involves the participant of specialists, while large traders prefer the pure limit order markets. The large stocks have more arrival rates which increase the execution probability of the limit orders. Then more limit orders come to and participate in the competition, and in the process of a small equilibrium spread forms in the trading of large stocks.

In these static modelsGlosten (1994); Rock (1996); Seppi (1997), one important assumption is that informed traders with private information have incentive to use aggressive orders or market order to realize their profits. These informed traders are impatient to obtain profit by trading according to their private information. They think that the value of their private information is short-lived. Compared to the informed trader, the uninformed trader (liquidity provider) are patient to use limit orders to reduce their cost and obtain profit by providing liquidity. However, the evidence of using limit orders by informed traders has been found. Kaniel and Liu (2006) show that limit orders are more informative. Similar results have been found by Cao et al. (2004) on the Australian Stock Exchange. Results from the experimental market constructed by Bloomfield et al. (2005) show that informed traders use more limit orders than do liquidity traders. When the private information is low and the market price move to the true value of the asset, the informed trader act as the dealer, who provide liquidity to the market by limit orders. They obtain profit by carrying less risk of being picked off. In other words, informed traders use limit orders better than uninformed traders. By analyzing data from NYSE, Anand et al. (2005) found the similar results in that the informed trader prefers to use market orders early in the trading day, while using more limit orders in the latter half day. They are better at using limit orders to trade than uniformed traders. In particular, Beber and Caglio (2005) find the evidence of the strategic behavior of the informed trader. They do not submit market orders to gain profits according to their private information immediately when they arrive at the market, but submit less aggressive limit orders on the other side of the limit order book to hide their information. All of these empirical evidence prove that the informed traders include limit orders in their order submission strategies to pursue their profits.

There are some limitation in using the static model to explain the continuous dynamic limit order market. First, it is impossible to observe order flow changes on the limit order book under the static model. Second, the assumption of the limit order trader as the liquidity provider and the market order trader as the informed trader prevent the decision of the trader on the order type choice. The optimal strategies for the trader should include the decision on the type of the order and then the price. However, the static models still provide some insights on the equilibrium of the order book and the adverse selection problems in the trading process.

3.2.2 Dynamic model for submission strategy

As we state above, the static model cannot draw the patterns of the order flow on the order book and cannot present the optimal strategies of traders when they include the changes of the order book in the future into their consideration. The dynamic model for the limit order market is a better system to capture these characteristics. One important characteristic of limit order market is that traders enter into the market asynchronously, which is different from the one of auction market in which all the traders exist in the market simultaneously. Then changes of the order flow can only be observed on the multi-stage. The systematic price patterns has been observed by some empirical work(Biais et al., 1995). Third, the dynamic model provides insights on the optimal order choice of the traders in the limit order market. The traders' decision has been effected by the future state of the order book and strategies of following traders. The dynamic model is a platform to investigate the optimal order choice of the traders conditional on the past, current and future state of the order book. They also extend the space of order decision, because at some decision point, the traders needs to decide whether or not to cancel the old limit order, or submit a new limit order.

Parlour (1998) construct a multi-stage model with multi-traders in the limit order market . The trader's consumption has two parts, consumption on day1 and consumption on day2. β is the trader's trade-off on his consumption between two days. Therefore, β is also the parameter for the trader's patience. When β is far from 1 that means the trader has the incentive to trade. When β is close to 1 that means the traders are patient to trade. Traders arrive in the market randomly. He knows that his order will affect the submission strategies of the following trader who inversely affect the execution probability of the existing orders on the order book. This assumption makes the execution probability of the order an endogenous fact in the submission strategies. His decision is also affected by the current status of both sides of the order book. The trader needs to choose to submit market buy/sell order or limit buy/sell order price. The limit order price is not continuous in this model. The price of limit buy order is equal to the market bid price and the price of limit sell order is equal to the market ask price. The market order is executed at once, while the limit order is executed with uncertainty. The execution of the limit order depends on the following arriving market orders. Only when there is enough market orders arriving to pick off the limit orders beyond his limit order before the end of the trading day, the limit order submitted by the trader could be executed with next market order. Different β type trader has different choice on the order type. The low β type trader tend to submit market sell order. The mid value of β trader prefer to submit limit order. They have no need to trade immediately, therefore they could use limit order to improve transaction price conditional on other waiting cost. The high β type trader will submit a market buy order because of their high valuation on the asset and the urgency of trade.

The limit order book affects the order submission strategies of the traders by the execution probability. The execution probability has positive effect on the trader's order choice. The higher the execution probability, the more trader types to choose limit order but not market order. Both sides of the limit order book have effect on the execution probability. On the same side of the book, the competition between the limit orders decrease the execution probability. The current limit orders with time priority and the following limit orders with price priority have higher execution probability. If the other side of book is thin, the trader prefers limit order. The arrival of market order will decrease, which in turn decreases the execution probability of limit order.

Empirical research provide sufficient support on the relation between the order book and the execution probability. Omura et al. (2000) find evidence from Tokyo Stock Exchange that the depth of the same side of the order book has negative effect on the order execution probability, while the depth of the other side of the book has positive effect on the execution probability. He also finds that when the spread is large, the execution probability of limit orders is small, but this does not mean that the large spread will attract more market orders. Although the large spread decrease the execution probability, however market orders become more expensive because of the increasing cost. Therefore, it is optimal to the traders to submit limit orders when the spread becomes large.

Further more, some empirical researches provide evidence on the relationship between the order book and order position. Ranaldo (2004) find that the order book depth on the same side increases the aggressiveness of the limit order and the depth on the other side of the book decreases the aggressiveness of the limit order. However the results of Beber and Caglio (2005) is a little bit different in this aspect. They find that traders submit more aggressive limit orders when the depth on the same side of the book is thick, while , the depth on the other side of the book has different effect on the limit orders. Buyers act less aggressively when the depth on the other side of the book is large. Sellers act more aggressively when they face a large depth on the other side of the book. Keim and Madhavan (1995) also get similar results on the different attitudes of the buyers and sellers. The possible explanation is that sellers are more impatient to stop their loss when the market price goes down and prefer to submit more aggressive limit sell order to reduce the cost of consuming liquidity under the condition that there is serious competition between limit buy orders. When address the question whether market depth conveys information for the traders with which they can use in their order submission strategies, some researchers announce that the limit order book is informative about the future order flow (Cao et al., 2004; Harris and Panchapagesan, 2005). Boehmer et al. (2005) show that there is price improvement due to information efficiency in the trading process. In particular, they find that open limit order book can increase the efficiency of the limit order submission strategies of the trader and increase the liquidity of the limit order market. However, Coppejans and Domowitz (1999) claim that information from the order book cannot explain the future order flow.

If the trader includes the current book status and the following status of the book in his decision process, there are some patterns of the order flow observed by this dynamic model. For example, the probability of observing a limit sell order at time $t + 1$ conditional on the transaction on a market buy order is larger than the the probability of observing a limit sell order at time $t + 1$ conditional on the transaction on other type of orders. On time t, the arriving of market buy order matches the limit sell order at ask, which decreases the queue of the limit sell order. The thin limit order book on the sell side attract more limit sell order than market orders. If the transaction happens on the other side of book, not only the trader will face more competing limit sell orders with time priority, but also the longer queue of the sell orders lures the following buyers to submit limit orders not market orders, while the limit sell order will be executed under the condition that there are more market orders coming to match the existing and following limit orders. In a word, the transaction follows the same side of transaction while the order is not followed by the same type of order. These predictions on the order flow pattern in the limit order market is very useful in reality to capture the abnormal order flows on the market. For example, the large orders have been split into small orders to decrease its effect on the market price. We will observe large series sequential orders on the same side of the order book. This is different from the results of the model that the probability of orders that followed by the same type of the orders is smaller than followed by the other side of the orders. The evidence for the correlations between the orders in the limit order market from New York Stock Exchange has been provided by Ellul et al. (2003).

There are some factors affecting the liquidity provision of limit orders. The reduced tick size decreases the liquidity provision of the limit orders (Declerck, 2000; Goldstein and Kavajecz, 2000; Jones and Lipson, 2001). The reduced tick decreases the spread of the market, then attract more traders to consume the liquidity, not providing the liquidity, then the liquidity provision has been reduced.

Another dynamic model to investigate the trader's optimal order choice behavior and to address the game theoretical problems faced by the trader is constructed by Foucault (1999). Comparing with the work of Parlour who concentrates on the choice of order type, this model focuses on the price quotation of the limit order traders in equilibrium. The traders arrive in the market randomly, submit long-lived order which could not been cancelled once submitted. Every trader just submit one unit of order. One restricted assumption is that only price improvement limit order can be submitted. This assumption turns the price priority problem into a problem of spread priority. The research is to investigates the price formation process and order placement strategies of the traders. They find that volatility plays a main role in the traders' order submission decision. When price volatility is high, the trader faces a high risk of being picked off, asking for more compensation. Therefore, the spread in the market is larger than the one with small volatility. The high spread increase the cost for the market order trader, then limit order become more attractive. When there are more limit order on the limit order book, the execution probability of limit orders decreases.There are similar results from the empirical research. In this three-stage model, the price quotation problem has been analyzed. Furthermore the risk of execution and the risk of being picked off have been included in the model. However, it cannot provide a clear picture of dynamic order flows in the limit order market.

Price volatility is relative to the execution probability and the expected profit of the traders. The limit order trader can get profit from the trade a temporary price fluctuation due to the imbalance of the order book, while lose from trading with informed trade who cause the perpetual price change. Handa and Schwartz (1996) claim that the trader will choose limit order when their profit from the liquidity provision can compensate the loss of being picked off by the informed trader. Therefore, when the price volatility is high, the profit of submitting limit orders as liquidity provision is higher than the loss by being picked off. Their explanation is different from Foucault's but gives the same results that price volatility attracts more limit orders. Not only empirical work finds evidence that high volatility increase the probability of limit order submissions, moreover, the trader will submit less aggressive orders under this condition. Beber and Caglio (2005) find that the increase in volatility drives less aggressive limit orders for the reason that limit order traders ask for more compensation for the risk of being picked off by the informed traders. Zovko and Farmer (2002) find that the limit order trader prefer to place a more aggressive limit order when the volatility of the asset price is low. Ranaldo (2004) also find that the high temporary volatility increases the probability of limit order submission and causing the limit order to be place more aggressively.

Ahn et al. (2001) tell the difference between the volatility rising from the different sides of the book and their effect on both sides of the orders. Their research shows that if the limit orders on both sides are calculated together, there is evidence on the relationship between the volatility and the order flow. They find that the increasing volatility on the ask(bid) attract more limit sell(buy) orders than market orders. They also observe the fact that market depth rises subsequent to an increase in transaction volatility, and the high volatility decreases the market depth. Their results and explanation coincide with the research of Handa and Schwartz (1996).

Foucault et al. (2005) construct a dynamic model for limit order book which contains the values of the formal model. The traders arrive sequentially in the market with different patience level because of the different waiting cost of them. Sellers and buyers come to the market alternately. Everyone just submit one unit of order. The time series is infinite and continuous, while the price line is discrete with one tick minimum. Once the limit order has been submitted, it cannot be canceled or resubmitted. One important assumption is that the trader must submit a limit order with an price improved. With this restriction, the order submission strategy of the trader depends only on the inside spread. The effect of the price priority in normal limit order market becomes the effect of spread priority. The limit order which shorten the spread more has the higher execution probability.

Their model predicts the strategies of traders in equilibrium, that is patient trader prefer limit order and impatient traders submit market orders. In equilibrium, the optimal strategies of the trader is that the margin value of the function of waiting cost for trader equals zero. The trader could not improve his limit order price without increasing the waiting cost. The equilibrium status is stationary in this model. This model also provides some predictions on the effects of variables such as spread, trading frequency, and market resiliency. The transaction frequency is weakly decreasing under wide spread. This is because that, when the spread is small, the market order prevail among the patient and impatient traders. When the spread become larger, the impatient traders prefer to submit limit orders. The resiliency is measured by the probability of the following limit orders pulling back the spread to its former level before the next transaction. When the traders act heterogeneously, the resiliency of the limit order book increases with the large proportion of patient traders and increasing the waiting cost. When the waiting cost or the number of patient traders increases, the bid-ask spread decrease with a large number of aggressive limit orders, which raises the market resiliency. When the order arrival rate increases, the traders act less aggressively, therefore, the market resiliency will decrease.

There are some other dynamic models to investigate the limit order market. Some of them focus on the optimal order submission strategies of various types of traders. Angel (1992) constructs a single-period model to investigate the choice of an informed trader who is forced to purchase a security. Harris (1998) develops a dynamic model to investigate the optimal order submission strategies for three types traders - uniformed trader, informed trader and value-motivated trader. Wald and Horrigan (2005) provides a two-period model to investigate the trader's decision of whether to place a limit order and at what price from the perspective of a risk-averse investor. He considers two cases - the forced-execution case and the optional-execution case, with some focus on the equilibrium of the order book. Rosu (2008) presents a continuous-time market, in which the trader could cancel or resubmit their orders. In equilibrium, the same type of traders have equal expected utilities. The dynamic model of Goettler et al. (2004) investigates the equilibrium status of the order book. This model extent the analysis with multi-unit and multi-type limit orders. These models are different in their treatment of the trader type, time interval and waiting cost etc.

3.2.3 Zero-intelligence traders

The assumption of rational traders is made in many microstructure model. Although the degree of rationality is different, the normal acknowledgement on the rational trader is based on their knowledge of how they could get rid of the systematic error to make the right decision. One question comes out from the assumption of the intelligent trader: does the efficiency of one market design come from the endogenous properties of the market or from the rational behavior of the trader? The market equilibrium is the cumulation of the effects of all the traders' behavior, which is different from the simple disconnected sum of strategic behavior of individuals, because of the probability that the systematic effects decrease the non systematic error. Gode and Sunder (1993) give the assumption of"zero-intelligence" traders in their theoretical model in double auction market. According their definition, ZI (zero-intelligence) traders are the traders who place their orders randomly but avoid loss from the trade and do not use the information of previous trade and ignore market rules to explain salient aspects of this data. The ZI trader model is useful in giving some insights into the problem of market efficiency, because the ZI trader model presents the performance of market separate from the performance of the traders. Comparing the difference between the experimental results of the ZI trader model and the ones of the human market, we could know the extent of the market performance. Gode and Sunder find that the difference between the results on the ZI trader model and the human schedules is very small and they do the same equilibrium price and quantity. They claim that the convergence of the price is coming from the property of the market under the condition of ZI traders. In other words, the market is efficient because of its own discipline, not from the rational behavior of the traders.

Gode and Sunderhe's work give some valuable insights on market efficiency. They also introduce the ZI traders into the research model in the markets. Cliff and Bruten (1997) gives some critique on their research. First, the mean transaction price in ZI trader model can be predicted from the expected value of the probability density function, which is applied to both the buyer's price probability density function and the seller's price probability density function, approach the equilibrium price under the condition that the magnitude of the gradient of supply and demand curve is equal. They also find that the price convergence in the ZI trader model does not happen in some special market design. They give the assumption of ZIP(zero-intelligence-plus) traders: stochastic traders with minimal intelligence, but they incorporate elementary machine learning techniques to alter their behavior on the basis of experience. They design the model in which the ZIP traders could adjust their offer price by the information of the market price. The market price is the only information that the trader use in his strategies. The margin profit is the difference between the traders's reservation price and the transaction price. According to the last transaction price, he needs to decrease or increase his margin profit restricted by the margin update rules. The experiment results show that the price convergence appears in most of the market design with ZIP trader model, although in some market, this convergence is not observed with ZI trader model. Furthermore, the equilibrium price and the expected profit from the ZIP model is more close to the ones of the human market, compared with the ZI model.

Therefore, the more complex ZIP traders act more like humans in the reality market than ZI traders. Even in a very simple market design, ZIP trade could complete price convergence.

4

Two-person single period double auction model

My research begins with the analysis of submission strategies of traders in double auction markets. The investigation of the informed traders' submission strategies in auction markets could help us to understand not only the decision process of investors, but also the mechanism efficiency of double auction markets, such as process of price formation and information dissemination. There are some questions attracting my interests. Which factors could affect the traders' submission strategies of order side and order prices, and what weight they have in the decision process? How do investors understand the submission strategies of the others, regarding their private value distribution as a common knowledge in markets? Different from other double auction markets, I posit a simple double auction market where traders can make choice between bids and asks according to their preference. Therefore, in my research, it is also important to analyse the probability of observing one buy and one sell in such markets.

In this chapter, I will introduce a static model in simple double auction markets. In this incomplete information market, two informed traders trade one unit of the asset with each other in one trading period. They have their own reservation price according to their individual signal and the information of the other one's value distribution. Also they are free to choose their order sides and order prices according to their preference. The trade is a single-period bargain with two stages. At stage I, two traders make their decision on their order side and order price, then submit their offers simultaneously, and at stage II, two orders match with each other. The trade could only happen following specific trading rules in which each trader has equal bargain power.

Before presenting my model in details, I would like to introduce the classic bargain model of Chatterjee and Samuelson (1983), on which my model based. Chatterjee and Samuelson investigated conditional trading strategies with incomplete information and uncertainty of success under the two-side bargain game. In their model, one seller and one buyer submit their offers simultaneously and the trade could only happen when the bid price is higher than the ask price. One important contribution of this model is that they treat the execution probability as an endogenous factor by modeling the interaction between the submission strategies of traders. The buyer assesses the seller's strategies according to the available information of the seller's private value distribution. Similarly, the seller has the information of the buyer's private value distribution and understand that the buyer has the information of his private value as well. In my research, the game-theoretical problem among traders is one of the important questions that I want to address. Therefore, I construct my model based on their model such that the execution probability is an endogenous factor in the model. Different from their model in which one buyer and one seller are fixed at the beginning of trade, I permit the traders to choose their own sides according to their own preference with underlying information. my research focuses on the optimal strategies of the traders and the price formation process in double auction markets rather than the conditions of bargain success.

4.1 Description of the model

I posit a double auction market where two traders with incomplete information trade with each other. These two informed traders are indifferent on every aspect, such as personal judgement, knowledge, and etc., such that they would take similar action under the similar condition. They enter into the market simultaneously to trade one unit of asset. According to the private signal they received separately, they get their own private reservation price of the asset. I use private-value rather than common-value in my model because of the research purpose of my model. In the simple two-trader model, the research focus is to observe the traders' submission strategies with private information and the traders' game theoretical competition between them, rather than the price formation. The private value assumption simplify the decision making process of traders, which easy the situation of subjects in the experiments. Except their private information of the asset, the distribution of reservation prices of each trader is a common knowledge in the market. The distributions of two reservation prices could be same or different from each other at the beginning of the trade. The private value distributions of the traders are assumed to follow normal distributions such that most of the private values cluster around a mean value. Then a couple of mean value and variance represent a specific private value distribution which is the available information of the opponent to the trader. Each trader could estimate possible situation and submission strategies of the other one according to the private value of his opponent. Specifically, if the private value distributions of the traders have the same mean value and variance, both of them are aware that his opponent's private value are drawn from the same distribution as his. I denote trader 1 as trader i and trader 2 as trader $3-i$ in my model. As the description above, trader 1 holds two parts of information: his own private reservation price V_i and the cumulative distribution function of trader $3-i$ regarded by trader i according to his information about trader $3-i$'s private value. $G_i(b_{3-i})$ denote the cumulative distribution function for trader $3-i$'s bid price regarded by trader i and $F_i(s_{3-i})$ denote the cumulative distribution function for trader $3-i$'s ask price. Meanwhile, because of the game-theoretical competition between two traders, trader 2 knows that trader 1 is informed by his private value distribution and assesses his submission strategies according to the information about him. And for the same reason, trader 1 is also aware that trader 2 knows that his opponent discriminates his private value. The knowledge of game-theoretical competition exists and goes on between two traders. With this assumption, the execution probability of offers becomes an endogenous variable in my model, which is influenced by the submission strategies of both sides of the traders. I must point out that, although the private value distributions are normal distributions $v \sim N(\bar{v}, \sigma)$, the offer price distributions derived from the underlying private value distributions do not always follow normal distributions. Normal distribution is useful to describe the random variables that cluster around a mean value. Since the normal distribution can be defined by two variables - mean and variance, it is easy to research the effects of the change of the distributions. I expect that the model with other distribution, the result of the model would be similar to the one with normal distribution.

4. TWO-PERSON SINGLE PERIOD DOUBLE AUCTION MODEL

The traders are assumed to be risk-neutral, who have no preference among the choices of risk and only concern the expected return. With this assumption, the probabilities of future cash flows are not included in my model and the expected value is exactly the value that the traders are care about. Meanwhile, the traders are rational, whose trading purpose is to maximize their profit. In a word, the rational risk-neutral traders in the model submit the optimal orders in order to maximize their expected return. Another important assumption is there is only one unit of the asset for trading in the market. In order to simplify my research and focus on the submission strategies and competition between the traders, I do not consider the quantity of orders. The order prices are derived from a continuous range without tick size. The continuous price range allows us to interpret the execution probability as a part of the continuous cumulative function of the other one's offer prices and to observe the every possible offer prices of the traders.

Because the traders need to choose both his order side and order price. There are two steps for the traders to make his decision. The first step is that he decides his order side, and secondly he figures out what price he submits according to his order side. We should notice that during the processing of order side decision, the traders are not informed the order side of the other one or the order price of the other one.

Firstly, the traders need to choose the order side of their orders after entering market. One feature of my model is that traders are allowed to choose the order side according to their preference. Here, I introduce a new variable, λ^b_{3-i} - the probability that trader 2 submits a buy order, regarded by trader 1 to denote the uncertainty of the order side choice of the traders. Chatterjee and Samuelson (1983) posit a bargain game with one seller and one buyer whose order sides are fixed in advance. Therefore, the unsuccessful trading could only happen when the bid price is lower than the sell price. The uncertainty of the opponent strategies is only refer to the opponent's offer price. Whereas, in my model, the traders are free to choose both the order price and the order side such that the uncertainty of the strategy increases under this assumption. Therefore, after introducing the new variable, the additional condition of the successful trading besides the one above is that there are one buy and one sell orders in the market. By allowing traders to choose order side, the difference between the double auction markets in my research and the limit order markets is reduced. As we all know, the traders' order side choice in limit order markets is affected by his preference and his trading purpose to pursue the profits. Here, the trading purpose of the informed traders is to realize the value of their private information and maximize their profits.

Secondly, the trader makes his decision on the order price, which are the optimal price that maximize his profit according to his order side. s_i denotes the offer price of trader 1 if he submits a sell order and b_i denotes the offer price of trader 1 if he submits a buy order. Similarly, s_{3-i} and b_{3-i} are the bid and ask prices of the trader 2 separately. Two traders submit their offers simultaneously without being informed of the submission strategies of the opponents. To reduce the difference between my double auction market model and limit order markets, I assume that every trader has equal weight to determine the transaction price. Therefore, the transaction price P is the average of the two offers if the trade happens: $P = \frac{b+s}{2}$. However, the execution of limit orders is not guaranteed in the market, and it only happens when there is one bid and one ask in the market and the bid price is higher than the ask price at the same time. In other words, transaction price must satisfy $s \leq P \leq b$ under the precondition of observing one buy and one ask in the market. When the transaction happens, the expected profit is equal to the difference between the true assets value and the transaction price. For a seller, his profit is equal to the transaction price minus his reservation price $\pi(s_i, v_i) = P - v_i$. For a buyer, his profit is equal to his reservation price minus the transaction price $\pi(b_i, v_i) = v_i - P$. As the function above, the profit of buyer is derived from the value above the transaction price to his reservation price under the condition that the market price would be equal to his private value. Similarly, the seller could obtain profits because his reservation price is below the transaction price.

Because they are the risk-neutral rational traders, their expected profits are important dependent variables in the model. Since two traders are similar on every aspect, I use trader 1 as the sample to describe the status of traders. When trader 1 submits a sell order, under the uncertainty of the other one's order submission strategies, his expected profit is given by:

$$E[\pi(s_i, v_i)] = \lambda^b_{3-i} \int_{s_i}^{+\infty} [(\frac{s_i + b_{3-i}}{2}) - v_i] g_i(b_{3-i}) db_{3-i} \tag{4.1}$$

This function expresses that the expected profit of trader 1 is decided by his price and the opponent's submission strategies including his order side and order prices. Because the bargain follows the rules that trade only happens when the bid is higher than the ask. The ask offers of trader 1 are only executed when the offer prices are in the area with the upper and lower supports of the bid offer distribution of trader 2, $+\infty$ and s_i, respectively. In other words, the execution probability of trader 1's asks is equal to the cumulative distribution function of trader 2's bids conditional on the bids being higher than the asks. In this function, $g_i(b_{3-i})$ is the density function of trader 2's bids, related to the cumulative function $G_i(b_{3-i})$. Because traders are free to choose their order side, the expected profits of the seller is reduced by multiplying the execution probability of the opponent submitting bid offers, λ^b_{3-i}.

Similarly, when trader 1 submits a buy order, his expected profit is:

$$E[\pi(b_i, v_i)] = (1 - \lambda^b_{3-i}) \int_{-\infty}^{b_i} [v_i - (\frac{b_i + s_{3-i}}{2})] f_i(s_{3-i}) db_{3-i} \tag{4.2}$$

As a buyer, trader 1's expected profits is realized by matching the sell orders of the opponents. The execution probability of trader 1's bid offer is decided by the cumulative function of trader 2's ask offers $F_i(b_{3-i})$. The bid offers are executed when the sell prices of the opponent are in the range from $-\infty$ to b_i, since asks are lower than bids for traders to make profits. $g_i(b_{3-i})$ is the density function of trader 2's bid offers. As the sum of the execution probability of buying and the execution probability of selling is equal to one, 1- λ^b_{3-i} is the execution probability of trader 2 submitting a sell order.

The Calculation of the function of trader i's expected profits is shown as following:

$$\begin{aligned} E[\pi(s_i, v_i)] &= \lambda^b_{3-i} \int_{s_i}^{+\infty} \frac{s_i g_i(b_{3-i})}{2} db_{3-i} + \lambda^b_{3-i} \int_{s_i}^{+\infty} \frac{b_{3-i} g_i(b_{3-i})}{2} db_{3-i} - \lambda^b_{3-i} \int_{s_i}^{+\infty} v_i g_i(b_{3-i}) db_{3-i} \\ &= \frac{\lambda^b_{3-i} s_i}{2} - \frac{\lambda^b_{3-i} s_i G_i(s_i)}{2} + \frac{\lambda^b_{3-i}\overline{X}}{2} - \frac{\lambda^b_{3-i} s_i G_i(s_i}{2} - \frac{\lambda^b_{3-i}}{2} + \frac{\lambda^b_{3-i} N_i(s_i)}{2} - \\ &\lambda^b_{3-i} v_i + \lambda^b_{3-i} v_i G_i(s_i) \\ &= \frac{\lambda^b_{3-i}}{2} [s_i - 2 s_i G_i(s_i) + \overline{X} + N_i(s_i) - 2v_i + 2v_i G_i(s_i) - 1] \end{aligned} \tag{4.3}$$

Where $N_i(s_i)$is the definite integral of $G_i(s_i)$, $N'(s_i) = G_i(s_i)$. $\overline{X}$ is the maximal value of bid offers. This general function of trader 1's expected profit shows that the trader's expected profit as a seller is decided by his reservation price, the distribution of the opponent's offer price, and the probability of the opponent submitting a buy order.

Similarly, by resolving the expected profits of trader 1 as a buyer, the general function of the expected value of buyer is:

$$\begin{aligned}
E[\pi(b_i, v_i] = (1-\lambda_{3-i}^b)\int_{-\infty}^{b} v_i f_i(s_{3-i})ds_{3-i} - (1-\lambda_{3-i}^b)\int_{-\infty}^{b} \frac{b_i f_i(s_{3-i})ds_{3-i}}{2} - \\
(1-\lambda_{3-i}^b)\int_{-\infty}^{b} \frac{s_{3-i} f_i(s_{3-i})ds_{3-i}}{2} \\
= (1-\lambda_{3-i}^b)v_i F_i(b_i) - \frac{(1-\lambda_{3-i}^b)b_i F_i(b_i)}{2} - \frac{(1-\lambda_{3-i}^b)b_i F_i(b_i)}{2} + \frac{(1-\lambda_{3-i}^b)M_i(b_i)}{2} \\
= (1-\lambda_{3-i}^b)v_i F_i(b_i) - (1-\lambda_{3-i}^b)b_i F(b_i) + \frac{(1-\lambda_{3-i}^b)M_i(b_i)}{2}
\end{aligned} \tag{4.4}$$

In this equation, $M_i(b_i)$ is the integral of $F_i(b_i)$, $M'(b_i) = F_i(b_i)$. It is expected that the expected profit of the buyer is affected by the reservation prices of the buyer, the distribution of the opponent's offer prices and the probability of the opponent submitting a sell order.

Next, I continue to investigate the traders' optimal submission strategies by the function of expected profits. As the assumptions above, the trading purpose of these traders is to maximize their expected profits. Their optimal strategy is to submit an order, which is equal to the cut off value that maximize their expected profits. In other words, they choose to submit an order, where the margin profit is equal to zero. Therefore, when the first order condition of the expected value equals to zero, the traders submit an optimal order to maximize their profits. The first order condition for a maximum for trader 1 as a seller is:

$$\frac{\partial E[\pi(s_i, v_i)]}{\partial s_i} = \frac{\lambda_{3-i}^b}{2} - \lambda_{3-i}^b G_i(s_i) - \lambda_{3-i}^b s_i g_i(s_i) + \frac{\lambda_{3-i}^b G_i(s_i)}{2} + \lambda_{3-i}^b v_i g_i(s_i) = 0$$

$$\frac{G_i(s_i)}{2} + g_i(s_i)(s_i - v_i) = \frac{1}{2} \tag{4.5}$$

Rewrite this function:

$$s_i = v_i + \frac{1 - G_i(s_i)}{2g_i(s_i)} \tag{4.6}$$

The equation 4.6 shows the relationships among the optimal sell prices and other independent variables. From the equation, s_i is equal to v_i and a half of the mills ratio of the value distribution function of trader 2's offer regarded by trader 1. This function explains that the seller's optimal order price is the sum of his reservation price and an amount that equals to a half of the mills ratio of the opponent's offer price distribution. Therefore, the private value and the opponent's offer price distribution determine the optimal offer price of trader 1. As I state above, the reservation prices of trader 1, v_i, are derived from his private signal and follow normal distributions. Besides his private value, he asks for an additional value, which is determined by the submission strategies of the opponent. $G_i(\bullet)$ is the cumulative distribution function of trader 2's buy offers underlying his private value distribution $X_i(\bullet)$ and $g_i(\bullet)$ is the related density function. I need to point out that $G_i(s_i^n) \ = X_i(v_i^n)$. In one word, to make an optimal offer price, the seller need to consider the submission strategies of the other one besides his own reservation price.

The second order condition of expected profits of trader 1 as a seller:

$$\frac{\partial^2 E[\pi(v_i, s_i)]}{\partial s_i^2} = -\lambda_{3-i}^b [g_i^{'}(s_i)(s_i - v_i) + \frac{3g_{i(s_i)}}{2}] \tag{4.7}$$

In order to confirm that the expected profits have one maximized value, the second order condition of expected profits is required to be less than zero. Only when $\frac{\partial^2 E[\pi(v_i,s_i)]}{\partial s_i^2} < 0$, there is a maximum value for sellers' expected profits. The proof is as following:
With function 4.6, we have:

$$g_i'(s_i) = -\frac{2g_i^2(s_{i)}}{1 - G_i(s_{i)}} \tag{4.8}$$

Put the function 4.8 into the seconde order condition 4.7 and rearrange it. The function 4.7 is proved always less than 0.

$$-\lambda_{3-i}^b [g_i^{'}(s_i)(s_i - v_i) + \frac{3g_{i(s_i)}}{2}] \quad < \quad 0 \tag{4.9}$$

$$g_i^{'}(s_i)(s_i - v_i) + \frac{3g_{i(s_i)}}{2} \quad > \quad 0 \tag{4.10}$$

$$\frac{g_i(s_i)}{2} \quad > \quad 0 \tag{4.11}$$

Similarly, I solve the equation of trader 1 when he submits a buy order. The first order condition of equation 4.4:

$$\frac{\partial E[\pi(b_i, v_i)]}{\partial b_i} = (1 - \lambda_{3-i}^b) v_i f_i(b_i) - (1 - \lambda_{3-i}^b) b_i f_i(b_i) + \frac{(1 - \lambda_{3-i}^b) F_i(b_i))}{2} = 0 \tag{4.12}$$

Then the function of optimal b_i for trader1 is:

$$b_i = v_i - \frac{F_i(b_i)}{2f_i(b_i)} \tag{4.13}$$

It is obvious that the bid prices depend on the reservation prices of trader 1 and the distribution of trader 2's offer prices regarded by trader 1. As a buyer, trader 1 asks for his bid offers which are lower than his reservation prices to obtain the profits and the discounts depending on the submission strategies of the other one. The complex function of the discount is similar to the mill's ratio, which is composed of the cumulative function and density function of the other one's offer prices.

The second conational of trader 1's expected profits as a buyer:

$$\frac{\partial^2 E[\pi(v_i, b_i)]}{\partial b_i^2} = -(1 - \lambda_{3-i}^b)[f_i^{'}(b_i)(b_i - v_i) + \frac{3f_i(b_i)}{2}] \tag{4.14}$$

To prove the expected profits of buyers have one maximized rather than minimum value, the second order condition of expected profits is expected to be less than zero. I present the process of proof as following:
According to function 4.13, we have:

$$f_i'(b_i) = -\frac{2f_i^2(b_{i)}}{F_i(b_{i)}} \tag{4.15}$$

Put the function 4.15 into the second order condition of 4.14and rearrange it. The function 4.14 is proved less than 0.

$$-(1 - \lambda_{3-i}^b)[f_i^{'}(b_i)(b_i - v_i) + \frac{3f_i(b_i)}{2}] \quad < \quad 0 \tag{4.16}$$

$$f_i^{'}(b_i)(b_i - v_i) + \frac{3f_i(b_i)}{2} \quad > \quad 0 \tag{4.17}$$

$$\frac{f_i(b_i)}{2} \quad > \quad 0 \tag{4.18}$$

As trader2's status is similar to that of trader1's, the functions of trader 2's ask and bid prices are similar to the functions of trader 1's. The only difference between these two is the subscripts in that these two traders are the same on other aspects except obtaining different signals. Their decision processes and the factors that affect their decisions are the same.

4.2 Results of single-period model

With equation 4.6 and 4.13, the optimal offer prices of traders as buyer or seller depend on the traders' reservation prices and the distributions of the opponents' offer prices. By solving these two functions of the optimal offer prices, I observe the relationship among the offer prices and the independent variables affecting the traders' submission strategies. With the help of software-Matlab, I use numerical method to investigate the relationships. It is an efficient method to provide useful figures to reveal the relationship among variables, and it is convenient for us to understand the submission strategies of traders in double auction markets.

As an implication of the functions of optimal offer prices, the variables, affecting the traders' submission strategies, include the traders' private reservation prices, the expected value and variance of the distribution of the opponents' offer prices, Furthermore, these variables have different weights and different effects on the offer prices of traders. Besides the offer prices, I also investigate the parameters which affect the traders' decision of order sides and the relationship between the correlation coefficient of two traders and their order prices. The results and implications obtained from the numerical method are presented as following.

4.2.1 The determinants of offer prices

Result 1 The reservation prices of trader 1 are positively related to his offers either he is a buyer or seller. Whereas, trader 1's offer prices are not affected by the private values of trader 2.

Traders make the offer price decision according to their own private information without the private information of the other one. Their own private information should be the main parameter that influence their offer price because the essential for earning profits for informed traders is to apply their private information properly. Figures 4.1 and 4.2 show the relationship between the valuation of two traders and the bid/ask prices of trader 1 as a buyer and seller separately. I observed that either the bids or the asks of trader 1 are positively related to his own reservation prices. Namely, trader 1 prefers to submit a bid/ask with higher price when his reservation price is higher.

Because of the high slopes of the curves in the figures, it is safe to say that the change of the offer prices is sensitive to the change of the reservation prices. The intuition behind the relationship is that when his reservation price is high, trader 1 would like to pay more for the asset as a buyer and ask for more compensation to sell the asset. Meanwhile, the figures also show that the valuation of trader 2 has no effect on the offer prices of trader 1. This result coincide with the implication of optimal offer price functions, in which the reservation prices of the opponent are non-disclosure to trader 1. In a word, the reservation prices of the traders are important to the traders' offer price decision, whereas the reservation prices of the other one are irrelevant to the traders' submission strategies.

Figure 4.1: The relationship between the reservation prices and bid prices
Note: $v_{i,b} = [105, 115]$; $v_{3-i,s} = [95, 105]$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = \sigma_{3-i} = 10$

Result 2 the expected values of the distribution of the opponent's offers prices have positive relationship with the bid/ask prices of the trader, either he is a buyer or seller.

Figure 4.2: The relationship between the reservation prices and ask prices
Note: $v_{i,b} = [105, 115]$; $v_{3-i,s} = [95, 105]$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = \sigma_{3-i} = 10$

The game-theoretical competition among traders is that traders consider the submission strategies of the opponents by assessing the opponents' private value distribution. The expected values of the value distribution are public information to traders. In figure 4.3 and 4.4, I observed that, when the expected values of the opponents' private value distribution are high, trader 1 submits the orders with high price whether he is a seller or buyer, and vice versa. The high expected value of the trader 2' value distribution increases the probability that trader 2 has a high private value. Trader 1 expects that his opponent prefers an order with high price. After taking the opponent's submission strategies into consideration, trader 1 would increase his offer prices to match the offer prices of the other one. In this way, he increases the expected profits by increasing the execution probability of his offers. I also find that the slopes in the figures of the expected values are flatter than the ones in the figures of the private values in that the expected values of the opponents take less weight on the offer price decision.

Result 3 The variances of the opponents' valuation distribution positively affect the traders' ask prices and negatively affect the traders' bid prices.

The variance of the opponents' private value distribution is another important parameter for traders to assess the submission strategies of their opponents. Figures 4.5 and 4.6 demonstrate the relationship between the variance of the opponents' private value distribution and the offer prices of traders. The special relationship can be observed that ask prices increase and bid prices decrease with the increase of variances. When trader 1 is informed of a high variance of the opponent's private value distribution, he expects that the private value distribution of the other one is flatter than the one with a low variance. Therefore, if trader 1 is a buyer, the probability of trader 2 with low private values increases such that the probability of trader 2 submitting a low ask increases. The optimal strategies of trader 1 is to decrease his offers to match the asks of opponents. Accordingly, if trader 1 is a seller, he expect that trader 2 moves up his bids as a result of the fat tail of his private value distribution. Trader 1 moves his asks to the same directions to increase the execution probability of orders.

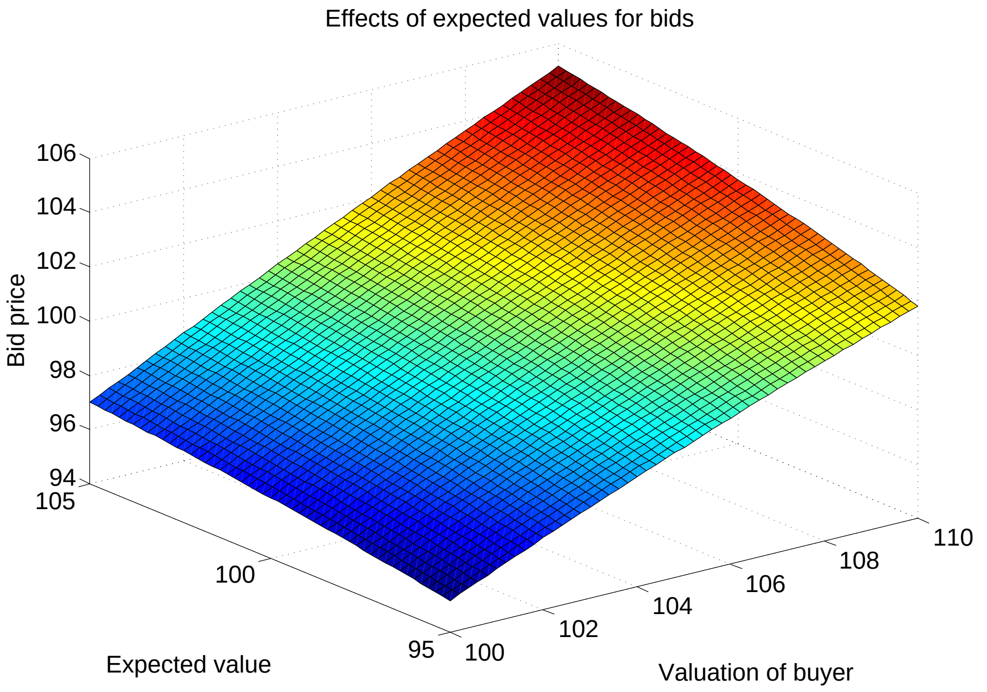


Figure 4.3: The relationship between the expected values of valuation distribution and bid prices

Note: $v_{i,b} = [100, 110]$; $v_{3-i,s} = 95$; $\mu_i = \mu_{3-i} = [95, 105]$; $\sigma_i = \sigma_{3-i} = 10$

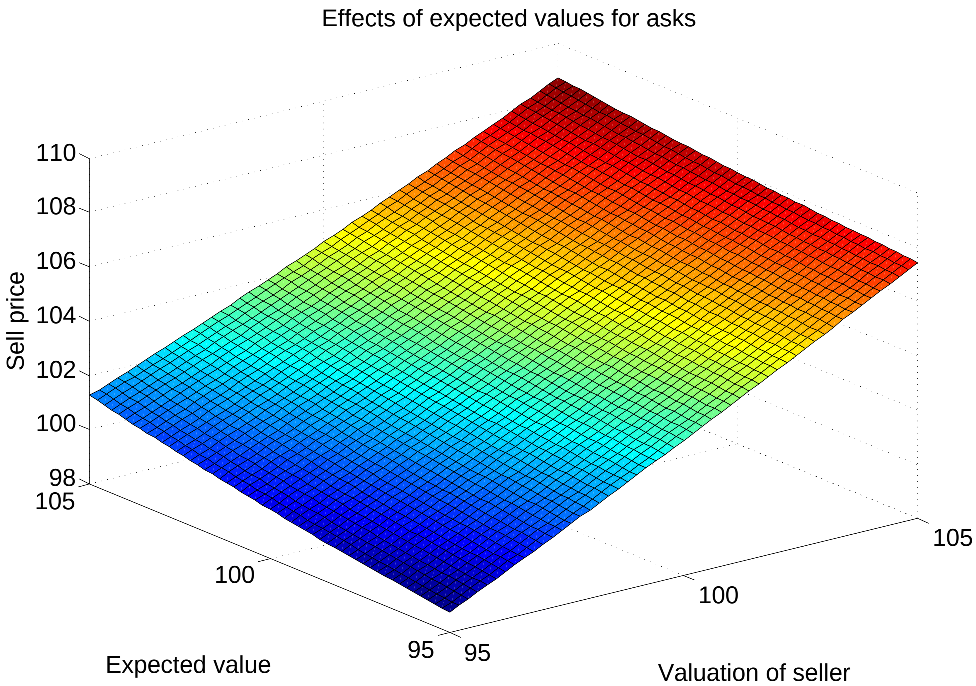


Figure 4.4: The relationship between the expected values of valuation distribution and ask prices

Note: $v_{i,b} = 105$; $v_{3-i,s} = [95, 105]$; $\mu_i = \mu_{3-i} = [95, 105]$; $\sigma_i = \sigma_{3-i} = 10$

Furthermore, when comparing the slopes of the private values, I find that the slopes of the variance is flatter, which means that the traders are less sensitive to the changes of the variance, and the weight of the variance is even less than the expected value on the offer price decision. To sum up, the private value is the most powerful parameter to affect the offer prices of traders, and the next one is the expected value of the opponents' private value distribution, and then the last one is the variance of the opponents' private value distribution.

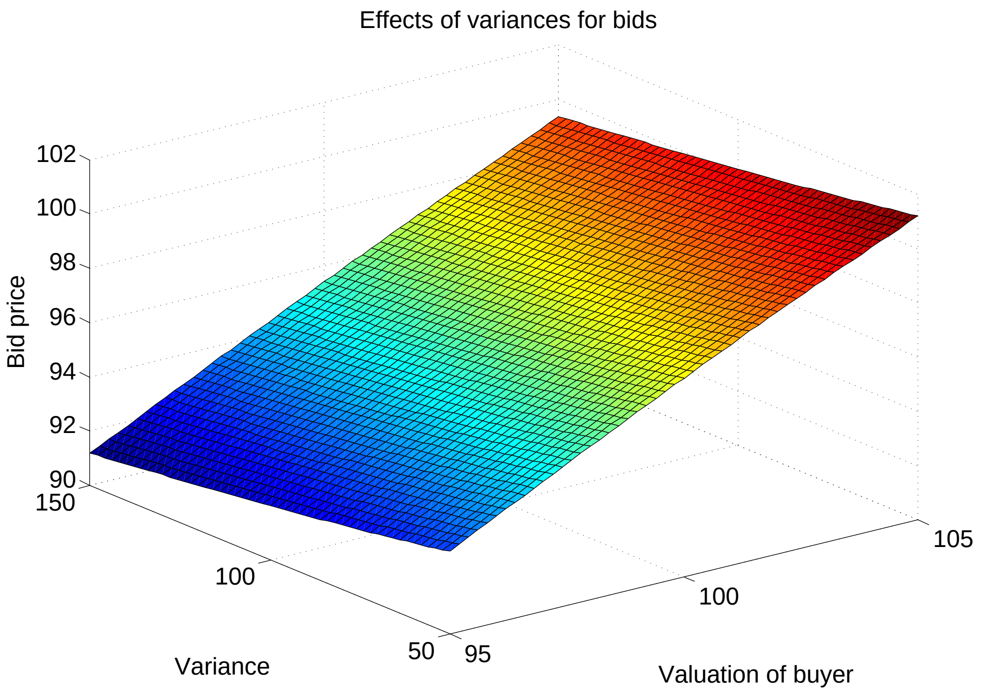


Figure 4.5: The relationship between the variance of valuation distribution and bid prices
Note: $v_{i,b} = [95, 105]$; $v_{3-i,s} = 95$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i^2 = \sigma_{3-i}^2 = [50, 100]$

Result 4 The change of transaction prices is positively related to the change of the traders' valuations.

The traders have equal power to decide the transaction prices. Therefore, the transaction prices are affected by the offer prices of both sides of traders. From figure

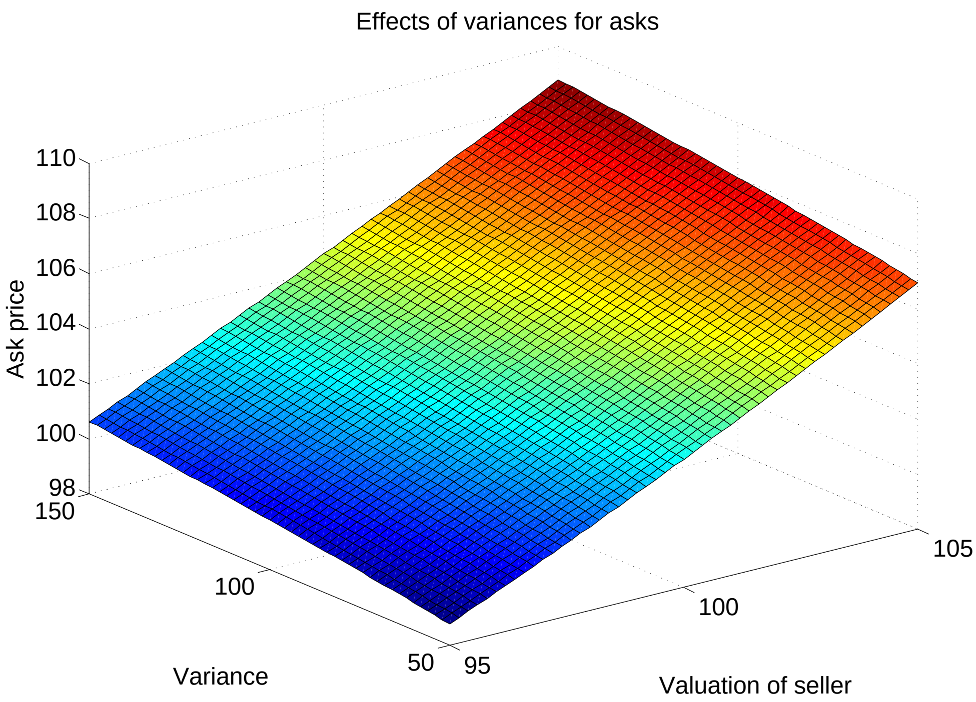


Figure 4.6: The relationship between the variance of valuation distribution and ask prices

Note: $v_{i,b} = 105$; $v_{3-i,s} = [95, 105]$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i^2 = \sigma_{3-i}^2 = [50, 100]$

4.7, I find that, following the trade rules that seller's valuations are lower than buyer's valuations, no transaction happens in this area. In the area where transaction occurs, the highest transaction price is on the point where both traders have highest private values and vice versa. The positive relationship between the transaction prices and the private values comes from the fact that the high/low valuations of two traders have positive effect on their offer prices and two offer prices decide the transaction prices. Note: $v_{i,b} = [100, 110]$; $v_{3-i,s} = [90, 100]$; $\mu_i = \mu_{3-i} = 101$; $\sigma_i^2 = \sigma_{3-i}^2 = 10$

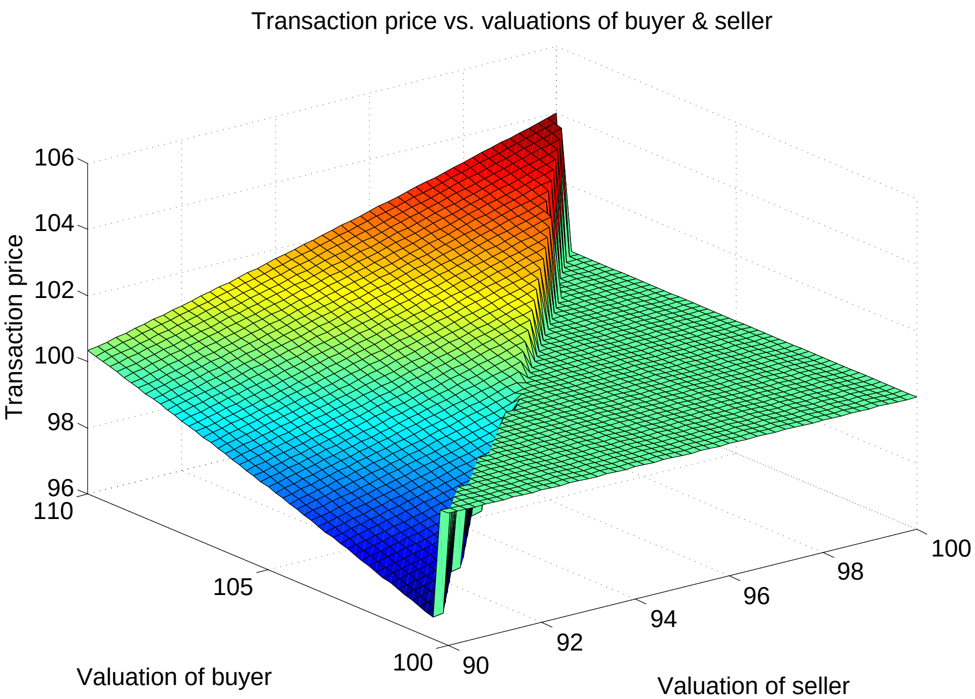


Figure 4.7: The relationship between the transaction prices and the valuations of two traders

4.2.2 The probability of observing one bid and one ask

In this section, I investigate the probability of the other one submitting a bid/ask and relevant parameters. As I state before, traders are free to choose their order side according to their own preference. Therefore, the expected profits of the traders are conditional with the other one submitting an order on the opposite side. To search the value of this probability and relevant parameters, I start my research from the conditional expected profit of traders.

Result 5 The conditional expected profit of trader 1 is positively related to his reservation price when he submits a buy order and negatively related to his valuation when he submits a sell order, under the assumption that the private value of traders derived from a common value distribution.

Normally, I suppose that two traders are similar in every aspect and their private values are derived from a common value distribution. The positive relationship between the bids and the expected profits of trader 1 in figure 4.8 shows that, with the increase of the private value, trader 1 prefers a bid to ask. On the contrary, figure4.9 demonstrates that trader 1 would be better off by choosing a ask than bid because of the negative relationship between the asks and the expected profits. The reason behind the different relationship is that when trader 1's private value is high relative to the common value distribution, the execution probability of bids is higher than asks and vice versa. The statement above is under the assumption that traders' private valuations come from a common distribution. If the traders' private valuations are belong to different value distribution, a high private value of trader 1 would choose different order side when he faces the different value distributions of the other one.

The order choice of traders depends on the results of the comparison of the expected profits between buyer and seller. Since the other one also has the right to choose the order side, I convert the conditional expected profits to the unconditional expected profits by embedding the probability of the other one submitting a bid/ask in the function. The unconditional expected profit of trader 1 when he submits a sell order is below:

$$E[\pi(s_i, v_i)] = \lambda^b_{3-i} * E[\pi|\text{trader 3-i submits a buy order}] \tag{4.19}$$

And the unconditional expected profit of trader 1 as a buyer is:

$$E[\pi(b_i, v_i)] = (1 - \lambda^b_{3-i}) * E[\pi|\text{trader 3-i submits a sell order}] \tag{4.20}$$

Figure 4.8: The relationship between the buyer's conditional expected profits and his valuations

Note: $v_{i,b} = [97, 103]$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i^2 = \sigma_{3-i}^2 = 10$

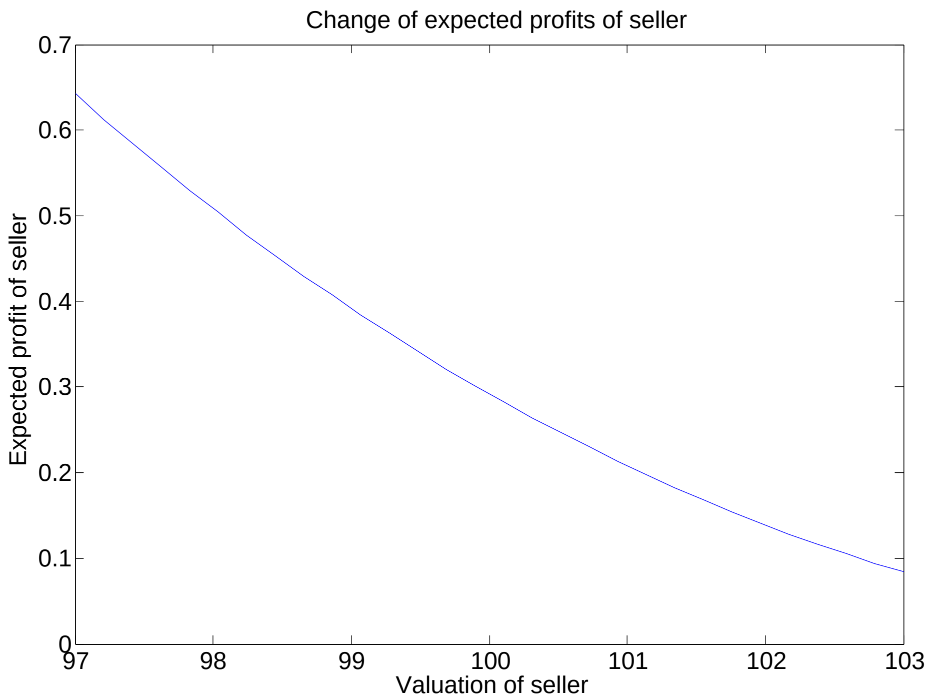


Figure 4.9: The relationship between the seller's conditional expected profits and his valuations

Note: $v_{i,b} = [97, 103]$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i^2 = \sigma_{3-i}^2 = 10$

When the unconditional expected profit of buying is higher than that of selling, trader 1 prefers a buy order. The numerical method is used to find the relationship among the parameters such as the expected value, the variance of the opponents' private value distribution and the probability of buying/selling. The results and analysis are below:

Result 6 Under the assumption that two traders are similar on every aspect and their private values are derived from the same value distribution, the probability of the other one submitting a bid/ask is 0.5 whatever the change of the expected value and variance of the common value distribution.

Figure 4.10 shows the relationship between the probability of submitting a sell order and the expected value of the opponents' value distribution. In this figure, the probability of selling is always equal to 0.5 with the change of the expected values of the other one. Because the private values of traders are derived from the common value distribution, there is no difference between the expected values of the traders' value distribution. On the other hand, two traders are similar on every aspect such that their behaviors are the same conditional on the same situations. Therefore, I expect the probability of the other one submitting a bid/ask regarded by the trader is 0.5 whatever the expected value of the opponent's value distribution. Figure 4.11 demonstrates the relationship between the probability of the other one's ask regarded by the trader and the variance of the common value distribution. The change of the variance does not change the value of the probability of selling which stick to 0.5. Together with the last result, I confirm that the change of the common value distribution does not influence the probability of the other one's bid/ask regarded by the trader.

Result 7 When the valuation distributions of the two traders are different, the probability of the trader's to sell increases as the expected value of the opponent's private value distribution moves beyond the his expected value and decreases as the expected value of the opponent's valuation distribution move back.

When trader 1 observes that the expected value of the trader 2's value distribution is higher than his, which means that the probability of trader 2 has a high private value is high, he expects that trader 2 prefers a bid to ask. On the contrary, the low

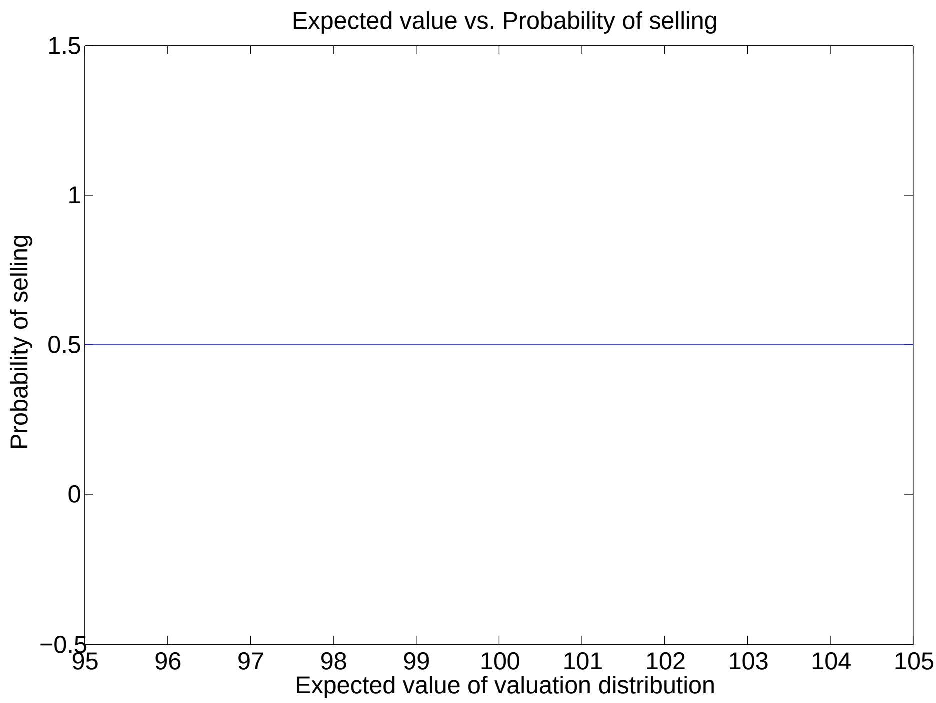


Figure 4.10: The relationship between the probability of submitting a sell order and the expected value

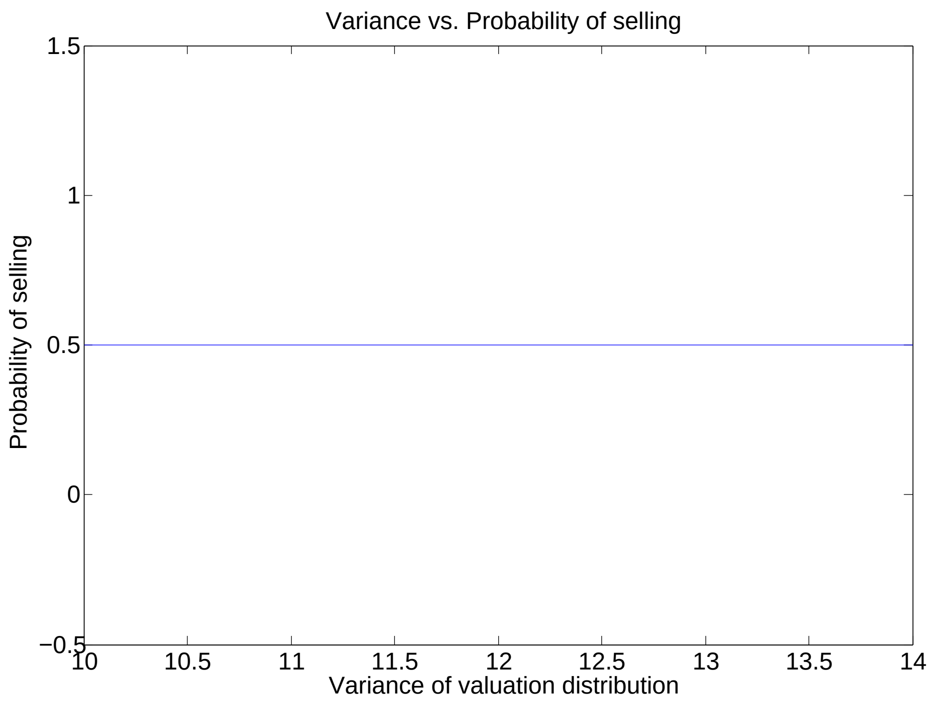


Figure 4.11: The relationship between the probability of submitting a sell order and the variance

expected value distribution of trader 2 relative to the expected value of trader 1's value distribution refers to a low private value, therefore trader 2 tends to submit an ask. In figure 4.12, the increase of the expected value of trader 2's private value distribution, in turn, is equivalent to the decrease of the expected value of trader 1. It is also coincide with the negative relationship between the probability of selling and the expected value of the private value distribution. The probability selling increase, and the probability of buying decreases. Then, the positive relationship between the expected value and the probability of buying could be confirmed.

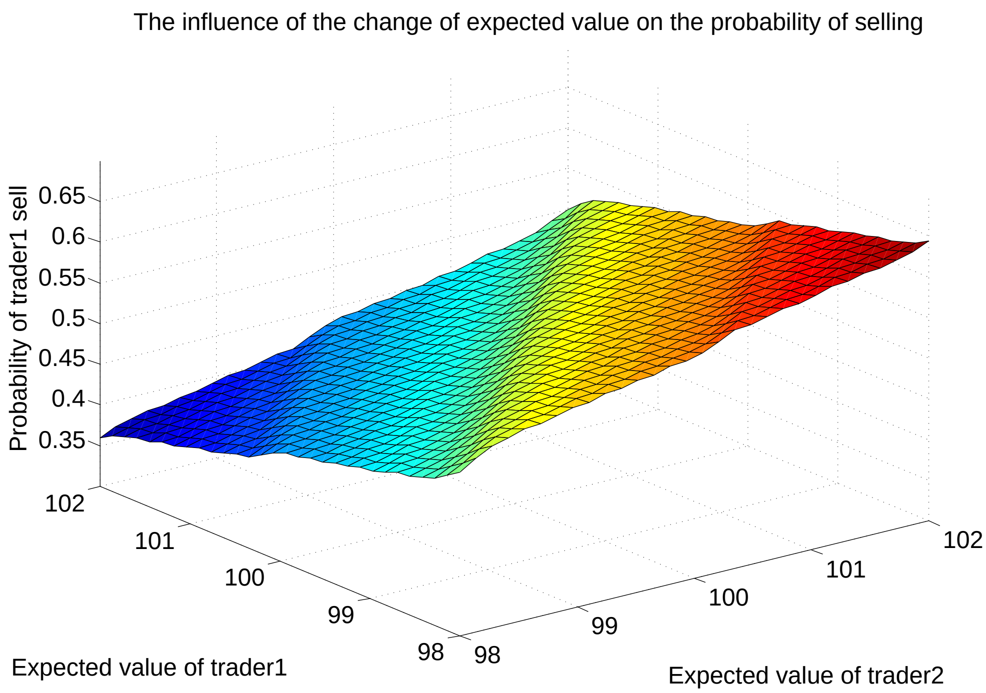


Figure 4.12: The effect of expected value on the probability of trader i submitting a sell order

Result 8: When the variances of two traders' valuation distribution are different, the probability of submitting a buy/sell order is till equal to 0.5.

Figure 4.13shows that the probability of the other one selling regarded by the trader is irrelevant to the variance of the other one's private value distribution. A basic insight into the relationship is that traders' decision of order sides is only dependent on this relationship between his private value and the expected value of the other one's private value distribution. Without being informed of the other one's private value, the traders consider the probability of the other ones' selling or buying by the relationship between his own expected value and the opponent's expected value.

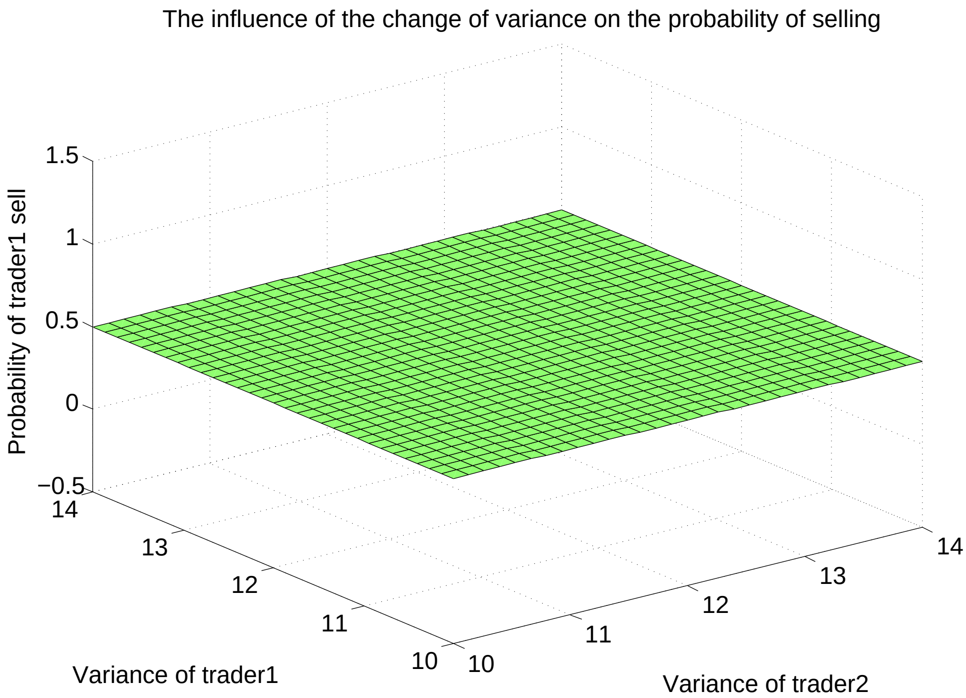


Figure 4.13: The effect of expected value on the probability of trader i submitting a sell order

4.2.3 Decision of order sides

The traders choose their order side according to their preference. Their order side choice is determined by the parameters such as their private value and the value distribution of the other one. The basic rule for rational traders is to choose the order side that obtains the higher expected profit. The factors deciding the expected profits of traders are the private value of traders, the valuation distribution/the offer price distribution of the opponents and the probability of the other one submitting an opposite order.

According to the corollaries above, it is the expected value of the valuation distribution of traders affecting the probability rather than the variance of the opponents' valuation distribution. Therefore, the relationship between the private value of the traders and the expected value of the valuation distribution of the opponents decide the order sides of traders.

Result 9: When the private value of the trader is lower than the expected value of the opponent's value distribution regarded by the trader, he prefers a sell order to a buy order. On the contrary, when his private value is higher than the expected value of the opponent's value distribution, he would submit a bid.

To obtain the highest expected profit, the trader is prone to choose the opposite order side of his opponent. For example, trader 1 expects that the high expected value of the opponent's valuation distribution implies that the other one's valuations distribute in an area beyond his valuation and a similar implication is expected by his opponent. Under this situation, trader 1 prefers to sell the assets and his opponent prefers to buy the assets. In the game-theories competition, traders know their own private value and the valuation distribution of the other one. Therefore, they judge their own position and their opponents by comparing the difference between these two values. Their expected profits are high as a buyer when their private value is high comparing to the valuation distribution of the other one. On the other hand, they attain more expected profits as a seller when their private value is low comparing to the valuation distribution of their opponents. Figure 4.14 compares the expected profits of traders as buyer and seller simultaneously. I observe that as the difference between the expected value of trader 2's valuation distribution and the private value of trader 1 changes from negative to positive, the expected profits of buying increases and the expected profits of selling decreases continuously. At the point that the private value equals to the expected value, there is no difference for trader 1 to choose buy or sell.

4.3 A special case

In this section, a special case of the single-period model is discussed. Normally, the reservation prices of traders are assumed to follow normal distributions. Under this

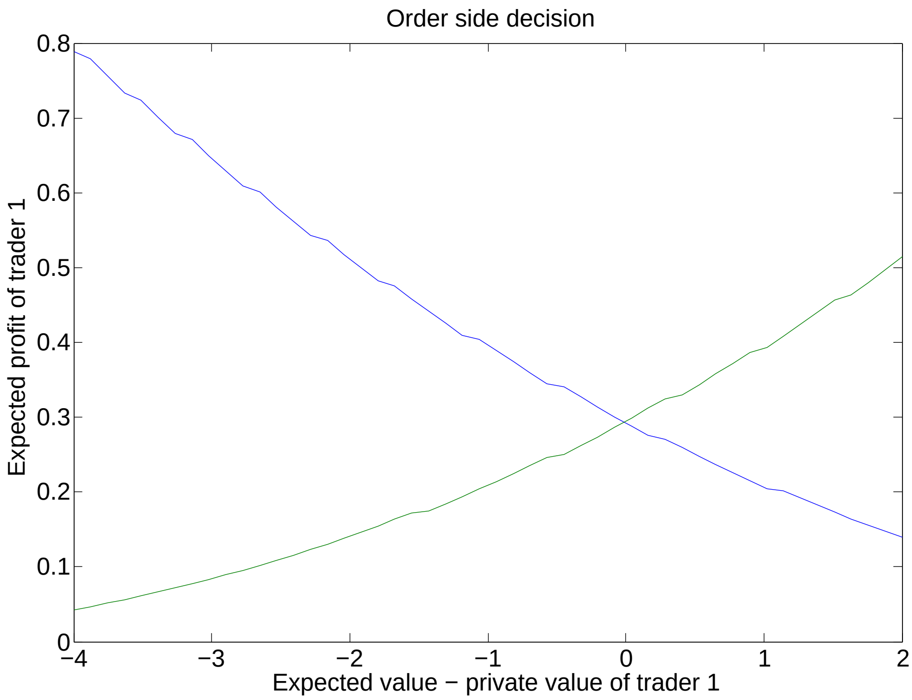


Figure 4.14: The expected profits of trader 1

assumption, the optimal order prices can not be solved directly by mathematically method because of the mill's ratio inside it. Here, I suppose that the traders' private value follows normal distribution. With the application of uniform distribution, the model is easier to be understood and solved directly, in that it is expected to provide meaningful results and supports to understand the traders' submission strategies in the simple double auction market.

In the special case, two traders' valuation distributions are derived from one uniform distribution with two boundaries a and b. Therefore, every point in the interval $[a, b]$ has equal probability, which means that each value of the reservation prices has equal probability to appear. According to the properties of uniform distribution, its density function and cumulative function are $F(x) = \frac{1}{b-a}$ and $g(x) = \frac{x-a}{b-a}$ separately. Other assumptions still follow the assumptions of previous model. Although the distribution of reservation prices follow uniform distribution, whether the distribution of the order prices following uniform distribution is unclear before calculation. However, it is true that for each reservation price, there is only one related optimal offer price. Meanwhile, in the previous section, I prove that the function of expected profit has only one maximum value. Therefore, the payability of each private value equals to the probability of its related optimal order price (Chatterjee and Samuelson, 1983). For example, the relationships between the probability of the order prices and the one of the reservation prices are demonstrated by the equations below:

$$G_{3-i}(b_i) = M_{3-i}(v_i) \tag{4.21}$$

$$g_{3-i}(b_i) = m_{3-i}(v_i) \tag{4.22}$$

In the equations, the $G(\bullet)$ and $g(\bullet)$ are the cumulative function and density function of the bid prices of the trader as buyer, following the notations before. $M_{3-i}(v_i)$ and $m_{3-i}(v_i)$ are the cumulative function and density function of the reservation prices for trader 1 when he submit bids, regarded by trader 2. Similarly, I have the cumulative and density function for trader 1 when he submits asks:

$$F_{3-i}(s_i) = N_{3-i}(v_i) \tag{4.23}$$

$$f_{3-i}(s_i) = n_{3-i}(v_i) \tag{4.24}$$

With these equations, we constructed the bridges between the distribution of order prices and the distribution of private value, which is useful to solve the function of optimal order prices.

4.3.1 The optimal offer prices

First, I present the solution of the function of optimal order prices. Optimal offer prices are the fundamental elements before solving the more complex functions and achieving most of the important results. Under the assumption of normal distribution, because of the mill's ratios in the function of optimal offer prices, the functions of optimal order prices can not be solved by mathematics method. Therefore, the results are achieved through Matlab programm. Benefiting from the simply cumulative function and density function of uniform distribution, the optimal order prices can be solved directly by mathematics method. In the previous section, the optimal order prices of trader 1 as a buyer or seller are presented as below:

$$b_i = v_i - \frac{F_i(b_i)}{2f_i(b_i)} \tag{4.25}$$

$$s_i = v_i + \frac{1 - G_i(s_i)}{2g_i(s_i)} \tag{4.26}$$

According to the equations above, the optimal order prices depend on the reservation price of traders and the order price distribution of the other one regarded by the trader. For each private value, there is a corresponding optimal order price: $S(v) = s$ and $B(v) = b$. Then, I have the other two equations: $S^{-1}(s) = v$ and $B^{-1}(b) = v$. $S^{-1}(\bullet)$ and $B^{-1}(\bullet)$ are the inverse function of $S(\bullet)$ and $B(\bullet)$ separately.

First I solve the optimal order price of buyer. The most difficult part in the equation of the optimal bid is the cumulative function and density function of the other one's asks. With the equation 4.23, for buyer, the distribution of the other one's asks can be converted into the distribution of the other one's private values:

$$F_i(b_i) = N_i(S^{-1}(b_i)) = N_i(Y) \tag{4.27}$$

In the function 4.27, Y defines a reservation price of seller whose corresponding ask equals to b_i. In another word, when the seller's optimal ask equals to the optimal bid price of buyer, Y is exactly the underlying private value of seller. Since Y is one of the private value of seller, put Y into the function of the other one's optimal sell price to get the function of Y. Meanwhile, according to the definition of Y, we can use b_i as the substitute for the s_{3-i} in the function of the other one's optimal ask. The solution is below:

$$\begin{aligned} Y &= b_i - \frac{1-G_{3-i}(b_i)}{2g_{3-i}(b_i)} \\ &= b_i - \frac{1-M_{3-i}(v_i)}{2m_{3-i}(v_i)} \\ &= b_i - \frac{1-\frac{v_i-a}{b-a}}{\frac{2}{b-a}} \\ &= b_i + \frac{v_i-b}{2} \end{aligned} \tag{4.28}$$

Then put the function 4.28 into the function of optimal bids for buyer and obtain the optimal bid for trader 1 with uniform distribution:

$$\begin{aligned} b_i &= v_i - \frac{F_i(b_i)}{2f_i(b_i)} \\ &= v_i - \frac{N_i(Y)}{2n_i(Y)} \\ &= v_i - \frac{\frac{Y-a}{b-a}}{\frac{2}{b-a}} \\ &= v_i - \frac{Y-a}{2} \\ &= v_i - \frac{b_i+\frac{v_i-b}{2}-a}{2} \\ &= \frac{v_i}{2} + \frac{b}{6} + \frac{a}{3} \end{aligned} \tag{4.29}$$

Then, we have the optimal bid price of trader 1 as a buyer. From the equations above, the traders' optimal bid prices are decided by their own reservation prices and the parameters of the other ones' private value distributions. Similarly, we could solve the function of optimal order prices for sellers. X is defined as a reservation price of the buyer, whose corresponding bid equals to s_i: $G_i(s_i) = M_i(B^{-1}(s_i)) = M_i(X)$. To solve the optimal ask of trader 1, we need solve the X in the probability function of trader 2.

$$\begin{aligned} X &= s_i + \frac{F_{3-i}(s_i)}{2f_i(s_{3-i})} \\ &= s_i + \frac{N_{3-i}(s_i)}{2n_i(s_{3-i})} \\ &= s_i + \frac{v_i-a}{2} \end{aligned} \tag{4.30}$$

Then, the optimal order prices for seller is:

$$\begin{aligned} s_i &= v_i + \tfrac{1-G_i(s_i)}{2g_i(s_i} \\ &= v_i + \tfrac{1-M_i(X)}{2m_i(X)} \\ &= v_i + \tfrac{1-\frac{X-a}{b-a}}{\frac{2}{b-a}} \\ &= v_i + \tfrac{b-X}{2} \\ &= \tfrac{v_i}{2} + \tfrac{b}{3} + \tfrac{a}{6} \end{aligned} \tag{4.31}$$

The functions 4.29 and 4.31 are the optimal order prices for trader 1 as buyer and seller separately. Similarly, we have the optimal order prices for trader 2:

$$b_{3-i} = \frac{v_{3-i}}{2} + \frac{b}{6} + \frac{a}{3} \tag{4.32}$$

$$s_{3-i} = \frac{v_{3-i}}{2} + \frac{b}{3} + \frac{a}{6} \tag{4.33}$$

The optimal order prices are the keys in my model to observe the traders' submission behaviors. The function of optimal order prices demonstrate that the optimal order prices depend on the private values of the trader and the parameters of the private value distributions of the other one. Furthermore, the upper boundary is more powerful to asks and lower boundary is more powerful to bids.

4.3.2 The relationship between the order prices and the variables

The variables that influence the traders' submission strategies in the special case are partially different from the ones in the previous model. The model with uniform distributions has three variables: private value, upper boundary a and lower boundary b. In these variables, the reservation price of traders is the most important factor to influence the order prices of traders. The other two variables - parameters of the private value distribution of the other one: a and b have different effects on the bids and asks. In this section, I present some results of the relationship between the order prices and these variables.

Results 1: The private values have positive effects on the order prices either the traders act as buyer or seller. However, the valuations of the other ones have no effects on the traders' order prices.

The private values are the most important variable that influence the traders' order prices. With the increasing of private values, the traders will to pay more as buyers and ask more compensation as sellers. It is not difficult to understand that the trader does not include the private value of the other one into his consideration because he is not informed the other one's private value. This result is consistent with the one of the model with normal distribution. The figure 4.15 demonstrates the relationship between the private values and the order prices of trader as buyer and seller.

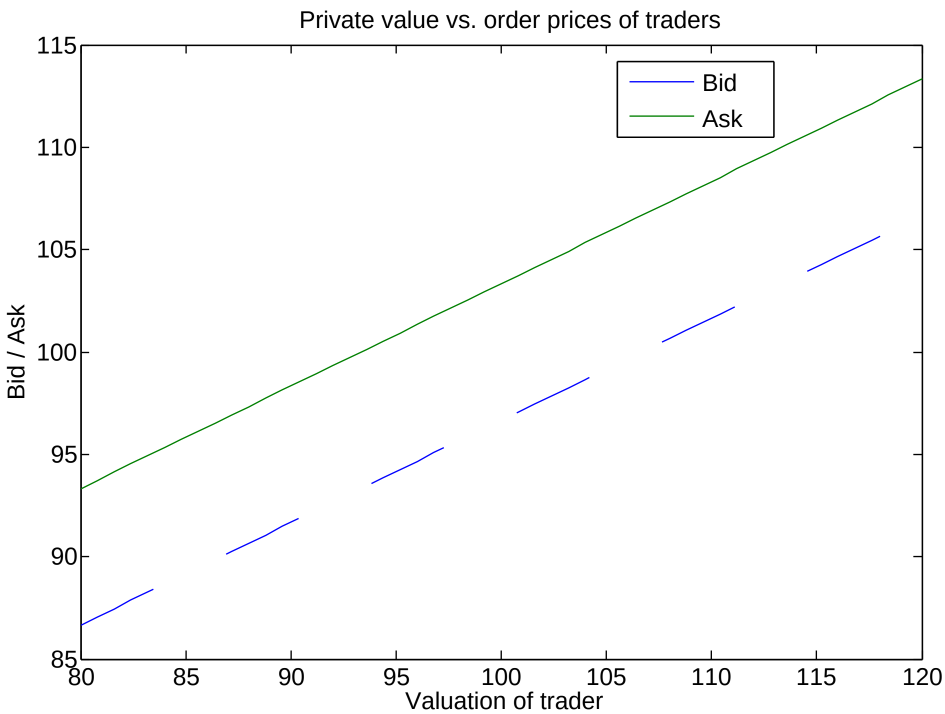


Figure 4.15: Private values vs. order price

Results 2: The lower boundary a positively influences the order prices of traders either for buyers or sellers.

The result that the lower boundaries a is positively related to the bid/ask prices is similar to the one of previous model. The result above implies that the traders with higher private value prefer higher bids/asks. When the traders observe that the lower boundaries a of the other one is high, he expects that the probability of the other one submitting an order with higher prices is high. His optimal strategies is to submit an order with higher prices to increase the execution probability, which in turn increases his own expected profits. Furthermore, from figure 4.16, I observed that with the increase of low boundary a, the bids increase more than the asks as the result of the higher weight of lower boundary a in the function of optimal bids. The explanation to this situation is that the lower boundary is related to the lowest ask of seller on the other side in that influence the execution probability of buyer greatly.

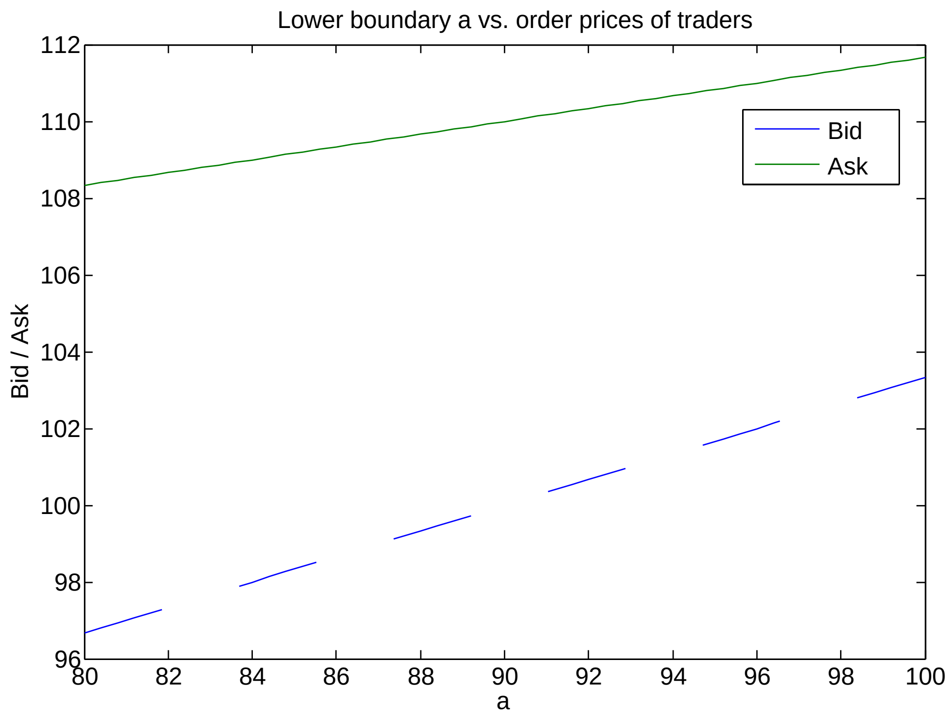


Figure 4.16: Lower boundaries a vs. order price

Results 3: The order prices for traders increase as the increase of upper boundary b and decrease as the decrease of upper boundary.

Similar with the results of low boundary a, the upper boundary b has positive effects on the order prices either for bids or asks. A higher upper boundary implies a higher order price of the other one, which in turn pushes the traders to increase their order prices to match the other ones' orders. In figure 4.17, the slope of bids is lower than the one of asks. Comparing with figure 4.16, the influence of the upper boundary on bids and asks is different from the one of lower boundary. It is caused by the different situation of buyers and sellers. For sellers, their opponents are buyers who buy the assets with the lower price than their reservation price. Therefore, the upper boundary is powerful to influence the execution probability of sellers through its effect on buyers' bids.

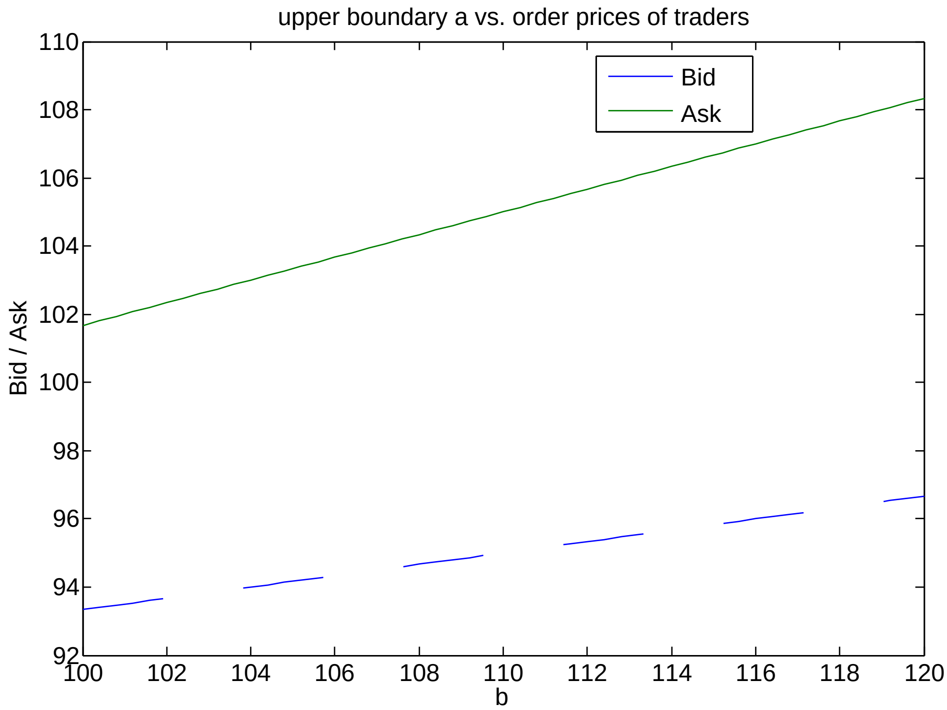


Figure 4.17: Upper boundaries b vs. order price

4.3.3 The order side decision of traders

There are two questions to answer in this section. One is the order side decision of the trader himself. The other one is the order side decision of the other one, regarded by the trader. In our model, we can find that the order side decision of the other one influences the order side decision of the trader himself and meanwhile, his order side decision also is involved in the order side decision of the other one. In my model, λ denotes the order side decision of the other one.

4.3.3.1 The expected profits of traders

The traders' order side decisions depend on the comparison between the expected profits as buyer and expected profits as seller. First, I start out from the expected profits of the trader as buyer and seller.

The traders' conditional profits is the profit conditional that the order is executed. I denote R_i^b is the conditional profits of trader 1 as a buyer and R_i^s is the conditional profits of trader 1 as a seller. Since the optimal order prices are obtained in the previous part, it is easy to solve the function of conditional expected profits. The conditional profit of trader 1 as a buyer is below:

$$\begin{aligned} R_i^b &= v_i - P \\ &= v_i - \frac{v_i + v_{3-i} + b + a}{4} \\ &= \frac{3v_i - (v_{3-i} + b + a)}{4} \end{aligned} \tag{4.34}$$

The conditional profit of trader 1 as a seller:

$$\begin{aligned} R_i^s &= P - v_i \\ &= \frac{v_i + v_{3-i} + b + a}{4} - v_i \\ &= \frac{v_{3-i} + b + a - 3v_i}{4} \end{aligned} \tag{4.35}$$

Similarly, I have $R_{3-i}^s = \frac{v_i + b + a - 3v_{3-i}}{4}$ and $R_{3-i}^b = \frac{3v_i - (v_{3-i} + b + a)}{4}$ for trader 2 as seller and buyer separately. Figure 4.18 display the change of the trader's conditional profits as the trader's private values change. There are positive relationship between the conditional profit of buyer and his private values, and negative relationship for seller.

Then I solve the functions of unconditional expected profit. The unconditional expected profit is the trader's expected profit, when the other one submitting an opposite side of order. I use trader 1 as an example to demonstrate the solution process of the unconditional expected profits. The equation 4.36 expresses the unconditional expected

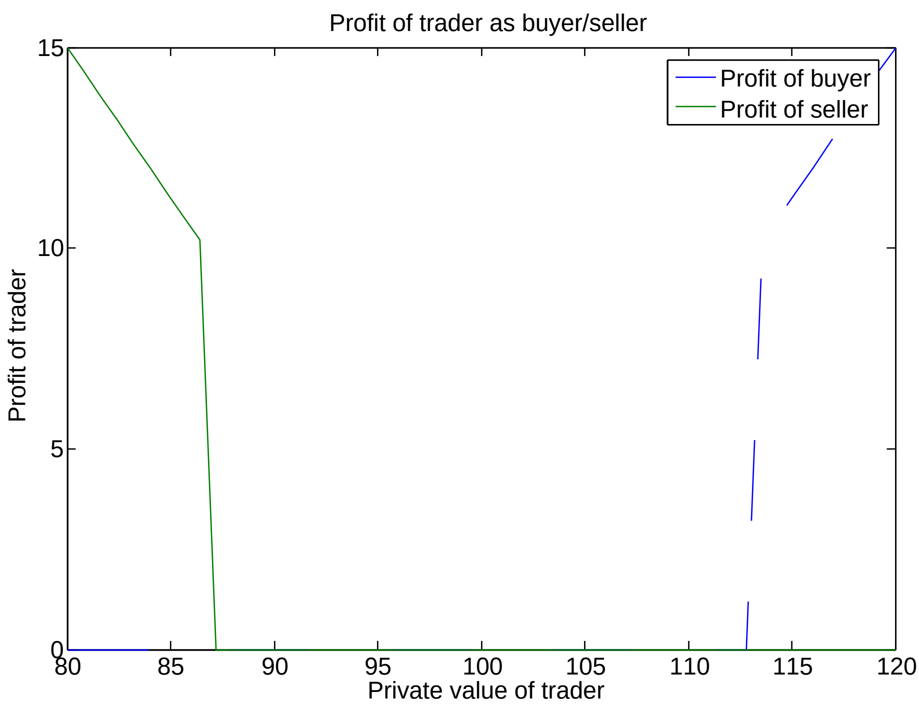


Figure 4.18: The relationship between the conditional profit of traders as buyer/seller and private values

profits for trader 1 when he bids:

$$\begin{aligned} E(\pi_i, b_i) &= \int_{-\infty}^{b_i} R_i^b f_i(s_{3-i}) ds_{3-i} \\ &= \int_a^{v_i+\frac{a}{3}-\frac{b}{3}} \frac{3v_i - v_{3-i} - a - b}{4} n_i(v_{3-i}) dv_{3-i} \\ &= \int_a^{v_i+\frac{a}{3}-\frac{b}{3}} \frac{3v_i - v_{3-i} - a - b}{4} \frac{1}{b-a} dv_{3-i} \\ &= \frac{(3v_i - b - a)(v_i + \frac{a}{3} - \frac{b}{3} - a)}{4(b-a)} - \frac{(v_i + \frac{a}{3} - \frac{b}{3})^2 - a^2}{8(b-a)} \\ &= \frac{45v_i^2 - 30bv_i - 60av_i + 20ab + 20a^2 + 5b^2}{72(b-a)} \end{aligned} \tag{4.36}$$

There is one important transformation in the equation above. The probability distribution of the seller's private value substitutes for the probability distribution of the seller's asks. The range of the definite integral of other one's asks is $[b_i, -\infty]$ for general case. In the special case with uniform distribution, the optimal order prices also follow uniform distribution. Therefore, it is safe to transform the cumulative function of the other one's asks into the cumulative function of the other one's reservation prices. Although the value of cumulative function and density function do not change, the range of the definite integral changes under this situation. It is clear that the other one submits the lowest ask when his reservation price equals to lowest boundary of the uniform distribution. Therefore, the lower boundary of the definite integral of other one's reservation price is a. The upper boundary of the new definite integral is just the sell's reservation price, whose related sell price is equal to the bid price of trader 1 in that the trade can happen.

$$\begin{aligned} s_{3-i} &= b_i \\ \frac{v_{3-i}}{2} + \frac{b}{3} + \frac{a}{6} &= \frac{v_i}{2} + \frac{b}{6} + \frac{a}{3} \\ v_{3-i} &= v_i + \frac{a}{3} - \frac{b}{3} \end{aligned} \tag{4.37}$$

The range of the definite integral of the other one's reservation prices is $[v_i + \frac{a}{3} - \frac{b}{3}, a]$.

The unconditional expected profit for trader 1 as a seller is:

$$\begin{aligned} E(\pi_i, s_i) &= \int_{s_{3-i}}^{+\infty} R_i^b g_i(b_{3-i}) db_{3-i} \\ &= \int_{v_{3-i}+\frac{b}{3}-\frac{a}{3}}^{b} \frac{v_{3-i} + b + a - v_i}{4} m_i(v_{3-i}) dv_{3-i} \\ &= \frac{(b + a - 3v_i)(\frac{2b}{3} + \frac{a}{3} - v_i}{4(b-a)} + \frac{b^2 - (v_i + \frac{b}{3} - \frac{a}{3})^2}{8(b-a)} \\ &= \frac{45v_i^2 - 60bv_i - 30av_i + 20ab + 20b^2 + 5a^2}{72(b-a)} \end{aligned} \tag{4.38}$$

In the equation above, the probability distribution of the other one's reservation price substitute for the one of the other one's bids. Therefore the upper boundary is b, with which the other one submits the highest bid. The lower boundary is the lowest bid price that is equal to the ask price to fulfill the transaction condition. Then the lower boundary is:

$$\begin{aligned} b_{3-i} &= s_i \\ \frac{v_{3-i}}{2} + \frac{b}{6} + \frac{a}{3} &= \frac{v_i}{2} + \frac{b}{3} + \frac{a}{6} \\ v_{3-i} &= v_i - \frac{a}{3} + \frac{b}{3} \end{aligned} \tag{4.39}$$

Figure 4.19 displays the relationship between the unconditional expected profits and the private values of buyers and sellers. The traders' private values have different influence on their unconditional expected profit. With the increase of the private value, the unconditional profits of the seller decrease and those of the buyer increase. We suppose that there is one point, on which there is no difference for the trader to choose bid or ask. Traders whose private value is lower than this point prefer asks than bids to achieve higher expected profits, vice versa.

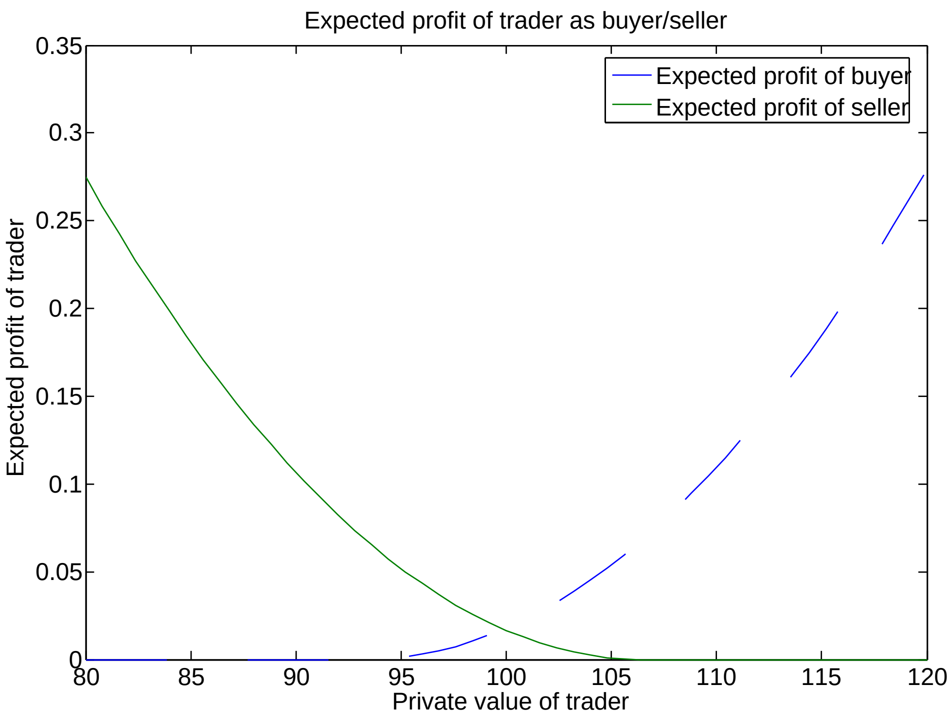


Figure 4.19: The relationship between the unconditional expected profit of trader as buyer/seller and private value

Table 4.1: The payoff matrix of two traders

		Trader 2	
		B ($\lambda_i^{3-i,b}$)	S ($1-\lambda_i^{3-i,b}$)
Trader 1	B ($\lambda_{3-i}^{i,b}$)	0, 0	$E(\pi_i, s_i), E(\pi_{3-i}, b_{3-i})$
	S ($1-\lambda_{3-i}^{i,b}$)	$E(\pi_i, b_i), E(\pi_{3-i}, s_{3-i})$	0, 0

4.3.3.2 The order side decision of the other one

There is a game-theoretical problem between two traders on their order side decisions. Here, I assume that the two traders have the same private value distribution. Each trader's payoff not only depends on his submission strategy, but also is decided by the submission strategy of the other one. If the other one's order side is same as the trader's order side, there is no any profits for the trader. If their order sides are different, their expected profits depend on what they choose. I use payoff matrix to solve the game theoretical problem.

Then, the payoff for trader 1 when he chooses to submit a bid: $(1-\lambda_i^{3-i,b}) * E(\pi_i, b_i) + \lambda_i^{3-i,b} * 0 = (1-\lambda_i^{3-i,b}) * E(\pi_i, b_i)$ The payoff for trader 1 as a seller: $(1-\lambda_i^{3-i,b}) * 0 + \lambda_i^{3-i,b} * E(\pi_i, s_i) = \lambda_i^{3-i,b} * E(\pi_i, s_i)$ To solve the Nash equilibrium matrix, when the two equations above equal to each other, there is no difference for trader 1 to buy or sell.

$$\begin{aligned} (1-\lambda_i^{3-i,b}) * E(\pi_i, b_i) &= \lambda_i^{3-i,b} * E(\pi_i, s_i) \\ E(\pi_i, b_i) - \lambda_i^{3-i,b} E(\pi_i, b_i) &= \lambda_i^{3-i,b}) E(\pi_i, s_i) \\ \lambda_i^{3-i,b} &= \frac{E(\pi_i,b_i)}{E(\pi_i,b_i)+E(\pi_i,s_i)} \\ \lambda_i^{3-i,b} &= \frac{45v_i^2-30bv_i-60av_i+20ab+20a^2+5b^2}{90v_i^2-90bv_i-90av_i+40ab+25a^2+25b^2} \end{aligned} \tag{4.40}$$

Because trader 2 is not informed with the reservation price of trader 1, he only knows the mean value of the reservation price of trader 1. Then $v_i = E(v_i) = \frac{a+b}{2}$. Therefore, the function 4.40 can be solved.

$$\begin{aligned} \lambda_i^{3-i,b} &= \frac{45a+45b-30ab-30b^2-60a^2-60ab+40ab+40a^2+10b^2}{90a+90b-90ab-90b^2-90a^2-90ab+80ab+50b^2+50a^2} \\ &= \frac{45a+45b-50ab-20b^2-20a^2}{90a+90b-100ab-40b^2-40a^2} \\ &= 0.5 \end{aligned} \tag{4.41}$$

Similarly, we have $\lambda_{3-i}^{i,b}$ equals 0.5 as well. This result is consistent with the previous results. The probability of the other one submitting a buy order expected by the trader is 0.5. Because these two traders are similar on every aspects, their order side choice should be half and half. Therefore, when the trader considers the other one's order side decision, he can suppose that the probability that his opponent will submit a bid or ask is half and half since their private value distributions are same.

4.3.3.3 The order side decision of the trader

The trader's order side decision depends on his private value and the order side decision of the other one. In the previous section, I prove that the probability of the other one submitting a bid is equal to 0.5 when their reservation price distributions are same. Then, the probability that the trader chooses to bid/ask can be obtained by comparing the trader's expected profits as buyer and seller. If we suppose that the trader prefers a bid than ask, there is a inequality below:

$$\begin{aligned} payoffofbuyer &> payoffofseller \\ (1-\lambda_i^{3-i,b}) * E(\pi_i, b_i) &> \lambda_i^{3-i,b} * E(\pi_i, s_i) \\ \lambda_i^{3-i,b} &> \frac{45v_i^2-30bv_i-60av_i+20ab+20a^2+5b^2}{90v_i^2-90bv_i-90av_i+40ab+25a^2+25b^2} \\ 0.5(90v_i^2-90bv_i-90av_i+40ab+25a^2+25b^2) &> 45v_i^2-30bv_i-60av_i+20ab+20a^2+5b^2 \\ 15av_i-15bv_i &> \frac{15a^2-15b^2}{2} \\ v_i &> \frac{a+b}{2} \end{aligned} \tag{4.42}$$

Therefore, the inequality shows that the trader prefers a bid than ask when his private value is higher than the middle value of the valuation distribution of the other one. He will submit an ask when his private value belongs to the lower valuation area, vice versa. On the point of mean value, there is no difference for him to buy or sell.

5

Two-person multi-period double auction model

In this chapter, I extend my models from single-period trading markets to multi-period trading markets to search the traders' submission strategies with information updating and the process of equilibrium transaction price formation. In double auction markets, traders submit their orders by their analysis of available private and public information. In the single-period model, the information available to traders is their private signal about the asset value and the opponent's private value distribution as public information. With the information, the traders would choose the order side, decide the price of their offers and submit the orders simultaneously to trade with each other. Although the static model provides useful insights into the traders' submission strategies in single-period trading markets, there are still some anomalies left, which require further research to understand the traders' behaviors in double auction markets. As we know, in financial markets, when the informed traders take advantages of their private information to obtain profits, their private information is revealed and captured by the markets though their order prices and volumes. The private information becomes available to the public with the continuous trading process. The high liquidity of the market is one of the important characteristics for assessing the efficiency of the market mechanism. Many researchers design dynamic models to search the traders' behaviors in the multi-period trading process and observe the process of market liquidity. In my research, the dynamic model in multi-period trading markets, which is designed to search the traders' submission strategies in multi-periods, makes my research closer to the realistic financial markets.

Under the condition of multi-period trading, I focus on the process of information updating of traders and the process of information disclosure. It is important to clarify the different strategies of traders with previous trading information, which is different from the strategies of traders in the single-period trading. Furthermore, understangding the equilibrium price formation process would help us understand the efficiency of the double auction mechanism. This multi-period model is constructed in a double auction market with two informed traders under similar assumptions like single-period model. However, in this model, trades would be repeated a finite number of times and the previous transaction data is available to the public. I expect that the multi-period model would not only reveal the relationship between the offer prices and the parameters, which affect the submission strategies of traders, but also clarify the process of information disclosure in double auction markets.

5.1 The description of the model

I design the multi-period double auction market similar to the single-period double auction market before, where there are only two informed traders to trade one unit of asset. At the beginning of the first trade, individuals receive their private signal of the asset value and are informed of the value distributions of their opponents. Similar to the single-period model, several assumptions are applied to simplify my analysis: the risk-neutral informed traders are rational, whose trading purpose is to maximize their profits; the private value distributions follow normal distributions; and there is no tick size. Their own private values are constant during the whole trading periods, because there is no new private information any more. Traders' submission strategies are based on their analysis of their own reservation price and the value distribution of the other one.

Different from single-period auction markets, traders could submit orders repeatedly whatever the transaction happens or not. They are allowed to adjust their offer prices to make trade happen or attain more profits according to their private information. Although one condition for transaction happening is that one private value is higher/lower than the other one, in my model, transactions are not guaranteed even if the difference between two traders' private values is significant. The multi-period trading provide the opportunity for traders to increase the execution probability by changing order sides or offer prices. On the other hand, because of the uncertainty of their private value to the other ones, they could make profit by their private information until their private information is revealed. At the end of each stages of trade, the offer prices of traders are available to everyone as the new information to make reassessment of the other one's valuation. Then traders could adjust their submission strategies according to all the information available. In the multi-period model, after each round of trade, there is an additional stage of belief updating. For example, trader 1 updates his belief on the value distribution of the other ones. He understands that the offer price of the other one is based on his private value and his assessment of the value distribution of trader 1. Therefore, the new distribution of trader 2's valuation regarded by trader 1 is the previous value distribution of trader 2 conditional with his latest offer price. Then at the beginning of next period of trade, trader 1 submits a new order according to his private valuation and the latest distribution of trader 2's valuation regarded by him. Next stage, two offers match with each other following the trading rule that two orders are on the different sides and bid price is no less than ask price. The rounds are repeated again and again until the private information of traders is revealed.

When two traders enter into the market initially, they face similar circumstance as the ones in the single-period market. They hold their own private information of asset value and make assessment of the opponent's valuation by his value distribution. At the end of the first trade, the transaction data, including the offer prices and transaction price if trade happened, is available to traders to update their beliefs about the other one's private value distribution. In every period t, I denote trader 1's private valuation distribution has the expected value $\mu_{i,v}^{t}(t=0)$ and variance $\sigma_{i,v}^{t}$ and trader 2's valuation distribution has the expected value $\mu_{3-i,v}^{t}$ and variance $\sigma_{3-i,v}^{t}$.

Then, the valuation distributions of traders regarded by the other one at time interval $t+1$ are equal to the valuation distributions at t conditional on his offer prices at t. For example, if trader 1 submits a sell order s at t, the expected value of the value distribution regarded by trader 2 should be:

$$\mu_{i,v}^{t+1} = \mu_{i,v}^{t} + \frac{cov(v_i, s)(s - \mu_{i,s}^{t})}{(\sigma_{i,s}^{t})^2} \tag{5.1}$$

Similarly, when trader 1 submits a bid at time t, the expected value and the variance of the value distribution of the bids are $\mu_{i,b}^{t}$ and $\sigma_{i,b}^{t}$. Then, the new conditional expected value of the value distribution regarded by trader 2 is:

$$\mu_{i,v}^{t+1} = \mu_{i,v}^{t} + \frac{cov(v_i, b)(b - \mu_{i,b}^{t})}{(\sigma_{i,b}^{t})^2} \tag{5.2}$$

On the other side of the market, trader 2's value distributions regarded by trader 1 are similar to trader 1's valuation distributions. The latest valuation distributions of trader 2 are equal to the previous valuation distributions conditional on his offer price. It is the updated value distribution of trader 2 which is considered by trader 1 to assess the private value of trader 2 when he makes his decision. The function of the conditional valuation distribution of trader 2 when he submits an ask is below:

$$\mu_{3-i,v}^{t+1} = \mu_{3-i,v}^{t} + \frac{cov(v_{3-i}, s)(s - \mu_{3-i,s}^{t})}{(\sigma_{3-i,s}^{t})^2} \tag{5.3}$$

The conditional expected value if trader 2 choose to be a buyer is:

$$\mu_{3-i,v}^{t+1} = \mu_{3-i,v}^{t} + \frac{cov(v_{3-i}, b)(b - \mu_{3-i,b}^{t})}{(\sigma_{3-i,b}^{t})^2} \tag{5.4}$$

At the beginning of each stage (except stage 1), traders would use the new information of the offer price at the previous stage to update his belief on the valuation distribution of the other one. The valuation distributions of the opponents is the only information for traders to make assessment of the submission strategies of the other one. As the functions of the updated expected values of the valuation distributions above, the valuation distributions of the opponents are updated after every trade with the arrival of the new information from the trade. I use numerical method to investigate the belief updating process of the traders and the change of their submission strategies. The relationship among the main parameters that affect the submission strategies of traders and the offer prices of traders could be observed by this method.

5.2 Results and implications

5.2.1 the relationship among the order prices and the variables

I am interested in clarifying the relationship between the offer prices and the factors that influence the submission strategies of traders. Similar to the single-stage model, there are two factors that influence the submission strategies of traders: the private value of traders and the value distribution of the other one. The private value of traders is stationary in the trading process as there is no new private information added. On the other hand, the traders update their belief on the value distribution of the other one.

Result 1: The private values of traders have positive effects on the traders' offer prices whether they are sellers or buyers during the whole trading process.

As informed traders, the private valuations of traders are the most important factor that decide the order prices of traders either in the single-period trading or the multi-period trading. As this is a private value model other than common value model, the private value of traders is stationary during the process of continuous trading. The results in the single-period model show that the relationship between the private values and the offer prices of traders is positive for all types of orders. Figures 5.1 and 5.2 display the relationship between the offer prices and the private values in the multi-stage trading for bids and asks separately. I find that the order prices increase with the increase of private value for buyers as well as for sellers. The linear positive relationship is observed at each stage in multi-stage trading as their regulation to assess optimal order prices and their private values do not change when the trades are repeated. The figures also demonstrate the change rate of the order prices decreasing as the number of stages increases. At the beginning of the trading stages, traders expect to earn more from their private value. However, they need adjust their order prices rapidly as the disclosure process of their private information. On the latter stages, because there is only a smaller difference between their private values and their order prices, buyers slowly increase their bids and sellers decrease their asks less. As the continues trading process, the traders' private information are fully revealed and their optimal strategies are to submit orders that equals to their private values.

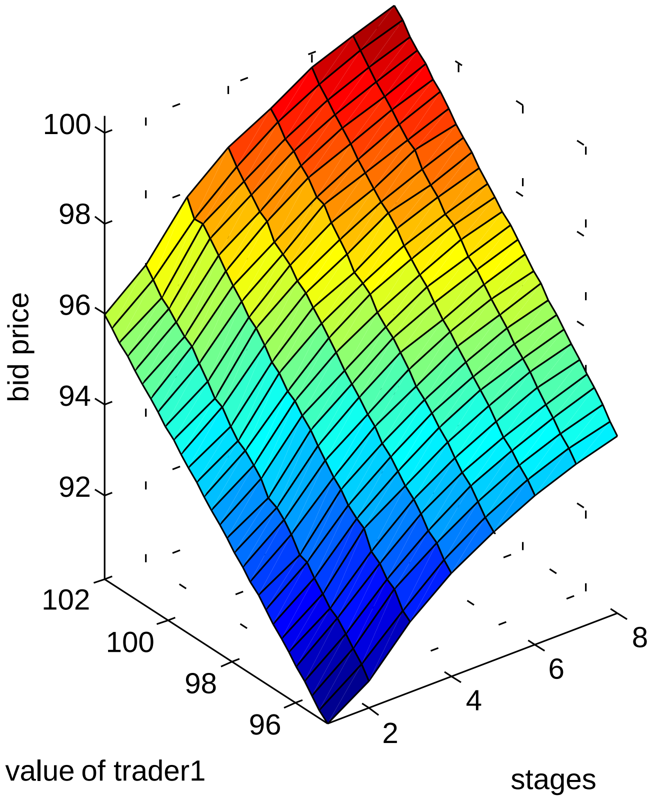


Figure 5.1: The relationship between the private value of trader 1 and his bid prices note: $v_i = [95, 102]$; $v_{3-i} = 98$; $\mu_i = 100$; $\mu_{3-i} = 100$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

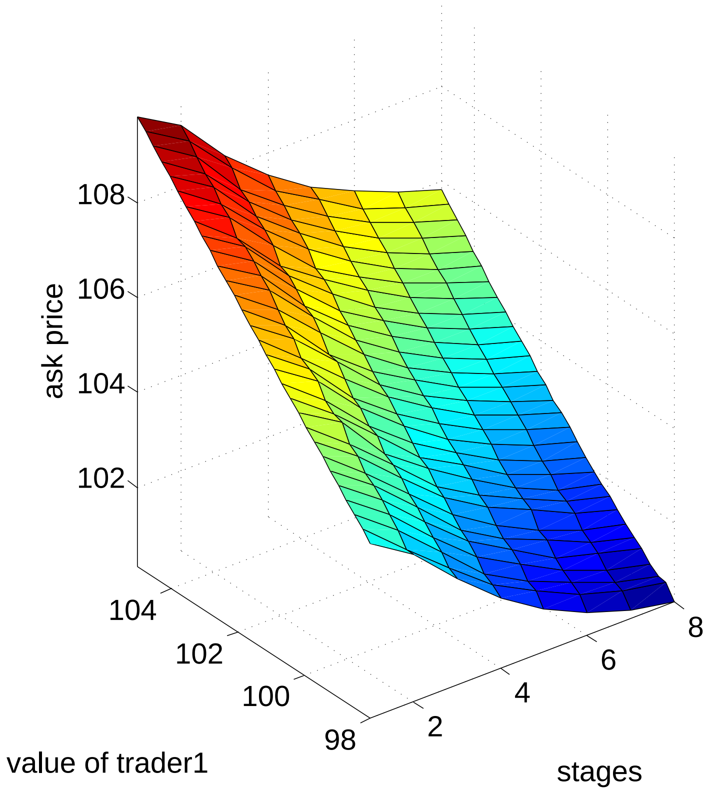


Figure 5.2: The relationship between the private value of trader 1 and his ask prices note: $v_i = [98, 105]$; $v_{3-i} = 102$; $\mu_i = 100$; $\mu_{3-i} = 100$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

Result 2: At the first time of trading, the trader's offer prices are positively related to the initial expected value of the opponents' valuation distribution regarded by the trader. Whereas, at the other stages of trading, the trader's offer price decision is irrelevant to the initial expected value of the other one's valuation distribution.

The expected values of the opponents' valuation distributions are the only information for traders to make assessment of the submission strategies of the other one. In the single-period model, the expected value of the valuation distribution of the other one has positive relationship with the offer prices of the traders whether they buy or sell. The traders increase their offer prices when they expect that the opponents would submit an order with high price because of the high expected value of their valuation distribution, vice versa. In the multi-stage trading, at the first time of trading, the traders submission strategies are similar to their strategies in the sing-stage trading. The expected value of the valuation distribution of the other one has positive effect on the trader's offer price. Figures 5.3 and 5.4 display the relationship between the offer prices and the expected values of the other one's valuation distribution for bids and asks over stages separately. The positive relationship between the initial expected value and the offer prices demonstrate that traders move their offers in the same direction with the change of the initial expected value at the first time of trade. Then, traders alter their beliefs of the expected values of the opponents' valuation distributions according to the information of the offer prices of the opponents, such that the initial expected values of the opponents' valuation distributions are irrelevant to the traders' offer price at the later stages of trading. Compared with the relationship of the private values, the slope of the expected value of the valuation distribution is flatter than that of the private values. The difference, which is similar to the explanation in the single-period market, is caused by the higher power of the private values to decide the offer price. Moreover, the changes of the offer prices over stages show that, to increase the execution probability, buyers increase bids and sellers decrease asks with the number of stages increases.

5.2.2 The changes of the order prices over stages

In the repeated trading game, the traders could adjust their offer prices to increase the execution probability, which in turn increase their expected profits under the new

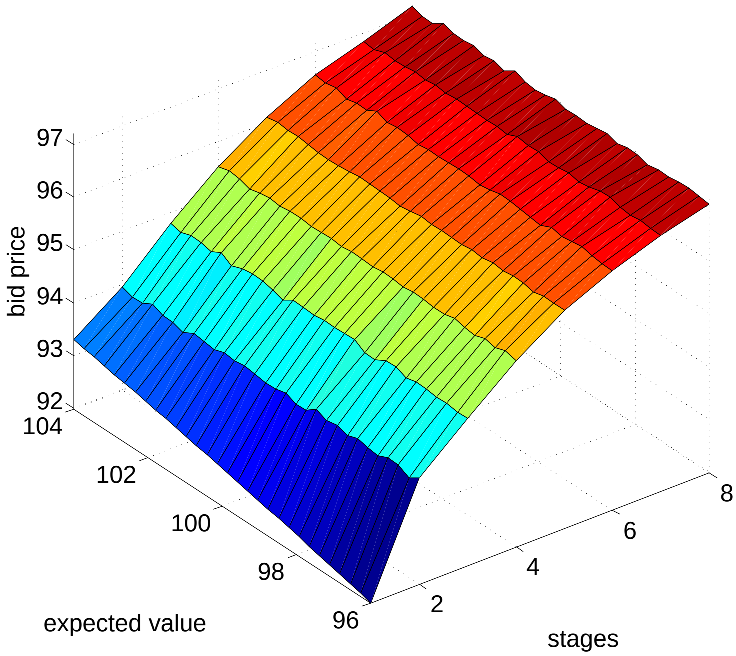


Figure 5.3: The relationship between the expected value of trader 1 and his bid price
Note: $v_{i,b} = 102$; $v_{3-i,s} = 98$; $\mu_i = \mu_{3-i} = [98, 105]$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

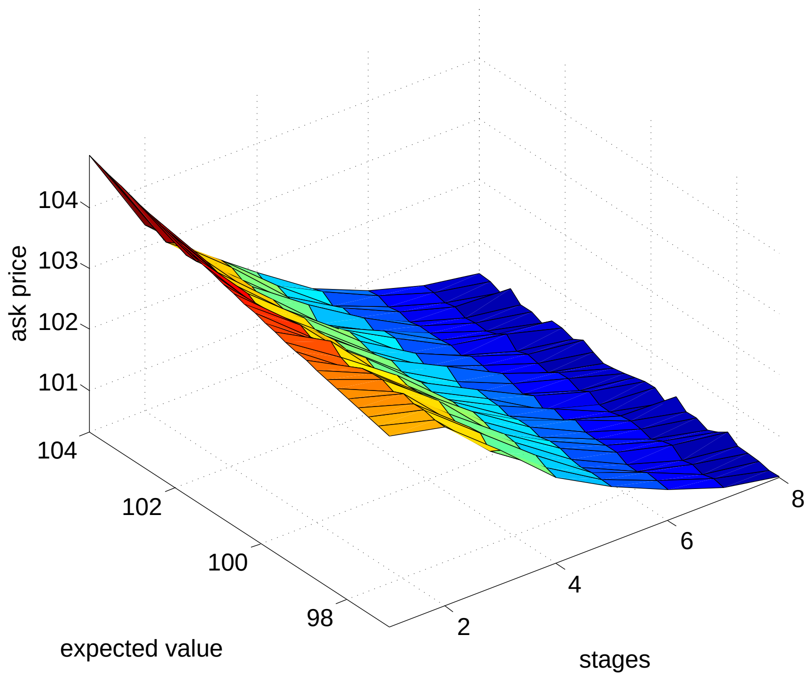


Figure 5.4: The relationship between the expected value of trader 1 and his ask price

Note: $v_{i,b} = 102$; $v_{3-i,s} = 98$; $\mu_i = \mu_{3-i} = [98, 105]$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

conditions. However, the changes of the offer prices are different because of the different relationship between the traders' private valuations. I classify the relationship between the buyers and sellers into two types: type I The ask price is higher than the bid price at the first stage because the valuation of the buyer is lower or slightly higher than that of seller. type II The ask price is lower than the bid price at the first stage because the valuation of the buyer is much higher than that of seller. Figure 5.5 and 5.6 display the changes of the offer prices for two types separately. From these two figures, I could tell the difference between them and the common implications behind them. In figure 5.5, the buyer increases his offer price to match the offer of the other one and the seller decreases his offer price to increase the execution probability on the other side over stages. During the process of repeated trading, the two offer prices cross at one point to make trade happen. In the process of continuous trading, the traders continually submit orders with prices closer to their redemption prices until their private values are revealed. For type II, at the first stage, as the reservation price of buyer is much higher than that of seller, the bid price is higher than the ask price and the transaction happens. Then traders would trade with the other ones continually and update their beliefs of the other ones' reservation prices as well. Both of them understand that they could obtain profits by trading until their private information is revealed completely. Figure 5.6 displays the process that traders' private information are revealed over stages after transactions occur. Specially, I expect that, similar to the traders' behavior in type I, traders decrease the difference between their private value and their order prices to increase their execution probability.

5.2.3 The change of the expected value of opponents' valuation distribution regarded by the traders

The expected values of the opponents' valuation distribution regarded by the traders are updated by the traders during the repeated trading process. By updating their beliefs of the other one's valuation distribution, the traders understand the private information of the opponents better and better. One of the functions of repeated trading is about information dissemination. The private information of the informed traders could be revealed fully by their orders during the process of trading. Even in the simple double auction market with two traders, the process of information dissemination is

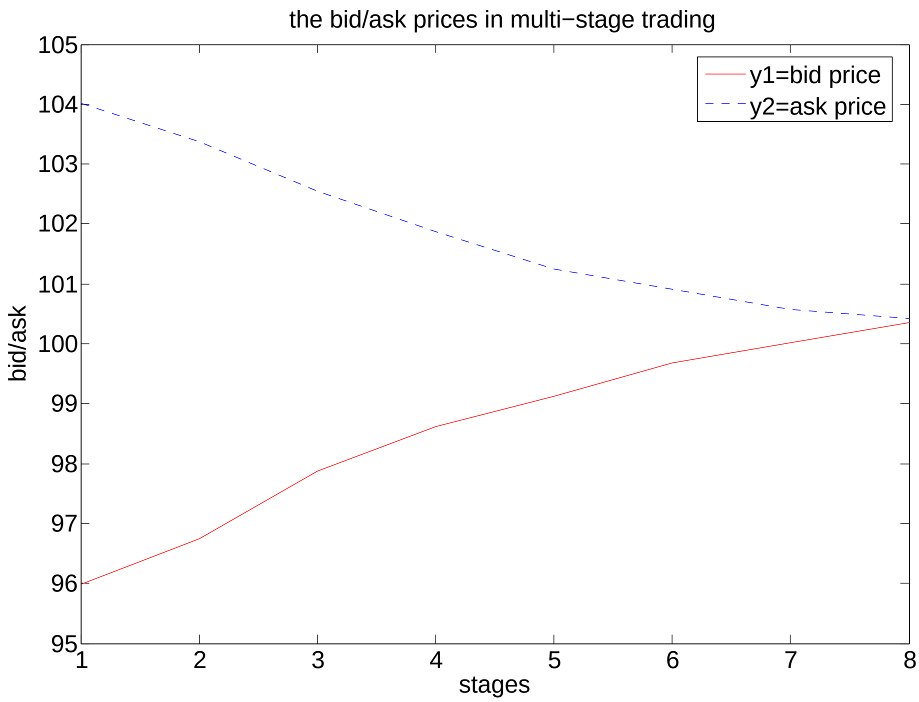


Figure 5.5: The bid and ask prices in the multi stage trading (type 1)
note: $v_{i,b} = 102$; $v_{3-i,s} = 98$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

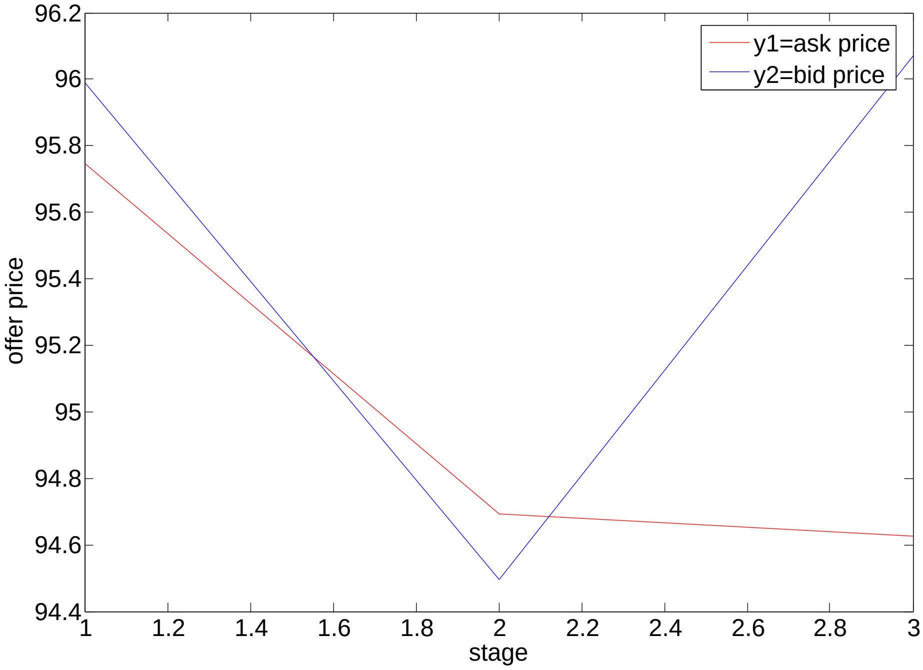


Figure 5.6: The bid and ask prices in the multi stage trading(type 2) note: $v_{i,b} = 102$; $v_{3-i,s} = 87$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = 15$; $\sigma_{3-i} = 15$

observed as the result of the game-competition between two traders. During the belief updating process, the valuation distribution of the opponent regarded by the trader should converge on his private value. Figure 5.7 shows the change of the mean values of the buyer's and seller's valuation distribution regarded by their opponents. In this figure, I observe that the expected values of the trader's valuation distribution regarded by the other one swing around the reservation price over stages and converge on his private valuation. On the other hand, because the variance of the conditional normal distribution is always smaller than the unconditional normal distribution, the variance of the valuation distributions decrease continuously with the increasing number of trading. We could figure out that, with repeated trading, the shape of the density function of the valuation distributions becomes steep with low variance until all the points of the valuation distributions concentrate on the line of expected value. Therefore, at last, the traders exactly know the opponents' private valuation and the two sides uncertainty in the markets diminishes.

5.2.4 The traders' submission strategies over stages with order side change

If the traders' decision of order sides is considered in the multi-period trading, there is a process that traders change their order sides into one buy and one sell before the process of order price converging. The traders would submit the same sides of orders under two situations: their private values are all lower than the mean values of valuation distributions and their private values are higher than the mean values of valuation distributions. After the traders submitting the two bids/asks at the first stage of trading, they would update their beliefs on the other ones' valuation distributions. Their updating behaviors make the valuation distributions of the other ones regarded by them diverge from each other in the process of repeated trading. Then, when one trader observe that his private value is lower than the mean value of his opponent's valuation distribution, he would change his order side to maximize his profits. When there are one bid and one ask on markets, the submission strategies of traders are similar with the ones without including the decision of order sides. Figure 5.8 displays the traders' submission strategies over stages when they could change their order side. We observe that the trader with lower private value change his order side to ask at the

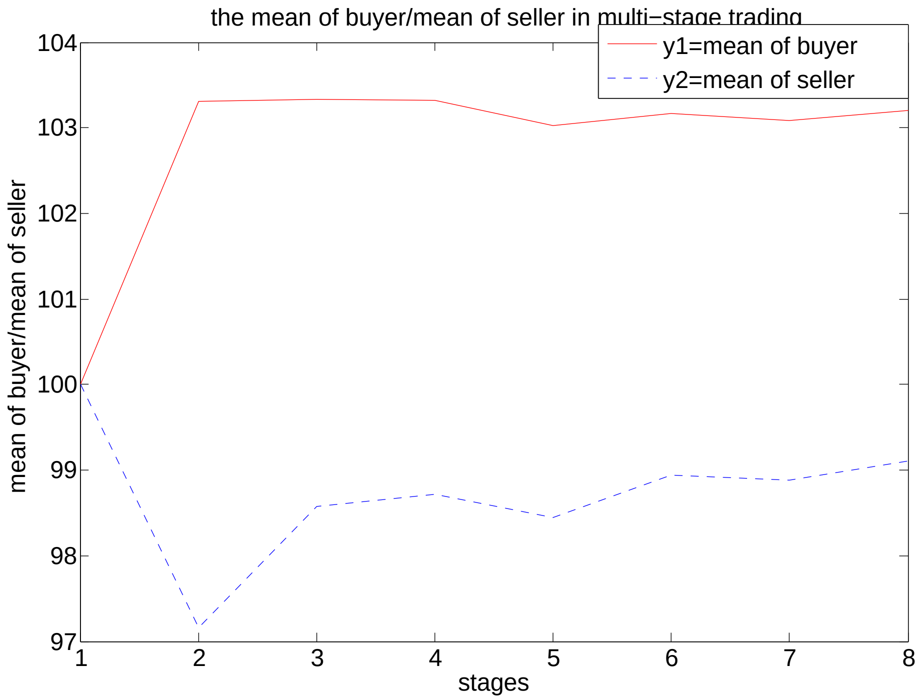


Figure 5.7: The mean value of the buyer/seller in the multi-stage trading note: $v_{i,b} = 102$; $v_{3-i,s} = 98$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = \sigma_{3-i} = 15$

second stage of trading. However, his ask is much higher than the bid. Then these two order prices converge in the later stages in the process of repeated trading. Meanwhile, their beliefs on their opponents valuation distribution become closer to their private values. Table 5.1 show the change of the order prices and decisions, the expected values of valuation distribution of two traders over stages.

Figure 5.8: The submission strategies of traders over stages
note: $v_{i,b} = 102$; $v_{3-i,s} = 98$; $\mu_i = \mu_{3-i} = 100$; $\sigma_i = \sigma_{3-i} = 15$

To sum up, I conclude the traders submission strategies in the multi-period model as following:

1. The trader increases his expectation on the mean value of his opponent's valuation distribution if the other one is a buyer, and decrease his valuation on the other one's mean value when the other one as a seller.

Table 5.1: The traders' order submission strategies over stages

	Stages					
	1	2	3	4	5	6
Order side-1	bid	bid	bid	bid	bid	bid
Order side-2	bid	ask	ask	ask	ask	ask
Order price-1	100.19	101.55	102.03	102.61	102.96	103.25
Order price-2	97.056	104.91	104.47	103.98	103.54	103.35
Expected value-1	100	105.89	106.62	106.21	105.72	105.9
Expected value-2	100	102.11	101.3	101.98	101.96	101.89

Note: private value-1=105; private value-2=101; expected value=100; variance=10

2. The trader submits a sell order when his private value is lower than the mean value of the other one's valuation distributions and a buy order when his private value is higher than the other one's mean value.

3. If the two traders submit the different sides of orders and the bid price is lower than ask, then two order prices converge gradually in the subsequent stages.

4. If the two traders submit two different sides of orders and the bid price is higher than ask, the buyer would increase his order continually and the seller would decrease his ask until their order prices is equal to their private values.

5. If the two traders submit the same side of orders, the one with lower (higher) private value would change his order side to ask (bid) in the later stages.

5.2.5 The comparison between the single-period model and the first-stage of multi-period model

The trader's submission strategies depend on his judgement on the information available to him either in the single-period market or in the multi-period market. In the single-period model, besides his private value, the trader is informed of the valuation distribution of his opponent's valuation distribution which follows normal distribution. There is no historical transaction data or any other useful information for the trader to assess the situation of his opponent. His optimal offer prices following the function 4.13 and 4.6 for buyer and seller.

At the first-stage of multi-stage market, the trader has his own private value and is informed of the opponent's valuation distribution. His initial situation at the first stage of trading is similar to the one in the single-period model. They need make his decision according to his private value and the initial value distribution of his opponent. By comparing the trader's submission behaviors in the single-period model and the ones at the first stage of the multi-period model, we link the single-period model to the multi-period model to understand the trader's submission behaviors.

Figure 5.9 and 5.10 display the relationship between the order prices in the single-period model and the ones at the first stage of the multi-period model when all the other variables are fixed. I observe that the slope of the fitted curve is on 45 degree. The result indicates the order prices in the single-period model are identical to the offer prices in the multi-period model when the other variables are the same. In another word, in theory, when all the other condition is same, the trader would adopt the same submission strategies, either he is in the single-period market or he is at the first stage of the multi-period market. The intuition behind this result is that the trader at the first stage of the multi-period model face the same information as what he has in the single-stage model. In my model, the traders only trade one unit of assets and maximize their profits at each stage, their order decision are not affected by their expectation in the future. Meanwhile, there is not any other cost related to the stages. In a word, the traders' submission strategies are only affected by the information at previous stage and not influenced by the circumstance in the future. Therefore, at the first stage of multi-period market, the information at the previous stage is equaled to zero, the traders' submission strategies is similar to their behaviors in the single-stage market.

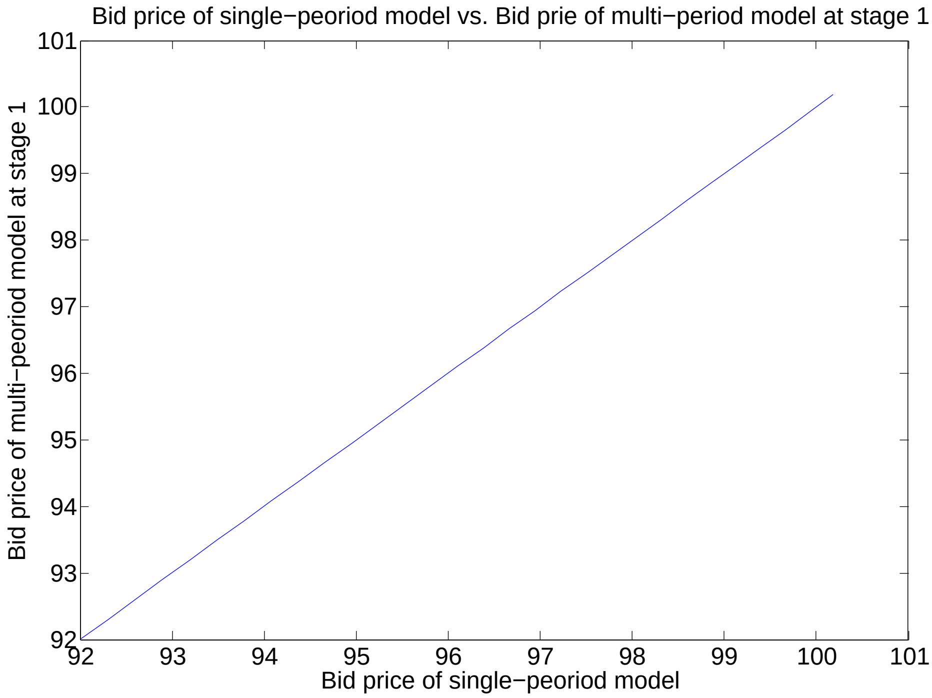


Figure 5.9: Bid prices of single-period model vs Bid prices at the first stage of multi-period model

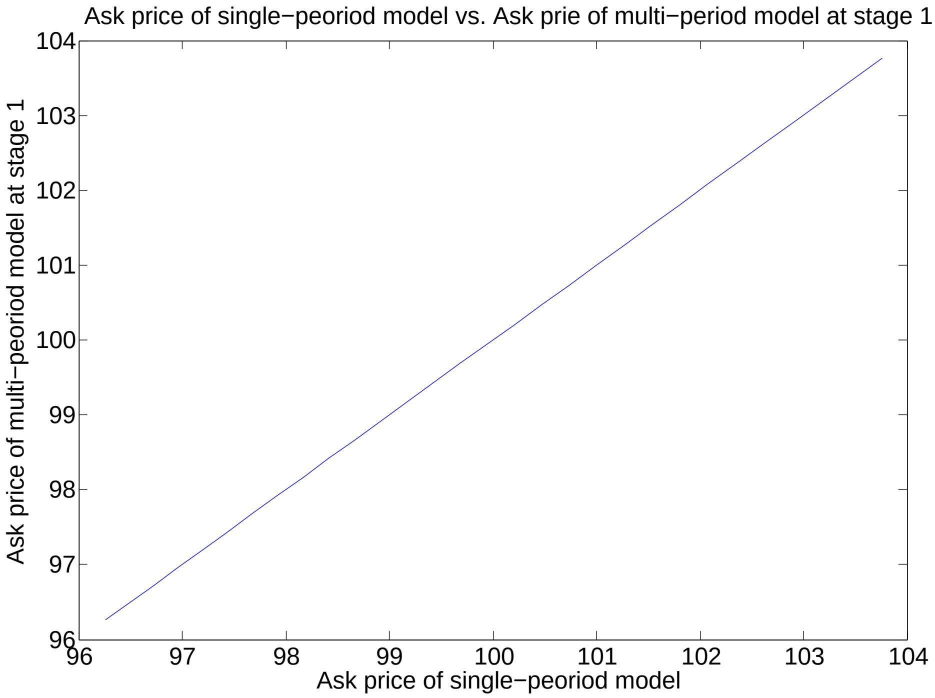


Figure 5.10: Ask prices of single-period model vs Ask prices at the first stage of multi-period model

6

Experiment

Before this chapter, I present the theoretical models where two-informed traders trading in a simple double auction market. The theoretical models display the informed traders' competition and submission strategies in double auction markets, also they show the process of private information dispersion. Based on these theoretical models, I design the experiments to research the traders' submission strategies in the double auction markets under experimental environment. Experiment is one of important methodologies in financial markets. Under the experimental environment, researchers are easier to observe the objects' behaviors and obtain financial data conditional with various specified conditions and limitations. It has been widely used to discover various financial phenomena and test the efficiency of theoretical models by providing empirical supports. There are two benefits of experiments in my research: on the one hand, the experimental results could provide useful supports to verify the efficiency of my theoretical models; on the other hand, under the experimental environment, by reducing the difference between my theoretical model and the real world, we could better understand the traders' behaviors in double auction markets and capture other important parameters that affect the traders' submission strategies. To compare the results of the experiment and theoretical model, it is important to construct the experimental environments following the requirements of theoretical models, . Whereas there are still some factors that could not be controlled in the experiments, such as the irrational behaviors of the subjects. I design the experiments carefully to simulate the procedures and conditions of theoretical model and minimize the difference between them.

The potential participants are the students from management and economic departments, who have some basic knowledge of financial markets. In the experimental markets, the subjects acted as the informed traders and traded with their opponents via the computer network following the specifical trading rules. They were divided into pairs automatically by the computer programm and each pair of traders submit their seal orders simultaneously. The computer system recorded every pairs of transaction data such as order sides, offer prices, transaction prices etc. The experimental data is aggregated and analyze to investigate the trading behaviors of subjects in the experimental markets. The experimental results would be compared with the theoretical results to clarify the different and common grounds of traders' submission behaviors in double auction markets. Not only the auction theory but also the behavioral finance theory would be involved to explain the traders' behaviors in the experimental markets.

6.1 Selection and background of subjects

I do the experiments based on the facilitation of University of Bath. The experiments are held in large computer rooms in the university. The volunteers are students from School of Management and Department of Economy. The computer program is the software from Comlabgames.com which is designed to conduct experiments with human subjects over via Internet.

In the theoretical model, I assume that the traders are rational investors, who maximize their profits in their trading action. It is expected that the person who has higher level of knowledge on financial markets can act more rational and professional. The most suitable subjects are the students in Accounting and Finance. Because the feedback rate of other projects is normally about 10% and there are only around one hundred students in this subject. I extend the range of potential participants to all the students from the undergraduate and postgraduate programs in School of Management and Economic department. Then the number of potential participants is over 1000, and these students are expected to have basic knowledge of financial markets.

I invite the students to participate in the trading game by emails. I sent an email to all the students in the two departments to invite them to participant this trading game. In this email, I said that for the research purpose on the trading behavior of traders in financial markets, a trading game would be held and call out participants. To encourage the students to participate, I specified that volunteers only need know basic knowledge of computer and no special professional skill required. I also provided the experiment timetable, in which several sessions are available for students to choose according to their timetable. These sessions would be held during two weeks. Each student is asked to make his first and second choice of the sessions in order to facilitate my arrangement on the number of students for each session. This email was sent again a few weeks later to inform the students who ignore the last email. Meanwhile, handouts were distributed in some classes to attract more participants. One hundred and twelve students show their interests to participate by email or by forms of handouts. The percentage of response is about 10%.

I allocate the students into the sessions according to their preference. The students from different subjects are mixed in the same secession to cancel out the potential effects of their background. The numbers of students expected to participate for each section are 38, 45, 29 respectively. In each session, there are undergraduate students and postgraduate students from different programs. We prepared one bilingual session, for international students, in which the introduction would be presented in both Chinese and English. After allocating all the volunteers into the sessions, another email was sent to inform the students of time and location. One day before each session, I also sent an email to remind the students again of all the details of the trading game respectively. Due to the reason of midsemester and others, the actual numbers of participants available for each session are 42,34,26.

The experiments are held in the same computer room to provide similar experimental condition. The large computer room provides enough spaces for students, in which normally students could not know who are their opponents. To run the game, one computer is used as the host computer to construct the whole network. The students could log in to play the game from any other computers. There is no any other specified requirement for the participants. To keep the participants active, each session are controlled around 1.5 hours.

At the beginning of each session, I gave a short introduction of this trading game. The introduction is presented to the participants as following. We simulate a simple double auction market. All the participants would be allocated in pairs to trade by the computer automatically. They act as informed traders to trade one unit of asset with his opponent. They do not need to know their opponents as the computer would reallocate the groups after the end of trade. They would be informed their private values, which is true value of the asset by their beliefs. Meanwhile, they would be also informed of their opponents' valuation distribution, from which their opponents' private values are drawn. The valuation distributions follow normal distributions and the mean values and variances of normal distributions are drawn from uniform distributions. They know that their opponents could also make assessment on their submission strategies by their valuation distributions. According to their information, first, they need to choose their order sides. Then they need to decide their order prices. These two sealed orders are submitted simultaneously to match with each other by computers. The transaction happens only if there are one bid and one ask, and the bid price is higher than ask price. The transaction price is the mean value of the bid price and ask price. Their profits are the difference between their offers and the related transaction price. It is equal to the private value minus the transaction price for buyers and the transaction price minus the private value for sellers. In the single-period trading game, they could only submit their orders once with the series of information. In the multi-period trading game, they play the trade several rounds with the same information. The multi-trading game provide their the chance to improve their orders to make trade happen.

6.2 Design of the experiments

Before the formal experiments, a pre-experiment was held to test the procedures of the trading game. The pre-experiment provides useful implication to improve the experiment design. First, I find that the number of the participants in each session should be controlled in a proper range. If there are too many participants in one session, the main computer becomes very slow to process the whole system. Second, variances should be drawn in a wide range to attract the attention of students. In the pre-experiment, the variances in a small range have not too much influence on the students' submission strategies. Therefore, in the large experiment, I controlled the number of participants in each sessions less than 50 and enlarge the range of uniform distribution for variances.

I design two experiments: the single-period trading game and the multi-period trading game, corresponding to the single-period model and the multi-period model. In each session, after the introduction, the participants play the preparatory game without the uncertainty of their opponents several times to familiar the computer programm and the conceptions in games well. Then the participants trade under the single-period trading game several rounds with different private values and valuation distributions each round. The multi-period trading game is played after the single-period game. Because of the time limitation, the multi-period trading game is only held in the last two sessions.

6.2.1 Experiment for single-period trading

In the single-period trading game, the computer would draw out private values and mean values and variances of valuation distributions at the beginning of every round. The students are allocated to couples automatically and randomly by the computer. The students could access these information during the whole process of trading. At the end of each round, the computer reports the transaction data: the order prices, the transaction prices and the profits of two parts. When the last round ends and the new round begins, the students would face the different series of information provided by the computers. Normally, the students are asked to process five rounds of single-period trading.

First, the students are informed of his private value about the asset, the mean value and variance of his opponent's valuation distribution. The mean values of the valuation distributions were drawn from a uniform distribution on the interval $[80, 120]$ and the variances of the valuation distributions were drawn from a uniform distribution on the inter val $[2, 15]$. The introduction page 6.1 shows the participants all the information available and initiate their decision process.

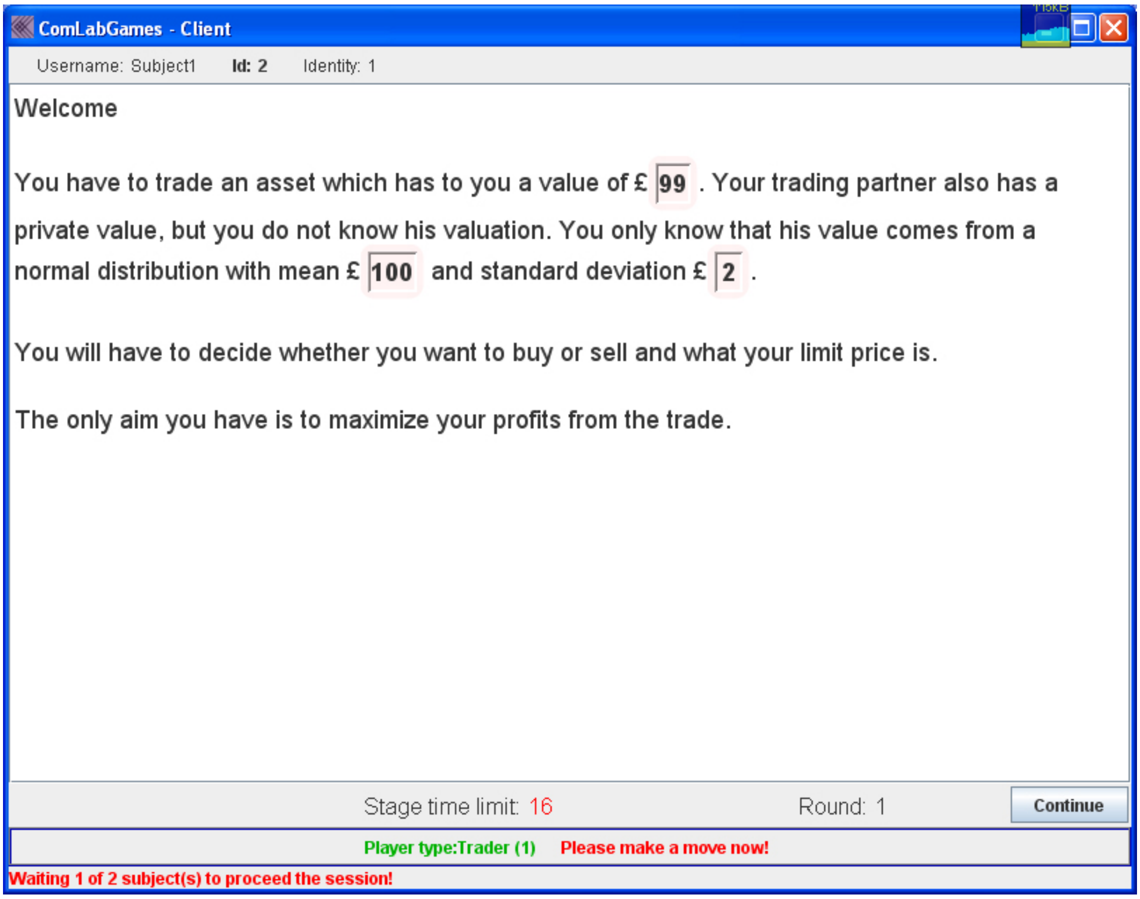


Figure 6.1: The introduction page for the experiment

Second, the students entered into the decision process. The first step is to make their decision on the order side according to the information they have. It is a common sense that, to make a profit, bid prices should be lower than reservation prices of buyers and ask prices should be higher than reservation prices of sellers. According to the results of theoretical model, the trader prefers a bid(ask) when his private value is higher(less) than the mean value of his opponent's valuation distribution. For the students' convenience to compare these two values, in the page of stage 1 6.2, we represent their private values and the valuation distributions of their opponents. According to the information, the students need to make their choice from two optional- buy or sell. Their decision of order sides could not change any more after this stage. Furthermore, their order price choices are base on their decision at this stage as well.

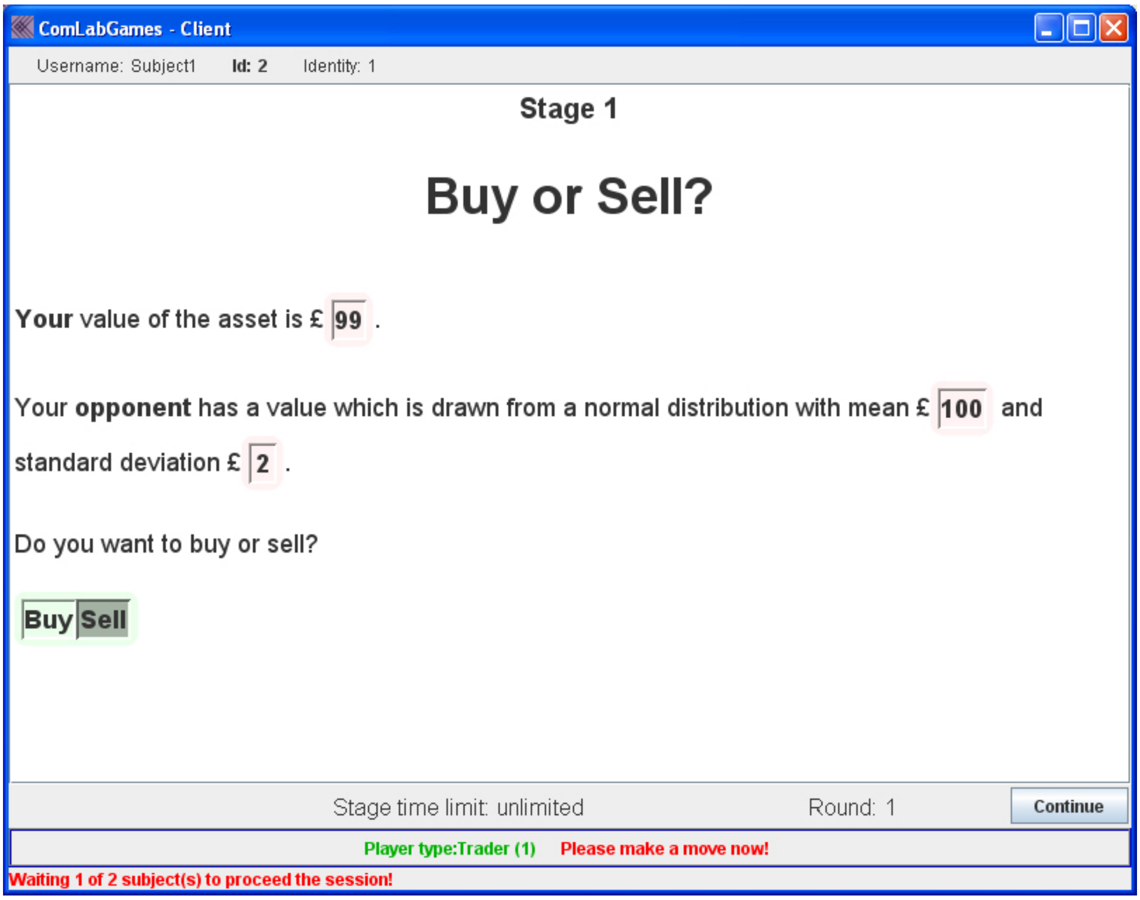


Figure 6.2: Stage 1 - order choice

Third, at this stage, the students need to make their decision on their offer prices, to maximize their profits. All the available information and their decision of order sides are listed in page 3 (see figure 6.3) above their offer price choices. Based on their decision of order sides, the students are required to fill the blank their bid/ask prices according to their preference . Before they press 'enter' key to confirm their order prices, they could change it in the blank. The order price decision is more complex than the decision of order sides, which is expected to spend more time to finish. The basic principle to make profit is that buyers should submit orders with the prices lower than their reservation prices and sellers should submit orders with the prices higher than their reservation price [1]. It is obviously that the students face the choice between the improvement on the order price and the execution probability. Lower (higher) bid(ask) prices give lower execution probability, which in turn reduce the execution probability. On the other hand, higher(lower) bids(asks) decrease the difference between the transaction prices and their order prices, and reduce the expected profits as well.

At last, the computer system received the orders of two traders and matched the orders automatically to give the last report. In the last page(see figure 6.4), besides indicating the relevant information, the computer reported the order prices of two traders and transaction result. If the transaction happened, the transaction price is the mean value of two orders and the profits of traders are the difference between their private values and transaction prices. If the transaction did not happen, the transaction is equal to zero and the profits of two traders are equal to zero as well. We provide these information to the students for their better understanding of the trading game and their behaviors. In the single-period model, these information is not used for the traders' trading decision in the later round because the private values and valuation distributions in each round are independent of the ones in other rounds. Therefore, the results of their trading behaviors have no effect on their later behaviors such that their trading purpose is to maximize their profits in the present trade.

[1] In the experiments, most students understood this principle properly, although we still found that few students submitted orders to make negative profits. It is the evidence of the existence of irrational behaviors in our experiments.

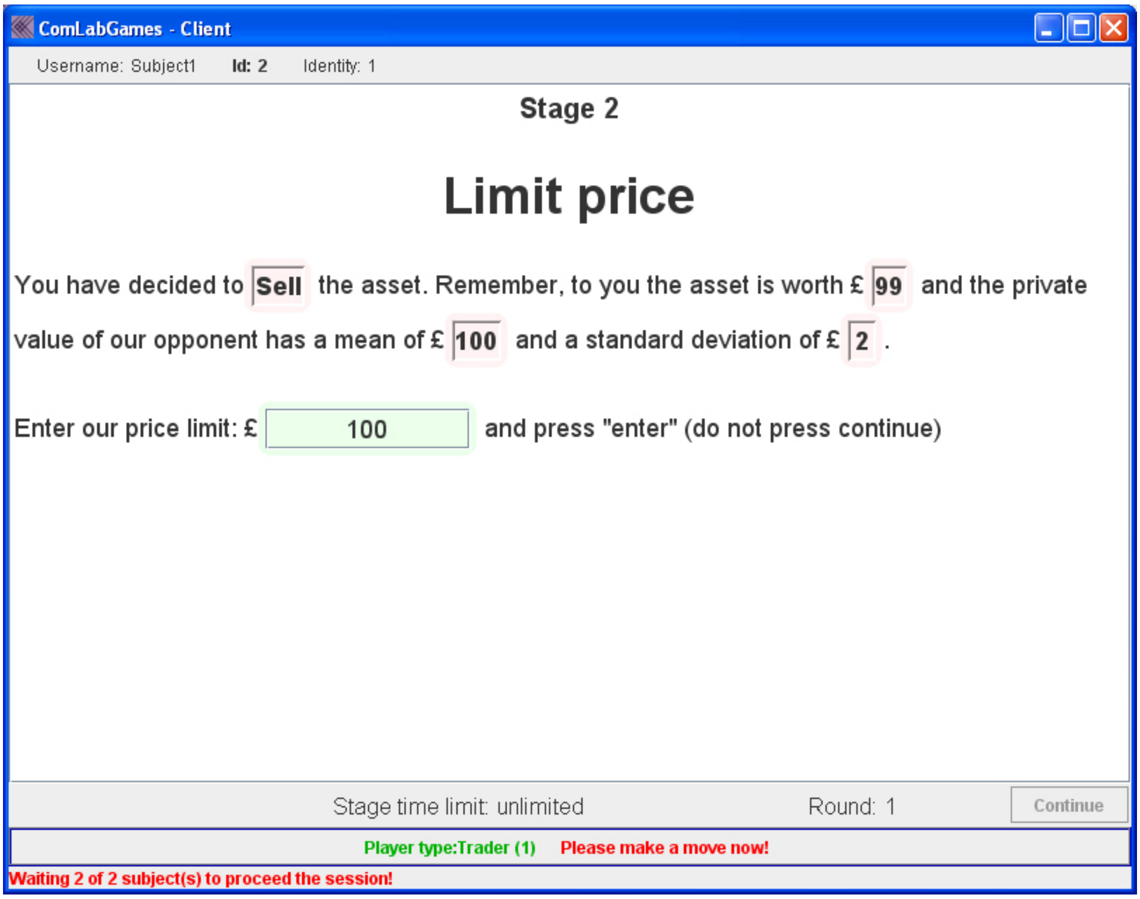


Figure 6.3: Stage 2 - order price decision

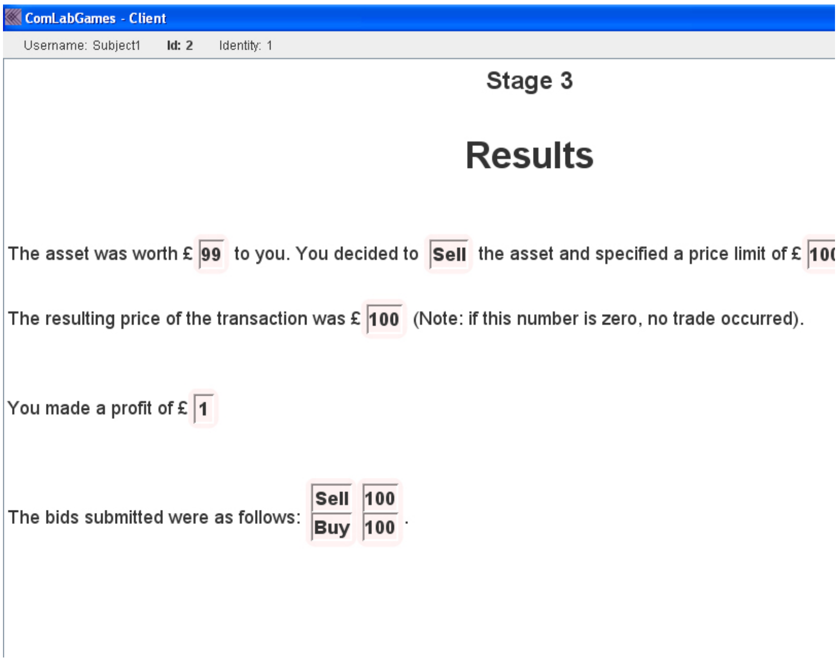


Figure 6.4: Stage 3 - transaction price and profit

The single-period experiments is designed to observe the submission strategies of participants in the single-period trading. Comparing with the corresponding single-period theoretical model, we should point out some difference. In the theoretical model, we assume that the traders are rational and risk-neutral. Actually, the students have no professional back ground of finance and their risk preference is uncertainty. We find some evidence of their irrational behaviors in experiments. Furthermore, the single-period theoretical model presents single-period markets, where all the factors in the markets are independent of all the factors in other markets. Whereas, in the single-period experiments, the information is dependent from the one in other rounds. However, we could not make sure that the students' behaviors could be independent from the ones in other rounds [1]. In the process of transaction, a series of data, such as the private value, the bid and ask price, and the transaction price, has been collected for analysis. Compared with the multi-period trading game, in the single-period trading game, the historical transaction data could not be considered in the submission deci-

[1] We can imagine the situation: a student maybe think to submit a relevant lower order prices because he regrets that he submit a higher order prices in the previous stage, although his decision on the last stage is proper. The irrigational behaviors of students affect the experimental results.

sion of traders, which makes the decision is simpler for the students. Next section, we introduce the design of multi-period trading game.

6.2.2 Experiment for multi-period trading

The multi-period trading experiments focus on the order submission strategies and the information dispersing in multi-period trading. In the progress of repeated trading, with the additional new information - the order prices, the traders could update their assessment on their opponents' valuation distributions to improve their own order prices. Therefore, they have different submission strategies in the process of repeated trading and the probability of transaction also increases with their adjustment. In the multi-period theoretical model, the results indicate that the offers of the traders converge to increase the probability of agreement if bids are higher than asks. When the trade happens, their offer price incline to their own private value, which indicate the process of private information disclosure. The main steps of multi-period experiments are similar to the ones of single-period experiments. The main difference is that, for multi-period trading game, a series of information was used in serval rounds and the students consider the transaction data at previous stages for their decision of order prices at the later stages.

Similar to the steps in the single-stage trading game, the students are allocated into pairs by the computer. Meanwhile, the computer provide each group with their private value and valuation distribution. The uniform distribution of the mean values of private valuation distribution has the interval $[80, 120]$ and that of the variances of private valuation distributions has the interval $[2, 15]$. Their private values are drawn stochastically from the same valuation distributions. The introduction page6.5 is same as the one in the single-period trading game, which includes the information of the trader's private value and the valuation distribution of his opponents.

The next steps is similar to the ones in the single-period trading game. The second page 6.6 requires the students to make their decision on the order side. And the third page6.7 asks the students to provide their order prices based on their order sides. In these two pages, the private value and the mean value and the variance of the valuation distribution listed above the black for their decision, providing fully access for students.

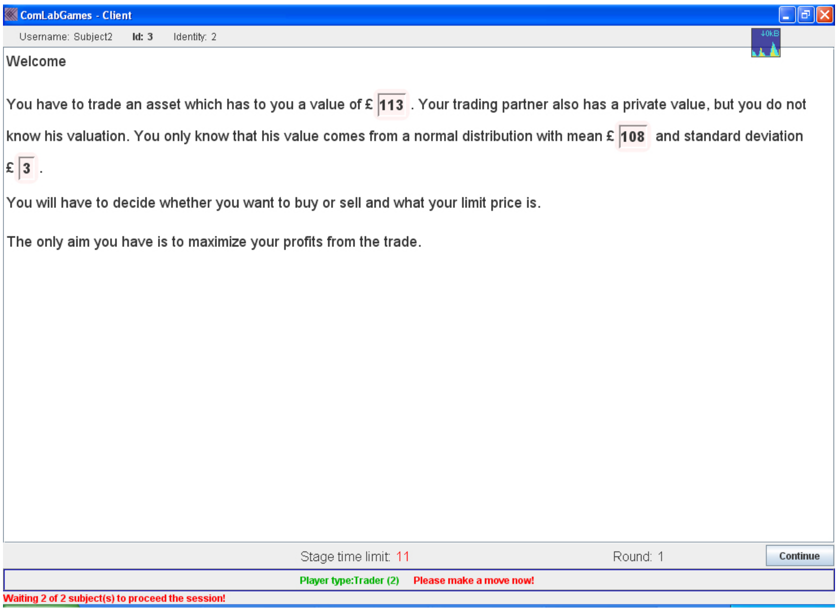


Figure 6.5: The introduction page for the experiment

Different from the single-period trading game, on the bottom of each page, there is a sign to show which round it is. In the multi-period experiment, the steps in every round are the same but there are several round for students to accomplish.

The last page6.8 reports the available information and the results of the transaction. The design of this page is same as the result page in the single-period experiments, but act more important role in the multi-period experiments, in that the transaction results provide useful information for the students to updating their belief on their opponents' valuation distribution and adjust their submission strategies in the later trading round.

The statement above present one round of trading in the multi-period trading game. Since the trade repeated several times with the same private values and the valuation distribution, the next round began with the previous private value and valuation distribution. Although the information of the valuation distribution is not change, the students' expectation on their opponents' valuation distribution is updated conditional to their opponents' order prices in the previous round as well as the decision of order sides of their opponents. Actually, the students know more about their opponents from

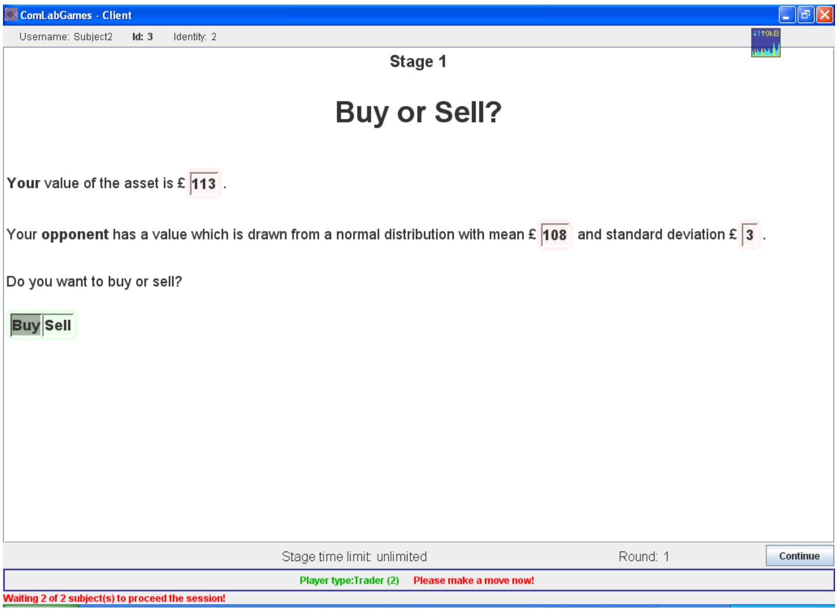


Figure 6.6: Stage 1 - order choice

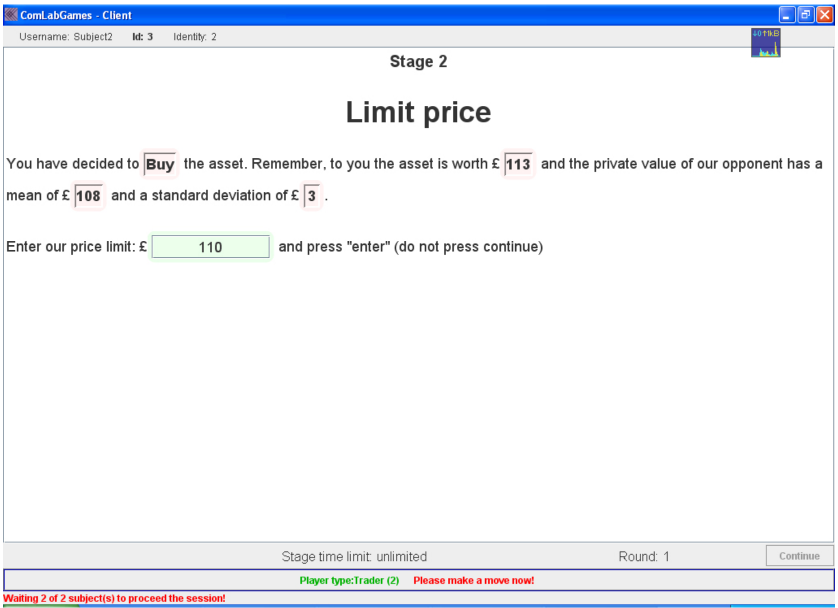


Figure 6.7: Stage 2 - order price decision

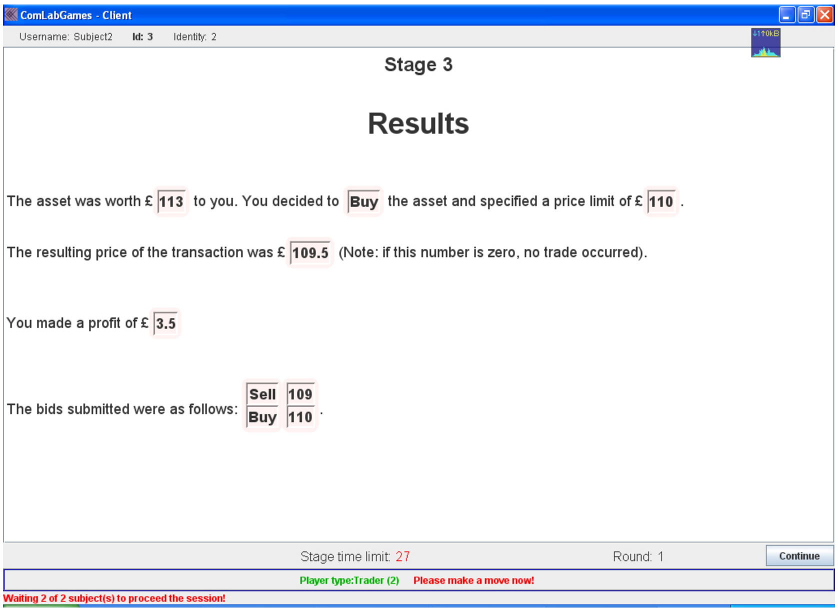


Figure 6.8: Stage 3 - transaction price and profit

their orders after the first time of trading. The decision procedure for the students is more complex than their decision in the single-period trading game. The new valuation distributions of their opponents are indirect to them, which ask their own assessment. Therefore, the results page at the end of each round is important for the students to obtain new information to update their belief on their opponents' valuation distribution and make their decision in the later rounds. Because of the time limit, we only run the multi-trading game once with three rounds for each session. In the multi-period game, the students have the chance to change their order sides and improve their order prices.

To design the experiments for the corresponding theoretical model, I put efforts to imitate the theoretical trading conditions in the laboratorial environment. For the uncertainty between two traders in the same group, the students are allocated into pairs by the computer automatically. The mean values and the variance of the valuation distributions are drawn from the uniform distributions and the private values are drawn from the normal distributions defined by these two values. In the whole process of trading, the students are only informed their own private value and unaware of their opponents' private values. One separate page for the decision of order sides is used to call the students' attention and control their decision steps as requirements. Each stage ended until two traders made their decisions, by which the orders are submitted simultaneously. Totally, the design of the experiments coincides with their corresponding theoretical model.

6.3 Results and implications of the experiments

As the statement above, the single-period experiments and the multi-period experiments are designed to investigate the subjects' behaviors in the laboratorial environment, following the trading rules of the theoretical model. The experiments properly designed provide the trading system for the students to trade in the similar trading environments of theoretical models. Meanwhile, these volunteers are limit on knowledge of financial market and have bias behaviors to submit optimal order prices. The comparison of the traders' submission behaviors in the experiments and in the theoretical models could provide consistent results as well as different results. The consistent experimental results support the results in the theoretical model. On the other hand, the difference between the results of experiments and theoretical models give insight into the traders' behaviors in the laboratorial environments. In the following section, I would discuss the results of the experiments by comparing with the ones in the theoretical model. Most of the experiments results are consistent with the ones in the theoretical models. The difference between these two could be explained by the traders' bias behaviors.

6.3.1 Experiment for single-period trading

The traders' behaviors in single-stage trading game include their order side choice and order price decision. As the description above, the subjects are required to submit their orders simultaneously according to their preference only once. The information available to them exists only in one period, then they face new opponents and information next time. The subjects are informed of the transaction results at the end of trade, but this information is just used to assess their previous submission behaviors. In every session, each student is asked to trade several times with different opponents who are selected automatically. In total, I collect 423 observations from these three sessions, in which the numbers of the buy offers and sell offers are 212 and 210 separately. The mean values of their valuation distributions are drawn from a continuous uniform distribution on the interval $[80, 120]$. The variances of the normal distributions are drawn from a continuous uniform distribution on the interval $[2, 15]$. For each pair of students, both of their private values are drawn from a common valuation distributions [1]. They are aware of that their opponents' private values come from the same valuation distributions as theirs'. In the single-stage trading, besides the uncontrolled factors such as personal traits, I do not provide other factors that influence the subjects' behaviors. The statistical analysis is approach from two aspects according to the traders' submission behaviors: the determinants of offer prices and the ones of order sides.

6.3.1.1 The determinants of offer prices in single-period experiments

As the statement in the single-period theoretical model, the main determinants of offer prices in my research are the private value, the expected value of valuation distribution and the variance of valuation distribution. The corresponding theoretical values are obtained by the theoretical functions, with the values of independent variables in experiments. Because the irrational subjects provide some extreme values, robust regression with the iteratively reweighted least squares algorithm is used to estimate coefficients.

Experimental result 1: the subjects' order prices increase with the increase of their private values either for buyers or sellers.

The positive relationship between the order prices and the traders' private values coincide with the results of the theoretical model. In the experiment, the information of private values is provided to the subjects at the beginning of trade. Most students understand the basic rule that bid prices should be lower than their reservation prices and ask prices should be higher than their reservation prices to obtain positive profits. Their submission behaviors show that they would move their offer prices in the same

[1]Although I permit that the valuation distributions are different for two traders, normally, I use the same valuation distribution for both traders. The main relationships between the variables for the common distributions are similar with the dissimilar distributions

direction as the change of their private values. The intuition behind this relationship is that as informed traders, the subjects would provide proper order prices according to their private information to maximize their profits. Their private information is the most important and essential factors to determine their profits from trading. The figure 6.9 and 6.10 demonstrate the relationship between the independent variable - private values and the dependent variable - order prices for buyers and sellers separately. In these figures, the slopes of the fitted curves for experimental order prices are positive and close to the ones of theoretical values. The estimated coefficients are in statistically significant level, which are listed in table 6.1.

Although the positive relationship between private values and order prices is confirmed by the experiments, there are statistically significant structure change between these two groups of data. I do the hypophysis test that the parameter vectors of these two is equality. The F test results are significant ($p < 0.01$) for buyers and insignificant ($p > 0.1$) for sellers. Meanwhile, I allow that these two groups of data have different constants and test the change of slopes. The significant change of slopes exists ($p < 0.01$ for buyer and $p > 0.1$ for seller). Therefore, there are some other important factors influence the traders' submission strategies. From the graphs, I find that the slope of the experimental fitted curve is flatter than that of theoretical values. The difference between the experimental results and the theoretical results is the result of the heuristic bias of the experimental subjects. The explanation are presented below in details.

Further complimentary to support the implications above is the relationship between the difference between the experimental offers and its related theoretical values. Different from the method above that observing the influence of the private values to the submission strategies of traders as a whole, we test the effects of the private values on the difference between the experimental price and related theoretical value in each pair. The figure 6.11and 6.12 demonstrate the relationship between the difference between experimental and theoretical bids and private value. Following the figures, table 6.2 displays the statistic results. The statistic results for the ask prices coincide with our expectation that the change of the difference between the experimental and theoretical values is dependent from the change of the private values in that the similar relationship for theoretical values and experimental values is supported. On the other

Table 6.1: The relationship between private value and order price

	Bid		Ask	
	Theory	Experiment	Theory	Experiment
Constant	17.3158***	43.8523***	25.6412***	25.8144***
	(5.899)	(6.8416)	(13.229)	(6.9494)
Private value	0.7699***	0.5116***	0.7888***	0.7915***
	(27.103)	(8.3135)	(39.5)	(20.654)
F - Coefficients difference	9.1108***		0.4325	
F - Slope difference	17.9388***		0.1488	

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

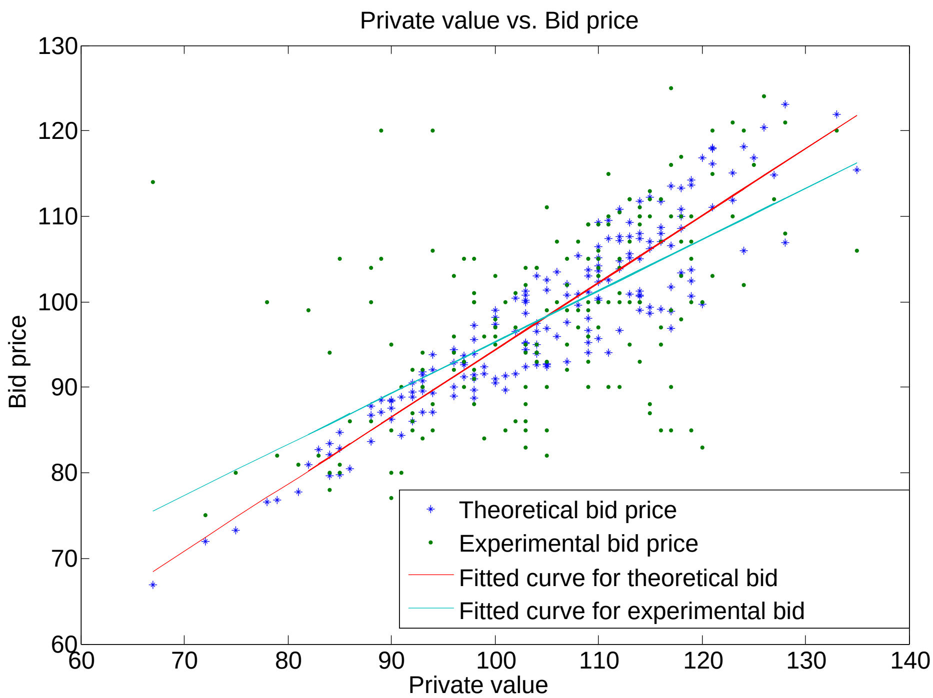


Figure 6.9: The relationship between private value and bid price of buyer

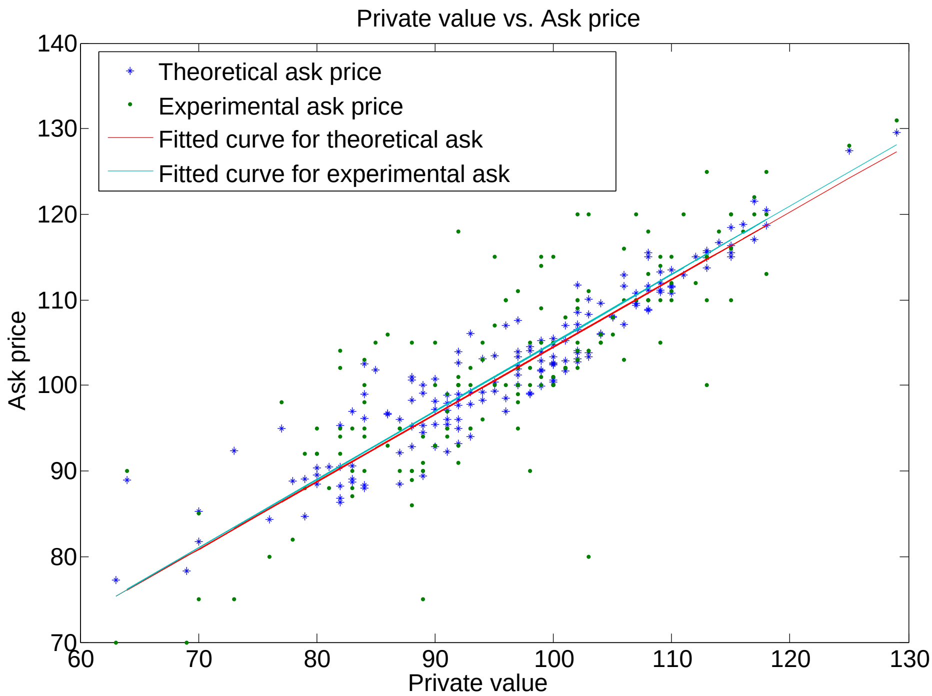


Figure 6.10: The relationship between private value and ask price of seller

hand, we observed that the coefficients for the buyers' bids are significant. The negative relationship between the private value and the difference between the experimental and theoretical values shows that the private values decrease as the difference between the experimental and theoretical values increase. In another word, as the changing of the the increasing of the private value, the experimental bid prices increase less than the related theoretical values. This result implies that the subjects in the single stage trading game prefer to increase their bid prices less than the related values in theory. Similar result is derived from the previous method of comparing the experimental bids and theoretical bids. Therefore, it is safe to say that positive relationship between the experimental offer prices and the private values exists in our experimental environment.

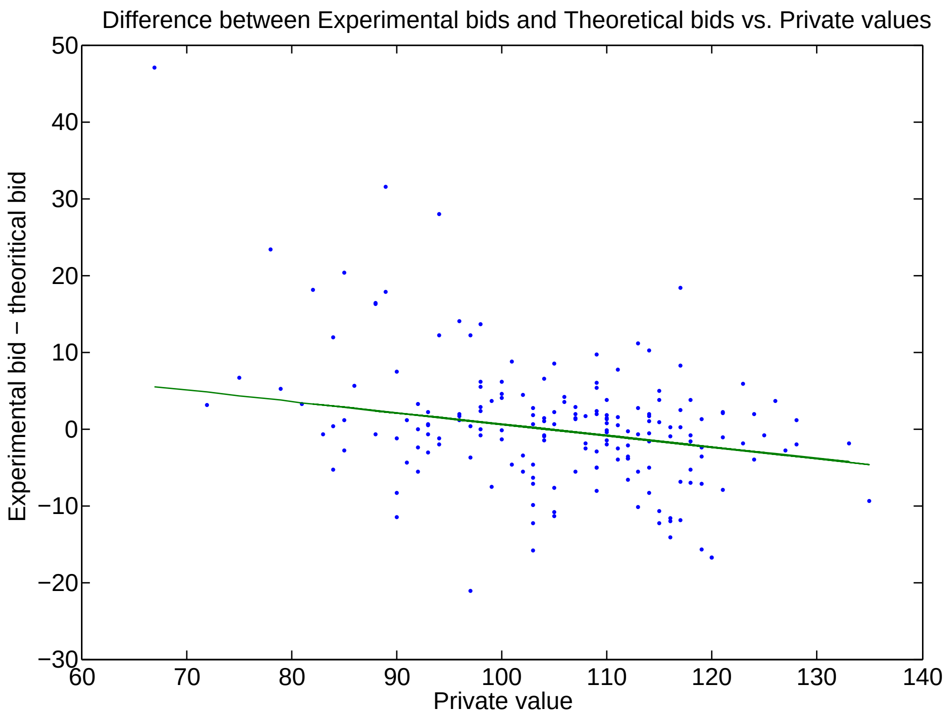


Figure 6.11: The relationship between the difference between experimental and theoretical bids and private values

Experiment result 2: The mean value of the valuation distributions positively in-

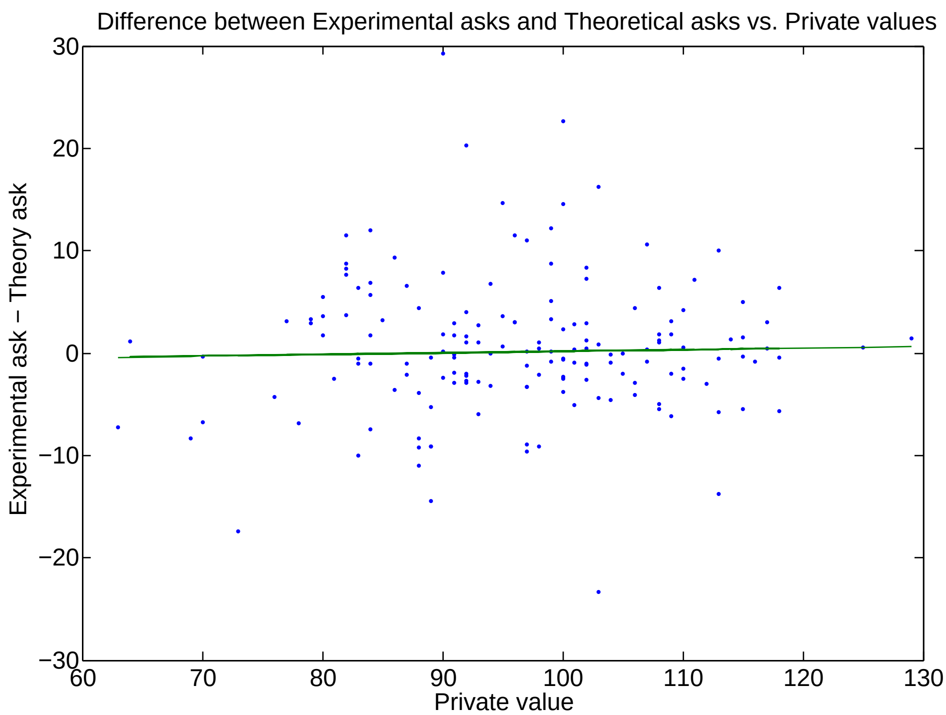


Figure 6.12: The relationship between the difference between experimental and theoretical asks and private values

Table 6.2: The relationship between private value and the difference between theoretical and experimental values

	Bid			Ask		
	Coefficient	t-stats	Prob.	Coefficient	t-stats	Prob.
Constant	15.442	3.741	0.000	-1.372	-0.385	0.701
Private value	-0.149	-3.815	0.000	0.015	0.420	0.675

fluence the subjects' order prices either for buyers or sellers.

The theoretical models demonstrate that the relationship between the expected value of value distributions and the order prices is positive. The subjects' behaviors in the experiments support the theoretical results. Based on their knowledge of economy, it is not difficult for the students to understand the mean value of the valuation distribution. Their submission strategies corresponding to the changes of expected value show that they consider the other one's situation during the process of decision making. As a consequence of this consideration, they move their order prices to the same direction as the moving of the valuation distribution of their opponents to increase the execution probability as well as their expected profits. Figure 6.13 and graph 6.14 display the relationship between the expected values of valuation distributions and the order prices for buyers and sellers separately. The statistical data is listed below in table 6.3. The estimated coefficients are statistically significant for buyers and sellers. Meanwhile, there is no structure changes between the experimental data and theoretical values.

I find that the experimental data fit the corresponding theoretical values better than they do for private values. As we know, to influence the order prices, the expected value of the opponent's valuation distribution has less power than the trader's private value. Therefore, compared with the figures for private values, the slopes of the fitted curve for expected values are flatter than the ones for private values. Then the effects of irrational behaviors are less significant. On the other hand, if the private value and the expected value are main determinants of order prices, when the traders weigh less on their own information, they must weigh more on the information of the other ones, vice versa. Combining the figures for private values and expected values, in the experiments, the buyer has a lower coefficient of slope for private values and a higher coefficients for expected values, compared with the theoretical values. The situation of seller is inverse. The implication behind this fact is that as my expectation the private value and the expected value are most important variables for order prices choice .

Similarly, we test the relationship between the expected values and the difference between the experimental and theoretical values. As we know, the expected value has less effects on the order prices of traders than the private value does, therefore the

Table 6.3: The relationship between expected value and order price

	Bid		Ask	
	Theoretical	Experiment	Theoretical	Experiment
Constant	44.474***	36.5753***	43.8431**	50.288***
	(9.2362)	(6.8821)	(6.6866)	(5.8985)
Expected value	0.5708**	0.6466***	0.5661***	0.5112***
	(11.622)	(11.928)	(8.8238)	(6.1281)
F-coefficients difference	1.4461		0.2088	
F-slope difference	2.7200		0.1797	

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

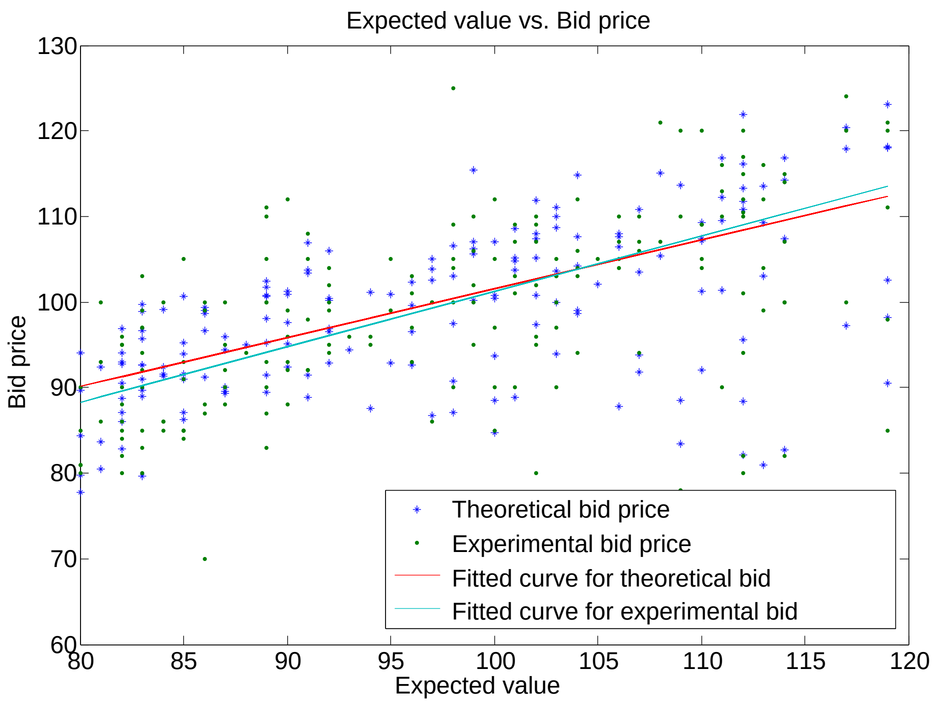


Figure 6.13: The relationship between expected value and bid price of buyer

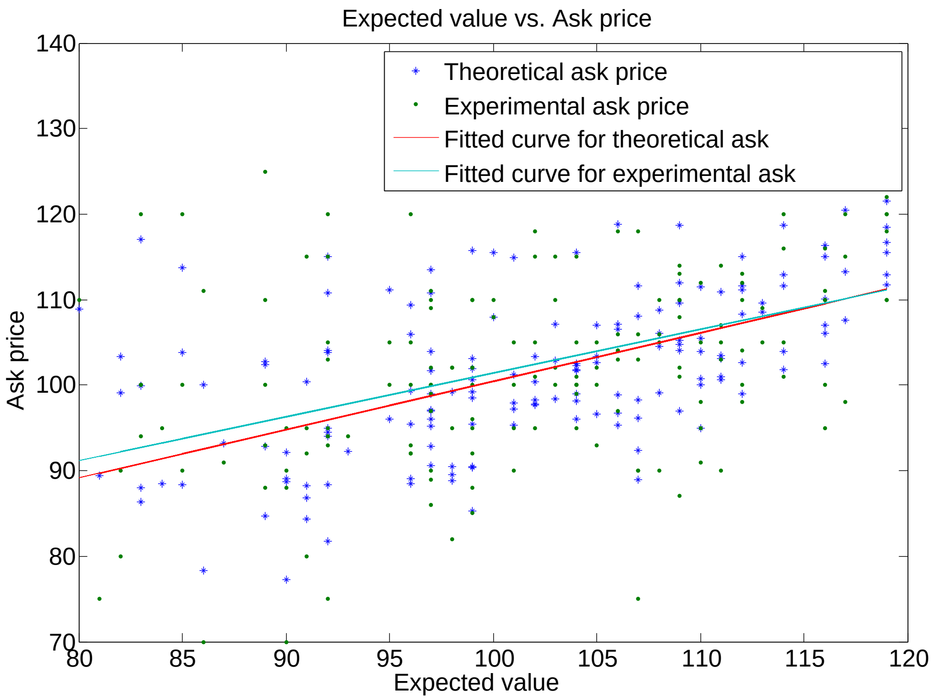


Figure 6.14: The relationship between expected value and ask price of seller

results are influenced by the private values and more difficult to observe. Figure 6.15 and figure 6.16 display the results with dependent variable - the difference between the experimental and theoretical bids and ask separately. The statistic results are listed in table 6.6. For buyers, the points of dependent variables surround the fitted curve with zero degree. The results of the positive relationship between the expected values and bid prices can not be rejected because that no negative relationship are observed in the figure. While, the results for sellers show that the subjects in the experiments increase their asks less with the increasing of the expected value of the valuation distributions. This implication is consistent with the observations above that the coefficient of the theoretical values is slightly higher than the one of the expected values. Therefore, the results for seller also can not discredited the result that there is the positive relationship between the expected values and ask prices.

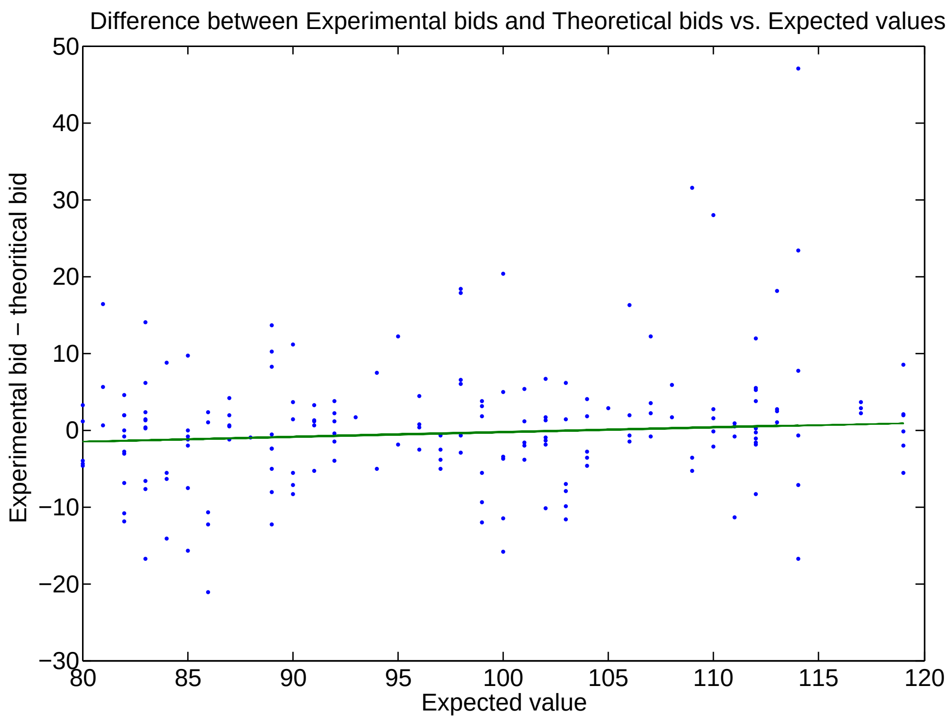


Figure 6.15: The relationship between the difference between experimental and theoretical bids and expected values

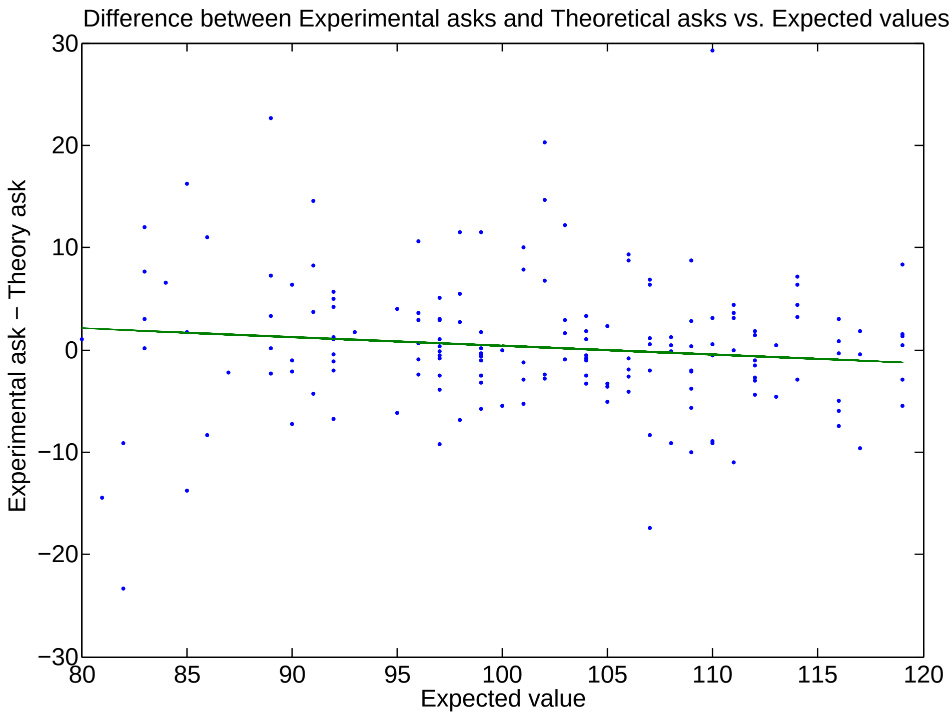


Figure 6.16: The relationship between the difference between experimental and theoretical asks and expected values

Table 6.4: The relationship between expected value and the difference between theoretical and experimental values

	Bid			Ask		
	Coefficient	t-stats	Prob.	Coefficient	t-stats	Prob.
Constant	-6.497	-1.590	0.114	8.897	1.945	0.053
Expected value	0.062	1.493	0.137	-0.085	-1.900	0.059

The variance of the valuation distribution is another independent variable that affects the order submission strategies of the traders in the theoretical model. The results of my theoretical model demonstrate that, when facing a high (low) variance, the buyer would decrease (increase) his order prices, to increase his expected profits. On the contrary, when facing a high (low) variance, the seller increase (decrease) his order prices to maximize his profits. Whereas, in the experiments, the observing of the influence of the variance on the order prices encounters the limitation of sample size [1]. Because the variance are least powerful to influence the order prices, in the experiments, the difference among the offer prices are derived from the difference between the private value and expected value. Therefore, with the effects of other variables, the estimated coefficients for variances are difficult to be statistically significant and the slope is close to 0. The graph 6.17 and 6.18 show the relationship between the bid/sell orders and the variance of the other one's valuation distribution. The statistical data is listed in the table 6.5. For bids, as the slope coefficient is not statistically significant, the different sign of slopes of experimental data and theoretical values could not prove the existence of diverging of experiment and theory for the effects of variance. For asks, the coefficients of slope for experiment and theory have same positive sign, which are statistically significant. The result demonstrates that the relationship between the variances and asks is similar in experiment and theory. Meanwhile, the coefficient equivalent test shows that there is no statistical difference between the experimental data and theoretical values. Furthermore, the reason that the asks fit better than bids is that the buyers prone to act irrationally than sellers. The implication coincides with the result of buyers' bias behaviors as the presentation of experimental data for private values.

As the statement above, variance is the least powerful variables that influence the offer prices of traders. Therefore, most of the observations above provide non-significant results for the relationship between the offer prices and variances. Using the dependent variable - the difference between the experimental and theoretical values, we could

[1] In theoretical model, to observing the effects of variance, all the other variables are constant. Because of the limitation of time and participants, I did not design the experiments for different variables respectively

Table 6.5: The relationship between variance and order price

	Bid		Ask	
	Theory	Experiment	Theory	Experiement
Constant	97.3733***	101.2201***	106.4098***	107.9211***
	(41.775)	(39.54)	(54.141)	(46.441)
Variance	0.1106	-0.2861	-0.555**	-0.6495**
	(0.4493)	(-1.0421)	(-2.4839)	(-2.4583)
F-coefficients difference	0.9187		0.3110	
F-slope difference	0.7920		0.1164	

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

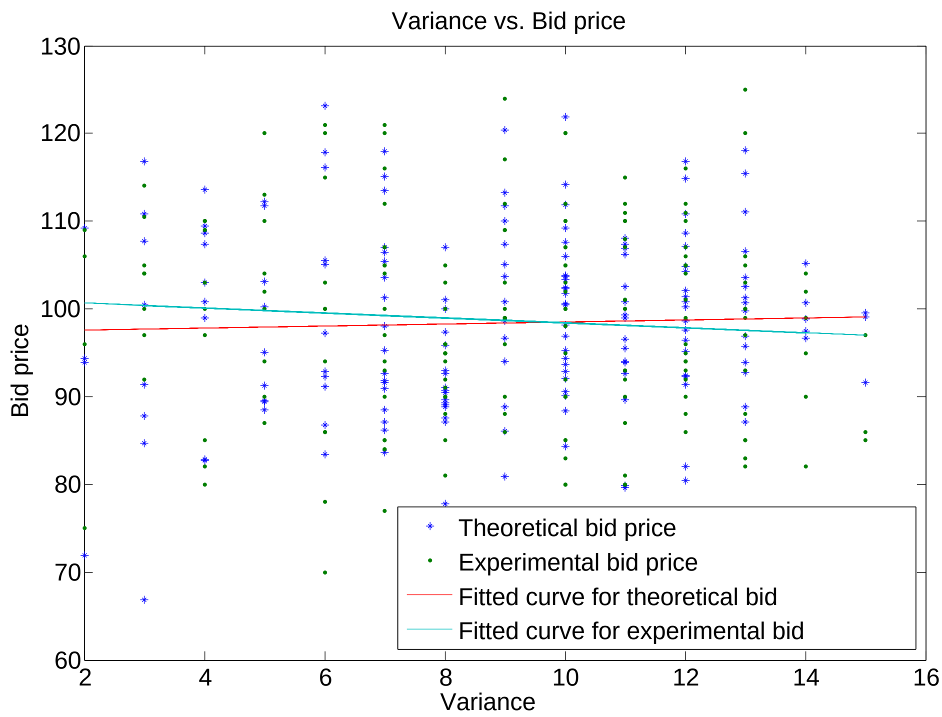


Figure 6.17: The relationship between variance and bid price of buyer

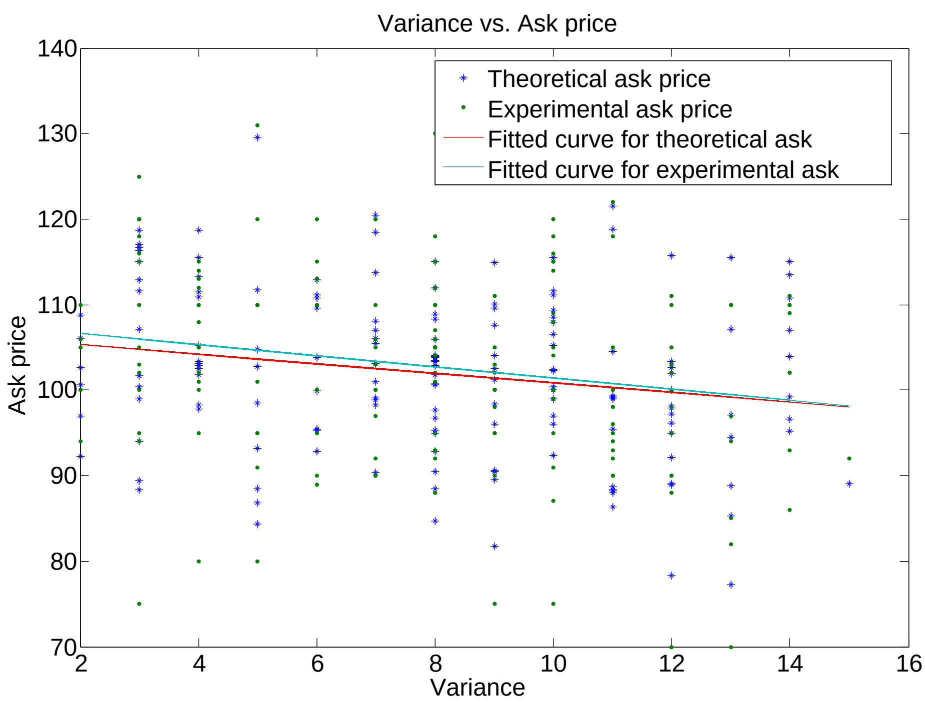


Figure 6.18: The relationship between variance and ask price of seller

observe if there are big negative relationship existing as the change of independent variable. The figures 6.19 and 6.20 demonstrate the relationship between the variance and the difference between the experimental and theoretical bids/asks. The insignificant coefficients show that there is no relationship between the variance and the differences between two values. This results supports our previous results in some extent.

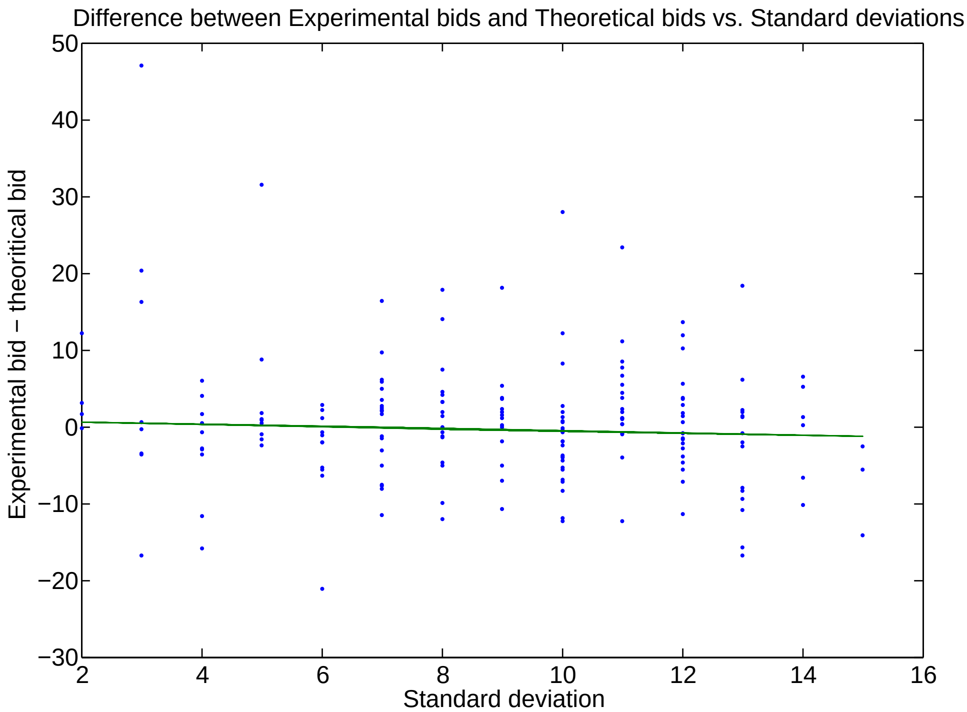


Figure 6.19: The relationship between the difference between experimental and theoretical bids and standard deviation

The multi-regression test results show that the independent variables have different power to influence the order prices. The multi-regression model includes three independent variables: $b/s = \alpha + \beta_1 * v_i + \beta_2 * \mu + \beta_3 * \sigma$. The statistical data is listed in the table. In the table, the coefficients of private values and mean values of valuation distributions are statistically significant for both buyers and sellers. Compared with the related theoretical values, the experimental coefficients of private values and mean values for bids are different from the theoretical coefficients statistically significant. These results

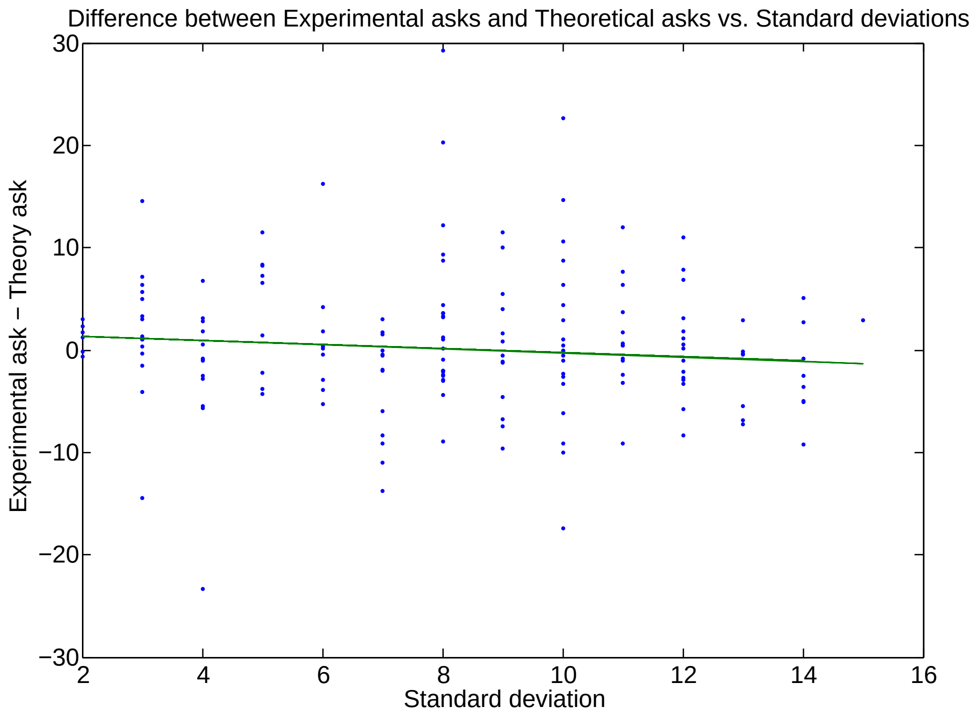


Figure 6.20: The relationship between the difference between experimental and theoretical asks and standard deviation

Table 6.6: The relationship between standard deviation and the difference between theoretical and experimental values

	Bid			Ask		
	Coefficient	t-stats	Prob.	Coefficient	t-stats	Prob.
Constant	0.883	0.624	0.534	1.789	1.589	0.114
Standard deviation	-0.145	-0.968	0.334	-0.205	-1.603	0.111

show that buyers put more weight on the information about their opponents and less weight on their own information. The results are caused by the bias behavior of buyer, and I will explain the bias behavior in details in a separate section. The experimental coefficients for sellers are closer to the theoretical coefficients, which indicate that the sellers' submission strategies in the single-period experiments are consistent with the strategies in the theoretical model. The coefficients of the variances of the valuation distribution are difficult to be statistically significant because of the trivial effects of the variance on the order prices and the ignorance of subjects with limited knowledge in financial markets. One useful observation is that the experimental coefficients and the theoretical coefficients for variance have same sign in the multi-regression. It indicates that the relationship between the variances and the order prices for both buyers and sellers are similar to some degree.

As indicated above, most of the relationships between the variables and order prices in the single-period experiments are consistent with the ones in the single-period model. The results imply that the subjects's submission strategies in the experiments are similar to the ones in the theoretical models. Next, I investigate the relationship between the experimental order prices and the related theoretical values. To reduce the influence of some outliers, I use robust regression to test the relationship between the theoretical value of order prices and the experimental order prices. The result indicates that there is a strong relationship between the theoretical values and experimental order prices (table 6.8). Figure 6.21 and 6.22 display that the slopes of fitted curves are close to 45 degree. The experiment results support that the theoretical model is efficient to capture the important variables that determine the order submission strategies of the trader in the simple double auction market. In the line of the results above, the asks fit the theoretical values better than bids because of the better performance of sellers.

6.3.1.2 The analysis of the heuristic bias behavior of the subjects

According to the analysis above, the experimental results are consistent with the ones in the theoretical models. Whereas, I also observed some difference between the subjects' submission strategies in the experiments and the traders' behaviors in the theoretical

Table 6.7: The multi-regression for order price in the single-period model

	Bid		Ask	
	Theoretical model	Experiment	Theoretical model	Experiement
Constant	-4.8518***	7.6929	4.14733**	8.8240
	(-2.6854)	(1.1849)	(2.4321)	(1.3297)
private	0.7540***	0.4500***	0.7119***	0.7365***
	(57.5397)	(9.5550)	(57.0973)	(15.1783)
expected value	0.2752***	0.4729***	0.2625***	0.2060***
	(19.5154)	(9.3331)	(18.0147)	(3.6330)
Variance	-0.33845***	-0.2675	0.3116***	0.2145
	(-6.6309)	(-1.4586)	(7.3773)	(1.3052)
F-private value	38.7001***		0.2681	
F-expected value	14.1333***		1.0352	
F-variance	0.1389		0.3638	

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table 6.8: The correlation between the experimental offers and the theoretical offers

	Bid	Ask
Constant	8.9139*	2.0387
	(1.9326)	(0.43195)
theoretical value	0.9065***	0.9814***
	(19.424)	(21.307)
theoretical value = 1	(-0.9533)	(-0.9539)

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

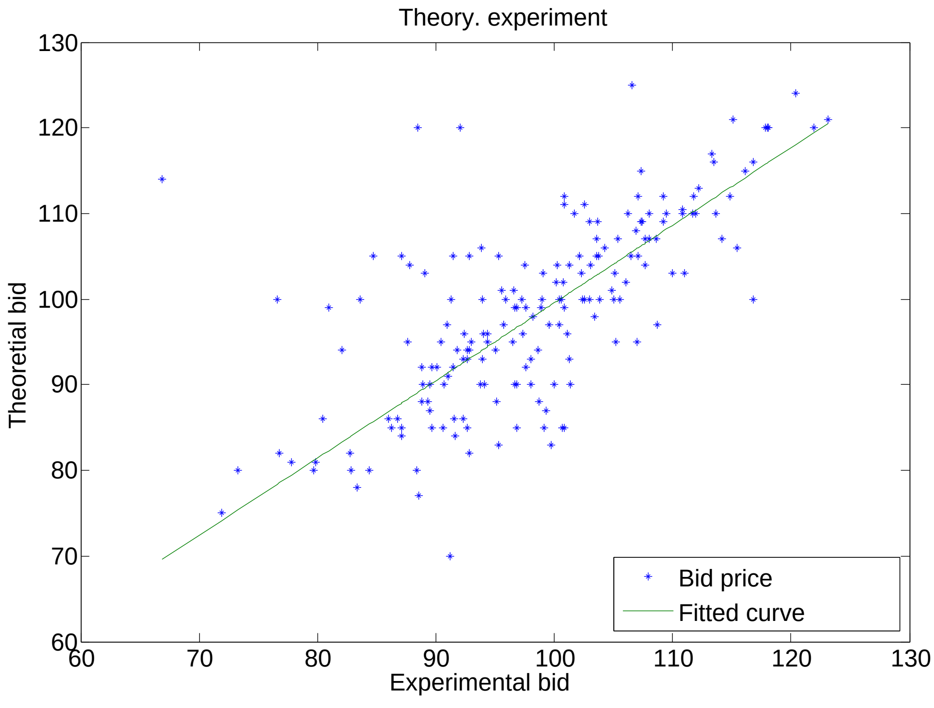


Figure 6.21: The relationship between the experimental bid price and the theoretical bid price

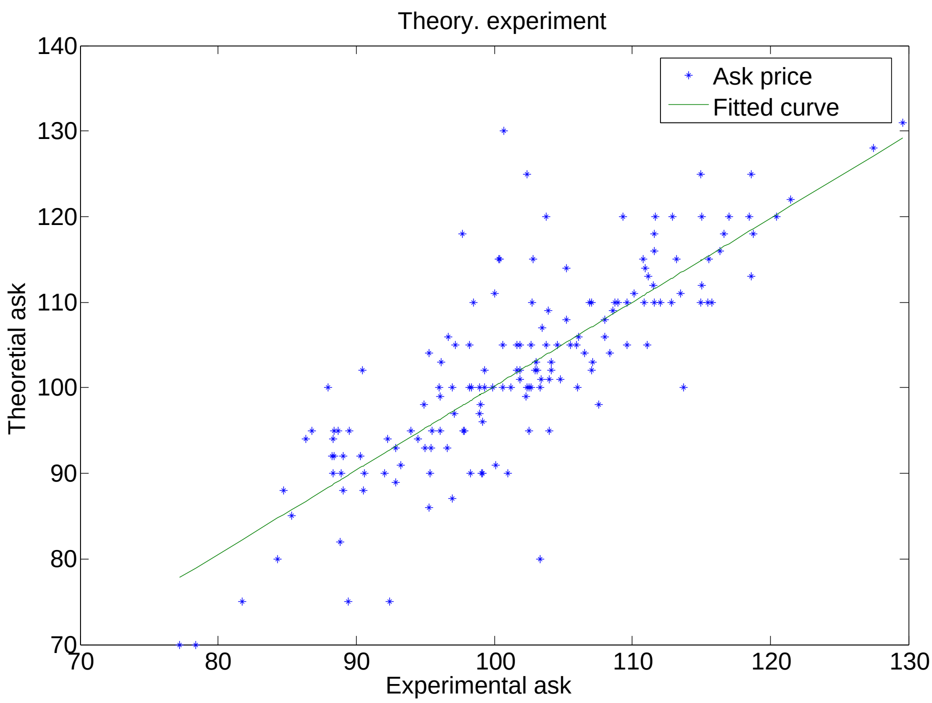


Figure 6.22: The relationship between the experimental ask price and the theoretical ask price

models. The difference is caused by the heuristic bias behaviors of subjects. I expected that they are overconfident to make their decision. Overconfident is a robust physiological phenomenon and attains a lot of research interest. A lot of research reported the observation of the overconfident behaviors in the experiment or the financial market. As a bias trait of the human, overconfident is explained as the bias on their own abilities or their judgement on the reliability of the events. In the financial market, the overconfident traders act more aggressively: they trade more frequently; they underreact to the information from rational traders (Odean, 1998); they overestimate the precision of their own information on the asset value and prefer to push the market price higher (Benos, 1998). Allen and Evans (2005) search the overconfident influence in the bidders and find that the overconfidence exist in 40% traders and are not reduced by experience. Because the overconfident traders have bias on the judgement of the expected value of their trade, they could not weigh their own private information properly (Hirshleifer, 2001). By observing the figure 6.9, I find that most traders submit bids lower than the related theoretical values and some outliers drive the slope of fitted curve flatter. Compared with the figure 6.10 for sellers, the buyers express bad performance to provide optimal order prices. The interpretation for the phenomenon is that the overconfidence of buyers are more serious than that of sellers. The buyers are overconfident on their ability to submit optimal orders to maximize their profits. Therefore, they take too much advantage from their private values by submit the orders with relatively low prices. They ignore the fact that because of the existence of uncertainty in markets, they should submit the relatively higher bids to increase the execution probability, which in turn increase their expected profits. On the other side, most of the sellers' asks are slightly higher than the related theoretical values. This indicates that sellers submit higher asks to take the advantages of their private information.

Overconfidence is observed by many researches in financial markets. In our experiments, the subjects are students with basic knowledge of financial market. The single-period experiments held at the beginning of each sessions. Although the students are not familiar with the trading game, their overconfidence of their ability to obtain profits lead them to submit orders with better price and lower execution probability. They overvalue the more direct determinant and undervalue the less direct determinant for the expected profits because of their overconfidence. However, as they realize the results of the trade after trading several times with different opponents, their submission behaviors change in the multi-period trading.

6.3.1.3 The determinants of decision of order sides

As indication in the theoretical model, the traders' decision of order sides depends on the relationship between their private values and the mean values of the valuation distribution. The variances of the valuation distribution are irrelevant to the decision of order sidess. To observe the subjects' decision of order sidess in the experiments, first, I analyze the relationship between their order prices and the difference between the private values and the expected value of valuation distribution. The figure 6.23 and 6.24 display the relationship between the offer prices and the values of private values minus expected values. Because the private values are more powerful to influence the order prices than the expected values, when the gap between the private values and the expected values increases, the order prices increase as well either for bids or asks. However, as indicated in table 6.9, the slope of the fitted curve for the buyers in the experiments are not statistically significant. The result shows that the change of the difference between the private values and the expected values does not influence the subjects' order prices. In another word, the effects of the private values are not statistically significant different from the ones of the mean values. The results coincide with the results above that buyers put less weight on their private values and more weight on the mean values of their opponents' valuation distributions. Whereas, the sellers' performance is better than buyers in understanding the difference between the private values and mean values. The slope of the fitted curve is in the significant level and there is no slope difference between the theoretical values and experimental data. Comparing the figure 6.23 with the figure 6.24, I could observe that the bids concentrate around the front of the fitted curve, while the asks allocate around the back of the fitted curve. This phenomena show that the subjects prefer to submit bids when they observe that their private values are higher than the expected values of the valuation distributions and submit asks when the private values are lower than the expected values. This observation is consistent with the results of theoretical model about the decision of order sides. In the theoretical model, the trader would submit a bid when his private value is higher than the expected value of his opponent's valuation distribution, vice versa. Therefore, the figures show that the subjects' decision of order sides depends on the relationship between the private values and expected values.

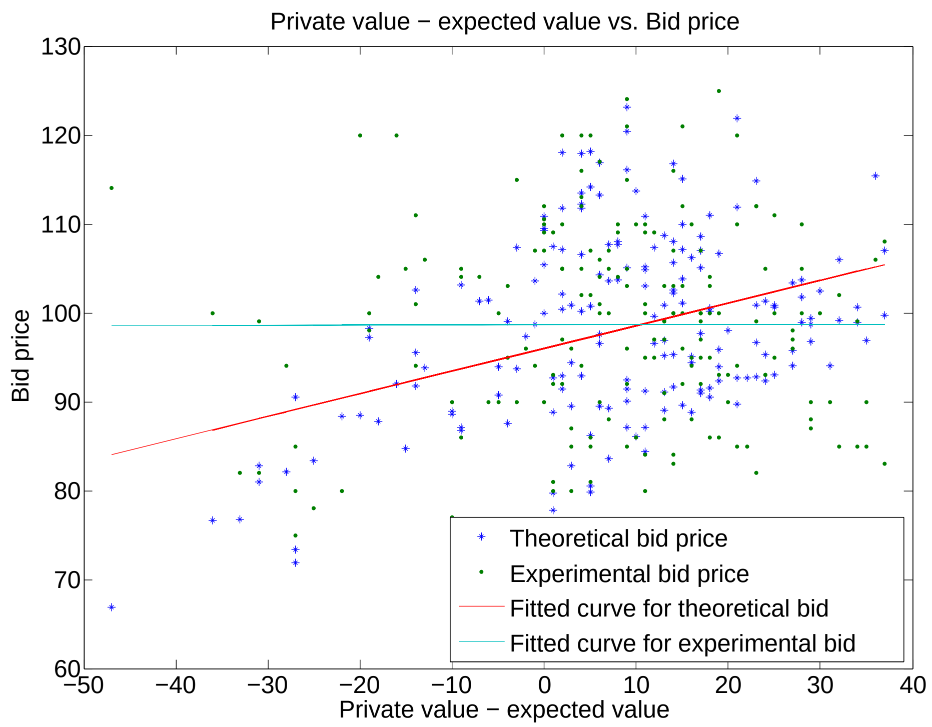


Figure 6.23: The relationship between the bid price and the difference between the private values and the expected values

Further more, I present the regression for the determinants of order sides. In table 6.10, the slope of the values of private values minus expected values is positive with statistically significant result, which implies that the bids increase with the increase of the difference between the private values and expected values. The private values are also positively proportional to the numbers of bids. Whereas, the expected values of the opponents' valuation distribution are negatively proportional to the numbers of bids. The subjects in the experiments prefer bids when their private values are relatively higher or the expected values of their opponents' valuation distribution are relatively lower, vice versa. This observation fits the traders' decision of order sides in

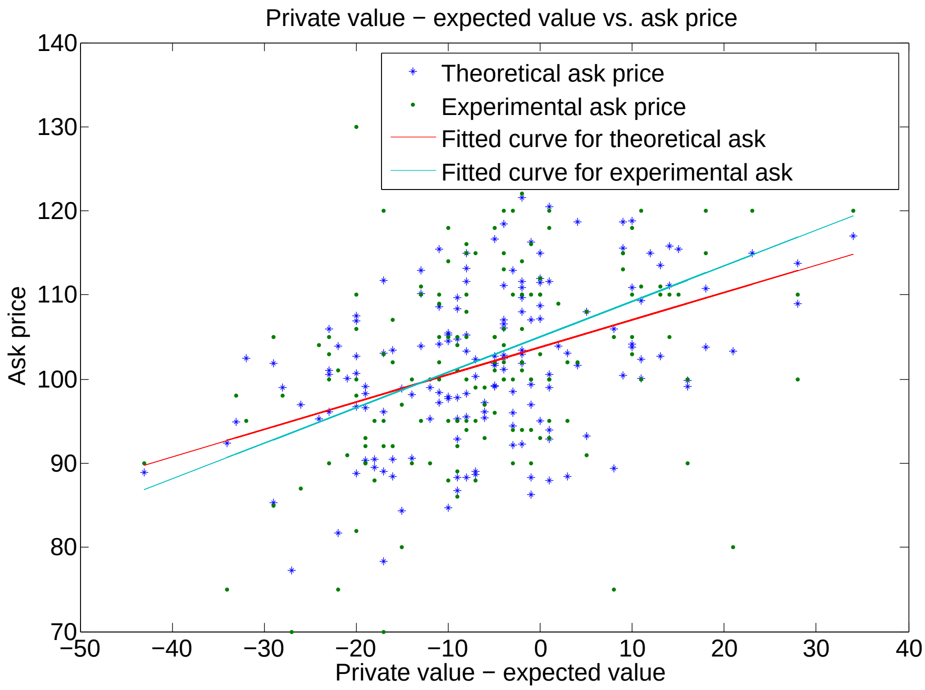


Figure 6.24: The relationship between the bid price and the difference between the private values and the expected values

Table 6.9: Private value - expected value vs. order price

	Bid		Ask	
	Theory	Experiment	Theory	Experiement
Constant	95.9895***	98.6729***	103.797***	105.0363***
	(115.54)	(100.94)	(136.74)	(120.66)
Private value - expected value	0.2551***	0.0017	0.3257***	0.4217**
	(5.3613)	(0.0296)	(6.0313)	(6.8085)
F-coefficient difference	13.1542***		0.5737	
F-slope difference	13.0007***		0.3071	

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

the theoretical model. More evidences for decision of order sides are presented below to support the strong relationship between the theoretical values and experimental data.

Table 6.10: The variables that affect the decision of order sides

	Private value-expected value	Private value	Expected value
constant	-0.0373	-7.9367	6.5208
	(-0.3189)	(-8.2707***)	(6.7823***)
slope	0.0806	0.0791	-0.0651
	(9.6488***)	(8.3527***)	(-6.8212***)

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

To investigate the subjects' decision of order sides, I would introduce Chi-square test to test the correlation between the experimental order sides and theoretical order sides. Before presenting the results of Chi-square test, we provide an intuitionistic evidence for the relationship between the experimental order sides and theoretical order sides. In the theoretical model, the results show that if two traders are the same on every aspect except different private values, the probability of submitting a buy/sell order is 0.5. Because there is no difference between two expected values of the valuation distribution, traders expected that the probability of their opponents to submit a bid/ask is equaled to 0.5 and their opponents have same expectation on their decision of order sides. In the experiment, there are 212 of buy orders and 211 of sell orders, then the percentage of buying is 0.5024, which is close to 0.5. This result supports the result in theoretical model that the percentage of observing a buy/sell order is equaled in simple double auction markets.

Chi-square tests are used here to investigate the goodness of fit of the experimental data and theoretical values in the aspect of decision of order sides. The statistical results are listed in table 6.11. The results show a strong relationship between the experimental data and theoretical values for decision of order sides. The order side choice of the subjects in the experiment are expected to follow the results of theoretical model. In another word, the theoretical model demonstrate the proper submission strategies of traders on their decision of order sides.

Table 6.11: Chi-square test for the decision of order sides

		Theory	
		Bid	Ask
Experiment	Bid	176	36
	Ask	44	166
		Chi-square test	
		χ^2	p-value
Chi-square		162.858***	0
Yates		160.38***	0

Note: χ in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

6.3.1.4 The transaction happening

Because of the existence of two sides uncertainty, the probability of transaction is low. As the descriptions in the theoretical model, the transaction only happens when the two traders submit the different sides of orders and meanwhile, the bid is higher than the ask. There are 423 observations in total. The chi-square test is used to test the goodness of fit of the experimental data to the theoretical values. The statistical results are listed in table 6.12. The critical value χ^2 equals 78.685 and the probability of insignificant is 0. The results show that the hull hypophysis of the independency has been rejected. The experiment data fits their theoretical values.

Table 6.12: Chi-square test for the transactions

		Theory	
		Trade	No trade
Experiment	Trade	80	30
	No trade	78	233
		Chi-square test	
		χ^2	p-value
Chi-square		78.685***	0
Yates		76.666***	0

Note: χ in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

6.3.1.5 Summary

First, in the aspect of the determinants of order prices, most of the estimated coefficients of the experimental offer prices are close to the ones of the corresponding theoretical values for privates and expected values. The only slope difference statistically significant is the regression for the bid prices and the private values. The slopes of the fitted curves of the experimental bids are lower than that of related theoretical values. The possible reason behind this difference is that the subjects's heuristic bias. The over confident subjects submitted the bids with higher unconditional profits and lower execution probabilities. For the variance, the estimated coefficient is not in the significant level for buyers, which means that buyers' behaviors is irrelevant to the variance of the valuation distributions. The insignificant result is caused by the bias behaviors of buyers, compared with the behaviors of sellers. The variance could explain parts of the offer prices of sellers and the slopes are close to each other. Furthermore, the slope of the fitted curves for the relationship between the theoretical value and experimental order prices are close to 45 degree. The strength of the relationship between the theoretical value and experimental order prices prove that the theoretical model could explain the subjects' submission strategies in the laboratorial environment.

Second, for the decision of order sides, the experimental data coincides with the theoretical values. The experimental result shows that 81% of order sides are in the same side as the expectation in theory. The Chi-square test confirm that the strong relationship between the decision of order sides in the experiments and the ones in theory. The students' decision of order sides follows the theoretical rules that traders choose bids (asks) when their private values are high (low) than the expected value of valuation distribution. Furthermore, as the subjects' behaviors in the experiments are very similar to the traders' behaviors in the theoretical model, the observations for the transaction happening are also close to the expectation in the theoretical model. The statistically test results support this implication. To sum up, the results of single-period experiments are consistent with the ones of single-period model.

6.3.2 The experiment for multi-period trading

The multi-period trading model explain the traders' repeated submission behavior in the continuous trading progress. Different from the trading behaviors of the traders in single stage, the traders in multi-period trading game have the opportunity to adjust their trading behaviors according to the new trading information. Meanwhile, the dynamic price pattern is observed during the continuous process. I design the multi-period experiment to observe the subjects submission behaviors in the multi-period trading. In the multi-stage trade experiments, the students were asked to trade three rounds for each pair of data, and also they can change their offer prices by updating their expectation on the value distribution of the other one according to the information of the other one's offer price from the previous stages. The multi-period trading experiment were held only on two sessions because of the time limitation. There are 19 pairs of effective data collected after delete several groups of unfinished trading. In this section, by analyzing the experimental data and comparing the difference between the experimental data and theoretical values, the experimental implication are given by analyzing the trading behavior of the traders in multi-stage trade. Meanwhile, the comparison between the theoretical model and the experiments by statistical analysis is made to observe the efficiency of my theoretical model.

6.3.2.1 Submission behaviors of the traders in experiment and theory

In this section, I discuss the results of the comparison between the subjects' submission strategies in the experiments and the traders' behaviors in the theoretical models. With statistical analysis, I could clarify the traders' submission strategies from the points of auction theory, as well as the subjects' bias behaviors in laboratory environments from the points of behavioral finance. To compare the results between the theoretical models and the experiments, the corresponding theoretical values underlying the same series of variables are obtained.

Traders' behaviors over stages As the description in my theoretical model, the trader updates his belief on his opponent's valuation distribution according to the information at pervious stage. His estimation of his opponent's private information is based on all the historical transaction data and information available to him. He changes his offer price because of the updated valuation distribution of his opponent, by which the trader could improve his execution probability. In the process of repeated trading, the opponent's valuation distribution regarded by the trader is expected to concentrate on its private value. Because the students are not required to provide their updated expected value of their opponents' valuation distribution, my observation focuses on the changes of their order prices and order sides. The related theoretical values could be calculated from two points of views. Method 1: the theoretical values are calculated by updating the expected value with the theoretical offer prices at the previous stage. Method 2: the theoretical values are calculated by updating the expected value with the experimental offer prices at the previous stage. With method 1, the theoretical values are calculated from the extent of whole period of trading. By these two method, I could observe the total difference between the strategies of subjects in laboratorial markets and ones in theoretical markets. My mainly focus is to analysis the difference between the pure theoretical data and pure experimental data. Meanwhile, to observe the individual updating behaviors at each stage, I introduce method 2 a complement to obtain the theoretical values by which the difference between the experimental data and the corresponding theoretical values is concentrated on each stage. I need to point out that the theoretical values with method 2 are influenced by the irrational behaviors of traders because of the underlying experimental order prices. Therefore, when observing the whole process of trading by theoretical values with method 2, the hybrid theoretical values would diverge the experimental order prices more than the theoretical values with method 1 do. Inversely, the similar results of these two methods would prove the close relationship between the subjects' behaviors in the experiments and the traders in the theoretical model.

The traders' behaviors are based on their own private value and the value distribution of the other one. As the statement in the theoretical model, their decision of order sides depend on the relationship between their private valuation and the valuation distribution of the other one. And their offer price are decided by their private value as well as their opponent's valuation distribution. Therefore, to observe the change of their submission strategies, I classify the groups into three types according to the relationship between the two private values in each pair.

- Type I: The private values of the subjects in one group are both lower than the expected value of their valuation distribution.
- Type II: The private values of the subjects in one group are both higher than the mean value of their valuation distribution.

- Type *III*: In the same group, one of the private values of the subjects in one group is lower than the mean value of his opponent's value distribution and the other one's private value is higher than his valuation distribution.

Because of the different relationship between the traders' private value and their opponent's valuation distribution, the transaction probabilities of these types are different. The situations of type *I* and type *II* traders are similar. The transaction probability of first two types of traders are relatively low, because the difference between their private values is low. They are expected to submit the same side of orders, whatever the difference between their private value is. To match their orders, the process of their adjustment has two stages: changing from the same order side to different sides and improving their order prices to make bids higher than asks. The transaction are easier to happen for the type *III* traders with bigger different private values. Actually, the main difference between the first two types of traders and type *III* traders is their decision of order sidess, which are influenced by the difference between their private values and the expected values of their opponents' valuation distributions. I collected 19 pairs of data to observe the subjects' submission strategies over stages [1]

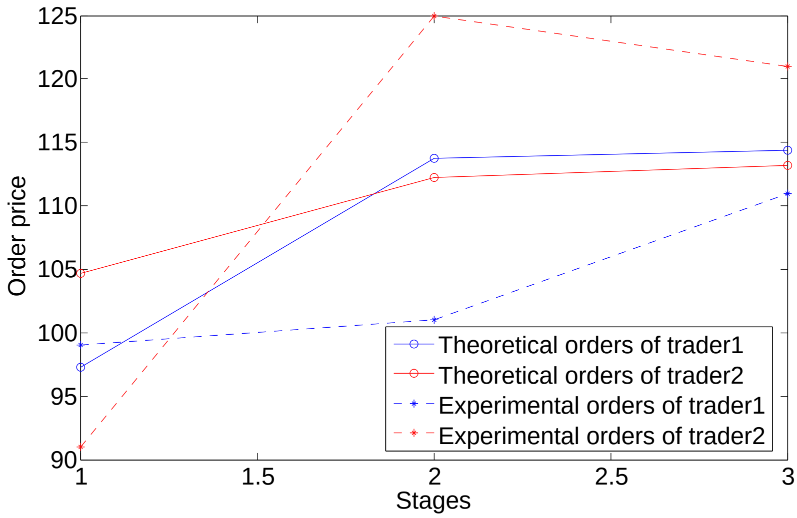
In theory, the type *I* traders submit sell orders simultaneously at the first stage because their private values are lower than the expected values of the valuation distributions. At the later stages, the traders update their assessment on the expectation valuation distribution of their opponents continuously. Conditional to the ask prices, both seller's expected value would decrease. Whereas, their private values are different such that their order prices are different at the first stage of trade. Then, their expectation on the other one's valuation distributions are diverge from the common valuation distribution with different speed. During the process of repeated trading, when one trader's assessment on the other one's expected value is lower than his private value. He would change his order side to bid. If his bid price is higher than the ask price, the trade happens. On the contrary, the traders will continue to adjust their order prices by updating information until the trade happens. Meanwhile, during the process of repeated trading, the trader's order prices would be close to their private values gradually. This is the process of private information disclosure.

[1] All the graphs for the comparison between the theoretical and experimental values over stages are listed in appendix *A*.

As the description in the theoretical model, the procedure of order match is complex. It depends on the traders' correct updating behaviors and their proper decision on the sides and prices of orders. Despite this, the subjects' performance in the experiments show their ability to match their orders somewhat.For example, figureA.3 displays the order match between two subjects with low private values. In this group, I observe two asks with different order prices at the first stage. Then, one of them change his ask to bid and both of them adjust their order prices to make the trade happen. The procedure of adjustment on the sides and prices of orders are consistent with the process in the theoretical model. I infer that with the increase of the number of stages, the probability of transaction would increase.

The trading behaviors of the type*II* traders with relatively high private values are similar to the type *I* traders. Because the traders' private values are higher than the mean value of the valuation distributions, they would prefer bids to asks. It is expected that, in theory, two bids would be submitted at the first stage of multi-period trading. Conditional to bid prices, the expected value of buyers' valuation distributions increase regarded by their opponents. Similarly, different private values produce different bids and different bids cause different expected values of the valuation distributions. Then, during the repeated trading process, I could observe that one trader would change his bid to ask when he finds that his private value is lower than the expected value of the other one's valuation distribution according to his belief updating of his opponent's valuation distribution. Next, one bid and one ask would converge to match order prices. The similar situation is observed in the subjects' behaviors in the experiments. For example, figure A.4 displays the submission behaviors of two subjects over stages, whose private values are relative high. I observed that the student with lower private value changed, his order side to ask at the third stage. Although the ask price is still a little bit higher than the bid price, it is expected that, with the increase of the stage number, the seller would decrease his ask and the buyer would increase his bid to make trade happen.

For the type *III* traders, the adjustment is only to match the bid and ask prices since they would submit one buy and one sell at the first stage of trading according to the rules of the theoretical model. During the process of repeated trading, the buyer would increase his bid and the seller would decrease his ask to make the trade happen.



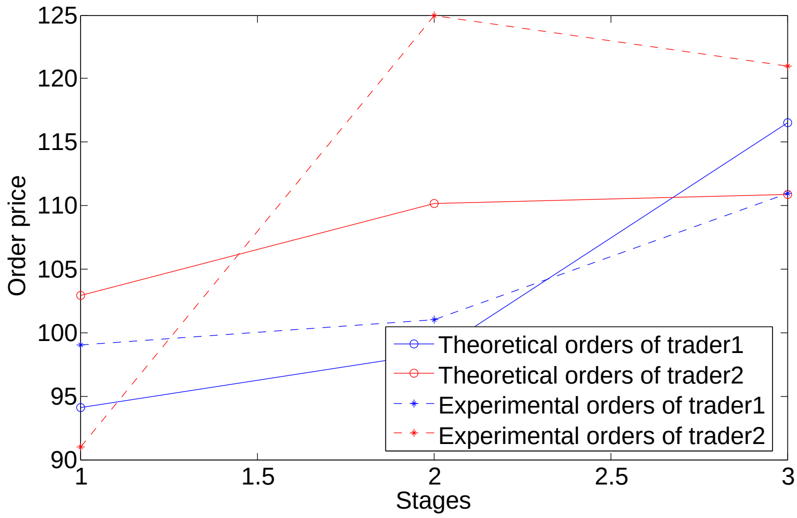


Figure 6.25: Offer prices of theoretical and experiment data (theoretical updating & experimental updating)

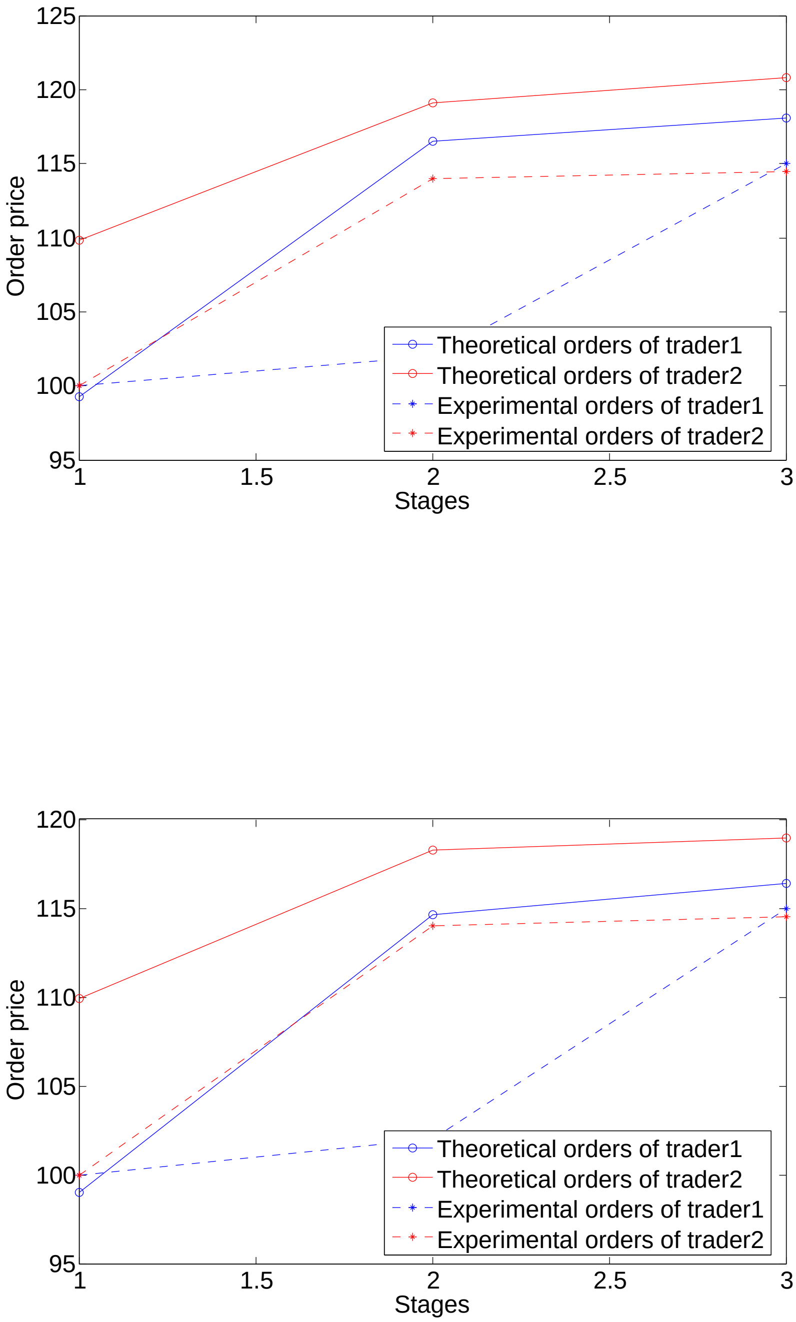


Figure 6.26: Offer prices of theoretical and experiment data (theoretical updating & experimental updating)

Once the trade happens, the buyer continually increases his bid and the seller decreases his ask as well until their order prices equal to their reservation prices. In fact, the procedures of order price change and information disseminate for the type *III* traders are similar to the rear part of the trading process for the type *I* and type *II* traders. In the experiments, many of the groups show the process of price converge (figure A.11). As for the aspect of private information disseminating, because there are only few stages, I could only observe that the subjects performance better to match their opponents' offer prices, but could not find obvious evidence that the traders prone to move their order prices to their private values after transaction happening. I find that the traders' behaviors become ambiguous after the transaction happens. Some of them may think that to increase the difference between their order prices and their private values could increase their profits. Some of them decrease this difference and then increase it at the next stage. To obtain a better understanding about the information dissemination in the multi-period trading, more stages of repeated trading game are needed in the future work.

Totally, the results show that there are some similarities between the subjects' submission strategies in the experiments and the trades' behaviors in the theoretical model. The subjects adjust their order prices and sides over stages to increase the probability of transaction happening. The bid and ask prices tend to converge during the process of repeated trading. There are two reasons that influence the correlation between the observations in the experiments and the theoretical model. On the one hand, the students' irrational behaviors diverge their submission strategies to the optimal ones. On the other hand, there are only a few stages for the students to understand the updating process and observe the private values of their opponents.

Experimental and theoretical order prices In the last section, I indicate the subjects' activities on updating their information, adjusting their order prices in the multi-period trading game and the comparison between the theoretical values and the experiment data. Next, I compare the bid/ask experiment data with their theoretical values at each stage respectively. At last, all the order prices are put together to be compared with their theoretical values.

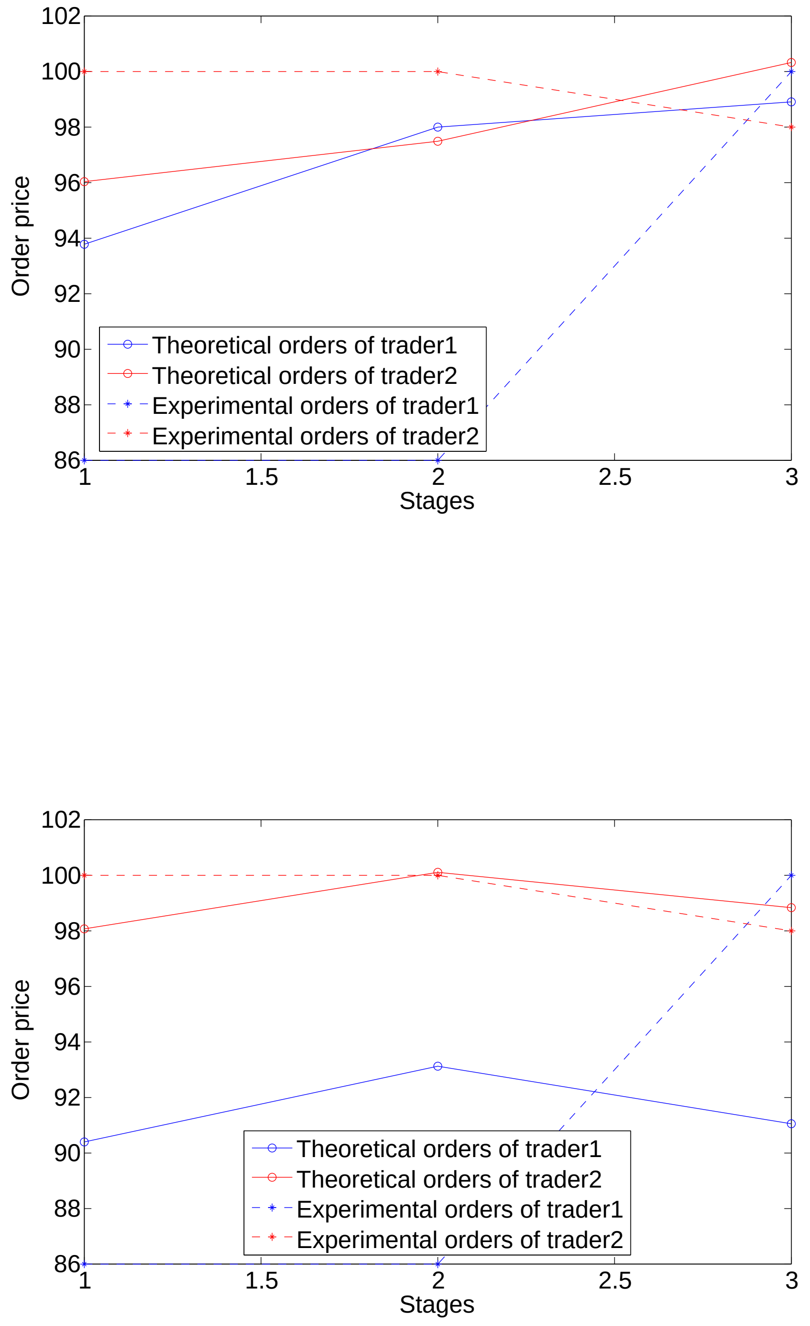


Figure 6.27: Offer prices of theoretical and experiment data (theoretical updating & experimental updating)

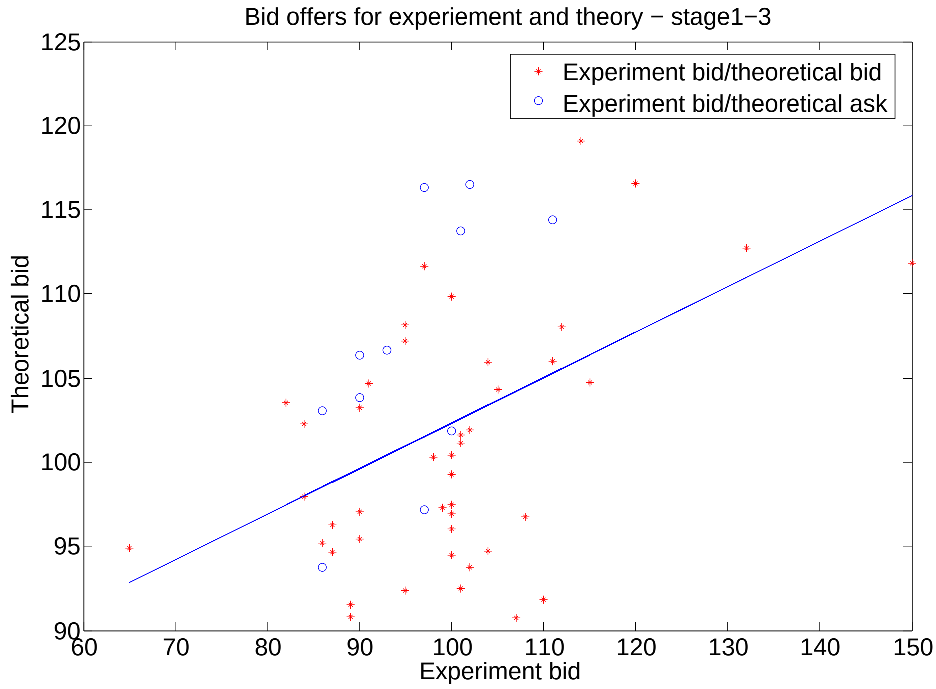
According to the methods above, the theoretical values are calculated by updating the theoretical order prices and the experimental order prices separately. Most of the statistical results show that the experimental order prices are consistent with the theoretical values. In another word, the theoretical value capture the most of the important experiment data and there is no statistical difference between the theoretical value and the experimental data. Figure B.7 and table B.7 display the comparison between the experiment and the theory for the order prices in the total stages. Other tables and figures for each stage are shown in appendix B. There is one exceptional insignificant result - the bid prices at stage 2 with theoretical updating. The explanation for this result is that traders' updating behaviors are influenced by their irrational behaviors especially at stage 2. Whereas, with the experimental updating, the slope coefficient is statistically significant, which in turn supports my inference above. Furthermore, I also observed that the traders' performance at the first stage and third stage is better than their performance at the second stage. There are two implications: first, the belief updating in the multi-period trading is complex to the subjects which in turn increases their irrational behaviors; second, with the increase of the number of stages, the traders' submission strategies are possibly closer to the optimal strategies in the theoretical model.

Table 6.13: Bid price of experiment & theory - total

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	75.276	9.523***	0.000	72.345	9.925***	0.000
Slope	0.271	3.425***	0.001	0.282	3.877***	0.000

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

The traders' choices on the order sides are decided by the relationship between the trader's private values and the value distributions of the other one. As indicated in the theoretical model, the traders prefer a bid to ask when their private values are higher than the mean value of their opponents' valuation distribution, vice versa. This rule is validated either in single-period markets or multi-period markets. In the multi-stage trading, when the relationship between their private values and the mean value of the



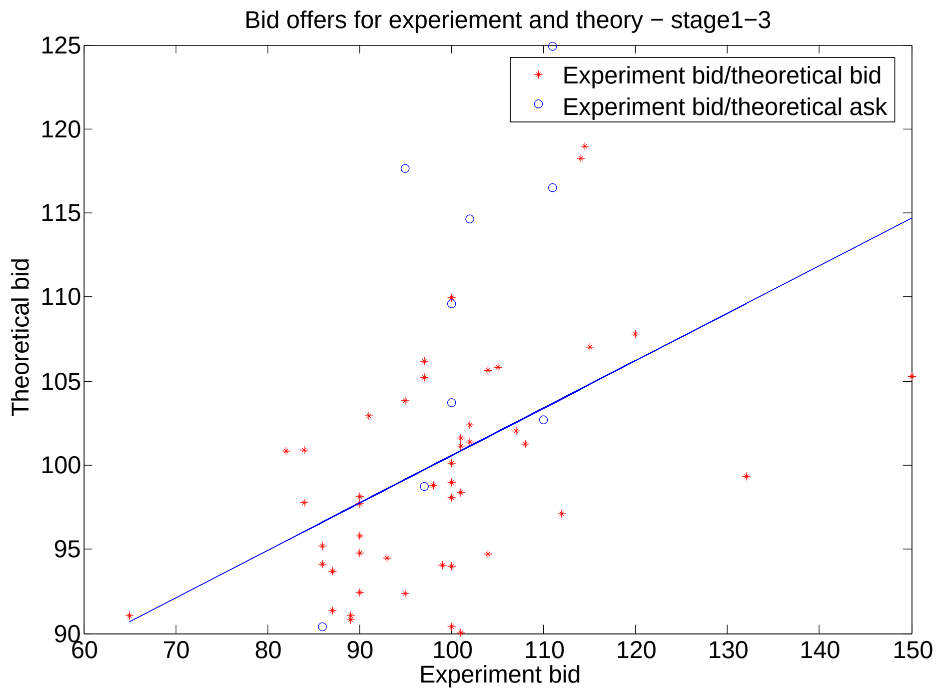
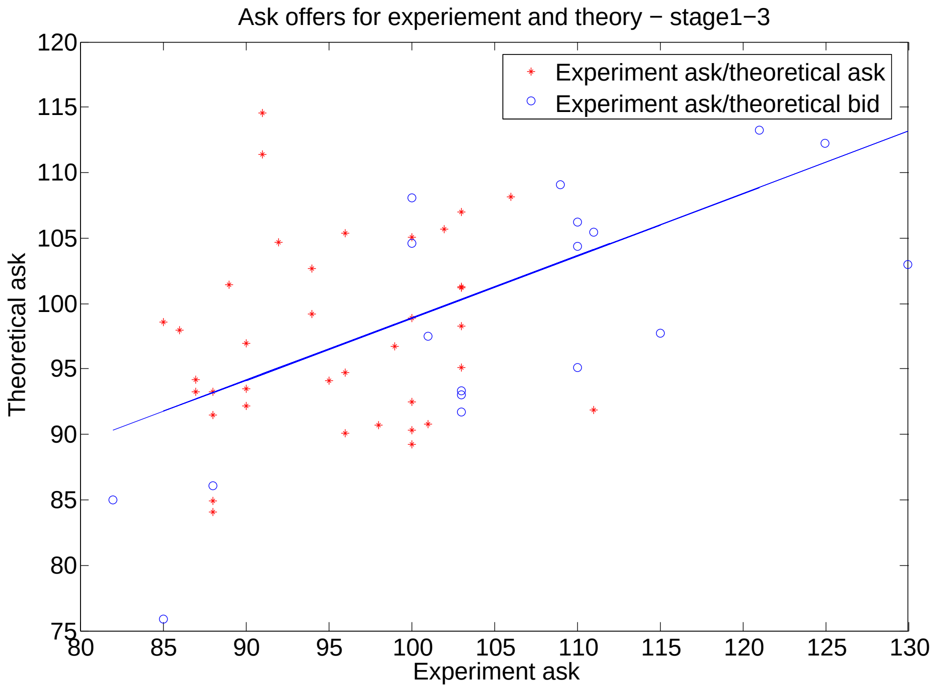


Figure 6.28: Bid offers at stage $1-3$ (theoretical updating & experimental updating)



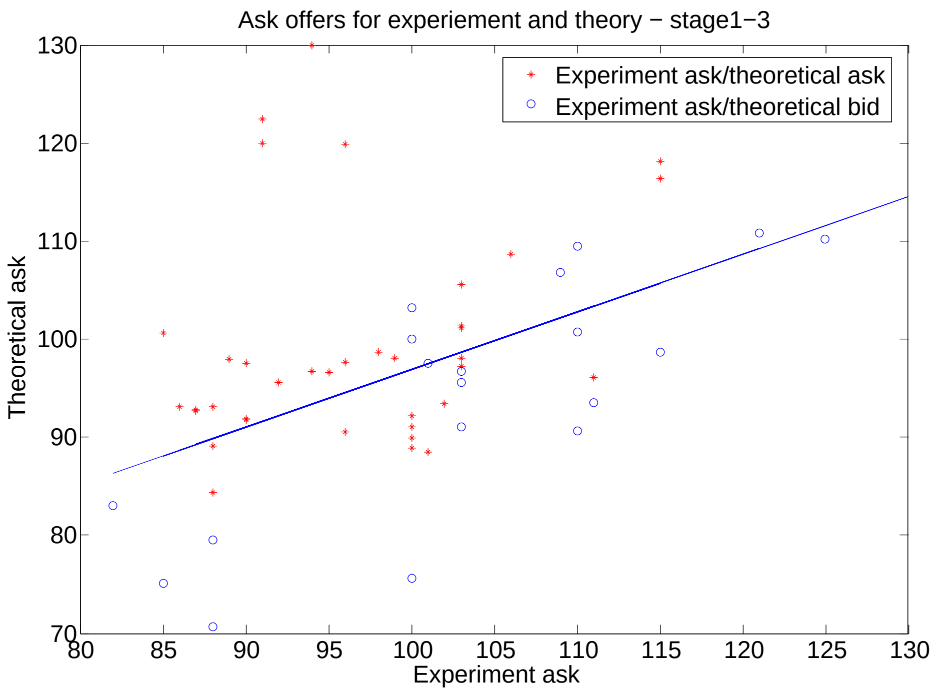


Figure 6.29: Bid offers in stage $1-3$ (theoretical updating & experimental updating)

Table 6.14: Ask price of experiment & theory - total

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	51.382	5.169***	0.000	38.179	3.214**	0.002
Slope	0.475	4.790***	0.000	0.587	4.924***	0.000

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

opponents' valuation distribution changes, the traders change their order sides during the continuous trading process. Especially, when both traders have relatively low(high) private values, one of them would change his order side to increase the probability of transaction happening. To investigate the relationship between the experimental order sides and theoretical order sides, 2×2 chi-square tests is used to test the goodness of fit for the experimental data to the corresponding theoretical values. The decision of order sides on bid/ask frequency are calculated for each stage separately and then for total stages. As for the decision of order sides, the statistical results prove that the experimental data fits the theoretical values well for each stages as well as the total stages. Therefore, I could confirm that the subjects' decision of order sides in the experiments are consistent with the traders' decision of order sides in the theoretical model. In another word, the theoretical model demonstrate the similar rules of the decision of order sides to the subjects' behaviors in the experiment. All the tables are listed in appendix B. Here, I present the table B.12 for total stages. We can observe that the critic values of Chi-square test are very high for the theoretical values with theoretical updating as well as the ones with experimental updating, which imply a strong relationship between the experimental data and theoretical values.

To compare the trader's behaviors in the experiment and the theory, the frequency of the transaction occurrence is an important variable to be investigated. In the single stage, the transaction frequency is low because of the strict trading rules. Whereas, in the multi-stage trade, the transaction frequency increases with the increase of the trade rounds, as the result of traders' adjusting their order prices and sides. Therefore, by comparing the experimental transaction frequency with the theoretical transaction one, I could observe the consistency of the subjects' updating behaviors as well as their

Table 6.15: Bid/ask decision in the experiment and theory - total

Theoretical updating				Experimental updating			
		Theory				Theory	
		Bid	Ask			Bid	Ask
Experiment	Bid	45	11	Experiment	Bid	47	9
	Ask	18	40		Ask	20	38
		χ^2	p-value			χ^2	p-value
Chi-square		28.035***	0.000	Chi-square		28.748***	0.000
Yates		28.035***	0.000	Yates		26.744***	0.000

Note: χ in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

submission strategies in multi-period model. Corresponding to the method above, the theoretical values are obtained with theoretical updating and experimental updating for comparison. The test for the transaction frequency is approached for each stages and total stages as well.

The contingency tables and statistical data are listed in appendix B. Here, I present the table B.16 for total stages to demonstrate the statistical results. Most of the critical values are not statistically significant. This result is expected to be caused by the irrational behaviors of the subjects in the experiments and the low number of observations. One exceptional test result is the Chi-square test for the data at total stages ($\chi^2 = 4.079, p = 0.043$). Therefore, with the increase of the number of observations, it is expected that the relationship between the experimental transaction data and the theoretical transaction one becomes stronger, as the influence of the individual's irrational behaviors decreases. Another significant result happens at stage one ($\chi^2 = 3.971, p = 0.046$). The traders behaviors in the single-period market are similar to their behaviors in the multi-period market. During the experiments, I found that the students' could better to underhand and adopt proper strategies in the single-period model. Whereas, in the multi-period model, they are in more complex situation. Especially, when the transaction happens at the first stage of trading, they confused and their strategies diverged. Therefore, the experimental transaction frequency is heavily influenced by the irrational behaviors of subjects at the later stages. Despite the

Table 6.16: Transaction and no transaction in the experiment and theory - total

Theoretical updating				Experimental updating			
		Theory				Theory	
		Trade	No trade			Trade	No trade
Experiment	Trade	11	5	Experiment	Trade	7	9
	No trade	16	25		No trade	11	30
		χ^2	p-value			χ^2	p-value
Chi-square		4.079**	0.043	Chi-square		1.525	0.217
Yates		2.974*	0.085	Yates		0.842	0.359

Note: χ in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

insignificant results at the rest stages, I could conclude in some degree that the experimental transaction frequency could interpret the theoretical transaction frequency.

Now I analyze the traders' updating behaviors in the multi-stage trading. To investigate their updating behaviors, I introduce the independent variable - the increas/decrease of the traders' order prices. The increasing and decreasing of offer prices are related to the changes of traders' submission strategies in the multi-stage trade according to their information. Although, I think it is not a very reliable parameter [1], it still could provide some useful information for the research of the subjects' behaviors in the laboratorial environment. Following the method above, the theoretical order prices are calculated by theoretical and experimental updating. The price trend is observed by the difference between the order prices at stage 1 and stage 2, the order prices at stage 2 and stage 3, and the order prices at stage 1 and stage 3.

[1] The increase/decrease of traders' order prices include the influence of order sides in the multi-period model. As the traders' change their order sides during the multi-period trading, an improvement of the transaction probability could be realized by decreasing the bid prices as well as increasing the ask prices.

The contingency tables and chi-square tests are listed in appendix B. The tests for the difference between stage 1 and stage 2 and the difference between stage 2 and stage 3 got insignificant results. The results demonstrate that the subjects' improvement on their orders are not consistent with the expectation in the theoretical model. Whereas, the test for the difference between the order prices at stage 1 and stage 3 is out of the confidential interval (table B.19), which imply the strong relationship between the experimental data and theoretical value in the aspect of price changes. The significant result is only available to the theoretical values with theoretical updating. The experimental updating theoretical values are greatly influenced by the irrational behaviors of subjects. There are two implications for the results of the price change. First, the irrational behaviors of subjects in the experiments are greatly influence the consistency between the experimental data and theoretical values. Because the subjects are limited to understand the information in the multi-period trading, their adjustment on the order prices according to their preference is difficult to follow the expectation in the theoretical model. Second, with the increase of the number of stages, the price trend in the experiments shows a closer relationship with the trend in the theory. Therefore, it is safe to say that the subjects express their ability to improve the transaction probability and could performance better with the increase of the number of stages. To sum up, with a large stage distance, the subjects' order prices change could explain the price change in the theoretical model in some degree.

6.3.2.2 The determinants of order prices in multi-stage trade

The traders' submission strategies include the choice of their order sides and the choice of their order prices. As the implication of the theoretical model, the order prices decision is more complex than the decision of order sides, which is influenced by more parameters. The traders' preference on the order prices is determined by their information. In the single stage, the information includes their private values, the expected values of the value distribution of the other one, and the variance of the value distribution of the other one. Normally, a common value distribution is used as a public information for both traders. The theoretical model demonstrates that the private value and the expected value are proportional to the offer prices of the traders, while variance is proportional to the ask price and inversely proportional to the bid price. In

Table 6.17: Increase and decrease in the experiment and theory - stage 1-3

Theoretical updating				Experimental updating			
		Theory				Theory	
		Increase	Decrease			Increase	Decrease
Experiment	Increase	18	2	Experiment	Increase	15	5
	Decrease	9	9		Decrease	9	9
		χ^2	p-value			χ^2	p-value
Chi-square		7.370***	0.007	Chi-square		2.545	0.111
Yates		5.553**	0.018	Yates		1.584	0.208

Note: χ in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

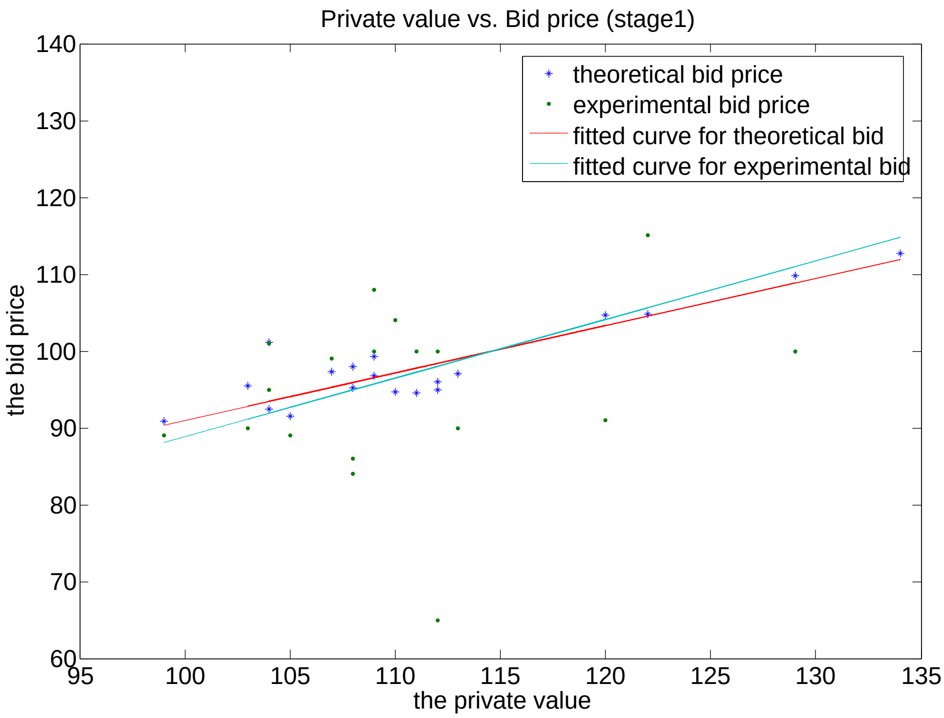
the multi-period model, at the first stage of trading, the traders' situation is similar to their situation in the single-stage market. However, they receive new transaction data at the end of the first stage. After that, the offer price of the other one in the previous stage are considered by the traders to make their decision of the order side and the order prices on the current stage. Meanwhile, the mean values and variances of the valuation distributions of the traders regarded by their opponent diverge from each other after information updating. Compared with the single-stage trading game, the effects of these determinants on the order prices of traders in the multi-stage trading would provide new implications to understand the submission behaviors of traders in double auction markets. These two methods are used to obtain the related theoretical values in this section.

In the multi-stage trading, the trader's private value does not change during the process of trade as no new private information is considered. The constant private value contributes heavily to the order price decision. The theoretical model demonstrates that the relationship between the private value and the offer prices is positive either for buyers or sellers. The positive effects of the private value on the order prices are expected in both the single-period trading game and multi-period trading game.

On stage 1, the information available for the subjects are their private value, the expected value and variance of the other one. The submission strategies of the subjects are similar with the trader behaviors in the single-stage market. Figures C.1 and C.2 show that the experimental data is close to the theoretical data. The only difference between the bids and asks is that the experimental bid prices are higher than their theoretical value and the ask prices are lower than their theoretical value. This difference is caused by the different characters between sellers and buyers from the points of behavioral finance theory. I would explain the reasons in details below. However, the statistical test shows that there is no slope change as the result of significant test. Therefore, the experimental results coincide with the theoretical model and confirm that the private value is a significant explanatory for the offer prices with positive relationship.

As we know, the traders' submission behavior changes from the second stage because of their updating behaviors. In the stage 2, the positive relationship between the private values and offer prices from the experimental data are similar to the one in the stage 1. Compared with the theoretical model, the slope of the fitted curve of experimental bid prices is higher than the slope of corresponding theoretical values, while there is no difference between the slopes of the experimental ask prices and theoretical ask prices. The negative experimental results of the higher bids show that some uncontrolled factors affect the judgement of the traders on their private values. Meanwhile, the slope of the experimental ask offers is slightly lower than it should be in theoretical model. The higher slope of the experimental bid prices implies that the traders submit higher bids than the related theoretical values. On the contrary, the sellers submit asks with relative low values. The explanation for traders' bias behaviors in details is presented below in separate paragraph.

In the stage 3, the positive relationships between the private value and offer prices are observed in this stage trade as well. It is the first time that the experimental bids are lower than their corresponding theoretical value attained by method 1. As we know, the method 1 is adopted to calculate the theoretical values from the points of whole progress, while the method 2 is adopted from the points of traders' updating behaviors on each stage. Therefore, the lower bids imply that after serval times of trading, the traders weigh less on their own private information. Similarly, I also find that subjects'



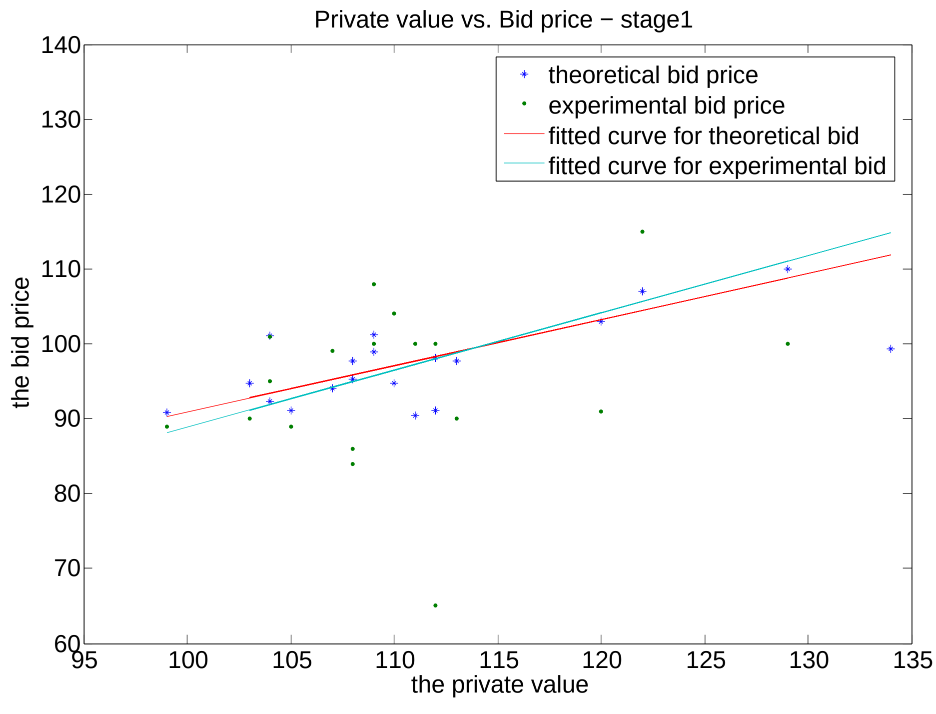
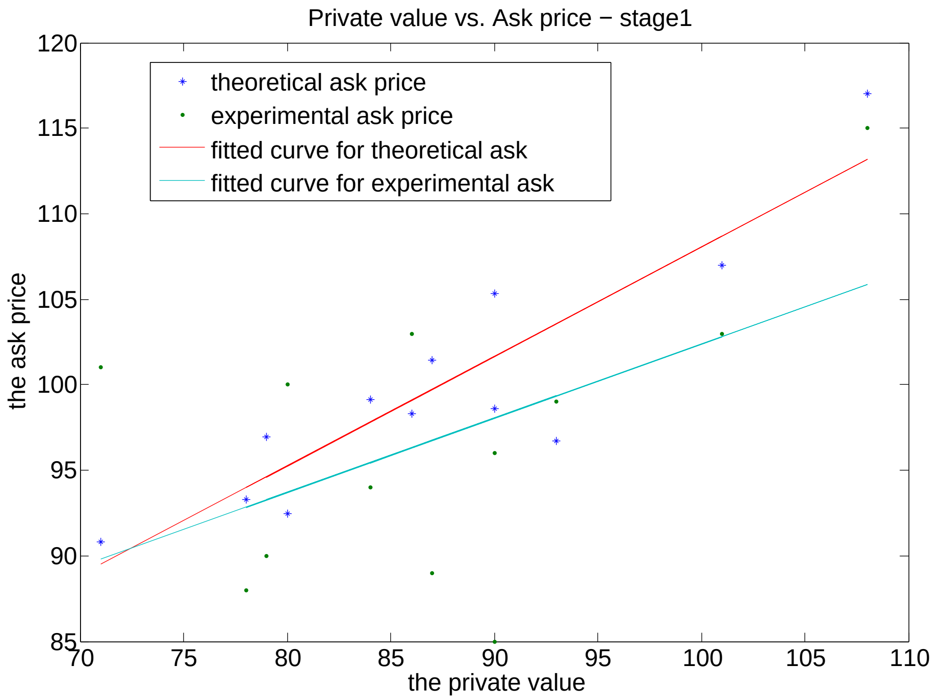


Figure 6.30: The effects of private values on bid prices - stage 1 (theoretical updating & experimental updating)



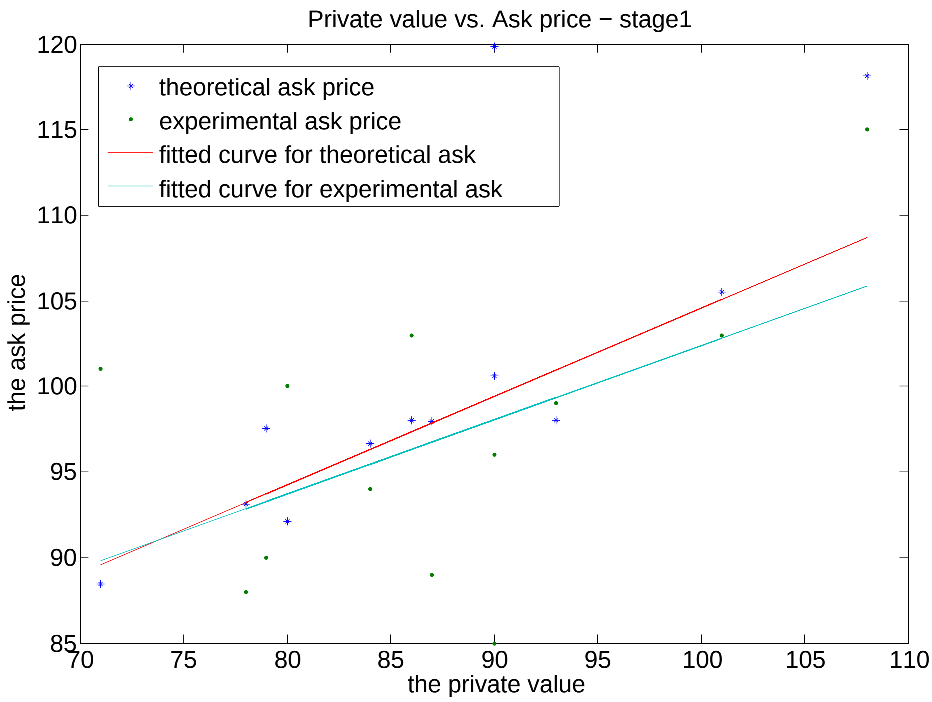


Figure 6.31: The effects of private values on ask prices - stage 1 (theoretical updating & experimental updating)

Table 6.18: Statistics for the private value vs. offer price - stage 1

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	29.50	3.77***	0.00	44.04***	4.85	0.00
		Slope	0.61	8.79***	0.00	0.64	6.19***	0.00
Theoretical updating	Experiment	C	12.55	0.37	0.71	59.00	2.89**	0.02
		Slope	0.767	2.55**	0.02	0.43	1.87*	0.09
F -slope difference			0.61			0.94		
	Theory	C	29.50	2.54**	0.02	52.88	5.48***	0.00
		Slope	0.61	5.93***	0.00	0.52	4.70***	0.00
Experimental updating	Experiment	C	12.55	0.37	0.71	59.00	2.89**	0.02
		Slope	0.76	2.55	0.02	0.43	1.87*	0.09
F -slope difference			0.58			-0.03		

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

ask prices are closer to their theoretical value after repeated trading and information updating. This result coincide with the point of behavioral finance that the subjects' behaviors could be reduced by experience.

At last, I combine the data on each stage to observe the relationship between the private values and the order prices. Most of the comparison between the experimental offer prices and the theoretical prices are not statistically significant expect the bid prices with method 2. The positive experimental results support the point that my theoretical model properly describes the relationship between the order prices and their determinants. The theoretical values attained by updating the previous experimental offers are greatly influenced by the irrational behaviors of traders. Therefore, with method 2, the big gap between the theoretical order prices is expected. However, with method 2, I could observe the bias behaviors of traders because the theoretical values describe their bias behaviors stage by stage. As the results listed in the table C.4, with method 2, the experimental bids have higher slope than the theoretical offers. In each stage, their reaction to their private value is bigger than it should be.

In sum, as one of the important variables that affect the traders' submission strategies, the private value is the most powerful to influence the order prices on each stages. The results of the experiment confirm that there is positive relationship between the traders' private values and the offer prices of similar to the inference in the theoretical models. Although some groups' experimental data affected by the bias behaviors of the subjects and show significant higher slope than the corresponding theoretical values, most of the results of experimental data coincide with those of the theoretical value.

In my experiment design, the traders' private values are drawn from the same distribution. Then, the initial expected values of the valuation distribution are the same for both traders. Different from the constant private value, the expected values are updated repeatedly by the traders according to the order prices of the opponents at the previous stages. As the implication of the theoretical model, the expected values of the opponents' valuation distribution have positive effects on the traders' order prices. In the process of continuous trading game, the trader's assessment on his opponent's valuation distribution is updated and the updated valuation distribution of the other one with new expected value provides the trader a clearer figure of the other one's private information. This process is one of the properties of the efficient double auction markets - the double auction market disseminate information efficiently as the demonstration in the experiments (Plott and Sunder, 1988). Although in this simple double auction market, the number of the buyers and the sellers is too small to achieve the competitive equilibrium, I find that this double auction market still has the function of information dissemination. The two dimension graphs C.9 and C.10 display the relationship between the expected value and bid/ask prices with comparison between the experimental data and theoretical values. For asks, the experimental coefficients are not in the significant level. This statistical result is probably caused by the small sample size and the big influence of some irrelevant observations.

At the second and third stages, the subjects could adjust their order prices by updating their assessment on their opponents' valuation distributions according to the order prices submitted by their opponents at the previous stages. The statistical results show that most of the slope coefficients are insignificant for the experimental data. Some of the slope coefficients of the theoretical values are also insignificant. There are three reasons behind these insignificant results. First, the expected values of the opponents' valuation distribution are less powerful to influence the order prices of the traders than their private values. Since the order prices are obtained with various combination of the variables' values, to observe the effects of the expected value, its effects would be influenced by other variables, especially the private value. Second, the updating process is complex for the students with basic knowledge of financial markets. Their bias and irrational behaviors diverge their order prices from the theoretical values at the later stages when they begin to update their assessment on the other ones' valuation distribution. On the other hand, the new expected value of the other one's valuation distribution is not intuitionistic for the students, the complex updating process make a gap between their assessment of the expected values and the corresponding theoretical values. Third, the observations in the experiments are small. As the number of observations are not enough, the influence of some outliers is great even with the robust regression.

In total stages, the bid orders show a positive relationship with the expected values of the opponents' valuation distribution, while the asks still are insignificant. The buyers' performance in the multi-period model is better than the sellers' behaviors as a result of sellers' bias anticipation on the expected value of their opponents' valuation distribution. The results show that the theoretical expected values of the opponents is irrelevant to the sellers' asks during the process of repeated trading. On the other side, the theoretical expected values have positive influence on the other ones' order prices.

The theoretical expected value is indirectly related with the subjects' order prices. Because of the complex process of valuation distribution updating, the subjects' private assessment on their opponents' valuation distributions is expected distinctively diverge from the theoretical valuation distributions, which results in the big difference between the experimental data and theoretical values. Furthermore, the small number of observations enlarges the influence of individual irrational behaviors of traders. Except the regression at stage 1, most of the regression results show that the theoretical expected values could not explain the traders' order prices at each stage. Whereas, I still find some evidences that the relationship between the theoretical expected values is positively related with the order prices in the theoretical model. The clue is expected to be clearer when the number of observations and stages increase.

As the results of theoretical model, the variances are less powerful to influence the trader's order prices and have no effects on the traders' decision of order sides. As indicated in the single-period model, there are two difficulties to investigate the effects of variances. It is difficult to observe the influence of variances since other powerful variables shade its contribution. Meanwhile, compared with the private values and the expected values, the variances are the most complex factors to be uderstood by the subjects. Therefore, I observed the irrelevant relationship between order prices and variance in the single-period trading game. In the multi-stage trading, the variance of the valuation distributions decreases continually conditional to the order prices of the other ones. Therefore, it always increases the bids and decreases asks. Similarly, their effects in the experiments are difficult to capture after combined with the effects of private values and variational expected values.

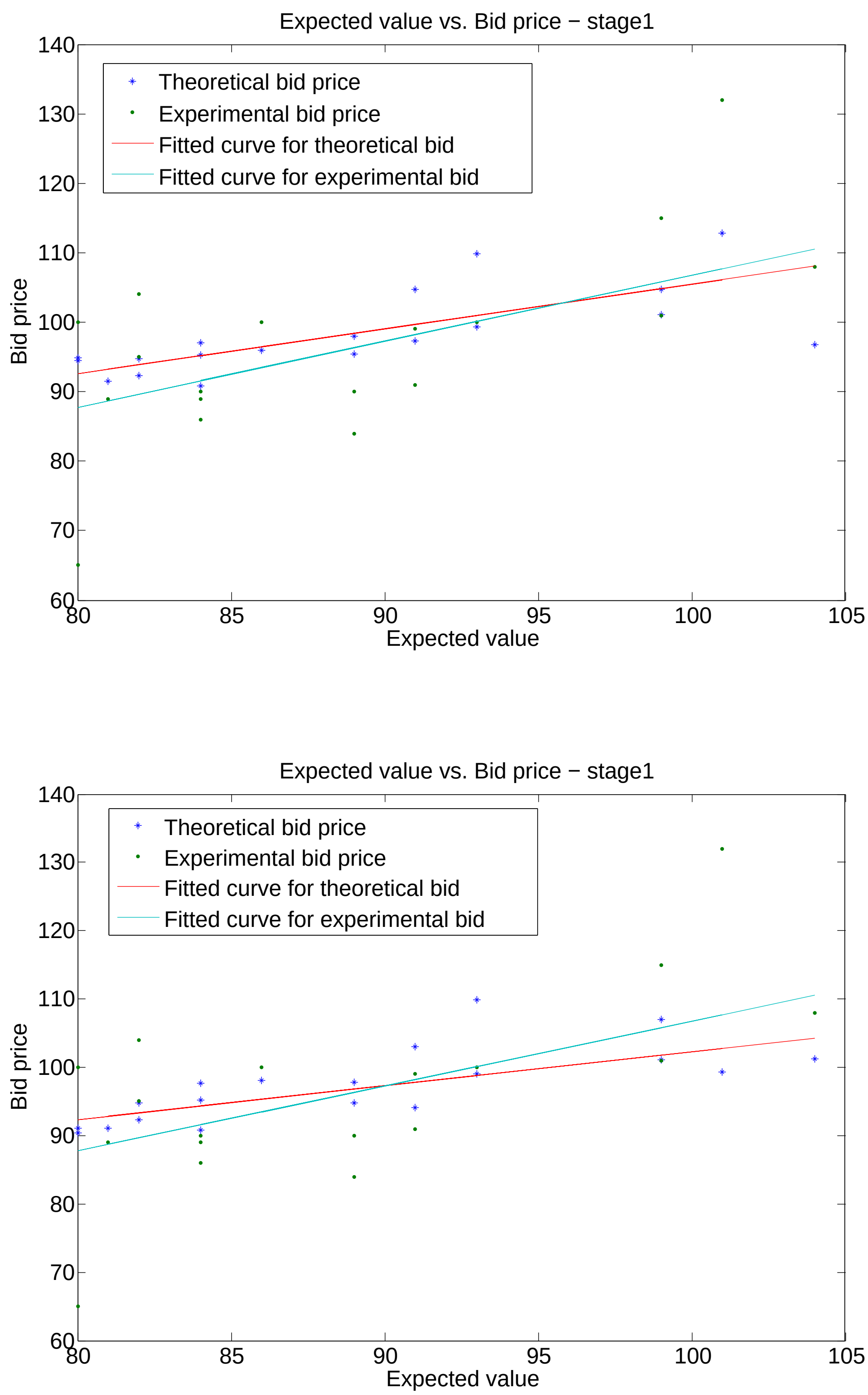
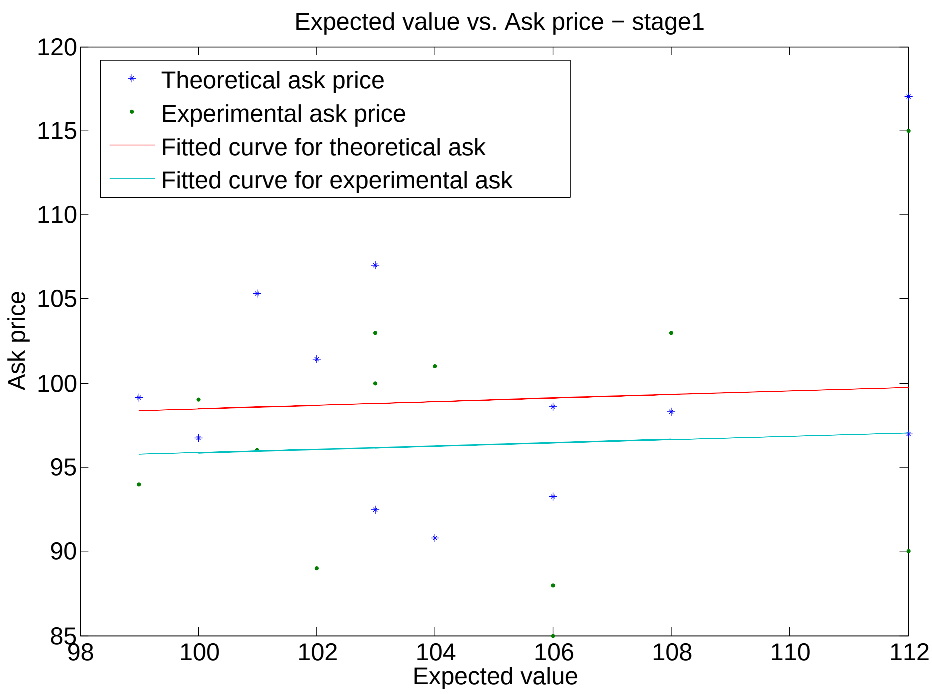


Figure 6.32: The effects of expected values on bid prices - stage 1 (theoretical updating & experimental updating)



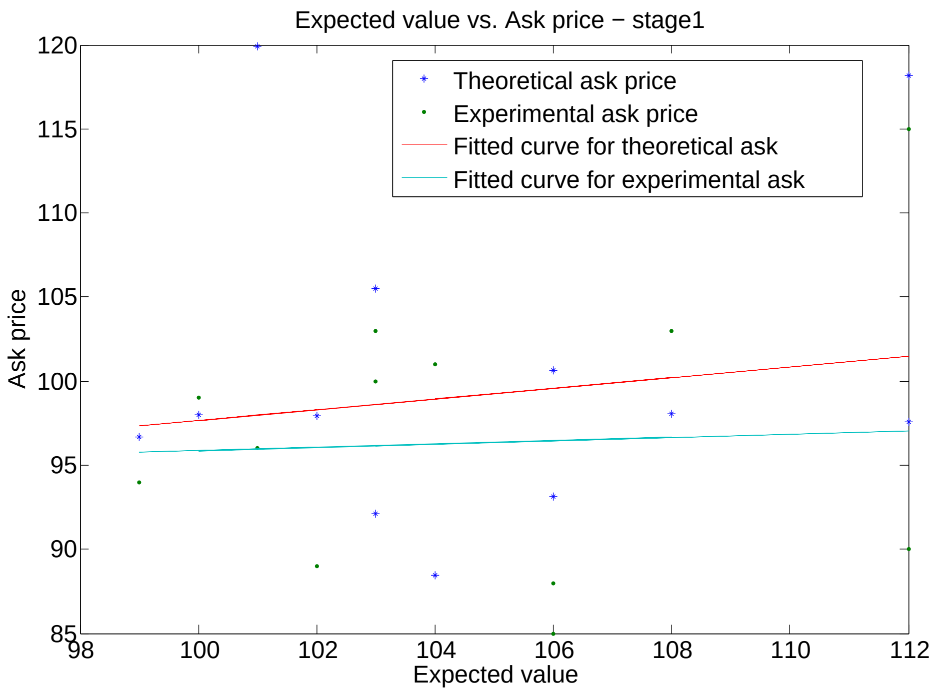


Figure 6.33: The effects of expected values on ask prices - stage 1 (theoretical updating & experimental updating)

The figures and tables are listed in appendix C. Most of the regressions show insignificant results. The experimental data for total stages has a positive slope in week significant level for bids, but the coefficients of theoretical data are insignificant. Therefore, there is no evidence to show the correlation between the experimental data and theoretical data in the aspect of variance.

As the complimentary of the observations of relationship between determinants and order prices, the regressions with the dependant variable - the difference between the theoretical order prices and related experimental ones show us the effects of the determinants from the points of each pair of order prices. The explanation is similar to the one for single-stage experiment. If the coefficient of slope is not statistical significant, the new regression supports the related one in that the effects of the determinant on the theoretical order prices is similar to the ones on the experimental prices. Therefore, we expect that the values of the difference between the theoretical prices and related experimental prices stochastically swing around the flat fitted line.

For the multi-stage experiment, I present the regression results in table 6.19 and 6.20 below. From the table, most of the regression results are statistic insignificant as we expected. Therefore, it is safe to say that there is no other statistic significant relationship between the determinants and the experimental order prices. In another word, the relationship between the determinants and the theoretical order prices reflects in the behaviors of the subjects in the experiment. Although there are some significant slope coefficient in some stages, the possible reason is the small number of observations in the multi-stage experiment. Therefore, we can observe that the sum of the data in total stages presents non-significant results.

Trader's bias behaviors in multi-stage trading In the multi-period, we observed that traders submit order with high execution probability and less optimal prices, compared with the related theoretical values. This observation is different from their bias behaviors in the single-period experiment, where the overconfidence dominates the subjects' behaviors. Figure C.1 displays that the buyers submit a higher bids than the theoretical values at stage 1. Their order prices increase the transaction probability but reduce their expected profits by decreasing the difference between the order prices and their private values. Figure C.2 shows that the sellers provide a lower

Table 6.19: The relationship between the determinants and the difference between the theoretical and experimental order prices - theoretical updating

		theoretical updating					
		Bid			Ask		
		Coefficient	t - stats	Prob.	Coefficient	t - stats	Prob.
Stage 1	C	-22.865	-0.687	0.501	14.611	0.682	0.511
	Private	0.197	0.664	0.516	-0.200	-0.821	0.431
	C	-51.980	-1.700	0.107	-2.420	-0.039	0.970
	Expected	0.573	1.674	0.113	-0.005	-0.008	0.993
	C	-7.913	-1.659	0.115	0.311	0.069	0.946
	Stdev	1.160	1.931	0.070	-0.488	-0.836	0.423
Stage 2	C	-59.707	-1.243	0.240	11.167	0.283	0.782
	Private	0.539	1.249	0.237	-0.143	-0.310	0.763
	C	-100.236	-2.350	0.038	-28.693	-0.738	0.476
	Expected	0.929	2.369	0.037	0.320	0.714	0.490
	C	-9.552	-1.084	0.302	10.591	2.111	0.058
	Stdev	1.124	0.906	0.384	-2.027	-2.698	0.021
Stage 3	C	32.357	1.060	0.312	3.888	0.315	0.758
	Private	-0.304	-1.126	0.284	-0.052	-0.381	0.710
	C	20.237	0.746	0.471	11.458	0.758	0.462
	Expected	-0.193	-0.822	0.429	-0.130	-0.814	0.430
	C	-9.152	-1.663	0.124	3.909	1.377	0.192
	Stdev	1.027	1.257	0.235	-0.767	-1.787	0.097
Total	C	10.618	0.551	0.584	7.675	0.624	0.536
	Private	-0.112	-0.649	0.520	-0.107	-0.767	0.448
	C	-2.071	-0.180	0.858	5.707	0.455	0.652
	Expected	0.004	0.039	0.969	-0.078	-0.591	0.558
	C	-8.953	-2.941	0.005	4.682	2.007	0.052
	Stdev	1.151	2.786	0.008	-1.045	-3.125	0.003

Table 6.20: The relationship between the determinants and the difference between the theoretical and experimental order prices - experimental updating

		experimental updating					
		Bid			Ask		
		Coefficient	t - stats	Prob.	Coefficient	t - stats	Prob.
	C	21.224	0.701	0.492	20.220	0.736	0.479
	Private	-0.210	-0.778	0.447	-0.268	-0.858	0.411
	C	-28.575	-0.913	0.374	2.874	0.036	0.972
Stage 1	Expected	0.304	0.869	0.397	-0.057	-0.075	0.942
	C	-7.878	-1.716	0.104	0.562	0.095	0.926
	Stdev	1.108	1.916	0.072	-0.520	-0.675	0.515
	C	11.864	0.372	0.717	10.016	0.199	0.846
	Private	-0.139	-0.480	0.640	-0.113	-0.194	0.850
	C	-112.169	-7.128	0.000	-30.238	-1.245	0.241
Stage 2	Expected	1.061	7.215	0.000	0.360	1.320	0.216
	C	-3.400	-0.502	0.625	9.672	1.154	0.275
	Stdev	0.063	0.058	0.955	-1.973	-1.631	0.134
	C	-13.204	-0.550	0.592	20.273	1.218	0.247
	Private	0.103	0.484	0.636	-0.228	-1.237	0.240
	C	-30.139	-3.265	0.006	8.402	0.382	0.709
Stage 3	Expected	0.272	3.140	0.008	-0.084	-0.353	0.730
	C	-9.463	-2.682	0.019	1.767	0.405	0.692
	Stdev	1.341	2.500	0.027	-0.253	-0.377	0.713
	C	-0.360	-0.023	0.982	18.070	1.297	0.203
	Private	-0.015	-0.109	0.914	-0.218	-1.382	0.176
	C	-26.563	-3.314	0.002	-1.492	-0.109	0.914
	Expected	0.250	3.114	0.003	0.010	0.070	0.945
	C	-7.654	-3.237	0.002	3.747	1.357	0.183
	Stdev	1.014	3.031	0.004	-0.687	-1.749	0.089

asks at stage 1, by which the transaction probability increase and the expected profits decrease. I consider this bias is caused by their overreaction because of their experience in the single-period model. Because the traders play the single-period trading game several time before play the multi-period trading game, their emotion is influenced by the experience in the single-period trading game. In the single-period trading game, they are overconfident to submit orders with better price and less execution probability. However, they found that the probability of making profits from successful transactions is lower than their expectation. They are pessimistic at the end of single-period trading. Therefore, they change their submission strategies to submit orders with worse order price and high execution probability. During the process of trading, they overreact to the new information by change their order prices dramatically in the multi-period trading.

I also observed the bias of conservatism in the repeated trading process. Some traders are slow to change their order prices after they get new information from previous stages. Some order prices even never change during the process of repeated trading. This phenomenon also can be explained by the less change of the market prices with the change of valuation caused by the new information. Kirchler (2009) claim that, when the fundamental value following a stochastic process, the change of market price is less than the change of fundamental value. Although, the subjects received the new information of the other ones' order prices. It looks that they did not consider the information entirely and update their belief on their opponents.

6.3.2.3 The comparison between the single-period trading game and the first stage of the multi-period trading game

In the theoretical model, the traders' submission strategies in the single-period model is identical to their strategies at the first stage of multi-period model. The basic intuitions behind the result are that the traders submit similar orders with similar private and public information, since their trading purpose is to maximize their profits at each stage. The relationship between the determinants and their order sides or order prices in the single-period model are similar to the ones in the multi-period model.

I observed that either in the multi-period experiment or at the first stage of single-period experiment, the subjects' order prices are positively related to their private value. The mean values of the opponent's valuation distributions positively influence the subjects' order prices in the single-period experiment. Whereas, because of the small number of observations, the positive effects of expected values at the first stage of multi-period experiment are only displayed by buyers. The slope of the fitted curve for the relationship between the asks and the the expected values of valuation distributions are not statistically significant. As for the variance, there is no significant result for the order prices at the first stage. Although some of the coefficients for the experimental data at the first stage of multi-period trading are weekly significant or insignificant, most of the relationships between the order prices and the variables are similar to the ones in the single-period trading. A closer comparison displays that the subjects in the single-period trading prefer to submit lower bids to higher asks relative to their private values. Whereas, the subjects prefer to submit higher bids and lower asks at the first stage of multi-period trading. In another word, they submitted orders with lower execution probability and better price at the single-period experiment and orders with higher execution probability and worse price at the first stage of multi-period trading. The explanation behind the different submission strategies is that the subjects are overconfident with their ability to assess their situation although uncertainty of their opponents dramatically decrease the execution probability of their orders. They pursue their profit by offering less or asking more compensation. The submission strategies in the multi-period experiment are slight difference because the students become pessimism after the single-period experiment. They overreact to the information in the multi-period experiments by putting more weights on the execution probability, instead of their order prices.

In the aspect of deciding of order sides, their behaviors in the single-period experiment and the ones at the first stage of multi-period experiment are similar according to the results of statistical tests, which is consistent with the results in the theoretical model. In theory, their decision of order sides depends on the relationship between their private values and the expected value of the opponents' valuation distributions. The chi-square test confirm a strong relationship between the experimental order sides and the theoretical order sides under both situations. Generally speaking, the subjects' submission strategies in the single-period experiment are similar to their strategies at the first stage of multi-period experiment on most aspect and consistent with the results in the theoretical model.

6.3.2.4 Summary

Although the observations of multi-period experiments are not as sufficient as the ones in single-period experiments, I still obtain useful results consistent with the theoretical models. First, I observed the converge of order prices in the process of repeated trading, by which the transaction probability increases. Most of the subjects show their ability to adjust their order sides and price to increase the transaction probability and make profits. However, because the decision making process in the multi-period model are more complex than in the single-period model, the divergence of the experimental order prices from the related theoretical values in the multi-period model is larger than in the single-period experiments, especially after the first stage. Whereas, the subjects' behaviors is close to the expectation in theory with the increase of the number of trading periods. The results of linear regression for the experimental order prices and the related theoretical value prove that the experimental data could explain the theoretical values in some degree.

Because there are only three periods, the tests for their updating behaviors does't provide strong supports for the correlation relationship between the theoretical values and experimental data. However, some tests provide significant results, such as the test of transaction frequency in total stages, the test of increase or decrease between stage 3 to 1. I conclude that with the increase of the observations and the number of trading, the experiment data have stronger relationship with the theoretical values.

In the aspect of the determinants of order prices, the main results of experiments are consistent with the ones of theory. As the influence of subjects' irrational behaviors, some of the estimated coefficients are not statistically significant, especially for the variables which have less power to influence the order prices. Similarly, I find that with the increase of the observations and the number of trade, the experimental data performance better.

6.3.3 Conclusion of the experiment

In this chapter, we introduce two experiments - single-period experiment and multi-period experiment to investigate the traders' submission behaviors in the laboratorial environment. With the help of experiments, not only I could understand the traders' submission strategies better by observing the subjects' behaviors in the experiments, but to verify the efficiency of the theoretical models by compared with the experiments.

In the single-period model, the experimental results are consistent with the theoretical results. The good results benefit from the large number of observations and the simple design for students to understand. In the aspect of decision of order sides, the experiment results support the rules in the theoretical model that traders prefer bids to asks when their private values are higher than the expected value of the other ones' valuation distribution, and vice versa. In the aspect of decision of order prices, the relationship between the order prices and their determinants in the experiments are similar to the ones in the theoretical models. I also observe that the subjects' bias behaviors cause the difference between the theoretical data and experimental data. For example, because the buyers weigh their own private value less, the slope of the fitted curve for the experimental orders is flat than the one for the theoretical values. The difference between the theoretical values and experiment data is results of the difference between the assumption of theoretical model and the conditions of experiments. Despite these small difference, there is a strong relationship between the experimental data and theoretical values in the single-period markets.

In the multi-stage experiments, the consistency between the theoretical values and experimental data only available on certain aspects. There are two difficulties in the multi-period experiments: the small number of observations and the complex process of making decision for the subjects. In the aspect of the decision of order sides, the chi-square tests provide positive results to support that the subjects' decision of order sides in the experiments are similar to the traders' decision of order sides in the theoretical model. The statistical tests for the transaction probability and price trend in the multi-period experiment do not make too much sense on their relationship with the corresponding theoretical values. In the aspect of decision of order prices, the relationship between the private values and the order prices is consistent with the results in the theoretical model. Whereas, the expected values and the variances of the other ones do not influence the order prices on most of the stages as the results of statistical test. However, I observed that the subjects' performance better when the number of stages increase. The results of multi-period experiment imply that the subjects have difficult to adopt optimal strategies in multi-period trading game, especially when they face more indirect and ambiguous information such as the variance and expected values. But their submission strategies are closer to the traders' submission strategies gradually with the number of stages increase. Totally, the subjects' submission behaviors in the multi-period experiments are similar to the traders' submission strategies in the theoretical models on some aspects.

The data at the first stage of multi-period trading are comparable with the one in the single-period trading, since the subjects are in the similar situations. The similarities between these two groups of data are that they display the same relationship between the variables and the order prices, which is consistent with the results in the theoretical models. The traders' behaviors are identical in the theoretical model. However, the traders' behaviors are slightly difference on these two stages. The subjects in the single-period trading prefer to submit lower bids and higher asks relative to their private values and prefer to submit higher bids and lower asks at the first stage of multi-period trading. The difference of their behaviors are the results of their heuristic bias.

7

Discussion and conclusion

The double auction markets arise much of interests because of their importance as the fundamental mechanism in most of the financial markets around the world. In the double auction markets, the highest bid order matches with the lowest ask offer automatically without involving intermediate between the buyers and sellers. There are many researches to investigate the double auction markets from the aspects of market efficiency, participants' submission strategies and market equilibrium, and etc. In my research, I focus on the traders' submission strategies in double auction markets to investigate their optimal strategies and the theoretical competition between them. My research starts from a single-period double auction market with two informed traders, and then extent the single-period model to the multi-period one. The models are designed to link the double auction markets and limit order markets, by which include the features of both markets to provide some new insights into the traders' behaviors in double auction markets. Different from the previous research on double auction markets, I allow the traders to choose their order sides as well as order prices. The traders' order side choices according to their information increase the uncertainty in the market and influence their optimal submission strategies. Comparing to the research in limit order markets, my models deal with the execution probabilities as an endogenous factor by involving the game theoretical competition between traders. In another word, I apply the bargain theory into the research of limit order market to investigate the optimal submission strategies of traders.

My research aim is to investigate the informed traders' submission strategies in a dynamic double auction market and the efficiency of the market to disseminate private information. The research start out from a single-period static model, in which two informed traders trade one unit of assets. One feature of the model is that I impose no restrictions on the traders' order sides. A parameter, the probability of the trader's opponent submitting a buy(sell) order, is introduced into the model, allowing traders to choose their order sides. Besides their decision of order sides, the optimal submission strategy is another question to address in the static model. With the assumption that their private valuation distributions follow normal distributions, the parameters that influence the traders' decision of order sides and prices include their private values, the expected value and the variance of opponents' valuation distribution. The static model illustrate the influence of these determinants on the traders' decision of order sides and optimal order prices. In order to investigate traders' submission strategies in multi-period markets and the process of information dissemination, I extend the single-period static model to the multi-period dynamic model. In multi-period markets, at the end of each stage of trading, the order prices of their opponents are available for traders to update their belief on their opponents' valuation distributions. They are allowed to change their order sides and prices by submitting a new order at next stage, according to their private values and their opponents' updated valuation distributions. With trade repeated continuously, their valuation distributions regarded by the others concentrate on their reservation prices. This is the process of information dissemination, by which traders' understanding of their opponents' private value become more and more clearly.

In order to provide evidence to support the results of theoretical models, I carried out the experiments: the single-period experiments and multi-period experiments, corresponding the static and dynamic model respectively. The subjects are students of the Economy department and School of Management of the University of Bath. In the experiments, traders were allocated into pairs automatically by the computer. They were require to trade with their opponents in the laboratorial environment to maximize their profits. In the single-period experiments, the students submitted their orders only once according to the information of private value and their opponents' valuation distributions. They played the single-period trading several times with different opponents. In the multi-period experiments, the students traded with the same opponents underlying the same series data serval rounds. In the process of repeated trading, the subjects changed their order sides and prices according to the updated order prices of their opponents. Although the irrational behaviors of traders had influence on the experiments results, I observed the improvement on the transaction probability and convergence of order prices during the process of repeated trading. Most experimental results are consistent with the results of theoretical models. The difference between the traders' submission strategies in theoretical models and the students' submission strategies in the experiments can be interpreted by the traders' heuristic bias, such as overconfidence, bounded rationality, etc. Overall, the experiments provide fair supports to the results of the theoretical models in the extent to explain the traders' submission strategies.

7.1 The two-trader trading in the single-period double auction market

The single-period model describes the situation when two informed traders trade one unit of asset in a single-period double auction market. The informed traders are supposed to be rational and risk neutral, whose trading purpose is to maximize their profits. After entering into the market, they are allowed to choose their order sides and price according to their private information of the asset's fundamental value and the information of the value distribution of the other one. Their private values are drawn from the normal distributions, which can be same or different. It is a common sense that every one in the market knows his opponent's valuation distribution, and his opponent knows that he knows, and continue. They submit their order simultaneously and have equal power to decide the transaction price. The single-period model investigates the influence of the determinants on the order side and price of traders.

In the aspect of decision of order prices, the static model illustrates that the traders' private values are most powerful to influence order prices. The traders' order prices increase with the increasing of their private values. It shows that traders will to pay more(less) as buyers the asset and ask more(less) compensation as sellers, when their private value is high (low). Similarly, their order prices increase with the increase of the expected value of their opponents' valuation distribution, and vice versa. The expected values of their opponents' valuation distributions imply that their opponents' private value are located in the area with relatively high or low values. Therefore, they need move their order prices closer to the mean values of their opponents' valuation distribution to improve the execution probability of their orders, which in turn increase their expected profits. The variances of their opponents' value distribution have different effects on traders' bids and asks respectably. With the increase of the variances of their opponents' valuation distributions, ask prices increase and bid prices decrease. The intuition behind the difference is that increase of asks or decrease of bids could improve traders' expected profits by increase the difference between their private values and order prices. Different variances display the shape of normal distribution' tail. For a buyer (seller), high variances of the other one's valuation distribution increase the execution probability of his orders and allow him to improve his order prices, that is increasing asks and decrease bids . Compared with the effects of variances, the influence of correlation coefficient between traders is similar to that of variances. The increase of correlation coefficient decreases traders' bid prices and increases their ask prices, and vice versa. The explanation is that the increase of correlation coefficient between traders decreases the level of uncertainty between traders. As a result, to improve their order prices, buyers decrease their bids and sellers increase their asks. When the uncertainty between them increases with the increase of correlation coefficient, traders decrease the difference between their private values and order prices to improve the execution probability.

The static model illustrates that their decision of order sides only depends on the relationship between the private value of traders and the expected values of their opponents' value distributions. When the traders' private values are higher than the mean value of their opponents' value distribution, traders prefer bids to asks. They expected that the probability of their opponents' private values lower (higher) than their private values is related to the positive values (negative values) of the difference between their private values and the mean values of their opponents' valuation distributions. Therefore, it is safe to infer that the probability to observe one buy and one sell is equal to 0.5, when every aspect of traders are similar and their private values are drawn from a common valuation distribution. If two traders' valuation distributions are different, the probability of observing a buy order on the order side increase with the increasing of the mean values of the opponent's value distribution, while the probability of observing a sell order increase with the decreasing of the opponents' mean values. Furthermore, variances have on effect on traders' decision of order sides, even under asymmetric distributions. I conclude that values of variables affect order prices, while the relationship between variables affect decision of order sides.

The experiment for the single-period model construct the similar double auction market, in which the subjects trade with their opponents by pairs, following the trading rules of the single-period model. The experiments investigate the subjects submission strategies in laboratorial environment. Most of the experimental results are consistent with the results of theoretical model, in the aspects of decision of order sides and prices. Whereas, the effects of variances are only statistically significant for asks. I interpret the insignificant result as the fact of the limited ability of non-professional students to understand the influence of variances and their irrational behaviors. It is also caused by the reason that the effects of variance are influenced by the effects of other powerful variables as all the variables change together. Given a closer observation, for private values, the slope of experimental order prices for bids is lower than that of the relative theoretical values. This difference could be explained by traders' overconfidence that they undervalue their uncertainty situation with other side and submit bids with low execution probability and high unconditional profits. Corresponding to buyers bias strategies, sellers' performance is better, although they are also overconfident to submit asks with low execution probability and high unconditional profits.

7.2 The two-trader trading in the multi-period double auction market

In multi-period double auction markets, traders submit their orders repeatedly until their private information is fully revealed. Compared with the single-period static model, the multi-period dynamic model includes the traders' adjustment on their order prices and sides by updating their information of their opponents' valuation distribution. At the end of each stage of trading, their assessment on their opponents' valuation distribution are conditional with their opponents' order prices. Traders increase their execution probability as well as expected profits by submitting a new orders based on the updated valuation distributions of their opponents. During the process of repeated trading, the information of their opponents' private values is delivered by their order prices continuously, until fully revealed. Therefore, with the repeated trading process, not only the transaction probability increases, but the private information is disseminated. The multi-period dynamic model illustrates that the simple double auction markets is efficient to disseminate information, even with two traders.

The multi-period model describe the submission strategies of traders in multi-period double auction markets. The determinants of order prices in the multi-period dynamic model are the same with the ones in the single-period model. The traders' private values are constant in the whole period of trading. The relationship between private values and order prices of traders are always positive at each stage. The relationship is in line with the one in the single-period model. The expected value of their opponents' valuation distribution are variable during the process of repeated trading, because that traders' updating their belief on their opponents' valuation with their order prices at previous stage. The mean values of their opponents' value distributions regarded by traders fluctuates around their private values and are closer to their private values gradually. In another word, the continuous updating process make the mean values of their opponents' value distributions equal the opponents' private values. In the multi-period trading, the variances conditional with the opponents' order prices continue to reduce. The relationship between the expected value of the conditional valuation distribution and the order prices is positive at each stage. Whereas, the initial expected values of the valuation distribution are irrelevant to the order prices at the later stages since the valuation distributions are updated by traders.

The change of order prices are different when the relationship between the traders' private values and the expected values of their opponents are different. If both traders' private values are higher(lower) than the expected values of the valuation distributions, they probably submit the same side of orders. The theoretical model demonstrates that the one with lower(higher) private values change his bid(ask) to ask(bid) very quick, when his private values is lower than the expected values of the other one. When the traders have different sides of orders but the bid is lower than the ask, their order prices converge as the buyer increases his bids and the seller decreases his asks. When the bid price is equal to the ask price, the transaction happens. However, the trade would continue until their private information is fully revealed. After the transaction happens, the buyer continuously increases his bids as the mean value of his valuation distribution regarded by the other one is closer to his private value, and the seller decreases his asks.

The multi-period experiments are designed to observe the subjects' order submission strategies in the multi-period double auction market, corresponding to the multi-period theoretical model. The students are required to trade with the same opponent several times by maximizing their profit at each stage. Most of the results in the multi-period experiments are consistent with the ones in the theoretical model. Because of the irrational behaviors of traders in the complex multi-period trading and the small number of observations, there are more insignificant results in the multi-period model. However, I find that the experimental data performance better at stage 3 or at total stages. Therefore, I believe that with the increase the number of stages, traders' submission stages in the multi-period trading are closer to the ones in the theoretical model.

The comparison of the traders' submission strategies in the single-period model and at the first stage in the multi-period model illustrate that their behaviors under these two situation are identical. Whereas, in the experiments, the subjects' behaviors at the first stage of multi-period trading are slight different from their behaviors in the single-stage experiment. In the single-period experiment, the overconfident subjects prefer to submit orders with low execution probability and high unconditional profits relative to their private values. In the multi-period experiment, after experiencing some unsuccessful trading, the pessimistic subjects prefer to submit orders with high execution probability and low unconditional profits.

7.3 Limitations and future research

My research investigates the traders' submission strategies in double auction markets. It provides some useful results to understand the relationship between the submission strategies of the traders and the determinants of their order sides and prices. It also observe the game-theoretical competition between traders with uncertainty and the information disseminating of the simple auction market. Here, I address some limitations in my research and remained questions from future research.

My model investigate the submission strategies and game-theoretical competition between two informed traders. My models do not address the submission behaviors of uniformed traders, who are important parts in the double auction market to provide liquidity. One possible extension is to investigate the competition between one uninformed trader and one informed trade in a two-person double auction market. The one-side uncertainty model is more meaningful under multi-period trading environment to observe the process of market liquidity.

Besides the type of the traders, in my model, there are only two traders in markets. I only observe the individual effects of traders and their competition. The question about the systematical effects of numerous traders remain unsolved. Given that, it is possible to observe the effects of order follows and the formation of market clear price. As we know, with the increase of the number of investors in markets, markets become more efficiently. Therefore, the model with numerous traders are useful to investigate the market efficiency and equilibrium. The extension of the model with numerous infinite or finite traders could address traders' behaviors from more aspects.

Moreover, in my model, there are only one unit of the asset for traders to trade. However, with this assumption, I rule out the traders' decision of order size. In my model, I only address the traders' decision of order prices and sides. Their behaviors would become more complex by considering their order size decision. The informed traders would consider to conceal their private information by their choice of order size and the uniformed traders observe and make assessment on the informed traders' private information through their order size. Therefore, the another extension is to allow traders to chose their order size.

It is also possible to relax some other assumptions of my model in further research. For example, it is possible to relax the limitation on the trader's risk preference, such as risk-adverse traders. The extension of my theoretical model in further research would provide more useful results to better understand the submission strategies of the traders in the limit order market.

Appendices

Appendix A

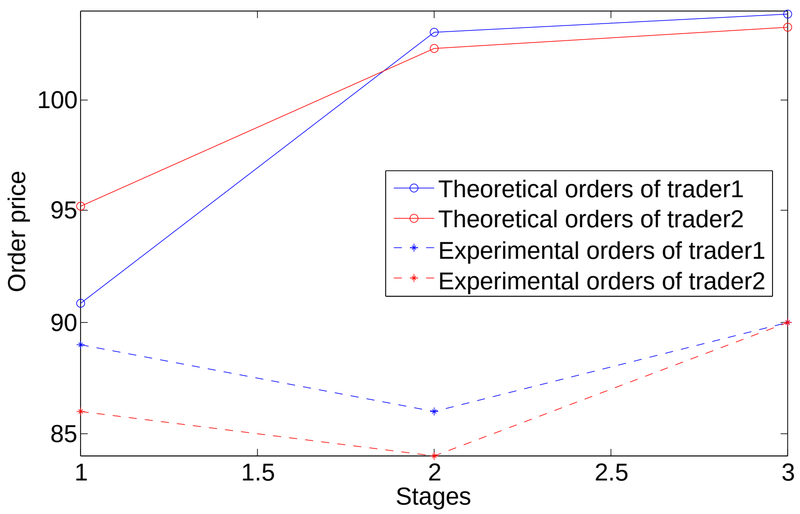
The graphs of multi-period trading experiment

In the multi-trading experiment, the multi-stage trading game are accomplished in the last two sessions. Because of the time limitation, the multi-stage trading game only round once for each session with three stages. Comparing to the single-stage trading game, the submission strategies in multi-trading game are more complex because of new information available. The students' reaction to the multi-stage trading game different from each other such that their decision times various compared with the single-stage game.

We got 19 pairs of efficient data for the subjects' order prices in multi-stage trading. As the statement before, the corresponding theoretical values for every offer price are calculated by two methods. Method 1: theoretical values are obtained by updating the theoretical offer prices. And method 2: theoretical values are calculated by updating the experiment order prices. The pure theoretical values with method 1 focus on the observing the diverges of trader's behaviors in the whole process. While the hybrid theoretical value with method 2 provide the opportunity to observe the difference of the traders' updating behaviors at each stage. For every group, two figures with different theoretical value list together for comparison. The comparison between the experimental and theoretical offer prices gives insights into the efficiency of theoretical models and other uncontrolled determinants for traders' submission behaviors in laboratorial environment. On the other hand, the comparison between t

A. THE GRAPHS OF MULTI-PERIOD TRADING EXPERIMENT

The following is the graphs for the 19 groups of data. We observe that the difference between the theoretical values and the experimental data is large because of the irrational behaviors of subjects. The irrational unsystematical behaviors of students is caused by their limitation of financial knowledge and ability to submit optimal orders. However, some groups' orders prices move towards the same direction with the theoretical model. To some degree, the subjects' submission strategies in the laboratorial environment are similar with our expectation in the theoretical models.



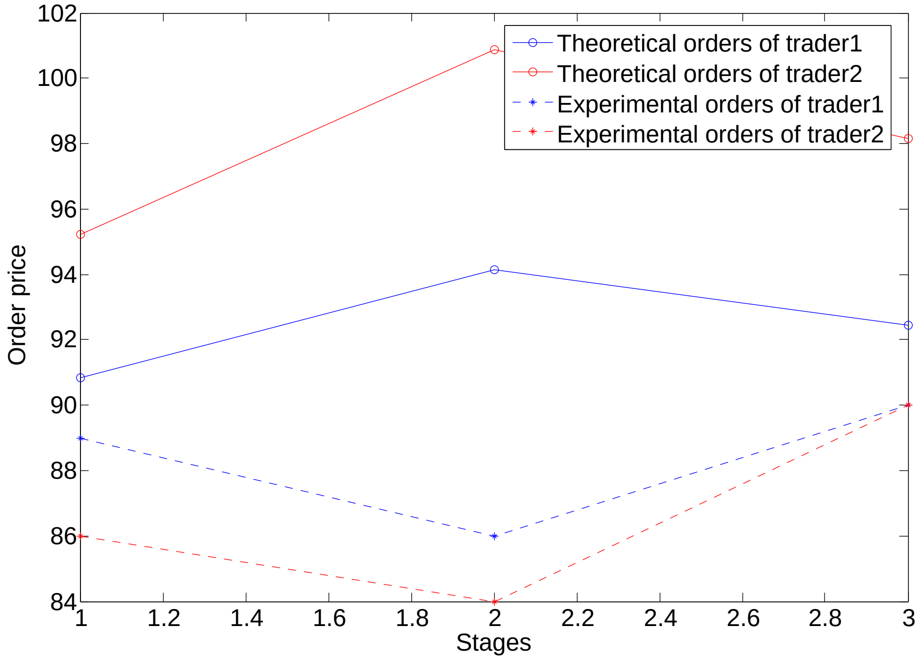
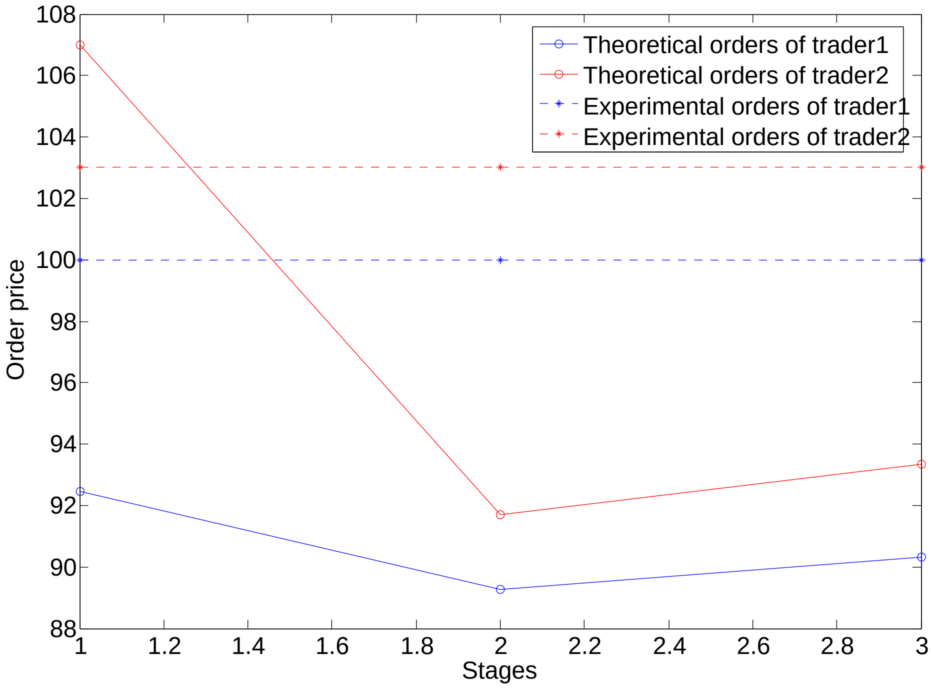


Figure A.1: Theoretical and experimental offer prices of group 1 (theoretical updating & experimental updating)



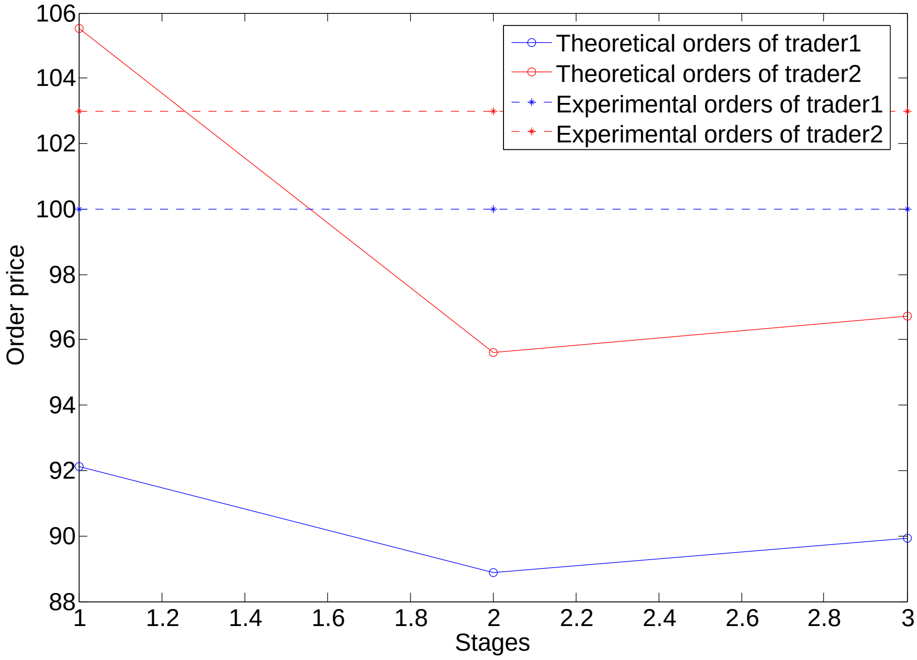
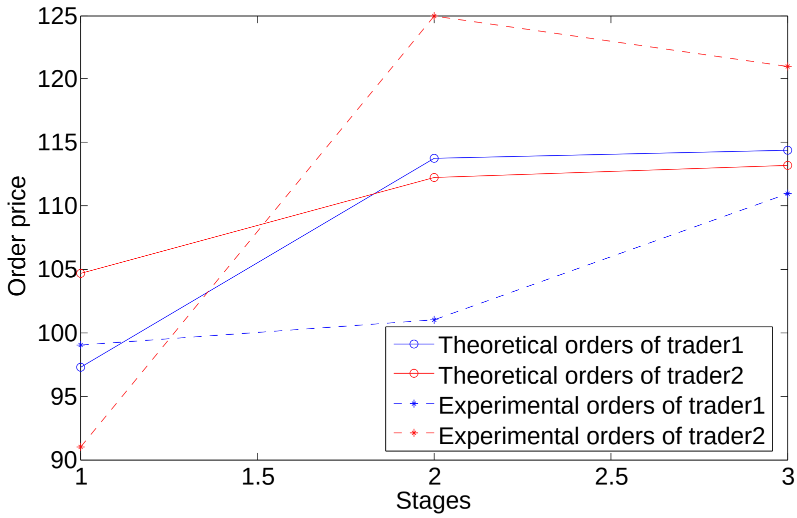


Figure A.2: Theoretical and experimental offer prices of group 2 (theoretical updating & experimental updating)



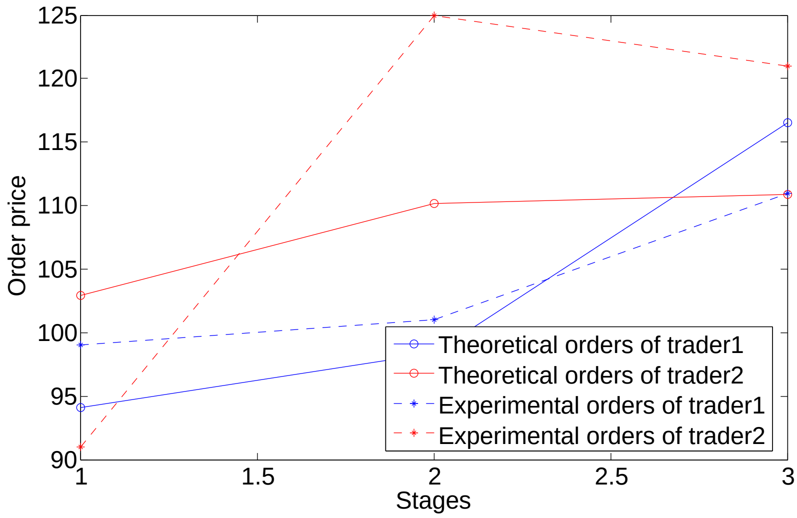
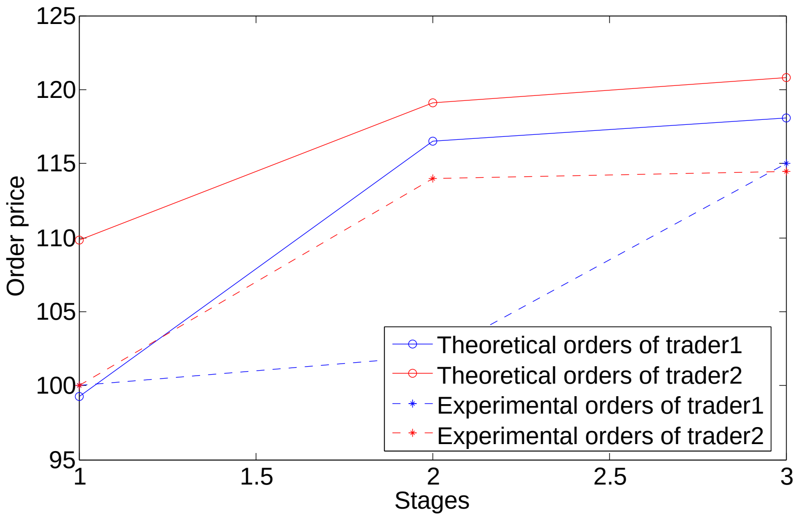


Figure A.3: Theoretical and experimental offer prices of group 3 (theoretical updating & experimental updating)



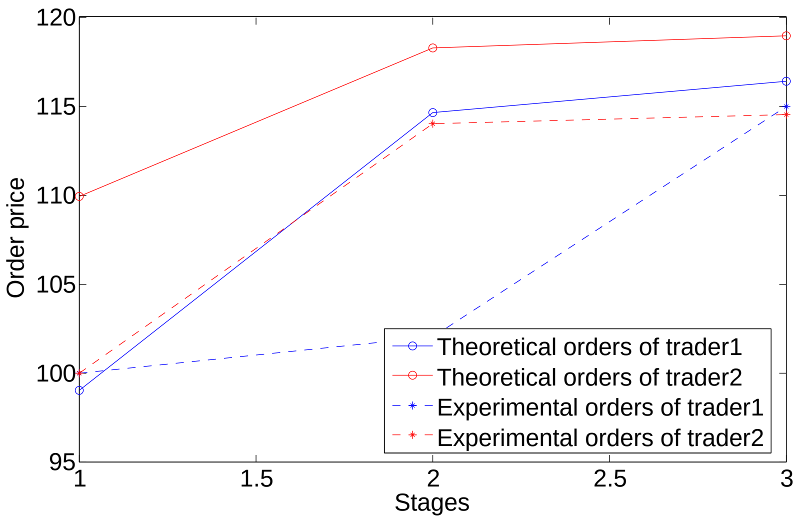
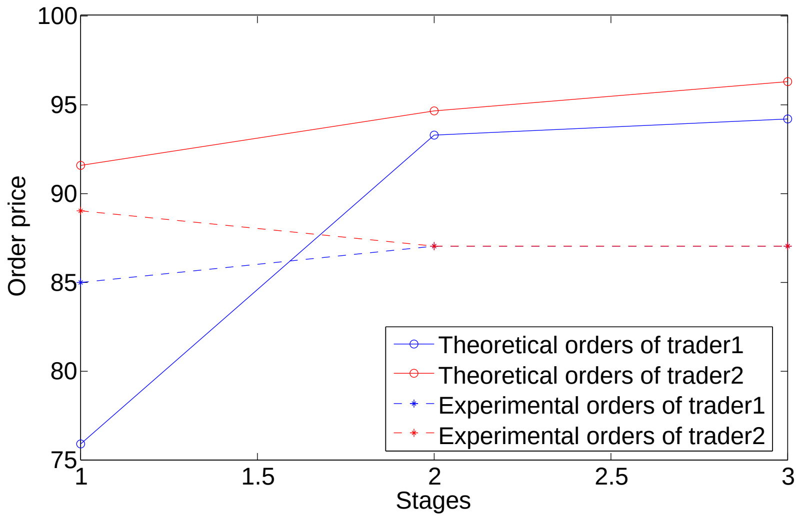


Figure A.4: Theoretical and experimental offer prices of group 4 (theoretical updating & experimental updating)



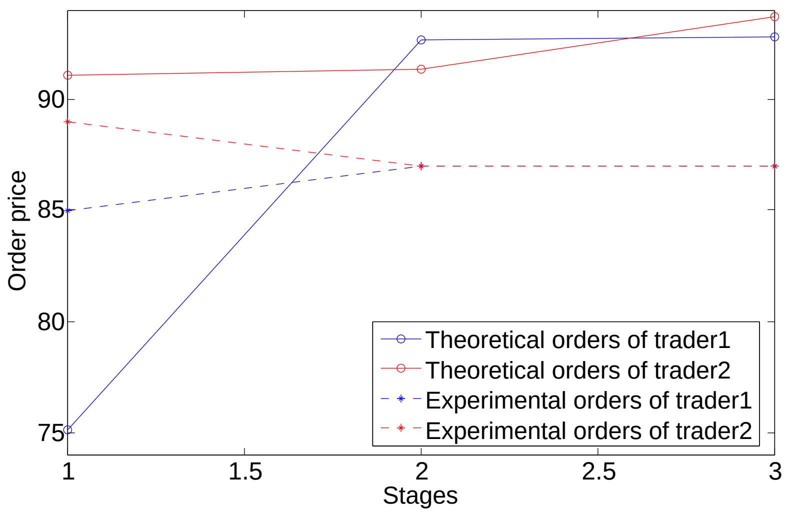
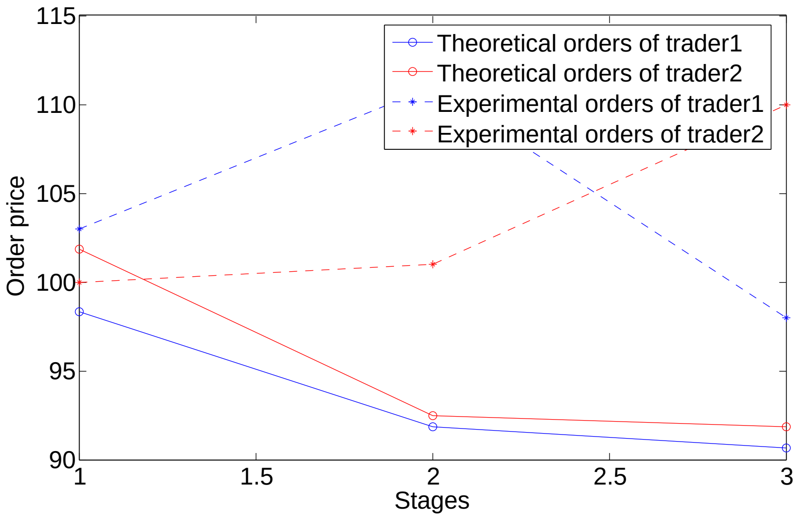


Figure A.5: Theoretical and experimental offer prices of group 5 (theoretical updating & experimental updating)



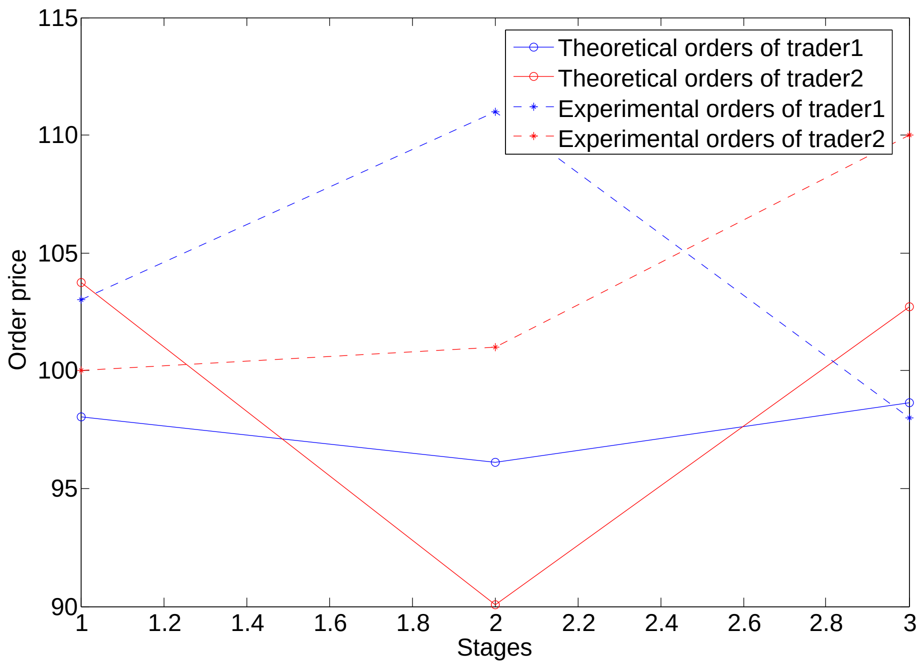
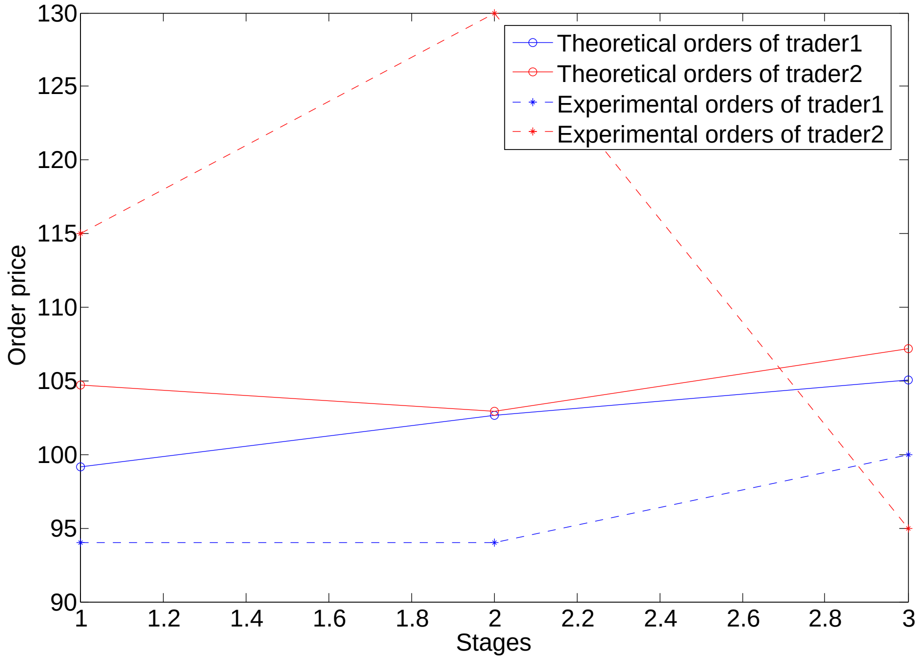


Figure A.6: Theoretical and experimental offer prices of group 6 (theoretical updating & experimental updating)



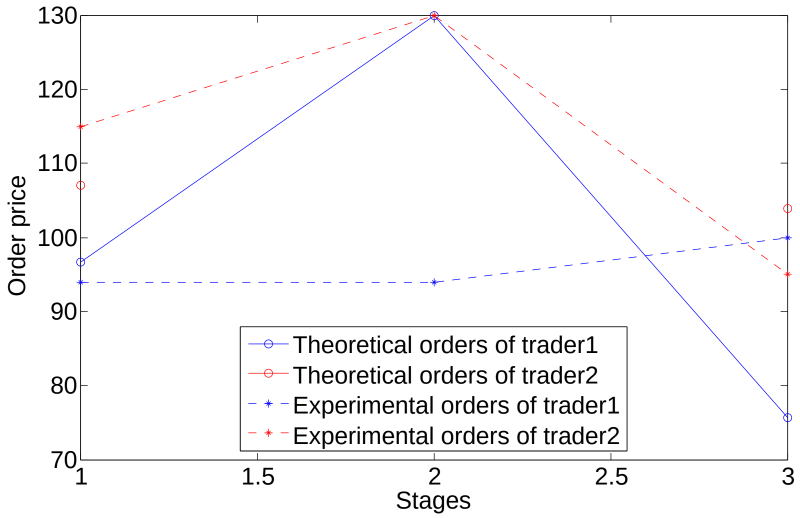
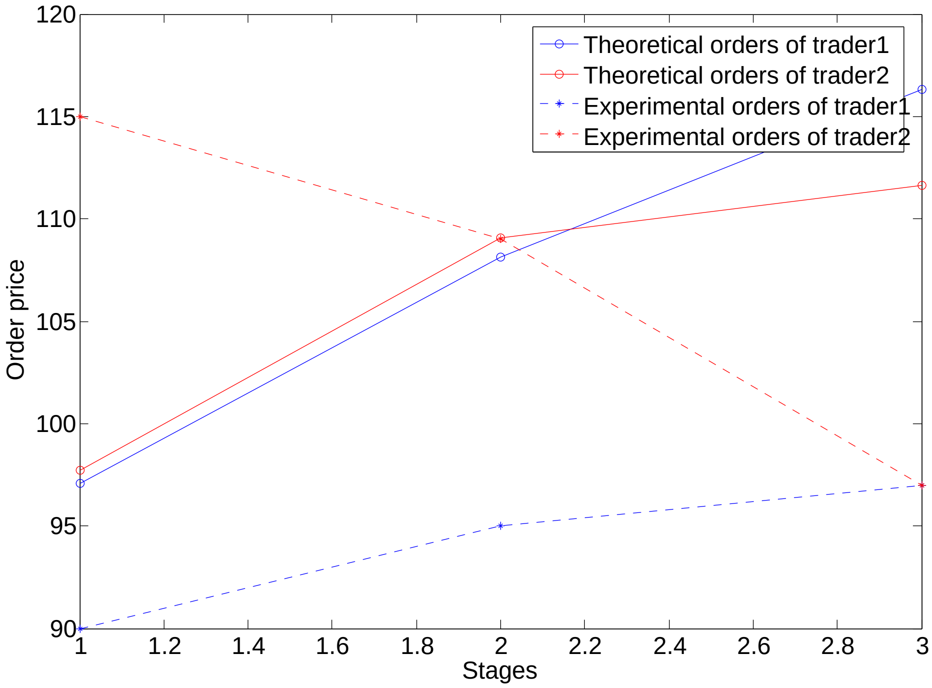


Figure A.7: Theoretical and experimental offer prices of group 7 (theoretical updating & experimental updating)



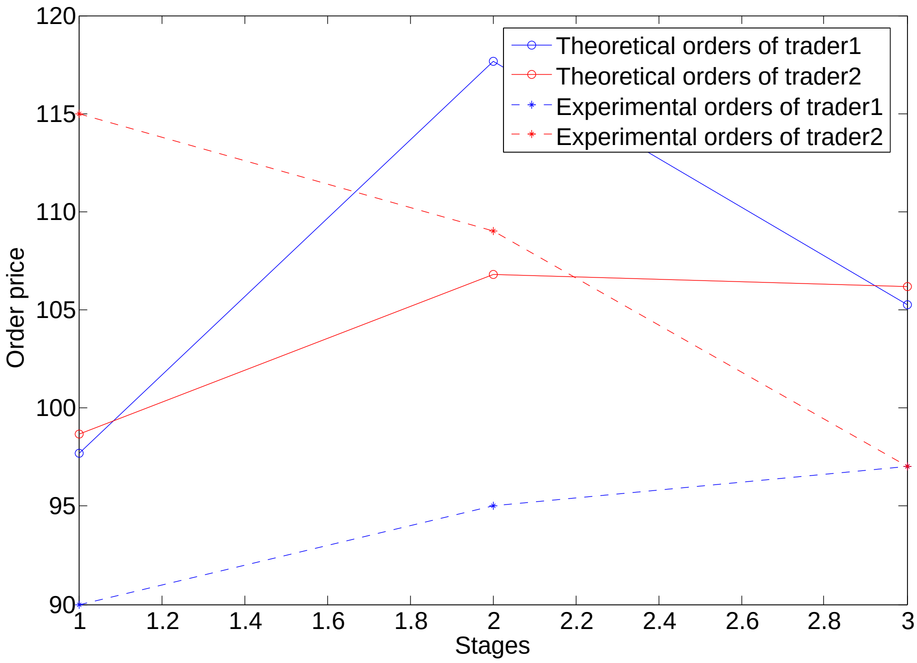
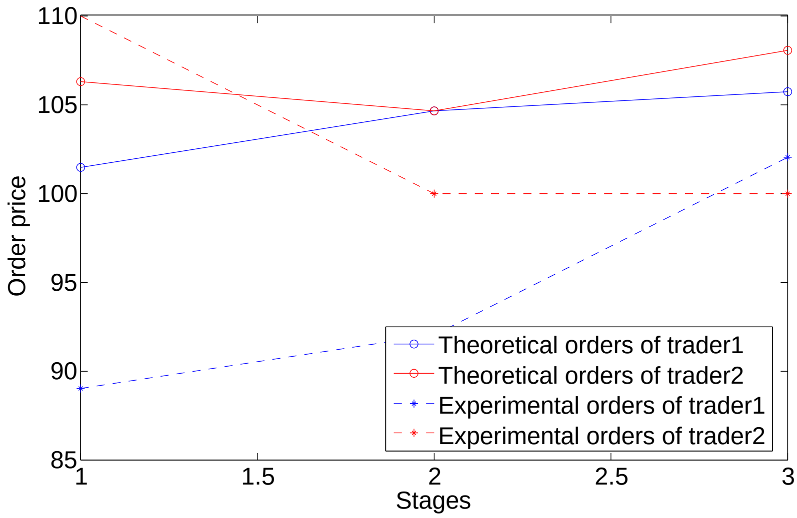


Figure A.8: Theoretical and experimental offer prices of group 8 (theoretical updating & experimental updating)



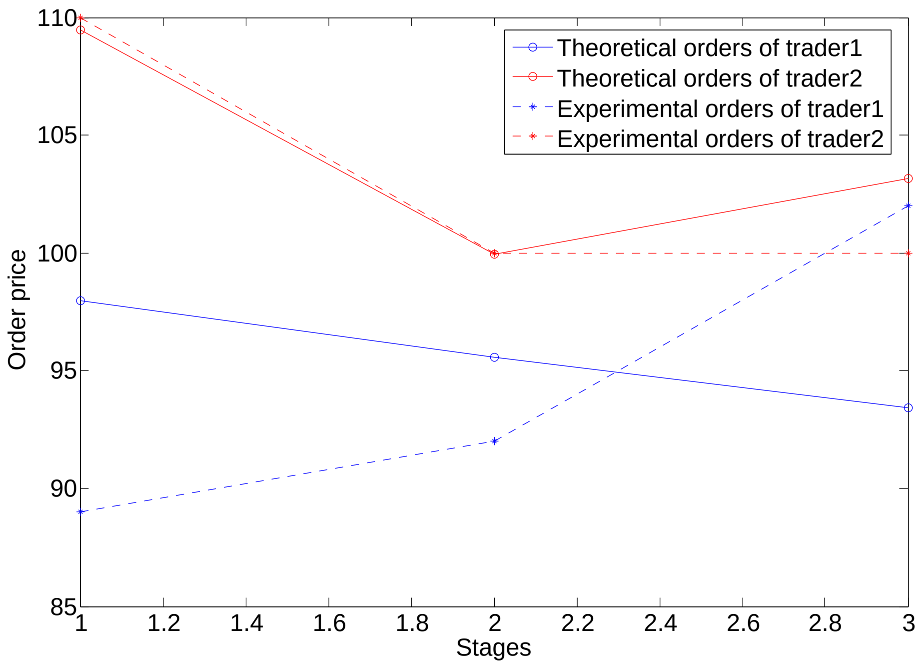
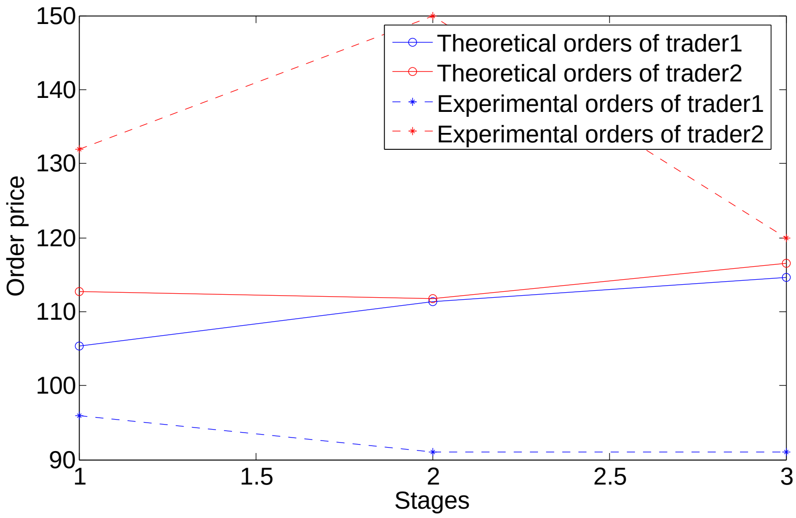


Figure A.9: Theoretical and experimental offer prices of group 9 (theoretical updating & experimental updating)



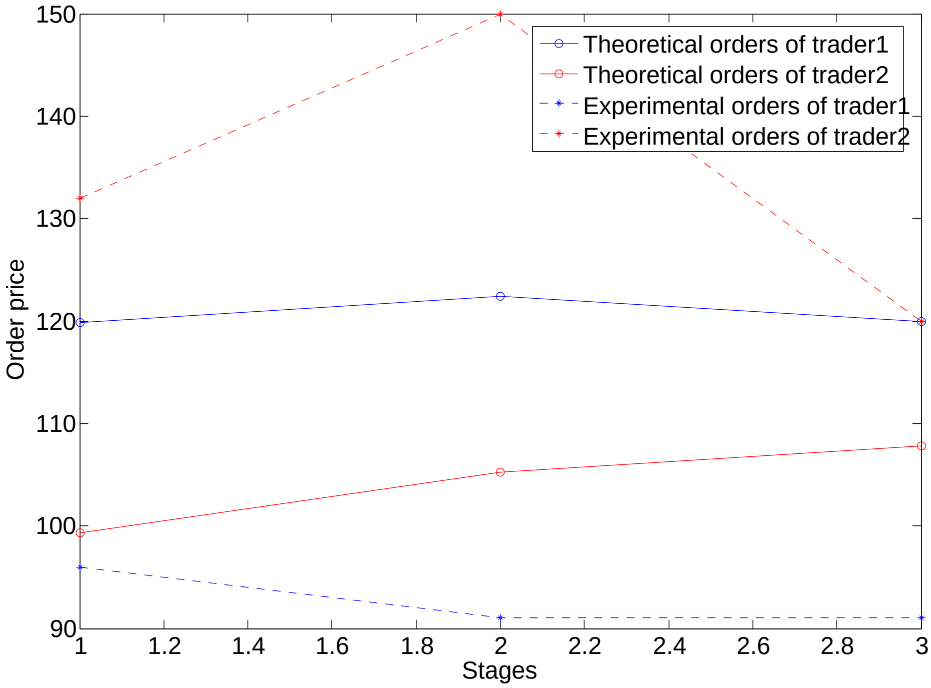


Figure A.10: Theoretical and experimental offer prices of group 10 (theoretical updating & experimental updating)

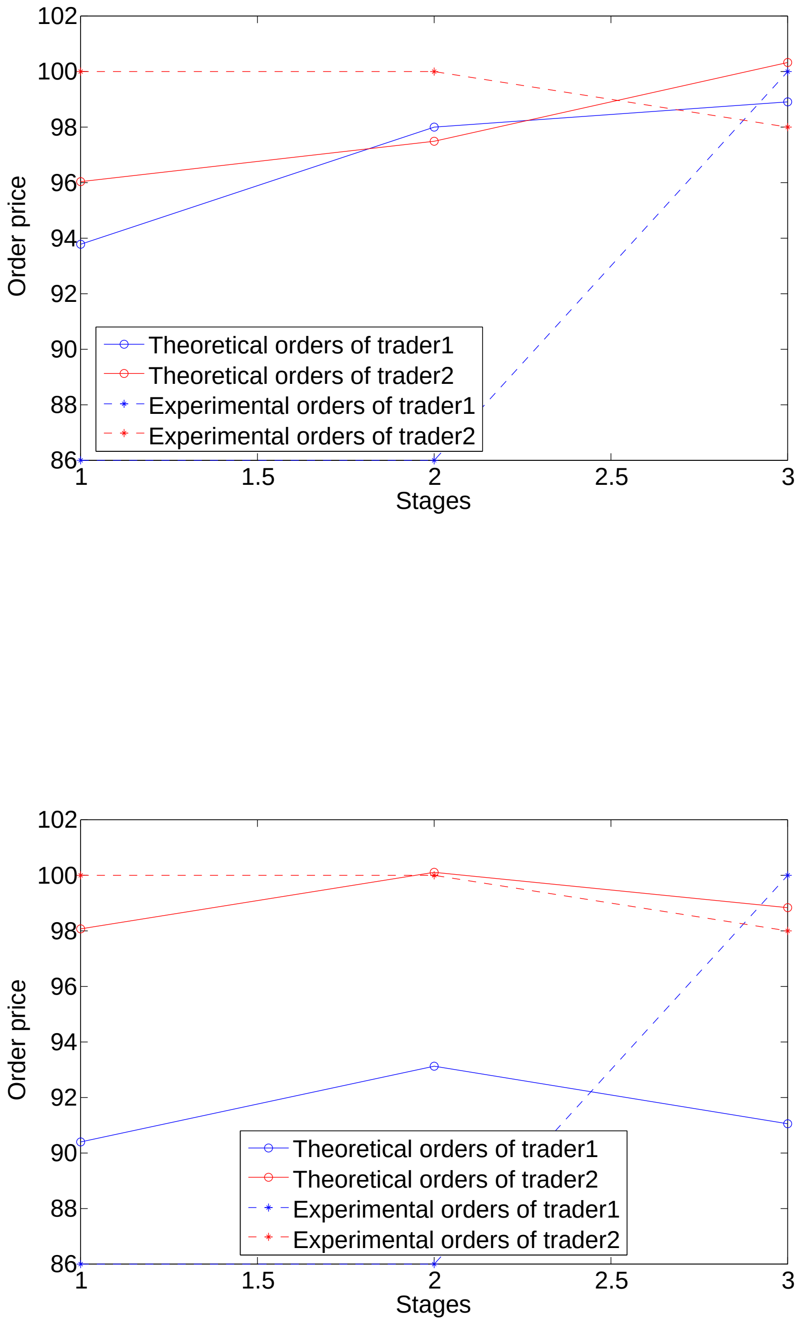
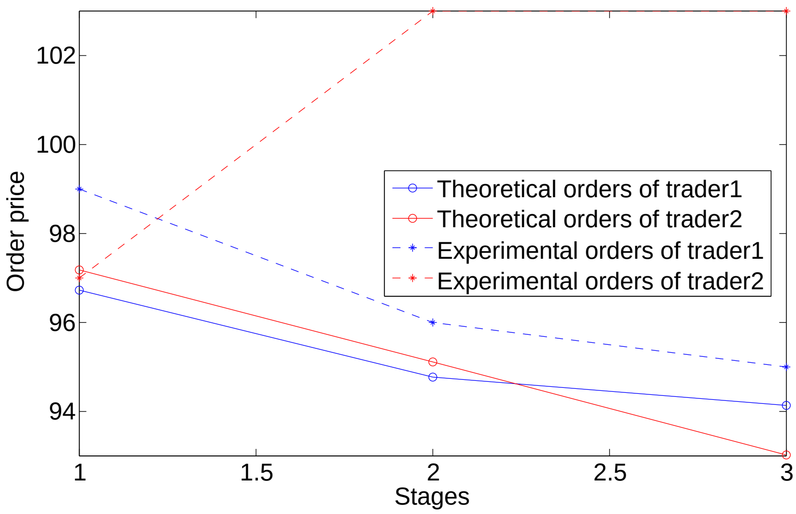


Figure A.11: Theoretical and experimental offer prices of group 11 (theoretical updating & experimental updating)



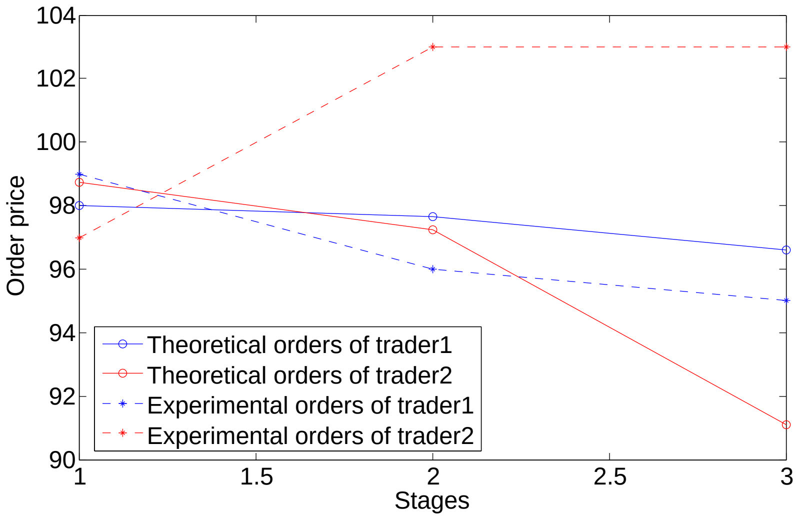


Figure A.12: Theoretical and experimental offer prices of group 12 (theoretical updating & experimental updating)

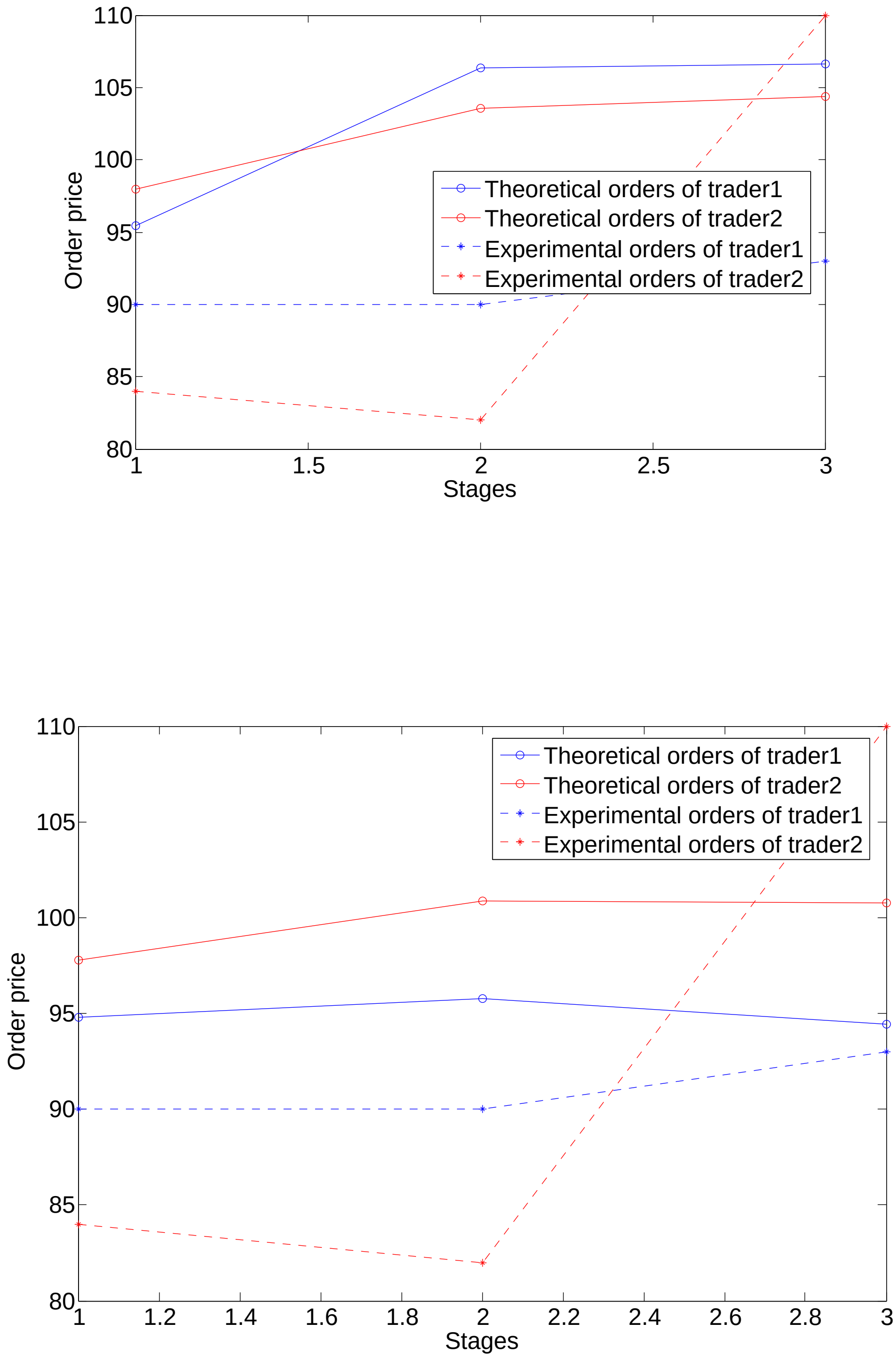


Figure A.13: Theoretical and experimental offer prices of group 13 (theoretical updating & experimental updating)

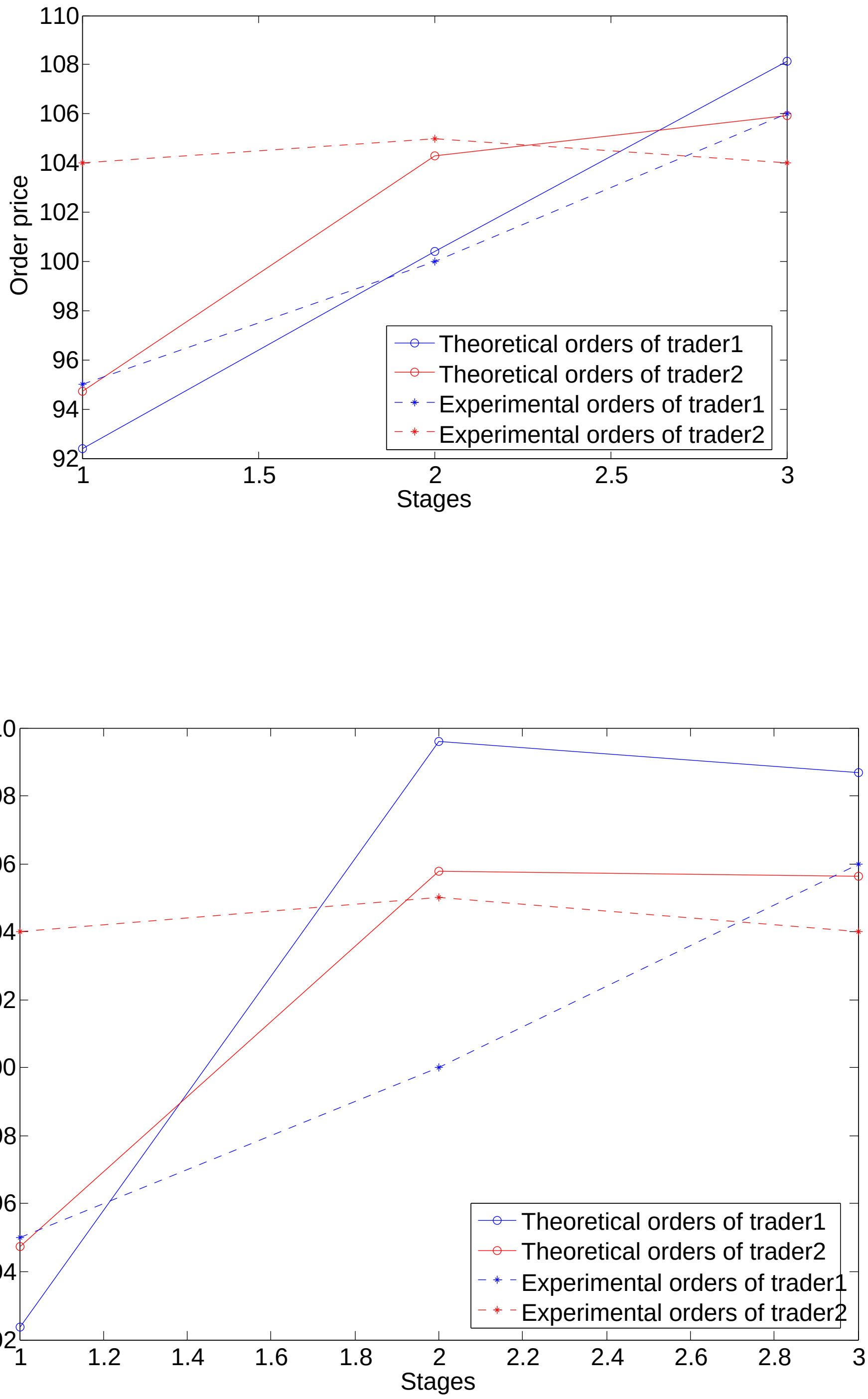
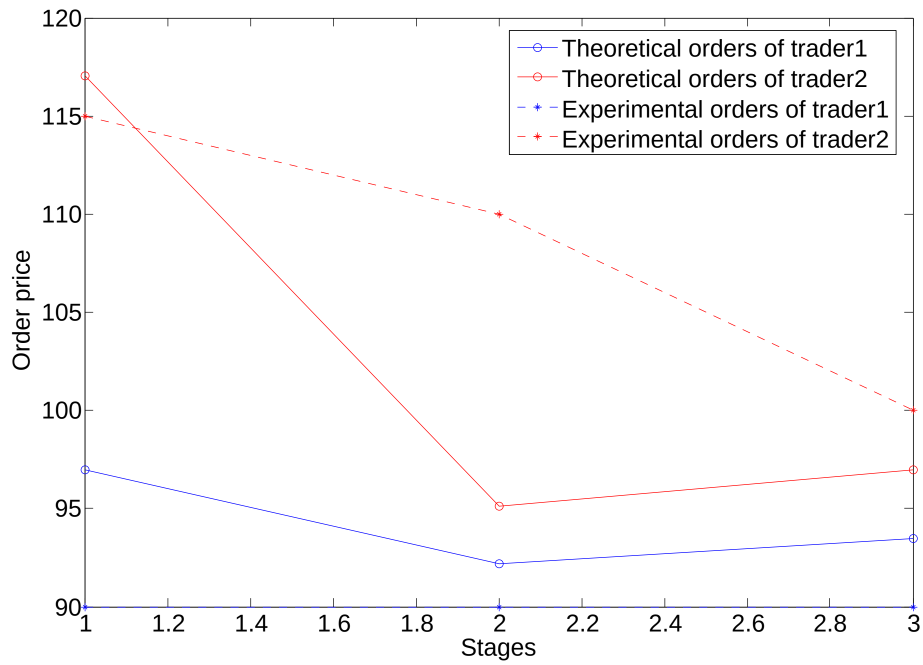


Figure A.14: Theoretical and experimental offer prices of group 14 (theoretical updating & experimental updating)



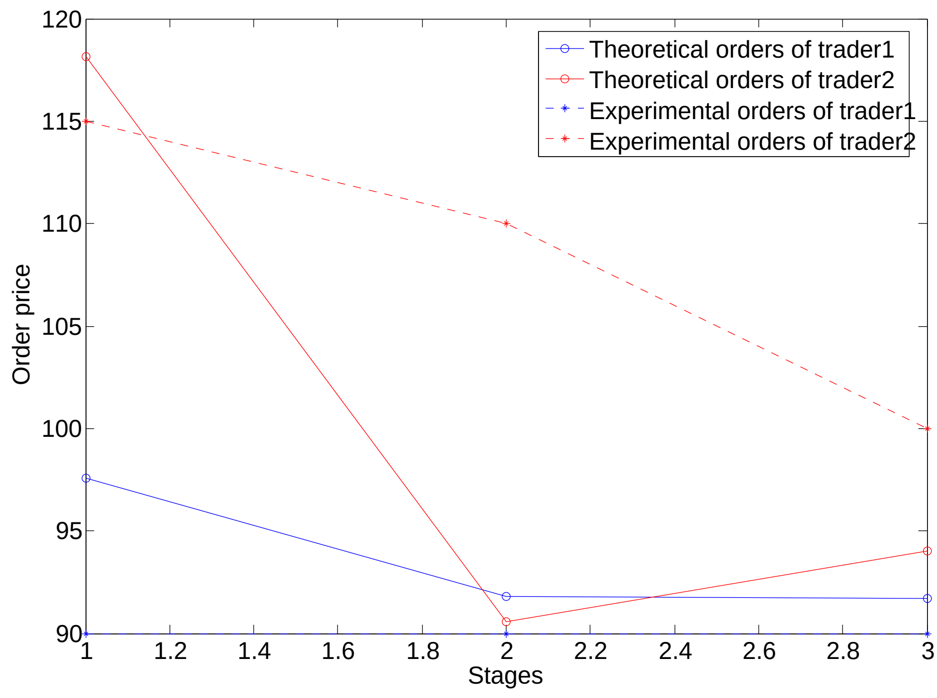
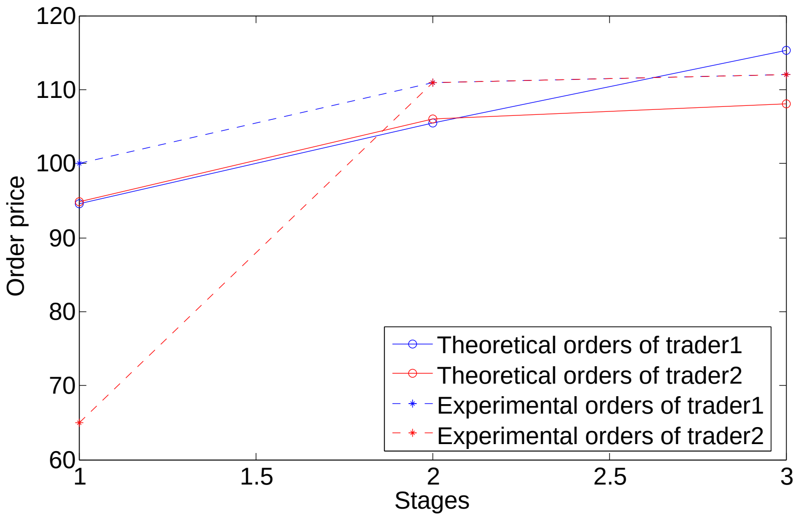


Figure A.15: Theoretical and experimental offer prices of group 15 (theoretical updating & experimental updating)



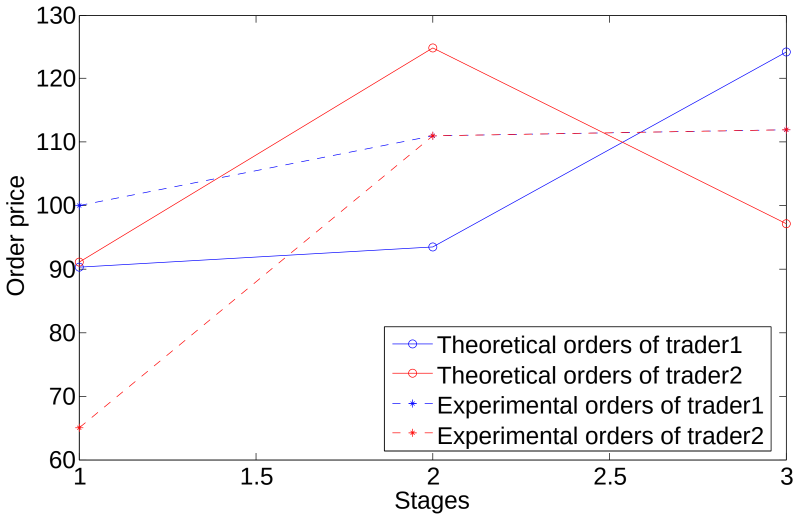
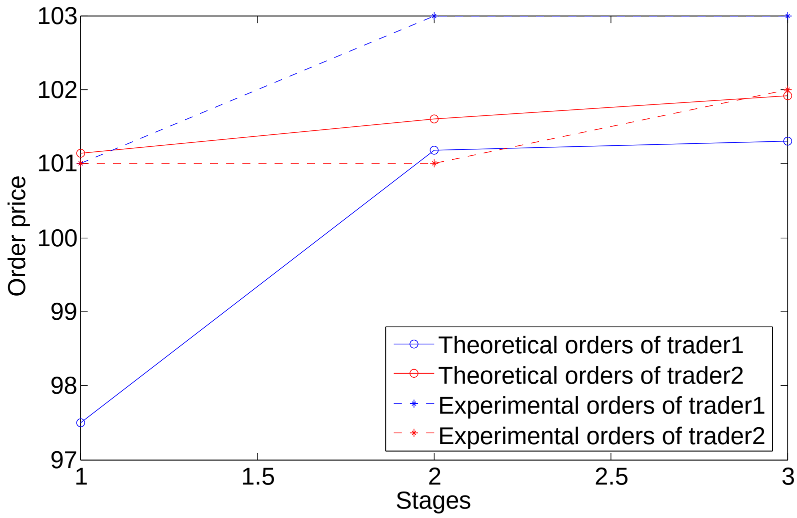


Figure A.16: Theoretical and experimental offer prices of group 16 (theoretical updating & experimental updating)



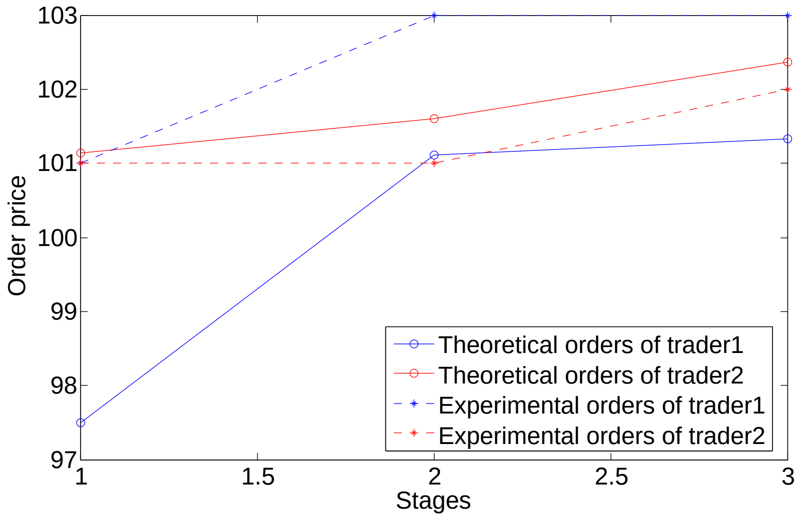
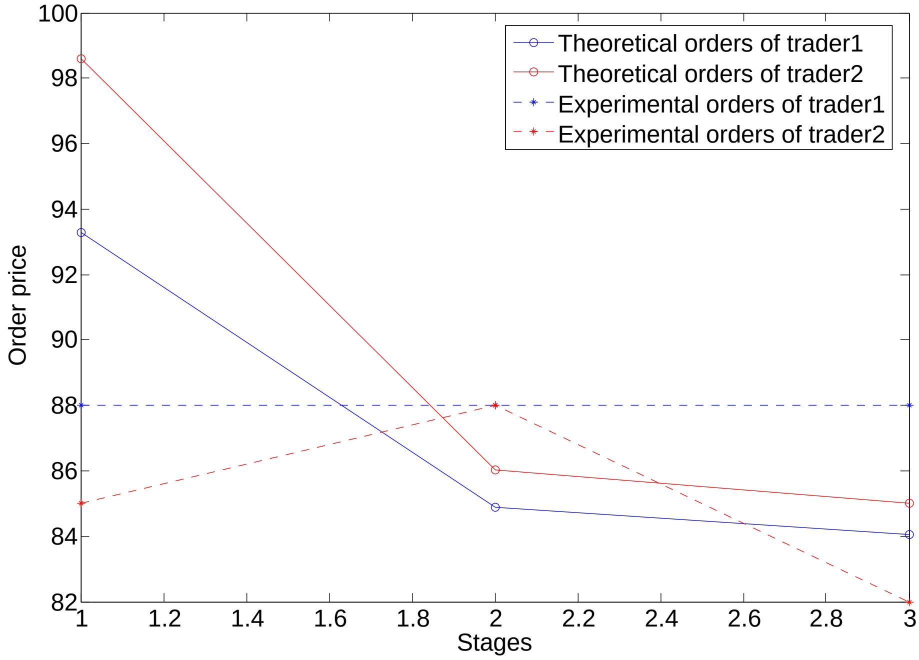


Figure A.17: Theoretical and experimental offer prices of group 17 (theoretical updating & experimental updating)



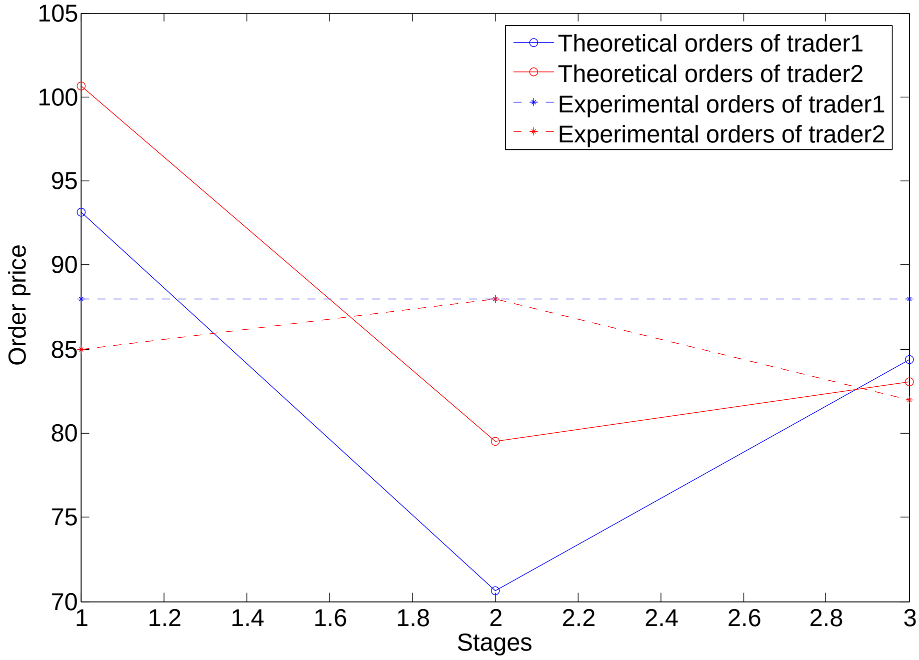
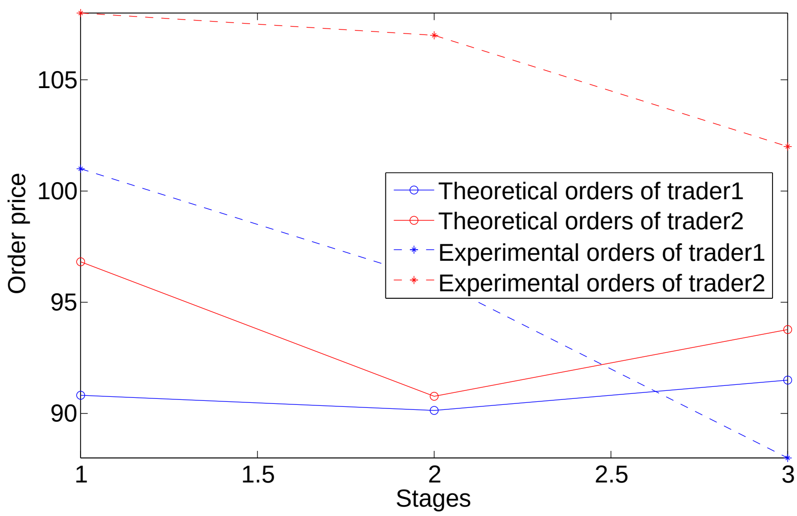


Figure A.18: Theoretical and experimental offer prices of group 18 (theoretical updating & experimental updating)



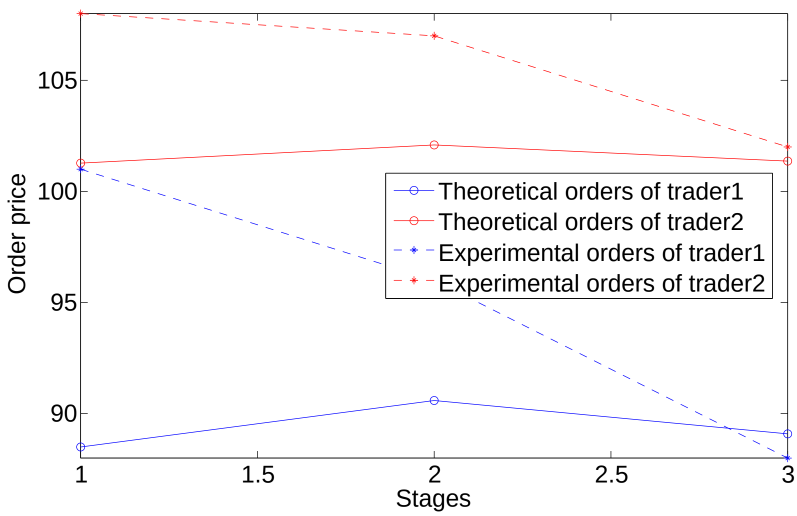


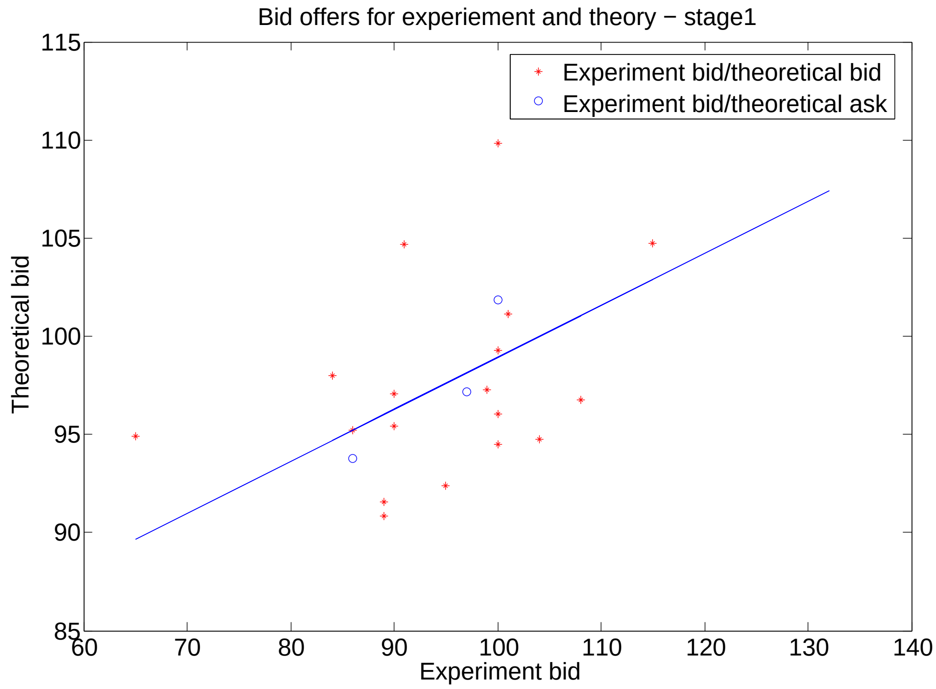
Figure A.19: Theoretical and experimental offer prices of group 19 (theoretical updating & experimental updating)

Appendix B

The comparison between the experimental data and related theoretical values

In chapter 6, we compare the experimental data with the related theoretical value to clarify the difference between the theoretical models and experiments. The statistical comparison is necessary for us to prove the efficiency of our theoretical models. Meanwhile, it helps us to better understand the traders' submission strategies in the laboratorial environment. First of all, we compare the experimental data and the related theoretical values at each stages separately and then put all the offer prices together. Secondly, we do the chi-square test for some important variables, such as buy or sell decision, trade or no trade, and etc. The analysis of the results are shown in chapter 6 in details.

The correlation between the experimental order prices and the related theoretical values:



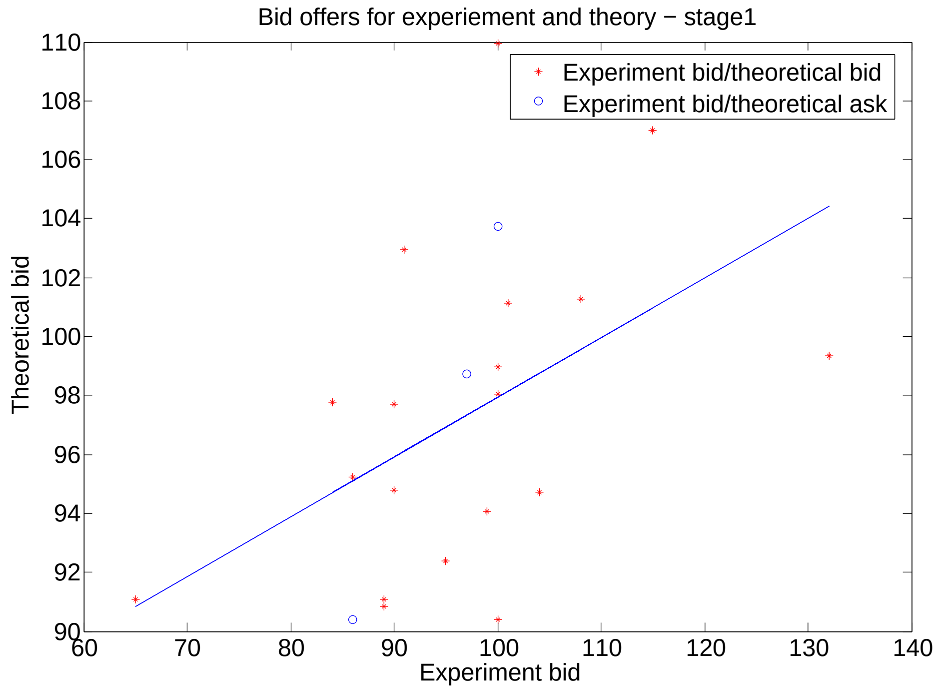


Figure B.1: Bid offers at stage 1 (theoretical updating & experimental updating)

Table B.1: Bid price of experiment & theory - stage 1

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	72.422	9.315***	0.000	77.697	9.310***	0.000
Slope	0.265	3.315***	0.003	0.202	2.358**	0.029

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.2: Ask price of experiment & theory - stage1

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	55.021	2.628**	0.020	50.237	1.909*	0.077
Slope	0.444	2.096*	0.055	0.494	1.855*	0.085

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.3: Bid price of experiment & theory - stage 2

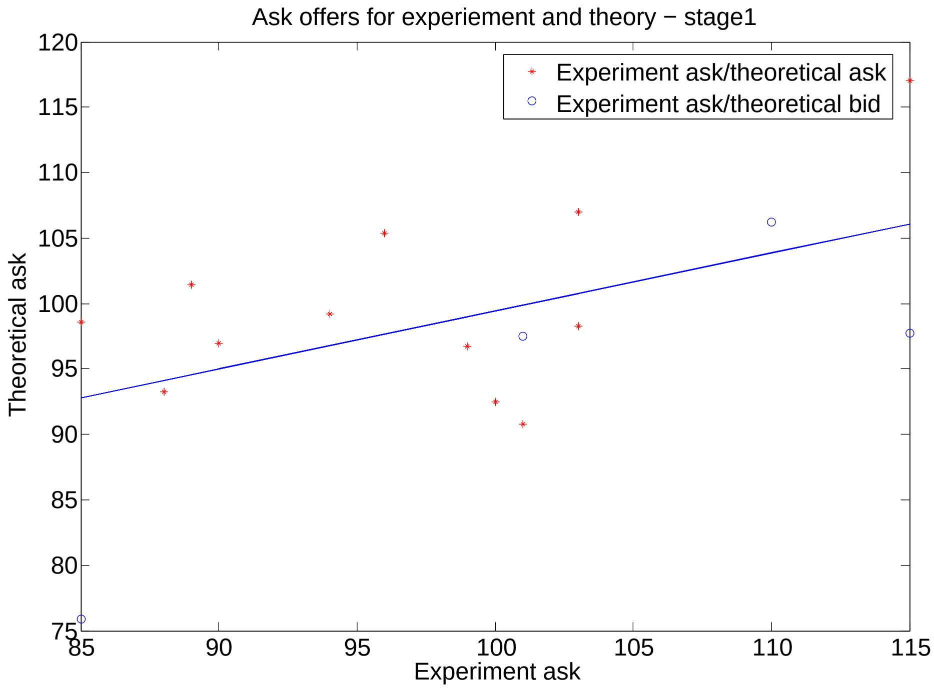
	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	87.898	6.271***	0.000	65.295	3.713***	0.002
Slope	0.163	1.185	0.254	0.390	2.267**	0.039

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.4: Ask price of experiment & theory - stage2

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	68.217	4.578***	0.000	47.197	2.409**	0.027
Slope	0.294	1.996*	0.061	0.474	2.413**	0.027

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.



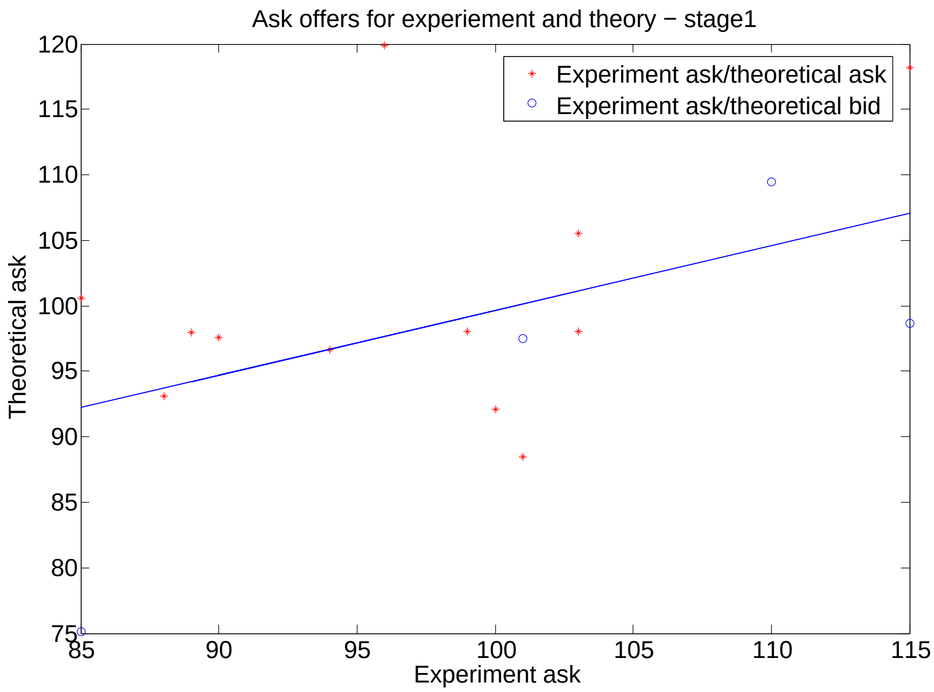


Figure B.2: Ask offers at stage 1 (theoretical updating & experimental updating)

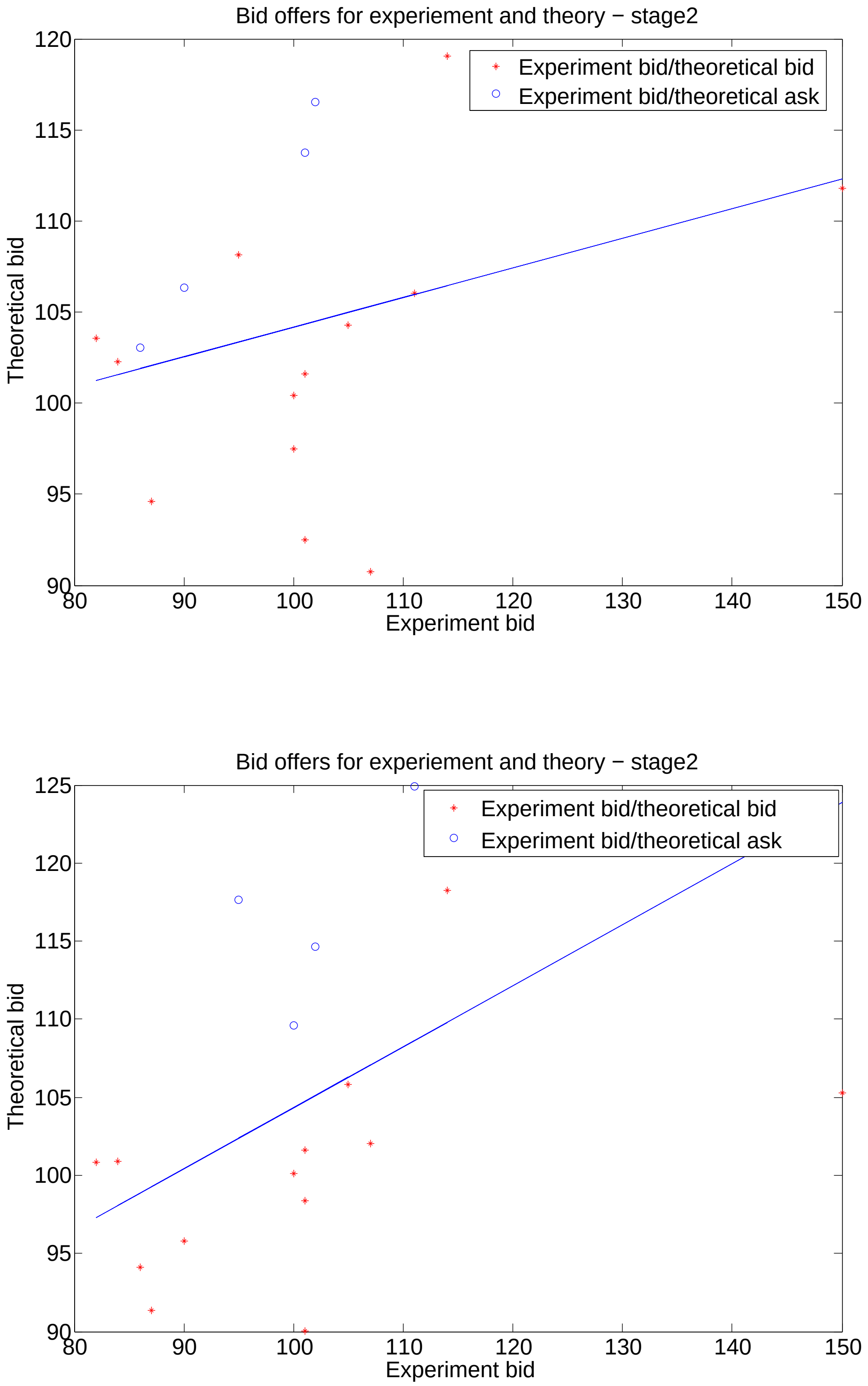
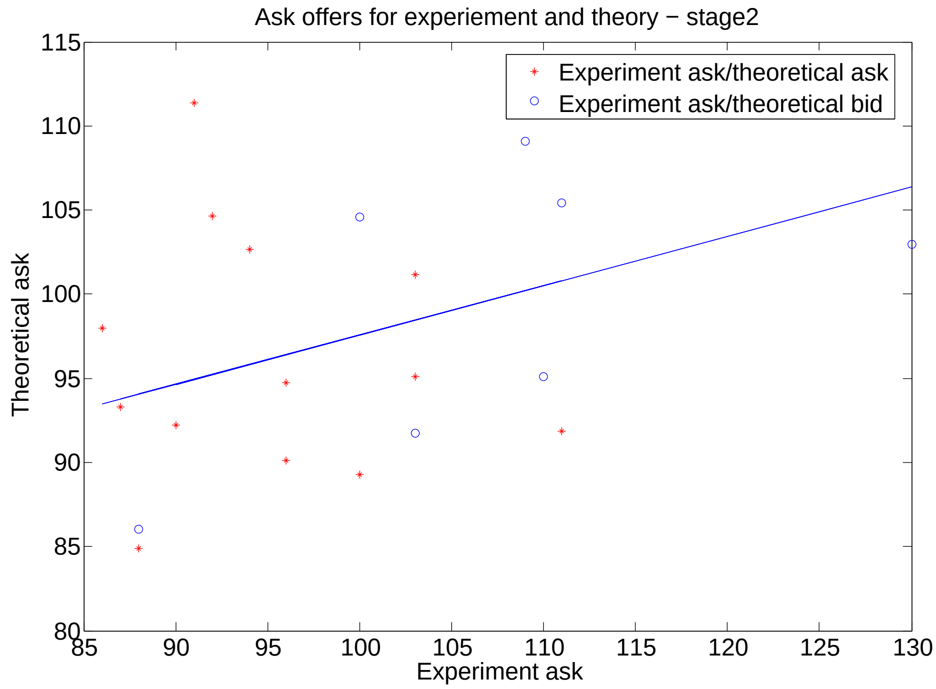


Figure B.3: Bid offers at stage 2 (theoretical updating & experimental updating)



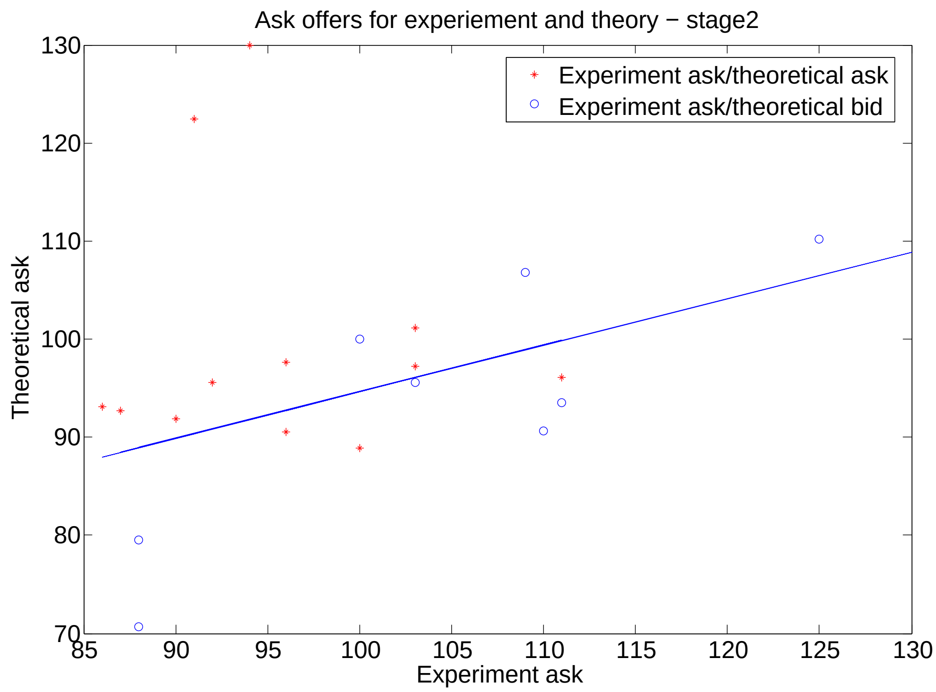
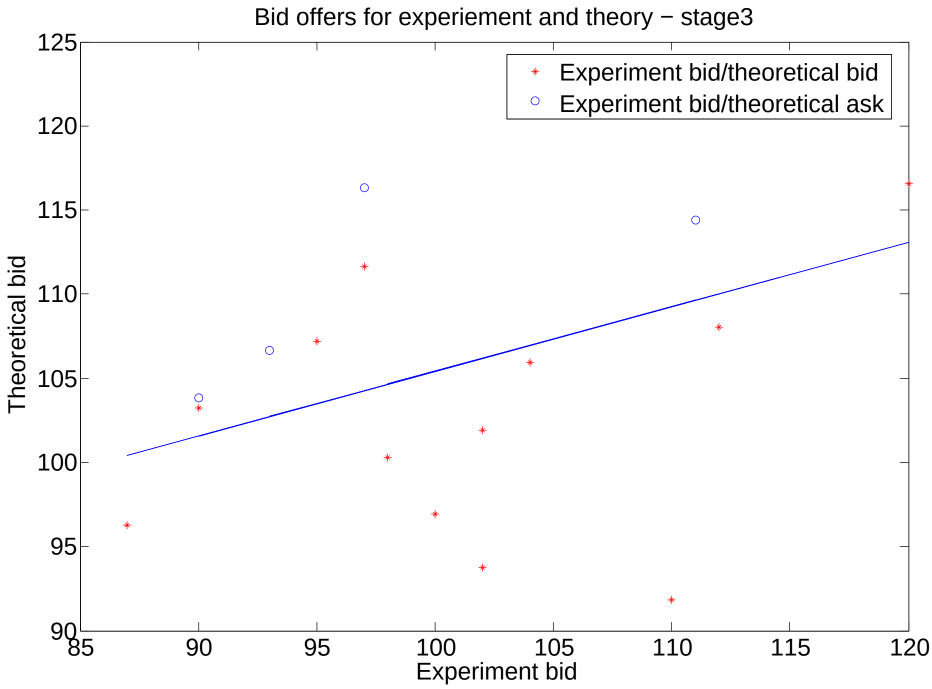


Figure B.4: Bid offers at stage 2 (theoretical updating & experimental updating)



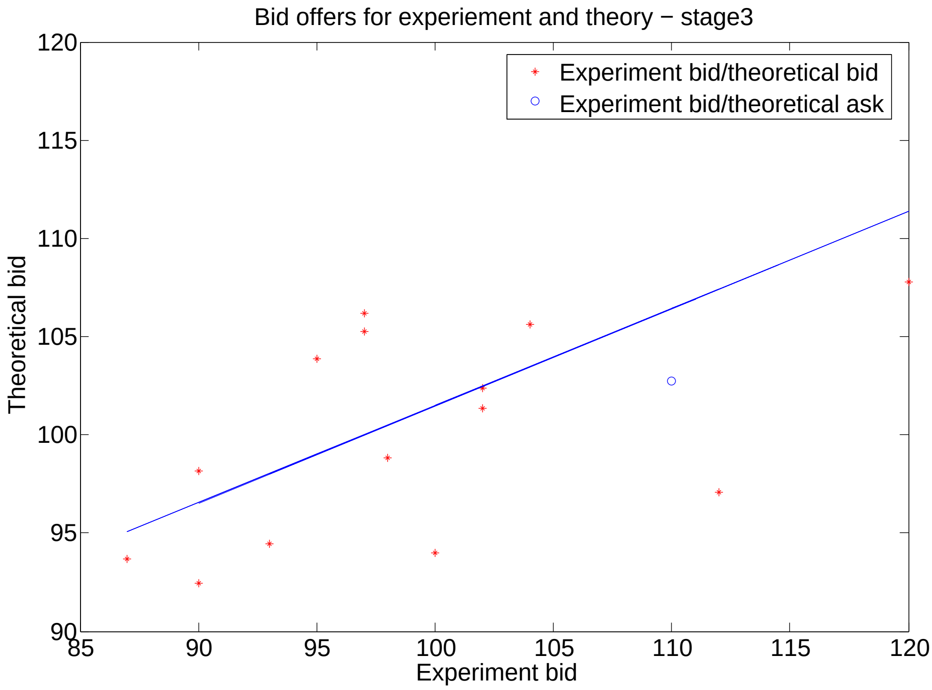


Figure B.5: Bid offers at stage 3 (theoretical updating & experimental updating)

Table B.5: Bid price of experiment & theory - stage3

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	67.033	3.041***	0.008	51.987	2.861**	0.012
Slope	0.384	1.772**	0.097	0.495	2.771**	0.014

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.6: Ask price of experiment & theory - stage3

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	22.993	1.426	0.170	18.391	0.944	0.357
Slope	0.769	4.779***	0.000	0.788	4.053***	0.001

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.7: Bid price of experiment & theory - total

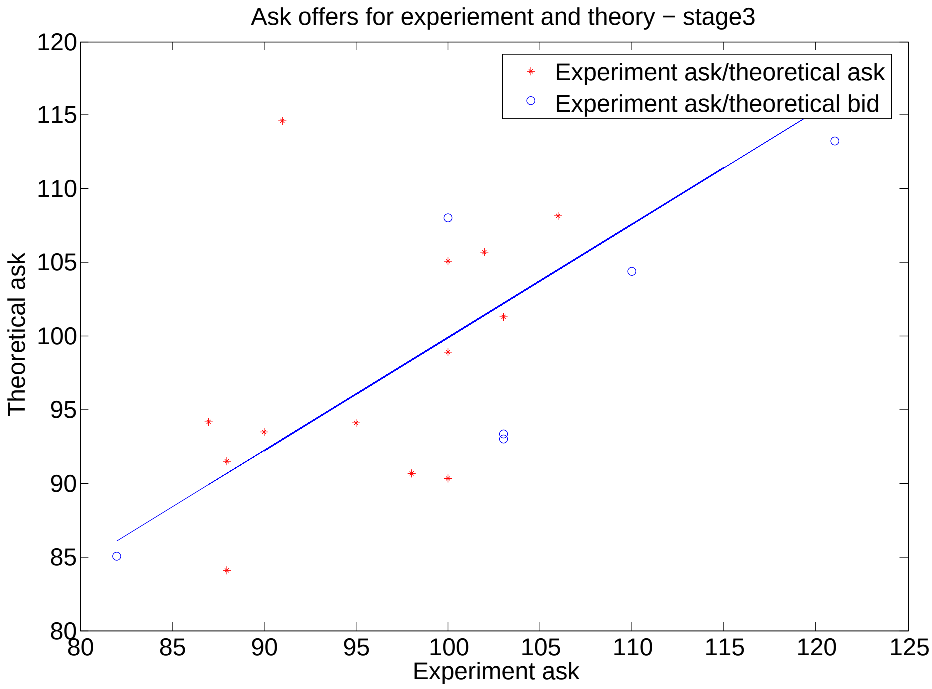
	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	75.276	9.523***	0.000	72.345	9.925***	0.000
Slope	0.271	3.425***	0.001	0.282	3.877***	0.000

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.

Table B.8: Ask price of experiment & theory - total

	Theoretical updating			Experimental updating		
Variable	Coefficient	t-Statistic	Prob.	Coefficient	t-Statistic	Prob.
C	51.382	5.169***	0.000	38.179	3.214***	0.002
Slope	0.475	4.790***	0.000	0.587	4.924***	0.000

Note: t-ratios in parentheses; *, **, *** indicate significance at 10% 5%,1% level.



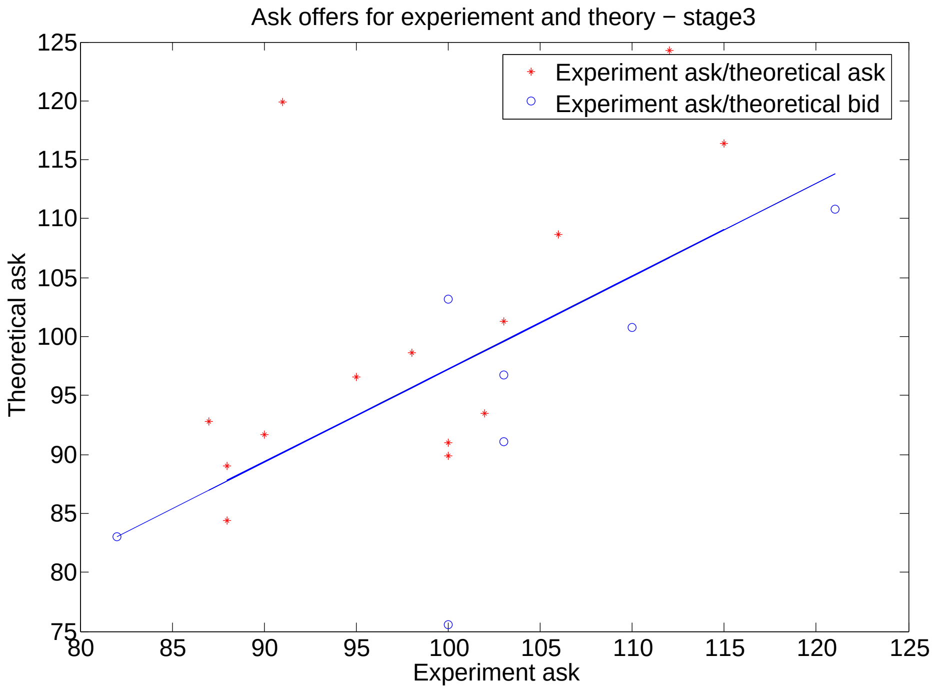


Figure B.6: Bid offers at stage 3 (theoretical updating & experimental updating)

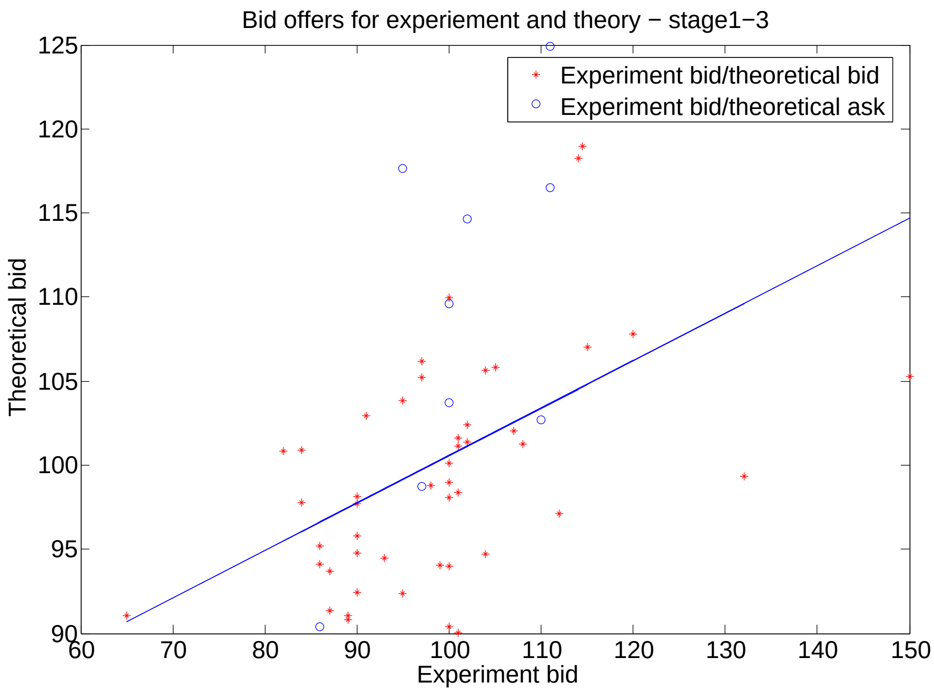
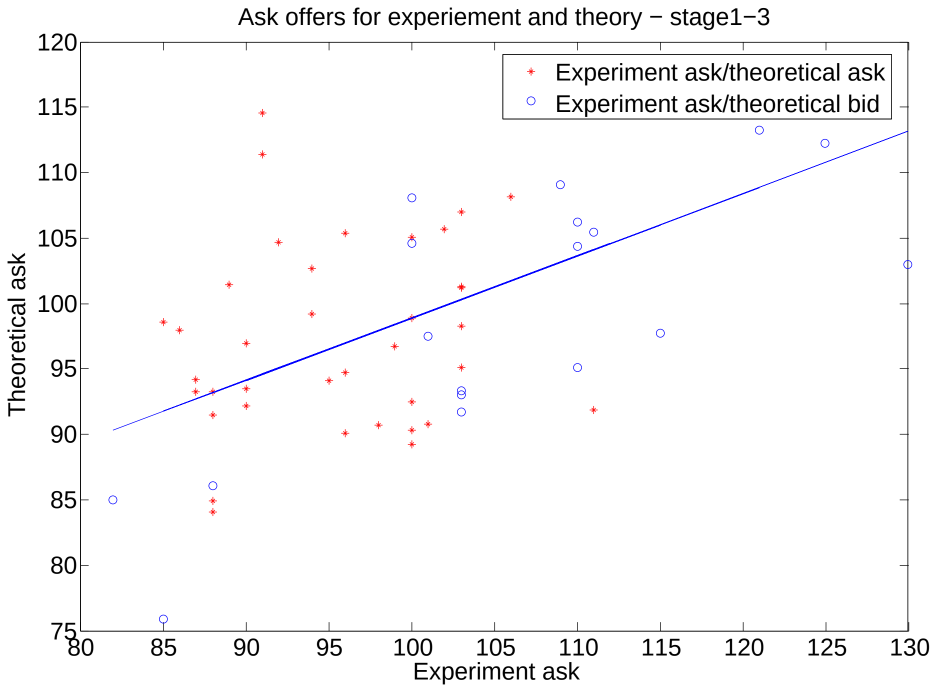


Figure B.7: Bid offers at stage $1 - 3$ (theoretical updating & experimental updating)



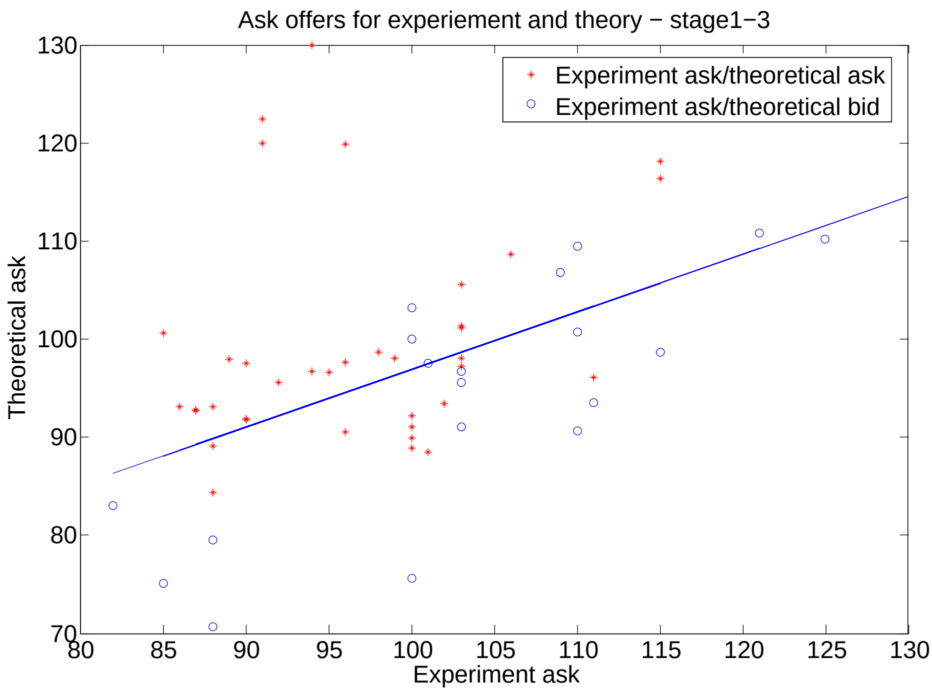


Figure B.8: Bid offers in stage 1 − 3 (theoretical updating & experimental updating)

The buy or sell decision for traders at each stages:

Table B.9: Bid/ask decision in the experiment and theory - stage 1

Theoretical updating				Experimental updating			
		Theory				Theory	
		Bid	Ask			Bid	Ask
Experiment	Bid	19	3	Experiment	Bid	19	3
	Ask	4	12		Ask	4	12
		χ^2	p-value			χ^2	p-value
Chi-square		14.599***	0.000	Chi-square		14.599***	0.000
Yates		12.144***	0.000	Yates		12.144***	0.000

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.10: Bid/ask decision in the experiment and theory - stage 2

Theoretical updating				Experimental updating			
		Theory				Theory	
		Bid	Ask			Bid	Ask
Experiment	Bid	13	4	Experiment	Bid	13	4
	Ask	8	13		Ask	9	12
		χ^2	p-value			χ^2	p-value
Chi-square		5.596**	0.018	Chi-square		4.354**	0.037
Yates		4.152**	0.042	Yates		3.085*	0.079

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.11: Bid/ask decision in the experiment and theory - stage 3

Theoretical updating				Experimental updating			
		Theory				Theory	
		Bid	Ask			Bid	Ask
Experiment	Bid	13	4	Experiment	Bid	15	2
	Ask	6	15		Ask	7	14
		χ^2	p-value			χ^2	p-value
Chi-square		8.622***	0.003	Chi-square		11.617***	0.001
Yates		6.812***	0.009	Yates		9.474***	0.002

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.12: Bid/ask decision in the experiment and theory - total

Theoretical updating				Experimental updating			
		Theory				Theory	
		Bid	Ask			Bid	Ask
Experiment	Bid	45	11	Experiment	Bid	47	9
	Ask	18	40		Ask	20	38
		χ^2	p-value			χ^2	p-value
Chi-square		28.035***	0.000	Chi-square		28.748***	0.000
Yates		28.035***	0.000	Yates		26.744***	0.000

Note: *, **, *** indicate significance at 10% 5%,1% level.

The transaction happens or not for each stages:

Table B.13: Transaction and no transaction in the experiment and theory - stage 1

Theoretical updating				Experimental updating			
		Theory				Theory	
		Trade	No trade			Trade	No trade
Experiment	Trade	3	2	Experiment	Trade	2	3
	No trade	2	12		No trade	2	12
		χ^2	p-value			χ^2	p-value
Chi-square		3.971**	0.046	Chi-square		1.466	0.226
Yates		1.963	0.161	Yates		0.327	0.567

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.14: Transaction and no transaction in the experiment and theory - stage 2

Theoretical updating				Experimental updating			
		Theory				Theory	
		Trade	No trade			Trade	No trade
Experiment	Trade	3	2	Experiment	Trade	2	3
	No trade	7	7		No trade	4	10
		χ^2	p-value			χ^2	p-value
Chi-square		0.148	0.700	Chi-square		0.223	0.637
Yates		0.019	0.890	Yates		0.008	0.929

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.15: Transaction and no transaction in the experiment and theory - stage 3

Theoretical updating				Experimental updating			
		Theory				Theory	
		Trade	No trade			Trade	No trade
Experiment	Trade	5	1	Experiment	Trade	3	3
	No trade	7	6		No trade	5	8
		χ^2	p-value			χ^2	p-value
Chi-square		1.534	0.216	Chi-square		0.224	0.636
Yates		0.529	0.467	Yates		0.001	0.975

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.16: Transaction and no transaction in the experiment and theory - total

Theoretical updating				Experimental updating			
		Theory				Theory	
		Trade	No trade			Trade	No trade
Experiment	Trade	11	5	Experiment	Trade	7	9
	No trade	16	25		No trade	11	30
		χ^2	p-value			χ^2	p-value
Chi-square		4.079**	0.043	Chi-square		1.525	0.217
Yates		2.974*	0.085	Yates		0.842	0.359

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.17: Increase and decrease in the experiment and theory - stage 1-2

Theoretical updating				Experimental updating			
		Theory				Theory	
		Increase	Decrease			Increase	Decrease
Experiment	Increase	12	6	Experiment	Increase	12	6
	Decrease	11	9		Decrease	13	7
		χ^2	p-value			χ^2	p-value
Chi-square		0.540	0.462	Chi-square		0.012	0.913
Yates		0.162	0.687	Yates		0.055	0.815

Note: *, **, *** indicate significance at 10% 5%,1% level.

The trader's order prices is increase or decrease at later stages:

Table B.18: Increase and decrease in the experiment and theory - stage 2-3

Theoretical updating				Experimental updating			
		Theory				Theory	
		Increase	Decrease			Increase	Decrease
Experiment	Increase	15	1	Experiment	Increase	6	10
	Decrease	17	5		Decrease	12	10
		χ^2	p-value			χ^2	p-value
Chi-square		1.891	0.169	Chi-square		1.080	0.299
Yates		0.855	0.355	Yates		0.504	0.478

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.19: Increase and decrease in the experiment and theory - stage 1-3

Theoretical updating				Experimental updating			
		Theory				Theory	
		Increase	Decrease			Increase	Decrease
Experiment	Increase	18	2	Experiment	Increase	15	5
	Decrease	9	9		Decrease	9	9
		χ^2	p-value			χ^2	p-value
Chi-square		7.370***	0.007	Chi-square		2.545	0.111
Yates		5.553**	0.018	Yates		1.584	0.208

Note: *, **, *** indicate significance at 10% 5%,1% level.

Table B.20: Increase and decrease in the experiment and theory - stage 1-2 and 2-3

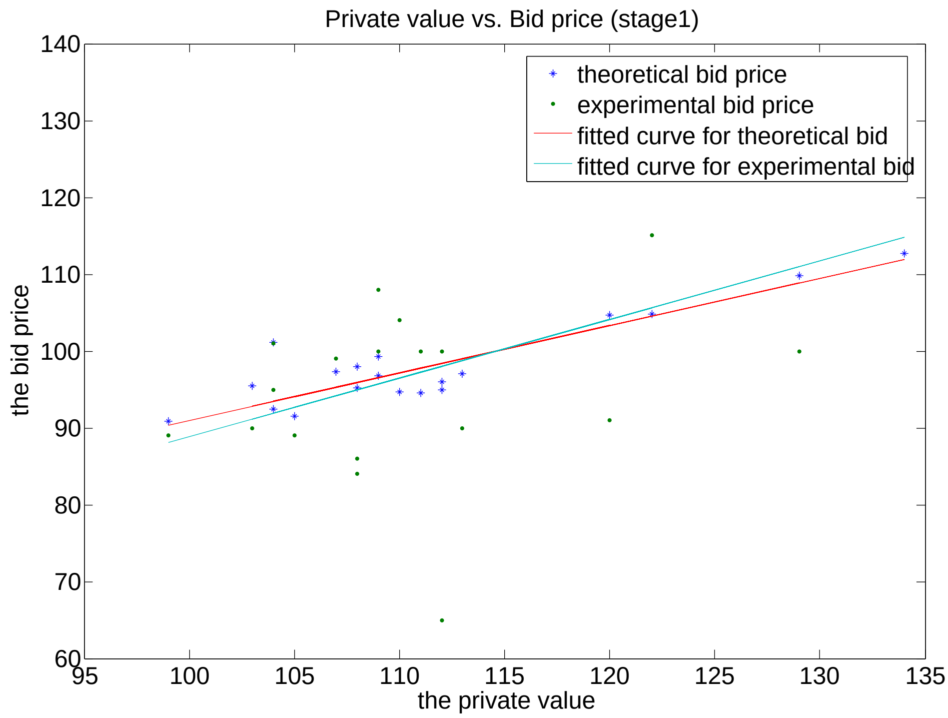
Theoretical updating				Experimental updating			
		Theory				Theory	
		Increase	Decrease			Increase	Decrease
Experiment	Increase	27	7	Experiment	Increase	18	16
	Decrease	28	14		Decrease	25	17
		χ^2	p-value			χ^2	p-value
Chi-square		1.526	0.217	Chi-square		0.331	0.565
Yates		0.955	0.328	Yates		0.118	0.731

Note: *, **, *** indicate significance at 10% 5%,1% level.

Appendix C

The determinants of order prices in the multi-stage trading experiment

In the multi-stage trading experiment, students were required to trade with the same opponent three times. We attempt to observe the relationship between the efficient variables and the offer prices of traders. The experimental data is compared with the related theoretical values to search the difference between the theoretical model and the experiment environment. The theoretical values are attained by two methods: updating the theoretical offer price on the last stage and updating the experimental offer price on the last stage. The comparison could make between these two methods as they address the question of traders' behaviors from different points of view. The analysis are approach stage by stage to observe any change of the traders' submission strategies in the continuous trading process.



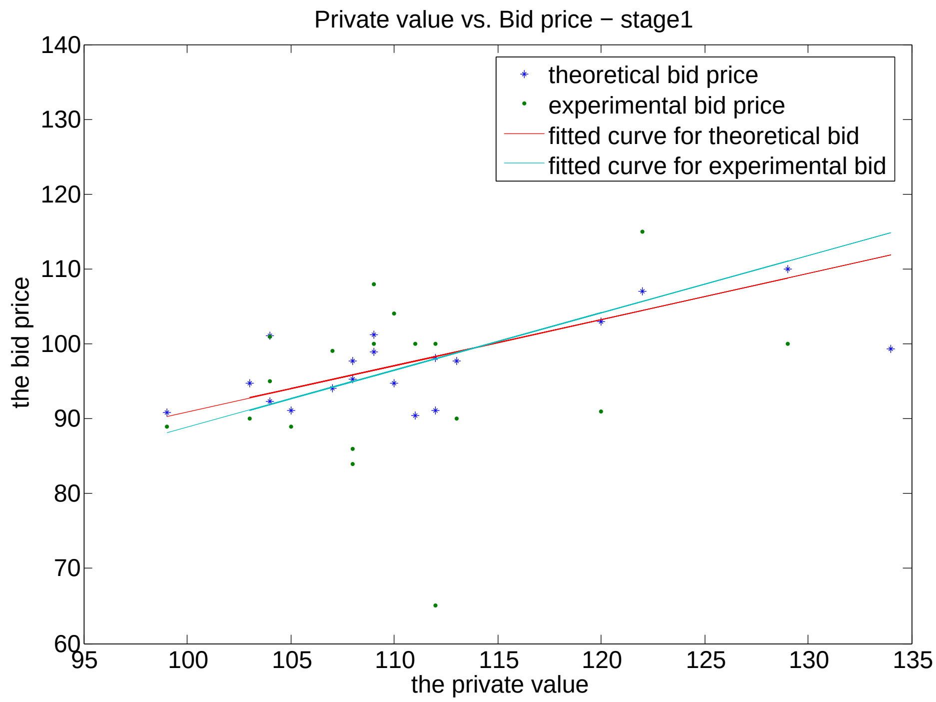
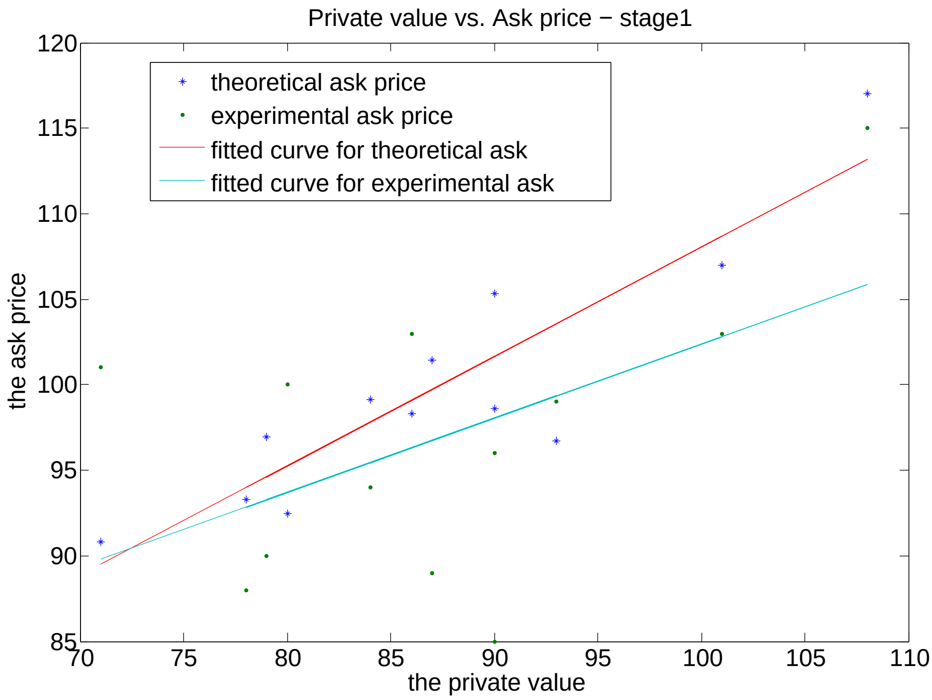


Figure C.1: The effects of private values on bid prices - stage 1 (theoretical updating & experimental updating)



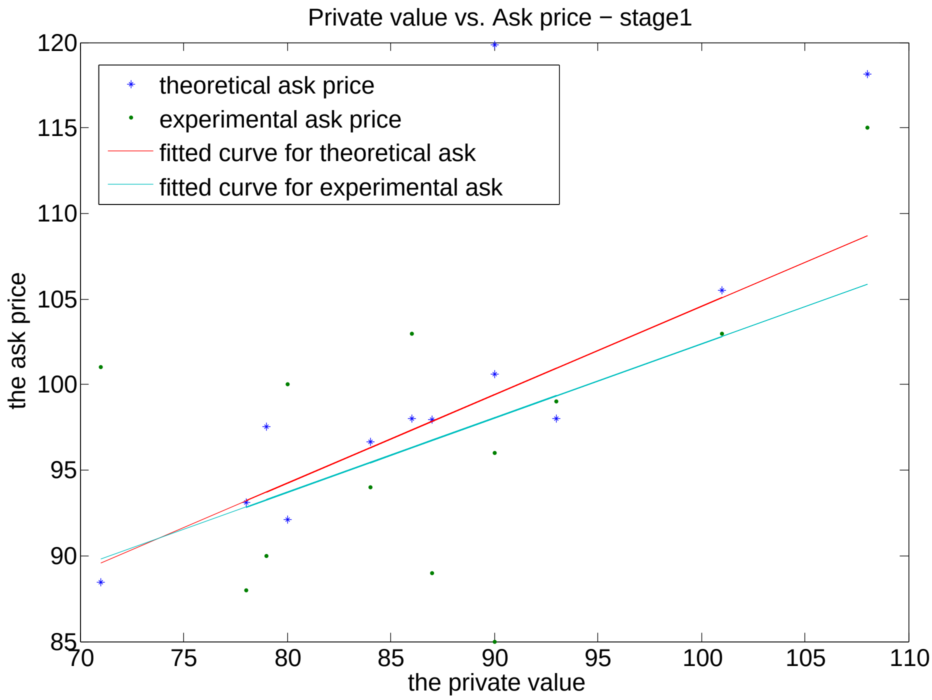
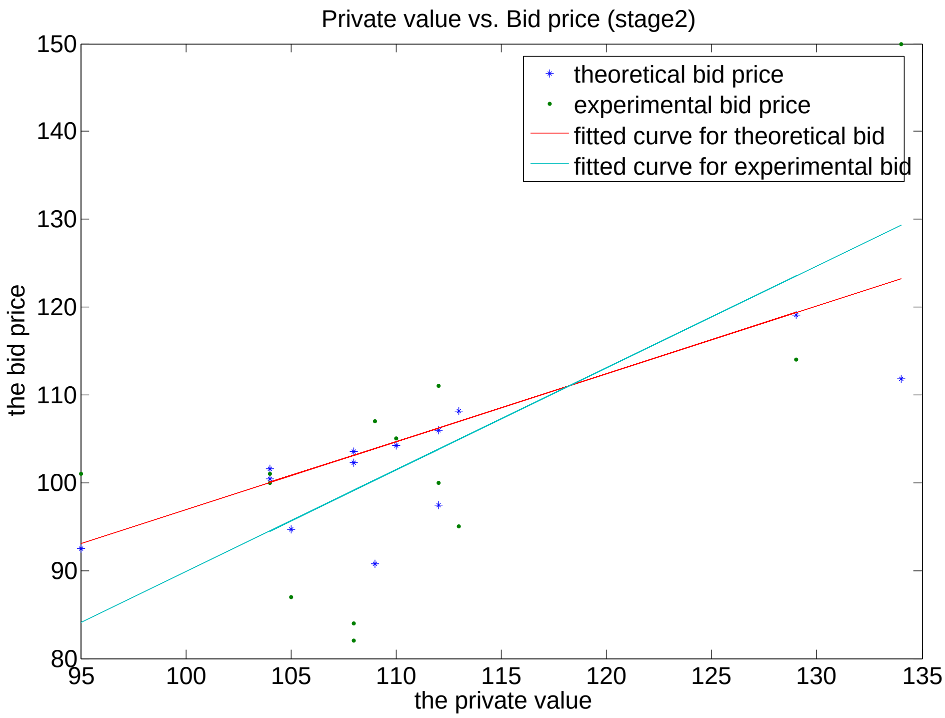


Figure C.2: The effects of private values on ask prices - stage 1 (theoretical updating & experimental updating)

Table C.1: Statistics for the private value vs. offer price - stage 1

		Bid				Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	29.50	3.77***	0.00	44.04	4.85***	0.00
		Slope	0.61	8.79***	0.00	0.64	6.19***	0.00
Theoretical updating	Experiment	C	12.55	0.37	0.71	59.00	2.89**	0.02
		Slope	0.767	2.55**	0.02	0.43	1.87*	0.09
F -slope difference			0.61			0.94		
	Theory	C	29.50	2.54**	0.02	52.88	5.48***	0.00
		Slope	0.61	5.93***	0.00	0.52	4.70***	0.00
Experimental updating	Experiment	C	12.55	0.37	0.71	59.00	2.89**	0.02
		Slope	0.76	2.55**	0.02	0.43	1.87*	0.09
F -slope difference			0.58			-0.03		

Note: *, **, *** indicate significance at 10% 5%,1% level.



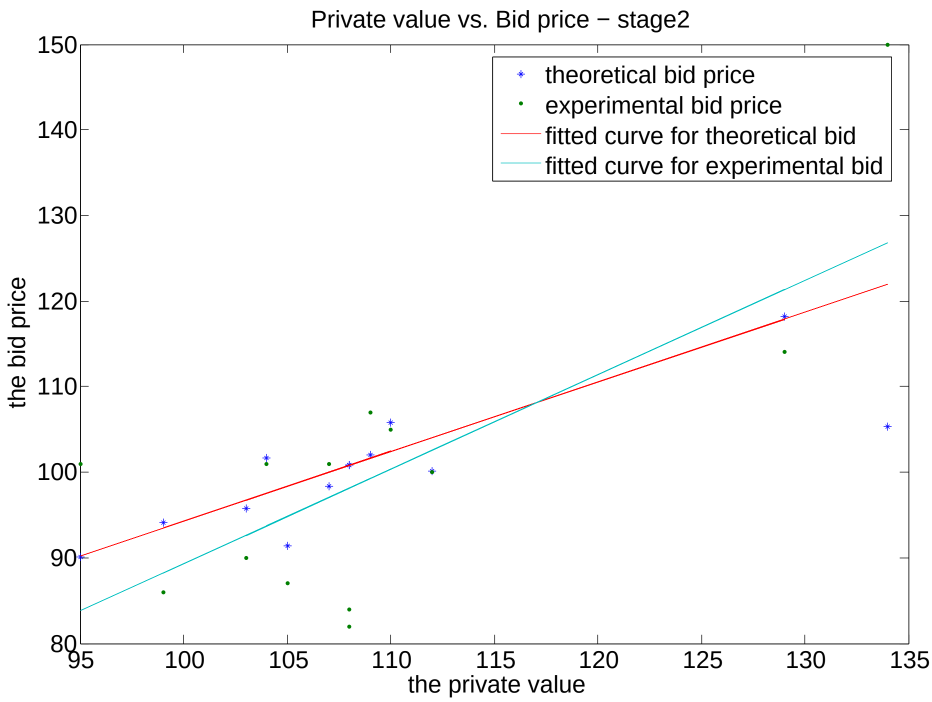
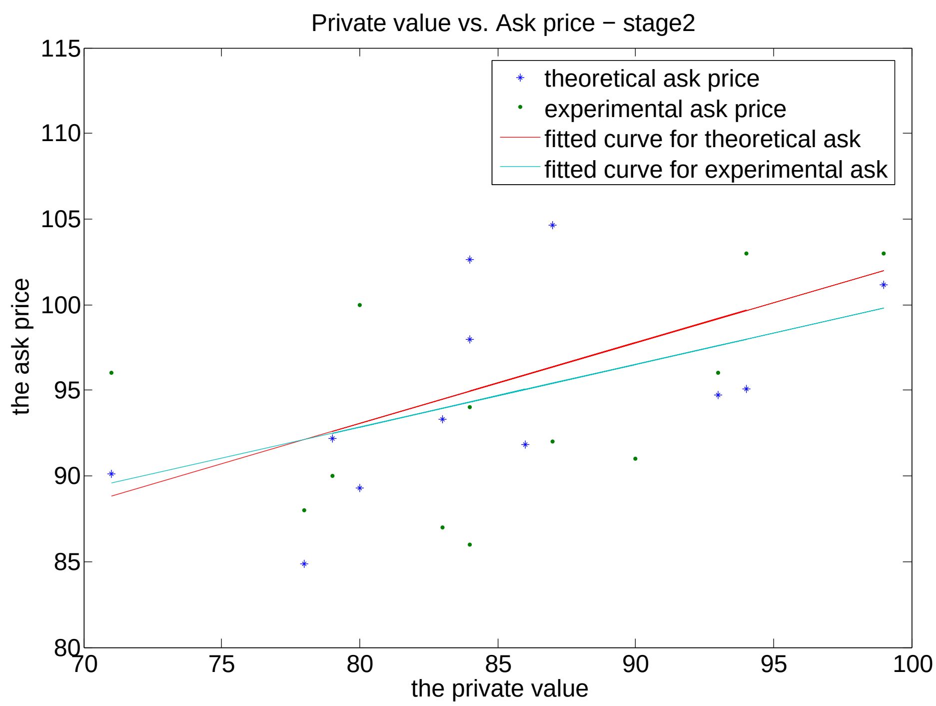


Figure C.3: The effects of private values on bid prices - stage 2 (theoretical updating & experimental updating)



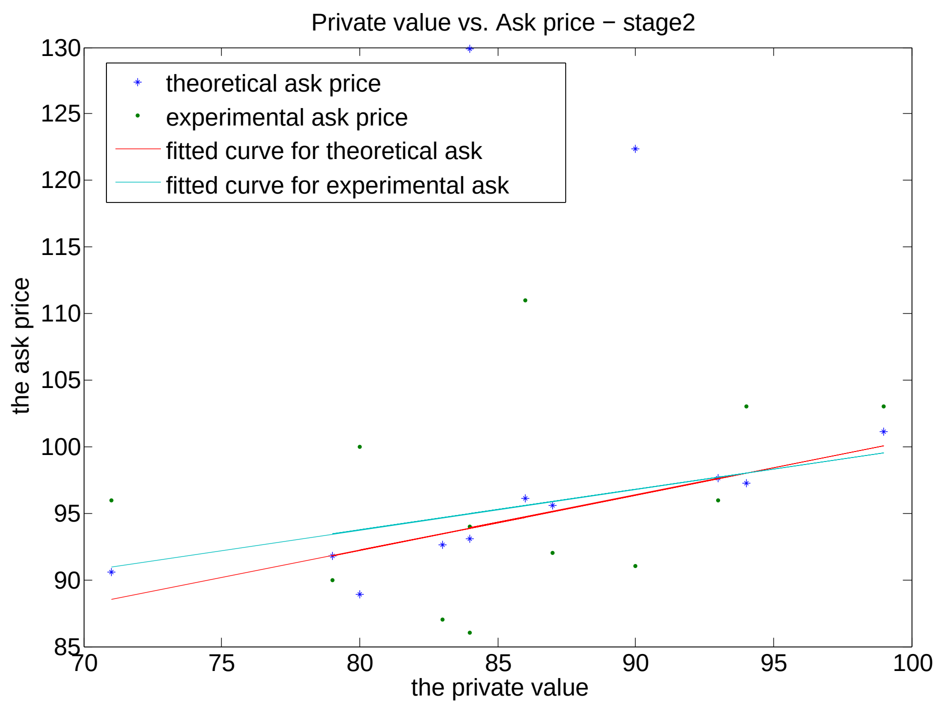
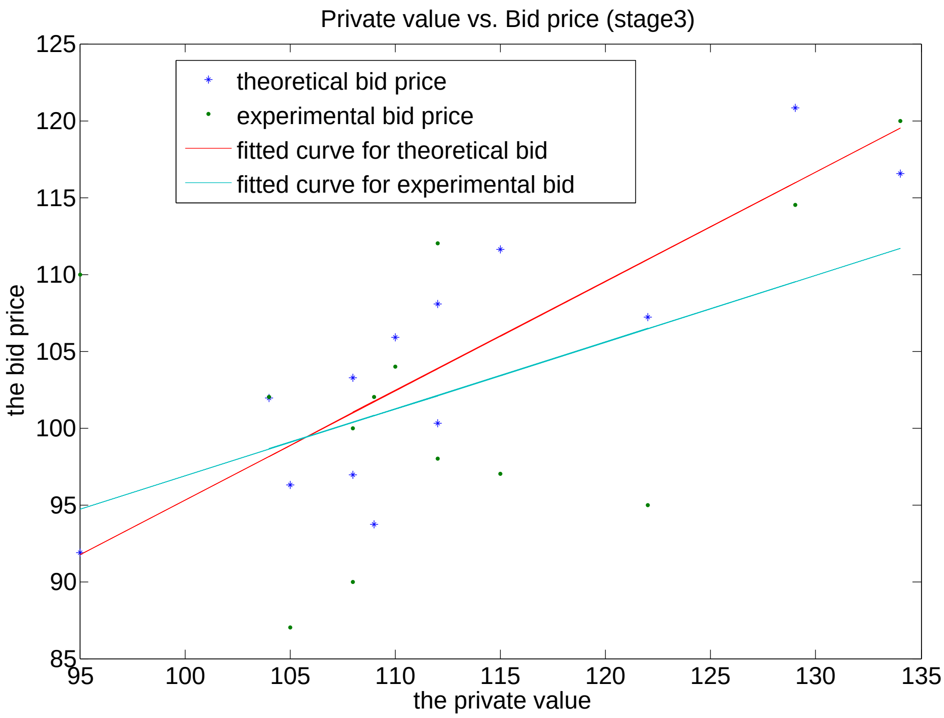


Figure C.4: The effects of private values on ask prices - stage 2 (theoretical updating & experimental updating)

Table C.2: Statistics for the private value vs. offer price - stage 2

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	19.49	2.29**	0.04	55.44	2.49**	0.03
		Slope	0.77	10.16***	0.00	0.47	1.81*	0.10
Theoretical updating	Experiment	C	-26.29	-0.65	0.53	63.59	2.64**	0.02
		Slope	1.16	3.19***	0.01	0.37	1.30	0.22
	F -slope difference		1.73*			0.11		
		C	13.18	1.27	0.23	59.16	2.58**	0.03
		Slope	0.81	8.61***	0.00	0.41	1.55	0.15
Experimental updating	Experiment	C	-21.05	-0.57	0.58	69.23	2.66**	0.02
		Slope	1.10	3.26***	0.01	0.31	1.01	0.33
	F -slope difference		2.48**			0.08		

Note: *, **, *** indicate significance at 10% 5%,1% level.



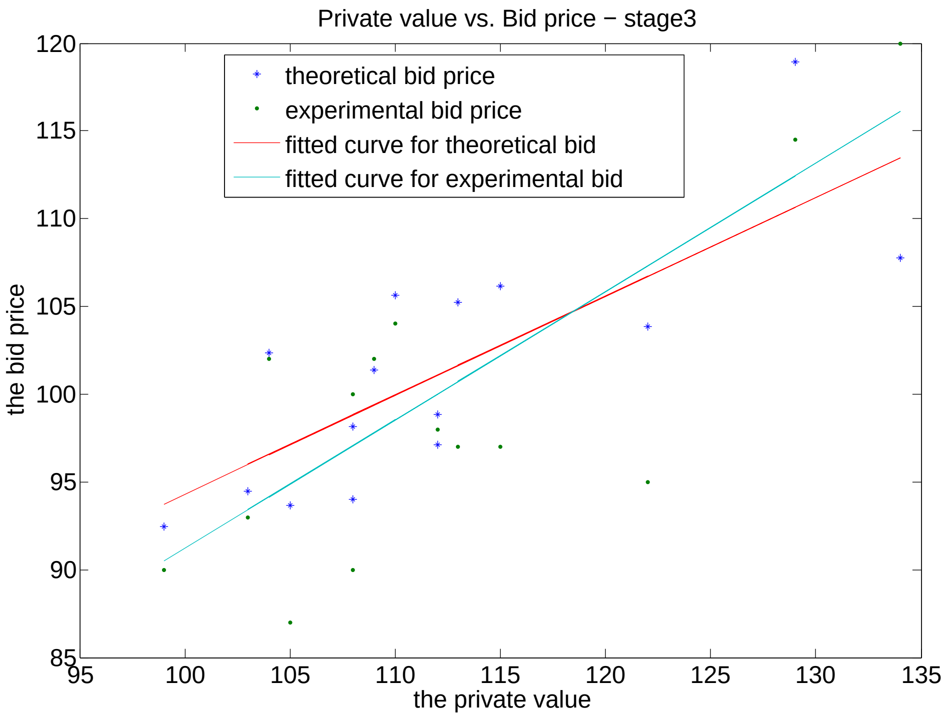
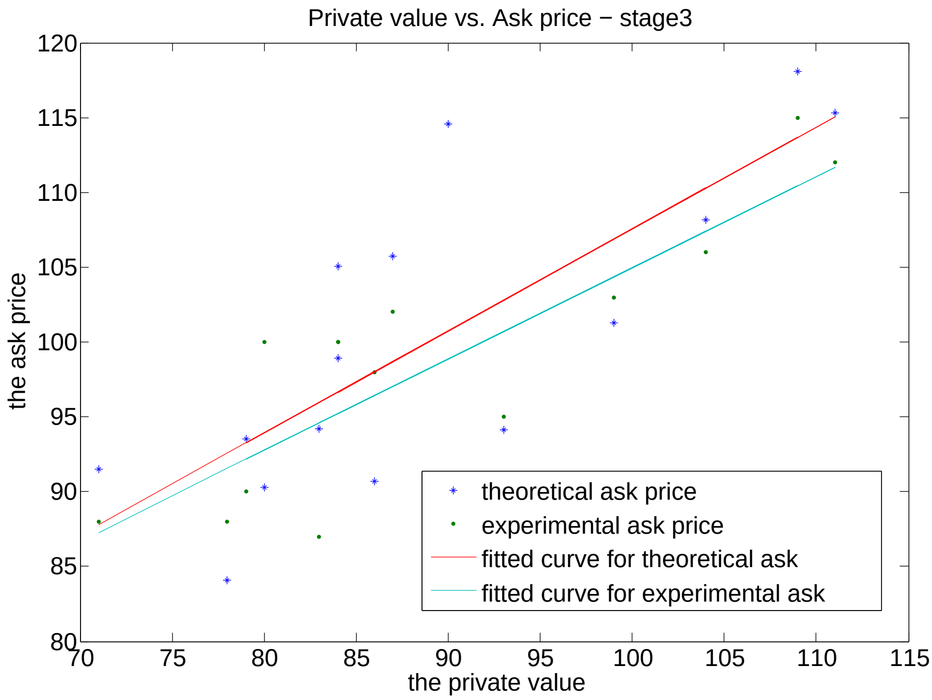


Figure C.5: The effects of private values on bid prices - stage 3 (theoretical updating & experimental updating)



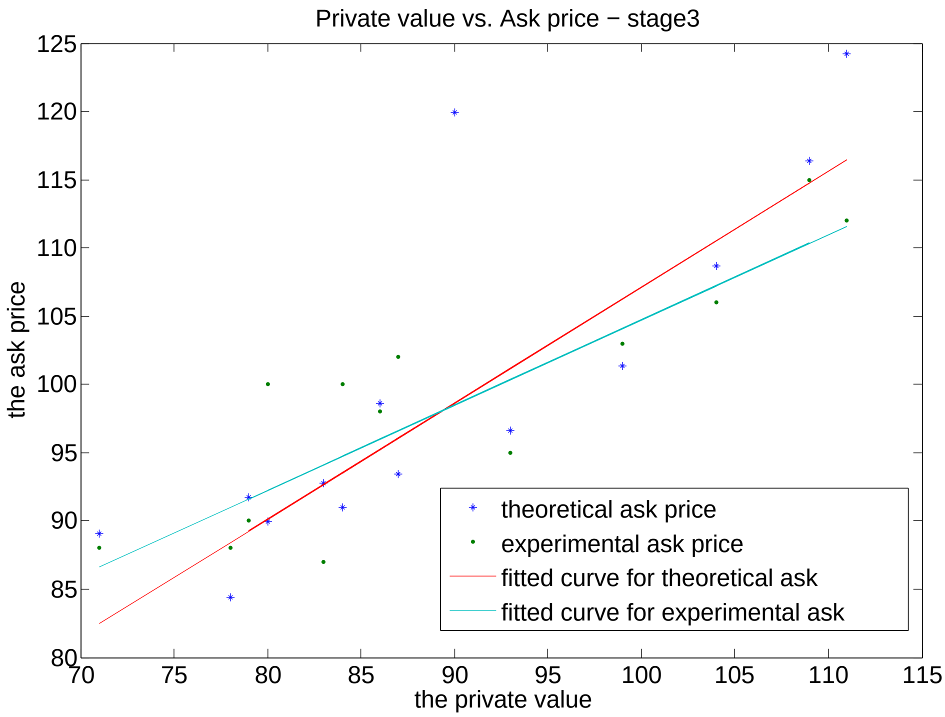
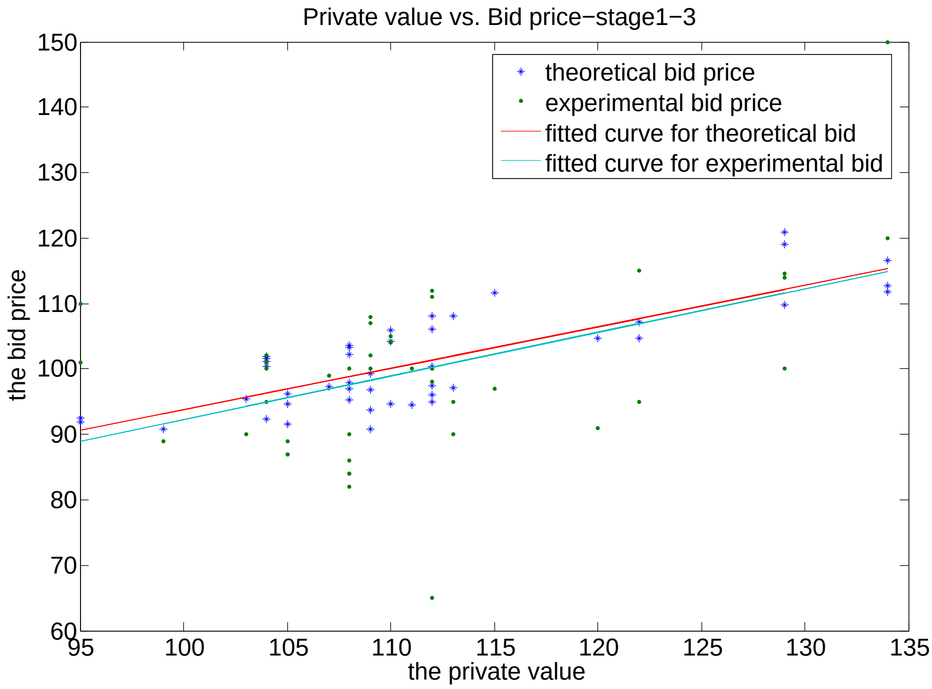


Figure C.6: The effects of private values on ask prices - stage 3 (theoretical updating & experimental updating)

Table C.3: Statistics for the private value vs. offer price - stage 3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	24.12	1.59	0.14	39.34	2.70**	0.02
		Slope	0.71	5.31***	0.00	0.68	4.21***	0.00
Theoretical updating	Experiment	C	53.43	1.81*	0.10	44.00	4.11***	0.00
		Slope	0.43	1.66	0.12	0.61	5.13***	0.00
F stats - slope difference			0.74			0.07		
	Theory	C	37.91	2.45**	0.03	22.19	1.89*	0.08
		Slope	0.56	4.11***	0.00	0.85	6.55***	0.00
Experimental updating	Experiment	C	17.98	0.81	0.43	42.30	3.98***	0.00
		Slope	0.73	3.69***	0.00	0.62	5.31***	0.00
F stats - slope difference			0.15			0.63		

Note: *, **, *** indicate significance at 10% 5%,1% level.



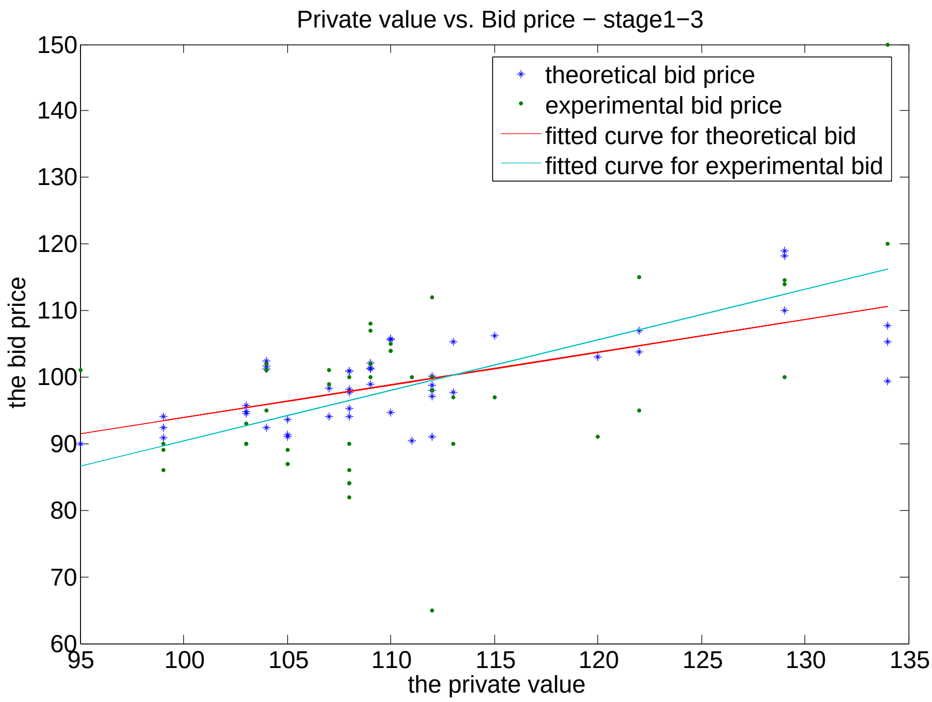
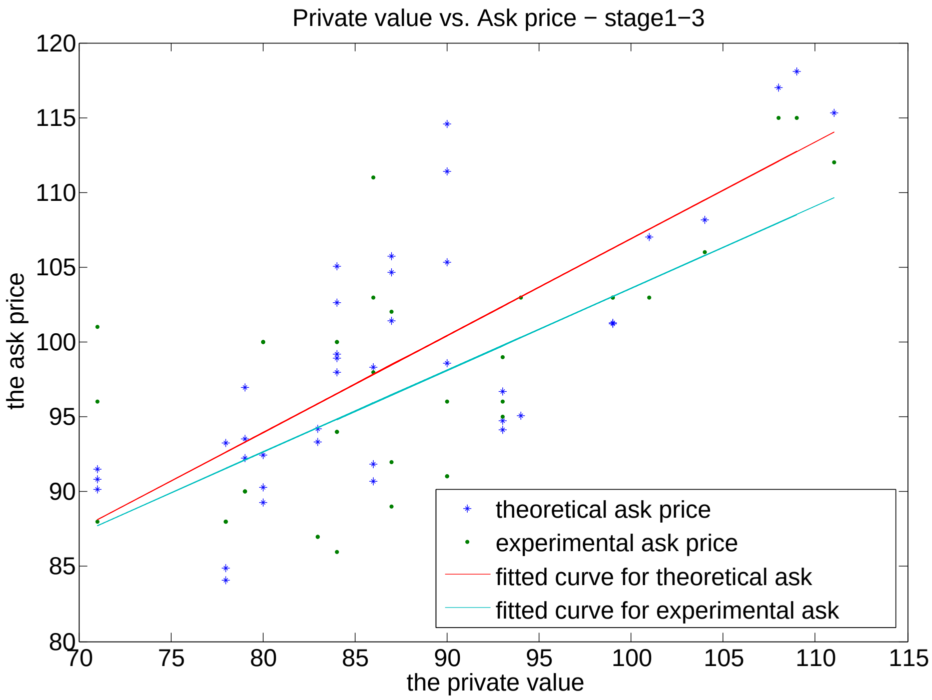


Figure C.7: The effects of private values on bid prices - stage 1-3 (theoretical updating & experimental updating)



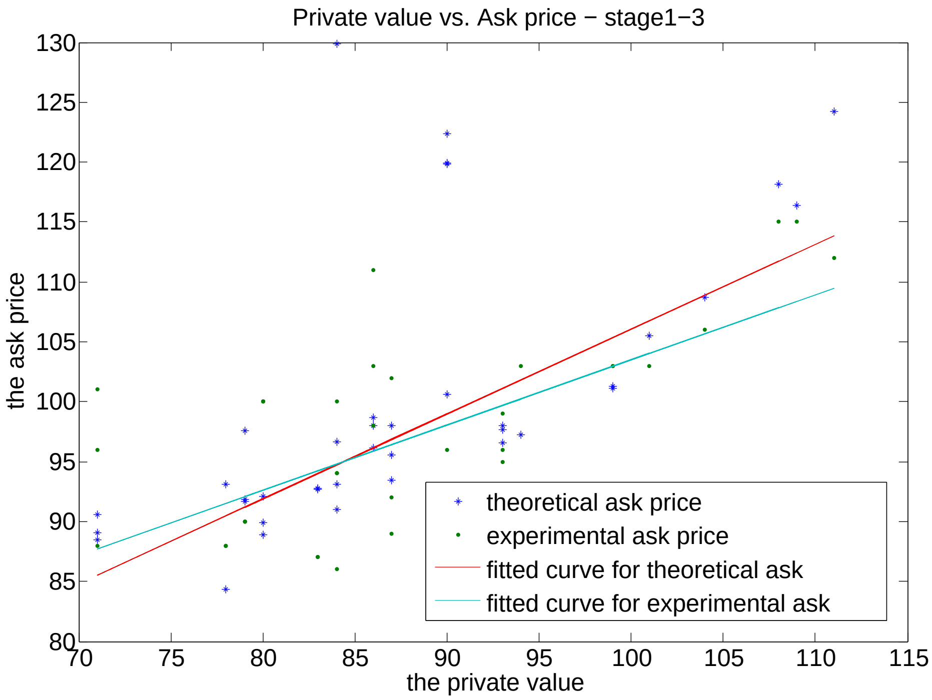


Figure C.8: The effects of private values on ask prices - stage 1-3 (theoretical updating & experimental updating)

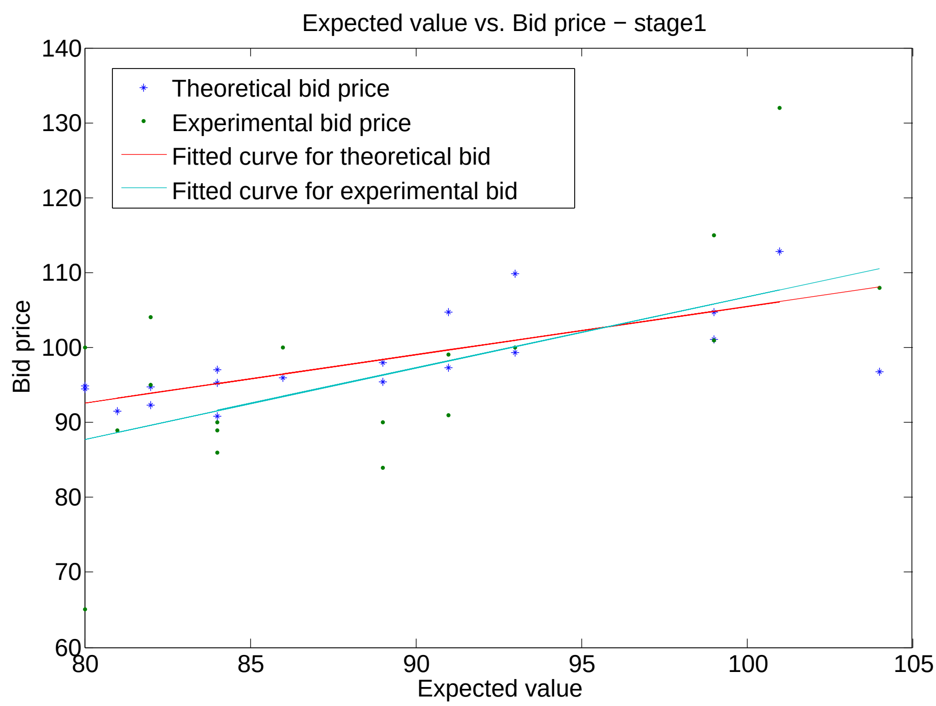
Table C.4: Statistics for the private value vs. offer price - stage 1-3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	30.50	3.55***	0.00	42.06	5.11***	0.00
		Slope	0.63	8.25***	0.00	0.65	6.93***	0.00
Theoretical updating	Experiment	C	25.63	1.37	0.18	48.79	5.19***	0.00
		Slope	0.67	3.99***	0.00	0.55	5.12***	0.00
F stats-slope difference			0.26			0.48		
	Theory	C	44.65	5.43***	0.00	35.31	5.04***	0.00
		Slope	0.49	6.68***	0.00	0.71	8.91***	0.00
Experimental updating	Experiment	C	14.48	0.90	0.37	49.12	5.04***	0.00
		Slope	0.76	5.26***	0.00	0.54	4.92***	0.00
F stats-slope difference			4.53**			0.47		

Note: *, **, *** indicate significance at 10% 5%,1% level.

C. THE DETERMINANTS OF ORDER PRICES IN THE MULTI-STAGE TRADING EXPERIMENT

The figures and table for the expected value are listed as following:



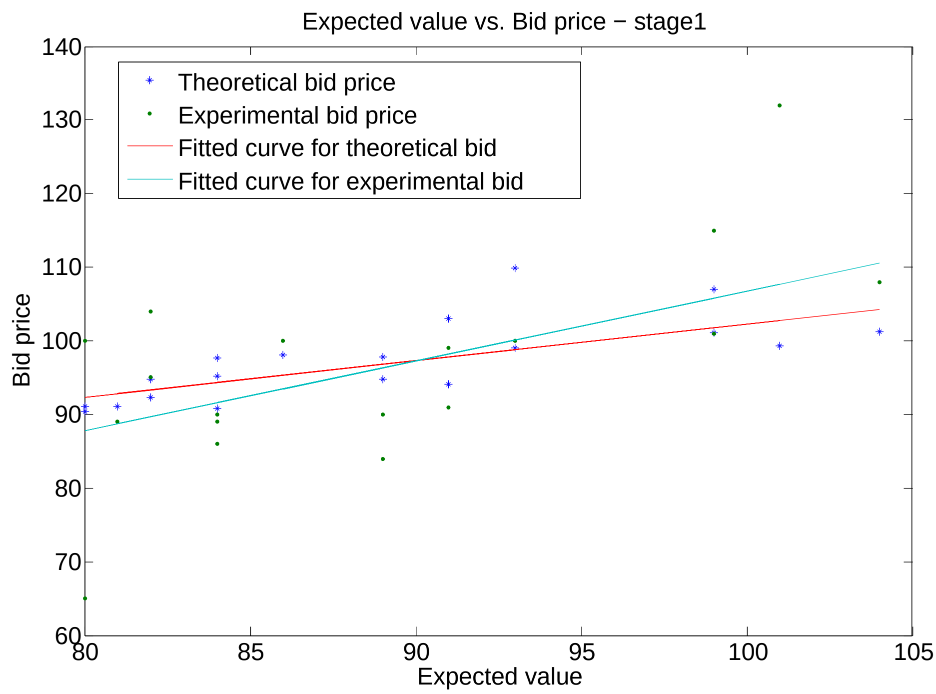
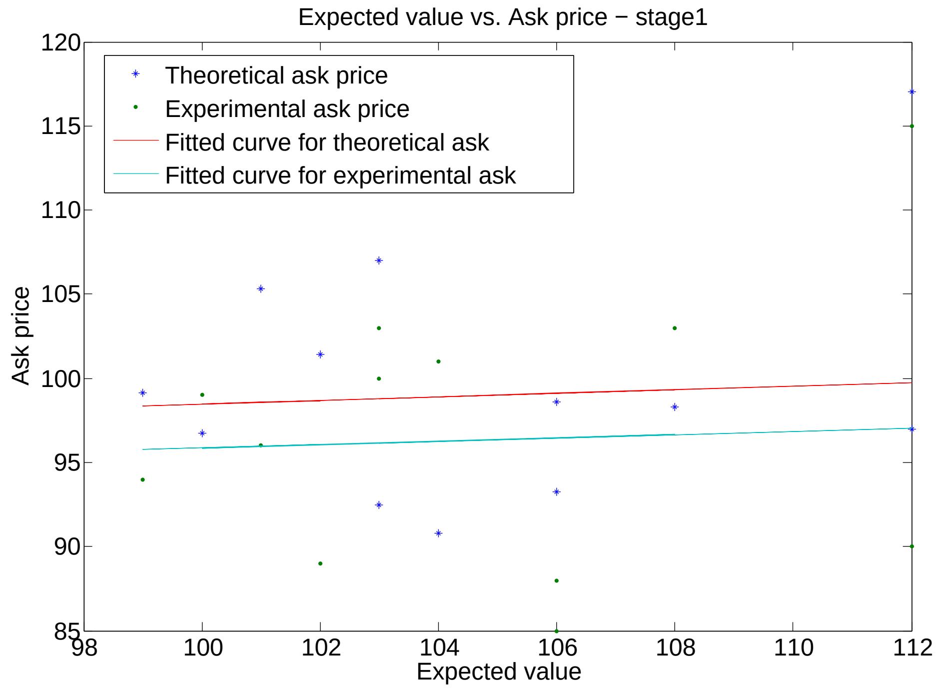


Figure C.9: The effects of expected values on bid prices - stage 1 (theoretical updating & experimental updating)



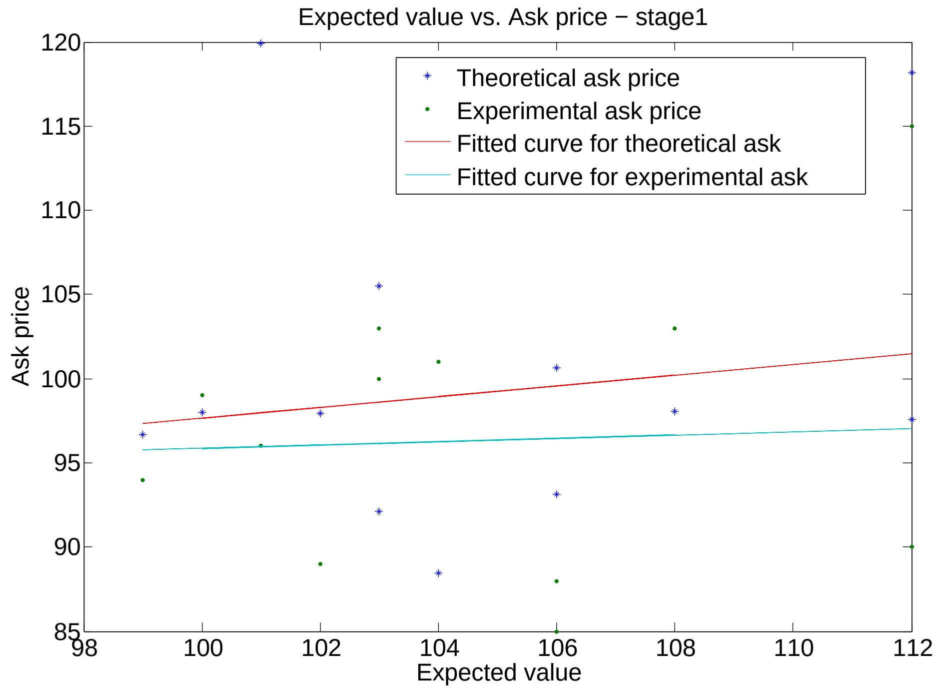
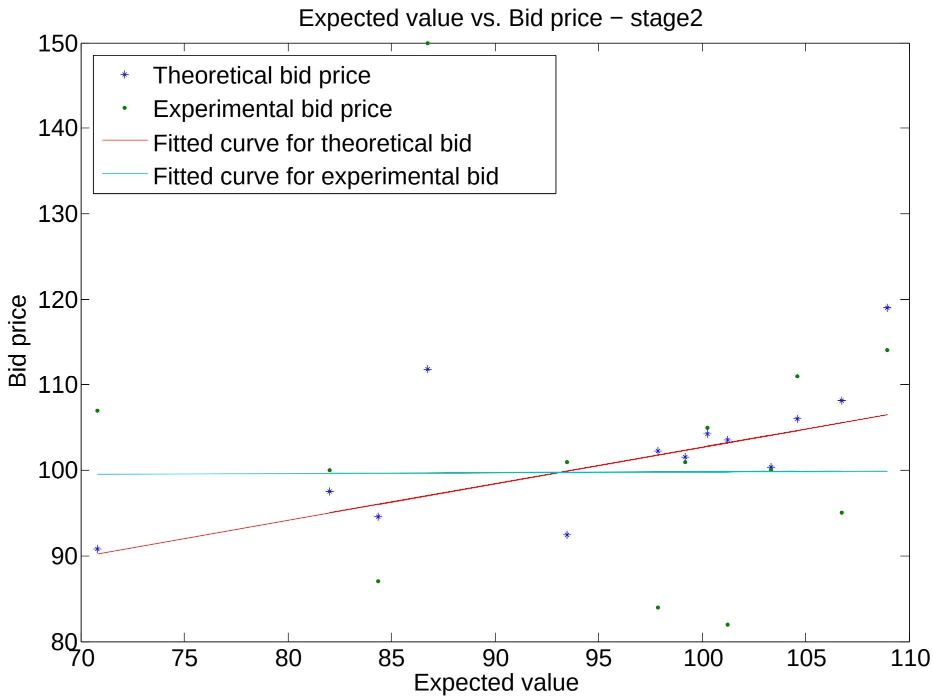


Figure C.10: The effects of expected values on bid prices - stage 1 (theoretical updating & experimental updating)

Table C.5: Statistics for the expected value vs. offer price - stage1

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	41.20	3.54***	0.00	87.73	1.63	0.13
		Slope	0.64	4.93***	0.00	0.11	0.21	0.84
Theoretical updating	Experiment	C	12.01	0.41	0.68	85.92	1.23	0.25
		Slope	0.95	2.92***	0.01	0.10	0.15	0.88
	F stats-slope difference		1.87		-0.29			
	Theory	C	52.53	5.14***	0.00	65.74	0.91	0.39
		Slope	0.50	4.35***	0.00	0.32	0.46	0.65
M2	Experiment	C	12.01	0.41	0.68	85.92	1.23	0.25
		Slope	0.95	2.92***	0.01	0.10	0.15	0.88
	F stats-slope difference		2.31*			-0.17		

Note: *, **, *** indicate significance at 10% 5%,1% level.



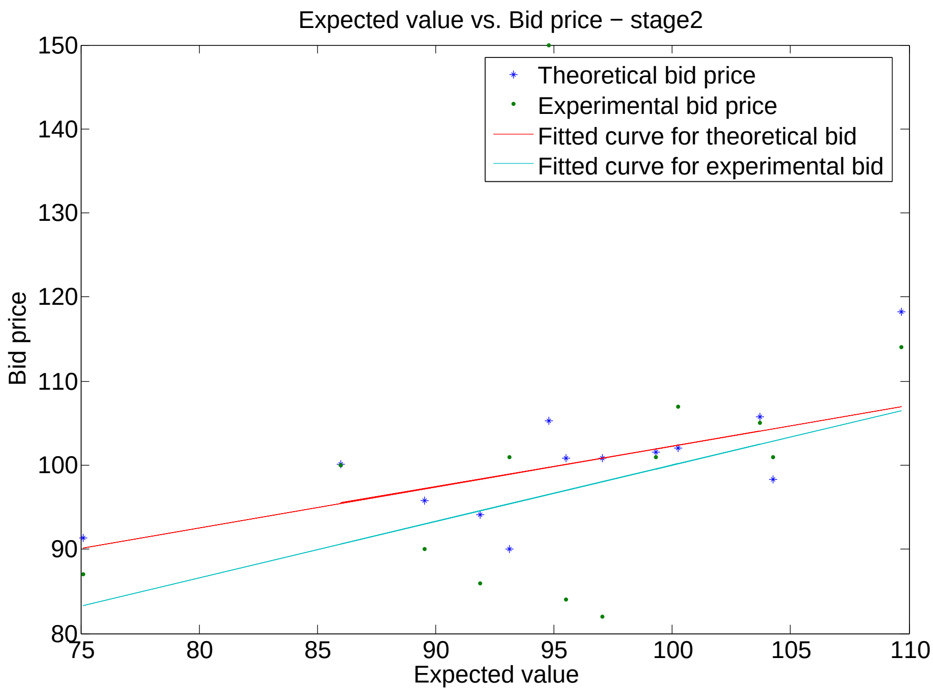


Figure C.11: The effects of expected values on bid prices - stage 2 (theoretical updating & experimental updating)

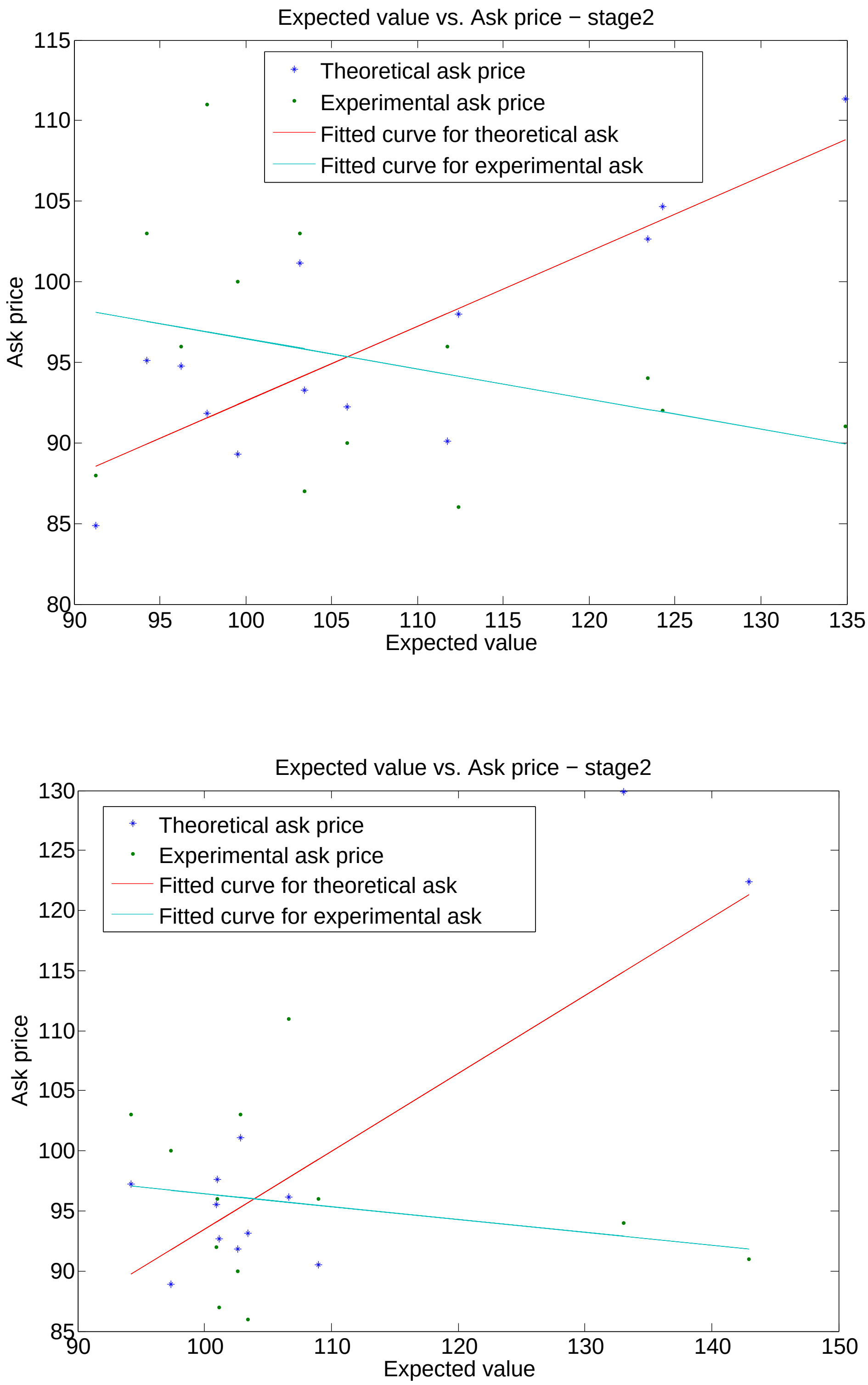
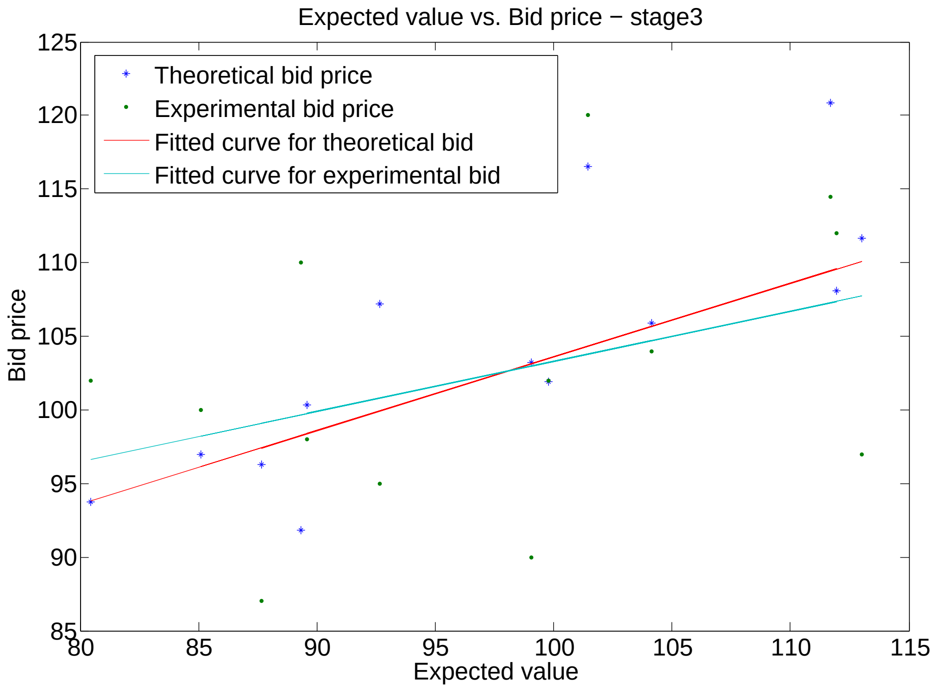


Figure C.12: The effects of expected values on ask prices - stage 2 (theoretical updating & experimental updating)

Table C.6: Statistics for the expected value vs. offer price - stage2

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	60.01	5.97***	0.00	46.25	4.41***	0.00
		Slope	0.43	4.08***	0.00	0.46	4.78***	0.00
Theoretical updating	Experiment	C	98.79	2.71**	0.02	115.18	6.35***	0.00
		Slope	0.01	0.03	0.98	-0.19	-1.12	0.29
F stats-slope difference			0.89			5.21		
	Theory	C	53.80	3.13***	0.01	28.64	1.98*	0.08
		Slope	0.48	2.70**	0.02	0.65	4.87***	0.00
Experimental updating	Experiment	C	-32.86	0.87	0.40	107.20	6.19***	0.00
		Slope	0.67	1.70	0.12	-0.11	-0.68	0.51
F stats-slope difference			-0.08			17.69**		

Note: *, **, *** indicate significance at 10% 5%,1% level.



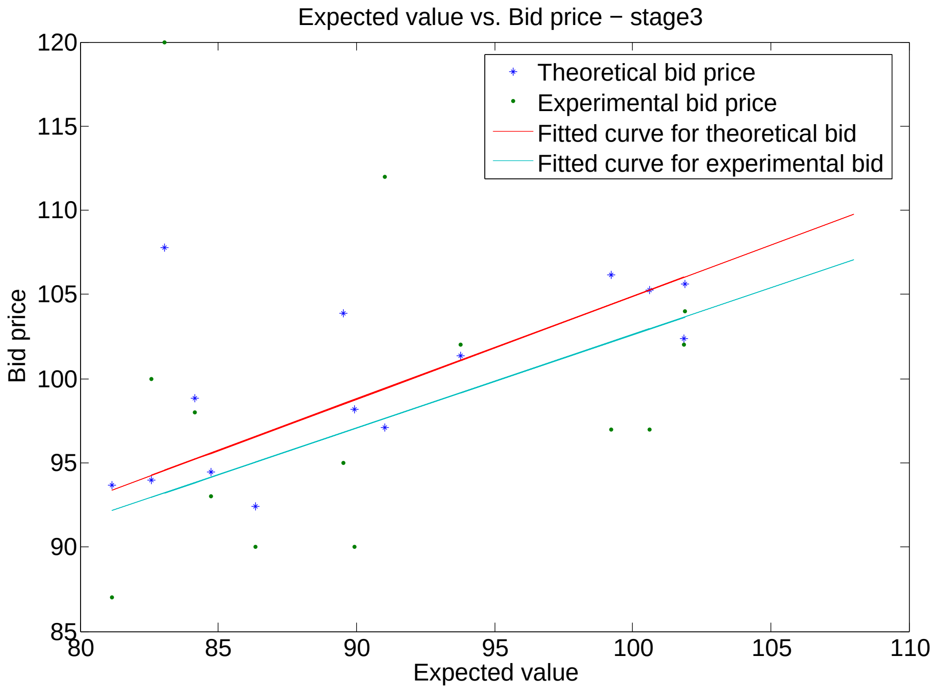
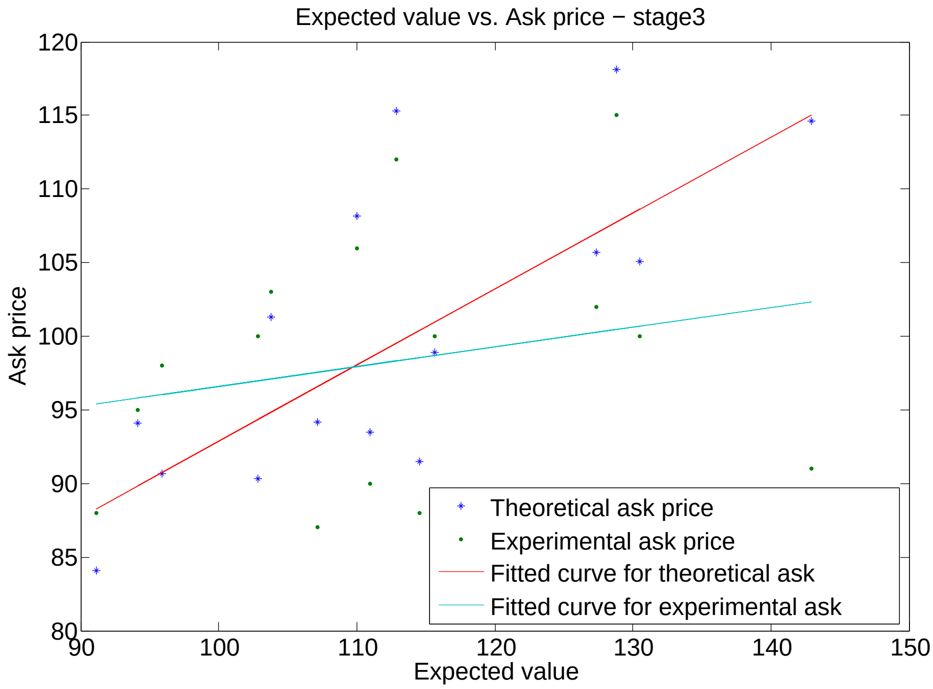


Figure C.13: The effects of expected values on bid prices - stage 3 (theoretical updating & experimental updating)



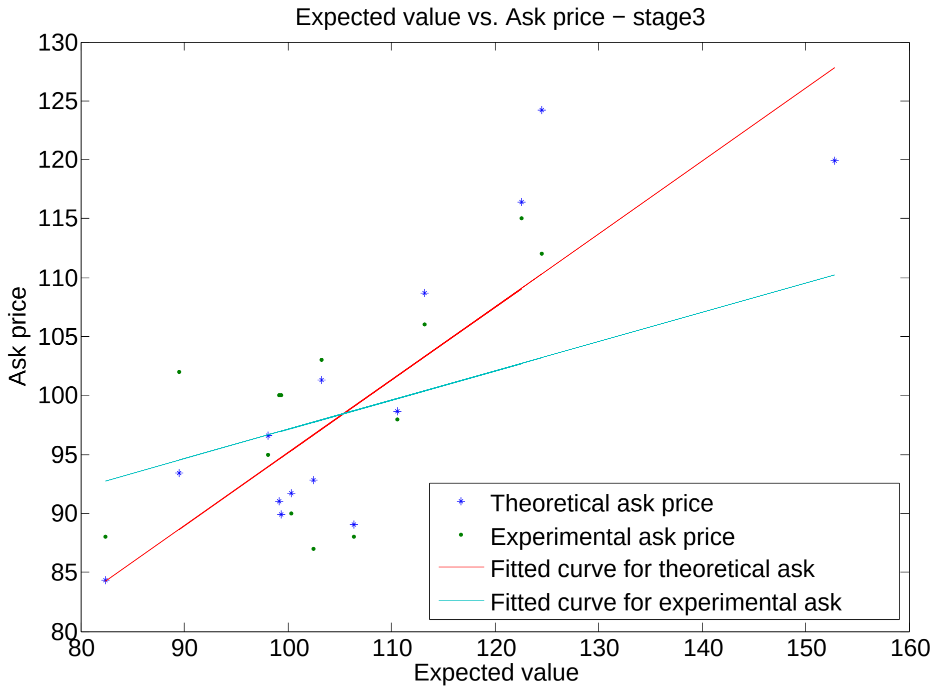
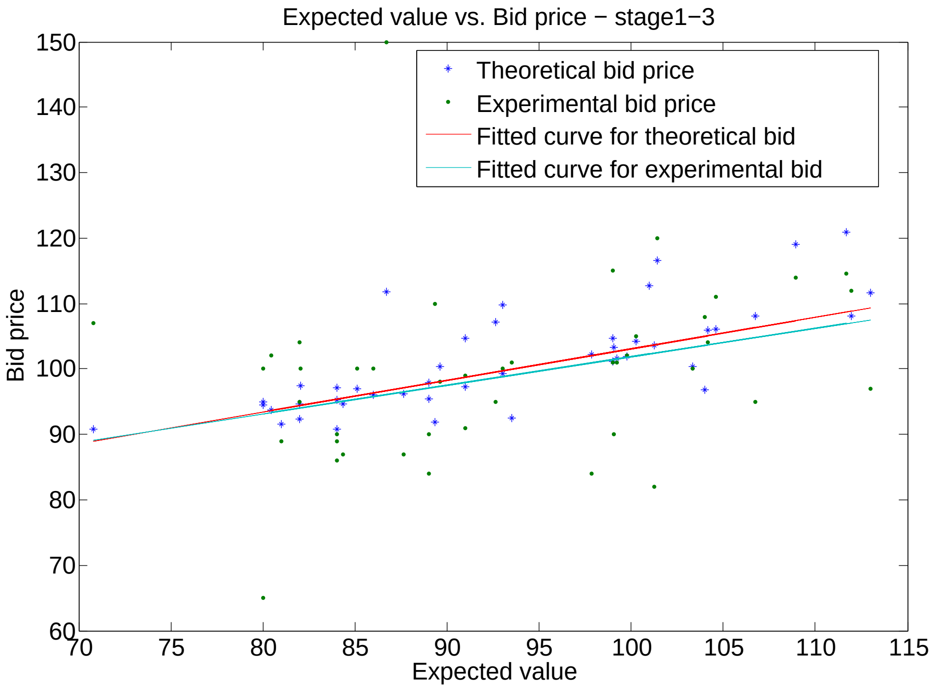


Figure C.14: The effects of expected values on ask prices - stage 3 (theoretical updating & experimental updating)

Table C.7: Statistics for the expected value vs. offer price - stage3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	53.56	5.36***	0.00	41.27	2.46**	0.03
		Slope	0.50	4.90***	0.00	0.52	3.49***	0.00
Theoretical updating	Experiment	C	69.29	2.71**	0.02	83.17	4.40***	0.00
		Slope	0.34	1.30	0.22	0.13	0.81	0.43
F stats -slope difference			-0.07			1.53		
	Theory	C	43.80	3.88***	0.00	33.29	2.66**	0.02
		Slope	0.61	5.00***	0.00	0.62	5.37***	0.00
Experimental updating	Experiment	C	47.08	2.01*	0.07	72.28	4.28***	0.00
		Slope	0.56	2.19**	0.05	0.25	1.60	0.14
F stats -slope difference			0.2			5.22**		

Note: *, **, *** indicate significance at 10% 5%,1% level.



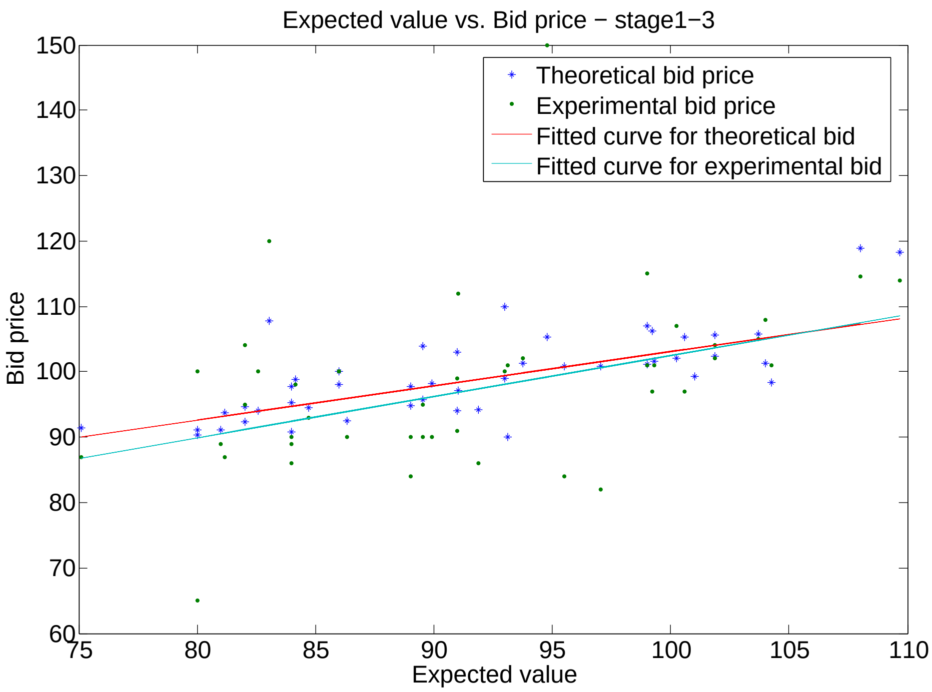
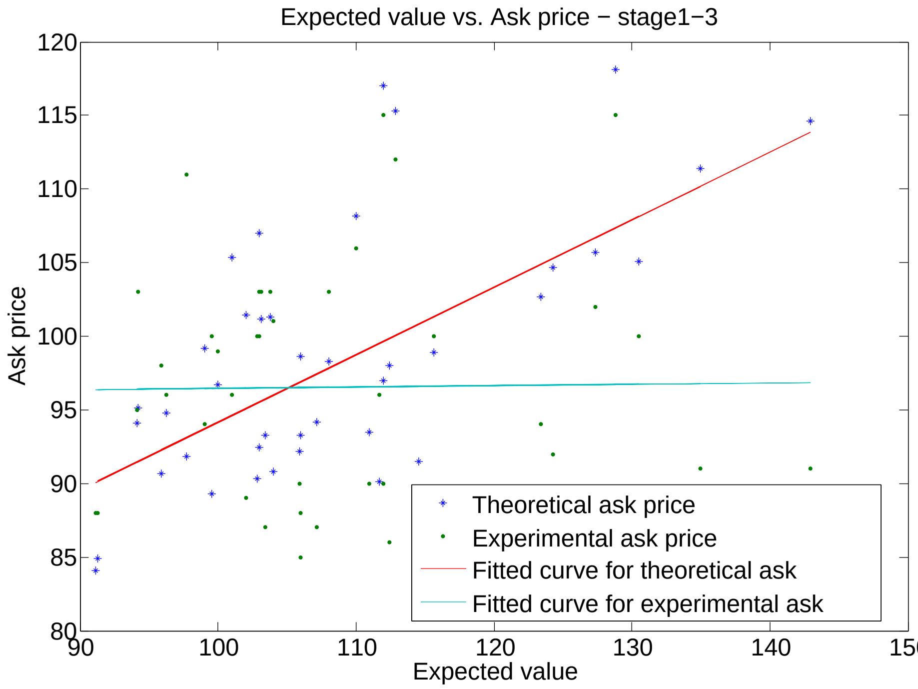


Figure C.15: The effects of expected values on bid prices - stage 1-3 (theoretical updating & experimental updating)



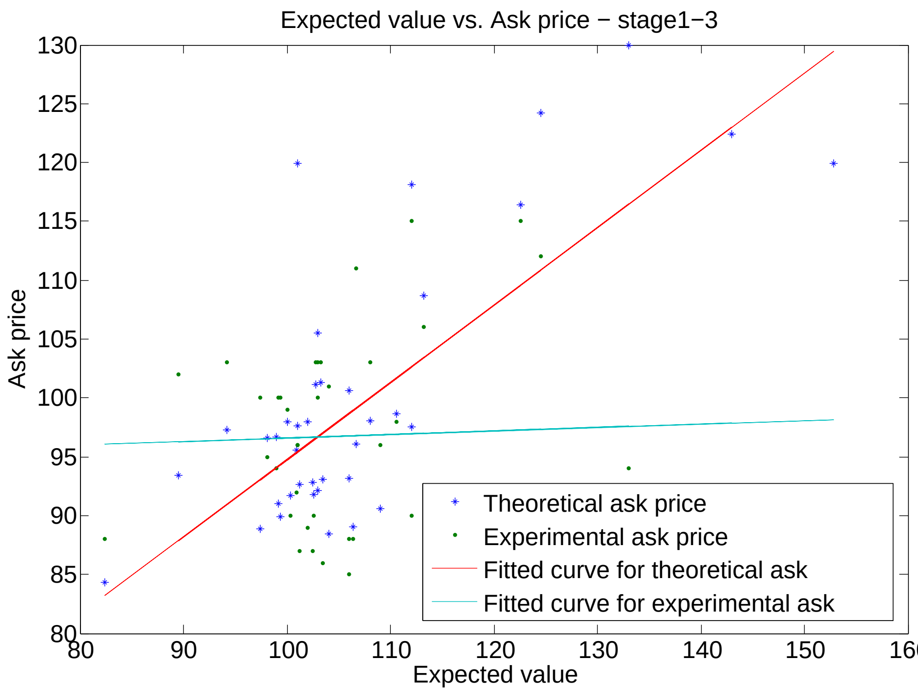


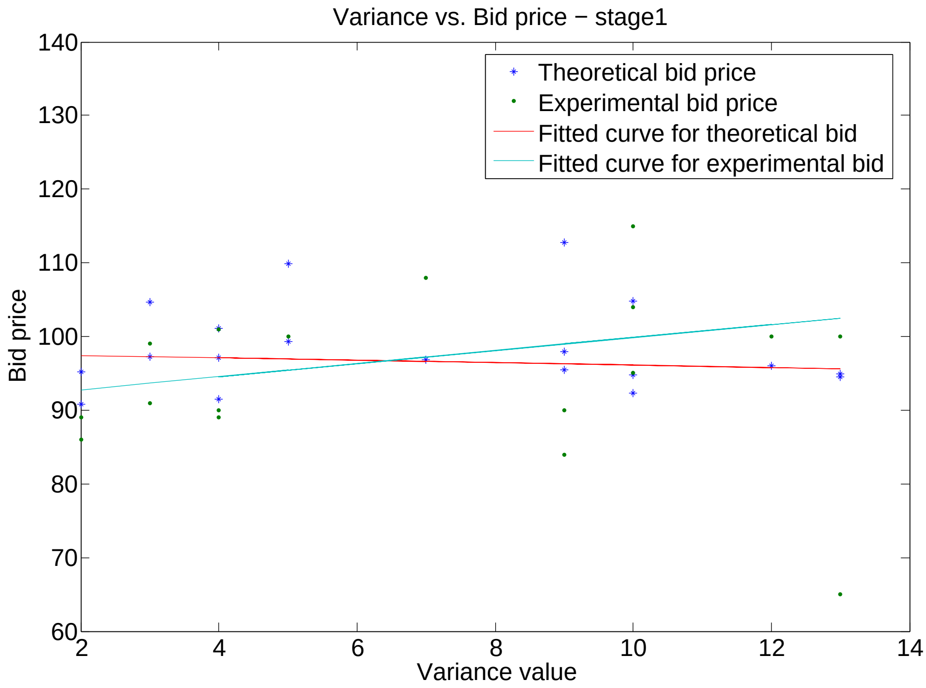
Figure C.16: The effects of expected values on bid prices - stage 1-3 (theoretical updating & experimental updating)

Table C.8: Statistics for the expected value vs. offer price - stage 1-3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	54.92	12.25***	0.00	48.31	5.19***	0.00
		Slope	0.48	10.07***	0.00	0.46	5.38***	0.00
Theoretical updating	Experiment	C	58.28	3.92***	0.00	95.51	7.77***	0.00
		Slope	0.44	2.75***	0.01	0.01	0.08	0.93
	F stats -slope difference		-0.2			4.65		
	Theory	C	50.80	7.81***	0.00	28.98	3.05***	0.00
		Slope	0.52	7.41***	0.00	0.66	7.43***	0.00
M2	Experiment	C	39.80	2.80***	0.01	93.59	8.21***	0.00
		Slope	0.63	4.06***	0.00	0.03	0.28	0.78
	F stats-slope difference		0			13.35**		

Note: *, **, *** indicate significance at 10% 5%,1% level.

The figures and table for the variance of the valuation distribution are listed as following:



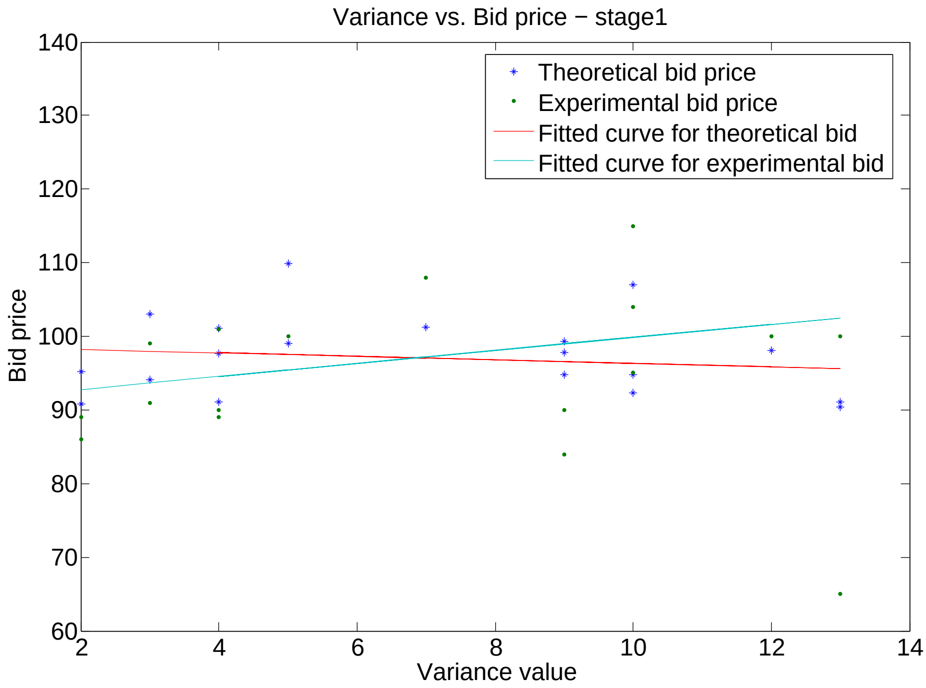
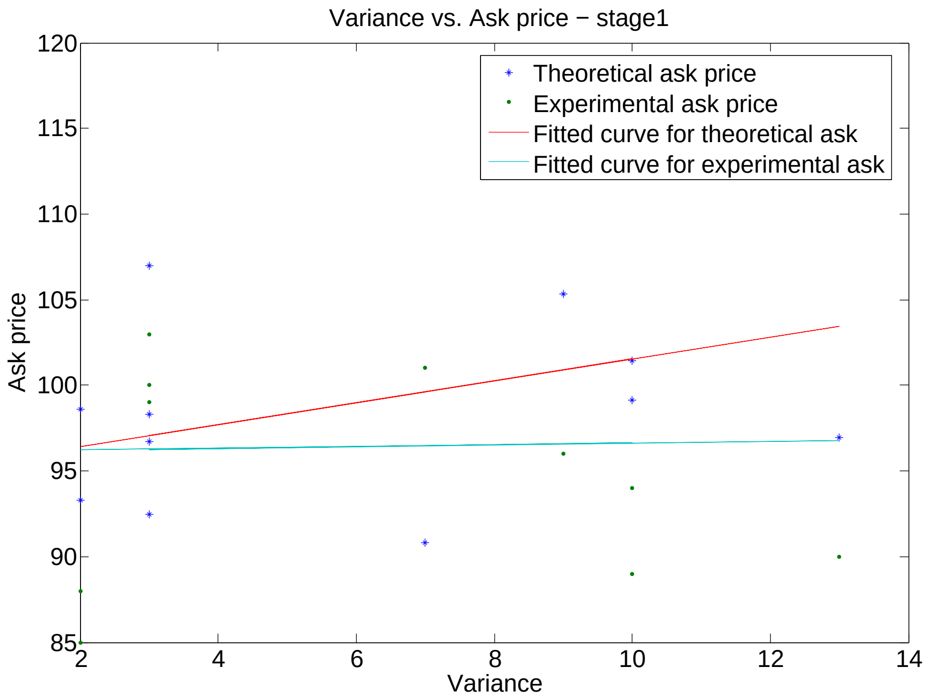


Figure C.17: The effects of variance on bid prices - stage 1 (theoretical updating & experimental updating)



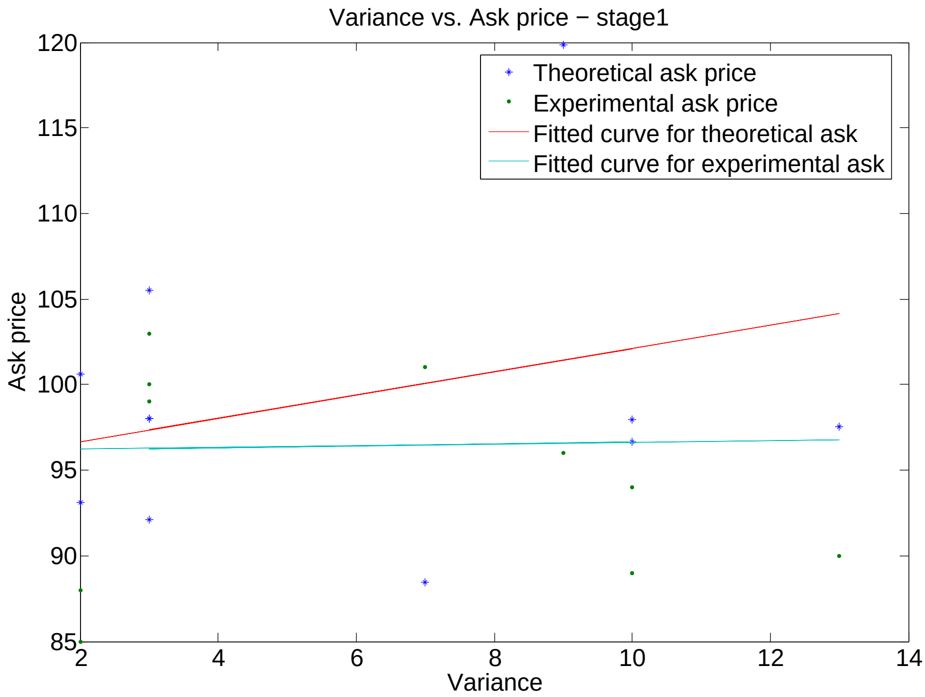
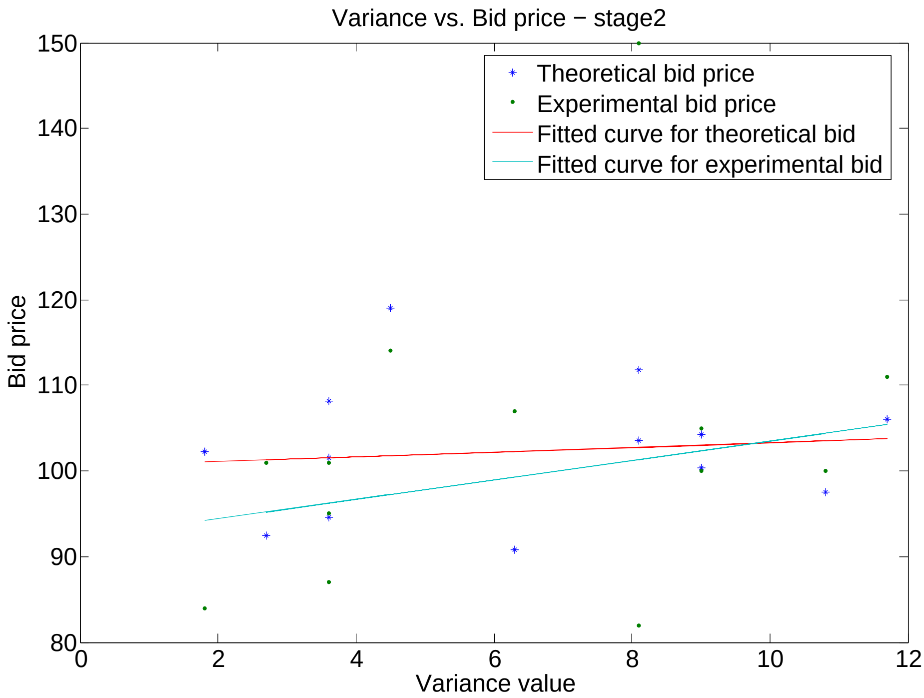


Figure C.18: The effects of variance on ask prices - stage 1 (theoretical updating & experimental updating)

Table C.9: Statistics for the variance vs. offer price - stage 1

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	97.79	37.91***	0.00	95.12	24.87***	0.00
		Slope	-0.17	-0.51	0.62	0.64	1.29	0.23
Theoretical updating	Experiment	C	91.02	16.06***	0.00	96.12	19.31***	0.00
		Slope	0.89	1.24	0.23	0.05	0.08	0.94
F stats-slope difference			-0.38			0.04		
	Theory	C	98.72	34.42***	0.00	95.31	17.43***	0.00
		Slope	-0.24	-0.67	0.51	0.68	0.96	0.36
Experimental updating	Experiment	C	91.02	16.06***	0.00	96.12	19.31***	0.00
		Slope	0.89	1.24	0.23	0.05	0.08	0.94
F stats-slope difference			-0.14			0.08		

Note: *, **, *** indicate significance at 10% 5%,1% level.



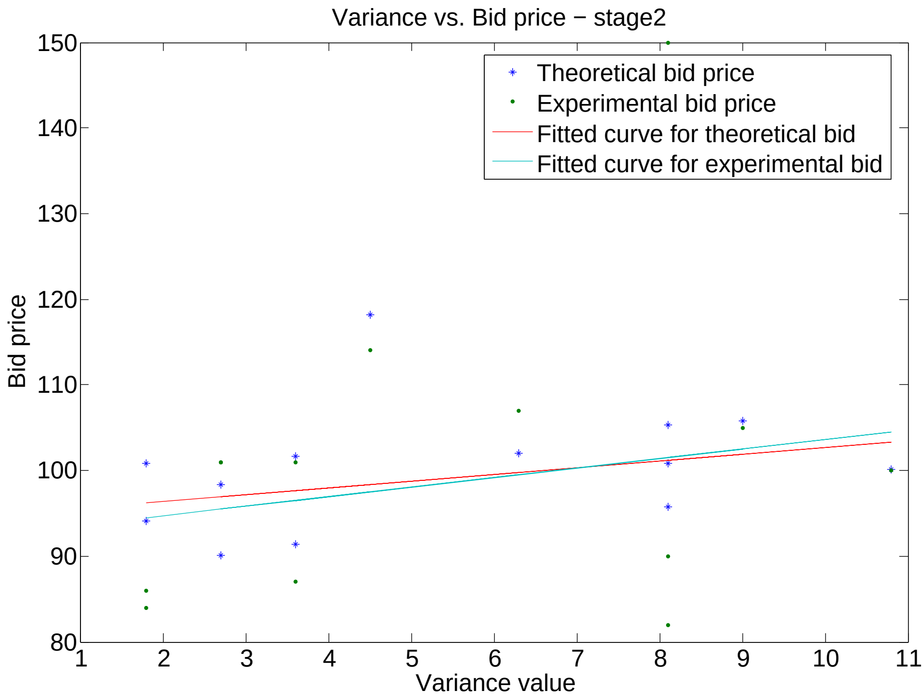
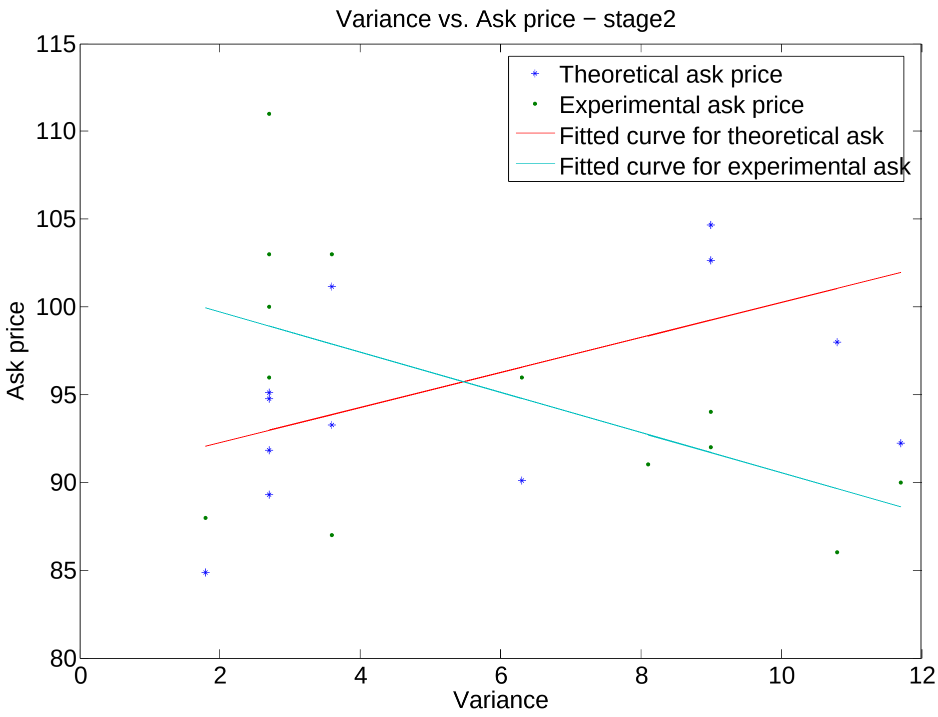


Figure C.19: The effects of variance on bid prices - stage 2 (theoretical updating & experimental updating)



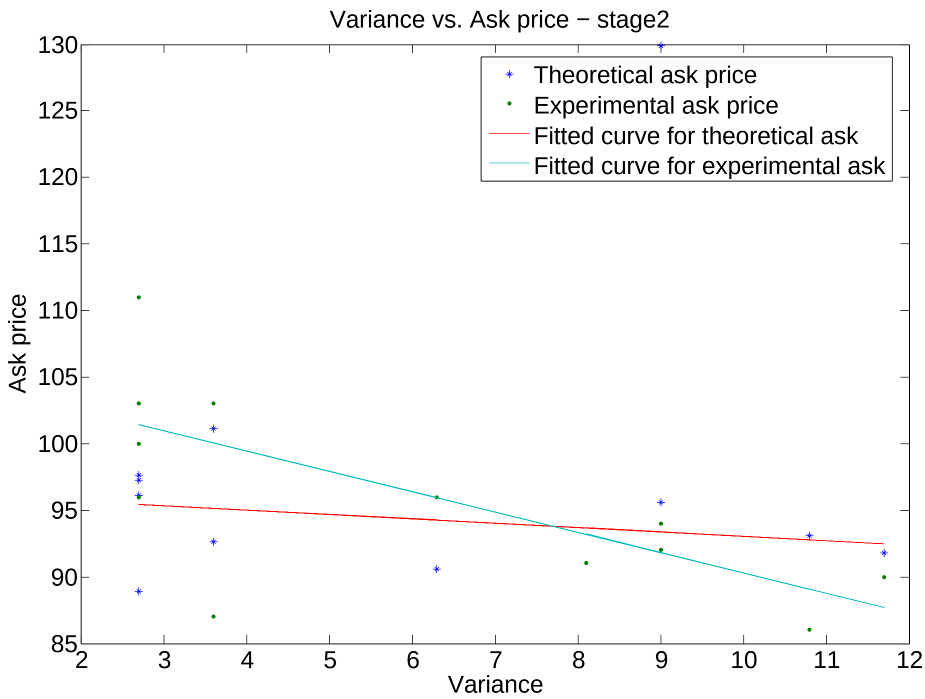
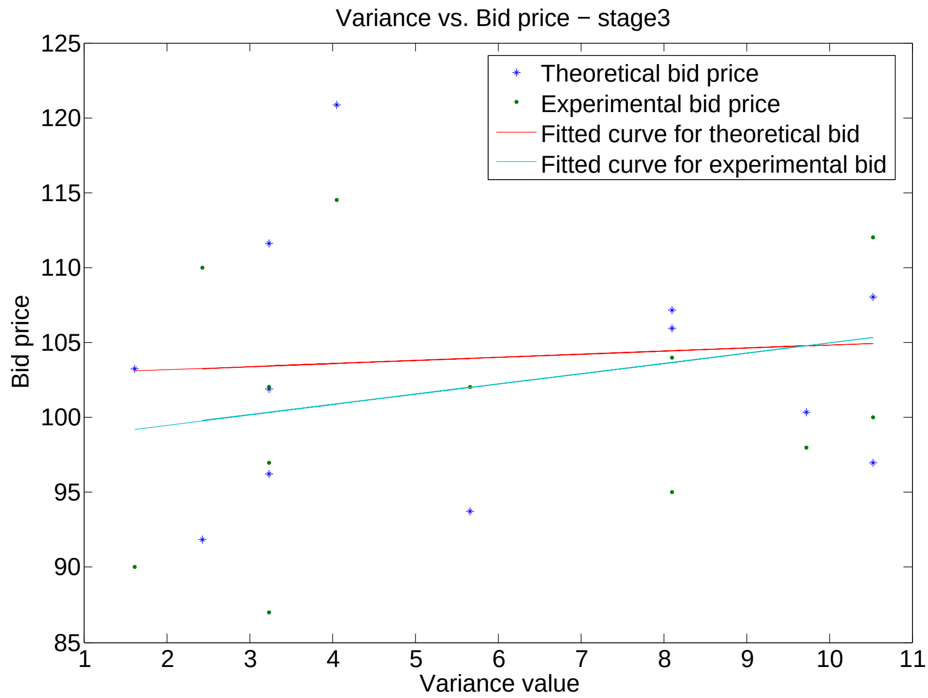


Figure C.20: The effects of variance on ask prices - stage 2 (theoretical updating & experimental updating)

Table C.10: Statistics for the variance vs. offer price - stage 2

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	100.52	19.36***	0.00	90.26	23.72***	0.00
		Slope	0.28	0.38	0.71	1.00	1.75	0.11
Theoretical updating	Experiment	C	92.17	11.43***	0.00	102.02	27.34***	0.00
		Slope	1.13	1.00	0.34	-1.15	-2.05*	0.06
	F stats-slope difference		0.27			2.29		
	Theory	C	94.87	25.44***	0.00	96.35	21.40***	0.00
		Slope	0.78	1.30	0.22	-0.33	-0.51	0.62
M2	Experiment	C	92.47	10.08***	0.00	105.59	33.11***	0.00
		Slope	1.11	0.75	0.47	-1.53	-3.32***	0.01
	F stats-slope difference		0.47		0.88			

Note: *, **, *** indicate significance at 10% 5%,1% level.



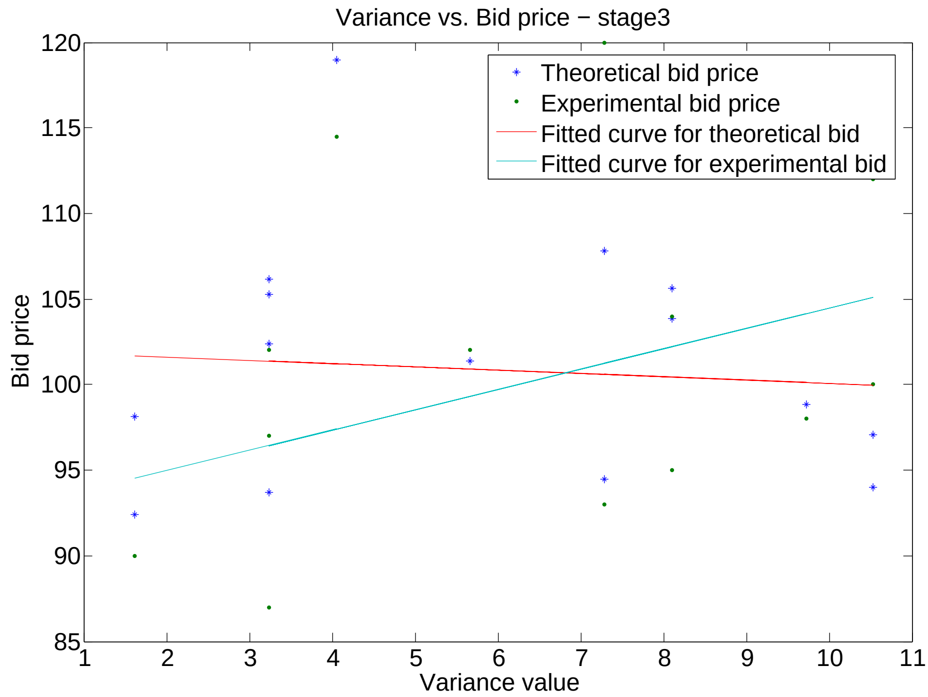
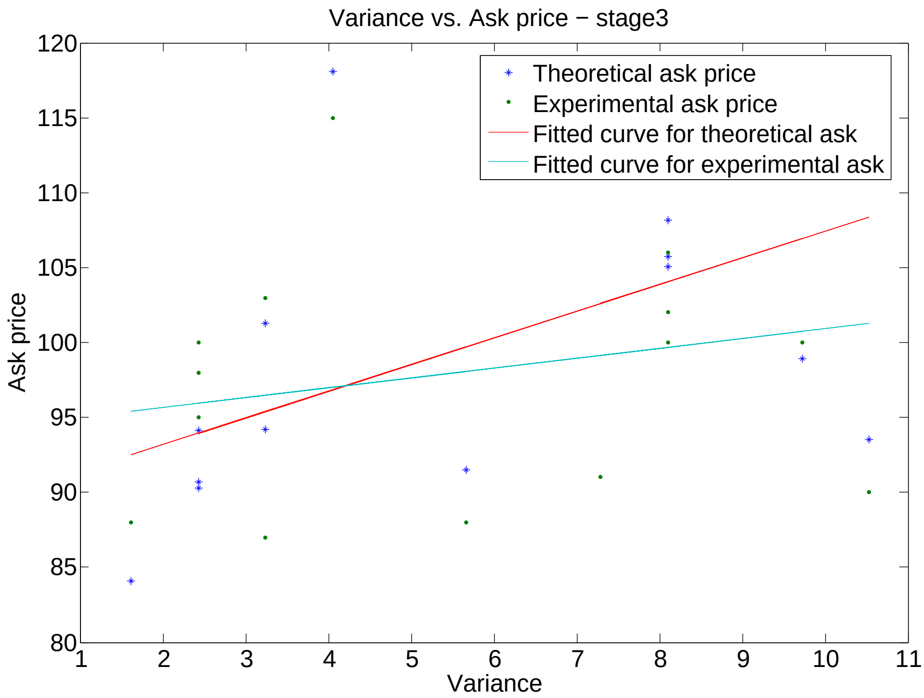


Figure C.21: The effects of variance on bid prices - stage 3 (theoretical updating & experimental updating)



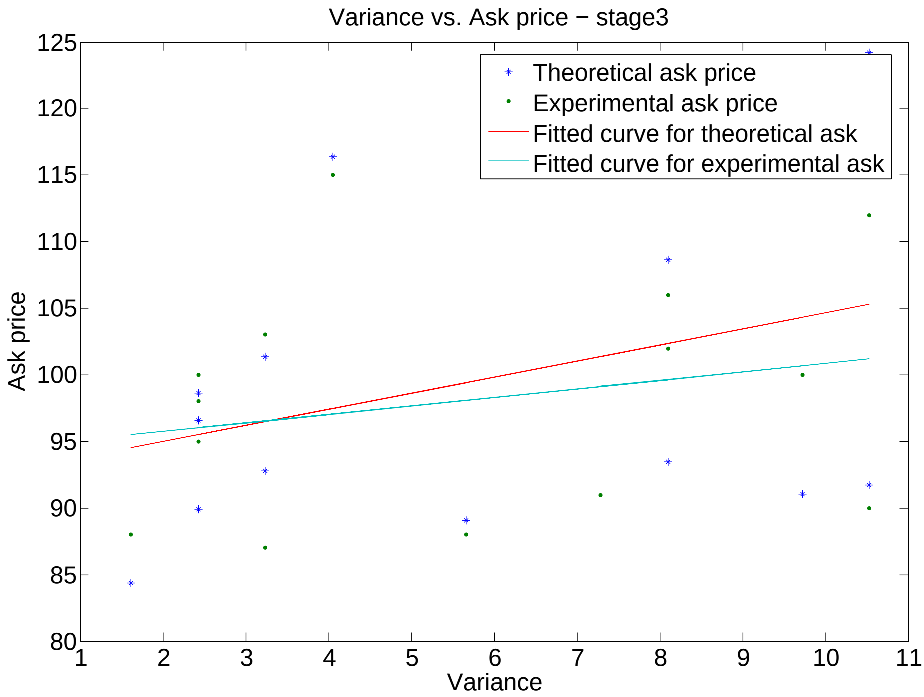
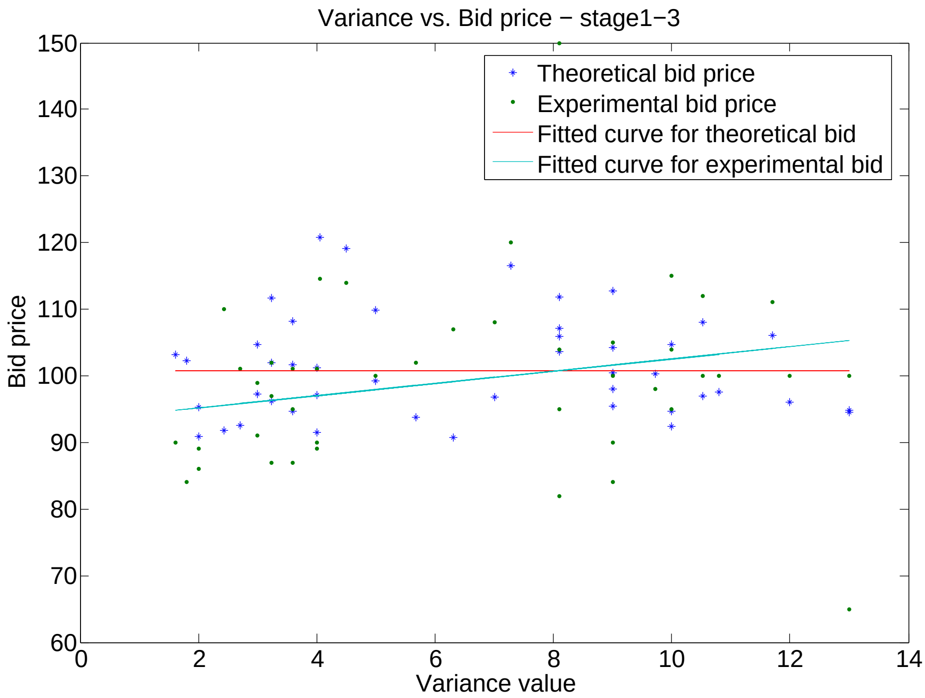


Figure C.22: The effects of variance on ask prices - stage 3 (theoretical updating & experimental updating)

Table C.11: Statistics for the variance vs. offer price - stage 3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	102.74	17.98***	0.00	89.60	17.27***	0.00
		Slope	0.21	0.25	0.81	1.79	2.28**	0.04
M1	Experiment	C	98.12	15.63***	0.00	94.34	18.96***	0.00
		Slope	0.69	0.74	0.48	0.66	0.87	0.40
F stats - slope difference			0.08			0.37		
	Theory	C	101.97	25.31***	0.00	92.58	12.44***	0.00
		Slope	-0.19	-0.31	0.76	1.21	1.05	0.31
M2	Experiment	C	92.58	18.24***	0.00	94.47	18.31***	0.00
		Slope	1.19	1.54	0.15	0.64	0.80	0.44
F stats - slope difference			0.96			0.12		

Note: *, **, *** indicate significance at 10% 5%,1% level.



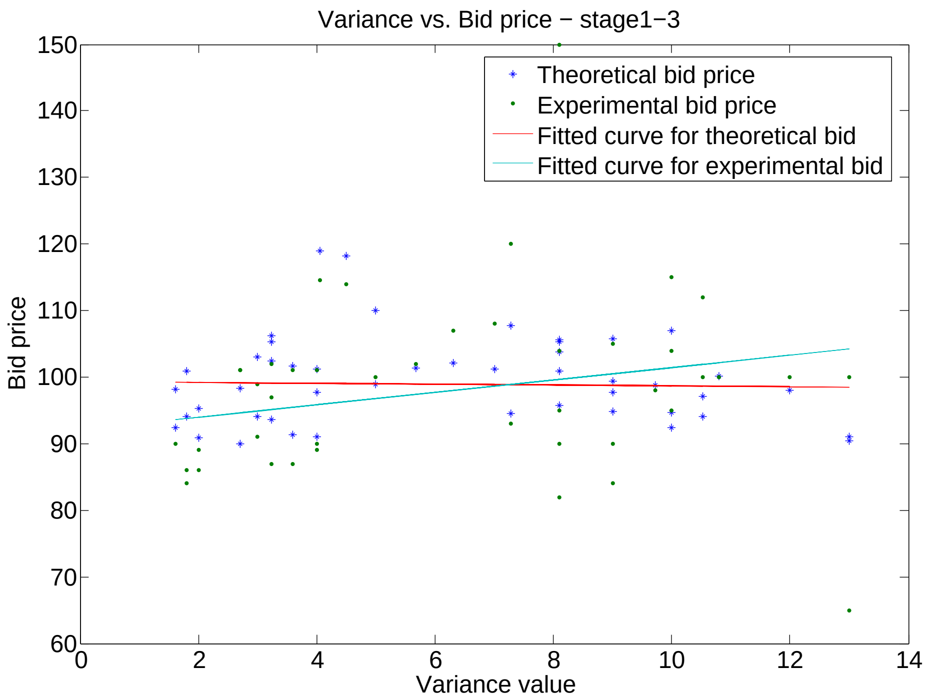
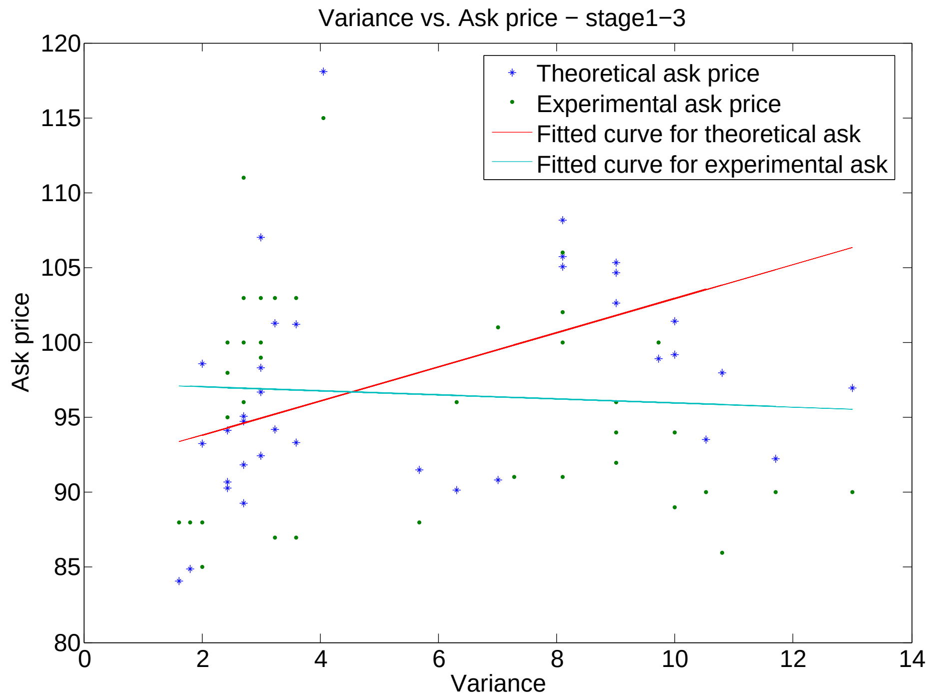


Figure C.23: The effects of variance on bid prices - stage 1-3 (theoretical updating & experimental updating)



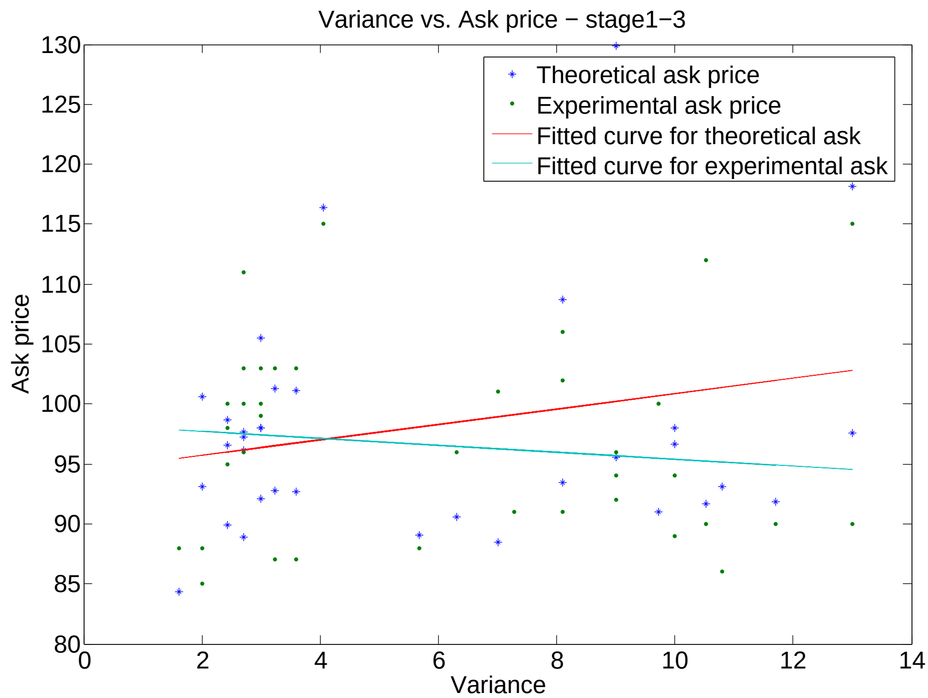


Figure C.24: The effects of variance on bid prices - stage 1-3 (theoretical updating & experimental updating)

Table C.12: Statistics for the variance vs. offer price - stage 1-3

			Bid			Ask		
		Variable	Coefficient	t-Static	Prob.	Coefficient	t-Static	Prob.
	Theory	C	100.69	39.13***	0.00	91.53	37.39***	0.00
		Slope	0.01	0.02	0.99	1.14	3.25***	0.00
Theoretical updating	Experiment	C	93.36	26.75***	0.00	97.31	37.07***	0.00
		Slope	0.92	1.93*	0.06	-0.13	-0.36	0.72
	F stats -slope difference		0.4			2.16		
	Theory	C	99.32	50.11***	0.00	94.46	26.51***	0.00
		Slope	-0.06	-0.22	0.83	0.64	1.26	0.22
M2	Experiment	C	92.09	29.09***	0.00	98.31	36.35***	0.00
		Slope	0.93	2.08*	0.04	-0.29	-0.76	0.45
	F stats -slope difference		0.92			1.09		

Note: *, **, *** indicate significance at 10% 5%,1% level.

C. THE DETERMINANTS OF ORDER PRICES IN THE MULTI-STAGE TRADING EXPERIMENT

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